Some Recent Existential Appeals to Mathematical Experience

Michael J. Shaffer
St. Cloud State University

Abstract

Some recent work by philosophers of mathematics has been aimed at showing that our knowledge of the existence of at least some mathematical objects and/or sets can be epistemically grounded by appealing to perceptual experience. The sensory capacity that they refer to in doing so is the ability to perceive numbers, mathematical properties and/or sets. The chief defense of this view as it applies to the perception of sets is found in Penelope Maddy's Realism in Mathematics, but a number of other philosophers have made similar, if more simple, appeals of this sort. For example, Jaegwon Kim (1981, 1982), John Bigelow (1988, 1990), and John Bigelow and Robert Pargetter (1990) have all defended such views. The main critical issue that will be raised here concerns the coherence of the notions of set perception and mathematical perception, and whether appeals to such perceptual faculties can really provide any justification for or explanation of belief in the existence of sets, mathematical properties and/or numbers.

1. Appeals to Mathematical Experience and the Existence of Sets and Numbers

§7 of Frege's *Foundations of Arithmetic* is famous for its unconvincing ridiculing of John Stuart Mill's empiricist philosophy of mathematics. As is well known, Mill held that our mathematical knowledge is acquired by inductive inference, though it is clear that he was not a realist with respect to mathematical entities or properties. In any case, mathematical empiricism of both the realist and anti-realist sorts sort fell on rather hard times for some time after Frege's attack,[1] at least until it was revived in a rather different form by Quine and Putnam.[2] Quine and Putnam, of course, are famous for having introduced the broadly empirical indispensability argument that is both popular and controversial.[3] However, the Quine/Putnam approach is not the only recent attempt to ground our knowledge of mathematics empirically and realistically.

Given Frege's rather superficial ridiculing of Millean empiricism, the apparent failure of reductivist logicism, and an important argument advanced by Paul

Principia, **10**(2) (2006), pp. 143–70. Published by NEL — Epistemology and Logic Research Group, Federal University of Santa Catarina (UFSC), Brazil.

Benacerraf (1973), a number of contemporary philosophers of mathematics have attempted to show that belief in the existence of at least some mathematical entities like numbers, mathematical properties and/or sets can be rationally grounded by appealing to perceptual experience. As a result, such knowledge is supposed to be grounded non-inferentially, at least in part. Of course it is not unusual to make appeals to perceptual experience when faced with difficult epistemological and ontological issues. Familiarity with the epistemological work of Reid and Moore, for example, amply demonstrates this point (Reid 1785/1969 and Moore 1939). As a result, this sort of approach has both some *prima facie* intuitive appeal and some appeal in virtue of the problems that afflict the alternatives mentioned above.

The contemporary philosophers obliquely referred to above who subscribe to this sort of mathematical empiricism and also to mathematical realism (hereafter MER) appeal to a *special* kind of experience intended to ground the rationality of belief in the existence of some mathematical objects and properties. This special kind of experience has been referred to variously as 'set perception' or, more generally, as 'mathematical experience' or 'mathematical perception'.[4] To strengthen the appeal to this particular and admittedly peculiar type of experience, Penelope Maddy, John Bigelow and Jaegwon Kim have tried to show how such perceptual faculties arise from and are related naturally to normal perceptual experiences. This is done, presumably, in order to show that by parity of reasoning if some beliefs formed on the basis of the ordinary perception are taken to be justified and rational to hold, then some beliefs formed on the basis of set, mathematical property and/or number perception should also be regarded as justified and rational to hold (at least to some degree). For convenience sake, this sort of general approach to the problem of mathematical knowledge will be referred to here as *the appeal to mathematical experience*.[5]

11. The "Appeal" of the Appeal

Maddy (1990a, 1990b), Kim (1981, 1982), Bigelow (1988, 1990) and Bigelow and Pargetter (1991) have all made more or less sophisticated justificatory appeals of roughly this sort, and they variously claim that such appeals provide, or help to provide, warrant for at least some basic mathematical beliefs, specifically for the belief that at least some sets, mathematical properties and/or numbers exist. The source of the *prima facie* appeal that this view has derives from the well-known objection to mathematical Platonism raised by Paul Benacerraf

(1973). Benacerraf's simple starting point is that given any sort of even minimally acceptable theory of knowing, in order to know anything about some type of things it must be the case that things of that type can causally interact with the knower in question. Benacerraf then reasons that given the Platonistic view that numbers are abstract objects it follows that we cannot have any knowledge of mathematical objects. This is simply because abstract objects so understood cannot causally interact with us. In being abstracta, numbers, sets, mathematical properties and the like are by their very nature causally inert. Abstract entities lack spatio-temporal location and are not materially concrete and so they cannot possibly enter into the sorts of causal relations with us necessary for our knowledge of them. However, we do seem to, in fact, possess mathematical knowledge, and so Benacerraf leaves us faced with a vexing problem about *how* knowledge of such mathematical objects could possible have come about.

The MERs approach to this problem is interesting because it would account for our knowledge of mathematically objects perceptually and this has a significant advantage over purely inferential accounts of the existence of mathematical objects (i.e. those that do not appeal to any direct experiences). Views that account for our putative knowledge of metaphysically unusual objects like mathematical objects (i.e. those that are not concrete) *inferentially* are likely to be, by their nature, controversial. To infer the existence of *concrete* objects (i.e. those with spatio-temporal properties) like electrons on the basis of observationally accessible data that they cause is one thing, but to infer the existence of *abstract* objects (objects without spatio-temporal properties and which cannot cause anything—including indirect empirical data) seems to be quite another thing altogether.

So some of these sorts of MERs have attempted to avoid the Benacerraf problem and to defend mathematical realism by denying the causal inertness of at least some mathematical objects, properties and/or sets. The view they then defend is that we are capable of directly perceiving some sets, mathematical properties and/or numbers via the use or ordinary perceptual mechanisms.[6] Thereby they attempt to side-step the Benacerraf problem and they avoid the need to appeal to any sort of special intuitive faculty attuned to the perception of numbers and/or sets such as that posited, for example, by Gödel (1947/1964). What these MERs claim is that we perceive some real sets, mathematical properties and/or numbers, for example, visually, and insofar as we often take successful visual experiences to play an important role in the (at least *prima facie*) justification of existential claims concerning objects like trees, chairs, coffee cups, etc., the MERs can then claim that we ought to treat existential claims concerning numbers, mathematical properties and/or sets in precisely the same manner.

12. Anticipation of the Critical Argument to Come

The main critical issue that will be raised here concerns the coherence of the notion of mathematical experience adopted by the MERs mentioned above, and, more specifically, it concerns what it is that differentiates mathematical or set perception from typical, garden-variety, perceptual experience. It will be argued that when the concepts of set and number perception are subjected to critical scrutiny, justificatory appeals based on these concepts turn out to be subject to serious problems, at least as set and number perception are typically conceived of by those MERs mentioned above.

