LOGICAL REALISM – A TALE OF TWO THEORIES*

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This paper compares two theories of the nature of logic: Maddy's (2007, 2014a) – henceforth "Theory 1" – and Sher's (1991, 2016) – "Theory 2".[1] The two theories share a significant element: they both diverge from the commonly held view that logic is grounded only in the mind (language, concepts, conventions, etc.). Instead, they argue that logic is crucially grounded in the world. But the two theories differ in significant ways as well. Most distinctly, one is an anti-holist, "austere naturalist" theory while the other is a non-naturalist "foundational-holistic" theory. This methodological difference affects their questions, goals, orientations, the scope of their investigations, their logical realism (the way they ground logic in the world), their explanation of the modal force of logic, and their approach to the relation between logic and mathematics.

The paper is not polemic. Its goal is *not* to compare the two theories with respect to their merits and deficiencies. One of its goal is a perspicuous description and analysis of the two theories, explaining their differences as well as commonalities. Another goal is showing that and how (i) a grounding of logic is possible, (ii) logical realism can be arrived at from different

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* I would like to thank Pen Maddy for helpful comments.

[1] While Maddy's *The Logical Must* (2014a) and Sher's *The Bounds of Logic* (1991) are fully devoted to logic, Maddy's *Second Philosophy* (2007) and Sher's *Epistemic Friction* (2016) are more general philosophical works. Each, however, has a substantial part devoted to logic: "A Second Philosophy of Logic" (Part III of Maddy 2007, pp. 197-302), and "An Outline of a Foundation for Logic" (Part IV of Sher 2016, pp. 237-338). Both authors presented their views in a number of papers as well. I shall refer to these according to need. In addition to "theory 1" and "theory 2", I shall use "theorist 1" and "theorist 2".

perspectives and using different methodologies, and (iii) grounding logic in the world is compatible with a central role for the human mind in logic. The discussion is divided into three parts: I. Basic Questions and Methodologies. II. Logic's Grounding in the World and Logical Necessity. III. Relation between Logic and Mathematics.

I. Basic Questions and Methodology

A. *Similarities.*

(a) *Questions.* Both theorist 1 and theorist 2 aim at a foundation or a grounding of logic, where by this I mean an investigation of the principles underlying logic, the source of the truth and reliability of logic, the nature of logical knowledge, the conditions of acceptance, rejection, revision of logical theories, the modal force of logic, and so on. Among the questions they ask are: "What is logic?" (Sher 1991: 8), "[W]hat is the subject matter of the science of logic?" (Maddy 2012: 484), "Is logic grounded in the mind or in the world?" (Sher 2016: 255)[2], "[W]hat is the ground of logical truth?" (Maddy 2014a: 121), "What specific features of the world is logic grounded in?" (Sher 2016: 271)[3], "[W]hat exactly are [the] logical facts about?" (Maddy 2012: 484), What is the source of "the *veridicality* of logic"? (Sher 2016: xiii), "[W]hat makes our logic ... reliable"? (Maddy 2014a: 109), "[What] underlies (and explains) the generality and strong modal force of logic[?]" (Sher 2016: 271)[4], What is "the so-called 'logical must'"? (Maddy 2014a:1), and so on. Both theorists focus on the contemporary heir(s) of Aristotelian syllogistic logic and Fregean quantificational logic, which I shall call here "mathematical logic".

(b) *The Philosopher's job.* An additional area of similarity concerns the more general

[2] Capital letters replaced by low-case letters.

[3] See fn. 2.

[4] Unitalicized.

aspects of the philosopher's job. Both theorists view themselves as conducting *theoretical inquiries*. More generally, as philosophers they view themselves as seeking the truth, or the facts, about whatever philosophical subject-matter they study, just as scientists and mathematicians seek the truth, or the facts, about the subject-matters *they* study.

(c) *Methodology.* Methodologically, both theorists are non-traditionalist in a number of ways: they reject the foundationalist methodology, "first philosophy", pure apriorism, the view that philosophy is isolated from other branches of knowledge, the view that logic is analytic, and so on. Both are neither nominalists nor relativists, instrumentalists, or radical pragmatists. Both regard all knowledge, including logical and philosophical knowledge, as fallible, revisable, subject to critical examination, and capable of improvement. Both believe that humans can correctly describe the world. Both reject the view that there is only one way of correctly describing the world – only "one true theory" of the world. Both reject the Kantian duality of thing-in-itself and appearance. Both regard philosophy as studying certain aspects of the world. And both view themselves as investigating classical philosophical questions using new methods.

(d) *Holism.* Another methodological similarity has to do with their attitude toward certain kinds of holism. Both theorists reject the kinds of holism that are connected with radical pragmatism, indispensability arguments, the view that our system of knowledge is confirmed or refuted only as a whole ("one-unit" holism (Sher 2016: 25)), the view that confirmation is of the same kind in all fields of knowledge and in all cases and that all items of knowledge equally partake in the confirmation of all items ("monolithic" and "homogeneous" holism (Maddy 2007: 275, 315)), the view that thin evidence is sufficient for confirmation, and so on.

B. *Differences.*

(a) *Questions and orientation.*

(i) Theorist 1 seeks to understand logic primarily by understanding the most elementary part of logic, while theorist 2 does not distinguish between the elementary and advanced parts of logic.

(ii) Theorist 1 says that she is not interested in questions concerning the distinctive features of logical truths/inferences (features that distinguish them from non-logical truths/inferences), questions concerning the scope of logic, the question of logicality as it applies to logical constants, and so on. Theorist 2 pursues all these questions.

(iii) Theorist 1 seeks an understanding of logic by picking up an example that *most people would regard as logical* and talking about that. Theorist 2 seeks an understanding of logic by focusing primarily on how it *should be* in order to perform its designated role/function.

(b) *Methodology: naturalism vs. figuring out.* Theorist 1 is an "austere" naturalist (as described in the inner jacket of Maddy 2014a). She identifies her naturalism as "methodological" (Maddy 2014a: 2) and describes it as close to "scientism" (Maddy 2007: 103). She follows Quine's lead in naturalizing all philosophical questions. For example, "[f]aced with the ... question ... how do we come to know about the world[, she] turns to contemporary cognitive science" (*ibid.:* 90), she "holds a roughly empirical account of logical truth" (Maddy 2014a: ix), and so on. Given any question, philosophical or scientific, she follows the same procedure in investigating it: "beginning from her ordinary perceptual beliefs, [she] gradually develop[s] more sophisticated observational and experimental techniques and correctives, eventually ascending to theory formation and confirmation, all in the sorts of empirical ways usually labeled 'scientific'" (*Ibid.:* 2). "[H]er inquiry takes place within ordinary science" (*ibid.:* 119). She "is born native to the laboratory" (Maddy 2007: 91). Science, for her, always comes first; philosophy – second. (She calls herself "Second Philosopher" and her philosophy "Second Philosophy" (*ibid.:* 19).) She is a bottom-top, rather than a top-bottom, theorist: "all standards, all truth must be laboriously built up from below" (*Ibid.:* 174).

