

Systems with Single Degree of Freedom and the Interpretation of Quantum Mechanics

Mehran Shaghghi

(ORCID iD: 0000-0002-0278-9936)

mehran@uic.edu

University of Illinois-Chicago, 60608-1264, IL, USA

Abstract

We show that the physical systems with absolutely one degree of freedom demonstrate a number of peculiarities, common in quantum mechanics (QM), including randomness, no-cloning, and non-commutativity. The central point in such systems is the limitation on their information storage capacity. Discussing the consequences of that limitation and the resulting parallels with quantum behavior we postulate that other quantum phenomena can also be explained by considering such systems. As an example, we offer an explanation for the EPR paradox. We also elucidate why counterfactual definiteness cannot be presumed in such systems. Thus we present an interpretation in which quantum physics can be understood as the physics of the systems with single degree of freedom. In the present work, we assume that the formalism of the QM is correct and well-supported by experimental verification and concentrate on the interpretational aspects of the theory.

Keywords Interpretation of quantum mechanics. Information theory. Quantum ontology. Entanglement. EPR paradox

1 Introduction

In his famous lectures, Richard Feynman decided to tell the “mystery” of quantum mechanics using the double-slit experiment “which has in it the heart of quantum mechanics” [1]. A prominent constituent in the double-slit experiment is the fundamental limitations on what properties of the photons can be measured: either extracting the which-way information (particle property) of the photons, or observing the interference (wave property) of the photons, but not both. Any effort to force observing both effects introduces an element of randomness that makes the results non-conclusive [2].

In practice, setting up a double-slit experiment to observe the “mysterious” quantum effects is not an easy task and requires certain prerequisites. For example, an ordinary light source cannot be used to form the interference patterns and thereupon the “mystery”. To see the quantum interference, all the incoming photons should be highly directional originate from the same location, and also be coherent so that their frequencies are the same. Using a laser source provides these conditions readily and has made demonstrating such experiments easily possible.

A quantum particle, such as a photon, has very limited degrees of freedom and therefore can only be loaded to carry a limited amount of information. For example, in using a photon as information carrier at most three pieces of information can be sent, using its position, frequency or spin. Indeed, the same degrees of freedom of the photon have been utilized in ‘polarized 3D systems’ to produce the shape, color and depth perception in many 3D movie theatres. In using electrons as information carriers too, information can be loaded on either of its similar limited

degrees of freedom. However, in some situations, not all the degrees of freedom of a system can be used to carry information. For example, in a setup that uses a stream of photons in which all are coming from the same source and having equal coherence length and equal energy, only the spin degree of freedom of the incoming photons remains not fixed. This is the setup followed in selecting the photons in the double-slit experiment, and this leaves the photons with only one degree of freedom for carrying information.

Physical systems with just one degree of freedom, due to their limited capacity to hold on pieces of information, as we will discuss, show behavior that are very different from what we are familiar with in ordinary systems and in classical physics. For example, they cannot hold two attributes simultaneously, as their information storage capacity is limited to one. The goal of this paper is to present a new interpretation of QM by discussing how quantum physics can be understood as the physics of the systems with only one bit of information capacity (Note that throughout this paper we use the term ‘bit’ in the sense of ‘one piece’ and not in the sense of ‘a binary digit’). Apart from its inherent interest, this interpretation can be used to solve a number of quantum dilemmas. One of these is the EPR paradox that we shall solve after presenting the interpretation.

2 1-bit information systems

Here we consider a physical entity which has left with only one degree of freedom and therefore can carry no more than one bit of information. In the physical world, this “1-bit information system” can be a structureless elementary particle such as an electron, a photon; or a big chunk of helium atoms in the Bose–Einstein condensate state, or each of the macroscopic molecules in the carefully prepared homogeneous beam of the macromolecules selected for performing the interference experiments with large molecules [3, 4] . For the sake of consistency, here we exemplify that “atom of information” as an electron whose spin state is the 1-bit system.

