# IMPOSSIBLE ODDS

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# Impossible Odds*

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A thesis (“weak *BCP*”) nearly universally held among philosophers of probability connects the concepts of objective chance and metaphysical modality: Any prospect (outcome) that has a positive chance of obtaining is metaphysically possible—(nearly) equivalently, any metaphysically impossible prospect has zero chance. Particular counterexamples are provided utilizing the monotonicity of chance, one of them related to *the four world paradox*. Explanations are offered for the persistent feeling that there cannot be chancy metaphysical necessities or chancy metaphysical impossibilities. Chance is objective but contrary to popular opinion it is also largely epistemic. Chancy necessities are analogous to necessary *a posteriori* truths.

## I

There is a common misconception concerning the relationship between metaphysical modality and *chance* (odds), in contrast to one’s individual degree of confidence (one’s “credence”).[1] The imagined connection is a trivial consequence of the so-called Basic Chance Principle, put forward by John Bigelow, John Collins, and Robert Pargetter:

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[1] The word ‘credence’ is generally used in the literature on Bayesian epistemology as a term for one’s “degree of belief” of a proposition, so that a subject’s *credence* toward $p = n$ if and only if he/she believes $p$ to degree $n$. The locution lacks clear sense. Whereas one’s confidence toward a proposition is a matter of degree, in at least most ordinary cases it is unclear what it means to say that belief is. If a subject has 80% confidence toward $p$ over $\sim p$, he/she does not thereby $\frac{1}{5}$-believe $\sim p$ (whatever partial belief is). One is inclined to say that the subject does not thereby believe $\sim p$ *at all*. Having a great deal of confidence favoring $p$ is not a way of believing $\sim p$ to some small degree. It is a way of *seriously doubting* $\sim p$, which is not the same thing as believing $\sim p$ a little bit.

If sense is to be made of believing to a degree, then $x$’s *degree of belief* function, $\boldsymbol{b}_x$, is better identified not with $x$’s degree of confidence (“credence”) function, $\boldsymbol{c}_x$, but with the alternative function: $\boldsymbol{b}_x(p) = 2\boldsymbol{c}_x(p) - 1$. If there is a direct numerical/quantitative relationship between confidence and belief, it is given by this equation, which defines degree of belief as a function of confidence. So understood, $x$’s degrees of belief are an appropriate recalibration of $x$’s confidence. A perfectly balanced degree of confidence of exactly 50%—leaning neither one way nor the other—translates into a degree of belief of exactly zero. A degree of confidence toward $p$ of 0.8 equates to belief of $p$ to degree 0.6, since confidence of degree 0.8 is 60% of the difference between full and zero belief. A degree of belief given by a negative number provides a measure of belief of the denial (disbelief), in accordance with the theorem that $\boldsymbol{c}_x(p) + \boldsymbol{c}_x(\sim p) = 1 \rightarrow \boldsymbol{b}_x(\sim p) = -\boldsymbol{b}_x(p)$. Thus , if $\boldsymbol{c}_x(\sim p) = 1 - \boldsymbol{c}_x(p)$, then one’s degree of belief of the denial is the additive inverse of one’s degree of belief. If so, confidence toward $p$ to degree 0.8 equates to less than zero belief of $\sim p$, to precisely the degree –0.6.

What is sometimes called ‘the Lockean thesis’ entails that there is a number $n$ such that always, $\forall x \forall p[x$ believes $p \leftrightarrow \boldsymbol{b}_x(p) \geq n]$. If there is such a number $n$, then degrees of belief greater than $-n$ and less than $n$ constitute a gradation of suspension of judgment. Arguably $n = 1$. *Cf.* the function ***cert*** defined below and note 32.

*BCP* If $chance_t(p) = x > 0$, and $h$ is the complete history of the world up to $t$, then the following are metaphysically compossible: (1) $p$ is true; (2) $h$ obtains; and (3) $chance_t(p) = x$.[2]

I here criticize the weak consequence of *BCP* that results by omitting both (2) and (3):

*weak BCP* Any prospect (outcome) that has a positive objective chance of obtaining is metaphysically possible, i.e., $\forall p[chance(p) > 0 \rightarrow \Diamond p]$.

The weak *BCP* thesis is traceable to Leibniz's 1678 dictum, '*Probabilitas est gradus possibilitatis*', interpreted (perhaps contrary to Leibniz's intent) as expressing that probability is a gradation of metaphysical possibility.[3] On this conception, a prospect has a positive probability if and only if it is metaphysically possible, has probability one if and only if it is necessary, and has probability zero if and only if it is impossible. This is a misconception. Metaphysical possibility does not admit of degrees of *more* and *less*—except in the perverse sense that whatever is possible is thereby "more possible" than whatever is impossible. The misconception entails the weak *BCP* thesis, but the thesis does not require the misconception. Given that objective chance is a probability, weak *BCP* is (nearly) equivalent to the thesis that the chance of any metaphysically necessary truth is one, or alternatively, that the chance of any metaphysical impossibility is zero. The weak *BCP* thesis is generally taken for granted in the philosophical literature and occasionally explicitly articulated.[4] Bigelow, Collins, and Pargetter state, "In general, if the chance of $A$ is positive there must be a possible future in which $A$ is true" (*ibid*., p. 459). Jonathan Schaffer says similarly, "if there is a non-zero chance of $p$, this should entail that $p$ is possible."[5] Antony Eagle says, "it seems clear that if an outcome has some chance of

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2 John Bigelow, John Collins, and Robert Pargetter, "The Big Bad Bug: What are the Humean's Chances?" *British Journal for the Philosophy of Science*, 44 (1993), pp. 443–62, at p. 459. The formulation of *BCP* provided here captures the intent behind the less precise formulation provided there.

3 Cited in Ian Hacking, "Equipossibility Theories of Probability," *British Journal for the Philosophy of Science*, 22 (1971), pp. 339–55, at p. 345.

4 The weak *BCP* thesis is entailed by variants of *BCP*, such as that proposed by Toby Handfield in *A Philosophical Guide to Chance* (Cambridge University Press, 2012), at p. 85. Weak *BCP* is also suggested (not entailed) by some of Saul Kripke's remarks in the preface to his *Naming and Necessity* (Oxford: Basil Blackwell, 1980), at pp. 16-20. See also D. H. Mellor, "Possibility, Chance and Necessity," *Australasian Journal of Philosophy*, 78, 1 (2000), pp. 16–27; Jonathan Schaffer, "Principled Chances," *British Journal for the Philosophy of Science*, 54 (2003), pp. 27–41; John L. Pollock, *Thinking About Acting: Logical Foundations for Rational Decision Making* (New York: Oxford University Press, 2006), at p. 94; Schaffer, "Deterministic Chance?", *British Journal for the Philosophy of Science*, 58 (2007), pp. 113–40; and Antony Eagle, "Deterministic Chance," *Noûs*, 45 (2011), pp. 269–99. In his entry in *the Stanford Encyclopedia of Philosophy* on "Chance versus Randomness" (supp. A, note 4) Eagle reports that Alan Hájek suggested the weak *BCP* thesis.

Weak *BCP* is a consequence of another thesis also widely taken for granted but not usually articulated: *Chance is modally intensional*, in the sense that $\Box(p \leftrightarrow q) \rightarrow chance(p) = chance(q)$. (Weak *BCP* follows, nearly enough, by letting $q$ be a contradiction.) In "Indicative *versus* Subjunctive Conditionals, Congruential *versus* Non-Hyperintensional Contexts," *Philosophical Issues*, 16 (2006), pp. 310-333, Timothy Williamson states the similar (stronger) principle that "if two outcomes could have differed in their chances, one of them could have occurred without the other" (p. 317). (I thank Antony Eagle for supplying this reference.)

5 "Deterministic Chance?" *British Journal for the Philosophy of Science*, 58, 2 (June 2007), pp. 113-140, at p. 124.

occurring, then it is possible that the outcome occurs" ("Chance versus Randomness," §1, *the Stanford Encyclopedia of Philosophy*) and "in the case of objective chance, it is hard to dispute that if *p* is necessary, then the chance of *p* should be 1" (*ibid*., supp. A, note 4). In a recent article Boris Kment asserts that "the connection between modality and chance is straightforward: There can be a non-zero chance at *t* that *P* only if it is metaphysically possible that *P*".[6] Schaffer and Kment (*ibid*., sec. 2.3) deem full *BCP* compelling.

Furthermore, any such sweeping connection as that stated in weak *BCP*—let alone that stated in full *BCP*—between the phenomena of chance and metaphysical modality cannot be mere coincidence. Rather, it must obtain, if at all, as a matter of the nature or logic of those very concepts. Thus, Bigelow, Collins, and Pargetter declare that "anything that failed to satisfy the *BCP* would not deserve to be called *chance*" (*op. cit*., p. 459).

