A THOROUGHLY MODERN WAGER

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ABSTRACT: Pascal's wager is a familiar heuristic designed to show that believing that God exists is of greater practical value than believing that God does not exist given the outcomes associated with those beliefs as understood in Christian theology. In this way Pascal argues that we that we ought to believe that God exists, independent of epistemic grounds. But, things are not easy, because he understands that belief is not subject to direct voluntary control. So, for purely practical reasons, he advises us to put ourselves in situations that will maximize our chances of acquiring the belief that God exists. In effect, he advises us to attempt to acquire that belief by indirect control. But, then the wager is not a proper decision problem since it does not involve a real choice. Additionally, there are at least two other problems that afflict the traditional wager: one involving the value of eternal damnation and one concerning the coherence of infinite utilities. In this paper the wager will be explored and a corrected version will be presented that yields a rather surprising, but theoretically correct, conclusion.

KEYWORDS: Pascal's wager, belief, acceptance, infinite utilities, decision

1. Introduction

Pascal's wager is a familiar heuristic designed to show that believing that God exists is of greater practical value than believing that God does not exist given the outcomes associated with those beliefs as understood in Christian theology. The wager has been presented in a variety of forms since its inception, but it is fundamentally based on the observation that there are only two possible factual states: God exists and God does not exist. We are to suppose also that there are two possible beliefs states we might have concerning those factual states: the belief that God exists and the belief that God does not exist. This yields four possible combinations of factual states and belief states: believing God exists and God exists, believing God exists and God does not exist, believing God does not exist and God exists, and believing God does not exist and God does not exist. The relevant outcomes that are consequences of those four states are, respectively, eternal salvation, the finite costs associated with belief in God's existence where He does not exist, eternal damnation,[1] and the finite benefits associated with

[1] See B. Pascal, *Pensées*, in *Pascal: Selections*, ed. Richard H. Popkin (New York: MacMillan, 1670/1989), 195-264. Pascal describes this outcome as "misery." As we shall see however, since it involves eternal damnation, this outcome is one that needs to be examined in greater detail.

believing God does not exist where He does not exist. On the basis of this simple looking heuristic Pascal concludes that it is better to believe that God exists than to believe that He does not exist. This is in part because the total expected value of the belief that God exists is essentially unaffected by the finite loss that one would suffer if that belief is false given the infinite value of that belief if it is true. Moreover, it is also supposed to be obvious that belief in God's existence has a greater expected value than the expected value of the belief that God does not exist. This is because whatever finite positive value that believing God does not exist has if it is true is swamped by the negative value it has if it is false (finite or not). So, according to the Wager, believing that God does not exist is clearly supposed to be the less valuable of the two beliefs in terms of total expected values. Thus understood, the wager is supposed to take the form of a standard decision-theoretic problem.

In this way Pascal argues that we that we ought to believe that God exists, independent of epistemic grounds. But, things are not so easy for two important reasons. First, there is a problem with this naïve construction of the wager because Pascal understands that belief is not subject to direct voluntary control. So, for purely practical reasons, he advises us to put ourselves in situations that will maximize our chances of acquiring the belief that God exists. In effect, he advises us to attempt to acquire that belief by indirect control. But, this suggestion raises a number of problems. For one, prior to acquiring the belief that God exists, what attitude(s) might we have with respect to the proposition that God exists? Additionally, how do those pre-belief attitudes relate to belief? Answering these questions has important implications concerning voluntarism with respect to propositional attitudes and rational commitment. In this paper it will be argued that the kind of commitment the wager involves is best modelled as a form of voluntary acceptance. This is because the kind of pre-belief attitude involved in Pascal's wager is governed by standards of pragmatic rationality that are of a different sort from those that apply to beliefs. Second, this naïve construction of the wager involves calculations of expected utility involving at least one outcome with an infinite utility. Specifically, Pascal supposes that belief in God's existence where He in fact exists has an infinite positive expected utility. But, standard decision theory is incompatible with outcomes having such utility values. As a result, here it will be suggested that in order to make sense of the wager in a thoroughly modern manner we need to introduce an alternative account of the nature of decision-theoretic rationality that allows for outcomes to have infinite values in a manner that does not raise any serious problems. Moreover, it will also be suggested that consistency demands that we treat the outcome involving eternal damnation as having infinite negative utility for the same sorts of reasons that we attribute infinite positive utility to the outcome involving eternal salvation. Ultimately, it will be shown that recognizing all of this yields a rather surprising, modern, and elegant re-construction of the wager with a very different conclusion.

2. Preliminaries: Fixing the Traditional Wager

Typical interpretations of Pascal's wager treat it as a decision problem involving a choice about expected utility of two competing beliefs. The simplest account of this choice is understood to be one between the belief that God exists and the belief that God does not exist.[2] This choice is then supposed to be evaluated in light of the expected outcomes determined by orthodox Christian theology as they depend on the possible factual states: God exists and God does not exist. The wager so understood is supposed to involve the following elements:

O: Options $\{Bp, B\neg p\}$.
S: States $\{(\exists x)(x = G), \neg(\exists x)(x = G)\}$.
C: Outcome Values $\{V_{\infty}, V_{-\beta}, V_{\alpha}, V_{-\alpha}\}$.[3]

This is the familiar expectation form of Pascal's wager.[4] The elements in O and S do not need to be more deeply analyzed at this point, but, at this juncture, we need to be clear what the elements of C represent. Most importantly, V_{∞} is meant to represent eternal salvation and so this is an infinite positive magnitude. Why is this supposed to be the case? As Pascal understands it what is being wagered in the wager's life and, in the case of this particular outcome, "…there is here an infinity of an infinite happy life to gain."[5] So, Pascal appears to be basing this contention that the outcome involves an infinity of positive value on the idea that it is an intrinsically positive outcome that is eternal in character. In Pascal's version of the wager $V_{-\beta}$ represents the *finite* loss associated with eternal damnation. V_{α} represents the positive value associated with having those experiences precluded by orthodox Christian theological practice and so it is a

[2] See, for example, Philip Quinn, "Moral Objections to Pascalian Wagering," in *Gambling on God: Essays on Pascal's Wager*, ed. Jeff Jordan (Lanham: Rowman & Littlefield, 1994), 61-81 and Jeff Jordan, "The Many Gods Objects," in *Gambling on God: Essays on Pascal's Wager*, 101-113.
[3] An outcome value is a function on outcomes. In decision theory these are treated as utilities. So, $V_i = u(O_i)$.
[4] See Ian Hacking, "The Logic of Pascal's Wager," *American Philosophical Quarterly* 9 (1972): 186-92 for discussion of the expectation argument and the related arguments from dominance and dominating expectation found in Pascal's notes.
[5] Pascal, *Pensées*.

finite positive magnitude. $V_{-\alpha}$ represents the value lost by failing to have those experiences precluded by orthodox Christian theological practice and so it is a finite negative value. These elements are then supposed to be related in terms of the following counterfactuals:[6]

CF1: $[Bp \,\&\, (\exists x)(x = G)] \Box\!\!\rightarrow V_{\infty}$.
CF2: $[Bp \,\&\, \neg(\exists x)(x = G)] \Box\!\!\rightarrow V_{-\alpha}$
CF3: $[B\neg p \,\&\, \neg(\exists x)(x = G)] \Box\!\!\rightarrow V_{\alpha}$.
CF4: $[B\neg p \,\&\, (\exists x)(x = G)] \Box\!\!\rightarrow V_{-\beta}$.

