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attitudes and epistemics

1.1 expressivism and epistemics

The semantic theory of expressivism has been applied within metaethics to evaluative words like 'good' and 'wrong', within epistemology to words like 'knows', and within the philosophy of language, to words like 'true', to epistemic modals like 'might', 'must', and 'probably', and to indicative conditionals. For each topic, expressivism promises the advantage of giving us the resources to say what sentences involving these words mean by telling us what it is to *believe* these things, rather than by telling us what it would be for them to be true. This, in turn, absolves these theories of the burden of holding that there *is* any general answer to what it is for these sentences to be true. However, expressivism is famously subject to a deep and general problem about how to account for the meanings of *complex* sentences – a problem variously known as the 'Frege-Geach' or 'embedding' problem. In this paper I will be interested in whether there are reasons to think that the embedding problem looks less difficult for some of these applications for expressivism, than for others.

In particular, in this paper I will be interested in the prospects for expressivism about what I will call *epistemics* – a class which I take to include epistemic modals like 'might' and 'must', sentential adverbs like 'probably', adjectives like 'likely' and 'improbable', and so-called 'open' indicative conditionals like 'if the Fed doesn't intervene, then the economy will enter a deflationary spiral'. There are several reasons to be particularly interested in expressivism about epistemics, relating both to the philosophical payoffs of such a view, and relating to the technical prospects for making it work. In other work I've touched on the especially interesting philosophical payoffs which make expressivism about epistemics interesting; in this paper I will be interested primarily in evaluating the possibility that there are better prospects for making expressivism about epistemics work than there are for making expressivism work about other topics.

There are two main reasons why one might suspect that expressivism about epistemics will have better prospects than expressivism about many other topics, including in metaethics. The first, which has been observed by a number of other authors,[1] is that because of the distinctive syntax that attaches to many epistemics, natural language appears to place limits on their arbitrary embeddability. For example, many constructions make it awkward to embed an indicative conditional in the antecedent of another indicative conditional, and epistemic modals embedded in one another require awkward constructions like 'it might be the case that Jack must be coming to the party'. If you are impressed by these syntactic complications, then you may hope that a compositional semantics for epistemics only needs to make sense of a limited set of embedded constructions – and so might be off of the hook for the full force of the difficulties otherwise facing an expressivist semantics. In contrast, for example, since 'wrong' can embed in all of the same ways that 'large' can, noncognitivist expressivists in metaethics are committed to making sense of the unrestricted embeddability of 'wrong'. This might make you think that in contrast, the prospects for a compositional semantics for epistemic modals are rosier.

Unfortunately for this source of optimism, however, epistemics have a striking semantic uniformity across a range of different syntactic constructions. For example, though it is difficult to embed conditionals in the antecedents of other conditionals using 'if…then', it is not so difficult if we mix 'if' and 'only if', as in 'If Jack will come to the party only if Jill is invited, then we should invite Jill.' It's also relatively easy to quantify into conditionals, as in 'We should avoid inviting anyone to the party who, if she comes is going to be obnoxious'. And though it requires special awkwardness to embed modal verbs arbitrarily, epistemic adverbs like 'probably' can embed in all of the same ways that any other adverb can, and epistemic adjectives like 'likely' can embed in all of the same ways as any other adjective. So while there may be some leverage here to reduce the burdens of the expressivist about epistemics, it is likely to be limited.

In contrast, in this paper I will be interested in a set of *semantic* reasons why expressivism about epistemic modals might be thought to turn out to be easier to defend than expressivism about other topics. My main goals are to (1) illustrate one of the main constraints involved with the Frege-Geach problem for expressivism, (2) show that a prominent expressivist treatment of epistemics in the current literature runs afoul of this constraint in a predictable way, and (3) show that the answer to whether this is a fatal problem for this view is more subtle for the case of epistemics, than it is for other applications for expressivism. Finally, I'll argue (4) that this complication is still not enough, by itself, to make the way safe for expressivism about epistemics, and hence that expressivism about epistemics, if it is to succeed, needs the same sort of *general* solution to the Frege-Geach problem that is needed by any other expressivist theory.

[1] See, for example, Edgington [1995] and Bennett [2003].

1.2 expressivist semantics and the frege-geach problem

Expressivist theories seek to give an account of the meaning of some sentences, 'P', without having to say what it would be, for it to be the case that P. Instead of characterizing what 'P' means by saying what it would be for it to be the case that P, the expressivist seeks to characterize what 'P' means by saying what it is to *believe* that P.[2] So, for example, noncognitivist expressivism in metaethics claims that believing that stealing is wrong is an importantly different kind of state of mind than believing that grass is green. Whereas the latter state has 'mind-to-world' direction of fit and keeps track of the way things are in the world – namely, whether grass is green – the former state has 'world-to-mind' direction of fit, and has the role simply of motivating you not to steal. According to this view, you *can't* characterize the meaning of 'stealing is wrong' by saying how the world has to be, in order for it to be the case that stealing is wrong, because believing that stealing is wrong *isn't* taking the world to be any particular way. So we have to understand the meaning of 'stealing is wrong' by instead understanding the distinctive state of mind that you need to be in, in order to believe that stealing is wrong.

Another – in my view, more interesting – application of expressivism is to indicative conditionals. On this view, there is no particular way that you have to be confident that the world is, in order to believe that P→Q[3] – you just have to have a high *conditional* credence in Q, conditional on P. Since there is no way that you are confident that the world is, when you believe that P→Q, we can't read off what it is to believe that P→Q, from a story about what it is for it to be the case that P→Q. So instead, we should give the meaning of 'P→Q' by understanding what it is to *believe* that P→Q – namely, to have a high *conditional credence*.[4]

Epistemic modals are an interesting application for expressivism for many of the same reasons that indicative conditionals are – indeed, according to many theorists, the two are closely related.[5] The basic

[2] See particularly Blackburn [1984], [1993], Gibbard [1990], [2003], and Schroeder [2008] both for general expositions of expressivism, and particularly for discussion of the metaethical case. Some formulations of expressivism eschew using the word 'believe' in this general way, preferring to use it only for a certain kind of belief. They substitute 'think' or 'judge' for the general case. For reasons why this is unnnecessary and problematic, see Horgan and Timmons [2006] and chapter 5 of Schroeder [2010].

[3] Throughout the paper I'll use 'P→Q' as a convenient shorthand for 'if P, then Q', '◊P' for 'it might be that P', '□P' for 'it must be that P', and following Yalcin [2007], '▵P' for 'probably, P'. Officially, I'll treat these as shorthand for natural-language sentences, rather than as part of their own formal language. Similarly, I'll use '&', and '∨' as shorthand for 'and', and 'or', and '~' as a shorthand for natural language negation of any kind.

[4] See particularly Adams [1975], Gibbard [1981], Edgington [1986], and Bennett [2003]. See also Schroeder [forthcoming] for discussion.

[5] In particular, many theorists follow Kratzer [1986] in treating indicative conditionals as compounds built out of epistemic modals, together with a restriction. For an alternative treatment of epistemic modals in terms of conditionals, see Gillies [2004]. See also Willer [forthcoming].

idea behind expressivism about epistemic modals is that there isn't anything you have to be confident in, in order to believe that it might be that P – there just has to be something that you have to be *unconfident* in – namely, that ~P. So on this view, believing that ◊P isn't being confident in anything, and hence we can't characterize the meaning of '◊P' by saying what it is for it to be the case that ◊P. Rather, we should characterize its meaning by saying what it is to believe that ◊P – namely, to have a positive credence that P. So much for expressivism, and expressivism about epistemics.[6]

The Frege-Geach problem for expressivism is the problem of generalizing the nice things that it is easy to say about simple, unembedded, sentences, into a compositional theory of meaning which tells us not just what it is to believe that P for simple sentences, 'P', but for *arbitrary* sentences, composed with the connectives of propositional logic, with quantifiers, modals, tense, generics, attitude reports, and so on.

Several factors conspire to make this problem difficult. First, as I'll go on to explain in a moment and justify at length later, truth-conditional techniques don't generalize straightforwardly to an expressivist semantics, so expressivists need to come up with their own compositional techniques, effectively treating the meanings of connectives as functions from the mental states expressed by component sentences to the mental state expressed by the whole. Second, these techniques need to work equally well, no matter what attitude is expressed by component sentences, including 'mixed' cases in which one component sentence expresses one kind of state of mind, and another expresses a different one. And third, despite all of these extra constraints, the resulting account needs to explain all of the right semantic properties of complex sentences, which they have in virtue of the way they are composed – including, famously, among others, logical and inferential properties. Finally, fourth, the account should be conservative in its treatment of purely 'descriptive' sentences – the ones that express ordinary beliefs, which by themselves don't require a special treatment. Each of these factors plays a key role in what has made the Frege Geach problem so tricky for expressivists to navigate, but in this paper I will be primarily interested in the first. So why *are* truth-conditional techniques unsuitable for developing an expressivist compositional semantics?

As we saw, the ultimate goal of an expressivist semantic theory is to assign each sentence, 'P', to what it is to believe that P. This structure makes the ordinary compositional tools applied by standard truth-conditional theories inapt for expressivists, because the semantic values employed by truth-conditional theories obey the principles of *complementation*, *intersection*, and *union*. That is, the conditions under which '~P' is true are just the complement of those under which 'P' is true, the conditions under which 'P&Q' is true are just the intersection of those under which 'P' is true and those under which 'Q' is

[6] See particularly Price [1983], Schnieder [2009], Swanson [forthcoming], and Yalcin [2007], [forthcoming].

true, and the conditions under which 'P∨Q' is true are just the union of those under which 'P' is true and those under which 'Q' is true.[7]

This places expressivist semantics under a special burden, because in contrast, belief-states do *not* obey the principles of complementation, intersection, or union. That is, the conditions under which you believe that ~P are *not* the complement of the conditions under which you believe that P, the conditions under which you believe that P&Q are *not* the intersection of the conditions under which you believe that P and the conditions under which you believe that Q, and the conditions under which you believe that P∨Q are *not* the union of the conditions under which you believe that P and those under which you believe that Q. So an expressivist semantics can't work by simply applying the same sorts of techniques to its ultimate semantic values as a truth-conditional semantics can.[8]

This is the first, and more superficial, of two important reasons why truth-conditional techniques do not suffice for an expressivist semantics. In part 3 we'll introduce an important complication to this lesson and discover the deeper of these two reasons, but first we need to take a detour, in part 2, to spell out in a little bit greater detail precisely why states of belief do not obey the principles of complementation, intersection, and union, and exactly what sorts of exceptions these principles allow.

