15 Points, Complexes, Complex Points, and a Yacht

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I comment here on two puzzling passages in Russell’s masterpiece, ‘On Denoting’, 1 One is the famous ‘Gray’s Elegy argument’, as it is usually called.2 Afterward, I discuss errors in the famous discussion of the small yacht and its touchy owner.

THE ‘GRAY’S ELEGY’ ARGUMENT

Russell’s famous argument, as I interpret it, is aimed against a popular theory of the semantics of definite descriptions:

ST: A definite description designates by virtue of the description’s semantic content, which fixes the designatum of the description to be (if anything) the individual or thing that uniquely answers to the description; further, when the definite description occurs in a declarative sentence, the description’s content represents the description’s designatum in the proposition expressed.

ST fleshes out the simple and seemingly innocuous thesis that definite descriptions are singular terms. It had been held by Russell in The Principles of Mathematics. It is also held by theorists as diverse as John Stuart Mill, Gottlob Frege, Alexius Meinong, and legions of others.

Here in a nutshell is Russell’s reductive argument against ST: The attempt to form a proposition directly about the content of a definite description (as by using an appropriate form of quotation) inevitably results in a proposition about the thing designated instead of the content expressed. I call this phenomenon the Collapse. In light of the Collapse, Russell argues, the ST theorist must accept that all propositions about a description’s content are about that content indirectly, representing it by means of a higher-level descriptive content. And this, according to Russell, renders our cognitive grip on definite descriptions inexplicable.

On my interpretation, Russell may be seen as arguing in eight separate stages (at least), as follows: At stage (I) he argues that there is some awkwardness in so much as stating the very theory ST. At stage (II) he argues that once a way of stating ST is found, the theory, so stated, gives
rise to a peculiar phenomenon: The attempt to form a singular proposition about the content of a definite description inevitably results instead in a general proposition about the individual designated by the description. This is the Collapse. At stage (III) the Collapse leads to a preferable formulation of ST. At stage (IV) Russell shows that the Collapse remains a feature of the reformulated theory. At stage (V) Russell argues that the Collapse commits ST to a very sweeping conclusion: that no singular term designating the content of a definite description can be what Russell will later call a logically proper name; instead any such term must be itself a definite description, or function as one. As Russell puts it, on our theory ST, ‘the meaning cannot be got at except by means of denoting phrases’ (p. 486). At stage (VI) he argues furthermore that the content of a definite description cannot be a constituent of the content of any definite description of it. Russell proceeds to complain at stage (VII) that the results of the preceding two stages are philosophically intolerable. At stage (VIII) he provides a complementary argument for the conclusion that ST ignores that which, by its own lights, is philosophically most significant about propositions.

In Russell’s terminology, a denoting phrase is a noun phrase beginning with what linguists call a determiner, like ‘every’, ‘some’, or ‘the’. Both definite and indefinite descriptions are denoting phrases, in Russell’s sense. A definite description of a given language is said to mean—in a more standard terminology, it expresses—a denoting complex c as its meaning. The denoting complex c, in turn, denotes—in Church’s terminology, it is a concept of—an object as its denotation. I here translate Russell’s term ‘meaning’ as ‘content’. Russell does not use any special term for the binary relation between a definite description and the object of which the expression’s content in the language is a concept. Instead Russell speaks of “the denotation of the meaning”, saying that a definite description α “has a meaning which denotes” an object x. Sometimes he says that α itself (as opposed to its content) denotes x. Here I avoid Russell’s term ‘denote’ altogether. Instead I use ‘determine’ for the relation between a complex c and the object x of which c is a concept, and I call x the ‘determinatum’ of c. I use ‘designate’ for the relation between the expression α and x, and I call x the ‘designatum’ of α.

Russell uses ‘C’ as a variable ranging over determining complexes, and sometimes instead as a metalinguistic variable ranging over determiner phrases. Frequently instead as a metalinguistic variable ranging over determiner variables, sometimes standing in for an arbitrary definite description, sometimes for a term designating an arbitrary determining complex. Any sentence in which ‘C’ occurs is a schematic letter (a substitutional variable), sometimes standing in for an arbitrary definite description, sometimes for a term designating an arbitrary determining complex. Any sentence in which ‘C’ occurs as a schematic letter is strictly speaking a schema, of which Russell means to assert every instance. With a little finesses, Russell’s intent can often be captured by taking ‘C’ as a variable either ranging over definite descriptions or ranging over determining complexes. I here use ‘α’ as a metalinguistic variable, and upper case ‘D’ as a schematic letter
standing in for an arbitrary definite description. I use lower-case ‘c’ as a determining-complex variable. I use Quine’s quasi-quotation marks, ‘¢’ and ‘†’ in combination with ‘α’. In quasi-quotation, all internal expressions are quoted, that is, mentioned (designated), except for metalinguistic variables, whose values are mentioned. I use single quotation marks for direct (expression) quotation. Following David Kaplan, I use superscripted occurrences of ‘m’ as indirect-quotation marks, and superscripted occurrences of ‘M’ as indirect-quasi-quotation marks (1971: 120–21). In indirect-quasi-quotation, the contents of all internal expressions are mentioned, except for determining-complex variables, whose values are mentioned. Here I avoid double quotation marks, except as scare-quotes when using another’s words. Departures from the original appear in boldface.

Analytical Translation of the Famously Obscure Passage

(A’) The relation of the content to the designatum involves certain rather curious difficulties, which seem in themselves sufficient to prove that the theory which leads to such difficulties must be wrong.

(B’) (I) When we wish to speak about, that is, to designate, the content of a determiner phrase, that is, of a definite description, as opposed to its designatum, the present mode of doing so is by indirect-quotation marks. Thus we say:

The centre of mass of the Solar System is a point, not a determining complex;

“The centre of mass of the Solar System” is a determining complex, not a point.

Or again,

The first line of Gray’s Elegy expresses a proposition.

“The first line of Gray’s Elegy” does not express a proposition.

Thus taking any determiner phrase, for example, taking any definite description . . . , α, we wish to consider the relation between α and "α" where the difference of the two is of the kind exemplified in the above two instances.5

(C’) We say, to begin with, that when α occurs it is the designatum of α that we are speaking about; but when "α" occurs, it is the content. Now the relation of content to designatum is not merely linguistic through the phrase, that is, it is not merely the indirect product of the semantic relations of being the content of a phrase and designating: there must be a direct, nonlinguistic, logico-metaphysical relation involved, which we express by saying that the content determines the designatum. But the difficulty which confronts us is that we cannot succeed in both preserving the connexion of content to designatum and preventing them—the content and the designatum—from being
one and the same; also that the content cannot be got at except by means of determiner phrases. This happens as follows.

