Meeting of the Aristotelian Society held online on 1 February 2021 at 5:30 p.m

VI—Paradoxes as Philosophical Method and Their Zenonian Origins

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In this paper I show that one of the most fruitful ways of employing paradoxes has been as a philosophical method that forces us to reconsider basic assumptions. After a brief discussion of recent understandings of the notion of paradoxes, I show that Zeno of Elea was the inventor of paradoxes in this sense, against the background of Heraclitus' and Parmenides' way of argumentation: in contrast to Heraclitus, Zeno's paradoxes do not ask us to embrace a paradoxical reality; and in contrast to Parmenides, Zeno shows common assumptions to be internally problematic, not just in light of Eleatic positions.

I

Introduction. Paradoxes have been employed in philosophy throughout its history in a number of ways—to silence an opponent, to call attention to an alleged inconsistency in reality, as games or jokes, and much else. Philosophically, we may distinguish between paradoxes intended to show that the objects of our inquiry are inconsistent (inconsistencies in reality or the world), paradoxes that are intended to show our conceptual tools to be problematic, and paradoxes intended to show a tendency in our language to become entangled in difficulties.[1] And paradoxes—of any of these three domains—can also be used as part of a *method* of investigation. By this I do not simply mean that they are systematically employed, but rather, as we will see below, that they can be used as a tool to test whether we may have chosen a problematic path in our investigation.

In this paper I want to show that one of the most fruitful ways of employing paradoxes has been as a kind of philosophical method, and that Zeno of Elea was the inventor of using paradoxes in this

[1] Kant, for example, thought that our reason is such that it entangles itself in paradoxes, the antinomies; vagueness paradoxes have varyingly been interpreted as displaying inconsistencies in our language, our concepts, or in reality.

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
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very sense.[2] We will first have a brief glance at different accounts of what is understood by a paradox, before we look at the structure of the paradoxes of Zeno that have been handed down to us. In a brief comparison between Zeno and thinkers before him, I will then show to what extent Zeno introduced paradoxes as a productive method that we are still employing today.

Understanding Zeno as introducing a new method into philosophy also fits with ancient testimonies that considered Zeno as a thinker who is relevant for questions of method: according to Diogenes Laertius, Aristotle called Zeno the inventor of dialectic, and Plato in his *Phaedrus* places Zeno in the context of practising sophistry and what he calls *antilogikê technê* (Diogenes Laertius VIII, 57; Plato, *Phaedrus* 261a–e.). The fact that the method Plato and Aristotle connect with Zeno is not the introduction of paradoxes is at least in part due to the fact that the Greek word *paradoxon* only starts being used as a noun, and as one relevant for philosophy, with Aristotle's *Organon*—and, as we will see, in Aristotle this term has a slightly different meaning from that which it has today.

II

What to Understand by a Paradox. We often distinguish between different kinds of paradox, such as epistemic, logical, ontological, or truth paradoxes;[3] but as paradoxes they all seem to share some basic structure. As a first pass we may follow Sainsbury's understanding of paradoxes as 'an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises' (Sainsbury 2009, p.1). With real paradoxes, it seems we have done everything right in choosing our premisses and correctly reasoned from them, but yet we end up with an unacceptable conclusion.

Rescher has argued against talking about 'acceptable' here, since being 'acceptable' is a matter of yes or no, while what we really face is different degrees of plausibility. For him 'a paradox arises when a

[2] While Zeno is often credited as the first thinker presenting us with paradoxes, usually we find neither reasons given for this claim nor any explanation specifying in which sense paradoxes are talked about (see, for example Cuonzo 2014, p. 143).

[3] Rescher (2001, pp. 72–3) classifies paradoxes as semantical (involving the idea of truth, falsity and reference), mathematical, physical (including Zeno`s paradoxes), epistemic (involving knowledge and belief), and philosophical.

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
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set of individually plausible propositions is collectively inconsistent', whereby the inconsistency at issue must be real rather than merely apparent (Rescher 2001, p. 6).

For our understanding of a paradox, we will leave in the reference to 'apparent', for two reasons. First, it sometimes takes centuries to judge whether a conclusion is indeed inconsistent, so what may have been a 'real' inconsistency with the conceptual tools available at one point in time may be understood only as an apparent inconsistency at another time. If we are not only talking about the current state of affairs but the development of paradoxes in the history of philosophy, and if we want to leave open the possibility that paradoxes concern our (changing) conceptual tools, it seems wise to talk about 'apparent inconsistencies'.[4] Secondly, in contemporary debates there is not always agreement on the question whether a paradox presents a 'real' inconsistency or not. There are quite a few strong paradoxes which according to some scholars have found a solution once and for all,[5] while other scholars point out that there is still a problem; and Zeno's paradoxes of motion continue to inspire possible replies from philosophers, mathematicians and physicists, most recently from quantum theory. So 'apparent' will also be left in to mark the possibility of an epistemic position of uncertainty.

The idea that the inconsistency a paradox displays may be 'a passing trait' (which the term 'apparent' can express) is also found in Quine. In 'The Ways of Paradox' (1966) he distinguishes three kinds of paradoxes: veridical paradoxes (which bring with them the air of absurdity but are in fact true),[6] falsidical paradoxes (the absurdity they display rests on an underlying fallacy),[7] and antinomies (which produce a self-contradiction by accepted ways of reasoning and thus require a reworking of our conceptual scheme).[8] Quine explicitly claims that what may seem to be an antinomy at one point in time may be seen as a falsidical paradox at another point in time: 'One

[4] Interestingly, Rescher takes up a lot of paradoxes from the history of philosophy without accommodating this fact in his definition of paradoxes.

[5] I will say more about the different strengths of paradoxes below.

[6] As an example, he names the fact that the hero of *The Pirates of Penzance*, Frederic, had only had five birthdays when he turned 21—a claim that sounds absurd, but can be seen as true once we know that Frederic was born on the 29th of February.

[7] For Quine, Zeno's Achilles and the tortoise is such a paradox, since Quine reduces this paradox to a false assumption about convergent series.

[8] These are Russell-like paradoxes (that is, paradoxes of class-membership) and those involving terms related to truth locutions.

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
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man's antinomy is another man's falsidical paradox, give or take a couple of thousand years' (1966, p. 11); and Zeno's paradoxes are for him a case in point.

If we now look at the reasoning used in paradoxical arguments, we see that Rescher's account does not explicitly mention our (apparently acceptable) *reasoning*—that is, the way in which we derive the conclusion from the premisses—as a separate point. However, some form of reasoning must be the implicit background for the collective inconsistency of the individually plausible propositions. For a collective inconsistency becomes clear through reasoning, unless we assume a divine point of view where the inconsistency of the individual premisses can 'just be seen'. Being human, I think we should make the reasoning involved explicit in our understanding of a paradox, since something can go wrong here. Thus in our attempts to solve paradoxes, we need to keep our reasoning in mind as one possible point requiring change (we may, for example, think that a wrong inference from the premisses to the conclusion plays a role in Zeno's arrow paradox—see Sattler 2020, ch. 3). That our reasoning is a central part of a paradox also becomes clear from Quine's claim that antinomies show that some 'tacit and trusted pattern of *reasoning* must be made explicit and henceforward be avoided or revised' (Quine 1966, pp. 7 and 13; my emphasis).

