Why ‘Might’?*

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Abstract

Why do we use epistemic modals like *might*? According to Factualism, the function of *might* is to exchange information about state-of-affairs in the modal universe. As an alternative to Factualism, this paper offers a game-theoretic rationale for epistemic possibility operators in a Bayesian setting. The background picture is one whereby communication facilitates coordination, but coordination could fail if there’s too much uncertainty, since the players’ ability to share a belief is undermined. However, *might* and related expressions can be used to reveal one’s uncertainty, and exploit this to coordinate despite the lack of a common epistemic ground. The final result is a way to articulate a non-Factualist view of epistemic possibility modals that builds on their standard semantics.

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1 Coordination in Times of Uncertainty

Shiv and Logan want to spend time together over the coming weekend. They prefer to go to the beach if it will be sunny and to a café if it will be raining, but they will only go to either place if the other goes. Their predicament is the familiar one of a coordination problem (Lewis, 1969). In a variant known as the signalling game, Shiv and Logan solve their coordination problem by sending a signal, namely an utterance that reveals information initially available to only one of the players, for instance about whether it will be sunny or rainy (Lewis, 1969; Skyrms, 2010).

It’s relatively well-understood how Shiv and Logan coordinate if the relevant information is public, or can be made public through communication. Roughly, q is public information within a group just in case all members of the group believe that q, all believe that all believe that q, all believe that all believe that all believe that q, and so on *ad infinitum.* A stronger condition than public belief is public knowledge (also called common knowledge), which plays a central role in many areas of inquiry, in philosophy as well as elsewhere (Lewis, 1969; Fagin et al., 1995). The stronger condition won’t be needed for the topic of this paper, and I’ll focus instead on public belief.

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Sometimes however, beliefs fail to be public. Even worse, sometimes a belief cannot be made public, and communication of ordinary matters of fact doesn't help. Such may be the case, for example, when too much uncertainty undermines the possibility of sharing a belief, or so I will argue below. Is there still a way to coordinate then?

There are things to say. For example, even if she is uncertain, Shiv could venture one of the following.

1. (a) It might be raining.
   (b) Perhaps it will be raining.
   (c) Maybe it will be raining.

In her context, if Shiv utters a sentence in (1) she would be naturally understood as suggesting to go to a café for the weekend, almost as if she simply asserted that it will be raining, while at the same time hedging that assertion. She is raising the possibility of rain, as it were, and that may be enough to persuade Logan to stay indoors. It is very natural for Shiv to continue (1b) as in (2a), much less natural (and not even so coherent) to continue as in (2b).

2. (a) Perhaps it will be raining. Let's stay in.
   (b) Perhaps it will be raining. Let's go out.

Even if the speaker is not in a position to outright assert that it will rain, the interlocutors might still coordinate on going to a café by means of something less committal than that assertion.

The present paper offers an analysis of epistemic possibility modals in signalling theory. Epistemic possibility modal auxiliaries and adverbs, and expressions of close kin (*might, perhaps, maybe, possibly, probably*, etc.) are natural resources to employ in case coordination is challenged by the lack of a common epistemic ground. I explore the intuitive idea that using epistemic possibility modals is a way of hedging an assertion, or, as I will also say, of expressing uncertainty. I will show that strategic hedging can be helpful with coordination problems, despite critical failures of public belief.

But I also have a broader goal. The project of this paper touches on a larger question, roughly: what is the function of epistemic modality in speech? What purpose does it serve? David Lewis's (1969) classic work on signalling provides a plausible rationale for linguistic conventions writ large: they arise to support coordination. Here I claim, in kindred spirit, that the *raison d'être* of epistemic possibility talk is to support coordination in times of uncertainty.

This claim is controversial. For an alternative view may seem straightforward: take the standard semantics for *might*, according to which epistemic modals are quantifiers over possible worlds, and combine it with a form of descriptivism, according to which words serve to describe reality. The result is Factualism about epistemic modality:

epistemic modal talk serves fundamentally to describe some feature of reality, to say how some aspect of the world is. ... there is a certain class of facts, the facts about what is (epistemically) possible, or probable, or necessary. (Yalcin, 2011, 296)

In other words, people resort to saying 'Might ϕ' in order to state that there is a possible world compatible with what someone (typically, the speaker) knows or believes. Accordingly, we use *might* to describe a corner of the modal universe.

This conclusion is puzzling: ordinary lives and practices are rarely concerned with otherworldly state-of-affairs. When Logan has to decide for the café or the beach, how does information about remote possible worlds help him? Yalcin (2007, 2011) has some objections to Factualism about epistemic modality, which I'll discuss in due course. He rejects it, but in doing so he rejects the standard semantics for *might*. I think, however, that we simply shouldn't buy into Factualism, while keeping the standard semantics. The semantics is a story about what a sentence 'Might ϕ' represents or describes the world as being like. The challenge is to articulate a view about how to go from the semantics to the dynamics of belief change and decision-making prompted by an utterance of 'Might ϕ'. I will provide a non-Factualist view that does this. I will build upon the standard semantics, but I will recruit game-theoretic resources to explain how the use of epistemic possibility modals factors in inference and action. The function of 'Might ϕ' is not to describe modal reality, but to coordinate action when public belief fails.[1]

2 Failures of Coordination

My focus is on sentences in which an epistemic possibility operator takes wide scope, as (1) above. I leave a few comments about embedded *might* for the last section, and a discussion of epistemic necessity for another day. I begin in this section by presenting two scenarios in which coordination between two agents is challenged by their uncertainty, which undermines the possibility of publicly sharing a belief. I then derive conditions for the existence of a coordination equilibrium despite their uncertainty. In §3, I'll add communication to the agents' interaction, and in §4 epistemic modality. Sections 5 and 6 make formal sense of the idea that *might* expresses uncertainty, and that this can help with coordination. This provides a strategic rationale for epistemic modality. Finally in §7, I'll come back to the question of Factualism.

