The Nothing from Infinity Paradox *versus* Plenitudinous Indeterminism

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Abstract: The Nothing from Infinity paradox arises when the combination of two infinitudes of point particles meet in a supertask and disappear. Corral-Villate claims that my arguments for disappearance fail and concedes that this failure also produces an extreme kind of indeterminism, which I have called plenitudinous. So my supertask at least poses a dilemma of extreme indeterminism within Newtonian point particle mechanics. Plenitudinous indeterminism might be trivial, although easy attempts to prove it so seem to fail in the face of plausible continuity principles. However, the question of its triviality is here moot, since I show that, except in one case, Corral-Villate's disproofs fail, and with a correction, the original arguments are unrefuted. Consequently, of the two contenders for the outcome of my supertask, the Nothing from Infinity paradox has won out.

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1. Introduction

The Nothing from Infinity paradox (in Shackel 2018) uses a supertask in which the combination of two infinitudes of point particles, a stationary infinitude and a moving infinitude (called the *M*s and *F*s respectively), meet and disappear, thereby constituting an instance of infinite indeterminism. I gave a number of proofs of the disappearance, starting with a simple one assuming the meeting can be treated as a composition whose outcome must be same as the outcome of each of its composing elements without thereby committing a fallacy of composition. This is then followed by four proofs appealing to four different continuity principles which produce the same result.

I must thank Amaia Corral-Villate for paying sufficiently close attention to my proofs to bring to light any weaknesses. She claims to disprove all my proofs and that the

> principle of mass conservation [gives] an independent…disproof. (2020:1)

Corral-Villate remarks, correctly, that if there is no disappearance

> this multiple collision is highly indeterministic (2020:3)

Insofar (but only insofar) as I was arguing that my supertask proved an extreme kind of indeterminism within Newtonian point particle mechanics this concedes my success. (The well-known cases of multi-point collisions in the literature produce only finite indeterminism.) For I may present the dilemma: either the infinitudes disappear, giving Nothing from Infinity; or they don't, giving continuum many futures for the universe that arise from a single initial condition. In their time reversals we have extreme indeterminism, in the latter case continuum many universes with different pasts giving rise to identical futures. Call the first kind *vanishing indeterminisms* and the second *plenitudinous indeterminisms.*

Is plenitudinous indeterminism trivial? Take an infinity of masses, the *n*th mass on the unit circle at π/n radians, all with unit velocity towards the origin.[1] Does this trivially produces infinite indeterminism? Not necessarily! For example, the within-world and across-worlds continuity principles in my original paper give deterministic outcomes for this case. The only proven cases of infinite indeterminism in the literature that I know of

[1] My thanks to a referee for this example.

are the paradoxes in my original paper and Perez Laraudogoitia's elegant extension (1998) of Black's machine (1950).

The possibility of plenitudinous indeterminism was evident when I wrote the original paper, but I did not point out the dilemma because I had proved its horn false by proving the paradox. Since Corral-Villate has raised it, the dilemma has appositely entered the dialectical situation: where relevant below, I trace how it might arise.

In this paper, however, I show that—except in one case—Corral-Villate's disproofs fail or do not prove what they appear to prove. The position at the end of the paper is that the second horn remains ruled out because my proofs of the Nothing from Infinity paradox, with a correction, have been sustained.

2. Corral-Villate §2.1 against the composition argument.

Corral-Villate does not distinguish the initial composition argument from the continuity arguments. My fault, since I see now that I failed to signpost adequately its distinction from the immediately following continuity argument. She does, however, address it before she addresses the first two continuity arguments. My composition argument:

> If…no fallacy of composition is involved in taking the meeting of the *M*s and *F*s to be constituted by an infinite repetition of the Beautiful Supertask then nothing gets to $x=0$ and the entire infinite energy of the *F*s has been absorbed by the *M*s. (2018:422)

Corral-Villate objects:

> The antecedent…is correct but the consequent does not follow….[There is] a substantial difference between…a finite repetition… and…an infinite repetition….a finite repetition of the Beautiful Supertask could not imply a different outcome….But an infinite repetition may….
>
> Thus, although no fallacy of composition is involved in assuming that the meeting…is constituted by an infinite repetition of the Beautiful Supertask, it is fallacious to assume that, because this is so, the evolution of the system could not imply a different outcome. (2020:4)

What justifies the antecedent is that the meeting of the *M*s and *F*s is nothing more than the repetition of infinitely many Beautiful Supertasks.[2] This is explained in the paper and is presumably why Corral-Villate grants no fallacy of composition: granting it, the objection as stated is not only mistaken, but is in danger of contradicting itself in the final remark.

What is supposed to be wrong with a fallacy of composition is that what is true of each member of a whole need not be true of the whole. Sometimes the reason for such inferences being fallacious is that the whole is more than the sum of its parts. Sometimes it depends on exactly what is true of each member. I cannot infer that a group of people has two arms just because each member of the group has two arms. However, extending this example in a way analogous to our case, it *does* follow from no member of the group getting to Paris that the group doesn't get to Paris.

Our case is not one in which the whole is more than the sum of its parts. This is partly why I was careful to speak of pluralities. There is nothing more to them than their parts. Likewise, there is nothing more to their meeting than the many meetings of their parts. Consequently, the outcome of the meeting of the pluralities is nothing more than the sum of the many meetings of their parts. Each such meeting is an instance of the Beautiful Supertask.

[2] Immaterial difference noted 2018:fn15. Beautiful Supertask: Perez Laraudogoitia 1996. Material for set-up worries: Shackel 2005.

In the Beautiful Supertask, the energy of a single incoming particle is absorbed by an infinity of particles similar to the *M*s and nothing gets to $x=0$. Granted that the meeting of the *M*s and *F*s is constituted by an infinite repetition of the Beautiful Supertask, then that just is for it to be constituted by an infinite repetition, one for each *F*, of the energy of an incoming particle of the *F*s being absorbed by the *M*s and nothing getting to $x=0$.[3] Hence the energy of the *F*s has been absorbed by the *M*s and nothing gets to $x=0$.

If at this point we add what had already been shown, that if the *M*s and *F*s exist at $t=1$ then they are at $x=0$,[4] then we have shown that at $t=1$ they have gone out of existence. We must be precise here and avoid an ambiguity in the conclusion. We have not shown that they exist at $t=1$ and exist at no time after; we have shown that they exist prior to $t=1$ but do not exist at $t=1$, nor any time after. That is to say, the period of their existence has no maximum but has $t=1$ as its least upper bound and the locations occupied during their existence have no minimum but have $x=0$ as their greatest lower bound. This is the conclusion not just of this argument but of all my arguments. This point will be important in §6.

The overarching argument, then, is this

1. The meeting of the pluralities of *M*s and *F*s is constituted by the individual meetings of the individual constituents of those pluralities.
2. The outcome of the meeting of the pluralities of Ms and Fs is composed of the meetings of the individual constituents of those pluralities.
3. Each such individual meeting is a Beautiful Supertask
4. Nothing gets to $x=0$ in a Beautiful Supertask
5. Therefore in the meeting of the pluralities of *M*s and *F*s nothing gets to zero.

This argument is valid and is so independently of whether the pluralities are finite or infinite. Line 2 merely spells out the relevant part of what there being no fallacy of composition amounts to in this case. So my consequent *does* follow from the granted antecedent and to deny the consequent is to contradict granting the antecedent. One more step takes us to the contradiction with her final remark.

Corral-Villate wishes at one and the same time to grant my no-fallacy-of-composition claim, grant the validity of this argument in the finite case but deny it in the infinite case. The overarching argument shows that, once the no-fallacy of composition claim is granted, there is no basis for drawing that distinction between the finite and infinite case. Consequently, when Corral-Villate says the argument is a fallacy she must reject the step to line 2 for the infinite case and that is to deny my no-fallacy of composition claim. Hence the danger of her contradicting herself.

