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n-Cylindrical Fuzzy Neutrosophic Topological Spaces

Sarannya Kumari R[1,*], Sunny Joseph Kalayathankal[2], Mathews George[3], Florentin Smarandache[4]

[1] Research Scholar, Catholicate College, Pattanamthitta, Kerala, India; saranya6685@gmail.com.

[2] Principal (Professor and Dean of Research), Jyothi College of Engineering, Thrissur, Kerala, India; sunnyjoseph2014@yahoo.com.

[3] Department of Mathematics, Providence College of Engineering, Alappuzha, Kerala, India; mathews.g@providence.edu.in.

[4] Department of Mathematics, University of New Mexico 705 Gurley Ave. Gallup, NM 87301, USA; smarand@unm.edu.

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Abstract

The objective of this study is to incorporate topological space into the realm of n-Cylindrical Fuzzy Neutrosophic Sets (n-CyFNS), which are the most novel type of fuzzy neutrosophic sets. In this paper, we introduce n-Cylindrical Fuzzy Neutrosophic Topological Spaces (n-CyFNTS), n-Cylindrical Fuzzy Neutrosophic (n-CyFN) open sets, and n-CyFN closed sets. We also defined the n-CyFN base, n-CyFN subbase, and some related theorems here.

Keywords: n-Cylindrical fuzzy neutrosophic sets, n-Cylindrical fuzzy neutrosophic open sets, n-Cylindrical fuzzy neutrosophic closed sets, n-Cylindrical fuzzy neutrosophic base.

1 | Introduction

Zadeh [1] laid the stepping stone to the field of uncertainties called fuzzy sets. The prime field of mathematics where the concepts and ideas of fuzzy sets drew a parallel was topology. Chang [2] enlivened the concept of fuzzy topological spaces using Zadeh's definition. Since then the various notions in classical topology have been extended to fuzzy topological spaces. Subsequently in the second half of 1970 and the beginning of 1980, many authors contributed a lot to this new field. Later Atanassov [3], [4] introduced a new set called Intuitionistic Fuzzy Set (IFS) in which the sum of both acceptance degree and rejection degree grades does not exceed 1. Later, intuitionistic fuzzy topological spaces via IFSs were obtained by Coker [5] in intuitionistic fuzzy topological spaces, Lee and Lee [6] discovered the properties of continuous, open, and closed maps. Yager [7] proposed the Pythagorean Fuzzy Set (PyFS) as a generalisation of IFS in 2013, which ensures that the value of the square sum of its membership degrees is less than or equal to 1. The concept of pythagorean fuzzy topological space was introduced by Olgun et al. [8]. Cuong [9] initiated the idea of the Picture Fuzzy Set (PFS). He utilized three indices (membership degree P (x), neutral-membership degree I (x), and non-membership degree N (x) in PFS with the condition that is $0 \leq P(x) + I(x) + N(x) \leq 1$. Obviously PFSs is more suitable than IFS and PyFS to deal with fuzziness and vagueness. The idea of picture fuzzy topological spaces was first initiated by Razaq et.al [10]. Later Spherical Fuzzy Sets (SFS) have been

Corresponding Author: saranya6685@gmail.com

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proposed by Kahraman and Gündogdu [11]. SFS should satisfy the condition that the squared sum of membership degree and non-membership degree and hesitancy degree should be equal to or less than one. Princy and Mohana [12] introduced spherical fuzzy topological spaces.

The neutrosophic set was introduced by Smarandache [13] and neutrosophic set is a generalization of IFS. Salama and Alblowi [14] introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. Smarandache [15] introduced the dependence degree of (also, the independence degree of) the fuzzy components, as well as the neutrosophic components, for the first time in 2006. Arockiarani and Jency [16] initiated the notion of fuzzy neutrosophic set as the sum of all the three membership functions does not exceed 3. Fuzzy neutrosophic topological space and basic operations on it was proposed by Veereswari [17]. Sarannya Kumari et al. [18] recently introduced n-Cylindrical Fuzzy Neutrosophic Sets (n-CyFNS), which have T and F as dependent components and I as independent components. Except for fuzzy neutrosophic sets, the n-CyFNS is the largest extension of fuzzy sets. In this case, the degree to which positive, neutral, and negative membership functions satisfy the condition, $0 \leq \beta A(x) \leq 1$ and $0 \leq \alpha A\ n(x) + \gamma An(x) \leq 1$, $n > 1$, is an integer. They also defined the distance between two n-CyFNS, as well as their properties and basic operations.

