ABSTRACT: This paper is a constructive response to Peter Baumann’s comments concerning the argument from inconsistency and explosion that was originally introduced in “Can Knowledge Really be Non-factive?” Specifically, this paper deals with Baumann’s two suggestions for how quasi-factivists might avoid this argument and it shows that they are both problematic. As such, his paper extends and strengthens the case against the view that knowledge is not factive, i.e. the view that knowledge implies that what is known is true or approximately true.

KEYWORDS: knowledge, factivity, approximate truth, dialethism, paraconsistency

1. Introduction

In a recent paper in this journal, Peter Baumann offers a friendly response to one of the criticisms leveled at quasi-factivism in “Can Knowledge Really be Non-factive?” Quasi-factivism about knowledge is the view that knowledge implies only approximate truth rather than strict truth, and, despite the seeming counter-intuitiveness of quasi-factivism and the orthodox nature of factivism, the view has been defended recently by some influential epistemologists. Specifically, Baumann focuses his attention on the argument from inconsistency and explosion (the AIE argument) from Shaffer 2021. This is because he takes it to be the strongest argument against quasi-factivism presented therein, and he offers two different ways that quasi-factivists might respond to the AIE argument. Respectively, he calls these the dialethism and paraconsistency (DP) response and the epistemic pluralism (EP) response. Here these two strategies for defending quasi-factivism about knowledge will be critically examined and rejected. So, the conclusion drawn here is that neither solution can save the quasi-factivist view of knowledge from the AIE.

1 Baumann 2021.
2 See, for example, Buckwalter & Turri 2020, Bricker (forthcoming), and Hazlett 2010.
2. The Argument from Inconsistency and Explosion

The orthodox conception of knowledge incorporates the following condition:

\[(\text{Factivity}) \text{ If } S \text{ knows that } p, \text{ then } p.\]

But quasi-factivists claim that one can know some propositions that are not strictly true, specifically one can know propositions that are only approximately true. But approximately true propositions are strictly false. So, the quasi-factivist holds that one can know at least some falsehoods. Quasi-factivists replace the factivity condition with this condition:

\[(\text{Quasi-factivity}) \text{ If } S \text{ knows that } p, \text{ then } p \text{ is true or } p \text{ approximates the truth.}\]

Bauman takes the most serious objection to quasi-factivism to be the one from “inconsistency and explosion” and Baumann helpfully reconstructs the AIE argument as follows.\(^3\) First, suppose quasi-factivity is correct and one can know some proposition \(p\) which is strictly false but approximately true. If this is true, then \(S\) can be in the following not uncommon epistemic state:

\[
\begin{align*}
(1) & \text{ } S \text{ knows that } p, \\
(2) & \text{ } S \text{ knows that } p \text{ is false.}
\end{align*}
\]

Given an ordinary principle of closure and (2) we get:

\[
(3) S \text{ knows that } \neg p.
\]

Given (1), (3) and closure under conjunction introduction we can derive:

\[
(4) S \text{ knows that } (p \text{ and } \neg p).\]

This is worrisome because we would have to attribute inconsistent beliefs to \(S\). In fact, we would have to attribute to \(S\) a belief in a contradiction. As Baumann notes, the important implication of quasi-factivism here is that if beliefs in contradictions can constitute knowledge, then we are dealing with a view that tolerates valorizing knowledge of inconsistencies. Quasi-factivists thus face a serious problem about opposing inconsistency in all cases of knowledge of approximate truth. On this basis he AIE raises an additional problem for quasi-factivists as well. This problem involves the logical principle of explosion (i.e. that anything follows from a contradiction).\(^5\) Specifically, a subject who simultaneously believes \(p\) and \(\neg p\) and

\(^3\) Shaffer 2021, sec.3.
\(^4\) See Baumann 2021.
\(^5\) See Shaffer 2021, 221.
who can acquire knowledge by deduction, can come to know any proposition. This seems patently absurd.

3. The Dialethism and Paraconsistency Solution

The dialethism and paraconsistency solution to the AIE is predicated on the idea that the undelaying logic of the propositions that are the objects of knowledge is classical and contains the notorious principle ex contradictione (sequitur) quodlibet (ECQ). This principle is the idea that contradictions imply every proposition. One reason that has motivated some thinkers to adopt paraconsistent logics is specifically that they do not treat ECQ as a valid form of inference. One reason behind dialethism is that this view allows that that some contradictions are true and that it is at least sometimes rational to believe contradictions. Thus, the DP solution is supposed to avoid the AIE argument by shifting the underlaying logic of knowledge from classical logic to paraconsistent logic and it allows for the idea that the relevant contradictions in question might be true and rational to believe. Baumann suggests this stratagem as one way for the quasi-factivist to avoid the unpalatable conclusion of the AIE. But this solution comes at an intolerably high price and this can be seen in looking at the consequences of this view for semantics and probabilistic justification.

