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Benardete paradoxes, patchwork principles, and the infinite past

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Abstract. Benardete paradoxes involve a beginningless set each member of which satisfies some predicate just in case no earlier member satisfies it. Such paradoxes have been wielded on behalf of arguments for the impossibility of an infinite past. These arguments often deploy patchwork principles in support of their key linking premise. Here I argue that patchwork principles fail to justify this key premise.

1 Benardete paradoxes

Suppose there's an infinite sequence of Reapers each of which is assigned a unique natural number and a designated time to post on Facebook. If no Reaper posts by Reaper *n*'s designated time, Reaper *n* posts at that time. But if an earlier Reaper *does* post by Reaper *n*'s designated time, Reaper *n* does nothing. Reaper 1's designated time is 60 seconds past noon; Reaper 2's designated time is 30 seconds past noon; more generally, for any *n*, Reaper *n*'s designated time is $60/2^{n-1}$ seconds past noon. No Reaper's designated time is at or before noon.

Reflection quickly reveals a contradiction. *Some* Reaper posts between 12:00 and 12:01, for if no Reaper posts until 12:01, Reaper 1 posts at 12:01. Suppose Reaper *n* posts. If so, then no Reaper before *n* posts, since a Reaper posts only if no earlier Reaper posts. But then no Reaper before Reaper (*n*+1) posts, in which case—since a Reaper posts if no earlier Reaper posts—Reaper (*n*+1) posts. Hence, some Reaper before *n* posts. So, if Reaper *n* posts, then no Reaper before *n* posts *and* some Reaper before *n* posts. From this it follows that Reaper *n* does not post. Since this reasoning holds for any arbitrary *n*, no Reaper posts. So, *no* Reaper posts, but—as shown earlier—*some* Reaper posts. Contradiction ensues, and everything explodes.

Benardete paradoxes like this variously involve deafening gongs, firing squads, grim reapers, gods erecting impassable walls, and more besides.[1] Some are supertasks; others are spread out over an infinite past or even an infinite future.[2] The paradoxes share a logically unsatisfiable structure: a beginningless, linearly ordered infinite set each member of which satisfies a predicate iff no earlier member satisfies it (Shackel 2005). The paradoxes have also been wielded on behalf of arguments for the impossibility of infinite pasts and infinite causal regresses (Koons 2020, 2014, Pruss 2018, Luna and Erasmus 2020, Erasmus 2018, Luna 2009). Here's the abstract, simplified form of these temporal finitist arguments:

[1] The paradoxes are named after their pioneer, José Benardete (Benardete 1964). For different variants, see Koons (2020, 2014), Pruss (2018), Luna (2009), Shackel (2005), Laraudogoitia (2003), and Hawthorne (2000), *inter alia*.
[2] For infinite future versions, see Cohen (2015) and Schmid (Forthcoming).

1. If infinite pasts are possible, then Benardete paradoxes are possible.
2. Benardete paradoxes are impossible.
3. So, infinite pasts are impossible.

A central motivation for premise (1) comes from *recombination* or *patchwork principles* (Koons 2020, 2014, Pruss 2018; see also Schmid 2023).[3] I will argue, however, that patchwork principles fail to support premise (1).

2 Patchwork principles and the infinite past

Patchwork principles trace back at least to Lewis (1983, 1986), but their Humean underpinnings are evident in their denial of necessary connections between spatiotemporally disjoint existences.[4] I will focus in particular on Koons' (2014) deployment of the patchwork principle on behalf of premise (1). The reasons for this focus are twofold. First, as far as I'm aware, Koons offers the only precise statement of the range of auxiliary assumptions needed to entail the possibility of a Benardete paradox from the possibility of an infinite past. Second—and as I shall explain—the criticism I'll develop applies *mutatis mutandis* to other deployments of patchwork principles on behalf of premise (1). My article is therefore of general relevance to Benardete paradoxes and their metaphysical implications (or lack thereof).

Let's begin with the hypothesis assumed for reductio:

> **H2. Possible Infinite Past, with Infinitely Many Parts (PIPIP)**. There is a possible world W and a region R and time *t* of W such R has a temporal part wholly earlier than *d* units before *t*, for every finite interval *d*. (*ibid*, p. 260)

We then add the patchwork principle:

> **P2. Infinitary Patchwork (PInf)**. If S is a countable series of possible worlds, and T a countable series of regions within those worlds such that T_i is part of W_i (for each *i*), and f is a metric and topology structure-preserving function from T into the set of spatiotemporal regions of world W such that no two values of f overlap, then there is a possible world W' and an isomorphism f' from the spatiotemporal regions of W to the spatiotemporal regions of W' such that the part of each world W_i within the region R_i exactly resembles the part of W' within region f'(f(R_i)). (*ibid*, p. 258)

[3] Typically, patchwork principles motivate (1) *in conjunction with* other claims like the individual possibility of a single Reaper and the intrinsicality of its powers and dispositions (Koons 2014, 2020)—more on this anon. Note that not all defenses of (1) (or premises relevantly similar to (1)) invoke patchwork principles. For instance, Luna and Erasmus (2020) rely on no patchwork principle. For criticisms of the use of patchwork principles and other defenses on behalf of (1), see Schmid and Malpass (2023) and Schmid (Forthcoming).

[4] For more on this denial and its connection to patchwork principles, see Wilson (2010, 2015).