Let us now move on to the question of how to characterize the premisses. As we just saw, for Rescher it is important that the premisses in question are (merely) plausible, since this explains why it is possible for us to get into inconsistencies and how we can solve paradoxes: for Rescher we get entangled in paradoxes since our modes of reasoning are valid when applied to true premisses, but may yield implausible or even contradictory conclusions when applied to merely plausible premisses. And Rescher's method of paradox-solution can be captured as finding the least plausible premiss (or premisses) in the argument, the removal of which will free us from the inconsistency in the conclusion (2001, pp. 26–27).[9] It is, however, worth pointing out, against Rescher, that not only plausibility but also acceptability can come in degrees—being acceptable need not be a matter of yes or no, since some things may be fully acceptable so that we endorse them, while others may be acceptable only under

[9] This focus on plausibility also seems to be the reason why Rescher refers solely to premisses in his account of how paradoxes arise, without explicitly mentioning our reasoning, as he thus has identified the 'culprit' to be one of the premisses.

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

certain conditions, for example, for the sake of an argument, and are thus acceptable to a lesser degree. And we also have to determine what counts as acceptable. But in whatever way we determine the exact relationship between plausibility and acceptability, what seems to be important for good arguments is that they work with premisses that *seem to be true* (see also Mackie 1973), they have the force of seeming truth, while plausibility does not need to have the same pull.[10] In most cases, not all of the premisses can be true,[11] so some premisses will only be in the neighbourhood of a true statement (see Sattler 2015). But they all at least seem to be true; for if a premiss neither was true nor seemed to be true , we would not be dealing with a paradox but straightforwardly with a bad argument.[12]

While talking about being acceptable in the case of the premisses seems to be unproblematic, we may want to specify the 'unacceptableness' of the conclusion. With paradoxes, the conclusions derived are 'apparently unacceptable' for Sainsbury; for Rescher, we get into inconsistencies. In Cuonzo, who does not give a definition of paradoxes, but rather a summary of other definitions, we find the conclusions to be 'obviously false or inconsistent' (Cuonzo 2014, p. 7).[13] According to Oms, who considers characterizations assuming paradoxes as an apparently valid argument with apparently true premisses and an apparently false conclusion as too narrow, the conclusion does not generate the kind of commitment in us 'that

[10] I take the difference between something seeming to be true and it being plausible to be that we are committed to something that seems true to us until disproven, while we may not feel equally committed to something sounding plausible.

[11] Having only true premisses in a paradox is only possible in cases where something has gone wrong in the reasoning, so that from true premisses we derive a false conclusion without this being obvious on the surface of the argument; or where the seemingly unacceptable conclusion turns out to be acceptable after all, in what Quine calls 'veridical paradoxes'.

[12] There may be cases in which a true premiss might not seem true, but unless we find out that this premiss is true after all on closer inquiry, we will simply understand the argument as resting on a problematic premiss and thus as a bad argument (as implicit in the discussion above, the seeming truth or falsity of a premiss can also be a passing trait). So if we have in our argument a clearly 'identifiable culprit', as Lycan (2010) calls it, then on my account we are not dealing with a paradox strictly speaking. Problematic assumptions generally have to seem to be true for them to be part of a paradox.

[13] She gives the following summary of a paradox, taking up the definitions of Rescher, Sainsbury and Mackie: 'A set of mutually inconsistent propositions, each of which seems true; an argument with seemingly true premisses, seemingly good reasoning, and an obviously false or contradictory conclusion; an unacceptable conclusion derived from seemingly true premisses and apparently valid reasoning' (Cuonzo 2014, p. 212).

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

should stem from the acceptance of the premisses and the validity of the argument'.[14]

While in some contexts a weaker understanding of paradoxes as not generating equal commitment to the conclusion and to the premisses may be useful,[15] I will concentrate here on paradoxes containing false and inconsistent conclusions. However, the falsity in question cannot simply be any old empirical falsity, as when an argument tells me it is sunny now, when this is not the case, since such an empirical falsity may simply be the result of a bad argument, but not yet a paradox. For a paradox, the falsity in question needs to be of a more fundamental kind: it shows not merely what happens to be false, but what is obviously false or absurd, and thus points out a problem in some of our core notions (so also Oms), so that we do not straightaway know how to fix it.[16] It will usually be an inconsistency.[17]

Summing up, the structure of a paradox can be said to be an apparently sound proof of an unacceptable conclusion. That is, the premisses worked with seem to be true and the reasoning from the premisses to the conclusion seems to be valid, and yet we end up with a problematic conclusion. I think such an understanding of a paradox is the one that has become most fruitful in philosophical discussions.[18]

We can locate the problem with the conclusion of the paradox either internally, within the paradox—if it is inconsistent in itself or inconsistent with one of the premisses used—or externally, if the conclusion is inconsistent with some known state of affairs, or with some principle or conviction we hold. We may think that such an

[14] In his talk 'Some Remarks on the Notion of Paradox' at the 2020 Joint Session of the Aristotelian Society and the Mind Association.

[15] This could, however, also include cases that are not paradoxes at all, but rather bring out the *unwelcome* consequences of different commitments we hold.

[16] This still allows for paradoxes to cover conflicts between a seemingly decisive a priori argument and a seemingly clear empirical finding, as it may have been found in conflicts between findings in quantum mechanics and classical logic; I owe this point to Guy Longworth.

[17] I will not deal with Whitehead and Russell's (1910, p. 37) analysis of paradoxes, since they are not sufficiently general in their discussion of paradoxes; their discussion is tailored to the class paradoxes, that is, to paradoxes concerning membership in classes or collections (logical or set-theoretical paradoxes). By contrast, the paradoxes of interest to them also fit the general understanding I give in the main text.

[18] This is not to say that this is how the word 'παράδοξος' was originally used in Greek; for this see below and Probst (1989). I am aware that some people understand paradoxes in a much broader sense, including all kinds of sensory puzzles, such as visual illusions; but in this paper I am interested in an understanding of paradoxes focused on conceptual questions.

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

additional principle or conviction we hold needs to function as an explicit premiss in the argument, otherwise we do not yet have a paradox as we do not have some *internal* inconsistency. While I agree that this is ultimately the case with real paradoxes, we will see such an additional principle playing a role in Parmenides and Aristotle. With them, what is called a paradox (in Aristotle) or may be seen as one (in Parmenides) is an argument that, when connected with some other principle or doctrine that a particular person or school may hold,[19] is shown to be problematic. The fact that this additional principle is not part of the argument as such will show that with Parmenides and Aristotle we do not find the kind of 'internal' paradox that Zeno establishes.

Paradoxes come in different strengths: there are weaker and stronger ones. The strength of a paradox is usually correlated with the amount of conceptual work necessary to solve them. If we count only those paradoxes as strong for which no solution seems possible or none has so far been found, this would rule out Zeno's paradoxes, since several solutions have been put forward. It would, however, also rule out most other paradoxes, since there are hardly any paradoxes where no solution has so far been suggested.[20] In fact, a situation without any suggested solution seems to be encountered only shortly after a paradox has been raised for the first time. So alternatively, we may think it is enough for a strong paradox in case it is convincing that people employ the assumptions it relies on and a solution is not simply obvious, irrespective of whether we have an agreed-upon resolution. In this sense, some of Zeno's paradoxes are clearly strong paradoxes. We should also judge them to be strong if we follow Sainsbury's criterion for strong paradoxes as those about whose solution we are still in severe and unresolved disagreement. For there is still severe and unresolved disagreement about how to solve Zeno's paradoxes—that merely employing mathematical tools does not fully address them (while it does allow us to deal successfully with motion and space in the sciences) has been pointed out by several researchers;[21] and the fact that new replies to Zeno continue to be suggested also points in this direction.

