

Necessity, a Leibnizian Thesis, and a Dialogical Semantics

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Abstract

In this paper, an interpretation of “necessity”, inspired by a Leibnizian idea and based on the method of dialogical logic, is introduced. The semantic rules corresponding to such an account of necessity are developed, and then some peculiarities, and some potential advantages, of the introduced dialogical explanation, in comparison with the customary explanation offered by the possible worlds semantics, are briefly discussed.

Keywords: Dialogical logic, Necessity, Strict Implication, Leibniz.

Introduction

The objective of the present note is to propose a dialogical explanation of *necessity*. Nowadays it seems to be very difficult to think about the modalities necessity and possibility outside of the possible worlds framework. However, it is still questionable whether such a framework is really to reveal the meaning of these modalities or it is just a technical method to serve as a semantics. If the latter is the case we should be careful not to consider what follows from our framework as the metaphysical characters of the notions in question. The fact that the possible world semantics (PWS hereafter) as a logical tool is very powerful is doubtless. Nevertheless, in order it to be taken as sufficient for the meaning explanation, something more than being technically fruitful is required. There are some philosophical arguments for or against the genuineness of PWS and its proper formulation. One notable alternative is the inferentialist approach to modality in which the meaning of, say, necessity is supposed to be given through an introduction rule. Such an approach is developed for example by Stephen Read [9] [8]. However, this approach has its own philosophical problems, more importantly it somehow equates necessity with the logical necessity, namely it assigns necessity only for those formulas which are valid according to some logical rules, thus denies any primitive significance for

necessary judgments. Besides, inferentialism uses a kind of “labelled” deductive system in order to develop a semantics for modality while such a system does reflect a kind of possible world attitude.¹

My aim is to offer an explanation, and correspondingly a method to serve as semantics, for necessity which is able to properly represent the intention behind this modality. In this respect I invite the reader not to think in terms of possible world semantics about necessity and possibility while studying this note, that is, provisionally suspend such an account. If you keep in mind the PWS account of necessity and expect some new technical results, there is nothing remarkable in this note. However, it would be an achievement for this note to demonstrate that there is at least an alternative way to understand modalities, and there is no need to assume PWS as intrinsic to the concept of necessity and possibility. The approach for which I am going to argue is that of dialogical logic.

The idea behind the dialogical logic, introduced by Lorenzen and Lorenz, is to explain the logical connectives by means of assigning to them certain roles in a dialogue. The standard dialogical logic had been designed to formulate the intuitionistic logic. Afterwards the method was extended to cover the classical logic and some other logics as well (by Lorenz, Rahman and others). Also, a dialogical semantics for modal logic has been developed by Rahman and Rückert [7]. However, the main idea of that work is rather to adapt the PWS into the dialogical framework, so that, though very fruitful in certain respects, it rather follows the PWS interpretation of modalities instead of offering a proper explanation. In the present investigation, I am going to use the framework of dialogical logic to develop a modal logic and to explain the nature of necessity, beginning with a Leibnizian account of this notion—which will be explained below.

I begin with a brief discussion to explain the Leibnizian thesis. Then I will briefly introduce the dialogical semantics. In the next part I will formulate the new dialogical rules. I will sketch out the main characteristics of the logic so obtained. It will be similar to **S4** with both classical and intuitionistic version. At the end I discuss the philosophical implications of the idea so explained. I will argue for the appropriateness of the introduced interpretation of necessity in respect with the intention behind this modality as it is recalled in the course of reasoning and argumentation.² I will also mention some possible, further works in the line of the offered approach in order to develop a comprehensive dialogical framework to deal with modalities and the issues around them.

1 Thesis: A Leibnizian account of necessity

Leibniz in several occasions defines necessity in contrast to contingency in a way that can be formulated as the following:

Necessary truths are those that are derivable, through finite steps, from the fundamental truths; while the steps of derivation for the contingent truths are infinite.³

It is quite common to understand “necessity” as the unconditional truth, namely that kind of truth which is unchallengeable according to its nature and according to the rules of reason. If we call those unconditional truths fundamental truths, according to Leibniz, anything derivable from such truths is also necessary.⁴ If we want to formulate such an idea in the object language, we will have that if q is a fundamental truth, p would be necessary provided that:

$$q \rightarrow_* p$$

The connective \rightarrow_* cannot be replaced with a material implication (\rightarrow). In both classical and intuitionistic logic we have the following axiom:

$$p \rightarrow (q \rightarrow p) \tag{1}$$

It says that if some proposition is true it is implied by any arbitrary proposition. Now if we take that notion of derivability which is used in the aforementioned Leibnizian thesis as represented by such a notion of implication used in the above axiom, every truth would turn out to be necessary, for a true proposition would be implied by any proposition including the fundamental truths, thus it would be considered as necessary. Therefore, we need another conditional than what validates (1).⁵

The formula (1) is sometimes called *the positive paradox of material implication*. Here, I am not claiming that it is “paradoxical”. Instead I argue that the material implication, due to the fact that it validates (1), is not capable to be used in formalizing the above mentioned idea in order to represent a logical definition of necessity. Here we need another kind of implication. It could be a *relevance implication*, since one of the motivations of relevance logic is to avoid (1). However, the connective \rightarrow_* is not a relevance implication for here we do not care about the relevance between antecedent and consequent. All we need is that in order to derive p we do not use something other than q (the idea is that we do not refer to the facts but to some fundamental truths); and it is possible that there would be no need to q itself so that p can be irrelevant to q . As it will be discussed in the following, the implication that we should use in lieu of \rightarrow_* is *strict implication*.⁶

The relation between strict implication and the modalities is well known. However, it is customary to take necessity or possibility as primitive then define strict implication by means of them. Accordingly it is usual to give a possible world semantics for strict implication. My approach here is the other way

around. I think, in accordance with the Leibnizian thesis, the notion of strict implication has priority over necessity. Moreover, I argue that the dialogical approach could better clarify the nature of strict implication and serve as a semantics for it in a way which is of less metaphysical presuppositions than the other alternative approaches.