To be more specific, it will be shown that such experiences have not been adequately individuated from other more mundane instances of perception employing the same faculties. As a result, appeals to mathematical experience do not provide adequate grounds to underwrite existential claims about sets, mathematical properties and numbers and so they cannot be used to advance the core position of MERs. In other words, what will be demonstrated in what follows is that there is a significant difference between the cases of making an existential claim on the basis of the perception of mundane physical objects and making an existential claim on the basis of perception of sets, mathematical properties and/or objects that constitutes a defeater with respect justification in the latter case. As a result, the justifying of beliefs in the existence of those things supposed to be perceived on the basis of the perception of numbers, mathematical properties and/or sets fails, but it is worth noting that this failure is neither the result of the Benacerraf problem itself nor is it the sort of flimsy dismissal of Millean mathematical empiricism offered by Frege. In any case, what will ultimately be demonstrated here is that the rationality of the belief in the existence of sets and other mathematical objects has not actually been justifiably established by this kind of appeal to mathematical experience.

The critical attack on the appeal to mathematical experience mounted here depends on drawing the distinction between the two methods for individuating mental and perceptual states: the narrow individuation of such states and the wide individuation of such states.[7] As it turns out, neither of these two methods for the individuation of perceptual contents is adequate for the satisfaction of the tasks that these MERs have set for themselves. In the case of narrow content individuation, this is true because appealing to set and/or number perception cannot possibly do the job that these MERs want it to do. In the case of wide content individuation, this is because the appeal that these MERs make to set and/or number perception either begs the question against the mathematical anti-realist and in so doing renders the appeal to mathematical experience superfluous to the MER project or is subject to defeat. So, at the very least, this sort of existential appeal to direct mathematical experience provides no independent justification for belief in the existence of sets, numbers, etc. This critical project will begin with an examination of the specific direct appeals to mathematical experience presented in Kim 1981 and 1982, and in Bigelow 1988 and 1990. After considering those views the somewhat more sophisticated version of this view presented in Maddy 1990a will also be addressed.

2. Kim on Perceiving Numbers

Kim's view of mathematical perception is laid out in his 1981 and 1982 and the pedigree of his view is unquestionably clear from his claim that, "Mill was right in his fundamental assumption that numerical properties and relations are accessible to observation and perception" (1982: 93). In his 1981 he nevertheless both elaborates and distinguishes his view from that of Mill in explaining that,

> Mill was surely wrong in using the model of induction to explain the epistemic role of experience for mathematical knowledge. But there is one aspect of that theory that we should salvage. It is this: we *see* in our perceptual experience *three* pebbles and *two* pebbles, and *see* also *that* they make up five pebbles. That is to say, *we perceive in our experience of the world, perhaps also within our minds, numerical properties instantiated, and we also perceive certain numerical or mathematical relationships to obtain* (1981: 344; emphasis in original).

So, it should be clear that Kim is making an epistemic appeal that is of a kind with those of the other MERs.[8]

What Kim focuses his attention primarily on is just the sort of parity argument noted in previous sections. In response to the Benacerraf problem, Kim simply asserts that our (perceptual) access to numbers is no better or worse off than our (perceptual) access to other more mundane physical properties. As we are often prepared to claim that we can see that an object is red or that it is a sphere and, hence, we are justified in believing that redness and sphericality exist, Kim implies that we ought to extend the same sort of treatment to numbers. In a key passage Kim asserts that,

> ... as objects of perceptual discrimination and judgment, there is nothing unusual, uncommon or mysterious about numerical properties and relations or, more generally, mathematical properties and relations... Numerical properties do not differ in respect of perceptual accessibility from sundry physical properties such as colors, shapes, odors, warmth and cold. They are among those 'sensible qualities' the Empiricists used to talk about; as may be recalled, number was thought to be a 'primary quality' of objects (1981: 345).

Moreover, he makes his realism yet more explicit when he tells us that,

> ... it is not clear why we should not say that the class of these dots, as well as the dots themselves, is right here on this piece of paper, that *this class came into existence when little Johnny made the dots with his ballpoint, that it moves when the piece of paper is moved, and that it goes out of existence when the paper is burned to ashes in the fireplace*. (Kim 1981, 349, my emphasis).

As Kim would appear to have it and *pace* Benacerraf mathematical properties exist literally and are then no less causally efficacious than any other, more mundane, physical property or object. Kim makes this point about our perceptual access to these properties clear in telling us that,

> Human perception is a causal process involving the feature of the object or situation perceived and the states of our sense organs and nervous system. Just as the character of our perceptual experience of there being a green dot is causally determined in part by the state of affairs of there being a green dot, so our perceptual experience of there being three dots out there, or that there are more green dots than red ones, is causally determined by there being three green dots, or there being more green dots than red ones (1981: 346).

As a result, Kim is prepared to assert that our perceptual faculties are attuned to the perception of numbers, mathematical properties and/or sets and that these things are not problematically abstract. With respect to the latter issue, Kim unabashedly asserts that numbers and sets have spatio-temporal location, that they can, for example, be moved or destroyed (1981: 349) and that they are generally causally efficacious (1981: 347). As such, the justification for believing in the existence of some numerical properties is supposed to be of a piece with other perceptually justified beliefs (1981: 347–48).

With respect to the former issue, Kim refers to Kaufmann, et al. 1949 and explains that,

> It is well known, from extensive psychological studies, that a normal human percipient can make accurate perceptual judgments of the number of dots in random patterns flashed on a screen for a short time (around one-fifth of a second) when the number is equal to or less than seven (Kim 1981: 345).

So it should be abundantly evident that Kim holds that mathematical properties are not ontologically unusual and so do not pose any special problem *qua* our perceptual and epistemic access to them. Moreover, he appears to hold both that our access to numerical properties is causal and that our perceptual ability with respect to those properties is a subject for empirical study.

21. Bigelow on Perceiving Structures

Bigelow, like Kim, is an unabashed realist about mathematical entities and the view he develops is an extension of D. M. Armstrong's (1973) account of the nature and epistemology of mathematics. The crux of the view is that mathematical entities (numbers in particular) are properties or relational structures and that these things exist in space and time. Such entities are kinds of universals and they are to be understood as existing in the same manner as other more mundane universals. The natural numbers then are recurrent universals *in rebus* and other mathematical entities are to be understood to be relations between relations (Bigelow 1988: 5).

In virtue of this mathematical entities are themselves taken to be physical and so are capable of being observed in precisely the same way that more mundane and familiar features of physical objects and relations between them are observable. As a result, Bigelow does not shy away from labeling his view as Pythagorean and it is clear that his view is much like those of the other MERs in both its ontological and epistemological aspects. In his 1988 Bigelow states his position on these matters as follows:

> The kind of realism I advocate is a descendent of David Armstrong's a posteriori realism. The theory is distinguished from traditional Platonism

> in a variety of ways. To keep your bearings, you should remember that Armstrong was a founding architect of modern materialism. He will have no truck with transcendent Platonic forms, and nor will I. Everything there is is *physical*—and this is worth saying, even though the term 'physical' sinks under scrutiny into a swamp of ambiguities. Hence universals, too, are physical. That is to say, the universal which exist are all real physical properties and relations among physical things. And thus, their existence cannot be deduced a priori, or founded simply on reflection about 'what every speaker knows'. Their existence is to be established by general scientific method—whatever that is. That is why Armstrong's view is called a posteriori realism.
>
> Thus, I argue that the world around us, the world of space and time, does contain mathematical objects like numbers. I portray these as no mere abstractions, existing separately from the physical things around us. Nor do I portray them as mere ideas in the mind; nor as empty symbols which refer to nothing beyond themselves (Bigelow 1988: 1).

