# Phonetic Possibility and Modal Logic

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## ABSTRACT

In this paper I propose a formalization, using modal logic, of the notion of possibility that phoneticians use when they judge speech sounds to be possible or impossible. I argue that the most natural candidate for a modal logic of phonetic possibility is the modal system T.

## I. INTRODUCTION[1]

Phoneticians sometimes make use of a notion of possibility that is specific to the science of phonetics. This notion comes into play when phoneticians judge various kinds of speech sounds to be possible or impossible. For example, there are cells in the IPA Chart [1] that are left empty, without a symbol, because the sounds which would go into those cells are deemed to be impossible to produce [1, p. 9].

This phonetic notion of possibility, like other notions of possibility found in science and philosophy, should be amenable to treatment by the methods of modal logic -- the branch of logic that deals with possibility, necessity, and kindred concepts [2]. In this paper I will sketch the beginnings of such a treatment. I will propose an analysis of the concept of phonetic possibility, and will use this analysis to decide which system of modal logic might capture the notion of phonetic possibility.

## II. PHONETIC POSSIBILITY: WHAT IS IT?

Philosophers and logicians recognize several different conceptions of possibility. Among the best-known of these conceptions are logical possibility and physical possibility. Is phonetic possibility reducible to any of these standard kinds of possibility, or is it something different? What kind of possibility might phonetic possibility be?

Any attempt to answer these questions must first reckon with the fact that phonetic possibility, as usually conceived, is a property of speech sounds. For example, a phonetician might say that a particular cell in the IPA consonant table corresponds to an impossible pair of sounds. (Such cells do exist in the table; see [1, p. 9].) This means that it is impossible for a speaker (or at least for a speaker with a reasonably standard vocal apparatus) to produce any sounds having the features specified by that cell in the chart.

The set of sounds with that particular combination of features is empty -- not only for contingent reasons, as if no speaker had happened to produce a sound of that kind yet, but because it is *impossible* for speakers to produce such a sound.

Typically, sounds of a certain kind are deemed to be possible if and only if sounds of that kind can be produced by speakers who have the vocal capacities typical of most human beings. Speakers with certain disorders might find some commonplace sounds to be impossible, but that fact does not count against the judgment that those sounds are possible. Similarly, there might conceivably be a single speaker with an idiosyncratic anatomy who is able to produce a sound normally deemed impossible. This fact would not count against the impossibility of that sound. Thus, the characterization of sounds as possible or not presupposes a choice of a class of typical or statistically "normal" speakers. The notion of phonetic possibility can only be defined relative to a class of speakers, whether or not this relativization is made explicit. If one wants to define the notion of phonetic possibility precisely, then one must first pick out a class of speakers. Let us call this selected class of speakers the class of *N-speakers*. Different choices of a class of N-speakers ultimately will lead to different judgments as to what is phonetically possible. For any judgment about the phonetic possibility of a sound, there is a class of N-speakers which either tacitly or explicitly forms a basis for that judgment. This class may or may not be precisely defined, but it is there. (Note that "normal," as used above, is not a valuation.)

## III. AN ANALYSIS OF PHONETIC POSSIBILITY

How is the class of N-speakers singled out? In practice, this is done by means of anatomical and physiological criteria. When one tries to decide which kinds of sounds are phonetically possible for typical speakers, one is implicitly selecting as N-speakers those speakers who lie within a certain range of physical makeup and capabilities. The ability to perform certain articulations, or to produce certain sounds that are in fact often produced by people, may be among the characteristics that are used to delimit the class of N-speakers. However, these characteristics cannot be limited to properties of the speaker's *internal* anatomy and physiology. There also will be relevant properties of the speaker's environment; these properties can affect the speaker's capabilities and can be considered part of the speaker's physiology, broadly conceived. For example, a speaker who is speaking in a helium-rich atmosphere is not a typical speaker.

We will not have to dwell for long upon the task of analyzing the concept of an N-speaker, or upon the details of the criteria for being an N-speaker. The important point is that phonetic possibility is relativized to a class of speakers defined by anatomical and functional criteria.

