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Dialectical Dispositions and Logic

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Abstract: According to *dialectical disposition expressivism* about conjunction, disjunction, and negation, the function of these connectives is to convey dispositions speakers have with respect to challenging and meeting challenges to assertions. This paper investigates the view's implications for logic. An interpretation in terms of dialectical dispositions is proposed for the proof rules of a bilateral sequent system. Rules that are sound with respect to this interpretation can be seen as generating an *intrinsic logic* of dialectical disposition expressivism. It is argued that such a logic will be very weak—weaker than the intersection of minimal logic and FDE.

Keywords: connectives, bilateralism, expressivism, pragmatism, logical consequence

1 Introduction

It is a familiar idea that the behavior of logical operators can be illuminated in terms of their use in circumstances of dialectical engagement—circumstances in which speakers challenge and meet challenges to each other's assertions. The best-known elaboration of this idea is the game-theoretic tradition of "dialogical logic" (Lorenzen & Lorenz, 1978); it has also been pursued within other approaches, such as inferentialism (Brandom, 2008; Lance, 2001) and expressivist pragmatism (Price, 1990, 1994, 2009). In each case, specifying the dialectical roles of operators has served as a way of providing a semantic grounding for a logical consequence relation. Thus Lorenzen aims to underwrite intuitionistic logic, Lance and Brandom arrive (respectively) at a weak relevance logic and classical logic, and Price claims that his account of negation's function supports classical logic. Questions have accordingly been raised about whether our discursive practices satisfy the assumptions that would be needed for deriving the claimed conclusions about logical consequence (Dutilh Novaes, 2021; Hodges, 2001; Marion, 2009).

[1] I would like to thank audiences at Logica 2023 and the University of Connecticut for helpful discussion. I am particularly grateful for comments by Julian Schlöder and Ryan Simonelli, and for suggestions by a reviewer.

The present paper concerns a less ambitious version of the dialectical approach, *dialectical disposition expressivism* (Shapiro, 2023, §5). This is a proposal about the functions of the types of logical complexity regimented in logician's English using sentential connectives 'and', 'or', and 'not'. The proposal, which will be presented in Section 2, is that these connectives let speakers convey certain dispositions with respect to dialectical engagement. Elsewhere (Shapiro, 2023, §4), I have argued that expressivist theorists have no reason to demand that their accounts of the functions of connectives should settle what is a logical consequence of what. As will be explained in Section 6, this is an upshot of *deflationism about logical consequence* (Shapiro, 2022). On that view, we should no more expect an expressivist account of logical vocabulary to settle questions about what follows logically from what than we would expect an expressivist account of moral vocabulary to settle questions about what is morally permissible.

Still, there remains a legitimate question: What logic, if any, *can* in fact be underwritten by this account of the connectives? We may understand the question as follows. Is there any formal consequence relation such that dialectical disposition expressivism endows that relation with something like the pragmatic significance logical consequence is thought to have?[2] If so, such a relation may be deemed an "intrinsic logic" of dialectical disposition expressivism.[3] Importantly, an advocate of dialectical disposition expressism can recognize such an intrinsic logic without identifying it as the relation of logical consequence.[4]

My aim is to assess whether dialectical disposition expressivism has an intrinsic logic. To this end, I exploit a parallel between clauses specifying the connectives' expressive functions and "bilateral" proof rules (Rumfitt, 2000; Smiley, 1996). Section 3 shows how such rules, when cast in sequent format, can be naturally interpreted in terms of dialectical dispositions. I then investigate which bilateral rules can be justified on this interpretation. Section 4 considers rules that involve connectives, while Section 5 considers "coordination rules," rules that (on the proposed interpretation) encapsulate principles concerning challenging and meeting challenges. My conclusion

[2] This question is raised, but not answered, in (Shapiro, 2023, p. 250 n. 29). I thank Julian Schlöder for pressing me to pursue it further.

[3] The phrase "intrinsic logic" is used in a loosely related sense by Brandom (2008, p. 139).

[4] Lance (2001, pp. 448–49) too suggests that some claims about what is a logical consequence of what may not "follow from considerations of the structure of ... linguistic practice per se." However, in contrast to deflationism about consequence, he says that any such claim would be a "substantive epistemic claim."

will be that the proof system consisting of justifiable bilateral rules yields a consequence relation weaker than the intersection of minimal logic and the paraconsistent and paracomplete logic FDE.

2 Expressive functions

Dialectical disposition expressivism holds that logical connectives serve to express dispositions with respect to moves in a "game of giving and asking for reasons" (Brandom, 1994).[5] These moves include both *asserting* and *rejecting* propositions. Asserting a proposition will be understood as assuming the responsibility to meet challenges, and thereby authorizing others to defer to one's assertion in meeting challenges to their own assertions (Brandom, 1994, pp. 171–72). Rejecting a proposition will be understood as expressing one's disposition to challenge assertions of the proposition.