2 Strict implication

The key point in our required relation is that when we say φ is strictly derivable from ψ , we mean that in order to derive φ we are allowed, if needed, only to use the claims stated by ψ ; we are not allowed to use other hypothesis or to look at the truths given outside of the assumption; however there is no obligation to use the assumption. In the other words, in our strict implication the consequence in order to be true does not need something more than the antecedent.

In order to stress the difference between material implication and the strict one, we may analyze the following formulas:

$$(\psi_1 \rightarrow (\psi_2 \rightarrow \varphi)) \rightarrow ((\psi_1 \wedge \psi_2) \rightarrow \varphi) \quad (2)$$

$$((\psi_1 \wedge \psi_2) \rightarrow \varphi) \rightarrow (\psi_1 \rightarrow (\psi_2 \rightarrow \varphi)) \quad (3)$$

Let us read the formula 2 as saying that if it is true that if ψ_1 holds then ψ_2 would imply φ , then if both ψ_1 and ψ_2 hold then φ would also hold. This is quite intuitive. Now let us see the conditional from the other side, namely the formula 3. Assume that if both ψ_1 and ψ_2 hold then φ would also hold, now would it imply that if ψ_1 is true then ψ_2 implies φ ? This also seems to be true. However if we use the notion of assumption, instead of premises, the case for the formula 3 would be different: it says if from the assumption that both ψ_1 and ψ_2 hold it is implied that φ holds, then from the assumption ψ_1 we would have that φ is implied by ψ_2 . If we accept this latter reading and take formula 3 as valid then we are using the material implication. However if by implication we understand a strict one, formula 3 would not be in general valid, for from the fact that A is strictly derivable from *both* B and C, it can not be concluded that if B holds then A would be strictly derivable from *only* C. So both material implication and strict implication validate the formula 2, but the formula 3 is admitted for the general implication and not for the strict one.

Assume that the formula 3 is valid and substitute ψ_1 by p , ψ_2 by q and φ again by p , so we will have:

$$((p \wedge q) \rightarrow p) \rightarrow (p \rightarrow (q \rightarrow p)) \quad (4)$$

The antecedent, namely $(p \wedge q) \rightarrow p$ is evident, so as a consequence we have:

$$p \rightarrow (q \rightarrow p) \quad (5)$$

It means that the above formula is valid for material implication; and it is usually considered as one of the paradoxes of material implication.⁷ Indeed it is one of the Hilbert's axioms for the classical logic and also one of the Heyting's axioms for the intuitionistic logic. Then as a criterion we can say that after formulating a strict implication according to our motivation, it should not validate the above formula. It is important to notice that in order to introduce a strict implication there is no need to revise the whole logical system. We can have a conservative extension of either classical logic or intuitionistic one. This will be clear after explaining our semantic rules.

3 A short introduction to dialogical semantics

The dialogical logic, introduced by Paul Lorenzen and then developed by Kuno Lorenz and others, is a framework to yield semantics for the logical systems; and also it can be seen as providing meaning-explanations for the logical elements. In dialogical semantics there are two parties, the proponent **P** and the opponent **O**. The proponent introduces a thesis and defends it against the attacks of the opponent. If there is a winning strategy for the proponent in respect to a statement, that statement is valid. The attacks and responses are to be performed according to two kinds of rules, particle rules and structural rules. Particle rules concern local moves, namely they determine that how each form of complex formulas can be attacked and defended. Structural rules govern the whole dialogue and determine the rights and the obligations of each party and that how a dialogue terminates and who is the winner. For a detailed explanation see, for example, [4] and [10].

The point is that such distinctions makes us able to deal with the meaning and the truth of an expression separately. The method grants that the meaning of a logical connective is not confused and is not dependent on the truth or falsity of its terms. The philosophical advantages of the dialogical approach, from a phenomenological point of view, have been discussed by the author in [13]. For a good overview of some philosophical features of dialogical logic see [11].

In what follows I introduce the rules for intuitionistic and classical logic, then I present the new rules for necessity which can be added to both systems..⁸

Particle Rules For any logical connective there is a particle rule which determines how to attack and defend a formula with a specific main connective.

These rules are standard in the literature:

	Attack	Response
$A \vee B$	$?_{\vee}$	A , or B (The defender chooses)
$A \wedge B$	$?_L$, or $?_R$ (The attacker chooses)	A , or B (respectively)
$A \rightarrow B$	A	B
$\neg A$	A	(No possible respond)
$\forall x A$	$?_{\forall x/c}$ (The attacker chooses c)	$A[x/c]$
$\exists x A$	$?_{\exists x}$	$A[x/c]$ (The defender chooses c)

Structural Rules Structural rules determines the structure of the interaction which form a certain argumentation. We have:

(SR-0) Starting Rule: The Proponent begins by asserting a thesis.

(SR-1) Move: The players make their moves alternately. Each move, with the exception of the starting move, is an attack or a defense.

(SR-2) Winning Rule: Player **X** wins iff it is **Y**'s turn to play and **Y** cannot perform any move.

(SR-3) No Delaying Tactics Rule: Both players can only perform moves that change the situation.⁹

(SR-4) Formal Rule: **P** cannot introduce any new atomic formula; new atomic formulas must be stated by **O** first. Atomic formulas can never be attacked.

(SR-5c) Classical Rule: In any move, each player may attack a complex formula uttered by the other player or defend him/herself against *any attack* (including those that have already been defended).