[1] The question of Factualism, as I understand it, is somewhat orthogonal to one's choice of semantic theory. However, some semantic theories that might be relatively better positioned to face the epistemological questions that are relevant here: for example, dynamic (Veltman, 1996; Yalcin, 2007; Willer, 2013), probabilistic (Lassiter, 2016; Moss, 2015), and proof-theoretic semantics (Incurvati and Schlöder, 2019). For lack of space, I do not broach the complicated general question how the game-theoretic account I present here maps onto these alternatives to standard semantics. On the more general question of Factualism, the present paper owes much to Price's (1990) work on negation.

I'll start by setting the stage. Consider the game of Table 1. Shiv (the column player) prefers to stay in, a, if Logan (the row player) stays in, and prefers to go out, b, if Logan goes out. Same for Logan. It's a coordination problem.

		S	
		a	b
L	a	$1,1$	$0,0$
	b	$0,0$	$1,1$

Table 1: Coordination Game

Given the numbers in Table 1, the players are indifferent as to staying in or going out so long as each does what the other does. Hence, they might as well toss a (fair) coin to coordinate, under some previously agreed upon stipulation: for example, they will stay in if and only if the coin lands heads. Suppose furthermore that only Shiv can see the coin: she will then report to Logan the outcome of the coin toss by sending a signal ('It's tails!'). Coordination comes easily (Rabin, 1990; Stalnaker, 2006).

We can imagine that Shiv and Logan coordinate by "pivoting" on the weekend being rainy or sunny, rather than the coin landing heads or tails, as well as on any other mutually exclusive propositions q and $\bar{q}$ (for all practical purposes, propositions are sets of possible worlds). The agents prefer a if q and b if $\bar{q}$, by previously agreed upon stipulation. If Shiv believes that q, she may signal so, and thereby share her belief with Logan. Neither Shiv nor Logan has reason to deceive the other, since either receives a positive payoff just in case the other does. Moreover, the players know this. Therefore, once a belief is shared by signalling, it becomes public: both believe it, both believe that both believe it, and so on. If q is public between Shiv and Logan, the rational (utility maximizing) choice for both is a. If $\bar{q}$ is public, the rational choice for both is b. This is a version of the traditional signalling game described by David Lewis (1969) in his foundational analysis of linguistic communication.[2]

Belief may fail to be public because people are distracted, uncurious, or unintelligent. The impasse here is "solved" by a sleight of idealization, of the kind that is ordinarily invoked in studies of rational agency (Fagin et al., 1995). I will assume that Shiv and Logan are Bayesian agents who do not suffer from such accidental shortcomings of rationality.[3]

[2] I am simplifying a bit. (1) Lewis talked about common knowledge, rather than common belief, but the stronger condition doesn't add much at this stage: we can coordinate on something false, so long as enough of us believe it. (2) Lewis assumed that the agents didn't speak a common language. He did so in order to show that conventional meanings can emerge as equilibria of the signalling game. I do not have this explanatory burden and drop the assumption. (3) Lewis worked with the notion of a Nash equilibrium, but the idea of solving the game by reference to an event with a given prior probability (the coin lands heads, the weekend will be rainy) leads in fact to a generalization known as *correlated equilibrium* (Aumann, 1987; Vanderschraaf, 1995), with which I work in the current paper.

[3] A Bayesian agent is someone whose credences are mathematically coherent, and who updates by Bayes rule. Logical omniscience is not needed for the analysis below, which makes it more plausible.

Belief may fail to be public because one or more players aren't opinionated, or even if everyone has the same opinion, it may not believed by everyone that everyone does. In these cases, sharing information leads to coordination, as we saw in the previous paragraph. Still, there are more complex cases of failure of public belief known to the literature, that are at least not obviously failures of rationality, and for which more information about ordinary matters of fact does not help. I'll discuss two structurally similar examples.[4]

The first scenario is a modified form of a popular story. Two generals will defeat the enemy only if they share the belief that they will attack, but since their communication channel is unreliable, they don't share that belief, and won't win.

> ***The Two Generals***. Two generals approach their common enemy from opposite directions. If a general attacks while the other doesn't, the attacker will be defeated with heavy casualties. If a general doesn't attack, nothing happens to him. If they attack simultaneously, they will win. However, neither can see what the other does, and they must rely on messengers crossing enemy lines to go from one general to the other.

Since there is no guarantee that a messenger reaches destination, even if a general sends 'I will Attack!', he will not attack without confirmation from the other general in the form of a message 'Copy. I will Attack!'. But there is no guarantee that the second message reaches destination either, for the same reasons. And so on for any finite number of messages. Therefore, neither general ever attacks, even after thousands of 'Attack' messages have been received by both sides! We can assume, without loss of generality, that each general thinks himself an attacker. But neither general believes that the other believes that both will attack. Failure of coordination results from the unreliability of the generals' communication channel, which prevents the belief that both will attack from becoming public.[5]

The second scenario is structurally similar in some relevant respects. This time, Shiv and Logan are planning for dinner. However, they are trying to coordinate on something vague.

> ***Vagueness***. Shiv and Logan just moved to a new town. They have been told about a great restaurant. If the restaurant is close, they would like to go there for dinner, but since they are quite tired, they prefer to eat in if the restaurant is far. They can either go out or stay in, but if either goes while the other doesn't, both will eat alone and be miserable.

Sometimes, a restaurant is neither definitely close, nor definitely far. There is no sharp boundary between close and far, and, in the "borderline area", Shiv and Logan may not

[4] A further step is taken by Harvey Lederman (2018), who argues that no belief can ever be public. This possibility need not detain us.

[5] The first version of the two generals' story appeared in Akkoyunlu et al. (1975). Gray (1978) is the most frequently cited source. There are many variants.

be very confident that the restaurant is close, so that, even if both believe that it is close, they do not believe that both believe that it is close: the other could easily have a different opinion. As for the two generals, coordination for Shiv and Logan is undermined by the failure of a particular belief to be public.

