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Research article

Puzzles for Recursive Reliabilism

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Abstract:
The recursive aspect of process reliabilism has rarely been examined. The regress puzzle, which illustrates infinite regress arising from the combination of the recursive structure and the no-defeater condition incorporated into it, is a valuable exception. However, this puzzle can be dealt with in the framework of process reliabilism by reconsidering the relationship between the recursion and the no-defeater condition based on the distinction between prima facie and ultima facie justification. Thus, the regress puzzle is not a basis for abandoning process reliabilism. A genuinely intractable problem for recursive reliabilism lies in the gap between the reliability of the entire path to a belief and that of its parts. Confronted with this puzzle, reliabilists can orient themselves toward 'reliable-as-a-whole reliabilism' instead of 'reliable-in-every-part reliabilism', including recursive reliabilism, which is found to be not well-motivated.

Keywords:
Process reliabilism, Defeaters, Infinite regress, Recursive definition

1. Introduction

The original version of process reliabilism proposed by Alvin Goldman (Goldman 1979) has been examined from various perspectives. However, most studies have focused on the externalistic, process-centric, or (at least seemingly) value-indifferent aspects of his theory. In contrast, little attention has been paid to the fact that the way his reliabilism accounts for epistemic justification is recursive. Nevertheless, the recursive structure has been handed down to subsequent reliabilists implicitly and explicitly without adequate examination. Fumerton's paper, which presents a puzzle for Goldman's recursive reliabilism concerning his

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no-defeater condition, is a valuable exception (Fumerton 1988). According to him, infinite regress arises when the recursive explanation is combined with his no-defeater condition. This point has sometimes been taken as providing one motivation for taking a different theory from Goldman's reliabilism (Bergmann 2006: 172–175; Beddor 2021).

The purpose of this paper is twofold. The first is to show that the regress puzzle presented by Fumerton is solvable within the framework of Goldmanian process reliabilism, and to argue that the existence of the regress puzzle does not provide a reason to abandon process reliabilism with the recursive structure. The second is to present another puzzle that upsets recursive reliabilism and to show the options that reliabilists can take in response to it. Sections 2 and 3 correspond to these two puzzles, respectively. To begin with, 2.1 confirms that Goldman's reliabilism provides a recursive explanation of justification and includes a no-defeater condition. Then, in 2.2, I look at the regress puzzle pointed out by Fumerton. Here, the infinite regress with which Goldman's account ends is illuminated by completing the details that he omits. In 2.3, I confirm that existing attempts to solve the regress puzzle from an externalist standpoint are founded on theories with different implications from process reliabilism, which appeal to concepts not found in process reliabilism. In 2.4 and 2.5, the regress puzzle is solved using only process reliabilist concepts. Section 3 is devoted to the second puzzle. In 3.1 the genuine puzzle is depicted through examples in which process reliabilism with a recursive structure leads to intuitively questionable conclusions. In 3.2, I argue that the problem spills over into the value of knowledge issue. In 3.3, the features of the second puzzle are revealed through comparisons with the preface paradox and discursive dilemma. In 3.4, two possible variants of reliabilism are illustrated, namely 'reliable-in-every-part reliabilism' and 'reliable-as-a-whole reliabilism', depending on how one responds to the puzzle. Finally, in 3.5, I show that Goldman's adoption of recursive reliabilism, a type of the former variant, is not well-motivated.

2. Regress puzzle

2.1 Goldman's process reliabilism

The basic idea of process reliabilism is that a belief is justified when it arises from a reliable belief-forming process, i.e., one which tends to produce true beliefs. Goldman explains justification recursively on the basis of the following idea (Goldman 1979: 13–14).[1]

1 To avoid complications, I omit details about the time at which S holds a belief henceforth.

BC:

S's belief in *p* is justified if S's believing *p* results from a belief-independent process that is unconditionally reliable.

RC:

S's belief in *p* is justified if S's believing *p* results from a belief-dependent process that is conditionally reliable, and if the beliefs on which this process operates in producing S's belief in *p* are themselves justified.

