Indeterminacy and Triviality

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1 Introduction: indeterminacy and cognitive role

Suppose that you're certain that (1) lacks a determinate truth value. I.e., you are fully confident that it is not determinately true and not determinately false.

(1) Frida is tall.

What attitude should you take towards (1)? Reject it? Suspend judgement? Adopt middling confidence? Adopt a special vagueness-related mode of uncertainty? Or what? Answering this question would be to take a stance on the *cognitive role of indeterminacy*.[1] It is the first step in building a theory of rational belief—and eventually an entire normative psychology—appropriate to sentences and propositions that lack determinate truth values.[2]

Call an answer to the cognitive role question *exclusionary* if the attitude to (1) it recommends is something that is rationally incompatible with belief. Middling confidence, suspension of judgement, rejection, a special mode of uncertainty—all these answers are exclusionary in the relevant sense. Despite their differences, exclusionary answers can reach consensus on the following: less than full confidence that A is determinate rationally requires less than full confidence that A. The contraposed form of this principle will be our starting point in what follows: certainty of A rationally requires certainty that A is determinate.

Since the exclusionary view is our starting point, we briefly motivate it (what follows idealizes away from the possibility of higher-order indeterminacy, which we discuss later). Suppose that an agent is less than fully confident that it's determinately true that Frida is tall. Assuming the usual link between credence and dispositions to bet, an agent who is fully confident that Frida is tall should be willing to take bets at any odds (or at least, at extremely unfavorable odds) on the proposition that Frida is tall. It seems strange to do so while

[1] For explicit attempts to answer this question, see Field 2000, 2004, 2008; Schiffer 2003; Dorr 2003; Smith 2008; Williams 2014a,b; Bacon 2018

[2] For attempts to talk about vague desire and rational belief, desire and decision in the context of vagueness, see for example Edgington 1997; Williams 2016.

having some credence that it's not determinately true that Frida is tall. If you agree, you should be sympathetic to the exclusionary starting point above that rules it out.

Among the various exclusionary positions, perhaps the least plausible is the following: when you're certain that it's indeterminate whether Frida is tall, it is rationally required that you utterly reject Frida being tall. This is *rejectionism*. It has defenders (cf. Field 2004), but it is a minority position. Most find it surprising as a thesis about indeterminacy associated with borderline cases of paradigmatically vague properties. As we point out below, rejectionism is intolerable for at least some applications of indeterminacy—for example, for thesis that the openness of the future consists in indeterminacy of future contingents.[3]

The main result of this paper is that the principle about certainty that we take as our starting point, together with some minimal side-premises, entails rejectionism. This is a surprising result, which requires abandoning at least some of our intuitive views about indeterminacy and cognitive role. As we point out in the final sections, there is at least a formal analogy between our argument and various triviality arguments that have been presented in the literature on conditionals and modals.[4] Perhaps this points towards a general solution. One tempting moral, which would apply both to our puzzle and to standard triviality arguments, is that standard Bayesian assumptions about learning need to be revised when we countenance modal, conditional, and determinacy operator. But what we say leaves open the idea that our puzzle requires a different kind of solution.

2 Formalizing the main claims

We start by formalizing the key theses in play. Let 'DET' stand for "it is determinate that". We work with a space of degrees of belief, but make only weak assumptions about it. There could be three degrees of belief (belief, agnosticism, full disbelief), or there could be infinitely many degrees of belief, modelled by the real numbers between $[0,1]$, or by intervals drawn from $[0,1]$. All we assume is that these degrees of belief are at least partially ordered by comparative strength ($\geq$), and that there is a strongest degree of belief (represented by 1, which we'll label certainty) and a weakest degree of belief (represented by 0, which we'll label rejection). We use '$\mathcal{C}$' to denote the set of all rational belief states. Also, we assume that degrees of belief are defined over propositions,

[3] Discussion of the indeterminate future dates back to Aristotle. For a clear recent articulation of the view on which future contingents are indeterminate, see Barnes and Cameron 2009.

[4] The literature on triviality results was started by Lewis 1976; see Hájek and Hall 1994 for an overview of early triviality results. For more recent results, see Bradley 2000, 2007; see also Charlow 2016; Russell and Hawthorne 2016; Goldstein forthcoming for extensions of triviality results beyond conditionals.

and we use Roman sans-serif capitals ('A', 'B', 'C', ...) as metavariables ranging over propositions. (All our arguments also carry over to a picture on which degrees of belief apply to sentences, *modulo* the usual qualifications concerning context dependence.)