(D') The one phrase \( \alpha \) was to have both content and designation. But if in an effort to designate the content, we speak of "the content of \( \alpha \)", that gives us the content (if any) of the designatum of \( \alpha \). "The content of the first line of Gray’s Elegy" designates the same complex as "The content of “The curfew tolls the knell of parting day”", and . . . not the same as "The content of “the first line of Gray’s Elegy”". Thus in order to get the content we want, we must speak not of "the content of \( \alpha \)", but of "the content of "\( \alpha "\)", which designates the same as "\( \alpha "\)" by itself.

(D') Similarly "the determinatum of \( \alpha "\) does not designate the determinatum we want, the determinatum of \( \alpha "\)'s content, but means something, that is, expresses a determining complex, which, if it determines anything at all, determines what is determined by the determinatum we want. For example, let \( \alpha \) be ‘the determining complex occurring in the second of the above instances’. Then "\( \alpha = "\)the first line of Gray’s Elegy" and "The determinatum of \( \alpha = "\)The curfew tolls the knell of parting day" are both true. But what we meant to have as the determinatum was "the first line of Gray’s Elegy". Thus we have failed to get what we wanted from "the determinatum of \( \alpha "\)."

(E') The difficulty in speaking of the content of a determining complex, that is, in using a phrase of the form "the content of "\( \alpha "\)”, may be stated thus: The moment we put the complex in a proposition, the proposition is about the determinatum; and hence if we make a proposition in which the subject component is "the content of \( c M "\), for some determining complex \( c \), then the subject represents the content (if any) of the determinatum of \( c \), which was not intended.

(E') This leads us to say that, when we distinguish content and determinatum of a determining complex, as we did in the preceding paragraph, we must be dealing in both cases with the content: the content has a determinatum and is a determining complex, and there is not something other than the content, which can be called "the complex “\( \alpha "\)”, and be said to have both content and a determinatum. The right phraseology, on the view in question, is that some contents have determinata.

(F') But this only makes our difficulty in speaking of contents more evident. For suppose \( c \) is our target complex, and let ‘\( D \)’ represent in what follows a determiner phrase that expresses \( c \) (for example, let \( c \) be “the centre of mass of the Solar System” and let ‘\( D \)’ stand in for the phrase ‘the centre of mass of the Solar System’); then we are to say that “\( D "\), that is, \( c \), is the content of the phrase ‘\( D \)’, instead of saying that “\( D "\) itself has a content. Nevertheless, whenever ‘\( D \)’ occurs without indirect-quotation marks, what is said is not about "\( D "\), the content of ‘\( D \)’, but only about \( D \), the designatum of ‘\( D \)’, as when we say:
The centre of mass of the Solar System is a point.

(F'ii) (V) Thus to speak of "D" itself, that is, to express a proposition about the content of ‘D’, our subject component must not be "D" itself, but something else, a new determining complex, which determines "D". Thus "D"—which iterated indirect quotation is what we use when we want to speak of the content of "D"—must be not the content of ‘D’, that is, not "D" itself, but something which determines the content.

(VI) And "D", that is, c must not be a constituent of this higher-level complex "D" (as it is of the content of c); for if "D" occurs in the complex, it will be its determinatum, not the content of ‘D’, that is, not "D" itself, that will be represented and there is no backward road from determinata to contents, because every object can be designated by an infinite number of different determiner phrases.

(VII) Thus it would seem that "D" and "D" are altogether different entities, such that "D" determines "D"; but this cannot be an explanation of "D", because the relation of "D" to "D" remains wholly mysterious; and where are we to find the determining complex "D" which is to determine "D"?

(VIII) Moreover, when "D" occurs in a proposition, it is not only the determinatum that occurs (as we shall see in the next paragraph); yet, on the view in question, "D" represents only the determinatum, the content (that is, the representing of "D" itself) being wholly relegated to "D". This is an inextricable tangle, and seems to prove that the whole distinction of content and designation has been wrongly conceived.

That the content is relevant when a determiner phrase occurs in a sentence expressing a proposition is formally proved by the puzzle about the author of Waverley. The proposition "Scott is the author of Waverley" has a property not possessed by "Scott is Scott", namely, the property that George IV wished to know whether it was true. Thus the two are not identical propositions; hence the content of ‘the author of Waverley’ must be relevant to the proposition as well as the designatum, if we adhere to the point of view to which this distinction belongs. Yet, as we have seen, so long as we adhere to this point of view, we are compelled to hold that only the designatum can be relevant. Thus the point of view in question must be abandoned.

Some previous interpreters do not so much as mention what I am calling the Collapse. Others have extracted the alleged phenomenon from (Ei), but place little or no importance on it. Some have depicted its occurrence in the ‘Gray’s Elegy’ passage as little more than a clever observation, characteristic of Russell but one that he makes only in passing and is of limited significance in the grand sweep of the overall argument. In sharp contrast, on my interpretation the Collapse is the very linchpin of the ‘Gray’s Elegy’
argument, and plays a pivotal role in later stages that constitute the heart of the argument.16

At stage (V) (the middle section of paragraph (F)), Russell argues by means of the Collapse that on ST, \( m\mathcal{D}m \neq \mathcal{D}m \), where ‘\( \mathcal{D} \)’ stands in for any definite description.17 We may designate a particular complex, say ‘the centre of mass of the Solar System’, in order to express a proposition about it. However, any proposition in which the complex itself occurs is about the centre of mass of the Solar System, that is, the determinatum of the target complex rather than the complex itself. A singular proposition about a determining complex is an evident impossibility; hence, any proposition that is about a complex must involve a second-level determining complex that determines the target complex. Hence, any term for a complex must function in the manner of a definite description. Even our indirect quotation, ‘the centre of mass of the Solar System’ (the closest thing there is to a standard name of the complex), must be a disguised definite description, expressing a second-level determining complex, \( m\text{"the centre of mass of the Solar System"} \), as its content. Furthermore, \( \text{"the centre of mass of the Solar System"} \) is distinct from, and in fact determines, \( \text{"the centre of mass of the Solar System"} \). It is in this very concrete sense that on ST, ‘the meaning cannot be got at except by means of determiner phrases’. The only way to designate a determining complex is by expressing a higher-level determining complex.18

