

The End is Near: Grim Reapers and Endless Futures

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Abstract. José Benardete developed a famous paradox involving a beginningless set of items each member of which satisfies some predicate just in case no earlier member satisfies it. The Grim Reaper version of this paradox has recently been employed in favor of various finitist metaphysical theses, ranging from temporal finitism to causal finitism to the discrete nature of time. Here, I examine a new challenge to these finitist arguments—namely, the challenge of implying that the future cannot be endless. In particular, I develop future-oriented Benardete paradoxes and examine their epistemic symmetry with past-oriented paradoxes.

1 Introduction

Suppose there's an infinite sequence of Grim Reapers bent on killing Fred. Each reaper is assigned a unique natural number and a designated time to kill him. If no earlier Reaper killed Fred at Reaper n 's designated time, Reaper n kills Fred at that time. But if an earlier Reaper killed Fred, Reaper n does nothing. Reaper 1's designated time is 60 seconds past noon; Reaper 2's designated time is 30 seconds past noon; Reaper 3's designated time is 15 seconds past noon; and so on *ad infinitum*.

Clearly, some Reaper must have killed Fred. If no Reaper killed Fred up to 12:01, for instance, then Reaper 1 would kill him. But reflection reveals that *no* Reaper could have killed him. If Reaper n killed Fred, then no earlier Reaper killed Fred. *A fortiori*, no Reaper killed Fred before Reaper $(n+1)$'s deadline—in which case, Reaper $(n+1)$ killed Fred. But then Reaper n didn't kill Fred, since Reaper n kills Fred only if no earlier Reaper kills him. So, there is no n such that Reaper n killed Fred. So, no Reaper killed Fred, and yet some Reaper must have killed him. Contradiction.

Versions of this paradox date back at least to Benardete (1964), and their manifold incarnations involve deafening gongs, firing squads, gods erecting impassable walls, and even sentences whose truth is conditioned on the truth of infinitely many other sentences.¹ These

¹ Paradoxical variants abound. See, *inter alia*, Koons (2020, 2014), Pruss (2018), Erasmus (2016, 2018), Cohen (2015), Luna (2009a, 2009b), Laraudogoitia (2003), Hawthorne (2000), Sorensen (1998), and Yablo (1993, 2000).

paradoxical incarnations—what I’ll call *Benardete paradoxes*—have also been employed in support of manifold metaphysical conclusions, ranging from the necessarily discrete nature of time to the necessary finitude of the past to the impossibility of infinite causal regresses. These metaphysical conclusions are typically *finitist* in nature—that is, they typically exclude certain infinities from the realm of metaphysical possibility.

My goal in this article is to level a new challenge to arguments for finitist metaphysical theses based on Benardete paradoxes—hereafter, *B-arguments*. The challenge targets a conditional premise in B-arguments that *links* the possibility of various infinities *to* the possibility of Benardete paradoxes. I begin in §2 by articulating the structure of Benardete paradoxes and explaining how they’re employed in favor of finitist theses. Then, in §3, I develop the challenge in the form of a dilemma: proponents of B-arguments must either embrace the impossibility of an endless future, or else justify B-argument linking premises in ways that don’t equally justify a new linking premise that, together with the impossibility of Benardete paradoxes, implies the impossibility an endless future.² I argue that if theism is true, then extant justifications for B-argument linking premises equally justify the new linking premise. I also consider extensions of my reasoning that discharge theistic assumptions. Finally, I address an objection in §4.

2 Benardete paradoxes, unsatisfiable pairs, and finitism

Benardete paradoxes share an abstract structure involving a pair of jointly logically unsatisfiable conditions (Shackel 2005). Let an *unbegun set* be an infinite set, linearly ordered by the abstract relation *before* (Bxy), with no first member (i.e., there is no member m such that no member n is before m). *Before* is not a *temporal* relation; it is, rather, an abstractly characterized ordering relation.³ The first condition in the unsatisfiable pair—the *Unbegun Condition*—can now be stated as follows, where x and y are members of a non-empty set S linearly ordered by the *before* relation:

$$\text{Unbegun Condition (UC): } \forall x \exists y (Byx)$$

According to *UC*, each member x of S is such that there’s a member y of S before x . Equivalently: there is no member x of S such that there are no members before x . S satisfies *UC* iff S is an *unbegun set*.⁴

The second condition in the unsatisfiable pair—the *At iff Nowhere Before Condition*—states that for each member x of a non-empty set S linearly ordered by *before*, x satisfies some

² As I use it, the future as of some time t is *endless* just in the case the number of non-overlapping intervals of equal, finite, non-zero, non-infinitesimal duration, each of which is later than t , is at least \aleph_0 . Thus, to say that the future *cannot* be endless is to say that, necessarily, as of any time t , the number of non-overlapping intervals of equal, finite, non-zero, non-infinitesimal duration, each of which is later than t , is finite. This could also be cast in *tensed* terms—simply replace talk of intervals later than t with talk of intervals that *will* occur.

³ Thus, even *after* or *later than* can serve as a *before* relation so long as they linearly order a given set. It’s the formal features of *before* that matter.