(SR-5i) Intuitionistic Rule: In any move, each player may attack a complex formula uttered by the other player or defend him/herself against *the last attack that has not yet been defended*.

Perhaps it would be good to give a simple example to show how this semantics works.¹⁰ Let us examine the formula $p \rightarrow (q \rightarrow p)$.

Remark on notation: The moves are brought in the order of utterance. The parenthesized information indicates the number of the move, whether it is attack (@) or response (Ⓙ) and to which move it is attack or response. The long dash indicates that there is no further move; and the participant on whose side it appears has been lost.

O		P	
		$p \rightarrow (q \rightarrow p)$	(0)
(1@0)	p	$q \rightarrow p$	(2Ⓙ1)
(3@2)	q	p	(4Ⓙ3)
	—————		

Here in the move 4, **P** is able to respond the attack, because **O** has asserted p before; and since there is no other move possible for **O**, **P** wins and the formula is demonstrated to be valid.

4 A new rule for the strict implication

In the following I use the notation \rightarrow to indicate the strict implication. As we saw, the dialogical rule for the material implication is based on the idea that if one asserts an implication then one should be able to assert the consequent if the challenger asserts the antecedent. Now I follow this idea and I add a restriction that once a strict conditional is challenged by asserting the antecedent, the defender is not allowed to refer to what have been asserted before this challenge, but he/she is allowed to use what have been accepted to be strictly implied by any thing granted by the antecedent. The rule corresponding to this observation goes as follows:

A formula in the form $\psi \rightarrow \phi$, i.e. strict implication, can be attacked by asserting ψ and the response is to assert ϕ ; but there is an extra condition:

*If in the move n , **X** asserts a strict implication and **Y** attacks it in the move m , $m > n$, then in the following moves **X** could not refer to any move l , $l < m$, of **Y** except that it was a strict implication; if **X** is the proponent this also means that he is not allowed to use an elementary assertion which is made by the opponent only before the move m .¹¹*

We said that the defender of a strict implication can not refer to any move before the attack unless it was itself a strict implication. This grants that if

X is able to assert α and if **Y** has accepted that β is strictly implied by α , **X** would be able also to request β and use it.

Now we can see the invalidity of the formulas 5:

O		P	
		$p \multimap (q \multimap p)$	(0)
(1@0)	p	$q \multimap p$	(2⊠1)
(3@2)	q	—————	

P has no other move because he can not use the elementary assertion p since it is asserted before the move 3, and also no counterattack is possible, then he loses.

Another formula which is problematic in respect with strict implication, $\neg p \rightarrow (p \rightarrow q)$, can be demonstrated, as it ought to be, that is not valid according to our rule:

O		P	
		$\neg p \multimap (p \multimap q)$	(0)
(1@0)	$\neg p$	$p \multimap q$	(2⊠1)
(3@2)	p	—————	

P loses because he can not use the elementary assertion p in order to attack the move $\neg p$ since it is asserted before the move 3; thus **P** has no winning strategy and the formula is not valid.

However a self-implication in the form $(p \multimap q) \multimap (p \multimap q)$ is valid as it should be:

O		P	
		$(p \multimap q) \multimap (p \multimap q)$	(0)
(1@0)	$p \multimap q$	$p \multimap q$	(2⊠1)
(3@2)	p	p	(4@1)
(5⊠4)	q	q	(6⊠3)
	—————		

Here in contrast to the previous dialogue, **P** can use the elementary assertion p in order to attack the move 1 because, although it is asserted before the move 3, it is a strict assertion; thus **P** wins and the formula is valid.

Although $\neg p \rightarrow (p \rightarrow q)$ is not valid, *ex falso* in the form $(\neg p \wedge p) \rightarrow q$ is valid for strict implication. This latter sometimes is considered as one of the paradoxes of the strict implication. However, there is nothing unintuitive here. Any argument in favor of *ex falso* for material implication should also work for the strict implication, for such arguments would progress in a way to show that from the absurdity *itself*, or from the contradiction itself, any arbitrary proposition follows; it means that the connection here is strict. For the strict implication it is not true that from any arbitrary false proposition every thing follows, but the original form of *ex falso* which is to say if absurdity holds then any arbitrary proposition would be true is valid. There are some arguments in the literature to show that *ex falso* is not valid at all, which are very interesting discussions in their own place, but here we can not deal with them and I just take the mentioned principle as correct.¹² My point is that if *ex falso* is to be considered as valid for the material implication, then it is also valid for the strict implication. Remember that the intention behind the strict implication was to reject the formula 3, therefore although we accept the following

$$(\neg p \wedge p) \rightarrow q$$

it does not lead us to accept the validity of the followings:

$$\neg p \rightarrow (p \rightarrow q)$$

$$p \rightarrow (\neg p \rightarrow q)$$

That one of these latters is not valid has been shown above. The other goes in a similar way. Here I demonstrate the validity of *ex falso* itself:

O		P	
		$(\neg p \wedge p) \rightarrow q$	(0)
(1@0)	$\neg p \wedge p$	$?_L$	(2@1)
(3@2)	$\neg p$	$?_R$	(4@1)
(5@4)	p	p	(6@3)

Here although **P** is not able to respond the attack posed by the move 2, he is able to counterattack; and finally there is no possibility for **O** to respond and since no counterattack is possible she loses. Therefore the thesis, namely the principle of *ex falso* is valid.

Another phenomenon which is sometimes called as a paradox of strict implication is that if ψ is a valid formula then for any arbitrary p , $p \rightarrow \psi$ is also

valid. However, there is nothing wrong here and this phenomenon does completely fit our intention behind the strict implication. Indeed our semantics must grant this phenomenon and it does so. Notice that by strict implication we intend that the consequent does not need more than the assumption to be true. Now if ψ is valid, namely it is a formal truth, then in the formula $p \rightarrow \psi$, it does not need anything other than p to be true, neither does it need p itself. It means that if we accept that something is true only according to the rules of pure reasoning, it should be true under any assumption.

