elements with large contributions to the negative elements with large contributions. Cosines and contributions for the punctuation example are given in Tables 3 and 4.

Multiple Correspondence Analysis

CA works with a contingency table that is equivalent to the analysis of two nominal variables (i.e., one for the rows and one for the columns). Multiple CA (MCA) is an extension of CA that analyzes the pattern of relationships among several nominal variables. MCA is used to analyze a set of observations described by a set of nominal variables. Each nominal variable comprises several levels, and each of these levels is coded as a binary variable. For example, gender (F vs. M) is a nominal variable with two levels. The pattern for a male respondent will be [0 1], and for a female respondent, [1 0]. The complete data table is composed of binary columns with one and only one column, per nominal variable, taking the value of 1.

MCA can also accommodate quantitative variables by recoding them as "bins." For example, a score with a range of -5 to $+5$ could be recoded as a nominal variable with three levels: less than 0, equal to 0, or more than 0. With this schema, a value of 3 will be expressed by the pattern 0 0 1. The coding schema of MCA implies that each row has the same total, which for CA implies that each row has the same *mass*.

Essentially, MCA is computed by using a CA program on the data table. It can be shown that the binary coding scheme used in MCA creates artificial factors and therefore artificially reduces the inertia explained. A solution for this problem is to correct the eigenvalues obtained from the CA program.

Hervé Abdi and Lynne J. Williams

See also Barycentric Discriminant Analysis; Canonical Correlation Analysis; Categorical Variable; Chi-Square Test; Coefficient Alpha; Data Mining; Descriptive Discriminant Analysis; Discriminant Analysis; Exploratory Data Analysis; Exploratory Factor Analysis; Guttman Scaling; Matrix Algebra; Principal Components Analysis; R

Further Readings

Benzécri, J. P. (1973). *L'analyse des données* [Data analysis] (2 vol.). Paris: Dunod.

Brunet, E. (1989). Faut-il ponderer les données linguistiques [Weighting linguistics data]. *CUMFID, 16,* 39–50.

Clausen, S. E. (1998). *Applied correspondence analysis.* Thousand Oaks, CA: Sage.

Escofier, B., & Pages, J. (1998). *Analyses factorielles simples et multiples* [Simple and multiple factor analysis]. Paris: Dunod.

Greenacre, M. J. (1984). *Theory and applications of correspondence analysis.* London: Academic Press.

Greenacre, M. J. (2007). *Correspondence analysis in practice* (2nd ed.). Boca Raton, FL: Chapman & Hall/CRC.

Greenacre, M. J., & Blasius, J. (Eds.). (2007). *Multiple correspondence analysis and related methods.* Boca Raton, FL: Chapman & Hall/CRC.

Hwang, H., Tomiuk, M. A., & Takane, Y. (2009). Correspondence analysis, multiple correspondence analysis and recent developments. In R. Millsap & A. Maydeu-Olivares (Eds.), *Handbook of quantitative methods in psychology* (pp. 243–263). London: Sage.

Lebart, L., & Fenelon, J. P. (1971). *Statistiques et informatique appliquees* [Applied statistics and computer science]. Paris: Dunod.

Weller, S. C., & Romney, A. K. (1990). *Metric scaling: Correspondence analysis.* Thousand Oaks, CA: Sage.

Correspondence Principle

The correspondence principle is generally known as the Bohr correspondence principle (CP), for Niels Bohr. It is considered one of Bohr's greatest contributions to physics, along with his derivation of the Balmer formula. Bohr's leading idea is that classical physics, though limited in scope, is indispensable for the understanding of quantum physics. The idea that old science is "indispensable" to the understanding of new science is in fact the main theme in using the concept of correspondence; therefore, the CP can be defined as the principle by which new theories of science (physics in particular) can relate to previously accepted theories in the field by means of approximation at a certain limit. Historically, Max Planck had introduced the concept in 1906. Bohr's first handling of the concept was in his first paper after World War I, in which he showed that quantum formalism would lead to classical physics when $n \to \infty$, where n is the quantum number. Although there were many previous uses of the concept, the important issue here is not to whom the concept can be attributed, but an understanding of the various ways that it can be used in scientific and philosophic research.