Vagueness has been linked to uncertainty, or lack of confidence, in various ways (Edgington, 1992; Smith, 2014; MacFarlane, 2016). In terms of credences, let's say that an agent i thinks that q just in case i has some positive degree of confidence that q is the case, that is, i assigns to q probability greater than the probability i assigns to its complement $\bar{q}$. In the borderline area, Shiv may think that the restaurant is close, although her confidence remains low: indeed, below a relevant threshold. Above the threshold, Shiv thinks that the restaurant is definitely close, or, as I shall say, she believes that it is close. At least notionally, belief is an all-or-nothing matter.

Of course, this is not an analysis of 'think' and 'believe', but a stipulation for describing a probability distribution over propositions. The stipulation plausibly fits with at least some informal uses of 'think' and 'believe', as we shall see. Accordingly, to think that q is to have some credence $> 1/2$ that q is the case, whereas to believe that q is for one's credence in q to be above a certain threshold, somewhere between 1/2 and 1. Credences have been understood differently in the philosophical literature, but for current purposes I set aside metaphysical questions about them (for discussion, see Weisberg, 2020).

What is a confidence threshold? In ordinary life, many factors contribute to one's level of confidence. In the context of the game, a qualitative characterization might help: i believes that q just in case i thinks that q, and thinks that others think that q as well. That is, one believes that q just in case one is confident enough that q is the case to think that others think that q as well. And so someone who thinks that the restaurant is close is uncertain (or, remains below the relevant confidence threshold) so long as she has a reasonable expectation that others are not of the same opinion. Similarly, a general who thinks they will attack but does not think that the other general thinks they will attack, is uncertain.

The terminology of thinking, believing, and being uncertain, may sound misleading. Does belief imply certainty? Not if 'certainty' means 'lack of Cartesian doubt', but it might if it means 'having high-ish confidence'. I mean the latter. Arguably this is a reasonable reading of at least some natural occurrences of the word 'belief', on which to say 'I believe that ϕ' indicates conviction. Likewise, there is a sense in which one could be confident that something is the case while believing that others disagree: but this just shows that there are other characterizations of the technical term 'confidence threshold' besides what I offered. That's fine. It's worth keeping in mind that Shiv and Logan, as well as the generals, are supposed to be rational epistemic peers: if they think that q expecting that someone else does not think that q, who is equally rational and in the same epistemic position, then they should be less confident about their judgement. I assume that they are. That's the sense of 'confidence' that is relevant here.

If confidence thresholds and shared attitudes line up in this way, the failures of coordination in The Two Generals and Vagueness can be seen as failures to share a belief about what both believe. The causes of these failures appear to differ: the unreliability of the general's communication channel in one case, and vagueness in the other. On reflection, these causes may not even be that far apart: language itself, in the borderline area, may be too unreliable a medium. I am not interested in pushing the analogies too far. It's enough to say that, given the descriptions of the two scenarios, the beliefs that aren't public cannot become so. It is built into the descriptions of the cases that more communication won't make the uncertainty disappear.

An abstract description of the two scenarios might help highlighting their structural similarities. Let there be at least three discrete states w_1, w_2, w_3 in the agents' environment. In w_1 the agents think that q, in w_3 they think that $\bar{q}$, and in w_2 an agent thinks that q while the other thinks that $\bar{q}$ (or vice-versa, but we need to worry about one case only, the other being analogous). The relevant possibilities for both generals are that they both attack, that neither does, and that only one does. If a general thinks he will attack, his problem is that he can't distinguish a world in which both attack from one in which the other doesn't. And while the relevant possibilities for Shiv and Logan are that the restaurant looks close to both, or far to both, or far to one and close to the other, if Shiv thinks that the restaurant is close, then her problem is to distinguish worlds in which the restaurant looks close to her and to Logan, from worlds in which the restaurant looks close to her but not to Logan.

We can represent this situation by letting the agents partition the logical space differently. A partition is a set of jointly exhaustive and mutually exclusive subsets of the space of relevant possibilities. Each agent i has their own partition, according to which worlds are sorted into q-worlds and $\bar{q}$-worlds. An agent i 'fails to distinguish' a world w from any w' that belongs to the same cell to which w belong, according to i's partition. Figure 2 represents two different partitions: the first general, or Shiv, on the left, and the second general, or Logan, on the right. In w_1 both think that q, in w_3 both think that $\bar{q}$, and they go separate ways in w_2. The use of partitions to represent an agent's mental state goes back to Aumann (1976).

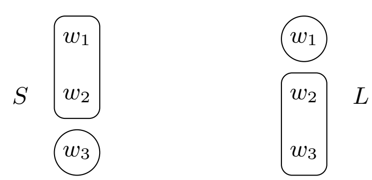

Figure 2: Different partitions of the logical space

Despite the lack of public belief, equilibria exist in which the players choose a if they believe that q, and choose b if they believe $\bar{q}$. As above, they prefer coordination, and they both know this. Each agent represents the beliefs that q and that $\bar{q}$, about themselves as well as the other, as in Figure 2. Therefore, each expects that their beliefs will be not aligned sometimes, and indeed they share a prior γ concerning the chance that this is the case. Let γ and δ be real numbers between 0 and 1:

$$p(w_1) = \delta(1-\gamma) \qquad p(w_2) = \gamma \qquad p(w_3) = (1-\delta)(1-\gamma)$$

That is, with chance γ the agents guess that they don't think what the other thinks, that is, they are in w_2. They assign complementary probabilities to the rest of the cases.[6]

Plausibly, an agent i chooses a only if i thinks q. If so, the expected utility of a for i is the sum of the utilities of choosing a while the opponent chooses a, and then b, weighted by the probabilities that these choices are made. Let u_i be i's utility function, and j be i's opponent.

$$eu_i(a) = p(q_i \;\&\; q_j) \cdot u_i(a,a) + p(q_i \;\&\; \bar{q}_j) \cdot u_i(a,b)$$

By table 1, according to which $u_i(a,b) = 0$ for either agent, and by Bayes rule, this simplifies to

$$eu_i(a) = p(q_i) \cdot p(q_j|q_i) \cdot u_i(a,a)$$

Consider S first, as in Figure 2. The probability that S thinks that q is $\delta(1-\gamma)+\gamma$, and the conditional probability that L thinks that q given that S thinks that q is just the proportion of cases in which L thinks q out of those in which L does: $\frac{\delta(1-\gamma)}{\delta(1-\gamma)+\gamma}$. Hence $eu_S(a) = \delta(1-\gamma)$.