Here is introduced the distinction between belief-independent and belief-dependent processes. Belief-independent processes do not involve any beliefs as input, such as vision and reflection. In contrast, belief-dependent processes, including reasoning and memory, require belief input. The justification of beliefs arising from these two types of processes is subject to different reliabilities: unconditional and conditional. Unconditional reliability simply means the tendency to produce true beliefs at a high rate. On the other hand, conditional reliability amounts to the high probability that a belief arising from the process is true when all the input beliefs are true.

Goldman intends these two conditions to be respectively sufficient and necessary when combined. However, when there are defeaters, there are cases in which justification does not seem to hold, even though these conditions are met.[2] For example, consider a situation in which I see a sculpture in a museum and believe that it is red based on my vision. Let us assume that my perceptual abilities are reliable and that there is nothing unusual about the

2 There are two types of defeaters: propositional defeaters, which cause a loss of knowledge simply due to them being true, and mental state defeaters, which cause a loss of justification for a belief due to them being held or being possible to hold by the subject (Bergmann 2006: 154–155). This paper is concerned with justification, so I will focus exclusively on the latter.

Mental state defeaters can be further distinguished into psychological defeaters, which a subject actually has as doxastic attitudes, and normative defeaters, which a subject should have in some sense (Lackey 2008: 44–55; Grundmann 2011: 158–159). Goldman's no-defeater condition, introduced at the end of this subsection, can be understood as an attempt to explain the latter by process reliability. In addition, there is a risk of inconsistency in adding a no-psychological-defeater condition to a simple reliability theory (BonJour and Sosa 2003: 32). For these reasons, I will follow Goldman and deal only with normative defeaters.

It should be noted here, however, that, precisely speaking, the defeaters that Goldman and I deal with are 'normative' in a different sense from Lackey's 'normative defeaters'. The normativity of our defeaters derives solely from the tendency of beliefs to be true and is not related to Lackey's 'ought' (Lackey 2008: 45) or, in other words, duty. Therefore, Graham and Lyons' criticism of Lackey's normative defeaters with regard to duty does not apply to the ones we are dealing with (Graham and Lyons 2021: 52–57).

lighting or other circumstances. Shortly afterwards, a trusted friend who accompanied me tells me that the area around the sculpture was illuminated by a red light, and I believe him.[3] In this case, according to the explanation above, the original belief that the sculpture is red is judged justified because BC is satisfied, although it seems no longer to be justified.

To deal with such cases, theories of justification often incorporate a no-defeater condition. Goldman is no exception. He suggests a version of BC that incorporates this type of condition as follows (Goldman 1979: 20).

> **BC(+ND):**
> S's belief in *p* is justified if S's belief in *p* results from a reliable process, and there is no reliable or conditionally reliable process available to S which, had it been used by S in addition to the process actually used, would have resulted in S's not believing *p*.

According to BC(+ND), even if a belief arises from a reliable process, it is not justified if there is a reliable alternative process available that would have prevented the belief from being held. In the previous example, BC(+ND) seems to explain nicely that the original belief that the sculpture is red is no longer justified because of the existence of the alternative process of believing the testimony of a credible friend. However, in presenting BC(+ND), Goldman 'omits certain details in the interest of clarity'. When these details are made explicit, the problem of infinite regress becomes apparent.

2.2 Regress puzzle

Epistemologists who present a theory of justification, including Goldman, are attempting to explain epistemic justification reductively, i.e., without appealing to epistemic concepts. However, Fumerton has pointed out that the expression 'conditionally reliable', which appears in BC(+ND), implicitly refers to the epistemic concept of 'justified belief'. This point becomes apparent when BC(+ND) is rewritten in a more complete form as follows (Fumerton 1988: 183).

> **BC(+ND)$_{comp}$:**
> S's belief in *p* is justified if the belief results from a reliable process, and there is no belief-independent unconditionally reliable process available to S which, had it been used by S in addition to the process actually used, would have resulted in S's not believing *p*, and there is no belief-dependent process which is conditionally reliable

3 I owe this example to Pollock and Cruz (1999: 44) and Lasonen-Aarnio (2010: 1).

that could have been used by S to process certain *justified* beliefs so as to result in him not believing *p*.

According to Fumerton, the contents of $BC(+ND)_{comp}$ and BC(+ND) are equivalent. However, the epistemic concept of 'justified' clearly appears in $BC(+ND)_{comp}$. Therefore, he diagnoses Goldman's theory as falling into infinite regress and failing as a project of reductive explanation.