In this paper, we introduce topological space in n-CyFNS environment. This is a new type of fuzzy neutrosophic sets in which T and F are dependent components and I independent components. Here we defined n-CyFN topological space, n-CyFN open sets. We also initiated n-CyFN base, n-CyFN subbase and some related results.

2 | Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions. Throughout this paper, $\cup$ denotes the universe of discourse.

Definition 1 ([1]). A fuzzy set A in $\cup$ is defined by membership function μ_A: A→ [0, 1] whose membership value $\mu_A(x)$ shows the degree to which $x \in \cup$ includes in the fuzzy set A for all $x \in \cup$.

Definition 2 ([2]). A fuzzy topological space is a pair (X, T), where X is any set and T is a family of fuzzy sets in X satisfying following axioms:

I. $\Phi, X \in T$.
II. If $A, B \in T$, then $A \cap B \in T$.
III. If $A_i \in T$ for each $i \in I$, then $\cup_i A_i \in T$.

Definition 3 ([3]). An IFS A on $\cup$ is an object of the form $A = \{(x, \alpha_A(x), \gamma_A(x) \mid x \in \cup)\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of membership of x in A, $\gamma_A(x) \in [0, 1]$ is called the degree of non-membership of x in A, and where α_A and γ_A satisfy (for all $x \in \cup$) $(\alpha_A(x) + \gamma_A(x) \leq 1)$ IFS ($\cup$) denote the set of all the IFSs on a universe $\cup$.

Definition 4 ([13]). A neutrosophic set A on $\cup$ is $A = < x, T_A(x), I_A(x), F_A(x) >$; $x \in \cup$, where T_A, I_A, F_A: A→ $]^-0,1^+[$ and $^-0 < T_A(x) + I_A(x) + F_A(x) < n\,3^+$.

Definition 5 ([16]). A fuzzy neutrosophic set A on $\cup$ is $A = < x, T_A(x), I_A(x), F_A(x) >$; $x \in \cup$, where T_A, I_A, F_A: A→ [0, 1] and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 6 ([13]). A neutrosophic set A on $\cup$ is an object of the form $A = \{(x, u_A(x), \zeta_A(x), v_A(x)):$ $x \in \cup\}$, where $u_A(x), \zeta_A(x), v_A(x) \in [0,1]$, $0 \leq u_A(x) + \zeta_A(x) + v_A(x) \leq 3$ for all $x \in \cup$. $u_A(x)$ is the degree

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of truth membership, $\zeta_A(x)$ is the degree of indeterminacy and $\nu_A(x)$ is the degree of non-membership. Here $u_A(x)$ and $\nu_A(x)$ are dependent components and $\zeta_A(x)$ is an independent component.

Definition 7 ([14]). A Neutrosophic Topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

I. (NT1) $0_N, 1_N \in \tau$.
II. (NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
III. (NT3) $\cup G_i \in \tau$ for all $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a Neutrosophic Topological Space (NTS) and any neutrosophic set in τ is known as Neutrosophic Open Set (NOS) in X. The elements of τ are called open neutrosophic sets. A neutrosophic set F is closed if and only if it C (F) is neutrosophic open.

Definition 8 ([17]). A Fuzzy Neutrosophic Topology (FNT) a non-empty set X is a family τ of fuzzy neutrosophic subsets in X satisfying the following axioms:

I. (FNT1) 0_N, $1_N \in \tau$.
II. (FNT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
III. (FNT3) $\cup G_i \in \tau$ for all $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a Fuzzy Neutrosophic Topological Space (FNTS) and any fuzzy neutrosophic set in τ is known as Fuzzy Neutrosophic Open Set (FNOS) in X. The elements of τ are called open fuzzy neutrosophic sets.

Definition 9 ([18]). An n-CyFNS A on U is an object of the form A= $\{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle \mid x \in U\}$ where $\alpha_A(x) \in [0, 1]$, called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$, called the degree of neutral membership of x in A and $\gamma_A(x) \in [0, 1]$, called the degree of negative membership of x in A, which satisfies the condition, (for all x$\in$ U) $(0 \leq \beta_A(x) \leq 1$ and $0 \leq \alpha_A{}^n(x) + \gamma_A{}^n(x) \leq 1$, n>1, is an integer. Here T and F are dependent neutrosophic components and I is 100% independent.

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-Cylindrical Fuzzy Neutrosophic Number (n-CyFNN) and is denoted as A= $\langle \alpha_A, \beta_A, \gamma_A\rangle$.

Definition 10 ([18]). (The Basic Connectives). Let $\mathcal{C}_N$ (U) denote the family of all n-CyFNS on U.