2.1 importation and exportation principles for belief

To simplify discussion, let's introduce some abbreviations. Where 'P' is a sentence, let '$[P]^{BF}$' be a term that refers to the set of possible individuals who believe that P. So $[P]^{BF}$ goes proxy for the property of believing that P, and hence is a fruitful way of modeling what an expressivist semantics needs to provide. It is possible that belief-properties may need to be individuated more fine-grainedly than sets of possible individuals, but nothing in this paper will turn on this idealization. This notation allows us to break each of the principles of complementation, intersection, and union into two components, which I call 'importation' and 'exportation' principles for obvious reasons:

~-exportation for belief	$[\sim P]^{BF} \subseteq Comp([P]^{BF})$	*(believing that ~P entails not believing that P)*
~-importation for belief	$Comp([P]^{BF}) \subseteq [\sim P]^{BF}$	*(not believing that P entails believing that ~P)*
&-exportation for belief	$[P\&Q]^{BF} \subseteq [P]^{BF} \cap [Q]^{BF}$	*(believing that P&Q entails believing each conjunct)*

[7] Note that in non-classical frameworks, the assumption that truth conditions are complementational for negation often gets dropped.

[8] For this insight into the source of the embedding problems facing expressivism, see particularly Hale [1993], Unwin [1999], [2001], and Schroeder [2008]. This characterization focuses on what makes it *hard* for expressivists to state compositional principles; in contrast, early discussions of the embedding problem simply assumed that they could not or pointed out that they had not. See particularly Ross [1938], Geach [1960], [1965], Searle [1962], and the response in Hare [1970].

&-importation for belief	$[P]^{BF}\cap[Q]^{BF}\subseteq[P\&Q]^{BF}$	*(believing each conjunct entails believing that P&Q)*
∨-exportation for belief	$[P\vee Q]^{BF}\subseteq[P]^{BF}\cup[Q]^{BF}$	*(believing that P∨Q entails believing some disjunct)*
∨-importation for belief	$[P]^{BF}\cup[Q]^{BF}\subseteq[P\vee Q]^{BF}$	*(believing either disjunct entails believing that P∨Q)*

We should start by observing that ~-exportation for belief and &-exportation for belief are very plausible principles. That is, it is very plausible that necessarily, if you believe that ~P, then you don't believe that P, and it is similarly very plausible that necessarily, if you believe that P&Q, then you both believe that P and believe that Q. But it is equally obvious that all four of the other principles are false.

Let's start with ~-importation first, since it is the clearest case. What ~-importation says, is that it is impossible to neither believe that P nor believe that ~P. Importantly, there are at least three different sorts of counterexamples to this claim. First, you can lack both beliefs because you don't have the requisite concepts to be able to consider whether P in the first place. If you aren't able to even consider whether P, then you aren't *able* to believe that P or believe that ~P, and so that's why you don't. Second, even if you have the requisite concepts to be able to consider whether P, you may never have actually considered whether P. In that case you don't believe that P – even if you would, if you were to consider it – and similarly for ~P. Third and finally, even if you have considered whether P, you may rationally remain agnostic as to whether P. So there are a number of ways in which you could believe neither that P nor that ~P.

In contrast, the case against &-importation for belief is more subtle. If you believe that P and you believe that Q, then you must have the requisite concepts to consider whether P, and the requisite concepts to consider whether Q – so our first sort of exception for the case of negation doesn't apply. Similarly, if you believe that P and you believe that Q, and you have considered whether P&Q, then it would appear to be some sort of breakdown in rationality for you to fail to believe that P&Q. So the case against &-importation for belief is based on the possibility of failing to consider whether P&Q – a kind of failure to *reflect* – and the possibility of this kind of *irrationality*. The difference between these kinds of exception to &-importation for belief will be important in what follows, because some philosophers have considered &-importation for belief to be a bullet worth biting, and because it will eventually be important to distinguish different kinds of counterexamples to our principles.

In contrast to the cases of negation and conjunction, the case for disjunction comes apart in both ways. Taking ∨-importation first: without loss of generality, if you believe that P but do not have the requisite concepts to consider whether Q, then you don't believe that P∨Q. Similarly, if you believe that P and have the requisite concepts to consider whether Q, but have never considered whether P∨Q, then you don't actually believe that P∨Q, even if you would if you considered it. And finally, if you are sufficiently irrational or confused about logic (perhaps for as non-culpable a reason as that you haven't studied your Grice), you may believe that P and/or believe that Q and consider whether P∨Q, and still fail to believe it. The case against ∨-exportation is even more straightforward. *Normal* cases of believing that P∨Q – every case in which pragmatic principles make it acceptable to say that P∨Q – are cases in which you are agnostic about P and agnostic about Q, and hence believe neither. Such cases need involve no failure either of reflection or irrationality – only the absence of complete information, which is, after all, our normal predicament.

2.2 importation and exportation principles for other attitudes

As noted earlier, one of the chief virtues of expressivism is that it allows us to characterize the meaning of a sentence, 'P', without saying what it is for it to be the case that P. This is because on an expressivist view, believing that Jack will come to the party and believing that Jack might come to the party are not a matter of having the same attitude toward different contents; rather they are distinct attitudes toward the same content. So there need be no such thing as the content of the belief that it might be that P. Small comfort to the expressivist, then, if hoping that Jack will come to the party and hoping that Jack might come to the party need to be understood as the same attitude toward distinct contents, or if supposing that Jack will come and supposing that Jack might come need to be understood as the same attitude toward distinct contents.

So expressivists really owe us an account of what it is to ϕ that P, for arbitrary attitude verbs 'ϕ' – not just an account of what it is to *believe* that P. This can be fruitfully thought of as a special case of the Frege-Geach problem more generally, because '*x* ϕs that P' is just one more kind of complex sentence in which 'P' can figure. Because this problem is ultimately very important for the fate of expressivism, it is worth pausing to see how or whether our observations about the importation and exportation principles for belief generalize to other attitudes.

So in general, let $[P]^{\phi}$ be the set of possible individuals who ϕ that (whether) P. So $[P]^{\text{HOPE}}$ is the set of possible individuals who hope that P, $[P]^{\text{DESIRE}}$ is the set of possible individuals who desire that P, $[P]^{\text{WONDER}}$ is the set of possible individuals who wonder whether P, and $[P]^{\text{SUPPOSE}}$ is the set of possible individuals who suppose that P. The exportation and importation principles that we considered for belief are just special cases of the following general principles:

~-exportation for ϕ	$[\sim P]^{\phi} \subseteq Comp([P]^{\phi})$
~-importation for ϕ	$Comp([P]^{\phi}) \subseteq [\sim P]^{\phi}$
&-exportation for ϕ	$[P\&Q]^{\phi} \subseteq [P]^{\phi} \cap [Q]^{\phi}$
&-importation for ϕ	$[P]^{\phi} \cap [Q]^{\phi} \subseteq [P\&Q]^{\phi}$
∨-exportation for ϕ	$[P \vee Q]^{\phi} \subseteq [P]^{\phi} \cup [Q]^{\phi}$
∨-importation for ϕ	$[P]^{\phi} \cup [Q]^{\phi} \subseteq [P \vee Q]^{\phi}$

Interestingly and very importantly, the plausibility of these principles fares differently for different attitudes. For example, though ~-exportation was a compelling principle for belief, it is hopelessly false for wondering and supposing. You can easily (and rationally!) suppose both that P and that ~P for the sake of argument. And plausibly, if you wonder whether P, so far from not wondering whether ~P, you *ipso facto* wonder whether ~P. It follows that the plausibility of ~-exportation for belief must turn on special features of *belief*, rather than on any general feature of attitudes as such.

Similarly, though &-exportation was a compelling principle for belief, it is hopelessly false for wondering and hoping. For example, it is rational to hope that P&Q without hoping that P, if you place high conditional odds on ~Q given P, provided that P&Q would be a particularly good outcome but P&~Q would be particularly bad. It is even possible to hope that P&Q without hoping that P *or* hoping that Q – if these conditions are symmetric. Moreover, not only is all of this perfectly possible, it is perfectly possible even if we restrict our attention to perfectly rational agents who have considered each of these possibilities.

&-exportation also fails for wondering. For example, suppose that Jill tells you, 'Jack will come to the party, but I'm not sure if I'll be able to join him – it depends on what time I get off of work'. Then you may wonder – perfectly rationally – whether Jack and Jill will both make it to the party, without wondering whether Jack will make it. Of course, in this case you do wonder whether Jill will make it to the party, so unlike with hoping, it isn't possible to wonder whether P&Q without either wondering whether P or wondering whether Q. So &-exportation is false for both hoping and wondering, but wondering at least obeys the weaker principle of &→∨-exportation, though hoping does not:

&→∨-exportation for ϕ	$[P\&Q]^{\phi} \subseteq [P]^{\phi} \cup [Q]^{\phi}$

Moreover, the plausibility of other exportation/importatation principles can also vary with the attitude that we consider. For example, when we discussed &-importation for belief, the only kinds of exceptions that we came up with had to do with either the failure to reflect, or an arguably quite strong sort of irrationality. And this made the case against &-importation for belief seem at least relatively subtle. But for wondering and hoping, it is easy to assemble a much wider range of exceptions which don't trade on either irrationality or a failure to reflect. For example, you may wonder whether P and wonder whether ~P without wondering whether P&~P. Similarly, if P&~Q and ~P&Q are both good outcomes, but P&Q is a bad outcome, then if you think P&~Q and ~P&Q are both much more likely than P&Q, then it makes rational sense for you to hope that P and hope that Q, without hoping that P&Q.[9]

Though we already noted that &-importation was implausible for belief, these last observations are important, because given the restricted range of exceptions to &-importation for belief, some philosophers have been tempted to endorse it. What the cases of hoping and wondering show, is that the features which make biting the bullet on &-importation for belief seem at least palatable do not extend to other attitudes, in general. I won't survey all of the possible combinations of attitudes with exportation/importation principles, but the foregoing remarks suffice to show that different attitudes are governed by different principles, and in particular that ~-exportation and &-exportation are not obeyed by every attitude, and that some attitudes provide even clearer exceptions to &-importation than belief does. These facts have important consequences for the semantics of attitude-verbs, and I'll return to this topic in 6.1.

3.1 gibbard's method of world-norm pairs

So far we've seen why expressivists cannot directly apply the tools of truth-conditional semantics – neither to their account of what it is to believe that P, for arbitrary 'P', nor to their accounts of what it is to hope that P, to suppose that P, or to wonder whether P. My next aim is to illustrate this general lesson by showing that a familiar attempt to apply truth-conditional techniques to an expressivist semantics is entirely compatible with the prediction that ∨-exportation is valid for cases of 'mixed' disjunctions, and in particular that a straightforward and natural interpretation of this framework actually predicts this conclusion.

The account that I'm going to use to illustrate this point is the formal method of world-norm pairs employed by Allan Gibbard in *Wise Choices, Apt Feelings*. Gibbard's system is illustrative both because it demonstrates a potential loophole in my explanation of why expressivists can't employ truth-conditional techniques, and because it has, I think, been poorly understood by a range of commentators exactly what does the work, in his treatment of the Frege-Geach problem. For purposes of this section I will be

[9] Famously but much more controversially, Michael Bratman [1987] has used similarly structured cases (his 'video game' case), to argue that it can even be rational to *intend* things that you know to be incompatible.

distinguishing between the *method of world-norm pairs*, which is the formal treatment that many commentators take to be constitutive of Gibbard's solution of the Frege-Geach problem, and Gibbard's solution full-stop, which I take it (and will be arguing) goes beyond (and needs to go beyond) the formal technique of the method of world-norm pairs.