The principle is important for the continuity in science. There are two ways of thinking about such continuity. A theory T covers a set of observations S. A new observation s_1 is detected. T cannot explain s_1. Scientists first try to adapt T to be able to account for s_1. But if T is not in principle able to explain s_1, then scientists will start to look for another theory T^* that can explain S and s_1. The scientist will try to derive T^* by using CP as a determining factor. In such a case, T^* should lead to T at a certain limit.

Nonetheless, sometimes there may be a set of new observations, S_1, for which it turns out that a direct derivation of T^* from T that might in principle account for S_1 is not possible or at least does not seem to be possible. Then the scientist will try to suggest T^* separately from the accepted set of boundary conditions and the observed set of S and S_1. But because T was able to explain the set of observations S, then it is highly probable that T has a certain limit of correct assumptions that led to its ability to explain S. Therefore, any new theory T^* that would account for S and S_1 should resemble T at a certain limit. This can be obtained by specifying a certain correspondence limit at which the new formalism of T^* will lead to the old formalism of T.

These two ways of obtaining T^* are the general forms of applying the correspondence principle. Nevertheless, the practice of science presents us with many ways of connecting T^* to T or parts of it. Hence it is important to discuss the physicists' different treatments of the CP. Moreover, the interpretation of CP and the implications of using CP will determine our picture of science and the future development of science; hence, it is important to discuss the philosophical implications of CP and the different philosophical understandings of the concept.

Formal Correspondence

In the current state of the relation between modern physics and classical physics, there are four kinds of formal correspondence between modern and classical physics.

Old Correspondence Principle (Numerical Correspondence)

Planck stressed the relation between his "radical" assumption of discrete energy levels that are proportional to frequency, and the classical theory. He insisted that the terms in the new equation refer to the very same classical properties. He formulated the CP so that the numerical value of

$$\lim_{h \to 0} [Quantumphysics] = [Classicalphysics]$$

He demonstrated that the radiation law for the energy density at frequency v,

$$u(v) = \frac{8\pi h v^3}{c^3(e^{hv/kT} - 1)}, \qquad (1)$$

corresponds numerically in the limit $h \to 0$ to the classical Rayleigh–Jeans law:

$$u(v) = \frac{8\pi k T v^2}{c^3}, \qquad (2)$$

where k is Boltzmann's constant, T is the temperature, and c is the speed of light. This kind of correspondence entails that the new theory should resemble the old one not just at the mathematical level but also at the conceptual level.

Configuration Correspondence Principle (Law Correspondence)

The configuration correspondence principle claims that the laws of new theories should correspond to the laws of the old theory. In the case of quantum and classical physics, quantum laws correspond to the classical laws when the probability density of the quantum state coincides with the classical probability density. Take, for example, a harmonic oscillator that has a classical probability density

$$P_C(x) = 1/(\pi\sqrt{x_0^2 - x^2}), \qquad (3)$$

where x is the displacement. Now if we superimpose the plot of this probability onto that of the quantum probability density $|\psi_n|^2$ of the eigenstates of the system and take (the quantum number) $n \to \infty$, we will obtain Figure 1 below. As

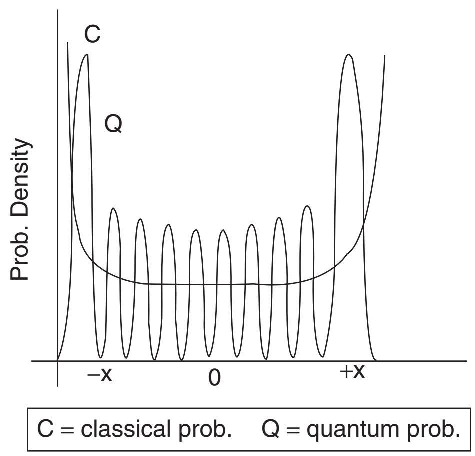

Figure 1 Classical Versus Quantum Probability Density

Richard Liboff, a leading expert in the field, has noted, the classical probability density P_C does not follow the quantum probability density $|\psi_n|^2$. Instead, it follows the local average in the limit of large quantum numbers *n:*

$$P_C(x) = \rangle P_Q(x)\langle = \rangle|\psi_n|^2\langle = \frac{1}{2\varepsilon}\int_{x-\varepsilon}^{x+\varepsilon} |\psi_n(y)|^2 dy. \quad (4)$$

Frequency Correspondence Principle

The third type of correspondence is the officially accepted form of correspondence that is known in quantum mechanics books as the Bohr Correspondence Principle. This claims that the classical results should emerge as a limiting case of the quantum results in the limits n (the quantum number) and $h \to 0$ (Planck's constant). Then in the case of frequency, the quantum value should be equal to the classical value, i.e., $\upsilon_Q = \upsilon_C$. In most cases in quantum mechanics, the quantum frequency coalesces with the classical frequency in the limit $n \to \infty$ and $h \to 0$.

