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Knowledge of Abstract Objects in Physics and Mathematics

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1 Introduction

The nature and reliability of our epistemic sources is a topic that has always been afforded a central place in philosophy, particularly in epistemology and the philosophy of science. But it is surely a topic that is also of considerable practical importance in non-philosophical contexts. For example, the issue of what constitutes good methodology has great practical significance in the contexts of both scientific and mathematical practice. So, while at times the debate between empiricists—those who hold that all knowledge is based on perceptual experience—and rationalists—those who deny empiricism—might seem to be rather divorced from practical applications, this is simply not true. Similarly, the metaphysical issue of just what the objects of scientific and mathematical knowledge are has, at times, been treated as a central issue in philosophy. However, unlike the issue of the reliability of our epistemic sources, it has often been taken to be an issue that has little or no effect on actual practice of those disciplines. This line of thinking is then that while it is clearly true that epistemic and metaphysical issues are central to philosophical practice, it is only the former types of concerns that have much in the way of practical implications for scientific and mathematical practice. Here, this particular bit of mythology about the significance of philosophy will be challenged in light of some deeply serious worries about the accessibility of the objects that are the putative subject matter of mathematics and the physical sciences. Let us then consider these particular matters in considerably more serious detail.

With respect to the methodological and epistemological issue just introduced, there are, for example, a number of critical applied debates that have occurred in both the history of mathematics and in the history of physical theory that demonstrate the

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significance of applied theoretical epistemology. For example, the debate between big bang theorists and steady-state cosmologists earlier in this century was explicitly framed in terms of whether or not scientific cosmology should be understood to be purely empirical or not, and resolving this issue had serious implications concerning both how we are supposed to do cosmology and how we are supposed to understand science itself.[1] Similarly, the debates between constructivists/intuitionists and Platonists in mathematics also raised serious issues concerning how mathematics is to be both practiced and understood.[2] So such epistemological issues should not merely be regarded as cases of philosophical hair splitting, and recently the debate about how we are to understand methodology, especially scientific methodology, and how it bears on scientific practice has been much revisited.

Methodological naturalists, in the spirit of Quine (1960 and 1969), have adopted the stance that there is but one way of knowing and that it is empirical and scientific, at least with respect to the issue of justification. This form of naturalism entails, at the very least, that the methods of the sciences are the only legitimate methods for inquiry in any domain, and it implies that the study of reason itself can be placed on an adequate scientific footing. Despite the obvious attraction of naturalized epistemologies in this respect, a growing number of philosophers continue to suggest that there is significant, i.e., synthetic, a priori knowledge. In most cases, such knowledge is supposedly limited to characteristically philosophical knowledge such as knowledge of the basic rules of deductive inference and it is supposed only to involve necessary truths, but there are notable and critical exceptions to this rule.

For example, it is clear that many philosophers of mathematics who follow the Platonic, Fregean, and Gödelian tradition continue to hold that our knowledge of mathematics is a priori in nature and it is important to emphasize at this juncture that this is neither per se a purely philosophical issue as such, nor is it a purely epistemic matter. The most common reason for adopting this view is, of course, the seemingly plausible metaphysical claim that mathematical objects are abstracta and so they are, by their nature, causally inert because abstracta are non-spatiotemporal entities. If this ontological point is granted, then the rationalists would have us believe that, for example, our mathematical knowledge and not just our philosophical knowledge can only be explained by appeal to some special non-empirical source. This is supposed to be the case because it is contended that we cannot possibly have acquired such knowledge perceptually. Moreover, adopting this metaphysical thesis indicates both that there are definite limits placed on mathematical inquiry that are fixed by the contents of the realm of abstract objects and that the proper method for mathematical inquiry involves the exercise of the intuitive faculty, that faculty that allows us to transcend the spatiotemporal world in order to acquire such knowledge.