From these counterfactuals we can generate the following decision matrix (DM1):

	Bp	$B\neg p$
$(\exists x)(x = G)$	V_{∞}	$V_{-\beta}$
$\neg(\exists x)(x = G)$	$V_{-\alpha}$	V_{α}

The expected value (EV) of an option is then defined as the sum of the expected values of the possible outcomes associated with each option. So we get the following expected values for B*p* and B¬*p*:

$EV(Bp) = V_{\infty} + V_{-\alpha}$.

$EV(B\neg p) = V_{-\beta} + V_{\alpha}$.

Since V_{∞} is an infinite magnitude the total expected value of B*p* is positive and infinite and since both $V_{-\beta}$ and V_{α} are finite the expected value of B¬*p* will be finite whatever magnitudes those values have, $EV(Bp) > EV(B\neg p)$. So, decision-theoretical considerations are supposed to favor belief over the alternative. But all is not kosher here and we can see that there are already problems with the original wager with respect to one element in O, specifically with respect to the outcome that is supposed to represent eternal damnation.

[6] See Michael J. Shaffer, "Decision Theory, Intelligent Planning and Counterfactuals," *Minds and Machines* 19 (2009): 61-92 for extensive and critical discussion of orthodox decision theory, especially with respect to the role that counterfactuals play in decision problems.

The problem is that the very same reasons that Pascal uses to support the contention that salvation (i.e. the outcome of B*p* & (∃*x*)(*x* = G)) should have an infinite positive value also support the view that eternal damnation (i.e. the outcomes of B¬*p* & (∃*x*)(*x* = G)) ought to have an infinite negative value, especially given Pascal's own commitment to orthodox Catholicism. By parity of reasoning the very eternality of eternal damnation implies that it involves an infinity of an infinitely *unhappy* life, whether or not Pascal himself acknowledges this or not.[7] So, in order then to be consistent we ought to replace the finitely valued outcome $V_{-\beta}$ with $V_{-\infty}$, an infinite negative magnitude.[8] To accommodate this insight we need to replace CF4 with CF4′:

CF4′: [B¬*p* & (∃*x*)(*x* = G)] □→ $V_{-\infty}$.

In light of this correction we get the following decision matrix (DM2):

	B*p*	B¬*p*
(∃*x*)(*x* = G)	V_{∞}	$V_{-\infty}$
¬(∃*x*)(*x* = G)	$V_{-\alpha}$	V_{α}

So we get the following expected values for B*p* and B¬*p*:

EV(B*p*) = $V_{\infty} + V_{-\alpha}$.

EV(B¬*p*) = $V_{-\infty} + V_{\alpha}$.

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[7] Duff, Lycan and Schlesinger and Hacking concur on this point about assigning an infinite negative utility to damnation. See Antony Duff, "Pascal's Wager and Infinite Utilities," *Analysis* 46 (1986): 107-109, William G. Lycan and George N. Schlesinger, "You Bet Your Life: Pascal's Wager Defended," in *Reason and Responsibility*, 7th ed., ed. Joel Feinberg (Belmont: Wadsworth, 1989), 82-90 and Ian Hacking, *An Introduction to Probability and Inductive Logic*, (Cambridge: Cambridge University Press, 2001), 118.

[8] It is important to note at this point that orthodox decision theory is actually incompatible with their being outcomes with infinite values (i.e. utilities). For details see Duff, "Pascal's Wager and Infinite Utilities," 107-109, Jeff Jordan, "Pascal's Wager Revisited," *Religious Studies* (1989) 34: 419-431, Edward McClennen, "Pascal's Wager and Finite Decision Theory," in *Gambling on God: Essays on Pascal's Wager*, 115-137, Alan Hájek, "Waging War on Pascal's Wager," *Philosophical Review* 112 (2003): 27-56 and P. Bartha, "Taking Stock of Infinite Value: Pascal's Wager and Relative Utilities," *Synthese* 154 (2007): 5-52. This issue will be more fully addressed in section 6.

Since $V_{¥}$ and $V_{-¥}$ are infinite quantities, intuitively it would seem to be the case that they respectively swamp V_{-a} and V_{a} and thus wholly determine the values EV(B*p*) and EV(BØ*p*). Moreover, it is still abundantly clear from the naïve perspective that in this corrected version of the wager EV(B*p*) > EV(B¬*p*), in fact the value of B*p* is *massively* greater than that of B¬*p*. So, from a pragmatic perspective, even with this small correction it appears to be the case that we ought to adopt B*p*.

However, Pascal also famously argued that belief is not voluntary and so one cannot on this basis simply choose to believe that God exists and thus make it so.[9] In light of this recognition he argued that the best that we can do is to attempt to indirectly bring about that belief state. This indirect approach is supposed to involve things like participating in Christian practice, mingling with believers and reading Christian texts. But, this fact about our lack of direct doxastic control introduces a crucial wrinkle into the traditional wager when it is understood as a decision problem. Specifically, this characterization of the wager wrongly assumes that the options involve are subject to direct control. In standard decision theory, the options an agent has must constitute a choice for the agent and as Levi points out,

> Having a choice presupposes having options. Having the option to perform some action entails having the ability to perform the action upon choosing it. Hence, having a choice presupposes having abilities to perform various actions upon choosing them.[10]

So, the wager, as traditionally understood, *is not really a real decision problem at all*. If the traditional construal of the wager involved a choice or a decision it would have to be the case that B*p* and B¬*p* constitute possible acts subject to the direct control of the agent. This is because, if it involves a real choice or decision, then it would have to be the case that the agent has the ability to perform those acts. But, according to Pascal (and many others), these states are not subject to our direct control. So, there is no decision problem here at all. The traditional wager simply is not a well-formed decision problem.

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[9] See Bernard Williams, "Deciding to Believe," in *Language, Belief and Metaphysics*, eds. Howard Evans Kiefer and Milton Karl Munitz (Albany: SUNY Press, 1970), 95-111 and Matthias Steup, "Doxastic Voluntarism and Epistemic Deontology," *Acta Analytica* 15 (2000): 25-56 for detailed discussion of direct doxastic voluntarism. Duff and Hacking both carefully emphasize this point in the context of Pascal's wager in, respectively, "Pascal's Wager and Infinite Utilities" and "The Logic of Pascal's Wager."

[10] Isaac Levi, *Hard Choices* (New York: Cambridge University Press, 1986), 47.

