

Numbers versus nominalists

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Nominalists maintain that a numerical-quantificational sentence like

(1_E) There are exactly two Martian moons

is more frugal ontologically than

(2_E) The number of Martian moons is two.

One who sincerely utters the latter, it is argued, is thereby committed, whereas one who utters the former is not, to there being besides the moons of Mars such a thing as their number. Therefore, by embracing (1_E) while disowning (2_E) the nominalist can consistently deny that there are numbers while accepting the sober pronouncements of mature science.¹

The tactic is immediately suspect. The observational sentence (1_E) logically entails a sentence that straightforwardly expresses that there is some number or other of a certain sort:

(3_E) $\exists n$ [there are exactly n Martian moons].

The ' n ' here is a variable that ranges over the natural numbers. In standard English without artificial variables, we have as an evident logical consequence of (1_E) 'Something is such that there are exactly that many Martian moons', or perhaps 'There is some exact number of Martian moons'. The validity of the inference of (3_E) from (1_E) is perfectly obvious, or at least

¹ See for example Rosen 2001, at 74–75. The example is Rosen's.

It is not clear, before going any further, whether the nominalist can consistently argue that (2_E) is committed to two (or to there being such a thing as two). It is not even completely clear that the nominalist can consistently argue that (1_E) is *not* so committed! Presumably, a sentence S is committed to x (or to there being x) if and only if S is committed to the proposition that there is such a thing as x (e.g. by logically entailing a sentence that expresses it or, less plausibly, by requiring x to be among the values of its variables to be true). And a sentence S can be committed to a proposition only if there is such a thing as that proposition itself. But it would seem that there is such a thing as the proposition that there is such a thing as two, if and only if there is such a thing as two. That is, there is such a proposition if and only if it is true. When the nominalist rejects (2_E) on the ground that it is committed to there being such a thing as two, does he/she inadvertently thereby commit him/herself to there being such a thing as two? Should the nominalist simply lay low?

obvious to virtually all but committed number-doubters. Even number-doubters should want to assert what (3_E) expresses. If a debate should arise among astronomers concerning exactly how many moons Mars has – with some astronomers insisting that a third moon has been discovered and other astronomers demurring – all sides of the debate will assent to (3_E). Mars uncontroversially has *some* exact number of moons; at issue is exactly how many. The stubborn nominalist who insists that (3_E) is false will only encourage those scientists on campus who already harbour deep suspicions toward their colleagues in the philosophy department.

If (1_E) entails (3_E), it evidently also entails something more specific:

(4_E) $\exists n[n \text{ is two and there are exactly } n \text{ Martian moons }]$.

In straight English we have as a logical consequence of (1_E) ‘Something is two and such that there are exactly that many Martian moons’. This consequence of (1_E) is every bit as specific, and every bit as anti-nominalist, as (2_E). Our sentences (2_E) and (4_E) are equivalent (or perhaps equivalent but for (2_E)’s supplementary specification that two is a number).²

Nominalists insist that (4_E) is no logical consequence of (1_E). Or at least so insist those nominalists who wish to acknowledge (1_E) along with the other firmly established results of astronomy and common sense. They will point out that the word ‘two’ occurring in (1_E) is not a noun but an adjective, and hence not a singular term subject to existential generalization. They claim that instead (1_E) is to be formalized in a familiar first-order manner as follows, where ‘M’ is a predicate symbol for being a moon of Mars:

(1_F) $\exists x \exists y [Mx \ \& \ My \ \& \ x \neq y \ \& \ \forall z (Mz \supset x = z \vee y = z)]$.

No single expression occurring in (1_F) corresponds to the ‘two’ occurring in (1_E). Alternatively, the nominalist will insist – much more plausibly³ – that (1_E) is to be formalized as:

(1_F) $\mathbf{II}x Mx$.

Here the boldface Roman numeral ‘**II**’ – which corresponds to the phrase ‘there are exactly two’ occurring in (1_E) – functions not as a singular term but as a variable-binding numerical quantifier. It makes little difference which formalization is proffered. No singular term corresponding to ‘two’ occurs in either. How is existential generalization supposed to take a foothold without a singular term to generalize upon?

² That is, given the observation that two is a number, the biconditional [(2_E) iff (4_E)] logically follows.

³ Cf. Salmon 1997.

Yet the inference of (4_E) from (1_E) feels quite legitimate. The intuitive appearance of validity has its source in a related feature of the inference. The inference appears valid because that is exactly what it is.