3.1 The Content Objection

The semantic content of a claim is what it rules out. Contradictions do not rule out anything. So, when one is in the sort of state that the AIE is based on (i.e. knowledge of contradictions) the subject is supposed to have knowledge that involves a proposition that does not rule out anything. This is one standard objection to dialethism, but there is more to be said here. It is not only that the proposition in question does not rule out anything, but also that, as a result, the proposition that the agent is supposed to believe has no content. This is because the semantic content of a proposition is what it rules out and contradiction rule out nothing. This can be seen most easily in terms of the widely accepted theory of possible worlds semantics, though the same point about meaning and “ruling out” is common to semantical theories.

Possible world semantics holds that the meanings of all well-formed declarative sentences in a language L are to be equated with the set of all possible
worlds at which that sentence \( P \) is true.\(^8\) In other words, a meaningful sentence imposes a partition on the space of possible worlds, thus dividing that space into the worlds where \( P \) is true and the worlds where \( P \) is false. This is what constitutes meaning. Specifically, where \( P_{\text{wwf}} \) are the well-formed declarative sentences of language \( L_i \), \( P \in P_{\text{wwf}} \), \( W \) is the set of worlds \( \{w_1, w_2, \ldots, w_n\} \) at which \( P \) is true, and \( W \) is the set of all possible worlds such that for each \( w_i \), \( w_i \in W \):

\[
\text{(Def. 1)} \quad \text{The meaning, } |P|, \text{ of any } P \text{ in a given } L = W.
\]

There are, of course, a variety of views concerning the nature of possible worlds, and, hence, a variety of views concerning how we ought to interpret Def. 1.\(^9\) Nevertheless, whatever one says about the ontological nature of possible worlds, according to this theory the meaning of a sentence \( P \) (i.e. the proposition \( p \) that \( P \) expresses) is exhaustively given by specifying the various ways the total world could have been such that the sentence in question is true. Here, we will refer to the set of \( M \) that constitute \( W \) for a given \( P \), as \( M \), or the ‘\( M \)-set’ of \( P \). The \( M \)-set of a given \( P \), is the semantic content of \( P \) in the Wittgensteinian and Popperian sense that the \( M \)-set specifies for \( P \) the “range that it leaves open to the facts.”\(^10\) The \( M \)-set is then just the meaning of \( P \). But contradictory propositions rule nothing out and hence have no meaning.\(^11\) As we have seen with respect to the AIE, the quasi-factivist is committed to the view that an agent \( S \) can know contradictions and, as we have just seen, is then committed to the idea that one can believe, be justified in believing, and know meaningless/contentless claims. Thus, the DP solution to the AIE is simply unacceptable and would come at far too high a price.

3.2 The Probability Objection

The probability calculus says that the probability of the negation of a claim is one minus the probability of that claim. More formally:

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(T1) \quad P(\neg p) = 1 - P(p).
\]

\(^10\) See Wittgenstein 1922, 41 and Popper 1959, 119-120.
\(^11\) It is important to note that this objection cannot be avoided by claiming that tautologies and contradictions are meaningful, but do not rule out anything or rule out everything as is suggested in Priest, et al. 2018. It is perfectly reasonable to hold that tautologies are necessarily true but have no semantic content (i.e. they are merely terminological synonymies) and that contradictions are necessarily false and have no semantic content (they are the negations of merely terminological synonymies).
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This follows from the axioms of the probability calculus (i.e. it is a theorem). But, if there are true known contradictions this cannot be the case. Suppose that S is in the sort of state at issue:

\[(4) \text{S knows that (p and not-p).}\]

As we have seen in terms of the AIE, the derivation of (4) involves the following claims:

\[(1) \text{S knows that p,}\]

and

\[(3) \text{S knows that not-p.}\]

If S knows that p, then (on the standard analysis of knowing) S’s belief that p must be adequately justified by S’s evidence e. Typical theories of justification model such justification in terms of probabilities understood in terms of the axioms of the probability calculus. This includes T1. Moreover, on typical probabilistic theories of justification, if S knows that p, then S’s belief that p is such that the P(p|e) ≥ k, where k is the probabilistic “threshold” for adequacy must be (significantly) greater than .5. Accordingly, in the kind of cases under consideration, S’s belief that p is such that P(p|e) ≥ k and S’s belief that ¬p is such that P(¬p|e) ≥ k, but according to T1 P(p) = 1 − P(¬p). To see the problem here, suppose that the probabilistic threshold for one’s justification rising to the level of knowledge is .92, that S knows that p, that S knows that ¬p, that P(p|e) for S is .93, and that P(¬p|e) for S is .93. But, given these assumptions, T1 implies the following claims:

\[(C1) \text{P(p|e) for S is .93 and P(¬p|e) is .07,}\]

and

\[(C2) \text{P(¬p|e) for S is .93 and P(p|e) is .07.}\]

But, the P(p|e) cannot be both .93 and .07 on the same evidence and P(¬p|e) cannot be both .93 and .07 on that same very evidence. So, the quasi-factivist’s view, when defended by appeal to the DP, yields probabilistic incoherence and is incompatible with the standard notion of justification. So again, defending quasi-factivism in terms of the DP defense has an intolerably high cost.

