THE JOURNAL OF PHILOSOPHY

VOLUME CVII, NO. 9, SEPTEMBER 2010

LAMBDA IN SENTENCES WITH DESIGNATORS: AN ODE TO COMPLEX PREDICATION*

In his classic studies of the "Logic of Sense and Denotation" (*LSD*), Alonzo Church contrasted three criteria or explications of the notion of *sense*, or semantic content, and of the resulting notion of synonymy, in the sense of *having the same sense*.[1] These are Alternatives (0), (1), and (2), numbered in order from the most strict to the least. On the most lax criterion, Alternative (2), synonymy is taken to be mere logical equivalence. On the intermediate Alternative (1), the criterion for synonymy is convertibility by means of Church's

*I am grateful to the participants in my seminars at the University of California, Santa Barbara during Winter 2007 and at the City University of New York Graduate Center during Fall 2009 and to my audiences at the University of Missouri Kline Workshop of Meaning, April 2009 and at the University of Buffalo, October 2009 for discussion of Kripke's critique and my response. I thank Luke Manning and Teresa Robertson for comments and suggestions. I am especially grateful to C. Anthony Anderson for valuable suggestions and for valuable discussion, both of the relevant issues and of the relevant and brilliant work of our remarkable former teacher, the late Alonzo Church.

[1] Church, "Abstract of 'A Formulation of the Logic of Sense and Denotation'," *Journal of Symbolic Logic*, XXI, 1 (March 1946): 31; "A Formulation of the Logic of Sense and Denotation," in Paul Henle, Horace Kallen, and Susanne Langer, eds., *Structure, Method and Meaning: Essays in Honor of Henry M. Sheffer* (New York: Liberal Arts Press, 1951), pp. 3–24; "Outline of a Revised Formulation of the Logic of Sense and Denotation," Part 1, *Noûs*, VII, 1 (March 1973): 24–33 and Part 2, VIII, 2 (May 1974): 135–56; "A Revised Formulation of the Logic of Sense and Denotation, Alternative (1)," *Noûs*, XXVII, 2 (June 1993): 141–57.

For some subsequent illuminating work on Church's *LSD*, see C. Anthony Anderson, "Alternative (I*): A Criterion of Identity for Intensional Entities," in Anderson and Michael Zelëny, eds., *Logic, Meaning and Computation: Essays in Memory of Alonzo Church* (Boston: Kluwer, 2001), pp. 395–427, at pp. 421–22. There is a valuable discussion of *LSD* and Church's three alternative criteria for synonymy in Anderson's "Alonzo Church's Contributions to Philosophy and Intensional Logic," *The Bulletin of Symbolic Logic*, IV, 2 (June 1998): 129–71. I thank Anderson for bibliographical references. (It is untrue that a psychedelic Lennon-McCartney composition contains a veiled reference to Church's classic study.)

0022-362X/10/0709/445–468

λ-abstraction operator.[2] Expressions are *λ-convertible* if one is obtainable from the other by a sequence of applications of the λ-conversion rules of λ-expansion, λ-contraction, and alphabetic change of bound variables. Since the λ-conversion rules are each logically reversible (that is, the reverse inference is equally valid), expressions that are λ-convertible are deemed "synonymous" on both Alternatives (1) and (2). On the strictest criterion, Alternative (0), the criterion for synonymy is Church's notion of synonymous isomorphism, an improvement over Rudolf Carnap's notion of intensional isomorphism.[3] Expressions are *synonymously isomorphic* if one is obtainable from the other by a sequence of alphabetic changes of bound variables or replacements of component expressions by syntactically simple synonyms. In particular, on Alternative (0) the inference rules of λ-expansion and λ-contraction are not deemed to preserve sense. Aside from interchange of synonyms, at least one of which is syntactically simple,[4] the only interchange of logical equivalents deemed strictly to preserve sense is alphabetic change of bound variables. Logically equivalent sentences that are not synonymously isomorphic are deemed not *strictly synonymous.*

Few language philosophers today stubbornly maintain that logically equivalent sentences have the same semantic content, in the sense of expressing the same proposition.[5] In my judgment, those philosophers who do, confuse the semantic content of a sentence with a different semantic value, which I call the *logical content.*[6] Logically

[2] The λ-abstraction operator is a variable-binding operator. The extension, with respect to an assignment s of values to variables, of a λ-abstract $\ulcorner(\lambda\alpha)[\zeta_\alpha]\urcorner$ is the function that assigns to any potential value v in the range of the variable α, the extension of ζ_α with respect to the assignment that assigns v to α and is otherwise the same as s. If ζ_α is a singular term, then $\ulcorner(\lambda\alpha)[\zeta_\alpha]\urcorner$ is a compound functor. If ζ_α is a formula, then $\ulcorner(\lambda\alpha)[\zeta_\alpha]\urcorner$ is a compound predicate. The λ-conversion rule of *λ-expansion* licenses the replacement within a formula of any occurrence of ζ_β by an occurrence of $\ulcorner(\lambda\alpha)[\zeta_\alpha](\beta)\urcorner$, where β is of the same syntactic type as α and ζ_β is the result of uniformly substituting free occurrences of β for the free occurrences of α throughout ζ_α. The λ-conversion rule of *λ-contraction* licenses the reverse replacement of $\ulcorner(\lambda\alpha)[\zeta_\alpha](\beta)\urcorner$ by ζ_β.

[3] Church makes this proposal in his masterly essay, "Intensional Isomorphism and Identity of Belief," *Philosophical Studies,* v, 5 (October 1954): 65–73; reprinted in Nathan Salmon and Scott Soames, eds., *Propositions and Attitudes* (New York: Oxford, 1988), pp. 159–68.

[4] Interchange of syntactically compound expressions is not permitted. Church's idea seems to have been that there are no primitive synonymy rules stating that two compound expressions are synonymous. Instead, the synonymy of compound expressions is determined by the compositional semantic rules that govern the contents of compound expressions. (Thanks to C. Anthony Anderson for discussion.)

[5] Two prominent contemporary philosophers who maintain something close to an adherence to Church's Alternative (2) are Jaakko Hintikka and Robert Stalnaker. David Lewis's disciples also belong to this camp. In general, followers of Frege and Russell do not.

[6] This may be what Frege intended by his notion of *Erkenntniswerte* ("cognitive value"). Cf. my "On Content," *Mind,* CI, 404 (October 1992): 733–51; reprinted in my *Metaphysics, Mathematics, and Meaning: Philosophical Papers I* (New York: Oxford, 2005), pp. 269–85.

equivalent sentences are exactly those that share the same logical content. The logical content of a sentence might be identified with the class of its models. Sentences that are exactly alike in logical content might yet differ in semantic content, in the propositions they semantically express. One example of such pairs of sentences is the logical theorem '$(p \supset q) \vee (q \supset r)$' and Peirce's law, '$[(p \supset q) \supset p] \supset p$'. Another example might be 'Snow is white' and 'The number that is one if snow is white, and is zero if snow is not white, is one'. One example not involving logical truths is 'If a trespasser is caught, then he/she is prosecuted' and 'No trespassers are caught unless they are prosecuted'. Since these sentences are perfectly understandable, the fact that their equivalence is confirmed only upon reflection would seem to indicate that there is a difference in semantic content, although there is no difference in logical content.

Under careful scrutiny Alternative (1) fares little better than Alternative (2). Another pair of nonsynonymous logical equivalents is obtained from the sentence,

(A_E) This yacht is larger than that yacht is.