Let it be conceded that the word ‘two’ does not occur in (1_E) as a singular term. Let it be conceded further that ‘there are exactly two’ in (1_E) functions instead as a quantifier of some sort. It does not follow that (4_E) is any stronger logically than (1_E). Quite the contrary, there is a short logical deduction from (1_F), by way of a crucial application of existential generalization, of a corresponding formalization of (4_E). The deduction proceeds as follows:

1. $\exists x\exists y(Mx \ \& \ My \ \& \ x \neq y \ \& \ \forall z[Mz \supset x = z \vee y = z])$ Premiss (1_F)
2. $\lambda F[\exists x\exists y(Fx \ \& \ Fy \ \& \ x \neq y \ \& \ \forall z[Fz \supset x = z \vee y = z])](M)$
1, 2nd-order λ -expansion
3. $\exists n[n = \lambda F[\exists x\exists y(Fx \ \& \ Fy \ \& \ x \neq y \ \& \ \forall z[Fz \supset x = z \vee y = z])] \ \& \ n(M)]$
2, 3rd-order logic (including EG)

If the sentence on line 1 of this deduction is a correct formalization of (1_E), then the sentence on line 3 is an equally proper formalization of (4_E). In any event, the sentence on line 3 is a logical consequence of (1_F), and certainly no less anti-nominalist than (4_E).

The λ -abstract occurring in the sentence on line 3 designates the Frege-Russell number two, or something closely akin to it. The sentence itself expresses that: (i) there is such a thing as the Frege-Russell number two; and (ii) the set of Martian moons is an instance. Indeed, the classical logicist account of the natural numbers is tailor-made to accommodate number words as numerical quantifiers rather than singular terms. The failure of the classical program of logicism does not discredit Frege’s and Russell’s genuine insights concerning the nature of the numbers. Following those insights, we may introduce a new second-order predicate ‘2’ (not to be confused with its Latin counterpart, the variable-binding numerical quantifier) to designate the Frege-Russell number two:

$$2 =_{def} \lambda F[\exists x\exists y(Fx \ \& \ Fy \ \& \ x \neq y \ \& \ \forall z[Fz \supset x = z \vee y = z])].$$

We then obtain by substitution in the sentence on line 3

$$(4_F) \exists n[n = 2 \ \& \ n(M)].$$

Alternatively, an analogous deduction proceeds from (1_F′) to

$$\exists n[n = \lambda F[\mathbb{I}x Fx] \ \& \ n(M)].$$

This conclusion is as good a formalization of (4_E) as $(1_F')$ is of (1_E) . In any event, it is certainly anti-nominalist if (4_E) is. If we now redefine our new second-order predicate '2' correspondingly,

$$2 =_{\text{def}} \lambda F[\mathbf{II}x Fx],$$

we alternatively obtain (4_F) as a logical consequence of $(1_F')$.⁴

Since nothing other than a single premiss, a definition, and logic is employed in these deductions, if either (1_F) or $(1_F')$ is a correct formalization of (1_E) , then one or the other of these deductions supports the intuition that (4_E) is exactly as it seems: a genuine existential-generalization consequence of (1_E) . It might come as a surprise that an intuitively valid natural-language inference employs higher-order logic. Surprising or not, the inference of (4_E) from (1_E) is a case in point. What is more significant is how easily the logical inference of (4_E) from (1_E) is justified. The nominalist therefore cannot consistently accept (1_E) while rejecting (4_E) . Score one goal for the numbers against their opponents.⁵

The initial premiss does not play a crucial role in these deductions. As Frege and Russell were well aware, that there are such things as the Frege-Russell numbers is provable in higher-order logic alone (with λ -abstraction), without the assistance of any empirical premiss.⁶ Small wonder it is logically deducible from (1_F) that there is such a thing as the Frege-Russell number of Martian moons and it is two. In so far as one's ontological commitments are a matter of the ontological consequences of

⁴ Either one of the numerical quantifier ' \mathbf{II} ' and the corresponding second-order predicate symbol '2' may be taken as corresponding to the phrase 'there are exactly two' occurring in (1_E) . The quantifier and the second-order predicate are inter-definable. Going in the other direction, a numerical generalization of the form $\lceil \mathbf{II}\alpha\phi_\alpha \rceil$, where α is an individual variable, may be taken as shorthand for $\lceil 2(\lambda\alpha[\phi_\alpha]) \rceil$, with the second-order predicate '2' taken as primitive. One way or the other, $(1_F')$ is definitionally equivalent to '2(M)'. (4_F) is then straightforwardly deducible by way of higher-order existential generalization.