Nevertheless, $n \to \infty$ and $h \to 0$. are not universally equivalent, because in some cases of the quantum systems, the limit $n \to \infty$ does not yield the classical one, while the limit $h \to 0$ does; the two results are not universally equivalent. The case of a particle trapped in a cubical box would be a good example: the frequency in the high quantum number domain turns out to be displaced as

$$\upsilon_q^{n+1} = \upsilon_q^n + h/2md,$$

where m is the particle's mass and d is the length of the box. Such a spectrum does not collapse toward the classical frequency in the limit of large quantum numbers, while the spectrum of the particle does degenerate to the classical continuum in the limit $h \to 0$ It can be argued that such correspondence would face another obvious problem relating to the assumption that Planck's constant goes to zero. What is the meaning of saying that "a constant goes to zero"? A constant is a number that has the same value at all times, and having it as zero is contradictory, unless it is zero. A reply to this problem might be that in correspondence, we ought to take the real limiting value and not the abstract one. In the case of relativity, the limit, "*c* goes to infinity" is an abstract one, and the real limit should be "*v*/*c* goes to zero." Now, when dealing with corresponding quantum mechanics to classical mechanics, one might say that we ought to take the limit $n \to \infty$ as a better one than $h \to 0$ The point here is that values like *c* and *h* are constants and would not tend to go to zero or to infinity, but *n* and v/*c* are variables—*n* = (0, 1, 2, 3, . . .) and v/*c* varies between 0 (when v = 0) and 1 (when vc). (Of course, this point can also count against the old CP of Planck, the first correspondence principle in our list, because it is built on the assumption that the limit is of Planck's constant going to zero.)

Form Correspondence Principle

The last type of correspondence is form CP, which claims that we can obtain correspondence if the functional (mathematical) form of the new theory is the same as that of the old theory. This kind of correspondence is especially fruitful in particular cases in which other kinds of correspondence do not apply. Let us take the example used in frequency correspondence (quantum frequency). As seen in the case of the particle in a cubical box, the outcome of $n \to \infty$ does not coincide with the outcome of $h \to 0$. Hence the two limits fail to achieve the same result. In cases such as this, form correspondence might overcome the difficulties facing frequency correspondence. The aim of form correspondence is to prove that classical frequency and quantum frequency have the same form. So, if υ_Q denotes quantum frequency, υ_C classical frequency, and E energy, then form correspondence is satisfied if $\upsilon_C(E)$ has the same functional form as $\upsilon_Q(E)$. Then, by using a dipole approximation, Liboff showed that the quantum transition between state $s+n$ and state s where $s \gg n$ gives the relation

$$\upsilon_Q^n(E) \approx n(E_s/2ma^2)^{1/2}. \qquad (5)$$

He also noticed that if we treat the same system classically (particles of energy E in a cubical box), the calculation of the radiated power in the *n*th vibrational mode is given by the expression

$$\upsilon_C^n(E) \approx n(E/2ma^2)^{1/2}. \qquad (6)$$

Both frequencies have the same form, even if one is characterizing quantum frequency and the other classical, and even if their experimental treatment differs. Hence, form CP is satisfied.

But such correspondence is not problem free; in the classical case, E denotes the average energy value of an ensemble of *n*th harmonic frequency, but in the quantum case, it denotes the eigenenergy of that level. Also, in the quantum case, the energy is discrete, and the only way to assert that the quantum frequency yields the classical one is by saying that when the quantum number is very big, the number of points that coincide with the classical frequency will increase, using the dipole approximation, which asserts that the distance between the points in the quantum case is assumed small. Hence the quantum case does not resemble the classical case as such, but it coincides with the average of an ensemble of classical cases.