As a naturalist, theorist 1 pursues her understanding of logic by pursuing empirical investigations within, or pertaining to, physics and psychology (cognitive science, neuroscience, biology). These include examination of whether different branches of contemporary physics share the world-view of elementary logic, psychological investigations of children's inferential capacities, and so on.

Theorist 2 is not a naturalist. She is not an anti-naturalist either, but she does not conduct her philosophical investigations by conducting scientific investigations. She believes that there is room for questions of multiple kinds about the world, including philosophical questions, and there are multiple methods for investigating such questions, including philosophical methods. Philosophical questions/methods are not superior to scientific questions/methods, but they are not inferior to them either. Philosophy is neither a "first" nor a "second" philosophy. It is simply philosophy. Theorist 2 appeals to scientific and/or mathematical developments when these contribute to her philosophical investigations, but she does not replace (what she takes to be) genuine philosophical questions by scientific questions or translate them to the latter. She does not limit herself to bottom-up investigations either. She is engaged in both bottom-up and top-down investigations.[5]

Theorist 2's main method of discovery/investigation is a general, widely used method she calls "figuring out" (Sher 2016: x, 85). The expression "figuring out" means what it means in everyday discourse: "putting two and two together", "configuring", "finding out by looking and drawing connections", and so on. Figuring out requires active *epistemic freedom*.[6] Theorist 2 conceives of it as a broadly *intellectual* method of investigation, one that can take multiple forms, is open to input of various kinds (including empirical input), and is often incorporated in other methods. Although figuring out requires freedom, it is subject to exacting standards of truth and justification. The activity of figuring out takes place both in experimental and in abstract fields of knowledge. For example, an experimental scientist has to figure out which experiment (which experimental activity among all the experimental activities he can engage in) will test a given

[5] See, e.g., Sher (1991): Ch. 3 is a top-down account and Ch. 4 a bottom-up account of logicality.

[6] For epistemic freedom see Sher (2016: Section 1.3).

scientific hypothesis. A theoretical scientist has to figure out whether two theories (such as quantum mechanics and relativity theory) are compatible and if not, whether and how they can be revised so as to be compatible. A meta-logician figures out whether a given theory is complete and how to prove its completeness/incompleteness. A philosopher figures out how to justify or refute a given philosophical argument (e.g., Hume's skeptical argument), how to solve philosophical problems (e.g., the Liar-paradox problem), and so on.[7]

When it comes to logic, theorist 2 is engaged in figuring out how to ground logic. This she does in a way that connects with her functional approach to logic: figuring out, first, what role or function logic is designed to play in our search for knowledge; figuring out, next, what features logic has to have in order to perform this role; figuring out whether having these features requires that logic be grounded in the world; figuring out what facets of the world logic has to, or can, be grounded in; and so on.

(c) *Levels of Inquiry.* Another methodological difference concerns levels of investigation. Theorist 1 affirms only one level of investigation, while theorist 2 affirms multiple levels.

Theorist 1 contrasts a *one-level* approach to inquiry with a *two-level* approach. Her paradigm of a two-level approach is Kant's empirical and transcendental levels of inquiry. "Kant's system ... epitomizes ... a striking structural feature: it involves two distinct levels of inquiry" (Maddy 2007: 47). The lower level is empirical and realistic, the higher level is apriori and idealistic. Theorist 1 "sees no point in the second [higher] level" (*Ibid.:* 81). "[O]ne [of her] recurrent theme[s] is the rejection of extra-scientific perspectives such as that of Kant's transcendental philosophy" (Maddy 1999: 109). She is "deaf and blind to the lure of the Kantian transcendental project" (Maddy 2007: 101). Her "investigations are pursued on one level, as part and parcel of the single mosaic of natural science" (*ibid.:* 47). She "makes no sense of transcendental analysis" (*Ibid.:* 225). "In place of Kant's two-level view, [she] seeks one unified

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[7] For more on *figuring out* see Sher (2016: 85-6, 192, 199, 243n, 254).

scientific account" (*Ibid.*).

Theorist 2 affirms multiple levels of inquiry. Given humans' significant cognitive limitations, they have to make full use of their cognitive capacities. And two especially important capacities they do have – "immanent" and "transcendent" thinking (Sher 2016: xi. 163, 166) – are associated with different levels of investigation. In *immanent* investigation we "direct... our mental gaze" (*ibid.:* 163) at the world in order to study it. This is what we do when we speak from *within* a theory. A theory studies some region of the world, asks what properties objects in this region have (what relations they stand in), what laws govern these properties (relations), and so on. Scientific, mathematical, and philosophical theories are all immanent in this sense. Epistemology, for example, is immanent in directing its gaze at that region of reality in which human knowledge is located. In *transcendent* investigation we go outside a given immanent inquiry and investigate this inquiry itself. We may investigate its relation to its subject-matter, its relations to other inquiries, and so on. Since our inquiries are part of the world, transcendent inquiry is also immanent. Not every immanent inquiry is transcendent, but every transcendent inquiry is immanent. Accordingly, we have multiple levels of inquiry: immanence, immanent-transcendence, immanent-double-transcendence, and so on. Epistemology is, for the most part, immanent-transcendent.

The standpoint of transcendent inquiries, as conceived by theorist 2, is *factual* and *rational* rather than *ideal* or *magical.* It is also *human* rather than *Godly*. Theorist 2 calls it "*HH [human-human] transcendence*: [t]ranscending one *human* standpoint, X, to another, *human*, standpoint Y" (*ibid.:* 167). One paradigm of transcendence of this kind is transcendence to a Tarskian meta-theory. There is nothing Godly, or mysterious, or magical, or absolute, or ideal about a Tarskian meta-theory, but it has certain capacities that a Tarskian object-theory (immanence without transcendence) lacks.[8] We can transcend the same theory to multiple other

[8] See Tarski (1933).

theories. For example, we can transcend science both to the sociology of science and to the philosophy of science. Transcendence (of this kind) "reflects the dynamic nature of human ... knowledge; in particular, our ability to move up and down, back and forth, from one [level of inquiry] to another, in cognizing the world (including ourselves in it)" (*Ibid.:* 232-3).