Consider L. The probability that L thinks that q is $\delta(1-\gamma)$, and the probability that S thinks that q given that L thinks that q is just 1, for all cases in which S thinks that q are cases in which L does too. It follows that $eu_L(a) = eu_S(a)$. By the same reasoning, $eu_S(b) = eu_L(b) = (1-\delta)(1-\gamma)$. Therefore, the coordination equilibrium (a,a) obtains only if $eu_S(a) > eu_S(b)$ and $eu_L(a) > eu_L(b)$, and so, only if $\delta > 1-\delta$. Similarly, the (b,b) equilibrium obtains only if $1-\delta > \delta$.

I have assumed in the preceding paragraph that $\gamma \neq 1$. This seems reasonable, since γ represents the chance that an agent doesn't think in the way the other does. In other words, so long as misalignment in belief isn't inevitable, coordination equilibria exist despite uncertainty. While reasonable, this conclusion is not very strong. For even if the agents are rational, and reason as we just did that coordination equilibria exist, it doesn't follow that they will coordinate. The uncertainty may still be too much for them to make a move.

An upper bound on γ would help. Well, it seems plausible to say that a sufficient condition for i to choose a is that both players think that q, that is, both consider q more likely than not. Then, a is a best response for both if $p(q_i \;\&\; q_j) > 1/2$. Then a is played if $\delta(1-\gamma) > 1/2$, namely if

$$\gamma < 1 - \frac{1}{2\delta} \tag{CT1}$$

[6] Technically, probabilities are only assigned to sets of possible worlds, in order to uniformly represent an agent's credal state. Therefore, '$p(w)$' is short for '$p(\{w\})$', but we spare the painful details.

and by similar reasoning, b is played if

$$\gamma < 1 - \frac{1}{2(1-\delta)} \tag{CT2}$$

The confidence thresholds CT1 and CT2 suffice for coordination on (a, a) and (b, b) respectively. These inequalities presuppose that δ is neither 0 nor 1, but the generality of the conclusion is not lessened, for if one has credence 0 or 1 in a proposition, there should be no failure of public belief among rational Bayesians. The numerical values in CT1 and CT2 have been chosen somewhat arbitrarily (if the confidence threshold should be higher than .5, for instance if it should be .75, it can be made so without changing the outline of the argument), but the conclusion is representative of a general point. Two conditions characterize the existence of a coordination equilibrium in conditions of uncertainty. For the (a, a) outcome, for example, it must be that $\delta > 1 - \delta$ and $\gamma < 1 - \frac{1}{2\delta}$.

In this section I have presented two intuitive scenarios in which coordination may fail due to the agents' uncertainty. If the agents' confidence that some proposition q is the case, a belief that q can't be public: given the descriptions of the cases, the belief that both believe that q cannot be shared. Furthermore, I have given analytic proof of the intuitive claim that, if the chance γ that their thinking differently is not too high, coordination may still obtain: the inequalities CT1 and CT2 specify what "not too high" means.

3 Assertion

The existence of a coordination equilibrium despite uncertainty does not depend on the agents communicating, but follows from their sharing a certain prior under constraints of rationality. Communication is an efficient method to find an equilibrium. For this, it suffices that the agents have two signals, ϕ and ψ: roughly, 'I will Attack!' and 'I will Not Attack!', or 'The restaurant is close' and 'The restaurant is far'. For clarity, I will write ψ as $\neg\phi$, although nothing depends on the presence of an overt scope-taking operator of negation. To study how signalling facilitates coordination, I start with a basic semantic framework.

Let W be a set of (contextually relevant) possible worlds, and $[\![\cdot]\!]^g$ an interpretation function relative to a variable assignment g, that maps a signal (an utterance) to its semantic content, itself a function from possible worlds to truth-values. (Henceforth, I drop the superscript to avoid clutter. I also ignore context-sensitivity, to keep things simple.) I make two assumptions: the interpretation of ϕ is q, and that of $\neg\phi$ is $\bar{q}$.

$$[\![\phi]\!] = \lambda w.w \in q$$
$$[\![\neg\phi]\!] = \lambda w.w \in \bar{q}$$

With respect to Figure 3 below, it may appear as though ϕ "means" $\{w_1, w_2\}$ for S, but $\{w_1\}$ for L.

$$S: \left| \begin{array}{l} [\![\phi]\!] = \{w_1, w_2\} \\ [\![\neg\phi]\!] = \{w_3\} \end{array} \right. \qquad\qquad L: \left| \begin{array}{l} [\![\phi]\!] = \{w_1\} \\ [\![\neg\phi]\!] = \{w_2, w_3\} \end{array} \right.$$

One is therefore tempted to conclude that ϕ and $\neg\phi$ mean different things when uttered by one or the other agent. But this conclusion equivocates. The agents do speak the same language, according to which ϕ and $\neg\phi$ mean q and $\bar{q}$ respectively. This is so, for both of them. In addition, by analyzing the propositions q and $\bar{q}$ as sets of worlds, we (in the metalanguage) can make distinctions that the agents don't make: recall that S does not distinguish w_1 and w_2, and L does not distinguish w_2 and w_3, prior to communication. The possible world analysis allows us to represent the agents' partitions of the logical space geometrically, as in Figure 3, which is an aid to intuition. But the fact that this representation assigns different sets of possible worlds to q and $\bar{q}$, relative to different partitions of the logical space, does not imply that ϕ and $\neg\phi$ "mean different things" to different agents.

