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The Paradox of Knowability and Factivity

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Fitch's paradox of knowability was first presented in Fitch 1963 and it is alleged to have serious implications for the realism/anti-realism dispute.¹ As a result of its importance for that issue, it has been the object of considerable, complex, and often heated debate. The paradox is fairly simple to grasp, however, and it is supposed to arise as follows. First, let us introduce an epistemic operator Kp that means very specifically that p is known by someone at some time. Suppose, then, that all truths are knowable by someone at sometime, and let us call the following principle the knowability principle:

$$(KP) \quad \forall p(p \rightarrow \diamond Kp).$$

Suppose then also that we are not omniscient and let us refer to this principle as the non-omniscience principle:

$$(NOP) \quad \exists p(p \bullet \neg Kp).$$

NOP then directly implies the following formula:

$$(F1) \quad p \bullet \neg Kp.$$

Moreover, we can construct the following substitution instance of KP by inserting F1 for p in KP, thus yielding:

$$(F2) \quad (p \bullet \neg Kp) \rightarrow \diamond K(p \bullet \neg Kp).$$

Since it is clearly possible to know F1, we get:

$$(F3) \quad \diamond K(p \bullet \neg Kp).$$

However, it is supposed to be easy to see then also that:

$$(F4) \quad \neg \diamond K(p \bullet \neg Kp).$$

¹ The generation of the paradox here closely follows Brogaard and Salerno (2009).

F4 is supposed to follow from the following principles of epistemic and modal logic:

- (CON) $K(p \bullet q) \vdash (Kp \bullet Kq)$.
 (K-ELIM) $Kp \vdash p$.
 (NEC-INT) If $\vdash p$, then $\Box p$
 (MOD) $\Box \neg p \vdash \neg \Diamond p$.

The proof of this is as follows:

- (1) $K(p \bullet \neg Kp)$ [assumption].
 (2) $Kp \bullet K\neg Kp$ [CON, 1].
 (3) $Kp \bullet \neg Kp$ [K-ELIM, 2].
 (4) $\neg K(p \bullet \neg Kp)$ [Reductio, 1-3].
 (5) $\Box \neg K(p \bullet \neg Kp)$ [NEC-INT, 4].
 (6) $\neg \Diamond K(p \bullet \neg Kp)$ [MOD, 5].

Thus we appear to have generated a contradiction in deriving F3 and F4. So, by reductio, and if one wants to maintain KP, then one must deny NOP. This amounts to the acceptance of:

- (OP) $\forall p(p \rightarrow Kp)$.

So every truth is supposed to be known. But this seems wildly paradoxical. It seems to be fairly obvious—almost a truism—that there might be unknown truths, and so something seems to be wrong here. But what exactly is wrong with the reasoning that leads to Fitch's conclusion? It is not at all clear and many complex solutions have been proposed to Fitch's paradox, but they are all controversial in one way or another.² For example, some responses involve restricting KP in various ways and others involve the introduction of intuitionistic logic.³ The view to be defended here is simpler and it is based on the contention that the knowability paradox isn't a paradox because the derivation of the paradox is faulty.⁴ So I suggest that the complex and often heated debate about how best to respond to the paradox is an unnecessary muddle. This is explained by showing that the K operator employed in generating the paradox is used equivocally and when the equivocation is eliminated, the derivation of F4 fails.

The standard accounts of epistemic logic treat the K operator as analogous to the \Box operator of alethic modality. Moreover, any such logic for the K operator that is at least as strong as the modal system K has K-

² See, Salerno (2009) and Kvanvig (2006).

³ See, for example, Eddington (1985) and Dummett (2001).

⁴ Williamson (1992, 2000) also claims that Fitch's paradox is not a paradox, but for different reasons.

ELIM as an axiom.⁵ Of course, typical accounts of the logic of the K operator involve epistemic analogs of the axioms of S4, and so standard epistemic logic has K-ELIM as an axiom. The problem that arises then in the derivation of F4 is that the K operator introduced in KP and NOP treats Kp as meaning that p is known by someone at sometime. However, as we shall see, on that reading the K operator appears to behave more like the \diamond of alethic modality than the \square of alethic modality. It is then deeply problematic to assume that the axioms of S4 (or even of the modal system K) apply to the K operator so understood. Specifically, the derivation of F4 requires that the K operator obey K-ELIM in order to generate the contradiction that leads to OP, but as we shall see momentarily, the specific K operator used in KP and NOP does not seem to obey this axiom.⁶ K-ELIM is of course an analog of \square -ELIM in modal logic:

$$(\square\text{-ELIM}) \square p \vdash p.$$

In alethic logic \square -ELIM makes sense because $\square p$ essentially asserts that if p is true in all possible worlds, p is true simpliciter because it is true at any given world. K-ELIM appears to assert that if p is known, p is true. This is often called factivity. But factivity is importantly ambiguous as it might be applied to the K operator used in generating Fitch's paradox. We can read it as a close analog of \square -ELIM and take it to assert something like the claim that for any world and any proposition, if p is known by someone at some time at that world, then p is true unrestrictedly (i.e. at any given world). Symbolically, this can be rendered as follows:

$$(\text{Factivity-1}) \forall w \forall p \exists s \exists t K_{\text{wst}} p \vdash p.$$

