Fabien Schang
University of Nancy 2

Beyond the Fregean myth: the value of logical values

Abstract: One of the most prominent myths in analytic philosophy is the so-called "Fregean Axiom", according to which the reference of a sentence is a truth value. In contrast to this referential semantics, a use-based formal semantics will be constructed in which the logical value of a sentence is not its putative referent but the information it conveys. Let us call by "Question Answer Semantics" (thereafter: **QAS**) the corresponding formal semantics: a non-Fregean many-valued logic, where the meaning of any sentence is an ordered *n*-tupled of yes-no answers to corresponding questions.
A sample of philosophical problems will be approached in order to justify the relevance of **QAS**. These include:
(1) illocutionary forces, and the logical analysis of speech-acts;
(2) the variety of logical negations, and their characterization in terms of restricted ranges of logical values;
(3) change in meaning, and the use of dynamic oppositions for belief sets.

1. The meaning of meaning

1.1. The "Fregean Axiom"

It is well known that, according to Frege (1892: 110), the meaning of a proper name is given by its *sense* and its *reference*. By a proper name, Frege means any expression corresponding to individual (*a*,*b*,*c*,…), predicate (*F*,*G*,*H*,…), or sentential (*p*,*q*,*r*,…) constants. According to his related principle of compositionality, the reference (or sense) of a sentential constant *p* is determined by any reference (or sense) occurring in *p*.
So far, so good: the reference is that which an expression refers to, and the sense is the way by which this expression comes to refer to it. But the peculiarity of Frege's theory lies in the sense and reference of a sentential variable. On the one hand, the sense of a sentence associated with a so-called "proposition" (*Gedanke*), which is not a grammatical proposition but an enigmatic abstract entity

mentioned by various expressions. On the other hand, the reference of a sentence could be expected to be a fact, or state of affairs, but Frege (1892: 34) opts for another entity: a *truth value*. Thus: “So werden wir dahin gedrängt, den Wahreitswert eines Satzes als seine Bedeutung anzuerkennen. Ich verstehe unter dem Wahrheitswerte eines Satzes den Umstand, daβ er wahr oder daβ er falsch ist. Weitere Wahreitswerte gibt es nicht. Ich nenne der Kürze halber den einen das Wahre, den anderen das Falsche. Jeder Behauptungssatz, in dem es auf die Bedeutung der Wörter ankommt, ist also als Eigenname aufzufassen, und zwar ist seine Bedeutung, falls sie vorhanden ist, entweder das Wahre oder das Falsche”.[1]

A consequence of this so-called “Fregean Axiom” (dubbed so by Roman Suszko) is that every true sentence refers one and the same thing: the True, in the sense that “all true (and, similarly all false) sentences describe the same state of affairs, that is, they have a common referent” (Suszko 1975: 170). It results in a fully extensional logic crediting the logical replacement theorem, according to which any subsentence can be freely substituted by another with the same truth value without changing the meaning of the whole. However, the Fregean Axiom may strike as counter-natural for whoever tends to associate the meaning of a sentence to its subject-matter, *i.e.* a single state of affairs. In this respect, Suszko’s view that the reference of a sentence is a *situation* appears as more natural, where two sentences are identical only if they refer to the same situation. By a situation, Suszko means the Wittgensteinian *Sachverhalt* that turns into a state of affairs (*Tatsache*) when made true. Without entering into the details of Suszko’s philosophy of logic, the following endorses his non-Fregean line while departing from his two-valued logic.

[1] The subsequent English translations are to Max Black, in M. Black and P. T. Geach (eds.), transl. from the *Philosophical Writings of Gottlob Frege*, Blackwell, Oxford (1960): “We are therefore driven into accepting the truth value of a sentence as constituting its reference. By the truth value of a sentence I understand the circumstance that it is true or false. There are no further truth values. For brevity I call the one the True, the other the False. Every declarative sentence concerned with the reference of its words is therefore to be regarded as a proper name, and its reference, if it has one, is either the true or the false.”