2.3 Responses by externalist theories other than process reliabilism

One way to avoid the regress puzzle is to explain defeat cases in a manner other than by adding a Goldman-style no-defeater condition. Bergmann's 'proper function theory', for example, imposes as a condition for justification that not only are the cognitive faculties that produced the belief reliable but also that they are functioning properly (Bergmann 2006: 153). He then explains that the reason why an agent's original belief in a case of defeat is not justified is that his cognitive functions are not functioning properly (Bergmann 2006: 174).[4] In this case, no regress occurs because the concept of justification is not used to explain the existence of defeaters. However, Bergmann's proper function theory is less thrifty than process reliabilism. Even if it has advantages in other respects, it is at least overly costly as a response to the regress puzzle as long as process reliabilism can deal with it.

A theory more akin to Goldman's reliabilism that can solve the regress puzzle is Beddor's 'reasons reliabilism' (Beddor 2021). The distinctive feature of his approach is that it accommodates defeaters by appealing to the concept of 'reason' to model the structure of justification and defeat.[5] According to Beddor's understanding, reasons are defined as inputs to reliable or conditionally reliable belief formation processes (Beddor 2021: 161). This conception of reason is more friendly to reliabilism than standard ones. However, strictly speaking, Beddor's theory has different implications from Goldman's. In formulating his view, Beddor asymmetrically identifies reasons with realized non-doxastic states or possible doxastic states (Beddor 2021: 161–162). Moreover, he makes the existence of reasons a necessary condition for defeating to hold (Beddor 2021: 163–164). These points make his theory different from process reliabilism. For example, suppose that I am convinced that I have passed an

4 This is an account of normative defeaters, the concern of this paper. He gives a different account of psychological defeaters (Bergmann 2006: 163–168).

5 With reasons reliabilism, Beddor also addresses the issue of alternative processes that should not be used (Beddor 2021: 166–167). There is insufficient space to discuss this in detail, but briefly, as Beddor states, the key to this problem lies in the accommodation of defeated-defeaters, and thus, it can also be addressed by my proposal as shown in 2.5.

exam, reasoning from my justified belief that I have studied hard enough. However, the fear of failing keeps me from opening my eyes in front of the screen displaying my result and I do not check it in the end. In fact, I have failed. In this case, the non-doxastic state of a possible field of view that includes the screen has not been achieved, but my belief in passing seems to be defeated. At least, there would be little relevant difference between this case and the case where I have checked the result but do not believe it. Indeed, it is still possible for a reasons reliabilist to argue that, intuitively speaking, defeating does not hold in this case, and thus that the original belief remains justified. This conflict of intuitions cannot be reconciled easily. However, at least, it can be said that reasons reliabilism is not merely a refined version of process reliabilism, but a theory with different implications. In the next subsection, I will show that it is possible to solve the regress puzzle within the framework of process reliabilism without appealing to the notion of reasons.

2.4 Prototype amendment: recursive explanation and the no-defeater condition

A process reliabilistic and direct alternative which can deal with the regress puzzle could be the following.

> BC_{pf}:
> S's belief in *p* is *prima facie* justified if S's believing *p* results from a belief-independent process that is unconditionally reliable.

> RC_{pf}:
> S's belief in *p* is *prima facie* justified if S's believing *p* results from a belief-dependent process that is conditionally reliable, and if the beliefs on which this process operates in producing S's belief in *p* are themselves prima facie justified.[6]

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6 It might seem that the prima facie justification of beliefs arising from a belief-dependent process requires that the input beliefs not only be prima facie justified, but also be ultima facie justified, and thus infinite regress cannot be avoided. In other words, the question may be raised, for example, whether S's belief in *a* is prima facie justified even in the situation where S believes *b* by a belief-independent process, and a belief in *a* arises from a belief-dependent process that has the belief in *b* as input, but there is an alternative process that prevents S from believing *b*. In proposing the amendment, I see no problem with the belief in question being judged to be prima facie justified as long as it is not ultima facie justified. This is because prima facie justification can be overturned by defeaters, and thus can be held without factoring in the existence of defeaters in advance. In this view, it is necessary for J to deny ultima facie justification of S's belief in *a*. To do so, we must be able to say, as a premise, that alternative processes that preclude the ultimate justification of the belief in *a* include alternative processes that prevent S from holding the belief in *b*. In other words,