So, it should be abundantly clear that Bigelow is both a realist and an empiricist concerning mathematical entities and much of what he says in support of this view also shows that he subscribes to just the sort of MER view described above.

In support of this attribution we can note that, while he is sympathetic to Kitcher's (1983) account of the transmission of mathematical knowledge he tells us that for the earliest mathematical knowledge of basic mathematical entities, "… a causal or perceptual story may apply" (Bigelow 1988: 4), and in distancing himself from Quinean holism and, by implication, the indispensability arguments he says that, "In epistemology, my tastes run rather more towards causal theories of knowledge. Causal theories tell us that, in order to know about something, there must be some sort of appropriate causal network linking you with that thing" (Bigelow 1988: 175).[9] Also, he is careful to make it clear both that he is "… a congenital realist about almost everything, as long as it is compatible with some sort of naturalism or physicalism, loosely construed" (Bigelow 1988: 123), and that "… there is no reason at all to think that universals are causally isolated from us" (Bigelow 1988: 175). So it is abundantly clear from these passages that Bigelow is subscribing to just the sort of MER view we have been considering, although as his interests in his 1988 and 1990 are more metaphysical than epistemological his view is somewhat less detailed than that of Kim.

Nevertheless, it is relatively clear that Kim and Bigelow all share in common the appeal to mathematical experience as ground for mathematical realism, and as this view does have many significant advantages over classical Platonism it is worth subjecting this aspect of their shared empirical epistemology of mathematics to critical scrutiny.

22. Maddy on Set Perception

Maddy's view of mathematical perception or, more specifically, her view of set perception is developed with certain important, specific, aims in mind. Her primary aim in her 1990a is to secure some form of realism about sets and other mathematical objects while, among other things, (1) avoiding the Benacerraf objection to Platonism about mathematical objects (Benacerraf 1973), and (2) maintaining a naturalistic view of perception. These desiderata are important components of her 1990a project of defending what she calls compromise Platonism and her view is based on the idea that while the Quine/Putnam indispensability arguments provide us with minimal reason to be realists about mathematical entities, they are not strong enough reasons to answer the Benacerraf problem. She agrees, as most philosophers of mathematics do, that when these arguments are taken in isolation, they do not provide sufficient reason to ground mathematical realism. So her reliance on the Quine/Putnam indispensability arguments leaves her with the Benacerraf problem and this is precisely where the appeal to mathematical experience comes into play.

Maddy (1990: 35) essentially argues that mathematical perception *explains* how we came to have mathematical knowledge in much the same way that Gödel (rather fancifully) thought that intuition explained our knowledge of mathematics. As a result, set perception plays an important explanatory/justificatory role in Maddy's more complicated 'two-aspect' epistemology of mathematics. The appeal to mathematical perception is then intended to buttress the 'thin' justification of the existence of mathematical entities supplied by the Quine/Putnam indispensability arguments. However, before we can subject her view of mathematical perception to critical scrutiny we need to more closely examine the details of her view of perception in general.

Maddy begins her line of argument by outlining a standard naturalistic (i.e. broadly causal) approach to perception along the lines of that which has been offered by Donald Hebb (1949, 1980), although she makes it clear that her own view *does not* depend on the correctness of that specific naturalistic view of perception.[10] What she intends to show in appealing to Hebb's theory of perception is that there is a perfectly good, physiologically realistic, theory of perception that explains the origination and development of our ordinary perceptual faculties sufficient to ground our beliefs in the existence of physical objects. Maddy comments that,

> The ability to perceive physical objects is not unlike the ability to perceive triangular figures, though it is more complex. The trick is to see a series of patterns as constituting views of a single thing. Just as the ability to see triangles develops over time, through the painstaking process of seeking out corners and comparing one triangle with another, the ability to see continuing physical objects develops over a period of experience with watching and manipulating them (Maddy 1990a: 57).

Moreover, in appealing to Hebb, Maddy notes that perceptual abilities are grounded in our possession of physiological/neurological mechanisms that serve as detectors for certain objects and/or properties some of which are native at birth, some of which are developmentally acquired and some of which are acquired via experience.

Most interestingly, it is clear that Maddy takes physical object perception to be an acquired skill (1990a: 50–67), and she explicitly subscribes to the naïvely realistic causal theory of perception in asserting that,

> … for Steve to perceive a tree before him is for there to be a tree before him, for him to gain perceptual beliefs, in particular that there is a tree before him, and for the tree before him to play an appropriate causal role in the generation of these perceptual beliefs (Maddy 1990a: 51).

This attribution to Maddy is further supported when, in referring to the major question raised by the Benacerraf problem, she asserts that,

> The question is what bridges the gap between what is causally interacted with and what is perceived, and the hope is that something like what does the bridging in the case of physical object perception can be seen to do the same job in the case of set perception. Notice that this way of putting the problem already assumes that we do in fact perceive physical objects … (Maddy 1990a: 50).

On this basis, Maddy then goes on to claim that the ability to perceive sets is just like our ability to perceive physical objects in that it is an ability to detect certain objects on the basis of our possessing certain neurological mechanisms and that this ability employs precisely the same sorts of neurological and perceptual mechanisms as visual perception. Consider her primary example:

> Steve needs two eggs for a certain recipe. The egg carton he takes from the refrigerator feels ominously light. He opens the carton and sees, to his relief, three eggs there. My claim is that Steve has perceived a set of three eggs. By the account of perception just canvassed, this requires that there be a set of three eggs in the carton, that Steve acquire perceptual beliefs about it, and that the set of eggs participate in the generation of these perceptual beliefs in the same way that my hand participates in the generation of my belief that there is a hand before me when I look at it in good light (Maddy 1990a: 58).

In discussing the details of such cases, she is clear in asserting that the ability to perceive the *set of eggs*, as distinct from merely perceiving the eggs individually, is an ability that is acquired via a combination of developmental processes and through experience; it is, in other words, a special acquired perceptual acuity. Given this view, it would seem to be perfectly reasonable to then infer that, at least some of us really do perceive sets and that our ability to come to have mathematical knowledge is *explained* by set perception. So this putative ability plays an important role in providing some additional empirical grounding for warranting belief in their existence, even if the existence of such entities is initially introduced by way of the Quine/Putnam indispensability arguments.

In anticipating some reactions to her view of set perception, Maddy is careful to note that one might simply respond either by accepting the Platonic view that sets are abstract and so cannot cause perceptual states to arise in us or by straightforwardly adopting anti-realism about sets. To these worries Maddy responds as follows. Concerning the Platonistic response, she is prepared to deny that (at least some) sets are abstract objects, and so they are not causally inert as assumed in the Benacerraf problem. Concerning the anti-realist response she simply appeals to the Quine/Putnam indispensability arguments and so suggests that sets are an indispensable part of our best ontological theories of the world. As such, they are on an ontological par with all sorts of more pedestrian objects. Given these views Maddy then can offer her more sophisticated 'two-aspect' approach to the problem of mathematical knowledge and the existence of mathematical entities. Her view is first predicated on the realist conclusion of the indispensability argument and her appeal to mathematical experience is then specifically designed to *explain* our knowledge of those entities in a way that naturalistically deals with the Benacerraf problem. The Quine/Putnam indispensability arguments and the theory of mathematical perception are thus combined to form an apparently compelling empiricist and realist epistemology of mathematics. However, we will see that even this more sophisticated MER view is problematic. In any case, having now laid out the essential aspects of her view of perception, we can now turn our attention to a critical appraisal of Kim's, Bigelow's and Maddy's related appeals to mathematical experience.