Gibbard's view is an expressivist one, because he holds that normative beliefs and non-normative beliefs are two fundamentally different kinds of state of mind. Normative beliefs, including beliefs about what is wrong and what is rational, are what Gibbard calls *norm-acceptance* states, which both contrast with ordinary descriptive beliefs in their psychological role and evolved for different reasons. The basic idea of the method of world-norm pairs is that just as we can represent ordinary belief-states by sets of worlds – the set of worlds that are consistent with that belief-state – we can also represent states of pure *norm-acceptance* with sets of *norms* – the norms that are consistent with that state of norm-acceptance.

The trick to Gibbard's compositional semantics, then, is to associate each sentence 'P' with a set of world-norm *pairs*, $|P|$ (my notation, not his). The basic idea of the trick is while there are *special cases* of sets of world-norm pairs that can be taken to characterize ordinary descriptive belief and pure states of norm-acceptance, respectively, sets of world-norm pairs provide a *more general* way of characterizing mental states, not all of which fall under these two special cases. The special case corresponding to ordinary descriptive beliefs are sets of world-norm pairs which are *norm-invariant*, meaning that every world that gets paired with any norm in the set also gets paired with every other norm ($\forall w{:}\exists n{:}\langle w,n\rangle \in |P| \rightarrow \forall n{:}\langle w,n\rangle \in |P|$). Because norm-invariant sets are equivalent to sets of worlds, they can be used to characterize pure states of ordinary descriptive belief, expressed by non-normative sentences.

The counterparts of norm-invariant sets are *world-invariant sets* ($\forall n{:}\exists w{:}\langle w,n\rangle \in |P| \rightarrow \forall w{:}\langle w,n\rangle \in |P|$). Because world-invariant sets are equivalent to sets of norms, they can be used to characterize pure states of norm-acceptance, expressed by normative sentences. But since not all sets of world-norm pairs are either norm-invariant or world-invariant, the method of characterizing states of mind by sets of world-norm pairs is actually more general, and so it can be used to characterize the states of mind associated with complex sentences which have both normative and non-normative parts.

With sets of world-norm pairs in hand, Gibbard's compositional semantics (for propositional connectives) is simple. It just applies the techniques of complementation, intersection, and union to these sets:

negation $|{\sim}P|=\text{Comp}(|P|)$
conjunction $|P\&Q|=|P|\cap|Q|$
disjunction $|P\vee Q|=|P|\cup|Q|$.
logic By subsets: $P_1\ldots P_n \vDash C$ iff $|P_1|\cap\ldots\cap|P_n|\subseteq|C|$.

Because the method of world-norm pairs is just a point semantics, it generates all and only the classical entailments. This has made it seem to a number of commentators that Gibbard's account solves the Frege-Geach problem *simply in virtue of* applying the method of world-norm pairs. I'll show in section 3.3 why this is not the case. But first, I need to clarify why Gibbard's account doesn't run directly afoul of my argument from section 1.2 that expressivists *can't* apply the truth-conditional techniques of complementation, intersection, and union.

3.2 exploiting a loophole?

In section 1.2 I claimed that expressivists can't simply apply the methods of complementation, intersection, and union, because that would lead them to validate all six of our importation and exportation principles. But Gibbard's method of world-norm pairs *does* apply the methods of complemenation, intersection, and union, and it does not validate all six of our importation and exportation principles. (Though it does validate four – all but ~-importation and ∨-exportation – at least the two that it doesn't validate are the two that are most obviously false.) So what went wrong with the argument?

There is no inconsistency, here – just a lesson about the scope of the argument in section 1.2. That argument didn't show that an expressivist semantics couldn't apply truth-conditional techniques *full-stop*; it just showed that it can't apply them to its *ultimate semantic values* – belief-states. The loophole exploited by Gibbard's method of world-norm pairs is to characterize belief-states *indirectly*. Norm-invariant sets of world-norm pairs are not themselves ordinary descriptive belief states, but they can be *associated* with ordinary descriptive belief states in a general way that allows us to use them to characterize states of ordinary descriptive belief. But even though the sets of norm-invariant world-norm pairs obey the principles of complementation, intersection, and union, the ordinary descriptive belief states which they characterize do not obey (all of) these principles. Similar points go for world-invariant sets of world-norm pairs – these are not themselves pure norm-acceptance states, but they can be used to *characterize* states of pure norm-acceptance, so that even though the sets obey (all of) these principles, the norm-acceptance states themselves do not.

In the next section, however, we'll see that this is not much of a loophole, because nothing about the method of world-norm pairs guarantees that the importation and exportation principles don't hold for *mixed* sentences.

3.3 how to validate ∨-exportation, lesson I

The problem with the idea that the method of world-norm pairs solves anything by itself, is that by itself, the method of world-norm pairs provides us with a formal system which outstrips any interpretation that we have given it. In constructing the method of world-norm pairs, we introduced sets of world-norm pairs by explaining how two of their special cases were to be interpreted – norm-invariant and world-invariant sets – without saying anything at all about how other cases were to be interpreted. The method of world-norm pairs, by itself, therefore, actually tells us nothing at all about what attitude is expressed by complex sentences with both normative and non-normative parts, unless we supplement it with a general way of interpreting the significance of arbitrary sets of world-norm pairs. And unfortunately, it is easy to show that on at least one of the most natural ways to interpret the system, it actually validates ∨-exportation in cases where one disjunct is normative and one is non-normative.

To see what this interpretation is and why it is a natural one, recall that Gibbard starts by explaining each of two ordinary states of mind – pure descriptive belief states, and pure states of norm-acceptance. Each of these states of mind evolved separately, and for different purposes, he claims. But if these are the only states of mind there are, then each person's total state of mind can be characterized by the combination of a descriptive belief state and a state of pure norm-acceptance – and hence by a pair of set of worlds and a set of norms, <W,N>.

The problem, however, is that if 'P' is normative and 'Q' is non-normative, then believing that P∨Q cannot be neatly factored into both having a certain descriptive belief and being in a certain norm-acceptance state. It's a matter, in the words of Simon Blackburn [1988], of being *tied to a tree* between a descriptive belief state and a norm-acceptance state. But since we're assuming that the only states of mind there are, are ordinary descriptive beliefs, ordinary states of norm-acceptance, and their combinations, there is no state of mind to be this state of being tied to a tree. So there is no categorical state of mind of believing P∨Q but not either believing P or believing Q. Since that state can't be either a state of purely descriptive belief, a pure state of norm-acceptance, or a combination of the two, on this view there is no room for such a state.

It's important to note that simply paying attention to the logic induced by the apparatus of world-norm pairs obscures this fact. And that is because we get the same logical connections among beliefs if we identify believing that $P \vee Q$ with the *disjunctive* state of either believing that P or believing that Q. In fact, this can easily be made more precise. To see how, all that we need to do, is to find a natural way of mapping sets of world-norm pairs $|P|$ to belief-states $[P]^{BF}$, under the assumption that each state of mind can be wholly characterized by a <W,N> pair. The following does the trick:

defn $\langle W,N\rangle$ *accepts* $|P|$ just in case $\forall w \in W{:}\forall n \in N{:}\langle w,n\rangle \in |P|$.
defn $x \in [P]^{BF}$ just in case *x*'s total state of mind, $\langle W,N\rangle^{x}$, accepts $|P|$.

The preceding characterization does exactly what we want, because when 'P' is a purely non-normative sentence, and hence $|P|$ is norm-invariant and therefore equivalent to a set of world, the definition tells us that believing that P only constrains W so that W is a subset of the set of worlds which characterizes $|P|$. Similarly, when 'P' is a purely normative sentence, it follows from the definition that 'P' only constrains N and in precisely the same intuitive way. Moreover, if 'P' is a conjunction of normative and non-normative sentences, it is easy to see from the definition that believing that P constrains both W and N, and in exactly the ways that they are constrained by each conjunct. So it is easy to see that this characterization of beliefs makes the right predictions about pure descriptive beliefs, that it makes the right predictions about states of pure norm-acceptance, and it treats conjunctions correctly.

More generally, it is easy to prove from these definitions, as would be expected from a possible-worlds characterization of belief, that if you believe the premises of a valid argument, then you believe its conclusion:

Observation 1 If $P_1 \ldots P_n \vDash C$, then $[P_1]^{BF} \cap \ldots \cap [P_n]^{BF} \subseteq [C]^{BF}$.

So this makes our definitions the 'right' way to characterize belief-states, under the assumption that anyone's total state of mind consists only in a pure descriptive belief state and a state of pure norm-acceptance, and shows that nothing about this characterization of belief undermines the claims about logic that come with the method of world-norm pairs.

But unfortunately, it is also easy to prove that this account gets the wrong results for mixed disjunctions, and in particular, that it predicts that if 'P' is normative and 'Q' is non-normative, then it is impossible to believe that $P \vee Q$ without either believing that P or believing that Q:

Observation 2 If $|P|$ is norm-invariant and $|Q|$ is world-invariant, then $[P \vee Q]^{BF} \subseteq [P]^{BF} \cup [Q]^{BF}$.

The proofs of these observations, which are both straightforward, are in Appendix 1, but we've already seen the intuitive reason to expect Observation 2. It follows because if P is normative and Q is non-normative, there is no state of pure descriptive belief that someone who believes that P∨Q is guaranteed to be in, and there is no state of pure norm-acceptance that she is guaranteed to be in. So *ipso facto* there is no combination of a descriptive belief state and state of pure norm-acceptance that she is guaranteed to be in.

Let me summarize what we've seen in this section, so that the lesson is not misinterpreted. I am not claiming that this is the *right* interpretation of Gibbard's solution to the Frege Geach problem, so I do not mean to be saying that Gibbard himself positively predicts that someone who believes that P∨Q has to either believe P or believe Q, if 'P' is normative and 'Q' is non-normative. What I am pointing out, is that the method of world-norm pairs does not suffice to avoid this problem, and in particular that this problem cannot be solved unless there is a distinct, *third* kind of state of mind, to correspond to mixed disjunctive beliefs – the kind of state that Simon Blackburn [1988] has evocatively referred to as being 'tied to a tree'.

The bottom line is that once we take the expressivist plunge and hold that believing that P and believing that Q are two quite different kinds of state of mind, no formal trick gets us off the hook for saying what it is to believe that P∨Q. Plausibly, it can't be just the same kind of state as believing that P, and it can't be just the same kind of state as believing that Q. But as we've seen, it can't just be the state of either believing that P or believing that Q, either. So it must be some other kind of state. Saying what this state is, is a *philosophical* problem, not a purely technical one, and no truth-conditional technique by itself is going to provide us the answer. This is the second, and deeper, reason why truth-conditional techniques do not suffice for expressivist semantics.

