Gila Sher

Invariance and Necessity

Abstract: Properties and relations in general have a certain degree of invariance, and some types of properties/relations have a stronger degree of invariance than others. In this paper I will show how the degrees of invariance of different types of properties are associated with, and explain, the modal force of the laws governing them. This explains differences in the modal force of laws/principles of different disciplines, starting with logic and mathematics and proceeding to physics and biology.

1 Introduction

Many philosophers are perplexed by necessity. What is the source of necessity – The world? Our mind (language, concepts, built-in categories)? Is necessity epistemic or metaphysical? Is it possible to explain the necessity of logical laws (logical truths, logical consequences), mathematical laws, physical laws? Are they necessary at all? Do they have the same kind of necessity? Are there many kinds of worldly (metaphysical) necessity or just one kind?

Humeans are deeply disturbed by the thought of necessary physical laws: They view the idea of such laws as an idea of secret, hidden, inexplicable, mystical powers that govern the world, as a remnant of the idea of laws created by God, or by some supernatural forces …

In this paper I will offer a philosophically systematic explanation of necessity in terms of invariance. This explanation does not purport to say everything there is to say about necessity or laws. But it captures something basic and significant about them, answers some of the above questions, and partly removes some of the worries. And it's down-to-earth in the sense of not invoking any mysterious traits or requiring any mysterious mental capacities.

2 The Idea of Invariance

Invariance in general is a relation: X is invariant under Y. Examples:

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1. Logical truths are invariant under changes in (Tarskian) models: They are not affected by such changes. You can replace one model by another, and the logical truths won't "notice", so to speak. If they hold in one model, they hold in all.
2. The laws of physics are invariant under changes of inertial reference frames. They are the same in all such frames. They are indifferent to or don't notice changes in such frames.
3. The laws of universal grammar are invariant under variations in natural language. They hold in all natural languages. They don't distinguish between one natural language and another. They don't even distinguish between actual and (merely) possible natural languages.
4. Logical constants are invariant under all isomorphisms. (Explanation: later on).

3 Fruitfulness and Explanatory Power of Invariance

As these examples suggest, the idea of invariance is very fruitful and has a strong explanatory power. Here are a few citations:

Science

1. Eugene Wigner: "There is a Structure in the laws of nature which we call laws of invariance. This structure is so far-reaching [that] in Some cases ... we guessed [laws of nature] on the basis of the postulate that they fit into the invariance structure. ... [L]aws of nature could not exist without principles of invariance." (Wigner 1967: 29)
2. Jim Woodward: "[E]xplanatory relations must be invariant relations, where a relation is invariant if it remains stable or unchanged as we change various other things. ... [L]aws describe invariant relationships". (Woodward 1997: 26 – 27)

Philosophy

1. Robert Nozick: "Questions about objectiveness depend upon the range of transformations under which something is invariant". (Nozick 2001: 10)
2. Kit Fine: Generality or abstractness are a matter of sensitivity to descriptive differences: More general elements are less sensitive and less general

elements are more sensitive. How do we understand sensitivity? – In terms of invariance. (Paraphrase, Fine 2011: 17)

Mathematics

1. Felix Klein: Developed the Erlangen Program, which offers a characterization of geometries in terms of the transformations under which their notions are invariant.
2. Kronecker: "When the concept of invariants ... is tied ... to the general concept of *equivalence*, then [it] reaches the most general realm of thought". (Cited in Mancosu 2016: 15)

Logic

Tarski: "I suggest that ... we call a notion 'logical' if it is invariant under all possible one-one transformations of the world onto itself". (Tarski 1966/86: 149)

4 Invariance of What?

Different things are invariant under different changes. In this talk I will focus on invariance of *properties* (relations) under certain changes. I will explain necessity in terms of such invariance.

Background Notes. Leaving aside, for the purpose of this paper, many questions concerning objects, properties, and the world, I will start with the following common-sensical picture:

1. The *world* contains: Individuals – level 0; n-place properties of individuals – level 1; properties of properties of level 1 – level 2; ...
2. *Object*: individual or property.
3. An individual can be either *actual* or *non-actual* (counterfactual). The world includes both actual and counterfactual individuals.
4. *Properties* are identified by their actual plus counterfactual extension.
5. The paper focuses on properties of the kind that science, mathematics, and logic are interested in. ("Properties" that lead to paradox, for example, are excluded.) Complete formal systematization: a later task. (Requires a solid idea to systematize).

