Strategic Set Theory — Preliminary Remarks

Abstract – An attempt to vindicate naive set theory against the perils of Russell's paradox by postulating a universal set V which is describable in two distinct description languages: predicative and extensional. The extensional description of a set consists of describing all its elements whereas its predicative description consists of describing what sets it is an element *of*.

Extensionally described V has an uncapturable description length, akin to its cardinality. But predicatively described, in virtue of being the set that is not contained in any set whatsoever, V has a minimal description length, a counterbalance to its cardinality.

Descriptive efficiency can genuinely be attributed to V (achieved in a third description language) only if it contains predicatively indiscernible but extensionally discernible sets.

1 – The description language implicated in axiomatic set theory is concerned with the extension of set (thus the set of all sets V has untamed description due to its famously uncapturable cardinality). Only if the elements of a set are specified it is uniquely specifiable as being contained in other sets. The extension of a set precedes its predication.

2 – Reversal of this ontological priority (constitutional role of a set preceding its extension) results in a description language in which V—postulated to be contained in no other set whatsoever—has a minimal KC in the new description language, a counterbalance to its cardinality. A set is uniquely identifiable by complete specification of what sets it is an element of not necessitating complete specification of what elements the set itself contains.

3 – If V contains sets which are predicatively identical but contain different elements then there is a third description language—heterogeneously drawing from both extensional and predicative descriptions—in which V is descriptively neither trivial nor untamed but efficient.

Appendix – The Kolmogorov complexity (KC) of an object is the length of the shortest possible description of the object in some fixed universal description language. Relative to a description language, a set A may be classified as information-theoretically admissible if the KC of the set in predicative terms is strictly less than its KC in extensional terms:

$$\sum_{a \in A} \mathrm{KC}(a) \; > \; \sum_{A \in \alpha} \mathrm{KC}(\alpha).$$

The set V is the set of all information-theoretically admissible sets in the language that incorporates the most efficient description of its elements.