# Special Relativity Anticipating Achilles Paradox

**1 –** Zeno's paradox of Achilles and Tortoise is relevant only when Achilles and the tortoise move at different speeds but not if they ever move at the same speed but different directions. Or not relevant if *there is* in addition a kind of space in which they move at the same speed contrary to appearances.

**2 –** According to a principle of special relativity the more an object moves in coordinate space the less it moves in coordinate time. If the relative speed between Achilles and the tortoise is $v$, after $\Delta T$ passage of time:

- from the point of view of Achilles, the tortoise moves $\Delta x = v\Delta T$ in coordinate space and $w\Delta T$ in coordinate time.
- and from the point of view of the tortoise, Achilles moves $\Delta x = v\Delta T$ in coordinate space and $w\Delta T$ in coordinate time.

**3 –** The framework of special relativity wholly describes the motion of both if $v^2 + w^2 = C^2$ where $C$ is the speed of light.

**4 –** Achilles and the tortoise *phenomenally* (ordinary 3-dimensional conception of space) move at different speeds, but *noumenally* (in 4-dimensional spacetime), along with *everything* else, move at the same speed $C$.

*Appendix –* Consider a 4-dimensional space $S$ in which *all* objects move at the same speed $c_S$ but in different directions. For the objects A and B that move in directions $\overrightarrow{u_A}$ and $\overrightarrow{u_B}$, respectively, if $\Delta x$ is the distance that B moves in the (3-dimensional) space orthogonal to $\overrightarrow{u_A}$ during a time interval $\Delta T$, the distance it moves in the direction of $\overrightarrow{u_A}$ will be

$$\sqrt{c_S^2 \Delta T^2 - \Delta x^2} = \sqrt{c_S^2 - v^2}\Delta T,$$

where $v = \Delta x/\Delta T$. Let $\Delta\tau$ be the quantity such that

$$c_S^2 \Delta T^2 = v^2 \Delta T^2 + c_S^2 \Delta\tau^2.$$

If $\Delta T$ is *interpreted* as coordinate time, $\Delta x$ as coordinate space, $\overrightarrow{u_A}$ as constituting the coordinate temporal axis, and the 3-dimensional space orthogonal to $\overrightarrow{u_A}$ as constituting the coordinate space, and if $c_S = C$, then the last equation describes the motion of the object B in a special relativity spacetime traveling with speed $v$ during the coordinate time interval $\Delta T$, and that $\Delta\tau$ is the proper time interval experienced for B:

$$\Delta\tau = \sqrt{1 - \frac{v^2}{C^2}}\Delta T.$$