We assume that agents have both categorical degrees of belief in propositions, and conditional degrees of belief in one proposition given another. If Cr picks out such a belief state we use $Cr(\mathsf{A})$ to pick out a degree of categorical belief in A and $Cr(\mathsf{B} \mid \mathsf{A})$ the degree of conditional belief in B given A. We understand conditional degree of belief in terms of update: $Cr(\mathsf{B} \mid \mathsf{A})$ denotes the posterior degree of belief in B had by a rational agent with prior credence function Cr, upon learning A (with certainty, as total information). On standard Bayesian accounts, the conditional credence $Cr(\mathsf{B} \mid \mathsf{A})$ is also set, by definition, to be equal to the ratio of the unconditional credences $Cr(\mathsf{A} \wedge \mathsf{B})$ and $Cr(\mathsf{A})$. But, on the current proposal, this is a substantial claim—indeed, one of the routes to block the argument will consist precisely in denying the ratio formula.

Let us now formalize our main claims. Our starting principle can be expressed as follows:

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\textsc{det}\ \mathsf{A}) = 1 \qquad (\textsc{certainty})$$

The rejectionist thesis is:

$$\forall Cr \in \mathcal{C} : Cr(\neg\textsc{det}\ \mathsf{A} \wedge \neg\textsc{det}\neg\mathsf{A}) = 1 \Rightarrow Cr(\mathsf{A}) = 0 \qquad (\textsc{rejectionism})$$

To prove REJECTIONISM, we prove a claim that entails it, namely the inequality:[5]

$$\forall Cr \in \mathcal{C} : Cr(\textsc{det}\ \mathsf{A}) \geq Cr(\mathsf{A}) \qquad (\textsc{equiv}1)$$

Let us also observe that EQUIV2 is pretty clearly true, and so we can strengthen our conclusion to the very informative identity EQUIVALENCE:[6]

$$\forall Cr \in \mathcal{C} : Cr(\textsc{det}\ \mathsf{A}) \leq Cr(\mathsf{A}) \qquad (\textsc{equiv}2)$$

$$\forall Cr \in \mathcal{C} : Cr(\textsc{det}\ \mathsf{A}) = Cr(\mathsf{A}). \qquad (\textsc{equivalence})$$

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[5] Two principles generate this entailment: (a) that A being indeterminate and it being determinate that A are inconsistent; and that if one is certain of one of a pair of inconsistent propositions, one is rationally required to reject the other. The antecedent of REJECTIONISM tells us that we are certain of A being indeterminate, so we must reject DET A which is inconsistent with it. Then by EQUIV1 we must reject A, i.e. the consequent of REJECTIONISM.

[6] We can also argue for it given a few more principles: (a) rational degree of belief doesn't drop over logical consequence; (b) determinacy is factive: $\textsc{det}\ \mathsf{A} \models \mathsf{A}$.

3 The proof

We are to prove EQUIV1 from CERTAINTY. Note that when $Cr(\mathsf{A}) = 0$ then EQUIV1 holds, so we may assume $Cr(\mathsf{A}) \neq 0$.

We assume three side premises. The first two are constraints on rational degrees of belief:

$$Cr(\mathsf{A} \mid \mathsf{A}) = 1 \qquad \text{(IDENTITY)}$$

$$Cr(\mathsf{B} \mid \mathsf{A}) = 1 \Rightarrow Cr(\mathsf{B}) \geq Cr(\mathsf{A}) \qquad \text{(BOUND)}$$

The third side premise is a closure principle, stating that that the result of updating a rational credence on proposition C (itself of non-zero credence) is a rational credence function:

$$\forall \mathsf{C} : Cr(\bullet) \in \mathcal{C} \wedge Cr(\mathsf{C}) \neq 0 \Rightarrow Cr(\bullet \mid \mathsf{C}) \in \mathcal{C} \qquad \text{(CLOSURE)}$$

We say more in defense of these side premises below. For the moment, let us flag that all of them are entailed by standard Bayesian tenets about credence—though one doesn't need to be a Bayesian to endorse them.

With these assumptions in place, it's simple to state the proof. Start with an arbitrary rational belief state Cr. By an instance of closure, $Cr(\bullet \mid \mathsf{A})$ is a rational belief state. We argue:

1. $Cr(\mathsf{A} \mid \mathsf{A}) = 1$ (from IDENTITY, applied to $Cr \in \mathcal{C}$)
2. If $Cr(\mathsf{A} \mid \mathsf{A}) = 1$, $Cr(\text{DET } \mathsf{A} \mid \mathsf{A}) = 1$ (CERTAINTY, applied to $Cr(\bullet \mid \mathsf{A}) \in \mathcal{C}$)
3. $Cr(\text{DET } \mathsf{A} \mid \mathsf{A}) = 1$ (from 1 and 2)
4. $Cr(\text{DET } \mathsf{A}) \geq Cr(\mathsf{A})$ (from 2 and BOUND, applied to $Cr \in \mathcal{C}$)

The last line is the relevant instance of EQUIV1, as required.

4 Reactions to the argument

The argument is valid, so there are just five things one can do in response:

i. Accept EQUIV1 and so accept REJECTIONISM

ii. Reject IDENTITY

iii. Reject BOUND

iv. Reject CLOSURE

v. Reject CERTAINTY

On the face of it, none of these options seems particularly plausible. Yet at least one of them has to be right. So the proof in §3 poses a puzzle.