3. Belief vs. Acceptance

As a result, the actual situation in which Pascal places us is really rather different than it has been traditionally understood. The wager does not involve acts B*p* and B¬*p* as options, because those states are not up to us (i.e. they are not directly subject to control). Hacking puts this nicely as follows:

> A decision problem requires a partition of possible actions. As Pascal sees it, you either act with indifference to God or you act in such a way that you will, in due course, believe in his existence and his edicts. There is no cant in Pascal. He accepts it as a piece of human nature that belief is catching: if you go along with pious people, give up bad habits, follow a life of 'holy water and sacraments' intended to 'stupefy one' into belief, you will become a believer. Pascal is speaking to one who is unsure whether to follow this path or whether to be indifferent to the morality of the church. The two possible acts are not 'Believe in God' and 'Do not believe.' One cannot decide to believe in God. One can decide to act so that one will very probably come to believe in God.[11]

But, Hacking does not apparently see what this actually implies about Pascal's wager. The first thing to note is that the belief states involved in the traditional construction of the wager are very much like $(\exists x)(x = G)$ and $\neg(\exists x)(x = G)$. In other words, they are better understood to be part of the set of factual states involved in the wager heuristic. What is up to us however, is whether we *accept p* or ¬*p*, and, in due course, we will explore what this entails. Nevertheless, as Pascal sees our situation, we cannot simply and directly choose to believe that God exists any more than we can choose directly that He exists, but we can commit ourselves to the proposition that He exists for prudential reasons. As he describes it, this appears to amount to *simulating* the life of a devout believer. In a moment of rhetorical flourish, Pascal describes this behavior as follows:

> You would like to attain faith and do not know the way; you would like to cure yourself of unbelief, and ask the remedy for it. Learn of those who have been bound like you, and who now stake all their possessions. These are people who know the way which you would follow, and who are cured of an ill which you would be cured. Follow the way by which they began; *by acting as if they believed, taking the holy water, having masses said, etc*. Even this will naturally make you believe, and deaden your acuteness (my italics).[12]

What is crucial to see at this point is that we can directly control this kind of commitment. One can directly, voluntarily and efficaciously choose to act as if one were a believer, even though one cannot in this way choose to be a believer.

[11] Hacking "The Logic of Pascal's Wager," 188.

[12] Pascal, *Pensées*, 259.

So, acceptance, unlike belief, is voluntary and accepting that God exists importantly involves engaging in religious activities of the familiar sort. Pascal, of course, crucially hopes that such acceptance will lead to bona fide belief and in due course we will return to discussion of the connection between such acceptance and belief. But first we need to see if the wager heuristic can be salvaged in light of this observation about the traditional wager and the nature of well-formed decision problems.

To begin, the standard propositional attitudes that are typically dealt with in epistemology and elsewhere are belief and knowledge and extant interpretations of the wager incorrectly treat it as involving only belief. But, it is also widely accepted that these are not the only propositional attitudes that can be had toward propositional contents, even if this appears to be an often forgotten or ignored point. We need only to consider the attitudes of considering *p*, grasping *p*, supposing that *p*, or of wishing that p be the case, and so on in order to see that the taxonomy of propositional attitudes is really quite diverse and complex. But, the fact that there has been relatively little discussion of these other propositional attitudes is a rather serious lacuna in philosophy, and it is likely that it has given rise to the tendency to over ascribe belief and knowledge to agents where other propositional attitudes are really at work in various cases. So, one core claim defended here is that this is just the sort of error that has afflicted traditional attempts to formally characterize the wager.

An important task then when we are considering situations or models that involve propositional commitments is to distinguish cases involving belief from those that do not involve belief. One effective way of doing this is to distinguish commitments that involve the norm of truth from those that do not involve the norm of truth. This is of course because it is widely agreed that the norm of belief is truth. By distinguishing such cases we can thereby avoid attributing inappropriate features to such situations, especially with respect to judgments of rationality. This can be effectively accomplished in the case of belief by looking at instances of the following argument scheme (*scheme 1a*):

> P1: The operant and appropriate norm in situation *x* involving *S*s attitude β toward the proposition that *p* is *y*.
>
> P2: *y* is not truth.
>
> P3: Truth is the norm of belief.
>
> Therefore, *S*s attitude β toward proposition *p* in situation *x* is not belief.

If we take seriously the claim that there are true (or even merely possible) substitution instances of this argument scheme, then it is reasonable to believe that we can make sense of the idea that there are propositions that are believable (i.e. it is possible to believe them) and even plausible (i.e. they are not known to be false and do not seem to be false), but that are not actually believed. This is because there can be non-truth-normed rational commitments. As we shall see, some of these commitments are pragmatic in nature and so the aim of committing in those cases is broadly pragmatic, others involve commitments based on plausibility. We know, of course, that $\Diamond Bp$ does not entail Bp as a matter of elementary modal logic, but it is also reasonable to suppose that we need not believe a proposition merely because it is plausible and rational to hold for some pragmatic reasons, or because it is merely plausible. So the upshot of this is that it is reasonable to believe that propositions can be rationally entertained but not believed, at least in the sense of plausibility or pragmatic rationality.[13] Once this possibility is seriously entertained it is apparent there are many cases of commitments that are not reasonably understood to be beliefs, but which allow us to achieve certain important and rational goals. In accordance with the recognition that many commonplace propositional commitments are not beliefs, L. J. Cohen in particular usefully distinguished belief from a particular form of acceptance.[14] He treated the latter as voluntary and pragmatically motivated, whereas the former is non-voluntary and epistemically motivated and showed how belief and acceptance have often been conflated with serious negative implications for a number of philosophical issues. Given this distinction the following argument scheme can be used to positively identify a commitment as a form of acceptance (*scheme 1b*):

> P1: If the operant and appropriate norm(s) in situation x involving Ss attitude β toward the proposition that p is plausibility and/or pragmatics, then β is a form of acceptance.
>
> P2: The operant and appropriate norm(s) in situation x involving Ss attitude β toward the proposition that p is plausibility and/or pragmatics.

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[13] See Richard Foley, "Pragmatic Reasons for Belief," in *Gambling on God: Essays on Pascal's Wager*, 31-46 and Eddy Zemach, "Pragmatic Reasons for Belief?" *Nous* 4 (1997): 525-527 for discussion of pragmatic and epistemic justification in the context of the Wager.

[14] See L. Jonathan Cohen, *An Essay on Belief and Acceptance* (Oxford: Clarendon Press, 1992), Michael J. Shaffer, "The Privacy of Belief, Morality and Epistemic Norms," *Social Epistemology* 20 (2006): 41-54, "Three Problematic Theories of Conditional Acceptance," *Logos & Episteme* (2011): 117-125, "Doxastic Voluntarism, Epistemic Deontology and Belief-contravening Commitments," *American Philosophical Quarterly* 50 (2013): 73-82 and "Epistemic Paradox and the Logic of Acceptance," *Journal of Experimental and Theoretical Artificial Intelligence* 25 (2013): 337-353 for various discussions of acceptance and belief.

Therefore, *S*s attitude β toward proposition *p* in situation *x* is a form of acceptance.

These kind of weaker but voluntary propositional commitments turn out to be quite commonplace attitudes to have toward propositions and they play roles in all sorts of behaviors like acting, exploring ideas, etc. More to the point, it will be argued here that a form of acceptance plays an important role in the proper understanding of the wager and the determination of the specific kind of acceptance that is at work in Pascal's wager is a crucial goal of this paper.

So, let us then begin by looking at the various concepts of acceptance in contrast to the concept of belief. The first important distinction to make with respect to the various attitudes of acceptance concerns the extent of such commitments. So, as we will understand it here, *S*s acceptance of *p* is *full*, if and only if *S*s commitment to *p* is governed by an appropriate closure principle.[15] A modest and reasonable version of such closure for acceptance can be simply rendered as follows:

(JBCM) If A*sp* and JB*s*($p \supset q$), then A*sq*.