At best, then, Corral-Villate has mis-stated her objection: she should have accused me of committing a fallacy of composition. The problem is that the accusation is hard to sustain for the very reasons that she felt obliged to grant me the no-fallacy-of-composition claim. It will not do merely to remind us of the known phenomenon that there are cases in which what is true of the finite cannot be assumed to be true of the infinite. That point must be shown to apply here. It is difficult to see how it *can* be applied here.

Indeed, I could have offered a straightforward argument going explicitly from the finite to the infinite: Since for any n in the natural numbers, the energy of the first n of the *F*s is absorbed by the *M*s and nothing gets to $x=0$, then for all n, the energy of the nth *F* is absorbed by the *M*s and nothing gets to $x=0$, hence the energy of all the *F*s has been absorbed by the *M*s and nothing gets to $x=0$. Note once again that this does not rely on a

[3] Details in Shackel 2018.

[4] There is no dispute over this conditional.

continuity claim (or if it does, then it is a different one from the four treated in my paper). Since Corral-Villate grants me the finite outcome premiss, she must follow me to the infinite outcome conclusion or explain where she finds a fallacy of composition. For that the infinite repetition of nothing getting to x=0 should produce a something getting to x=0 is a piece of unexplained magic.

The magic may appear to appeal because of an analogy to Thompson's lamp, whose infinitely many switchings on and off fails to determine the final state of the lamp. The mechanism by which the infinite repetition in Thompson's lamp produces indeterminism is that we have alternating repetitions of mutually exclusive outcomes, but in our supertask we have repetitions of a single outcome. So the analogy fails and does not support the fallacy of composition claim.

The reason the analogy fails would appear to be the reason why any attempt to show I commit a fallacy of composition must fail. My argument is, in brief, that denumerably many nothings produce nothing. Corral-Villate's objection appears to be that from a finitude of nothings producing nothing (which she grants), I move illicitly to the infinitude of nothings producing nothing. Given the nature of the supertask, however, stopping at bald asseveration is arbitrary. There remains a burden of explaining exactly how the denumerably many nothings of the supertask add up to produce something. I cannot find an approach to that burden in Corral-Villate's objection and I do not believe it can be fulfilled.

In conclusion, to resist the composition argument one must insist that, contrary to appearances, it does in fact commit a fallacy of composition. The nature of the composition, however, gives no reason in itself for the result of the composition to differ from the individual results that compose it. Consequently the burden is on the opponent to justify the claim of fallacy. A final point to make, then, is that the four continuity arguments (to which we now turn) are therefore also ways to *support* the absence of a fallacy of composition, since each shows a way in which the fallacy claim is false.

3. Corral-Villate §2.1 against the within-world and across-world continuity arguments

For the first two continuity arguments I show that: *lemma* 1: at t=1 nothing is at x=0, *lemma* 2: nor anywhere in $(0,\infty)$.[5] Since the initial locations of the *M*s and *F*s are in $(0,\infty)$, adding the continuity of paths gives the conclusion that nothing is anywhere in $(-\infty,\infty)$.

It counts as two arguments because *lemma* 1 is proved using an across-world continuity principle and I point out the evident availability of a similarly structured proof using a within-world continuity principle. Taking '$F\text{s}|_n$' to designate a plurality of the first n particles of the *F*s:

> If we…assume a continuity principle, then the meeting of the *M*s and *F*s is the limit as $n\to\infty$ of the $F\text{s}|_n$ meeting the *M*s….Since for all nearby worlds no particle is at x=0 at t=[1], for any particle to be at x=0 at t=1 in our universe would be a discontinuity and that is ruled out by continuity. Hence for our universe, there is no particle is at x=0 at t=[1].[6] (2018:422-3)

Corral-Villate objects to *lemma* 1:

> The incorrectness…is clear [from] an observation…[made by]…Shackel:

[5] Not labelled as lemmas in the original paper

[6] Corral-Villate (2020:fn.4) charitably notes and corrects two typos, which correction I have gratefully adopted here.