Weak *BCP* is in fact neither analytic nor trivial. Although initially attractive, it is just the sort of thesis that requires a philosophical defense if it is to command rational credence. Some believe that weak *BCP* is a theorem of the standard probability calculus.[7] That is a myth. As will be shown, something very close to the opposite is true: There are *chancy impossibilities*, i.e., prospects that have a positive chance yet are metaphysically impossible.[8]

How can it be that on the one hand, something could not happen at all, and on the other hand, there is a positive chance of its occurrence? The assertion by Bigelow, Collins, and Pargetter that such a prospect is precluded by the very meaning of the word 'chance' is natural if one is in the grip of a false theory or flawed understanding of

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[6] Boris Kment, "Chance and the Structure of Modal Space," *Mind* (forthcoming), https://pdfs.semanticscholar.org/8365/e42eaa7434816d7cff2160b9d26938230d8c.pdf (accessed online on and before May 15, 2018), section 1.

[7] See note 4. Antony Eagle informs me that Alan Hájek suggested to him in correspondence in 2005 that weak *BCP* can be taken as a corollary of an axiom of probability theory. Pollock, *op. cit.*, argues that the thesis follows from "the probability calculus." His argument assumes that every metaphysically necessary truth (including the theorems of mathematics) is equivalent to '$p \vee \sim p$'. In a similar vein, Eagle believes that the sorts of propositions that have objective chance are sets of metaphysically possible worlds.

See David Lewis, "A Subjectivist's Guide to Objective Chance," in Richard Jeffrey, ed., *Studies in Inductive Logic and Probability, Vol. II* (Berkeley, CA: University of California Press, 1980), pp. 263-293. Numerous philosophers influenced by Lewis evidently take probability to be mathematically defined over propositions as sets of metaphysically possible worlds, in such a way that, by definition, if such a proposition is metaphysically impossible (i.e., the empty set of possible worlds) it thereby has probability zero, and if it is metaphysically necessary it thereby has probability one. In fact, standard probability theory does not invoke the notion of a metaphysically possible world. The concepts of metaphysical modality are metaphysical, not probability-theoretical. In standard probability theory, a probability space includes an assignment of specific probabilities to "events"—basically disjunctions of "outcomes" (e.g., of rolls of a die)—and is entirely neutral whether those outcomes are states of affairs, worlds, or otherwise, and whether they are metaphysically possible or not. The notion of probability invoked throughout the present essay conforms to the standard (and metaphysically neutral) probability-theoretical modeling. (See the following note.) By contrast, the weak-*BCP* proponent's notion arguably deviates from that modeled in standard probability theory. Excessively briefly, some *BCP* proponents see chance in some sense as a gradation of metaphysical unpreventability, and thereby misunderstand 'chance' to mean something like *degree of metaphysical unpreventability*—a concept of questionable significance and even dubious legitimacy. (This apparent misunderstanding is despite, and in addition to, the fact that insofar as weak-*BCP* proponents use 'chance' as a word of Standard English, they also mean by it what it means.) See note 32 below.

[8] By 'chancy' I mean *having a chance greater than zero and less than one*. I draw no distinction here among prospects—metaphysically possible or not—and propositions, outcomes, events, or states of affairs (although distinctions among these might be legitimately drawn for some purposes).

chance. Once it is seen how the weak *BCP* thesis is falsified, one can see why a chancy impossibility is not only a viable phenomenon but in fact commonplace.

Preliminarily we consider a modal paradox I presented in a previous millennium, *the four world paradox*.[9] Consider Theseus's ship. Suppose that it originated entirely from exactly one hundred interlocking wooden planks, $P_1$, $P_2$, ..., $P_{100}$, each the same size and mass. (See note 10 below.) Suppose further (what seems surely correct) that any ship of its design is such that it could have originated from some different matter but could not have originated from entirely different matter. Suppose for illustration that, as a matter of metaphysical necessity, any ship of this design could have originated from matter that is at least 98% of the actual original matter, but could not have originated from as many as three different planks, $P_{101}$, $P_{102}$, and $P_{103}$, which are, we may suppose, molecule-for-molecule duplicates of $P_{98}$, $P_{99}$, and $P_{100}$, respectively. I have argued extensively that this set-up by itself discredits *S4* as the logic of metaphysical modality. Very closely related considerations refute the weak *BCP* thesis.

Suppose that swapping any one or two of the planks with replicas preserves the identity of the ship so constructed (more precisely, it preserves the ship's *haecceity*), while swapping three or more planks yields a different ship. Suppose further that this is metaphysically necessary, necessarily necessary, necessarily necessarily necessary, and so on. Let $W_1$ be a possible world in which a ship, Woody, is the only ship constructed from $P_1$, $P_2$, ..., $P_{100}$. Let $W_2$ be the world that is possible according to $W_1$ and that results by interchanging $P_{98}$, $P_{99}$, and $P_{100}$ as they are in $W_1$ with $P_{101}$, $P_{102}$, and $P_{103}$ (with whatever other changes this requires, while preserving the other 97 planks as they are in $W_1$). The ship constructed in $W_2$ using $P_{101}$, $P_{102}$, and $P_{103}$ in place of $P_{98}$, $P_{99}$, and $P_{100}$ is not Woody. It is a replica, Woody′. Let $W_3$ be the world that is possible according to $W_1$ and that results from $W_1$ by interchanging $P_{99}$ and $P_{100}$ as they are in $W_1$ with $P_{102}$ and $P_{103}$ as they are in $W_2$. In $W_3$, Woody is constructed using $P_{102}$ and $P_{103}$ in place of $P_{99}$ and $P_{100}$. Finally, let $W_4$ be the world that is possible according to $W_2$ and that results from $W_2$ by interchanging $P_{101}$ as it is in $W_2$ with $P_{98}$ as it is in $W_1$ (with whatever other changes this requires, while leaving $P_{102}$ and $P_{103}$ as they are in $W_2$).

The worlds $W_3$ and $W_4$ are exactly like in their material configuration; there is no purely material difference whatsoever between them. Every piece of matter—molecule for molecule, atom for atom, quark for quark—has exactly the same material history in $W_4$ that it has in $W_3$. Yet they are different worlds. In particular, in $W_4$ it is not Woody, but the distinct ship Woody′, that is the ship constructed from the relevant planks.[10] This apparent result evidently contradicts the extremely plausible thesis that the identities of

9 "How *Not* to Derive Essentialism from the Theory of Reference," *Journal of Philosophy*, LXXVI, 12 (December 1979), pp. 703-725, at pp. 722-724. *Cf.* my *Reference and Essence* (Amherst, NY: Prometheus Books, 1981, 2005), at pp. 229-252, 268-271, 273-344. Both the paradox and its proposed solution were based on an important and insightful argument of Hugh Chandler's in "Plantinga and the Contingently Possible," *Analysis*, 36, 2 (January 1976), pp. 106-109. Chandler credited Robert Stalnaker, reporting (pp. 107-108) that Stalnaker had suggested a problem similar to (albeit also significantly different from) the four world paradox. Stalnaker has not received sufficient credit in the literature and does not accept my proposed solution. (Curiously, Chandler considers the solution but pointedly stops short of endorsing it.)

10 Kment poses the paradox using a different example and different labeling. (He takes $W_3$ to be the actual world. It is irrelevant which, if any, of the worlds is taken to be actual.) On the assumptions of the set-up (and *modulo* the differences between the examples), the worlds that Kment labels '$w_1$', '@', and '$w_2$' conflict with *S4*.

the ordinary physical objects—the ships, the tables, the mountains, the trees—necessarily supervene on the world's complete material history, the complete history of the exact configuration of all the world's matter.[11]

The solution I deem correct exploits the fact (which regrettably remains controversial) that the correct logic of metaphysical modality is weaker than *S5* and weaker even than *S4*, on which the accessibility relation $\boldsymbol{R}$ between worlds, besides being reflexive, is invariably transitive. (On *B*, $\boldsymbol{R}$ is invariably reflexive and symmetric. On *S5*, $\boldsymbol{R}$ is invariably an equivalence relation.) That is, on the solution I endorse there are worlds, $w$, $w'$, and $w''$, such that $w'$ is possible according to (or "possible relative to," or "accessible to") $w$, and $w''$ is possible according to $w'$, but $w''$ is impossible according to $w$. The paradox is solved by noting that while $W_4$ is possible according to $W_2$, which is itself possible according to $W_1$, there is no reason, aside from mere prejudice in favor of *S4*, to suppose that $W_4$ is possible according to $W_1$. Indeed, the supervenience thesis provides a compelling reason to hold that $W_4$ is impossible according to $W_1$. Furthermore, the same considerations that establish that there is such a world as $W_4$ also establish that there is a world $W_5$ that is possible according to $W_3$ and that results from $W_3$ by interchanging $P_{98}$ with $P_{101}$ as it is in $W_3$. In $W_5$, Woody is constructed from $P_1$, $P_2$, . . ., $P_{97}$, $P_{101}$, $P_{102}$, and $P_{103}$. Since it is impossible according to $W_1$ for Woody to have been constructed from those planks, despite being possible according to $W_3$ (which in turn is possible according to $W_1$), $W_5$ is impossible according to $W_1$. It is not only logically coherent, it is in fact demonstrable that in the scenario described, the accessibility relation $\boldsymbol{R}$ for metaphysical modality is not transitive.[12]

## II

The first argument I shall provide against the weak *BCP* thesis is in effect an inversion of an objection that Kment offers to the solution just described. (See note 20 below.) Although I cite Kment's argument as a foil, the burden of my comments is neither critical nor defensive but constructive. My principal objective is to make a plausible and persuasive case for chancy impossibilities and chancy necessities.