The 1-bit atom of information system here should not be confused with the classical one bit systems, which in fact possess many more than one degree of freedom. For example, a fair coin with the head and tail states is not a true 1-bit system. In labeling a fair coin as a one bit state one simply ignores many of its other attributes, such as its temperature, position or direction in the x-y plane, and takes such degrees of freedom as redundancies; however, physically those each constitute different states.

2.1 Randomness

When performing a measurement on a 1-bit information system, e.g. measuring the spin state of an electron – our example for the 1-bit information systems– in a certain direction, all the information content of the 1-bit system gets extracted and the system is left with no more information. But what happens if we keep performing experiments and extracting data from that zero bit state? The system is left with no more information to extract; meaning, unless we

perform the same experiment on that spin, we will only collect random data. Random data is an expression of zero information as we will see.

Mathematically the information gained in a process is defined as the change in the Shannon entropy [5]

$$I_{12} = H(X_1) - H(X_2), \quad (1)$$

in which Shannon entropy is defined as

$$H(X) = -\sum P(x_i) \log_2 P(x_i), \quad (2)$$

where

$$X = \{x_1, x_2, x_3, \dots\}, \quad (3)$$

is the set of probable outcomes with $P(x_i)$ being their probabilities.

In two cases the information gain in the measurements is zero. The first case is when there is no change in the obtained data, that is, when $H(X) = 0$. The other case is when there is no change in the Shannon entropy, $H(X_1) = H(X_2)$, and therefore $I_{12} = 0$, which is the case of collecting random results (=infinite conflicting data). In layman terms, zero information means either no new data, or continuously receiving random data, essentially receiving conflicting news. Remember an older TV set, ‘no data’ corresponds to the TV being off, while complete randomness corresponds to when the TV is on and displays static (noise/TV snow). Note that a black screen while the TV is on does not mean zero information but rather, the information that the TV displays a black scene.

In this description, the unpredictable random result that we collect in measuring the spin state of the above-mentioned electron in a different direction is a true manifestation of dealing with a state with zero information content. Feynman, during his talk on what would later be coined nanotechnology famously, stated: “there is plenty of room at the bottom” [6]. For the case of the 1-bit systems however we are pretty close to the bottom; after performing a measurement on the system, there is absolutely no more information to look for and seek to extract.

Trying to extract data from a zero information state does result in a series of conflicting random data (note that measurements always result in values). This situation cannot be dealt with by devising an underlying hidden variable. It is even hard to envision how a structure-less electron could carry machinery to produce the sets of results described with complex hidden variable theories. To put it simply, in performing measurements on a 1-bit information system, the randomness arises since there is no more information in the system’s storage to be gained. The information that does not exist cannot be measured and the randomness in the collected data is the pure result of hitting the information null ground and is not removable.

2.2 The singularity of the zero information state

Reaching the zero-information state brings more peculiarities to the behavior of a system. Consider a 1-bit system with no information loaded on it, a zero-information state. What state is the system in? Because this is a totally unfamiliar case, the answer is far from obvious. Discussing some fabricated situations may help in getting some ideas.

In trying to understand the zero information state, imagine having to install a clock on a city tower, with only the hour hand on the face. During the installation, there should be no information loaded on the clock so the bystanders won't be misled. There are different ways to accomplish this; for example, temporarily detaching the hand off the clock face so no information gets conveyed to the onlookers or, during installation, placing twelve hands on the clock pointing to all of the 12 positions so no time information can be collected. Obviously, the aforementioned aren't the only options, any number of hands can be placed on the clock, provided they don't all point to the same position. In short, the clock can be in all these states and carry no time-information.