We should observe that the puzzle holds for every single proposition. So, for example, suppose you think that CLOSURE fails as a general thesis for arbitrary propositions, because some of those propositions simply are not rationally learnable. That gets you out of one instance of the argument, but does nothing to help you get out of other instances of the argument involving propositions that are rationally learnable.

So we should distinguish two ways of reacting to the puzzle. On the one hand, we might defend an across-the-board solution. For example, we might endorse REJECTIONISM for every single proposition; or one may resist the argument by denying that IDENTITY ever holds. On the other, we might defend a piecemeal solution. For example, we might hold that CERTAINTY fails for certain propositions, and REJECTIONISM is true for others.

In the next sections, we discuss options (i)–(v). We won't try to settle definitively which of them is right, but we will steer the debate in directions that seem plausible to us.

4.1 Accepting REJECTIONISM

As we said, some cognitive role theorists endorse REJECTIONISM. So one might think that we just provided an argument for an across-the-board endorsement of this position. We want to resist this conclusion, which we find particuarly implausible.

Why is REJECTIONISM so implausible? Notice that there are many different kinds of indeterminacy. There are borderline cases of paradigm gradable adjectives (*tall*, *bald*, *red*), but there are also borderline cases of the relation *being the same person as*. Believers in the open future hold that future contingents are indeterminate. The conditional *if I roll a fair die, it will land even* is classified as indeterminate on many theories. In many of these cases, rejectionism straightforwardly conflicts with common sense. One obvious case is that of future contingents. My attitude to the indeterminate future contingent *I will catch the train this afternoon* is uncertainty, not utter disbelief. More in general, our processes of deliberation about the future seem to presuppose that propositions about the future should receive positive credence, even though it is indeterminate whether they are true. So endorsing REJECTIONISM about future contingents would be disruptive both for our ordinary deliberations and for our philosophical theorizing about them. Similar considerations apply to other kinds of indeterminacy. For example, assigning positive credence to indeterminate claims about personal identity is arguably central to understanding moral and self-interested concern for the future.[7]

[7] For an argument for an exclusionary answer to the cognitive role question on this sort of basis, see Williams 2014b.

Perhaps these considerations can be overridden via decisive theoretical arguments. But, absent these, we think that REJECTIONISM should not be invoked as an across-the-board solution. We need to explore further options.

4.2 Rejecting IDENTITY

Rejecting IDENTITY (repeated below), whether across the board or for some cases, also seems implausible.

$$Cr(\mathsf{A} \mid \mathsf{A}) = 1 \qquad \text{(IDENTITY)}$$

All we need to derive IDENTITY are two principles that seem very safe. The first is simply the logical validity of iteration. The second is a principle saying that, if A entails B, the conditional probability of the latter given the former is 1.

$$\mathsf{A} \vDash \mathsf{A} \qquad \text{(ITERATION)}$$

$$\text{If } \mathsf{A} \vDash \mathsf{B}, \text{ then } Cr(\mathsf{B} \mid \mathsf{A}) = 1 \qquad \text{(ENTAILMENT)}$$

At the very least, it's extraordinarily implausible that we will be able to appeal to the rejection of IDENTITY as an across-the-board response to the puzzle.[8]

4.3 Rejecting BOUND

Consider now BOUND:

$$Cr(\mathsf{B} \mid \mathsf{A}) = 1 \Rightarrow Cr(\mathsf{B}) \geq Cr(\mathsf{A}) \qquad \text{(BOUND)}$$

Let us observe first that BOUND is entailed by the classical construal of conditional probability, spelled out in RATIO, together with the principle that one's credence in a conjunct is an upper bound in one's credence in a conjunction.[9]

$$Cr(\mathsf{A} \wedge \mathsf{B}) = Cr(\mathsf{B} \mid \mathsf{A}) \times Cr(\mathsf{A}) \qquad \text{(RATIO)}$$

$$Cr(\mathsf{A}) \geq Cr(\mathsf{A} \wedge \mathsf{B}) \qquad \text{(CONJUNCTION)}$$

CONJUNCTION seems extremely plausible. Of course, we could find a weird enough logic for conjunction that invalidates it. But for current purposes we won't question it. Assuming that CONJUNCTION is safe, then, rejecting BOUND entails rejecting RATIO.

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[8] Though see the discussion of degrees of determinacy and conditional probability in Williams 2016 for a precedent.

[9] Proof: assume $Cr(\mathsf{B} \mid \mathsf{A}) = 1$. Then, via RATIO, $Cr(\mathsf{A} \wedge \mathsf{B}) = Cr(\mathsf{A})$. Via CONJUNCTION, $Cr(\mathsf{B}) \geq Cr(\mathsf{A} \wedge \mathsf{B})$; by replacing $Cr(\mathsf{A} \wedge \mathsf{B})$ with $Cr(\mathsf{A})$ in the inequality it follows that $Cr(\mathsf{B}) \geq Cr(\mathsf{A})$.