Where *S*s commitment is not full in this sense we will call such acceptance *limited*. The second important distinction to make among the various forms of acceptance concerns the norm that governs such cases of acceptance and thus fixes the kind of rationality that such commitments involve. So, if *S*s acceptance of *p* is *strong*, then *S*s commitment to *p* is such that *p* should be maximally plausible for *S*. Here plausibility will be understood in the following sense. *S* is plausible for *p*, if and only if, *S* does not know that $\neg p$ and *p* does not *prima facie* seem to be false to *S*. Where *S*s commitment is not strong in this sense we will call *S*s commitment *weak* and the norm that governs such weak forms of acceptance will be understood to be pragmatic utility. So, if *S*s acceptance of *p* is weak, then *S*s commitment to *p* is such that *p* should be maximally pragmatically justified for *S*. that Adopting the attitude of weak acceptance towards a proposition *may* involve propositions that are taken to be plausible or doing so may involve propositions that are in fact be plausible despite the agent's not taking them to be so, but

[15] Here we do not need to settle the issue about whether closure principles should be understood as involving logical or material implication, whether such closure principles should be objective rather than subjective and whether the closure principle should involve closure under belief or justified belief. So, these matters will be ignored for the purposes at hand. For further discussion of these issues see Shaffer, "Doxastic Voluntarism, Epistemic Deontology and Belief-contravening Commitments" and "Epistemic Paradox and the Logic of Acceptance." What matters here is that we understand that full acceptance involve commitment to all of the implications of an accepted proposition.

neither of these conditions are required to weakly accept a proposition. An agent might be pragmatically entertaining a proposition that happens to be plausible, but the plausibility of that proposition may not be the *rational basis* on which it is being entertained. In other words, plausibility may not be among the ultimate reasons for the adoption of that proposition. So, many such pragmatic commitments involve propositions the adoption of which is not motivated by plausibility and many commitments that aim at the adoption of plausible propositions may not be adopted for pragmatic reasons. But, where we have commitments that aim at both plausibility and pragmatic utility we have cases of what we can call *mixed acceptance* and in such cases we must be clear that the rational basis for accepting a proposition is *both* plausibility and pragmatic utility. So understood these two important distinctions yield six important categories of acceptance: strong full acceptance, weak full acceptance, strong limited acceptance, weak limited acceptance, mixed full acceptance and mixed weak acceptance. Further, more-refined versions of each of these forms of acceptance can then be determined by specifying additional features definitive of each of these types of propositional attitude. But, for the purposes at hand we can ignore these more fine-grained characterizations and focus directly on determination of which of these form(s) of acceptance are involved in the wager.

To begin, let us consider the weakest form of acceptance so understood, weak limited acceptance. As it is to be understood here, *weak limited acceptance* is a propositional attitude like belief and knowledge. Its main features are as follows:

WL1. Accepting *p* is purely voluntary.

WL2. Accepting *p* is non-evidential.

WL3. Accepting *p* is a form of supposition.

WL4. Accepting *p* is a pragmatic matter.

WL5. Accepting *p* is contextual.

WL6. Accepting *p* is not a commitment to the literal truth of *p*.

WL7. Accepting *p* is not governed by any closure principle.

More specific versions of scheme 1b arguments will then allow us to discriminate truth-normed commitments like belief from non-truth-normed commitments like this particular form of acceptance on the basis of the norm(s) it does involve. In any case, the view endorsed here is that accepting a proposition in this particular weak and limited way is a sort of voluntary, non-evidential but suppositional, pragmatic and contextual commitment that is something like epistemically "trying out" or "using" a proposition and *some of its implications in some contexts*, and while the account of weak limited acceptance offered here shares some features in common with Cohen's account it is appreciably different because on Cohen's account acceptance is characterized by subjective closure under material implication.[16] This principle is typically understood as follows:

(SCM) If A*sp* and B*s*($p \supset q$), then A*sq*.

This closure principle is however too weak and as full acceptance is characterized here it will be understood to involve JBCM. This is simply because SCM is far too subjective in closing acceptance only under what are *believed* to be the material implication of an accepted proposition. Nevertheless, Cohen's form of acceptance is still a form of weak *full* acceptance since it does obey a form of closure. In any case, limited forms of acceptance can be distinguished from forms of full acceptance in virtue of the following general argument schemes (*schemes 2a* and *2b* respectively):

P1: In any situation *x* involving *S*s acceptance of *p*, if *S*s commitment to *p* is governed by some closure principle *k*, then that commitment is a form of full acceptance.

P2: *S*s attitude β toward *p* in situation *C* is not governed by some closure principle *k*.

Therefore, *S*s attitude toward proposition *p* in situation *x* is not a form of full acceptance.

P1: In any situation *x* involving *S*s acceptance of *p*, if *S*s commitment to *p* is governed by some closure principle *k*, then that commitment is a form of full acceptance.

P2: *S*s commitment to *p* in situation *C* is governed by some closure principle *k*.

Therefore, *S*s attitude toward proposition *p* in situation *x* is a form of full acceptance.

So, we can demonstrate that a given commitment is/is not a case of full acceptance by exploring whether an agent's acceptance satisfies some appropriate closure principle.

[16] One might also believe that such attitudes are governed by other closure principles such as closure under logical implication. Since this matter plays no role in the context of this paper, it will be ignored here. See Shaffer, "Epistemic Paradox and the Logic of Acceptance" for some discussion of the issue of closure in the context of different forms of acceptance.

We are then able distinguish cases of weak acceptance from cases of strong acceptance and from cases of mixed acceptance by determining whether they involve the requirement that S's acceptance of *p* is motivated by consideration of plausibility, whether *S*s commitment to *p* is merely pragmatically motivated, or whether *S*s commitment to *p* is motived both considerations of both plausibility and pragmatics. Given this distinction, *strong full acceptance* can be understood to be characterized in terms of the following principles:

SF1. Accepting *p* is purely voluntary.

SF2. Accepting *p* is non-evidential.

SF3. Accepting *p* is a form of supposition.

SF4. Accepting *p* requires that *S* takes *p* to be plausible.

SF5. Accepting *p* is not a commitment to the literal truth of *p*.

SF6. Accepting *p* is governed by JBCM.

Weak full acceptance can, similarly, be characterized as follows:

WF1. Accepting *p* is purely voluntary.

WF2. Accepting *p* is non-evidential.

WF3. Accepting *p* is a form of supposition.

WF4. Accepting *p* is a pragmatic matter.

WF5. Accepting *p* is not a commitment to the literal truth of *p*.

WF6. Accepting *p* is governed by JBCM.

Finally, *mixed full acceptance* can be characterized as follows:

MF1. Accepting *p* is purely voluntary.

MF2. Accepting *p* is non-evidential.

MF3. Accepting *p* is a form of supposition.

MF4. Accepting *p* requires that *S* takes *p* to be plausible.

MF5. Accepting *p* is a pragmatic matter.

MF6. Accepting *p* is not a commitment to the literal truth of *p*.

MF7. Accepting *p* is governed by JBCM.