Suppose that, unbeknownst to the speaker, the occurrences of both complex demonstratives 'this yacht' and 'that yacht' are uttered with reference to the very same yacht. Even so, (A_E) contrasts sharply in semantic content with

(B_E) That yacht is a thing that is larger than it itself is.[7]

If the guest in Russell's famous example, on finally seeing the yacht, had compared it ("that yacht") to the one shown to him in a deceptive photograph ("this yacht"), he might well have believed the proposition expressed by (A_E) without thereby believing the proposition expressed by (B_E). This consideration is, by itself, already sufficient to prescribe contrasting semantic analyses of (A_E) and (B_E). The contrast between (A_E) and (B_E) also provides an explanation, or at least the beginning of an explanation, of how one can believe of the relevant yacht that it is larger than it is without believing that something is larger than itself. Distinguishing the contents of (A_E) and (B_E)

[7] I would formulate this as 'That yacht is a thing that is larger than oneself', except that use of the personal reflexive pronoun 'oneself' here, rather than the impersonal 'itself', is of questionable grammaticality. Still, the latter reduces the temptation to read the reflexive-pronoun occurrence as designating the yacht in question and instead encourages reading it as a bound variable. (I strongly suspect the former construal is incorrect. See the following note.) The English language does not present a happy alternative. As will soon become apparent, the reading I intend is best captured using a recurrent bound variable in lieu of a reflexive pronoun: 'That yacht is a thing x such that x is larger than x is'.

also provides the beginning of an explanation of why it is that one who believes the content of (A_E) is in no position to see that his belief is inconsistent. If (A_E) is read instead as expressing no more or less than what is expressed in (B_E), no such explanations are forthcoming.[8]

The differing semantic analyses of (A_E) and (B_E) reflect some of the structural differences between the sentences. Sentence (A_E) attributes a binary relation between a pair of objects—the mentioned yacht and that same yacht—whereas (B_E) attributes an impossible property to the yacht: being self-larger. The proposition expressed by (B_E)—the proposition about the yacht, that it is a thing-larger-than-it-itself—applies a notion of *reflexivity*.[9] The proposition expressed by (A_E) does not invoke reflexivity or anything else out of the ordinary.

The special semantic properties of (B_E) that distinguish it from (A_E) are captured by using the λ-operator:

(λ*x*)[*x* is larger than *x* is](that yacht).

This is to be read:

That yacht is a thing *x* such that *x* is larger than *x* is.

If the complex demonstratives in (A_E) and (B_E) are now replaced by a single proper name '*a*' for the yacht in question, the resulting sentences respectively preserve the propositions expressed:

(A) *a* is larger than *a* is.
(B) (λ*x*)[*x* is larger than *x* is](*a*).

These sentences therefore do not express the same proposition. The λ-abstract in (B) expresses a property or concept not expressed in (A): that of being self-larger. Yet (A) and (B) are logically equivalent, by the rules of λ-expansion (which licenses the inference from (A) to (B)) and λ-contraction (which licenses the reverse inference). This result discredits Alternative (1) as an analysis of propositional content.[10]

[8] Cf. Salmon, "Reflexivity," *Notre Dame Journal of Formal Logic*, XXVII, 3 (July 1986): 401–29, reprinted in Salmon and Soames, eds., *op. cit.*, pp. 240–74; "Reflections on Reflexivity," *Linguistics and Philosophy*, XV, 1 (February 1992): 53–63; "Illogical Belief," in James E. Tomberlin, ed., *Philosophical Perspectives 3: Philosophy of Mind and Action Theory* (Atascadero, CA: Ridgeview, 1989), pp. 243–85; and "The Resilience of Illogical Belief," *Noûs*, XL, 2 (June 2006): 369–75. All are reprinted in *Content, Cognition, and Communication: Philosophical Papers II* (New York: Oxford, 2007), pp. 30–64, 191–227. Cf. also my "Pronouns as Variables," *Philosophy and Phenomenological Research*, LXXII, 3 (May 2006): 656–64, reprinted in *Metaphysics, Mathematics, and Meaning: Philosophical Papers I*, pp. 399–406; and "Constraint with Restraint," in Gary Ostertag, ed., forthcoming festschrift for Stephen Schiffer.

[9] By 'reflexivity' I mean the notion of reflexivization expressed by '(λ*R*)[(λ*x*)[*R*(*x*, *x*)]]'.

[10] As a referee for this JOURNAL points out, there are examples of the same phenomenon from pure mathematics. One such example is '17 is evenly divisible by an

To reiterate: Where the two complex demonstratives occurring in (A_E) and the proper name '*a*' occurring in (A) all designate the same yacht, (A) expresses the same proposition as (A_E), (B) the same proposition as (B_E); yet (A) and (B) express different propositions.[11]

Saul Kripke is dubious.[12] He raises three considerations that disincline him to accept this account of the semantic content of

---

integer n iff $n = 17 \vee n = 1$' and '17 is prime'. One could come to believe what is expressed by the former without thereby believing that 17 is evenly divisible only by itself and 1. Therefore, different propositions are expressed.

[11] Kit Fine, in *Semantic Relationism* (Malden, MA: Blackwell, 2007), at p. 69, evidently misunderstands me as claiming that (A_E) and (A) semantically express different propositions while (A) and (B) express the same proposition—the reverse of my actual view. Fine defends and develops a view first proffered by Hilary Putnam in "Synonymy, and the Analysis of Belief Sentences," *Analysis*, XIV, 5 (1954): 114–22, reprinted in Salmon and Soames, eds., *op. cit.*, pp. 149–58, and later championed by David Kaplan in "Words," *Proceedings of the Aristotelian Society, Supplementary Volumes*, LXIV (1990): 93–119, at p. 95n6. The basic idea is that, where α and β are exactly synonymous terms (terms having the very same semantic content), $\phi_{\alpha\beta}$ is a sentence containing free occurrences of both terms, and $\phi_{\alpha\alpha}$ is the result of substituting free occurrences of α for free occurrences of β in $\phi_{\alpha\beta}$, the two sentences semantically expresses different propositions—as, for example, 'Bachelors socialize with other bachelors' and 'Unmarried men socialize with other bachelors' (assuming that 'bachelor' and 'unmarried man' are exactly synonymous). Putnam et al. contend that $\varphi_{\alpha\alpha}$ (at least normally) expresses a proposition that in some manner reflects additional material (additional structure, information, or something) that normally results from α's recurrence whereas $\phi_{\alpha\beta}$ does not. Kaplan and Fine maintain that (A_E) and (A) likewise (at least typically) express different propositions.

Church leveled a powerful criticism of the position of Putnam et al. in "Intensional Isomorphism and Identity of Belief": the position has the unwelcome consequence that the proposition expressed by the English sentence 'Unmarried men socialize with other bachelors' is inexpressible in a language that has only a phrase but no single word (or additional phrase) corresponding to the English 'unmarried man' (or else, at best, the proposition is expressible in such a language only by means of an allegedly ambiguous construction, and then only by means of an allegedly strained reading). To my knowledge, none of the view's adherents have addressed Church's observation that this consequence is excessively implausible. (The criticism, as presented here, involves minor extrapolation in conformity with Church's intent.)

In "Recurrence" (unpublished), I make further criticism of Fine's theory.

[12] Kripke, "Russell's Notion of Scope," *Mind*, CXIV, 456 (October 2005): 1005–37, at p. 1025n45.

I take this opportunity to correct Kripke's characterization at p. 1022 of our communication concerning Russell's example. In that discussion I emphasized the distinction in semantic content that I draw, and of which Kripke is dubious, between the binary-relational predication '*a* is larger than *a* is' and the monadic-predicational '*a* is a thing larger than itself'. I used the distinction not to solve a particular problem Kripke had noticed in Russell's discussion of his example, but rather to support my contention (which Kripke does not accept) that it is possible for one to believe, concerning a particular yacht *a*, that *a* is larger than *a* is while not thereby believing that *a* is self-larger (that is, a thing *x* larger than *x*). I was aware that this distinction (even if it is legitimate, as I maintain) does not solve the problem Kripke had noticed. For more on the relevance to Russell's example, see my "Points, Complexes, Complex Points,

λ-abstraction. Kripke's objections, as well as discussions I have had with him, indicate that he strongly favors Alternative (1) (or something close to it). The disagreement between us is pointed and fundamental. It is my considered judgment that Alternative (1) is incorrect and its advocacy a significant leap backward.

I

One of Kripke's objections is that my account of the semantic content of λ-abstraction threatens to call into question the validity of the standard proof in modal logic of the necessity of identity:

$$(x)(y)[x = y \supset \Box(x = y)].$$

Equally, Kripke argues, my account calls into question the validity of my own disproof of the popular thesis that, for some pairs of objects, there is no fact of the matter concerning whether they are one and the same or numerically distinct.[13] The necessity of identity is proved as follows: First, one proves a trivial lemma by applying the modal rule of necessitation to the reflexive law of identity, to obtain '$\Box(x = x)$'. The theorem is then easily proved by assuming '$x = y$', then invoking Leibniz's law of substitution to replace the second occurrence of 'x' in the lemma with 'y'. The disproof of the indeterminacy of identity invokes an application of Leibniz's law using an analogous lemma of the logic of determinacy: that there is a fact of the matter concerning whether x is x.[14] Kripke's objection to my account of the content of abstraction is the following:

> Someone might argue against the necessity of identity by claiming that only the self-identity of x is necessary, while the identity of x and x is contingent. Similarly, he or she might "refute" Salmon's own argument against vague [that is, indeterminate] identity by a parallel argument. Surely Salmon should be wary of this. I am.[15]

---

and a Yacht," in Nicholas Griffin and Dale Jacquette, eds., *Russell vs. Meinong: The Legacy of "On Denoting"* (New York: Routledge, 2008), pp. 343–63.