Russell has thus far argued that the theory ST is committed, by the Collapse, to denying the very possibility of singular propositions about contents. Some commentators have construed this argument as an objection to Frege’s theory, which rejects singular propositions.19 Such an argument would be a howler. On the contrary, Fregeans should welcome the conclusion derived at stage (V), which provides a reductio argument against ST in conjunction with singular propositions of unrestricted subject matter—a theory like Mill’s or that of Russell’s Principles. The incoherence of these non-Fregean versions of ST may even be given a kind of proof, using the principle of Compositionality (which Russell relied on at least implicitly and Frege explicitly endorsed), according to which the content of a compound expression is an effectively computable function of the contents of the contentful components. Compositionality is subject to certain restrictions. For example, the content of a compound expression containing a standard (syntactic) quotation is a function of the content of the quotation itself, together with the contents of the surrounding subexpressions, but not of the content of the quoted expression. Subject to such restrictions as this, Compositionality evidently entails a similarly restricted principle of Synonymous Interchange, according to which substitution of a synonym within a larger expression preserves content. (I here call a pair of expressions synonymous if there is something that is the content of both.)

To give the argument its sharpest focus, we consider Russell’s example:
(1) The centre of mass of the Solar System is a point.

According to ST, the grammatical subject of (1), ‘the centre of mass of the Solar System’, expresses the determining complex "the centre of mass of the Solar System" as its English content. According to the non-Fregean version of ST, the content of the indirect quotation "the centre of mass of the Solar System" itself is this same determining complex, and sentences containing the indirect quotation express singular propositions about the complex. Hence, the description and the indirect quotation are synonymous according to the non-Fregean version of ST. Therefore, by Synonymous Interchange, so also are (1) and

(2) "The centre of mass of the Solar System" is a point.

But (1) is true while (2) is necessarily false, indicating that they do not express the same thing. The content of (2) must invoke the second-level complex "the first line of Gray’s Elegy" to represent the first-level complex. (The same argument may be given using the free variable ‘c’ in place of the indirect quotation. On the supposition that the content of the variable under the established assignment is its value, the variable has the very same content as the definite description ‘the first line of Gray’s Elegy’. The Collapse then follows directly by Synonymous Interchange. This refutes the assumption that the variable under its assignment is a logically proper name for the complex in question.) The theory ST is thus committed to extending its content/designation distinction for definite descriptions to all terms that designate determining complexes.

The argument can be repeated in connection with the content of the indirect quotation itself. The argument is thus converted into an argument by mathematical induction for an infinite hierarchy of contents associated with ‘the first line of Gray’s Elegy’. Indeed, the postulated second-level complex "the first line of Gray’s Elegy" is, for Frege, the content that the description expresses when occurring in ungerade (‘oblique’) contexts, like the contexts created by ‘believes that’ and by indirect quotation marks. He called this the indirect sense of ‘the first line of Gray’s Elegy’. The series beginning with ‘The curfew tolls the knell of parting day’, followed by "the first line of Gray’s Elegy", "the first line of Gray’s Elegy", and so on, is precisely Frege’s infinite hierarchy of senses for the definite description (treating designation as the bottom level in the hierarchy). Not all of Frege’s disciples have followed the master down the garden path to Frege’s jungle. Two noteworthy deserters are Carnap and Dummett. But Church has followed Frege even here. In fact, at least one of the loyal opposition has as well. Russell’s argument via the Collapse for ST’s commitment to the hierarchy was independently reinvented closer to the end of the previous century by Tyler Burge.
Russell clarifies the nature of the hierarchy at stage (VI), which makes up the final third of (F). A feature of (F_{iii}) that is typically overlooked is that it again invokes the Collapse. Russell observes that the target complex is not only distinct from the postulated second-level complex we seek; it is not even a constituent of the latter complex (as it is of “Russell has memorized the first line of Gray’s Elegy”, and of “the content of the first line of Gray’s Elegy”). Here Russell pursues the obvious question: Given that the indirect quotation “the first line of Gray’s Elegy” must express a second-level complex that determines our target complex, which second-level complex does it express? The best way to identify the sought after second-level complex would be to provide a definite description of the form ‘the determining complex that is such-and-such’ which is fully understood (independently of indirect-quotation), and which is synonymous with “the first line of Gray’s Elegy”. Given Compositionality, it might be hoped that the suitable definite description will incorporate something expressing the designated target complex itself. We would thus construct the postulated second-level complex using the target complex. However, the desired description cannot be ‘the complex that determines the first line of Gray’s Elegy’, for there are infinitely many and varied complexes each of which determines the words ‘The curfew tolls . . . ’. Let us try a different tack. Let ‘c’ name the target complex, and consider: the determining complex that is c. Russell observes that this will not do either. Indeed, no description of the form ‘the determining complex that bears relation R to c’ will succeed. Or to put the same point somewhat differently, our postulated second-level complex cannot be “the determining complex that bears R to c”, for some binary relation R. (Note the indirect-quasi-quotation marks.) For the Collapse occurs with determining complexes just as it does with propositions. The content of the description collapses into: “the determining complex that bears R to the first line of Gray’s Elegy”. The problem here is that there is no ‘backward road’ from the words ‘The curfew tolls . . . ’ to their particular representation by “the first line of Gray’s Elegy”, and likewise no backward road from the Solar System’s centre of mass to its particular representation as such. That is, there is no relevantly identifiable binary relation R whose converse is a ‘choice’ function that selects exactly our target complex, to the exclusion of all others, and assigns it, and only it, to its determinatum. If R is taken to be the relation of determining, then the collapsed second-level complex fails to determine a unique complex because there are too many complexes (infinitely many, in fact) that bear this relation to the first line of Gray’s Elegy. And if R is taken to be the relation of identity, then the resulting second-level complex fails to determine a unique complex because there are too few complexes that bear this relation to the first line of Gray’s Elegy. More generally, if c is our target complex, the postulated second-level complex cannot be of the form $Mf(c)^M$, where ‘f’ designates a choice function that selects a distinguished or privileged determining complex from the class of all complexes that determine a given object. It is important to notice that
the missing choice function $f$ goes not at the level from the target complex to the second-level complex, but at the bottom level from the determinatum to the complex itself. A ‘low’ backward road might enable us to construct the postulated second-level complex from the target complex. But high or low, no backward road is forthcoming.