5 Central Claims Concerning Invariance and Necessity

1. Every property is invariant under some 1–1 [and onto] replacements of individuals.
2. Some properties have a higher degree of invariance than others.
3. The greater the degree of invariance of a given property, the greater the degree of necessity of the laws governing it.
4. This is generalizable to fields of knowledge. It explains why (and in what sense) logical laws have a greater degree of necessity than physical laws, physical laws have a greater degree of necessity than biological laws/regularities, etc.

5.1 Claim 1: *Every property is invariant under some 1–1 [and onto] replacements of individuals.*

Let me begin with an observation about properties. Properties in general are *selective*. They "pay attention" or "are attuned" to *some* features of objects but not others. For example:

The 1st-level property *is-a-human* is not attuned to differences in gender. Gravity (the property of being subject to gravitational forces) is not attuned to differences between living and non-living objects. This introduces the possibility of characterizing, or comparing, properties in terms of what they are attuned and not attuned to. What changes in the world they "notice" and what changes they are "blind" to. This is what my characterization/comparison of properties in terms of invariance will do.

Heuristic notes

1. For ease of explanation I help myself to the language of set-theory and assume bivalence. But this is not essential for the claims made in this paper.
2. For ease of presentation, I think of the total collection of actual plus counterfactual individuals as divisible into domains.
3. In these terms I think of properties as characterized by which objects they hold of in different (actual plus counterfactual) domains.

Notation
D (Domain): a non-empty set of individuals.
r (Replacement Function): any 1–1 function from some domain D onto a domain D' (possibly $D = D'$).

Using this notation, the claim is: $(\forall P)(\exists r)(P$ is invariant under $r)$. r is indexed to some domain D, or a pair of domains, $\langle D, D' \rangle$. When we index it just to D, the understanding is that D' is its range, whatever this is.

The claim itself is trivially true, since every property is invariant under *identity replacements*: functions r that replace each individual in D by itself. But my claim is stronger: properties are commonly invariant under more than just identity r's.

Case 1: *1st-level Properties*
Example: *is-a-human*.

Consider:
$D = \{\text{Obama, Tarski}, 1, 2\}$.
$r : r(\text{Obama}) = \text{Trump}, r(\text{Tarski}) = \text{Frege}, r(1) = 4, r(2) = \text{Mt. Everest}$.
$(D' = \{\text{Trump, Frege}, 4, \text{Mt. Everest}\}.)$

Claim: The 1^{st}-level property *is-a-human* is invariant under this r.

Why? Because r replaces each individual that has the property *is-a-human* by an individual that also *has* the property *is-a-human* and each individual that *doesn't have* the property *is-a-human* by an individual that *doesn't have* the property *is-a-human*.

Note that *is-a-human* is not invariant under *all* 1–1 r's. For example, it is not invariant under:

$$r' : r'(\text{Obama}) = 1, r'(\text{Tarski}) = 2, r'(1) = \text{Tree1}, r'(2) = \text{Tree2}.$$

Because under this r, the property *is-a-human* is changed to (replaced by) the property *is-a-number*. So, *is-a-human* is invariant under *some*, but *not all* r's.

In Fact: *Every* n-place 1^{st}-level property is invariant under *some* r, but *many* 1^{st}-level properties are *not* invariant under *all* r's.

Case 2: 2^{nd}-level properties
Example: IS-A-PROPERTY-OF-MAMMALS. (This is a property of all 1^{st}-level properties that are applicable in principle to Mammals.)

Among the 1st-level properties that have this 2nd-level property are the properties *is-a-human* and *is-a-horse*. Among the 1st-level properties that don't have this 2nd-level property are the properties *is-a-number* and *is-a-tree*.

Explanation: Consider the domain D as above. I.e., $D = \{Obama, Tarski, 1, 2\}$. Let r be a 1-1 function on D where:

$$r(\text{Obama}) = \text{Horse } 1,\ r(\text{Tarski}) = \text{Horse } 2,\ r(1) = \text{Tree } 1,\ r(2) = \text{Tree } 2.$$

Claim: IS-A-PROPERTY-OF-MAMMALS is invariant under this r.

Why? Because r induces a replacement of the 1st-level property *is-a-human* by the 1st-level property *is-a-horse*. But the 2nd-level property IS-A-PROPERTY-OF-MAMMALS does not notice this change. From the point of view of IS-A-PROPERTY-OF-MAMMALS there is no difference bet the 1st-level properties *is-a-human* and *is-a-horse*.

