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The Accuracy and Rationality of Imprecise Credences[1]

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Abstract

It has been claimed that, in response to certain kinds of evidence ("incomplete" or "non-specific" evidence), agents ought to adopt *imprecise credences*: doxastic states that are represented by sets of credence functions rather than single ones. In this paper I argue that, given some plausible constraints on accuracy measures, accuracy-centered epistemologists must reject the requirement to adopt imprecise credences. I then show that even the claim that imprecise credences are *permitted* is problematic for accuracy-centered epistemology. It follows that if imprecise credal states are permitted or required in the cases that their defenders appeal to, then the requirements of rationality can outstrip what would be warranted by an interest in accuracy.

1. Introduction: Precise and Imprecise Credences

An agent has precise credences if her belief state is representable by a credence function that assigns numbers to propositions. These numbers represent how confident the agent is in each of the propositions that the credence function is defined over. Representing the belief states of a rational agent using a precise credence function has a variety of virtues. But some have thought that the precise model should be abandoned and replaced with *the imprecise model*—a model according to which an agent is represented by *a set* of credence functions, called "a representor" (the term comes from van Fraassen (1990)).

Why move to an imprecise model? I will be focusing on what I will call "the standard imprecise view"[2] (or "the standard view" for short). On the standard view, some bodies of evidence—"incomplete" or "non-specific" evidence – warrant a doxastic state that is not representable by any single credence function, but is, representable, instead, by a set of credence functions. Not much is said about what exactly it means for evidence to be "incomplete" or "non-specific," but the examples that are appealed to in defense of the standard view consist of pairs of cases like the following:

FAIR COIN: The only evidence you have that is relevant to whether the coin in front of you will land heads (we'll call this proposition "Heads") is that the coin is fair.

MYSTERY COIN: The only evidence you have that is relevant to whether Heads is that the objective chance of Heads is between 0.05 and 0.95.

All parties agree that it's rational[3] to have a 0.5 credence in Heads in FAIR COIN. But what about MYSTERY COIN? It may be tempting to think that you should assign a 0.5 credence to Heads in MYSTERY COIN as well. After all, you might think that, in a case like MYSTERY COIN, your credence in Heads should be the average of the credences you assign to the different chance hypotheses, weighted by your credences that these hypotheses obtain. Furthermore, one might argue, you should have an even probability distribution over the different chance hypotheses in this case, since you have no reason to privilege any one over any of the others. If that's right, then the "weighted" average, and therefore the rational credence in Heads, will be 0.5.

But the defenders of the standard imprecise view think that the above reasoning is mistaken. Joyce (2005) argues that a 0.5 credence in MYSTERY-COIN is irrational. He claims that an agent who assigns a 0.5 credence in such a case is "acting as if he has some reason to rule out those possibilities in which [the objective chance is not .5], even though none of his evidence speaks on the issue" (170).[4] So, according to the standard view, in cases like MYSTERY COIN the evidence warrants a doxastic response that is best represented as a *set* of credence functions rather than a single one. Joyce claims that an agent in the evidential situation described in MYSTERY COIN is being "epistemically irresponsible unless, for each x between [.05 and .95], his credal state **c** contains at least one credence function such that **c**[Heads] = x" (171). In sum:

> The Standard Imprecise View: Given certain bodies of evidence—"incomplete" or "non-specific" evidence—any doxastic state representable by a single credence function is irrational. Instead, agents with incomplete evidence are required to adopt a doxastic state that is representable by a *set* of credence functions. In particular, if the only evidence you have concerning whether P is that the objective chance function for {P, ~P} is in the set of probability functions *S*, then your evidence requires you to adopt the doxastic attitude represented by *S*.

Two quick notes that will be important for later: First, a weak version of David Lewis's Principal Principle—the claim that agents should match their credences to the chances when they are known—is entailed by the standard imprecise view. Second, I will assume that the standard impreciser thinks that for any set of probability distributions *S* over the partition {P, ~P}, one might have as one's evidence only the proposition that the chance function is a member *S*. Given this assumption, it follows from the standard imprecise view that:

> PROBABLE CAN BE RATIONAL: For any set of probability distributions *S* over a partition: {P, ~P}, there is some evidential situation that requires one to adopt a doxastic state represented by *S*.

The aim of this paper is to argue that an accuracy-centered approach to epistemology is inconsistent with the standard imprecise view.[5] At the end I'll argue that an attempt to modify the standard view so that it only requires imprecise credences to be *permitted* is also problematic from the accuracy-centered perspective. Since the aim of this paper is to argue that accuracy-centered epistemologists must reject the standard imprecise view, it will be helpful to begin by getting clear on what exactly *accuracy-centered epistemology* amounts to. I turn to this in the next section.

2. Accuracy-Centered Epistemology

It is common to think that there must be some connection between epistemic rationality and accuracy. Accuracy-centered epistemologists take this thought very seriously. According to accuracy-centered epistemologists, *all* epistemic norms are rooted fundamentally in an agent's rational pursuit of accuracy. The accuracy-centered epistemologist's project involves showing how rational requirements can be derived from accuracy-based considerations.

As it happens, Joyce himself endorses an accuracy-centered epistemology. In his characterization of the view, accuracy-centered epistemology "sets up the minimization of gradational inaccuracy as the paramount epistemic end and puts epistemologists in the business of telling believers how to most rationally pursue it" (ms., 4). Easwaran and Fitelsen (2012) argue that Joyce's accuracy-based arguments are in tension with the idea that one is rationally required to adopt the credences that *one's evidence supports*. In responding to these arguments Joyce claims that evidential and accuracy considerations cannot compete. This is because, says Joyce, "rules of evidence have no *independent* normative status. They are ancillary norms that regulate beliefs for the purpose of achieving doxastic accuracy" (19). Indeed, he continues, "in a fully-articulated accuracy-based epistemology all norms of evidence will be underwritten by rationales which show how they contribute to the rational pursuit of accuracy" (22).

Accuracy-centered epistemologists have, in recent years, provided arguments for a host of rational requirements: probabilism (Joyce 1998, 2009), coherence requirements for full belief (Easwaran and Fitelson forthcoming[6]) conditionalization (Greaves and Wallace 2006), the Principal Principle (Pettigrew 2012), deference principles (Pettigrew and Titelbaum 2015), and requirements concerning how to respond to disagreement (Moss 2011, Levinstein 2015 and Staffel forthcoming). Accuracy-centered considerations in favor of adopting imprecise credences in cases of "incomplete" or "non-specific" evidence have been offered as well (see Konek (forthcoming)). But I will argue that there cannot, in principle, be any accuracy-based rationale for the requirements imposed by the standard imprecise view. Since accuracy-centered epistemologists think that *all* rational requirements are underwritten by accuracy-based justifications, accuracy-centered epistemologists must reject the standard imprecise view. At the end of the paper I will also consider whether an accuracy-centered epistemologist can accept the claim that imprecise credences are *permitted* in cases of incomplete evidence. I will argue that if the accuracy-centered epistemologist wants to maintain the Principal Principle (which has been defended by Pettigrew (2012) on accuracy grounds), she must also reject the claim that imprecise credal states are *permitted* in cases of incomplete evidence.