I will presuppose a conception of *challenging* where a challenge stands in a strong tension with the challenged assertion. To start with, challenging an assertion places the asserter under an obligation to defend the assertion, by adducing warrant or neutralizing the challenge, on pain of having to withdraw their assertion. But this needn't suffice to constitute a challenge. For example, merely telling someone that there is *insufficient evidence* for their assertion won't count as a challenge. Here is how I propose to understand the difference. If one asserts a proposition for which one also claims there is insufficient evidence, this needn't undermine the authority of one's pronouncement about the lack of evidence. By contrast, when one asserts a proposition that one also challenges, this undermines the authority of the challenge as well as of the assertion.[6]

Simplifying for present purposes, I will speak of asserting/rejecting sentences rather than propositions, and consider only sentences of a language whose logical complexity is exhausted by the three sentential connectives $\wedge$, $\vee$, and $\neg$. Concerning conjunction, the hypothesis is that one who asserts $A \wedge B$ expresses a disposition she has with regard to that very assertion. Specifically, she expresses her being disposed thus:

[5] This section abridges, with some additions, the proposal in (Shapiro, 2023, §5).

[6] There is an affinity here with Brandom's characterization of *incompatible* propositions, on which commitment to both propositions precludes entitlement to *either* proposition (Brandom, 1994, p. 169). Unlike Brandom, however, I won't invoke a relation of incompatibility between propositions, or the notion of commitment to a proposition.

($\wedge$-$\mathrm{c_i}$) She is prepared to recognize an interlocutor's rejection of A (likewise of B) as a challenge to her assertion.

($\wedge$-$\mathrm{m_s}$) When an interlocutor has challenged her assertion, she is prepared to adduce, as a way to meet the challenge, any pair of available assertions of A and B.

Let me explain the terminology. In the labels for the clauses, 'c' and 'm' stand for *challenging* and a *meeting* a challenge, while subscripts 'i' and 's' specify whether this is being done by *interlocutor* or *speaker*. An (actual or potential) assertion counts as "available" to a speaker if it is either an assertion she is prepared to make, or an assertion by another speaker she is prepared to defer to, in meeting a challenge.

Importantly, the above clauses don't specify the *only* kinds of challenge an asserter of a conjunction will recognize, or the *only* way she will be prepared to meet challenges to her assertion. For example, one who asserts $A \wedge B$ will be prepared to recognize an interlocutor's rejection of $A \wedge B$ as a challenge, yet the function of $\wedge$ isn't explained in terms of its conveying that disposition. One might wonder why both ($\wedge$-$\mathrm{c_i}$) and ($\wedge$-$\mathrm{m_s}$) are needed to explain the function of $\wedge$. The reason is that it's possible to have disposition ($\wedge$-$\mathrm{c_i}$) without disposition ($\wedge$-$\mathrm{m_s}$), and also vice versa. The former possibility is often exemplified by asserters of $(A \wedge B) \wedge C$. The latter possibility is often exemplifed by asserters of $(A \wedge B) \vee C$.

Making an assertion that expresses the above-specified disposition can facilitate dialectical engagement. Consider a circumstance in which one would be prepared to meet a challenge to one's assertion of C by asserting both A and B. With the resource of conjunction, dispute about one's defense of C can take the form of the assertion and rejection of the sentence $A \wedge B$. An interlocutor may be prepared to reject this conjunction without being prepared to reject either conjunct.

A parallel benefit is provided by a disjunction connective. In asserting $A \vee B$, a speaker expresses her being disposed thus:

($\vee$-$\mathrm{c_i}$) She is is prepared to recognize an interlocutor's pair of rejections of A and of B as a challenge to her assertion.

($\vee$-$\mathrm{m_s}$) When an interlocutor has challenged her assertion, she is prepared to adduce, as a way to meet the challenge, any available assertion of A (likewise of B).

Expressing this disposition, too, is useful in dialectical engagement. Consider a circumstance in which one could meet a challenge to one's assertion of C by an assertion of A (if available) as well as by an assertion of B (if available). Without asserting either A or B (perhaps one would reject A, while the interlocutor would reject B), one may meet the challenge by asserting their disjunction. With the resource of disjunction, dispute about one's defense of C can take the form of assertion and rejection of $A \vee B$.

Finally, following Price (1990), we can see negation as providing a means of rejecting a sentence by asserting another sentence. In asserting $\neg A$, a speaker expresses her being disposed thus:

($\neg$-$\mathrm{c_s}$) She is prepared to challenge any assertion of A.

($\neg$-$\mathrm{m_s}$) When her assertion is challenged, she is prepared to adduce, as a way to meet the challenge, any available assertion she would recognize as a way to challenge assertions of A.

Whereas the dispositions expressed by asserting a conjunction or disjunction all concern *that very assertion*, only one of the two dispositions expressed by asserting a negation does so, namely ($\neg$-$\mathrm{m_s}$). A connective may also be such that asserting a sentence with it as major connective expresses dispositions that concern only the sentence's proper constituents. In (Shapiro, 2018), I propose that the dialectical dispositions expressed by a type of conditional concern only assertions and rejections of the *antecedent* and *consequent*.

The above expressive clauses help us understand the *raison d'être* of propositional logical complexity. Being able to assert logically complex sentences serves a purpose that would otherwise require including, as moves in the game of giving and asking for reasons, a hierarchy of distinct speech acts involving pluralities of sentences, starting with acts of asserting/rejecting pairs of sentences taken conjunctively or disjunctively. According to Humberstone (2000, p. 367–68), advocates of "bilateral" accounts of connectives in terms of assertion and rejection must explain why they don't invoke additional act of asserting conjunctively and disjunctively, acts standing to *conjunction* and *disjunction* the way rejection stands to *negation*. The present proposal yields a reply. The point of conjunction/disjunction is to let us do without asserting and rejecting conjunctively/disjunctively, whereas negation doesn't let us do without rejecting, which was invoked in explaining the functions of all three connectives.[7] Negation does, however, let us do without a distinct speech act of (say) disjunctively asserting one sentence together with another sentence taken negatively.