This problem concerning our knowledge of abstract objects as it is realized in the philosophy of mathematics has been dubbed the Benacerraf problem, as it was Benacerraf who first formulated the problem in what we might call contemporary terms.[3] However, in a related manner, some philosophers of science have argued, roughly, that since the objects with which the *sciences* deal are idealized they are also abstract. Thus,

[1] See Kragh 1996.
[2] See Maddy 1990, Maddy 1997 and especially Kline 1980.
[3] See Benacerraf (1973). Mark Steiner (1975), however, made roughly the same point at the same time.

our knowledge of the objects of the sciences cannot be empirical for what seem to be the same sort of reasons given for the adoption of rationalism in the context of the Benacerraf problem and mathematical objects. More specifically, because the theoretical descriptions of perfectly round spheres, frictionless media, and perfectly inviscid fluids incorporate idealizations and, hence, do not describe actual (or perhaps even possible) objects, we are invited to suppose that they must describe *abstract objects*.[4] In honor of one of the most famous defenders of this closely related line of argument, Alexander Koyré, let us refer to this problem as the Koyré problem.[5] Notice that this constitutes a rather startling and serious threat to naturalism and that this position essentially endorses the view that at least some *physics* and not just philosophy or mathematics can be practiced in the armchair via the exercise of the intuitive faculty. As such, naturalists ought to take this threat seriously, for what it asserts is that *science itself is not itself naturalizable*.

It will become clear in the course of this discussion that both the Benacerraf and Koyré problems are special cases of a more general problem that—for lack of a better designation—we can refer to as the *causal isolation problem* and it is simply the problem of explaining our knowledge of abstract objects given some mild restrictions on what we should take to be a minimally plausible epistemology. As a result, it is reasonable to suppose that there is much to be gained from looking at the Benacerraf problem if we are to rebut Koyré's contention that *scientific* method is rationalistic. Here, the causal isolation (CI) argument will be critically examined, and more specifically it will be shown that even if we grant the main contention of Koyré's more specific argument—i.e., that at least some of the objects about which the sciences speak are idealized—our knowledge of those objects requires appeal neither to the a priori nor to the faculty of intuition.[6] As a result, a fully naturalized epistemology will be shown to be in principle compatible with scientific knowledge of idealized objects and thereby the practice of armchair physics that Koyré and others seem to believe to be possible will be challenged.[7] It should also be noted at this point that this paper will not engage with the contention that knowledge of abstract objects is inferential. This is primarily because of the notorious problems associated with the most plausible inferential approach to ontological questions based on inference to the best explanation.[8] As such, the issue here is addressed in terms of direct, non-inferential solutions to the causal isolation problem.

[4] See Quine (1960), Yablo (1998), and Psillos (2011) for an explicit statement of the view that many of the objects of the sciences are abstracta.

[5] Duhem (1954/1991) also defends the point about the idealized nature of physical theory, but he seems to resist the temptation to endorse realism about abstracta in adopting a thoroughgoing conventionalism. Interestingly, this does not seem to be entirely incompatible with naturalism as conventions are not knowledge.

[6] The position advocated here constitutes a further articulation of the view developed in Shaffer 2012.

[7] This response to Koyré's argument might also seem to suggest that the Benacerraf problem might be amenable to a similar solution, but there are important differences between the two problems that indicate why we should resist the temptation to believe that a solution to the Koyré problem will entail a similar solution to the Benacerraf problem. Specifically, while it is clear that the idealized objects of physics are abstract, it is clear that they are intended to be idealizations of actual concrete objects. This is not obviously true of mathematical objects, as many of them are not objects that could have concrete existence. The possibility of developing this line of argumentation as it applies to mathematical objects will not, however, be pursued here.

[8] See Van Fraassen 1980 and 1989.