In the physical systems, however, the constituents of a system do not change when they reach the zero-information state. For example, electrons always have spin¹ so we cannot have, an electron with no spin (cf. the exemplified clock with no hand) or a different total spin (cf. the exemplified clock with more than one hand). But how can a zero-information state have attributes with no values? Back to the clock example, imagine a clock in utterly empty space, with no number on its face. In such a condition, the position of the hand indicates and implies no information. This resembles the case in which a 1-bit system is in before it gets in touch with another system. When a 1-bit system is informationally insulated from the outside world there is no reference to define any state; the state of the system is indefinite and not localized in the state-space. With no reference point, there is no way to differentiate among the different possible states. Mathematically speaking in such a situation all x_i 's in Eq. (3) lose their distinction. Hence, the zero-information state can be regarded as being in all possible states and none at the same instant².

The point that we want to make is that digging down to the deepest level of reality, an isolated system's state which can hold only a bit of information, with no reference point is not absolute; its state is indefinite and only will possess meaning within a frame of reference. To have a reference at least another system in the universe is needed. After all, what can a piece of data imply in an absolutely empty space, or equivalently when it is informationally insulated from the outside world? To quantify a state, a reference point always needs to be invoked. The value of any attribute gets defined in comparison. In an empty world, a piece of data is void of meaning. A 1-bit system possesses a definite state only after it interacts with another system.

¹ This does not imply that the spin necessarily "has" a value, the measured property, before the measurement.

² The longitude of the North Pole is a perceptible example of the singularity; it can be simultaneously viewed as all degree values and none.

2.2.1 Qubit

As mentioned, a 1-bit system whose information content is not yet measured, that is informationally isolated from the world, is in an indefinite state. This singular state can be thought of as a state that is in no state, or alternatively, as a state that is in all the state-space. This description matches with what is known as a qubit [7] in quantum physics. A qubit, compared to a classical bit (which is always in either 0 or 1 state), can be in any mixture of those states before being observed. As described, this property is a peculiarity of any unmeasured 1-bit system.

2.2.2 Collapse

We saw that a 1-bit information system is in no defined state before being in contact with another system, a reference. Only after a 1-bit system gets in contact with a reference its state acquires value. Before that, the 1-bit system carries zero information and is singular as its information content is undefined. The singularity of the 1-bit system collapses and its value gets defined only after the state gets measured by an apparatus with a reference. This process is the same as what in quantum physics is described as the collapse.

2.2.3 No-cloning

As discussed, a 1-bit system before getting measured is in an indefinite state. An indefinite state cannot be duplicated in principle. Part of the peculiarity of the 1-bit system is the system's one bit of information capacity. The system jumps from the completely unspecific state, when it is in zero-information state, to a fully known state, when its one-bit value is measured. A 1-bit system moves between its extreme informational configurations by a single act of measurement. In general, there is no constraint on copying a state, but in the case of the 1-bit system, before the act of measurement, the state is basically undefined and cannot be duplicated. In quantum physics this peculiarity has been known as the no-cloning theorem [8, 9] in which it is proven that to create a copy of an unknown quantum state is impossible.

2.2.4 Non-commutativity

A consequence of having only one degree of freedom in the 1-bit system is that the system has a tight limit on how much information it can hold and transmit. Therefore, it is the case that the first measurement on a 1-bit system extracts the information content of the system and a second different measurement will read a random outcome. This makes the order of operations important for a 1-bit system. Unlike in classical physics, here the order of operations and the parameter which is measured first can make difference for the 1-bit systems. Again, similar behavior has been known in quantum physics.

3 Entanglement

We have found that the behavior of the 1-bit system in zero-information state is both peculiar and unfamiliar; its state is fundamentally unknown before performing a measurement and it can be regarded as being anywhere in the state-space, that is in superposition since no distinction

exists among those states in an informationally isolated system. This situation changes, however, as soon as one goes from a 1-bit system to a 2-bits system. In a 2-bits system, the system cannot be in all possible configurations since each subsystem can be evaluated with respect to the other in principle. Nevertheless, as we will see it is possible to have 2-bits systems made up of two 1-bit subsystems that still display all the peculiarities described in the 1-bit system case.