1.2. Toward "non-Fregean logics"

Suszko was not the sole logician to be somehow surprised by Frege's sentential reference. The pioneer of tense logic, Arthur Norman Prior, assailed the Fregean view that sentences refer to truth values: "The theory with which Frege's name is especially associated is one which is apt to strike one at first as rather fantastic, being usually expressed as a theory that sentences are names of truth values."

But a clear-cut difference is to be made between Prior's and Suszko's stances, however: the former proposed a many-valued logic for tensed sentences in Prior 1957, while the latter always blamed the introduction of many-valued logics. He argued for his position with strong words against Jan Lukasiewicz: "Lukasiewicz is the chief perpetrator of a magnificent conceptual deceit lasting out in mathematical logic to the present day" (Suszko 1977: 377).

According to Suszko, every value beyond the true and the false is not a logical, but an *algebraic* value that is the referent of a sentence. Why does Suszko make a distinction between logical values and any further value as a sentential referent?[2] This may have to do with the structural properties of logic: truth is the only value that counts to define logical consequence, so that any further value is merely counted as false or untrue. But it remains to see why any further value is considered by him as an algebraic value or *referent*.

1.3. Between Frege and Suszko: a Question Answer Semantics

Suszko went on saying that "any multiplication of logical values is a mad idea and, in fact, Lukasiewicz did not actualize it." (Suszko 1975: 378). Such a statement is as surprising as right, if one considers a logic to be two-valued whenever the relation of logical consequence is characterized by only two logical values: one *designated* value (the true), and one *non-designated* value (the untrue). By doing so, Suszko argued that any further value beyond the true could be taken as a non-designated value (including Lukasiewicz's third value of indeterminacy). But we disagree with Suszko in this respect: not only may logical consequence be

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[2] "Thus, the logical valuations and algebraic valuations are functions of quite different conceptual nature. The former relate to the truth and falsity and the latter represent the reference assignments." (Suszko 1977: 378).

characterized in more than only one model-theoretical way, in terms of truth-preservation; but we maintain with Frege that logical values are sentential referents. In a nutshell: our coming semantics appears as a trade-off between Frege's and Suszko's views of reference.

For one thing, our insistence upon the questions-answers game leads to a semantics that agrees with Frege's theory of judgment while departing from his two-valued characterization of any judgeable content (*i.e.* the sense of a sentence, or proposition). Similarly, it agrees with Suszko's view that there can be more than two referents or semantic correlates for sentences while maintaining against him that these so-called algebraic values are properly logical values. We borrow this semantics from Stanislas Jaskowski's technique of *product systems*, where logical values are a *n*-ordered combination of classical values 1 and 0.[3]

Let us call by *Question Answer Semantics* (thereafter: **QAS**) the subsequent formal semantics, where questions give the sense of a sentence while answers convey their reference. Questions and answers essentially contribute to the meaning of a sentence in any scientific inquiry, as argued in Frege (1918, 62-3): "Ein Fortschritt in der Wissenschaft geschieht gewöhnlich so, daβ zuerst ein Gedanke gefaβt wird, wie er etwa in einer Satzfrage ausgedrückt werden kann, worauf dann nach angestellten Untersuchungen dieser Gedanke zuletzt als wahr erkannt wird. In der Form des Behauptungssatzes sprechen wir die Anerkennung der Wahrheit aus." And just as in Frege's theory of judgment, a difference is to be made between the sentential content of a judgment and the judgment itself: the thought that is expressed by a declarative sentence is primarily considered and, then, judged to be true or false by a thinking subject. Thus in Frege (1919, 143): "Eine Satzfrage enthält die Aufforderung, einen Gedanken entweder als wahr anzuerkennen, order als falsch zu verwerfen. (…) Die Antwort auf eine Frage ist eine Behauptung, der ein Urteil zu Grunde liegt, und zwar sowohl, wenn die Frage bejaht, als auch wenn sie verneint wird".[4]

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[3] About the roots of product systems in the area of many-valued logics, see Jaskowski, S.: "Investigations into the System of Intuitionist Logic" (1936), in Storrs McCall (ed.), *Polish logic 1920-1939*, Oxford University Press, 1967, 259-263.