(d) *Holism.* We have seen that certain types of holism are rejected both by theorist 1 and by theorist 2. But while theorist 1 regards these types as exhausting holism, and as a result views herself as an anti-holist, theorist 2 does not. Theorist 2 sees new possibilities for holism, and she develops a new type of holism, "foundational-holism" (*ibid.:* viii, 20). Foundational-holism is neither a one-unit holism nor a monolithic/homogeneous holism, neither a radically pragmatic holism nor an indispensabilist or a thin-evidence holism. Foundational-holism is motivated by the belief that the classical philosophical project of constructing a foundation/grounding for knowledge, truth, logic, and so on is worthwhile, but the traditional method for carrying it out, foundational*ism,* is self-defeating. In particular, foundationalism's requirement that the grounding of knowledge be a strict partial ordering with minimal elements (a tree or a pyramid), where each unit of knowledge is grounded in units lower than it in the foundationalist hierarchy, is self-defeating. The problem is that the entire foundation depends on the minimal elements, but the minimal elements themselves cannot be properly grounded.

Foundational-holism discards the strict ordering requirement. It says that "there is no inherent connection between grounding our system of knowledge in reality and doing so in a strictly ordered manner" (*ibid.:* 23). Instead, "[o]ur system of knowledge is connected to reality by a multifaceted relation (or network of relations), strictly-ordered in some sections, not strictly-ordered in others. All branches of knowledge, qua branches of *knowledge,* must be grounded in reality, but their grounding need not follow a single, strict, and rigid pattern" (*Ibid.*). This, however, does not mean that foundational-holism's conception of the grounding of knowledge is thin or watered-down. On the contrary. It is designed for making substantial grounding possible, as we shall see in the case of logic below. We may say that "foundational holism puts the holistic method in [the] service of a robust, world-oriented, universal foundational project" (*ibid.:* ix).

One of the problems that undermines Quine's holism, according to theorist 1, is circularity: "the Quinean holist ... can[not] carry out his empirical testing of logical laws without presupposing at least some of those laws" (Maddy 2007: 203). Foundational-holism is especially adept at handling circularity of this kind. Rejecting destructive as well as trivializing ("P; therefore P") circularities, it explains away circularities of the kind theorist 1 attributes to Quine as *partial* and *temporary*. They are partial since in revising logic we use other resources than just logic: we use mathematical resources, philosophical resources, scientific resources, and so on, so the overall revision is only partly circular. And this partial circularity is in principle temporary: every revision requires holding something fixed (assuming something), but we are not required to hold the same thing fixed, either in the course of a given revision or in future revisions. In critically revising logic we use some logical resources but we hold off others, we switch back and forth between assuming X and leaving X open to revision (while assuming Y), and so on. Furthermore, upon completing a given revision, we turn to examining its assumptions, using any resources that are available to us, including resources that are external to these assumptions and resources that we created based on earlier revisions of our theories. This may lead us to reexamine some of our past revisions, but this is as it should be. No illusory promise of a final, ultimate, infallible examination is given, but progress is in principle achieved.[9]

II. Grounding Logic in the World and the Standing of Logical Necessity

Both theory 1 and theory 2 ground mathematical logic in the world, and in a sense they both ground it in the *formal structure* of the world. But they differ in the scope of their

[9] For more on the foundational-holistic treatment of circularity, see (Sher 2016: 30-4, 323).

grounding, their understanding of "formal structure", the way they ground logic in the world, and their treatment of the necessity of logic.

Theory 1. In grounding a basic discipline such as logic, we face a special challenge: finding an effective starting point. Although theorist 1's grounding of logic is supposed to be empirical, she starts with Kant. Why Kant? – Because

> [t]here's considerable appeal [both] to the suggestion that logic depends on very general structural features of the world, and to the quite different idea that logic is embodied in our most primitive forms of conceptualization. Kant's combination of transcendental idealism and empirical realism accomplishes the neat trick of giving us both these at once: transcendentally, logic is dictated by the forms of judgment and pure concepts of the discursive intellect; empirically, logic describes the underlying structure of the world. [Maddy 2007: 225]

Now, theorist 1, as we have seen above, rejects large parts of the Kantian account, including Kant's entire transcendental approach. What she is after is a "second-philosophical [i.e., *naturalist*] version of the Kantian position that preserves its merits while bypassing the transcendental" (*ibid.*).

Theorist 1 divides logic into two parts: "*rudimentary logic*" (*ibid.:* 231, my italics) and what I shall call *full-scale logic*. Rudimentary logic is pre-logic, proto-logic, or the most elementary part of logic; full-scale logic is standard[10], bivalent, 1st-order mathematical logic in its entirety. What is included and not included in rudimentary logic? There is a "considerable distance between rudimentary logic and modern, first-order predicate logic" (*ibid.:* 283). Rudimentary logic includes a fair number of the primitive rules applicable to disjunction, conjunction, and to some extent negation, identity, and the universal and existential quantifiers. But its conditional is the syllogistic rather than the truth-functional conditional. And it has no provisions for avoiding indeterminacy, non-denoting names, paradoxical predicates, and so on. As a result, rudimentary logic is a *trivalent* rather than a classical (bivalent) logic, and is to some

[10] "Standard" in the sense of being restricted to the standard logical constants (excluding other logical constants such as the quantifier "Most").

degree fuzzy as well. Very few of the classical logical truths hold in rudimentary logic, and many classical logical inferences fail as well. For example, whereas Modus Ponens and Disjunctive Syllogism hold in rudimentary logic, Modus Tollens and Reductio ad Absurdum fail. Furthermore, rudimentary logic lacks the systematicity of full-scale logic.[11] Another distinctive feature of rudimentary logic is that it includes some basic arithmetic, such as very simple additions and multiplications.[12] Epistemically, rudimentary logic can be characterized as the kind of logic-mathematics that is available to infants and very young children.