There are at least three reasons why these results should be of interest even to those who are not committed accuracy-centered epistemologists. First, the arguments will show that endorsing the requirement to adopt imprecise credences doesn't allow one to remain *neutral* on the accuracy-centered approach to epistemology. The standard view commits one to the claim that the requirements of rationality can outstrip what would be warranted by an interest in accuracy. Second, if one thinks that some rational requirements are grounded in accuracy-based considerations, but some are not, it may be of interest to realize that the requirement to adopt imprecise credences, if it exists, is of the latter sort. Finally, along the way, I will also prove some interesting results about the kinds of accuracy measures that defenders of imprecise credences might use. These results will point to serious problems with all of the measures for imprecise credal states that have, as far as I know, been defended in the literature. The arguments for these results do not rely on a commitment to accuracy-centered epistemology.

2.1. Measuring Accuracy

Accuracy-centered epistemologists claim that rational requirements are ultimately grounded in an agent's rational pursuit of accuracy. But what is accuracy, and how do we measure it?

Let's start by thinking about the accuracy of an agent's precise credence towards a proposition P. Intuitively, we can think of the accuracy of an agent's credence in P as measuring its "closeness to the truth." You're maximally accurate with respect to P if you have credence 1 in P and P is true, or if you have credence 0 in P and P is false. You're maximally *in*accurate with respect to P if you have credence 1 in P when P is false or credence 0 in P when P is true. The greater your confidence in P, the *more* accurate you are with respect to P when P is true and the *less* accurate you are with respect to P when P is false.

We can also talk about the accuracy of *a credence function* defined over a partition, given the truth values of the members of that partition.[7] In general, the higher your credences are in truths and the lower your credences are in falsehoods, the more accurate your credence function will be.

Here's a more formal characterization: let *an assignment of truth values to the members of a partition* $\mathcal{X}$ be a function that assigns to each $X_i \in \mathcal{X}$ a number: either 0 or 1, representing the truth value of that proposition. Since $\mathcal{X}$ is a partition, a *consistent* truth-value assignment (one that represents a genuine possibility, or a possible world) will be a truth-value assignment to the members of $\mathcal{X}$ that assigns 1 to exactly one proposition in $\mathcal{X}$ and 0 to the rest. For this reason, I'll sometimes abuse notation and refer to the truth value assignment that assigns 1 to X_i and 0 to everything else as simply "X_i."

Let $C_{\mathcal{X}}$ be the set of all credence functions defined on $\mathcal{X}$ and let $V_{\mathcal{X}}$ be the set of all consistent truth-value assignments to the elements of $\mathcal{X}$. What we're looking for is a function that takes a member of $C_{\mathcal{X}}$ and a member of $V_{\mathcal{X}}$ and assigns to the credence function/truth value assignment pair a number (which will be between 0 and 1) representing the accuracy of that credence function given the truth-value assignment. We'll call such a measure, over a partition $\mathcal{X}$, "$\mathcal{G}_{\mathcal{X}}$."

$$\mathcal{G}_{\mathcal{X}}: C_{\mathcal{X}} \text{ X } V_{\mathcal{X}} \rightarrow [0,1]$$

$\mathcal{G}_{\mathcal{X}}$ tells us how accurate a credence function defined over $\mathcal{X}$ is given any consistent truth-value assignment over $\mathcal{X}$.

2.2. *The Permission Principle*

The accuracy-centered epistemologist I have in mind endorses the following principle:

> PERMISSION: For any belief state **b** defined over $\mathcal{X}$, if, on every acceptable accuracy measure, there exists a belief state **b′** defined over $\mathcal{X}$, that is no less accurate than **b**, for every v $\in V_{\mathcal{X}}$, then there can be no rational requirement to adopt **b**.

Many arguments in the accuracy-centered literature rely on PERMISSION and on much stronger principles.[8] And although there may be *some* way of adopting the accuracy-centered approach endorsed by Joyce and others that doesn't require commitment to PERMISSION, when I talk about an accuracy-centered epistemology, I am going to be assuming an approach to epistemology that accepts, at minimum, this principle. So it is worth noting why the principle flows naturally from the thought that all rational requirements are grounded in a rational agent's pursuit of accuracy.

Here's the idea: If PERMISSION were false, then there could be a requirement for agents with E to adopt **b** even though, for every acceptable accuracy measure, there is a **b′** that does no worse than **b** accuracy-wise. But if there were such a requirement, then any agent with E, who uses an acceptable accuracy measure, will be able to reasonably ask the following question of some **b′**: "Why should I adopt **b** rather than **b′**? After all, **b′** will *never* do worse than **b** when it comes to accuracy!" The accuracy-centered epistemologist who thinks that **b** is required is committed to thinking that there must be *some* accuracy-based reason that could motivate the agent (assuming she's rational) to prefer **b** to **b′**. But if **b′** is no more accurate than **b** in *any* world, it's hard to see how there could be such a reason.[9]

In what follows I will consider two ways of measuring the accuracy of imprecise credal states. I will first consider using *numerical* accuracy scores (I'll call this the "precise" way of measuring accuracy for imprecise credences). I will then consider using non-numerical accuracy scores (I'll call this the "imprecise" way of measuring accuracy for imprecise credences). I will argue that whether we measure accuracy precisely or imprecisely, on an accuracy-centered approach to epistemology, the standard imprecise view is false. I will do this by arguing for the following two claims:

(1) If PERMISSION is true, and accuracy for imprecise credal states is measured *precisely*, there is *no* imprecise credal attitude that can be rationally required in MYSTERY-COIN.
(2) If PERMISSION is true, and accuracy for imprecise credal states is measured *imprecisely*, then the imprecise credal states recommended by the standard view cannot be required in MYSTERY-COIN.

Since MYSTERY-COIN is a case of "incomplete" evidence, it follows from (1) and (2) that PERMISSION (a commitment of accuracy-centered epistemology) entails the falsity of the standard imprecise view.

3. Measuring Accuracy Precisely

In this section I argue for the first of the two claims: that on an accuracy-centered approach we can't be rationally required to adopt imprecise credences in cases like MYSTERY-COIN if we measure the accuracy of imprecise credal states using numbers.