So ends stage (VI). Because there is no backward road from ‘The curfew tolls . . . ’ to “the first line of Gray’s *Elegy*”, it follows *via* the Collapse that the second-level complex “the first line of Gray’s *Elegy*” is not constructed from the target complex “the first line of Gray’s *Elegy*”. Indirect quotations thus constitute a restriction on a principle of *Strong Compositionality* (also endorsed by both Frege and Russell), according to which the content of a compound expression is not only a function of, but is in fact a complex composed of, the contents of the contentful components.

Russell might have taken the argument a step further. Continuing and embellishing the argument on Russell’s behalf, although the indirect quotation “‘the first line of Gray’s *Elegy*’ expresses, and thereby uniquely fixes, the postulated second-level complex, the target complex designated by the indirect quotation does not itself uniquely single out the second-level complex. It is a serious mistake, for example, to suppose that “‘the first line of Gray’s *Elegy*’ can be described as the content of “the first line of Gray’s *Elegy*”. (Russell believes he has shown that on *ST*, this description designates the target complex itself, whereas the description actually designates nothing. The alternative phrase, ‘the content of “the first line of Gray’s *Elegy*’ does designate the target complex itself. Still, we do not get at the postulated second-level complex.) But neither can “‘the first line of Gray’s *Elegy*’ be described as the complex that determines “the first line of Gray’s *Elegy*”. For any given object there are infinitely many complexes that determine it. Our target complex is also determined by such second-level complexes as “the determining complex occurring in the second of Russell’s instances” and “the determining complex that has given Russell’s readers more headaches than any other”—neither of which is suited to be the content expressed by “‘the first line of Gray’s *Elegy*’”. Thus not only is it the case, as Russell explicitly argues, that the target complex is altogether different from the postulated second-level complex. The target complex does not even uniquely fix the second-level complex. Never mind the Collapse. If there is no backward road from determinata to determining complexes, then not only is there no low road from the first line of Gray’s *Elegy* to “the first line of Gray’s *Elegy*”; there is likewise no high road from “the first line of Gray’s *Elegy*” to “the first line of Gray’s *Elegy*”.

We have no way to go from the content of a definite description to the content of its indirect quotation. Our indirect quotation marks thus yield a restriction also on the weaker principle of Compositionality: The content of an indirect quotation is not even a computable function of (let alone a complex composed partly of) the content of the expression within the quotes. This result is stronger than the conclusion that Russell explicitly draws. If the
target complex were a constituent of the postulated second-level complex, presumably it would single out the latter complex. But the mere fact that the target complex is not a constituent of the second-level complex does not yet rule out the possibility that the target complex uniquely fixes the second-level complex in some other manner. The fact that there is a multiplicity of complexes determining any given object seems to do just that. (By contrast, the indirect quotation ‘“the first line of Gray’s Elegy”’ singles out the second-level complex, as its English content.)

SIZE MATTERS

Having disposed of ST once and for all with his ‘Gray’s Elegy’ argument (so he believes), Russell moves on to illustrate the important distinction between primary occurrence and secondary occurrence with his famous example of the touchy yacht owner:

I have heard of a touchy owner of a yacht to whom a guest, on first seeing it, remarked, ‘I thought your yacht was larger than it is’; and the owner replied, ‘No, my yacht is not larger than it is’. What the guest meant was, ‘The size that I thought your yacht was is greater than the size your yacht is’; the meaning attributed to him is, ‘I thought the size of your yacht was greater than the size of your yacht’. (p. 489)

Speaking on ‘Russell’s Notion of Scope’ at Rutgers University in May 2005 (and also in conversation some years earlier), Saul Kripke pointed out a significant snag in Russell’s treatment of his yacht example. The meaning Russell attributes to the guest cannot be correct. Indeed, the guest might well have thought that the yacht was larger than it turned out to be without there being any particular size the guest thought the yacht was. Much more likely, the guest, on the basis of the owner’s boasts, had merely judged the yacht to be, at a minimum, grander than it turned out on visual inspection to be, that is, to be some size or other among a range of sizes (perhaps indeterminately delineated), each noticeably greater than the yacht’s actual size. In that case, the description, ‘the size I thought your yacht was’ is improper. Russell’s distinction of primary and secondary occurrence therefore appears to be of no help in removing the misunderstanding. Indeed, it seems that definite descriptions, as such, are entirely irrelevant to the example. In his talk Kripke said, ‘So Russell’s analysis in terms of his theory of descriptions, as stated, is incorrect . . . How to fix up Russell’s example is a little complicated, and not clear. Maybe it has relatively little to do with the definite descriptions themselves.’

Any such conclusion robs Russell of credit he richly deserves. The correct conclusion is that although Russell’s theory of descriptions indeed applies to the case at hand, he misapplied it. Correctly applied, Russell’s theory
provides insight into an example that, at least on the surface, does not appear to invoke definite descriptions, or indeed any determiner phrases of the sort to which Russell’s theory directly applies.27

The sentence that is supposed to be subject to the primary/secondary occurrence ambiguity is:

\[ S: I \text{ thought your yacht was larger than it is.} \]

Now in general, a statement of the form,

\[ L: \alpha \text{ is larger than } \beta \text{ is} \]

plausibly analyses into ‘\( \alpha \text{ is greater in size than } \beta \text{ is}\)', which plausibly analyses into ‘\( \alpha \text{ has a greater size than } \beta \text{ has}\)’. This last, in turn, plausibly analyses into ‘The size that \( \alpha \text{ has} \) is greater than the size that \( \beta \text{ has}\)’, or more simply:

\[ L': \text{The size of } \alpha \text{ is greater than the size of } \beta. \]

This analysis uncovers two definite-description occurrences that remain concealed in \( L \)'s surface form. Plugging this analysis of \( L \) into \( S \), letting both \( \alpha \) and \( \beta \) be ‘your yacht’, yields the following:

\[ S': I \text{ thought the size of your yacht was greater than the size of your yacht.} \]