2 (Mild) Naturalism and the Benacerraf Problem

The essence of the Benacerraf problem concerns the co-tenability of broadly causal theories of knowledge—what many take to be our best theories of knowledge—and the claim that we, in point of fact, have knowledge of at least some mathematical claims. Benacerraf originally presented the problem as the more specific issue of the co-tenability of reliabilist theories of knowledge and mathematical knowledge, but as many commentators have noted, the problem is easily generalized to any theory of knowledge that accepts even some mild form of the claim that our knowledge of a given domain of objects presupposes the possibility of causal interaction with the objects in that domain.[9] The sentiment here seems to be based on what Colyvan (2001) calls the eliatic principle, and he finds it to be a consequence of the sort of naturalism endorsed by David Armstrong.[10] As Colyvan notes, on this sort of view, "We should believe in only causally active entities (2001, p. 32)." The problem concerning our knowledge of mathematics then follows readily when we accept the generally accepted metaphysical claims that mathematical objects are abstract and that abstract objects are causally inert.

What is of great interest in the context of the discussion to follow is the answer to the Benacerraf problem based on the revival of a more or less pure form of Platonism in the tradition of Kurt Gödel (1947/1983), for this seems to be the kind of answer to the problem of our knowledge of the abstracta of the sciences that Koyré appears to endorse. What philosophers of mathematics like Hale and Wright (2001) have recently offered in their attempts to answer to the Benacerraf problem is a view that invites us to accept the existence of some form of intuitive faculty that yields a priori knowledge of the truths of mathematics.[11] This is by no means an unusual suggestion when we confront problems concerning knowledge of metaphysically unusual entities as even casual familiarity with the history of philosophy demonstrates. Often, in considering what we might call these sorts of "hard problems" in epistemology, philosophers have succumbed to the temptation to appeal to such mysterious faculties, for the alternative seems to be nothing more than a concession to the skeptic. For example, it seems to be a relatively common practice in moral theory to appeal to intuition as the faculty that supplies us with knowledge of normative moral principles and this stratagem is pursued because we do not seem to be obviously able to acquire knowledge of what ought to be the case from perception of what is the case. Consequently, intuition is then often posited as the faculty that allows us to bridge the Moorean gap between fact and value, for the alternative is to concede to the skeptic that we do not have such knowledge and thus to strengthen the moral skeptics position. The same sort of problem confronts us in both the case of our knowledge of mathematical objects and the case of our knowledge of the idealized objects of physics. It would appear to be the case that the choice is either that we accept that there is some intuitive faculty that will allow us to a priori acquire knowledge of those objects or we must accept some form of skeptical anti-realism about those domains of inquiry. The problem is then, at least prima facie, that it

9 See Maddy (1990), Kim (1981), and, especially, Field (1989, 1980).
10 See Armstrong (1989).
11 Specifically see Hale and Wright (2001).

looks like that in attempting to avoid the peril of the skeptical conclusion we must adopt an epistemological view that unhappily implies the rejection of naturalism.

3 Koyré's Argument

While this sort of reasoning might seem to be prima facie plausible in the contexts of moral theory and mathematics, it seems to be entirely out of place in the context of the physical sciences. Nevertheless, as noted above, Alexander Koyré is famous—or rather infamous—for his defense of the position that theoretical claims in the sciences are accepted a priori, and this alternative might be seen to be particularly appealing in light of the perception that the entities and situations that are quantified over in theoretical claims that depend on idealizing assumptions do not exist in the actual, i.e., concrete, world. Koyré claims, for example, that

It is impossible in practice to produce a plane surface which is truly plane; or to make a spherical surface which is so in reality. Perfectly rigid bodies do not, and cannot, exist *in rerum natura*; nor can perfectly elastic bodies; and it is not possible to make an absolutely correct measurement. Perfection is not of this world: no doubt we can approach it, but we cannot attain it. Between empirical fact and theoretical concept there remains, and always will remain, a gap that cannot be bridged (Koyré 1960, p.45).

Elsewhere, he concludes on the basis of this observation that

> Good physics is made *a priori*. Theory precedes fact...Fundamental laws of motion (and of rest), laws that determine the spatio-temporal behaviour of material bodies, are laws of a mathematical nature. Of the same nature of those which govern relations and laws of figures and of numbers. We find and discover them not in Nature, but in ourselves, in our mind, in our memory, as Plato long ago has taught us (Koyre 1943, p. 347).