The *entanglement* is what makes it possible to get a 1-bit system made up of more than one 'atom of information'. In its simplest case, two coupled electrons, with the total maximum capacity of two bits, can be correlated so that the entangled system is a 1-bit state. In this entanglement, one bit of information is already registered in the correlation they share (the subsystems have complete covariance, for example, they have opposite spins) leaving only one bit of information capacity for the system. The entangled electrons jointly hold the one bit capacity of the system and this composite 2 components system encompasses all the same peculiarities described in the 1-bit systems.

This construct can be generalized for making a 1-bit system out of the system of n electrons, as in Greenberger–Horne–Zeilinger (GHZ) states [10], in which all the n components jointly hold the one bit of information capacity (Appendix).

In the light of the similarities we found between behavior of the 1-bit information systems and quantum particles next we attempt to use this picture for better understanding of some quantum riddles.

4 The Spooky EPR

Probably the most discussed entangled pair in physics is the EPR case. In a thought experiment proposed by Albert Einstein, Boris Podolsky and Nathan Rosen (EPR) [11] two entangled particles, S_1 & S_2 , are spatially separated. When the particles are so far apart that any classical interaction between the two would be impossible, a measurement on one particle nonetheless determines the corresponding result of the measurement of the other. How is it possible for the particles to coordinate the outcomes of the measurements?

This experiment poses another challenge to quantum physics too. In this setup, it seems possible to measure non-commuting variables (for example S_x and S_y) on each particle: the values of S_{1x} and S_{2y} , can be measured directly on the corresponding particle without the classical disturbance from the other and at the same time the values of S_{1y} and S_{2x} can be determined due to the particle's correlation. Einstein and colleagues argued that "every element of the physical reality must have a counterpart in the physical theory" [11], and pointed out that in terms of quantum mechanics formalism "when the operators corresponding to the two physical quantities do not commute the two quantities cannot have simultaneous reality" [11] and therefore by this proposed experiment they questioned the completeness of quantum theory.

Later Einstein restated the situation as "the real factual situation of system S_1 is independent of what is done with system S_2 , which is spatially separated from the former" [12]. In quantum mechanics, if you measure S_1 's spin, the state gets "set" by the measurement, but somehow S_2 also instantly, in a *spooky way*, "feels" what spin it is supposed to take on. To Einstein, this is a clear violation of the principle of locality and he argued against the notion that the theory provides "a complete description of a real factual situation." [12]

Here we discuss the situation according to the picture presented in the current paper. The two entangled electrons in the EPR pair already share one bit of information, namely, their sum of spins is zero. The remaining one bit information capacity is jointly shared by the two subsystems. *A priori* this joint 1-bit system has the same properties as any other isolated 1-bit system; its state is not defined, so before the measurement an assumption cannot be made regarding its value. The pairs carry zero information and it is only after the registration that they possess a value. Bell inequalities show clearly that presumption of value on the pairs leads to contradictions [13].

In short, the EPR pair is two spatially separated electrons that jointly carry one bit of information, a physically expanded 1-bit system. We advocate the view that the solution to the EPR paradox is to note that I) informational correlations are nonlocal, and II) a system in zero-information state holds no element of reality.

Informational correlations are mathematical in nature and while mathematical correlations can be shared between two physical systems, this doesn't make the mathematical correlations local. For example, if my brother and I jointly have \$10,000.00 in an account, it does not matter whether we reside in the same location or not. Assuming I withdraw nothing as soon as I look up the account balance, I know instantaneously the amount he has taken regardless of his physical distance. Physical systems can jointly share correlations between themselves, but it is not that one forces the other to be correlated. (cf. temperature does not prompt the molecules to go fast).

A distinction should be made between nonlocal enforcement of correlations, as in the EPR pair, and nonlocal communication, which, although sometimes confused with the former, is a far stronger condition. It has been made clear that EPR's nonlocality of correlations cannot be exploited for nonlocal observer-to-observer communication [14-16].