> "One might think that by the same strategy (appealing to a principle of continuity), because in the finite case no particle disappears, at the limit no particle disappears. That would create an interesting antinomy with the argument just given were it correct."
>
> In fact, the particle disappearance conclusion implies a…counterexample to Shackel's…continuity principle-based criterion, consequently refuting…his argument. But Shackel…neglects the relevance of his observation….In addition, he assumes the antinomy…does not apply because…the particles would have to be located somewhere and, right before introducing this observation, he has reasoned why this cannot be. (2020:5)

Indeed, in the next sentence I stated:

> Particles, however, have to be located whereas we have just proved that the set of points where they can be located is empty. (2018:423)

If my proof is valid and its soundness obscure, then we have an interesting antimony. Yes, it might be wielded against the continuity principle. Yet the continuity principle is defensible and we would want some reason why Newtonian worlds are not continuous in this way—on this point, rejecting the within-worlds principle in particular looks hard to justify. On the other hand, granted the principle and my supertask, the vanishing and plenitudinous indeterminisms are neatly united by each finding its place on its own half of the antimony.

If my proof is sound, I'm neither neglecting nor assuming anything. Furthermore, the reason for the apparent discontinuity between the finite cases in which no particles disappear and the infinite case in which they all disappear is explained by the analysis of the dynamics of what happens to the location of the *M*s and the first *n* of the Fs given in proving *lemma* 2 (2018: 423-4). It is impossible for there to be such an explanation of the other discontinuity (that between the finite cases in which no particle is at $x=0$ at $t=1$ and the infinite case of infinitely many particles being at $x=0$ at $t=1$) since, as we know, that discontinuity implies plenitudinous indeterminism, which is the evolution for $t\geq 1$ *being unexplainable by the dynamics*, and hence that discontinuity itself being unexplainable by the dynamics.

Corral-Villate quotes my argument that starts from *lemma* 1 and summarises the argument for *lemma* 2 :

> First, nothing gets to 0…nor, therefore, further left by…continuous paths for particles. Second, the *M*s are confined to the limit $n\rightarrow\infty$ of $(0,1/(n+1)]$=…the empty set, so there is no point in the space at which they can be. (2018:423)

Corral-Villate objects to *lemma* 2:

> The interpretation of the limit of an interval…is irrelevant as it is a purely formal operation with no correlation with any dynamically relevant aspect of the analysed configuration. This signifies that no consequence can be extracted from it other than the purely metrical one, namely, that the resulting set is empty. Thus, it is unfounded to assert that the particles are confined to that limit. (2020:5)

Corral-Villate doesn't address the proof of the disappearance of the *F*s (which is similar, but requires additional technicalities in setting it up) but states in footnote 5 that she objects to it for the same reason.

In addition to the remark about 'a purely formal operation', the reason we must take this to be an objection to *lemma* 2 is that it is irrelevant to the argument for *lemma* 1.

The claim that this is 'a purely a formal operation…[etc].' is false. First, we need to be clear exactly where the accusation of 'a purely formal operation' is being applied. It is not an objection to the continuity principle that the meeting of the *M*s and *F*s is the limit as $n\rightarrow\infty$ of the first *n* of the *F*s the meeting the *M*s.[7] The objection being discussed is one where she is granting the continuity principle and arguing that it doesn't suffice to prove the disappearance. Her accusation of formally but not dynamically justified is, rather, that proving (0,1/(n+1)] tends to the empty set as $n\rightarrow\infty$ doesn't prove the particles disappear. This objection is neglecting the fact that *the set of particle locations* is a subset of (0,1/(n+1)].[8]