Incidentally, notice that RATIO assumes that multiplication is well-defined on degrees of belief. So, in order to claim that BOUND follows from RATIO, we need more substantial assumptions about degrees of belief than the ones we have taken up in §2.

Of course, classical Bayesians take RATIO to be definitional of conditional probability. But recall that in §2 we explicitly disavowed this construal. Rather, we defined conditional credence in terms of update, and left it as an issue to be adjudicated whether RATIO holds. So one option is to deny that the notion of conditional probability that captures update can be defined in the usual way. We come back to this option in §7.

4.4 Rejecting CLOSURE

CLOSURE says that the result of updating a rational credence function on proposition C (itself of non-zero credence) is also a rational credence function.

In §2, we defined conditional probabilities just as the probabilities that are reached by a rational agent via update. So we are guaranteed that, for any proposition C that captures an agent's total evidence, conditonalizing on C has to lead to a rational credal state. This leaves room for one way in which CLOSURE could fail. It might be that some propositions cannot serve as a rational agent's total evidence. In particular, we might claim that one can only learn perfectly determinate propositions: learning A always entails also learning DET A. We will return also to this claim later on.

4.5 Rejecting CERTAINTY

CERTAINTY (repeated below) is the principle we have introduced in this paper.

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\textsc{det}\ \mathsf{A}) = 1 \qquad (\textsc{certainty})$$

We have already defended it informally in the introduction. Here we give two explicit arguments for it.

First, we revisit the idea that any *exclusionary* cognitive role for indeterminacy is committed to CERTAINTY. Suppose for reductio that CERTAINTY fails. Then there is a rational credence function Cr such that $Cr(\textsc{det}\ \mathsf{A}) < 1$ and $Cr(\mathsf{A}) = 1$. So there must be some possibility w given non-zero credence such that $Cr(\textsc{det}\ \mathsf{A} \mid w) = 0$. But we also know that $Cr(\mathsf{A} \mid w) = 1$, and since $\textsc{det}\neg\mathsf{A}$ is incompatible with A, $Cr(\textsc{det}\neg\mathsf{A} \mid w) = 0$, and so $Cr(\textsc{det}\ \mathsf{A} \vee \textsc{det}\neg\mathsf{A} \mid w) = 0$. Absent higher order indeterminacy, which will be considered more fully below, this means $Cr(\neg\textsc{det}\ \mathsf{A} \wedge \neg\textsc{det}\neg\mathsf{A} \mid w) = 1$. So a failure of CERTAINTY commit us (via CLOSURE) to a rational belief state in which we are certain that a proposition is indeterminate and certain of that proposition itself. That is absurd on its face, but more specifically, it is incompatible with any exclusionary conception of the cognitive role of indeterminacy, on which something incompatible with full belief (e.g. middling confidence, suspension of judgement etc) is required when we are certain a proposition is indeterminate.

Second, a number of theorists hold that assertability requires determinacy:

if a subject is in a position to assert A, then it is determinately true that A.[10]

$$\textsc{assert}\ \mathsf{A} \vdash \textsc{det}\ \mathsf{A} \quad (\textsc{p}1)$$

It is very plausible, moreover, that complete confidence in a proposition suffices for complete confidence that that proposition is assertable.[11]

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\textsc{assert}\ \mathsf{A}) = 1 \quad (\textsc{p}2)$$

From (p1) and (p2), together with a basic principle about monotonicity of credence[12], we get CERTAINTY.

5 Roadmap

The rest of the paper expands the discussion in three directions. In §6 we show how the argument can be generalized in various ways, to sidestep different kinds of resistance maneuvers. In §7, we discuss in detail the possibility of blocking the argument by rejecting closure, in the light of a result developed by *[name omitted for blind review]*. In §8, we explore the analogy with triviality arguments for modals and conditionals.

6 Generalizations

This section aims at generalizing the reach of the argument. Some theorists might be tempted to block the argument by giving up on CERTAINTY, appealing to one of a number of motivations. In this section, we show that these attempts still lead to results that are unacceptable to non-rejectionists.

6.1 Hedging Certainty

Here is one reason why one might think that CERTAINTY, while on the right lines, is too strong. Suppose you are walking through a rose garden, looking down a line of roses that incrementally change from clear red to clear orange. You might think that you can remain certain that a rose is red in cases where we have at least some doubt whether the rose is determinately red.[13] That is: while confidence in a rose being determinately red cannot be dramatically lower than your confidence in it being red, one might think that it is rationally permissible for there to be a slight drop between the latter and the former.

[10] For example, we interpret in this way the notion of determinate truth in McGee and McLaughlin 1995.

[11] At least, in an idealized sense of assertability, that abstracts from Gricean considerations of conversational relevance, informativity, etc.

[12] The principle, which follows from ENTAILMENT and BOUND, is that, if $\mathsf{A} \vDash \mathsf{B}$, then $Cr(\mathsf{A}) \leq Cr(\mathsf{B})$.