Let $B_{\mathcal{X}}$ be the set of belief states in the imprecise model defined over the partition $\mathcal{X}$. This will be the set of *sets of credence functions* defined over $\mathcal{X}$. Our accuracy measure, $\mathcal{G}_{\mathcal{X}}$*, will be a function that takes as input a belief state in $B_{\mathcal{X}}$ defined over $\mathcal{X}$, and a consistent truth-value assignment over $\mathcal{X}$, and outputs a number representing the accuracy of the belief state given the truth value assignment.

$$\mathcal{G}_{\mathcal{X}}^*: B_{\mathcal{X}} \text{ X } V_{\mathcal{X}} \rightarrow [0,1]$$

$\mathcal{G}_{\mathcal{X}}$* tells us how accurate a belief state defined over $\mathcal{X}$ is, given a consistent truth-value assignment for $\mathcal{X}$.

I propose the following three constraints on $\mathcal{G}$*:

First,

EXTENSION: $\mathcal{G}$* should be an extension of a plausible accuracy measure, $\mathcal{G}$, for precise credal states.

What makes $\mathcal{G}$ a plausible accuracy measure for precise credal states? All I will assume is that $\mathcal{G}$ gives the maximal score (score 1) to credence functions that assign 1 to all the truths (that the function is defined over) and 0 to all falsehoods, it gives the minimal score (score 0) to credence functions that assign 1 to all falsehoods and 0 to all the truths, and that it is continuous through the space of probability functions. By "continuous through the space of probability functions" I mean the following: for any partition $\mathcal{X}$, and any member of that partition X_i, small differences between probability functions defined over $\mathcal{X}$ should result in small differences in the accuracy of those functions as evaluated at X_i.[10]

What EXTENSION says is that if we looked only at the scores that $\mathcal{G}$* gives to *precise* credal states, this restricted $\mathcal{G}$* should be equivalent to a plausible accuracy measure for precise credal states. The motivation for EXTENSION is that we don't want our more general accuracy measure to deliver verdicts that we find unacceptable when comparing the accuracy of precise credal states with one another.

The second constraint is:

BOUNDEDNESS: $\mathcal{G}$* should be bounded by 0 and 1, and take the values 0 and 1.

The motivation for BOUNDEDNESS is that even in the imprecise model there is, intuitively, a maximally accurate belief state and a minimally accurate belief state. The maximally accurate belief state contains only the function that assigns 1 to all truths and 0 to all falsehoods. Vice versa for the minimally accurate belief state. Since both the maximally accurate and the minimally accurate state are precise, and $\mathcal{G}$* is an extension of a plausible accuracy measure $\mathcal{G}$, $\mathcal{G}$* should have the same bounds as $\mathcal{G}$: 0 and 1.[11]

The final constraint is:

PROBABILISTIC ADMISSIBILITY: Take any partition $\mathcal{X}$ and any probabilistic belief state **b** defined over $\mathcal{X}$. (A belief state is probabilistic if it is a set that contains only probability functions.) There can be no belief state **b′** defined over $\mathcal{X}$ such that **b′** is more accurate than **b** for some $X_i \in \mathcal{X}$, and no less accurate than **b** for any $X_i \in \mathcal{X}$. (In other words: probabilistic belief states cannot be *dominated.*)

Why accept PROBABILISTIC ADMISSIBILITY? Joyce (2009) argues that this is a constraint on any plausible accuracy measure.[12] But it is especially plausible in the current dialectical context because I am interested in an imprecise credence defender who accepts the view that, for any probabilistic precise or imprecise belief state, **b**, there is some body of evidence that requires adopting **b**. (This is what PROBABLE CAN BE RATIONAL said.) But if we had an accuracy measure according to which there exists a **b′** that is more accurate than a probabilistic **b** in some worlds, and is no less accurate than **b** in *any* world, then it's hard to see how there could be a requirement to adopt **b**. For in requiring an agent to adopt **b**, we could be requiring her to adopt a belief state that is *accuracy dominated.*[13] Certainly an accuracy-centered epistemologist should reject the possibility of such a requirement, but I think that the idea that an agent could be *required* to adopt an accuracy-dominated state should be implausible even to those who are not committed accuracy-centered epistemologists. So, unless we're willing to claim that agents using an acceptable accuracy measure are sometimes required to adopt accuracy-dominated states, we cannot maintain PROBABLE CAN BE RATIONAL without accepting the PROBABILISTIC ADMISSIBILITY constraint.

3.3. Imprecision-1

With these constraints in mind, let's return to MYSTERY-COIN. I will now prove the following:

IMPRECISION-1: For any probabilistic imprecise belief state **i** defined over the partition {Heads, Tails} (from here on "H/T"), and any numerical accuracy measure $\mathcal{G}^*$ for imprecise belief states that satisfies EXTENSION, BOUNDEDNESS and PROBABILISTIC ADMISSIBILITY, there will be a precise probability function **p,** that is no less accurate than **i,** for any v in $V_{H/T}$.

To prove IMPRECISION-1 I first need to prove a lemma. (The proof of both the principle and the lemma may be skipped without losing the main thread, but the proofs are relatively painless and, I think, illuminating.) The lemma, in essence, says the following: *If* our accuracy measure gives the same score to a precise and an imprecise state when the coin lands heads, it had better also give those states the same accuracy score when the coin lands tails.

LEMMA:

For any probabilistic imprecise belief state **i** and any probabilistic precise belief state **p** in $B_{H/T}$ (the set of belief states defined over H/T):

If $\mathcal{G}^*_{H/T}$ (**i,** Heads) $=$ $\mathcal{G}^*_{H/T}$(**p**, Heads), then

$\mathcal{G}^*_{H/T}$ (**i,** Tails) $=$ $\mathcal{G}^*_{H/T}$(**p**, Tails)

PROOF OF LEMMA:
Suppose that:

$\mathcal{G}^*_{H/T}$ (**i**, Heads) = $\mathcal{G}^*_{H/T}$(**p**, Heads) but

$\mathcal{G}^*_{H/T}$ (**i**, Tails) ≠ $\mathcal{G}^*_{H/T}$(**p**, Tails).

Then, either:

$\mathcal{G}^*_{H/T}$ (**i**, Tails) > $\mathcal{G}^*_{H/T}$(**p**, Tails) or

$\mathcal{G}^*_{H/T}$ (**i**, Tails) < $\mathcal{G}^*_{H/T}$(**p**, Tails).