Kment describes a scenario in which an ideally rational agent, $A$, knows at time $t_0$ in advance that the construction at $t_1$ of a ship is to be settled by the spin of a fair wheel of fortune with three open possibilities: (1) $P_1$, $P_2$, . . ., $P_{100}$ are to be assembled into a ship; (2) $P_1$, $P_2$, . . ., $P_{97}$, $P_{101}$, $P_{102}$, and $P_{103}$ are to be assembled; or alternatively, (3) $P_1$, $P_2$, . . ., $P_{97}$, $P_{98}$, $P_{102}$, and $P_{103}$ are to be assembled. (The wheel omits the prospect of any other assembly.) $A$ is also 100% certain that if a ship is constructed from $P_1$, $P_2$, . . ., $P_{100}$ (alternative (1)), then that same ship could not have been constructed using $P_{101}$, $P_{102}$, and $P_{103}$ instead of $P_{98}$, $P_{99}$, and $P_{100}$ (alternative (2)), and *vice versa*. Suppose the wheel lands on option (1) and at $t_1$ Woody is constructed. Thus $W_1$ is realized while each of $W_2$ and $W_3$ is merely possible. $A$ does not know the outcome of the spin when at some time $t_2$ after the plan is implemented, $A$

11 The supervenience thesis is modal, not to be confused with the stronger, anti-haecceitist thesis that the identities of ordinary physical objects in a world *reduce to* the purely material facts.

12 Antony Eagle observes that as a corollary of the multiplication rule for conditional probability, $pr[pr(p) > 0] > 0 \rightarrow pr(p) > 0$, i.e., any prospect $p$ having a positive probability of having a positive probability itself has a positive probability. (For a proof apply the multiplication rule to '$pr[p \,\&\, pr(p) > 0]$'.) Besides entailing weak *BCP*, the Leibnizian dictum that misidentifies metaphysical possibility with positive probability thus also entails '$\Diamond\Diamond p \rightarrow \Diamond p$', a formulation of the characteristic axiom of *S4* as the (putative) logic of metaphysical modality.

is introduced to Woody as the ship that resulted at $t_1$ from the spin of the wheel in accordance with the plan. *A* is certain that exactly one of three prospects ("outcomes")—either (1), or (2), or else (3)—was realized. The specifics of Kment's objection are not relevant here but for one: Kment asserts the weak *BCP* thesis without argument (*op. cit.*, first page), and then argues from it against my solution to the four world paradox.[13]

The mere logical coherence of the view that necessarily, Woody might have been constructed using slightly different matter but could not have been constructed using entirely different matter is beyond reasonable, reflective doubt. The mere logical coherence invalidates *S4* modal logic.[14] A valid argument for the conclusion that the mere coherence of $W_1$, $W_3$, and $W_5$ conflicts with a particular theory of chance, insofar as the premises are accepted, does not provide reasonable grounds for doubting the coherence; it provides grounds for rejecting the theory.[15]

Consider how someone in *A*'s epistemic situation might arrive at the correct judgment in $W_1$ that the chance that Woody was constructed from $P_1, P_2, \ldots, \mathrm{P}_{100}$ is one-in-three. It might appear as if at least part of the process goes as follows: Given *A*'s knowledge about the plan involving the fair wheel of fortune and *A*'s ignorance of the outcome, *A* knows at $t_2$ in $W_1$ that the chance that the ship constructed at $t_1$ in accordance with the plan was constructed from $P_1, P_2, \ldots, P_{100}$ is exactly one-in-three. Given *A*'s introduction to Woody, *A* also knows at $t_2$

13 I have altered the specifics of Kment's example to fit the original four world paradox. Specifically, relying crucially on weak *BCP* Kment argues for the erroneous conclusion that my favored solution to the four world paradox conflicts with a variant of David Lewis's Principal Principle.

14 See my "The Logic of What Might Have Been," *Philosophical Review*, 98, 1 (January 1989), pp. 3-34; reprinted in my *Metaphysics, Mathematics, and Meaning* (Oxford University Press, 2005), pp. 129-149.

15 Kment favors an alternative account of the modal profile of ordinary physical objects (*op. cit.*, sec. 2.6). According to that account, where a ship *S* is constituted by matter *m*, there are very (perhaps infinitely) many additional objects also constituted by *m* and located exactly where *S* is, but differing from *S* in their modal profiles. A minor variant of this alternative putative solution is defended at length by Sarah-Jane Leslie in "Essence, Plenitude, and Paradox," in J. Hawthorne and J. Turner, eds, *Philosophical Perspectives 25: Metaphysics, 2011*, pp. 277-296. Teresa Robertson has refuted proliferation accounts of this sort, in "Things Undreamt Of: Chandler's Paradox, Modal Properties, and Plenitude," presented to the University of California, Irvine, 2013; the Latin American Analytic Philosophy Conference and Conference of the Brazilian Society for Analytic Philosophy, Fortaleza, Brazil, 2014; and the State University of Campinas, São Paulo, Brazil, 2014. Specifically, Robertson demonstrated that such accounts provide no viable alternative solution to modal paradoxes like that of the four worlds, indeed that, absent unspecified, *ad hoc*, question-begging restrictions, such accounts tacitly postulate coincident ship-shaped objects that (if they exist) violate *S4* and even *B* modal logics. The alleged existence of a plenitude of coincident ship-shaped non-ships is in fact all but irrelevant to the modal paradoxes. Defenders of *S4* need to argue that the very idea of an artifact that necessarily (and necessarily necessarily, etc.) might have been constructed from a fixed portion of different matter but could not have been constructed from more than that fixed portion of different matter is incoherent, indeed analytically inconsistent. Turning one's attention toward a plenitude of non-ships is not a way of solving a philosophical problem that arises for ships. To divert attention is to change the subject rather than to solve the problem.

Kment goes so far as to count the putative modal variants of a ship as themselves ships. Leslie is more cautious, characterizing the modal variants as "ships (or ship-like entities)" (*op. cit.*, p. 281) and "axes (or axe-like entities)" (p. 290)—though the use of parentheses would seem to indicate that she unofficially sides with Kment. The claim that there are trillions of ships coincident with *S* and constituted by *m* flagrantly violates Bertrand Russell's admonition against "a failure of that feeling for reality which ought to be preserved even in the most abstract studies" (*Introduction to Mathematical Philosophy*, London: Allen and Unwin, 1919, at p. 169). Far worse than a carefully reasoned rejection of *S4*, the claim is altogether unacceptable. The postulated trillions of extra entities are not ships (tables, axes, etc.). Given their deviant modal profiles, they are (assuming they exist) metaphysically peculiar, artifact-shaped *things*, which unobtrusively accompany the genuine artifact. The matter *m*, though not peculiar, is also ship-shaped yet not a ship.

that the ship constructed at $t_1$ in accordance with the plan = the ship $A$ sees = Woody. Applying Leibniz's law, $A$ infers that the chance that Woody was constructed at $t_1$ from $P_1$, $P_2$, . . ., $P_{100}$ is exactly one-in-three. But is such an inference valid? The propositional operator 'the chance that ___' is non-extensional. For example, the result of filling the operand-position with 'The next roll of the die does not land on 3' designates five-in-six, a safe bet. Supposing that the next roll of the die does indeed land on three, the numeral '3' is co-designative with 'the number on which the next roll of the die lands'. But the result of substitution of the latter for the former is 'the chance that the next roll of the die does not land on the number on which the next roll of the die lands', which designates the worst possible odds. Inferring the chance of a singular proposition from the chance of a general proposition together with an identity proposition is risky business and calls for due diligence.

Let us follow a more judicious strategy. The notion of chance as applied to propositions may be straightforwardly extended to apply to sentences in a language: The chance of a sentence is the chance of its expressed proposition. (Sentential chance may be relativized to a context. Both sentential and propositional chance may be relativized to a proposition-guise. See below.) Let the functor '*chance*( )' be understood as designating a semantic measure defined over sentences of a particular language, and let $\vDash$ be logical entailment between sentences of that language. The following meta-principle is fundamental to chance and yields considerable philosophical purchase:

*Monotonicity* Any logical consequence of a sentence has at least as great a chance as that sentence has, i.e., where $\ulcorner chance(\phi)\urcorner$ and $\ulcorner chance(\psi)\urcorner$ are both defined, $\phi \vDash \psi \rightarrow chance(\phi) \leq chance(\psi)$.

Monotonicity follows from three principles fundamental to probabilities: (*i*) Chance is non-negative; (*ii*) Logically equivalent sentences have the same chance; (*iii*) If $\phi$ and $\chi$ are logical contraries, then $chance(\ulcorner\phi \vee \chi\urcorner) = chance(\phi) + chance(\chi)$.[16] Alternatively, Monotonicity may be taken as a basic principle concerning chance. (It entails (*ii*).) No prospect $p$ has a greater chance than any of its logical consequences; the chance of any logical consequence of $p$ is an upper bound on the chance of $p$. Equivalently, no logical consequence of $p$ has less chance than $p$ itself; the chance of $p$ is a lower bound on the chances of $p$'s consequences.