[4] Hence the Fregean split of a declarative statement into three main steps, in

But unlike Frege, we claim that the so-called reference of a sentence, while being a logical value, is not a *truth value* (between truth and falsehood) but an *answer* to an initial question (between *yes* and *no*). We depart with Frege's theory of judgment in at least two respects: on the one hand, not every judgment is an *assertion*, contrary to what the German logician assumed throughout his works. Frege's identity (judgment = assertion) is mainly due to the goal-aimed activity of scientific investigation that purports to attain the *truth*. A corollary of the preceding difference is that not every denial is a negative assertion, on the other hand.
While Frege took truth to be an ideal object which the scientist strives to reach, nothing prevents ones from doubting about this Platonist picture and preferring a pragmatist view of truth as common agreement in a scientific community. If so, then truth is not an mythical object but a down-to-earth construction that is expressed through an assertion and relies upon the speaker's arguments.
Let $\mathbf{AR_4} = \langle \mathcal{L},\mathcal{Q},\mathcal{M}\rangle$ be a logic of acceptance and rejection, in order to account for this Non-Fregean view of truth. Its structure includes a formal language $\mathcal{L}$ of sentential variables $Var = \{p_1, \ldots, p_n, q_1, \ldots q_n, \ldots\}$, a thought-forming operator of question $\mathcal{Q}$ upon $\mathcal{L}$, and a matrix $\mathcal{M}$ that is an interpretation model of $\mathcal{L}$.
The semantics $\mathcal{M} = \langle \sim,\wedge,\vee,\rightarrow, \mathcal{A}, \mathcal{A}_4 = \{\{0,1/3,2/3,1\},\{1,2/3\}\}\rangle$ includes:
- a set of logical constants: $\sim$ for negation, $\wedge$ for conjunction, $\vee$ for disjunction, and $\rightarrow$ for conditional;

Frege (1918, 62): "Wir unterscheiden demnach: 1. das Fassen des Gedankens - das Denken. 2. die Anerkennung der Wahrheit eines Gedankens - das Urteilen, 3. Die Kundgebung dieses Urteils -das Behaupten". Frege uniquely refers here to complete interrogative sentence, that is, questions without interrogative pronouns (*who*, *where*, *what*, etc.) and whose answer is either yes or no. These questions help to introduce a thought through a sentential content.

- an interpretation function $\mathcal{A}$ from L to $\mathcal{A}_4$, that turns any sentence $\mathcal{Q}(p)$ into a statement (or judgment) $\mathcal{A}(p)$;
- a set $\mathbf{A}_4$ of four logical values, with a subset of two designated values.

Following Frege, each statement about *p* is an answer to a corresponding question about its sentential content. But unlike Frege, not every judgment is an assertion in $\mathbf{AR}_4$. The question $\mathcal{Q}$ that any thinker implicitly wonders about the thought that *p* is a twofold one: $\mathcal{Q}(p) = \langle q_1;q_2\rangle$, where q_1 = "do I hold *p* to be *true*?" and q_2: "do I hold *p* to be *false*?".[5] The ensuing answer $\mathcal{A}(p) = \langle a_1(p);a_2(p)\rangle$ includes either an *affirmation* expressing acceptance: "yes" ($a(p) = +$), or a *denial* expressing rejection: "no" ($a(p) = -$). The four logical values are thus a combination of answers A to questions about a sentence; these correspond to a decreasing variety of judgments: positive assertion for $\mathcal{A}(p) = \langle +;-\rangle = 1$, conjecture for $\mathcal{A}(p) = \langle +;+\rangle = 2/3$, doubt for $\mathcal{A}(p) = \langle -;-\rangle = 1/3$, and negative assertion for $\mathcal{A}(p) = \langle -;+\rangle = 0$.