So following Koyré, one might think that our empirical access to such systems is absolutely barred because they are abstract rather than concrete entities; i.e., they do not and often cannot possibly exist in the space-time continuum. Because they are abstracta, objects like perfectly plane surfaces, frictionless spheres, space-time points, instantaneous velocities, etc., are not the sorts of things with which we can ever come into any sort of causal contact. Most importantly, they are not the kinds of things that we can perceive, as perception is itself a form of causal contact. If this is true, then it would appear to be the case that we must reject empiricism and concede that our acceptance of such claims *must* be based on pure thought alone, if it is rational at all. Unlike Koyré, Pierre Duhem (1954/1991) and, more recently, Nancy Cartwright (1983) drew from this observation the conclusion that we ought to accept a skeptical and anti-realist stance with respect to physics and the other sciences when it comes to theoretical claims. However, for obvious reasons, this move should be resisted as it would imply that many of the claims of our best physical theories, say the quantum theory of fields for example, are neither true nor do they describe actual phenomena.

So there *is* a very substantial temptation to follow Koyré in his acceptance of Platonic rationalism even in the context of physics. However, Koyré's argument appears to be especially damaging to those with both empiricistic and naturalistic leanings as it directly conflicts with the core epistemic thesis of the most orthodox version of naturalism, i.e., that there is only one form of knowing, scientific inquiry, and that it is empirical. It should be apparent, however, how similar Koyré's epistemological view is to that of contemporary mathematical Platonists like Bob Hale and Crispin Wright. More importantly, we should also notice how implausible this sort of position is on closer inspection. Most glaringly, the defenders of these Platonistic positions are forced into the straightforwardly ad hoc maneuver of positing an intuitive faculty merely in order to avoid the relevant skeptical conclusion, and so the rationalist position in these various contexts looks to be a glaring case of question begging against skeptical anti-realists. Moreover, absent some independent reasons to believe in the existence of such faculties and in the existence of the abstracta in question, the Platonistic response is based on nothing more than a mere possibility and thus it cannot constitute a reasonable *actual* response to the skeptic. It is utterly ineffective to offer as a non-skeptical solution to the problem of the possibility of knowledge of some class of abstract objects the claim that we might have some such mysterious faculty that permits such knowledge. The interesting question is then whether naturalists can do better in explaining our knowledge of these alleged abstracta by appeal to some actual and scientifically legitimate faculty, thus preserving the purely empirical character of such knowledge without begging the question against the skeptical anti-realist.

4 The Causal Isolation Argument and the Platonic Solution

At this point, it should be apparent that, at least prima facie, what we are really facing is a single type of very traditional problem that happens to be manifested in two quite disparate domains, i.e., in mathematics and in the physical sciences. The fundamental problem here is simply the problem of accounting for our knowledge of abstract objects and their nature. As Koyré notes, this is of course a problem that Plato confronted and to be sure it is a problem that has driven much of the development of epistemology and metaphysics throughout the history of philosophy. In any case, we can generalize from the cases of the more specific Benacerraf and Koyré problems and see that the common problematic is constructed as follows. First, we are asked to assume that some types of object (e.g., Ox) are abstract in nature. Second, that these objects are abstract is implicitly taken to entail that objects of type Ox are causally inert. This is because we are invited to accept the metaphysical principle that only concrete types of objects that exist spatiotemporally can have causal powers. However, we are then asked to recognize that we, in fact, have knowledge of properties and relations involving objects of type Ox. Schematically, where "Ox" is some type of object, "Ax" is the predicate "x is abstract," "Ix" is the predicate "x is causally inert," and "Kxy" is the grossly simplified predicate "y has knowledge of x," we can then frame the CI theses that constitute CI problems simply as follows:

1. $(\forall x)\ Ox \supset Ax$. (ABST)
2. $(\forall x)\ Ax \supset Ix$. (ONT)
3. $(\exists x)(\exists y)\ Ox \bullet Kxy$. (KN)

We are then also invited to assume (CTK) that our best epistemological theories require that the knowability of a certain type of objects and the properties of those objects requires the possibility of our being causally connectable to those objects. We can thus construct a number of domain-specific CI problems concerning alleged abstracta and our knowledge of them simply by specifying various appropriate O*x*s. It is important at this point to note the modal aspect of ONT and of I*x* in particular. The claim being made is that such entities are in principle casually inert. That is to say that abstracta are such that it is not *possible* in some important sense for them to exert causal influence. Thus, even an ideally poised epistemic agent could not know about such an object. This is not a trivial point either. For example, in our world here are regions of our actual space-time which are causally disjoint from other regions of that very same space-time. So it is *physically* impossible that an agent in the first region could be causally connected to an object in the second region due to constraints imposed on us by general relativity. Nevertheless, if that agent were sufficiently closer to the region in question she would be able to be causally connected to such an object. What ONT principles say is that it is *metaphysically* impossible to be in causal contact with the objects in question.

What is of primary interest here are the details of the characteristic solutions favored by many (contemporary) Platonic rationalists and the alternative responses that naturalists might make to CI problems. As we have seen, rationalists like Hale and Koyré, in considering very different domains, appear to respond to these skeptical Benacerrafian worries by adopting the following view:

(RAT1) We must possess some non-perceptual faculty that allows us to come into causal contact with abstract objects.

So, they endorse CTK and propose a new causally efficacious faculty to bridge the gap between the concrete world and abstracta. As a result, they are committed to the view that we should ultimately also revise our thinking about ONT. This particular response then involves accepting that abstracta are not causally inert even though they do not exist spatiotemporally.

Given this rationalist response, our knowledge of objects of types O*x* is to be explained by appeal to some more or less well-specified *causal* faculty that puts us into appropriate contact with abstracta that exist non-spatiotemporally. This faculty is supposed to allow us then to bridge the gap between what exists concretely and what exists abstractly. However, this specific response is not one that appears to be available to naturalists, especially as we currently have no good scientific reasons to believe that any such faculty exists and it is, for the reasons given above (i.e., that the positing of the intuitive faculty is both ad hoc and only a mere possibility), totally ineffective as a response to skeptical and anti-realist worries. Predictably, there are however a number of other characteristic responses that one might make to CI problems that are based on the plausibility of the theses involved and two that appear to be open to naturalists offer far more plausible answers to CI-type problems and more specifically to Koyré's problem.

5 An Unacceptable Naturalistic Response

The first sort of non-Platonist solutions to the Benacerraf problem that we will consider and which might be extrapolated to the more general CI-type problems shares at least one point in common with the Platonic rationalists' response. Specifically, this response also involves accepting CTK. Jaegwon Kim (1981), John Bigelow (1988), and Penelope Maddy (1990) have independently answered the Benacerraf problem by asserting that at least some supposedly abstract objects exist spatiotemporally and hence can be perceived in accord with the CTK condition on admissible theories of knowledge. This quasi-Aristotelian—or perhaps Pythagorean—response involves the core claim that numbers, sets, and/or mathematical properties are on an ontological par with objects like rocks and properties like colors. There is then no special problem explaining how we come to know of them. At least some such entities and/or properties are known empirically via our ordinary perceptual faculties. As a result, the quasi-Aristotelian response allows for a thorough-going naturalized response to the Benacerraf problem and it is supposed to avoid the worries that afflict the Platonistic response qua the existence of the faculty by which the objects in question are known. This is achieved by denying ABST with respect to those types of entities. Thereby, these naturalists do not need to adopt RAT1 or anything like it. The means by which we come to know of abstract objects does not involve any sort of new or special faculty that allows us to be in causal contact with abstracta.

