Russell's-Paradox-Intercepting Corollary to the Axiom of Extensionality

1 – It follows from the axiom of extensionality that

$$\forall \mathtt{A}\ \forall \mathtt{B} \neq \mathtt{A}\ \Big[\forall \mathtt{x} \neq \mathtt{y}\ (\mathtt{x} \in \mathtt{A} \Leftrightarrow \mathtt{x} \in \mathtt{B}) \quad \Longrightarrow \quad \mathtt{y} \in \mathtt{A} \Leftrightarrow\ \mathtt{y} \notin \mathtt{B}\Big].$$

2 – A corollary to the axiom of extensionality:

$$\forall \mathtt{A}\ \forall \mathtt{B}\ \left[\forall \mathtt{x} \neq \mathtt{y}\ \exists \mathtt{C} \neq \mathtt{A} \quad \begin{pmatrix} & \mathtt{x} \in \mathtt{A} \Leftrightarrow \mathtt{x} \in \mathtt{C} \\ \wedge & \mathtt{x} \in \mathtt{B} \Leftrightarrow \mathtt{x} \in \mathtt{C} \\ \wedge & \mathtt{y} \in \mathtt{B} \Leftrightarrow \mathtt{y} \in \mathtt{C} \end{pmatrix} \ \Longrightarrow\ \mathtt{y} \in \mathtt{A} \Leftrightarrow\ \mathtt{y} \notin \mathtt{B}\right].$$

3 – The set of all sets that are not members of themselves appears to be a member of itself if and only if it is not a member of itself:

$$\mathtt{R} = \{\forall \mathtt{x} \mid \mathtt{x} \notin \mathtt{x}\} \ \Rightarrow\ \mathtt{R} \in \mathtt{R} \Leftrightarrow \mathtt{R} \notin \mathtt{R}.$$

However we must read the putatively paradoxical conclusion as

$$\mathtt{R} \in \mathtt{R} \quad \Leftrightarrow \quad \mathtt{R} \notin \{\forall \mathtt{x} \mid \mathtt{x} \notin \mathtt{x}\}$$

and that $\mathtt{R}$ is not simply $\{\forall \mathtt{x} \mid \mathtt{x} \notin \mathtt{x}\}$ but

$$\mathtt{R} = \Big\{\forall \mathtt{x}\ \Big|\ \Big(\mathtt{R} \in \mathtt{R} \quad \Leftrightarrow \quad \mathtt{R} \notin \{\forall \mathtt{x} \mid \mathtt{x} \notin \mathtt{x}\}\Big) \wedge \mathtt{x} \notin \mathtt{x}\Big\}.$$

4 – The emendation in $\mathtt{R}$ is dictated in compliance with the corollary where $\mathtt{y} = \mathtt{A} = \mathtt{R}$ and $\mathtt{B} = \{\forall \mathtt{x} \mid \mathtt{x} \notin \mathtt{x}\}$ and $\mathtt{C} = \mathtt{x}$.