Notice that in all of these cases are cases of voluntary, complete and total commitments and that the completeness and totality of these attitudes is due to the fact that they are governed by closure principles, specifically by JBCM. They are all suppositional, non-evidential and non-truth-normed kinds of commitments and they differ only in terms of the non-evidential norms which govern them. Strong full acceptance has plausibility as a norm. Weak full acceptance has practical utility as a norm and mixed full acceptance has both plausibility and practical utility as norms.

So, let us then turn to the issue of the identifying the specific type of propositional commitment at work in the wager. Recall that, in his insightful discussion of Pascal's wager, Hacking noted the following crucial point: "The two possible acts are not 'Believe in God' and 'Do not believe.' One cannot decide to believe in God. One can decide to act so that one will very probably come to believe in God."[17] In accordance with this observation we can now establish quite easily that the commitment involved in the wager is not belief and that it is acceptance in the following manner. For the wagering agent S,

> P1: The operant and appropriate norms in the wager involving Ss attitude β toward the proposition that God exists are practical gain and/or plausibility.
>
> P2: practical gain and plausibility are not truth.
>
> P3: Truth is the norm of belief.
>
> Therefore, Ss attitude β toward the proposition God exists in situation the wager is not belief.

> P1: If the operant and appropriate norms in the wager involving Ss attitude β toward proposition that God exists in the wager is plausibility and/or pragmatics, then β is a form of acceptance.
>
> P2: The operant norm in the wager involving Ss attitude β toward proposition that God exists in the wager is plausibility and/or pragmatics.
>
> Therefore, Ss attitude β toward the proposition that God exists in the wager is a form of acceptance.

So on this basis it should be clear that the kind of commitment involved in the wager is not belief. This is because it is not motivated by a commitment to literal truth. Rather, it is aimed at some other target, and given what Pascal says about the wager the agent's options are best understood to involve a form of acceptance because that choice is motivated by pragmatic considerations. Moreover, we can also see that the wager cannot reasonably be taken to involve limited acceptance via the following consideration. If the agent's attitude involved in the wager were a form of limited acceptance, then it would not be governed by

[17] Hacking, "The Logic of Pascal's Wager," 188.

a closure principle and might also be contextual. But, this just won't do in the case of the wager. As Pascal sees it, in wagering the commitment we must have towards the proposition that God exists must be extensive enough to yield a high probability that such acceptance will lead to bona fide belief and it must be sufficiently extensive to include all of the implications of Christian practice. But, if this acceptance were limited it would allow for the agent to reject many of the implications of accepting Christian principles and practices and it would not require committing to any of them in all contexts. Thus, it seems rather unlikely that such limited acceptance of those principles would suffice to do what Pascal has in mind, the conversion of acceptance into belief by systematically feigning belief. It is simply not reasonable to believe that half-hearted, incomplete and contextually limited acceptance of those principles and practices will likely bring about bona fide belief. Given this more nuanced understanding of the propositional attitudes at work in the wager let us then return to the matter of formally characterizing the wager.

4. The Wager as a Decision Problem

The wager in all of its forms arises out of the observation that the epistemic evidence and arguments relevant to the matter of God's existence are, at best, inconclusive. In a more forceful and pessimistic frame of mind Pascal appears to believe, in fact, that they are totally ineffective and epistemically inert. For example, he says of the epistemic attempt to ground commitment to God's existence that, "Reason can decide nothing here."[18] So, the real, pragmatically motivated, wager is supposed to supplant those failed attempts to epistemically justify belief in God's existence. It does so, however, by changing the standards of rationality from epistemic rationality to a form of non-epistemic rationality. Specifically, it changes the issue from one that involves epistemic reasons to one that involves specifically pragmatic considerations. But Pascal and those who defend the wager heuristic have not appreciated all of the important implications that this entails. Since weak full acceptance and mixed full acceptance are both voluntary, governed by closure, do not have truth as a norm and do have pragmatic utility as a norm, these forms of acceptance are the only really plausible candidates for the attitudes at work in a defensible form of the wager as a decision problem. Given what Pascal says it is simply not possible that the wager involves belief because the reasons he is trying to use to motivate the disbeliever to adopt the commitment to Christian practice are pragmatic and involve a real choice.

[18] Pascal, *Pensées*, 257.

In any case, we are now in a position to see that a properly constructed version of the wager involves the following elements:

O′: Options {Ap, A¬p}.

S′: States {(∃x)(x = G) & Bp, (∃x)(x = G) & ¬Bp, ¬(∃x)(x = G) & Bp, ¬(∃x)(x = G) & B¬p}.

C′: Outcome Values {V_{∞},$V_{-\infty}$, V_{α}, $V_{-\alpha}$}.

The first thing to note about these elements that will be used to formulate a coherent construction of the wager as a decision problem is that that the elements in S′ are rather different than those used to construct the traditional wager. They are the following more complex compound factual states: God exists and the agent believes that He does exist; God exists and the agent believes that He doesn't exist; God does not exist and agent believes that He does exist and God does not exist and the agent believes that He does not exist. Secondly, the elements of O′ in this construction involve some form of full acceptance that is subject to the direct control of the wagering agent. So understood, this problem does constitute a real decision problem. It involves a real choice: the choice between committing to p or not for reasons that are pragmatic and/or related to plausibility and this is just what Pascal had in mind. In this alternate construction of the wager the elements of O′, F′ and S′ are related in terms of the following counterfactuals:

CF1: [Ap & Bp & (∃x)(x = G)] □→ V_{∞}.

CF2: [Ap & B¬p & (∃x)(x = G)] □→ $V_{-\infty}$.

CF3: [Ap & Bp & ¬(∃x)(x = G)] □→ $V_{-\alpha}$

CF4: [Ap & B¬p & ¬(∃x)(x = G)] □→ $V_{-\alpha}$.

CF5: [A¬p & Bp & (∃x)(x = G)] □→ V_{∞}.

CF6: [A¬p & B¬p & (∃x)(x = G)] □→ $V_{-\infty}$.

CF7: [A¬p & Bp & ¬(∃x)(x = G)] □→ V_{α}.

CF8: [A¬p & B¬p & ¬(∃x)(x = G)] □→ V_{α}.