⁴ Per the formal features of linear ordering relations, the fact that S satisfies *UC* entails that S has infinitely many elements (assuming, of course, that S is non-empty—an assumption I’ll hereafter leave implicit).

predicate E if and only if no member before x satisfies E. Where x and y are members of a non-empty set S linearly ordered by *before*:

At iff Nowhere Before Condition (ANBC): $\forall x (Ex \leftrightarrow \neg \exists y (Ey \ \& \ Byx))$

Shackel (2005) and others have shown that these two conditions are simply inconsistent—they jointly entail a contradiction. Suppose that S is an unbegun set (i.e., a set satisfying *UC*) that satisfies *ANBC*. Now suppose (for reductio) that a member x of S satisfies E. Then, by *ANBC*, no member before x satisfies E. But then every element y before x is such that no member before y satisfies E—in which case, by *ANBC*, every member y before x satisfies E. Since there *is* some member before x (per *UC*), some member before x satisfies E. Contradiction. So, by reductio, no member of S satisfies E. But then some arbitrary member z of S is such that no member before z satisfies E—in which case, by *ANBC*, z satisfies E. Hence, some member of S satisfies E. Contradiction. Hence, *UC* and *ANBC* are jointly unsatisfiable.

We might wonder why any substantive metaphysical conclusion is called for in response to Benardete paradoxes. After all, we shouldn't blame one of the *conjuncts* of an unsatisfiable pair simply because the *pair* is unsatisfiable. At this juncture, those who employ Benardete paradoxes in support of finitist metaphysical theses offer the following B-argument schema, where the notion of possibility at play is *metaphysical possibility*:

1. If there could be unbegun sets ordered by relation R (where R functions as a *before* relation), then there could be unbegun sets that satisfy *ANBC*. (*Linking premise*)
2. But there cannot be unbegun sets that satisfy *ANBC*.⁵
3. So, there cannot be unbegun sets ordered by R .

Filling in R with *causes*, Pruss (2018, pp. 47–48) offers an argument in favor of causal finitism that (implicitly) fits this schema.⁶ Temporal finitists have also employed Benardete paradoxes in support of their view (Koons 2014, 2020, Luna 2009a). The abstract form of the temporal finitist argument runs:

4. If there could be an unbegun set of non-overlapping, finite, non-zero, non-infinitesimal, equal intervals of past time ordered by the *earlier than* relation, then there could be an unbegun set that satisfies *ANBC*. (*Linking premise*)
5. But there cannot be an unbegun set that satisfies *ANBC*.
6. So, there cannot be an unbegun set of non-overlapping, finite, non-zero, non-infinitesimal, equal intervals of past time ordered by the *earlier than* relation.

⁵ This just falls out of the facts that (i) *UC* and *ANBC* are jointly logically unsatisfiable, and (ii) an unbegun set definitionally satisfies *UC*. Also, I follow defenders of B-arguments in assuming classical logic.

⁶ For B-arguments in favor of causal finitism—the thesis that, necessarily, nothing has infinitely many causes—see also Koons (2020), Erasmus (2016, 2018), and Luna (2009a).

Other B-arguments roughly follow suit. With all this background in hand, we can now turn to the new challenge for B-arguments.

3 A grim dilemma

To explain the challenge, I'll articulate a *future-oriented* Benardete paradox and construct an argument symmetric to standard B-arguments whose conclusion is that the future cannot be endless (§3.1). The challenge is a dilemma: either embrace the impossibility of an endless future, or else justify B-argument linking premises in ways that don't equally support the symmetric argument's linking premise. I'll then argue that if theism is true, extant justifications for B-argument linking premises equally support the symmetric argument's linking premise (§3.2). The significance of this conclusion will be discussed in due course, as will extensions of my reasoning that discharge theistic assumptions.

3.1 A future-oriented paradox

Suppose there's an endless series of Grim Reapers, each of which has a scythe, a unique natural number, and a designated future day to swing its scythe. Through acts of divine revelation, God (truthfully) informs each Reaper whether a future Reaper will swing its scythe. If God informs Reaper n that *no* Reaper in the future of Reaper n 's designated day will swing its scythe, then Reaper n swings its scythe. But if God informs Reaper n that *some* Reaper in the future of Reaper n 's designated day will swing its scythe, then Reaper n does nothing. Reaper 1's designated day is (today + 1 day); Reaper 2's designated day is (today + 2 days); more generally, for each natural number n , Reaper n 's designated day is (today + n days).

It should be evident that this scenario instantiates the unsatisfiable pair—each Reaper is such that there is a Reaper in its future (giving us *UC*), and each Reaper swings its scythe iff no Reaper in its future swings its scythe (giving us *ANBC*).⁷ This scenario is therefore structurally identical to past-oriented Benardete paradoxes employed in the context of standard B-arguments for finitist theses. The only differences, of course, are the reversed temporal ordering and the addition of God. Given their structural identity and reversed temporal ordering, we can construct a symmetric argument for the impossibility of an endless future:

7. If there could be an unbegun set of non-overlapping, finite, non-zero, non-infinitesimal, equal intervals of future time ordered by the *later than* relation, then there could be an unbegun set that satisfies *ANBC*.⁸ (*Linking premise*)
8. But there cannot be an unbegun set that satisfies *ANBC*.

⁷ Cohen (2015) develops a future-oriented Benardete paradox along these lines. However, my present contribution builds significantly on Cohen's, since—unlike Cohen—I will examine how the principal lines of support for B-argument linking premises equally support a future-oriented linking premise.

⁸ The 'unbegun set' to which (7)'s consequent refers is the set of Reapers in my future-oriented story.

9. So, there cannot be an unbeginning set of non-overlapping, finite, non-zero, non-infinitesimal, equal intervals of future time ordered by the *later than* relation.

This is where the challenge for proponents of B-arguments arises. They cannot deny (8) on pain of denying a premise in B-arguments. They therefore face a dilemma: either embrace (9), or else explain why their justification for B-argument linking premises does not carry over to (7). For if such justification *does* carry over—that is, if it equally justifies (7)—then if one *accepts* B-argument linking premises on its basis, one should also accept (7) on the same basis.