[19] Such as Parmenides' own doctrine.

[20] This does not imply that scholars agree that these solutions will work—for some scholars these paradoxes will be solved, for others not.

[21] See Code (1982a, 1982b), Hasper (2006), and Sattler (2020, ch. 3); cf. Sainsbury (2009, pp. 16–18).

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

There is, however, an additional problem when dealing with these ancient paradoxes, in that in several cases not only do we not have an agreed upon solution, we also do not have an agreed upon diagnosis; there is disagreement about what exactly the paradox is. This is the case, for example, with Zeno's moving rows paradox (Sattler 2015). In these cases it is often hard to keep solution and diagnosis clearly apart. Nevertheless, we should be clear that introducing paradoxes as the method I think Zeno establishes is not the same as a method for paradox solution (as, for example, Rescher tries to establish). While a method of paradox solution will help us figure out whether we have to deny the validity of an argument, the truth of its premisses or the alleged inconsistency of the conclusion, and may provide us with means to do so, a method of establishing paradoxes will be a way of presenting what we take for granted as implying some form of (conceptual) problem.

Let me finally stress one point that is in the background of any discussion of philosophical paradoxes as I discuss them here: in order for paradoxes to work as paradoxes, the law of non-contradiction (in one form or other) has to hold (*pace* Priest 2006). The conclusion of a paradox is, as just noted, either inconsistent in itself or inconsistent with some further claim we are committed to. Thus for a paradox to work it has to be clear that getting entangled in a contradiction is proof that something is wrong in our argument, and that we need to find a way to change it (see also Mackie 1973). Paradoxes thus understood do not undermine the principle of non-contradiction, but rather use it as the most important standard for our reasoning—a point we will see to be of crucial importance later on (and central for the difference between Heraclitus and Zeno).[22] Since the principle of non-contradiction is a central principle of reasoning in all paradoxes as I understand them, I will not count it as an (internal) element in a paradox set, but rather as a general criterion.

The principle of non-contradiction was not always understood in the way we understand it today—an understanding that was first formulated in outline in Aristotle and prepared for in Plato.[23] In

[22] We may think that in extreme cases the other elements in the paradox could emerge as being more plausible than the principle of non-contradiction, so that we are prepared to give it up, and thus the paradox would undermine it. However, if we encounter such a case (and not simply a case that requires some adjustment in the understanding of the principle of non-contradiction, as can also be seen in the history of philosophy; see Sattler 2020, ch. 1), it seems to me we are giving up the very notion of a paradox.

[23] Aristotle states it, however, only in terms of internal negation; see Horn (2001); cf. Sattler (forthcoming b).

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

Parmenides and Zeno, we find a less fully developed version according to which the principle of non-contradiction is understood as 'not (*S* is *P* and *S* is not-*P*) regardless of respect, and *S* has to be either *P* or not-*P*', whereby *P* and not-*P* can be understood as what we would call contraries or contradictory opposites.[24] Interestingly, however, there seems to be only one paradox of Zeno where this specific version of the principle of non-contradiction features in deriving the paradox: in the arrow paradox, Zeno infers the conclusion that the arrow is unmoved from premisses giving the conditions for something being at rest ('Everything is at rest whenever it is in a space equal to its own size'). Accordingly, we have to assume that Zeno equates 'being at rest' and 'being unmoved'. This only works if we assume that everything has to be either in motion or at rest, excluding the third possibility that some things may simply be unmoved (for example, in the way we may claim eternal truths to be). In none of the other paradoxes, however, will this specific interpretation of the principle of non-contradiction play a role.

III

The Structure of Zeno's Paradoxes. Zeno was the inventor of numerous paradoxes.[25] The paradoxes that have been handed down to us can be divided into three series, the paradoxes of *topos* (place or space), the paradoxes of plurality, the paradoxes of motion, and in addition, the single paradox of the falling millet seed.[26] While we cannot be sure whether, in addition to the paradoxes we possess,

[24] Thus the so-called principle of excluded middle simultaneously has to hold but is interpreted in terms of contradictories or contraries. This version of the principle of non-contradiction starts being changed implicitly by post-Parmenidean thinkers like the atomists, and explicitly by Plato. The Eleatic understanding of the principle of non-contradiction allows Zeno to take disjunctions like *F* and non-*F* as exhausting logical space, so that we can infer the truth of one disjunct from the inconsistency of the other: given that plurality leads to inconsistency, we have to assume a (simple) One (Lee 1936, frs. 1 and 2).

[25] See DK 29A15, Lee (1936), Barnes (1982, p. 233), Kirk, Raven and Schofield (hereafter KRS) (1983, pp. 264–5). In citations, 'DK' indicates the Diels-Kranz numbering used in the standard edition of Presocratic texts, Diels and Kranz (1951–2).

[26] The paradoxes of *topos* can be found in DK 29A24 and B4, and in Lee (1936, frs. 13–18); the paradoxes of plurality in DK 29B1–3 and A21–23, and Lee (1936, frs. 1–12); the paradoxes of motion in DK 29A25–28, and Lee (1936, frs. 19–36); and the paradox of the falling millet seed in DK 29A29, and Lee (1936, frs. 37–38). For the division, see Lee (1936, p. 9). We hardly have any of Zeno's own words—there are a few sentences with the paradoxes of plurality in Simplicius, and a single sentence with the paradoxes of *topos* in Diogenes; see DK 29B1–4; KRS (1983, p. 266); and Lee (1936, p. 29).

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

Zeno may have come up with paradoxes of a completely different kind , we can at least look at whether the paradoxes handed down to us have a common structure and what kind of a function this structure (or these structures) suggests—whether Zeno saw them as pointing out inconsistencies in reality, in language, or in our conceptual tools.

There have been frequent attempts in the scholarship on Zeno to fit his paradoxes into a systematic programme. Often these programmes focus only on the four paradoxes of motion,[27] but sometimes they are meant to embrace all of them. In the English-speaking world, Owen (1957–8) most prominently sought to reconstruct one system for all the paradoxes we know of by interpreting them as the different manifestations of the supposition that the real world is divided. If all of Zeno's paradoxes fitted such a programme, this may also suggest a common conceptual structure. However, so far, all these schemes have shown themselves to be wanting in their interpretations of the paradoxes.[28]

While the paradoxes do not all seem to fit into one systematic programme, in that they do not all contain the very same problematic assumption (or pair of assumptions), they may, however, all follow the same general paradoxical structure (in the way discussed in the last section). In his *Parmenides*, Plato has Socrates summarize the first argument of Zeno as 'If the things that are, are many, they must be both like and unlike, which is impossible' (*Parmenides* 127e; Lee 1936, fr. 12). And in what follows, Plato seems to suggest something along these lines as the general structure of Zeno's paradoxes: 'If we assume that there is x (for example, that there is a plurality), then Zeno shows that x is F as well as not-F'.[29] 'x is F and not-F' can be

[27] See, for example, Salmon (1980) and Heath (1921, vol. 1, p. 275). For an argument against the scheme employed by them, see KRS (1983, p. 265). Brochard (1954) sees a couple of such schemes at work.