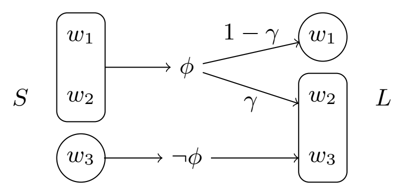

Figure 3: Signalling game under uncertainty

Sending signals ϕ and $\neg\phi$, interpreted as q and $\bar{q}$ respectively, reveals to the signal receiver what the signal sender thinks, and this is how they coordinate—though not always, for they might not think the same. Suppose first that S thinks that $\bar{q}$. Then she sends $\neg\phi$. Moreover, upon receiving $\neg\phi$, L thinks that S thinks that $\bar{q}$, for S has no reason to deceive L. Therefore, a signal $\neg\phi$ is how L can distinguish a possibility in which L thinks that $\bar{q}$ and S does too, from one in which L thinks that $\bar{q}$ and S doesn't. In addition, L thinks that $\bar{q}$ in any circumstance in which S does, hence if $\neg\phi$ is sent, S and L believe that $\bar{q}$. Moreover, since they can reason to this point, they believe that they believe that $\bar{q}$, and so on. Therefore, if $\neg\phi$ is sent, $\bar{q}$ is public.

If instead S thinks that q, she sends ϕ and L thinks that S thinks q, but L thinks either q or $\bar{q}$ with probability $1-\gamma$ and γ respectively. So, S signals ϕ if she believes that q, and signals $\neg\phi$ if she believes that $\bar{q}$, for in both cases she thinks that something is the case with enough confidence that she also thinks the other thinks the same. The remaining cases obtain with γ frequency and can be interpreted as those in which S wrongly guesses what L thinks.

It's convenient to call assertion the action of an agent who signals ϕ while believing that q, as opposed to merely thinking that q. That is, an agent asserts ϕ just in case she utters a sentence whose content she believes. Such use of the term 'assertion' may be justified by loose reference to Grice's (1975) Maxim of Quality. The Maxim prescribes, roughly, 'Assert only what you believe'. Reference to Quality is loose because I am employing 'belief' as a quasi-technical term, and while Grice's informal use is arguably compatible with mine, it might just be no more than that.

With communication (and relative to the game in Figure 3), S and L coordinate on doing a if ϕ is asserted, and on doing b if $\neg\phi$ is asserted. With γ chance, S utters ϕ as she thinks that q, but L thinks that $\bar{q}$, and therefore S chooses to do a while L chooses to do b. This may be as good as it gets, if their common language is indeed limited to ϕ and $\neg\phi$. Shiv and Logan, or the generals, will then have to learn to live with the occasional failures of coordination.

4 Hedging an Assertion

If the agents' language includes epistemic vocabulary, then they could make their uncertainty manifest. This could potentially matter because, intuitively, an utterance of 'Might ϕ' allows interlocutors to make finer-grained distinctions in speech about their attitudes towards ϕ, indicating their commitment or degree of confidence. For example, Shiv and Logan could attempt to coordinate in the Vagueness scenario by uttering one of (3).

3. (a) The restaurant might be close.
   (b) Perhaps the restaurant is close.
   (c) Maybe the restaurant is close.

Rather than an outright assertion that the restaurant is close, if they are uncertain they could more strategically reveal what they think by hedging. Using epistemic language to disclose one's attitude more precisely adds information that can be potentially exploited for coordination. Importantly, this is not semantically encoded information, but something that can be retrieved inferentially, by observing how language is used in the circumstances in which it is used. The agents reflect upon what has been said, and what else could have been said, to derive conclusions about each other, and figure out how to take action.[7]

Suppose that, besides the two signals ϕ and $\neg\phi$, the agents' language includes epistemic vocabulary. They can utter sentences such as 'Perhaps the restaurant is close' and 'Perhaps the restaurant is far', or 'Maybe I will Attack' and 'Maybe I will Not Attack'. First, we need to extend the semantics to the modality with respect to the game under discussion. The general idea is that a sentence $\Diamond\phi$ is true just in case ϕ is compatible with some body of information, indexed to some agent. We can intuitively say that $\Diamond\phi$ is true at a world w

[7] It may well be that the sentences in (3) are not equivalent in all respects, for reasons that have to do, for example, with the embeddability of *might* but not of *perhaps* under operators on semantic content such as *if* and *believe* (Incurvati and Schlöder, 2019). Everything I say is meant to apply to all epistemic possibility modals. The natural linguistic kind that is my target in the present paper is what Incurvati and Schlöder (2019) call weak assertion: a speech act typically performed by a speaker who utters a sentence whose main operator is an epistemic possibility modal.

just in case there is some agent i in the game who, in w, thinks that ϕ. With appropriate idealization about the rationality of the agents in the game, this is plausible enough. By this light, in w_2 one may truly say, 'It might be that ϕ and it might be that $\neg\phi$'. That seems the right thing to say when one is uncertain.

Formally, the semantic framework is expanded with a function $E : W \to 2^W$ from worlds to sets of them. Intuitively, Ew is the set of worlds (in the context) accessible to a world w, according to some proper epistemic accessibility relation. The standard semantics for $\Diamond$ is in terms of E.

$$[\![\Diamond]\!] = \lambda Q \lambda w. \exists w' \in Ew : w' \in Q$$

The interpretation function is compositional. Since $[\![\phi]\!] = \lambda w.w \in q$, it follows that $\Diamond\phi$ is true at w just in case there is a w' accessible from w such that q is true at w'. Finally, we should to say what 'accessible' means. Recall that a partition is a set of worlds that, intuitively, an agent cannot distinguish prior to communication. For convenience, we might simply say that a world in a partition is accessible to any other world in the same partition. In other words, w' is epistemically accessible from w just in case there is someone who cannot distinguish them.

On this semantic sketch, $\Diamond\phi$ is strictly weaker than ϕ, for $\Diamond\phi$ is true so long as at least someone thinks that ϕ is true. This proposal comes close to the influential discussion of Kratzer (2012), and von Fintel and Gillies (2011). Epistemic possibility modals are quantifiers over a (contextually restricted) set of possibilities that are salient to some agents. Use of $\Diamond\phi$ indicates a certain epistemic perspective on ϕ, namely, that ϕ is true as far as someone in the relevant group thinks (typically, the speaker). I skip the finer points of epistemic modality in natural language, which would require a discussion of phenomena that go beyond the scope of this paper, such as modal subordination and evidentiality (von Fintel and Gillies, 2010; Lassiter, 2016; Mandelkern, 2019; Roberts, 2019).