Embracing (9) may be quite costly. For if (9) is true, then the future cannot be endless. But to many, it seems obvious that the future *could* be endless.⁹ For the sake of modesty, however, I will not rely on the falsity of (9). Instead, my challenge will remain in dilemma form, and the remainder of my discussion will focus on the second horn. Failing to meet the second horn (and consequently being impaled by the first) is significant in itself.

Before turning to the extant justifications for B-argument linking premises, it's worth noting that such justifications may not equally support (7) simply because the future-oriented paradox whose possibility is asserted in (7)'s consequent involves God. After all, motivations for B-argument linking premises certainly don't motivate the possibility of *God's existence* conditional on some anti-finitist assumption. But given the way I've constructed the future-oriented paradox—and given that the possibility of this paradox is asserted in (7)'s consequent—motivating (7) *does* require motivating the possibility of God's existence conditional on the possibility of an endless future. So, motivations for B-argument linking premises may not carry over to (7).

To circumvent this worry, I will argue that *if* theism is true, *then* motivations for B-argument linking premises carry over to (7). To put it differently, while motivations for B-argument linking premises may not *by themselves* carry over to (7), they *do* carry over *when conjoined* with theism. The significance of this contention is twofold.

First, many B-arguments are wielded on behalf of premises in first-cause arguments *for theism*. For instance, Pruss (2018), Koons (2014, 2020), and Erasmus (2016, 2018) all employ B-arguments on behalf of the Kalam cosmological argument for God's existence. It's significant, then, that *conditional* on God's existence, motivations for B-argument linking premises equally motivate (7). This, in turn, commits those who deploy B-arguments on behalf of theistic arguments to the impossibility of an endless future. Moreover, if B-argument proponents wish to *avoid* this commitment, then they should reject theism—a significant result in itself.

One could, of course, affirm theism but hold that God's revealing future events to individuals is impossible. But this too is a significant result, and it also seems costly—plausibly, an omnipotent and omniscient being would be able to do as much. Alternatively, one might affirm theism but seek to avoid future-oriented paradoxes by adopting *open theism*. According to open theism, some causally possible future events are presently undetermined, and for any such event

⁹ Other modal epistemological tools can be adduced here too, such as conceivability and the fact that degreed differences don't normally make for a categorical difference in modal status. On the latter, see Rasmussen (2014).

E, God knows neither that E will occur nor that E will not occur (Rhoda 2008). So, if it's presently undetermined whether some future Reaper swings its scythe—say, because this depends on God's or Reapers' future libertarianly free choices—then God does not know whether some future Reaper swings its scythe. This, in turn, prevents the future-oriented paradox (as stated) from arising—or so one might think. Now, if averting my challenge pushes one to adopt open theism, this alone is significant. But more importantly, it's not clear open theism will help here. Open theists grant that God knows his single, present, already-formed infallible intentions. But this is all we need to run the Benardete paradox above. In particular, we can simply suppose that each Reaper is a deterministic automaton whose (in)actions are ordained by one of God's present infallible intentions. In that case, there's no indeterminacy concerning whether some future Reaper will swing its scythe. Further, God need only know his already-formed intention and reveal to each Reaper, based on his knowledge of said intention, whether some Reaper in its future swings its scythe. Open theism, it seems, won't help.

Second, the assumption of theism is dischargeable so long as *other* assumptions are conditioned upon. After all, future-oriented Benardete paradoxes can be constructed without God. For instance, suppose that each Reaper in the future-oriented paradox is causally sensitive *not* to acts of divine revelation but rather the states of any *subsequent* Reaper. Suppose, further, that each Reaper enjoys the power and disposition to scythe-swing iff no future Reaper scythe-swings. Alternatively, suppose that it's simply a brute fact about the Reapers that each will scythe-swing iff no future Reaper scythe-swings (where this biconditional is purely truth-functional and devoid of any causal sensitivity). In both cases, *UC* and *ANBC* are satisfied, but no theistic assumptions are operative. Instead, the relevant auxiliary assumptions include (i) the metaphysical possibility of mechanisms that are causally sensitive to certain future states, and (ii) the metaphysical possibility of mechanisms whose states are brutally correlated with certain future states. But because these assumptions are controversial and rejected by some of the main proponents of B-arguments,¹⁰ I will focus primarily on establishing the parity between B-argument linking premises and (7) conditional on theism. I will only briefly discuss conditioning on non-theistic assumptions.

Having explained the nature and significance of my contention, let's turn to the extant support for B-argument linking premises.

3.2 Extant support for B-argument linking premises

Proponents of B-arguments have offered two principal lines of support for the linking premises. The first line—developed in Koons (2014, 2020)—appeals to patchwork (or recombination) principles. The second line—developed in Pruss (2018, ch. 3), Luna (2009b), and Luna and Erasmus (2020)—appeals to the fact that (something like) a mysterious force would be required to prevent paradoxes from arising if the relevant linking premises were false. Given the

¹⁰ For instance, Pruss and Koons (2021) argue for a version of the principle of sufficient reason and hence would likely reject (ii). Koons (2014, pp. 264–265) holds that 'forward-looking dispositions', i.e., future-sensitive causal dispositions, are impossible and hence would reject (i).

implausibility of such a force, we're justified in accepting the relevant linking premises. I'll consider each line of support in turn. Once more, my main contention is that each line of support equally motivates (7) given theism.