23. The Problem of MERkiness

As things stand Maddy, Kim and Bigelow all say relatively little about the details of the perceptual process involved in the perception of mathematical entities *qua* its physiological basis (although Maddy clearly says the most about this), but they are fairly explicit in their accepting that our perception of sets and mathematical entities and relations is a relatively ordinary causal process just like that which occurs in the more or less mundane perception of physical properties like that of color (Kim 1981: 346). However, unlike the cases of color perception or shape perception, it is not clear at all what the physiological basis of number perception really is. The human visual system is, for example, most obviously attuned to the detection of light; the eye is photoreceptive to electromagnetic radiation in the range of wavelengths 400–700nm (McIlwin 1996: 3).

Our best neurophysiological theories of vision explain then that because the eye possesses this sensitivity it is capable of directly detecting oriented contours, velocity and spectral composition, types of information contained in the retinal image.[11] However, these sorts of explanatory accounts of vision do not attribute to the eye the basic capacity to detect numbers or sets. Maddy at least wisely admits that set perception is an acquired skill that is supposed to be *parasitic* on, for example, the eye's basic photoreceptive functions (1990a: 50–67), but Kim and Bigelow simply remain silent on the particular physiological ground for number perception and they say nothing about whether it is an acquired facility or not. But given the fact that our best scientific account of human vision, it appears that if we do possess the capacity to directly see numbers, mathematical properties and/or sets, then the view that this is an acquired and parasitic ability seems to be the most reasonable option. That this is so is of course because our best theories of vision,[12] for example, do not treat the eye as having the basic function of directly detecting sets, numbers and/or mathematical properties.

The issue of whether this perceptual ability is acquired or not aside, it is then absolutely crucial for the agenda of all of these MER's that they provide us with some account of the physiological basis for such special perceptual acuities. However, as it turns out, we do not have such an account and this has devastating consequences for these MER's appeal to mathematical experience because, in the cases of Kim and Bigelow, it undermines the *prima facie* justification for the existential claims that such appeals are aimed at securing,[13] and in the case of Maddy this lacuna leaves her with nothing more than the Quine/Putnam indispensability arguments as the basis for her realism and thus with no answer to the Benacerraf problem.

24. Perception, Special Acuities and Community Agreement

In considering Kim's, Bigelow's and Maddy's views that set, mathematical property and/or number perception are special sensory acuities, acquired or native, a particular and difficult problem arises, especially in cases where there are disagreements about existential claims and where there is perceptual variation across individuals. The seemingly obvious suggestion is that in such cases appealing to inter-subjective community agreement could be appealed to in order to circumvent any problems. However, adopting this strategy as a general practice unfairly biases the issue of which segment of the population is correct in favor of the majority group and ignores the possibility that the perceptions of the members of the majority are those that are non-verific. Recall that Maddy's explicitly naturalistic take on the issue of set perception, including her appeal to the Hebbian theory of perception, suggests that we have good reason to believe that if we work to develop it we are capable of possessing a reliable, and rather ordinary, belief-forming mechanism with respect to sets and given what we know about the neurophysiology of the visual system and Kim's and Bigelow's subscriptions to the causal theory of perception, Kim and Bigelow should believe something like this as well.

The substantive point is then that we could ignore the problematic issue of inter-subjective agreement as having any bearing on the existential question of the existence of the objects of special perceptual faculties if there were an adequate explanatory theory of the perceptual mechanism sensitive to sets, mathematical properties and/or numbers that accounted for the reliability of this faculty, its mechanics, and that also permitted us to individuate (for example) mathematical (visual) perceptions from other (visual) perceptual states. The latter point is especially important as Kim, Maddy and Bigelow all want to claim that our ordinary perceptual faculties like vision often provide us with or explain mathematical knowledge. So we are owed an account of the method that allows us to discriminate specific perceptual states with mathematical content from specific perceptual states without mathematical content. In other words they are obligated to account for the special content of mathematical perceptions in order to distinguish them from non-mathematical perceptual states of the very same sense organs.

In order to illustrate this line of thought consider the analogous case of taste sensitivity with respect to the substance phenol. The general population happens, as a matter of fact, to be partitioned into two groups with regard to tasting phenol. One group, the minority, reports that phenol tastes bitter. The other group, the majority, reports that it is tasteless. The natural ontological question to ask is then whether or not phenol is really bitter; whether bitterness is really an objective property of phenol. Can we simply assume in this case that the majority is correct, and that phenol is not bitter? Surely we cannot respond in this naïve manner. We do not, and should not, automatically impugn the claims of those who appear to be sensitive to phenol because the majority of us are not sensitive to this apparent property of phenol. Problems *can and often do* arise, however, both when we try to account for such differences in perceptual abilities and when we attempt to ascertain the ontological significance of such perceptual states. Typically, what we look for in such cases is some physiological reason to suppose that phenol tasters possess relevantly different sensory organs and so possess a reliable faculty for detecting real properties that most of us cannot detect.

So what we might reasonably believe in this case is that phenol is bitter and there is some difference in the sensory modalities of the two groups. As it turns out, despite the fact that the majority may not possess the ability to detect such properties, there are many cases of minorities that possess special sensory acuities that we take to be accurate precisely because we have detailed understanding of the physiological basis of those special acuities. So, while it may or may not be the case that the individuals in the different partitions actually have different perceptions because they actually have different sensory modalities, one lesson is clear, the size of the partitions tells us nothing about which partition is having sensations that are verific. The real worries that might then arise, for example, in the case of the phenol tasters are that, first, there may be no objective property being identified at all, and, second, that it may not be the objective property of tasting bitter, that is being detected by the phenol tasters. Absent sufficient independent reasons to believe, e.g., that the phenol tasters are really accurately detecting some objective feature of phenol, it is surely possible that this is just the result some subjective quirk in the phenol tasters' physiology or that they are merely detecting some pedestrian property of phenol and not its actual bitterness.