The semantics, however, is not be the end of the story. Contrary to Factualism, the function of *might* is not to state that there is a possible world of some kind, or that some relations obtain somewhere in the modal universe. Its function is to raise a possibility, albeit with some hesitation or uncertainty. I'll give a more precise characterization of this idea in the next section.

5 Inference

Following Stalnaker (1978, 1999, 2002), conversation is a cooperative enterprise whereby interlocutors narrow down the common ground (the set of possibilities that are "live" for them, at that point in the conversation). The task for the listener is to figure out which world is actual, given what the speaker said: the listener knows what else could have been said, and the information semantically encoded in the utterance, and infers what the speaker thinks the actual world is like. I will describe this process as a Bayesian inference to explain how an utterance of 'Might ϕ' raises a possibility, and expresses uncertainty.

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Suppose that, according to Shiv, it might be raining: she thinks it will rain, but she can't tell if Logan thinks so as well. In this situation, Shiv has ruled out the possibility that both her and Logan think that it will be sunny, but the belief that it will be raining is not public, for she is uncertain about what Logan thinks. Logan has to figure out which world is actual, according to Shiv. I will indicate with $cg(0)$ Shiv and Logan's common ground at time 0, that is, at the beginning of their interaction. The times to consider are 0 before communication, 1 after an utterance is sent, and 2 after the listener has guessed what the actual world is. It's helpful to understand this temporal progression as a listener-centered process, though something analogous might be going on for the speaker, insofar as the speaker can think through what the listener does.

Since Shiv believes that $\Diamond\phi$, she asserts so. An assertion is a proposal to update the common ground (Stalnaker, 1978). Assuming the proposal is accepted by the listener, a new common ground is defined by the update. Since $[\![\Diamond\phi]\!]$ is the information compatible with $\Diamond\phi$, we calculate the effect of an assertion of $\Diamond\phi$ by intersection.

$$cg(1) = cg(0) \cap [\![\Diamond\phi]\!] = \{w_1, w_2, w_3\} \cap \{w_1, w_2\} = \{w_1, w_2\}$$

Thus, an assertion of $\Diamond\phi$ rules out the possibilities incompatible with ϕ. This is all standard, both the semantics (Kratzer, 2012; von Fintel and Gillies, 2011), and the dynamics of assertion (Stalnaker, 1978). Accordingly, the effect of asserting $\Diamond\phi$ is typically informative: intuitively, an assertion of $\Diamond\phi$ tells you that the actual world is not a $\neg\phi$-world. What can the interlocutors deduce from this?

A simple hypothesis is that all possibilities compatible with what interlocutors mutually accept are equally likely to be the actual world. This general idea gives us a simple formula for base-rate probabilities for which world is actual.

$$\text{For all times } t \text{ and for all } w \text{ in } cg(t) : p(w) = \frac{1}{|cg(t)|}$$

Once the agents narrow down the common ground to $\{w\}$, the probability that w is the actual world is 1. In $cg(0)$, we have $p(w_1) = p(w_2) = p(w_3) = 1/3$. These priors are easily definable given the notion of common ground, conceived as a set, as in Stalnaker's classic work. It is implausible to assume that ordinary speakers would be able to volunteer these likelihoods, because they can't count possible worlds to begin with (especially since they cannot distinguish them sometimes), but this account gives us a natural description of the interlocutors' credal state, during a conversation.

Once the assertion of $\Diamond\phi$ is accepted, $cg(1) = \{w_1, w_2\}$, and therefore $p(w_1) = p(w_2) = 1/2$. At this point, the listener may reason as follows. Assuming the speaker is truthful, a world in $cg(1)$ is the actual world, but there are only two signals compatible with this information, namely ϕ and $\Diamond\phi$. The speaker wouldn't send ϕ unless she believes that q, in which case we are in w_1 (see Figure 3). And she didn't send ϕ. Therefore, she does not think that we are in w_1. Since w_2 is the only other world in $cg(1)$, w_2 must be the actual world according to the speaker.

This reasoning can be formalized in a Bayesian setting. At time 1, after the update, L has equal priors for the worlds in $cg(1)$. Moreover, L expects S to be truthful. Since a truthful speaker could send both ϕ and $\Diamond\phi$ in $cg(1)$, and nothing else, L holds even priors for the events that these signals are sent.

$$p(\phi) = p(\Diamond\phi) = 1/2$$

Finally, L expects S to send ϕ in w_1, not in w_2. For, an assertion that ϕ reveals the speaker's belief that q, but the speaker believes that q only in w_1. Therefore, L's conditional probability for the event that ϕ is sent, given that w_1 is the actual world, is nearly 1. As for $\Diamond\phi$, it will be sent with probability approximately 0 in w_1, since in w_1 we know that ϕ is sent with probability approximately 1. So, $\Diamond\phi$ will be sent in the only other world compatible with its content, namely w_2.

$$\begin{array}{ll} p(\phi|w_1) \approx 1 & \quad p(\Diamond\phi|w_1) \approx 0 \\ p(\phi|w_2) \approx 0 & \quad p(\Diamond\phi|w_2) \approx 1 \end{array}$$

The last step is for L to update by Bayes rule. The posterior probability that a world is actual is calculated by conditionalizing on the evidence, namely the observation that $\Diamond\phi$ was sent.

$$p'(w_2) = \frac{p(\Diamond\phi|w_2) \cdot p(w_2)}{p(\Diamond\phi)} \approx \frac{1 \cdot 1/2}{1/2} \approx 1$$

In this sense, by signalling $\Diamond\phi$ the speaker raises the possibility that w_2 is the actual world: she quite literally raises the probability that this is so!