[28] See Barnes (1982, pp. 233–4) and KRS (1983, p. 265); I discuss this in Sattler (2020, ch. 3).

[29] We may at first think that this is a structure that works only for the plurality paradoxes, but in *Parmenides* 128e ff., Plato mentions motion and rest as a central pair of concepts, and may thus hint at the motion paradoxes: '"But if someone first distinguishes as separate the forms, themselves by themselves, of the things I was talking about a moment ago—for example, likeness and unlikeness, multitude and oneness, rest and motion, and everything of that sort—and then shows that in themselves they can mix together and separate, I for my part", Socrates said, "would be utterly amazed, Zeno"' (*Parmenides* 129d–e, translation by Gill and Ryan with alterations); see also *Phaedrus* 261c–e. Accordingly, the structure given by Plato may be seen as an attempt to combine at least the plurality and motion paradoxes by showing that the basic structure is the same in both series: to make the same thing seem both F and not-F.

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

understood as the conclusion that characterizes a paradox—a conclusion that is unacceptable because it is inconsistent, as suggested above.[30]

In order to figure out whether Plato's suggestion does indeed capture all of Zeno's paradoxes, so that we can understand Zeno as claiming that if we attempt to give an account of motion, plurality, *topoi*, or the sound of falling millet seeds, then we will always end up with the claim that *x* is *F* and not-*F*, I will give a brief overview of Zeno's paradoxes.[31]

Of the plurality paradoxes, some can obviously be understood along these lines: if we assume a plurality, the many things are *F* and not-*F*, for example, like and unlike in Plato's *Parmenides* 127e1–4, finitely and infinitely many according to DK 29B3. With others, however, it seems less clear whether they present indeed an inconsistency, or more generally, an absurd result—for example, DK 29B1 shows that if a plurality exists, each of the things of the plurality has to be so small as to have no magnitude at all and so large as to be infinite.[32] We may understand this conclusion indeed as a logical inconsistency—something cannot at the same time be of some (infinite) and of no magnitude. Alternatively, we may think that given the meaning of small and large, something cannot simultaneously be excessively small and excessively large in extension, while the paradox seems to force us into this absurd claim. Finally, we may think that the paradox presents a conflict between showing things of a plurality to be infinitely small or large and our experience that such things are of a finite size (and thus neither infinitely small nor large)—while such an experience is not explicitly mentioned and thus may be seen as being external to the paradox, it would be internal to the belief system of any pluralist (not just to a particular school).

Of the motion paradoxes, the dichotomy and the Achilles paradox present us with the proof that in order to cover a finite distance

[30] We may wonder whether *x* is a proposition or a thing characterized by *F* and what kind of an item *F* is. For the ancient context, we should not restrict ourselves to propositions, but may also consider individual terms.

[31] I can give only a very rough overview of the basic paradoxes here; for a defence of the interpretation of the plurality and motion paradoxes, see Sattler (2015; 2020, ch. 3); I discuss the *topos* paradoxes in my book manuscript on space (Sattler MS a); and the millet seed paradox in my unpublished paper 'A New Beginning of the Vagueness Paradoxes' (Sattler MS b).

[32] This is also how Cuonzo (2014, p. 148) reads it.

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

we have to cover infinitely many smaller distances, and that we have to cover these infinitely many smaller distances in a finite time.[33] This may be understood as showing that the same *x*—a distance to be covered in a motion—has to be both finite and infinite, and thus holds inconsistent attributes. Accordingly, we may understand these two paradoxes as showing that *x* has to be *F* and not-*F*.[34] The arrow paradox is meant to show that the flying arrow is at rest.[35] In this paradox, rest is seen as the contradictory opposite of motion (see Sattler 2020, ch. 3);[36] thus the paradox shows that the arrow is both *F* and not-*F*, both moving and not moving. The fourth paradox of motion, the paradox of the moving rows, aims to show that the set-up of three rows of equal size, two of which move in opposite directions, leads to the conclusion that 'half the time equals double the time'.[37] Again, it is not fully clear here whether this paradox presents us with an absurd consequence in the sense of being analytically false (as a straightforward way of understanding the explicit claim we get from Aristotle that half the time is double the time may suggest) or a consequence that can be expressed as a logical contradiction (for instance, that half the time is not half the time).

The millet seed paradox shows that if a whole bushel of millet seeds makes a sound when falling, then also an individual millet seed has to make a sound when falling (DK 29A29). This paradox can also be understood along the lines of *x* being *F* and not-*F*. For we end up with the conclusion that a millet seed does make a sound when falling, which contradicts our experience that a falling millet seed does not make a sound. So the seed makes a sound and does not make a sound: *x* is *F* and not-*F*.

[33] DK 29A25 (Lee 1936, fr. 19; Aristotle, *Physics* 233a21–23, 239b9–14) and DK 29A26 (Lee 1936, fr. 26; Aristotle, *Physics* 239b14–29). Aristotle argues in the latter passage that the main paradoxical point is the same in these two paradoxes.

[34] Even if the two paradoxes also raise other points; and the question whether the two attributes are indeed inconsistent or not depends on what we understand by 'finite' and 'infinite', as already Aristotle shows in his treatment of Zeno's paradoxes.

[35] DK 29A27 (Lee 1936, fr. 29; Aristotle, *Physics* 239b5–9) and DK 29A27 (Lee 1936, fr. 28; Aristotle, *Physics* 239b30–33).

[36] As we saw above, Zeno does not distinguish between being unmoved and being at rest, which allows for motion and rest to be understood as contradictories. We do not find the distinction between 'resting' and 'not-moving' clearly drawn before Plato's *Parmenides*, and systematically employed before Aristotle.

[37] DK 29A28 (Lee 1936, fr. 35; Aristotle, *Physics* 239b33–240a17).

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

Finally, the paradox of *topos* presents us with an infinite regress (DK 29A24):[38] if everything that exists is in a place and place itself exists, then place will be in a place, ad infinitum.

Accordingly, we see that while many conclusions of Zeno's paradoxes have the structure '*x* is *F* and not-*F*', not all can be characterized like this. Besides logically inconsistent consequences, we also find analytically false ones and infinite regresses—thus different structures, not all of which fit the general characteristic given above, even if we may associate them all with the paradoxical genre more generally.

A characteristic that we do, however, find in all of these paradoxes is that they start from positions that are not Zeno's own, but instead are widely held—that there is plurality, motion, *topoi*, and bushels of millet seeds making a sound—and show these to be problematic in themselves. Zeno is here not demonstrating convictions held by himself to be paradoxical, as can be seen, for example, from Simplicius' account of Zeno's plurality paradoxes, according to which Zeno shows that the person who claims there to be a plurality gets into an inconsistency (τῶι πολλὰ εἶναι λέγοντι συμβαίνει τὰ ἐναντία λέγειν) (DK B2; Simplicius, *Phys.* 139.5). And three fragments make it explicit that Zeno himself did indeed not share the assumption of plurality, divisibility, and so on (Lee 1936, frs. 1–3, the second of which I quote here):[39]

> For, he argues, if it were divisible, then suppose the process of dichotomy to have taken place: then either there will be left certain ultimate magnitudes, which are minima and indivisible, but infinite in number, and so the whole will be made up of minima but of an infinite number of them; or else it will vanish and will be divided away into nothing, and so be made up of parts that are nothing. Both of which conclusions are absurd. It cannot therefore be divided, but remains one. Further, since it is everywhere homogeneous, if it is divisible it will be divisible everywhere alike, and not divisible at one point and indivisible at another. Suppose it therefore everywhere divided. Then it is clear again that nothing remains and it vanishes, and so that, if it is made up of parts, it is made up of parts that are nothing. For so long as any part

[38] Sedley (2017) suggests understanding the *topos* paradox in a way that fits the '*x* is both *F* and not-*F*' structure. For an argument against this reconstruction, see Sattler (MS a, ch. 2). I am leaving aside here what can be understood as a second *topos* paradox (DK 29B4).