Now we will propose a semi-rigorous analysis of the concept of phonetic possibility relative to a class of speakers. In view of the above discussion, we restrict the scope of this analysis in two ways:

(1) We restrict attention to the phonetic possibility of *classes of sounds*, and specifically of classes defined (explicitly or implicitly) in terms of anatomical and physiological characteristics of the speaker. This encompasses all the familiar classes of sounds that one finds on the IPA Chart: bilabials, plosives, voiced fricatives, central vowels, etc.

(2) We restrict attention to certain classes of N-speakers: classes that are defined in terms of anatomical and physiological characteristics of speakers.

Throughout the rest of the paper, K is a variable ranging over classes of the kind specified in (2) above.

Here is the proposed analysis:

> K is phonetically possible if and only if it is possible that some N-speaker produces a sound belonging to K.

This analysis appears to reduce phonetic possibility to some other, as yet unspecified, kind of possibility. Consider the word "possible" on the right hand side of the "if and only if." What kind of possibility does that word "possible" express? Physical possibility is the most plausible choice. Intuitively speaking, a sound is possible if some normal speaker who obeys all physical laws is able to produce the sound. However, if a normal speaker *not* subject to normal physical laws existed and could produce the sound, that would *not* make the sound phonetically possible. Following standard practice in modal logic and philosophy, we can use arguments about possible worlds to back up this claim. For example, we can imagine a logically possible world (not physically possible) in which the physical laws differ from the physical laws of the actual world in such a way that the speed of sound in air fluctuates rapidly and randomly in time. In such a logically possible world, a normal speaker -- as defined by familiar anatomical and physiological criteria -- might produce some wildly implausible sounds. But that does not make those sounds phonetically possible; it only means that such sounds would be phonetically possible if the laws were physics were different. (No reasonable phonetician would use the logical consistency of such an imagined world as evidence that the wildly implausible sounds uttered in that world are phonetically possible!) Thus, for a sound to be phonetically possible, it is not sufficient that a normal speaker can produce that sound in some logically possible world. A sound is phonetically possible if and only if a normal speaker can produce it while subject to the physical laws that hold in our actual world. In other words, a sound is phonetically possible if and only if a normal speaker might produce that sound in a *physically* possible world.

Thus, we may say that

> K is phonetically possible if and only if it is physically possible that some N-speaker produces a sound belonging to K.

We can translate this into symbolic notation as follows.

Logical and class-theoretic symbols:

$\leftrightarrow$ equivalence

$\exists$ existential quantifier

$\wedge$ and

$\Diamond$ physical possibility

$\in$ is a member of

Nonlogical symbols:

Ph(K) for "K is phonetically possible"

N(x) for "x is an N-speaker"

Prod(x, y) for "x produces y"

Formalization of our analysis of phonetic possibility:

$$Ph(K) \leftrightarrow \Diamond(\exists x)[N(x) \wedge (\exists y)[y \in K \wedge Prod(x, y)]].$$

It is more convenient to state this in terms of production of classes of sounds rather than production of sounds. Abbreviate $(\exists y)[y \in K \wedge Prod(x, y)]$ to Pr(x, K). Then we get

$$Ph(K) \leftrightarrow \Diamond(\exists x)[N(x) \wedge Pr(x, K)].$$

## IV. CONSTRUCTING A MODAL SYSTEM

If we want to develop a modal logic for phonetic possibility, then we must be able to represent phonetic possibility as a modal operator on sentences, not merely as a property of kinds of sounds. That is, we must be able to make statements like "It is phonetically possible that P," where P is a statement, instead of only making statements like "K is phonetically possible," where K is a class. To take this new step, we will follow the customary possible worlds semantics for modal logic, and we will try to understand what it means for a world (in the modal logician's sense of the word "world") to be phonetically possible.

A world w is phonetically possible if and only if in w, every sound produced by an N-speaker is of a phonetically possible kind. Thus,

> w is a phonetically possible world if and only if:
> in w, $(\forall x)(\forall K)[N(x) \wedge Pr(x, K) \rightarrow Ph(K)]$ is true,

where $\forall$ is the universal quantifier and $\rightarrow$ is material implication.

Note that we are quantifying over the classes K of sounds here. This is second-order quantification of a relatively simple kind. Perhaps we could replace it with quantification over phonetic properties or articulatory positions if we wished to do so. We will not do so here.