Note too that like *is-a-human*, IS-A-PROPERTY-OF-MAMMALS is *not* invariant under every r. For example, it is *not* invariant under r':

$$r'(\text{Obama}) = \text{Tree } 1,\ r'(\text{Tarski}) = \text{Tree } 2,\ r'(1) = \text{Snake } 1,\ r'(2) = \text{Snake } 2.$$

r' induces a replacement of *is-a-human* by *is-a-tree*, and *is-a-tree* is not A-PROPERTY-OF-MAMMALS.

In a similar way, any 2nd-level property is invariant under *some* r's, but *many* 2nd-level properties are *not* invariant under *every* r. This applies to properties of any level.

5.2 Claim 2: *Some properties have a higher degree of invariance than others.*

While properties in general are invariant under *some* 1–1 replacements of individuals, *not all* properties are invariant under *the same* 1–1 replacements. This suggests that *some* properties are invariant under *more* 1–1 replacements of individuals than others. "More" can be measured in several ways. As a starting point we understand "more" in the sense of *inclusion*:

$$P_1 \text{ is invariant under more } r\text{'s than } P_2$$
$$\text{iff}$$
$$\{r\colon P_1 \text{ is invariant under } r\} \supsetneq \{r\colon P_2 \text{ is invariant under } r\}.$$

Terminology

1. "Degree of invariance of P". The degree of invariance of P – $DI(P)$ – is the class of all r's such that P is invariant under r : $DI(P) =_{Df} \{r : P \text{ is invariant under } r\}$. Clearly, every property has a degree of invariance.
2. "Greater degree of invariance":

$$DI(P_1) > DI(P_2)$$
$$\text{iff}$$
$$P_1 \text{ is invariant under } \textit{more} \text{ 1–1 replacements of individuals than } P_2$$
$$\text{(in the inclusion sense).}$$

Clearly, greater degree of invariance – $DI(X) > DI(Y)$ – is a partial ordering.

Back to claim 2: Some properties have a higher degree of invariance than others. To establish this claim we observe that some properties are invariant under *all* r's. The properties we have discussed so far are not of this kind, but some properties are. For example, *is-identical-to* is invariant under *all* r's. For any r (on any D) and any a, b in D:

$$a = b \rightarrow r(a) = r(b),$$
$$a \neq b \rightarrow r(a) \neq r(b).$$

Clearly, properties that are invariant under *all* r's have a *higher degree* of invariance than other properties. We will say that these properties have a *maximal* degree of invariance.

5.3 Claim 3: *The greater the degree of invariance of a given property, the greater the degree of necessity of its laws.*

Let's look at identity (*is-identical-to*) again. More specifically, let's look at what its *maximal degree of invariance*, by itself, tells us (or determines) about it. (This is informative, because whatever holds of identity due to its maximal invariance holds of *all* properties with maximal invariance.) Let us think of the *idea* of *laws of identity* (intuitively, principles that govern/describe the behavior of identity in all areas, actual and counterfactual, to which it applies), *not* of the *specific* laws of identity. The maximal invariance of identity determines that if a statement/fact is a law of identity, then it holds in *all* domains of individuals, *actual and counterfactual*. Why? Because: Given that identity itself doesn't distinguish between any individuals, its laws cannot be limited to just some domains of individuals. In particular: if the laws of identity hold of *any* individuals, they hold of *all actual and counterfactual* individuals. That is, the laws of identity apply to the *totality* of individuals, *actual and counterfactual*. Conclusion: whatever the laws of identity are, they hold of the totality of individuals, actual and counterfactual. I.e., the laws of identity have a maximal (actual and) counterfactual scope.

Now, given the common understanding of *necessity* in terms of (*actual-*) *counterfactual* scope – namely, for any fact/statement/law *X*: necessary (*X*) iff *X* holds in all (actual-) counterfactual domains – this means that the *laws of identity* (whatever they are) have *maximal necessity*. So:

Maximal invariance → Maximal necessity.

P is maximally-invariant → Laws of *P* have maximal actual-counterfactual scope → Laws of P have maximal necessity.

This can be generalized in two directions:

1. The laws of all properties with maximal invariance have maximal necessity.
2. The correlation between degrees of invariance and necessity holds in general, i.e., for laws of all properties (not just for laws of maximally-invariant properties).