[7] Ripley (2020, p. 59 n. 7) likewise replies to Humberstone by noting the explanatory priority that bilateralism accords to rejection.

3 Interpreting sequent rules

In the cases of conjunction and disjunction, the above pairs of expressive clauses bear a suggestive resemblance to rules of a bilateral natural deduction system, specifically the negative and affirmative introduction rules. Here are these rules for conjunction:

$$\frac{-A}{-A \wedge B} \quad \frac{-B}{-A \wedge B}\,(-\wedge\mathrm{I}) \qquad \frac{+A \qquad +B}{+A \wedge B}\,(+\wedge\mathrm{I})$$

Such rules are usually interpreted in terms of conditions on warranted or coherent assertion and rejection (Ripley, 2017). But their resemblance to our expressive clauses ($\wedge$-$\mathrm{c_i}$) and ($\wedge$-$\mathrm{m_s}$), respectively, motivates pursuing an alternative interpretation in terms of dialectical dispositions. For the purpose of using such an interpretation to build a consequence relation, it will be useful to reformulate the rules in sequent style. Here Γ is a set of signed sentences:

$$\frac{\Gamma \Rightarrow -A\;[-B]}{\Gamma \Rightarrow -A \wedge B}\,(-\wedge\mathrm{R}) \qquad \frac{\Gamma \Rightarrow +A \qquad \Gamma \Rightarrow +B}{\Gamma \Rightarrow +A \wedge B}\,(+\wedge\mathrm{R})$$

To begin with, we define two relations between Γ and a signed sentence ϕ. Both relations will be relativized to an agent a, and for convenience both will be expressed in our metalanguage using the same ambiguous notation '$\Gamma \vdash_a \phi$'. The turnstile will receive different interpretations depending on whether ϕ carries positive or negative sign. In this respect, the current approach resembles the use of "dual" turnstiles for proof and refutation by Wansing (2017) and Ayhan (2021). The first relation, between Γ and negatively signed ϕ, corresponds to an agent's disposition with regard to *recognizing challenges* to their assertion.

Definition 1 $\Gamma \vdash_a -C$ iff a is disposed to recognize the following combination of speech acts by an interlocutor as challenging any assertion by a of C: the assertion of each positively signed member of Γ together with the rejection of each negatively signed member of Γ .

To see how this relation may apply, suppose that a is an "ideal discursive agent," one who (i) always *exhibits* all dialectical dispositions she expresses and (ii) knows which dialectical dispositions any assertion by her or by her interlocutors would express. I will now argue for the following conditional:

(1) If $\Gamma \vdash_a -A$ or $\Gamma \vdash_a -B$, then $\Gamma \vdash_a -A \wedge B$.

Suppose that a asserts $A \wedge B$. According to clause ($\wedge$-c$_\mathrm{i}$), she thereby expresses a disposition to recognize an interlocutor's rejection of A as a challenge. As an ideal discursive agent, she will know that she in fact possesses this disposition. Now suppose, in addition, that there is a certain combination of assertions and rejections such that if a were to assert A, she would be prepared to recognize that combination as jointly challenging her assertion of A. Knowing this about herself, a will presumably be prepared to recognize an interlocutor who makes that combination of assertions and rejections as challenging her assertion of $A \wedge B$.

A second definable relation, this time between Γ and a positively signed ϕ, corresponds to a disposition with regard to adducing assertions to *meet challenges*. Here a bit of additional complexity is needed to accommodate an asymmetry between assertion and rejection. Whereas both assertions and rejections can count as challenging an assertion, such challenges can be met by further assertions, but not by rejections.

Definition 2 $\Gamma \vdash_a +C$ iff a is disposed to recognize the following combination of available assertions as meeting any challenge to any assertion by a of C: assertions of each positively signed member of Γ together with assertions, for each negatively signed member of Γ, of some sentence(s) that a regards as challenging that sentence's assertion.

An argument parallel to the one previously given uses clause ($\wedge$-m$_\mathrm{s}$) to show, for any ideal agent a

(2) If $\Gamma \vdash_a +A$ and $\Gamma \vdash_a +B$, then $\Gamma \vdash_a +A \wedge B$.

In terms of the dual relations just defined, we can specify a pragmatic interpretation of an arbitrary sequent rule with n premises.

$$\frac{\Gamma_1 \Rightarrow \phi_1 \qquad \ldots \qquad \Gamma_n \Rightarrow \phi_n}{\Delta \Rightarrow \psi}\ (\mathrm{R})$$

Definition 3 Sequent rule R is *DDE-sound* iff for all its instances and any ideal discursive agent a, if $\Gamma_i \vdash_a \phi_i$ for all $i \leq n$, then $\Delta \vdash_a \psi$.

The reasoning sketched above for (1) and (2) supports the claim that the rules $-\wedge$R and $+\wedge$R are DDE-sound. Here I should call attention to the fact that the reasoning assumed, in effect, that the two relations written '$\Gamma \vdash_a \phi$' exhibit a transitivity in virtue of which the following rule is DDE-sound.

$$\frac{\Gamma \Rightarrow \phi \qquad \Gamma, \phi \Rightarrow \psi}{\Gamma \Rightarrow \psi}\ (\mathrm{Cut})$$

The claim that Cut is DDE-sound is hardly indisputable. However, in the interest of investigating how strong an intrinsic logic can be defended using plausible assumptions, I propose to recognize Cut as DDE-sound unless we find that doing so would stand in tension with claiming DDE-soundness for other rules for which that claim is no less plausible. (One such consideration will emerge in Section 4.3.)