Here, 'exact resemblance' is understood as *exact duplication*, where exact duplicates share their intrinsic properties (*ibid*). In rough and simple terms, P2 says that so long as there's a possible world with enough spatiotemporal 'room' to accommodate (without overlap) an arbitrary arrangement of individually possible spacetime regions (including their contents), then there's a possible world—the 'patched-together world'—containing exact intrinsic duplicates of those regions in precisely that arrangement.[5]

Next up are two auxiliary assumptions needed for H2 and P2 to imply the possibility of a Benardete paradox:

> **P1. Possible Grim Reaper (PGR)**. There is a possible world W and a region R such that R has a finite temporal duration d seconds, there is a Grim Reaper wholly contained within R, and throughout R the Grim Reaper has the power and disposition to create a particle and place it at a designated position d meters from the plane P if there is no Fred particle closer to the plane than d meters, and otherwise to maintain any Fred particle that is within d meters of the plane in its initial position. (*ibid*, p. 257)

> **P3. Intrinsicality of the Grim Reapers' Powers and Dispositions (PDIn)**. The powers and dispositions ascribed to each Grim Reaper are properties intrinsic to that Reaper in its corresponding region and world. (*ibid*, p. 258)

P1 states the individual possibility of a Reaper, while P3 states the intrinsicality of Reapers' powers and dispositions. Koons argues that (H2 & P1 & P2 & P3) entails the possibility of a Benardete paradox involving infinitely many Reapers spanning the infinite past each of which creates and places a particle iff no earlier Reaper does so. Here's the basic idea. If P1 and P3 are true, then there's an individually possible spacetime region containing a Reaper endowed with intrinsic powers and dispositions to place a particle in the manner described. If H2 is true, then there's a possible world with an infinite past providing enough spatiotemporal 'room' to accommodate (without overlap) a beginningless arrangement of infinitely many non-overlapping spacetime regions containing (exact intrinsic duplicates of) those Reapers. Given P2, it follows that there's a possible world containing exact intrinsic duplicates of precisely those Reaper-containing regions in precisely that beginningless arrangement. But *that* implies the possibility of a Benardete paradox. Since such a paradox is contradictory (and hence impossible), (H2 & P1 & P2 & P3) is false. Koons urges us to affirm (P1 & P2 & P3) and hence to reject H2.

3 The problem

[5] In using 'patched-together', I don't mean to imply that the patchwork principle is a *world-making* principle (as it is in Armstrong's combinatorialism). My usage of 'patched-together' is for ease of exposition and is compatible with possible worlds simply being accurately *described by* (rather than *constructed from*) the patchwork principle. Note that the 'sample patches' are the individually possible spacetime regions (including their contents).

This argument, however, is problematic. To begin, note that patchwork principles are generally restricted to *intrinsic duplicates* (Lewis 1986, p. 89, Bricker 2020, p. 283). Thus, the only properties of objects and regions that we're licensed to infer are preserved from sample-patch-worlds to the patched-together world are *intrinsic* properties of those objects and regions.[6] So, for instance, if an apple is intrinsically massive in sample-patch-world w_1, and if a banana is intrinsically massive in sample-patch-world w_2, then any patched-together world w_3 involving (a duplicate of) that apple from w_1 and (a duplicate of) that banana from w_2 will preserve the fruits' masses from w_1 and w_2, respectively. But w_3 *may not* preserve their respective *extrinsic* properties from w_1 and w_2. For instance, if the banana in w_2 exists alongside no apples in w_2, the banana's duplicate clearly *lacks* this property in w_3. Koons (2014, p. 258) follows suit in restricting his patchwork principle (P2) to exactly intrinsically resembling duplicates.

This restriction, however, is an important plank in a problem for Koons' reductio of H2. As we've seen, P2 doesn't license us to infer that extrinsic properties are preserved from sample-patch-worlds to the relevant patched-together world. The only extrinsic properties that P2 licenses us to infer are instantiated in a patched-together world are those whose instantiation is *entailed* by (a) the instantiation of the intrinsic properties of the relevant individual sample patches, together with (b) the spatiotemporal arrangement of those sample patches in the patched-together world.

Here's why this matters. As Koons recognizes, the paradoxical scenario involving Reapers requires the successful transmission of some signal among the Reapers. Otherwise, later Reapers wouldn't be sensitive to the acts of previous Reapers. But this sensitivity is needed for each Reaper to act iff no previous Reaper acts, which in turn is needed to generate the contradiction.[7] As Koons writes:

> What's really required for the argument to work is an assumption about the persistence of signals of a certain kind. When a Grim Reaper fails to find a "Fred" particle in the appropriate region of space, he is in effect receiving a null signal from his predecessors. He is then supposed to send a signal (in the form of an appropriately placed Fred particle) to all of his successors to the effect, "I, GR number *n*, am the first to have acted." We can re-formulate the argument in a way that removes all reference to the particle. What's essential is that each "Grim Signaler" (to change the name) has the passive power of receiving any signal sent by a predecessor (if there is in fact one), and the active power of sending a signal (of the form "Grim Signaler *n* was the first to initiate this signal") to a

[6] In P2, the 'sample-patch-worlds' are the W_i, each of which contains the relevant individual possibilities (i.e., each of which contains an individually possible region, T_i, and its contents). The 'patched-together' world is W'.