In point of fact, in the case of phenol and a whole host of other compounds, e.g., 6-n-propylthiouracil, phenylthiocarbamide, etc., the difference in ability to taste the bitterness of such chemicals is genetically determined.[14] So persons in one genetic partition are tasters and those in the other are non-tasters. Tasters have larger numbers of fungiform papillae that hold our taste buds and determine taste sensitivity. Interestingly, there is also a sub-group of tasters who are what are known as supertasters of these substances, those who report not only that they taste bitter but also that they are overwhelmingly bitter, and they have the largest number of fungiform papillae (Duffy and Bartoshuk 2000). As a result, those of us who are non-tasters (whether we constitute a majority or not) and those of who are tasters and supertasters are all justified in believing that the tasters and supertasters of these substances are really identifying a property of those substances in question. The justification, however, is provided *only* in virtue of our possessing an adequate scientific explanation of the variation in perceptual apparatus between tasters and non-tasters that accounts for the special acuity attributed to tasters and supertasters.[15]

Hebb's theory and its more sophisticated modern decedents—and by association Maddy's, Kim's and Bigelow's general theories of perception—in being broadly naturalistic (i.e. causal) theories of perception, depend then on our specifying some neuropsychological difference between phenol tasters and non-phenol tasters that would provide reason to believe that our perceptual organs are actually detectors with respect to that type of substance and which would be sufficient to individuate the contents of instances of such special perceptual acuities from ordinary perceptions using those same detectors.[16] This applies equally in the case that mathematical perception is a native ability and in the case that it is an acquired ability. Absent any such identifiable differences of this sort, we would certainly have no good reason to believe the existential claim that there is some such objective property/substance and that it is being detected by those with such a special sensory acuity merely based on their direct experience, and the same point would arise even if the partition sizes were reversed. Similarly, absent such an account, we would have no good reason to believe that our mathematical knowledge had been successfully explained.

The lesson is then that if we are to naturalistically ground and/or explain belief in the existence of objects of type Ox on the basis of this special sort of perception, there must be an epistemically adequate account of the difference between those who perceive O*x* and those who do not sufficient to underwrite the individuation of such perceptual states. That this is required of us is especially important in cases where there is variation in perceptual ability and where there is serious disagreement about the existence of the alleged objects of perception in question.[17] In the case of mathematical and set-theoretical objects it seems reasonable to hold that both such conditions are met, and so MERs are obligated to provide an adequate account of the special sensory acuity that their view depends on. However, it is not clear that this can be done while preserving the fundamental insight of the appeal to mathematical perception.

3. The Individuation of Perceptual Contents and Mathematical Perception

Consideration of the MER justificatory and explanatory appeals shall begin, as noted in the introduction, with a consideration of the coherence of the notion of mathematical or set perception in terms of the narrow/wide content distinction as it applies to perceptual states. As McGinn (1989), and others, have pointed out we can usefully apply this distinction to perceptual states as well as higher order cognitive states such as beliefs, e.g. to beliefs regarding natural kinds. This distinction finds its origin in Putnam 1975, but Block makes it particularly clearly in the following passage.

> One can think of narrow and wide individuation as specifying different aspects of meaning, narrow and wide meaning. (I am not saying that narrow and wide meaning are *kinds* of meaning, but only aspects or perhaps only *determinates* of meaning.) Narrow meaning is 'in the head,' in the sense of this phrase which it indicates supervenience on physical constitution, and narrow meaning captures the semantic aspect of what is in common in utterances of (e.g.) (1) [I am in danger of being run over] by different people. Wide meaning, by contrast, depends on what individuals outside the head are referred to, so wide meaning is not "in the head." The type of individuation that gives rise to the concept of narrow meaning also gives rise to a corresponding concept of narrow belief content (Block 1994: 85, bracketed material added from 84 for clarification).

This particular distinction between aspects of meaning arose with the causal theories of meaning proposed originally by Putnam (1975) and Kripke (1972) with respect to natural kind terms. Putnam's initial claim was that meanings, at least of natural kind terms, just are not "in the head". In fact, for Putnam, no parts of the meaning of such terms are in the head. That is to say, that the assumptions of methodological solipsism are wrong. Instead, meanings are determined by causal chains traced back to entities in the world external to the subject in question. In any case, what the view amounts to is that psychological states do not determine extensions (see Devitt 1990). The upshot of this preference for theories of wide content, or wide meaning, is that the relevant states in question are individuated by appeal to entities external to the believer or perceiver. Consequently, two such states are different if the objects that caused them are different. For example, the perception that a dog is speaking to me is to be individuated from the perception that a policeman is speaking to me because one was caused by a dog and the other by a policeman.

The alternative position, known loosely both as methodological solipsism and as meaning holism, claims that meanings are in the head, and that they are determined by the conceptual role a term or state plays in the cognitive architecture of an individual belief system. In effect, this position, in its pure forms, remains silent about the entities external to the believer or perceiver. As a result, the system, or lattice, of beliefs 'in the head' of the subject need only be empirically adequate rather than true in its fullest sense.[18] In any case, on this view such states are individuated by reference to the conceptual scheme of that individual. With respect to perceptual states this position says that a perceptual state is a state internal to the perceiver, and it is an instantiation of a perceptual concept whose content is *determined* by the role of that concept in the conceptual scheme that the perceiver holds. As a result, for those who hold such theories, perception is radically plastic and state contents are not determined or individuated by their causal history or by their relation to anything external to the perceiver. On this notion of perceptual content perceptions are *interpreted*, at the base level, as being of such-and-such a type as determined by the conceptual scheme of the perceiver. As such, they do not in any way assume or imply the external existence of the objects mentioned in the content of such perceptual states independent of the concept under which that object falls, and, thus, typically such theories preclude the possibility of simple, direct, verification of the external existence or properties of the putative external objects. This preference for narrow content individuation implies that perceptual states are individuated by the concepts that they fall under. Thus, the perception that a dog is speaking to me is individuated from the perception that a policeman is speaking to me because the first involves the perceiver's concept of dogs and the second the perceiver's concept of policeman.

Now we can ask what this distinction implies for appeals to mathematical experience of the sort given by Maddy, Kim, and Bigelow, and it is clear that regardless of which side of the distinction the advocate of the appeal to mathematical experience falls on they are in trouble. As this distinction exhausts the field of theories of content and the individuation of contents, such theorists like those we have been considering must opt for one or the other approach and so they face the horns of a troubling dilemma.[19]

First let us consider the more simple appeals to mathematical experience offered by Kim and Bigelow. On the one hand, in the case in which the advocate of this sort of appeal to mathematical experience adopts the former position, the preference for individuation of perceptual states via wide content, it seems that we ought, at least in principle, to be able to give a causal account of the content of a mathematical or set-theoretical perception that would allow us to individuate such experience from ordinary perceptions. However, if this sort of advocate of the appeal to mathematical experience adopts the wide approach without a detailed theory of the neurophysiological differences between set perceivers and non-set perceivers, or between number perceivers and non-number perceivers, and absent some other proof to support the claim that sets or numbers exist, then they simply beg the question against the mathematical or set-theoretical anti-realist.

On the other hand, if the advocate of the Kim and Bigelow style MER-type appeal adopts the narrow content approach to the individuation of perceptual states, then they radically weaken their position with respect to the justification of existence claims concerning the objects of such states. In fact, in doing so these philosophers of mathematics undermine their claim that there are direct mathematical or set-theoretical perceptions at all. Given purely narrow content theories, perceptions are more properly regarded as being conceptual interpretations of perceptions that may or may not correlate with real things. Perceptions of the world construed 'as if' sets or numbers existed, just cannot do the job that MERs require of them whether it is to justify or explain mathematical knowledge. Such concepts may not be concepts of things or properties actually instantiated in the world, and presumably we do not just want *any* description of the phenomena but rather we want the correct one, the true one. In the case of the solipsistic approach to content individuation, we would need some independent argument or evidence for the grounding of the belief in the existence of those particular external objects rather than those implied by some alternative conceptual scheme.