Contrary to appearances, this analysis of \( S \) does not remove all ambiguity. The ambiguity is evidently preserved intact; \( S' \) is evidently ambiguous in the same way that \( S \) is. What is significant is that the ambiguity of \( S' \) is evidently one of scope. The definite description ‘the size of your yacht’ occurs twice in \( S' \). Russell should probably be seen as maintaining that the same description therefore implicitly occurs twice in \( S \) itself. The touchy yacht owner deliberately misinterprets the guest’s remark, precisely as Russell indicates, by giving both the left-hand (first) and right-hand (second) description occurrences in \( S' \) their secondary-occurrence readings:

\[ I \text{ thought: that there is a size } s \text{ that was uniquely a size of your yacht and} \]
\[ \text{a size } s' \text{ that was also uniquely a size of your yacht and } s \text{ was greater than } s'. \]

or more simply,

\[ S_{2,2}: I \text{ thought: that a unique size of your yacht was greater than a unique size of your yacht.} \]

By contrast with \( S_{2,2} \), and by contrast also with Russell’s careless stab at capturing the guest’s intent (and contrary to the thrust of Kripke’s remark),
giving the right-hand description occurrence its primary-occurrence reading while still giving the left-hand description occurrence its secondary-occurrence reading yields precisely what the guest did mean:

There is a size \( r \) that is uniquely a size of your yacht and I thought: that there is a size \( s \) that was uniquely a size of your yacht and \( s \) was greater than \( r \),

or more simply,

\[ S_{2,1}: \text{There is a size } r \text{ that is a unique size of your yacht and I thought: that a unique size of your yacht was greater than } r. \]

This result should not be underappreciated. It appears to vindicate Russell on two counts: (i) the deep structure of the original problem sentence \( S \) evidently involves a definite description, ‘the size of your yacht’, hidden in the surface form; and as a result (ii) Russell’s theory of descriptions, with its distinction of primary and secondary occurrence, is indeed evidently applicable to the case, roughly as he says. Kripke is correct that \( S \) does not involve the improper description, ‘the size that I thought your yacht was’, but everything Russell says about the example is correct once his sloppy ascription is replaced with a more careful formulation, like ‘The size of your yacht is such that I thought the size of your yacht was greater than that’.

There remains a problem of a sort rather different from the problem that Kripke noticed, but one that genuinely calls Russell’s theory into question. According to that theory, the left-hand description occurrence in \( S' \) is subject to the same two options of primary and secondary occurrence. Giving both description occurrences their primary-occurrence readings yields a result, \( S_{1,1} \), analogous to \( S_{2,2} \). Giving the left-hand description occurrence its primary-occurrence reading while giving the right-hand its secondary-occurrence reading yields yet another unintended interpretation, but this one is truly bizarre:

\[ S_{1,2}: \text{There is a size } r \text{ that is a unique size of your yacht and I thought: that } r \text{ was greater than a unique size of your yacht.} \]

This says, in effect, that the guest thought the yacht was smaller than it is! The main problem with \( S_{1,2} \) is not merely that it is a misinterpretation of the guest’s intent (although it is certainly that). The main problem is not even merely that it imputes to the guest exactly the opposite of the guest’s intent (although it does that as well). The main problem with \( S_{1,2} \) is that it is not a possible reading of the original sentence \( S \) at all. Instead, it is perhaps a natural reading of a very different sentence:

\[ S": \text{Your yacht is larger than I thought it was.} \]
Any theory entailing that $S$ may legitimately be read as $S''$ is incorrect. What remains unclear is whether Russell's is such a theory. Insofar as Russell might have held that $S$ may be read as $S'$, there is ample cause for worry on this score. Perhaps Russell may avoid the difficulty by analysing $L$ differently in terms of ‘the size of your yacht’. More promising, perhaps some argument can be provided that whereas $S_{1,1}$ and $S_{2,2}$ are indeed possible readings of $S$—and hence the ‘misunderstanding’, precisely as Russell holds—$S_{1,2}$ by contrast is precluded by considerations extraneous, and complementary, to the theory of descriptions.

More plausible still, $S_{1,2}$ may not be a legitimate reading of $S$ any more than $S_{1,1}$ is. The ‘was’ in the original sentence $S$ (‘was larger’) might be seen as somehow incorporating subjunctive mood, the ‘is’ (‘than it is’) as incorporating indicative mood. The contrasting moods may be taken as indicating that the left-hand occurrence of ‘the size of your yacht’ in $S'$ is to be a secondary occurrence (‘the size your yacht had’), the right-hand a primary occurrence (‘the size your yacht has’), thereby unequivocally yielding $S_{1,1}$. Instead, the owner misreads $S$ as ‘I thought your yacht was larger than it was’, yielding $S_{2,2}$.

NOTES

1. I have had the essentials of the interpretations provided here since 1972, but many others have greatly influenced my thought on the topic, too many others to list here. No one influenced me more than David Kaplan. The Santa Barbarians Discussion Group patiently worked through my edited version of the crucial passage in 1997. I am indebted to them, especially C. Anthony Anderson, for their comments and our efforts. By not venturing to challenge the interpretation, the group shares some responsibility for the final product—how much responsibility depending upon the success or failure of the project. I am also grateful to Alan Berger, Saul Kripke, Teresa Robertson, the participants in my seminars at UCSB and UCLA during 1998–99, notably Roberta Ballarin, Stavroula Glezakos, David Kaplan, and D. Anthony Martin, and my audience at the McMaster University 2005 conference on ‘Russell vs. Meinong: 100 Years After On Denoting’, for their insightful comments, notably Matt Griffin. Finally, I am grateful to Oxford University Press for permission to incorporate portions of my article ‘On Designating’.

2. Previous discussions include the following, chronologically: Alonzo Church (1943: 302); Ronald J. Butler (1954); John Searle (1958); Peter Geach (1959); Ronald Jager (1960); David Kaplan (1969); A. J. Ayer (1971: 30–32); Christine E. Cassin (1971); Michael Dummet (1973: 267–68); Herbert Hochberg (1976); Simon Blackburn and Alan Code (1978); Geach (1978); Blackburn and Code (1978); A. Manser (1985); Peter Hylton (1990: 249–64); Pawel Turnau (1991); Michael Pakaluk (1993); Russell Wahl (1993); Michael Kremer (1994); Harold Noonan (1996); Gregory Landini (1998); William Demopoulos (1999); Gideon Makin (2000: 22–45, 206–22); James Levine (2004).

3. Fregeans may substitute the word ‘sense’ wherever I use ‘content’.

4. Kaplan there calls indirect-quotation marks meaning-quotation marks. Indirect quotation quotes not expressions but their content.
5. That is, we wish to consider the relation between ‘the centre of mass of the Solar System’ and “the centre of mass of the Solar System”, between ‘the first line of Gray’s Elegy’ and “the first line of Gray’s Elegy”, and so on.