[7] I'm here setting aside the option, considered in Schmid (Forthcoming), that a Reaper brutely satisfies this biconditional in a purely truth-functional way devoid of any causal sensitivity to other Reapers. I set this aside for three reasons. First, those who defend the temporal finitist argument from (1)–(3) tend to think this option is impossible. Second, if the option *is* possible, then the endless future problem for the temporal finitist argument becomes far more pressing (cf. Schmid Forthcoming). Third, those who defend the temporal finitist argument tend also to adopt *causal* finitism as a unified solution to Benardete paradoxes. But causal finitism cannot solve the full breadth of Benardete paradoxes if we allow for brute non-causal sensitivity of this sort. And in any case, my argument would still show that patchwork-principle-based defenses of (1) fail for those who *do* think this option is impossible.

successor GS (again, if there is one). This pair of passive and active powers is intrinsic to each Grim Signaler. (*ibid*, pp. 263–264)

Recall that *sample patches* are individually possible regions and their contents. In the current context, then, an individual sample patch is a spacetime region R containing a Signaler S and any signal L traveling within R at some possible world *w*. Importantly, though, whether L successfully propagates *from* R *to* some R-disjoint region R* at *w* is *extrinsic* to R, S in R, and L in R at *w*.[8] Such propagation depends on whether a distinct, non-overlapping spacetime region receives L and hence depends on *more* than the intrinsic features of R, S in R, and L in R—e.g., *whether there even is* a disjoint region into which L can propagate, whether something *else* in *w* (either something non-spatial or something in a region disjoint from R and R*) causally prevents L's propagation at the R–R* boundary, and whether L inexplicably ceases at that boundary. This is true even if L's *capacity* to propagate into a disjoint region is intrinsic to L in R. What matters is whether that capacity is *exercised* or *manifested*, and *that* depends not just on the intrinsic properties of L in R (or the intrinsic properties of R itself or S in R) but also on broader facts about *w* beyond R and its contents.

So, whether L propagates into an R-disjoint region is *extrinsic* to R, S in R, and L in R at *w*.[9] But since (i) Benardete paradoxes *require* the successful propagation of a signal from one Signaler-containing region to a disjoint region—which, again, is *extrinsic* to that Signaler-containing region and the Signaler and signal contained therein—and since (ii) the only extrinsic properties that P2 licenses us to infer are instantiated in a patched-together world are those whose instantiation is *entailed* by (a) the instantiation of the intrinsic properties of the relevant sample patches, together with (b) the spatiotemporal arrangement of those sample patches in the patched-together world, it follows that (iii) P2 only licenses the inference to the possibility of a *paradoxical* patched-together world if the successful propagation of a signal from one Signaler-containing region to a disjoint region is *entailed* by (a) the instantiation of the intrinsic properties of the relevant Signaler-containing regions and the Signalers and any signals contained therein, together with (b) the spatiotemporal arrangement of those regions in the patched-together world.

But whether a signal successfully propagates from one Signaler-containing region to a disjoint region is *not* entailed by the instantiation of the intrinsic properties of the relevant Signaler-containing regions and the Signalers and any signals contained therein, *regardless* of the spatiotemporal arrangement of those regions. Consider two such Signaler-containing regions, R_1 and R_2. Even if R_1 and R_2 are adjacent (though non-overlapping, as P2 requires), and even if the *intrinsic* properties of R_1 and R_2 allow any signal from their respective Signalers to propagate *within* the respective regions uninterrupted, whether a signal successfully propagates *between* them depends on whether the signal ceases at their exact boundary. But nothing about the *intrinsic* properties of either adjacent region (or their Signalers, or the signal itself) entails that the signal

[8] My points could also be made in terms of a signal *being successfully received* from a disjoint region rather than *successfully propagating* to a disjoint region. For simplicity, I'll stick with the latter.

[9] For simplicity, I will hereafter leave implicit the indexing to worlds. Since *w* was an arbitrary world, the relevant property is extrinsic in *all* worlds, and hence omitting the indexing is unproblematic.

won't stop at their exact boundary. Whether a signal stops at their exact boundary is a function not just of intrinsic facts about the Signalers, signals, regions, and their arrangement but also of whether *additional facts obtain*—e.g., whether something outside of R_1 and R_2 interrupts the signal at their exact boundary, or whether logical constraints non-causally explain the failure of signal transmission across the boundary, or whether the signal just inexplicably ceases at the boundary. Whether such facts obtain is plainly not entailed by the arrangement and intrinsic features of R_1, R_2, their Signalers, and the signal. Since R_1 and R_2 were chosen arbitrarily—and since the reasoning doesn't rely on adjacency—the point generalizes to any Signaler-containing regions and arrangement thereof.