4.1 yalcin's semantics for epistemics

In part 3 I drew out the deeper reason why truth-conditional techniques do not suffice for expressivist semantics by considering Gibbard's method of world-norm pairs. We now turn to a prominent semantics for epistemic modals from the recent literature offered by Seth Yalcin [2007], which Yalcin [2007], [forthcoming] interprets as a kind of expressivist semantics. As we'll see, Yalcin's semantics has much in common with Gibbard's method of world-norm pairs, and it makes predictions that are very similar to the interpretation of Gibbard offered in section 3.3, and for very similar reasons. In this section and the next we'll consider the basic semantics for epistemic modals and conditionals stated in Yalcin [2007], which I'll simply refer to as *Y*. Yalcin [forthcoming] makes one small modification to this view, which results in an improvement and we'll briefly consider in section 4.3. But for now we'll focus on the basic view.

Officially, Yalcin describes his semantics in terms of an assignment of sentences to 'truth' values relative to contexts of utterance, worlds, and a third parameter which he calls *information spaces*. In order to avoid the question of whether he means *true* by 'true', however, I'll adopt the equivalent characterization of taking *Y* to map sentences to sets of world/information-space pairs, relative to contexts of utterance. I'll use '$|P|_c$' – my notation, not Yalcin's – to denote the set of world/information-space pairs expressed by 'P' in context c, and modifying my terminology to allow for context-dependence, I'll use '$[P]_c^{BF}$' to denote the set of possible individuals who satisfy 'x believes that P' relative to c.[10] In what follows I'll use 'i' and 'j' to range over probability spaces, 'c' to range over contexts, and 'w' and 'v' to range over worlds, sometimes dropping 'c' when it is clear from context.

Just as for Gibbard, some sets of world-norm pairs were *norm-invariant* and hence equivalent to sets of worlds, so that they could be associated with ordinary descriptive beliefs and hence with non-normative sentences, for Yalcin some sets of world/information-space pairs are *information-invariant*, and hence equivalent to sets of worlds. These can be associated with ordinary beliefs, and hence with sentences that are free of epistemics. Similarly, just as for Gibbard, some sets of world-norm pairs were *world-invariant* and hence equivalent to sets of norms, for Yalcin some sets of world/information-space pairs are world-invariant and hence equivalent to sets of information-spaces. Yalcin's semantic clauses for epistemics guarantee that every sentence with a widest-scope epistemic ('might', 'must', 'probably', or 'if…then') is associated with a world-invariant set.[11]

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[10] It's hard to give an intuitive characterization of the significance is, of expressing a particular set of world/probability-space pairs. This is no different than the case of the method of world-norm pairs. The best way to think about it, I think, is that their significance is given by the fact that each determines a unique state of mind (though a method introduced below), but applying the techniques of complementation and union to them doesn't create the same problems as applying those techniques to states of mind directly. The only hitch with this way of interpreting what they mean, is that there are distinct sets of world/probability-space pairs which determine the same state of mind – so it is not exactly accurate to say that their significance is exhausted by the states of mind that they determine.

[11] Since it it is the world-invariance of sentences with widest-scope epistemic modals that interests me here, rather than their details, I won't need to actually state Yalcin's semantic clauses for the modals. But for reference, here are the official clauses from Yalcin [2007] for '◊', '□', and '▵', as reformulated in my notation (for explanation of the notation, see section 4.2):

$$|\Diamond P|_c=\{<w,i>:\exists x\in\pi_i:\forall v\in x:<v,i>\in|P|_c\}$$
$$|\Box P|_c=\{<w,i>:\forall x\in\pi_i:\forall v\in x:<v,i>\in|P|_c\}$$
$$|\triangle P|_c=\{<w,i>:Pr_i(\{v:<v,i>\in|P|_c\})>½\}$$

It is easy to see that none of these formulas constrain w in any way. Hence every sentence with a widest-scope '◊', '□', or '▵', and all sentences composed out of such sentences by '~', '&', and '∨', expresses a world-invariant set of w-i pairs.

Similarly, just as with the method of world-norm pairs, Yalcin applies the truth-conditional techniques of complementation, intersection, and union:

negation	$\vert{\sim}P\vert_c = \text{Comp}(\vert P\vert_c)$
conjunction	$\vert P\&Q\vert_c = \vert P\vert_c \cap \vert Q\vert_c$
disjunction	$\vert P\vee Q\vert_c = \vert P\vert_c \cup \vert Q\vert_c$
logic	By subsets: $P_1 \ldots P_n \vDash C$ iff $\forall c{:}\, \vert P_1\vert_c \cap \ldots \cap \vert P_n\vert_c \subseteq \vert C\vert_c$

4.2 how to validate ∨-exportation, lesson 2

As with Gibbard, nothing about this apparatus so far tells us what sort of state of mind is characterized by an arbitrary set of world/information-space pairs. But whereas in section 3.3 I had to introduce a plausible interpretation, in order to get an answer out of the method of world-norm pairs, Yalcin goes on to offer an answer to this question, at least implicitly, because he actually gives a semantics for belief ascriptions.[12] His answer has the same two-part structure as the one we used in section 3.3, but in order to state it precisely, we first need to get on the table a little bit more of Yalcin's picture of what information spaces are.

Officially, information spaces are to be understood as triples, $\langle\Pi,\pi,Pr\rangle$, where Π is a partition of the space of possible worlds, π is a (non-empty[13]) subset of Π, and *Pr* is a probability function defined over the cells of π. Intuitively, Π is a 'resolution' of the space of alternatives that are 'recognized' by the information space, and π is the set of alternatives that are recognized as 'live' by $\langle\Pi,\pi,Pr\rangle$. He also assumes that each person's total belief state can be characterized by such an information space:

defn: i *accepts* $\vert P\vert_c$ just in case $\forall x\in\pi_i{:}\forall w\in x{:}\langle w,i\rangle\in\vert P\vert_c$.[14,15]

Yalcin never gives an official clause for '→', in part because there are complications with doing so, since it would require defining a way of 'shifting' from one information space i, to a corresponding information space 'updated' by $\vert P\vert_c$. But what he does say makes it clear that however the semantics for '→' is to work, it will be world-invariant:

> Conditionals express properties of probability spaces: an indicative conditional ($\phi\rightarrow\psi$) in context is true with respect to a probability space **P** just in case a certain other probability space (determined as a function of **P**) which accepts the antecedent also accepts the consequent. [2007, 1018]

Clearly a 'property of probability spaces' is invariant with respect to worlds, and in fact the ensuing characterization makes no mention of worlds.

[12] Yalcin's answer to how to interpret arbitrary sets of world-norm pairs is only implicit, rather than explicit, because we need to read it off of the semantics that he provides for attitude ascriptions. On the assumption that his semantics is intended to be a semantics for the language we are actually speaking, we can therefore say the things about belief that it licenses.

[13] Yalcin doesn't state this condition explicitly, but it is required in order for *Pr* to be a probability function, so I'll assume it in what follows.

[14] Note that Yalcin writes 'accepts ϕ_c', where 'ϕ' is his schematic letter for sentences. But since it's not clear what 'ϕ_c' refers to, I take it that treating 'accepts' as expressing a relation to sets of w-i pairs – which, after all, are the semantic values of sentences in context – to be a harmless clarification.

defn: $x \in [P]_c^{BF}$ just in case the information space characterizing *x*'s total belief state accepts $|P|_c$.

In other words, you believe that P just in case the probability space which characterizes your total belief state accepts P. As with the interpretation of the method of world-norm pairs in section 3.3, these definitions have the natural properties of getting the intuitively right results when 'P' is free of epistemics, getting the intuitively right results when 'P' has a widest-scope epistemic, and guaranteeing that if you believe the premises of a valid argument, you also believe its conclusion.

But as with the view discussed in section 3.3, Yalcin's semantics validates our importation and exportation principles in a range of special cases. In particular:

Observation 3 *Y* validates ~-exportation, &-importation, &-exportation, and ∨-importation, for all substitution instances for 'P' and 'Q'.[16]

Observation 4 *Y* validates ~-importation for all substitution-instances where $|P|_c$ is world-invariant, and validates ∨-exportation for all substitution-instances where either $|P|_c$ or $|Q|_c$ is world-invariant.

Observation 5 *Y* validates the negations of ~-importation and ∨-exportation for all substitution-instances where $|P|_c$ and $|Q|_c$ are both information-invariant, and $|P|_c$ and $|Q|_c$ are independent – i.e., neither $|P|_c \subseteq |Q|_c$ nor $|Q|_c \subseteq |P|_c$.

Since if 'P' has any widest-scope epistemic, then $|P|_c$ is world-invariant, it is an immediate consequence of observations 3 and 4 that *Y* validates all six of our importation/exportation principles whenever either 'P' or 'Q' has any widest-scope epistemic – which is exactly the reason, as we saw in section 1.2, that truth-conditional techniques are problematic for the expressivist. But in contrast, since if 'P' is free of epistemics, $|P|_c$ is information-invariant, observation 5 shows that *Y* avoids at least the most egregious problems that would arise, if we applied the techniques of complementation and union directly to possible belief-states, $[P]_c^{BF}$. As with Gibbard's method of world-norm pairs, this is the payoff of applying the truth-conditional techniques of complementation, intersection, and union only *indirectly*, to w-i pairs, rather than directly to the sets of probability-spaces that characterize belief-states.

From the point of view of our observations in section 2.2 that &-importation and ∨-importation are false for belief, observation 5 looks like small consolation. But from the point of view of a theorist

[15] Although it looks different in his framework, Yalcin's definition of *acceptance* is borrowed directly from Veltman [1996], on whose system Yalcin's is closely modeled.

[16] The proofs of observations 3, 4, and 5 are straightforward and given in Appendix 2.

who is already comfortable with the commitments of a possible-worlds semantics for attitude verbs, observations 3 and 5 make things look just right, at least for sentences that are free of epistemics. That's because a standard possible-worlds semantics for belief validates &-importation and ∨-importation, as well as ~-exportation and &-exportation. Though I myself would describe this as a failure of possible-worlds semantics for attitude verbs, from a different perspective it suggests that so far as the epistemic-free portion of the language goes, *Y* at least fares no worse, in virtue of being expressivist, than a standard possible-worlds semantics. And that, in turn, is some kind of achievement.[17]

So in this section we've seen that Yalcin's trick of applying truth-conditional techniques to primary semantic values which determine, but are not determined by, the secondary semantic values that describe states of mind does well to avoid validating all six of our importation and exportation principles, across the board. But – completely predictably, from the perspective of thinking about the obstacles facing an expressivist semantics – it still validates all six as applied to sentences containing epistemics. And it still validates &-importation and ∨-importation across the board as well – both of which we've observed to be false. This further illustrates the abstract points from sections 1.2 and 3.3 about the constraints facing an expressivist semantics, and our two reasons why truth-conditional techniques do not suffice for expressivist semantics.[18]

4.3 refining yalcin's semantics

In Yalcin [forthcoming], Yalcin himself notes a special case of the difficulty that we are observing with importation and exportation principles – namely, that ~-importation is implausible, even where 'P' has a widest-scope epistemic modal. Yalcin also observes (in effect) that ∨-importation is not generally valid, and has exceptions particularly in cases in which the subject does not have the concepts to consider one of the disjuncts. He then offers an intuitive idea about how to refine his view, in order to avoid predicting

[17] Hale [1993], Kölbel [2004], and Schroeder [2008] all argue extensively that the sharpest way to see the force of the problems expressivists encounter in offering a compositional semantics, is to see how those problems arise for ordinary sentences free of the terms to which the expressivist seeks to give a special account. This corresponds to the fourth factor contributing to the difficult of the Frege-Geach problem discussed in section 1.2.