Whatever the merits of this response in the context of the Benacerraf problem, it is not at all plausible as an answer to the Koyré problem. Where in the case of mathematical entities or properties we might suppose that at least some such objects are members of the spatiotemporal world, this does not seem to work in the case of the idealized objects of the physical sciences. The quasi-Aristotelian response in the context of mathematics involves adding mathematical entities and/or attributing mathematical properties to the totality of the world, thus resulting minimally in the view that concrete objects have (abstract) numerical properties in addition to their physical properties, but consider the analogous maneuver in the context of the sciences. Were we, for example, to attribute the property of being frictionless to a certain class of relevant objects in the concrete world in this manner, we would be forced to accept the view both that these actual objects are subject to frictional forces and that they are not subject to frictional forces. Of course, examples can easily be multiplied here. Thus, while this general strategy might look appealing in the context of the Benacerraf problem, it would, in the context of the more specific Koyré problem, imply that physical objects have incompatible properties and so would thereby commit us to the acceptance of a vast set of actual contradictions. As we shall see, this is fundamentally the result of the fact that idealizations are counterfactual in nature and involve simplifying actual systems, whereas this is not true of the mathematical case. So while this naturalizing strategy might perhaps succeed as a naturalistic account of mathematics, it will not suffice as a naturalistic account of our knowledge of idealized objects like those frequently discussed in the sciences.[12]

[12] In point of fact, this is not really a very good naturalistic account of mathematical knowledge for the following reason. Perceptual contents must either be individuated widely or narrowly following McGinn (1989). The problem, although it is a bit more complicated, is essentially that if they are widely individuated then existential claims about mathematical objects based on direct perception are question begging and if they are narrowly individuated then such appeals to mathematical perception are insufficient to ground existential claims without some further independent verification that such things exist and cause the perceptual states in question.

6 The Second Naturalistic Response

The second and perhaps less obvious response that naturalists might make to CI-type problems, and more specifically to the Koyré problem, involves a fundamental disagreement with ONT principles while retaining ABST for the relevant entities. In effect, this response involves denying both (1) that we have some special faculty that allows us to be in causal contact with abstracta and (2) that abstracta exist spatiotemporally in the ordinary sense of that expression. So, it is neither Platonic nor quasi-Aristotelian. Moreover, unlike the first unacceptable naturalistic strategy with respect to this skeptical problem, it allows the naturalist to offer a unified account of our knowledge of mathematical entities and of the idealized entities often employed in physical theory. In these regards, it promises to be a better solution and this strategy amounts to rethinking CI problems altogether in both the context of physical theory and the context of mathematics.

What this solution ultimately amounts to is that view that both mathematical and idealized entities are conceptual or imaginary entities—they are the objects/contents of certain thoughts. In adopting this view, the naturalist can then avoid the positing of speculative faculties like the Gödelian faculty of intuition as it has been adopted by Platonists. But such naturalists can also at the same time deny that mathematical and idealized entities exist spatiotemporally in the ordinary sense. Thus, they can avoid the problem of the attribution of incompatible properties to actual physical objects. They can then also avoid the issue concerning the Koyré problem that the quasi-Aristotelians face that was raised in the immediately preceding section. On this view, all abstracta are just mental entities. In being objects of thought, they are accessible to us without the need for any special faculty to bridge the gap between spatiotemporally located entities and non-spatiotemporally located abstracta, but they do not exist concretely as physical objects or properties of physical objects. Rather, they are objects of conception and so conceptualism about both sorts of entities provides for a unified solution to the problem of our knowledge of abstract objects.

The first step in constructing a solution of this sort is to note that properly empirical theories and mathematical theories share a basic justificational structure in common. As a result, the methodologies of both the physical sciences and mathematics are importantly similar. In both cases, conjectures are generated and these conjectures are tested against less general (i.e., more basic) statements. The real difference then between properly empirical theories and mathematical theories can then be drawn in terms of the nature of the basic statements that serve as the potential falsifiers for these two types of theories. The only real difference then between the methodology of pure mathematics and that of the properly empirical sciences is the nature of the experiments that characterizes work in pure mathematics and work in the properly empirical sciences. Nevertheless, this difference in experimental character can be elegantly used in order to account for our knowledge of abstract objects sometimes appealed to in the empirical sciences as well.