Notice, too, that even if each region's intrinsic features are perfectly conducive to the transmission of signals therein—and even if the Signalers therein intrinsically have the power and disposition to send a signal (and act upon it once received)—this doesn't guarantee that a signal will pass *between* the regions. Imagine that the spatial coordinates of R_1 are represented by the clopen interval [0, 1) while those of R_2 are represented by the clopen interval [1, 2). Imagine also that R_1 temporally spans [12:00, 1:00) while R_2 temporally spans [1:00, 2:00). R_1 and R_2 are thus adjacent but non-overlapping spacetime regions. Notice that even if the intrinsic features of R_1 and R_2 are perfectly conducive to signal transmission—such that for any signal in R_1, that signal transmits smoothly throughout R_1, and ditto for R_2—this doesn't imply that a signal originating from one must propagate into the other. For a signal might exactly span the spatial interval [0, 1) and the temporal interval [12:00, 1:00) while failing to span any of [1, 2) or [1:00, 2:00). This plainly does not compromise either region's intrinsic conduciveness to signal transmission, since the signal originating in R_1 perfectly spans the entirety of R_1 but simply fails to ever enter R_2. Thus, in the situation at hand, any signal in R_1 transmits smoothly throughout R_1, and ditto for R_2 (trivially, since no signal is ever present in R_2). Nor does any of this compromise the Signalers' intrinsic powers and dispositions. Quite clearly, the Signalers can retain their intrinsic power and disposition to send and act upon a signal even though a signal originating from one fails to successfully propagate into the other's region. The Signalers are not deprived of this intrinsic power and disposition, for instance, if something outside R_1 and R_2 interrupts the transmission of the signal at the R_1–R_2 boundary. Finally, even if the signal itself has the intrinsic *capacity* to travel between regions, whether that capacity is *manifested* depends not just on the intrinsic features of the signal (or R_1 and R_2) *but also* on broader facts about the world, such as whether something outside R_1 and R_2 prevents the signal from transmitting. Consequently, nothing about the intrinsic character of the *signal* guarantees successful signal transmission between regions either.

Of course, one might *stipulate* that the signals in question are *special* signals which have some intrinsic property or properties that guarantee successful propagation into disjoint regions (if such exist). But such signals are simply incompatible with P2, since they involve necessary connections between the intrinsic characters of spatiotemporally non-overlapping realities. That's just *what it is* for the intrinsic properties of something in one region to *guarantee* that something happens in another, disjoint region. Consequently, the aforementioned stipulation would *by itself*

establish my thesis that patchwork-principle-based defenses of premise (1) fail, as the stipulation implies the *falsity* of such principles.[10]

So, the successful propagation of a signal from one Signaler-containing region to a disjoint region is *not* entailed by (a) the instantiation of the intrinsic properties of the relevant Signaler-containing regions and the Signalers and any signals contained therein, together with (b) the spatiotemporal arrangement of those regions in the patched-together world. Since the successful propagation of a signal from one Signaler-containing region to a disjoint region is *not* entailed by (a) together with (b), and since P2 only licenses the inference to the possibility of a *paradoxical* patched-together world if such successful propagation *is* entailed by (a) together with (b), it follows that P2 does *not* license the inference to the possibility of a *paradoxical* patched-together world. This holds even assuming the possibility of infinite pasts (H2) and individual Signalers endowed with intrinsic powers and dispositions to send (and act upon) signals (P1 and P3 corollaries).[11] Koons' reductio of H2 is therefore blocked, since the reductio requires P2 (together with H2, P1, and P3) to license the inference to the possibility of a *paradoxical* patched-together world. As that inference is *not* licensed, Koons' reductio of H2 fails.

Here, then, is a summary of the problem. The only extrinsic properties that P2 licenses us to infer are instantiated in a patched-together world are those whose instantiation is entailed by (a) the instantiation of the intrinsic properties of the relevant sample patches, together with (b) the spatiotemporal arrangement of those sample patches in the relevant patched-together world. Since *successfully propagating a signal to a disjoint region* is *extrinsic* to a Signaler and its region, P2 only licenses us to infer that a Signaler instantiates this property in the relevant patched-together world if its instantiation is entailed by (a) the instantiation of the intrinsic properties of the relevant sample patches—that is, the relevant Signaler-containing regions and the Signalers and any signals contained therein—together with (b) the spatiotemporal arrangement of those regions in the relevant patched-together world. But the instantiation of this extrinsic property is *not* entailed by (a) together with (b). Hence, P2 does *not* license us to infer that any Signaler instantiates this property in the relevant patched-together world. But the Signalers instantiating this property is *required* for the presence of a Benardete paradox, and so if P2 doesn't license us to infer that any Signaler instantiates this property in the relevant patched-together world, then P2 *also* doesn't license us to infer the possibility of a *paradoxical* patched-together world. And this holds even when P2 is conjoined with H2, P1, and P3. Hence, P2 doesn't license us to infer the possibility of

[10] There's yet another problem with this stipulation: it's not clear whether special signals of this sort are even possible to begin with, which calls into question Koons' needed assumption that the sample patches (containing the Signalers and their signals) are individually possible. At the *very* least, it's not clear why we should reject the possibility of infinite pasts (H2) instead of rejecting the possibility of such strange signals on the basis of the paradox.

[11] Even if a Signaler has that power and disposition intrinsically, *whether it succeeds in transmitting a signal to a wholly spatiotemporally disjoint Signaler* is *extrinsic* to the former Signaler and the region it occupies (and also not entailed by their intrinsic features, as we've seen). Thus, even if (as Koons suggests) we're entitled to "assume that whether or not a power is exercised successfully, and whether or not some disposition is followed in exercising it, is a matter intrinsic to the situation in which the exercise occurs" (*ibid*, p. 263), the successful exercise of the power to send a signal is different from the successful *transmission* of that signal, *once* that power is exercised, to a disjoint region. Even if the power and its exercise are intrinsic, *whether the signal resulting from that exercise successfully passes into a disjoint region* is neither intrinsic nor entailed by that intrinsic (exercise of) power.

a *paradoxical* patched-together world even when conjoined with H2, P1, and P3. Since Koons' reductio of H2 requires that P2 *does* license this, that reductio fails.