It's impossible that $\mathcal{G}^*_{H/T}$ (**i**, Tails) > $\mathcal{G}^*_{H/T}$(**p**, Tails). For if this were the case, then **i** would be just as accurate as **p** in Heads worlds, but *more* accurate than **p** in Tails worlds. This means that **i** would dominate **p**, a probabilistic belief state, but PROBABILISTIC ADMISSIBILITY forbids this from happening.

Similarly, it's impossible that $\mathcal{G}^*_{H/T}$ (**i**, Tails) < $\mathcal{G}^*_{H/T}$(**p**, Tails). For if this were the case, then **p** would be just as accurate as **i** in Heads worlds, but *more* accurate than **i** in Tails worlds. This would mean that **p** dominates **i**, a probabilistic belief state, but PROBABILISTIC ADMISSIBILITY forbids this from happening.

It follows that *if* **i** and **p** are equally accurate in Heads worlds, they must be equally accurate in Tails worlds as well.[14] This is what LEMMA says. I'll now prove IMPRECISION-1.

PROOF OF IMPRECISION-1:
Take any probabilistic imprecise belief state, **i**, defined over H/T. And let:

(1) $\mathcal{G}^*_{H/T}$(**i**, Heads) = r.

By BOUNDEDNESS, $r \in [0,1]$.

By EXTENSION, $\mathcal{G}^*_{H/T}$ is an extension of a plausible accuracy measure for precise credences, $G_{H/T}$. Since the credence functions defined over H/T that have the maximal and minimal accuracy scores are probability functions (they assign 1 to the truth and 0 to the falsehood, and vice versa), the set of accuracy scores for *probability* functions is bounded by 0 and 1. We are also assuming that $\mathcal{G}_{H/T}$ is continuous through the space of probability functions. So it follows from the intermediate value theorem that, for any $r \in [0,1]$, there exists a precise probability function **p**, such that

(2) $\mathcal{G}_{H/T}$(**p**, Heads) = r.[15]

By EXTENSION, it follows from (2) that

(3) $\mathcal{G}^*_{H/T}$(**p**, Heads) = r

It follows from (1) and (3) that:

(4) $\mathcal{G}^*_{H/T}(\mathbf{i}, \text{Heads}) = \mathcal{G}^*_{H/T}(\mathbf{p}, \text{Heads})$.

Finally, it follows from LEMMA and (4) that

(5) $\mathcal{G}^*_{H/T}(\mathbf{i}, \text{Tails}) = \mathcal{G}^*_{H/T}(\mathbf{p}, \text{Tails})$.

Recall that **i** represents any probabilistic imprecise belief state in $B_{H/T}$. So what (4) and (5) tell us is that, *on any acceptable accuracy measure* $\mathcal{G}^*$, there will be, for any probabilistic imprecise **i,** a precise function **p** that is just as accurate as **i** in every world. This proves IMPRECISION-1.[16]

IMPRECISION-1, in combination with PERMISSION, entails that no imprecise probabilistic state **i** defined over Heads/Tails can ever be rationally required if we measure the accuracy of imprecise states using numbers.

4. Measuring Accuracy Imprecisely

Perhaps the defender of the standard view will think that measuring the accuracy of imprecise credences using *precise numbers* is wrongheaded. After all, the whole point of imprecise credences, say their defenders, is that traditional credences are too precise to represent the appropriate doxastic attitude in response to certain bodies of evidence. Once we allow for these "imprecise" attitudes, you might think that it would be best to represent the accuracy of an imprecise credal state with some other sort of object—perhaps a *set* of numbers. What exactly the non-numerical object is won't make a difference for what follows.

The argument I gave using *numerical* accuracy measures was an argument against requiring *any* imprecise state defined over a two-cell partition: for any such state, I showed, there is a precise state that is at least as accurate as it in every world. The result below, which allows us to measure accuracy using some non-numerical object, will not be that for *any* imprecise credal state defined over a two-cell partition, there is a precise state that is always at least as accurate as it. Rather, I will show something more specific.

To state the claim, we first need a way of describing an imprecise agent's attitude towards a single proposition. To this end, we will represent the agent's confidence towards a proposition X_i, by the set of credences assigned to that proposition by each credence function in the agent's representor, **b**. We'll call this set of credences towards X_i "$\mathbf{b}(X_i)$." When this set forms an interval $[a,b]$, we'll say that an agent has the "interval-valued credence $[a,b]$ in X_i."[17] I will show in this section that, no matter what sort of object we use to represent accuracy, the credence of 0.5 in each cell of a two-cell partition is no less accurate than any imprecise state that assigns to each cell in the partition an interval-valued credence centered at 0.5. For the purposes of uniformity, I'll now represent a .5 credence as the (trivial) interval: [0.5, 0.5]. The argument I provide may generalize, but I will not attempt to provide a generalization here. For if I can show that the imprecise credences that the evidence purportedly supports in MYSTERY-COIN are not required, then I will have shown that the standard view is false. These credences are centered at 0.5.

Let's call our non-numerical accuracy measure $\mathcal{G}^{**}$. For this result, I need to impose the following two constraints on $\mathcal{G}^{**}$. The first is PROBABILITIC ADMISSIBILITY. The second is what I will call, following Pettigrew (forthcoming) "STRONG EXTENSIONALITY." The basic thought behind STRONG EXTENSIONALITY is that the accuracy of a belief state at a world should depend only on: (a) the truth-values at that world of the propositions on which the belief state is defined and (b) the agent's confidence in each of the propositions on which the belief state is defined.

Here's a more formal characterization: Following Pettigrew, we can define the *accuracy profile* of <**b**, v> (that is, the accuracy profile of a belief state **b** given some consistent truth value assignment, v), as the following multiset:

$$\{<\mathbf{b}(X_i), v(X_i)> | X_i \in \mathcal{X}\}$$

(A multiset is like a set, in that it is unordered, but unlike a set, there can be repeated elements.) The second constraint then says:

STRONG EXTENSIONALITY: For any belief states b_1 and b_2 defined over $\mathcal{X}$, and any consistent truth value assignments v_1 and v_2 defined over $\mathcal{X}$: If $<\mathbf{b}_1, v_1>$ and $<\mathbf{b}_2, v_2>$ have the same accuracy-profile, then $<\mathbf{b}_1, v_1>$ and $<\mathbf{b}_2, v_2>$ have the same accuracy score.

In other words: The accuracy of a belief state **b** given a consistent truth value assignment v is fully determined by the accuracy profile of <**b**, v>.