Here I show the dialogue for the case of $p \rightarrow p$. This is valid in general, so $q \rightarrow (p \rightarrow p)$ must be valid:

O		P	
		$q \rightarrow (p \rightarrow p)$	(0)
(1@0)	q	$p \rightarrow p$	(2@1)
(3@2)	p	p	(4@1)

In general it is easy to see that if **P** has a winning strategy for ψ , for any arbitrary q he will have a winning strategy for $q \rightarrow \psi$.

It is worth mentioning that the strict implication is transitive, namely the following formula is valid:

$$(p \rightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r))$$

The dialogue to demonstrate the validity of this formula is brought in the appendix.

It is worth that the material implication and the strict implication can both be used in a same logical system and they are not reducible to each other; a strict implication implies a material implication and not vice versa. Let's see the dialogues.

$(p \rightarrow q) \rightarrow (p \rightarrow q)$ is valid:

O		P	
		$(p \rightarrow q) \rightarrow (p \rightarrow q)$	(0)
(1@0)	$p \rightarrow q$	$p \rightarrow q$	(2@1)
(3@2)	p	p	(4@1)
(5@4)	q	q	(6@3)

$(p \rightarrow q) \rightarrow (p \rightarrow q)$ is not valid:

O		P	
		$(p \rightarrow q) \rightarrow (p \neg q)$	(0)
(1@0)	$p \rightarrow q$	$p \neg q$	(2@1)
(3@2)	p	_____	

Now, having explained the logical character of the strict implication, I shall go to investigate the logical character of the fundamental truths in order to formulate necessity.

5 A rule for the fundamental truths and a logical definition for necessity

Now, we have the half of what we need in order to formulate necessity according to the Leibnizian thesis. The other half we need is a logical representation of the canonical truths. It is quite easy. In our dialogical semantics a complex formula can be challenged according to the particle rules, and atomic formulas are not challengeable but the proponent can not assert them unless the opponent has asserted them before. Now the fundamental truths according to their nature are agreed upon and thus unchallengeable. Let us introduce a new constant to represent the set of fundamental truths: “+”. Such a constant should be treated as an atomic formula in the course of a dialogue since no attack is allowed against it. However it has a peculiar feature that the proponent can assert it every where in the dialogue, if according to other rules he needs such an assertion, without it necessarily having been asserted by the opponent. Therefore, the semantic rule concerning + is this:

*Both the proponent and the opponent are permitted to assert +.
No attack against it is possible.*

Now, as it was our intention, we define necessity as follows:

$$\Box p \equiv (+ \neg p) \tag{6}$$

In formal dialogues we deal with + as if it is an elementary proposition, granted by the both parties, and we do not care about its content. However, as mentioned above it is the conjunction of all fundamental truths. In regard to the latter we should say that they all are, according to Leibniz, identity judgments. There are two kinds of truths, truth of reason and truth of fact. The two kinds are finally based upon two main principles, that is, respectively, the principle of identity (or that of non contradiction) and the principle of sufficient reason. According to Leibniz, all truths of reason, namely the fundamental or

eternal truths, are based upon identities (of monads and of ideas) and thus belong to the realm of God’s understanding, whereas all truths of facts depend on, besides the eternal truths, God’s will. Therefore, any necessary truth can be traced back to an identity. In our notation, $+$ is the collection of those identities. However we don’t need to analyze the notion of identity itself here as far as we are concerned with the formal dialogues and thus with the formal definition of necessity.¹³

There is a constant which may seem similar to our new constant $+$, i.e. \top . However, in regard to their meanings they are different, though the logical rule corresponding to them are the same and they are logically equivalent. \top may read “there is a truth”, while $+$ is the set of all fundamental truths. It is easy to see that we have both $+\rightarrow\top$ and $\top\rightarrow+$, so as I said they are logically equivalent. Nevertheless, in regard to the dialogical framework their meanings are different. As far as we perform a *formal* dialogue they may be used technically interchangeably, but in a *material* dialogue which is to examine a logical consequence of some given premises in a specific context, $+$ must be declared as the basic truths of the field, beside other truths that are given as premises but not as necessary. Imagine a thought experiment about a physical problem; we set an assumptive situation then depart to examine a thesis. Here, beside the possible facts assumed, we should observe some principles as unchallengeable. So according to our rule these latter judgments can be recalled by the proponent everywhere in the dialogue, while other truths can not be recalled under an attack toward a strict implication. While the content of \top is a particular judgment “there is a truth”, or, as some would prefer to say, an arbitrary tautology, or, in some interpretations, the set of the truths of the theory within which the argumentation takes place, the content of $+$ is the set of all unchallengeable truths, which are taken as known or declared for every dialogue.

The fact that in a particular dialogue, or a particular argumentation, in this or that field, the participants only declare some fundamental truths of that field, namely some part of $+$ whose content concern the fundamental facts of that field, should not be understood as if we have different kinds of $+$ each for every particular field. Rather, $+$ is universal and it can be used every where, though if it would be required to be explicit before entering a dialogue, it is suitable to just mention those parts which would be related to the thesis under discussion.¹⁴

Now let us go back to the discussion on the consequences of the introduced account of necessity.

It is easy to show that necessity entails truth, namely the formula \mathbf{T} holds:

$$(+ \neg p) \rightarrow p$$

O		P	
(1@0) $+ \neg p$		(+ $\neg p$) $\rightarrow p$ (0)	
(3@2) p		+	(2@1)
(3@2) p		p	(4@1)
—————			

In the above dialogue, **P** is able to assert $+$ in the move 2 in order to counterattack **O**'s move 1; then she has no choice but to assert p and so the proponent use it to respond the first attack. There is no other move possible and **P** wins.