Of course, one could always *add* to the reductio the auxiliary assumption that for every natural number *n*, the signal from S_{n+1} in R_{n+1} in the patched-together world successfully propagates to S_n in R_n in the patched-together world. But adding this assumption simply concedes my central point: namely, that patchwork principles *themselves*—even together with H2, P1, and P3—do not imply the possibility of a Benardete paradox and hence do not support premise (1) of the temporal finitist's argument. What's more, we could easily take the reductio as a reductio *not* of H2 but of *this auxiliary assumption*. At the very least, we've been given *no reason* to reject H2 instead of this auxiliary assumption. And yet to successfully mount a reductio of H2, such a reason is needed.

Finally, notice that my problem here generalizes. Benardete paradoxes (at least among causally linked concreta[12]) require *some* sort of transmission of information between disjoint spacetime regions—otherwise, later members of the beginningless set wouldn't be sensitive to the features of previous members. This sensitivity, in turn, is needed for each member to satisfy a predicate (e.g., perform some action) iff no earlier member satisfies it. Benardete paradoxes therefore require each member to have at least one extrinsic property specifying the successful propagation (or reception) of information to (or from) a disjoint region. But the only extrinsic properties that patchwork principles license us to infer are instantiated in patched-together worlds are those whose instantiation is entailed by (a) the instantiation of the intrinsic properties of the relevant sample patches, together with (b) the spatiotemporal arrangement of those sample patches in the patched-together world. But extrinsic properties specifying the successful propagation (or reception) of information to (or from) a disjoint region are *not* entailed by the intrinsic features of regions, the intrinsic features of any signals and mechanisms *contained* in those regions, and the spatiotemporal arrangement of those regions. Patchwork principles therefore do not license us to infer that such extrinsic properties are instantiated in patched-together worlds. Since Benardete paradoxes *require* the instantiation of such extrinsic properties, patchwork principles do not license us to infer the possibility of Benardete paradoxes. This holds even assuming that infinite pasts and individual mechanisms (with intrinsic capacities to send, receive, and act upon information-encoding signals) are possible. But since licensing that inference is needed to support premise (1) of the temporal finitist's argument, patchwork principles fail to justify premise (1) of the temporal finitist's argument.

4 Two objections

In what follows, I address two objections to my case. The first objection seeks to redescribe Benardete paradoxes without appeal to extrinsic features (§4.1). The second objection seeks to turn extrinsic properties into intrinsic properties via conditionalization (§4.2).

[12] I'm thus setting aside Benardete paradoxes involving one sentence being true iff none of infinitely many other sentences (along some linear ordering relation) are true (cf. Yablo 1993, Shackel 2005).

4.1 Redescription

Objection. There's a simple way to redescribe the construction of Benardete paradoxes that does not require any appeal to extrinsic features. I argued, in brief, that Benardete paradoxes require that the behavior of each Signaler (or Reaper, or etc.) is sensitive to the behavior of disjoint Signalers (or Reapers, or etc.),[13] which requires each Signaler and its containing region to instantiate certain extrinsic properties in the patched-together world that patchwork principles *do not imply* will be instantiated in said world. Patchwork principles, then—even together with the possibility of an infinite past (H2) and Signalers (P1) endowed with the relevant intrinsic powers and dispositions (P3)—do *not* imply the possibility of Benardete paradoxes. But—the objection continues—we need only appeal to the *intrinsic* dispositions of how a given Signaler will react to a *signal*, not necessarily a *disjoint signaler*.

In more detail, let R_n be a spacetime region one second in duration and one meter in width, with a closed border on the right side and an open border on the left side. Likewise, assume the temporal duration of this region is topologically open in the earlier-than direction and closed in the later-than direction. Suppose there's a Signaler S_n in the middle of this region with the following intrinsic power and disposition: if a signal comes from the left before a designated time t_n, then I propagate that signal to the right, and if *no* signal arrives from the left by t_n, then I send the signal 'N' to the right. Assume further that the intrinsic properties of this spacetime region are such as to allow any signal to smoothly transmit throughout the region without any loss.

Assuming there's a possible world with enough spatiotemporal room to accommodate it—as there is, if infinite pasts are possible—by the patchwork principle we can arrange these regions such that each R_{n+1} occurs immediately to the left of and immediately temporally prior to R_n. This will deliver the possibility of a Benardete paradox without directly relying on any Signaler's sensitivity to disjoint Signalers. Instead, we only rely on the *intrinsic* powers and dispositions of the Signalers to act (directly) on *signals*.