Let $\chi_1$ be 'The ship constructed at $t_1$ in accordance with the relevant plan was constructed from $P_1$, $P_2$, . . ., $P_{100}$', which is true with respect to $W_1$. By hypothesis, in $A$'s epistemic situation at $t_2$ in $W_1$, the chance that some ship is uniquely constructed at $t_1$ from those planks is one-in-three. Thus $chance(\chi_1) = \frac{1}{3}$. Let $\delta$ be the identity statement 'Woody is the ship constructed at $t_1$ in accordance with the relevant plan'. In $A$'s epistemic situation, $\delta$ has a chance of one. (It is a logical consequence of 'Woody is the ship before me' and 'The ship before me is the ship constructed at $t_1$ in accordance with the relevant plan'. By hypothesis, each of these premises is certain in $A$'s epistemic situation.) Let $\phi_1$ be the conjunction $\ulcorner\chi_1 \ \& \ \delta\urcorner$, and let $\psi_1$ be 'Woody was constructed using $P_1$, $P_2$, . . ., $P_{100}$'. By the multiplication theorem, $chance(\phi_1) = chance(\delta) \cdot chance(\chi_1|\delta)$. Since $chance(\delta) = 1$, $chance(\chi_1|\delta) = chance(\chi_1) = \frac{1}{3}$. Since $\phi_1 \vDash \psi_1$, it follows by

16 Proof: Assume $\phi \vDash \psi$ and let $\chi$ be $\ulcorner(\sim\phi \ \& \ \psi)\urcorner$, so that $\phi$ and $\chi$ are logical contraries and $\ulcorner\phi \vee \chi\urcorner$ is logically equivalent to $\psi$. Then $chance(\psi) = chance(\ulcorner\phi \vee \chi\urcorner) = chance(\phi) + chance(\chi)$. Since $chance(\chi) \geq 0$, it follows that $chance(\phi) \leq chance(\psi)$. See note 7 above.

Given that *chance*('$p \vee \sim p$') = 1, by (*ii*) (which is a consequence of Monotonicity), every logical truth has chance one. It does not follow that every metaphysically necessary truth does.

Monotonicity that $chance(\psi_1) \geq \frac{1}{3}$. In $A$'s epistemic situation the chance that Woody was constructed using $P_1, P_2, \ldots, P_{100}$ is at least one-in-three.

The same argument applies, *mutatis mutandis*, to the part-sets $\{P_1, P_2, \ldots, P_{98}, P_{102}, P_{103}\}$ and $\{P_1, P_2, \ldots, P_{97}, P_{101}, P_{102}, P_{103}\}$ in place of $\{P_1, P_2, \ldots, P_{100}\}$. Each of the three jointly exhaustive and mutually exclusive prospects therefore has a chance in $W_1$ of at least one-in-three. If one of the three prospects had a chance greater than one-in-three, then the disjunction of the remaining two prospects would have a compensatory chance of less than two-in-three, so that the chance of the disjunction remains one. In that case, at least one of the remaining two prospects would have to have a chance of less than one-in-three. But that has been disproved. Therefore, in $A$'s epistemic situation at $t_2$ in $W_1$—i.e., given $A$'s background information—each of the three prospects has a one-in-three chance.

This application of Monotonicity yields a significant result for the theory of chance. Weak *BCP* entails that in $A$'s epistemic situation in $W_1$ at $t_2$, insofar as Woody could not have been constructed using $P_{101}$, $P_{102}$, and $P_{103}$, the chance of that prospect is zero. That assessment is incorrect. Let $\delta$ be as before, and let $\chi_2$ be 'The ship constructed at $t_1$ in accordance with the relevant plan was constructed using $P_{101}$, $P_{102}$, and $P_{103}$', which is true with respect to $W_2$. Let $\phi_5$ be $\ulcorner \chi_2 \ \& \ \delta \urcorner$ and let $\psi_5$ be its logical consequence 'Woody was constructed using $P_{101}$, $P_{102}$, and $P_{103}$'. ($\phi_5$ and $\psi_5$ are so-called because, although false with respect to $W_2$, they are true with respect to $W_5$.) Substituting $\chi_2$ for $\chi_1$, $\phi_5$ for $\phi_1$, and $\psi_5$ for $\psi_1$ in the argument given in the preceding two paragraphs, it is established that even in $W_1$, $chance(\chi_2) = chance(\phi_5) = chance(\psi_5) = \frac{1}{3}$. Although in $W_1$ Woody could not have been constructed using $P_{101}$, $P_{102}$, and $P_{103}$, the chance in $W_1$ that Woody was thusly constructed is greater than zero. In $W_1$, although $W_5$ is impossible it has a positive chance of having been realized.[17]

Using Monotonicity and an intuitive assessment of the chance of particular propositions, we have deduced the existence of chancy impossibilities from the plausible metaphysical hypothesis that at least one physical artifact necessarily might have been constructed from slightly different matter but could not have been constructed from entirely different matter:

> *CI* Some metaphysically impossible prospects (outcomes) have a positive objective chance of obtaining, i.e., $\exists p[\sim\Diamond p \ \& \ chance(p) > 0]$.[18]

Likewise, some metaphysically necessary prospects have a positive chance less than one, i.e., there are *chancy necessities*. This result delivers a blow to the hope (for those who

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17 See note 13. Kment's agent $A$ is ideally rational in each of $W_1$, $W_2$, and $W_3$. $A$ therefore knows Monotonicity, and has complete confidence in its deliverances. $A$ also has complete confidence in the disputed metaphysical view. $A$ should therefore have complete confidence that there are chancy impossibilities, and that $A$'s own epistemic situation in relation to Woody likely involves one. As a consequence, $A$ will have a degree of confidence at $t_2$ of exactly $33\frac{1}{3}\%$ for each of the prospects concerning Woody's construction, conditional on the chance at $t_0$ of that prospect being one-in-three. This is in accordance with Lewis's Principal Principle. (But see note 1 above concerning "credence").

Agent $A$ has complete confidence that necessarily, Woody might have been constructed using just two different planks but could not have been constructed using as many as three different planks. According to Kment, $A$ also has complete confidence that a proposition has a positive chance only if it is a metaphysical possibility. Yet $A$ is supposedly also ideally rational. I submit that such an agent is a logical impossibility.

18 More precisely, there are worlds $w$ and propositions $p$ such that $p$ is impossible in $w$ and yet even in the epistemic situation of an ideally rational subject who is certain in $w$ that $p$ is impossible in $w$ (and is therefore completely confident, given that $w$ is realized, that the chance of $p$ is zero), $p$ has a positive chance in $w$. In $w$, $p$ is a chancy impossibility.

harbor it) of an analysis of chance as the frequency of the metaphysically possible worlds in which the prospect in question obtains.[19]

Kment maintains that the weak *BCP* thesis is significantly more compelling than my preferred solution to the four world paradox. On the contrary, even if the relevant metaphysical hypothesis is incorrect, it is at least logically coherent. That coherence alone establishes both: that *S4* modal logic is fallacious regarding metaphysical modality; and that weak *BCP* is not analytic.[20] The derivation of *CI* reveals that the weak *BCP* thesis is the likely result of an oversight. Insofar as there are (or could be) objects with essential (metaphysically necessary) properties of a certain kind—as is quite plausible—some logically possible propositions that are thereby ruled out by metaphysical constraints rather than by the vicissitudes of fortune nevertheless have genuine positive chance, by virtue of the additive property of probability.

## III

Further examples of chancy impossibilities are easily adduced. Suppose, as seems at least coherent, that the sex of a human zygote is an essential property. The human primary sex ratio (the ratio of human males to females at conception) is approximately one-to-one. Then for any human zygote of unknown sex, the chance of it being male is approximately one-in-two, even though in roughly one-half of such cases the prospect is metaphysically impossible. An argument analogous to the previous argument using Monotonicity yields this alternate counterexample to weak *BCP*. Suppose the zygote in the test tube is female. Let $\chi'$ be 'The zygote in the test tube is male', which has a chance of approximately one-in-two; let $\delta'$ be '*That zygote* (the one in the test tube) is the zygote in the test tube' (uttered while pointing to the relevant zygote), which is certain. Let $\phi'$ be $\ulcorner\chi' \,\&\, \delta'\urcorner$. Let $\psi'$ be '*That zygote* (the one in the test tube) is male' (uttered while pointing to the relevant zygote). Its content is impossible but it evidently has approximately a one-in-two chance.