Thus, the listener takes a guess at the speaker's credal state, estimating that it's one of uncertainty. We can also describe the inference in the ideology of possible worlds, but we must be careful: it would be wrong to say that by uttering 'Might ϕ' the speaker says (semantically, as it were) that the world is w_2 and not w_1. The difference between these worlds is one whereby both believe something, or only one does. Recall, however, that the speaker does not distinguish between these two worlds, prior to communication. Indeed, the distinction the agents can draw is between q and $\bar{q}$. This is a distinction about matters of fact. There is a further distinction about confidence levels that we use worlds to represent. The agents are sensitive to it, because their payoffs depend on it, but it is not tracked by the semantics of the language fragment they use. (Of course, such distinction is tracked by the semantics of other sentences, such as 'I am confident that ϕ', and so on. But we have not assumed that the agents have these sentences in their vocabulary.)

Nevertheless, the agents become aware of a distinction between confidence levels by using epistemic language. From the observation that 'It might be raining' was uttered, with the semantics it has, and given what else could have been uttered, the listener draws a conclusion about the speaker's confidence level. This conclusion may be plausible, but need not be more than that. It could well turn out that the speaker is unduly cautious, and that both believe that it will be raining. That is, the listener's inference is a defeasible one, and not a semantic entailment. Like ordinary pragmatic reasoning, the conclusion of the reasoning above is information not packaged in the semantic content of the sentence that was uttered by the speaker. The sentence ‘It might be raining’ is (semantically) not about the speaker’s, or anybody’s credal state: though information about one’s confidence level can be defeasibly derived from the observation that it was used.

Indeed, the listener’s inference as I described it is closely related to the derivation of a scalar implicature (Grice, 1975, 1989). Scalar reasoning too can be analyzed in Bayesian terms in a closely analogous way (Franke, 2011; De Jaegher and van Rooij, 2014; Franke et al., 2012; Lassiter and Goodman, 2017). My suggestion, however, is not that epistemic possibility modals trigger scalar implicatures: such claim is difficult to evaluate, for it does not seem that ‘Might ϕ’ is less informative than ϕ, in the way in which ‘At least two Fs are G’ is less informative than ‘Exactly two Fs are G’, other things being equal. (It’s not as if ‘Might ϕ’ is “less true” than ϕ.) It’s more promising to take scalar implicatures to be instances of more general inferential abilities, which may be seen elsewhere along the boundary between semantics and epistemology (Goodman and Stuhlmüller, 2013; Goodman and Frank, 2016).

In this section I began reaping the benefits of a Bayesian approach to epistemic modality, built on a game-theoretic setting. I have shown in what sense the use of *might* and related expressions can serve to raise a possibility, and how doing so may reveal information about an agent’s confidence level, that need not be packed into the semantics of ‘Might ϕ’.

6 Action

Finally, it’s time to turn to action. If ‘Might ϕ’ is asserted, how do agents react? I have assumed so far that agents do a if they think that q, and b if they think that $\bar{q}$. This is, however, a rather crude assumption, for we might want to say that uncertainty comes with indecision (cf. MacFarlane, 2016). Consider the following interaction:

> Logan: ‘I can’t find Fred’.
> Shiv: ‘Perhaps he’s in the garden’.
> Logan goes to the garden looking for Fred.

This seems to be a perfectly rational sequence of events. If Logan’s credence that Fred is in the garden is just below 0.5, Logan doesn’t think that Fred is in the garden, but may or may not go there looking for him. Logan is undecided. However, if Shiv were to tell Logan that Fred might be in the garden, then that could be a reason to go out. An account of how this works is straightforward, under the general assumptions I have already made regarding the rationality of linguistic agents.

Suppose that Shiv thinks that the restaurant is close and that Logan thinks that it is far. Shiv’s basic expectation is that she will go out and Logan will stay in. But then, they won’t coordinate. However, if she says ‘The restaurant might be close’, she thereby signals her uncertainty to Logan, and might therefore reasonably expect this to have some consequences. In particular, she will have expectations about Logan's reactions to her manifestation of uncertainty. And then she will have higher-order expectations about what her reaction is to what she expects Logan's reaction to her utterance is, and so on. The result is essentially an instance of iterated back-and-forth reasoning between speaker and listener.[8]

It helps to imagine this expectation-building process as speaker-centered (though a reflective listener could go through the steps as well). For simplicity, I focus on a case in which S thinks that q and that L thinks that $\bar{q}$, the converse being analogous (that is, we are in w_2). The process begins with the speaker manifesting her uncertainty by signalling 'Might ϕ'. Doing so, she expects that the listener would at least hesitate before taking action. I indicate with $p_i(x)$ the probability that agent i performs action x.[9] The process is broken down in various steps.

Step 0: Prior to communication. S thinks that q, so she chooses action a. There is no hesitation here. The speaker's prior is p_S at step 0.

$$p_S(a) = 1 \qquad p_S(b) = 0$$

At this stage, the listener also acts unhesitatingly and coordination fails (in w_2).

$$p_L(a) = 0 \qquad p_L(b) = 1$$

Step 1: Taking communication into account. The speaker signals $\Diamond\phi$ and expects that the listener wouldn't stubbornly choose action b, but rather be at least somewhat undecided. Plausibly, L's expected hesitation is a matter of taking a random action between a and b. The listener's prior is p_L at step 1.

$$p_L(a) = p_L(b) = 0.5$$

Step 2: Building expectations upon expectations. The speaker now reflects about what L's expectations are regarding S's action in response to L's priors. Each next step from now on is obtained by normalizing an agent's prior with the opponent's.

$$p'_S(a) = \frac{p_S(a)}{\Sigma_{i \in I} p_i(a)} = \frac{p_S(a)}{p_S(a) + p_L(a)} = \frac{1}{1 + 1/2} = 2/3 \approx 0.666$$

[8]There are two slightly different frameworks one could use to reconstruct the expectation-building process: iterated best response models of pragmatic reasoning (Franke, 2011), and rational speech act models (Frank, 2017). See Qing and Franke (2015) for a comparison between the two. The discussion in this section is inspired mainly by rational speech act models, but could be carried out in other settings with some adjustments.

[9]More precisely, $p_i(x)$ is a conditional probability, of doing x given that one thinks that q. But now I am simplifying both notation and discussion, by holding fixed that we are in w_2.