In the earlier analysis of the waves of collisions (2018:421-2) the effect on the *M*s has been shown by dynamic analysis. Initially, the *n*th particle of the *M*s, m_n, is positioned at $x=1/n$. The effect of the wave of collisions caused by the impact of the first of the *F*s is to move all the *M*s one place to the left (i.e. m_1 from $x=1$ to $x=1/2$, m_2 from $x=1/2$ to $x=1/3$, and so on), of the second to move them another place to the left, and so on. So the effect of first *n* of the *F*s is to move to move m_1 from $x=1$ to $x=1/(n+1)$, m_2 from $x=1/2$ to $x=1/(n+2)$, and so on. So initially the *M*s occupy positions in (0,1] and the effect on the *M*s of the waves of collisions caused by the first *n* of the *F*s is to have moved them to positions in (0,1/(n+1)]. Granted that the meeting of the *M*s and *F*s is the limit as *n* tends to infinity of first *n* of the *F*s the meeting the *M*s, the result follows: at $t=1$ the subset of $(0,\infty)$ containing the locations of the *M*s is the limit as $n\rightarrow\infty$ of (0,1/(n+1)]=the empty set. Hence, *lemma* 2 is proved. So my treatment here is no mere formalism but the routine use of an isomorphism between our one-dimensional Newtonian space and the real line.[9]

Corral-Villate then offers an obviously unsound argument to which she thinks I am committed by the argument for the disappearance of the *M*s.

> consider.…particles…at points $x_n=1/n$ for…$n\in\mathbb{N}$. These particles have velocity $v_n=-1/n$ at time t=0 and are named p_n
>
> …now apply [Shackel's] argument… "*The Ps are confined to the limit as* $n\rightarrow\infty$ *of the spatial interval* (0,1/*n*]…[=] *the empty set. Thus, at t=1 there is no point in the space at which the particles can be.*" Clearly, this is wrong.…the correct conclusion…is…that all the…particles collide at…$x=0$ at…$t=1$.
>
> There is no significant difference whatsoever between this…configuration and…Shackel['s].…thus, there is no…justification…why Shackel's argument…is not sound for my configuration whilst it is…for his original system. Consequently, this first argument he uses to justify the particle disappearance is not conclusive at all. (2020:5-6 my emphasis)

That Corral-Villate presents this as an embarrassment to me appears to be based on a misunderstanding of my dialectic.

The argument in italics is valid. The way she misunderstands my dialectic is thinking I am committed to its premiss for the *P*s because its analogy for the *M*s is a premiss for my argument that the *M*s disappear. I am not. The mistake turns on the word 'confined'. Crucially, the putative premiss for the *M*s is in fact an intermediate conclusion. I used

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[7] My thanks to a referee for raising this possibility.

[8] *Lemma* 2 is about ruling out $(0,\infty)$ for particle locations. *Lemma* 1 and continuous paths rule out $(-\infty,0]$.

[9] On isomorphisms between physical and mathematical spaces see Ketland 2021.

lemma 1 and the continuity of paths, plus the fact that initially the particles are all in $(0,\infty)$, to prove that the *M*s are nowhere in $(-\infty, 0]$. It was only on this basis that I said 'the *M*s are *confined* to the limit as *n* tends to infinity of $(0,1/(n+1)]$'. There is no such basis for ruling out $(-\infty, 0]$ for the *P*s in Corral-Villate's example and this is why there *is* a significant difference between 'this…configuration' and mine..

This example from Corral-Villate is followed by a discussion in which, having earlier faulted the reasoning for *lemma* 2 as 'a purely formal operation…[etc]', she now grants that it was correct:

> The…reasoning Shackel gives…is:
>
> "To labour the point, since the *M*s are confined to the limit as *n* tends to infinity of $(0,1/(n+1)]$, for any $x>0$ there is an $n\in\mathbb{N}$ such that $1/(n+1)<x$ and so no constituent of the *M*s can be at *x*."
>
> Although it is true that for any $x> 0$ there is an $n\in\mathbb{N}$ such that $1/(n+1)<x$, the straightforward conclusion that follows is simply that the particles cannot be at any point $x> 0$. This itself does not imply that the particles need to have disappeared. (2020:6)