We can now define the notion that will be our central interest:

Definition 4 Let $\mathcal{G}$ be a set of unsigned sentences, and Γ the set of their positively signed counterparts. Then the sentences in $\mathcal{G}$ have A as a *DDE-intrinsic consequence* iff the "all-positive" sequent $\Gamma \Rightarrow +A$ is derivable in a system of DDE-sound rules.

Our question, now, concerns what additional sequent rules are DDE-sound, and thus how strong an intrinsic logic dialectical disposition expressivism will yield via our pragmatic interpretation of sequent rules.

4 Connective rules and initial sequents

It makes sense to consider first the full set of standard bilateral connective rules for $\wedge$, $\vee$, and $\neg$. These rules in natural-deduction format are known to yield the Belnap-Dunn logic FDE (Tamminga & Tanaka, 1999).[8] I will instead use corresponding Gentzen-style rules. The following system G_0 consists of one structural rule giving us initial sequents, in addition to four rules for each connective: positively and negatively signed left and right introduction rules.

$$\frac{}{\Gamma, \phi \Rightarrow \phi}\ \text{(Id)}$$

$$\frac{\Gamma, +A\ [+B] \Rightarrow \phi}{\Gamma, +A \wedge B \Rightarrow \phi}\ (+\wedge\text{L}) \qquad \frac{\Gamma \Rightarrow +A \qquad \Gamma \Rightarrow +B}{\Gamma \Rightarrow +A \wedge B}\ (+\wedge\text{R})$$

$$\frac{\Gamma, -A \Rightarrow \phi \qquad \Gamma, -B \Rightarrow \phi}{\Gamma, -A \wedge B \Rightarrow \phi}\ (-\wedge\text{L}) \qquad \frac{\Gamma \Rightarrow -A\ [-B]}{\Gamma \Rightarrow -A \wedge B}\ (-\wedge\text{R})$$

[8] They are the connective rules in (Rumfitt, 2000, pp. 800–802). Rumfitt is ultimately interested in a system for classical logic. But, as Gibbard (2002) observes, Rumfitt's connective rules, in the absence additional "coordination principles" to be considered later, yield a "non-classical *constructive logic with strong negation*," namely Nelson's N4, of which FDE is the nonimplicative fragment (Omori & Wansing, 2017).

$$\frac{\Gamma, +A \Rightarrow \phi \qquad \Gamma, +B \Rightarrow \phi}{\Gamma, +A \vee B \Rightarrow \phi}\ (+\vee\mathrm{L}) \qquad \frac{\Gamma \Rightarrow +A\ [+B]}{\Gamma \Rightarrow +A \vee B}\ (+\vee\mathrm{R})$$

$$\frac{\Gamma, -A\ [-B] \Rightarrow \phi}{\Gamma, -A \vee B \Rightarrow \phi}\ (-\vee\mathrm{L}) \qquad \frac{\Gamma \Rightarrow -A \qquad \Gamma \Rightarrow -B}{\Gamma \Rightarrow -A \vee B}\ (-\vee\mathrm{R})$$

$$\frac{\Gamma, -A \Rightarrow \phi}{\Gamma, +\neg A \Rightarrow \phi}\ (+\neg\mathrm{L}) \qquad \frac{\Gamma \Rightarrow -A}{\Gamma \Rightarrow +\neg A}\ (+\neg\mathrm{R})$$

$$\frac{\Gamma, +A \Rightarrow \phi}{\Gamma, -\neg A \Rightarrow \phi}\ (-\neg\mathrm{L}) \qquad \frac{\Gamma \Rightarrow +A}{\Gamma \Rightarrow -\neg A}\ (-\neg\mathrm{R})$$

Proposition 1 *G_0 derives all and only the all-positive sequents corresponding to the consequences of FDE, and the signed Cut rule is admissible.*

Proof sketch. Consider the translation from signed to unsigned sequents that removes positive signs and replaces negative signs by $\neg$ (Humberstone, 2000, p. 365). Let G_0' be the unsigned system consisting of the translations of the rules of G_0, except for the (trivially redundant) translations of $+\neg$L and $+\neg$R. G_0' is a system for FDE, a close variant of the cut-free system $\mathbf{LE}_{\text{fde2}}$ in (Anderson & Belnap, 1975, p. 179).[9]

We show that an all-positive sequent is derivable in G_0 iff its translation is derivable in G_0'. Left to right: any G_0-derivation translates into a G_0'-derivation. Right to left: take any G_0'-derivation of the unsigned sequent that translates the given all-positive sequent. For each step, consider the inference resulting from affixing positive signs to all sentences. It suffices to show that this inference is admissible in G_0. The only non-trivial cases are instances of *negated connective rules* of G_0'. Here we use the fact (shown by induction on derivation height) that $+\neg$L and $+\neg$R of G_0 are invertible. For example, consider the rule of G_0' that derives $\Gamma \Rightarrow \neg(A \wedge B)$ from $\Gamma \Rightarrow \neg A$. Let Γ^+ be the result of affixing positive signs to all members of Γ. Since $+\neg$R is invertible, the inference from $\Gamma^+ \Rightarrow +\neg A$ to $\Gamma^+ \Rightarrow -A$ is admissible in G_0, and the latter sequent derives $\Gamma^+ \Rightarrow +\neg(A \wedge B)$. The admissibility of Cut in G_0 likewise follows from the admissibility of unsigned Cut in G_0'. $\square$