Step 3: As in the previous step, one's expectation for the opponent's action is obtained by normalizing the opponent's prior at the previous step with one's own.

$$p'_L(a) = \frac{p_L(a)}{p_L(a) + p'_S(a)} = \frac{1/2}{1/2 + 2/3} = 3/7 \approx 0.428$$

And so on. By proceeding in this way, the probability that the speaker chooses a tends approximately to 0.6, and the probability that the listener chooses a tends approximately to 0.4. Conversely for b. Throughout this process, the speaker thinks that q and the listener thinks that $\bar{q}$, and they make no mistake about this: they share no public belief about q or $\bar{q}$, at the beginning nor at the end of this process. (Probabilities get normalized at each step so that their beliefs remain complementary.)

Yet, just a few steps into expectation-building the agents' expected utility increases by a significant margin. Recall from §2 the value of an agent's expected utility for action a:

$$eu_i(a) = p(q_i \ \& \ q_j) \cdot u_i(a, a) = \delta(1 - \gamma)$$

This assumes that one gets a payoff for a just in case both think that q. But one's payoff for a increases, by the reasoning above, also in proportion to the probability that S thinks that q, L thinks that $\bar{q}$, but both do a. So we define a revised notion of expected utility, indexed by the number of steps taken in the back and forth reasoning.

$$eu_i^n(a) = eu_i(a) + p(q_S \ \& \ \bar{q}_L) \cdot p_S^n(a) \cdot p_L^n(a) \cdot u_i(a, a)$$

At Step 0, the listener doesn't think that q, and so $p_L^0(a) = 0$. Therefore, the overall expected utility of a at 0 is simply $eu_i(a)$, as in §2. But for all actions x, and for all steps n beyond 0, the agents' expected utility increases with respect to the sequence of steps.

$$eu_i^0(x) \leq eu_i^n(x)$$

The general conclusion is this. An assertion of 'Might ϕ' triggers a process of mutual reflection about what one expects the other will do, and higher-order expectations based on that, and so on. The base step is the intuitively plausible idea that the speaker, having signalled her uncertainty, expects the listener to react properly, by hesitating between a and b. Based on this, the speaker reflects on how to react to the listener's hesitation. As a result, speaker and listener's expected utilities improve.

Note that the agents need not have perfect powers of reasoning. They need not follow the induction to infinity. It suffices that one or two steps are taken, and already the use of 'Might ϕ' proves to be useful, in the very concrete sense that expected utility is higher. The strategic advantage of 'Might ϕ' can be shown well within a bounded conception of rationality. Public belief fails, but the chances of coordination improve by the use of *might*. This is why epistemic possibility modals are used in speech: this is a game-theoretic rationale for *might*.

7 The language game of *might*

A strong motivation for the account of *might* I presented comes from a question about the function of this and similar words. What do we use *might* for? What could be a reason why human language contains words of this kind? The truth-conditional semantics tells us that *might* is a quantifier over possible worlds. I don't think that it also gives us an answer to these questions. Factualism is the view that it does. But Factualism is not very convincing: we don't say 'Might ϕ' to tell each others how things are in the modal universe. Very few of us (mostly philosophers) care about that. On my view, we say 'Might ϕ' when we are not confident about ϕ, to express our uncertainty and thereby improve our chances to coordinate despite the failure of public belief.

Rather than building the case against Factualism, my main goal in this paper has been to offer an alternative explanation (among potentially many) of the role of epistemic possibility in our cognitive lives. The case against Factualism has been brought in some detail by Yalcin (2007, 2011, 2012), who then develops a form of Expressivism about epistemic modals on which *might* operates on the structure of the common ground, without contributing a proposition to it. Yalcin's account has some similarities with my proposal, and some interesting differences. I will conclude with some remarks on this.[10]

A big difference between the two accounts is, as I hinted above, that Yalcin does not think that 'Might ϕ' expresses a proposition, in the canonical Stalnakerian sense. My account is based on the premise that it does. Its standard semantics is important. As we saw, an update by $\Diamond\phi$ does contribute information to the common ground, by virtue of shedding possibilities: those in which it's definitely the case that $\neg\phi$. In this respect, an assertion of $\Diamond\phi$ is informative, just like the assertion of any declarative sentence (except tautologies). The information carried by $\Diamond\phi$ narrows down the agents' possibilities, so that an inference to the conclusion that the speaker is not very confident can be made, by a simple application of Bayes' rule.

Yalcin motivates his Expressivism, in part, by discussing Yalcin's puzzle. The puzzle is based on the observation that (4b) is infelicitous, but (4a) is not.

4. (a) Suppose it's raining and I don't know that it's raining.
   (b) # Suppose it's raining and it might not be raining.

This is unexpected if 'Might ϕ' meant, more or less, 'I don't know that not-ϕ'. However, we can explain the puzzle while remaining within standard semantics. What we need is a story about the interlocutors' mental states. The inconsistency in (4b) is one about the attitudes of the speaker. On the account I presented, one asserts that it's raining if one believes that it's raining, whereas one asserts that it might not be raining if one rules out

[10] Yalcin explicitly presents his account as a form of Expressivism, in the tradition of meta-ethical expressivists such as Gibbard (1990). Whether the account I presented is a form of expressivism depends on a discussion of the (rather large) debate on expressivism in meta-ethics. I leave this question for another time.

that it's raining, that is, thinks that it's not raining but need not think that others think so as well. Therefore, the conjunction 'ϕ and $\Diamond\neg\phi$' is an inconsistency at the level of the logic of attitudes, for it commits one to both believe and rule out that ϕ. This explanation is essentially the one given by Incurvati and Schlöder (2019) in a proof-theoretic setting. They put it as follows: "assenting to ϕ *and it might not be that* ϕ immediately reduces to both assenting and dissenting from ϕ" (Incurvati and Schlöder, 2019, 764). Since to assent and dissent, or (as I said) to believe and rule out, are incompatible attitudes, the infelicity of (4b) is predicted simply because one cannot suppose an inconsistency.

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