In speaking about theory 1's grounding of rudimentary logic we may distinguish between its grounding of the *veridicality* of rudimentary logic and its grounding of its *necessity*. The *veridicality* of rudimentary logic is grounded, metaphysically, in the *physical world*, and epistemically in the *human mind*. What, in the *physical world*, is the veridicality of rudimentary logic grounded in? It is grounded in what theorist 1 calls, after Kant and Frege, the "KF-structure" (*ibid.:* 229) of the physical world, which consists of "a domain of objects that bear properties and stand in relations, perhaps some universal properties, plus compounds of these involving conjunctions, disjunctions and negations", where "some interconnections between these situations are robust ground-consequent dependencies". (*Ibid.:* 228-9) This structure behaves in a way that supports the logical inferences of rudimentary logic. For example, a physical object that has the property P or the property Q but does not have the property Q has the property P. The veridicality of Disjunctive Syllogism is grounded in this fact.

How does theorist 1 establish the claim that the world has this KF-structure? She establishes it in a process of the kind described in Part I, namely, one that begins with common sense and then turns more and more deeply into science. Here is how she establishes the reality of KF-objects:

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[11] See (*ibid.:* 229-31, 283).

[12] See (*ibid.:* 318).

> Common sense clearly endorses the idea that the world contains many medium-sized physical objects. Such things cohere, have boundaries, and move continuously as units; examples range from apples, chairs, and people, to boulders, books, and baseballs. When the Second Philosopher examines these beliefs more closely, she finds both confirmation and explication. Such objects are indeed distinct from their surroundings; they are composed of intricately arranged atoms dotted throughout largely empty space, and those atoms and arrangements differ starkly from the atoms and arrangements in the space nearby. Their cohesiveness comes from the bonds between their atoms; their solidity comes from the electromagnetic fields they generate; they move according to certain principles of motion, and so on. On further, more specialized investigation, she uncovers further such objects: planets, blood cells, spider mites. She concludes that ordinary physical objects are real structures in the world. [*ibid.:* 234]

But science also limits the reality of the KF-world. Theorist 1 learns from science that the KF-world is real on the *macro-level* yet not on the *micro-level*:

> Our examination of the extent to which the world's structuring into objects can be pushed into the microscopic, turns up a... breakdown... . The micro-world is not structured into things of the familiar sort; though the world does contain numerous ordinary objects, it also contains phenomena that are not so structured. Despite our scientific predisposition to see the world in these terms, our pursuit of science itself has taught us that the world is not as we expect it to be, not in all its parts. This portion of our empirical hypothesis – that the world consists of coherent objects that move as units along continuous spatiotemporal paths – must be qualified. The world is structured into such objects at the macro-level, but at the micro-level, all current evidence suggests that it is not. ... Here not only the spatiotemporal features of objects are undermined, but the pure notion of an object as an individual thing. Thus it seems the micro-world cannot be said to display an abstract KF-structure of individual objects. [*Ibid.:* 236-7]

The veridicality of rudimentary logic is grounded only in the macro-world, hence is limited to it.

Turning to the grounding of the veridicality of rudimentary logic in the *human mind* (epistemic grounding), the underlying idea is that for us to have a logic that simulates the KF-world, our mind has to be appropriately structured. For this, theorist 1 appeals to contemporary psychology: "[t]here can be little doubt that ordinary adults see the world in terms of individual objects" (*ibid.:* 245). But "there is more ... than this" (*ibid.*): contemporary psychology tells us that this way of seeing the world is not "acquired or learned"; it is "the product of [humans'] primitive conceptual mechanisms" (*ibid.*). Contemporary researchers of infant and child cognitive development, such as Elizabeth Spelke, have shown that infants cognize the world in terms of object unity:

> [I]infants as young as 4 months are able to individuate and identify objects using spatiotemporal criteria: spatial contiguity, common fate, and continuous motion. No such object can be in two places at once; two such objects cannot occupy the same location; all such objects travel on continuous paths ... . They are 'complete, connected, solid bodies that persist over occlusion and maintain their identity through time'. [*ibid.:* 253]
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> [O]ther principles [include] continuity, solidity, and contact ... . [C]ohesion is fundamental. [*ibid.:* 254]

Objects of this kind are often called in the literature "Spelke-objects". Theorist 1 says:

> [I]t seems humans are so configured, biologically, that they come to perceive a world of Spelke objects, without instruction, given ordinary maturation in a normal environment. Or, in the Second Philosopher's terms, the ability to perceive Spelke objects is part of a human being's most primitive cognitive equipment. [*ibid.:* 258]

What connects the epistemic grounding of rudimentary logic to its metaphysical grounding is: "humans are so configured *because* the macro-world is so structured" (*ibid.:* 264, my italics). In the case of objects:

> [T]he spatiotemporal information central to the notion of a Spelke object is generally more reliable than the information infants eschew. ... And the types of spatiotemporal information infants prefer – cohesion, solidity, continuity of motion – are more reliable than those they tend to ignore – similarity, good continuation, good form. [*ibid.:* 264-5]

Theorist 1 is aware of the fact that the new theory of child development may not be completely accurate. Nevertheless, she says, it is clear that

> human beings have primitive cognitive mechanisms capable of detecting and representing KF-structures because they live in a largely KF-world and interact almost exclusively with its KF-aspects. The evidence clearly suggests that we – like the chicks, pigeons, and monkeys – are responding directly to some of the world's most elementary features. [*Ibid.:* 270]

We may sum up theory 1's grounding of the veridicality of rudimentary logic in the world on the one hand and in the human mind on the other thus:

A. "[R]udimentary logic is true of the world insofar as it's a KF-world, and in many but not all respects it is".

B. "[H]uman beings believe the simple truths of rudimentary logic because their most primitive cognitive mechanisms allow them to detect and represent the KF-structure in the world".

C. "[T]he primitive cognitive mechanisms of human beings are this way because we live in a largely KF-world and interact almost exclusively with its KF-structures". (*Ibid.:* 271)

But while the KF-structure of the physical world, at least on the macro-level, grounds the veridicality of rudimentary logic, it does *not* ground its *necessity*, according to theorist 1. The reason is that it is a *contingent* physical fact that the macro-world is a KF-world.[13] Why do people regard the inferences of rudimentary logic as necessary? Theorist 1's answer to this question is similar to Hume's answer to the question why the laws of nature are usually regarded as necessary: our psychology. But while Hume's answer is given in terms of "habit" and "custom", theorist 1 appeals to the results obtained by contemporary developmental psychology. The logical inferences of rudimentary logic, contemporary psychology tell us, are built into the primitive mechanisms of human cognition. As a result, these inferences appear "obvious" to us, and this, in turn, leads us to view them as necessary.[14]

It should be noted that in a certain respect, theory 1's account of rudimentary logical inferences does not distinguish between logical and non-logical inferences. Depending on what contemporary physics and psychology say about the basic features of the world and the basic features of human cognition, it may very well be the case that there is no difference between the grounding of non-logical rudimentary inferences (possibly: "This is a dog; therefore it barks")

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[13] See (*ibid.:* 273).