Response. The problem, however, remains: *whether the signal to which S_n in R_n reacts was successfully transmitted from S_{n+1} in R_{n+1} is extrinsic to S_n and R_n.* And, crucially, the presence of a Benardete paradox in the relevant patched-together world requires such successful transmission between regions, as Koons (*ibid*, pp. 263–264) rightly observes when he notes the need for the persistence of signals across the spacetime regions containing Signalers. For suppose that *no* successful transmission of signals between the R_i (containing Signalers, or Reapers, or etc.) occurs in the patched-together world. Then no Signaler receives a signal from its left before its designated time. Each Signaler therefore sends 'N' to its right—a signal which, of course, doesn't successfully transmit into the adjacent region but stops at the exact boundary between the regions. *Notice that there's nothing contradictory here*. We don't actually have a Benardete paradox on our hands;

[13] Again, this is something *proponents* of Benardete-paradox-based arguments for finitist theses grant—consider again Koons' passage from earlier or Pruss' (2018) causal finitist solution to Benardete paradoxes. Causal finitism is a *solution* to Benardete paradoxes precisely because each member of the paradoxical beginningless chain is causally sensitive to the prior members.

there's no beginningless set of items each member of which satisfies a predicate iff no earlier member satisfies it. For it is simply untrue that each member sends a signal iff no earlier member sends a signal; instead, each member sends a signal *even though* earlier members all send signals. Successful transmission of signals between regions *is* therefore needed for a Benardete paradox to arise. And, crucially, *whether a signal successfully transmits to R_n from R_{n+1}* is plainly *extrinsic* to R_n and the Signaler (S_n) and signal therein. And as I argued in the previous section, the instantiation of the relevant extrinsic property by R_n (or S_n) is not entailed by the intrinsic properties of R_n, S_n, R_{n+1}, S_{n+1}, and S_{n+1}'s signal together with their adjacent arrangement. Thus, the patchwork principle—even together with H2, P1, and P3—does *not* imply the possibility of a Benardete paradox, since the patchwork principle doesn't license us to infer that the signal from S_{n+1} in R_{n+1} successfully propagates to the disjoint region R_n in the patched-together world. And yet such successful propagation is required for there to be a Benardete paradox.

Once more, we can always *add* to the reductio the auxiliary assumption that for every natural number *n*, the signal from Signaler S_{n+1} in region R_{n+1} successfully propagates to Signaler S_n in region R_n in the patched-together world. But again, this only concedes my central point: that patchwork principles—even together with the possibility of an infinite past and Signalers endowed with the relevant intrinsic powers and dispositions—do not imply the possibility of Benardete paradoxes and hence do not themselves justify premise (1) of the temporal finitist's argument. And again, rather than inferring the negation of H2, we could instead use the reductio to infer the negation of *this auxiliary assumption*. At the very least, we've been given *no reason* to reject H2 instead of this auxiliary assumption. And yet to successfully mount a reductio of H2, such a reason is needed.

It may be worth considering a (potential) reason here, which I'll hereafter call '*Reason*': if the auxiliary assumption is false, then the cessations of the signals at the boundaries of the relevant regions in the infinite patched-together world would be *uncaused*. But—*Reason* continues—uncaused events are impossible.

Fully assessing *Reason* extends beyond the scope of this paper. In fact, even if *Reason* is right, my central point stands: patchwork principles *themselves* fail to support the crucial premise of the temporal finitist's argument. My point—one that hasn't been uncovered in the literature—is *precisely* that something like *Reason* will be needed in addition. Assessing *Reason* is therefore excess to requirement for purposes of my paper. Still, though, there's value in considering its merits, and doing so may facilitate future research. I'll therefore offer two tentative responses.

First, *the patchwork principle itself* entails the possibility of *precisely* these sorts of uncaused cessations *even in finite contexts*.[14] Simply take as our sample patches (i) a spacetime region R_A containing a signal being propagated to its boundary, and (ii) a spacetime region R_B wholly devoid of any such signal and anything that could obstruct any such signal. Since there are clearly possible worlds (e.g., ours) with enough spatiotemporal room to fit R_A and R_B adjacently—and since R_A and R_B are each clearly possible—the patchwork principle entails the possibility that R_A and R_B *are* adjacent. But in that case, R_A's signal propagated to the boundary between R_A and

[14] Koons (2014, pp. 266–267) makes a similar point.

R_B ceases exactly at that boundary (and fails to pass into R_B) even though nothing obstructs its transmission. We therefore have an uncaused cessation on our hands. Thus, the proponent of the patchwork principle (and hence the proponent of the patchwork-principle-based defense of premise (1)) cannot consistently employ *Reason*. If one objects that something *else* in this patched-together world—i.e., something *outside* of R_A and R_B—would interrupt the signal at their boundary (resulting in a *caused* cessation), then the same can be said about the *original* signal cessations in the *infinite* patched-together world. But then *Reason* fails.

Second, even if the cessations would be *uncaused*, they wouldn't thereby be *unexplained*; there may be a *non-causal* explanation of their cessation in terms of logical constraints. After all, if the signals *didn't* cease in the infinite patched-together world, then we'd get a contradictory Benardete paradox, which is impossible. By my lights, this (non-causally) explains their failure to transmit in the patched-together world, just as the mathematical impossibility of the contrary (non-causally) explains one's failure to cross the seven bridges of Königsburg without doubling back on oneself (Baron and Colyvan 2019). And at least speaking for myself, I only find uncaused events problematic if there's no *non-causal* explanation for those events.[15]

Much more could be said here, but that suffices for present purposes. Again, neither of these responses affect my ultimate conclusion: patchwork principles fail to support the crucial premise of the temporal finitist's argument.