Explanations

1. We have seen that due to identity's maximal degree of invariance, its laws, whatever they are, have a maximal degree of necessity. Since this is due just to the maximal degree of invariance of identity, it holds of all properties with a maximal degree of invariance: if they are governed by any laws, they are governed by laws with a maximal degree of necessity. Let us call such properties "maximally-invariant properties", or for short "maximal properties". So: the fact that

Degree of invariance of maximal properties
>
Degree of invariance of non-maximal properties,
→:
Degree of necessity of laws governing maximal properties
>
Degree of necessity of laws governing non-maximal properties.
(The actual-counterfactual scope of the former is greater than that of the latter).

2. The second generalization says that the correlation between degree of invariance and degree of necessity is general. Take, for example, the following properties:

(1) Gravity: x-is-subject-to-gravitational-forces
(2) Evolution: x-is-subject-to-evolutionary-forces.

Intuitively, $DI(1) > DI(2)$.

What leads us to think that this is the case?

Consider an r that replaces animate objects by inanimate objects.
(1) – gravity – is invariant under this r, but (2) – evolution – is not.
→: (1) remains the same in more actual-counterfactual domains than (2).
→: The actual-counterfactual scope of laws of (1) > the actual-counterfactual scope of laws of (2).

And, in general:

$DI(P_1) > DI(P_2)$
→: P_2 distinguishes between more actual-counterfactual individuals than P_1
→: Actual-counterfactual scope of laws of P_1 > actual-counterfactual scope of laws of P_2
→: $DN(P_1) > DN(P_2)$.

Note: Technically we have to adjust our conception of “>” for pairs of properties which are both not maximally invariant. (Thanks to an anonymous participant in the Wittgenstein Symposium for pointing this out.) For a proposal see Sher, work in progress.

5.4 Claim 4: *The relation between greater property invariance and greater necessity of laws is generalizable to fields of knowledge.*

Take physics, for example. It is quite clear that generally:

$$DI \text{ of maximal properties} > DI \text{ of physical properties.}$$

And, given the systematic connection between invariance and necessity:

DN of laws of maximal properties
>

DN of laws of physical properties.

If we identify a field of knowledge *X* whose defining properties are maximally invariant, this will establish the claim that

DN of laws of Field *X*
>
DN of laws of Physics

and explain why this is the case. And if (following the refinement of the definition of “>”) we can show that *DI*(the most highly invariant physical properties) > *DI*(the most highly invariant biological properties), we will be able to show that, and explain why, the degree of necessity of physical laws (associated with highly invariant physical properties) is greater than the degree of necessity of biological principles/laws.

6 Application to Logic and Mathematics

We have seen that identity is maximally invariant. It is easy to see that all the properties denoted by the standard logical constants of predicate logic (of any order) have maximal invariance. For example: Consider “∃”, viewed as denoting the 2nd-level property of NON-EMPTINESS [“$(\exists x)Px$” says that the (1st-level) property P is not empty].

Clearly, given *any* domain D and a 1–1 replacement r of the Individuals of D, P is replaced by a property P' such that NON-EMPTY (P) iff NON-EMPTY (P'). So: IS-NONEMPTY (∃) is invariant under *all* r’s, i.e., is maximally invariant. Similarly, NEGATION (~), which in predicate-logic contexts (“$\sim Px$”) denotes the 2nd-level property of COMPLEMENTATION, is maximally invariant: If r takes P in D to P' in D', then it takes COMPLEMENT(P) in D to COMPLEMENT(P') in D'. So: COMPLEMENTATION(~) has maximal invariance. And the same holds for all the standard logical constants of predicate logic.

From Invariance under r to Invariance under Isomorphisms. Now, if we talk in terms of structures, $\langle D, P\rangle$, $\langle D', P'\rangle$, where D, D' are any domains and P, P' are extensions of properties in D, D', respectively, invariance under all 1–1 r’s from D onto D' becomes *invariance under all isomorphisms*. This is the basis for the claim that logical constants are invariant under isomorphisms (in various forms: Mostowski 1957, Lindström 1966, Tarski 1966/86, Sher 1991, and so on). I.e., invariance under isomorphisms is a *criterion* for logical operators (properties), and with a few additional requirements (see Sher 1991), also a criterion for logical constants.