6. It might have been more perspicuous for Russell to formulate his objection this way: We cannot succeed in both preserving the connection of content to designatum and allowing the content and the designatum to be one and the same. Moreover, we cannot even succeed in both preserving the connection of content to designatum and disallowing the content and the designatum from being one and the same unless the content cannot be got at except by means of determiner phrases. That is, if we preserve the connection whereby the designatum of a definite description is determined by the description’s content which is distinct from the designatum itself, then the content cannot be designated by means of a ‘genuine name in the strict, logical sense’.

7. This yields the awkward result that $\text{content of } \alpha^\pi = \text{true}. I am here attributing to Russell a serious equivocation, resulting from his dual use of inverted commas both as direct–quotation marks and as indirect-quotation marks. He appears to believe that he has derived from the theory that definite descriptions have a content/designation distinction the consequence that in order to designate the centre of mass of the Solar System we must use ‘the content of the centre of mass of the Solar System’ (which Russell fails to distinguish from the perfectly appropriate ‘the content of the centre of mass of the Solar System’), thus ascribing a content to a determining complex itself. As a criticism of the content/designation theory, or even as a neutral description, this is a red herring. Instead, the theory entails that one may designate the centre of mass of the Solar System using the functor ‘the content of’ in combination with ‘the centre of mass of the Solar System’ and direct quotation, not indirect. Russell has a stronger criticism to make of the theory, though his presentation is coloured somewhat by this red herring.

8. In the original text, Russell here uses ‘C’ as a schematic letter standing in for a term designating a determining complex. The preceding two sentences should read: For example, let ‘C’ [stand in for] ‘the determining complex occurring in the second of the above instances’. Then $C = \text{the first line of Gray’s Elegy}$, and the determinatum of $C = \text{The curfew tolls the knell of parting day’}. I have reformulated this in the metalinguistic mode using ‘$\alpha$’, quasi-quotation, and the predicate ‘is true’.

9. Pace Russell, his apparent observation that in order to designate the designatum of $\alpha$ we should use ‘the determinatum of “$\alpha”’ rather than ‘the determinatum of $\alpha$’, though correct, provides no support whatever for his apparent conclusion that in order to designate the content of $\alpha$, rather than using ‘the content of $\alpha$’ we must use ‘the content of “$\alpha”’, which is in fact equally inappropriate. Instead we can designate $\alpha$’s content by using ‘the content of “$\alpha”’ or $\text{content of } \alpha^\pi$. Analogously, we can equally designate $\alpha$’s designatum using ‘the designatum of “$\alpha”’ or $\alpha$ itself.

10. That is, as soon as we put a determining complex in a proposition, by using a sentence involving a singular term whose content is the complex, the proposition is about the complex’s determinatum. This generates what I call the Collapse. As Russell will argue below, this same phenomenon arises even when designating the complex by using the simple indirect quotation $\text{content of } \alpha^\pi$.

11. Roughly, a proposition component represents an object $x$ in a proposition $p$ if $p$ is about $x$ in virtue of that component. This marks the first use by
Russell of his variable ‘C’ as ranging over determining complexes rather than definite descriptions. Moreover, the quotation marks here are indirect-quasi-quotation marks. The quotation “the content of c” designates the determining complex consisting of the content of the functor ‘the content of’ joined with the complex c. Let c be the particular determining complex, “the first line of Gray’s Elegy”. When we attempt to form a proposition about it by using a sentence containing the indirect quotation “the first line of Gray’s Elegy” (Russell supposes that one way to do this on the theory he is criticizing is by means of the sentence “The content of the first line of Gray’s Elegy” is intriguing’), if the quotation functions as a logically proper name of the determining complex, then the resulting proposition is that (the content of) the first line of Gray’s Elegy is intriguing, rather than a proposition about the intended determining complex itself. This is one particular form of the Collapse: In attempting to form a proposition about a determining complex c by using a sentence containing an indirect quotation “α”, where α is a definite description that expresses c, we generate a proposition not about c but about its determinatum. We might use “the content of ‘α’” instead of the indirect quotation “α”, but having assimilated this to “the content of “α””, or failing to distinguish the two, Russell believes he has just shown that use of such a phrase inevitably comes to grief, via the Collapse. In any event, the objective in (D) was to form a singular proposition about a determining complex, not a proposition in which the target complex is represented as the content of this or that phrase.

12. In this sense, “the meaning cannot be got at except by means of determiner phrases”; it cannot be genuinely named, in the strict, logical sense.

13. For example, let us attempt to name a particular complex, say “the first line of Gray’s Elegy”, in order to express a proposition about it. Any proposition in which the complex itself occurs is about the first line of Gray’s Elegy, i.e. the determinatum of the target complex rather than the complex itself. Any proposition that is about the complex itself will involve a second-level determining complex that determines the target complex. For example, the indirect quotation “the first line of Gray’s Elegy” itself must express a second-level determining complex, “the first line of Gray’s Elegy”, as its content. Moreover, the target complex is not a constituent of the postulated second-level complex, as it is of “the content of the first line of Gray’s Elegy”. The second-level complex cannot, for example, be of the form “the determining complex that bears relation R to c”, for some relation R and where c is our target complex, “the first line of Gray’s Elegy”. For the Collapse occurs here just as it does with propositions; the complex just formed collapses into “the determining complex that bears R to the first line of Gray’s Elegy”. If R is the relation of determining, then this second-level complex fails to determine a unique complex because there are too many complexes that bear this relation to the first line of Gray’s Elegy (infinitely many, in fact). And if R is the relation of identity, then this second-level complex fails to determine a unique complex because there are too few complexes that bear this relation to the first line of Gray’s Elegy (none, in fact).

14. We have no idea which determining complex “D” is of the infinitely many complexes that determine “D”.

15. The inextricable tangle does indeed seem to prove that the whole distinction of content and designation has been wrongly conceived . . . by Russell. On the theory that definite descriptions are singular terms, whereas the proposition is about the description’s designatum and not about the content, the content itself is relevant to the proposition’s identity, and especially to its distinctness from other propositions involving determining
complexes with the same determinata. This is the very point of the theory (and Russell knows it).