[14] See (*ibid.:* 273-4).

and logical rudimentary inferences ("This dog barks; therefore, it barks or meows").[15] In this sense, it is not specifically a grounding of logic. This, however, would not be problematic for theorist 1 who, as we have noted above, has no interest in explaining the difference between logical and non-logical truths/ inferences.

Turning to full-scale logic, i.e., standard classical (bivalent) 1st-order logic, the move to this logic, according to theory 1, is motivated by the theoretical limitations or shortcomings of rudimentary logic. "In light of these ... shortcomings, ... logicians have sought stronger, more manageable systems of logic. The considerable distance between rudimentary logic and modern, 1st-order predicate logic ('classical logic') [is] bridged by a number of restrictions and idealizations, all more or less traceable to Frege" (*ibid.:* 283). These include the devising of an idealized, "logically perfect language'" (*ibid.*) under which "the names nam[e], the predicates classify" (*ibid.:* 288), the conditional is truth-functional, there is no limit on the complexity of formulas, and so on. "Here, just as in the rest of science, we [are] sensitive to the benefits and dangers of idealization, and satisfy ourselves that the idealization in question is appropriate to the case at hand" (*ibid.:* 287). The result is a "vastly more effective instrument" (*ibid.:* 288) than rudimentary logic. "The justification [of the idealizations underlying this instrument is] that they make it possible to achieve results that would otherwise be impossible or impractical, and that they do so without introducing any relevant distortions" (*ibid.*). Under these conditions "the familiar logic can be trusted" (*ibid.*). In this way, full-scale logic is "grounded in a rudimentary logic that's both true of the world and embedded in our most primitive modes of cognition and representation" (*ibid.*). In contexts where classical logic is inappropriate, we may resort to various "deviant logics" (*ibid.*).

Summing up we may say that theory 1 grounds the veridicality of logic both

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[15] In this respect, theory 1's account of logic may be similar to Quine's account of analyticity in terms of *stimuli* – "stimulus analyticity" – which does not distinguish between"Dogs bark" and "Bachelors are unmarried".

(metaphysically) in the world and (epistemically) in the mind, and its necessity it grounds only (epistemically) in the mind. The veridicality of logic is grounded metaphysically in the KF-structure of the physical world and epistemically in our primitive cognitive mechanisms. These groundings are direct in the case of rudimentary logic, indirect in the case of full-scale logic. But the veridicality of full-scale logic is also grounded in our theoretical abilities, including our ability to engage in appropriate idealization, systematization, and so on.

Since the KF-structure is limited to the macro-level of the physical world, the veridicality of logic is limited to that level.

The seeming necessity of logic, unlike its veridicality, is limited to rudimentary logic. Epistemically, it is grounded in the same primitive cognitive mechanisms as its veridicality. These mechanisms make rudimentary logic appear obvious to us, hence seemingly necessary. Metaphysically, however, the necessity of rudimentary logic is illusory since its veridicality is contingent. It is contingent on the basic structure of the physical world and on the primitive cognitive mechanisms of the human mind.

Theory 2. While theorist 1 has one starting point – Kant – theorist 2 has two starting points. The first is the question with which she actually started her investigation of logic and which gave her a clue to the grounding of logic. The second is the first step in her subsequent rational presentation of the grounding itself. The first is the question of logical constants, which is the focus of Sher (1991). The second is the question of logic's task (function) in the human pursuit of knowledge, with which her functional outline of a foundation for logic in Sher (2016: Ch. 10) begins. Here I shall start with the latter.

A. *What is Logic's Task in the Pursuit of Knowledge?* To answer this question, theorist 2 begins with a few observations on the "basic human cognitive-epistemic situation" (Sher 2017: 373). These include the observations that (i) for one reason or another, humans seek to know the world in its full complexity, and not just practically but also theoretically, (ii) their cognitive capacities are considerable in some respects, limited – and sometimes severely limited – in others, (iii) the latter makes the pursuit of knowledge highly problematic for them, while the former enables them to find solutions to, avoid, or compensate for, some of these problems. As a result of this situation, it is both worthwhile and feasible for humans to develop methods for expanding their knowledge, such as methods of inference that use knowledge they already have to obtain new knowledge (employing means within their reach).

What methods of inference would provide this benefit? Given humans' aspiration to know the world as it is, rather than as they imagine or wish it to be (or are told, or are manipulated to believe, it is), one of their most important standards of success is *truth.* They would greatly benefit from methods of inference that *transmit truth from premises to conclusions*. And a method of inference that is *highly general* and *modally-strong* (i.e., guarantees the transmission of truth in all fields of knowledge with a strong modal force) would be especially beneficial. This, theorist 2 suggests, is logic's task (or one of its main tasks):

> *Logic's task is to develop a method of inference which is both highly general and has an especially strong modal force. More specifically, its task is to develop a method for constructing inferences that transmit truth from sentences to sentences with an especially strong modal force and regardless of field of knowledge*. [Sher 2016: 255]

B. *What constraints does this task set on an adequate logical system and its foundation/grounding?* Here, theorist 2 begins with the observation that the kind of truth that logical inference is required to transmit is the kind of truth that serves as a standard for successful knowledge. Since the aim is to know the world as it is, this truth is *truth-in-the-world*, i.e., truth as correspondence with the world (though not necessarily, and probably usually not, correspondence in the traditional, naive sense of copy, picture, mirror image, or even direct isomorphism)[16]. But if logical inference is to transmit truth-in-the-world, it has to take the world (or something relevant in the world) into account. To see this, suppose someone comes to you and says: "I invented/constructed/put-together a beautiful logical system, L. You should adopt it

[16] For discussion of non-naive correspondence, see Sher (2016, Ch. 8).

as your logic." To adopt L, you have to ascertain that it adequately performs the task of logic, i.e., that its inferences transmit truth-in-the-world from premises to conclusion and that they do this with a strong modal force. Now, suppose L says that the sentences $S_1,...,S_n$ logically imply the sentence S, but *the world is such* that the conjoined truth of $S_1,...,S_n$ *rules out* the truth of S. In that case, you should not adopt the logic L. And even if the world is such that all of $S_1,...,S_n$ and S are true or at least one of $S_1,...,S_n$ is false, but this is merely an accidental matter (there is no strong modal connection between the circumstances in the world under which $S_1,...,S_n$ are true and those under which S is true), you should not adopt L. Conclusion: for a logical system to be adequate, it cannot say that S follows logically from $S_1,...,S_n$ *unless* there is an appropriate connection between the *worldly conditions* that would render all of $S_1,...,S_n$ true and the *worldly conditions* that would render S true.