[39] Porphyry attributes fragment 2 to Parmenides, but Alexander and Simplicius think it to be more likely by Zeno; see also Lee (1936, p. 12). For this ascription to Zeno, see also Sattler (2020, ch. 3), and Lee (1936, fr. 1), Simplicius 139.19–22.

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

> having magnitude is left, the process of division is not complete. And so, he argues, it is obvious from these considerations that what is is indivisible, without parts, and one. (Lee 1936, fr. 2; Simplicius 139.27, Lee's translation)

This paradox claims that if we assume some one thing (such as, for example, a finite distance) to be divisible and thus to have some kind of parts, it will not only at the same time be a plurality, but also leave us with no way to give a consistent account of these parts. Accordingly, having shown the paradoxical result of this assumption, Zeno argues that we should assume what truly is to be indivisible, without parts, and so really one; it is thus the opposite of what is commonly assumed.

We may think that showing an opponent to get into problems with her argument is a common feature in any sort of agonistic argument culture and found in philosophy right from its very beginning. Thus it would not be a feature specific to Zeno's paradoxes. What is, however, special is that Zeno does not show these problems by starting from his own assumptions. Rather, he attempts to show that, independent of any positions he may have hold, the assumptions of plurality and motion can be shown to be inconsistent or absurd from within, and thus from the very starting point of the people who assume them; and only after this inconsistency has been shown does Zeno, in some cases, suggest the opposing position as an alternative.[40] All of Zeno's paradoxes start from the position of the opponent, take up the opponent's claims and then demonstrate that accepting these claims entangles the opponent himself in a contradiction or an absurdity.[41] Locating the paradox completely within the position of an opponent is a move first introduced with Zeno, as a brief comparison with his predecessors will show.[42]

[40] We may think that Plato's *Parmenides* 128a–d shows Zeno with an agenda of his own, namely, to support Parmenides' philosophy. However, the second part of this dialogue claims to use Zeno's method in order to train Socrates in a scientific inquiry that aims for truth independently of any further agenda, see 135d ff.

[41] This also holds true for interpretations which do not see Zeno as ultimately arguing for a particular position.

[42] Zeno's paradoxes also show that our everyday experience is not in fact an adequate criterion for judging an explanation of what there truly is; see Lee (1936, fr. 8), Philoponus, *Phys*. 42.18–21.

Proceedings of the Aristotelian Society, *Vol*. cxxi, *Part* 2
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IV

Possible Predecessors. Paradoxes seem to be have been introduced into philosophy by Heraclitus, who was born roughly sixty years before Zeno. Heraclitus' whole philosophy may be seen as being based on paradoxes or paradoxical thinking. He is well known for claims such as 'God is day night, winter summer, war peace, satiety hunger'.[43] While this claim does not as such form a contradiction, we do indeed find in Heraclitus real contradictions which seem to suggest a paradox:

> In the same river we step and do not step, we are and are not. (DK 22B49a)[44]

With claims like this, Heraclitus seems to be the first philosopher to introduce paradoxes as an important point into philosophy: we step and do not step in the same river, we are and are not; without any further distinctions of respects or time, this seems to be a clear contradiction.[45] If I went swimming at a certain spot in the Danube yesterday and again today, it seems I went into the Danube twice, and thus have stepped into the same river. However, when I went into the Danube today, it seems I didn't encounter the very same river, for if I put in some green powder to dye the water yesterday, it will have disappeared today. The river is constantly changing, since what makes a river a river—its water—is constantly changing, so that we may ask, in the Platonic variation of the paradox, whether we step into the same river even once (DK B91; Plato, *Cratylus* 402a).[46]

[43] This is the first part of fragment 67.

[44] There are two further versions of this paradox (DK 22 B12 and 91). For a discussion of these three fragments, see McCabe (2015, ch. 2). For the debate about their authenticity, see Marcovich (1967, pp. 206–14), Barnes (1982, p. 66), and McCabe (2015, p. 35 nn.2–4).

[45] DK 22B49a employs a contradiction, as we saw above, but more often Heraclitus employs contraries, for example in DK 22B67 and 22B88. In the scholarly literature, there is a big debate over whether Heraclitus violates the law of non-contradiction—as, for example, Barnes (1982, pp. 71, 79–81) claims— or not—as, for example, Graham (2010, p. 190) argues.

[46] Aristotle (*Metaphysics* 1010a13–15), however, takes this intensification to be a criticism of Cratylus, who didn't think Heraclitus radical enough; and Kirk (1962, pp. 369–79) argues that Plato's version is a misinterpretation of Heraclitus, who in his river fragments is meant to show the preservation of identity in spite of change, due to the regularity of the change. Wiggins (1982, pp. 25–6) thinks that the fact that Heraclitus talks of rivers and the like, and thus of 'continuants', shows that he does not think of rivers themselves as processes, which, in contrast to rivers, 'are regular, or gradual or fitful, take time, have temporal parts'.

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

Heraclitus' paradox suggest that it gives an accurate description of reality.

Some interpreters, like McCabe, have understood Heraclitus as offering resolutions to these paradoxes himself: for her, the paradox quoted above is resolved by the claim in the variation of this paradox in fragment 12 that 'to those who step into the same rivers, different and different waters flow'—a river qua the outlines of the river banks can be stepped into twice; while a river qua the waters which constitute it is constantly changing. So in general, we may understand Heraclitus as raising paradoxes that can then be resolved by Heraclitus' introduction of the appropriate respects—for example, the same thing, such as sea water, is the purest and the foulest, because it produces opposite effects on different objects or perceivers: it is deadly for human beings, but life-preserving for fish (DK fr. 61).[47] But if we understand Heraclitus like this—and I am not claiming that it is false to do so—then we do not really find in his work any strong paradoxes in the sense specified above, but only riddles in paradoxical-sounding formulations that can be resolved with the help of different respects.[48] So either Heraclitus does not really deal with paradoxes or, if we do not follow McCabe's interpretation and take the river fragment as intending to present a genuine paradox, his paradoxes concern the object of investigation, and Heraclitus suggests that these objects, and more broadly the world as such, are paradoxically structured. (In the following, I will deal only with this second option, since it is the only one relevant for our discussion.)

Both Heraclitus and Zeno ask us to change our common belief system because of their philosophy—Heraclitus in that we should endorse that reality is constantly changing and paradoxical, Zeno by claiming that central common beliefs lead to paradoxes and thus have to be given up. But in contrast to Heraclitus' paradoxes, the inconsistencies deployed in Zeno's paradoxes are not to be embraced, but meant to be avoided. This can be done in different ways: in some of his plurality paradoxes, as in Lee (1936, frs. 1–3), Zeno explicitly suggests avoiding the paradoxical result by accepting the opposite position (the position introduced by Parmenides), that there

[47] So these would not be cases of paradoxes that were initially strong, but resolved by later thinkers due to available distinctions; rather, Heraclitus himself would give us the necessary differences of respects and distinctions, and thus would show them not to be strong paradoxes.