[13] See my *Reference and Essence* (Amherst, NY: Prometheus Books, 1981, 2005), at pp. 241–45; and "Identity Facts," in Christopher Hill, ed., *Philosophical Topics*, XXX, 1 (Spring 2002): 237–67, reprinted in *Metaphysics, Mathematics, and Meaning*, chapter 10.

[14] *Proof*: Assume for a *reductio* that there is a pair of objects, x and y, such that there is no fact concerning whether x and y are the same object. By the lemma, there is a fact concerning whether x is x. Therefore x and y are not exactly alike in every respect. The latter lacks x's feature that there is a fact concerning whether x is it. Therefore, by Leibniz's law, x and y are distinct. In that case, there is a fact after all concerning whether x is y.

[15] Kripke, "Russell's Notion of Scope," p. 1025.

So I am. The attempted refutations fail. But their failure casts no doubt on my account of the semantic contents of λ-abstracts. The claim that propositions p and q are not the same—as I make, for example, in connection with (A) and (B)—entails nothing whatsoever about whether p and q differ in truth-value, or even in modal status, or even in logical content. It is essential to my view that λ-converts like (A) and (B) are logically equivalent but nonsynonymous. In particular, on my account '$x = x$' and '$(\lambda y)[y = y](x)$' are equivalent. (Indeed, both are logically valid.) It follows that the propositions expressed (under the assignment of a value to 'x'), although distinct, share the same metaphysical status. In particular, one is a necessary truth if and only if the other is. Insofar as the objector to the necessity of identity wishes to conform to my account of abstraction, the concession that the self-identity of x is a necessary truth about x is self-defeating. Likewise, the concession that there is a fact of the matter concerning whether x is self-identical is one concession too many. The objector to the proof of the determinacy of identity is thereby committed, on my account, to the equivalent observation that there is equally a fact of the matter concerning whether x is x. The attempted refutations collapse; the proofs go through without a hitch.

Of course, a different objector to the necessity of identity might uphold part of my account of the content of abstraction while disrespecting the rest by holding that the identity of x with x is neither necessary nor equivalent to the self-identity of x (which is agreed to be necessary). Such a stance is transparently unacceptable. But this is no objection to my account, which explicitly rejects the stance in question. My account entails that, in light of their equivalence, the identity of x with x and the self-identity of x cannot differ in modal status: both are necessary. This account offers no comfort or solace to the envisioned enemy.

Indeed, the proof of the modal theorem is also (among other things) a disproof of the offending stance. And the proof is perfectly compatible with my account of the content of abstraction. The proofs of both the necessity and the determinacy of identity in fact make no assumptions whatsoever concerning the content of abstraction. They do not even invoke abstraction.

A stickler about Leibniz's law might insist otherwise. It is not implausible that, insofar as Leibniz's law is based upon Leibniz's observation that things that are one and the same are exactly alike in every respect (the indiscernibility of identicals), the substitution rule should be restricted to monadic predications. If the λ-abstraction operator is available, such a restriction is not as severe as it might seem. To guarantee validity of the proofs under such a restriction, both λ-expansion and λ-contraction would be required. From '$\Box(x = x)$', one must first prove '$(\lambda z)[\Box(x = z)](x)$' by λ-expansion before the restricted version of Leibniz's law can be applied. One will also need to perform λ-contraction on '$(\lambda z)[\Box(x = z)](y)$' to obtain '$\Box(x = y)$'. Exactly analogous modifications might also be required in the proof of the determinacy of identity. But all these modifications are easily accommodated if one is going to be that way about it.[16] The legitimacy of the proofs requires only that these instances of λ-conversion preserve truth in any model.[17] There simply is no further requirement that they should also preserve semantic content. Indeed, such a further requirement in general would be crippling to logic.

II

A second objection of Kripke's is that, in drawing a distinction in content between (A) and (B), I am committed to an implausible proliferation of propositions. For if my account is correct, Kripke argues, each of the following sentences expresses a proposition closely related to but distinct from that expressed by each of the others:

$$
\begin{array}{ll}
 & \phi_a \\
\Pi_1(a): & (\lambda x_1)[\phi_{x1}](a) \\
\Pi_2(a): & (\lambda x_2)[(\lambda x_1)[\phi_{x1}](x_2)](a) \\
\Pi_3(a): & (\lambda x_3)[(\lambda x_2)[(\lambda x_1)[\phi_{x1}](x_2)](x_3)](a) \\
 & \vdots \\
\Pi_{i+1}(a): & (\lambda x_{i+1})[\Pi_i(x_{i+1})](a) \\
 & \vdots
\end{array}
$$

Kripke asks, "Is all this really plausible?"[18]

[16] Details are provided in my "Identity Facts," at pp. 170–71, 176 of *Metaphysics, Mathematics, and Meaning*.

[17] Some modal instances of unrestricted λ-conversion are invalid. It is illegitimate, for example, to infer by λ-expansion from '□(the number of planets is exactly how many planets there are)' to '(λn)[□(n is exactly how many planets there are)](the number of planets)' (that is, 'The number of planets is such that, necessarily, *that many* is exactly how many planets there are'). It is equally illegitimate to infer by λ-contraction from '(λn)[◊~(n is exactly how many planets there are)](the number of planets)' to '◊~(the number of planets is exactly how many planets there are)'. Leibniz's law must be restricted to disallow substitution between 'the number of planets' and 'eight' in modal contexts. (Or is it 'nine'? Modal logic does not say.) By contrast, the instances of λ-conversion involved in the proofs of both the necessity and the determinacy of identity involve only abstraction on a variable (a rigid designator) and are consequently valid.

[18] Kripke, "Russell's Notion of Scope," p. 1025. 'Π_1' abbreviates the λ-abstracted predicate $\ulcorner(\lambda x_1)[\phi_{x1}]\urcorner$. For $i > 1$, 'Π_i' abbreviates the complex predicate $\ulcorner(\lambda x_i)[\ldots(\lambda x_3)[(\lambda x_2)[(\lambda x_1)[\phi_{x1}](x_2)](x_3)]\ldots(x_i)]\urcorner$. (Kripke does not use abbreviations for the λ-abstracted predicates.)

The short answer is that it is not especially implausible. (Perhaps the question is rhetorical.) A more significant question is this: Is it really so clear that the sentences in Kripke's sequence all express exactly the same proposition?

Kripke does not provide the rationale for his apparent inclination to judge that the sentences do express exactly the same proposition. The pattern of progression would seem to suggest that either all of the sentences express the same proposition or none of them do. If it is denied that the initial sentence, ϕ_a, expresses the same proposition as its successor, $\ulcorner\Pi_1(a)\urcorner$, then by parity of form, Kripke might have reasoned, it must also be denied that $\ulcorner\Pi_1(a)\urcorner$ and its own successor, $\ulcorner\Pi_2(a)\urcorner$, express the same proposition, and also $\ulcorner\Pi_2(a)\urcorner$ and its successor, $\ulcorner\Pi_3(a)\urcorner$, and so on. The opposite judgment that each consecutive pair of sentences, $\ulcorner\Pi_i(a)\urcorner$ and $\ulcorner\Pi_{i+1}(a)\urcorner$, express the same proposition is encouraged by the fact that their logical equivalence is utterly trivial. Even more important, in addition to sharing a common logical content, all of the sentences $\ulcorner\Pi_i(a)\urcorner$, for $i \geq 1$, also share a common logical *form*, consisting of a compound monadic predicate attached to the particular name '*a*'. Furthermore, each compound predicate Π_{i+1} is trivially logically equivalent to its predecessor Π_i, and thus they evidently express the very same property. Perhaps Kripke infers from these commonalities by mathematical induction that *all* of the sentences express the very proposition expressed by the initial sentence ϕ_a.