In the EPR pair, we should note that any separated measurements of the properties of an extended 1-bit system should be treated as parts of the same informational state, regardless of the degree of separation of the measurements in time and/or space. Considering the nonlocal nature of the informational correlations, the relative time ordering of the observations on the two systems, as well as their relative spatial arrangement, are irrelevant to the result. It has to be noted too that prior to the measurement, no value pre-existed on the system and there is no physical reality to be changed. There is no magical communication between the EPR pair; the informational correlation that is shared between the pair secures the combined value of

information on the system. When compared, the results on the subsystems match if the measurement decisions were consistent; otherwise, there is randomness. In any case, no message can be transmitted between the two. The problem of completeness postured in the EPR paper is therefore ill-posed. There exists no reality in the case of zero-information states to be concerned about it.

5 Null-information state

Many of such confusions arise since the zero-information state is not part of our natural familiar world and grasping its peculiarities needs some contemplation. We mentioned that the 1-bit information systems are not the same as the classical one bit systems since the latter still carry information in their other degrees of freedom. A similar distinction exists between the described zero-information state and the classical zero information states, which we are more familiar with. In a classical zero information state, regardless of its nomenclature, there is still other information held by the system. For example, assume your keys are lost and there is zero information about their location; regardless, the keys still have certain colors, temperatures, etc. That is while certain information is lacking about the system, the information content of the system is not zero and the system is still loaded with a myriad of other information. For an ‘atom of information’ however, the case is different as it can hold at most only one piece of information. When it is loaded with a piece of information, no other information can be assumed about it. The system possesses no capacity then to hold any other attribute such as temperature and color; it is in absolute zero, or a null-information state.

This is the crucial difference between the classical zero information state and the null-information state. The classical zero-information case is due to the lack of knowledge that the omniscient God has access to, but in the null-information case there is nothing to know about the state, even for the omnipotent, omniscient God. This distinction was not evident to Einstein when he rejected the probabilistic interpretation of quantum mechanics by saying that “God does not play dice with the universe” [17]. Zero information is about unknown information, while null-information is undefined information.

6 State function

Following all the aforementioned points what can be said about a system in the null-information state? To be exact nothing definite can be said about such a system, but to be smart one may still be able to say a few things about the system, probabilistically though. Concerning the null-information state, no information can be asserted, but possibly some general statements can be made about a system in such states using “peripheral” knowledge and boundary conditions. This approach can help in getting some idea about the situation, typically statistical. While having some general idea does not give definite predictive power, it is still better than no information.

In this approach, a mathematical construct can be employed to represent the statistical knowledge of the behavior of a 1-bit system. This state function portrays all the possible outcomes of the measurement on a 1-bit system in the null-information state. This is similar to contemplating all the options when there is no information about a case and all scenarios are therefore imagined. A big difference is that in the statistical analysis of such no information cases, it is taken into account that for the zero information cases all the alternatives are not possible simultaneously; however, for the null-information cases the singularity of the state means that all of the options coexist in superposition. One should note, however, that in the null-information cases there is no underlying reality and the state function should be understood as an epistemic state (state of knowledge) rather than an ontic state (state of reality).

The aforementioned approach can help us to understand the general framework that is followed in the mathematical formalism of quantum mechanics. The mathematics that is employed no longer represents the behavior of the system but rather the statistical knowledge of its behavior. This is the wave-like interpretation, the outcome is unpredictable and no absolute knowledge is possible. Unlike classical physics, where seeking deterministic results is the common practice, only probabilistic knowledge is possible in this realm.

In quantum mechanics, physicists have recognized a function which yields statistical knowledge for quantum systems. In the picture we presented the mathematical formulation of quantum mechanics can be seen as a recipe to construct the set of all possible states that expand a null information state. Schrödinger's equation can be derived using this idea along with a few general conservation laws (details will be presented elsewhere). Feynman's path integrals formulation [18] also follows a similar notion that when no information is known about a 1-bit system, use the null-state expansion and sum over all the possibilities rather than remaining silent about the situation.