[9] $\mathbf{LE}_{\text{fde2}}$ is identical to G_B in (Pynko, 1995). The differences between it and G_0' are just that $\mathbf{LE}_{\text{fde2}}$ has sequences as antecedents and succedents, and that the unsigned rules corresponding to $+\wedge$L, $-\wedge$R, $+\vee$R and $-\vee$L take "multiplicative" form. It is straightforward to show that if Σ is a sequence containing all and only members of the set Γ, then $\Gamma \Rightarrow \phi$ is derivable in G_0' iff $\Sigma \Rightarrow \phi$ is derivable in $\mathbf{LE}_{\text{fde2}}$.

4.1 A starting point

Which rules of G_0 are DDE-sound? Six rules are directly underwritten by our expressive clauses. We have already seen, in discussing claims (1) and (2), how the clauses for conjunction support $+\wedge$R and $-\wedge$R. Parallel reasoning uses expressive clauses ($\vee$-m$_\mathrm{s}$) and ($\vee$-c$_\mathrm{i}$) to justify the corresponding rules for disjunction, $+\vee$R and $-\vee$R. Additionally, in the case of negation, ($\neg$-c$_\mathrm{s}$) will justify $+\neg$L, while ($\neg$-m$_\mathrm{s}$) will justify $+\neg$R.

Admittedly, the rationale for including side formulas Γ in the initial sequents given by (Id) may not seem compelling. One counterexample might seem to be $-A, +A \vdash_a +A$. Will an asserter of A ever recognize, as meeting a challenge to her assertion, the puzzling combination of an assertion of A together with an assertion of something she takes to challenge A? I'll assume that if she won't do so, this will be because that pair of assertions won't count as available to her. Indeed, it isn't clear that they will ever be available. Even if she is prepared to assert both A and $\neg A$, perhaps in response to paradox, she may take both assertions' authority to be compromised.[10]

A more problematic case might seem to be $+A, -A \vdash_a -A$. Will an asserter of A be disposed to recognize an interlocutor who rejects A, while also asserting it, as having issued a challenge? I'll assume that if the asserter won't be so disposed, this is because she views the interlocutor's rejection of A as an act whose authority is undercut by his simultaneous assertion of A. However, it will be convenient not to modify Definition 1 to require that the interlocutor's challenge be recognized as one that carries authority, thereby avoiding $+A, -A \vdash_a -A$ while making the turnstile nonmonotonic. My goal is to argue that dialectical disposition expressivism generates at most a very weak consequence relation. As was the case with transitivity, this goal is served by not questioning the monotonicity of the relations defined in Definitions 1 and 2, unless we find that preserving monotonicity prevents us from recognizing additional rules as DDE-sound.[11]

[10] Priest (2006a) holds that while it's impossible to assert and reject the same sentence, we may sometimes be rationally required to. On the present view, doing so is possible, but only at the price of undercutting the authority of both acts—something we may find rationally unavoidable. However, the view is also compatible with holding (with Field, 2008) that we should reject a Liar sentence but *not* do so by asserting its negation.

[11] Still, the potential for nonmonotonicity shouldn't be exaggerated. For example, the mere fact that an agent a is disposed, *in some context*, to meet a challenge to her assertion of 'The match will light' by asserting 'The match was struck' doesn't make it the case that 'The match was struck' $\vdash_a$ 'The match will light'. Thus the failure of 'The match was struck', 'The match was wet' $\vdash_a$ 'The match will light' is no violation of monotonicity.

4.2 Three other defensible rules

What can we say about the remaining six rules of G_0? A strong case can be made for the DDE-soundness of three of them: $+\wedge$L, $+\vee$L, and $-\neg$R. For the first two rules, a full discussion would require separate arguments for the cases where ϕ is positively and negatively signed. Here I will instead choose one case for each rule, as the relevant considerations extend to the case involving the other sign.

Starting with $+\wedge$L, take the case where ϕ is negatively signed. Our task is to justify the claim that if $\Gamma, A \vdash_a -C$, then $\Gamma, A \wedge B \vdash_a -C$. Suppose that an ideal discursive agent a would recognize an interlocutor's assertion of A as constituting part of a challenge to her assertion of C. Now, since she is an ideal discursive agent, a recognizes that someone who asserts $A \wedge B$ thereby overtly undertakes responsibility to meet any challenges to an assertion of A. Hence she will presumably take an interlocutor's status as having challenged her assertion of C to be preserved if, in place of asserting A, the interlocutor instead asserts $A \wedge B$.[12] Similar reasoning applies in the case of instances of $+\wedge$L where ϕ is positively signed.

For $+\vee$L, choose this time the case where ϕ is positively signed. Our task is to justify the claim that if $\Gamma, +A \vdash_a +C$ and $\Gamma, +B \vdash_a +C$, then $\Gamma, +A \vee B \vdash_a +C$. Suppose that an ideal discursive agent a takes it that a challenge to her assertion of C would be met (in the context of some other speech acts) by an available assertion of A, and that the challenge would likewise be met if that assertion were replaced by an available assertion of B. Now a will recognize that someone who asserts $A \vee B$ thereby overtly undertakes responsibility to meet any concurrent challenges to *both* A and B. But she recognizes that assertion of *either* of these sentences would suffice to meet a challenge to her assertion of C. Hence, she will presumably count an available assertion of $A \vee B$ as likewise meeting the challenge to C.[13] Again, similar reasoning applies in the case of instances of $+\vee$L where ϕ is negatively signed.