[13] Thanks to *[name omitted for blind review]* for this kind of case.

If we take this suggestion on board, we will replace CERTAINTY with the following modified principle:

$$Cr(\mathsf{A}) = 1 \Rightarrow Cr(\textsc{det}\ \mathsf{A}) \approx 1 \qquad \text{(HEDGED CERTAINTY)}$$

Here $x \approx y$ presupposes a new relation among degrees of belief: that of being near one another. If degrees of belief are modelled by real numbers in $[0,1]$, $x \approx y$ may be read as $|x - y| < \varepsilon$ for some small ε.

The following variant of BOUND is just as plausible as the original:[14]

$$Cr(\mathsf{B} \mid \mathsf{A}) \approx 1 \Rightarrow Cr(\mathsf{B}) \gtrsim Cr(\mathsf{A}) \qquad \text{(HEDGED BOUND)}$$

But now, a result follows that is similar to our original. Putting HEDGED CERTAINTY and HEDGED BOUND together with IDENTITY and CLOSURE, the argument proceeds as before, with the conclusion:

$$Cr(\textsc{det}\ \mathsf{A}) \gtrsim Cr(\mathsf{A}) \qquad \text{(HEDGED EQUIV1)}$$

And this establishes a hedged version of rejectionism:

$$Cr(\neg\textsc{det}\ \mathsf{A} \wedge \neg\textsc{det}\neg\mathsf{A}) = 1 \Rightarrow Cr(\mathsf{A}) \approx 0 \qquad \text{(HEDGED REJECTIONISM)}$$

Dialectically, this is just as bad for anti-rejectionists as the original result. Their thesis was that the appropriate response to indeterminacy was some state of uncertainty—middling credence, agnosticism, or whatever—that is incompatible with hedged rejection.

6.2 Higher Order Indeterminacy

Our interlocutor may at this point withdraw even from HEDGED CERTAINTY, and reframe her worry. Perhaps the real worry with our almost-borderline red rose was higher order vagueness. If there are higher order borderline cases—borderline cases between it being indeterminate whether a rose is red and it being determinate whether a rose is red—then, she reasons, it must be possible to have a determinately red rose that is not determinately determinately red. Certainty that such a rose is red may be appropriate (since the rose *is* determinately red). But *ex hypothesi*, the rose is not determinately determinately red. Hence the appropriate attitude towards the proposition *the rose is determinately red* could be the very kind of uncertainty appropriate to propositions that are indeterminate. (Note too that, suspiciously, we idealized away from higher order indeterminacy in motivating CERTAINTY).

We might question our interlocuteur's case, since it is not clear that we can (with rational certainty) identify a case as one of determinate but not

[14] Here we're understanding $\gtrsim$ as follows: $x \gtrsim y \iff x \geq y \vee x \approx y$.

determinate-determinate redness, as see supposes. We think she should not have given up on HEDGED CERTAINTY so quickly. But rather than push this point, we develop another route to something tantamount to our original conclusion. One of the reasons for interest in the variant presented below is that—like the original, but unlike the hedged version just given—it is very neutral on the quantity and structure of degrees of belief, not requiring notions like "near certainty" in its formulation.

This variant of our argument uses a weakened determinacy operator, $\textsc{det}_w$. We drop CERTAINTY and instead start from:

$$Cr(\mathsf{A}) = 1 \Rightarrow Cr(\textsc{det}_w\mathsf{A}) = 1 \qquad \text{(WEAK CERTAINTY)}$$

What is weak determinacy? If determinacy means: has degree of truth 1, then weak determinacy may be: having degree of truth at least 0.75. If determinacy requires that a proposition be true on every sharpening, then weak determinacy may be: being true on three quarters of the sharpenings.

We run the argument exactly as before, substituting $\textsc{det}_w$ for DET throughout and using only the original side premises IDENTITY, BOUND, and CLOSURE. We obtain:

$$Cr(\textsc{det}_w\mathsf{A}) \geq Cr(\mathsf{A}) \qquad \text{(WEAK EQUIV1)}$$

And this establishes another variant of rejectionism:

$$Cr(\neg\textsc{det}_w\mathsf{A} \wedge \neg\textsc{det}_w\neg\mathsf{A}) = 1 \Rightarrow Cr(\mathsf{A}) = 0 \qquad \text{(WEAK REJECTIONISM)}$$

$\textsc{det}_w$ is entailed by DET, but does not entail it, so it is easier to be confident that $\textsc{det}_w$ applies to a proposition than that DET applies to it. WEAKENED CERTAINTY is indeed a weaker claim that CERTAINTY. Dually, the notion of indeterminacy that is defined out of weak determinacy is stronger than straight indeterminacy: only when you are certain that A is a 'central case' of indeterminacy should you be certain that neither it nor its negation is even weakly determinate. But of course, rivals to rejectionism who think that uncertainty is called for when you are certain that something is indeterminate will a fortiori think that uncertainty (rather than rejection) is called for in these central cases of indeterminacy. So WEAK REJECTIONISM is not something they can live with.[15]

[15] Higher order weak determinacy is little explored, but of obvious relevance here. For example, on Williamson's fixed-width margin of error models for higher order vagueness (1992; 1994) nothing is higher-order weakly determinate at all orders. This could form the basis for an independent objection to WEAK CERTAINTY. However, our initial investigations show that there are natural variants of these models that avoid this feature. An objection from this quarter would have to dig into the plausibility of the various detailed modelling assumptions in play.