And, of course, I never claimed that *lemma* 2 alone did. In the concluding remarks of the section written against the within-world and across-world continuity arguments (Corral-Villate 2020:§2.1) she then immediately acknowledges that I didn't and instead returns to her objection to *lemma* 1:

> But, of course, considering that Shackel…previously concluded that no particle can be at $x=0$ nor further left, he then reasons:
>
> "To have proved that there is no position at which any *M*s can be appears to be sufficient proof that the *M*s have gone out of existence."
>
> Still, my previous discussion showed that the collision of all infinite particles at point $x=0$ is so far not precluded. Thus,…given Shackel's…reasoning…relies…on…the…continuity principle…rul[ing] out…this multiple collision, it does indeed fail to prove that particles go out of existence. Consequently, the results…Shackel infers from…the…continuity principle…are shown to be not conclusive at all. (2020:6)

The problem now is that these concluding remarks of the section concede that the critique she gives on pages 5 and 6 directed against *lemma* 2, whilst appearing to introduce further objections to these continuity arguments, in fact add nothing to her earlier objection to *lemma* 1. Obviously, if *lemma* 1 were false then the proof of the disappearance of the *M*s would be unsound. But it would be unsound only for that reason, not because *lemma* 2 is false nor because its proof was 'a purely formal operation with no correlation with any dynamically relevant aspect of the analysed configuration'—both of which claims she has now given up—nor, since the analogy does not apply, because my argument commits me to the *P*s disappearing.

4. Corral-Villate §2.2 against the continuous paths argument

Corral-Villate's objection to the continuous paths argument is that it is 'not valid' (2020:7) because it depends on the within or across-worlds continuity principles, and so, additionally, my claim that it is an independent argument is false. Her argument for the dependency claim is this:

> Shackel [says] “It would be arbitrary to assume that what is constituted of an infinite repetition of the Beautiful Supertask could achieve what no individual or finite repetitions can.”
>
> Clearly, this quote…shows that Shackel is not…prescinding from the…continuity principle. Thus, his continuous path principle-based reasoning is not independent from the previous continuity principle. (2020:7)

Prima facie, my arbitrariness premiss makes no assumption of the within-world or across-worlds continuity principles and Corral-Villate gives no more argument for its dependence than is here quoted. I see no grounds for such an argument.

What is arguable is that continuous paths argument and the composition argument are not wholly independent. The basis for this would be an argument that the arbitrariness premiss and the no-fallacy-of-composition premiss are in some sense dependent, one being a necessary part of the support for the other. Of course, independent arguments may share premisses, so I am not granting this point. Nevertheless, although I doubt that this is correct, this is why I gave 5 arguments but only claimed 4 independent arguments. Since Corral-Villate did not distinguish the composition argument from the within-world or across-worlds continuity arguments, this is perhaps why she thinks there is a dependence on those principles.

5. Corral-Villate §2.3 against the fourth continuity argument

The dialectic of my fourth argument is more complex than the others, being:

> a dilemma. Either the oscillating velocities from the infinite sequence of starts and stops matter [or]…don’t matter (2018:426-7)

In the paper the starts-and-stops-don’t-matter horn is treated first:

> If the particles exist at $t=1$ they have an instantaneous velocity, which…is the limiting average velocity, and so they all simultaneously reach universe escape velocity at $t=1$, which amounts to going out of existence for all $t>1$. (2018:426)

Corral-Villate objects:

> this…conclusion is based on the interpretation of instantaneous velocity…[that]…is not exactly right. The correct calculation of the instantaneous velocity at…*t* is…the limit of the average velocity over any interval of time containing…[*t*]…and not simply any one bounded by it. Furthermore, for this limit to exist…it must give the same result independently of which interval containing…[*t*]…it is calculated in. (2020:7)

This is correct, so far as it goes. But it does not prove what it is claimed to prove. I have shown that the left velocity (limiting average velocity as $t\rightarrow1$ from below) is unbounded and therefore the particles cease to exist. In doing this I am following the other cases in the literature that operate in this way (Benardete 1964; Moore 1990; Perez Laraudogoitia 1997; Oppy 2006), for all of which there are no intervals to determine limiting average velocities with endpoints *after* the time at which their limiting average velocity is unbounded, because the correlate objects no longer exist at any such times.