STRONG EXTENSIONALITY is probably the most controversial of the constraints discussed so far, but I think that, at least in the cases I am interested in, it is highly plausible. For although there might be some contexts in which one wants to assign accuracy to an agent's belief state as a whole in a more complex way, by taking into account holistic features of the belief state, or giving different weights to different propositions, it will suffice for my purposes that, in at least some very simple cases, involving only a proposition and its negation, and in which neither of the two propositions is, in any way, more important than the other, the accuracy of an agent's belief state as a whole is determined solely by the truth values of the propositions the belief state is defined over, and the credal values assigned to those propositions. Since, the standard view applies to very simple two-cell cases like MYSTERY-COIN, I can narrow my focus to such cases. I will now prove:

IMPRECISION-2: For any probabilistic imprecise belief state **m** defined over H/T, that assigns to each proposition in H/T an interval-valued credence $[a,b]$, where $[a,b]$ is centered at 0.5, and any accuracy measure $\mathcal{G}^{**}$ for imprecise belief states that satisfies PROBABILISTIC ADMISSIBILITY and STRONG EXTENSIONALITY, the precise belief state, **s**, that assigns [0.5, 0.5] to each of Heads and Tails is no less than accurate than **m,** for any v in $V_{H/T}$.[18]

PROOF OF IMPRECISION-2:

Let **s** be the (precise) belief state that assigns [0.5, 0.5] to each of Heads and Tails. And let **m** be any imprecise belief state defined over H/T which assigns to each proposition in H/T an interval-valued credence $[a,b]$ centered at 0.5.[19]

Since **m** assigns [*a,b*] to each of Heads and Tails, it follows from STRONG EXTENSIONALITY that **m** will get the same accuracy score in Heads worlds as it does in Tails worlds. For, in each of these worlds, **m** will be a state that assigns [*a,b*] to a truth and [*a,b*] to a falsehood. (And thus, <**m**, Heads> and <**m**, Tails> have the same accuracy profile.) **s** gets the same accuracy score in Heads worlds and Tails worlds for the same reason: no matter how the world is, **s** will be assigning [0.5, 0.5] to a truth and [0.5, 0.5] to a falsehood. Thus,

(1) $\mathcal{G}^{**}$ (**m**, Heads) = $\mathcal{G}^{**}$ (**m**, Tails)
$\mathcal{G}^{**}$ (**s**, Heads) = $\mathcal{G}^{**}$ (**s**, Tails).

It follows from (1) that if **m** were more accurate than **s** in Heads worlds, it would also have to be more accurate than **s** in Tails worlds. For if **m** were more accurate than **s** in Heads worlds, we would have:

(2) $\mathcal{G}^{**}$ (**m,** Heads) > $\mathcal{G}^{**}$ (**s**, Heads).

And by the equalities in (1), we can substitute into (2) to get

(3) $\mathcal{G}^{**}$ (**m,** Tails) > $\mathcal{G}^{**}$ (**s**, Tails).

But now note that **m** *can't* be more accurate than **s** in Heads worlds, for (2) and (3) together entail that **m** is *always* more accurate than **s**, a probabilistic belief state. So if **m** is more accurate than **s** in Heads worlds, **m** dominates the probabilistic belief state **s**. But PROBABILISTIC ADMISSIBILITY forbids this from happening. Thus **m** cannot be more accurate than **s** in Heads worlds.

m also cannot be more accurate than **s** in Tails worlds. For suppose it were. Then:

(4) $\mathcal{G}^{**}$ (**m,** Tails) > $\mathcal{G}^{**}$ (**s**, Tails)

Substituting, using the equalities in (1) gives us:

(5) $\mathcal{G}^{**}$ (**m,** Heads) > $\mathcal{G}^{**}$ (**s**, Heads)

Thus, if **m** were more accurate than **s** in Tails worlds, it would have to be more accurate than **s** in every world and so **s**, a probabilistic belief state, would be dominated. But PROBABILISTIC ADMISSIBILITY forbids this from happening. Thus **m** cannot be more accurate than **s** in Tails worlds.

It follows that **m** cannot be more accurate than **s** in Heads worlds or Tails worlds.[20] So **s** is no less accurate than **m** for any v ∈ $V_{\mathrm{H/T}}$. [21]

In sum, IMPRECISION-1 and IMPRECISION-2 tell us that, whether we represent the accuracy scores of imprecise credences using numbers or using some other object, we can always find a precise credal state that is, in every world, no less accurate, than the imprecise credal state recommended by the standard view in MYSTERY-COIN. Thus, the accuracy-centered epistemologist must reject the claim that there are some bodies of evidence in response to which an agent is rationally required to adopt such a state.

5. Can Imprecise Credences be Permitted for Agents with Incomplete Evidence?

I have shown that if we take an accuracy-centered approach to epistemology, the standard imprecise view, which requires imprecise credences in cases of incomplete evidence, is false. The imprecise credal state recommended by the standard view in MYSTERY-COIN (call it **i**) can't be rationally required because, on every acceptable accuracy measure, there is a precise probability function (call it **p**), defined over H/T, that is no less accurate than **i** in every world. This means that there can be no accuracy-based rationale for the requirement to adopt imprecise credences. But, in fact, the arguments show something stronger: that if we wish to maintain the Principal Principle, and we think that dominated belief states are *forbidden* we must also reject the claim that an agent with incomplete evidence is *permitted* to adopt imprecise credences. Here's why:

First, note that the proof I gave for IMPRECISION 2 can be "flipped" by replacing all instances of "**m**" with "**s**" and vice versa, so as to yield the result that, not only is the imprecise **m** never less accurate than the precise **s**; the precise **s** is also never less accurate than the imprecise **m.** That is:

> REVERSE IMPRECISION-2: For any probabilistic imprecise belief state **m** defined over H/T, that assigns to each proposition in H/T an interval-valued credence [*a,b*], where [*a,b*] is centered at 0.5, and any accuracy measure $\mathcal{G}^{**}$ for imprecise belief states that satisfies PROBABILISTIC ADMISSIBILITY and STRONG EXTENSIONALITY, **m** is no less accurate than the precise belief state, **s**, that assigns [0.5, 0.5] to each of Heads and Tails, for any v in $V_{\mathrm{H/T}}$.

If we want to maintain the Principal Principle, this looks like bad news. After all, the Principal Principle tells us that agents whose only evidence contains the proposition that the coin is fair are *required* to assign a .5 credence to each of Heads and Tails. But if some imprecise state **m** is never less accurate than this precise state **s**, then it follows from PERMISSION that **s** is *never* required—no matter what the evidence.