6.3 Regularity

We consider one final attempt at sidestepping the argument. Some Bayesians say that it is irrational to ever become certain of any proposition that is not a logical truth: rationality requires that we always remain open to the possibility of error. This is a controversial general thesis about rationality, known as 'regularity'.[16] In the context of our argument, regularity is relevant because it clashes badly with CLOSURE.[17] CLOSURE states that, if a credence function Cr counts as rational, the credence function $Cr(\bullet \mid \mathsf{A})$ that we obtain by conditonalizing on A also counts as rational. Regularity states that conditionalizing on a proposition A is rational only if A is a logical truth.

To address these concerns, we make use of notions of approximate certainty again. Assume that we cannot rationally learn contingent propositions with certainty. Plausibly, though, we are able to become *nearly* certain of them: we write $Cr(\bullet \uparrow \mathsf{C})$ for the result of updating on C in the sense of becoming almost certain of it. In the typical Bayesian framework where degrees of belief are modelled by the unit interval $[0,1]$, $Cr(\bullet \uparrow \mathsf{C})$ can be characterized as the result of Jeffrey-conditionalizing on a partition that includes C, and where C's coefficient is $1-\varepsilon$, where ε is the very constant used to characterize $\approx$ earlier.

We can now run a variant of our argument with the following premises, built around a notion of APPROXIMATE CLOSURE specifically designed to appeal to fans of the regularity constraint. For this, we need not just notions of approximate equality $\approx$, but approximate approximate equality $\approx\approx$, approximate approximate approximate equality $\approx\approx\approx$ etc.[18] The argument runs:

$$Cr(\mathsf{A}) \approx 1 \Rightarrow Cr(\textsc{det}\ \mathsf{A}) \approx 1 \qquad \text{(APPROXIMATE CERTAINTY)}$$

$$Cr(\mathsf{A} \uparrow \mathsf{A}) \approx 1 \qquad \text{(APPROXIMATE IDENTITY)}$$

$$Cr(\mathsf{B} \uparrow \mathsf{A}) \approx 1 \Rightarrow Cr(\mathsf{B}) \gtrapprox Cr(\mathsf{A}) \qquad \text{(APPROXIMATE BOUND)}$$

$$\forall \mathsf{C} : Cr(\bullet) \in \mathcal{C} \wedge Cr(\mathsf{C}) \neq 0 \Rightarrow Cr(\bullet \uparrow \mathsf{C}) \in \mathcal{C} \qquad \text{(APPROXIMATE CLOSURE)}$$

The argument then proceeds exactly as before, with the conclusion:

$$Cr(\textsc{det}\ \mathsf{A}) \gtrapprox Cr(\mathsf{A}) \qquad \text{(APPROX EQUIV1)}$$

And this establishes an approximate version of rejectionism:

$$Cr(\neg\textsc{det}\ \mathsf{A} \wedge \neg\textsc{det}\neg\mathsf{A}) \approx 1 \Longrightarrow Cr(\mathsf{A}) \approx\approx\approx 0 \qquad \text{(APPROX REJECTIONISM)}$$

This is no better for rivals to rejectionism than was the original conclusion. If you think that agnosticism or middling credence is the right response to

[16] For discussion and references, see **?**.

[17] This response was first put to us by *[name omitted for blond review]* Compare Lewis 1986.

[18] If credences are real numbers, we have $x \approx y$ is true iff $|x-y| \leq \varepsilon$, and analogously, $x \approx\approx y$ iff $|x-y| \leq 2\varepsilon$, $x \approx\approx\approx y$ $|x-y| \leq 3\varepsilon$, etc. $x \gtrapprox y := x \geq y \vee x \approx\approx y$.

indeterminacy, then you shouldn't think that if we're nearly certain that something is indeterminate, we're forced to be within a small distance of 1 (approximately approximately approximately equal to 1)—but that is what approximate rejectionism tells us.

Even for those who do not insist on regularity, the above form of our argument holds interest. One reaction to the argument that we discuss in §7 below holds that some propositions could be rationally learned with certainty (*pace* regularity), but that they have to be perfectly determinate. But the discussion in this section shows that, to resist all versions of the argument, one must hold that possibly vague propositions are unlearnable in a much stronger sense: we cannot even learn them in the Jeffrey-conditionalization sense.