J:
S's belief in *p* is *ultima facie* justified if and only if
S's belief in *p* is prima facie justified and
there is no belief-independent unconditionally reliable process available to S which, had it been used by S in addition to the process actually used, would have resulted in S's not believing *p* and
there is no belief-dependent process which is conditionally reliable that could have been used by S to process certain *prima facie* justified beliefs so as to result in him not believing *p*.

Here, I introduce a distinction between ultima facie and prima facie justified beliefs. The former are what I have simply called 'justified beliefs' thus far, which Goldman and other epistemologists attempt to explain. The latter merely refer to beliefs that are produced from reliable processes and do not imply the former.

Based on this distinction, the alternative first explains prima facie justification recursively, and then ultima facie justification by the no-defeater condition, which is explained using the concept of prima facie justification. In other words, Goldman's theory adds the no-defeater condition to the base clause, whereas the alternative adds it to the entire base and recursive clause. Therefore, in this amendment, the undefined notion of ultima facie justification does not appear in the part of J corresponding to the no-defeater condition, but instead the notion of prima facie justification appears, which has already been explained recursively by BC and RC. Therefore, infinite regress is avoided.

2.5 Final amendment: coping with defeated-defeaters

However, there is an apparent difficulty with the amendment above: the problem of defeated-defeaters (cf. Lyons 2009: 124). As a precondition, defeaters themselves also have justificational status of their own. Defeated-defeaters are defeaters that are not ultima facie justified because of the existence of other defeaters, although they may possibly be prima facie justified. For instance, in the case of the red sculpture, another trusted friend tells me that the friend who informed me of the red illumination was wearing red sunglasses when looking up at the light. At this point, my defeating belief that a red light illuminated the area is not ultima

S would not have believed *a* if he had been prevented from believing *b* by an alternative process. If there is any doubt about this premise, it arises not so much from the recursive structure as from the counter-factual conception of defeaters. See note 10 for a brief perspective on how to deal with the problem of counter-factual conception.

facie justified. Now that I have reliable testimony that my friend was looking at the light through red sunglasses, it seems that there is still a chance that my original belief that the sculpture was red will be ultima facie justified. This is because, standardly speaking, defeated-defeaters are not considered to prevent justification of the target beliefs (Lackey 2008: 46).[7] Rewording in reliabilist terminology, I might say that even if there is a reliable and available process (P_1) that causes a subject to cease to hold a target belief, i.e., even if there is a prima facie justified defeater, it is not ultima facie justified if there is another reliable process (P_2) that prevents her from holding the belief that could result from P_1, and it seems that the possibility of the original belief being ultima facie justified has not been lost. According to the amendment above, however, the mere existence of a prima facie justified defeater, even if it is not ultima facie justified, would mean that the original belief could not be ultima facie justified.

Naturally, there can be not only defeaters for defeaters for the original belief, but also defeaters for defeaters for defeaters, and this relationship may continue further. Then, I take p to be the original belief whose justification is in question and define the defeating belief for d_n as d_{n+1}; for example, the defeating belief for p is defined as d_1, that for d_1 is d_2, that for d_2 is d_3, and so on. In this regard, when an available process can prevent a subject from having another belief x by giving rise to a belief y, that belief is called a 'defeating belief y for x'.

For defeat to hold, the defeater itself must be ultima facie justified. Intuitively, d_1 is ultima facie justified iff d_2 is absent, or d_2 is present but d_3 is also present and d_4 is absent, or…. In other words, d_1 is ultima facie justified iff one of the flows shown by the dashed lines on the following flowchart (Fig. 1) is realized. For the present, the term 'reliable complex process' in Fig. 1 refers to a series of processes consisting of one or more belief-independent processes and zero or more belief-dependent processes that produce justified beliefs, e.g., a memory process that outputs a belief given input from a visual process.[8]

The following is a more formal expression of the intuitive explanation above.