On a standard American roulette wheel there are a total of 38 numbered pockets, 11 of which are numbered by (i.e., labelled by an Arabic numeral that designates) a prime number. Suppose that the individual numerical pocket-labels on a fair American roulette wheel are covered with removable opaque tape. The wheel is spun and the ball lands on a concealed number. What is the objective chance that it is a prime number? Answer: Exactly 11-in-38, just shy of 29%. Suppose it happened that the ball landed on 26. Although it is impossible for 26 to be prime, in this setup the chance of that very number being prime is positive. Let $\chi''$ be 'The number on which the ball has landed is prime'. This sentence has a chance of 11-in-38. Let $\delta''$ be '*That number* (the one on which the ball has landed) is the number on which the ball has landed', uttered while gesturing toward the concealed numeral. This sentence has a positive chance. Let $\phi''$ be $\ulcorner\chi'' \,\&\, \delta''\urcorner$, and let $\psi''$ be '*That number* (the one on which the ball has landed) is prime', uttered

19 A variant of the example seems to establish that some prospects with odds very near certainty (e.g., 99.99%) are not only metaphysically impossible but not even possibly possible. The example employs the additional assumption (roughly) that necessarily, Woody's construction requires more than one-half of the planks (matter) from which it in fact originated, supplemented by sufficiently many additional planks.

20 Kment says (*ibid*., sec. 2.3) that the disputed metaphysical position is inconsistent with full *BCP*. His arguments could be recast, contrary to his intent, as *reductio* arguments for *CI*. So reconfigured, Kment's arguments are considerably more complicated than the present argument using Monotonicity.

while gesturing toward the concealed numeral. In the setup its content is evidently a chancy impossibility.[21]

Analogously, suppose, as seems plausible, that it is an essential property of any conscious human being that he/she did not originally come into the world as an inorganic robot devoid of consciousness. Now suppose that collected together in a room is an Argentinian truck driver and four inorganic, unconscious robotic replicas made in Japan. One of the room's inhabitants is selected entirely at random. What is the objective chance that it is an inorganic robot? Exactly four-in-five. As it happens, the selected inhabitant is the Argentinian.

Even in the absence of potentially controversial essentialist theses, and relying instead solely on the modal logic of 'actually', Monotonicity generates an unlimited number of counterexamples to the weak *BCP* thesis. Let $\phi$ be any sentence that expresses a false proposition having a positive chance, e.g., 'The next roll of the die will land on three'—or if that happens to be true, then 'The next roll of the die will land on four'. Then since $\phi$ logically entails $\ulcorner$Actually $\phi\urcorner$, the latter expresses an impossibility which, by Monotonicity, has a chance at least as great as that of $\phi$ itself.

This formulaic version can be given the same structure as the versions involving essentialism. Let $\chi'''$ be the sentence, $\ulcorner$In the possible world that is realized, $\phi\urcorner$, which is false but has the same positive chance as $\phi$. Let $\delta'''$ be the modal-logical truth 'This world (the one that is realized) is the possible world that is realized', whose chance is one. Let $\phi'''$ be $\ulcorner\chi''' \,\&\, \delta'''\urcorner$, which has the same chance as $\chi'''$. Let $\psi'''$ be the conclusion, $\ulcorner$In this world (the one that is realized), $\phi\urcorner$. Though it expresses a metaphysical impossibility, by Monotonicity $\psi'''$ has at least as much chance as $\phi'''$. Teresa Robertson observes that this argument for *CI* can also be run contrapositively: Let $\phi$ be any true but chancy sentence, e.g. $\ulcorner$The next roll of the die will land on $n\urcorner$ where $n$ is the Arabic numeral

21 This example has certain features in common with an example given by John Hawthorne and Maria Lasonen-Aarnio in "Knowledge and Objective Chance," in Patrick Greenough & Duncan Pritchard, eds., *Williamson on Knowledge* (Oxford University Press, 2009), pp. 92-108. Hawthorne and Lasonen-Aarnio consider fixing the reference of the name 'Lucky' by the description 'the winner of the lottery' to generate *a priori* knowledge of the contingent and low-chance fact concerning the winner that he/she will win the lottery. However, no contingent *a priori* knowledge is involved in the present case—or, for that matter, in Hawthorne and Lasonen-Aarnio's. *Cf.* my *Frege's Puzzle* (Atascadero, CA: Ridgeview, 1986, 1991), at pp. 138-142; and "How to Measure the Standard Metre," *Proceedings of the Aristotelian Society* New Series, 88 (1987/1988), pp. 193-217, reprinted in my *Content, Cognition, and Communication* (Oxford University Press, 2007), chapter 7, pp. 141-158.

It is tempting to object that the singular fact expressed by $\delta''$ is not certain, since it is unknown (in the relevant epistemic situation) on which number the ball has landed. The objection is misplaced but the issues involved are quite delicate. Several facts are relevant here: (*i*) In ordinary conversation one may refer directly to a whole number simply by gesturing toward its canonical Arabic numeral and uttering 'that number'; (*ii*) one can know of a person (*de re*) that he/she is F (e.g., committed a particular theft) without any knowledge or opinion who it is; (*iii*) one can know that some person or other is uniquely hidden behind a curtain, and know of that person (*de re*) with certainty that *he or she* (whoever it is) is the person on the other side of the curtain; and (*iv*) the demonstrative phrase '*that number* (the one on which the ball has landed)', with its accompanying demonstration, may be replaced throughout with the artificial-language term '*dthat* [the number on which the ball has landed]'. In any event, (*v*) $\delta''$ expresses (with respect to the relevant context) the singular proposition about 26 that it is the number on which the ball has landed. The chance of that true singular proposition is objectively at least 1-in-38. Monotonicity requires that insofar as $\phi''$ has some positive chance, $\psi''$ does as well. Finally (*vi*), even if the roulette-wheel example is found unconvincing because of the *a priori* certainty that 26 is not prime, there remain the other examples of chancy impossibilities.

such that the resulting sentence is true. Since ⌜Actually $\phi$⌝ logically entails $\phi$, by Monotonicity if weak *BCP* is correct, then *chance*($\phi$) = 1. Weak *BCP* is therefore incorrect.[22]

A defender of weak *BCP*, the referee for this journal suggests that those who will oppose *CI* in the face of these examples should reject Monotonicity in favor of the metaphysically modal monotonicity principle that results by replacing logical entailment with the subset relation between sets of metaphysically possible worlds. (See note 4.) The referee cites the entailment $\phi$ by ⌜Actually $\phi$⌝ as one for which Monotonicity should be rejected, since the modal intension of ⌜Actually $\phi$⌝ is the set of all possible worlds even when the modal intension of $\phi$ is a proper subset. Yet few philosophers, if any, will go so far as to deny that it is knowable by reason alone that if actually the next roll of the die will land on *n*, then the next roll of the die will land on *n*. My claims are that: (*a*) chance aligns with logic, in the sense that logical equivalents have the same chance; but (*b*) partly as a consequence of (*a*), chance does not align in the analogous manner with metaphysical modality. Although perhaps initially surprising, claim (*b*) survives critical scrutiny. By contrast, the opposite claims that chance instead aligns with metaphysical modality and thereby fails to align with logic strikes the present author as unacceptable on philosophico-conceptual grounds. Faced with a choice between aligning chance with logic (Monotonicity) and aligning chance with metaphysics (modal monotonicity), we should go with logic. I regard the rejection of Monotonicity in response to my objections as telling against weak *BCP*.

The choice between monotonicity principles is a distraction from the crux of the matter. For most of the examples the $\models$ in Monotonicity may be taken as classical-logical entailment instead of modal-indexical-logical entailment. For present purposes this weaker version of Monotonicity coincides closely enough with the metaphysical-modal variant. The principal bone of contention lies instead in the assessment of the chances of the various singular propositions whose chance is at issue. The referee believes that, whereas $\psi_5$ has a one-in-three chance in $W_1$ of being true in English, the chance of $\psi_5$ itself (in contrast to its being true) is zero, since by hypothesis it is metaphysically impossible in $W_1$ for Woody to have been constructed using $P_{101}$, $P_{102}$, and $P_{103}$. The *CI* opponent is committed to the judgment that: (1) any appearance or intuition that *chance*($\phi_5$) = *chance*($\psi_5$) = ⅓ in $W_1$ is erroneous; and (2) solely because of a seemingly extraneous, metaphysically necessary fact, which is *a posteriori* and unknown to *A* in $W_1$, *chance*($\phi_5$) = *chance*($\psi_5$) = 0. That judgement in turn forces the *CI* opponent to judge furthermore that *chance*($\psi_1$) is not ⅓ but fully ½, in order that the laws of probability not be violated. These assessments of chances are decidedly unintuitive, at least in the

22 Upon seeing a draft of the present paper Daniel Nolan informed me that he independently thought of a related argument in connection with 'actually'. See his "Chance and Necessity" in J. Hawthorne and J. Turner, eds, *Philosophical Perspectives, 30, 1 Metaphysics, 2016* (published 2017), pp. 294-308. Nolan does not accept that there can be chancy impossibilities. If I understand his recommended position with respect to my roulette-wheel example (see *op. cit.* especially pp. 304-306), he proposes that whereas the proposition that *that number* [pointing toward the pocket into which the ball has landed] is prime has a chance of 11-in-38, the distinct but necessarily equivalent proposition that 26 is prime has no chance at all, not even a chance of zero. (Alan Hájek has argued that there are chance gaps in "What Conditional Probability Could Not Be," *Synthese*, 137, 3, December 2003, pp. 273-323.) I do not accept Nolan's proposal; I take it that the propositions in question are one and the same, hence with the same disparate chances. Nolan is inclined to reject the thesis that chance is modally intensional. (See note 4 above.) On this he and I agree. Chancy impossibilities are co-intensional with outright contradictions, which invariably leave nothing to chance. Insofar as chance obeys Monotonicity, however, logically equivalent prospects invariably have the same chances.

standard, primary sense of 'chance', and are defensible only by appealing to weak *BCP* itself. The *CI* opponent is blind to the important differences between certain false sentences like $\psi_5$ or $\psi'$ and an outright contradiction. It should be noted also that, regardless of what the *CI* opponent may judge, assuming that agent *A* understands $\psi_5$, the chance in *A*'s epistemic situation of the Tarskian *T*-sentence ⌜'Woody was constructed using $P_{101}$, $P_{102}$, and $P_{103}$' is true in English if and only if $\psi_5$⌝ is one. Monotonicity is a sidebar.