6.1 The collapse of the wave function

In the picture provided, the wave function is solely a mathematical expansion of a null-information state, and not a physical entity. Therefore, the wave function collapse is the collapse of that mathematical expansion of an unknowable to a piece of information. The collapse happens when the 1-bit system gets in touch with another system that has a set point and the singularity of the null-information state spontaneously breaks. Such spontaneous breaking is not an unfamiliar occurrence in physics and constitutes the underlying concept of a vast number of physical phenomena ranging from ferromagnetism and superconductivity in condensed matter physics to the Higgs mechanism in the standard model of elementary particles.

It is the act of measurement therefore that creates a definite “element of reality” and a value for the null-information state. It is therefore a physical process (as opposed to some weird suggestions) at which the measurement leads to the collapse of the null-information state to a definite state and gaining a certain value.

7 Concluding remarks

“I remember discussions with Bohr which went through many hours till very late at night and ended almost in despair; and when at the end of the discussion I went alone for a walk in the neighbouring park I repeated to myself again and again the question: Can nature possibly be so absurd as it seemed to us in these atomic experiments?” W. Heisenberg [19]

About a century has passed since the development of quantum physics and it still remains somewhat mysterious. Despite the unparalleled predictive capacity of quantum mechanics, an interpretation of its formalism that can be regarded as uncontroversial has not been available and among the scientific community, there is no consensus on the interpretational aspect of the theory [20-22].

In this paper, we showed that the physics of the 1-bit systems has many similarities with quantum physics. Furthermore, we discussed how concepts like superposition and entanglement can be interpreted in a comprehensible fashion. In the picture presented, we used two principal elements:

1. There are physical systems at the extreme that have only one unoccupied degree of freedom and accordingly just one piece of information capacity.
2. Zero information means either no new data, or conflicting data (randomness).

There are also four points that we benefited from: I) Measurements always result in values; II) The extraction of more information from a system than its information content is not possible; III) Informational correlations are mathematical and not necessarily local; and IV) Information can be shared and held jointly by physical systems.

The presented picture, quantum physics as the physics of the 1-bit information systems, can be the unification model for explaining quantum mechanics. Thus explaining away the apparent oddness and confusion of quantum physics in general and helping to finally clearly grasp the physical meaning of the theory.

A novel point in the picture we presented is the concept of null information. Similar to the historical concepts of zero, negative numbers and imaginary numbers, at first this concept can also be accompanied by paradoxes and misunderstandings. It is not unexpected that controversies in interpreting the philosophical and epistemological implications of null information arise; however, as in the other cases, soon this concept can be a part of common scientific knowledge.

An important corollary of the picture we presented is the rejection of the counterfactual definiteness for the 1-bit systems. The familiarity of counterfactual definiteness such as ‘the moon is there even if no one looks’ [23] is not applicable in 1-bit systems. To make it concise, in systems that can carry only one piece of information, there can be no more than one piece of “definiteness” and thus “counterfactual definiteness” cannot be presumed. This feature has been debated in many discussions that contrasted quantum mechanics with classical physics and many

paradoxes are rooted upon [11, 24, 25]. As mentioned, given that a corresponding measurement for an attribute is not already performed, the 1-bit systems possess no value for that attribute, and hereupon ‘the unperformed experiments have no results’ [26]. In this realm when measurements are performed, values result, but these values should not be considered to be a disclosure of pre-existing values.

In conclusion, we showed that many peculiarities of quantum physics can be understood as the properties of the systems which have just a single degree of freedom. Furthermore, by presuming that quantum systems are systems with single degree of freedom we could explain topics such as EPR paradox and impossibility of counterfactual definiteness in QM. The interpretation we presented in this paper can help to explain where quantum physics comes from. It is suggestive that this new picture sheds new lights on the meaning and philosophical implications of concepts such as entanglement, information, reality, and quantum computation.

Funding No relevant funding.

Compliance with ethical standards

Conflicts of interest The author states that there is no conflict of interest.

Ethical statement This work did not involve any active collection of human data.

Data accessibility statement This work does not have any experimental data.