In the case of properly empirical sciences, the theoretical conjectures which are introduced about objects and their properties are intended to explain singular observation statements (i.e., singular spatiotemporal statements) and so can be potentially falsified by them. In the case of the theoretical conjectures of pure mathematics, those conjectures are introduced to explain *other sorts of basic statements*. In this case, they are designed to explain lower-level theorems that can be usefully understood to be kinds of observation statements, albeit *not* as statements about singular spatiotemporal events. Rather, the basic statements of pure mathematics are derivatives of our imagination and since they are supposed to be explained by the theoretical conjectures of pure mathematics, those observation statements are about imaginary or conceptual objects. It is these imaginary objects then that serve as potential falsifiers of the conjectures of pure mathematics. The experiments in which this is done are not then experiments involving observations of spatiotemporal singular objects, but rather they are kinds of *thought experiments* involving imaginary objects that are of a logical level that is less general than the conjectures with respect to which they serve as potential falsifiers.[13]

As we have seen, empirical theories that do not deal with abstracta have potential falsifiers that are singular spatiotemporal statements. So, purely empirical theories are statements that can apparently only be directly tested by properly empirical and concrete experiments designed to decide the truth of certain singular spatiotemporal statements and thereby to test properly empirical theories. This is not the case, however, for the theories of pure mathematics and for scientific theories concerning abstract systems/objects. The statements of pure mathematics deal only with abstracta and so they can be potentially falsified only by lower-level statements about imaginary or conceptual objects. They can only then be potentially falsified by something other than properly empirical experiments. Given the view in question, they can only be potentially falsified by conceptual objects. A most interesting possibility is then that the methodological account of how we deal with scientific theories that concern abstract entities shares some features of both the methodology of pure mathematics and that of the properly empirical sciences.

The second step in developing this solution involves recognizing that idealized theories are counterfactuals about abstracta. As an example of an idealized theory that concerns an abstract system, consider the Euler equation for fluid flow:

$$(\mathrm{E}_1)\ \rho D\boldsymbol{u}/Dt = -\nabla p$$

Here, ρ is the fluid's mass density, $D\boldsymbol{u}/Dt$ is the hydrodynamic derivative of the fluid velocity, that is, $D\boldsymbol{u}/Dt = d\boldsymbol{u}/dt + \boldsymbol{u} \cdot \nabla\boldsymbol{u}$, and ∇p is the pressure gradient. In E_1, it is also assumed that external forces are absent. This equation holds true only of perfect, inviscid, fluids, even though it is often applied to real systems. So, strictly speaking, E_1 is false; it is true only of an abstract class of systems. Since they are abstract, these sorts of systems do not exist concretely. In the context of applying the Euler equation, it is falsely assumed to be the case that there are no forces parallel to the surfaces of contact with the rest of the fluid; that is to say, there are no viscous forces that oppose the motion of the fluid along the direction of flow. E_1 actually is then better understood as the following assertion (*the Euler counterfactual*):

> (EC_1) If x is a fluid and there are no viscous forces that oppose the motion of the fluid along the direction of flow, then x's behavior obeys $\rho D\boldsymbol{u}/Dt = -\nabla p$ (i.e., the Euler equation).

[13] See Lakatos (1976).