The elements of Kripke's sequence are all logically equivalent, indeed λ-convertible. That is not in dispute. I explicitly distinguish particular instances of the initial pair, ϕ_a and $\ulcorner\Pi_1(a)\urcorner$, as expressing different propositions despite being λ-convertible. It is precisely this to which Kripke objects. It does not immediately follow from my making this distinction, however, that I am committed to distinguishing likewise between $\ulcorner\Pi_1(a)\urcorner$ and $\ulcorner\Pi_2(a)\urcorner$ in regard to semantic content, nor indeed between any pairing of $\ulcorner\Pi_i(a)\urcorner$ and $\ulcorner\Pi_j(a)\urcorner$ for i, $j \geq 1$. The initial sentence, ϕ_a, unlike the rest of the sequence, need not in general have the special logical form of a predication $\ulcorner\Pi_0(a)\urcorner$ with Π_0 a monadic predicate. The asymmetry between the initial pair—which I explicitly claim need not be synonymous—and the other consecutive pairs, $\ulcorner\Pi_i(a)\urcorner$ and $\ulcorner\Pi_{i+1}(a)\urcorner$ (for $i \geq 1$), leaves ample room for a distinction between the two sorts of cases as regards the issue of preservation of semantic content.

One important basis for distinguishing the semantic contents of the initial pair is precisely the fact that $\ulcorner\Pi_1(a)\urcorner$ is a monadic predication, whereas ϕ_a need not be. This justification is utterly lacking with all the other pairs. The supposed parity of form is an illusion generated by a hasty overgeneralization. Any inclination one might have to see the relation between ϕ_a and $\ulcorner\Pi_1(a)\urcorner$ as fully on a par with that between $\ulcorner\Pi_1(a)\urcorner$ and $\ulcorner\Pi_2(a)\urcorner$ might be traced to a tempting, albeit unjustified, focus on the very special case where ϕ_a is $\ulcorner\Pi_0(a)\urcorner$ with Π_0 a monadic predicate. Admittedly, this special case is not one that motivates the general distinction in content between ϕ_a and $\ulcorner(\lambda x_1)[\phi_{x1}](a)\urcorner$, or at least not very forcefully. Perhaps it is arguable that the compound monadic predicate $\ulcorner(\lambda x_1)[\Pi_0(x_1)]\urcorner$ is exactly synonymous with Π_0 itself, making $\ulcorner(\lambda x_1)[\Pi_0(x_1)](a)\urcorner$ exactly synonymous with $\ulcorner\Pi_0(a)\urcorner$.[19] No such argument is forthcoming with regard to the general case of $\ulcorner\Pi_1(a)\urcorner$ and ϕ_a.[20]

There is an ironic analogy that aptly illustrates this point. In another context entirely, Kripke distinguishes between the propositions expressed by a pair of sentences, $\ulcorner\psi[(\iota x)\phi_x]\urcorner$ and $\ulcorner\psi(\alpha)\urcorner$, where these sentences are related in that α is a name whose "reference is fixed" by the definite description $\ulcorner(\iota x)\phi_x\urcorner$. Kripke concedes that the identity statement $\ulcorner\alpha = (\iota x)\phi_x\urcorner$ is then *a priori*, so that there is an epistemological equivalence between $\ulcorner\psi[(\iota x)\phi_x]\urcorner$ and $\ulcorner\psi(\alpha)\urcorner$. That is, their biconditional is *a priori*, according to Kripke, so that one of the sentences is *a priori* if and only if the other is as well.[21] He nevertheless distinguishes between the two sentences in regard to semantic content, on the ground that they can, and often do, differ in modal status despite their epistemological equivalence. One of Kripke's examples is the pair 'The planet that gravitationally perturbs the orbit of Uranus exerts an attractive force on Uranus' and 'Neptune exerts an attractive force on Uranus'. The former is true with respect to every

[19] This argument is rejected on Alternative (0). Anderson points out that in "A Revised Formulation of the Logic of Sense and Denotation. Alternative (1)," Church explicitly considers expanding the rules of λ-conversion to include substitution of $\ulcorner(\lambda\alpha)[\Pi(\alpha)]\urcorner$ by Π and *vice versa*, calling the resulting modification of Alternative (1) as a criterion for synonymy 'Alternative (1′)' (p. 149). Alternative (1′) identifies the semantic contents of the predicates 'is seaworthy' and '$(\lambda x)[x$ is seaworthy$]$' (in English, 'is seaworthy' and 'is a thing that is seaworthy').

[20] Anderson points out that there is another sequence of sentences analogous to Kripke's in which each consecutive pair are trivially logically equivalent: $\ulcorner\Pi_0(a)\urcorner$; $\ulcorner(\lambda F)[Fa](\Pi_0)\urcorner$; $\ulcorner(\lambda\Phi)[\Phi(\Pi_0)]((\lambda F)[Fa])\urcorner$; and so on. Here it is clear that the sentences do not all express a single proposition. Each successive sentence is constructed from components of higher type than the corresponding components of its predecessor.

[21] This aspect of Kripke's account has been disputed. Cf. my "How to Measure the Standard Metre," *Proceedings of the Aristotelian Society*, n.s., LXXXVIII (1987–1988): 193–217; reprinted in my *Content, Cognition, and Communication*, pp. 139–56. Kripke has modified his view somewhat in light of the criticism. See his "A Puzzle about Belief," in Avishai Margalit, ed., *Meaning and Use* (Boston: D. Reidel, 1979), pp. 239–83, especially at p. 281n44; reprinted in Salmon and Soames, eds., *op. cit.*, pp. 102–48, at p. 147n44; also reprinted in Matthew Davidson, ed., *On Sense and Direct Reference* (Boston: McGraw-Hill, 2007), pp. 1002–36, at p. 1034n44.

possible world in which the orbit of Uranus is perturbed by a planet. The latter is not.[22]

Imagine now a clever critic who raises the following objection against Kripke:

> Let us introduce an infinite sequence of names: $\alpha_1, \alpha_2, \alpha_3, \ldots$ (for example, 'Neptune$_1$', 'Neptune$_2$', and so on). We let the reference of α_1 be fixed by the description $\ulcorner(\iota x)\phi_x\urcorner$, and for each $i \geq 1$ we let the reference of α_{i+1} be fixed by its predecessor α_i. We now construct a corresponding infinite sequence of sentences: $\ulcorner\psi[(\iota x)\phi_x]\urcorner$, $\ulcorner\psi(\alpha_1)\urcorner$, $\ulcorner\psi(\alpha_2)\urcorner$, $\ulcorner\psi(\alpha_3)\urcorner$, ... Kripke concedes that these sentences are all epistemologically equivalent. If he is right, each of these sentences nevertheless expresses a proposition that is distinct from that expressed by each of the others. This is quite implausible. Rather, each of the sentences evidently expresses the very same proposition. Kripke's semantic distinction between the initial pair, $\ulcorner\psi[(\iota x)\phi_x]\urcorner$ and $\ulcorner\psi(\alpha_1)\urcorner$, is therefore bogus, a distinction without a difference.

The objection is certainly misguided. In distinguishing between the initial pair as regards the propositions expressed, Kripke is in no way committed to claiming that each of these sentences expresses a unique proposition distinct from those expressed by the others. Quite the contrary, there is a glaring difference between the specially introduced name that Kripke proposes—the special case of α_1—and all the succeeding names α_{i+1} proposed by the critic. The reference of the initial name α_1 is fixed by a *description*, $\ulcorner(\iota x)\phi_x\urcorner$—which is typically nonrigid—whereas the reference of any name α_i with $i > 1$ is fixed by *another proper name just like it.* The description that fixes the reference of α_1 is not just another *proper name*, α_0. Such a name would be rigid even when the description is not. This is precisely the point. The ground for distinguishing the initial pair of sentences—their potential for differing in modal status—is consequently utterly lacking with any pairing of the $\ulcorner\psi(\alpha_i)\urcorner$.

The situation here is exactly analogous to Kripke's own objection to my account of the content of abstraction, but for the substitution of modal for structural properties.

III

I have argued against Kripke that my distinguishing ϕ_a and $\ulcorner\Pi_1(a)\urcorner$ as regards content does not commit me to distinguishing the rest. But the mere fact that there is conceptual room for a particular position does not entail that the position is sound.

[22] Cf. Kripke's *Naming and Necessity* (Cambridge: Harvard, 1972, 1980), at pp. 14–15, 79n.