The same applies here. I will, however, concede an inaccuracy in stating the conclusion of the argument. I stated a weaker conclusion than I proved. Indeed, I need not have conditioned on the particles existing at $t=1$ but simply noted that their left velocity is unbounded. This last suffices, since the moment of unbounded velocity is not a moment

of existence (since to have unbounded velocity is impossible) but is rather the moment of going out of existence. So the conclusion should have been that the particles do not exist for all t≥1. Having gone out of existence, there are no subsequent intervals of their existence over which to take limiting average velocities that conflict with the unbounded left velocity, so no proof that they don't have an instantaneous velocity.

Corral-Villate then moves on to 'assuming that particles continue to exist for any t≥1' (2020:8). Of course, since this is the very point at issue it can't simply be assumed. Nevertheless, she argues on this assumption for divergent limiting average velocities, the substance of which appears in footnotes 6 and 7.

The objections of footnote 6 are not correct. Corral-Villate proffers a configuration like mine except that the initial velocity of the *F*s is '$v_{fn}=-(n+1)$' i.e. f_2 has velocity –3, etc. Whichever particle is carrying the momentum of the *n*th particle of the *F*s of her configuration arrives at $x=0$ at $t=n/(n+1)$, i.e. each arrives at different times. So her conclusion:

> all the…particles collid[*e*] at $x=0$ at $t=1$ (2020:fn 6)

is false. Even if it were not false, it would not

> impl[y] a general critique to the…adequateness of Shackel's system to…model the Immovable Object meeting the Irresistible Force…because configurations…sufficient to represent the paradox give drastically different outcomes depending on…the parameters. (Corral-Villate 2020:fn 6)

There is no reason why modelling that paradox shouldn't require some parameters rather than others.

For the evolution of the configuration in her footnote 7, and all similar cases, we have the left velocity unbounded but the right velocity finite. This means that each individual particle (not just the collection) has unbounded momentum prior to $t=1$, since for each finite momentum, *p*, and for each particle, there exists a time $t<1$ at which the particle has momentum greater than *p*. And yet each particle has finite momentum for $t>1$. This strikes me as a reductio of the continued existence assumption and hence an alternate argument for the first horn.

So the objections to the starts-and-stops-don't-matter horn of the fourth argument do not succeed. Corral-Villate misrepresents the dialectic of the starts-and-stops-matter horn as

> simply abandoning the conclusion previously defended, namely, that this instantaneous velocity is infinite (2020:fn.10)

That they might matter is, in fact, raised as a possible way round the other horn's argument for particle disappearance (2018:426). For that reason, on the assumption that starts and stops *do* matter, I offered two argument for inexistence.

Corral-Villate says a premiss of the first argument, that indeterminate instantaneous velocity for a particle implies its inexistence, is false:

> indefiniteness of…instantaneous velocity characterises any…collision of particles...[because] at the instant…of…collision the left velocity…is different from the right velocity. (2020:9 and fn.9)

This is correct. What I should have said is that the *left* velocity, *is itself indeterminate*, and used that indeterminacy for an—admittedly controversial but not necessarily false—premiss instead. I won't go further because Corral-Villate's point has given me a better option.

The assumption that starts and stops matter implies the crucial premiss used in the second argument: that the alternating sequence of starts and stops determines the state of

the particles at the limit. The argument using that premiss is valid, but Corral-Villate can object that it is unsound because the crucial premiss is false, false because it implies a falsehood, namely, the very premiss for which Corral-Villate faulted the first argument! I concede that the second argument is unsound for that very reason.