C. *Is there a logical system and a foundation/grounding for such a system that satisfy these constraints?* Theorist 2's answer to this question is positive: an appropriately built logical system satisfies these constraints. What is an appropriately built logical system? Assuming the common conception of logical inference as taking into account only the logical structure of the sentences involved, where this is a matter of the identity and distribution of logical constants, we may characterize an appropriately built logical system as one that satisfies the following conditions: (i) its logical constants denote *formal* properties (relations, functions), and (ii) its inferences are based on laws governing the formal properties denoted by its logical constants.[17]

To see why such a logical system is grounded in the world and how its special grounding ensures its strong modal force, we need to turn to theorist 2's systematic conception of

[17] For more on an "appropriately built" logical system see Sher (1991: Ch. 3, 2016: Ch. 10). For the purpose of this paper we can think of standard 1st-order mathematical logic as a paradigmatic example of such a system.

formality.[18]

Formality as Strong Invariance. Theory 2 offers a systematic explanation of *formality* in terms of *invariance*, a highly fruitful concept that is widely used in science and mathematics, and has also been used in certain developments of meta-logic.[19] There are various types of invariance; theorist 2 employs a notion of invariance that may be called *property-invariance.*

The basic idea underlying property-invariance is that properties, in general, are selective in character, i.e., they "notice", "distinguish", or "are affected by" some differences between objects but not others. A property is *invariant* under all and only replacements of objects that it does not distinguish between. For example, the 1st-level property *is-a-human*[20] distinguishes between men and dogs but not between men and women or between dogs and horses. It is invariant under replacements of men by women (and *vice versa*) and of dogs by horses (and *vice versa*) but not under replacements of men by dogs (or *vice versa*). Precisifying, theory 2 says that *is-a-human* is invariant under *all 1-1 and onto replacements* of men by women[21] (and *vice versa*) and of dogs by cats (and *vice versa*) *in any equinumerous domains*, but not under any 1-1 and onto replacement of men by dogs (or *vice versa*) in any equinumerous domains. Other properties have different invariances.[22]

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[18] In a sense, theorist 1 also grounds logic – or at least its veridicality – in the formal structure of the world, since the KF-features she talks about are intuitively formal. And in one of her works (Maddy 2014b) she talks about the "formal" structure of the world instead of its "KF" structure. But she does not offer a systematic characterization of formality and seems averse to such a characterization.

[19] For relevant examples, see Mostowski (1957), Lindström (1966), and Tarski (1966, Published 1986).

[20] I use italics for 1st-level properties.

[21] More precisely, replacements of individuals which are men by individuals which are women.

[22] For example, *is-a-woman*, unlike *is-a-human*, is not invariant under replacements of men by women (or vice versa).

One significant result of the application of invariance to properties is that some properties have the distinctive feature of being *maximally-invariant* (Sher 2021), i.e., invariant under *all* 1-1 and onto replacements of individuals – *actual and counterfactual* – in any (equinumerous) domains. For example, the 1st-level property (relation) *is-identical-to* is of this kind. Expanding this feature to 2nd- (and higher-) level properties[23] and using current logical and mathematical terminology, it is easy to see that all the other properties denoted by the standard logical constants – IS-NOT-EMPTY, IS-UNIVERSAL, COMPLEMENT, INTERSECTION, UNION, INCLUSION (denoted by "$\exists$", "$\forall$", "~", "&", "$\vee$", "$\supset$")[24] – are also maximally-invariant.

The notion of *maximal invariance* coincides with that of *invariance under all isomorphisms*.[25] Noting that the latter notion intuitively captures, or precisifies, the idea of objectual or worldly *formality*,[26] theorist 2 introduces the following definition or criterion of formality:

> *Criterion of Formality: A property is formal* iff *it is maximally-invariant* iff *it is invariant under all isomorphisms.*

Take IS-NOT-EMPTY (the denotation of "$\exists$") as an example. Its argument-structures have the form <D, B>, where D is a domain of individuals and B is the extension of a 1-place 1st-level property P in D. IS-NOT-EMPTY is formal because it is invariant under any replacement of <D,B>

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[23] A (1-place) 2nd-level property **P** is maximally-invariant iff for any 1-1 and onto replacement *r* of individuals from any domain, D_1, by individuals from any equinumerous domain, D_2, and any 1st-level property P on D_1, P has the property **P** iff its image under *r* has the property **P** (or, more precisely: the extension of P in D_1, B, has the property **P** iff its image under *r* has the property **P**).

[24] (i) I use small capital letters for 2nd-level properties. (ii) The listed denotations of the logical connectives are in typical predicative contexts (e.g., Φx & Ψx), but they are expandable to other contexts as well. (See Sher 2016: Section 10.4.)

[25] For "invariance under (all) isomorphisms" see Sher (1991, Ch. 3; 2016, 10.4). For its relation to "maximal invariance" see Sher (2021).

[26] See Sher (1991, Ch. 3; 2016, 10.4-5).

by an isomorphic structure <D', B'>: B is not empty (in D) iff B' is not empty (in D').

It follows from the above criterion that the properties denoted by the standard logical constants are *formal*. And it further follows that, given a well-built system of logical inference, its inferences are grounded in laws governing formal properties. As an example, take the logical inference

(1) $(\exists x)(Px \vee Qx), \sim(\exists x)Px \vDash (\exists x)Qx$.

This inference is grounded in a law that connects the combination of (i) non-emptiness of a union of two properties and (ii) emptiness of one of these properties to (iii) non-emptiness of the other. I.e., this inference is grounded in a law that connects one combination of formal properties (of objects) to another. That is to say, it is grounded in a law governing formal properties, i.e., a *formal law* governing the world. More generally, logical inferences are grounded in *formal facets* of the world, or in the *formal structure* of the world.[27]

But this is not all. Not only is the veridicality (transmission of truth) of logical inferences, but also their necessity, is grounded in the formal structure of the world.