Ultima Facie Justified Defeating Belief:
d_1 is ultima facie justified if and only if
there exists a natural number k such that there is no reliable available complex process which, if used by S, would have produced d_{2k}, and for all natural numbers l satisfying

7 To be precise, if a defeated-defeater does not cause a belief to lose its justification, then it falls outside the characterization of defeaters as 'causing a loss of epistemic positive statuses'. Although it is at most an apparent defeater, I call it a 'defeater' for convenience.

8 As will be discussed in 3.5, there can be another conception of the reliability of a complex process. In the terminology I introduce in 3.5, it can be expressed that what I do here is to formulate reliability in every part.

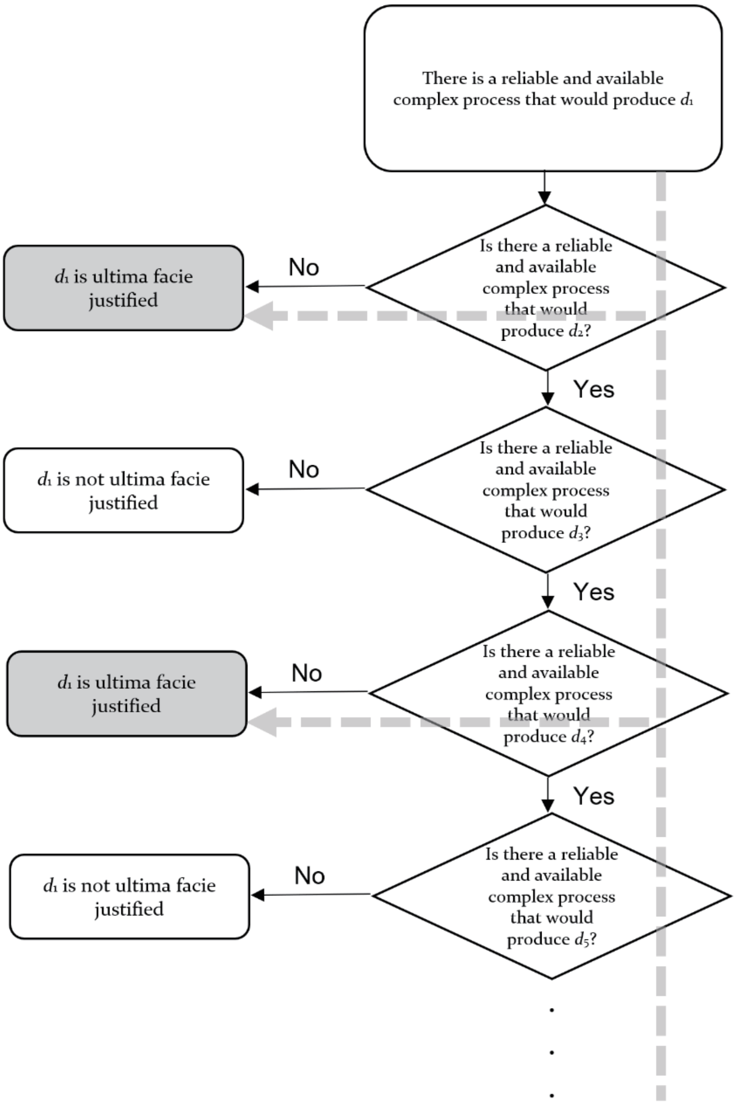

Fig. 1 Ultima facie justification of d_1.

$1 < l < 2k$, there is a reliable available complex process which, if used by S, would have produced d_l.

For example, in the case of the red sculpture, the process of believing my friend's testimony allows me to obtain a defeating belief (d_1) that 'the area around the sculpture was illuminated by a red light',[9] but the process of believing another friend's testimony allows me to obtain a defeating belief (d_2) that 'my friend was looking up at the light while wearing red sunglasses'. At this point, if there is no process that produces a defeating belief (d_3) for d_2, such as a belief that my friend who gave the second testimony was absent when the first testifier was around the sculpture, d_1 is not ultima facie justified, and thus p is ultima facie justified; if there is, then the existence of a process that produces d_4 is to be focused on.