Finally, consider $-\neg R$ and the claim that if $\Gamma \vdash_a +A$, then $\Gamma \vdash_a -\neg A$. Since part of what an assertion of $\neg A$ expresses is the speaker's disposition

[12] Here I assume the plausibility of the following principle. When someone has, in asserting, overtly committed themselves to meeting challenges to A, their assertion would count as challenging any assertions that would be challenged by asserting A.

[13] Here I assume the plausibility of the following principle. When someone has, in asserting, overtly committed themselves to meeting concurrent challenges to A and B, then their assertion (if available) meets challenges that could be met by available assertions of A as well as by available assertions of B.

to challenge assertions of A, it's reasonable to expect that one who asserts $\neg A$ will in turn recognize an interlocutor's assertion of A as a challenge to her own assertion. In reaching this conclusion, I'm not assuming that speakers regard the relation of challenging as symmetric—indeed, I'll offer a counterexample to that generalization in Section 5. Rather, the pair A and $\neg A$ is a special case. That's because I'm not merely relying on the claim that an ideal discursive agent who asserts A will in fact recognize assertions of $\neg A$ as challenges. Rather, I'm using the stronger claim that ideal discursive agents will recognize the asserting of $\neg A$ as a *way to express* that one is prepared to challenge all assertions of A.

4.3 Three problematic rules

That leaves three rules of G_0 to consider, namely the negative left introduction rules $-\wedge$L, $-\vee$L, and $-\neg$L. Each of these rules derives a conclusion about how ideal speakers are disposed to regard rejections of logical compounds. For example, justifying the DDE-soundness of $-\vee$L requires justifying the claim that if $\Gamma, -A \vdash_a \phi$, then $\Gamma, -A \vee B \vdash_a \phi$. But our account of the expressive role of disjunction doesn't appear to have any consequences for how speakers will regard their own, or an interlocutor's, *rejection* of a disjunction. The prospects of a direct pragmatic justification of these rules look dim.

There is a general point here: the expressive function attributed to a connective by dialectical disposition expressivism is served in *assertions* of sentences with that major connective, not in rejections. In that sense, the position isn't fully "bilateral" — in understanding the role of the disjunction connective, asserting disjunctions has explanatory priority over rejecting them, even though what is expressed by asserting disjunctions is in turn understood in terms of the *assertion and rejection* of their disjuncts.

If the three negative left introduction rules can't be justified as DDE-sound, how weak will this leave the consequence relation generated by the remaining ones? Let G_1 be given by all rules of G_0 except for $-\wedge$L, $-\vee$L, and $-\neg$L.

Proposition 2 *The consequence relation corresponding to the derivability of all-positive sequents in G_1 is weaker than the intersection of FDE and minimal logic.*

Proof sketch. Each of the rules in G_1, in addition to $-\vee$L, is derivable in the system for minimal logic obtained by omitting the initial sequent

$+A, -A \Rightarrow \phi$ from Humberstone's signed system for intuitionistic logic (Humberstone, 2000, p. 365). On the other hand, that system derives all-positive sequents that are not derivable in G_1. For example, an easy induction shows that a sequent with $+\neg A$ in the antecedent is derivable in G_1 only when A is a subformula of the succedent. Hence G_1 fails to derive the triple-negation elimination sequent $+\neg\neg\neg A \Rightarrow +\neg A$. Furthermore, G_1 fails to derive the De Morgan sequent $+\neg(A \vee B) \Rightarrow +\neg A$. □

4.4 An additional connective rule?

So far, we have only been looking at connective rules based on standard bilateral proof systems. Might dialectical disposition expressivism motivate additional connective rules on the present interpretation? The obvious candidate would be a form of disjunctive syllogism:[14]

$$\frac{\Gamma, +B \Rightarrow \phi}{\Gamma, -A, +A \vee B \Rightarrow \phi}\ (+\vee\mathrm{L}')$$

We might seek to justify this rule by hypothesizing that in asserting $A \vee B$, a speaker expresses her being disposed thus:

($\vee$-$\mathrm{m_s}'$) When an interlocutor has challenged her assertion of B, she is prepared to recognize, as a way to meet the challenge, any available assertion she would recognize as a way to challenge A.

Does $+\vee\mathrm{L}'$ plausibly reflect the expressive role of 'or'? If so, and we maintain our supposition that Cut is DDE–sound, then $+\vee\mathrm{L}'$ can't be DDE-sound. To see why, consider the derivation

$$\frac{\dfrac{+A, +B \Rightarrow +B}{+A, -A, +A \vee B \Rightarrow +B}\ (+\vee\mathrm{L}') \qquad \dfrac{-A, +A \Rightarrow +A}{-A, +A \Rightarrow +A \vee B}\ (+\vee\mathrm{R})}{+A, -A \Rightarrow +B}\ (\mathrm{Cut})$$

Do we really wish to say that an ideal agent will be disposed to take a challenge to their assertion of any sentence B to be met by available assertions of A together with a sentence they take to challenge the assertion of A? Even