There might even be impossible prospects with positive chance independently of Monotonicity. Goldbach's conjecture might be correct and it might be incorrect. For "sufficiently large" integers, the greater the integer, the more ways there are of designating it as the sum of two smaller numbers. Statistical considerations based on the probabilistic distribution of prime numbers arguably favor the weaker conjecture that for some integer *n* or other, every even integer greater than *n* is the sum of two primes, so that if the original Goldbach conjecture is false, then there is a greatest counterexample.[23] If that is correct, then: (*i*) if Goldbach's conjecture is correct for sufficiently large even integers, then the proposition that there are infinitely many even integers that are not Goldbach numbers is a metaphysically impossible prospect that in the current state of ignorance has a positive chance; (*ii*) if on the other hand Goldbach's conjecture has infinitely many counterexamples, then the proposition that it holds for sufficiently large even integers is itself such a prospect.

There is a chance-like distribution that, unlike chance, assigns probabilities in conformity with weak *BCP*. The *modally-adjusted chance* of a prospect *p* is the conditional chance of *p* given the entirety of chancy metaphysically necessary truths, known or unknown. Modally-adjusted chance is a crude variant of genuine chance, indifferent to differences of chance among necessary truths. To say that a prospect has a modally-adjusted chance of one is to say that either it has a chance of one, or else it does not but is metaphysically necessary. Likewise, to say that a prospect has a modally-adjusted chance of zero is to say that either it has a chance of zero, or else it is a chancy impossibility. The adjustment for non-contingent prospects impacts the assessments of contingent prospects. In $W_1$, whereas $chance(\psi_1) = chance(\psi_5) = \frac{1}{3}$, $modally\text{-}adjusted\text{-}chance(\psi_1) = \frac{1}{2}$, in compensation for the fact that $modally\text{-}adjusted\text{-}chance(\psi_5) = 0$.

If there is a sense of the word 'chance' in Standard English that corresponds to modally-adjusted chance—a big 'if'—it is not the primary sense. (See note 7.) The examples of chancy impossibilities forcefully illustrate that the *CI* opponent's assessments of chance are squarely at odds (if the reader will pardon the expression) with the very practice of making educated estimates concerning outcomes of which we are uncertain. Consider again the example of the zygote. Hopeful parents sometimes spend substantial sums to centrifuge the sperm used to fertilize an ovum, thereby dramatically increasing the odds that their *in vitro* zygote will be a particular sex. They would be none too amused if their local philosophy professor should inform them that, notwithstanding their efforts and expenditures, and contrary to the fertility clinic's promises, the centrifuge procedure had absolutely no influence or impact whatsoever on the *de re* chance that their zygote is the desired sex. For according to weak *BCP*, insofar as the sex of a zygote is an essential

[23] Henry F. Fliegel and Douglas S. Robertson, "Goldbach's Comet: the Numbers Related to Goldbach's Conjecture," *Journal of Recreational Mathematics*, 21, 1 (1989): pp. 1–7.

property, the *de re* chance that an existing zygote is a particular sex is invariably either zero or one, depending simply on the zygote's sex. Fortunately, a second opinion from a clinical statistician will confirm that the actual chance is slightly less than one, in line with the fertility clinic's estimate.

## IV

Insofar as the chances of various impossibilities are what they are, the weak *BCP* thesis fails. Why, then, is the thesis at least initially attractive? More to the point, how does it happen that a metaphysical impossibility can have a positive chance of obtaining?

One reason for the initial pull of weak *BCP* has already been noted: It is easy to overlook that a logically possible prospect that is precluded by metaphysical constraints rather than by the vicissitudes of fortune is still a matter of chance insofar as it follows logically from a chancy falsehood. Another possible reason that weak *BCP* is initially attractive is that one might be tempted by a fallacious inference from one or both of a couple of truisms: (1) the chance of $p$, given that $p$ is necessary, is one, i.e., $chance(p|\Box p) = 1$; (2) the chance of $p$, given that $p$ is impossible, is zero, i.e., $chance(p|\sim\Diamond p) = 0$. Although they sound similar, the claim that the chance of $p$, given $q$, is $n$ is not equivalent to (and does not even entail) the conditional claim that given $q$, the chance of $p$ is $n$—not even for a Bayesian. That is, $chance(p|q) = n \not\models [q \rightarrow chance(p) = n]$. While there is a conceptual connection between metaphysical modality and objective chance, it is far more tenuous than philosophers have generally recognized.[24]

A more explanatory reason for the initial attraction of the weak *BCP* thesis has to do with the very nature of chance. It is extremely dubious that the chance of a prospect $p$ is a *propensity*.[25] In rolling a pair of fair dice, the chance of obtaining a composite number is exactly $^{21}/_{36} = {^7}/_{12}$. We should be wary of inferring that two dice, together as a duo, are causally, physically, or metaphysically predisposed or inclined, precisely to the degree $^7/_{12}$, to deliver a composite number on being rolled. It is not even clear what such a claim means exactly, or even that it has any definite meaning. Certainly the claim that there is a $^7/_{12}$ chance of rolling a composite number with a pair of fair dice cannot be identified with such a propensity claim—except insofar as by 'propensity' one simply means *chance*, so that "propensity theory" is not a substantive theory in any robust sense but an analytic truism. The chance of a prospect is a measure of something but not of some mysterious physical or metaphysical nudge that influences dice rolls and the like. Whatever is the most compelling theory of the kind of chance possessed by coin tosses, die rolls, lotteries, wheels of fortune, dreidels, and phenomena

24 *CI* does not rule out that $\forall p[chance(p) = 1 \rightarrow p]$, nor (nearly equivalently) that $\forall p[p \rightarrow chance(p) > 0]$, which arguably follows from the factivity of knowledge. (The analogs for so-called credence are not plausible.) In that case, since metaphysical necessity is also factive, $\forall p[chance(p) = 1 \rightarrow \Diamond p]$ and $\forall p[\Box p \rightarrow chance(p) > 0]$. This would establish a tenuous connection between metaphysical modality and chance, much weaker than the connection posited by weak *BCP*. Analogously, metaphysical modality and knowledge are tenuously connected in that whatever is known is metaphysically possible and whatever is metaphysically necessary is epistemically possible.

25 Lewis, *op. cit.*, identifies objective chance with "propensity, understood as making sense in the single case" (p. 263). Kment says, "It is natural to think of the present chance that *P* as something like the strength of the universe's present tendency to evolve into a state where *P*" (*op. cit.*, section 2.3). See note 7 above. See also Antony Eagle, "Twenty-one Arguments against Propensity Analyses of Probability," *Erkenntnis*, 60, 3 (May 2004), pp. 371-416.

sufficiently like these, it does not include the assertion that chance is a causal, physical, or metaphysical propensity.

The chance of a prospect is objective. It does not follow that chance is not at all an epistemic matter—unless the word 'objective' is redefined in a non-standard manner.[26] In fact, with regard to phenomena like coin tosses, dice rolls, and roulette, *chance is epistemic*.

Objective chance is relative to an inclusive (and typically broader) range of prospective outcomes or scenarios (pairs of whole numbers one-through-six, heads or tails, etc.). The chance of a prospect is simply a percentage or ratio (proportion, quotient, fraction), in effect a numerical comparison of the scenarios which are stipulated to include the prospect in question—and which need not even be metaphysically possible—to the antecedently understood full range of prospective scenarios. It is something like the proportion of the subclass of suitably (possibly weighted) epistemically possible scenarios in which the prospect obtains to the full range of relevant epistemically possible scenarios. In the most straightforward type of case—a finite equiprobable space representing a fair coin, a pair of fair dice, a fair lottery, or the like—the chance of $p$ is the classical Laplacian ratio of the number of epistemically "equipossible" (none weighted more heavily than any other), mutually exclusive, scenarios in which $p$ obtains to the number of the jointly exhaustive totality of relevant, epistemically equipossible, mutually exclusive scenarios.[27] Relative to a fixed epistemic situation with regard to outcome, the Laplacian ratio is exactly as objective as the odds of winning a fair lottery or of rolling snake eyes with a pair of fair dice. Roughly, the Laplacian ratio is to propensity theory as natural selection is to creationism.