Let us return for a moment to the special case where ϕ_a is $\ulcorner\Pi_0(a)\urcorner$ with Π_0 a monadic predicate. Is the contention correct that $\ulcorner(\lambda x_1)[\Pi_0(x_1)](a)\urcorner$ expresses exactly the same proposition as $\ulcorner\Pi_0(a)\urcorner$? Consider an English analog of this. Let Π_0 be the predicate 'is seaworthy', and consider the expanded sentence '*a* is a thing that is seaworthy'. Is this strictly synonymous with the contracted '*a* is seaworthy'? An exactly similar question arises about the semantic contents of $\ulcorner\Pi_2(a)\urcorner$, $\ulcorner\Pi_3(a)\urcorner$, $\ulcorner\Pi_4(a)\urcorner$, and so on. Is the proposition that *a* is a thing that is a thing that is seaworthy (the content of $\ulcorner\Pi_2(a)\urcorner$) exactly the same as the proposition that *a* is seaworthy? What about the proposition that *a* is a thing that is a thing that is a thing that is seaworthy ($\ulcorner\Pi_3(a)\urcorner$)? Are these all exactly the same, as Kripke is inclined to believe?

These are not at all easy questions with easy answers. Kripke cannot legitimately claim that intuition squarely supports his favored judgment that they are all a single proposition. This verdict is no more intuitive than its rival. As already noted, all these sentences do have a common logical content and a common logical form, consisting of a monadic predicate attached to '*a*'. And the predicates are all trivially equivalent. For this reason, I am prepared to assert that they stand or fall together. Either no two of them express exactly the same proposition, or they all do; it is either all or none. On the other hand, considerations of propositional attitude should give one serious pause before declaring that these sentences express exactly the same proposition with not a hair's difference, as Kripke implicitly does. It should be noted also that on Alternative (0) these same sentences are deemed not strictly synonymous.[23] I have taken no explicit stand on the issue. I am somewhat inclined to judge that each does indeed express a unique proposition distinct from the others, but I am prepared to be persuaded either way.

Even if Kripke's implicitly proposed identification in content between ϕ_a and $\ulcorner\Pi_1(a)\urcorner$ is correct in the special case where ϕ_a is $\ulcorner\Pi_0(a)\urcorner$ for some monadic predicate Π_0—a big 'if'—it cannot be argued plausibly that such identification extends to the case where ϕ_a does not have this special form. We have already seen this in one particular case, where ϕ_a is (A) and $\ulcorner\Pi_1(a)\urcorner$ is (B).[24]

[23] See note 19 above. Church says of the proposed modification of Alternative (1) mentioned there—allowing that interchange between $\ulcorner(\lambda\alpha)[\Pi(\alpha)]\urcorner$ and Π preserves sense—that whether such modification is needed "in the end…may depend on somewhat doubtful judgments as to whether given (declarative) sentences convey exactly the same item of information, or whether instead it is closely related but different such items" (pp. 148–49). Interestingly, Church does not consider emending Alternative (0) similarly.

[24] Does Kripke simply overlook the case in which ϕ_a involves multiple occurrences of '*a*'? He explicitly points out, "If *n*-place relations are involved, the situation comes

Consider also the case where ϕ_a is a simple conjunction, for example, 'a is large and a is seaworthy'. Here, $\ulcorner\Pi_1(a)\urcorner$ expresses the monadic-predication proposition that a is a thing that is both large and seaworthy. The next sentence, $\ulcorner\Pi_2(a)\urcorner$, expresses that a is a thing that is a thing that is both large and seaworthy, the next that a is a thing that is a thing that is a thing that is both large and seaworthy, and so on. Are all these propositions, strictly speaking, different propositions from one another? Are they all one and the very same proposition? I do not endorse either verdict. What I do claim is that, whatever Kripke's inclinations might be, the conjunctive proposition that a is large and also a is seaworthy is surely different from (even though logically equivalent to) the monadic-predication proposition that a is a thing that is both-large-and-seaworthy. One who is mistaken about the identity of the yacht in question might well believe the former proposition ("This yacht is large whereas that yacht is seaworthy") without thereby believing the latter.[25]

I also claim that '$(\lambda x)[x$ is large & x is seaworthy$](a)$' expresses the latter proposition rather than the former. Similarly with regard to the contents of (A_E) and (B_E). And I claim that intuition decidedly supports these judgments. Even if these verdicts, or their grounds, commit me to distinguishing each of the sentences in the sequence as regards semantic content, as Kripke asserts, the latter issue is far too delicate to decide the significantly easier issue concerning ϕ_a and $\ulcorner\Pi_1(a)\urcorner$ when ϕ_a involves significant structure (for example, multiple occurrences of 'a')—an issue that is decided on philosophically intuitive grounds through consideration of a range of such cases.

IV

Kripke's most forceful counter-consideration concerns Russell's account of (B) in contrast to mine. Having once embraced the classical

to involve complicated infinite trees." It is precisely in such cases that the potential non-synonymy of ϕ_a and $\ulcorner\Pi_1(a)\urcorner$ is laid bare.

Kripke's observation is strictly true even for the special case where ϕ_a is of the form $\ulcorner\Pi_0(a)\urcorner$ with Π_0 a monadic predicate. This sentence yields not only $\ulcorner(\lambda x)[\Pi_0(x)](a)\urcorner$ by λ-expansion, but also $\ulcorner(\lambda x)[\Pi_0(a)](a)\urcorner$. Is the proposition that a is a thing that is seaworthy the same as, or different from, the proposition that a is such that a is seaworthy? Is either the same as the (logically equivalent) proposition that a is seaworthy? Do considerations of propositional attitude shed any light?

[25] Distinguishing the contents of 'a is large & a is seaworthy' and its λ-convert, '$(\lambda x)[x$ is large & x is seaworthy$](a)$', provides the beginning of an explanation of why it is that one who believes the content of the former might not yet be in a position to be able to infer that a is a thing both large and seaworthy. The situation here is exactly like Kripke's own case of Pierre, in "A Puzzle about Belief," *loc. cit.* See notes 8 and 21 above.

Fregean distinction between semantic content ("meaning") and designation, by 1905 Russell came to reject it.[26] He regarded sentences, roughly, as designators of propositions. Furthermore, he regarded λ-abstracts like '$(\lambda x)[x$ is larger than x is$]$' as functors that designate propositional functions. His view supports a stronger version of λ-conversion rules, on which a sentence and its λ-convert are not only alike in truth-value but also co-designative on logical grounds alone. In particular, on Russell's view, (B) designates the proposition obtained by applying the function designated by '$(\lambda x)[x$ is larger than x is$]$' to the yacht designated by 'a'. This, on Russell's account, is exactly the proposition designated by (A).[27] Thus, as Kripke puts it, Russell did not intend any distinction between (A) and (B).

Kripke says that Russell's account of (A) and (B) strikes him as correct. He adds, "Nor does a mathematician analogously intend any distinction between $\lambda x(x!)(3)$ and the number 6. Nor did Church, inventor of the lambda notation, intend any such distinction."[28] It must be noted in response that, quite the contrary, the mathematician's understanding and use of the λ-calculus in fact casts serious doubt on Russell's account of (A) and (B) while simultaneously supporting my account. The result of applying the factorial function $(\lambda x)[x!]$ to the number three is indeed the number six. The 'is' here is the 'is' of strict identity. The complex expression '$(\lambda x)[x!](3)$' and the numeral '6' are co-designative, hence co-extensional. Indeed, they are mathematically equivalent, in the sense that '$(\lambda x)[x!](3) = 6$' is a mathematical theorem. But as Church would have observed, these two expressions for the number six differ dramatically in semantic content, in "sense." To begin with, although they are mathematically equivalent, the two expressions are arguably not logically equivalent, let alone λ-convertible. If so, they are not deemed synonymous even on Alternative (2), let alone on Alternatives (1) or (0). This is a striking point of disanalogy with (A) and (B), which are λ-converts.[29]

[26] While calling the distinction 'Fregean', it should be acknowledged that philosophers before Frege also upheld the distinction—including Russell's godfather, John Stuart Mill, who distinguished "connotation" from "denotation." Russell himself also upheld the distinction, before his 1905 breakthrough.

[27] See note 2 above. Russell's account is obtained from the explanation provided there by assuming that formulae designate propositions, under assignments of values to variables, and that where ϕ_α is a formula, the designatum, with respect to an assignment s of values to variables, of the λ-abstract $\ulcorner(\lambda\alpha)[\phi_\alpha]\urcorner$ is the function that assigns to anything o in the range of the variable α, the designatum of ϕ_α with respect to the assignment that assigns o to α and is otherwise the same as s.