As it is already known strict implication should be equal to necessity of a material implication. That is:

$$(p \neg q) \equiv (+ \neg (p \rightarrow q)) \quad (7)$$

Or:

$$(p \neg q) \equiv \Box(p \rightarrow q) \quad (8)$$

The formula (8) is normally considered as the definition of the strict implication. But in the present approach, the strict implication has priority over necessity. And necessity is defined on the basis of the strict implication as stated in the formula (6). Therefore the formula (8), or its equivalent the formula (7), is not a definition and needs a proof if it is true. Indeed it is true and I bring the dialogue to demonstrate its validity in the appendix.

It can be shown that the modal system so defined validates **S4** (but not **B** nor **S5**). First of all our semantics grants the *Necessitation Rule*. That is, any theorem of logic is necessary. This can be seen from the fact, which we mentioned before, that if **P** has a winning strategy for ψ , for any arbitrary q he will have a winning strategy for $q \neg \psi$. Thus, if **P** has a winning strategy for ψ , he will have a winning strategy for $+ \neg \psi$. It means that if ψ is a theorem then so is $\Box\psi$. Also our semantics validates the principles of **K**: $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$, or $(+ \neg (p \rightarrow q)) \rightarrow ((+ \neg p) \rightarrow (+ \neg q))$. I bring the core dialogue to demonstrate this principle in table 3 of the appendix. Moreover our dialogical rules validates the axiom of **S4**, that is, $\Box p \rightarrow \Box\Box p$, or $(+ \neg p) \rightarrow (+ \neg (+ \neg p))$. This is shown in table 4 of the appendix.

In order to examine that whether the presented semantics is equal to **S4** we need completeness results; but I postpone this task to further works. My aim here was not to provide a semantics for an existing system, rather to introduce a new approach capable to be employed in order to explain the logical nature of necessity. Moreover, I focused on necessity; and although possibility may be defined in an easy way as $\neg\Box\neg$, it is not so favorable according to the attitude

that has motivated the current investigation. Possibility should be considered, at least in the first step, as having its own independent meaning. Indeed there are some significant notions recalled by the term “possibility” which are not able to be indirectly explained in term of necessity.¹⁵

However, if we just define possibility as $\neg\Box\neg$, what would be given is the classical version of **S4**, not **S5**.¹⁶ Let us see why the present dialogue semantics does not validate the **5** axiom, given that one defines \diamond as $\neg\Box\neg$ that is the **5** axiom reads $(\neg(+\text{-}\neg p)) \rightarrow (+\text{-}(\neg(+\text{-}\neg p)))$.

O		P	
		$(\neg(+\text{-}\neg p)) \rightarrow (+\text{-}(\neg(+\text{-}\neg p)))$	(0)
(1@0)	$\neg(+\text{-}\neg p)$	$+\text{-}(\neg(+\text{-}\neg p))$	(2@1)
(3@2)	$+$	$\neg(+\text{-}\neg p)$	(4@3)
(5@4)	$+\text{-}\neg p$	$+$	(6@5)
(7@6)	$\neg p$	—————	

Here **P** has no further move because he cannot attack the move 1 (he obviously cannot attack the move 7 since it requires to assert an elementary formula which has not been asserted by **O**) since he is under the attack toward a strict implication posed in the move 3 the he would be able to refer to the move 1 only if the main connective of that move would be strict implication but now it is negation.

The meaning of necessity explained in the discussed dialogical way would be same for both classical and intuitionistic logic; and it is, as always, the difference between a structural rule of them which causes the differences. Therefore the problem for the interpretation of intuitionistic modal logic which arises in the possible world semantics has no place here. By choosing (SR-5i) instead of (SR-5c) we may obtain intuitionistic version of, say, **S4**. One can use the rules for quantifier and develop a first-order modal logic. But I drop that job here and postpone it to further works. As the main objective of the present discussion is to sketch out the main idea of an alternative way of explanation of the logical behavior of necessity, I stop the formal studies here and go to clarify the insights that the introduced approach may bring forward.

6 Analysis: modality and dialogue

To summarize, in order to formulate necessity we appeal to the notion of fundamental truths and that of strict implication. Fundamental truths and those are strictly implied by them are necessary.

A necessary truth is not contingent and in this sense it is independent of the perceptual experience. However, we are not to say that a necessary truth is always a formal or a logical truth. The fundamental truths may not be,

and mostly they are not, analytical—in the Kantian sense. For example that 2 is greater than 1 is a fundamental truth, thus necessary. However, it is synthetic in the sense that it not a result of a mere logical proof. In any case it should be *a priori*, namely it is not the subject of perceptual examination. Nevertheless, this does not mean that they are independent of the experience in general, for they may be grasped by (categorical) intuition while this latter is a non-sensuous experience.¹⁷ Also we should be careful if we say that necessary truths are *a priori*, for they are so in the sense that once the essence of an object is grasped there is no need for further, *a posteriori*, examination to obtain those truths. However, this does not mean that *a priori* truths are there and can be obtained immediately and without any effort. Indeed most of *a priori* truths are gradually discovered as a science progresses. In that sense, as I have stated before, the content of + can be enriched in the course of time.

So we should be careful not to equate necessity with logical validity or analyticity, though every logical truth is necessary.

However, what is important is the role of necessity in the dialogue. We may understand this modality as a way we deal with a truth modified as necessary in the course of dialogue. The explanation goes as follows:

Imagine that we are to argue about a thesis. There are a set of premises which are agreed upon by the participants. However, we have two kinds of premises, a set of fundamental principles governing the field about which we are arguing, and a set of assumptions which are relevant to our current discussion. Now assume during the dialogue someone claims that A is necessary, and then the other asserts “O.k. show it!”. Here, the defender may no longer use the assertions made before this claim nor the assumptions, however he/she can use the fundamental principles since they are supposed to constitute all knowledge belonging to the field. So, necessity is conceived in relation to argumentation: necessary truths are those that can be recalled under any assumption, and can be proved under any assumption,¹⁸ whereas contingent truths should be suspended under some assumptions.