The main thrust of form correspondence is that it can relate a branch of physics to a different branch on the basis of form resemblance, such as in the case of superconductivity. Here, a quantum formula corresponds to classical equations if we can change the quantum formula in the limit into a form where it looks similar to a classical form. The case of Josephson junctions in superconductivity, which are an important factor in building superconducting quantum interference devices, presents a perfect demonstration of such concept.

Brian David Josephson proved that the relation between the phase difference and the voltage is given by $\frac{¶\delta}{¶t} = \frac{2e}{\hbar}V$, that is, the voltage $V = \frac{\hbar}{2e}\frac{¶\delta}{¶t}$. Now, by the assertion that the Josephson junction would behave as a classical circuit, the total current would be

$$I = I_c \sin\delta + \frac{\eta}{2\mathrm{Re}}\frac{d\delta}{dt} + \frac{\eta C}{2e}\frac{d^2\delta}{dt^2}. \qquad (7)$$

This equation relates the current with the phase difference but without any direct reference to the voltage. Furthermore, if we apply form correspondence, Equation 7 is analogous to the equation of a pendulum in classical mechanics. The total torque τ on the pendulum would be

$$\tau = M\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + \tau_0 \sin\theta, \qquad (8)$$

where M is the moment of inertia, D is the viscous damping, and τ is the applied torque.

Both these equations have the general mathematical form

$$Y = Y_0 \sin x + B\frac{dx}{dt} + A\frac{d^2x}{dt^2}. \qquad (9)$$

This kind of correspondence can be widely used to help in the solution of many problems in physics. Therefore, to find new horizons in physics, some might even think of relating some of the new theories that have not yet applied CP. Such is the case with form corresponding quantum chaos to classical chaos. The argument runs as follows: Classical chaos exists. If quantum mechanics is to be counted as a complete theory in describing nature, then it ought to have a notion that corresponds to classical chaos. That notion can be called quantum chaos. But what are the possible things that resemble chaotic behavior in quantum systems? The reply gave rise to quantum chaos. However, it turns out that a direct correspondence between the notion of chaos in quantum mechanics and that in classical mechanics does not exist.

Therefore, form correspondence would be fruitful here. Instead of corresponding quantum chaos to classical chaos, we can correspond both of them to a third entity. Classical chaos goes in a certain limit to the formφ, and quantum chaos goes to the same form at the same limit:

$$\lim_{n \to \infty} classical\,choas = \varphi$$

$$\lim_{n \to \infty} quantum\,choas = \varphi,$$

but because we have only classical and quantum theories, then the correspondence is from one to the other, as suggested by Gordon Belot and John Earman.

In addition to these four formal forms of correspondence, many other notions of correspondence might apply, such as *conceptual correspondence,* whereby new concepts ought to resemble old concepts at the limited range of applicability of such concepts. In addition, there is *observational correspondence,* which is a weak case of correspondence whereby the quantum will correspond to what is expected to be observed classically at a certain limit. *Structural correspondence* combines elements from both form correspondence and law correspondence. Hence, scientific practice might need different kinds of correspondence to achieve new relations and to relate certain domains of applicability to other domains.

Philosophical Implications

Usually, principles in science, such as the Archimedes principle, are universally accepted. This is not the case with CP. Although CP is considered by most physicists to be a good heuristic device, it is not accepted across the board. There are two positions: The first thinks of development in science as a mere trial-and-error process; the second thinks that science is progressive and mirrors reality, and therefore new theories cannot cast away old, successful theories but merely limit the old ones to certain boundaries.

Max Born, for example, believed that scientists depend mainly on trial and error in a shattered jungle, where they do not have any signposts in science, and it is all up to them to discover new roads in science. He advised scientists to rely not on "abstract reason" but on experience. However, to accept CP means that we accept abstract reason; it also means that we do not depend on trial and error but reason from whatever accepted knowledge we have to arrive at new knowledge.

The philosophical front is much more complex. There are as many positions regarding correspondence as there are philosophers writing on the subject. But in general, realists are the defenders of the concept, whereas positivists, instrumentalists, empiricists, and antirealists are, if not opposed to the principle, then indifferent about it. Some might accept it as a useful heuristic device, but that does not give it any authoritarian power in science.