[28] Kripke, "Russell's Notion of Scope," p. 1025.

[29] I assume that the numeral '6' is an individual constant, defined perhaps by 'the successor of 5' or taken as primitive. An example better suited to Kripke's purpose

Even aside from such considerations, the nonsynonymy of '$(\lambda x)[x!]$ (3)' and '6' is obvious. The numeral '6' is the canonical designator of six in Arabic notation. One does not understand the numeral—one does not grasp what its semantic content is—unless one knows which number is designated thereby. By contrast, calculation is required to determine that '$\lambda x[x!](3)$' designates six. These mathematically equivalent designators are thus co-extensional but nonsynonymous. On my account, (A) and (B) are analogously logically equivalent, hence co-extensional, yet nonsynonymous. This is not so on Russell's account, which wrongly deems (A) and (B) strictly synonymous. I embrace Kripke's analogy, modulo that different species of equivalence are involved. Contrary to the thrust of Kripke's objections, however, the defender of Alternative (1) cannot do so.

It might be replied that, just as classical logic is concerned with extensions and not with intensions, so pure mathematics is concerned only with the extensions of mathematical notation. The mathematician *qua* mathematician writes the equation '$3! = 6$' but does not also say whether this sentence is informative, or trivial, or according to Kant to be labeled 'analytic', or containing a very valuable extension of knowledge. When Frege addressed these matters, he was wearing his philosopher hat, not his mathematician hat. In particular, pure mathematics does not say whether '$3! = 6$' differs in semantic content, or in cognitive value, from '$6 = 6$'. Strictly speaking, it may be argued, pure mathematics does not say anything at all about particular expressions ('3!', '6', '$\lambda x[x!](3)$', and so on). In particular, the mathematician *qua* mathematician does not draw any semantic distinction between '$\lambda x[x!](3)$' and '6'.

I do not believe this reply accords with Kripke's intent, but it requires rebuttal all the same. Gödel's method of arithmetizing syntax already demonstrates that there is a sense in which pure mathematics is not altogether free of reference to the very notation in which its theorems are formulated. Some mathematical theorems explicitly concern overtly semantic notions—for example, Tarski's theorem about truth. The very point of Church's *LSD* is that it is a mathematical theory about the entities that serve as semantic contents. Church's intention was undoubtedly that this mathematical theory ultimately was to be combined with a mathematical theory of structures (models) to yield a mathematical theory that does for the

---

would be something like '$(\lambda x)[2(x!) - x^2 + x](3)$', as contrasted with its λ-convert '$2(3!) - 3^2 + 3$'—except that such genuinely analogous examples strongly support my account of (A) and (B) over the Russellian account that Kripke favors. (See note 37 below.)

semantics of sense (content) roughly what Tarski's theory of truth-in-a-model does for extensional semantics.

Furthermore, it is simply false that the mathematician (*qua* mathematician) does not discriminate among different mathematical expressions for the number six. It is a mathematical question, for example, how to designate six using a sequence of occurrences of '1' and '0' ("ones and zeroes") in binary notation. (It is not an interesting mathematical question, but it is a mathematical question.) There is no similar mathematical issue of how to designate six in Arabic notation. (There is an issue, but it is not a mathematical one.)

More to the point, when the mathematician (*qua* mathematician) endeavors to calculate the factorial function for the number three as argument—a purely mathematical task if anything is—there are infinitely many expressions for six that do *not* qualify as solutions. One expression that is absolutely disqualified is '$(\lambda x)[x!](3)$'. In fact, among expressions for six, this one, along with its λ-convert '3!', are among the least qualified of all. (Other expressions for six that are also absolutely disqualified include '$3! + 0$', '$3! - 0$', '$3!/1$', and so on.) The most qualified expression in Arabic notation—maybe the only absolutely and fully qualified expression—is precisely the Arabic numeral '6'. Thus, although the mathematician draws no distinction between six and the result of applying the factorial function to three, the mathematician (*qua* mathematician) draws a very sharp distinction between '6' and '$(\lambda x)[x!](3)$'.[30] Among expressions for six, those that qualify as solutions to the equation '$x = 3!$' and those that do not are separate but equal. By definition, this discrimination—the bisection of expressions for six into disjoint sets of those that qualify as solutions to the factorial of three and those that do not—is not based solely on designation. It is based, in fact, on semantic content. The mathematician *qua* mathematician, whether he/she realizes it or not, is thereby concerned with matters of semantic content, even if only implicitly.

Though the mathematician might not explicitly mention the semantic contents of mathematical expressions, it is plainly a *fact* (even if it is not asserted) that the mathematically equivalent expressions '$(\lambda x)[x!](3)$' and '6' differ in semantic content. They are mathematically equivalent and therefore co-extensional, but they are not strictly synonymous (nor λ-convertible).

[30] In his 1992 Alfred North Whitehead Lectures on "Logicism, Wittgenstein, and *De Re* Beliefs about Numbers" at Harvard University, Kripke made a closely related point, specifically concerning recursive-function theory.

V

Kripke's depiction of Church is historically inaccurate. Church explicitly preferred Alternative (0) to the less strict Alternatives (1) and (2) as an explication of the objects of the propositional attitudes. He wrote that he "attaches the greater importance to Alternative (0) [than to Alternative (1)] because it would seem that it is in this direction… that a satisfactory analysis is to be sought of statements regarding assertion and belief."[31] This characteristically judicious comparison of the relative merits of the alternatives is exactly correct. Whether Alternative (0) is a step in the right direction or not, certainly Alternatives (1) and (2) go in the wrong direction.

Further, in correspondence with C. Anthony Anderson in 1973, Church argued that Alternative (1) is unsuitable for a "logic of belief statements." Church notes there that the apparatus in his paper, "A Formulation of the Simple Theory of Types," provides a notation suitable for arithmetic—with canonical designators for the natural numbers, a sign for multiplication, and so on—in which the notation, **k×l**, for the result of multiplying the numbers canonically designated by **k** and **l**, is in general λ-convertible with the canonical designator, **m**, of the resulting product.[32] Nevertheless, Church argues, someone might erroneously believe the proposition expressed by ⌜**m** is prime⌝ without thereby believing the proposition expressed by ⌜**k×l** is prime⌝.[33] (See also note 23 above.) Some twenty years later, Church said that taking λ-convertibility as a criterion for synonymy "may be thought counterintuitive if propositions in the sense of Alternative (1) are to be taken as objects of assertion and belief."[34]

Although they are logically equivalent and, indeed, λ-convertible, (A) and (B) are not *strictly* synonymous on Alternative (0). Church regarded (A) and (B) as expressing different propositions. Curiously, Kripke's example and his appeal to Church's authority thus strongly support my account of the semantics of (A) and (B) over the Russellian account that Kripke prefers, while these same considerations in fact discredit the latter.[35]

[31] Church, "A Formulation of *LSD*," at p. 7n7.

[32] Church, "A Formulation of the Simple Theory of Types," *Journal of Symbolic Logic*, V, 2 (June 1940): 56–68.

[33] Cf. Anderson, "Alternative (I*): A Criterion of Identity for Intensional Entities," at pp. 421–23. I am grateful to Anderson for sharing with me Church's letter, dated May 5, 1973. I have taken liberties in adapting Church's formulation to the notation of the present essay. The canonical number-designators in the Church apparatus are not in general synonymous with their Arabic-numeral counterparts. See note 29 above.

[34] Church, "A Revised Formulation of the Logic of Sense and Denotation. Alternative (1)," at p. 156n2.

[35] Whereas Kripke evidently misunderstands Church's position, he might well have Russell correct. If so, then Russell evidently changed his mind after *The Principles of*

The argument for the nonsynonymy of (A) and (B) by analogy to '$(\lambda x)[x!](3)$' and '6' exploits the Fregean semantic distinction between content (*Sinn*) and extension (*Bedeutung*). In the short passage quoted in the preceding section, Kripke is pointing out that "the mathematician," and in particular Church, holds that the factorial of three just *is* six, so that '$(\lambda x)[x!](3)$' and '6' are co-designative. Kripke is not claiming that mathematicians regard these expressions as synonyms. By analogy, Kripke believes (A) and (B) should be seen as designating the same proposition.