Definition 11. Inclusion: For every two A, B $\in \mathcal{C}_N$ (U), A $\subseteq$ B iff (for all x $\in$ U, $\alpha_A(x) \leq \alpha_B(x)$ and $\beta_A(x) \leq \beta_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$) and A = B iff (A $\subseteq$ B and B $\subseteq$ A).

Definition 12. Union: For every two A, B $\in \mathcal{C}_N$ (U), the union of two n-CyFNSs A and B is A$\cup$B(x) = $\{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle \mid x \in U\}$.

Definition 13. Intersection: For every two A, B $\in \mathcal{C}_N$ (U), the intersection of two n-CyFNSs A and B is A$\cap$B(x) = $\{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle \mid x \in U\}$.

Definition 14. Complementation: For every A $\in \mathcal{C}_N$ (U), the complement of an n-CyFNS A is $A^c = \{\langle x, \gamma_A(x), \beta_A(x), \alpha_A(x)\rangle \mid x \in U\}$.

Definition 15. Sum: For every two A, B $\in \mathcal{C}_N$ (U), the sum of two n-CyFNSs A and B is A$\oplus$B(x) = $\{\langle x, (\frac{\alpha_A(x).\alpha_B(x)}{\alpha_A\ x) + \alpha_B(x)}, \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma B(x))\rangle \mid x \in U\}$.

Definition 16. Difference: For every two A, B $\in \mathcal{C}_N$ (Ʊ), the difference of two n-CyFNSs A and B is A⊖B(x) = {⟨ x, max ($\alpha_A(x)$, $\alpha_B(x)$), min ($\beta_A(x)$, $\beta_B(x)$), $\frac{\gamma_A(x).\gamma_B(x)}{\gamma_A(x)+\gamma_B(x)}$ ⟩ | x ∈ Ʊ}.

Definition 17. Product: For every two A, B $\in \mathcal{C}_N$ (Ʊ), the product of two n-CyFNSs A and B is A⊗B (x) = {⟨ x, ($\alpha_A(x).\alpha_B(x)$, $\beta_A(x).\beta_B(x)$, $\gamma_A(x).\gamma_B(x)$) ⟩ | x ∈ Ʊ}.

Definition 18. Division: For every two A, B $\in \mathcal{C}_N$ (Ʊ), A⊘B is A⊘ B(x) = {⟨ x, min ($\alpha_A(x)$, $\alpha_B(x)$), $\beta_A(x)$. $\beta_B(x)$, max ($\gamma_A(x)$. $\gamma_B(x)$) ⟩ | x ∈ Ʊ}.

Results ([18]):

I. If A ⊆ B and B ⊆ C then A ⊆ C.
II. A∪B = B∪A & A∩B = B∩A.
III. (A∪B) ∪C = A∪ (B∪C) & (A∩B) ∩C = A∩ (B∩C).
IV. (A∪B) ∩C = (A∩C) ∪ (B∩C) & (A∩B) ∪C = (A∪C) ∩ (B∪C).
V. A∩A = A & A ∪ A = A.
VI. De Morgan's Law for A & B ie, $(A\cup B)^c = A^c \cap B^c$ & $(A\cap B)^c = A^c \cup B^c$.
VII. (A⊕B) = (B⊕A).
VIII. (A⊗ B) = (B ⊗A).

3 | n-Cylindrical Fuzzy Neutrosophic Topological Spaces

Definition 18. Let $\{A_i: i \in I\}$ be an arbitrary family of n-CyFNS in Ʊ.

Then $\cap A_i$ = {⟨ x, inf ($\alpha_{Ai}(x)$), inf ($\beta_{Ai}(x)$), sup (($\gamma_{Ai}(x)$) ⟩ | x ∈ Ʊ}.

$\cup A_i$ = {⟨ x, sup ($\alpha_{Ai}(x)$), sup ($\beta_{Ai}(x)$), inf (($\gamma_{Ai}(x)$) ⟩ | x ∈ Ʊ}.

Definition 19. 0_{CyN} = {⟨ x, 0, 0, 1 ⟩ | x ∈ Ʊ} and 1_{CyN} = {⟨ x, 1, 1, 0 ⟩ | x ∈ Ʊ}.

3.1 | n-Cylindrical Fuzzy Neutrosophic Topological Spaces

In this part, we give a definition of n-Cylindrical Fuzzy Neutrosophic Topology (n-CyFNT) and its related properties according to Chang's FTS.