[18] For the initiated, there is an alternative way of seeing why it is an essential feature of *Y*, given Yalcin's central motivations, that it fails to make all of the distinctions among states of mind that we need. This is because to make all of the distinctions that we need, the system has to assign a *distinct* state of mind to each set of world/information-space pairs. Like the interpretation of Gibbard's system in section 3.3, *Y* fails to do this in part because it assigns the *same* state of mind to some distinct sets of such pairs. But this isn't an incidental feature for Yalcin; on the contrary, as those familiar with his paper will observe, one of Yalcin's central motivations is to deal with what he calls 'epistemic contradictions', of the form 'P&◇~P'. He deals with these precisely by identifying $[P]_c^{BF}$ with $[\sim\Diamond\sim P]_c^{BF}$ even though he does not identify $|P|_c$ with $|\sim\Diamond\sim P|_c$ – in other words, by assigning some distinct sets of world/information-space pairs to the same mental state.

these results, at least given some auxiliary assumptions. Yalcin also observes that &-importation is not in general valid, though it may be valid when restricted to fully reflective and rational agents, and offers a different intuitive idea about how to refine his view, in order to avoid predicting this result. Ultimately, he offers a single formal refinement of the view which achieves all three of these goals. In this section we'll look briefly at these refinements, in order to see what they are, just in order to see that they offer a *piecemeal* approach to what we've seen is a unified problem, and in order to see that they leave Yalcin's predictions about ∨-exportation intact. For convenience, I'll refer to the refined view as *Y2*.

Yalcin's first intuitive idea, is that if you don't have the concepts to consider whether P, then you shouldn't automatically count as believing that ◊P. So he proposes, effectively, that in order to believe that P, the question whether P must be 'on your radar' (my expression, not his). On the assumption that having the question whether ◊P on your radar requires having the question whether P on your radar, this further constraint on belief explains why it is possible to neither believe that ◊P nor believe that ~◊P. Similarly, since the question whether P can be on your radar without the question whether Q being on your radar, on the assumption that having the question whether P∨Q on your radar requires having the question whether Q on your radar, this same constraint explains why it is possible to believe that P without believing that P∨Q – invalidating ∨-importation.[19]

It would be relatively easy to simply strengthen Yalcin's original definition of 'accepts' to yield just these results, but that is not what he does. Instead, what he does is effectively to keep the original definition of 'accepts' and instead give up the assumption that each agent's complete belief state can be characterized by a single information space. In its place, he assumes that each agent's belief state can be characterized by a *function* from resolutions, Π, to information spaces. This is in order to make good on his second intuitive idea, about how to avoid validating &-importation.[20]

The way to avoid validating &-importation, Yalcin observes, following remarks by both Stalnaker and Lewis, is to treat an agent's state of mind as potentially *fragmented*, with each fragment capturable by an information space, but with the possibility that there may be no single information space which captures the whole. This is where the idea to model belief states by functions from resolutions to information

[19] Note that this still fails to explain the possibility of irrationally failing to believe that P∨Q, for someone for whom both whether P and whether Q are on her radar, so while rightly invalidating ∨-importation, it still fails to capture the full range of cases in which ∨-importation fails.
[20] Note that this can be thought of as a strict generalization of the earlier assumption that an agent's total belief state can be characterized by a single information space, by treating that as a special case either of a constant function or a function that is defined for only one resolution.

spaces comes from – each resolution over which this function is defined is one of the 'fragments' of the agent's beliefs.[21] The resulting view, for *Y2*, looks like this:

defn: i *accepts* X just in case $\forall x \in \pi_i : \forall w \in x : \langle w,i \rangle \in X$ (the same definition as for *Y*).
defn: $x \in [P]_c^{BF}$ just in case the information space to which *x*'s belief function maps the resolution given by the question whether P accepts $|P|_c$.

On the assumption that an agent's belief function is undefined over questions that are not on her radar, this account can explain why it is possible to neither believe that ◊P nor that ~◊P, as well as why ∨-introduction is invalid.[22] But it can also explain why &-importation fails. The idea is that &-importation can fail if an agent's belief-function maps the question whether P to an information space which accepts |P| and maps the question whether Q to an information space which accepts |Q|, but maps the question whether P&Q to an information space which does not accept |P&Q|. As Yalcin notes, this is irrational, and can only happen for an agent with a fragmented belief function, which assigns different questions to different information spaces. But intuitively, that is the right result about &-importation – violating it requires either irrationality or a lack of reflection.

What Yalcin [forthcoming] never addresses, because he treats each of the issues about ~-importation with epistemic modals, ∨-importation, and &-importation as separate issues rather than as symptoms of a single underlying feature of the view, is whether this revised treatment can also avoid validating ∨-exportation when epistemics are involved. But it is easy to see that his account will treat this in the same way as &-importation. Even though when 'Q' has a widest-scope epistemic, every information space that accepts $|P \vee Q|_c$ also either accepts $|P|_c$ or accepts $|Q|_c$, the refinement in Yalcin's view leaves it open that an agent's belief function maps the question whether P∨Q to an information space that accepts $|P \vee Q|_c$ (and hence either accepts $|P|_c$ or accepts $|Q|_c$), but maps the question whether P to an information space that does not accept $|P|_c$ (and hence does not accept $|P \vee Q|_c$), and maps the question whether Q to an information space that does not accept $|Q|_c$ (and hence does not accept $|P \vee Q|_c$). But as with the case of &-importation, Yalcin's account simply predicts that such an agent's psychology is necessarily fragmented, in the sense that her belief state must map different questions to different

[21] It is natural to read Stalnaker and Lewis as holding that there are relatively few 'fragments', but note that Yalcin's treatment allows for a distinct 'fragment' for every possible question.
[22] Note that Yalcin doesn't actually spell out how his revised account captures the result that it is possible to neither believe neither that ◊P nor that ~◊P or why ∨-importation fails. The *obvious* way to do this, however, would be to treat an agent's belief function as undefined over some resolutions and use that to do the work.

information spaces. Consequently, as with &-importation, Yalcin's account predicts that when 'Q' has a widest-scope epistemic modal, anyone who believes that P∨Q without either believing that P or believing that Q must be suffering either from a failure of reflection or a failure of rationality.

What we've seen in this section is that, through a creative refinement of his view, Yalcin can avoid a number of the most obvious flaws apparent in *Y*, the semantics from Yalcin [2007]. But we've also seen that these refinements leave intact the prediction that ∨-exportation is valid for *reflective, rational* agents, when either disjunct has a widest-scope epistemic. But ordinarily, as we observed in section 2.1, exceptions to ∨-exportation need involve no failure in reflection or rationality whatsoever. Could epistemics be any different? We turn to that question in part 5.

5.1 apparent counterexamples to ∨-exportation with epistemics

Unfortunately for Yalcin, there at least appear on the face of it to be intuitive counterexamples to ∨-exportation where epistemics are involved which don't trade on any lack of reflection or rationality of the agents involved. It's easy to construct such examples for 'must', 'probably', and 'if…then' (though there are reasons to think there are no such cases for 'might'):

Jack's Vacation

> Jill finds out that Jack took a vacation last year to one of the non-contiguous U.S. states. Since she knows that the only non-contiguous U.S. states are Alaska and Hawaii, she concludes that either Jack went to Alaska, or he must have gone to Hawaii.

In *Jack's Vacation*, Jill's conclusion appears to be warranted, she does not believe that Jack went to Alaska, and she does not believe that he must have gone to Hawaii. Since she believes the disjunction without believing either disjunct, she is a counterexample to ∨-exportation. But her situation does not require any failure of reflection or of rationality – no further rational reflection, given her information, is going to lead her to conclude that Jack went to Alaska, to conclude that he must have gone to Hawaii, or to give up on her belief that either he went to Alaska or he must have gone to Hawaii. And similar points go for the following cases:

Shieva's Deadline

> Last night Shieva calls me to express frustration with the paper that she is working on, and tells me that if she hasn't finished by this morning, she's going to consult her magic 8-ball about whether to give up and follow its advice. Since I know that most of the answers on

her magic 8-ball are positive, when I recall our conversation from last night, I conclude that either Shieva finished her paper by this morning, or she probably gave up.

Karen's Degree

Phil overhears at a party that Karen has recently gotten a PhD in philosophy from a top-ranked program in Manhattan. Since Phil knows that the only top-ranked philosophy departments in Manhattan which have recently granted PhDs are Columbia, NYU, and CUNY, he concludes that either Karen went to Columbia, or if she didn't go to NYU, then she went to CUNY.

Each of these cases constitutes a *prima facie* problem for Yalcin's continuing difficulty distinguishing beliefs in disjunctions from the disjunctive state of believing one of the disjuncts. But fortunately for Yalcin (and the reason why, as advertised up front, epistemics complicate the issues surrounding the Frege-Geach problem in a way that other applications don't), the tenability of each of these counterexamples is open to question.

5.2 what about disjunctive syllogism?

The complication, is that if these cases constitute counterexamples to the validity of $\vee$-exportation for epistemics under a restriction to reflective, rational agents, then they are also counterexamples to disjunctive syllogism. Take *Jack's Vacation* first. If Jill is rational and reflective, then she believes that Jack might not have gone to Hawaii, and hence that it's not the case that Jack must have gone to Hawaii. But she's in no position to conclude, on the basis of her belief that either Jack went to Alaska or he must have gone to Hawaii, and her belief that it's not the case that he must have gone to Hawaii, that Jack went to Alaska. So if this counterexample is to do the work required of it, then it must also be a counterexample to disjunctive syllogism.

Similar points go for Shieva's Deadline. If I'm 50% confident, given what I know about Shieva and her paper project, that she will have finished by this morning, then if I am reflective and rational, I will believe that probably, either she finished her paper by this morning or she is still working on it. So if I'm reflective and rational, I'll infer that it is not the case that she probably gave up. But just because I believe that either she finished her paper by this morning or she probably gave up, and believe that it's not the case that she probably gave up, it doesn't follow that I can rationally infer that she finished her paper by this morning – for my evidence only makes that conclusion 50% likely. So again, if this counterexample is to do the work required of it, then it must also be a counterexample to disjunctive syllogism.

Finally, the same problem arises for Karen's Degree. If you ask Phil whether, if Karen didn't go to NYU, then she went to CUNY, he will deny it – for after all, she might have gone to Columbia. So if Phil is rational and reflective, then he believes that it is not the case that if Karen went to NYU, then she went to CUNY. But that and his belief that either Karen went to Columbia or, if she didn't go to NYU, then she went to CUNY don't allow Phil to rationally conclude that Karen went to Columbia – for all he knows is that she got her degree in Manhattan, which is perfectly compatible with both the conclusion that she went to NYU and the conclusion that she went to CUNY. So again, it looks like the counterexample can do the right sort of work only if it is also a counterexample to disjunctive syllogism.