But, this decision problem gives rise to a very different and perhaps rather surprising decision matrix (DM3):

	$\mathrm{A}p$	$\mathrm{A}\neg p$
$\mathrm{B}p \,\&\, (\exists x)(x = \mathrm{G})$	V_{∞}	V_{∞}
$\mathrm{B}\neg p \,\&\, (\exists x)(x = \mathrm{G})$	$V_{-\infty}$	$V_{-\infty}$
$\mathrm{B}p \,\&\, \neg(\exists x)(x = \mathrm{G})$	$V_{-\alpha}$	V_{α}
$\mathrm{B}\neg p \,\&\, \neg(\exists x)(x = \mathrm{G})$	$V_{-\alpha}$	V_{α}

The values assigned as the outcomes in CF1, CF3, CF4, CF6, CF7 and CF8 are straightforwardly unproblematic. But, the values assigned as the outcomes in the consequents of CF2 and CF5 are worthy of some additional commentary. The antecedent of CF2 describes the situation where the agent accepts that God exists and the agent does not believe that God exists, but God exists. In this case the agent is voluntarily committed to God's existence for practical reasons, but the agent is really in the state of disbelief about God's existence. This is the principle case that Pascal is ultimately concerned with in his discussion of the prospects for those who believe that God does not exist. However, he sees some hope here. This is because this could be a case where the disbeliever might bring about belief through acting as if God exists. The posited outcome of this state is, however, still eternal damnation, just as in the case of CF6, if such acceptance is not actually converted into true belief. This is because the disbeliever is then merely simulating in his acting as if God exists in the possible case where He exists. If the simulation is unsuccessful and does not bring about true belief, the agent still suffers eternal damnation. Such agents are ultimately not *earnest* believers and so are no better off than those who fail even to act as if God exists in the possible case where God exists. The antecedent of CF5 describes the situation where the agent accepts that God does not exist, the agent believes that God exists and God exists. In this case we have an agent who is a true believer who acts as if God did not exist in the possible situation where He exists. This could be the case of an agent who is attempting to reject God's existence by simulation of the life of a disbeliever, despite his actually believing otherwise, in much the same sort of manner that the agent who disbelieves in the case described by the antecedent of CF2 might be attempting to bring about true belief. But, since this agent is in fact a true believer he stands to gain eternal salvation unless his accepting that God does not exist brings about true disbelief for basically the same reasons that the agent in CF2 faces eternal damnation.

However, what is most important to notice about this decision problem is that based on CF1-8 it looks like we get the following surprising expected values for A*p* and A¬*p*:

$EV(Ap) = V_{\infty} + V_{-\infty} + V_{-\alpha} + V_{-\alpha}$.

$EV(A\neg p) = V_{\infty} + V_{-\infty} + V_{\alpha} + V_{\alpha}$.

Since V_{∞} are infinite quantities $V_{-\infty}$ they cancel out and the values EV(A*p*) and EV(A¬*p*) are wholly determined by the values of the other finitary outcomes.[19] It should be clear that in this corrected construction of the wager as a well-formed decision problem $EV(A\neg p) > EV(Ap)$. So, from a purely pragmatic perspective, if we entertain Pascal's invitation to wager so understood we ought to adopt A¬*p*! According to the properly constructed wager we should not accept that God exists. In other words, from the perspective of practical rationality, according to the properly reconstructed wager it is irrational to behave as if God exists.

5. From Acceptance to Belief

However, it is clear that what Pascal has in mind is that in adopting A*p* we will thereby come to adopt B*p*, or, at least, there will be very likely that accepting *p* will lead to belief that *p*. That is the unavoidable implication of his advice to the disbeliever that they should emulate believers and, "Follow the way by which they began; by acting as if they believed, taking the holy water, having masses said, etc. Even this will naturally make you believe, and deaden your acuteness."[20] Notice that if the principle that acceptance guarantees belief is included in the set of factual states that characterize the wager, then CF2 and CF4 both reduce to one of CF1 or CF3 and CF5 and CF7 will reduce to one of CF6 or CF8. Effectively, a robust enough connection between A*p* and B*p* will eliminate the act/state combinations involving A*p* and B¬*p* and A¬*p* and B*p*. If this is the case, then it would restore the result of the traditional wager because it would eliminate the outcomes of CF2 and CF4. Thereby EV(A*p*) would be changed from $V_{\infty} + V_{-\infty} + V_{-\alpha} + V_{-\alpha}$ to $V_{-\infty} + V_{-\alpha}$. Similarly, it would eliminate the outcomes of CF5 and CF7,

[19] This result also depends on being able to partition the relevant outcome values as follows: $(V_{\infty} + V_{-\infty}) + (V_{-\alpha} + V_{-\alpha})$ and $(V_{\infty} + V_{-\infty}) + (V_{\alpha} + V_{\alpha})$. This is the natural way to do the calculations however as it treats the outcomes corresponding to God exists and God does not exist as the primary basis on which to partition the outcomes in both cases.

[20] Pascal, *Pensées*, 259.

yielding a value of $V_{-\infty} + V_{\alpha}$ for EV(A¬*p*). If this is the case, then EV(A*p*) > (A¬*p*) and we get the result that it is pragmatically rational to accept that God exists in the sense that it is pragmatically rational to simulate the life of a believer. But, deriving this result is totally dependent on showing that the connection between acceptance and belief is sufficiently robust and we are now in a position to ask what we might say about the connection between A*p* and B*p* on which the traditional wager then critically hinges.[21]

As it turns out, what is specifically and crucially important for this attempt to recapture the traditional wager and its theistically inclined result is establishing that the probability of B*p* given A*p* is 1. This is easy to see based on the following consideration of the corrected version of the wager presented above. Suppose that weak/mixed full acceptance of a proposition renders belief in that proposition likely with a very high probability but not with probability 1. So the probability that an agent will believe a proposition at some later time, given that it is accepted in the weak or mixed sense at an earlier time is close to but not equal to 1 (i.e. $P(B_{t+n}p|A_tp) \approx 1$). If this is true, then (relatively speaking) the outcomes in CF2, CF4, CF5 and CF7 would be very unlikely scenarios, and CF1, CF3, CF6 and CF8 would be very likely scenarios. But, given the nature of the outcomes themselves this would not change the outcome that EV(A¬*p*) > EV(A*p*). This is because while considerations of probability can impact the expected utility values of outcomes involving finite expected values, they have no impact on the expected utility values of the outcomes involving infinite values. The infinitary nature of those magnitudes swamps any non-unitary probability no matter how close to 1 it is. For example, the value associated with CF5 is still V_{∞} even if that outcome is only infintiessimally probable due to the fact that acceptance almost always leads to belief. This is because the expected value of A¬*p* in that case is just the product of the probability that the outcome in question will come about and the magnitude of the expected value of that outcome of A¬*p* in the world state B*p* & $(\exists x)(x = G)$. So, the expected value of A¬*p* in this case is still V_{∞}. The same thing goes for all of the values and probabilities associated with CF1, CF2, CF5 and CF6. Since all of this is the case we still get the cancellation of the infinitary outcomes for both A*p* and A¬*p* in the calculation of the total expected values for those options. This makes the expected value of A*p* dependent only on the outcomes of CF3 and CF4

[21] This is not, of course, true if we were to reconstruct the wager in such a way that we replace the infinite losses and gains with vast but finite losses and gains, and this has been suggested by Jordan in "Pascal's Wager Revisited," for other reasons. In that case, the wager can be salvaged without it being the case that $P(B_{t+n}p|A_tp) = 1$, but that is not Pascal's wager. Pascal's wager clearly involves some infinitary expected values.

and the expected value of $A\neg p$ dependent only on the outcomes of CF7 and CF8. As should be clear then, it is still the case that $EV(A\neg p) > EV(Ap)$. So, unless $P(B_{t+n}p|A_tp) = 1$ we still get the surprising result that the corrected and well-formed version of the wager shows that from the perspective of pragmatic rationality we should not simulate the life of the believer.