The rationale behind the two rules will be clear by considering a purely knowledge oriented dialogue. The implication is supposed to be treated in the way that if someone asserts an implication he/she is obliged to assert the consequence if the other party gives the antecedent. Now in the strict implication one may use only what are asserted after the assertion of the antecedent, namely any assumption which is asserted before the challenge should be suspended. However if the challenger has asserted some claims as unsuspendable, namely as necessary, the defender is allowed to recall them if needed. Accordingly if one claims that a judgment is necessary, one should be able to show it only using the fundamental truths and the claims that are accepted as necessary by the challenger. Following such an idea and formulating the rules to

represent them we have obtained a semantic framework to deal with a logic containing necessity without appealing to the notion of possible world or any model-theoretic framework and without reducing the notion of necessity to its proof-theoretic aspect.

7 Conclusion and further works

The dialogical logic provides us with a tool to understand the meaning of necessity and also to formulate it within a logical system. The PWS is not the indispensable way of interpretation of necessity, or other modalities; and the metaphysical problems arisen within it are not the innate problems pertained to the modalities themselves.

Here I just discussed the issue of necessity for the propositional logic. Nevertheless, it is plausible to follow the presented line of thoughts to formulate “possibility” (if there is an independent notion of it), first-order modal logic and the relation of identity. The key point is that instead of simulating a metaphysical model we begin with the role of the notions in question in the dialogue. Of course this is not to say that the PWS is flawed, rather that we should not overestimate its philosophical power however technically fruitful it is. The dialogical method is more appropriate when the objective is to explain the meaning of the logical issues and to represent their phenomenological characters.

Notes

¹ Even putting such an objection aside, one may still has doubts about the appropriateness of the inferentialist theory of meaning, doubts of the kind that have been explained by Prior.

²By “intention” I recall the idea of intentionality as explained within the transcendental phenomenology. Indeed the philosophical motivation behind the present work is the transcendental phenomenology, and the declination of PSW and inferentialism can be seen in this respect. For detailed phenomenological arguments against inferentialist and PWS (and in general model-theoretic) meaning explanations see [13].

³See *Monadology* paragraphs 33–36, [5, p. 646].

⁴For Leibniz every fact is ultimately based on the fundamental truths. However, those truths that we consider as contingent are involved in a continuity of uncountable, co-existing phenomena so that the chain of reasons behind them are infinite; and from the point of view of human being to trace back the reasons to the fundamental truths can not be done.

⁵One may object here that the validity of (1) is not due to the character of the material implication but to a structural rule; and then what should be modified in order the Leibnizian thesis not to fall in triviality is not the kind of implication. Such an argument goes as follows:

The validity of (1) relies on two things, namely the semantics of \rightarrow and weakening.

$p \vdash p$ (axiom)

$p, q \vdash p$ (weakening)

$p \vdash q \rightarrow p$ (\rightarrow intro)

$\vdash p \rightarrow (q \rightarrow p)$ (\rightarrow intro)

So, the mentioned argument claims, that perhaps what should be taken as problematic is the weakening not the semantic of the material implication; for there are other proofs in which the same semantic for the implication is used and it seems not to be problematic for our main thesis. For example:

$(p \wedge q) \vdash (p \wedge q)$ (axiom)

$(p \wedge q) \vdash p$ (\wedge elimination)

$\vdash (p \wedge q) \rightarrow p$ (\rightarrow intro)

We agree that $(p \wedge q) \rightarrow p$ is not problematic. In a close look we will see that the application of the introduction rule in the two proofs are different. In the former just one of the premises is chosen as the antecedent of the introduced implication, but in the former all premises (which is a single formula here) are brought as the antecedent. The difference is exactly here. I argue that for the Leibnizian account of necessity the former case would be problematic if we take the implication as representing the relation of derivability. There is no reason to think that weakening is undesirable. The problem is that the semantic of material implication is not suitable in order to formulate our idea about necessity. Thus we need another conditional. This does not mean that the material implication is unsuitable at all or that it should be replaced. Rather, we can expand our logic by adding a new connective so that necessity can be defined on the basis of this new connective.

⁶In some occasions Leibniz gives a seemingly different definition for the necessity. However, this second definition is also based on strict implication, and following the classical logic the two definitions would turn out to be the same. Since I prefer intuitionistic logic I have chosen the above one. The second definition can be stated as follows:

It is necessary what its contrary implies a contradiction.

Leibniz gives such a definition in the paragraph 13 of *Discourse on Metaphysics*:

In order to meet the objection completely, I say that the connection or consecution is of two kinds; one is absolutely necessary, whose contrary implies contradiction, and that deduction occurs in the eternal truths like the those of geometry; the other is necessary only ex hypothesi, and so to speak by accident, but in itself it is contingent since the contrary is not implied. This latter sequence is not founded upon ideas wholly pure and upon the pure understanding of God, but upon his free decrees and upon the processes of the universe.