Formality implies Necessity. This is another result that theorist 2 arrives at based on the connection between formality and invariance. Formal properties are maximally-invariant, i.e., they do not distinguish between *any* individuals, *actual or counterfactual*. In other words, they behave in the same way regardless of whether the individuals they apply to are actual or counterfactual.[28] As a result, the laws governing/describing formal properties, i.e., *formal laws*, cannot distinguish between *any* individuals, *actual or counterfactual*, either. (If they did they would not accurately describe the behavior of formal properties.) That is to say: formal laws hold in all domains, regardless of whether their individuals are *actual or counterfactual*, and as such *have a strong modal force* or are *necessary*. Indeed, they hold even of counterfactual

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[27] For further details see (*ibid.*).

[28] Or, if they are higher-level properties, they behave in the same way with regard to any properties of any actual or counterfactual individuals, and so on.

individuals that are not physically possible, hence they have an *especially* strong modal force or are *highly* necessary.

Thus, the strong modal force of

(1) $(\exists x)(Px \vee Qx), \sim(\exists x)Px \vDash (\exists x)Qx$,

is grounded in a law that governs the formal properties of non-emptiness, union, and complementation. Since this law describes the behavior of these properties in any domain of any individuals, actual or counterfactual, it is highly necessary.

Theorist 2 sums up her grounding of logic as follows:

> *Logic (logical consequence) is grounded in formal laws governing reality – laws governing formal features of objects and properties – and it is the ... applicability and strong modal force of such laws that underlies (and explains) the [veridicality] and strong modal force of logic.* [Sher 2016: 271]

Theorist 2, we see, regards the formal laws governing the world as necessary. How does she respond to theorist 1's claim that the KF-structure of the world, which is theorist 1's version of theorist 2's "formal structure of the world", is contingent? Theorist 2 does not say that the formal structure of the world, including its formal properties and formal laws, could not have been different than they are. (She leaves this an open question.) What she says is that regardless of what the formal laws are, they are laws that govern formal properties which, being formal, are maximally-invariant, and that, as a result, these laws have a larger actual-counterfactual scope than whatever laws govern the physical, biological, psychological properties of objects in the world, which are not maximally-invariant. It is in this sense that formal laws have a higher degree of necessity than physical, biological, psychological and other laws, and as such are highly necessary.[29]

Theory 2's grounding of logic is, thus, not dependent either on a particular logic or on

[29] Figuratively, we can describe this in terms of formal "multiverses". Different formal multiverses have different formal laws, but in each formal multiverse, the formal laws have a stronger modal force (a larger actual-counterfactual scope) than any of its physical laws (or any of the physical laws within the physical multiverses that are included in this formal multiverse).

particular formal properties and laws. Theory 2 says that whatever the formal properties and their laws turn out to be, they will ground a logical system that satisfies the task of logic in a world governed by those laws.

One result of grounding logic in the formal structure of the world, where formality is defined in terms of maximal invariance, is a delineation of the *scope* and *distinguishing characteristics* of (mathematical) logic. Suppose the formal properties are (the 2nd-level correlates) of those definable by contemporary (1st-order) set-theory. Then the scope of (mathematical) logic extends to all logical systems with logical constants denoting such properties. This includes logical systems with such logical quantifiers as MOST, FINITELY-MANY, INDENUMERABLY-MANY, IS-WELL-ORDERED, etc., i.e., logics with so-called "generalized quantifiers" of the kind studied in Barwise & Feferman (1985). The distinguishing characteristic of all logics of this kind, including standard 1st-order logic, is their *formality:* their having modally-strong truth-transmitting inferences grounded in formal laws.

I have said that theory 2, like theory 1, grounds logic both in the world and in the human mind. What in the human mind is logic grounded in according to theory 2? It is grounded in (i) humans' interest in an especially strong method of inference centered on the transmission of truth from premises to conclusion, (ii) humans' cognitive abilities, in particular their ability to figure out what the formal laws are[30] and how to construct a logical system grounded in these laws[31], and so on.

To sum up: Theory 2 grounds logic – full-scale mathematical logic – in the formal structure of the world, and this grounding encompasses both the veridicality and modal force. Formality is explained in terms of maximal invariance, which is tantamount to invariance under isomorphisms. The formality of logical inferences distinguishes them from non-logical

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[30] See discussion of the relation between logic and mathematics in Part III below.

[31] See Sher (1991, 2016: Ch. 10).

inferences and determines the scope (maximal extendability) of (mathematical) logic.[32]

III. Relation between Logic and Mathematics

Theorist 1, as we have seen above, places the elementary parts of arithmetic under the same category as the elementary parts of logic:

> [W]e've seen ... that the world ... enjoy[s] a considerable amount of logical structuring, and we now note that this includes the facts underlying such elementary arithmetical claims as 2+2=4... . Likewise it appears that any KF-structure validates the worldly instances of simple multiplications, like 2×3=6... . Exponentiation is more complex, but still within the range of KF-structuring. Thus, insofar as it is KF-, the world reflects the structure of elementary arithmetical equalities and inequalities. [Maddy 2007: 318-9]

But more advanced parts of arithmetic are beyond the scope of this connection:

> [E]lementary arithmetical claims like 2+2=4 are answerable to the logical structure of the world – which means they have the same status as rudimentary logic – but ... the '...' of mathematical number theory is another matter. ... [A]s soon as the '...' comes into play, we've entered the realm of higher mathematics. [*Ibid.:* 361-2]

Other parts of mathematics, such as higher set-theory (set-theory beyond number theory) are still farther away from logic. These parts of mathematics seem not to be grounded in the KF-world at all in the way rudimentary logic or rudimentary arithmetic are.

Leaving aside most of theorist 1's rich work on mathematics (see, e.g., Maddy 1988 and 1997), I focus here only on her view of the relation between logic and what she calls "deep mathematics", which includes higher set theory.

Theorist 1 emphasizes the "sharp contrast" between her "robust worldly" grounding of "logic" and her "objective but metaphysically neutral" grounding of "set theory" (Maddy 2014c: 222-3). In the case of logic her focus is on "the ground of logical *truth*", in the case of set theory on "the ground of set theoretic *practice*" (*ibid.:* 223, my emphasis). Whereas metaphysical

[32] For theory 2's approach to deviant logics, see Sher (2016: 10.10).

questions come first (or at least early) in her understanding of logic, "the question of set theory's metaphysics comes last" (*ibid.*: 228). And while she is a *robust realist* with respect to rudimentary logic, she is a *thin realist* or even an *irrealist* with respect to set theory. But mostly she sees set theory as pursuing goals *internal* to mathematics, goals that are objective but have very little, if anything, to do with the world or with truth. Indeed, in her view, there is "no fact of the matter ... whether or not the notions of truth and existence... should be extended to set theory and the rest of [deep] mathematics" (*ibid.:* 230). What grounds deep mathematics is neither objects nor truth: "What we have here is a form of Objectivism without objects, and even without truth. ... [W]hat grounds mathematics isn't mathematical objects ... [or] even [mathematical] truth, it's mathematical depth." (*Ibid.:* 231).