[14]See (Mares, 2004, pp. 184–85). The symmetric rule for conjunction discussed there would derive $\Gamma, +A, -A \wedge B \Rightarrow \phi$ from $\Gamma, -B \Rightarrow \phi$. This is an unpromising candidate for DDE-soundness, due to the asymmetry between assertion and rejection explained in Section 4.3.

granting that such a combination of assertions may never be available, the dispositional claim is dubious.[15]

It appears, then, that at least one step in the above derivation isn't justified by the dialectical dispositions of ideal discursive agents. In support of $+\vee\mathrm{L}'$ rather than Cut as the culprit, clause ($\vee$-$\mathrm{m_s}'$) can be disputed. In certain contexts, asserting $A \vee B$ may not convey that a speaker is disposed to recognize available assertions that challenge A as meeting challenges to assertions of B. Such contexts include ones where it's presupposed that the speaker's only way to meet a challenge to $A \vee B$ would be to assert A.

5 Coordination rules

At this point, a natural thought is that we may need to take a different route to justifying a stronger intrinsic consequence relation. So far, we have only considered *connective rules* in addition to (Id). But standard bilateral systems also employ also *non-connective* rules that Rumfitt (2000, p. 804) calls *coordination principles*: they "co-ordinate the assignment of positive and negative signs to particular contents." Is it possible that we can justify as DDE–sound coordination principles that are inadmissible in G_1, and perhaps even inadmissible in systems for FDE and/or minimal logic?

Following Smiley (1996), Rumfitt himself focuses on a coordination rule he calls "Smileian Reductio." Here $*$ reverses a sentence's sign.

$$\frac{\Gamma, \phi \Rightarrow \psi \qquad \Gamma, \phi \Rightarrow \psi*}{\Gamma \Rightarrow \phi*}\ \text{(SRed)}$$

The system resulting from adding SRed to G_0 is sound and complete with respect to classical consequence.[16] We can also consider the restricted version where ϕ is positively signed, which belongs to the rules of Humberstone's system for intuitionistic logic. This rule derives $+A, -A \Rightarrow -C$, yet it's

[15] If we add $+\vee\mathrm{L}'$, Cut is no longer admissible, since there is no Cut-free derivation of $+A, -A \Rightarrow +B$. Extending G_0 with Cut and $+\vee\mathrm{L}'$ has the effect of adding $\Gamma, +A, -A \Rightarrow \phi$ as an initial sequent. That's because the latter yields a proof system for the logic K_3, one that renders Cut and $+\vee\mathrm{L}'$ admissible. To see that extending G_0' with $\Gamma, A, \neg A \Rightarrow B$ yields a system for K_3, note how G_0' and the extension mirror, respectively, the tableau systems for FDE and K_3 in (Priest, 2008); cf. (Beall, 2011, pp. 333–35).

[16] For completeness, it suffices to show that G_0+SRed renders admissible the natural deduction rules which, together with SRed, yield classical logic (Rumfitt, 2000, p. 804). To show this for the elimination rules, one can use Cut, which is derivable using SRed, Id, and a weakening rule admissible in G_0+SRed (Humberstone 2000, p. 351). For soundness, one can check that each rule is sound respect to the semantics in (Smiley, 1996).

hard to see how it will be the case that $+A, -A \vdash_a -C$. Surely an agent needn't be disposed to recognize the joint assertion and rejection of the same sentence as a challenge to any other assertion they have made.

More likely candidates for a DDE-sound coordination rule would be restricted versions of "Smileian Reversal," whose unrestricted formulation is as follows:[17]

$$\frac{\Gamma, \phi \Rightarrow \psi}{\Gamma, \psi* \Rightarrow \phi*} \text{ (SRev)}$$

Of the four rules this formulation encompasses, only the first two are admissible in Humberstone's system for intuitionistic logic.

$$\frac{\Gamma, +A \Rightarrow +B}{\Gamma, -B \Rightarrow -A} \text{ (R1)} \qquad \frac{\Gamma, +A \Rightarrow -B}{\Gamma, +B \Rightarrow -A} \text{ (R2)}$$

$$\frac{\Gamma, -A \Rightarrow +B}{\Gamma, -B \Rightarrow +A} \text{ (R3)} \qquad \frac{\Gamma, -A \Rightarrow -B}{\Gamma, +B \Rightarrow +A} \text{ (R4)}$$

None of these rules is admissible in G_0. Consider however their restrictions to empty Γ, henceforth R1′–R4′.

Proposition 3 *Each of R1′–R4′ is admissible in G_0, but each is inadmissible in G_1.*

Proof sketch. Each of R1′–R4′ is admissible in G_0 just in case its unsigned translation is admissible in G_0'. And those contraposition rules are all admissible in systems for FDE (cf. Priest, 2008, p. 162). As for G_1, adding R1′ would derive $-A \vee B \Rightarrow -A$, adding R2′ would derive $+\neg(A \vee B) \Rightarrow -A$, adding R3′ would derive $-\neg A \Rightarrow +A$, and adding R4′ would derive $+\neg\neg A \Rightarrow +A$. □

Can we justify any of R1′–R4′ as DDE-sound? Interpreted as here proposed, these rules link *challenging* with *meeting challenges*. I'll argue that the linkages fail, and that we can understand why by understanding how the room left open by their failure is exploited by certain vocabulary. Specifically, I propose that epistemic modal operators "It might be the case that A" ($\Diamond A$) and "It must be the case that A" ($\Box A$) yield counterexamples to each of the restricted reversal rules.