3.2.1 First line

The first line of support appeals to *patchwork principles*. These principles trace back at least to Lewis (1983, pp. 76–77), but their Humean underpinnings are evident in their denial of necessary connections between distinct existences (or between the intrinsic characters of distinct existences in non-overlapping spacetime regions). For present purposes, I'll simply grant the patchwork principles used to support the relevant linking premises.

Let's focus in particular on the patchwork principle Koons (2020) employs:

The principle relies on two assumptions. First, we assume that some particular, localized situation, S , is metaphysically possible (and so contained in some possible world w_1). Second, we assume that there is a second possible world w_2 with a spatiotemporal or causal structure that provides enough 'room' for S to be repeated κ [times] (where κ is a cardinal number, either finite or infinite). On these two assumptions, the patchwork principle licenses us to conclude that there is a third possible world, w_3 , in which a situation intrinsically identical to S has been repeated κ times (in the arrangement corresponding to the structure of w_2). The picture is that w_2 provides the frame, w_1 the sample patch, and w_3 the completed quilt. (*Ibid*, pp. 5–6)

My goal in this subsection is to show that this principle equally supports (7) if theism is true.¹¹ To do this, I simply need to show how a quilted world (w_3) instantiating my future-oriented Benardete paradox can result from applying Koons' principle to a framework world (w_2) and a world (w_1) containing an individual, intrinsically-specified localized sample patch. Now, if the future could be endless, then there's a possible world w_2 with (say) infinitely many future days and so enough 'room' to accommodate a unique Reaper (together with its scythe) for each day of the endless future.¹² Thus, to construct a quilted world (w_3) instantiating a future-oriented Benardete paradox, all we need is the localized sample patch—i.e., a metaphysically possible particular situation wherein a single Reaper implements the rule to swing its scythe iff no Reaper in its future swings its scythe.

By my lights, it's eminently plausible—assuming God can reveal future events to individuals—that an *individual* Reaper following the relevant rule in light of its received revelation

¹¹ Koons (2014) uses a similar principle, *Infinitary Patchwork*, to deliver the relevant linking premise. I will here consider only the principle in Koons (2020), since what I say thereon applies *mutatis mutandis* to *Infinitary Patchwork*.

¹² We can also innocuously suppose that (i) there's enough space in such a world to fit a single scythe-equipped Reaper on each day of the future, and (ii) there's enough space and time in such a world to fit God and some revelation of his on each day of the future (which, in God's case, is trivial given that he isn't located in spacetime). Clearly, if the future could be endless, some possible world satisfies (i) and (ii).

is possible (i.e., contained in some possible world w_1). Suppose w_1 is a world in which Reaper 1 is the only Reaper to ever exist. (We can also suppose time both begins and ends in w_1 .) In w_1 , the following localized situation, S, obtains. S includes God, Reaper 1 with its scythe, and God's revealing (on a particular day) to Reaper 1 whether some future Reaper swings its scythe. (In the case at hand, God reveals that no future Reaper swings its scythe.) In S, the Reaper has the power and disposition to swing its scythe if it's divinely revealed that *no* future Reaper swings its scythe, and to *refrain* from swinging its scythe if it's divinely revealed that *some* future Reaper swings its scythe. (In the case at hand, Reaper 1 proceeds to swing its scythe.¹³) The powers in question—both God's and the Reaper's—are intrinsic to their bearers in S and w_1 , just as the Reapers' powers in Koons' Reaper paradox are intrinsic to them in their localized patch and world.

Now, if God exists and can reveal future events to individuals, S (and so w_1) seems clearly possible. There's nothing contradictory or absurd about it—it doesn't instantiate any Benardete paradox, and nor does it involve infinitely many causes affecting a target state.¹⁴ It's also perfectly conceivable and imaginable, even upon reflection and detailed specification. Plus, those who accept the *possibility* of divine foretelling tend to accept that God has *in fact* revealed future events to individuals—and, further, that individuals have acted in light of those revelations.¹⁵ Moreover, it may seem intolerably arbitrary and inexplicable if S were *impossible* while *extremely* similar situations (wherein individuals act in light of divine revelation of future events) are entirely possible. There doesn't seem to be any relevant difference between such possible situations and S that could account for diverging modal statuses. By my lights, then, S (and so w_1) is clearly possible on the assumption that God can reveal future events to individuals. At the very least, this possibility—under the aforementioned assumption—isn't *less plausible* than the individual possibility of a single, past-sensitive Reaper used in Koons' (2020) B-argument.

We therefore have all the ingredients for justifying (7) using Koons' patchwork principle. If the future is endless, then we have a framework world w_2 with enough room to accommodate infinitely many duplications of S for each day of the endless future. And the localized situation S is metaphysically possible (and so contained in some possible world w_1). Koons' patchwork principle implies that there is thus a third possible world w_3 in which a situation intrinsically identical to S has been repeated infinitely many times extending into the endless future.

¹³ I assume (quite innocuously) that God cannot reveal a falsehood. Note that I don't simply *build into* S the extrinsic fact that God's revelation is true. Instead, this is simply entailed by God's nature, which is *intrinsic* to S (since *God* is intrinsic to S). This isn't any more problematic than the fact that some extrinsic properties of Koons' past-sensitive Reapers (e.g., *is such that 1+1=2*) are necessary concomitants of their intrinsic properties.

¹⁴ In fact, notice that—because the future has an end in w_1 —*even if* we suppose (contrary to what I've argued) that God's revelation to a Reaper depends on God's knowledge of what happens throughout the whole future (as opposed to depending on God's knowledge of a single divine intention), and *even if* we suppose that this entails that <such divine knowledge, if the future is endless, is dependent on infinitely many distinct future days> in violation of causal (or, rather, *dependence*) finitism, w_1 *itself* doesn't violate any finitist strictures. To be sure, the *patched-together world* might involve some such violation. (Though, I'll argue below that it doesn't.) But the possibility of said world *simply follows* from Koons' patchwork principle together with the possibility of each of S, w_1 , and an endless future.