From Logical Constants to Logical Laws. The laws of logic are the laws governing the denotations of the logical constants: the laws of identity, negation (complementation), disjunction (union), existential and universal quantifiers (non-emptiness and universality or empty complement), finite cardinality quantifiers, etc. Since the properties denoted by these constants have a maximal degree of invariance, the relation, established above, between invariance and necessity, determines that the laws governing logical constants/properties have a maximal degree of necessity. Note: this will not change if we add more constants denoting maximal properties to standard logic as logical constants (e.g., MOST, FINITLEY MANY, INDENUMERABLY MANY, IS-SYMMETRIC, IS-WELL-ORDERED, etc.). As a result, the laws governing the resulting logics (their logical truths and logical consequences) will be just as necessary.

We can now close the circle:

Degree of necessity of logical laws
$>$
Degree of necessity of physical laws
[$>$
Degree of necessity of biological laws (principles)].

The distinction between types of necessity gives rise to a distinction between types or spaces of possibility:

Space of logical possibility
$>$
Space of physical possibility
[$>$
Space of biological possibility].

Ex.: an object that is not subject to gravity is not physically possible, but it is logically possible.

There is much more to say about invariance, necessity, and logic. My most thorough discussions of this topic appears in Sher (1991 and 2016, Part IV). Here I will limit myself to one point, concerning the relation between logic and mathematics:

7 Relation Between Logic and Mathematics

Going back to maximally-invariant properties we notice two interesting things:

1. Not just logical properties, but also many mathematical properties (2^{nd}-level complementation, intersection, inclusion, cardinality properties, ...), are maximally invariant. In fact: the totality of maximally-invariant properties is the totality of mathematical properties, construed (in the large majority of cases) as higher-level properties.
2. Yet: in mathematics itself, mathematical properties are often construed as 1^{st}-level properties, and 1^{st}-level mathematical properties are, for the most part, not maximally-invariant. Examples: Consider the 1^{st}-level properties *is-the-number-1* ($= 1$), where 1 is an individual, and *is-an-odd-number*. These 1^{st}-level properties are not invariant under all *r*'s. E.g., they are not invariant under any *r* such that $r(1) =$ Barack Obama. But they are correlated with higher-level maximally-invariant mathematical properties. (Indeed, even mathematical individuals are correlated with higher-level maximally-invariant mathematical properties. For example, the individual 1 is correlated with the 2^{nd}-level cardinality property ONE.) So, what we have is:
   a. Maximally invariant properties include both higher-level mathematical properties and logical properties (which coincide with certain mathematical properties, such as NON-EMPTINESS ($\exists$)).
   b. Maximally-invariant higher-level mathematical properties are systematically correlated with lower-level mathematical properties which are not maximally invariant.

This suggests that there is something in common to logical properties and maximally-invariant mathematical properties, something that also underlies the correlation of non-maximally-invariant mathematical properties with maximally-invariant mathematical properties. Elsewhere I suggested that this common feature is *formality*. Maximal invariance is a criterion of formality, and formality is the common basis of logic and mathematics.

Questions:

1. Is this logicism? No: mathematics is not reducible to logic.
2. Is this identity (mathematics = logic)? No.

What, then, is the relation between logic and mathematics. My answer is that there is a division of labor between mathematics and logic. Mathematics studies formal properties and their laws. Logic builds formal properties into our language as logical-constants, to be used as tools of reasoning, based on the laws of the formal properties they denote: laws of identity, complementation, intersection, inclusion, non-emptiness, finite cardinalities, possibly infinite cardinalities, etc.

One possible objection is: You argue for a close connection between logic and math. But logic is trivial, while mathematics is not. Response: Both logic and mathematics have trivial and non-trivial parts. Examples: both $1 + 1 = 2$ and $Pa \& Qa \rightarrow Pa$ are trivial; both the content of Cantor's (mathematical) theorem (which says that the cardinality of the power set of s is larger than the cardinality of s) and its *logical* derivation from the axioms of ZFC/Z are not trivial.