4.2 Conditionalization

Objection. My case rests on the claim that a *paradoxical* patched-together world requires Signalers (or Reapers) to instantiate certain extrinsic properties—properties that P2 (even together with H2, P1, and P3) does *not* imply are instantiated in the patched-together world. But this overlooks that some extrinsic properties can be turned into intrinsic ones via *conditionalization*. In particular, we can describe a Signaler's power in *conditional* form as the power to successfully send the appropriate signal to the next Signaler in a disjoint region *if* such a Signaler exists, there is no spatiotemporal discontinuity between them, and so on. Importantly, this conditional power is *intrinsic* to each Signaler; its possession does not depend on there actually *being* a disjoint Signaler, etc. Consequently, P2 *does* license us to infer its instantiation in a patched-together world (when the relevant sample patches are Signaler-containing regions). But then P2 (together with H2, P1, and P3) *does*, after all, license the inference to the possibility of a paradoxical patched-together world.

[15] For more on non-causal explanation—and for many examples in philosophy of science and philosophy of mathematics—see (*inter alia*) Colyvan (2001, pp. 47–51), Lipton (2004), Lange (2013), Baron and Colyvan (2016), Reutlinger (2017), Reutlinger and Saatsi (2018), and the discussion and references in Baron and Colyvan (2019). Alternatively, we might follow Koons (2000, chs. 7 and 15) in proposing that logical facts—like (in our case) the narrow logical impossibility of the unsatisfiable pair (cf. Shackel 2005)—are causally efficacious in constraining the character of the spatiotemporal world. As Koons writes, "Facts about the logical laws impinge on the concrete world by actively preventing contrary-to-logic situations from developing" (2006, p. 247). We might then offer a *causal* explanation of the failure of signal transmission in terms of logical laws, *pace Reason*.

Response. While we can describe a Signaler's *power* in conditional form—and while this does imply that the *power* is intrinsic to each Signaler—whether that power is *exercised* or *manifested* is nevertheless extrinsic to each Signaler. For whether a Signaler S_{n+1} *exercises* that power depends on (i) whether there *is* another Signaler S_n in a disjoint spacetime region, (ii) whether S_{n+1}'s signal successfully propagates between S_{n+1}'s region and S_n's region, etc. And these, of course, are extrinsic to S_{n+1}. So, the *exercise* of a Signaler's (intrinsic) conditional power is *itself* extrinsic to that Signaler and its containing region. But then we can simply run the same reasoning from §3 applied to the *exercise* of this power: since the power's exercise is *extrinsic* to a Signaler, P2 only licenses us to infer that the power is exercised in a patched-together world if the power's exercise therein is *entailed* by the intrinsic properties of the relevant sample patches together with their spatiotemporal arrangement in the patched-together world. But this entailment does *not* hold for the reasons given in §3—the intrinsic properties of the regions and the Signalers and signals they contain do *not* guarantee that any signals successfully propagate between disjoint regions, and this is true *irrespective* of the arrangement of those regions. After all, a signal might be prevented from successfully propagating between regions by something *else* in the world (e.g., some non-spatial entity); or a signal might inexplicably cease at the boundary between regions; or any number of things might happen which are independent of the arrangement and intrinsic properties of the regions and the Signalers and signals they contain.

Of course, one might respond that the Reaper has the (intrinsic) conditional power to successfully send a signal *if* no such signal-propagation-precluding facts obtain. But *the patchwork principle alone* does not license us to infer that no such signal-propagation-precluding facts obtain in the patched-together world.[16] That no such facts obtain in the patched-together world is an *auxiliary assumption* that needs to be *added* to Koons' assumptions (H2, P1, P2, and P3) to deliver the possibility of a *paradoxical* patched-together world. (If we *don't* add this auxiliary assumption, then there's no guarantee that signals will successfully propagate between regions in the patched-together world. But such successful propagation is *needed* for a Benardete paradox to obtain, as explained in §3.) But once more, adding an auxiliary assumption only concedes my central point: that patchwork principles—even together with the possibility of infinite pasts and Signalers endowed with the relevant intrinsic powers and dispositions—do not imply the possibility of Benardete paradoxes and hence do not themselves justify premise (1) of the temporal finitist's argument. And again, rather than inferring the negation of H2, we could instead use the reductio to infer the negation of *this auxiliary assumption*. At the very least, we've been given no reason to reject H2 instead of this auxiliary assumption. And yet to successfully mount a reductio of H2, such a reason is needed.

A reviewer suggests that the only auxiliary assumption needed is that no causal factor impedes with logical necessity the successful transmission of signals among Reapers, and that this is a plausible assumption since causal factors never act with logical necessity.

[16] For instance, the patchwork principle certainly does not imply that there is no non-spatial entity in a patched-together world that prevents signal propagation. Nor does the principle imply that signals do not inexplicably cease at the boundaries of regions in a patched-together world. Nor does the principle imply that no logical facts *non-causally* explain signal propagation failure in patched-together worlds. And so on.

But *pace* the reviewer, it is not true that this is the only auxiliary assumption needed. First, even if no *causal* factor explains the failure of signal transmission in a patched-together world, there may be a *non-causal* explanation for its failure. Alternatively, there may be *no* explanation for its failure. Thus, the reviewer's auxiliary does *not* suffice to entail (together with H2, P1, P2, and P3) the possibility of Benardete paradoxes in patched-together worlds.