[17] In systems where SRed and weakening are admissible, SRev is admissible as well, as its conclusion is derivable using SRed from the weakened premise $\Gamma, \psi*, \phi \Rightarrow \psi$ together with $\Gamma, \psi*, \phi \Rightarrow \psi*$.

$$\frac{+A \Rightarrow +\Box A}{-\Box A \Rightarrow -A} \qquad \frac{+\neg A \Rightarrow -\Diamond A}{+\Diamond A \Rightarrow -\neg A}$$

$$\frac{-A \Rightarrow +\Box\neg A}{-\Box\neg A \Rightarrow +A} \qquad \frac{-A \Rightarrow -\Diamond A}{+\Diamond A \Rightarrow +A}$$

Consider first the counterexample to the DDE-soundness of R1′. Arguably, an ideal discursive agent will be disposed to recognize available assertions of A as meeting a challenge to her own assertion of $\Box A$. Yet she won't be disposed to recognize an interlocutor's rejection of $\Box A$ as challenging her own assertion of A.

The counterexample to the DDE-soundness of R2′ is used by Lennertz (2019) to argue that the relation two agents stand in when *one disagrees with the other* can be asymmetric. In the present context, it amounts to an asymmetry pertaining to the *act of challenging*.[18] An ideal agent who asserts "It might snow in August" will be disposed to recognize an interlocutor's assertion of "It won't snow in August" as a challenge she needs to meet on pain of withdrawing her assertion. By contrast, an ideal agent who asserts "It won't snow in August" needn't be disposed to recognize an interlocutor's rejoinder of "It might snow in August" as a challenge she needs to meet on pain of withdrawal. The interlocutor has expressed his unwillingness to concede that it won't snow in August. However, he needn't be regarded as having challenged the asserter of "It won't snow in August" to defend her assertion on pain of withdrawal.[19]

6 Conclusion

We have been asking whether the expressive functions of connectives proposed by dialectical disposition expressivism endow any relation between sets of sentences and sentences with consequence-like pragmatic significance. The strongest consequence relation we have found a way to defend as being

[18] Simonelli (2023) argues that if challenging is understood, following Brandom, as asserting an *incompatible content* (commitment to which precludes entitlement to the challenged assertion's content), challenging must be symmetric. Responding to Lennertz's example, he leaves open the possibility that epistemic modals lack the kind of content that figures in Brandomian incompatibility. The present account doesn't appeal to incompatibility between contents.

[19] Compare Incurvati and Schlöder (2019, pp. 754–55, 759) on the speech act of "weak assertion" of A, which they describe as one that "prevents [$\neg A$] from being added to the common ground." They argue that assertions of $\Diamond A$ license A's weak assertion. As challenging is understood here, weakly asserting A doesn't amount to challenging assertions of $\neg A$.

(in this sense) *intrinsic* to dialectical disposition expressivism is generated by proof system G_1. This is a very weak consequence relation. It fails to validate any arguments invalid in minimal logic, including ones that are valid in FDE such as $\neg(A \wedge B) \vdash \neg A \vee \neg B$ or $\neg\neg A \vdash A$; it likewise fails to validate any arguments invalid in FDE, including ones that are valid in minimal logic such as $A, \neg A \vdash \neg B$. Furthermore, it even fails to validate some arguments that are valid in both minimal logic and FDE, such as $\neg\neg\neg A \vdash \neg A$ and $\neg(A \vee B) \vdash \neg A$.

Is the apparent weakness of its intrinsic logic an objection to dialectical disposition expressivism? It would only be an objection if that view aimed to give an explanation of the functions of connectives that accounts for logical consequences. Elsewhere, I argue that this aim would be out of place (Shapiro, 2023). That's because according to the deflationism about logical consequence defended in (Shapiro, 2022), talk of consequence is fundamentally *not* metalinguistic talk about sentences and their relations. Rather, consequence talk serves to let us generalize over *logical conditionals*. These are sentences whose major connective can be expressed in English using locutions like 'that ... entails that ... ' (Anderson & Belnap, 1975, p. 491) or 'if ... then logically ... ' (Priest, 2006b, p.82).[20] For example, by saying that every sentence of the form 'It isn't the case that it isn't the case that p' has 'p' as a logical consequence, we achieve the effect of generalizing over an infinite class of logical conditionals:

> If it isn't the case that snow is not white, then logically snow is white.
> If it isn't the case that theft is not wrong, then logically theft is wrong.
> etc.

On this view, inquiry into what is a logical consequence of what is only superficially about relations between sentences; it is more fundamentally inquiry into matters formulated using a logical conditional rather than a consequence predicate.

Now just as expressivists about the function of 'wrong' shouldn't be expected to show that their account settles whether *theft is wrong*, expressivists about the function of 'not' shouldn't be expected to show that their account settles whether *if it isn't the case that theft is not wrong, then logically theft is wrong*. Settling the former question involves doing ethics; settling the latter involves doing logic. In neither case should we expect that the

[20] Here 'logically' is to be understood as part of the conditional connective, rather than as expressing a modal operator that is part of its consequent.

question can be settled by studying the functions of words. Hence, on the deflationist approach to logical consequence, expressivists about the function 'not' shouldn't be expected to show that their account of that function settles whether instances of double negation elimination are cases of logical consequence.

In short, even if dialectical disposition expressivism yields only a very weak intrinsic logic, the position is compatible with holding that *logical consequence* is far stronger.

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