Church indeed did hold, with Frege, that (A) and (B) may be seen as co-designative—but of a truth-value, not of a proposition. In this, I fully agree with Frege and Church and disagree with Russell and Kripke. If sentences are designators, they designate truth-values. Propositions are the semantic contents of sentences, not the designata. Russell, of course, was dubious of the Fregean distinction between semantic content and designation, and strove to avoid it. As is well known, Kripke too finds much in Frege's distinction to dispute, especially in regard to proper names. While I am completely convinced by Kripke's critique of the Fregean distinction as applied to proper names, I am equally convinced that it is a mistake to follow Russell rather than Frege-Church with regard to sentences.

Let us shift attention for a moment from sentences to singular terms. The mathematical analog of a Russellian propositional function in connection with the factorial function is a function that assigns to a

---

Mathematics (Cambridge: University Press, 1903). An adherent of Alternative (1) will hold that '$(\lambda xy)[x > y](ab)$' and '$(\lambda yx)[x > y](ba)$' are synonymous, both expressing simply that a is greater than b. Anderson points out, however, that in *The Principles* Russell takes up the question, deciding that these are not synonymous. Russell writes:

> A question of considerable importance to logic, and especially to the theory of inference, may be raised with regard to difference of sense. Are $a\boldsymbol{R}b$ and $b\check{\boldsymbol{R}}a$ really different propositions, or do they only differ linguistically? It may be held that there is only one relation $\boldsymbol{R}$, and that all necessary distinctions can be obtained from that between $a\boldsymbol{R}b$ and $b\boldsymbol{R}a$. It may be said that, owing to the exigencies of speech and writing, we are compelled to mention either a or b first, and that this gives a seeming difference between "a is greater than b" and "b is less than a"; but that, in reality, these two propositions are identical. But if we take this view we shall find it hard to explain the indubitable distinction between *greater* and *less*. These two words have certainly each a meaning, even when no terms are mentioned as related by them. And they certainly have different meanings, and are [i.e., denote] certainly relations. Hence if we are to hold that "a is greater than b" and "b is less than a" are the same proposition, we shall have to maintain that both *greater* and *less* enter into each of these propositions, which seems obviously false.... Hence, it would seem, we must admit that $\boldsymbol{R}$ and $\check{\boldsymbol{R}}$ are distinct relations...and "$a\boldsymbol{R}b$ implies $b\check{\boldsymbol{R}}a$" must be a genuine inference. (Section 219, pp. 228–29).

natural number n a particular numerical concept: *the product of the natural numbers less than or equal to* n. This function assigns to three not six, but a concept: *the product of the natural numbers less than or equal to three*. This function is decidedly not what the mathematician takes '$(\lambda x)[x!]$' to designate. The λ-abstract designates the factorial function—a mathematical function from numbers to numbers—not a function from numbers to numerical concepts involving those numbers. Insofar as the latter function is a semantic value of '$(\lambda x)[x!]$', it is a semantic value at the level of semantic content, not one at the level of designation.

Shift back to sentences. Before his immersion in *LSD*, Church had proffered a proof of sorts, from a principle of *compositionality of designation*, for the Fregean conclusion that, if sentences are designators (and if trivially mathematically equivalent designators are co-designative), then sentences that are alike in truth-value are co-designative even if they express different propositions. At nearly the same time as Church, Gödel provided a similar proof, also inspired by Frege's arguments.[36] The general form of the argument has been called *the slingshot* because of its supposedly disarming power to slay philosophical giants (for example, the thesis that sentences designate propositions). The principle of compositionality of designation is that (in the absence of such deviant devices as quotation, intensional operators, 'believes that', and so on) the designatum of a compound designator is a function of the designata of any component designators. As Anderson points out, a severely restricted version of compositionality of designation suffices for the proof:

> (*Comp*) Assuming that sentences are designators, the designatum of an identity sentence $\ulcorner \alpha = \beta \urcorner$ is a function of the designata of its singular terms, α and β.

A simple formulation of the proof goes as follows: Assume (*Comp*) and that trivially mathematically equivalent designators are co-designative. It follows that, if sentences are designators, then all sentences that are alike in truth-value are co-designative. *Proof*: Assume that sentences are designators. Let ϕ and ψ be any sentences that are alike in truth-value, for example, 'Snow is white' and 'Water runs downhill' or 'Snow is green' and 'Water runs uphill'. Then ϕ and the complex sentence

[36] Church, "Review of Carnap's *Introduction to Semantics*," *The Philosophical Review*, LII, 3 (May 1943): 298–304; and *Introduction to Mathematical Logic I* (Princeton: University Press, 1956), pp. 24–25, reprinted in Davidson, ed., *op. cit.*, pp. 54–81, at p. 75; Gödel, "Russell's Mathematical Logic," in Paul Arthur Schilpp, ed., *The Philosophy of Bertrand Russell* (La Salle, IL: Open Court, 1989), pp. 123–53, at pp. 128–29.

$\ulcorner(\iota n)([\phi \supset n = 1] \wedge [\sim\phi \supset n = 0]) = 1\urcorner$ are co-designative, since they are trivially mathematically equivalent. By (*Comp*), the latter co-designates with $\ulcorner(\iota n)([\psi \supset n = 1] \wedge [\sim\psi \supset n = 0]) = 1\urcorner$, since the definite descriptions contained within the two sentences are co-designative. Since this last sentence is trivially mathematically equivalent to ψ, they are co-designative. Therefore, ϕ and ψ are co-designative.[37]

A straightforward variant of the proof shows that if predicates are designators, then co-extensional predicates are co-designative even if they are not synonymous, as with Quine's example of 'is a cordate' and 'is a renate'.[38] This result supports Church's contention that the λ-abstract, $\ulcorner(\lambda x)[\phi_x]\urcorner$, designates not a propositional function but something fully extensional—the class of things satisfying ϕ_x or alternatively, with Church (following Frege), this class's characteristic function.

Kripke knows the Church-Gödel proof well. (The particular proof presented here is derived from improved versions of the argument which David Kaplan had formulated in his 1964 doctoral dissertation on "Foundations of Intensional Logic" and which Kripke presented independently in an undergraduate course I took in 1972.) The argument's conclusion supports the thesis—which I accept—that insofar as '$(\lambda x)[x$ is larger than x is]' is a designator, it designates the empty class (more accurately, the constant function to falsehood) rather than a Russellian propositional function. To resist the Church-Gödel argument, Kripke is ultimately committed to rejecting even the minimal compositionality principle (*Comp*). This rejection strikes the present author as excessively implausible. (See the Appendix below.)

Kripke evidently believes that '$(\lambda x)[x$ is larger than x is]' designates a propositional function, which assigns to anything the proposition

[37] David Braun correctly points out that the equivalence between ϕ and $\ulcorner(\iota n)([\phi \supset n = 1] \wedge [\sim\phi \supset n = 0]) = 1\urcorner$ is not logical in the usual sense but mathematical. The bi-conditional formed from these two sentences is not a first-order-logical truth but a first-order-logical consequence of the truism, '$1 \neq 0$'. On the other hand, '$1 \neq 0$' is arguably a trivial second-order logical truth, so that the two sentences are trivially second-order-logically equivalent. Cf. my "Numbers versus Nominalists," *Analysis*, LXVIII, 3 (July 2008): 177–82.

[38] Assume that predicates are designators. Let Π and Π' be any co-extensional monadic predicates, for example, 'is a cordate' and 'is a renate'. Then Π co-designates with the complex predicate $\ulcorner(\lambda x)[(\iota n)([\Pi(x) \supset n = 1] \wedge [\sim\Pi(x) \supset n = 0]) = 1]\urcorner$, since they are trivially mathematically equivalent. By compositionality of designation, the latter predicate co-designates with $\ulcorner(\lambda x)[(\iota n)([\Pi'(x) \supset n = 1] \wedge [\sim\Pi'(x) \supset n = 0]) = 1]\urcorner$ since for every value of 'x', the definite descriptions contained within the two complex predicates are co-designative. Since this last predicate is trivially mathematically equivalent to Π', they are co-designative. Therefore, Π and Π' are co-designative. Cf. my *Reference and Essence*, pp. 48–52; and my *Frege's Puzzle* (Atascadero, CA: Ridgeview, 1986, 1991), pp. 22–23.

that the thing in question is larger than that same thing is, rather than the proposition that it is self-larger. In light of the Church-Gödel argument, it is rather the semantic content of the λ-abstract, not the designatum, which is closely related to a Russellian propositional function. I do not deny that the propositional function is a semantic value of the λ-abstract. On the contrary, I would insist that it obviously *is* a semantic value of some sort—at the level of semantic content, not at the level of extension. What I deny is that the semantic content is just this function. It is significantly more plausible that (B) expresses the monadic proposition that *a* is self-larger rather than the binary-relational proposition that *a* is larger than *a* is. Kripke's misgivings notwithstanding, I know of no convincing reason to doubt this. And there are good reasons to acknowledge it.