Definition 20. An n-CyFNT on a non-empty set X is a family, τ_X, of n-CyFNS in X which satisfies the following conditions:

I. 0_{cyN}, $1_{cyN} \in \tau_X$.
II. $A_1 \cap A_2 \in \tau_X$.
III. $\cup A_i \in \tau_X$, for any arbitrary family $A_i \in \tau_X$, i∈ I.

The pair (X, τ_X) is called an n-cylindrical fuzzy neutrosophic topological space n-Cylindrical Fuzzy Neutrosophic Topological Spaces (n-CyFNTS) and any n-CyFNS belongs to τ_X is called an n-Cylindrical Fuzzy Neutrosophic Open Set (n-CyFNOS) and the complement of n-CyFNOS is called n-Cylindrical Fuzzy Neutrosophic Closed Set (n-CyFNCS) in X. Like classical topological spaces and fuzzy topological spaces, the family {0_{cyN}, 1_{cyN}} is called indiscrete n-CyFNTS and the topology containing all the n-CyFN subsets is called discrete n-CyFNTs.

Remark: Obviously any fuzzy topological space or intuitionistic fuzzy topological space or pythagorean fuzzy topological space is an n-CyFN topological space as any subsets of the fuzzy space, intuitionistic fuzzy space, and pythagorean fuzzy space can be viewed as n-CyFN subsets. But the converse of the above doesn't follow and it can be evident from the following example:

Example 1. Let X = { p, q} and τ_X = { 1_{cyN} , 0_{cyN}, A, B, C, D}, where,

A = {<p; 0.5, 0.5, 0.7>, <q; 0.2, 05, 0.4>}, B = {<p; 0.6, 0.5, 0.5>, <q; 0.3, 0.5, 0.9>}, C = {<p; 0.6, 0.5, 0.5>, <q; 0.3, 0.5, 0.4>}, D = {<p; 0.5, 0.5, 0.7>, <q; 0.2, 0.5, 0.9>}, is clearly an n-CyFNTS.

Definition 21. Let (X, τ_{X1}) and (X, τ_{X2}) be n-CyFNTSs.

I. τ_{X2} is finer than τ_{X1} if $\tau_{X2} \supseteq \tau_{X1}$.
II. τ_{X2} is strictly finer than τ_{X1} if $\tau_{X2} \supset \tau_{X1}$.
III. τ_{X2} *and* τ_{X1} are said to be comparable if it holds $\tau_{X2} \supseteq \tau_{X1}$ or $\tau_{X1} \supseteq \tau_{X2}$.

Example 2. Consider the *Example 1*.

X = {p, q}, τ_X = {1_{cyN}, 0_{cyN}, A, B, C, D} and τ_{X1} = {1_{cyN}, 0_{cyN}, A} are two n-CyFN topologies on X. Clearly we can see that $\tau_X \supset \tau_{X1}$.

Definition 22. Let (X, τ_X) be a CyFNTS on X.

$\mathscr{B} \subseteq \tau_X$, a sub family of τ_X is called an n-CyFN base for (X, τ_X), if each member of τ_X may be expressed as the union of members in $\mathscr{B}$.

$\mathscr{S} \subseteq \tau_X$, a sub family of τ_X is called a n-CyFN sub-base for (X, τ_X), if the family of all finite intersections of $\mathscr{S}$ forms a base for (X, τ_X). Here it can be said that $\mathscr{S}$ generates (X, τ_X).

Theorem 1. Let (X, τ_X) be an n-CyFNTS and $\mathscr{B} \subseteq \tau_X$, be a n-cylindrical fuzzy neutrosophic base for τ_X. Then τ_X is the collection of all union of members of $\mathscr{B}$.

Proof: The definition of the base of an n-CyFNTS clearly proves the theorem.

Theorem 2. Let (X, τ_X) be an n-CyFNTS and $\mathscr{B} \subseteq \tau_X$. Then $\mathscr{B}$ is an n-cylindrical fuzzy neutrosophic base for τ_X if and only if for any x∈X and any G∈ τ_X containing x, there exists B∈ $\mathscr{B}$ such that x∈B⊆G.

Proof: Suppose $\mathscr{B}$ is an n-cylindrical fuzzy neutrosophic base for τ_X.

Let G∈ τ_X and x∈G. Now G = $\bigcup_i B_i$, $B_i \in \mathscr{B}$, x∈G ⇒x∈ $\bigcup_i B_i$ ⇒x∈ B_i for some B_i and let $B_i = B$.

That is, x∈B = $B_i \subseteq \bigcup_i B_i \subseteq$ G, hence x ∈ ⊆ B G.