Unfortunately for our counterexamples, therefore, it's not much of a recommendation of a counterexample, that it would also be a counterexample to disjunctive syllogism. This is what I meant at the opening of the paper, when I advertised that I was going to be interested in exploring one interesting way in which epistemics are relevantly different from other applications for expressivism, in a way that mitigates a core feature of what makes the Frege-Geach problem difficult. Given that potential counterexamples to ∨-exportation for epistemics would also be counterexamples to disjunctive syllogism, that would appear to put Yalcin, whose view predicts that such counterexamples necessarily involve a failure of either reflection or rationality, in good company.

5.3 things are still complicated!

Despite this complication, for a number of reasons, I still don't think that Yalcin should rest easy. Some of these reasons concern more general versions of the Frege-Geach problem, and I'll get to them in part 6. But one of them is that despite the fact that my putative counterexamples would also need to be counterexamples to disjunctive syllogism in order to do the work that is required of them, I still don't think that it is at all obvious that they *aren't* such counterexamples. This is because these cases are in general very difficult to explain, but there does exist an attractive and powerful explanation of all of them. This explanation is given by a close competitor to expressivism about epistemics – the *dynamic* treatment given in *update semantics*.

One natural initial reaction to the putative counterexamples is to suppose that scope is doing the work, so that the examples are not examples of a belief that $P \vee \oplus Q$ at all, but rather examples of a belief that $\oplus(P \vee Q)$ (where '$\oplus$' is the epistemic). For example, on this view when we say that Jill believes that either Jack went to Alaska or he must have gone to Hawaii, what we are saying is just that Jill believes that it must be that either Jack went to Alaska or he went to Hawaii. Because 'must' has widest scope in this belief, rather than 'or', this hypothesis would both explain why the case involves no counterexample to disjunctive syllogism, and why it involves no counterexample to ∨-exportation. So this hypothesis sounds good for the case of Jack's Vacation.

But this hypothesis can't be right, in general, because it gets the wrong results in the case of Shieva's Deadline. To see why, consider a slight variant on the case, in which rather than believing that Shieva's magic 8-ball provides mostly affirmative answers, I believe that it provides mostly negative answers. Since given when I know about Shieva and the paper that she was working on, I am 50% confident that she finished her paper by this morning, and since I believe that at least some significant fraction of the answers given by her magic 8-ball are positive, I believe that it is probably the case that either she finished her paper by this morning or she gave up. But intuitively, I claim, in this case I don't believe that either Shieva finished her paper by this morning or she probably gave up – rather, I believe that either Shieva finished her paper by this morning or she is probably still working on it. The problem this case raises, is that the wide-scope reading is too weak to capture the intuitive force of the disjunctive belief report.

So if these cases can't be interpreted as involving unexpected scope readings, then how *are* they to be interpreted? The theorist who wishes to maintain that they are not counterexamples either to ∨-exportation or to disjunctive syllogism must suppose that they receive some other sort of non-standard reading that is not to be predicted by straightforward compositional principles. For example, she might hypothesize that there is a special rule whereby '$P \vee \oplus Q$' gets interpreted as '$P \vee (\sim P \rightarrow \oplus Q)$', whenever '$\oplus$' is an epistemic. But there is something very suspicious about needing to postulate special *ad hoc* rules for special readings that arise only in certain contexts. In general, the linguistic principle of *complementarity* suggests that if 'or' sentences get one reading when their disjuncts are free of epistemics, and a different reading when their disjuncts contain epistemics, then the best explanation will treat these as arising from a single meaning for 'or', together with the effects of interacting with epistemics.

This sounds very abstract, but there is a view in the literature that is actually a natural and close competitor to expressivism about epistemics, which makes *exactly* these predictions about our cases – the *dynamic* theory given by *update semantics*.[23] Without getting into the details, we can see how a dynamic theory could have this effect by observing that all of our cases involve epistemics which appear in the *second*

[23] See in particular Veltman [1985], [1996] for the underlying mechanics of a dynamic theory of 'might' and 'must', and in particular Gillies [2004] and Willer [forthcoming] for versions of the dynamic theory which yield the initially intuitive predictions about these cases (Veltman's theory actually agrees with Yalcin [2007], which is closely modeled on it). For influential dynamic approaches to other topics, see in particular Kamp [1981], Heim [1982], and Groenendijk and Stokhof [1991]. For a discussion of how to treat attitude verbs in dynamic semantics, see Heim [1992].

disjunct, rather than the first. This means that if they *are* counterexamples to disjunctive syllogism, they are counterexamples to the rule which allows inferring P from P∨Q and ~Q, and not, at least directly, to the rule which allows inferring Q from P∨Q and ~P. Ordinarily, we don't distinguish these two rules, because ordinarily we assume that 'or' is commutative – that is, that P∨Q and Q∨P are equivalent. And indeed, 'or' *does* seem to be commutative, in cases where both 'P' and 'Q' are free of epistemics. But notice that 'or' does *not* seem to be commutative, in any of our cases. It was actually *essential* to the intuitive plausibility of the cases as counterexamples to ∨-exportation that the epistemics appeared in the second disjunct. So this suggests that generalizing the observation that 'or' is commutative from cases that are free of epistemics to cases involving epistemics may be hasty.

Strikingly, it is typical of dynamic semantic theories to deny that 'or' is commutative, in general. Such accounts typically define 'or' in terms of 'and' and 'not', and define 'and' in a way that guarantees that it commutes for conjuncts that are free of essentially 'dynamic' language (including epistemics, on such accounts), but does not commute when one conjunct contains such dynamic language. So given the definition of 'or' in term of 'and', the same properties go for 'or'. In fact, it is easy to supplement Anthony Gillies' [2004] treatment of 'must' and of indicative conditionals with a natural treatment of 'believes that' in order to yield exactly the predictions that our putative counterexamples require for the cases of Jack's Vacation and of Karen's Degree, and natural extensions of Gillies' view generalize these predictions to the case of Shieva's Deadline.[24] Ironically, Yalcin's expressivist semantics is itself very closely modeled on the dynamic treatment of Frank Veltman [1996], which happens to be unusual among dynamic theories in taking care to give an account of conjunction which ensures that it is commutative. So Veltman's account makes the same predictions as Yalcin's.[25]

In this section I don't mean to have argued outright for the conclusion that 'or' fails to be generally commutative, or that Gillies' account yields a treatment of these cases that is preferable to Yalcin's – though I am to some extent sympathetic to this view. All I mean to have argued, is that the issues are complex, and it is far from obvious that the fact that these cases would also be counterexamples to disjunctive syllogism, if they are counterexamples to Yalcin's view, entails that they are not, nevertheless, counterexamples to both. This question deserves much more consideration than I've been able to give it, here, but with more territory yet to cover, we must move on.

[24] See Schmitt [work-in-progress] for an extension of Gillies' framework that allows treatment of 'probably'.
[25] Peculiarly, in note 23, Gillies [2004] addresses the question as to whether 'or' would be commutative, if introduced into his system by the standard definition from 'and' and 'not', and erroneously insists that it would be.

6.1 problems remaining

So here's where we are: so far I've covered three of the four main goals I advertised up front, (1) illustrating one of the main constraints involved with the Frege-Geach problem, (2) showing that Yalcin's expressivist semantics for epistemics runs afoul of this constraint in a predictable way, and (3) showing that whether this is a fatal problem for Yalcin's view is much more subtle for the case of epistemics, than it is for some other prominent applications for expressivism, including within metaethics. This leaves us with task (4): to argue that this complication is still not enough to make the way safe for expressivism about epistemics. In this section I'll note two serious remaining (and totally predictable) problems for Yalcin's view, and then in the closing section I'll look at what an approach to the Frege-Geach problem that is more informed by a direct appreciation of the structure of the problem might look like.

The first problem I want to discuss is that the propositional connectives ('not', 'and', and 'or') are just the tip of the iceberg for the Frege-Geach problem, and a truly compositional semantics needs to aspire to be able to treat *all* of these correctly. As the case of 'or' shows us, however, even though employing standard truth-conditional compositional methods can straightforwardly yield the right predictions about the entailments among first-order sentences, it doesn't, by itself, explain what the corresponding *states of mind* are, and depending on the details, we can actually substantively make intuitively surprising predictions about the logical relationships among those states of mind – in particular, validating special cases of ∨-exportation. But once we appreciate the shape of this problem, it is easy to see that it will arise again for other constructions.

In particular, problems exactly analogous to those for 'or' are going to arise with quantifiers. These are particularly easy to illustrate with the case of the existential quantifier, but I believe that similar problems go for the universal quantifier. Of course, Yalcin doesn't tell us exactly how his account treats quantifiers, but that is because since it is structured as an ordinary truth-conditional account, it is to be understood as employing the standard truth-conditional treatment. So in other words, what we need to do, is to relativize to assignments, assigning (possibly open) *formulas* to sets of w-i pairs, relative to both a context of utterance and an assignment. The compositional rules for '~', '&', and '∨' can stand from before, supplemented by the following rules for '∃' and '∀', where a\x ranges over assignments which differ from a at most in what they assign to x.

negation $|{\sim}P|_{c,a}=\text{Comp}(|P|_{c,a})$
conjunction $|P\&Q|_{c,a}=|P|_{c,a}\cap|Q|_{c,a}$
disjunction $|P\vee Q|_{c,a}=|P|_{c,a}\cup|Q|_{c,a}$

existential	$\vert\exists x Fx\vert_{c,a} = \cup_{a\backslash x}\vert Fx\vert_{c,a\backslash x}$
universal	$\vert\forall x Fx\vert_{c,a} = \cap_{a\backslash x}\vert Fx\vert_{c,a\backslash x}$
logic	By subsets: $P_1 \ldots P_n \vDash C$ iff $\forall c{:}\forall a{:}\vert P_1\vert_{c,a}\cap\ldots\cap\vert P_n\vert_{c,a}\subseteq\vert C\vert_{c,a}$

For the same reasons as before, this system makes all of the right predictions about what follows from what, but it also straightforwardly generalizes to predict that it is impossible to believe that $\exists x\oplus Fx$, for any epistemic '$\oplus$', without there being some *x* such that you believe that $\oplus Fx$.

Unfortunately, this does not look like a good prediction. Suppose that Mary tells Kim that Jack, one of her sons, must have a girlfriend, because she's seen lipstick on his collar. Kim remembers what Mary told her about lipstick, but forgets which son she was talking about. So she remembers that one of Mary's sons must have a girlfriend. Here I think it's clear that she isn't just remembering that it must be that one of Mary's sons has a girlfriend – that's too weak, because she remembers that there is one in particular who must have a girlfriend; she just doesn't remember which. So what she remembers is of the form $\exists x\Box Fx$, rather than $\Box\exists xFx$. But there clearly need be no one of whom Kim believes that *he* must have a girlfriend, because since she knows that Mary has three sons and can't rule any of them out, any of them might be the one.