While this may look like a problem for any accuracy-centered epistemologist who wants to endorse the Principal Principle, in fact, it is only a problem for those who think that the imprecise credal states recommended by the standard view are sometimes permitted. The reason for this is that if you were a hardcore defender of precise credences, and thought that imprecise credal states were always forbidden, you would reject the motivation I provided for the PROBABILISTIC ADMISSIBILITY constraint.[22] For recall that I motivated the constraint by pointing out that the standard impreciser, who thinks that all probabilistic imprecise states are sometimes *required* had better not allow these states to be accuracy dominated. That's all well and good for those sympathetic with the standard imprecise view. But if, for example, one thought that imprecise credal states were always forbidden, then there would be no reason to think that any probabilistic *imprecise* state must be non-dominated. (Indeed, defenders of precise credences should rejoice if it turned out that, on any plausible accuracy measure, imprecise credal states were dominated!) So once the standard imprecise view is rejected we need to rethink PROBABILISTIC ADMISSIBILITY, at least as applied to imprecise states.

Once we do this rethinking, though, we'll see that it's not *only* the standard imprecisers who should accept this constraint. Those who think that, for any probabilistic imprecise state **i**, there is some body of evidence, E, such that agents with E are *permitted* to adopt **i**, should also accept the PROBABILISTIC ADMISSIBILITY constraint. For on the commonly accepted accuracy-centered picture, if **i** is dominated on every acceptable accuracy measure, **i** won't just fail to be required: it will be *forbidden*, no matter what the evidence. Therefore if you think that, for every probabilistic imprecise state, there is some body of evidence that permits it, you will want the probabilistic states to be non-dominated, and thus, you should still accept the PROBABILISTIC ADMISSIBILITY constraint. The motivations for STRONG EXTENSIONALITY (as applied to the very simple cases we are concerned with) are orthogonal to any issues concerning imprecise credences. Thus, those who accept a modified version of the standard view according to which, for every probabilistic imprecise state **i**, there is some body of evidence that *permits* agents with that evidence to adopt **i**, should accept the two constraints I proposed in section 4.

So we now face the following two facts: first, that if PROBABILISTIC ADMISSIBILITY and STRONG EXTENSIONALITY are plausible constraints on accuracy measures for imprecise credences, then, on an accuracy-centered picture, the Principal Principle must be rejected (this follows from PERMISSION and REVERSE IMPRECISION 2), and second, that these constraints should remain plausible to accuracy-centered epistemologists who thinks that every probabilistic imprecise state is sometimes permitted. Putting these facts together we get an argument for the claim that accuracy-centered epistemologists who want to maintain the Principal Principle should deny, not only the claim that incomplete evidence always requires these imprecise states, but also that it permits them.

At this point one might wonder: can it really be true that imprecise credences aren't ever *permitted*? I think that this concern becomes especially pressing when we consider cases in which we have a large amount of very complex evidence. (To my mind, these cases provide a stronger motivation for jettisoning the requirement to have precise credences than cases like MYSTERY COIN, as I suggest in my (2012)). For example, if we consider propositions like the proposition that Hilary Clinton will be the next president (C), it simply seems implausible that, given our current evidence, we have (or ought to have) a doxastic attitude towards C that is represented by some precise number, like .63451. So are we not forced to countenance, at very least, the *permission* of adopting imprecise credences if we want to account for the rationality of our attitudes in such cases? I think not. For one might think that agents aren't always required to adopt precise credences without thinking that they are required, or permitted, to adopt *imprecise* credences.

One reason for adopting such a view is that there are, I think, some grounds for skepticism concerning the hypothesis that sets of credence functions actually represent genuine psychological alternatives to precise credal states. For example, if one thinks that what an agent's doxastic state is like is grounded by facts about the agent's dispositions to act (I am not endorsing this view—it's simply used as an illustration), then one will think that the imprecisers owe us a story about what sorts of actions are rationalized by imprecise credences. Without such a story, one might think, we should be skeptical of their existence. The problem is that there is, at the moment, no agreed upon decision theory for imprecise credences that distinguishes the behavior of agents with imprecise credences from the behavior of agents with precise credences.[23] So, at least at this point, giving a characterization of *what it is* to have imprecise credences in terms of one's dispositions to act is no simple task. More generally, very little has been said about exactly what the psychological reality is that underlies the representor model.[24] This makes it difficult to determine whether we should, in fact, accept that the existence of doxastic states that are representable by sets of credence functions. I don't intend to argue here that these states don't exist—only to caution us from inferring too quickly from the claim that we're not always required to adopt precise credences to the claim that we're required, or permitted, to adopt what have been called "imprecise credences."

One alternative to the imprecise credence model is to think that it can sometimes be *indeterminate* what doxastic attitude an agent takes (or should take) towards a proposition. Note that this is arguably different from thinking that there is some attitude that, in these cases, the agent *determinately* has (or should have): one that is representable by a (precise!) set of credence functions. A discussion of how to best understand the nature of such indeterminacy, and an evaluation of such a proposal is well beyond the scope of this paper (see Rinard (ms.) for an interesting discussion of indeterminacy in one's confidence levels). My point is just that denying that sets of credence functions represent genuine doxastic alternatives doesn't *entail* that all agents have, or ought to have, precise credences in every proposition.[25] There are, I think, many other options worth exploring. Whether these other options can be given an accuracy-based motivation remains to be seen.[26]

6. Conclusion

I have argued that accuracy-centered epistemologists must reject the standard view of imprecise credences. This is because, whether we measure the accuracy of imprecise credences using numbers or using some other object, there is, on every acceptable accuracy measure, a precise credal state that is never less accurate than the imprecise state that the standard view recommends in cases of incomplete evidence like MYSTERY COIN. I have also argued that if the Principal Principle is accepted, the accuracy-centered epistemologist should reject the claim that any agent with incomplete evidence is *permitted* to adopt the imprecise states recommended by the standard view. I conclude that if imprecise credal states are made rational by a certain kind of evidence, this is not a fact that can be explained by our interest in having doxastic states that accurately represent the way the world is.

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[2] Defenders of the standard imprecise view include Levi (1974, 1985), Kaplan (1996, 2010), Joyce (2005, 2010), Sturgeon (2010), Moss (forthcoming) and Konek (forthcoming). For a survey of other motivations see Bradley (2015).

[3] For the purposes of this paper, I assume that the rational doxastic state is the one supported by the agent's evidence. "Rational" and "supported by the evidence" are used interchangeably.

[4] Joyce also mentions well-known problems with this line of reasoning stemming from particular concerns about the principle of indifference (in particular: "cube-factory" cases), but he thinks that these problems are potentially solvable and are far less serious than the considerations he raises (see p.169–70).

[5] For other arguments against the standard imprecise view see White (2009), Schoenfield (2012), Rinard (2014) and Elga (2010).

[6] Actually, Easwaran and Fitelson don't count as accuracy-centered epistemologists given the way I am using the term since they deny that *all* epistemic norms are rooted in the rational pursuit of accuracy.