Even among realists there is more than one position. Some such as Elie Zahar claim that the CP influence ought to stem from old theory and arrive at the new through derivative power. Heinz Post is more flexible; he accepts both ways as legitimate and suggests a *generalized correspondence principle* that ought to be applied to all the developments in science. In doing so, he is replying to Thomas Kuhn's *Structure of Scientific Revolutions,* rejecting Kuhn's claim of paradigm shift and insisting on scientific continuity. In doing so, Post is also rejecting Paul Feyerabend's concept of incommensurability.

So is CP really needed? Does correspondence relate new theories to old ones, or are the new theories deduced from old theories using CP? Can old theories really be preserved? Or what, if anything, can be preserved from the old theories? What about incommensurability between the new and the old? How can we look at correspondence in light of Kuhn's concept of scientific revolution? What happens when there is a paradigm shift? All these questions are in fact related to our interpretation of CP.

CP, realists say, would help us in understanding developments in science as miracle free (the no-miracle argument). Nevertheless, by accepting CP as a principle that new theories should uphold, we in effect are trapped within the scope of old theories. This means that if our original line of reasoning was wrong and still explains the set of observations that we obtained, then the latter theory that obeys CP will resolve the problems of the old theory within a certain limit, will no doubt continue to hold the posits of the wrong theory, and will continue to abide by its accepted boundary conditions. This means that we will not be able to see where old science went wrong. In reply, conventional realists and structural realists would argue that the well-confirmed old theories are good representations of nature, and hence any new theories should resemble them at certain limits. Well, this is the heart of the matter. Even if old theories are confirmed by experimental evidence, this is not enough to claim that the abstract theory is correct. Why? Mathematically speaking, if we have any finite set of observations, then there are many possible mathematical models that can describe this set. Hence, how can we determine that the model that was picked by the old science was the right one?

But even if we accept CP as a heuristic device, there are many ways that the concept can be applied. Each of these ways has a different set of problems for realists, and it is not possible to accept any generalized form of correspondence.

The realist position was challenged by many philosophers. Kuhn proved that during scientific revolutions the new science adopts a new paradigm in which the wordings of the old science might continue, but with different meanings. He demonstrated such a change with mass: The concept of mass in relativity is not the same as Newtonian mass. Feyerabend asserted that the changes between new science and old science make them incommensurable with each other. Hence, the realist notion of approximating new theories to old ones is going beyond the accepted limits of approximation.

The other major recent attacks on realism come from pessimistic metainduction (Larry Laudan) on one hand and new versions of empiricist arguments (Bas van Fraassen) on the other. Van Fraassen defines his position as *constructive empiricism*. Laudan relies on the history of science to claim that the realists' explanation of the successes of science does not hold. He argues that the success of theories cannot offer grounds for accepting that these theories are true (or even approximately true). He presents a list of theories that have been successful and yet are now acknowledged to be false. Hence, he concludes, depending on our previous experience with scientific revolutions, the only reasonable induction would be that it is highly probable that our current successful theories will turn out to be false. Van Fraassen claims that despite the success of theories in accounting for phenomena (their empirical adequacy), there can never be any grounds for believing any claims beyond those about what is observable. That is, we cannot say that such theories are real or that they represent nature; we can only claim that they can account for the observed phenomena.

Recent trends in realism tried to salvage realism from these attacks, but most of these trends depend on claiming that we do not need to save the old theory as a whole; we can save only the representative part. Structural realists, such as John Worrell and Elie Zahar, claim that only the mathematical structure need be saved and that CP is capable of assisting us in saving it. Philip Kitcher asserts that only presupposition posits can survive. Towfic Shomar claims that the dichotomy should be horizontal rather than vertical and that the only parts that would survive are the phenomenological models (phenomenological realism). Stathis Psillos claims that scientific theories can be divided into two parts, one consisting of the claims that contributed to successes in science (working postulates) and the other consisting of idle components.

Hans Radder, following Roy Bhaskar, thinks that progress in science is like a production line: There are inputs and outputs; hence our old knowledge of theories and observations is the input that dictates the output (our new theories). CP is important in the process; it is a good heuristic device, but it is not essential, and in many cases it does not work.