¹⁵ For instance, in Matthew 26:23–25 and John 13:26–27, Jesus reveals to his apostles (including Judas) that Judas will betray him. This is a revelation of a future event to individuals. These individuals, in turn, acted in light of this revelation.

But, of course, w_3 is *not* possible, as it contains a future-oriented Benardete paradox. Koons' patchwork principle thus facilitates the following argument for (7), which is identical to (14):

10. If there could be an unbegun set of non-overlapping, finite, non-zero, non-infinitesimal, equal intervals of future time ordered by the *later than* relation, then there is a possible world w_2 with enough room to accommodate infinitely many duplications of S spanning the endless future.
11. There's a possible world w_1 containing S.
12. If there are possible worlds w_1 and w_2 (as characterized above), then there is a possible world w_3 in which a situation intrinsically identical to S has been repeated infinitely many times extending into the endless future. (*Koons' patchwork principle*)
13. If there is a possible world w_3 in which a situation intrinsically identical to S has been repeated infinitely many times extending into the endless future, then there could be an unbegun set that satisfies ANBC.
14. So, if there could be an unbegun set of non-overlapping, finite, non-zero, non-infinitesimal, equal intervals of future time ordered by the *later than* relation, then there could be an unbegun set that satisfies ANBC.

The payoff is that the first line of support for B-argument linking premises equally motivates (7) given theism. Thus, if theism is true, then B-argument proponents should either jettison this first line of support, or else embrace the impossibility of an endless future.

Now, one might object that my case for thinking <Koons' patchwork principle equally supports (7) given theism> mistakenly treats time and causation as if they were independent, with time as a space-like dimension upon which causal connections can be imposed willy-nilly. But if causal connections provide time with its unity, and if the asymmetry of causation provides time with its directionality, then the very idea of a backward causation (in the form of a future-directed causal regress) can be excluded at the outset.

But I don't find this objection convincing for two reasons. First, there needn't be any backward causation in the case at hand. Each Reaper's act(s) isn't future-sensitive in virtue of being *causally affected* by infinitely many future Reapers; instead, it's future-sensitive in virtue of a *single act of divine revelation*. And we needn't suppose that each act of divine revelation is *itself* causally impacted by the activities of future Reapers. Perhaps God simply wills that each future Reaper does such-and-such, and God reveals to each Reaper what God wills each *future* Reaper to do. Here, it's not as though a single state of God is causally affected by future Reapers; the direction of causation is the other way around: a single state of God causally affects the future Reapers. In such a situation, neither the action of any Reaper, nor any of God's acts of revelation, nor God's willing of the future, involves any backward causation.

Now, one might reply that we still have aspects of the future explaining aspects of the present in the case at hand. For instance, the future fact that some (else: no) future Reaper swings

its scythe (partly) explains God's knowing said fact; God's knowing said fact (partly) explains God's revelation; and God's revelation (partly) explains the present Reaper's (in)action.

But there are three things to say in response. First, *explanatory* connections, unlike *causal* connections, do not provide time with its directionality and unity. Backward explanation, then, is not problematic—at least not for the reason the objection at hand articulates. Second, if backward explanation *is* problematic, this seems to be a problem *for theism*, not a problem for my case that the extant lines of support for B-argument linking premises, *together with* theism, equally support (7). For $\langle S \text{ knows that } p \rangle$ is plausibly (partly) explained by the truth of p , in which case God's foreknowledge is (partly) explained by the future facts known. If God exists and has foreknowledge, then backward explanation seems unavoidable.¹⁶ Third, I've already challenged the idea that God's revelation to a present Reaper is explained by God's knowledge of whether some future Reaper acts (an assumption which is crucial for securing the backward explanation from the future fact known to the present knowledge thereof). God's revelation to the present Reaper can instead simply be based on God's single, occurrent intention to determine whether some future Reaper swings its scythe. So, backward explanation is avoided altogether.

But let's not lose the dialectical forest for the trees. I promised two reasons why I find the original objection wanting. We just explored the first, so let's explore the second. The second reason is a dilemma: either acting on the basis of divine revelation of the future is possible or impossible. Suppose it's possible. Then S and w_1 are possible. Since a world containing an endless future is (we're assuming) similarly possible, and since such a world has enough room to accommodate infinitely many duplications of S spanning the endless future, *it simply follows*, per Koons' patchwork principle, that the patched-up, quilted-together world (w_3) instantiating a future-oriented Benardete paradox is possible. It's no use objecting, then, that such a world is *impossible* because it problematically divorces causation and time. The world is already impossible by dint of being contradictory. If it's impossible for another reason, so be it. But since (i) the world's possibility *follows from* the possibility of each of S , w_1 , and an endless future *together with* Koons' patchwork principle (given theism), and since (ii) each of S , w_1 , and an endless future is possible, the *impossibility* of said world simply entails that (iii) Koons' patchwork principle is false (given theism).

But now suppose that acting on the basis of divine revelation of the future is impossible. This alone is significant. But it's also, I suspect, an unwelcome commitment for most theists. Intuitively, an omnipotent and omniscient God would be able to reveal future facts, and intuitively, recipients thereof can act in light of said revelation. Some of the world's major religions, moreover, seem to entail its possibility.

Anyway, let's get back on track. We just finished examining whether the first line of support for B-argument linking premises equally supports (7) given theism. Before turning to the second line of support, I will highlight one way the theistic assumption might be discharged.