The same prediction is problematic for other epistemics. For example, taking the case of conditionals, Mary might tell Kim that if Jack comes to the party, he's going to bring a friend. Again Kim remembers what Mary said, but forgets which son she said it about. The same points go as before. In particular, there is no one of whom Kim believes that if he comes to the party, then he'll bring a friend, but she does believe that there is someone who, if he comes, will bring a friend.[26]

So even though the issues about disjunctive syllogism make Yalcin's predictions about the way in which epistemics embed under 'or' seem less obviously problematic than they would be, if he were defending a view about some other topic (for example, in metaethics), it is important to understand that they are just the tip of the iceberg. This is a simple reminder that the Frege-Geach problem is a *general* problem. A real solution, as opposed to an *ad hoc* patch, should match the full generality of the problem.

So much for the first significant problem still standing in Yalcin's way. The second problem that I want to note here, is that Yalcin's approach still fares very poorly in its treatment of attitudes other than belief. As we observed in section 2.2, different attitudes are governed by different import/export principles, and each principle fails for at least some attitudes. So whatever explains why &-exportation and

[26] For more discussion of cases of quantifiers scoped over epistemic modals, see Swanson [2010]; for discussion of quantifiers scoped over conditionals, see especially Kölbel [2000].

~-exportation are valid for belief had better be a feature specifically of *belief*, rather than a feature of the semantics for attitude verbs in general. But Yalcin [2007] explicitly offers a semantics for attitude verbs other than 'believes' which is exactly analogous, working by assuming that an agent's total supposition state, for example, can be characterized by an information space. So his account predicts exactly the same importation and exportation principles as for belief, and in particular that it is just as impossible to both suppose that P and suppose that ~P, as it is to both believe that P and believe that ~P (see Appendix 2 for the details). If this were true, it would place a substantial damper on our ability to demonstrate anything by *reductio*. It's small comfort that pointing out that this would itself seem to be a *reductio* of this position would be dialectically ineffective.

You might wonder whether the refinement in Yalcin [forthcoming] has anything to offer to this problem, and it may. But if it does, it can't be by simply generalizing what goes for 'believes' to 'supposes', by also treating an agent's supposition state as a function from questions to information spaces. That would again only yield exactly the same behavior for supposition as for belief. Perhaps supposition could be treated in some other way, but I see no reason to be particularly optimistic, given that Yalcin's attempt to extend his treatment of 'believes that' to 'supposes that' is not a merely incidental feature of his approach, but is actually closely connected to one of his primary motivations in Yalcin [2007], which is to explain why sentences like 'suppose that P but ◊~P' seem to be incoherent.

6.2 expressivism with real contents?

What these remaining problems for Yalcin's approach show, I believe, is that what an expressivist needs – even an expressivist about epistemic modals, despite the admitted complications – is a full appreciation of the generality of the structural problem facing expressivist semantics. Only with such a full appreciation on board, are we going to be able to construct a solution that fits the full generality of the problem, rather than one that looks good on various sub-problems but needs numerous amendments and adjustments along the way to deal with increasingly more difficult technical problems. This is because, as I've been aiming to show all along in this paper, these *aren't* just a series of technical problems – they're symptoms of an underlying *conceptual* problem. That's what makes them so predictable. To come up with these problems for Yalcin's view, I didn't have to cleverly try out lots of random predictions; all I needed to do was to look in the predictable places for it to have problems, given the central constraints on the Frege-Geach problem.

One of the main constraints on expressivist theorizing, I believe and have argued in numerous places,[27] is that it can't really deal in mental states directly, but rather should deal in some kind of *contents*. This is so for a number of reasons. Among the most important of these reasons is the fact, as we saw in section 3.2, that taking an *indirect* approach to our compositional account of what it is to believe that P, we can make it much easier to give such an account. Indeed, I take it that the standard view is that there is nothing complicated about logically complex belief at all – it is just ordinary belief, merely distinguished by its distinctive contents. A second important reason why even an expressivist should traffic in contents is that we ultimately need an account not just of what it is to *believe* that P, but of what it is to *hope* that P, to wonder whether P, and so on, for each of the other attitudes. If these attitudes don't have a common set of contents to traffic in, this would be an extraordinarily difficult problem.

So that leaves us with an important question: where should expressivists look for contents? What kind of thing should they take them to be? This question is complicated by the fact that on the face of it, it is central to the expressivist's position that different beliefs can be fundamentally different kinds of attitude. For example, the metaethical expressivist says that believing that grass is green and believing that stealing is wrong are totally different kinds of states of mind, and the epistemic expressivist says that believing that Jack will come and believing that Jack might come are *different* attitudes toward the same content – in contrast, apparently, to being the *same* attitude toward *different* contents. This makes it look very hard to see what neutral contents could turn out to be.

Yalcin's approach to the question of contents is to start with a preconceived notion of what contents need to be like – they need to behave in the standard ways that the sorts of contents attributed by ordinary truth-conditional approaches to semantic theory take contents to behave – obeying the principles of complementation, intersection and union. This core conception of contents is preserved in both versions of his account. Starting under this constraint, Yalcin defines a *mapping* from these contents to belief-states – starting with an obviously inadequate mapping in his [2007] paper, and moving to a more complicated but still incorrect mapping in the [forthcoming] version. In order to preserve the fundamental character of the truth-conditional approach, he is forced to keep the conception of contents, and reinterpret the conception of what relation you need to bear to a content, in order to count as believing it. And on both versions of the view, this relation is motivated primarily by the predictions that it gives us about the conditions under which it counts someone as having a given belief, rather than on the basis of

[27] See especially Schroeder [2008], [forthcoming b].

any natural way of understanding why believing is just a matter of bearing such an arcane relation to such a content.

In contrast, I believe that if we want to make progress for expressivism, it is wiser to go in unconstrained by preconceptions about what contents need to be like, and instead to start by thinking harder *philosophically* about whether there is a natural level of description at which all beliefs can be conceived of as attitudes toward the same underlying kind of content. At first this seems like it is inconsistent with the epistemic expressivist's idea that there *is* no content that $\Diamond P$ or that $P \rightarrow Q$, in which an agent needs to have a high credence, in order to believe that $\Diamond P$ or that $P \rightarrow Q$. But it is not so inconsistent; it only follows that whatever neutral contents are, they are a different sort of thing from the objects of credence.[28]

A good *philosophical* understanding of how the belief that P and the belief that $\Diamond P$ can be *conceptually* understood as involving the same attitude toward distinct contents would give us something important that Yalcin's approach doesn't and can't, constrained as it is by its adherence to truth-conditional techniques – it would give us a natural way of understanding what it *means* to say that a sentence has the semantic value that it does. Unfortunately, there is no space to pursue this idea at length here. My point in raising this possibility in closing, however, is that this is exactly the strategy that I have argued extensively elsewhere that expressivists must pursue in order to make good on their semantic program. Since many of my arguments elsewhere have often focused on the case of noncognitivist expressivism in metaethics as central, and since there *are* complications about epistemic terms, it has been the project of this paper to show that the problems facing existing expressivist accounts of epistemic modals are still striking, that they are entirely predictable given the nature of the underlying problems, and that in contrast, the same tools that I've elsewhere argued are key to making progress for other forms of expressivism have exactly the right structure to fit the underlying problem. Epistemics are no exception to this rule.[29]

Appendix I

In this appendix we prove observations 1 and 2 about the system stated in section 3.3 as an interpretation of Gibbard's method of world-norm pairs. For reference, we start by re-stating the key definitions, along with some other useful terminology:

[28] See in particular the discussion in Schroeder [forthcoming a] and [forthcoming b].

[29] Special thanks to Jake Ross, Johannes Schmitt, Lewis Powell, Scott Soames, Ben Lennertz, Jamie Dreier, Malte Willer, Kit Fine, Thony Gillies, Branden Fitelson, Ángel Pinillos, Julia Staffel, and to audiences at New York University, the University of California Santa Barbara, and the University of Toronto.

Semantics: $|{\sim}P| = \mathrm{Comp}(|P|)$

$|P\&Q| = |P| \cap |Q|$

$|P \vee Q| = |P| \cup |Q|$.

Logic: By subsets: $P_1 \ldots P_n \vDash C$ iff $|P_1| \cap \ldots \cap |P_n| \subseteq |C|$.

Terminology: $\langle W,N\rangle$ *accepts* $|P|$ just in case $\forall w \in W{:}\forall n \in N{:}\langle w,n\rangle \in |P|$

$x \in [P]^{BF}$ just in case x's total state of mind, $\langle W,N\rangle^{x}$, accepts $|P|$

$\langle W,N\rangle^* = \{\langle w,n\rangle{:}w \in W \& n \in N\}$

$|P|$ is *norm-invariant* just in case $\forall w \exists n{:}\langle w,n\rangle \in |P| \supset \forall m{:}\langle w,m\rangle \in |P|$

$|P|$ is *world-invariant* just in case $\forall n \exists w{:}\langle w,n\rangle \in |P| \supset \forall v{:}\langle v,n\rangle \in |P|$

Observation 1: If $P_1 \ldots P_n \vDash C$, then $[P_1]^{BF} \cap \ldots \cap [P_n]^{BF} \subseteq [C]^{BF}$.

Lemma 1: $\langle W,N\rangle$ accepts $|P|$ iff $\langle W,N\rangle^* \subseteq |P|$.

Proof: $\langle W,N\rangle^* \subseteq |P|$ iff $\forall w \in W{:}\forall n \in N{:}\langle w,n\rangle \in |P|$

…iff by defn, $\langle W,N\rangle$ accepts $|P|$.

Lemma 2: If $P_1 \ldots P_n \vDash C$, and $\forall \langle W,N\rangle{:}\langle W,N\rangle^* \subseteq |P_1| \& \ldots \& \langle W,N\rangle^* \subseteq |P_n|$, then $\langle W,N\rangle^* \subseteq |C|$.

Proof: Assume that $\forall \langle W,N\rangle{:}\langle W,N\rangle^* \subseteq |P_1| \& \ldots \& \langle W,N\rangle^* \subseteq |P_n|$.

So $\forall \langle W,N\rangle{:}\langle W,N\rangle^* \subseteq |P_1| \cap \ldots \cap |P_n|$.

So by definition of $P_1 \ldots P_n \vDash C$, $\forall \langle W,N\rangle{:}\langle W,N\rangle^* \subseteq |C|$.

Main Result: If $P_1 \ldots P_n \vDash C$, then $[P_1]^{BF} \cap \ldots \cap [P_n]^{BF} \subseteq [C]^{BF}$.

Proof: Let $x \in [P_1]^{BF} \cap \ldots \cap [P_n]^{BF}$. So by defn of $[P]^{BF}$, $\langle W,N\rangle_x$ accepts $|P_1| \ldots |P_n|$.