[5, p. 310]

That is p is necessary if

$$\neg p \rightarrow_* \perp$$

But it should be obvious that what Leibniz means here by “implies” is the strict implication not the material one, for otherwise every truth would be necessary. Assume that p is true, so in accordance with an axiom of classical and intuitionistic logic (the formula 5 above), we will have:

$$\frac{p}{\neg p \rightarrow p}$$

From the other hand, by means of self-implication we have:

$$\overline{\neg p \rightarrow \neg p}$$

Putting these two together, it follows that:

$$\frac{p}{\neg p \rightarrow (p \wedge \neg p)}$$

It means that if p is true then its negation would lead in a contradiction. So if the implication in the above definition is understood as the material implication, every truth should be considered as necessary. This shows that we really need a special kind of implication which blocks such a triviality. It should be clear that Leibniz's intention is that kind of implication which does not use other than the assumption declared. For example, if from the assumption that the sky is not blue, and not using any other information, a contradiction is derived then that the sky is blue would be necessary. So, both definitions, though they may seem different, are based on a same and more fundamental notion which is the strict implication. However, this latter definition is primarily to define $\Box\neg\neg p$. Since in the classical logic we have $\neg\neg p \leftrightarrow p$ this would be equal to $\Box p$. But in the intuitionistic logic we only have $p \rightarrow \neg\neg p$, so there is a difference between the two definitions and the one I have chosen is the appropriate one if we want to work within the intuitionistic attitude.

⁷Though there is no paradox by itself here. Paradox would arise if one takes the material implication as the strict one, namely confuses the two and expect that it satisfies the peculiarities of the strict implication.

⁸The following rules are standard within dialogical studies. However in the current manner of presentation I particularly benefited from the representations given in [6].

⁹In order to observe this rules it should be determined that each party how many times may repeat a same attack (repetition of defense is redundant in any case and it is not allowed). This is called *repetition ranks*. Clerbout [2] has shown that it would be sufficient to assign the rank 1 to the opponent and 2 to the proponent, namely there would be a winning strategy for a formula if and only if there is a winning strategy for that formula while the proponent is allowed to attack twice against a same move and the opponent is allowed to do so only once. Therefore, I do not specify the ranks in the following dialogues, and one can suppose that it is 1 for **O** and 2 for **P**.

¹⁰In the following dialogues I just sketch out the core play which can be used to build a winning strategy. For the sake of simplicity I do not discuss all possible plays to show that in any case there is a winning strategy for **P** (or for **O** if I show the thesis is not valid); it is not difficult to show and I omit it in order to focus on the core of the argument.

¹¹We know that in general the proponent can not assert an atomic unless it has been asserted by the opponent before. Now if the opponent attacks a strict implication asserted by the proponent, the proponent is obliged to *forget* all atomics asserted before this attack.

¹²Beside the attitude of paraconsistence logicians, there are arguments within intuitionistic tradition to reject the validity of *ex falso*, see for instance [15]. However for the reasons, that I discuss in [13], I think that the mentioned principle is indeed justified.

¹³As it is clear there are significant difficulties around the notion of identity; and it is beyond the task of this paper to deal with them. I tried to put first steps to clarify some of those difficulties in [12].

¹⁴In relation to the mentioned point, I shall emphasize that the definition of necessity suggested here has nothing to do with the idea of *relative necessity* which is known in the relevant literature. Although the formula (6) may seem to be similar to the definition of relative necessity introduced by Smiley [14], we should notice that both in regard to the motivation behind them and to their proper meanings, they are essentially different. First of all, Smiley just take as primitive a notion of absolute necessity and then define relative necessity on the basis of it. I do not want here mention the possible philosophical objections against such an idea nor the formal problems of the kinds that are shown by Humberstone [3]. However, it would be worth mentioning that Humberstone's own alternative is based on

the possible world semantics, then again the philosophical conception he develops is not that much different from the customary one.

Another thing which perhaps I should stress concerns the notion of absolute necessity. For those familiar with the concept of relative necessity, absolute necessity is taken to be equal with the logical necessity. This is not the case for the notion of necessity as understood in this paper. To explain, recall the difference between a physical truth and a logical truth. The fact that we have a logic in which we can also deal with physical truths does not convert them to logical truths. For a truth to be logical means that it is obtainable only according to the logical rules. The case is the same for necessity. We introduce a logic to deal with necessity, but this does not mean that the necessity at work is merely the logical necessity.

¹⁵For instance, under the term “possible” we may intend the notion “conceivable” or “intelligible”; also we may use it as for the notion of “imaginable”. Yet imaginability and conceivability can be considered as having their own logical characteristics alongside with that of possibility; and in neither case there is no such a straightforward definition on the basis of necessity. A fruitful discussion about the conceptual difference between possibility, imagination and conception can be found in [1].

¹⁶In the standard classical version of **S4** we have the theorem $\diamond\diamond p \rightarrow \diamond p$, beside the theorem $p \rightarrow \diamond p$. In the appendix I bring dialogues to demonstrate that the present system validates the mentioned formulas, once one choses to define \diamond as $\neg\Box\neg$.

¹⁷Notice that Leibniz’s use of the word analytic is different from that of Kant and that in our time. Leibniz uses this word in order to indicate that a truth is not result of a factual coincidence but it is intrinsic in its subject. In this sense also Kant would admit that, for example, $5+7=12$ is analytic. But Kant uses the term analytic when we just work with formal definitions and formal relations. So he considers $5+7=12$ as synthetic for we deal with the contents, although he admits that the truth of the mentioned claim rests only on the contents of its subjects. We follow Kantian terminology and say, in a complete agreement with Leibnizian attitude, that there are synthetic necessities.

¹⁸One may say they are unsuspendable and indeed serve as ground in an objectively oriented argumentation.

Appendix

1

$$(p \multimap q) \multimap ((q \multimap r) \multimap (p \multimap r))$$

O		P	
		$(p \multimap q) \multimap ((q \multimap r) \multimap (p \multimap r))$	(0)
(1@0)	$p \multimap q$	$(q \multimap r) \multimap (p \multimap r)$	(2@1)
(3@2)	$q \multimap r$	$p \multimap r$	(4@3)
(5@4)	p	p	(6@1)
(7@6)	q	q	(8@3)
(9@8)	r	r	(10@5)

Here in the moves 6 and 8, **P** is able to attack, respectively the moves 1 and 3 because they are strict implications.