Note also that the quantifier over K quantifies into a modality (inside of Ph(K)). This, however, should pose no big problems, because once we settle on the criteria for selecting the N-speakers, we can expect to get the same classes K in all worlds. Here is why we should expect this. We took K to be a variable, not over all classes of sounds indiscriminately, but only over those classes which are defined solely in terms of the anatomical and physiological properties of speakers. (Classes defined in terms of articulatory features, like plosive, nasal, open vowel, etc., are of this general kind.) Also, we took the class of N-speakers to be defined by anatomical and physiological properties of speakers. Thus, if we try to enumerate all the values of K, we will find that the range of values depends only upon the anatomy and physiology of our N-speakers -- and we have fixed this in advance once and for all. Thus, our choices for the class K are fixed too, since each class K is specified by a class definition framed in terms of anatomical and physiological characteristics.

This is not to say that each class K has the same members in each world. I am using the term "class" in a conventional philosophical sense, according to which the extension of a property is a class. For all we know, each value of K might have slightly different members in different worlds. For example, voiced labiodental fricatives in one world might sound a little different from those in some other world, thereby giving the class K defined by this property different extensions in different worlds.

The above definition of "w is a phonetically possible world" involves a notion of physical possibility; this notion is built in via the definition of Ph. There is more than one possible choice for this notion of physical possibility; should we use physical possibility at w, or physical possibility at the actual world? My answer is that we should use physical possibility at the actual world. If we use physical possibility at w, then we might get phonetically possible worlds in which ordinary human speakers make very odd sounds due to laws of physics different from the ones that hold in the actual world. (Recall my earlier example of the wild sounds.) Such worlds should not be regarded as phonetically possible. However, our definition of phonetically possible world does not rule out phonetically possible worlds that are physically impossible. It is not hard to imagine such a world: for example, a world in which all speakers produce all and only the sounds in the actual IPA Chart, and yet in which some physical laws are quite different from the way they are now (maybe gravity is Newtonian instead of Einsteinian). Thus, there can be physically impossible but phonetically possible worlds. A physically possible world, for our purposes, may be taken to be a world with the same physical laws as the actual world.

Thus, in the domain of all phonetically possible worlds, there will be some worlds that are physically impossible from the standpoint of our actual world -- that is, physically impossible *at* the actual world.

This dependence of the phonetic possibility of a possible world on the physical laws of the actual world makes it clear that the phonetic possibility of a given world is relative to the choice of an actual world. We cannot simply proclaim that a world w is phonetically possible. Instead, we can only say that w is possible relative to the actual world -- or, more generally, relative to some possible world v. This relativeness has an important consequence for the modal logic of phonetic possibility: it implies that the possible-worlds semantics of this logic involves an *accessibility relation* among possible worlds. We can define this accessibility relation as follows:

> wRv (read: "v is accessible from w") if and only if:
> in v, $(\forall x)(\forall K)[N(x) \wedge Pr(x, K) \rightarrow Ph_w(K)]$ is true,

where $Ph_w(K) \leftrightarrow \Diamond_w(\exists x)[N(x) \wedge Pr(x, K)]$ and $\Diamond_w$ denotes physical possibility relative to w (that is, conformity to the physical laws that hold in w).

The relation R is an accessibility relation that we can use to construct a modal logic for phonetic possibility. Informally, wRv means that any kind K of sound produced by an N-speaker in v would be physically possible for an N-speaker to produce in w. Roughly speaking, this means that v only contains speech sounds that would be physically possible for an N-speaker to produce in w. Thus, if w is a world, a statement P is phonetically possible at w if and only if for some world v, wRv and P is true at v. For example, a statement is phonetically possible in the actual world if and only if there is some phonetically possible world (phonetically possible *relative to* the actual world) in which the statement is true. This way of characterizing the possibility of a statement will be familiar to modal logicians; the only novelty is that we are using an accessibility relation suited to *phonetic* possibility, instead of some other, more familiar accessibility relation.

What sort of modal logic do we finally obtain? First, note that there will be two modal operators in the system. First, there will be a phonetic possibility operator; since possibility operators conventionally are written with the diamond symbol, we will use $\Diamond_{\phi}$ for phonetic possibility. The semantics of this operator are given by the following rule, which I already have stated in another form:

$\Diamond_{\phi}$P is true at a world w if and only if P is true at some world v such that wRv.