However, this is *not* the reading of K that best fits the concept of knowledge used in generating the knowability paradox, and it is obviously unacceptable. To render K more sensible and to capture the modally pregnant sense of knowability used in generating the paradox, K needs to be indexed to a world so that it asserts that if p is known *at world* w_1 , then p is true *at* w_1 (and those worlds that are epistemically accessible from w_1).⁷ But on this reading it simply does not follow from the fact that p is known in some world, that it is true at all worlds. This is the case because it is

⁵ See Mayer (2001).

⁶ Kvanvig briefly considers this type of strategy in chapter 4 of his 2006, but he takes the strategy to be the denial of factivity for the knowledge operator. It will be suggested here that denying K-ELIM need not amount to the rejection of the factivity condition for knowledge.

⁷ See Brogaard and Salerno 2007 for a more complex world indexing strategy based on considerations from the philosophy of language.

possible that p is only contingently true. Symbolically, the idea that someone knows that p at some time is better understood to obey the following principle:

(Factivity-2) $\forall w \forall p \exists s \exists t K_{wst} p \vdash p$ is true at w and at every world accessible from w .

Suitably understood this is just the principle we need: for every proposition p and world w , if p is known by someone at some time at w , then p is true at that world and those worlds that are epistemically accessible from w_1 .

To see the important implications of factivity-2, consider the following basic model theory for standard epistemic logic. Let W be a set of worlds such that each $w_i \in W$, and R be the relation of epistemic possibility relating worlds. $\langle W, R \rangle$ is then a Kripke frame in the usual sense and propositions will be subsets of W such that p is true in w if and only if $w \in p$. Let $R(w)$ be defined as follows: $R(w) = \{x \in W: Rwx\}$. Then p is known if and only if p follows from $R(w)$. In other words, p is known at w_i if and only if p is true in all worlds that are epistemically accessible from w_i . Notice then that p can be known in this sense while being false at some worlds, and that factivity really can't mean anything more than that if Kp is true at w_1 , then p is true at w_1 and all worlds epistemically accessible from w_1 . That this is so is virtually trivial to see. More importantly, this structure shows more clearly what factivity-2 asserts. More precisely, factivity-2 can then be understood as follows:

(Factivity-2) $\forall w \forall p \exists s \exists t K_{wst} p \vdash p$ is true for $R(w) \subseteq p$.

According to factivity-2, if p is known at a world w_1 , then p is only true at that world and those worlds that are epistemically accessible from w_1 . But, from this it does not follow that p is true unrestrictedly, i.e. in every model, as is the case with the corresponding alethic principle.

What is then crucially important for Fitch's alleged paradox is that the reading of K used in KP and NOP obeys factivity-2, and so K -ELIM needs to be understood in this way. In KP and NOP , K is used to indicate that *there is some possible world where p is known by someone at sometime*. It would be blatantly silly to read the K operator as asserting that p is known by someone at some time in every world, and this is because some truths are obviously not known by someone in some worlds at some times. It is easy to construct arbitrary maximal and consistent state descriptions where there is at least one unknown truth and so many such scenarios are possible. Let us then introduce subscripts to make the different uses of K clear. In KP and NOP , K_{Fp} (Fitch knowledge) is used and it asserts the modally infused notion that p is known by someone at sometime—understood to mean that there is a possible world where p is known by someone at some time,

whereas in standard epistemic logic we have K_{Sp} (standard knowledge), understood as simply asserting that p is known without any world indexing.

The supposed generation of the paradox can then be suitably clarified as follows by making the distinction between the two concepts of knowledge explicit:

$$(KP') \quad \forall p(p \rightarrow \diamond K_{Fp}).$$

Again, we can then also suppose that we are not omniscient and introduce:

$$(NOP') \quad \exists p(p \bullet \neg K_{Fp}).$$

NOP then still implies the following formula:

$$(F1') \quad p \bullet \neg K_{Fp}.$$

We can then construct the following substitution instance of KP in the same way as before yielding:

$$(F2') \quad (p \bullet \neg K_{Fp}) \rightarrow \diamond K_F(p \bullet \neg K_{Fp}).$$