Appendix: GHZ and W entangled states

Greenberger–Horne–Zeilinger state is a type of entangled quantum state that involves at least three subsystems. In simple words, it is a superposition of all subsystems being in state $|\uparrow\rangle$ with all of them being in state $|\downarrow\rangle$. The 3-qubit GHZ state can be written as

$$|GHZ\rangle = \frac{|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle}{\sqrt{2}}, \quad (\text{A.1})$$

and in the general form with $n \geq 3$ subsystems

$$|GHZ\rangle = \frac{|\uparrow\rangle^{\otimes n} + |\downarrow\rangle^{\otimes n}}{\sqrt{2}}. \quad (\text{A.2})$$

For the general n -particle $|GHZ\rangle$ entangled state the information capacity of the system can be found by these considerations: the n subsystems can hold n bits of information. However, $n-1$ bits are already set in the correlations among the subsystems in the type of: $|S_1\rangle = |S_2\rangle, |S_2\rangle = |S_3\rangle, \dots, |S_{n-1}\rangle = |S_n\rangle$. That leaves only 1 bit of information capacity for this n -particle system.

The W state involves another class of a multipartite entangled state. For three qubits it has the following form

$$|W\rangle = \frac{|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle}{\sqrt{3}}. \quad (\text{A.3})$$

The notion of W state can be generalized for n -particles [27] as the superposition state with equal expansion coefficients of all possible pure states in which exactly one of the particles is in an “excited state”, $|\uparrow\rangle$, while all other ones are in the “ground state”, $|\downarrow\rangle$:

$$|W\rangle = \frac{1}{\sqrt{n}} (|\uparrow\downarrow\downarrow\dots\downarrow\rangle + |\downarrow\uparrow\downarrow\dots\downarrow\rangle + \dots + |\downarrow\downarrow\dots\uparrow\rangle). \quad (\text{A.4})$$

For general n -particle $|W\rangle$ state also the information capacity of the system is only 1 bit; from the n bits of information that can be carried by the system $n-1$ bits are already set: 1 bit of information is embedded in $\sum_k \langle \uparrow | S_k \rangle = 1$ (*i.e.* exactly one of the subsystems is in the “excited state”). The $n-2$ correlations of the form $|S_i\rangle = |S_j\rangle$ among the $n-1$ subsystems (*i.e.* all these subsystems are in the same state) fix $n-2$ bits of information. Thus, only 1 bit of information remains as the information capacity of the system.

A.1 Pairwise entanglement

Note that the GHZ state can be written as

$$|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle = (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \otimes |\rightarrow\rangle - (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle) \otimes |\leftarrow\rangle \quad (\text{A.5})$$

where the third particle is written as a superposition in the X basis (in contrast with the Z basis) in which $|\uparrow\rangle = |\rightarrow\rangle - |\leftarrow\rangle$ and $|\downarrow\rangle = |\rightarrow\rangle + |\leftarrow\rangle$. In this case, measurement of the GHZ state along the X basis for the third particle then results in a maximally entangled Bell state.

In writing the GHZ according to this expansion one bit of information is fixed by $|S_1\rangle = |S_2\rangle$. For the remaining two bits of information, measurement in the X basis yields one bit of information, and finally, the remaining 1 bit of information is shared between the first two particles, in a Bell state.

References

- [1] R. Feynman, R. Leighton, and M. Sands, “The Feynman Lectures, Vol. III,” *Addison and Wesley, New York MATH Google Scholar*, 1965.
- [2] W. K. Wootters, and W. H. Zurek, “COMPLEMENTARITY IN THE DOUBLE-SLIT EXPERIMENT - QUANTUM NON-SEPARABILITY AND A QUANTITATIVE STATEMENT OF BOHR'S PRINCIPLE,” *Physical Review D*, vol. 19, no. 2, pp. 473-484, 1979.
- [3] M. Arndt *et al.*, “Wave-particle duality of C 60 molecules,” *nature*, vol. 401, no. 6754, pp. 680, 1999.