While we agree with Frege that "Eine Satzfrage enthält die Aufforderung, einen Gedanken entweder as wahr anzuerkennen, order als falsch zu verwerfen", we find it oversimplifying to add that "Die Antwort auf eine Frage ist eine Behauptung, der ein Urteil zu Grunde liegt, und zwar sowohl, wenn die Frage bejaht, als auch wenn sie verneint wird".[6] For the content of a question may be denied without its opposite to be thereby affirmed: the third logical value of doubt means that the thinker denies both the truth and the falsehood of a sentential content, but that doesn't mean that the sentence is not true or false *per se*.

The Fregean Axiom led to this objective myth of truth values as embedding the referents of sentences; but nothing compels to accept such a realist view of logical values, and our own interpretation of logical values consists in viewing them as mere epistemic attitudes

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[5] Another formulation for q_2 is "do I hold not-*p* to be true?", where the truth of not-*p* is not equivalent with the untruth of *p*. See Schang 2009 about the resulting distinction between *inconsistency* and *incoherence*.

[6] "A propositional question contains a demand that we should either acknowledge the truth of a thought, or reject it as false. (…) The answer to a question is an assertion based upon a judgment; this is so equally whether the answer is affirmative or negative."

without any ontological commitment. Hence the subsequent distinctions between:
- judgment and assertion
The latter is just one among four possible sorts of judgment $\mathcal{A}(p)$;
- assertion and affirmation
Assertion is just one sort of affirmation, in addition with the weaker answer of conjecture; *Behauptung* is read as a synonym of assertion in Frege's texts, whereas it is read in $\mathbf{AR_4}$ as the single component a_1 = + of the complete attitude of assertion $\langle +;- \rangle$;
- denial and negation
The former concept is a no-answer while the latter commonly refers to the sentential content and expresses a thought, not an answer about it. Following the terminology of Searle and Vanderveken 1985, denial is an *illocutionary* negation and negation is a *locutionary* operator; but both equally come from the Latin verb *negare*, which means "denying" or "saying no".

2. Meaning in use

The way in which meaning can be redefined is related to the way in which judgments can be used. Let us return to Frege's arguments for his theory of judgment, before turning to our own one that pays a good deal of attention upon the concept of *negation*.

2.1. Negation and denial

Needless to say that Frege's logic was only concerned with the foundation of mathematics and, consequently, the use of declarative sentences. Frege repeatedly said that assertion purports to tell the truth by means of such a sentence, so that any other use of a sentence is to be ruled out from his mathematical logic. But the very practice of scientific research could lead to a more fine-grained theory of judgment, especially concerning the role of negation.

Frege 1919 attempted to answer to two intertwined questions. The first one is about the different sorts of judgments there are: "Gibt es zwei verschiedene Weisen des Urteilens, von denen jene bei der bejahenden, diese bei der verneinenden Antwort auf eine Frage gebraucht wird? Oder ist das Urteilen in beiden Fällen dasselbe?"[7]

[7] "Are there two different modes of judgment, the one being employed when the

The second one about the place of negation in a judgment: “Gehört das Verneinen zum Urteilen? Oder ist die Verneinung Teil des Gedankens, der dem Urteil unterliegt?”[8]. To the former question, Frege claimed that there is only one sort of judgment: assertion. Accordingly, he replied to the latter that negation is a property of the sentential content only, thus making irrelevant any distinction between affirmative and negative judgments.

The Fregean *Begriffschrift* intended to bring out the assertive force of a judgement by means of the vertical stroke |, in addition with the horizontal stroke - for sentential contents. Thus |- *p* means that the thought (or proposition) that *p* is asserted by the speaker; furthermore, the view that there could be only one sort of judgment entails that any negative judgment amounts to a negative assertion: |- ¬*p*. Turning the strokes into capital letters, let us symbolize by A and R the opposite acts of *affirmation* and *rejection,* with A(*p*) for $a_1(p) = +$ and R(*p*) for $a_1(p) = -$.[9] Accordingly, Frege claims that R(*p*) and A(¬*p*) don't make any difference, since whoever rejects or denies *p* thereby affirms its opposite negation ¬*p*. A lexical way to make this point is to argue that every denial is an affirmation like “It is false that *p*”, where being false for a thought that *p* means that the opposite thought ¬*p* is true. This so-called Equivalence Thesis has been challenged by Parsons (1984), and we do the same here: every negative assertion is a denial, but the converse needn't hold.[10]

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answer is yes and the other when the answer is no? Or is this the same judgment in both cases?”