¹⁶ You could, of course, say that God is *timeless* and so strictly speaking doesn't *foreknow*. But the problem remains, since surely God can reveal future facts to present people, landing us once more in backwards explanation—a person's present belief is explained by the revelation, which is explained by the divine knowledge, which is explained by the future fact. The person's present belief is therefore (partly) explained by the future fact.

Recall that our individually possible, localized sample patch S contained a single Reaper with the intrinsic power and disposition to implement its rule in response to divine revelation. But we could also take a localized sample patch S* containing a single Reaper with intrinsic powers and dispositions that are sensitive to the states of future Reapers (if there are any).¹⁷ In particular, suppose S* contains a Reaper with the intrinsic power and disposition to swing its scythe iff no later Reaper swings its scythe. So long as S* is individually possible, we can run essentially the same argument for (7) by swapping ‘S’ with ‘S*’. Doing so replaces theistic assumptions with the assumption that, possibly, some mechanism has the intrinsic power and disposition to ϕ iff no later mechanism ϕ s.¹⁸ Speaking for myself, an individual mechanism of this sort *seems* possible. You, however, may not share my reaction to the case. For this reason, I will not put much stock in it. Its purpose is to highlight that theistic assumptions aren’t strictly necessary and to encourage future work on the matter.

3.2.2 Second line

The second line of support for B-argument linking premises appeals to the implausible ‘mysterious forces’ allegedly implied by denying such premises. If we suppose that the members of a particular unbegun set obtain (whether sequentially or all at once)—ordered, say, by the relation *earlier than* or *causes*—there doesn’t seem to be anything that could *prevent* them from satisfying *ANBC*. Logic, after all, isn’t some causal force that could prevent all the members from satisfying some predicate just in case no earlier member does. Since there’s nothing that could plausibly *prevent* *ANBC* from being satisfied for the relevant unbegun sets *if* such sets were possible, it’s concluded that *ANBC* is possibly satisfied for the relevant unbegun sets *if* such sets were possible. And *that* delivers the linking premise in B-arguments.

One version of this second line of support is given in Luna (2009b), who (in the passage below) is responding to a Benardete paradox described by Yablo in which there’s an unbegun set of demons, each of whom intends to say ‘yes’ if all previous demons say ‘no’, and ‘no’ otherwise. What stops the demons from collectively implementing their intentions? For Yablo, ‘Logic stops them’ (2000, p. 150). To which Luna replies:

The problem is that logic is no causal force that could intervene as an overall ontological factor to stop the demons. To see how unlike any ontological factor logic is, just ask *exactly which demons are stopped by logic*, for there is no logical necessity that a particular group of them be. (Luna 2009b, p. 95, emphasis in original)

¹⁷ Suppose that the world from which S* is ‘extracted’ contains only one Reaper.

¹⁸ Note that there’s considerable leeway in how the relevant future-sensitivity gets cashed out. For instance, à la Koons’ (2014) version of the paradox, the Reapers need only be sensitive to a single future fact: whether there will be a particle in a certain plane. (Suppose each Reaper’s power and disposition is to place a particle in the plane if no particle will be placed on that plane in the future, and otherwise to prevent there being a particle on the plane.) Alternatively, à la Koons’ (2020) version of the paradox, each Reaper *n* might have the power and disposition to send a signal to every previous Reaper iff Reaper *n* does not receive a signal from any future Reaper. Some of these forms of backward influence may be more plausibly possible than others.

Luna and Erasmus (2020) make essentially the same point in reference to the deafening peal Benardete paradox:

[L]ogic is not a causal force that could step in and stop Smith from ringing the peal on certain days. Indeed, if logic could stop Smith from ringing the peal, *on which days would it stop him?* There is simply no logical necessity that Smith cannot ring the peal on certain days in all worlds in which [time is beginningless]. (*Ibid*, p. 167, emphasis in original)

While the quoted passages are primarily responding to Yablo, they can clearly be employed on behalf of linking premises in B-arguments. Now, as with the first line of support, I won't directly challenge the mysterious force line of support. Instead, I'll argue that the support, if successful, equally justifies (7) given theism.

To show this, we can simply mount entirely symmetric considerations as applied to unbegun sets (of, say, Reapers) ordered by the *later than* relation. If the *past* could be beginningless, they say, surely there could be a beginningless sequence of (say) demons. And if there could be a beginningless sequence of demons, surely it could be the case that they all implement the intention to say 'yes' if all earlier demons say 'no', otherwise 'no'. After all, there doesn't seem to be anything that could *prevent* them from doing so. *Logic* certainly has no causal force to prevent as much. And yet there cannot be a beginningless collection of demons all of whom implement an intention like that. Hence, they conclude, the past cannot be beginningless. But we could equally argue as follows. If the future could be endless, surely there could be an endless sequence of demons. And if there could be an endless sequence of demons, surely it could be the case that they all implement the intention to say 'yes' if all later reapers say 'no', otherwise 'no'.¹⁹ After all, there doesn't seem to be anything that could *prevent* them from doing so. *Logic* certainly has no causal force to prevent as much. And nor does *metaphysics*—there's no spooky metaphysical 'force' that causally prevents Benardete paradoxes, or ungrounded causal chains, from arising.²⁰ And yet there cannot be such an endless collection of demons all of whom implement their intentions. Hence, the future cannot be endless. *Mutatis mutandis* for other variants of the Benardete paradox.