[48] Such puzzles may still be seen as a forerunner to what Quine calls veridical paradoxes.

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

is only one thing that is unmoved. Other paradoxes, like the arrow or the stadium paradox, however, do not explicitly derive any opposing assumption from the paradox. Here the paradoxes may be seen to suggest either that we should simply not assume that there is any motion of what truly is or that we should give up our philosophical inquiry into motion—for even if there is motion we will either be incapable of comprehending it, or at least incapable of giving a consistent account of it—or that we need to look for other ways out of this predicament. In any case, Zeno's paradoxes are meant to *stop* our usual beliefs as a first step, because they violate the principle of non-contradiction. Heraclitus also questions common assumptions, but he does so by claiming that to truly grasp these assumptions means thinking about what is assumed, things such as motion, in a contradictory way—his statements *endorse* a structure of reality that violates the principle of non-contradiction and thus seems strange to common sense.

That Heraclitus and Zeno employ the principle of non-contradiction in crucially different ways is also supported by Aristotle's investigation of this principle in his *Metaphysics*. There he claims that 'it is impossible for anyone to believe that the same thing is and is not, as some consider Heraclitus said' (DK 22A7; *Metaphysics* 1005b23–25). While Aristotle himself leaves it open whether Heraclitus did indeed attempt to violate the principle of non-contradiction, some people whom Aristotle refers to did in fact think Heraclitus went against it. This possible violation of the principle of non-contradiction suggests that Heraclitus' paradoxes are not paradoxes in the sense specified above, according to which paradoxes do not question the principle of non-contradiction, but rather use this principle as an important criterion: only because we attempt to adhere to this principle do paradoxes seem to be problematic. In contrast to Heraclitus, most of us assume that in order to make progress in philosophy we need to show how paradoxes can be resolved or avoided. Paradoxes are a warning sign that in some way we are at a dead end; they tell us that we have to go back to the crossing and choose a different turning.

We may think that Zeno's use of paradoxical structures is not in fact that different from Heraclitus', since Zeno is engaged in setting up paradoxes, not in solving them. But in Zeno's paradoxes it is made clear that if we show something to violate the principle of non-contradiction, something has to change: we need to rethink or stop

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

our investigation, or at least some conversation about it has to take place. Zeno's paradoxes work because it is clear that getting entangled in a contradiction is a proof that the argument or account involving the contradiction cannot stand as it is. While Zeno uses the principle of non-contradiction as the crucial criterion for philosophical inquiries in his paradoxes, for Heraclitus this principle is one that does not capture what we ultimately inquire about, and that thus is no reliable guide for our inquiries.[49]

Summing up, we can say that if Heraclitus is indeed presenting us with full-fledged paradoxes, he uses them as a genre quite different to the one Zeno employs. Heraclitus suggests endorsing his paradoxes, while Zeno's paradoxes are meant to stop our common assumptions, such as that there is motion and plurality, on which these paradoxes build. This difference is also displayed in the different role the principle of non-contradiction plays in their philosophy. Finally, Heraclitus' paradoxes concern the *content* of investigation—they show the world to be paradoxical—while Zeno's paradoxes work as a *tool* for changing our assumptions. While both approaches may lead us to changes in our view about the world, Heraclitus' paradoxes give us such a (contradictory) world view, while Zeno's paradoxes do not give us the content of the new world view, but are meant to make us change our assumption about what truly is; thus they work as a philosophical tool, what we may see as a method, to make us pause in our normal investigation and rethink our assumptions and reasoning.

Given that Parmenides uses the principle of non-contradiction as a central and systematic criterion of inquiry in his poem, and that his poem is indeed the first place where a version of the principle of non-contradiction is methodically employed as such a criterion (see Sattler 2020, ch. 2), we may think that we find Zeno's method already in Parmenides.[50] If we look briefly at the argumentative structure in Parmenides' poem, we find a couple of passages that may seem to anticipate this method. Parmenides argues for a certain

[49] Nevertheless, Heraclitus has to assume that other people take the principle of non-contradiction at least implicitly as a guideline, for otherwise his fragments would not produce their provoking effect. If we follow a McCabian reading of Heraclitus, we see him respect the principle of non-contradiction by resolving the apparent contradiction with the help of different respects; but, as mentioned above, in this case he would in fact not be involved in raising strong paradoxes, at least not if we think his solutions to be obviously acceptable.

[50] Parmenides, too, thinks that if we get into a contradiction owing to our common assumptions, we have to reject these assumptions. Part of the following discussion of Parmenides overlaps with Sattler (2020).

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

understanding of what is (*eon*, Being) by showing that the contrary assumption leads to absurdities or contradictions—'what-is' is not-*F* because it cannot be *F*. For example, he argues in DK 28 B8, lines 3–14 for the claim that 'what-is' is ungenerated, by examining the two possibilities of generation (and destruction) there are—what has come into being could come either from what-is-not or from what-is. And by showing that both possibilities lead to contradictions, it is demonstrated that 'what-is', Being, cannot be generated.[51]

Prima facie, such argumentative structures in Parmenides seem to correspond to Zeno's paradoxical method. What Parmenides in fact claims, however, is rather that 'you cannot think *p* because this conflicts with *q*, which I have established before'. Assuming 'what-is' to have been generated would not be consistent with what he himself has established so far (namely, that 'what-is' has to be and that 'what-is-not' cannot be), and thus the assumption that 'what-is' is generated has to be rejected. So Parmenides uses a standard external to what somebody accounting for generation would use, while Zeno in his paradoxes takes up the opponent's claims and shows that accepting these claims entangles the opponent herself in a contradiction or an infinite regress.

Parmenides' poem may, however, in other parts anticipate what Zeno does in his paradoxes, for example, in its account of the two-headed mortals in fragment 6 for whom

> 'what-is' and 'what-is-not' is regarded as the same
> and not the same, and of all things there is a backward turning path. (fr. 6, lines 8–9)

Here it seems that the mortals themselves claim that 'what-is' and 'what-is-not' are the same and not the same. Thus their claims seem to be inconsistent in themselves, and not simply in conflict with Parmenides' account. However, it is on the basis of Parmenides' strict account of what we should understand by 'what-is' that their claims seem to confuse what Parmenides considers to be 'what-is' and 'what-is-not' (what they themselves would probably consider to be all 'what-is', Being), which is thus also seen as being the same and not the same.

By contrast, Zeno goes a step further than Parmenides when he shows that some assumptions, such as that there is a plurality, are

[51] For a more detailed analysis of this passage, see Sattler (2020, ch. 3).