[8] “Does denial belong to the judgment? Or is denial a part of the thought that the judgment assumes?”

[9] Our symbolism makes clearly appear that denying *p* needn't be the same as affirming its sentential negation ¬*p*: it merely means a no-answer concerning the truth of *p*. Any conflation of R(*p*) and A(¬*p*) comes from the assumption of bivalence, and the latter is not assumed in our four-valued logic.

[10] In symbols: A(¬*p*) → R(*p*) is valid in $\mathbf{AR_4}$, while R(p) → A(¬p) is not; see section 2.2. Vernant (2003) also called this inference the “Russell's law”, where the latter already argued that not every sentence is either asserted or denied by a speaker. But what prevented Russell from making a step further was his contemporary context against *psychologism*: “Logically speaking, the notion of denying a proposition *p* is not relevant; only the truth of non-*p* concerns logic”. (Russell (1904: 41). The following wants to show that such a distinction between asserting non-*p* and denying *p* doesn't lead to psychologism.

2.2. The variety of negations

A distinction between denial and negation has already been claimed throughout the history of logic: from the Four categoricals in Aristotle's logic to illocutionary forces in Searle and Vanderveken (1985), through Arnauld and Nicole's theory of judgment, the manifold use of negation suddenly passed under silence with the rise of mathematical logic in the late 19th century.[11] Actually, the reduction of logical negation to the Stoic sentential negation seems to be counterbalanced by the very use of negative expressions in natural language. Moreover, the same can be said within the very practice of science: despite Frege's view that only sentential negation does matter for the scientific language of truth-search, assumptions equally count in addition with axioms in the elaboration of reasoning.

If so, why did Frege restrict denial to negative assertion? Parsons (1984) notes that he did so mainly for sake of notational economy: the simpler a logical symbolism is, the more valuable it is. Frege (1919, 155) clearly argues for this connection between simplicity and efficiency: "Bei der Annahme von zwei verschiedenen Weisen des Urteilens haben wir nötig:

1. die behauptende Kraft im Falle des Bejahens,
2. die behauptende Kraft im Falle des Verneinens, etwa in unlöslicher Verbindung mit dem Worte 'falsch',
3. eine Verneinungswort wie 'nicht' in Sätzen, die ohne behauptende Kraft ausgesprochen werden.

Nehmen wir dagegen nur eine einzige Weise des Urteilens an, haben wir dafür nur nötig:

1. die behauptende Kraft,
2. eine Verneinungswort.

Eine solche Ersparung zeigt immer eine weitergetriebene Zerlegung an, und diese bewirkt."

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[11] Vanderveken claimed that his formal semantics cannot make a crucial use of logical values, because these denotations are irrelevant to the meaning of a speech-act. But he says so because of his natural assumption of the Fregean Axiom, according to which logical values cannot be but truth values. Our rejection of the Fregean Axiom does justice to algebraic semantics and logical values.

To the contrary, we see in the Fregean truth-valuations a reductive limitation in the expression of judgments. The simplicity of one unique judgment (i.e. affirmation) doesn't entail an efficient account of how we use negation in our daily judgments. In order to show the explanatory value of our logical values and to defend the use of a negation "without affirmative force", let us exemplify the results of $\mathbf{AR_4}$ and its applications in philosophy.