Consequently, it seems that the advocates of the Kim Bigelow MER appeal *must* adopt the first tactic, the wide content approach to the individuation of perceptual states, and as we have seen that approach is deeply problematic as it begs the question against the anti-realist. This last contention, that such MERs must accept the wide theory of content individuation, is, in any case, supported by much of what Kim and Bigelow say about perception and knowledge. It seems particularly perspicuous and appropriate given that these two MERs are highly sympathetic to causal theories of knowledge that are by their very nature externalist or wide theories of perceptions, belief and justification.[20]

However, given what has been critically noted about this horn of the dilemma it then would become crucially important for these defenders of MER to offer an account of the neurological and physiological differences referred to above but as this account is wholly absent as things stand. So just like those MERs who might opt for the narrow approach to content individuation these externalist MERs must apparently offer some independent *inferential* proof to support the claim that sets or numbers exist if their existential claims based on the appeal to mathematical experience are to have any epistemic purchase whatsoever. However, *ipso facto*, *this renders the direct appeal to mathematical experience impotent as a source of independent justification for the existence of sets and other mathematical objects*.[21] To establish that the appeal to mathematical experience actually justifies realism about mathematical entities given an externalist approach to the individuation of perceptual contents and without begging that question against the anti-realist, we would need some account of the proper functioning of our perceptual apparatus sufficient to justify our belief that we are actually in causal (i.e. perceptual) contact with numbers. However, as things stand there is no such extant account of this sort and so the externalist MER appeal does not *actually* justify mathematical realism.[22] That such an account *might be true* cannot possibly do the *de facto* epistemological work required to warrant belief in mathematical realism. Moreover, it is curious to note that should the MER actually provide such an account in an attempt to secure *de facto* justification for mathematical realism on the basis of the appeal to mathematical realism, it would be by its very nature inferential and hence would seem to render the appeal to direct experience redundant. The perceptual states themselves would no longer provide *independent* justification for mathematical realism.

So we have what looks like a serious objection to the Kim and Bigelow MER appeal to mathematical experience no matter what form of theory they adopt concerning the individuation of perceptual contents. There is then no extant good reason to believe that we perceive numbers, numerical properties or sets, or that we come to know that they exist in the direct manner that these defenders of MER assert. This is because adopting the narrow means of perceptual content individuation does not underwrite such existential claims and adopting wide means for individuating perceptual contents when such perceptual states have not been properly, i.e. naturalistically, individuated from those perceptions that are not mathematical or set-theoretical also does not do the trick either. If this is the case, then it is surely improper to claim that the truth of any fundamental mathematical and set-theoretical claims, especially those concerning the existence of the objects of such claims, can be grounded in appealing to special forms of direct perception that employ our ordinary perceptual faculties, at least as things currently stand. What externalists would be required in order to establish that such a claim actually secures the sort of justification in the existence of mathematical entities is a reasonable causal account of the mechanism by which such perceptions are produced and we admittedly do not yet have any such account. Moreover, should such an account be provided it would undermine the very appeal to justification by appeal to direct mathematical experience that these two MERs share in common. However, Maddy's more sophisticated version of MER might appear to more appealing in light of these problems, as it is not merely a simple and direct appeal to mathematical experience like the ones examined to this point.

In the case of Maddy's version of MER things are slightly more complicated and this is because she does appeal to an additional argument to ground her empirical realism with respect to mathematics. Specifically, as we have see above, she appeals to the Quine/Putnam indispensability arguments to initially—and presumably only partially—ground belief in the existence of mathematical entities. The problem that then arises for Maddy's view is similar—although not identical to—that which afflicts the view of Kim and Bigelow. The problem for Maddy is that absent an account of mathematical perception that is sufficient to allow the individuation of mathematical perceptions from non-mathematical perceptions using those same perceptual organs we *have not actually explained* how we have mathematical knowledge and so have not actually answered the Benacerraf problem.

On the one hand, if Maddy were to opt for a wide theory of perceptual content individuation we ought to be able to give a causal account of the content of a mathematical or set-theoretical perception that would allow us to individuate such experiences from ordinary perceptions. However, absent a detailed and *true* theory of the neurophysiological differences between set perceivers and non-set perceivers we simply have no extant and adequate explanation at all of how this sense modality does what it is alleged to do. Merely *speculating* that some theory, like Hebb's or one its successors, might explain set perception is nothing more than wishful thinking. Theories only explain—and hence only add justificatory support—when they are formulated in detail, empirically tested and true. The lack of such a theory constitutes a defeater for the claim that our knowledge of mathematics has been explained. Moreover, given this horn of the dilemma Maddy faces the charges that her view is only as strong as the Quine/Putnam indispensability arguments, which she herself explicitly (1997) takes to be inadequate to justify a full-blooded realism when taken in isolation and that she has not offered an answer to the Benacerraf argument.

On the other hand, if the Maddy adopts the narrow content approach to the individuation of perceptual states, then she radically weakens her position with respect to the justification of existence claims concerning the objects of such states and with respect to the Benacerraf problem. Again, given a purely narrow content theory of individuation perceptions may not correlate with real things. A theory that explains perceptions of the world construed 'as if' sets or numbers existed just cannot do the job that Maddy requires, not to mention that—as in the case of the first horn of the dilemma she faces here—no such articulated theory has been offered to us. As a result, we have a defeater here as well for the claim that our mathematical knowledge has been explained. In any case narrow content theories *cannot possibly* explain mathematical knowledge in such a way as to deal with the Benacerraf problem and so if Maddy were to opt for a theory of content individuation of the narrow sort she cannot meet one of the two major desiderata she sets out to meet. As a result, she seems to be forced onto the first horn of this dilemma and so must concede the failure of her early view.

Our current understanding of perception, both acquired and native, does not explain how we perceive mathematical entities and at least as far as things stand with respect to our current science of perception, we do not have an adequate explanation of mathematical perception. As a result, Maddy's 1990a view amounts to little more than pure speculation about an empirical matter and adherence to the, at least controversial, Quine/Putnam indispensability arguments. The main point raised here, however, is that due to the lack an adequate neurophysiological theory of set perception Maddy's view fails to explain our mathematical knowledge and this is a significant and damning weakness for her early views, *whatever* one might say about the adequacy of the Quine/Putnam indispensability argument.

4. Conclusion

So neither the more simple and direct appeals to mathematical experience made by Kim and Bigelow, nor the more complex appeal to mathematical experience made by Maddy are ultimately successful in grounding or supporting MER. So, those of us who would like to defend a form of MER must look to the actual science of perception of numbers for a theory that would satisfy the requirements noted above or we must look for other sorts of arguments to ground empiricism and realism about mathematical entities. The simple lesson that we should take to heart from all of this seems to be that the empirical issue of whether and how sets, mathematical entities and/or mathematical properties can be perceived is a serious scientific matter that needs scientific attention *before* we can make such appeals in the context of the philosophical discussion of the epistemology of mathematics.[23] This is especially important for those of us who abide by the tenets of naturalism. Naturalists, in particular, must be careful not to speculate about the relevant science that may or may not support broadly philosophical views.