Second—and ignoring my first point—even if no causal factor impedes signal transmission of logical necessity, it may still be logically necessary that *some causal factor or other* impedes signal transmission.[17] Thus, once again, the reviewer's auxiliary does not suffice to entail (together with H2, P1, P2, and P3) the possibility of Benardete paradoxes in patched-together worlds.

Third, even if it's logically necessary that some causal factor impedes the transmission of signals among the infinitely many Reapers in patched-together worlds, this *does not imply* that it is logically necessary *full-stop* that some causal factor acts. It only implies that it is logically necessary that *if* other conditions hold which jointly entail that some causal factor acts, *then* some causal factor acts. But *that* isn't absurd; in fact, it's clearly true! Compare: it is logically necessary that *if* (i) all beginnings have causes, and (ii) an Arsenal match begins, *then* something causes the Arsenal match. The same holds for the causal factors that (we are supposing) impede signal transmission. It is only logically necessary that they act *if* various other conditions hold which jointly entail that they act—e.g., (i) there's a beginningless arrangement of infinitely many Reapers each of which has the power and disposition to send a signal iff it does not receive a signal from any preceding Reaper, (ii) the Reapers *successfully exercise* their powers and dispositions to send signals, (iii) if those signals fail to transmit between Reapers, there's no *non-causal* explanation thereof, and (iv) if those signals fail to transmit between Reapers, there is *some* explanation thereof. Consequently, I see no issue in rejecting the reviewer's auxiliary assumption. Rejecting it does not imply any implausible sense in which causal factors act with logical necessity.

Finally, even if my preceding replies fail, my ultimate conclusion remains: patchwork principles alone fail to support the crucial premise of the temporal finitist's argument. Defending an auxiliary hypothesis in response amounts to *recognizing* this conclusion.

5 Conclusion

According to a crucial premise in a popular temporal finitist argument, the possibility of Benardete paradoxes follows from the possibility of infinite pasts. Many temporal finitists have deployed patchwork principles in support of this premise. These principles roughly state that so long as there is a possible world with enough spatiotemporal 'room' to accommodate an arbitrary arrangement of individually possible, non-overlapping spacetime regions and their contents, then some possible world includes intrinsic duplicates those regions and their contents in that arrangement. If we assume that infinite pasts are possible, then there is a possible world with enough spatiotemporal

[17] The scope of the necessity operator is crucial here. Even if it is logically necessary that some causal factor impedes signal transmission, it doesn't follow that some causal factor F is such that it is logically necessary that F impedes signal transmission. To suppose otherwise is to illicitly swap the modal operator with the existential quantifier.

'room' to accommodate a beginningless arrangement of non-overlapping regions containing Reapers (or Signalers) each of which has the intrinsic power and disposition to (say) create and place a particle iff no earlier Reaper does so. Since regions containing Reapers of this sort are *individually* possible, patchwork principles imply that if infinite pasts are possible, then some possible world includes a beginningless arrangement of duplicate Reapers with the aforementioned powers and dispositions. Since such an arrangement entails the existence of a Benardete paradox, patchwork principles imply that if infinite pasts are possible, then Benardete paradoxes are possible.

Against this reasoning, I argued that patchwork principles do *not* support the crucial premise of the temporal finitist's argument. Focusing on a representative patchwork principle P2 and Signalers instead of Reapers, I argued that P2 does *not* license the inference to the possibility of a paradoxical world even assuming the possibility of infinite pasts (H2) and regions containing Signalers with the relevant intrinsic powers and dispositions (P1 and P3). My argument for this conclusion runs as follows.

For any extrinsic property P, P2 licenses us to infer the instantiation of P in a patched-together *only if* P's instantiation therein is entailed by (a) the instantiation of the intrinsic properties of the relevant sample patches, together with (b) the spatiotemporal arrangement of those sample patches in the relevant patched-together world. But whether a signal in region R successfully propagates to an R-disjoint region is *extrinsic* to R, that signal in R, and the Signaler in R. Hence, a property specifying the signal's successful propagation to a disjoint region is *extrinsic*—in which case, P2 licenses us to infer its instantiation in a patched-together *only if* its instantiation therein is entailed by (a) the instantiation of the intrinsic properties of the relevant sample patches, together with (b) the spatiotemporal arrangement of those sample patches in the relevant patched-together world. But the instantiation of this property in a patched-together world is *not* entailed by (a) together with (b). So, P2 does *not* license us to infer its instantiation in a patched-together world. But P2 licensing the inference to its instantiation in a patched-together world is *required* for P2 licensing the inference to the possibility of a *paradoxical* patched-together world. So, P2 does not license the inference to the possibility of a paradoxical patched-together world. Since assuming the possibility of infinite pasts (H2) and regions containing Signalers with the relevant intrinsic powers and dispositions (P1 and P3) does not change this conclusion, it follows that P2 does *not* license the inference to the possibility of a paradoxical world even assuming H2, P1, and P3. But P2 supports the crucial premise of the temporal finitist's argument only if P2 *does* license this inference. So, P2 does not support the crucial premise of the temporal finitist's argument. As the same problem will afflict any deployment of patchwork principles on that premise's behalf, patchwork principles fail to support the temporal finitist's key premise.

While my conclusion is negative in character, I hope to have paved the way for future research on Benardete paradoxes, temporal finitism, patchwork principles, and the intimate connections among them.[18]

[18] Many thanks to three anonymous reviewers for excellent feedback on a previous draft.

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