A first application is an investigation into the meaning of this peculiar logical constant: negation. Whereas Frege only paid attention to the classical sentential negation, other logical uses of negation may be rendered within the conceptual frame of **QAS** and our logic of acceptance and rejection. Just as Parsons (1984: 140) argued that Frege (1919) "limits his argument to sentences which have truth values (…) sentences or propositions without truth values are exactly the cases in which [the Equivalence Thesis] is most doubtful", our answer-values help to render the non-classical values (beyond truth and falsehood) in a more intuitive way that avoids any mention about truth values. Thus, a "gappy" sentence *p* is taken to be *neither true nor false* when the answerer denies both q_1 and q_2, that is: $\mathcal{A}(p) = \langle -;- \rangle = 1/3$; and a "glutty" sentence *p* is taken to be *both true and false* when the answerer affirms both q_1 and q_2, that is: $\mathcal{A}(p) = \langle +;+ \rangle = 2/3$.
Correspondingly, a difference between the logical values doesn't entail any difference in the general features of logical negation: it doesn't turn the true into the false in $\mathbf{AR_4}$, insofar as truth and falsehood don't appear any longer in the logical values but are contained into the questions and then contribute to the sense of a sentence (and not its reference). Rather, logical negation proceeds by reversing the ordered pair of a logical value.

For any pair $\mathcal{A}(p) = \langle a_1(p),a_2(p) \rangle$, $\mathcal{A}(\neg p) = \langle a_2(p),a_1(p) \rangle$

This can explain why intuitionist logic doesn't validate excluded middle or double negation, or why paraconsistent logic doesn't validate Duns Scot's law or disjunctive syllogism: their divergent view of truth is such that their subset of logical values doesn't result in a designated value for any interpretation of the classical validities.[12]

2.3. Change in meaning

A second efficient application of our logical values is the *theory of opposition*, recalling in the same time the Four categorical sentences of the Aristotelian traditional logic. On the one hand, an objection can be made to our preceding logical values, regarding the values of *conjecture* and *doubt*: there hardly seems to be any difference between affirming or denying both the truth and falsehood of a sentence, in the sense that they commonly lead to a similar state of indecision. This requires a change in the characterization of the logical values in **QAS**. On the other hand, this can be done if we change the content of the questions $\mathcal{Q}$. Instead of the two preceding questions, another ordered set of three question can be suggested in turn: q_1(p): "is there *no* evidence for *p*?", q_2(p): "is there *some* (but not every) evidence for *p*?", and q_3(p): "is there *every* evidence for *p*?". It results in a larger set of eight logical values $\mathcal{A}(p) = \langle a_1(p);a_2(p);a_3(p)\rangle$. This device helps to make a preliminary distinction between conjecture: $\mathcal{A}(p) = \langle -;+;- \rangle$ and doubt: $\mathcal{A}(p) = \langle -;-;- \rangle$; but it also helps to give a recursive specification of logical oppositions.

Following seminal works by Piaget and Gottschalk,[13] our logical values can be used to express oppositions by means of various negations. Let $\mathcal{A}(p) = \langle -;-;+ \rangle$ meaning that there is all evidence for *p*. Assuming that this logical value is an appropriate counterpart of *necessary truth*, we can reconstruct the Aristotelian square of modalities by this way and compare it with his Four categorical sentences.

The *contrary* opposite of necessary truth is necessary falsehood, or

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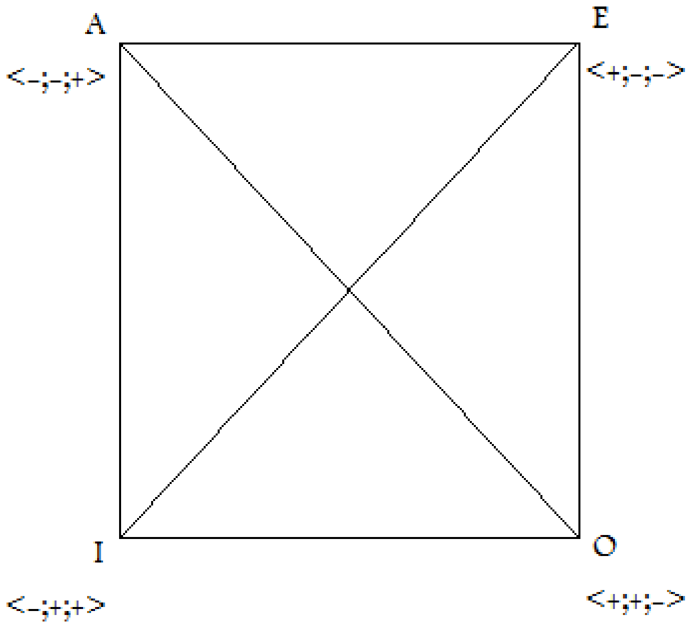
[12] Each of these non-classical logics can be depicted by a restricted range of logical values for their sentences. Thus intuitionist (gappy) logic has a restricted domain of valuation $\mathcal{A}_3$ = {{0,1/3,1},{1}, while paraconsistent (glutty) logic has another restricted subset $\mathcal{A}_3$ = {{0,2/3,1},{1,2/3}}. But logical negation proceeds in the same way throughout these logical systems. For an intuitive account of their various valuations, see Schang (2009).