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

not merely inconsistent with a position that he or Parmenides may hold, but are indeed *inconsistent in themselves*—a point we find made explicitly in Simplicius' account of Zeno's plurality paradoxes, referred to above:

> In his book, in which many arguments are put forward, he shows in each that a person who says that there is a plurality is stating something self-contradictory (*ta enantia legein*). One of these arguments is that in which he shows that, if there is a plurality, things are both large and small, so large as to be infinite in magnitude, so small as to have no magnitude at all. (DK 29 B2; Simplicius, *Phys.* 139.5; Lee's translation)[52]

In this passage, Simplicius claims that Zeno shows the very assumption of a plurality to be self-contradictory. Thus from the position of somebody assuming plurality, Zeno shows how this assumption undermines itself, without relying on Eleatic claims.[53]

Finally, we may think that Parmenides' treatment of the realm of *doxa* also shows the opponents' position to be inconsistent in itself (even if it is left to the reader to draw this conclusion), so that we would already get this important methodological move from Parmenides. However, the inconsistency Parmenides points out is again that the realm of *doxa* is inconsistent with what Parmenides has established before, in the realm of *alêtheia*. In itself, without the Parmenidean background, we do not have any reason to assume light and night as principles to be problematic (fr. 8, lines 51–59). The way of *doxa* could not be understood as being inconsistent if the way of *alêtheia* had not been sketched beforehand. Zeno, by contrast, attempts to show that independent of what is established by Parmenides in his poem, the pluralist assumptions do not get off the ground, because by assuming plurality they have to make inconsistent claims. While Parmenides shows that the opponent of Eleatic philosophy contradicts what Parmenides himself has already established, Zeno tries to show that the opponent of Eleatic philosophy

[52] Cf. also the account in Plato's *Parmenides* 127e1–4.

[53] This is the general way Zeno's paradoxes work. We saw above that there are also some paradoxes of plurality, however, where from this 'neutral' starting point Zeno aims to establish an Eleatic One.

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

contradicts himself. Thus Zeno's method of creating paradoxes allows for a stronger refutation.[54]

Parmenides' argument may nevertheless be seen as paradoxical in a way, if we take into account the usage of the word '*paradoxon*' in ancient Greek. In everyday Greek, the adjective '*paradoxos*' refers to what is uncommon or different from what is expected; it is against (*para*) the usual expectation (*doxa*), that is, not fitting common opinion or judgement, strange or shocking, and thus in need of explanation. It is the opposite of what is *endoxos*, that is, what is generally approved or acknowledged. Parmenides' austere account of what there is—only the One, absolutely simple Being—may be seen as *para-dox* in this original sense of the adjective, namely, strange and in need of explanation.

The first person to use the noun in Greek philosophy as a technical term is Aristotle in his *Organon*.[55] In his *Sophistical Refutations* 165b13–22, he tells us that those who argue as competitors and rivals have five aims: refutation, falsity, paradox, solecism, and reducing the opponent in the discussion to babbling. Thus he lists paradoxes in a competitive context, where they are the third best after refutation and showing the opponent to utter a falsehood; he does not understand paradoxes as a neutral tool or mode of inquiry. And in 172b29–34, Aristotle suggests paradoxical statements to be drawn with respect to specific school doctrines and ways to solve them. So here we are not dealing with 'strong' paradoxes in the sense described above, and they are tied to assumptions of specific schools. While Parmenides does not show specific school doctrines to be paradoxical, he demonstrates the thinking of the mortals to be inconsistent with respect to his own thoughts, and thus in relation to a certain doctrine. By contrast, what we find in Zeno is the attempt to show motion, plurality and *topos* to be inconsistent independent of any specific school or philosophical doctrine.

Given Aristotle's understanding of the Greek word 'paradoxon', we should not be surprised that in his discussions of Zeno, Aristotle does not talk about 'paradoxes'. Rather, in most cases he talks fairly

[54] In contrast to a recent trend in the secondary literature, I understand Zeno as supporting Parmenides' position, and argue in detail for this in Sattler (2020, ch. 3). My main claims in the paper here are, however, independent of this understanding.

[55] In Plato, we find it in the sense that the adjective in ordinary Greek also carries ('what is strange and contrary to common opinion').

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

neutrally of Zeno's *logos*,[56] sometimes of his *axioma* (*Metaphysics* 1001b) or his *aporia* (*Physics* 209a; in 210b he uses the verb *êporein*); and only once does he claim that Zeno *paralogizetai* (reasons falsely), with respect to the arrow paradox (*Physics* 239b).

We may also think of Gorgias as the inventor of philosophical paradoxes as a method, since it is a common methodological move of Gorgias to provide us with seemingly exhaustive dichotomies of a certain realm of which he then shows that none of the disjuncts are possible. For example, in the *Apology of Palamedes* we are told that Palamedes either betrayed the Greeks on his own or with the help of others, but he cannot have done it on his own nor with the help of others. Furthermore, the second part of Gorgias' *On What Is Not* has been understood as a *reductio ad absurdum* of the Parmenidean thesis. And in his *Helen*, it is not only from his own position that we are made to see that she both went to Troy and yet was innocent. Gorgias is, however, younger than Zeno, and I have tried to show elsewhere (Sattler MS a, ch. 2) that at least in some of his paradoxical arguments he is clearly building on Zeno. Thus while we may want to understand part of Gorgias' oeuvre as using a paradoxical method similar to that which Zeno does, with Gorgias we find some further refinement of the paradoxical structure established by Zeno.

Finally, we may think that a methodological employment of paradoxes derives from outside of philosophy—from the realm of mathematics.[57] Mathematical *reductio ad absurdum* proofs—as we see paradigmatically in the proof of the incommensurability of the diagonal and the side of a square—function similarly to many of Zeno's paradoxes: that is, the contradiction or absurdity is taken to show that the position implying it is false, that is, false absolutely, not merely false given someone's position.[58] Such proofs seem to have been used in ancient Greek mathematics quite early on. Our evidence for the incommensurability proof is Euclid, but since knowledge of the incommensurability is a

[56] *Physics* 233a, 239b, 250a, 263a; *Prior Analytics* 65b; *Topics* 160b; *Soph. elech.* 172a, and 179b; in 182b it is the *logos* of both Parmenides and Zeno.

[57] For example, Cuonzo (2014, p.141) claims that 'ancient mathematics provides the foundation on which paradoxes emerged in Greece', without, however, giving any proof or references for this claim.

[58] I take arguments by *reductio* to be (typically) paradox-involving as long as the targeted assumption seems true, in contrast to uses of *reductio* meant to undermine assumptions that do not even seem true, or which only seem true to a particular group of people.

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

good deal older,[59] *reductio ad absurdum* proofs may be as well. Exactly how old they are, however, seems impossible to say. They probably date to the fifth century BCE,[60] but this still leaves it open whether—and if so, to what degree—Zeno may have been influenced by mathematicians (or whether the influence may even have been the other way round).[61] And while in *reductio ad absurdum* proofs one particular assumption is clearly targeted as leading to inconsistencies, in Zeno this is sometimes, but not always the case—with paradoxes like the falling millet seed, it is not clear which (if any single one) of the seemingly true claims are to blame for the inconsistency.

V

The Characterization of Paradoxes in Zeno—A Method. Heraclitus, Parmenides and Zeno all attempt to change our common assumptions with their philosophy. But in contrast to Heraclitus and Parmenides, Zeno's paradoxes do not encourage us to embrace a paradoxical reality or show that our common assumptions seem paradoxical in the light of a particular position we hold. We can say that what is specific for Zeno's paradoxes are two points. First, they show the philosophical assumptions of an opponent to be problematic from within this opponent's position, and thus on its own ground. Zeno is not, like Parmenides, showing that somebody is in trouble with her assumptions from the point of view of a certain doctrine held by someone else. So without bringing his own conviction to the table, Zeno puts himself into the shoes of his opponent and demonstrates from within this position that there are problems.

[59] It is said to come from the Pythagoreans, but our first text testifying to it is in fact Plato's *Theaetetus*; see the appendix to Book X of Euclid's *Elements* and Knorr (1975, ch. 2).