We can also parody (with no seeming loss of plausibility) the two quotations from earlier:

¹⁹ Again, we can add to the picture a God capable of revealing to a demon whether some future demon says 'no'. All the same points about mysterious forces apply: there isn't some ghostly 'force' preventing God from revealing this fact to all the demons.

²⁰ It's therefore no use responding that the metaphysical impossibility of ungrounded causal chains prevents the future-oriented Benardete paradoxes from arising in the endless future. For then: (i) the detractor of B-arguments can equally claim, with no seeming loss of plausibility, that the *logical impossibility* (and hence *metaphysical impossibility*) prevents the past-oriented Benardete paradoxes from arising in the beginningless past; and (ii) the detractor can simply apply (again, with no seeming loss of plausibility) the mysterious force objection to the link between an endless future and ungrounded chains: what *prevents* such chains from arising given that there's enough room to accommodate such chains and a God able to grant any finite number of things future-sensitive powers? *Metaphysics*, like logic, has no force to prevent as much. (But suppose that metaphysical impossibilities *do* have the ability to prevent as much. Then, since every logical impossibility is also a metaphysical impossibility, it simply follows that logical impossibilities have the relevant ability, too.)

The problem is that logic and metaphysics aren't causal forces that could intervene as overall ontological factors to stop the demons and divine foretelling acts. To see how unlike any ontological factors logic and metaphysics are, just ask *exactly which future demons or divine foretelling acts are stopped by logic or metaphysics*, for there's no logical or metaphysical necessity that a particular group of them be.

And again:

Neither logic nor metaphysics are causal forces that could step in and stop either Smith from ringing the peal on certain future days or stop God from foretelling stuff to Smith on certain future days. Indeed, if logic or metaphysics could stop Smith from ringing the peal or God from foretelling stuff, *on which days would they stop them?* There's simply no logical or metaphysical necessity that Smith cannot ring the peal on certain future days or that God cannot foretell to Smith on certain future days in all worlds in which time is endless.

Pruss (2018, pp. 48–49) also levels a version of the mysterious force line of support. He argues, first, that if causal finitism is false, then there could be a benign (non-contradictory) *Reversed Grim Reaper* story. This story involves infinitely many Reapers whose designated times (beginning at 10:30) converge from the *earlier than* direction to 11:00. In such a case, the set of Reapers (ordered by the *earlier than* relation) is no longer unbegun—it has a first member at 10:30. And as with the original past-oriented story, each Reaper performs a task iff no earlier Reaper does. The Reaper at 10:30 performs the task, and all other Reapers do nothing. No contradiction or absurdity here. But, continues Pruss, if the *Reversed Grim Reaper* story is possible, then it would surely be possible for there to be infinitely many 'tinkerers' with indeterministic free will, each of whom independently adjusts the dials on a given Reaper (for *each* Reaper) around 9:30. Pruss then adds:

[S]urely it would be *possible* for them all to set the dials to the settings in the original story. For *each* individual tinkerer could set the dial on *her* Grim Reaper to the setting that it would need to have in the original story. But since the tinkerers are independent and indeterministically free, what other tinkerers are doing doesn't affect what one of them can do. So there should be no difficulty about them *all* setting their Grim Reapers to the values needed for the original paradox. Otherwise, we have to suppose some strange metaphysical force preventing some settings. (*Ibid*, emphasis in original)

However, this too can be parodied (with no seeming loss of plausibility) to the endless future. If the future could be endless, then there could be a scenario in which God creates a unique Reaper on each day of the endless future, where each Reaper can form and implement intentions but nevertheless refrains from doing so. In this scenario, there is no divine revelation; there are no scythe-swinging intentions; there aren't any Reaper exercises of causal power; the Reapers'

intentions or powers aren't causally linked in any way; there are no ungrounded causal chains; and so on. If the future could be endless and God exists, there should be no difficulty about such a scenario, and so such a scenario would be possible.

But if *this* scenario is possible, then the following scenario would surely be possible, too. Each of the aforementioned Reapers has an indeterministic coin, independent of any other coin, and implements the following rules:

- (i) Toss the coin two times.
- (ii) If the first toss lands heads, then form (and implement if its condition is met) the intention to scythe-swing if God reveals that no future Reaper swings its scythe.
- (iii) If the first toss lands tails, do not form the intention in (ii).
- (iv) If the second toss lands heads, then form (and implement if its condition is met) the intention to *refrain* from scythe-swinging if God reveals that some future Reaper swings its scythe.
- (v) If the second toss lands tails, do not form the intention in (iv) or any other intention.

And, further, God has an indeterministic coin, independent of any other coins, and implements the following rules:

- (vi) Toss the coin.
- (vii) If the coin lands heads, then for each Reaper, reveal to it whether some Reaper in its future swings its scythe.
- (viii) If the coin lands tails, then do not reveal anything to any Reaper.

We can now echo Pruss:

If *this* scenario were possible, then it would surely *also* be possible for all the Reapers and God to have all their coin flips land heads. For *each individual* Reaper could flip heads on both occasions, and God (individually) can flip heads; each such result would make its associated individual Reaper (and God) satisfy its original conditions in the future-oriented Benardete paradox. But since the coin flips are independent and indeterministic, the results of *other* coin flips don't affect the results of an individual Reaper's (or God's) coin flips. So there should be no difficulty about them *all* landing heads—which, in turn, would suffice for the original future-oriented paradox. Otherwise, we have to suppose some strange metaphysical force preventing some coins from landing heads.