Before moving on, let us we point out that the resources that we have deployed throughout this section can be brought together. Approximate and hedged versions of our argument can be combined; we discuss the resulting principles in a footnote.[19]

7 Denying closure or ratio: restricting principles to perfectly determinate propositions

In §4, we saw that two of the most promising strategies for resisting the argument were linked to changing our understanding of conditional probability and update. In this section, we investigate these routes in further detail.

Let us first consider denying CLOSURE (repeated below).[20]

$$\forall \mathsf{C} : Cr(\bullet) \in \mathcal{C} \wedge Cr(\mathsf{C}) \neq 0 \Rightarrow Cr(\bullet \mid \mathsf{C}) \in \mathcal{C} \qquad \text{(CLOSURE)}$$

Each instance of CLOSURE follows from two claims. First: a rational agent with prior belief state Cr, who learns C as total information with certainty, has posterior (categorical) beliefs given by $Cr(\bullet \mid \mathsf{C})$. Second: the particular C involved

[19]Someone might have the concern that a belief in A being within ϵ of 1 doesn't guarantee that our belief in DET A is within ϵ of 1. But this interlocutor may endorse a suitably hedged variant of the principle: that the consequent follows if A meets some tighter bound–within some δ of 1, where $\delta < \varepsilon$. Writing $\simeq$ for this tighter approximation, we can combine approximate and hedged versions of our argument via the following premises:

$$Cr(\mathsf{A}) \simeq 1 \Rightarrow Cr(\text{DET}_w \mathsf{A}) \approx 1 \qquad \text{(HEDGED WEAK APPROX CERTAINTY)}$$

$$Cr(\mathsf{A} \uparrow \mathsf{A}) \simeq 1 \qquad \text{(HEDGED APPROX IDENTITY)}$$

$$Cr(\mathsf{B} \uparrow \mathsf{A}) \approx 1 \Rightarrow Cr(\mathsf{B}) \gtrapprox Cr(\mathsf{A}) \qquad \text{(APPROXIMATE BOUND)}$$

$$\forall \mathsf{C} : Cr(\bullet) \in \mathcal{C} \wedge Cr(\mathsf{C}) \neq 0 \Rightarrow Cr(\bullet \uparrow \mathsf{C}) \in \mathcal{C} \qquad \text{(APPROXIMATE CLOSURE)}$$

There are contexts where this variant of our argument—strengthened in several dimensions—is required.

[20]For discussion of this material, we are indebted to *names omitted for blind review* .

in the instance of closure is learnable: it is possible to learn it, with certainty, as total information.

Resistance on the first point is ruled out, given the way we are understanding conditional probability in the present context. In §2, we have simply stipulated that $Cr(\mathsf{B} \mid \mathsf{A})$ denotes the posterior degree of belief in B had by a rational agent with prior credence function Cr, upon learning A with certainty as total information. So the only route to deny CLOSURE is to target the second condition: we might deny that some propositions can be learned as one's total information.

Even though CLOSURE involves universal quantification over propositions, all we need to run an instance of our argument is a particular instance of CLOSURE. So, if we want to pursue an across-the-board solution to the problem via this route, we need to deny all instances of CLOSURE that involve propositions that are possibly indeterminate. That is, we should maintain that all propositions that can rationally be learned with certainty are perfectly determinate: they are of the form $\ulcorner \textsc{det}^* \ldots \mathsf{A} \urcorner$ (where '$\textsc{det}^*$' stands for an infinite stack of determinacy operators). In fact, as [*name omitted for blind review*]has shown in correspondence, *modulo* standard classical Bayesian assumptions CERTAINTY is simply equivalent to the claim that all learnable propositions are perfectly determinate.[21]

On this option, then, CLOSURE fails because $Cr(\bullet \mid \mathsf{C})$ only picks out a rational credence function when C is perfectly determinate. (Strictly speaking, $Cr(\bullet \mid \mathsf{C})$ is simply undefined, since $Cr(\bullet \mid \mathsf{C})$ represents the result of *rationally* updating Cr on C.)

This is not the place to adjudicate the suggestion that every learnable proposition is perfectly determinate. Let us just notice that this claim is highly controversial, and that it has been forcefully denied recently. For example, Andrew Bacon (2018) argues that the totality of what we learn through perception, reflection and testimony is inexact and potentially vague information.

Now, let us turn to the other option: denying RATIO (repeated below), with the goal of invalidating BOUND.

$$Cr(\mathsf{A} \wedge \mathsf{B}) = Cr(\mathsf{B} \mid \mathsf{A}) \times Cr(\mathsf{A}) \qquad (\textsc{ratio})$$

Building on our discussion of the failure of CLOSURE, there is a natural way to motivate the failure of RATIO. This time we grant that subjects may rationally update on propositions that are not perfectly determinate, and hence that $Cr(\bullet \mid \mathsf{C})$ is well-defined for all C with positive credence. But we claim that RATIO holds if and only if the proposition that is updated on is perfectly

[21] More precisely: *[name]* introduces the notion of a *determinacy fixed-point*, defined as follows:

A is a determinacy fixed-point just in case $\textsc{det}\ \mathsf{A} = \mathsf{A}$.