How do we explain the relation between higher- and lower-level mathematics? In spite of the fact that lower- and higher- level mathematics have different degrees of invariance, there is a systematic connection between them. To understand both the connection and the difference between them, let's note that to understand any field of knowledge, we have to distinguish two questions:

1. What does it study?
2. How does it study it?

What a given field of knowledge studies depends on:

1. What there is to be studied.
2. What the field in question is interested in.

How the given field studies what it is interested in depends on:

1. Humans' cognitive resources.
2. Pragmatic and methodological considerations.

On the present proposal:

1. What does mathematics study? – Formal properties and the laws governing them.
2. How does mathematics study these? – Often, by constructing 1^{st}-level "models" of these properties and laws, i.e., 1^{st}-level structures representing (higher-level) formal properties, and studying these structures (e.g., studying finite cardinality properties by studying numbers (numerical individuals)).
3. Why does mathematics do this? There are many possible explanations. For example: humans may be better at figuring out formal relations when they think in terms of structures of individuals rather than in terms of structures

> of properties, properties of properties, and so on. (This is something for cognitive science to investigate.) But the question which is more relevant to us, as philosophers, is: Is it possible, in principle, to adequately study an object of one kind by studying an object of a different kind that systematically represents it? Answer: Yes. (Analogy: designing a big skyscraper, made of steel and concrete by working with a small plastic model of the skyscraper.)

What about necessity? Are the laws of 1^{st}-order arithmetic and set-theory necessary? Answer: Yes. They acquire their necessity from the fact that they represent formal laws, laws that, due to the high degree of invariance of the properties they hold of, are highly – indeed, maximally – necessary.

8 Ramifications for Metaphysical Necessity and Scientific Laws

Necessity is usually viewed as a metaphysical subject-matter. But we explained several basic things about necessity without referring to metaphysics. Why? And what are the degrees of invariance and necessity of metaphysical properties and laws (principles)? Answer: The subject-matter of metaphysics is much less homogeneous, and much more difficult to demarcate, than that of logic, mathematics, physics, or biology. Focusing on invariance, metaphysics deals both with very broad subject matters, such as objects in general, and narrower subject matters with principles such as:

– An object cannot be both all of one color and of another color (at the same time).
– An object cannot have two different temperatures at the same time.

This suggests that metaphysics can be divided into sub-fields: 1. Those whose degree of invariance, hence necessity, is as high as that of logic and mathematics. 2. Those whose degree of invariance, hence, necessity, is lower. Overall, metaphysical invariance, hence necessity, seem to lie in between those of logic/mathematics and physics.

Turning to laws of nature, I mentioned earlier that philosophers of science are especially worried about the necessity of scientific laws, including physical laws. They are also concerned that general philosophy may usurp science. So let me focus on explaining what the philosophical investigation of invariance does, and does not, tell us about the necessity of physical laws. It tells us that the fact that many physical properties have fairly strong degrees of invariance shows that there are objective grounds for modally strong physical laws. But it does not tell us to what extent the world is governed by physical laws and what specific laws it is governed by.

Invariance shows that the world is ready, so to speak, or has an appropriate infrastructure, for being governed by necessary laws. Example: Invariance says that gravity is a candidate for physical laws, while the property of being Mount Everest is not, and it says that if gravity is governed by laws, they have a fairly strong modal force. But it doesn't determine whether such laws would be Newtonian or relativistic. There are also types of necessity that invariance doesn't explain, though they are arguably less philosophically significant than those it does explain. For example: it doesn't explain (1) necessity as fixity or (2) singular necessity (unless they are based on invariance considerations).

1. *Necessity as Fixity*. In mathematics, science, and everyday life, we often identify a property by connecting it to other properties. We often say: "let *P* be …" . Say: "Let *P* be the property of being a set (or a natural number)". Then, this determines the identity of *P* in advance, and in a sense makes the principles characterizing it necessary (in the given discourse or theory). Similarly, often – either before, or during, or after a given investigation – we identify a property by saying what it is constituted of. Example: "Let rest-energy be energy that does not vanish at 0 speed". Or we can just stipulate: "Let *is-a-dagnet* be the property of being a crazy politician". Then, it is necessary, in the fixity sense, that

(i) Rest energy does not vanish at 0 speed.
(ii) Dagnets are crazy.

This kind of necessity is not explained by invariance (unless there are theoretical reasons, and in particular, reasons involving invariance, for deciding what to hold fixed and how to fix it).

2. *Singular Necessity*. There are a number of fundamental constants in physics: c – the speed of light in a vacuum, h – the Planck constant: a number that the energy of any body must be a multiple of.

These constants are embedded in physically-necessary laws, but they cannot be explained by invariance, unless they follow from considerations of invariance. So if, and to the extent that "Nothing moves faster than c" is required for the invariance of physical laws, its necessity is explained by invariance; otherwise, it is not.

There is much more to say about invariance, including the relation between invariance and abstraction, apriority, and generality. But I leave this for another occasion.

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