Again, since it is possible to know F1', we get:

$$(F3') \quad \diamond K_F(p \bullet \neg K_{Fp}).$$

However, importantly, we now have:

$$(F4') \quad \neg \diamond K_S(p \bullet \neg K_{Sp}).$$

Here F4 follows from the following clarified principles of epistemic and modal logic:

$$(CON') \quad K_S(p \bullet q) \vdash (K_{Sp} \bullet K_{Sq}).$$

$$(K-ELIM') \quad K_{Sp} \vdash p.$$

$$(NEC-INT') \quad \text{If } \vdash p, \text{ then } \Box p$$

$$(MOD') \quad \Box \neg p \vdash \neg \diamond p.$$

We are then supposed to conclude that,

$$(OP') \quad \forall p(p \rightarrow K_{Fp}).$$

The problem now, however, is that in this case there is no contradiction. The conjunction of F3' and F4' is not a contradiction and the paradox does not arise because of the equivocation on K . It might however be supposed that the paradox can be restored by replacing the standard K operator with Fitch's K operator in CON' and K-ELIM', and thereby doing the same in F4' as follows:

$$(F4'') \quad \neg \diamond K_F(p \bullet \neg K_{Fp}).$$

$$(CON'') \quad K_F(p \bullet q) \vdash (K_{Fp} \bullet K_{Fq}).$$

$$(K-ELIM'') \quad K_{Fp} \vdash p.$$

However, this will not work because it is easy to see that K_{Fp} as it is understood more clearly does not obey a standard K-ELIM axiom and the step from (2) to (3) fails. We cannot validly derive the unrestricted truth of $\neg Kp$ from $K\neg Kp$. All that follows from $K\neg Kp$ given the sort of factivity-2 understanding of K-ELIM'' is that $\neg Kp$ is true at the world we are indexing to and at those worlds that are accessible from that index world. So, as noted at the outset and as more clearly understood in terms of the model theory introduced above, K_{Fp} appears to behave much more like a \diamond operator in some important respects, and the following sort of principle is manifestly false for the \diamond operator:

$$(\diamond\text{-ELIM}) \diamond p \vdash p.$$

The truth of p does not follow from its mere possibility. By the same token, the truth of p does not follow from its mere knowability. So the same problem arises for K_{Fp} and this is similarly easy to see. If we accept that K_{Fp} means that there is a world in which someone knows p at some time, then that p is true at all worlds does not follow from K_{Fp} . It only follows that (where p is contingent) p is true at the world where p is known in the Fitch sense and at worlds that are epistemically accessible from that world.⁸ So the proposition p may be unknown and false in some or all other worlds. Thus F4'' cannot be soundly derived from CON'' and K-ELIM''. This shows that there is in fact no paradox here at all. The perception that Fitch's paradox is paradoxical is only an appearance, and it is based on a fundamental equivocation on the concept of knowledge and a fundamental ambiguity in the understanding of the factivity condition on knowledge. Since there is no paradox, there is no need for the sorts of elaborate and controversial solutions that are currently extant in the literature on knowability.

⁸ Incidentally, when we read the K operator in the Fitch sense NOP' turns out to be false. Recall that it is rendered as follows: (NOP') $\exists p(p \bullet \neg K_{Fp})$. But if K_{Fp} means that someone in some possible world knows p at some time, then NOP' is false. There will always be a world where, for any true p , p is known, although at any given world p may not be known. This principle fails for much the same reason that K-ELIM fails for K_{Fp} . This is because the K operator so understood has implicit modal content that is much like alethic possibility. While it is true that we are not omniscient when understood relative to the actual world, this is not incompatible with the idea that every proposition is known by someone at some world or other. The omniscience principle is as follows: (OP) $\forall p(p \rightarrow Kp)$. So we can see that OP is true for Fitch knowledge and given what has just been said this is obviously true.

References

- Brogaard, B. and Salerno, J. (2007). Knowability, Possibility and Paradox. In Hendrix, V., Pritchard, D. (eds.) *New Waves in Epistemology*. (pp. 270-299 New York: Palgrave-MacMillan.
- Brogaard, B. Salerno, J. (2009). Fitch's Paradox of Knowability. *The Stanford Encyclopedia of Philosophy (Fall 2009 Edition)*, Edward N. Zalta(ed.), URL=<<http://plato.stanford.edu/archives/fall2009/entries/fitch-paradox/>>.
- Dummett, M. (2001). Victor's Error. *Analysis* 61, 1-2.
- Eddington, D. (1985). The Paradox of Knowability. *Mind* 94, 557-568.
- Fitch, F. (1963). A Logical Analysis of Some Value Concepts. *The Journal of Symbolic Logic* 28, 135–142.
- Kvanvig, J. (2006). *The Knowability Paradox*. Oxford: Oxford University Press.
- Meyer, J.-J. Ch. (2001). Epistemic Logic. In *The Blackwell Guide to Philosophical Logic*. (ed.) Gobel, L. (pp. 183-202) London: Blackwell.
- Salerno, J. (ed.) (2009). *New Essays on the Knowability Paradox*. Oxford: Oxford University Press.
- Williamson, T. (1992). On Intuitionistic Modal Epistemic Logic. *Journal of Philosophical Logic* 21, 63-89.
- Williamson, T. (2000). *Knowledge and its Limits*. Oxford: Oxford University Press.