But, are there any good reasons to suppose that $P(B_{t+n}p|A_tp) = 1$? There are four obvious positions one might take on the matter, and they involve treating the connection between Ap and Bp as (1) a strong modal connection, (2) a matter of natural law, (3) a logical implication, or (4) a brute unitary conditional probability. Let us begin by considering (1). First, is it reasonable to suppose that there is a strong modal tie between accepting and believing that would entail that $P(B_{t+n}p|A_tp) = 1$? Clearly, if $\Box(A_tp \supset B_{t+n}p)$, then $P(B_{t+n}p|A_tp) = 1 = 1$. But, $\Box(A_tp \supset B_{t+n}p)$ seems simply to be false. It is far too strong to even be remotely plausible. There is nothing at all impossible about the existence of cases where an agent has the following attitudes: A_tp & $B_{t+n}\neg p$. They are simply cases where simulating belief in p for practical reasons does not successfully result in later believing that p. There is nothing at all contradictory about such cases. Moreover, surely part of the gravity that Pascal attaches to the wager is that it is no sure thing that this kind of acceptance will lead to belief, certainly not as a matter of alethic necessity. He seems to be acutely aware that actual failure in this regard is a real possibility and that our efforts at simulation of belief thus require work and earnest hope that our efforts to bring about belief are successful. Otherwise we face eternal damnation. Second, is it reasonable to believe that there is a strong nomological tie between accepting and believing that would entail that $P(B_{t+n}p|A_tp) = 1$? To this end, suppose that the necessitarian view of laws of nature is correct and that $N(A_tp, B_{t+n}p)$ is true (i.e. that acceptance nomologically necessitates belief).[22] If this were true then, it would be the case that $P(B_{t+n}p|A_tp) = 1$ in the actual world and in those possible worlds characterized by the same laws. But, is it reasonable to suppose that an agent's having the attitudes A_tp & $B_{t+n}\neg p$ is nomologically impossible? Surely it is not, and it is simply not reasonable to suppose that cases involving A_tp & $B_{t+n}\neg p$ are precluded by the laws of nature in the actual world or in close possible worlds characterized by the same laws. This is simply because there are, in fact, actual cases where simulating belief in p does not successfully result in later believing that p. Consider for example, any number of cases

[22] For elaboration of the necessitarian view see David M. Armstrong, *What is a Law of Nature?* (Cambridge: Cambridge University Press, 1983), Fred Dretske, "Laws of Nature," *Philosophy of Science* 44 (1997): 248-268 and Michael Tooley, "The Nature of Laws," *Canadian Journal of Philosophy* 7 (1977): 667-698.

involving acting, or pretending, or indoctrinating where the agent accepts a set of propositions and their implications for some period of time but does not ultimately come to believe them. So, it is not reasonable to suppose that acceptance will lead to belief as a matter of nomological necessity and there is no reason to believe that $P(B_{t+n}p|A_tp) = 1$ is true on that basis. Suppose then that one were to adopt the yet weaker view that $Ap \supset Bp$. If this implication were true at the actual world, as a matter of mere regularity, then it would also be the case that $P(B_{t+n}p|A_tp) = 1$. But, again, this is totally implausible because there are clearly actual cases where simulating belief in a proposition does not lead to belief in that proposition at a later time. So, this suggestion fares no better than the two stronger alternatives we have considered. Finally, let us consider $P(B_{t+n}p|A_tp) = 1$ itself. Is there any good reason to suppose that the key proposition about the probabilistic relationship between Ap and Bp is itself true? Certainly the answer is no and this claim is not true for exactly the same sorts of reasons we have just examined in the context of stronger attempts to yield that result. It simply isn't true that acceptance always leads to subsequent belief. $P(B_{t+n}p|A_tp) \neq 1$. The real problem with the wager then, however, is that, given any interpretation of the connection between acceptance and belief, when the wager is properly rendered as a decision problem involving voluntary acceptance it favors $A\neg p$ as a matter of pragmatic rationality. This is because there is no plausible way to justify the claim that $P(B_{t+n}p|A_tp) = 1$ and this is necessary for recapturing the result of the original but ill-formed version of the wager. As a result, when the wager is properly constructed as a well-formed decision problem involving acceptance rather than belief $EV(A\neg p) > EV(Ap)$. So, if pragmatic considerations are all we have to go on, then we should not accept Christian practice. We should behave as if the proposition that God exists is false. This is what prudence actually advises if this construction of the wager is theoretically sound.

6. Infinite Utilities, Maximin and the Modernized Wager

However, there is still one deeply serious problem with wager arguments that must be contended with. Specifically, as mentioned in section 1, standard decision theory is notoriously incompatible with the idea that there can be outcomes with infinite valued utilities.[23] This renders the results of the original wager and the modernized re-construction moot. None of these decision problems can be framed in terms of the standard theory of utility and this looks to be essential to these

[23] See Duff, "Pascal's Wager and Infinite Utilities," Jordan, "Pascal's Wager Revisited, McClennen, "Pascal's Wager and Finite Decision Theory," Hájek, "Waging War on Pascal's Wager," and Bartha, "Taking Stock of Infinite Value."

sorts of arguments. Absent some way to incorporate infinite utilities into decision theory, we simply cannot meaningfully apply the principle of maximizing expected utility in these problems. A simple solution to this problem will be introduced here that appeals to the minimax principle, but it still allows for infinite utilities and corresponding preferences. It avoids this problem by completely avoiding appeals to probabilities in the argument.

In order to set the stage for the first possible solution to the problem of infinite utilities let us examine why the standard theory of utility involved in orthodox decision theory rules out outcomes with such utilities. Standard utility theory is based on the idea that if an agent's preferences obey a certain set of axioms, then they can be represented as a utility function that exhibits certain supposedly desirable algebraic features. These axioms are introduced on the basis of their supposed intuitive (i.e. a priori) plausibility. Let "x ≼ y" mean "x is weakly preferred to y", "x ≺ y" mean "x is strictly preferred to y" (i.e. x is weakly preferable to y but x is not indifferent relative to y) and "x ∽ y" mean "x is indifferent relative to y" (i.e. x is weakly preferred to y and y is weakly preferred to x). Let O_i, O_j and O_k represent distinct outcomes and **p**, **q**, **r**,… represent distinct probability values. Finally, let $u(O_i)$ be a function representing a real numbered valuation of O_i. Given these basic representations we can then represent a gamble with a probability p of winning O_1 and a probability q of winning O_2 as [**p**O_1, (1 – **p**) O_2]. In terms of these representations, the axioms are used to characterize what is intuitively taken to be rational preference orderings are as follows.[24] First we have the ordering axiom:

(U1) The preference relation ≽ is a total ordering that is reflexive and transitive.

Second, we have the better prizes axiom:

(U2) For a fixed probability, prefer the gamble with a greater prize.

Third, we have the better chances axiom:

(U3) For a fixed prize prefer the gamble with a greater probability.

Fourth, we have the reduction of compound gambles axiom:

(U4) Compound gambles are to be evaluated in terms of the probability calculus.

Finally, we have the Archimedean or Continuity axiom:

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[24] This is the standard presentation of this representation theorem and it closely follows Bartha, "Taking Stock of Infinite Value." See Michael D. Resnik, *Choices* (Minneapolis: University of Minneapolis Press, 1987) and Gerald Gaus, *On Philosophy, Politics and Economics* (Belmont: Wadsworth, 2008) as well.