Conversely suppose the given condition holds, ie, Let G ∈ τ_X. For each x∈G, there exists $B_x \in \mathscr{B}$ such that x ∈ B_x ⊆ G.

B_x⊆ G for all x. Then,

$$\cup_{x\in G} Bx \subseteq G. \tag{1}$$

But from the assumption G∈ τ_X and, x ∈ B_x for all x∈G and B_x⊆ G Since G is n-CyFNOS in X, G can be expressed as:

$$G \subseteq \cup_{x\in G} Bx \quad \text{where } Bx \in \mathscr{B} \subseteq \tau_X. \tag{2}$$

From *Eqs. (1)* and *(2)*; G = $\cup_{x\in G}$ B_x ; $B_x \in \mathscr{B}$ thus $\mathscr{B}$ is an n-CyFN base for τ_X.

Definition 23. Let (X, τ_X) be an n-CyFNTS and Y⊆ X. Then the collection $\tau_Y = \{X_i \cap Y: X_i \in \tau_X, i \in I\}$ is called n-cylindrical fuzzy neutrosophic subspace topology on Y. Hence (Y, τ_Y) is called n-cylindrical fuzzy neutrosophic topological subspace of (X, τ_X).

Theorem 3. Let (X, τ_X) be an n-CyFNTS and Y⊆ X, then τ_Y, an n-CyFN subspace topology on Y is an n-CyFNTS.

Proof: Certainly 0_{cy}, $1_{cy} \in \tau_Y$ since $0_{cx} \cap Y = 0_{cy}$ and $1_{cx} \cap Y = 1_{cy}$.

Also $\tau_X = \{X_i \subseteq X, \ i \in I\}$.

Hence it is closed under arbitrary n-cylindrical fuzzy neutrosophic union.

$$\bigcup_{i}(X_i \cup Y) = (\bigcup_{i} X_i) \cup Y. \tag{3}$$

Also it is closed under finite n-cylindrical fuzzy neutrosophic intersection.

Hence the theorem follows.

$$\bigcap_{i=1}^{n}(X_i \cap Y) = (\bigcap_{i=1}^{n} X_i) \cap Y. \tag{4}$$

Example 3. Let X be the set of all integers. Consider f ∈ n-CyFNS such that $f(x) = <1, \frac{1}{x}, 0>$; x≥ 1 and $x \in X = <0, -\frac{1}{x}, 1>$; x≤ −1 = < 1, 1, 0 > ; x = 0, then (X, τ_X) is an n-CyFNTS with $\tau_X = \{1_{cyN}, 0_{cyN}, f\}$.

Let Y denote set of all even integers ie, $y = 2x \in Y g(y) = <1, \frac{1}{y}, 0>$; y≥ 1 = $<0, -\frac{1}{y}, 1>$; y≤ −1 = < 1, 1, 0 >; y = 0. Clearly (Y, τ_Y) is a sub space topology, $\tau_Y = \{1_{cyN}, 0_{cyN}, g\}$.

Theorem 4. If $\mathscr{B}$ is an n-CyFN base for (X, τ_X) and Y⊆ X, then $\mathscr{B}_Y = \{B \cap Y \mid B \in \mathscr{B}\}$ is an n-CyFN base for (Y, τ_Y).

Proof: Let G is n-CyFN open in X and y∈G∩Y. Now choose $B \in \mathscr{B}$ such that y∈B⊆G.

Thus y∈B∩ Y ⊆G∩Y. Hence $\mathscr{B}_Y$ an n-CyFN base for (Y, τ_Y) by *Theorem 2*.

Theorem 5. Let (X, τ_X) be a CyFNTS and (Y, τ_Y) be an n-cylindrical fuzzy neutrosophic topological subspace. If $\subseteq Y$ Zis n-cylindrical fuzzy neutrosophic open in Y then Z is n-cylindrical fuzzy neutrosophic open in X.

Proof: It is evident from the definition of n-cylindrical fuzzy topological subspace.

4 | Conclusion

Our goal with this paper is to broaden the scope of n-CyFNS to topological spaces. Here we introduce the fundamental definitions of n-CyFNTS, n-CyFN open sets, and n-CyFN closed sets, as well as examples. The terms n-CyFN base, n-CyFN sub base, and related theorems were also defined. This paper is the first to investigate n-CyFNTS. This research will undoubtedly be the basis for the further development of n-CyFNTS and their applications in various fields. Evidently, these ideas have the potential to inspire additional research in the future.

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Conflicts of Interest

The authors declare no conflict of interest.

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