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12 See, for example Howson & Urbach 1993.

13 See Shaffer 2018.


4. The Epistemic Pluralism Solution

The other solution to the AIE that Baumann suggest on behalf of the quasi-factivist is the epistemic pluralism solution. This solution is considerably less radical than the DP solution and so it is much more plausible. Rather than introducing an implausibly radical revision of logic, the EP solution attempts to avoid the problem for quasi-factivism that the AIE raises by introducing multiple concepts of knowledge that can be represented as different indexed knowledge operators. Baumann characterizes the view as follows:

(Pluralism) There is more than one knowledge relation: for instance, knowledge of strict truths (“knowledge-s”) and knowledge of approximate truths and strict falsehoods (“knowledge-a”).

So rather than there being one such operator Kp, there can be different kinds of knowledge and each such operator that represents a different kind of knowledge will have different properties. Most importantly, there can be factive and quasi-factive knowledge operators K-sp and K-ap respectively. The upshot is then that the possibility of contradiction on which the AIE is built can be avoided by showing that when the two different knowledge operators are properly substituted in AIE, we find that there are no actual contradictions involved. The relevant re-workings of the claims involved in the AIE are then as follows, where p is only approximately true:

(1*) S knows-a that p,
(2*) S knows-s that p is false,
(3*) S knows-s that ~p.

But there is no problematic analog of

(4) S knows that (p and ~p),

in terms of know-s or in terms of knows-a. There is no contradiction in terms of knows-a or in terms of knows-s that follows from (1*) and (2*). In other words, there is nothing contradictory about the conjunctive claim S knows-a that p and S knows-s that ~p. The contradiction identified in the original AIE argument is thus supposed to be the result of failing to see that the distinct knowledge claims that give rise to (4) in the AIE actually have the different forms K-ap and K-s~p. So, given the EP solution, there is no need to reject classical logic and adopt a paraconsistent logic and there is no need to endorse dialethism in order to avoid the conclusion of the AIE. This is simply because there is no contradiction involved in the sorts of examples

14 Baumann 2021, 459.
used to support the AIE and the threat of ECQ is thus supposed to be only apparent rather than real. This is a solution to AIE that is clearly preferrable to the DP solution simply due to its being less radical. But is this solution really one that we should adopt rather than rejecting quasi-factivism? The answer defended here is a forceful “no”.

4.1 The Perils of Epistemic Pluralism

So, what exactly is wrong with the EP solution to the AIE? Essentially, it is easy to see that EP solution will not save quasi-factivism. There are several reasons why this is so. First, as Buamann notes, the solution to the AIE that employs the EP strategy is utterly ad hoc. The EP solution depends on the idea that there are at least two importantly distinct knowledge concepts and that the knowledge operator in (1) of the AIE is the $K_s$-a operator, while the knowledge operator involved in (2) and (3) of the AIE is the $K_s$-s operator. While this might be the case, it is certainly not obviously true. Why accept that this is actually the case? That it is possible that there are two separate knowledge operators involved in the AIE does nothing to eliminate the paradox in anything like a serious manner. Third, as Baumann notes, the pluralizing maneuver opens the door to further pluralization of the concepts of knowledge, with no obvious limitation. We might then, for example, consider adding Baumann’s knowledge-1 concept to our conceptual arsenal or knowledge-i, where believing in the belief condition is replaced with imagining.\(^{15}\) Again, as Baumann notes, his begs the obvious question concerning why these various concepts are knowledge concepts, especially if they do not share any essential feature(s) in common.\(^{16}\) Third, pluralizing the concept of knowledge is a sure invitation to semantic confusion. Why not simply acknowledge that there are other concepts that are related to but distinct from knowledge? So, all of this indicates the inadequacy of both the DP and EP solutions to the AIE and suggests that rejecting quasi-factivism is the correct response to the AIE. Moreover, this importantly supports the ideas that epistemologists should explore knowledge-like states in addition to bona fide knowledge states and that we ought to be sensitive to the possibility of confusing knowledge with quasi-knowledge.

5. Conclusion

So, while Baumann’s suggestion of these two possible ways for quasi-factivists to avoid the AIE are interesting, they are ultimately unsuccessful as substantive

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\(^{15}\) See Baumann 2021, 461.

\(^{16}\) Baumann 2021, 461.
defenses of quasi-factivism. Quasi-factivists then need to look elsewhere for a solution to the AIE or they simply need to concede quasi-factivism. As things stand though, the latter option is strongly motivated. Knowledge is factive, but there are likely a host of knowledge-like, factive, non-factive and quasi-factive, propositional attitudes. This suggest that a bit of conceptual engineering is needed in order to distinguish such states and we may need to introduce more fine-grained terminological distinctions between these different states in order both to avoid the appearance of contradiction and to avoid the confusions that arise from our failing to have such tools in hand.

References

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