This does not leave me without a fourth continuity argument, however. The second horn of the dilemma arose only by contemplating the possibility of starts and stops mattering. As we just saw, the crucial premiss supplied thereby implies a falsehood and so Corral-Villate's helpful critique *rules out* the second horn altogether, leaving the successful argument of the first horn in place as the totality of the fourth argument.

6. Corral-Villate §3 the principle of mass conservation against all five arguments

Corral-Villate concludes by claiming that in addition to her objections to my specific arguments she has a knockdown argument against the Nothing from Infinity paradox:

> the…principle of mass conservation…states that particle world lines do not have beginning or end points and that mass is constant along a world line….. [Shackel's] particle disappearance conclusion…violates the principle ..because [his] particle disappearance…supposes that all the infinite world lines end at point x=0 at instant t=1. (2020:9-11)

This argument has the form of a reductio: suppose Shackel's arguments are sound, then the world lines of his particles have an endpoint, but that contradicts the principle of mass conservation, therefore his arguments fail.

I mentioned above the need for precision in stating my conclusion that the *M*s and *F*s go out of existence at t=1. Corral-Villate's claim that 'all the infinite world lines end at point x=0 at instant t=1' is exploiting the ambiguity here. What she needs for my conclusion to violate the principle of mass conservation is for it to be that the particles exist at t=1, when they are located at x=0, but exist at no time after that. Were that the case my conclusion would indeed imply that their world lines have an endpoint at (x,t)=(0,1). But, of course, that is not my conclusion. My conclusion is that they exist prior to t=1 but do not exist at t=1 nor at any time after, so their world lines have no endpoint but are open lines.

The ambiguity here is clearly disambiguated by the standard mathematical distinctions I used above. For all particles, the greatest lower bound of their position is x=0 but there is no minimum and the least upper bound on the times of their existence is t=1 but there is no maximum. Since for their world lines to have an endpoint would be for this minimum and maximum to exist, they do not have an endpoint.

7. Conclusion

We may present my supertask as creating a dilemma between vanishing and plenitudinous indeterminism. Whichever way it goes, my supertask proves an extreme kind of indeterminism within Newtonian point particle mechanics. If even only one of my arguments for the Nothing from Infinity paradox is sound, the second horn does not arise. Nevertheless, plenitudinous indeterminism strikes me as interesting in its own right. It is possible that it can arise too easily to be other than trivial, although two of the continuity principles in my original paper undermine an apparently straightforward example attempting to demonstrate the triviality. So that question needs further examination. Here, however, is not the place to do it.

My arguments for the Nothing from Infinity paradox are the composition argument and the four continuity arguments. A result of this helpful dialogue with Corral-Villate, in particular the question of independence of the third continuity argument, was to convince me that the thoughts underlying the composition argument are more fundamental to the

Nothing from Infinity paradox than the continuity principles. Indeed, one might wield them as a ground for the first two continuity principles. For that reason I regret not making its role as a separate argument much clearer in the original paper and I am grateful for the opportunity to give it more attention here.

Corral-Villate claims to disprove the four continuity arguments. She also claims to give an independent general argument against the Nothing from Infinity paradox from the principle of mass conservation. Her only successes were against the fourth continuity argument, in which she found an inaccuracy in its first horn and an error in its second horn. Yet, as we saw, the inaccuracy is correctable, and her success in the second horn gives me the ground on which to avoid entirely that horn and leave the first horn standing alone as a fourth argument.

Consequently, in this paper I have refuted Corral-Villate's attempt to refute the Nothing from Infinity paradox. The soundness of my arguments depends in part on the principles of composition and continuity I have used, some of which I readily concede could bear further analysis, most especially concerning their extent and domain of application. Corral-Villate's objections, however, were directed at validity and the principles are intuitively appealing if we are to deal with infinite collections of particles. For these reasons, I maintain that my arguments for the Nothing from Infinity paradox (as herein corrected) are sound. Of the contenders for the outcome of my supertask, the Nothing from Infinity paradox versus plenitudinous indeterminism, the Nothing from Infinity paradox has won out.

8. References

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