(If I were attempting a fully formal treatment, I would phrase this and subsequent definitions in terms of truth at a world in a model instead of just truth at a world. Interested readers can correct this omission if they wish.) Second, there will be a *phonetic necessity* operator; since necessity operators normally are written with boxes, we will denote this by $\Box_{\phi}$. The phonetic necessity operator is defined in terms of the phonetic possibility operator in a conventional manner, as follows:

$\Box_{\phi}P \leftrightarrow \sim\Diamond_{\phi}\sim P$

The idea of phonetic necessity is not ordinarily used in phonetics (at least not explicitly). However, its introduction adds nothing fundamentally new to phonetic reasoning. A statement is phonetically necessary if and only if its *negation* is phonetically *impossible*. For example, if it is phonetically impossible that anyone produces a certain sound, then it is phonetically necessary that no one produces that sound.

Note that the statements which are phonetically possible or phonetically necessary will not always be statements about sounds. Our definitions of these operators will make many non-phonetic statements either possible or necessary. For example, it is phonetically possible that the earth is round. This is the case because in some phonetically possible world (the actual world), the earth is round. This extension of the notion of phonetic possibility to statements outside of phonetics may seem strange, but it does no harm, and it is inevitable if we are going to define phonetic possibility and necessity operators that act on arbitrary statements.

So far, our phonetic modal logic does not look much different from well-known modal logics. What will the logic of phonetic possibility be like? To answer this question, we must find a system of modal logic for which the system of possible worlds described above forms a model. According to well-known results in modal semantics (see for example [2, ch. 3]), the structural properties of the accessibility relation R can tell us which axioms of modal logic the possibility and necessity operators will obey. We will now investigate these structural properties.

From the definition of R, it is obvious that R is reflexive. However, R is not symmetric. To see this, imagine a case of a world v which has physical laws slightly different from the actual world w. Suppose that the N-speakers in w and in v are physically able to produce exactly the same kinds of sounds, except that there are a few sounds that only the N-speakers in v can produce (due to differences in physical laws). If the speakers in w and v produce all sounds possible to them, then vRw but not wRv.

Further, R is not transitive. To see this, imagine the actual world w, a world v accessible from w, and a world u accessible from v. Any kind of sound (value of K) produced by an N-speaker in u is physically possible for an N-speaker to produce in v, and any kind of sound produced by an N-speaker in v is physically possible for an N-speaker to produce in w. However, it might be the case that a kind of sound that is produced in u just never happens to be produced in v, even though its production is physically possible in v. In that case, wRv and vRu do not guarantee that this kind of sound is physically possible for an N-speaker in w to produce. Thus, the transitivity of R is not guaranteed.

Also, R is not euclidean. Imagine worlds u, v and w, all with slightly different physical laws. Consider a scenario in which N-speakers can produce slightly different sets of sounds in worlds w and v, and N-speakers in u can produce all of the sounds that N-speakers can produce in either w or v. Perhaps w and v each lack one sound relative to u. Then we will have uRv and uRw, but not vRw.

Some well-established results in modal logic (see for example [2, p. 80]) allow us to say what axioms of modal logic our system must obey if R has the properties described above. According to these results, our modal system should have axiom T because R is reflexive, but should not have axioms B, 4 or 5 because R is not symmetric, transitive, or euclidean. We also can safely assume that the rule of modal consequence [2, p.19] holds in this logic, since any statement that follows logically from phonetically necessary statements should be phonetically necessary too. From these considerations, we can deduce that the logic of phonetic possibility -- or at least the version of that logic that we have developed here -- is a superset of the system T, but is not as strong as the system S4. Hence the most natural candidate for such a modal logic is T.

## NOTES

1. In writing this paper I have relied upon some general background information about phonetics and modal logic. This information is common knowledge in those fields and can be found in introductory textbooks on those subjects.

## REFERENCES

[1] International Phonetic Association, *Handbook of the International Phonetic Association* (Cambridge, UK: Cambridge University Press, 1999).

[2] Brian F. Chellas, *Modal Logic: An Introduction* (Cambridge, UK: Cambridge University Press, 1980).