16. Blackburn and Code mention the Collapse only after presenting their rival interpretation, which does not rely on the Collapse (1978a: 76; crediting David Kaplan for showing them that the Collapse refutes the earlier theory of designating in Russell’s *Principles*). In sharp contrast to my interpretation, they express uncertainty whether Russell is even aware of the Collapse by the time he writes ‘On Denoting’. As against the hypothesis that he was, they say that ‘although this is a problem as to how one refers to senses [contents], the obvious solution is not to attack Frege, but rather to insist that his three-entity view [distinguishing among an expression, its content, and its designatum] applies to all referring [designating] expressions’. There are at least five problems with this. First, Russell was explicitly aware of the Collapse already in the lengthy and rambling ‘On Fundamentals’, begun not two months prior to ‘On Denoting’ and posthumously published in *The Collected Papers of Bertrand Russell* (1994a: 363, 382, and passim). Indeed, some passages of ‘On Fundamentals’ appear virtually verbatim in the ‘Gray’s Elegy’ argument, which is in certain respects a streamlined version of the convoluted reasonings of the former. Second, whereas one might hope to solve the problem by insisting that any singular term that designates a content always has its own content distinct from its designatum, the same distinction does not have to be extended to all terms (including names for concrete objects) in order for the solution to work. Third, though Russell was aware of the possibility of a theory like the one Blackburn and Code call ‘the obvious solution’ (as is shown by a passage they quote from *Principles*), he did not unequivocally endorse it. Fourth, on the contrary, a central purpose of ‘On Denoting’ is precisely to reject Frege’s ‘three-entity view’ in regard to all singular terms, and replace it with a two-entity view. Finally, and most importantly, the very point of paragraphs (F) and (G) appears to be precisely that the very proposal in question utterly fails to solve the problem.

17. The expression \( "m \text{ mm } D \text{ mm } " \) may stand in for the iterated indirect quotation \( "m \text{ mm } \) the centre of mass of the Solar System \( "m \text{ mm } \) \)'', which designates the content of the indirect quotation, “the centre of mass of the Solar System”.

18. This does not rule out that the content can also be “got at” by means of an indefinite description, even if it is deemed not a singular term. Since ST is neutral regarding indefinite descriptions, it is equally consistent with the view that definite and indefinite descriptions alike are singular terms. The latter view makes indefinite descriptions subject to the argument from the Collapse. On the Theory of Descriptions, by contrast, a definite description is analysed as a special kind of indefinite description, neither being a singular term. The interpretation of this stage of Russell’s argument is strongly supported by the fact that he also gives this argument in writings just prior to ‘On Denoting’ (posthumously published). Cf. his ‘On Fundamentals’ and ‘On Meaning and Denotation’, also in *The Collected Papers of Bertrand Russell* (1994b: 322).

19. Searle (1958: 139–40) depicts Russell as arguing that in order for a term to designate, the designated object must, if we are not to “succumb to mysticism”, occur in the propositions expressed with the help of the designating term; but then the Collapse excludes the possibility of designating determining complexes. Searle complains that the whole point of Frege’s theory, which Russell is attacking, is to deny Russell’s premise. It is possible that Church construes the argument similarly.

20. In ‘Über Sinn und Bedeutung’ (1994: 149), Frege identified the indirect sense of a sentence \( \varphi \) with the customary sense of ‘the thought that \( \varphi \)’, which phrase may be presumed synonymous with \( "m \varphi " \).
21. Carnap (1947: 118–37, especially 129–33) may be profitably interpreted as rejecting singular propositions about individuals, while accepting that ungerade constructions (as occur in belief attributions, modal claims, etc.) express singular propositions about the contents of their complement clauses. Cf. Dummett (1973: 267–68) and Parsons (1981).

22. Church disagrees with Frege on some details, and as I argue in (Salmon 1993), he may have been inconsistent regarding the issue of the hierarchy.

23. Burge argues (1979: 271–72), as follows, specifically that Frege’s theory of Sinn and Bedeutung is committed to hierarchies of sense, when coupled with Church’s methodology of eliminating ambiguity-producing devices (like ‘believes that’) that shift expressions in their scope into ungerade mode in favour of fully extensional operators applied to univocal names of senses: Suppose for a reductio that the true proposition that Opus 132 is a masterpiece does not contain a second-level complex that determines the proposition that Opus 132 is a masterpiece, and that instead the latter proposition represents itself in the former proposition. In accordance with Church’s methodology, we introduce an artificial extensional two-place operator ‘Believes’ for the binary relation of belief (between a believer and the object believed), so that ‘Bela Believes (“Opus 132 is a masterpiece”)’ expresses that Bela believes that Opus 132 is a masterpiece. Then according to Frege’s theory, the quasi-artificial expression E, ‘Bela Believes (Opus 132 is a masterpiece)’, expresses the bizarre proposition that Bela believes a particular truth-value—to wit, the truth-value that is truth if Opus 132 is a masterpiece, and is falsity otherwise. But by our reductio hypothesis, E expresses a content consisting of the very components of the proposition that Bela believes that Opus 132 is a masterpiece, composed the very same way.

24. Striking evidence that the central thrust of the ‘Gray’s Elegy’ argument has been lost on Russell’s readers is provided by Burge’s remark (280, n8) that to his knowledge, the argument presented above was nowhere explicitly stated before. Burge’s argument employs a sentence in place of a definite description, but this difference from Russell’s examples is completely inessential to the general argument. Burge also frames his argument in terms of a Fregean conception whereby an artificial notation should be used to avoid natural-language ambiguities produced by ungerade devices (e.g., ‘Believes’ in place of ‘believes that’). This introduces additional complexity, also inessential to the general point and leading to an unnecessarily restricted conclusion. Burge’s argument may be strengthened as follows: Suppose for a reductio that the true proposition that “the centre of mass of the Solar System” is a sense does not contain a second-level complex that determines “the centre of mass of the Solar System”, and that instead the complex “the centre of mass of the Solar System” represents itself in the proposition. The English sentence S, ‘The centre of mass of the Solar System is a sense’—which contains no artificial notation—then expresses a proposition consisting of the very components of the proposition that “the centre of mass of the Solar System” is a sense, and composed the very same way. By Compositionality, S therefore expresses our target proposition. But this conflicts with the fact that S is false.