(U5) For any outcome that is ranked between two others there is a gamble between the more preferred and less preferred outcomes such that the agent is indifferent between it and the outcome ranked in between the more preferred and less preferred outcomes.

Formally, in terms of $\preccurlyeq$ these axioms can be presented as follows:

(U1.0) For any O_i and O_j either $O_i \preccurlyeq O_j$ or $O_j \preccurlyeq O_i$,

(U1.1) For any O_i, $O_i \preccurlyeq O_i$.

(U1.2) For any O_i, O_j and O_k, if $O_i \preccurlyeq O_j$ and $O_j \preccurlyeq O_k$, then $O_i \preccurlyeq O_k$.

(U2) $O_i \preccurlyeq O_j$, iff, for any $0 \leq \mathbf{p} \leq 1$ and any O_k, $[\mathbf{p}O_k, (1 - \mathbf{p})O_i] \preccurlyeq [\mathbf{p}O_k, (1 - \mathbf{p})O_j]$ and $[\mathbf{p}O_i, (1 - \mathbf{p})O_k] \preccurlyeq [\mathbf{p}O_j, (1 - \mathbf{p})O_k]$.

(U3) If $O_i \preccurlyeq O_j$, then for any $0 \leq \mathbf{p}, \mathbf{q} \leq 1$, $\mathbf{p} \geq \mathbf{q}$ iff $[\mathbf{p}O_i, (1 - \mathbf{p})O_j] \preccurlyeq [\mathbf{q}O_i, (1 - \mathbf{q})O_j]$

(U4) For any O_i and O_j and $\mathbf{p}, \mathbf{q}, \mathbf{r}$ such that $0 \leq \mathbf{p}, \mathbf{q}, \mathbf{r} \leq 1$, $[\mathbf{p}[\mathbf{q}O_i, (1 - \mathbf{q})O_j], (1 - \mathbf{p})[rO_i, (1 - \mathbf{r})O_j]] \backsim [\mathbf{r}O_i, (1 - \mathbf{t})O_j]$ for $\mathbf{t} = \mathbf{pq} + (1 - \mathbf{p})\mathbf{r}$.

(U5) If $O_i \preccurlyeq O_j$ and $O_j \preccurlyeq O_k$, then there is a $\mathbf{p}$ such that $0 \leq \mathbf{p} \leq 1$ and $O_j \backsim [\mathbf{p}O_i, (1 - \mathbf{p})O_k]$.

If an agent's preferences satisfy these axioms then those preferences can be represented by a real valued utility function $u(O_i)$ obeying the following two important conditions:

(C1) $O_i \preccurlyeq O_j$ iff $u(O_i) \leq u(O_j)$.

(C2) $u([\mathbf{p}O_i, (1 - \mathbf{p})O_j]) = \mathbf{p}u(Oi) + (1 - \mathbf{p})u(O_j)$.

The Expected Utility Theorem, the core idea behind utility theory, is then simply this claim that if one's preferences satisfy U1-U5, then those preferences can be represented as a real valued utility function satisfying C1 and C2.[25] In other words, formal utilities are a real-valued measure of preference and the value V_i of an outcome O_i is just $u(O_i)$. What is key here is that U1-U5 implicitly rule out infinite utilities and thus rule out a priori that agents can have corresponding

[25] This is just the standard way of introducing utility theory via a representation theorem. This approach takes it as given a priori that U1-U5 are true. Recently, this approach to legitimizing decision theory has been challenged in Kenny Easwaran, "Decision Theory without Representation Theorems," *Philosophers' Imprint* 14 (2014): 1-30 and by Christopher J. G. Meacham, C. and Jonathan Weisberg, "Representation Theorems and the Foundations of Decision Theory," *Australasian Journal of Philosophy* 89 (2011): 641-663.

preferences. Specifically, U2, U3 and U5 are incompatible with there being infinitely valued outcomes.[26]

As a result, a simple way to address the problem of infinite utilities in the modernized wager involves treating that wager as a decision under total ignorance/uncertainty involving infinite utilities. In such decision situations it is acknowledged that no probabilities can be meaningfully assigned to the outcomes and so the advice about what to do in such situations is wholly a function of the utilities involved. Given this approach we simply acknowledge that there are no probabilities that can be meaningfully assigned in the wager and so there are no expected utilities defined as products of probabilities and utilities involved in the wager. As we have seen this comports well, however, with Pascal's own understanding of the problem about which he makes the following claim: "Reason can decide nothing here."[27] As a result, the standard rule of maximizing expected utility does not apply. Rather, in cases where the potential loses are great and where we have no information about probabilities other than that the probabilities of all the outcomes are non-zero many decision theorists suggest that we use the maximin rule to determine what to do.[28] This has some additional appeal to it as well given Pascal's comments about our lack of epistemic reasons that pertain to the question of God's existence that were examined earlier and which can be usefully extrapolated to the modernized wager. If this is the case, then as long as the outcomes associated with [Ap & Bp & $(\exists x)(x = G)$] and [Ap & B$\neg p$ & $(\exists x)(x = G)$] and with [A$\neg p$ & Bp & $(\exists x)(x = G)$] and [A$\neg p$ & B$\neg p$ & $(\exists x)(x = G)$] are finite and symmetric the necessary cancellation occurs and the verdict that we should reject the life of the believer holds. This is because according to the maximin rule we are to maximize the minimum. So we look at the decision table and look at the worst outcomes for the two acts A$\neg p$ and Ap. It turns out that this is the case for A$\neg p$ where B$\neg p$ & $(\exists x)(x = G)$ and this is the case for Ap also where B$\neg p$ & $(\exists x)(x = G)$. But these maximal minima are equal (i.e. $-\infty$). So, according to the lexical maximin rule we are to look at the next lowest outcome(s) of A$\neg p$ and Ap. In the case of Ap we have the next lowest minima where Bp & $\neg(\exists x)(x = G)$ and where B$\neg p$ & $\neg(\exists x)(x = G)$. This value is $-\alpha$ in both cases. In the case of A$\neg p$ the next lowest minima are where we have Bp & $\neg(\exists x)(x = G)$ and B$\neg p$. The value

[26] See McClennen, "Pascal's Wager and Finite Decision Theory" and Bartha, "Taking Stock of Infinite Value" for details.

[27] Pascal, *Pensées*, 257.

[28] See Abraham Wald "Contributions to the Theory of Statistical Estimation and Testing Hypotheses," *The Annals of Mathematics* 10 (1939): 299-326, "Statistical Decision Functions Which Minimize the Maximum Risk," *The Annals of Mathematics,* 46 (1945): 265-280 and Resnik, *Choices*.

in both cases is α. So, the lexical maximin rule tells us to do A$\neg p$ in the case where we cannot assign probabilities to the states of the world involved and the surprising verdict of the modernized reconstructed wager still holds.[29]

[29] There is also a related, simple and obvious way to yield the same result in terms of a simple dominance argument, if one objects to the maximin argument offered here. An act A dominates an act B if for every outcome the utility of A is equal to or greater than the utility of B and for at least one outcome the utility of A is greater than that of B. The dominance rule, then says something like, where probabilities cannot be meaningfully assigned, do the dominant act. The corrected version of the wager presented here then suggests two arguments in favor of the non-acceptance conclusion.