[60] Von Fritz (1970) dates it to the last quarter of the fifth century at the latest.

[61] Lee (1936, p. 112) thinks it to be 'quite possible that Zeno`s method may have suggested to geometers the proof by *reductio ad absurdum*'. The degree of any mutual influence between mathematics and philosophy is difficult to trace back exactly, since our mathematical sources before Euclid are very sparse—after Euclid, most of the earlier mathematical texts seem not to have been handed down any longer. Given that Plato and Aristotle use mathematical concepts in order to solve problems of a conceptualizing nature, mathematics seems to have had an influence on philosophy at least at that time, but this does not help with Zeno. It used to be the case that scholars read Zeno's paradoxes as putting the mathematical notion of the infinitesimal into question, which was in turn seen as leading to Eudoxus' development of proportion theory. This story was, however, put into doubt by Owen (1957–8), and since Knorr (1975) most philosophers of ancient mathematics argue against the assumption that philosophy had any influence on ancient mathematics at that time. I remain agnostic on this question.

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

Second, Zeno does not embrace his paradoxes, as Heraclitus does, but takes them as a clear sign that at least some of the assumptions on which they rest have to change (as he shows in fragments 1 and 2).

Thus, in contrast to bad arguments, such paradoxes leave us in a situation in which we do not even know where to start in order to deal with them, especially if they concern some of our most basic assumptions. Such situations may lead into scepticism, as arguments of the Pyrrhonian and Academic Sceptics suggest; and they can be used for sophistical purposes or lead to nihilism, as the example of Gorgias shows. But by forcing us to pause in our usual assumptions and to stop our inquiry in the way it has proceeded so far, such paradoxes can also be fruitful for further philosophical investigation, since they make us rethink what we have taken for granted. And this seems to be the aim of Zeno's paradoxes. They are the beginning of a use of paradoxes of which Quine claims that 'more than once in history the discovery of paradoxes has been the occasion of major reconstruction at the foundation of thought' (1966, p. 3). Zenonian paradoxes prevent us from simply accepting common, seemingly unproblematic assumptions, such as that there is motion and plurality, and instead force us either to accept an opposing assumption, such as that what truly is is one and unmoved, or to show how we can avoid the problems raised by his paradoxes and thus keep these common assumptions. In any case, it will make our own assumptions and their implications clearer for ourselves.

David Sedley (2017) has recently suggested that the central claim of Zeno's *topos* paradox—'Everything that exists is somewhere; but if place is an existent, where would it be? Presumably in another place, and that place in another place, *and so on and so forth* (καὶ οὕτως εἰς τὸ πρόσω)', which Aristotle translated into an infinite regress—in fact consciously leaves open how he proceeds. According to Sedley, 'Zeno's book offered no more than skeletons of arguments, to be developed in oral debate according to his audience's responses' (2017, p. 29). If Sedley is correct, Zeno drafted his paradoxes in a way that could be adjusted to the respective opponent and thus worked as a method to show different opinions to be problematic.

Heraclitus' paradoxes can be seen as concerning the content of inquiry—they attempt to persuade us that consistent claims of the world give us merely a surface impression but not its deep structure; grasping this deep structure means embracing Heraclitus' paradoxes and agreeing that the reality we are trying to investigate is itself

Proceedings of the Aristotelian Society, *Vol.* cxxi, *Part* 2
doi: 10.1093/arisoc/aoab003

paradoxical. By contrast, Zeno's paradoxes can be seen as the beginning of a method for philosophical inquiry. Of course, we may think that showing the objects of our inquiry to be inconsistent and there being (alleged) inconsistencies in reality is also an important move in philosophy—and one that we not only find in Heraclitus, but also, for example, with scholars of dialetheism. Be this as it may, this is clearly a different use of paradoxes from the methodological one I think is at work with Zeno.

Furthermore, we may think that Zeno, like Heraclitus, is attempting to correct a mistaken assumption we hold about reality, and thus about a certain content: instead of assuming what truly is to be a plurality that moves and changes and is in places, we should assume a Parmenidean unmoved one, just as Heraclitus is trying to show that the stable and consistently describable objects in our reality are in fact undergoing constant change and can be grasped only with the help of contradictions. However, by making it clear that the paradoxes are not to be embraced, but are rather a stopping point which should make us rethink our assumptions, and by leaving at least some ends open to adjustment depending on the interlocutor, Zeno employs these paradoxes not as an account about reality but more generally as a tool for investigation.

In this way, Zeno shows what can be really fruitful about paradoxes in philosophy, and prepares the ground for using them as a method of inquiry that has been employed in philosophy ever since Zeno established them. Taking one's starting point from the opponent in order to show the opponent to be internally inconsistent without bringing in one's own position is a feature characteristic of the Platonic Socrates (and presumably a method further developed by the historic Socrates) (see also Bobzien 2020). Socrates frequently shows that the assumptions of his interlocutors are inconsistent in themselves or incompatible with some of their other claims (without thus necessarily having an agenda of his own at this point in the conversation).[62]

[62] For example, Socrates refutes Euthyphro's second attempt to give an account of piety as what is dear to the gods by showing that this account leads into what we can see as a paradox:

(1) Pious is what is dear to the gods.
(2) The gods do not agree on what is dear to them; the same thing can be hated by some gods and loved by others.
(3) Conclusion: the same thing is pious and not pious.

The Platonic Socrates, too, does not use the word 'paradox' for these occurrences; for example, what we call the 'Meno paradox' Socrates calls an *eristikos logos* (*Meno* 80e), an argument eager for strife.

Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
doi: 10.1093/arisoc/aoab003

Discussing paradoxes today, we do not necessarily think of a particular opponent; but the main ideas of Zeno's paradoxes—showing a position to be inconsistent from within this position, and showing that this is the case for some of the most basic philosophical assumptions—are still the mark of a strong paradox.

Given that a good deal of philosophy does not concern itself with experiments and does not proceed in a way that would allow us to derive corrections immediately from the empirical realm (in the way we may do with models in the sciences, where we can, for example, test predictions derived from our model), paradoxes are a central way of correcting our theories in philosophy. In this way, paradoxes may be seen as a kind of *via negativa*—they do not positively tell us anything about our object of inquiry, but they do tell us that our current assumptions about our object are problematic—either because they *are* or because they *seem to be*[63] mutually inconsistent with each other.

Accordingly, paradoxes stop us in our usual investigations and force us to reconsider basic assumptions we hold about our object of inquiry. In this way, they can act like an enzyme for further conceptual developments.[64] For example, in ancient times, Zeno's paradox of *topos* led Plato to spell out in his *Timaeus* why not everything that exists has to be in a place, while it made Aristotle, in his *Physics*, explicitly distinguish the different senses of *en* ('in') in the Greek language.[65] And Zeno's paradoxes of motion are employed by Aristotle in his *Physics* exactly in order to prove the possibility of a science of motion that can avoid these paradoxes (see Sattler forthcoming a). In this way, Zeno can be seen a founder of philosophical paradoxes as a method, as a fruitful and essential corrective for philosophical investigation.[66]

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[63] In case something has gone wrong in our reasoning.

[64] Even if some paradoxes may never be solvable—perhaps our conceptual tools may always display some problems in our attempt to understand the world.

[65] See Sattler (MS a, ch. 2).

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Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
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Proceedings of the Aristotelian Society, *Vol.* CXXI, *Part* 2
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