⁵ Rosen (2001: 78) says that he believes that the classical objection to nominalism, that a numerical sentence like (1_E) analytically entails some anti-nominalist sentence like (2_E) , can be answered. There is no hint there, however, which steps he would challenge in the deduction of the Frege-Russell number. (Perhaps all of them.)

⁶ The failure of the classical logicist program, in combination with effete authoritative abuse, has created the misimpression among some philosophers that the Frege-Russell numbers cannot be proved in higher-order logic. Quite the contrary, things very much like the Frege-Russell numbers are directly provable. Given the earlier definition of '2', in third-order logic with λ -abstraction $\vdash \exists n(n = 2)$. Similarly for the other Arabic numerals.

one's theory, the Frege-Russell numbers are a commitment of all of us, friends of numbers and foes alike. Number deniers unintentionally commit themselves, through inconsistency, to there being things of every sort whatsoever, even round squares. Score another goal for the numbers.

This is not to say by any means that the game is over for the nominalists, but their fight is uphill. Quine denied that higher-order logic is logic – and this precisely because of the ontology that higher-order logic embraces.⁷ He would have balked at the very first inference in the proposed deduction, which he would have regarded as invoking an ontologically expensive theory, one that in this very instance goes beyond pure logic. For the predicate symbol 'M' occurring in line 2 of the deduction evidently there designates the set of Mars's moons – something whose existence Quine accepted only reluctantly, and not as a matter of pure logic. Quine reckoned that there is no designation of the set of Martian moons in (1E), and that line 2 is therefore no logical consequence of line 1. Even more extreme, in Quine's view, is the designation in line 3 of the Frege-Russell number two. Quine maintained that there is no mention of the Frege-Russell number (or any of its next of kin) in line 2, let alone in line 1, and that therefore line 3 is no logical consequence of line 1.

Quine would have rejected the proposed deduction of (4F) from (1F) as therefore not being purely logical, precisely because new entities are posited at each new line of the deduction. But then Quine was hardly a neutral agnostic with regard to the question of whether there are numbers or other abstracta. Consequently, he was not an unbiased arbiter concerning the validity of natural-language arguments whose formal deductions proceed by way of higher-order logic. Arguments that persuade agnostics are philosophically helpful even if they do not persuade sceptics. Genuine non-sceptics are reluctant to renounce the inference of (3E) from (1E); to the extent one is prepared to forego this inference one is not genuinely neutral. In fact, all of us – those who embrace numbers, those who deny them, the cautiously agnostic, and the indifferent – should be extremely reluctant to disown the inference. The ultimate arbiter concerning logical validity is logical intuition. We should be extremely reluctant to forego the inference not merely because, *pace* Quine, it is valid, but because it is *intuitively* valid.

During the infamous Watergate scandal, the then President Richard Nixon defended himself by saying that he accepted responsibility for the wrongdoing of his subordinates but he accepted none of the blame. The analytical validity of the inference he defied eventually overcame his

⁷ For example, Quine 1970: 27–28, 66–68.

stubborn refusal to draw that inference. There is a lesson here: the power of logic is underestimated only at one's peril.

It is not enough for number-deniers to announce that they judge the inference of (4_E) from (1_E) logically invalid and to declare this verdict perfectly reasonable. Nor can number-deniers legitimately argue that since (4_E) entails the existence of something not mentioned in (1_E), the former is no logical consequence of the latter. The deduction of (4_F) from (1_F) may be taken as showing to the contrary that (1_E) logically entails the existence of two no less than (4_E) does. Those who deem the reasoning involved in the proposed deduction of (4_F) not purely logical (contrary to my assessment) owe a convincing explanation, or at least a plausible explanation, of the argument's apparent validity. They need to make it plausible that the apparent validity is an illusion, due perhaps to an understandable oversight – as when the Aristotelian logician, in fallaciously inferring 'Some trespassers will be prosecuted' from 'All trespassers will be prosecuted', had simply overlooked the case where the threat has the desired effect. In the meantime, nominalists who go so far as to deem (1_E) literally true and (3_E) literally false perform a valuable service to the discipline by avoiding service on committees with scientists. Though a nominalist may generally speak with the vulgar and think with the learned, it can only be a matter of time in such an environment before one's secret doctrines are revealed – for example, that despite Mars having exactly two moons there is no exact number of Martian moons.⁸

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⁸ I urged many of these ideas at a meeting of the Santa Barbarians, organized by C. Anthony Anderson. I am grateful to the participants for discussion, especially Anderson and Michael Rescorla (neither of them nominalists). I am grateful also to my audiences at the University of Kansas and the National Autonomous University of Mexico for their reactions, to Alan Berger for discussion, and to Gideon Rosen for correspondence.