25. The argument just given on Russell’s behalf purports to prove that, in Frege’s terminology, the sense of an indirect quotation is not an effectively computable function of the customary senses of the expressions within the indirect quotes. Frege concedes that the sense of a compound expression is not always composed of the customary senses of the component expressions. Frege would insist, however, that indirect-quotation marks do not violate Compositionality, or even Strong Compositionality as he intends these principles, since an expression does not have its customary sense when occurring within indirect-quotation marks and instead expresses its indirect sense, which does uniquely fix the sense of the indirect quotation. He says something analogous in connection with direct quotation. Direct quotations of customary synonyms are not themselves synonyms.

26. [Added after original submission of this article.] Kripke’s criticism has since been published (2005: see 1021–23). I take this opportunity to correct Kripke’s characterization there (1022) of our communication concerning Russell’s example. In my earlier discussion with Kripke I emphasized a distinction in semantic content that I draw, and of which Kripke is dubious, between the binary-relational predication ‘a is larger than a’ and the monadic-predicational ‘a is a thing larger than itself’—the latter symbolized as ‘(\(\lambda x\))x is larger than x’. I used the distinction not to solve the problem Kripke noticed in Russell’s discussion of his example, but rather to support my contention (which Kripke does not accept) that it is possible for one to believe concerning a particular yacht \(y\), that \(y\) is larger than \(y\), while not thereby believing that \(y\) is self-larger (i.e., a thing \(x\) larger than itself). Cf. my ‘Reflexivity’ (1986) and my ‘Reflections on Reflexivity’ (1992). I was aware that this distinction (even if it is legitimate, as I maintain) does not solve the problem Kripke had noticed. I have known the corrected “purely Russellian” analysis, and have so interpreted Russell’s intended treatment of the example, since I first studied ‘On Denoting’ in 1971–72 (in undergraduate courses given by Alonzo Church, David Kaplan, Kripke and others). I had given the example essentially the same Russellian analysis on first reading ‘On Denoting’. Each of Kripke’s explicit misgivings (1025, n45) concerning my former proposal can be met. In particular, on my proposal, although a formula \(\varphi_{\beta}\) and its lambda-convert \((\lambda \alpha)\varphi_{\alpha}(\beta)\) differ in semantic content, the two remain coextensional and indeed logically equivalent (at least in the absence of nonextensional devices). The distinction in content in no way undermines the observation that there is always a fact of the matter concerning whether \(x = x\) (just as there is a fact concerning whether \(x\) is self-identical— the two matters being equivalent), any more than it undermines the observation that it is a necessary truth that \(x = x\). Furthermore, Kripke’s claim that “Church, inventor of the lambda notation, did not intend any such distinction” in semantic content between a formula \(\varphi_{\alpha}\) and its lambda-convert \((\lambda \alpha)\varphi_{\alpha}(\beta)\) (as between ‘a is larger than a’ and ‘(\(\lambda x\))x is larger than x’) is historically incorrect. On Church’s Alternative (0), which he explicitly preferred over Alternatives (1) and (2) as an explication of having the same sense, such lambda converts are, as Church recognized, although logically equivalent, not synonymous— just as the mathematical expressions ‘3!’ (alternatively, ‘(\(\lambda x\))x!\(\beta\)’) and ‘6’ are co-designative but not synonymous. I am in agreement with Church in this. (\(\lambda\)-converts are regarded as synonymous on the other two alternatives.) Cf.
David Kaplan and Terence Parsons have noted related difficulties in Russell's discussion of the example. I am grateful to Kripke, Kaplan, and Parsons for discussion. Kripke is now persuaded, whereas I believe Parsons remains unconvinced, that Russell's distinction of primary and secondary occurrence, properly applied, nevertheless provides an insightful diagnosis of the ambiguity. See the following note concerning Kaplan. Russell correctly observes in the same paragraph of 'On Denoting' that the primary-occurrence reading of 'George IV wondered whether Scott is the author of Waverley' is true if King George glimpses Scott at a distance and asks 'Is that Scott?'. Russell was likely assuming, at least for the purposes of illustration, a commonsensical epistemology on which visual perception of an object is sufficient to enable one to apprehend singular propositions about it, and thus to bear de re propositional attitudes towards it. However, Russell's observation thus seems incompatible with his claim that 'an interest in the [reflexive] law of identity can hardly be attributed to the first gentleman of Europe'. Kaplan, Kripke, and others have independently also noticed this flaw in Russell's presentation. (See Kripke, 2005: 1023–24.) Russell's observation can be made consistently with Russell's insistence that King George had no interest in the reflexivity of identity by distinguishing, as I do, the singular proposition about Scott that he is Scott from the singular proposition about Scott that he is self-identical (a thing identical with itself), so that one can in fact wonder about Scott whether he is him without thereby wondering whether he is self-identical. See the preceding note. Kripke argues (1024–25) that this distinction was not available to Russell given his logical apparatus. I never asserted that Russell's account of sentences involving \(\lambda\)-abstraction is compatible with my own. I believe, however, that Russell could have distinguished (even if not through his apparatus for propositional-functional abstraction) between the singular propositions about Scott that he is him and that he is self-identical (a thing identical with itself). He might also have interpreted the reflexivity of identity as not involving the property of being self-identical. In any event, King George's wondering about Scott whether he is him (as glimpsed from a distance) should not be misrepresented as a concern about the reflexivity of identity. King George knows about Scott all the while (even while glimpsing him from a distance) both that he is him and that he is, as with everything else, self-identical.

Kaplan, in (1973) observes, ‘The yacht owner’s guest who is reported by Russell to have become entangled in ‘I thought that your yacht was longer than it is’ should have said, ‘Look, let’s call the length of your yacht ‘a russell’. What I was trying to say is that I thought that your yacht was longer than a russell.’ If the result of such a dubbing were the introduction of ‘russell’ as a mere abbreviation for ‘the length of your yacht’, the whole performance would have been in vain’ (501). The measurement term ‘russell’ in Kaplan’s disambiguation of \(S\) serves much the same purpose as the anaphoric pronoun in ‘The size of your yacht is such that I thought the size of your yacht was greater than that’. So does the variable ‘\(r\)’ in \(S_{2,1}\). This suggests that Kaplan had in mind the same correction proposed here.

A potential difference between \(S_{1,1}\) and \(S_{2,2}\) is that one might easily come to believe of a single thing \(x\), de re, that \(x\) is greater than \(x\) is. I take it to be clear, however, that the guest did not believe this of the yacht’s size. See n26 above.
REFERENCES

Points, Complexes, Complex Points, and a Yacht