References

Armstrong, D. 1973. *Belief, Truth and Knowledge*. Cambridge: Cambridge University Press.

Bartoshuk, L. 2000. 'Comparing Sensory Experiences Across Individuals: Recent Psychophysical Advances Illuminate Genetic Variation in Taste Perception.' *Chemical Senses* **25**: 447–60.

Bartoshuk, L., V. Duffy, D. Reed and A. Williams. 1996. Supertasting, Earaches and Head Injury: Genetics and Pathology Alter our Taste Worlds.' *Neuroscience and Behavioral Reviews* **20**: 79–87.

Benacerraf, P. 1973. 'Mathematical Truth.' *Journal of Philosophy* **70**: 661–79.

Bigelow, J. 1988. *The Reality of Numbers*. Oxford: Clarendon Press.

——. 1990. 'Sets are Universals.' In A. D. Irvine (ed.), *Physicalism in Mathematics*. Dordrecht: Kluwer.

Bigelow, J. and R. Pargetter. 1990. *Science and Necessity*. Cambridge: Cambridge University Press.

Block, N. 1993. 'Holism, Hyper-analyticity, and Hyper-compositionality.' *Mind & Language* **8**: 1–25.

——. 1994. 'Advertisement for a Semantics of Psychology.' In S. Stich and T. Warfield (eds.), *Mental Representation*. Cambridge: Blackwell.

Bonevac, D. 1982. *Reduction in the Abstract Sciences*. Indianapolis: Hackett Publishing.

Boothe, R. 2002. *Perception of the Visual Environment*. New York: Springer-Verlag.

Bueno, O. 2000. 'Empiricism, Mathematical Truth and Mathematical Knowledge.' In N. Shanks and R. Gardener (eds.), *Logic, Probability and Science*. Amsterdam: Rodopi.

——. 2003. 'Quine's Double Standard: Undermining the Indispensability Argument Via the Indeterminacy of Reference.' *Principia* **7**: 17–39.

Cheyne, C. and C. Pidgen. 1996. 'Pythagorean Powers or a Challenge to Platonism.' *Australasian Journal of Philosophy* **74**: 639–45.

Colyvan, M. 2003. *The Indispensability of Mathematics*. Oxford: Oxford University Press.

Coren, S., L. Ward and J. Enns. 2004. *Sensation and Perception* 6^{th} ed. New York: John Wiley and Sons.

Devitt, M. 1990. 'Meaning Just Ain't in the Head.' In G. Boolos (ed.), *Meanings and Method: Essays in Honor of Hilary Putnam*. Cambridge: Cambridge University Press.

Duffy, V. and L. Bartoshuk. 2000. 'Food Acceptance and Genetic Variation in Taste.' *Journal of the American Dietetic Association* **100**: 647–55.

Field, H. 1980. *Science Without Numbers*. Oxford: Blackwell.

——. 1989. *Realism, Mathematics and Modality*. Oxford: Blackwell.

Gödel, K. 1947/1964. 'What is Cantor's Continuum Problem?' In P. Benacerraf and H. Putnam (eds.), *Philosophy of Mathematics* 2^{nd} Ed. Cambridge: Cambridge University Press.

Hebb, D. 1949. *The Organization of Behavior*. New York: John Wiley and Sons.

——. 1980. *Essay on Mind*. Hillsdale, NJ: Lawrence Earlbaum.

Kaufmann, E. L., et al. 1949. 'The Discrimination of Visual Numbers.' *American Journal of Psychology* **62**: 498–525.

Kim, J. 1981. 'The Role of Perception in A Priori Knowledge: Some Remarks.' *Philosophical Studies* **40**: 339–54.

——. 1982. 'Perceiving Numbers and Numerical Relations (abstract).' *Nous* **16**: 93–4.

Kitcher, P. 1983. *The Nature of Mathematical Knowledge*. Oxford: Oxford University Press.

Kripke, S. 1972. *Naming and Necessity*. Cambridge, MA: Harvard University Press.

LePore, E. and B. Loewer. 1987. 'Dual Aspect Semantics.' In E. Lepore (ed.), *New Directions in Semantics*. London: Academic Press.

Maddy. P. 1980. 'Perception and Mathematical Intuition.' *Philosophical Review* **89**: 163–96.
——. 1990a. *Realism in Mathematics*. Oxford: Clarendon Press.
——. 1990b. 'Physicalistic Platonism.' In A. D. Irvine (ed.), *Physicalism in Mathematics*. Dordrecht: Kluwer.
——. 1992. 'Indispensability and Practice.' *Journal of Philosophy* **89**: 275–89.
——. 1995. 'Naturalism and Ontology.' *Philosophia Mathematica* **3**: 248–70.
——. 1997. *Naturalism in Mathematics*. Oxford: Oxford University Press.
McGinn, C. 1989. *Mental Content*. Cambridge, MA: Blackwell.
McIlwin, J. 1996. *Introduction to the Biology of Vision*. Cambridge: Cambridge University Press.
Moore, G. E. 1939. 'Proof of an External World.' *Proceedings of the British Academy* **25**: 273–300.
Pargetter, R. 1990. 'Experience, Proper Basicality, and Belief in God.' *International Journal for the Philosophy of Religion* **27**: 141–63.
Pryor, J. 2000. 'The Skeptic and the Dogmatist.' *Nous* **34**: 517–49.
Putnam, H. 1972. *Philosophy of Logic*. New York: Allen and Unwin.
——. 1975. 'The Meaning of "Meaning".' In K. Gunderson (ed.), *Minnesota Studies in the Philosophy of Science,* vol. VII. Minneapolis: The University of Minnesota Press.
Reid, T. 1785/1969. *Essays on the Intellectual Powers of Man*. Cambridge: MIT Press.
Quine, W. V. O. 1961. *From a Logical Point of View*, 2nd ed. New York: Harper and Row.
——. 1966. *The Ways of Paradox*. New York: Random House.
Shaffer, M. 2004. 'A Defeater for the Claim that Belief in God is Properly Basic.' *Philo* **7**: 57–70.
——. 2007. 'Taste, Gastronomic Expertise and Objectivity.' In F. Allhoff and D. Monroe (eds.), *Food & Philosophy*. Oxford: Blackwell Publishing.
Van Fraassen, B. 1980. *The Scientific Image*. Oxford: Clarendon Press.

Keywords
Epistemology, mathematics, realism, perception, empiricism.

Michael J. Shaffer
Department of Philosophy
St. Cloud State University
CH 365
720 4th Ave. South
St. Cloud, MN 56387
USA
mjshaffer@stcloudstate.edu

Resumo

Alguns trabalhos recentes de filósofos da matemática tiveram o objetivo de mostrar que nosso conhecimento da existência de pelo menos alguns objetos matemáticos e/ou conjuntos pode ser epistemicamente fundamentado por meio de um apelo à experiência perceptiva. A capacidade sensorial a que eles se referem ao fazer isso é a capacidade de perceber números, propriedades matemáticas e/ou conjuntos. A principal defesa dessa concepção, no que se aplica à percepção de conjuntos, pode ser encontrada no livro Realism in Mathematics, de Penelope Maddy, mas vários outros filósofos recorrem a argumentos similares, se bem que mais simples, desse tipo. Por exemplo, Jaegwon Kim (1981, 1982), John Bigelow (1988, 1990), e John Bigelow e Robert Pargetter (1990) todos defenderam tais concepções. A questão crítica central que será levantada aqui diz respeito à coerência das noções de percepção de conjuntos e percepção matemática, e se os orgumentos em favor de tais faculdades perpceptivas realmente podem dar uma justificação para a crença na existência de conjuntos, propriedades matemáticas e/ou números, ou uma explicação para tal crença.