At first sight, one might fear that there is a different kind of infinite regress here since this verification procedure could continue indefinitely. In other words, an infinite number of matters need to be considered to ascertain whether or not the natural number k exists. However, unlike the regress pointed out by Fumerton, this one is finite. This is because the number of matters that should be taken into account depends on the number of reliable complex processes available to the subject at a given time. First, the number of belief-independent and belief-dependent processes available to the subject at a given time is thought to be finite. Of course, by combining these finite processes, it would be possible to create an infinite number of complex processes. Even if there were only two processes available, a belief-independent process A, which outputs a single belief, and a belief-dependent process B, which takes a single belief as input and outputs a single belief, there would be an infinite number of possible series such as A, A-B, A-B-B, A-B-B-B, and so on. However, the subject at a particular point in time cannot use a complex process that continues indefinitely nor such an infinite number of complex processes. Therefore, the number of complex processes that a subject can actually use is still finite. In conclusion, it is sufficient to consider a finite number of matters to ascertain the existence of k, and this definition does not fall into an infinite regress.

To be precise, however, in addition to the finiteness of the number of complex processes available to the subject, I also assume some less counter-intuitive assumptions: that the number of defeating beliefs for a belief is finite and that the number of beliefs that can arise from a belief-forming process is also finite. These assumptions, which I follow, are implicit in Goldman's BC(+ND). I also understand the concept of 'available' in the same way as Goldman. That is, 'available processes' do not include the gathering of new evidence nor the use of methods that will become available in the future as a result of scientific progress but are limited to what the subject can do with the resources she already has at the time (Goldman 1979: 20). It seems plausible to accept these assumptions and the conception because, without such

9 It is also possible that there are processes that give rise to other defeating beliefs, for example, when I look in the mirror and realise that 'I was looking at the sculpture wearing red sunglasses'. Furthermore, the same is true for defeating beliefs at each stage. I omit the details about these cases, but as we shall see below, as long as some assumptions are allowed for, this is not a problem for the argument.

limitations, the hurdle imposed for justification would be so high as to be almost unattainable in everyday situations.

Using the ultima facie justification of defeating belief defined in this way, J can be reformulated as follows.

J′:
S's belief in *p* is ultima facie justified if and only if
S's belief in *p* is prima facie justified and
there is no reliable complex process available to S which, had it been used by S in addition to the process actually used, would have resulted in S's not having defeating belief d_1 for *p*, or
there is a reliable complex process available to S which, had it been used by S in addition to the process actually used, would have resulted in S's not having defeating belief d_1 for *p* and d_1 is not ultima facie justified.

Here appear only the predefined 'prima facie justification' and 'ultima facie justification of a defeating belief', instead of undefined epistemic concepts. Thus, the version of the reliabilist theory that consists of BC_{pf}, RC_{pf}, and J' does not suffer from the regress puzzle and can adequately explain defeated-defeaters at the same time.[10]

3. Another puzzle

3.1 Puzzle cases

As I have shown, process reliabilism can solve the regress puzzle. However, Goldman's and my version of process reliabilism above, which shares a recursive structure, faces another puzzle. This puzzle arises independently of the no-defeater condition. Therefore, in the following, the cases without a defeater will be considered, and Goldman's simple version of

10 In addition to the regress puzzle and the derivative problem of defeated-defeaters, there might be other difficulties with appealing to the counter-factual absence of beliefs (Beddor 2015: 150–156). As the interest of this paper is in the recursive structure, I cannot respond to this point fully here. To give a brief perspective, Graham and Lyons' idea demonstrated in their recent work (Graham and Lyons 2021: 57–65), which regards a defeater for a belief as a warrant to believe the negation or to believe that the warrants for the original belief are inadequate, seems to be consistent and complementary with my suggestion about defeaters and recursive explanations. By replacing the counter-factual conception of defeaters in J' and in the definition of ultima facie justified defeating belief with their explanation appealing to the notion of warrant, we might obtain a better theory in the manner of process reliabilism.

process reliabilism consisting only of BC and RC, without the no-defeater condition, will be discussed as a representative of recursive reliabilism.[11] The theory is restated below.

Simple process reliabilism:

BC:

S's belief in *p* is justified if S's believing *p* results from a belief-independent process that is unconditionally reliable.