The relevant kind of possibility that the relevant scenarios have is epistemic, akin to the non-metaphysical possibility expressed by 'for all we know'. Epistemic possibility—the propositional attribute expressed by the phrase 'for all we know'—is the dual of knowledge: A proposition is *epistemically possible for x* if and only if its denial is not known by $x$ (e.g., by those engaged in the present inquiry). Epistemic possibility is easily confused with metaphysical possibility. (Or is it the other way around?) Notoriously, epistemic possibility is not a reliable indicator of metaphysical possibility. In our current state of ignorance Goldbach's conjecture and its denial are each epistemically possible,

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26 In *Ars Conjectandi* (posthumous 1713), Jacob Bernoulli conceived probability "as a measurable degree of certainty." By contrast, in "Chance versus Randomness," §1, Antony Eagle takes it that by definition, if there is any such phenomenon as chance, it is objective and thus not epistemic. See note 32 below.

27 "The probability of an event is the ratio of the number of cases that are favorable to the number of all the possible cases, where there is no reason to believe that one of these cases must happen rather than the others, which makes them equally possible for us. Full appreciation of these cases is one of the most delicate points in the analysis of chance."—Pierre-Simon Laplace, *Analytic Theory of Probability* (1812), Second Book, *General Theory of Probability*, chapter 1, "General Principles of This Theory," in *œuvres de Laplace* (Paris: Imprimerie Royale, 1847, at p. 195)

"Probability has reference partly to our ignorance, partly to our knowledge. . . . The theory of chance consists in reducing all the events of the same kind to a certain number of cases equally possible, that is to say, to such as we may be equally undecided in regard to their existence, and in determining the number of cases favorable to the event whose probability is sought. The ratio of this number to that of all the cases possible is the measure of this probability, which is thus simply a fraction whose numerator is the number of favorable cases and whose denominator is the number of all the possible cases."—Laplace, *A Philosophical Essay on Probability* (1814) (New York: Dover Publications, 1951)

I am not here proposing the classical Laplacian ratio as a definition of 'chance'. However, chance must coincide with the classical Laplacian ratio in those cases where the latter is defined.

although one of the two is metaphysically impossible. In most contexts, the kind of possibility involved in chance is epistemic possibility in a rarefied sense, something like the concept expressed by 'for all that follows logically from what we know'. This attribute is also epistemic.[28]

The concepts of metaphysical necessity and possibility are purely metaphysical. Metaphysical modality is unaffected by how one takes the proposition in question, by the guise under which a proposition is apprehended. Chance is otherwise; one's knowledge is affected by the proposition-guise. A proposition $p$ is *epistemically possible for $x$ under* a particular proposition-guise $y$ if and only if $x$ grasps $p$ by means of $y$ and $\sim p$ is not known by $x$ under $Neg(y)$.[29] David Lewis writes, "to the question of how disparate chances can be reconciled with one another, my answer is: *it can't be done*" (*op. cit*., p. 118). My own answer is more upbeat. Disparate chances relative to a fixed epistemic situation are reconciled by the fact that the chance of a proposition is relative to a suitable proposition-guise. The chancy impossibility expressed by $\psi''$ in the roulette-wheel example is also expressed by '26 is prime', whose chance is at least normally zero. The prospect of 26 being prime thus has disparate chances. Under one proposition-guise it is epistemically possible; under another it is not.

For this reason the distribution that assigns to each sentence and/or proposition its chances is not a function. By contrast, chance relative to both a body of background information and an assignment of proposition-guises to sentences/propositions is arguably a function. The functor '*chance*( )' that occurs in Monotonicity may be understood accordingly, as implicitly relativized to a body-of-information-*cum*-assignment-of-proposition-guises. For many purposes—'Paderewski'-type cases aside[30] —one may regard a sentence as carrying a unique proposition-guise with it. Alternatively, the proper objects of chance may be taken to be sentences themselves (in a language) in lieu of their associated propositions. (The chance of a sentence is not invariably the same as the chance that the sentence is true, which is the chance of something meta-linguistic.) For non-'Paderewski' cases, Monotonicity is sufficiently precise as formulated above. It is allowed, for example, that the propositions that 26 is prime and that *that number* is prime are the same and yet *chance*('26 is prime') $\neq$ *chance*('*That number* is prime'). This assumes that

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28 A prospect that is epistemically possible in the ordinary sense might be epistemically impossible in the logically omniscient sense. Though its denial is not known, that denial might nonetheless follow from what is known. Potentially, one and the same prospect might be assessed in one context as having better than even odds (in a given epistemic situation) and in another context as having zero chance (in the same epistemic situation).

29 *Cf.* my "Relative and Absolute Apriority," *Philosophical Studies*, 69, (1993), pp. 83-100.

30 Saul Kripke, "A Puzzle about Belief," in A. Margalit, ed., *Meaning and Use* (Boston: D. Reidel, 1976), pp. 239-283. Arguably, in the epistemic situation of Kripke's character Pierre, *chance*('London is pretty') is low whereas *chance*('*Londres est jolie*') is high, despite the fact that the two sentences are exactly synonymous. More to the point, arguably it could happen that in the epistemic situation of Kripke's Peter: (*i*) *chance*('Paderewski was honored with a U.S. postage stamp') is low under the guise (i.e. perhaps, under the presentation of the relevant proposition as), *That man (the pianist and composer named 'Paderewski') was honored with a U.S. postage stamp*; whereas (*ii*) *chance*('Paderewski was honored with a U.S. postage stamp') is one under the guise, *That man (the former Polish prime minister named 'Paderewski') was honored with a U.S. postage stamp*.

chance for the demonstrative sentence is understood by means of an appropriate guise of 26, as *whichever number it is on which the ball has landed*.[31]

Recall Leibniz's dictum. Chance is not a gradation of metaphysical modality, which is in any case not a matter of degree, but it is a gradation of *something*, some commodity that a prospect possesses to the maximum degree if its chance is 100% and altogether lacks if its chance is only 50%. Probability of 100% is called 'certainty' rather than 'necessity', for good reason. For many purposes, chance is a gradation of epistemic possibility and partial-to-full certainty—precisely the kind of certainty or uncertainty that is typically conferred upon a prospect by the chance that it obtains. Specifically, for many purposes, chance may be conceived as providing a prospect's *degree of certainty*, ***cert***, as defined by:

$$\boldsymbol{cert}(p) = 2chance(p) - 1$$

The function, ***cert***, assigns to a proposition a degree of partial-to-full certainty. Exactly even odds (e.g., the chance that a fair coin lands tails) translates into zero certainty one way or the other. A chance of 0.8 (four-in-five) equates to certainty to degree 0.6, i.e., 60% of full certainty. A degree of certainty given by a negative number provides a measure of certainty of the denial, according to the theorem that $\boldsymbol{cert}(\sim p)$ is the additive inverse of $\boldsymbol{cert}(p)$, i.e., $\boldsymbol{cert}(\sim p) = -\boldsymbol{cert}(p)$. The chance of $p$ is 0.8 if and only if the degree of certainty of $\sim p$ is $-0.6$. Odds greater than zero and less than one constitute a gradation of epistemic *uncertainty*.[32]

## V

There appears to be a connection between chance and future contingents. Many writers are under the misimpression that chance is concerned exclusively with the future, and as there is before the toss not yet any fact about how the coin will land, the perfect balance

31 Something analogous might be true also of logical entailment. In the standard sense of 'logically valid', although 'Hesperus = Hesperus' and 'Hesperus = Phosphorus' express the same singular proposition, the former is valid (and its chance therefore one), the latter invalid. Likewise, '26 is prime' and $\psi''$ uttered with reference to 26, are not logically equivalent sentences though they express the same proposition. See notes 16 and 30 above. *Cf.* Handfield, *ed. A Philosophical Guide to Chance*, at pp. 5-7. (I thank Daniel Nolan for pressing me to address these matters).

David Chalmers and Antony Eagle have independently suggested that since proponents of weak *BCP* hold that chance is non-epistemic and not relative to proposition-guises, weak-*BCP* proponents and I merely mean different things by 'chance'. See note 26 above. Foreshadowing points to be emphasized shortly, it cannot be concluded from the fact that the verdicts of a critique clash with the theory in question, that the critique therefore targets a different phenomenon. The assessments of chance made by *BCP* are at odds with the primary sense of 'chance', as indicated by the actual practice of making educated estimates concerning uncertain outcomes. It should be noted also that the *de re* chance of 26 being prime is objectively zero under an arithmetical guise, e.g., *26 is prime*, and is equally objective under the alternative guise, *That number (the one on which the ball has landed) is prime*. The chances of '26 is prime' and of $\psi''$ are thereby exactly as objective as the chances of '$2 \times 13$ is prime' and $\chi''$.