Palavras-chave

Epistemologia, matemática, realismo, percepção, empirismo.

Notes

[1] Of course, Mill, in being a nominalist, is best interpreted as being an anti-realist about mathematical entities. Empiricist accounts of mathematical knowledge offered after Mill shared in this assumption and they generally involved the logicist/anti-realist maneuver of reducing mathematical entities away. This project, however, turned out to be an abject failure. In opposition to this view the form of empiricism with which we shall be primarily concerned here is devoutly welded to realism.

[2] Notably, Kitcher has also recently defended a rather different form of empiricism in his 1983.

[3] See Colyvan 2003 for a defense of such arguments and Cheyne and Pidgen 1996, Maddy 1992, 1995 and 1997, Field 1980 and Bueno 2003 for criticism of the indispensability arguments.

[4] Here the expression 'set perception' will be reserved for the specific ability to perceive sets, whereas 'mathematical perception' will be used to designate the more general ability to perceive abstract mathematical and set-theoretical objects.

[5] The choice to use the term 'appeal' rather than 'argument' is intentional. Direct appeals to perceptual experience as grounds for existential claims are not typically taken to be

inferential. Moreover, the expression 'mathematical experience' will be used to refer to both number and set perception.

[6] As it is not my purpose to solve the general problem of whether we perceive objects or only the properties of objects, I shall remain neutral on this issue. However, readers should be cautioned that what is really at issue is not only whether numbers exist qua objects, but also whether it is true that concrete physical objects have real causally efficacious abstract set-theoretical or numerical properties.

[7] Specifically, the criticism of the arguments from mathematical experience raised here will depend heavily upon the narrow/wide content distinction with respect to mental and perceptual states that Putnam (1975), Kripke (1972), McGinn (1989), Block (1993, 1994), and others have employed to great effect in various contexts.

[8] As a matter of fact, Kim notes in footnote 12 of his 1981 that his view is importantly similar to that defended by Maddy in her 1980, even though he also suggests that his epistemological views on the matter are somewhat different.

[9] Bigelow, however (Bigelow 1988: 3, 178), does not rule out the possibility that his view might be strengthened by an additional appeal to the Quine/Putnam indispensability arguments as Maddy (1990) also suggested. However, as has been frequently noted, Maddy (1992, 1995, 1997), Bueno (2003), Field (1980) and many others have rejected the adequacy of the indispensability arguments.

[10] See Maddy 1990: 67. 'Naturalistic' is here meant to simply imply that the theory is scientifically legitimate in the sense that it is physiologically realistic and purely causal.

[11] See Boothe 2002 for extensive discussion of the neurophysiology of vision, especially chapter 5.

[12] See for example, McIlwin 1996 and Coren, Ward, and Enns 2004.

[13] That is unless one simply accepts the sort of blatantly question-begging perceptual dogmatism defended by, for example, Pryor 2000 in the context of perceptually odd entities. This would be especially troubling, however, in cases involving the positing of such controversial and not obviously perceivable entities. Consider, by analogy, the case of expert wine or food perception. Ought we to accept that certain people can perceive tastes that the average person cannot and hence that there are such taste properties because they simply claim to do so and when there is absolutely *no* theory concerning how they do so? The same point applies in the case of disagreement between groups of experts and ordinary people as well in this case. Why ought we to believe that they can make some perceptual discrimination that we cannot make using the same perceptual systems simply because they claim to be able to do so. See Shaffer (2007) for discussion of this related case. It is also important to note that this kind of case is rather different than one person's being able to *interpret* what he or she sees in a different way than someone else. A doctor's 'seeing' a shadow on a radiograph as a pathology when a novice sees only a shadow is not the same kind of case as that involving sets. Both the doctor and the

novice see the same thing (i.e. a shadow). The difference between perceivers in cases like this one is simply one of learning to *interpret* the shadow as a tumor and neither lacks the ability to detect shadows with their eyes. In the case of set perception it is not a matter of learning to *interpret* our ordinary perceptions in terms of set vocabulary, but it is supposed to be possession of a new perceptual ability to actually perceive some content that the novice cannot even perceive.

[14] See Bartoshuk, Duffy, Reed and Williams 1996, Duffy and Bartoshuk 2000, and Bartoshuk 2000.

[15] See Shaffer (2007) for more on taste perception and special acuties.

[16] Notice that this argument does not imply any sort of general skepticism with respect to perception. This is, of course, the case because we do have explanatorily adequate accounts of our perception of many kinds of properties and/or objects.

[17] A nice parallel, noted by Pargetter (1990), can be drawn between the cases of the alleged perception of numbers and of God. Shaffer 2004 contains a critical argument against the employment of perception in grounding beliefs in such religious entities that parallels the argument given here with respect to numbers and sets and Shaffer (2007) treats a related case involving taste perception along the same lines.

[18] See Van Fraassen 1980 for details concerning empirical adequacy versus truth and see Bueno 2000 for a defense of a form of mathematical empiricism coupled with anti-realism about mathematical objects explicitly based on Van Fraassen's views.

[19] One might suspect that hybrid views, those views that accept some form of dual aspect approach to content individuation, might avoid this dilemma, but this is not the case. Whatever else is wrong with dual aspect views views (see Lepore and Loewer 1987), they cannot help with this problem in the context of individuating perceptual states. Any theory that accepts a role for narrow content individuation of perceptual states will entail that the object as perceived may not be like the object itself. Narrow content approaches, in this way, always allow for the 'pollution' of content in a skeptical way that will undermine any assumption of the veracity of such perceptions. In effect then, the opposition is really between pure wide content theories of perceptual state individuation and any theory that affords any role to narrow content in individuating perceptual contents.

[20] See Maddy 1990, chapter 2, Armstrong 1973, and Kitcher 1992.

[21] In point of fact, Maddy actually anticipates this sort of maneuver in her dealing with the anti-realist response to her Steve example (1990: 59). She suggests that the real justification for existential claims about the existence of numbers is really just the Quine/Putnam indispensability argument. This, as we shall see is however, problematic with respect to answering the Benacerraf problem in terms of MER. Subsequently, Maddy (1997) has given up on MER in the sense that she now rejects as interesting existential questions about mathematical objects (233) and seems to shift her focus entirely from offering perceptual justifications for mathematics to pure pragmatic justification of

mathematics in the spirit of a modified Quinean mathematical naturalism and which focus primarily on accounting for actual mathematical practice.

[22] It is almost not worth noting, but appeal to some promissory note about the relevant science here is surely insufficient to generate the requisite justification.

[23] My own response to this sort of dilemma is to endorse the sort of naturalistic reductionism suggested by Bonevac (1982).