But is CP a necessary claim for all kinds of realism to account for developments in science? Some, including Shomar, do not think so. Nancy Cartwright accepts that theories are mere tools; she thinks that scientific theories are patchwork that helps in constructing models that represent different parts of nature. Some of these models depend on tools borrowed from quantum mechanics and account for phenomena related to the microscopic world; others use tools from classical mechanics and account for phenomena in the macroscopic world. There is no need to account for any connection between these models. Phenomenological realism, too, takes theories as merely tools to construct phenomenological models that are capable of representing nature. In that case, whether the fundamental theories correspond to each other to some extent or not is irrelevant. The correspondence of theories concerns realists who think that fundamental theories represent nature and approximate its blueprint.

Currently, theoretical physics is facing a deadlock; as Lee Smolin and Peter Woit have argued, the majority of theoretical physicists are running after the unification of all forces and laws of physics. They are after the theory of everything. They are convinced that science is converging toward a final theory that represents the truth about nature. They are in a way in agreement with the realists, who hold that successive theories of "mature science" approximate the truth more and more, so science should be in quest of the final theory of the final truth.

Theoretical representation might represent the truth about nature, but we can easily imagine that we have more than one theory to depend on. Nature is complex, and in light of the richness of nature, which is reflected in scientific practice, one may be unable to accept that Albert Einstein's request for simplicity and beauty can give the correct picture of current science when complexity and diversity appear to overshadow it. The complexity of iphysics forces some toward a total disagreement with Einstein's dream of finding a unified theory for everything. To some, such a dream directly contradicts the accepted theoretical representations of physics. Diversity and complexity are the main characteristics of such representations.

Nonetheless, CP is an important heuristic device that can help scientists arrive at new knowledge, but scientists and philosophers should be careful as to how much of CP they want to accept. As long as they understand and accept that there is more than one version of CP and as long as they accept that not all new theories can, even in principle, revert to old theories at a certain point, then they might benefit from applying CP. One other remark of caution: Scientists and philosophers also need to accept that old theories might be wrong; the wrong mathematical form may have been picked, and if they continue to accept such a form, they will continue to uphold a false science.

Towfic Shomar

See also Frequency Distribution; Models; Paradigm; Positivism; Theory

Further Readings

Fadner, W. L. (1985). Theoretical support for the generalized correspondence principle, *American Journal of Physics, 53,* 829–838.

French, S., & Kamminga, H. (Eds.). (1993). *Correspondence, invariance and heuristics: Essays in honour of Heinz Post.* Dordrecht, the Netherlands: Kluwer Academic.

Hartmann, S. (2002). On correspondence. *Studies in History & Philosophy of Modern Physics, 33B,* 79–94.

Krajewski, W. (1977). *Correspondence principle and growth in science.* Dordrecht, the Netherlands: Reidel.

Liboff, R. (1975). Bohr's correspondence principle for large quantum numbers. *Foundations of Physics, 5*(2), 271–293.

Liboff, R. (1984). The correspondence principle revisited. *Physics Today, February,* 50–55.

Makowski, A. (2006). A brief survey of various formulations of the correspondence principle. *European Journal of Physics, 27*(5), 1133–1139.

Radder, H. (1991). Heuristics and the generalized correspondence principle. *British Journal for the Philosophy of Science, 42,* 195–226.

Shomar, T. (2001). Structural realism and the correspondence principle. *Proceedings of the conference on Mulla Sadra and the world's contemporary philosophy,* Kish, Iran: Mulla Sudra Institute?.

Zahar, E. (1988). *Einstein's revolution: A study in heuristics.* LaSalle, IL: Open Court.

Covariate

Similar to an independent variable, a covariate is complementary to the dependent, or response, variable. A variable is a covariate if it is related to the dependent variable. According to this definition, any variable that is measurable and considered to have a statistical relationship with the dependent variable would qualify as a potential covariate. A covariate is thus a possible predictive or explanatory variable of the dependent variable. This may be the reason that in regression analyses, independent variables (i.e., the *regressors*) are sometimes called covariates. Used in this context, covariates are of primary interest. In most other circumstances, however, covariates are of no primary interest compared with the independent variables. They arise because the experimental or observational units are heterogeneous. When this occurs, their existence is mostly a nuisance because they may interact with the independent variables to obscure the true relationship between the dependent and the independent variables. It is in this circumstance that one needs to be aware of and make efforts to control the effect of covariates. Viewed in this context, covariates may be called by other names, such as concomitant variables, auxiliary variables, or secondary variables. This