By my lights, this is no less plausible than Pruss' mysterious force line of support for the linking premise of his B-argument. If I'm right about this, then we have equally powerful reason to accept (7) on theism. And as before, we can discharge any theistic assumptions if we assume that mechanisms with certain future-sensitive powers and dispositions are possible. Simply imagine infinitely many such mechanisms spread throughout the endless future, each of which is equipped

with an indeterministic coin (independent of the rest), and each of which implements rules relevantly similar to (i)–(v).

In what follows, I address an objection to my new challenge for B-arguments.

4 Objection and riposte

Objection. The future-oriented Benardete paradox doesn't show that the future cannot be endless (under theism); it simply lends further support to causal finitism. After all, the paradox is killed if we debar infinite (ungrounded) causal chains from the realm of metaphysical possibility, since each Reaper's action is sensitive to (and hence causally dependent on) the actions of infinitely many future Reapers.

What this translates to is that (7) isn't true *as stated*. It needs to be modified to the following:

7*. If (i) there could be an unbegun set of finite, non-zero, non-infinitesimal, equal intervals of future time ordered by the *later than* relation, *and if* (ii) there could be a backward-directed ungrounded causal chain spanning that infinite series of future time intervals, *then* there could be an unbegun set that satisfies *ANBC*.

But even if extant justifications for B-argument linking premises equally support (7*) given theism, causal finitists can simply *grant* (7*) while retaining the possibility of an endless future. It's just that backward-directed ungrounded causal chains *spanning* said future cannot exist. And my case only shows that such extant justifications equally support (7*) given theism, *not* (7).

Riposte. I have three responses. First, the objection simply doesn't engage with the *reasons* I gave for thinking that the possibility of future-oriented Benardete paradoxes *simply follows* from the possibility of an endless future. In particular, I argued that the principal lines of support in the literature for B-argument linking premises *equally support* (7) given theism, which connects the possibility of an endless future to the possibility of future-oriented Benardete paradoxes. As it stands, the worry in question doesn't address my case for the symmetry between the respective linking premises. It simply *asserts* that my case only shows symmetric support for (7*) rather than (7). It gives no grounds for questioning the *reasons* I gave for thinking (7) *is* symmetrically supported.

Second, even if we grant that future-oriented Benardete paradoxes violate causal finitism, the objection neglects the fact that my case, if successful, is *precisely* a case to the effect that a violation of causal finitism *follows from* the possibility of an endless future. In other words, (7*)'s antecedent's second conjunct, (ii), *simply follows from* its first conjunct, (i). And the reasons I gave for this entailment mirror *precisely* those that *proponents* of B-arguments use in support of *their* linking premises. In particular, it simply *falls out of* (i) the individual possibility of S, (ii) the possibility that the future is endless, and (iii) Koons' patchwork principle that (iv) causal finitism

is violated.²¹ Per Koons' patchwork principle, we can simply duplicate the intrinsically possible S infinitely many times into the possible framework of an endless future, *thereby entailing* the possibility of a backward-directed ungrounded causal chain. Furthermore, it simply *falls out* of the mysterious force line of support that an objectionable mysterious force is required to *prevent* violations of causal finitism *if* the future could be endless (and theism is true).

Third, and as previously explained, it's not true that future-oriented Benardete paradoxes require a violation of causal finitism. Each Reaper need not be future-sensitive in virtue of being *causally affected* by infinitely many future Reapers; instead, it's future-sensitive in virtue of a *single act of divine revelation*. And again, we needn't suppose that each act of divine revelation is *itself* causally impacted by the activities of infinitely many future Reapers. Perhaps God simply wills that each future Reaper does such-and-such, and God reveals to each Reaper what God wills each *future* Reaper to do. This doesn't violate causal finitism, since causal finitism only denies the possibility of causal chains in which a target state is directly or indirectly causally affected by infinitely many causes. Thus, in such a situation, neither the action of any Reaper, nor any of God's acts of revelation, nor God's willing of the future, involves any violation of causal finitism.

One might object to my third reply that we still have here an unbegun set of items, each of whose acts (or states) explanatorily depends on infinitely many items 'earlier' in the set ordered by the dependence relation (though, of course, *later* in the *temporally* ordered set). I have two rejoinders. First, even if this objection succeeds, my first two replies stand. Second, it's not true that the future-oriented Benardete paradox I've constructed violates explanatory dependence finitism. I've already explained why there need be no *causal* dependence here, and the same reasons (*mutatis mutandis*) show why there need be no *grounding* dependence here either (or *functionally realizing* dependence, or *constitutive* dependence, or any other explanatory dependence relation). The explanatory structure, instead, need only have a single root (God) explaining infinitely many distinct acts or states. But this structure alone isn't paradoxical or contradictory—it doesn't posit an ungrounded explanatory chain, and there are entirely benign cases of a single root explaining infinitely many distinct items.²²

5 Conclusion

After setting up the dialectical context of B-arguments, I leveled a new challenge thereto. The challenge is a dilemma: proponents of B-arguments must either accept the impossibility of an endless future, or else justify B-argument linking premises in ways that don't equally justify a symmetric, future-oriented linking premise. I argued that if theism is true, then extant justifications for B-argument linking premises equally justify the symmetric linking premise. I also sketched ways to motivate the parity between B-argument linking premises and the symmetric linking

²¹ Again, *granting* that future-oriented Benardete paradoxes involve temporally backward infinite causal regresses.

²² Consider, e.g., the benign T-schema regress wherein p explains $T(p)$, and $T(T(p))$, and $T(T(T(p)))$, and so on *ad infinitum*.

premise without any theistic assumptions. Most of all, I hope to have served and advanced debates concerning Benardete paradoxes, finitism, and patchwork principles.²³

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²³ Many thanks to two anonymous referees for very helpful feedback.