[7] Frequently we talk about the accuracy of credence functions defined over the *Boolean algebra* generated by a partition (which will include the propositions in the partition as well as propositions formed from those in the partition using negation, conjunction and disjunction). However, I will be restricting myself to *probability* functions and focusing on *two-cell* partitions. Since the standard view applies to *all* cases of incomplete evidence, it applies, in particular, to cases involving a two-cell partition, like MYSTERY-COIN. So if I can show that imprecise credences are not required in the two-cell case, then I will have shown that the standard view is false and, more generally, that if imprecise credences are ever required, it can't be because of some feature of an evidential situation that can apply in a two-cell case. Since I am focusing on a two-cell partition, there will be no difference in the accuracy ordering of probability functions defined over the partition, and the accuracy ordering of probability functions defined over the algebra generated by the partition. This is because the only additional propositions that will be in the algebra generated by a two-cell partition: {P, ~P} will be equivalent to P, ~P, the tautology or the contradiction. The tautology is always true and will have probability 1, the contradiction is always false and will have probability 0, and any proposition equivalent to P will be true whenever P is and will have the same probability as P. The same holds for ~P.

[8] For example Joyce's amended (2009) argument for probabilism (the amended version is on his website) requires a strengthened version of PERMISSION according to which there can be no requirement to adopt **b** so long as there is *some* acceptable accuracy measure on which **b′** is never less accurate than **b**. Here's why: Joyce's amended "Coherent Admissibility" principle (which he needs for the argument) says that, on any acceptable accuracy measure, if there exists some **c′** that is no less accurate than **c** in any world, then **c** is not probabilistic. The motivation, as I understand it, (and as it is understood in Pettigrew (forthcoming), Ch. 3) is as follows: If **c** *is* probabilistic, then, by the Principal Principle, **c** may sometimes be rationally required. But if, on *some* acceptable accuracy measure, **c′** is no less accurate than **c** in any world, then **c** *can't* be rationally required. Why? The thought is that **c** can't be rationally required given the existence of such a **c′**, because there cannot be a requirement to adopt some credence function, when there is an alternative that never does worse accuracy-wise according to some acceptable measure. Thus, we get the amended Coherent Admissibility principle. Since Coherent Admissibility is appealed to in justifying the sorts of measures that are used in a large swath of the accuracy-centered arguments, (a stronger version of) PERMISSION is being relied on implicitly in much of these arguments.

[9] Note that PERMISSION fares much better than the related *dominance* principle (which is appealed to in a host of accuracy-based arguments, such as the argument for probabilism) when it comes to what is known as "the Bronfman objection." Dominance says that **b** is *forbidden* if, on every acceptable accuracy measure, there is a **b′** that is more accurate than **b** for some $X_i \in \mathcal{X}$, and no less accurate than **b** for any $X_i \in \mathcal{X}$. Bronfman (ms.) thinks that **b** shouldn't be forbidden merely because, on every accuracy measure there is *some* **b′** that dominates **b**, for there may be no **b′** that dominates

b on *every* acceptable accuracy measure. Furthermore, the thought goes, agents may permissibly be undecided between accuracy measures, or it may be indeterminate which accuracy measure represents the right way of valuing accuracy. So if there isn't a particular belief state that dominates **b** according to *all* acceptable accuracy measures, says Bronfman, we can't claim that **b** is forbidden. Pettigrew (forthcoming) Ch.5 provides a detailed discussion of the objection and a response, and although I won't delve into these details here, I should note that the accuracy-centered epistemologist is committed to finding *some* response to this objection because the core results of accuracy-centered epistemology rely on the dominance principle. My argument for the claim that imprecise credences can't be required, however, does not rely on dominance. The reason that PERMISSION fares better than dominance when it comes to responding to the Bronfman objection is that PERMISSION never entails that some belief state is *forbidden*—only that some belief state is *permitted*. Thus, as long as it is not *required* that agents be undecided, in the relevant sense, between accuracy measures, there will be some rational agents who value accuracy in a way that is represented by a particular acceptable accuracy measure. At very least, then, those agents shouldn't be required to adopt a belief state **b** that is no less accurate, on their measure, than some **b′** in every world. It is also worth noting that if one accepts, in a limited range of cases, the "strong extensionality" constraint on accuracy measures that I will defend later, I will be able to rely on an even weaker principle that completely sidesteps the Bronfman objection: that **b** cannot be required if there exists *some particular **b′*** such that *on every acceptable accuracy measure* **b′** is no less accurate than **b** (See note 21).

[10] See Pettigrew (forthcoming) section 4.2 for a precise characterization of the continuity requirement. Continuity is frequently assumed through the space of all *credence* functions. This entails continuity through the space of probability functions as well. For we can represent every credence function defined over an n-membered partition $\mathcal{X}$ as a vector in $\mathbb{R}^n$. de Finetti (1974) proved that the set of *probability* functions is the set of linear combinations of consistent truth value assignments defined over X whose coefficients sum to 1. So the set of probability functions will form a continuous hyperplane through $\mathbb{R}^n$. (Why? Because small differences in the coefficients of the linear combinations will lead to small differences in the resulting vector.) Because the space of probability functions is a continuous hyperplane through the space of credence functions, continuity over credence functions entails continuity over probability functions.

[11] In fact, for the results that follow, I can rely on something weaker: that for every imprecise state, there is some precise state that is more accurate than it, and some precise state that is less accurate than it. Thanks to Catrin Campell-Moore for this point.

[12] Actually Joyce's constraint is stronger: his (amended) "coherent admissibility" also rules out accuracy measures according to which there is some **b′** that *is tied* for accuracy to a probabilistic **b** in every X_i.

[13] **b** may not be dominated according to *every* acceptable accuracy measure. But if there is some permissible accuracy measure on which **b** is dominated, and this accuracy measure represents the way some rational agent values accuracy, then in requiring the agent to adopt **b** we would be requiring her to adopt a belief state that is accuracy-dominated *according to her acceptable measure of accuracy*.

[14] The only numerical scoring rules for imprecise credences in the literature that I am aware of are the modified Briar score in Seidenfeld, Schervish and Kadane (2012) and a cluster of rules described in Konek (forthcoming). Seidenfeld et al. point out that their rule fails to be strictly proper. But they do not seem to have realized that their rule also violates PROBABILISTIC ADMISSIBILITY. (The rule violates PROBABILISTIC ADMISSIBILITY because, according to their rule, sometimes precise and imprecise states get the same score in Heads worlds but different scores in Tails worlds.) Konek's rules violate PROBABILISTIC ADMISSIBILITY for the same reason. I do not assume anywhere in this paper that the scoring rule in question must be proper or strictly proper. This remains a controversial matter that I do not wish to take a stand on here. See Seidenfeld et al. (2012) and Mayo-Wilson and Wheeler (forthcoming) for discussion of scoring rules for imprecise credences and the relation to propriety.