2-1

$$(p \neg q) \rightarrow \Box(p \rightarrow q)$$

O		P	
		$(p \neg q) \rightarrow (+ \neg (p \rightarrow q))$	(0)
(1@0)	$p \neg q$	$+ \neg (p \rightarrow q)$	(2@1)
(3@2)	$+$	$p \rightarrow q$	(4@3)
(5@4)	p	p	(6@1)
(7@6)	q	q	(8@5)

2-2

$$\Box(p \rightarrow q) \rightarrow (p \neg q)$$

O		P	
		$(+ \neg (p \rightarrow q)) \rightarrow (p \neg q)$	(0)
(1@0)	$+ \neg (p \rightarrow q)$	$p \neg q$	(2@1)
(3@2)	p	$+$	(4@1)
(5@4)	$p \rightarrow q$	p	(6@5)
(7@6)	q	q	(8@3)

3

$$\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$$

O		P	
		$(+ \neg (p \rightarrow q)) \rightarrow ((+ \neg p) \rightarrow (+ \neg q))$	(0)
(1@0)	$+ \neg (p \rightarrow q)$	$(+ \neg p) \rightarrow (+ \neg q)$	(2@1)
(3@2)	$+ \neg p$	$+ \neg q$	(4@2)
(5@4)	$+$	$+$	(6@1)
(7@6)	$p \rightarrow q$	$+$	(8@3)
(9@8)	p	p	(10@7)
(11@10)	q	q	(12@5)

4

$$\Box p \rightarrow \Box \Box p$$

O		P	
		$(+ \neg p) \rightarrow (+ \neg (+ \neg p))$	(0)
(1@0)	$+ \neg p$	$+ \neg (+ \neg p)$	(2@1)
(3@2)	$+$	$+ \neg p$	(4@3)
(5@4)	$+$	$+$	(6@1)
(7@6)	p	p	(8@5)

5

$$p \rightarrow \Diamond p$$

O		P	
		$p \rightarrow \neg(+ \neg \neg p)$	(0)
(1@0)	p	$\neg(+ \neg \neg p)$	(2@1)
(3@2)	$+ \neg \neg p$	$+$	(4@3)
(5@4)	$\neg p$	p	(6@5)

6

$$\Diamond \Diamond p \rightarrow \Diamond p$$

O		P	
		$\neg(+ \neg \neg(+ \neg \neg(+ \neg \neg p))) \rightarrow \neg(+ \neg \neg p)$	(0)
(1@0)	$\neg(+ \neg \neg(+ \neg \neg(+ \neg \neg p)))$	$\neg(+ \neg \neg p)$	(2@1)
(3@2)	$+ \neg \neg p$	$+ \neg \neg(+ \neg \neg p)$	(4@1)
(5@4)	$+$	$\neg(\neg(+ \neg \neg p))$	(6@5)
(7@6)	$\neg(+ \neg \neg p)$	$+ \neg \neg p$	(8@7)
(9@8)	$+$	$\neg p$	(10@9)
(11@10)	p	$+$	(12@3)
(13@12)	$\neg p$	p	(14@13)

References

- [1] J. Y. Beziau. Possibility, imagination and conception. *Principios*, 23, 2016.
- [2] N. Clerbout. Finiteness of plays and the dialogical problem of decidability. *IfCoLog Journal of Logics and their Applications*, 1:1:115–130, 2014.
- [3] L. Humberstone. Relative necessity revisited. *Reports on Mathematical Logic*, 13:33–42, 1981.
- [4] L. Keiff. Dialogical logic. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2011 edition, 2011.
- [5] G. Leibniz. *Philosophical Papers and Letters*, Ed. L. E. Loemker. Kluwer Academic Publishers, 1989.
- [6] S. Rahman and L. Keiff. On how to be a dialogician. In Daniel Vanderveken, editor, *Logic, Thought and Action*, volume 2 of *Logic, Epistemology, and the Unity of Science*, chapter 17, pages 359–408. Springer Netherlands, 2005.
- [7] S. Rahman and H. Rückert. Dialogische modallogik (für t, b, s4, und s5). *Logique Et Analyse*, 167(168):243–282, 1999.
- [8] S. Read. Harmony and modality. In C. Dgremont, L. Kieff, and H. Rückert, editors, *Dialogues, Logics and Other Strange Things: Essays in Honour of Shahid Rahman*, pages 285–303. College Publications, 2008.
- [9] S. Read. Necessary truth and proof. *Kriterion: Revista de Filosofia*, 51(121), 2010.
- [10] J. Redmond and M. Fontaine. *How to Play Dialogues: An Introduction to Dialogical Logic*. College Publications, 2011.
- [11] H. Rückert. Why dialogical logic? In H. Wansing, editor, *Essays on Non-Classical Logic*, pages 165–185. River Edge: World Scientific, 2001.
- [12] M. Shafiei. A higher-order dialogical logic to demonstrate Leibniz’s law of identity of indiscernibles. *Journal of Humanities of Valparaiso*, 0(9):73–88, 2017.
- [13] M. Shafiei. *Meaning and Intentionality; A Dialogical Approach*. College Publications, forthcoming.

- [14] T. Smiley. Relative necessity. *Journal of Symbolic Logic*, 28(2):113–134, 1963.
- [15] M. van Atten. The hypothetical judgement in the history of intuitionistic logic. In C. Glymour, W. Wang, and D. Westerstahl, editors, *Logic, Methodology, and Philosophy of Science XIII: Proceedings of the 2007 international congress in Beijing*, page 122–136, London, 2009. College Publications.

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