[13] See Piaget, J. 1949. *Traité de Logique Opératoire* (2nd ed., 1972); and Gottschalk, W.H. 1953. "The Theory of Quaternality", *Journal of Symbolic Logic* 18, 193-6.

impossibility, according to which there is no evidence for *p*: $\mathcal{A}(p) = \langle +;-;- \rangle$. Its *contradictory* opposite is negative possibility, according to which there either no or some (but not all) evidence for *p*: $\mathcal{A}(p) = \langle +;+;- \rangle$. And its *subaltern* opposite is positive possibility, according to which there is either all or some (but not all) evidence for *p*: $\mathcal{A}(p) = \langle -;+;+ \rangle$.

THE LOGICAL VALUES IN OPPOSITION



We thus obtain a group of four opposite-forming operators O^X, where X designates a type of transformation within the theory of opposition. Given that the answers of affirmation ($a = +$) and denial ($a = -$) proceed by involution, the denial of a denial is an affirmation[14] and gives rise to the following group of oppositional transformations. Let $\langle x;y;z \rangle$ be the general form of an answer $\mathcal{A}$, and let *x*' be the denial of a given answer *x*. Thus:

- (Sub)Contrariety: $O^{(S)CT}(\langle x;y;z \rangle) = \langle z;y;x \rangle$
- Contradiction: $O^{CD}(\langle x;y;z \rangle) = \langle x';y';z' \rangle$

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[14] The entrenched rules of *bivalence* and *involution* are thus preserved in a certain sense, within our many-valued logic of acceptance and rejection: every answer is either an affirmation or a denial, *tertium non datur*; and the denial of a denial is an affirmation. However, it must be noticed that these properties are not properties of logical negation (which is a sentential operator) but denial (which is the component of a logical value).

- Subalternation: $O^{CT}(\langle x;y;z\rangle) = \langle z';y';x'\rangle$

Applying these dynamic operations to logical values helps to give a semantic calculus for changing beliefs, since our valuations correspond to belief attitudes. Furthermore, they give a dynamic interpretation of the theory of opposition in turning a belief state into one of its logical opposites: a speaker *contradicts* another one by turning an assertion into a negative conjecture, for instance.

Conclusion: the explanatory value of logical values

Our examination of Frege's theory of sense and reference attempted to bring two main results:

- firstly, to show that identifying the reference of a sentence to a truth value isn't taken to be so granted and can be challenged by means of a non-Fregean logic;
- secondly, to construct an algebraic semantics that departs from Frege's truth-valuations while making use of logical values as the referents of sentences.

The distinction between referential semantics and use-based semantics is therefore tighter than it could first appear: the so-called referents of sentences are equally determined by the use of a question-answer game. Such a game has been clearly advocated by Frege as the basis of any scientific practice, but his objective myth of truth values prevented him from conceiving any other logic than a bivalent one.

Finally, this paper has opposed two sorts of values for the logical values: an *economical* value in Frege's theory of judgment, where denial is equated with positive assertion; an *efficient* value in our logic of acceptance and rejection, where the technique of product systems helped to give a more fine-grained analysis of denial and to account for the plurality of logical negations.

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Key words: affirmation, denial, negation, Question Answer Semantics, logical values, reference, sense, truth values