The antecedent of such a statement is a statement of the relevant idealizing conditions, and the consequent is an idealized theoretical claim holding true only of some class of abstracta that are physically impossible. In order to incorporate some of the types of forces that have been idealized away in E_1 into the description of the motion of a fluid, one must turn to the Navier-Stokes equation:

$$(E_2)\ d\boldsymbol{u}\Big/dt + \boldsymbol{u}\bullet\nabla\boldsymbol{u} = -1\Big/\rho\nabla p + v\nabla^2\boldsymbol{u}$$

Here, v is the kinematic viscosity and $\nabla^2\boldsymbol{u}$ is the Laplacian of the fluid velocity, and it is again being assumed that there are no external forces. E_2, however, is considerably more difficult to solve, because it is a second-order equation. The Euler equation is a first-order equation, and, in point of fact, it is not even known whether the Navier-Stokes equation is solvable over long periods of time. Nevertheless, as in the case of E_1, with E_2, we are actually dealing with a counterfactual because the idealizing conditions under which it holds are never actually met and the system it describes is abstract, even though it is closer to a correct description of actual fluid flow. Of course, we could make those idealizing assumptions explicit as we did in the case of E_1. In any case, we retain Euler's equation and use it in practical applications despite this because it is computationally tractable in a way that the Navier-Stokes equation is not, and Euler's equation holds true only in a model that is an intentional simplification of that in which the Navier-Stokes equation holds. Both are idealizations of fluid flow in the actual world, and it is often the case that we must settle for using the less realistic alternative of two idealizing theories as a matter of satisficing. In many cases, the Euler equation provides us with results that are sometimes pragmatically acceptable, and we must often employ such theories because we *cannot*, as a matter of computational complexity, use the more realistic theory at all.[14] E_1 and E_2 are, of course, intimately related in the sense that the models in which they hold are importantly similar, and despite the fact that both theories involve degrees of considerable idealization E_1 and E_2 and so only apply to abstract systems, they are believed to be true on the basis of empirical evidence. They are contingent counterfactual conjectures about how a specific type of real system would behave if it were simpler in some explicitly specified respects. So idealized theories can then be treated as kinds of counterfactuals about abstract systems.

7 Idealized Systems as Fictions

What is important here then is that this raises the perplexing question of how it is possible for us to gather actual evidence that bears on the truth claims about what appear to be causally isolated abstract objects. Robert Stalnaker identified this general problem as it applies to counterfactuals, and we can refer to it here as *the epistemic access problem*. Stalnaker's sketch of a solution to this problem is based on the following observation:

[14] See Chorin and Marsden (1990) and Tritton (1977) for a more detailed discussion of this case.

> It is because counterfactuals are generally about possible worlds which are very much like the actual one, and defined in terms of it, that evidence is so often relevant to their truth (Stalnaker 1968, 53).

Notice that this worry is very much like the problem about our access to abstracta raised by Benacerraf. However, this very general and schematic suggestion concerning the manner in which counterfactuals are confirmed is in need of some elaboration, but it does promise to allow for a reasonable naturalistic solution to the Koyré problem. The core contention involved in this solution is that non-actual possible worlds and their contents are (partially) fictions and that fictions are artifacts created by the operation of human imagination. So, the solution offered here is closely related to the view of the metaphysics of fiction developed by Tomasson (1999).[15]

The final step needed for the development of this suggestion about the methodology of the empirical sciences involves a mild expansion of the typical conception of what counts as an observation statement. But this view is broadly consonant with empiricism and requires only that we understand the faculty of imagination as something like a fallible form of internal perception that is intimately connected to our more well-understood visual systems. However, this expansion of the category of observation reflects the actual scientific accounts that we currently have of image-like thinking and its roles in both science and mathematics.[16] As such, this maneuver accords well with typical forms of epistemological naturalism.

Two things are then of great importance here. The first is that there is nothing to this account of our knowledge of scientific abstracta involved in the empirical sciences that needs trouble naturalists too much, at least when properly understood. The second is that this approach to the problem of our knowledge of abstract objects is that it allows for a uniform treatment of the Benacerraf problem and the Koyré problem. So, on those bases it is to be preferred to both the quasi-Aristotelian naturalistic alternative and the Platonistic rationalist alternative.

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[15] See Sainsbury (2010) for a useful discussion of factionalism.

[16] See Pylyshyn (2003) for example.

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