On the other side, advocacy of Alternative (1), and rejection of (*Comp*), seems to be based on confusion. It is possible using the λ-operator to designate a singular proposition as the value of a propositional function. The following expression presents the proposition that *a* is larger than *a* is precisely as the result of applying to *a* the very propositional function mentioned in the preceding paragraph:

(C) (λ*x*)[the proposition that *x* is larger than *x* is](*a*).

(See note 2.) This expression thus designates the very proposition expressed by (A). But (C) is not thereby synonymous with (A). Let alone is (C) synonymous with (B), which expresses a different proposition from the one that (A) expresses and (C) designates. In fact, (C) does not *express* a proposition at all. It is not a sentence; it is a term. It designates the proposition expressed by (A) by *describing* it, as the value of a particular propositional function for a specified argument. As such, (C) is a compound descriptive term for a proposition, in the same way that '(λ*x*)[*x*!](3)' is a compound descriptive term for six. The content of (C) is not a proposition; it is a proposition concept. This contrasts rather sharply with (B).

The λ-abstraction operator is essentially a device for forming compound functors and other operators from open expressions, in particular, compound predicates from open formulae. A predicate, whether simple or compound, expresses an attribute or concept as its semantic content, one that determines a class as extension.[39] The predicate's content is a component of the contents of the typical sentences in which the predicate occurs. A predication sentence, $\ulcorner\Pi(\alpha_1, \alpha_2, \ldots\alpha_n)\urcorner$, thereby expresses a content of a special sort peculiar to predication sentences.

[39] This observation might be credited to John Stuart Mill, who observed that a "general name" is invariably "connotative" as well as "denotative." See note 26 above.

Were it not that its predicate expresses a class-determining attribute or concept, a predication sentence would not express a proposition. If Π were simply a Millian name (a logically proper name) for a class, for example, without connotation (intensional content), then the string of symbols, $\ulcorner \Pi(\alpha) \urcorner$, would not express a proposition; it would not *assert* anything. Alternative (1) is essentially blind to the special predication role of the λ-abstracted predicate. In its blindness, Alternative (1) reads (B) as a descriptive designation of a proposition rather than as a predication sentence. In effect, Alternative (1) mistakes (B) for a notational variant of (C)—or worse, for a proposition name whose designatum is fixed by a description, *to wit*, by (C).

NATHAN SALMON

University of California, Santa Barbara and
City University of New York Graduate Center

APPENDIX: COMPOSITIONALITY AND THE CHURCH-GÖDEL ARGUMENT

Russell's theory of descriptions endeavors to sidestep the Church-Gödel "proof" concerning the designata of sentences. That theory contradicts the argument's plausible assumption that a definite description $\ulcorner(\iota\alpha)\phi_\alpha\urcorner$ designates the thing that satisfies its matrix ϕ_α if exactly one thing does. This attempt to block the Church-Gödel argument is ultimately inadequate. The Fregean assumption that a proper definite description designates the thing that uniquely answers to it is in fact inessential. Even if definite descriptions are regarded as restricted existential quantifiers (on the model of $\ulcorner$some unique *NP*$\urcorner$) rather than as singular terms, the two definite descriptions invoked in the argument are still co-extensional, so that a restricted version of extensionality of designation would still be applicable:

(*Ext*) Assuming that sentences are designators, the designatum of a sentence of the form $\ulcorner(\iota\alpha)\phi_\alpha = \beta\urcorner$ is a function of the extensions of both $\ulcorner(\iota\alpha)\phi_\alpha\urcorner$ and β.

Strictly speaking, Russell regards definite descriptions neither as singular terms nor as restricted quantifiers but as altogether content-less (having no "meaning in isolation"). I doubt Kripke wishes to go so far. (He has surprised me on occasion.) Indeed, in *Naming and Necessity*, Kripke explicitly acknowledges that a natural-language definite description is a *designator* (typically nonrigid) of the thing that uniquely answers to it.[40]

[40] Kripke is somewhat more circumspect in "Speaker's Reference and Semantic Reference," in P. French, T. Uehling, and H. Wettstein, eds., *Contemporary Perspectives in the Philosophy of Language* (Minneapolis: Minnesota UP, 1979), pp. 6–27. But see note 42 below.

Relying on extensionality of designation in lieu of (*Comp*), definite descriptions are eliminable altogether from the proof in favor of λ-abstracted predicates. The original sentence ϕ is trivially mathematically equivalent to $\ulcorner(\lambda n)[(\phi \supset n = 1) \wedge (\sim\phi \supset n = 0)](1)\urcorner$ (that is, $\ulcorner$One is a thing that is one if ϕ and is zero if not-$\phi\urcorner$). Replacing the predicate $\ulcorner(\lambda n)[(\phi \supset n = 1) \wedge (\sim\phi \supset n = 0)]\urcorner$, in turn, by the co-extensional predicate $\ulcorner(\lambda n)[(\psi \supset n = 1) \wedge (\sim\psi \supset n = 0)]\urcorner$ yields $\ulcorner(\lambda n)[(\psi \supset n = 1) \wedge (\sim\psi \supset n = 0)](1)\urcorner$, which is trivially mathematically equivalent to ψ. The following minimal extensionality principle suffices:

> (*Ext′*) Assuming that sentences are designators, the designatum of a monadic-predication sentence, $\ulcorner\Pi(\alpha)\urcorner$, is a function of the designatum of its singular term α and the extension of its predicate Π.

Russell and Kripke (assuming they will concede that trivially mathematically equivalent designators are co-designative) are thus committed to rejecting (*Ext′*). Evidently, on their view, the designatum of a compound designator containing a predicate—even if it is a predication sentence, $\ulcorner\Pi(\alpha_1, \alpha_2, \ldots\alpha_n)\urcorner$—depends on the propositional function semantically associated with that predicate, rather than on the predicate's extension. Otherwise, the sentences '*a* is a renate' and '*a* is a cordate' will be co-designative (as Frege, Church, and I take them to be).

Whereas Russell denied that definite descriptions are contentful, he nevertheless regarded proper definite descriptions as simulating designation, whereby a proper definite description pseudo-designates the thing that uniquely answers to it. Russell used the term 'denotation' to cover (among other things) both the designatum of a singular term and the pseudo-designatum of a description. A variant of (*Comp*) is sufficient for the proof:

> (*Comp′*) Assuming that sentences are designators, the designatum of a sentence of the form $\ulcorner(\iota\alpha)\phi_\alpha = \beta\urcorner$ is a function of the "denotations" (that is, the designata or pseudo-designata) of $\ulcorner(\iota\alpha)\phi_\alpha\urcorner$ and β.

Russell and Kripke are committed to rejecting this restricted compositionality principle.

The Church-Gödel proof can make do instead with the original minimal compositionality principle (*Comp*) by using an artificial variable-binding operator—which might as well be inverted iota—and which, it is stipulated, forms a compound singular term, in contrast to a restricted quantifier, from an open formula. This device makes for the strongest version of the argument, since it relies on the weakest assumptions. Although Russell had proposed exactly such a device himself, by the time of the publication of "On Denoting" he needed to maintain that such a device is somehow impossible. He seems to have believed exactly this.[41] It is extremely doubtful that Kripke wishes to go so far. Indeed, in a discussion removed from the present one, Kripke postulated a natural-language analog to the very device in question, explicitly arguing that interpreting the English definite article 'the' by means of this device yields a language that might even be English.[42] In accepting the possibility of this device, Kripke thus rejects (*Comp*) or is committed to doing so—just as Russell must reject (*Comp′*).

[41] Cf. my account of Russell's notorious "Gray's *Elegy*" argument in "On Designating," *Mind*, CXIV, 456 (October 2005): 1069–133; reprinted in my *Metaphysics, Mathematics, and Meaning*, pp. 286–334.

[42] Cf. Kripke's so-called *weak Russell language*, set out in "Speaker's Reference and Semantic Reference." This possible language "takes definite descriptions to be primitive designators" (p. 16).