[name] shows that CERTAINTY is equivalent to the claim that evidential propositions are determinacy fixed-points.

determinate. This gets the result that $Cr(\mathsf{A}) = Cr(\textsc{det}\ \mathsf{A})$ if A is a perfectly determinate proposition, but not otherwise.

Choosing this route might be a plausible option for those who want to explore a solution similar to the denial of closure, but want to allow that we may learn not perfectly determinate propositions. So far as we can see, the main hurdle for this route is to develop a plausible philosophical justification for the restriction of RATIO. We leave this task to future work.

8 Analogies with modal triviality

Our proof has close relatives in the literature on conditionals and modality. A number of theorists (Stalnaker 1970, Adams 1975, Edgington 1995) have pointed out an intuitive constraint on credences in conditionals: a subject's credences in a conditional should line up with their conditional credences in the consequent, given the antecedent.

> **Stalnaker's Thesis.** For all A, B, and for all $Cr \in \mathcal{C}$: $Cr(\mathsf{A} > \mathsf{B}) = Cr(\mathsf{B} \mid \mathsf{A})$

The unrestricted endorsement of Stalnaker's Thesis is notoriously problematic. Appealing to Stalnaker's Thesis and to standard Bayesian principles, Lewis (1976) shows that we can prove that the probability of a conditional $\mathsf{A} > \mathsf{B}$ has to be identical to the probability of its consequent—an unacceptable result.

Recent literature on triviality has pointed out that similarly unacceptable consequences can be reached via assumptions that are strictly weaker and no less intuitive. Also, it has been pointed out that triviality is not confined to conditionals, but rather generalizes to modalized statements of various sort.[22] *[reference omitted for blind review]* lays out a template for generating triviality results of this kind. This template starts from a constraint of the following form, for specific A and B:

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\mathsf{B}) = 1 \qquad \text{(TRIVIALITY SCHEMA)}$$

From TRIVIALITY SCHEMA, using standard Bayesian principles, we can prove that $Pr(\mathsf{A}) \leq Pr(\mathsf{B})$. Our CERTAINTY is, of course, a particular instance of TRIVIALITY SCHEMA, and EQUIV1 is the local instance of the schematic consequence mentioned. The proof we gave in section 2 can be run schematically, and refines the premises needed for this schematic connection.

For concreteness, let us consider the following way of instantiating TRIVIALITY SCHEMA: we replace MUST A (with MUST understood epistemically) for B.

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\textsc{must}\ \mathsf{A}) = 1 \qquad \text{(MUST CONSTRAINT)}$$

[22] The first point is due to Richard Bradley (see e.g. 2000; 2007); for examples of triviality arguments applied to epistemic modals, see e.g. Russell and Hawthorne 2016, Goldstein forthcoming.

The consequence of this (in a Bayesian setting, or at least assuming the weaker side-premises we have employed in this paper) is that credence in A is a lower bound on the credence of MUST A. But of course, whenever you're uncertain whether A is true or not, your credence in A should be higher than your credence in MUST A, since the latter should be zero or near-zero.

Once we see the analogy between the indeterminacy and the modal cases, it is tempting to seek a unified solution to the two puzzles. Different theorists will have different inclinations on this issue.

On the one hand, a uniform solution seems *prima facie* desirable. Once we see TRIVIALITY SCHEMA, the puzzle appears to be generated by some abstract, shared features of the logic of determinacy and epistemic modality. On the other, it might be that the explanatory resources we need to appeal to are going to be different from case to case. For example, it seems plausible to us that for the case of modal and conditionals the solution will involve denying CLOSURE or RATIO. In fact, this response follows from a natural idea: rational learning about the world is invariably accompanied by learning about our own epistemic response: hence e.g. rationally learning A is invariably accompanied by learning MUST A. But, as we saw in the previous section, the corresponding claim for the case of determinacy is at least very controversial. So it is unclear that a uniform response is desirable.

9 Conclusion

We have given an argument that starts from a plausible principle about determinacy and credence, i.e. CERTAINTY, and, via three plausible side-premises, leads to a controversial claim about cognitive role, i.e. REJECTIONISM.

Seeing this outcome, one might start questioning CERTAINTY. But, as we have argued, CERTAINTY is plausible, and moreover reasonable variants of it lead to equally damaging results. Alternatively, one might want to question one of the three side-premises. IDENTITY seems unassailable. We have seen that there are routes to denying BOUND (via denying RATIO) and CLOSURE, but that this strategy leads into controversial territory. Other solutions, like switching to weaker variants of CERTAINTY or endorsing regularity, also won't defeat all versions of the argument.

We conclude that our puzzle raises a substantial challenge, which is not easily addressed by any extant account of belief and indeterminacy. We hope that thinking about it will prove enlightening for theories of cognitive role, as well as for theorists interested in triviality results of various sorts.

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