32 Compare the definition given in note 1 above of degree of belief $\boldsymbol{b}_x$ as a function of degree of confidence ("credence") $\boldsymbol{c}_x$. The scalar quantity ***cert*** is the same recalibration as $\boldsymbol{b}_x$, but of chance in lieu of confidence. Confidence is typically classified as epistemic and chance as non-epistemic. In their primary senses, it is precisely the other way around: Where confidence is principally doxastic rather than epistemic, chance is primarily epistemic. See notes 24 and 26. A critical question: Is there a number $n$ such that always, $\forall p[p \text{ is known} \leftrightarrow \boldsymbol{cert}(p) \geq n]$? If there is, then degrees of certainty greater than $-n$ and less than $n$ constitute a gradation of ignorance. (Is $n$ equal to 1?).

between opposing propensities is tipped the moment the coin is tossed and lands tails. Thus Lewis writes, "What's past is no longer chancy. The past, unlike the future, has no chance of being any other way than the way it actually is" (*op. cit*., p. 273). That chance exclusively concerns the future is built into *BCP* and, Lewis argues (*ibid*., pp. 276-278), also into his so-called Principal Principle.[33] On the contrary, chance concerns the present and the past as much as it concerns the future. We routinely assess the chances of past outcomes that are unknown to us. A second ago Jones drew a card completely at random from a thoroughly shuffled deck and without looking at it placed the card on the bottom. The chance that Jones drew clubs—a past outcome—is neither one nor zero but one-in-four. It is in line with the chance that when the card Jones drew is checked immediately afterward—a future outcome—it will be seen to be clubs. Contrary to Lewis and others, for each of the four suits—(clubs, diamonds, hearts, and spades)— hence including the suit Jones actually drew—the present chance is three-in-four that Jones drew a suit different from it. With regard to coin tosses, lotteries, and the like, the chance of a prospect, whether actual or counterfactual, is relative to background information, effectively to one's epistemic situation. The abrupt change of odds when the coin lands merely reflects the advance in knowledge, which augments the background information. Even after the toss, in the absence of a specification of the outcome the chance that the coin landed heads is one-in-two. By the same token, the pre-toss conditional chance of heads is zero given that the coin will land tails, i.e., given information that no one but a clairvoyant has before the toss. If in the fullness of time the coin lands tails, then it is already true pre-toss—even though it is still preventable and not yet knowable by natural means—that the coin will land tails.[34]

The referee for this journal suggests that since proponents of *BCP* believe that chance at a time takes account of all chance-relevant matters up to that time, *BCP* proponents and I merely mean different things by 'chance'. In general, it cannot be legitimately concluded from a proposed counterexample to a theory that since the theory delivers a different verdict, the objection therefore concerns a different phenomenon. For example, even if pre-Gettier proponents of the traditional analysis of '*S* knows *p*' in some sense misunderstood the verb 'know' to mean *justified, true belief*, it cannot be concluded that the famous Gettier objection concerns a different phenomenon from traditional epistemology. Even if pre-Gettier epistemologists misunderstood 'know', insofar as they spoke Standard English they also meant by 'know' what it means, and the Gettier examples refuted their mistaken theory concerning it.

There is a chance-like probability distribution that, unlike chance, assigns one or zero to all present and past outcomes. The *historically-adjusted chance* at time *t* of a prospect *p* is the conditional chance of *p* given the entire history of the world right up to *t*, including all the unknown facts about the past. If *p* is a prospect that occurs prior to or at *t*, then $\textit{historically-adjusted-chance}_t(p) = 1$; and if *p* is a prospect whose failure to occur is prior to or at *t*, then $\textit{historically-adjusted-chance}_t(p) = 0$. *Cf.* the notion of modally-adjusted chance defined above. Historically-adjusted chance does not discriminate among the chances of prior and contemporaneous outcomes. A prospect has an historically-

33 Again see note 7 above. See also James M. Joyce, "Epistemic Deference: The Case of Chance," *Proceedings of the Aristotelian Society*, New Series, Vol. 107 (2007), pp. 187-206, at p. 199; and Schaffer, "Deterministic Chance?" at pp. 124-125.

34 *Cf.* Handfield, *ed*, *A Philosophical Guide to Chance*, pp. 7-8, 28-30, 85.

adjusted chance at $t$ of one if and only if either it has a chance at $t$ of one, or else it is chancy at $t$ but occurs prior to or at $t$. Although several of the arguments of the present paper involve prior outcomes, they are extendable with slight modification to historically-adjusted chance.

There is a third chance-like probability distribution, an amalgam of modally-adjusted chance and historically-adjusted chance. While it is sensitive to the chances of future contingents, *modally/historically-adjusted chance* is blind to differences of chance among metaphysically unpreventable outcomes—outcomes that are past or present or metaphysically necessary. A prospect has a modally/historically-adjusted chance at $t$ of one if and only if either it has a chance at $t$ of one, or else it is chancy but metaphysically unpreventable at $t$, because it is metaphysically necessary or simply occurs prior to or at $t$. Modally/historically-adjusted chance conforms to many current philosophical misconceptions concerning chance, including full *BCP*. (Again, see note 7.) However, examples of chancy impossibilities illustrate that it is significantly cruder than genuine chance.

As Teresa Robertson points out, there is an even more distant relative of chance. The *factually-adjusted chance* of $p$ is the conditional chance of $p$ given the truth, the whole truth, and nothing but the truth, including the unknown. Factually-adjusted chance is the omniscient distribution: the minimally discriminating probability function that assigns one to all truths and zero to all falsehoods (and is otherwise undefined). Consider a philosopher who says, "The chance of rolling a prime number with a pair of fair dice is either one or zero, depending on how the dice land." If such a philosopher means something different by 'chance', that is not all there is to the matter. Insofar as he/she uses 'chance' as a word of Standard English, the philosopher also means by 'chance' what it means. Suppose we point out that the espoused view misjudges the risk of betting on rolling a prime number with a pair of fair dice: Absent a specification of the outcome, we warn, the chance of rolling a prime is actually $^5/_{12}$, not a good bet. Suppose now the philosopher replies, "You're merely operating with a different notion. Your notion evidently makes discriminations among uncertain propositions independently of their actual truth-value. My notion is concerned exclusively with actuality, and as such it is entirely objective. Objectively speaking, whether it is a good bet that a prime number will be rolled depends entirely on how the dice actually land. What's real is not chancy. Reality, unlike unreality, has no chance of being any other way than the way it actually is." Something is very wrong. Such a philosopher is not merely operating with an alternative notion of chance. Factually-adjusted chance trivially obeys the laws of probability, but it is no kind of chance, properly so-called. It deviates from genuine chance to such an extreme as to be completely disengaged from the practice of making estimates—perfectly objective estimates—concerning uncertain outcomes. To be sure, modally/historically-adjusted chance deviates more moderately from genuine chance. The deviation is of the same kind and along the same lines as that of so-called factually-adjusted chance.

It is initially plausible that any metaphysically necessary truth is knowable *a priori*. For experience being contingent, it seems that if experience is required to know $p$, then if experience had gone the other way, $p$ would be false. However, we have learned from Kripke that insofar as there are *a posteriori* essential properties, metaphysical necessity is no guarantee of apriority. Where there are *a posteriori* essential properties, necessity is likewise no guarantee of certainty in the sense of a chance of one. In $W_1$, Woody is essentially not constructed using $P_{101}$, $P_{102}$, and $P_{103}$. Yet that it was not so constructed is *a posteriori*. Agent $A$ knows at $t_2$ that there is a one-in-three chance that Woody was constructed using $P_{98}$, $P_{99}$, and $P_{100}$, and that if it was, then it is impossible for it to have been constructed instead using $P_{101}$, $P_{102}$, and $P_{103}$. But in $A$'s epistemic situation at $t_2$ in $W_1$, there is a one-in-three chance that Woody *was* constructed using $P_{101}$, $P_{102}$, and $P_{103}$, impossible though that prospect is in $W_1$. Indeed, $A$ knows at $t_2$ that there is a two-in-three chance that Woody was constructed in such a manner that there is a one-in-three chance of Woody having been constructed in a manner that would be, as a matter of fact, impossible.

It has not been argued here that there is no legitimate, non-epistemic concept of objective probability that attaches in an absolute manner to singular propositions. The common notion of objective chance as a measure of the likelihood of outcomes of dice rolls, coin tosses, and the like is epistemic, and just as there are *a posteriori* essential properties, there are chancy impossibilities.[35]

Philosophical orthodoxy is supported less by reason and insight than by faith and inertia. The odds are long that *CI* will gain currency in the foreseeable future. Still, they are not impossible odds. Hope springs eternal. Or at least hope springs for the remainder of this heretic's lifetime.

[35] Consider the semantic measure *least-chance*($\phi$), defined as the smallest chance of the proposition expressed by $\phi$. In Pierre's epistemic situation, *least-chance*('London = London') < 1. (See note 30 and the preceding note.) Least-chance is not monotonic with regard to logical entailment between sentences. Sentence $\psi''$ uttered with reference to 26 expresses an impossibility, and whereas *chance*($\psi''$) is positive, *least-chance*($\psi''$) = 0. Hence, although $\phi'' \models \psi''$, *least-chance*($\phi''$) > *least-chance*($\psi''$). An ideally rational subject will have complete confidence that London = London, conditional on *p*—for any proposition *p*, even that *least-chance*('London = London') < 1.