- [4] S. Gerlich *et al.*, "Quantum interference of large organic molecules," *Nature communications*, vol. 2, pp. 263, 2011.
- [5] C. E. Shannon, "A mathematical theory of communication," *Bell system technical journal*, vol. 27, no. 3, pp. 379-423, 1948.
- [6] R. P. Feynman, "There's plenty of room at the bottom: An invitation to enter a new field of physics," *Handbook of Nanoscience, Engineering, and Technology, Third Edition*, pp. 26-35: CRC Press, 2012.
- [7] B. Schumacher, "Quantum coding," *Physical Review A*, vol. 51, no. 4, pp. 2738, 1995.
- [8] W. K. Wootters, and W. H. Zurek, "A single quantum cannot be cloned," *Nature*, vol. 299, no. 5886, pp. 802, 1982.
- [9] D. Dieks, "Communication by EPR devices," *Physics Letters A*, vol. 92, no. 6, pp. 271-272, 1982.
- [10] D. M. Greenberger, M. A. Horne, and A. Zeilinger, "Going beyond Bell's theorem," *Bell's theorem, quantum theory and conceptions of the universe*, pp. 69-72: Springer, 1989.
- [11] A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?," *Physical review*, vol. 47, no. 10, pp. 777, 1935.
- [12] P. A. Schilpp, "Albert Einstein: Philosopher-Scientist, Library of Living Philosophers," *Evanston, Illinois*, 1949.
- [13] J. S. Bell, "On the einstein podolsky rosen paradox," *Physics Physique Fizika*, vol. 1, no. 3, pp. 195, 1964.
- [14] P. H. Eberhard, "Bell's theorem and the different concepts of locality," *Il Nuovo Cimento B (1971-1996)*, vol. 46, no. 2, pp. 392-419, 1978.
- [15] P. H. Eberhard, "Constraints of determinism and of Bell's inequalities are not equivalent," *Physical Review Letters*, vol. 49, no. 20, pp. 1474, 1982.
- [16] G.-C. Ghirardi, A. Rimini, and T. Weber, "A general argument against superluminal transmission through the quantum mechanical measurement process," *Lettere Al Nuovo Cimento (1971-1985)*, vol. 27, no. 10, pp. 293-298, 1980.
- [17] M. Born, "The Born-Einstein Letters, with comments by M," *Born. New York NY: Walker*, 1971.
- [18] R. P. Feynman, and L. M. Brown, *Feynman's thesis: a new approach to quantum theory*: World Scientific, 2005.
- [19] W. Heisenberg, "Physics and philosophy," 1958.
- [20] M. Tegmark, "The interpretation of quantum mechanics: Many worlds or many words?," *Fortschritte der Physik: Progress of Physics*, vol. 46, no. 6-8, pp. 855-862, 1998.
- [21] M. Schlosshauer, J. Kofler, and A. Zeilinger, "A snapshot of foundational attitudes toward quantum mechanics," *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, vol. 44, no. 3, pp. 222-230, 2013.
- [22] T. Norsen, and S. Nelson, "Yet another snapshot of foundational attitudes toward quantum mechanics," *arXiv preprint arXiv:1306.4646*, 2013.
- [23] N. D. Mermin, "Is the moon there when nobody looks? Reality and the quantum theory," *Physics today*, vol. 38, no. 4, pp. 38-47, 1985.
- [24] L. Hardy, "Quantum mechanics, local realistic theories, and Lorentz-invariant realistic theories," *Physical Review Letters*, vol. 68, no. 20, pp. 2981, 1992.

- [25] H. P. Stapp, "Nonlocal character of quantum theory," *American Journal of Physics*, vol. 65, no. 4, pp. 300-304, 1997.
- [26] A. Peres, "Unperformed experiments have no results," *American Journal of Physics*, vol. 46, no. 7, pp. 745-747, 1978.
- [27] W. Dür, G. Vidal, and J. I. Cirac, "Three qubits can be entangled in two inequivalent ways," *Physical Review A*, vol. 62, no. 6, pp. 062314, 2000.