So, while the main focus of theorist 1's investigation of logic is its connection to the basic structure of the world on the one hand and the human mind on the other, this is not the focus of her investigation of mathematics. And while she approaches logic from the outside so to speak, asking what physics and psychology – which are external to it – teach us about its grounding, she does not approach deep mathematics in this way. She does not draw a systematic connection between logic and mathematics.

Unlike theorist 1, theorist 2 does draw a systematic connection between logic and mathematics. She connects logic and mathematics, including set theory beyond number theory, *in principle*, based on their joint *formality*. She is a realist with respect to the formal structure of the world, which she views as central to both logic and mathematics. The formality of mathematics, like the formality of logic, is an *invariance result*: all higher-level mathematical properties, including all higher-level correlates of 1st-level mathematical properties, are maximally-invariant, or invariant under all isomorphisms, hence formal. This result systematically connects logic and mathematics. Logic and mathematics, however, are not identical. Neither is one reducible to the other. There is a division of labor between them. *Mathematics studies the laws governing formal properties while logic utilizes these laws to* *construct a powerful method of inference*. Theorist 2 calls this joint conception of logic and mathematics "semantic formalism" (Sher 2016: 321) and "formal-structuralis[m]" (Sher 2001, Sher 2016: 321n). Logic and mathematics, on this view, develop in tandem: starting from a minimal logic-mathematics, we use its resources to develop a more advanced mathematics, which we use to develop a more advanced logic, which we use to develop a still more advanced mathematics, and so on.[33]

Theory 2's realist account of logic is thus connected to a realist account of mathematics. This realism, however, is not Platonic. It starts with regular objects and proceeds, based on straightforward invariance considerations, to their formal properties, the modally-strong laws governing these properties, and the modally-strong inferences based on these laws.

IV. The World[34]

Let me conclude with a clarification of theorist 2's counterpart of theorist 1's KF-world, which I shall refer to simply as "the world". What the basic constituents of the world are is, for theorist 2, an open question, determined jointly by investigations of different disciplines. She does not distinguish between a "rudimentary" or "straightforward" structure of the world (which theorist 1 associates with KF-structure) and the rest of its structure. Whatever the structure of the world is, it contains both formal and non-formal elements, and in discussing logic theorist 2 is interested in its formal elements. Due to its formality, a correct description of the formal structure of the world is likely to be given by mathematics. One candidate for a mathematical theory of formal structure is ZFC. If ZFC, or another theory of a similar kind, is a correct theory

[33] For further discussion, including the way 1st-order mathematics is related to logic through 2nd-order mathematics, the apparent tension between the unbounded infinitude of mathematics and the (presumed) finitehood of the world, and the multiplicity of goals of mathematics of which tracking the formal structure of the world is only one, see Sher (2016, Sections 8.4 and 10.8).

[34] I would like to thank Pen Maddy for a question that led me to add this section.

of the formal structure, then the basic elements of the world (as far as its formal structure is concerned) are individuals and properties (where by properties she means also relations and functions). Given her understanding of formality, the formal elements of the world are properties, not individuals. Their formality is explained in terms of invariance. Among its formal properties she does not distinguish between "simple" properties on the one hand, such as identity, finite cardinality, intersection, complementation, etc., and "higher" properties such as infinite cardinality, being an ordering relation, a well-ordering relation, well-founded, etc. All of these are on a par as far as the formal structure of the world and its ability to ground logical consequences are concerned.

But not only does theorist 2 include in her counterpart of theorist 1's KF-world "higher" formal properties, she does not rule out that this world is not straightforwardly connected to our language. For example, she does not rule out the possibility that 1st-order mathematical statements that on the surface refer to mathematical individuals are true, yet the world does not contain mathematical individuals. For her mathematical truth is grounded in the formal structure of the world, but since the formal structure of the world arises largely on the level of properties of properties, it does not require mathematical individuals. One possibility is that what we think of as mathematical individuals are posits representing higher-level formal properties. (For example, numerical individuals represent cardinality properties.) 1st-order arithmetic and set-theory are, thus, true or false of the world, but indirectly so. (The reason we introduce such posits may be that we, humans, figure out things better when we think about them in terms of individuals and their properties than in terms of properties and their properties.) This, from her perspective, does not conflict with robust mathematical realism: an indirect connection with the world is just as real and just as robust as a direct connection. By being a mathematical realist, though, theorist 2 does not say that the only goal of mathematics is to describe the formal structure of the world. Mathematics has a variety of goals, and describing the laws governing formal properties that hold of individuals and their properties in the world is just one of these, albeit a philosophically significant one.

Two additional notes: (a) Although theorist 2 talks in terms of individuals and properties, she does not rule out the possibility that the basic elements are of a different kind. In that case, the notions of formality and invariance will have to be adjusted. (b) For theorist 2 it is an open question whether the macroscopic and microscopic world have the same formal structure and if not, how much their formal structures differ, and whether their difference can be represented within a single theory of formal structure or not. She considers various possibilities here, including, but not limited to, ones that are similar to theorist 1's understanding of the situation. (c) Unlike theorist 1, theorist 2 is not committed to her counterpart of the KF-world being trivalent rather than bivalent. Whether the world is formally trivalent or bivalent is an open question, where the formal difference between the two is a matter of such things as whether the basic form of properties in a given domain of individuals is that of two complementary subsets of the domain, or three complementary subsets.

We have seen how two different approaches to logic, the one austerely naturalist, the other not, the one non-holist the other foundational-holist, the one having a KF-conception of the world, the other another, lead to, and sanction, logical realism. Both ground the veridicality of logic in the formal structure of the world and one extends this grounding to its necessity as well. Both also emphasize the connection between the human mind and the world as central to this grounding. One focuses on evolutionary connections between the basic structure of the world and humans' cognitive mechanisms, the other on humans' interest in knowledge of the world and their ability to expand it by building a method of inference that tracks its formal structure. Logical realism is a minority view in contemporary philosophy. I hope that the tale of two theories that arrive at it from different directions will contribute to philosophers' understanding of the motivation, grounds, and richness of this view.

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