So by Lemma 1, $\langle W,N\rangle_x^* \subseteq |P_1| \& \ldots \& \langle W,N\rangle_x^* \subseteq |P_n|$.

And by Lemma 2, $\langle W,N\rangle_x^* \subseteq |C|$.

So by Lemma 1 and defn of $[P]^{BF}$, $x \in [C]^{BF}$.

Observation 2: If $|P|$ is norm-invariant and $|Q|$ is world-invariant, then $[P \vee Q]^{BF} \subseteq [P]^{BF} \cup [Q]^{BF}$.

Lemma 3: If $\langle w,n\rangle \in \langle W,N\rangle^*$ and $\langle v,m\rangle \in \langle W,N\rangle^*$, then $\langle v,n\rangle \in \langle W,N\rangle^*$.

Proof: Let $\langle w,n\rangle \in \langle W,N\rangle^*$ and $\langle v,m\rangle \in \langle W,N\rangle^*$. So $v \in W$ and $n \in N$. So $\langle v,n\rangle \in \langle W,N\rangle^*$.

Main result: Assume that $|P|$ is norm-invariant, $|Q|$ is world-invariant, and $\langle W,N\rangle$ accepts $|P \vee Q|$, to show that $\langle W,N\rangle$ accepts $|P|$ or $\langle W,N\rangle$ accepts $|Q|$.

Proof: By Lemma 1, $\langle W,N\rangle^* \subseteq |P \vee Q| = |P| \cup |Q|$.

Suppose for *reductio* that $\langle W,N\rangle^* \not\subset |P|$ and $\langle W,N\rangle^* \not\subset |Q|$.

So $\exists \langle w,n\rangle \in \langle W,N\rangle^*{:}\langle w,n\rangle \in |P| \& \langle w,n\rangle \notin |Q|$ and $\exists \langle v,m\rangle \in \langle W,N\rangle^*{:}\langle v,m\rangle \in |Q| \& \langle v,m\rangle \notin |P|$.

Since $|P|$ is norm-invariant and $\langle v,m\rangle \notin P$, $\langle v,n\rangle \notin P$.

Similarly, since $|Q|$ is world-invariant and $\langle w,n\rangle \notin |Q|$, $\langle v,n\rangle \notin |Q|$.

So $\langle v,n\rangle \notin |P| \cup |Q|$. But by Lemma 3, $\langle v,n\rangle \in \langle W,N\rangle^*$.

So $\langle W,N\rangle^* \not\subset |P| \cup |Q|$, contradicting our assumption.

Appendix 2

Here we prove observations 3-5, mentioned in the main body of the paper, about system *Y*, the semantics for epistemics stated in Yalcin [2007]. Since it plays no role, I'll drop the 'c' subscript throughout for convenience. We start by re-stating key definitions, for reference, and introduce angle brackets as a helpful new abbreviation:

Semantics: $|{\sim}P|=\text{Comp}(|P|)$
$|P\&Q|=|P|\cap|Q|$
$|P\vee Q|=|P|\cup|Q|$

Terminology: i accepts $|P|$ iff $\forall x\in\pi_i:\forall w\in x:<w,i>\in|P|$
$\langle P\rangle=\{i: \text{i accepts } |P|\}$
$i^{\phi(x)}$ = the information state which characterizes x's complete ϕ state
$[P]^{\phi}=\{x: i^{\phi(x)}\in\langle P\rangle\}$
$|P|$ is *information-invariant* just in case $\forall w\exists i:<w,i>\in|P|\supset\forall j:<w,j>\in|P|$
$|P|$ is *world-invariant* just in case $\forall i\exists w:<w,i>\in|P|\supset\forall v:<v,i>\in|P|$

Since $x\in[P]^{\phi}$ iff $i^{\phi(x)}\in\langle P\rangle$, I'll show throughout that the required set-inclusion relations hold for the values of $\langle P\rangle$, taking what goes for $[P]^{\phi}$ to follow immediately. So trivially, what goes for one attitude ϕ goes for all, as noted in section 6.1. Another trivial observations that will come in handy: if $|P|$ and $|Q|$ are both world(information)-invariant, then $|{\sim}P|$, $|P\&Q|$, and $|P\vee Q|$ are all world(information)-invariant.

And now for the proofs of our main observations:

Observation 3 *Y* validates ~-exportation, &-importation, &-exportation, and ∨-importation, for all substitution instances for 'P' and 'Q', for any attitude ϕ.

Proof: To show that $\langle{\sim}P\rangle\subseteq\text{Comp}(\langle P\rangle)$, $\langle P\rangle\cap\langle Q\rangle\subseteq\langle P\&Q\rangle$, $\langle P\&Q\rangle\subseteq\langle P\rangle\cap\langle Q\rangle$, and $\langle P\rangle\cup\langle Q\rangle\subseteq\langle P\vee Q\rangle$.

Taking ~-exportation first. Let $i\in\langle{\sim}P\rangle$. So $\forall x\in\pi_i:\forall w\in x:<w,i>\in|{\sim}P|$. So by definition of $|{\sim}P|$, $\forall x\in\pi_i:\forall w\in x:<w,i>\notin|P|$ – so i does not accept $|P|$, and hence $i\in\text{Comp}(\langle P\rangle)$.

Next, &-importation and &-exportation. $i\in\langle P\rangle\cap\langle Q\rangle$ iff $\forall x\in\pi_i:\forall w\in x:<w,i>\in|P|$ and $\forall x\in\pi_i:\forall w\in x:<w,i>\in|P|$ iff $\forall x\in\pi_i:\forall w\in x:<w,i>\in|P|\cap|Q|$, so by definition of $|P\&Q|$, iff $\forall x\in\pi_i:\forall w\in x:<w,i>\in|P\&Q|$, which by definition is true iff $i\in\langle P\&Q\rangle$.

Finally, ∨-importation. Without loss of generality, let $i\in\langle P\rangle$. So $\forall x\in\pi_i:\forall w\in x:<w,i>\in|P|$, so trivially $\forall x\in\pi_i:\forall w\in x:<w,i>\in|P\vee Q|$, i.e., $i\in\langle P\vee Q\rangle$.

Observation 4 Y validates ~-importation for all substitution-instances where $|P|$ is world-invariant, and validates ∨-exportation for all substitution-instances where either $|P|$ or $|Q|$ is world-invariant, for any attitude ϕ.

Proof: To show that when $|P|$ is world-invariant, $Comp(\langle P\rangle)\subseteq\langle \sim P\rangle$, and when either $|P|$ or $|Q|$ is world-invariant, $\langle P\vee Q\rangle\subseteq\langle P\rangle\cup\langle Q\rangle$.

First, ~-exportation. Assume that $|P|$ is world-invariant, from which it follows by our trivial observation (above) that $|\sim P|$ is world-invariant as well – i.e., that $\forall i\exists w{:}<w,i>\in|\sim P|\supset\forall v{:}<v,i>\in|\sim P|$. Now let $i\in Comp(\langle P\rangle)$, so $i\notin\langle P\rangle$. So $\exists x\in\pi_i{:}\exists w\in x{:}<w,i>\notin|P|$. So by definition of $|\sim P|$, $\exists x\in\pi_i{:}\exists w\in x{:}<w,i>\in|\sim P|$. So since $|\sim P|$ is world-invariant, $\forall v{:}<v,i>\in|\sim P|$. But then trivially, $\forall x\in\pi_i{:}\forall w\in x{:}<w,i>\in|\sim P|$, so $i\in\langle\sim P\rangle$.

Second, ∨-exportation. Without loss of generality, assume that $|P|$ is world-invariant – i.e., that $\forall i\exists w{:}<w,i>\in|P|\supset\forall v{:}<v,i>\in|P|$. And let $i\in\langle P\vee Q\rangle$ – so $\forall x\in\pi_i{:}\forall w\in x{:}<w,i>\in|P\vee Q|$. So since by definition $|P\vee Q|=|P|\cup|Q|$, $\forall x\in\pi_i{:}\forall w\in x{:}<w,i>\in|P|\vee<w,i>\in|Q|$. So now take two cases. Either (i) $\exists w{:}<w,i>\in|P|$ or (ii) $\forall w{:}<w,i>\notin|P|$.

Case (i): assume that $\exists w{:}<w,i>\in|P|$. Since we are assuming that $|P|$ is world-invariant, it follows that $\forall w{:}<w,i>\in|P|$, and hence that $\forall x\in\pi_i{:}\forall w\in x{:}<w,i>\in|P|$ – i.e., that $i\in\langle P\rangle\subseteq\langle P\rangle\cup\langle Q\rangle$.

Case (ii): Assume that $\forall w{:}<w,i>\notin|P|$. But then, since $\forall x\in\pi_i{:}\forall w\in x{:}<w,i>\in|P|\vee<w,i>\in|Q|$, it follows that $\forall x\in\pi_i{:}\forall w\in x{:}<w,i>\in|Q|$ – i.e., that $i\in\langle Q\rangle\subseteq\langle P\rangle\cup\langle Q\rangle$.

Observation 5 Y validates the negations of ~-importation and ∨-exportation for all substitution-instances where $|P|$ and $|Q|$ are both information-invariant.

Proof: To show that if $|P|$ and $|Q|$ are both information-invariant and independent, then $Comp(\langle P\rangle)\not\subset\langle\sim P\rangle$ and $\langle P\vee Q\rangle\not\subset\langle P\rangle\cup\langle Q\rangle$.

First, ~-importation. Assume that $|P|$ is information-invariant – i.e., that $\forall w\exists i{:}<w,i>\in|P|\supset\forall j{:}<w,j>\in|P|$. So $|\sim P|$ is also information-invariant. So $\forall w\exists i{:}<w,i>\in|\sim P|\supset\forall j{:}<w,j>\in|\sim P|$. So let i be any probability space for which $\exists x\in\pi_i\exists w\in x<w,i>\in|P|$ and $\exists x\in\pi_i\exists w\in x<w,i>\notin|P|$. Then $i\notin\langle P\rangle$ and $i\notin\langle\sim P\rangle$.

Second, ∨-exportation. Assume that $|P|$ and $|Q|$ are both information-invariant, and that $|P|$ and $|Q|$ are independent – i.e., that neither $|P|\subseteq|Q|$ nor $|Q|\subseteq|P|$. So $\forall w\exists i{:}<w,i>\in|P|\supset\forall j{:}<w,j>\in|P|$ and $\forall w\exists i{:}<w,i>\in|Q|\supset\forall j{:}<w,j>\in|Q|$. Then let i be any probability space such that $\forall x\in\pi_i\forall w\in x<w,i>\in|P\vee Q|$ but for which $\exists x\in\pi_i\exists w\in x<w,i>\notin|P|$ and $\exists x\in\pi_i\exists w\in x<w,i>\notin|Q|$ – in other words, i is a probability space which distributes probability over both P-worlds and Q-worlds. We know there is some such space, because we assumed that neither $|P| \subseteq |Q|$ nor $|Q| \subseteq |P|$. By stipulation, $i \in \langle P \vee Q \rangle$ but $i \notin \langle P \rangle$ and $i \notin \langle Q \rangle$.

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