[15] The intermediate value theorem is usually stated as the claim that if a continuous real-valued function *f* defined over an interval [a,b] takes values f(a) and f(b), then, for any $r \in [f(a),f(b)]$, there exists an $x \in [a,b]$ such that $f(x) = r$. Since we're dealing with probability functions defined over a two cell partition, the domain of our accuracy measure is not the set of real numbers, so application of the theorem is not totally straightforward. More carefully, then, here's what we do: First, we represent

probability functions defined over {Heads, Tails} as vectors in the unit square (for example: (.5, .5) represents the probability function that assigns .5 to Heads and .5 to Tails). We then consider the continuous function that maps these vectors in the unit square (the probability functions), to their accuracy score as evaluated at Heads worlds. Finally, we appeal to a generalization of the intermediate value theorem, which says that if X is a connected topological space and $(Y, <)$ is a totally ordered set, and f is a continuous function from X to Y, then, if a and b are two points in X and u is a point in Y lying between $f(a)$ and $f(b)$ with respect to $<$, then there exists c in X such that $f(c) = u$. Since the unit square is a connected topological space, and the unit interval is a totally ordered set, the generalized theorem applied unproblematically.

[16] I have since found a related result in the statistics literature by Lindley (1982). He proves that there is a known transformation of the values that represent an agent's *estimate* for some event, given by upper and lower bounds, to probabilities.

[17] Note that this does not require that all information about an agent's credal state be encoded in the set of credences assigned to each proposition. It requires only that all of the information that's relevant to the agent's *degree of confidence in a particular proposition* be encoded by the set of credences assigned to that proposition by each credence function in the representor. However, if we are considering probabilistic states defined over a two-cell partition, then, in fact, *all* information about the agent's credal state *is* encoded in the credal assignments given to each proposition. For the set of credences assigned to each of P and ~P determines a unique set of probability functions over {P, ~P}.

[18] When I say that, on any acceptable measure, **s** is no less accurate than **m** in any world, I mean that given any acceptable accuracy measure, we won't be able to find a world in which, according to that measure, **s** is less accurate than **m**. It is compatible with this that on some acceptable accuracy measures **s** and **m** are *incomparable* in some worlds. But that doesn't matter in the larger scheme of things, since if it's true in every world that **s** and **m** have equal accuracy scores, incomparable accuracy scores, or that **s** is more accurate than **m**, there can still can be no accuracy based rationale for a requirement to adopt **m**. For we still won't be able to find a world in which **m** has some accuracy-based *advantage* over **s**. Since the accuracy-centered epistemologist thinks that all rational requirements are "underwritten by rationales which show how they contribute to the rational pursuit of accuracy" (Joyce (ms.), 22) accuracy-centered epistemologists won't be able to accept the claim that there is a requirement to adopt **m.**

[19] Since **i** and **p** are being used throughout to stand for generic imprecise and precise belief states, I am using the first letter of the alternative terminology: *mushy* and *sharp* to describe the particular belief states in question.

[20] The only non-numerical measure for imprecise credences described in the literature (so far as I know) is a measure that Seidenfeld et al. (2012) call the "two-tier lexicographic IP-Brier score." This measure violates PROBABILISTIC ADMISSIBILITY. For on the lexicographic IP-Brier score **s** will be more accurate than **m**, in both Heads worlds and Tails worlds. Thus **m** will be dominated.

[21] Note that the same proof can be used to show that if we measure imprecise credences using *numbers*, then the credence function that assigns .5 to each of Heads and Tails is never less accurate than an imprecise credal state that assigns to each of Heads and Tails $[a,b]$ where $[a,b]$ is centered at 0.5. (Simply substitute G* for G** in the proof.) So if we accept PROBABILISTIC ADMISSIBILITY and STRONG EXTENSIONALITY, it follows that whether we measure accuracy with numbers or not, the credence function that assigns .5 to each of Heads and Tails does no worse than the imprecise state recommended in MYSTERY-COIN for *every* acceptable accuracy measure. This is the second reason that the main line of argument in this paper is not susceptible to "the Bronfman objection" (see note 9).

[22] Thanks to Jennifer Carr for pointing this out to me.

[23] For discussion of decision theory for imprecise credences see, for example, Elga (2010), Joyce (2010), Bradley and Steele (2014), Chandler (2014), Sud (2014), Rinard (2015), Moss (forthcoming) and Bradley (ms.).

[24] See Rinard (ms.) and Bradley (forthcoming) for discussion on this issue.

[25] Indeed, it's worth noting that the imprecise credal model as it is traditionally understood doesn't seem well suited to allay potential concerns about the excessive degree of *precision* in the precise credal model. This is because, as a number of authors have pointed out, it's not clear that it's any more plausible

to claim that my degree of confidence in C is aptly represented by some *interval* like [.3423, .8746] than it is to claim that my degree of confidence in C is represented by some number like .63452. (This worry is discussed in Sturgeon (2008), p.158, Maher (2006), Kaplan (2010) and Rinard (ms.).) Nor do I think that the standard imprecisers intended their view to serve primarily as a way of addressing these sorts of concerns (Kaplan (2010), in responding to Maher, says as much). For note that the defenders of imprecise credences much more commonly appeal to cases like MYSTERY-COIN in defending imprecise credences, than to cases in which our body of evidence is large and complicated, like our evidence concerning the proposition that Clinton will be the next president (Moss (forthcoming) is an exception). I think that this is because it would be much harder for the imprecise credence defender to motivate her view by appealing to cases like the Clinton case: Our discomfort in requiring agents to adopt a precise credence in a proposition like C given our evidence isn't much assuaged by requiring, instead, a precise interval. On the other hand, in cases like MYSTERY-COIN, which *are* commonly used to motivate imprecise credences, we're given a set of information about the chances (say that they are between .05 and .95) that non-arbitrarily fixes the intervals' boundaries. Thus, imprecise credences (perhaps ironically) are not especially well poised to address concerns about arbitrariness in the precise credal model.

[26] In my (2012), I argued that cognitive limitations may motivate a permission for limited agents like ourselves to sometimes adopt doxastic states that aren't representable as precise credence functions. Although, at the time, I wrote that *imprecise credences* were motivated by these considerations, I am now skeptical of this for the reasons given in note 25. I still, however, think that such a strategy may successfully motivate a permission for cognitively limited agents to adopt doxastic attitudes that aren't representable by precise credence functions.

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