RC:

S's belief in *p* is justified if S's believing *p* results from a belief-dependent process that is conditionally reliable, and if the beliefs on which this process operates in producing S's belief in *p* are themselves justified.

The following example illustrates the puzzle.

Example 1:

S believes propositions *a*, *b*, *c*, *d*, *e*, *f*, and *g* by a belief-independent process that produces true beliefs with 90% probability. The events which these propositions express are independent of each other. Moreover, S has a belief-dependent reasoning process which, given a set of S's doxastic attitudes about some propositions, outputs their conjunction belief when S believes all of them and outputs a belief of the negation of the conjunction when S does not. S believes that $a \wedge b \wedge c \wedge d \wedge e \wedge f \wedge g$ by this process. In this regard, S never suspends beliefs about *a* to *g*.

Simple process reliabilism judges the justificational status of S's beliefs as follows. As a premise, process reliabilism admits the justification of a belief when the rate at which the process that produced it produces true beliefs exceeds a certain threshold, whether in BC or RC. In this case, if we suppose that the level of reliability required for justification is 80%, S's beliefs in *a*, *b*, *c*, *d*, *e*, *f*, and *g* are all justified because BC is satisfied. Furthermore, the belief-dependent reasoning process is conditionally reliable because its outputs are always true as long as all the input beliefs are true. Therefore, combined with the fact that S's beliefs in *a* to *g* are justified, RC is satisfied, and the belief that $a \wedge b \wedge c \wedge d \wedge e \wedge f \wedge g$ is judged justified.

However, the probability that *a* to *g* are all true, namely, the probability that $a \wedge b \wedge c \wedge d \wedge e \wedge f \wedge g$ is true, is $0.9^7 \approx 0.48$, which is far from the required level of reliability we have

11 Of course, because of its recursive structure, my version of process reliabilism including BC_{pf} and RC_{pf} suffers from the same puzzle.

assumed. Thus, it is not certain whether the conclusion of simple process reliabilism is desirable.

Another example shows the opposite situation.

Example 2:
S believes propositions a, b, c, d, e, f, and g by a belief-independent process that produces true beliefs with 60% probability. The events which these propositions express are independent of each other. Moreover, S has a belief-dependent reasoning process which, given a set of S's doxastic attitudes about some propositions, outputs their disjunction belief when S believes at least one of them and outputs a belief of the negation of the disjunction when S believes none of them. S believes that $a \lor b \lor c \lor d \lor e \lor f \lor g$ by this process. In this regard, S never suspends beliefs about a to g.

As long as we accept the 80% reliability requirement above, simple process reliabilism does not admit justification of S's belief in $a \lor b \lor c \lor d \lor e \lor f \lor g$ because the inputs to the reasoning process are not justified due to the low reliability of the belief-independent process which produced them. Nevertheless, the probability that $a \lor b \lor c \lor d \lor e \lor f \lor g$ is true is $1 - (1 - 0.6)^7 \approx 0.998$, which exceeds the criterion. Here again, the judgement of simple process reliabilism is not self-evidently true.

3.2 Inconvenient implication in the value of knowledge issue

The problem goes beyond the counter-intuitive consequences in some cases: these consequences make it difficult for reliabilists to account for the extra value of knowledge. The extra value of knowledge, i.e., that knowledge has a value that mere true belief does not, is explained by typical process reliabilists as below. First, knowledge is defined as follows: S knows that p iff p is true, S believes p to be true, S's belief that p was produced through a reliable process, and a suitable anti-Gettier clause is satisfied (Goldman and Olsson 2009: 22). The difference between the value of knowledge defined in this way and that of merely true beliefs is explained by appealing to conditional probability (Goldman and Olsson 2009: 27–31). Consider the case where, in a given situation, S knows p, which implies that S believes p by a reliable process, and the case where S believes p by an unreliable process, but it happens to be true in the same situation. Suppose that in the future, S is placed in a similar situation. Then, under the condition that the former case holds, S can expect to be able to use the same reliable process in the future and therefore has a high probability of acquiring a true belief. In contrast, since it is mere chance that S has the true belief p in the latter case, S cannot be expected to form a true belief with a high probability in the future situation. The extra value of knowledge consists of this difference in conditional probability.

However, if reliability demanded for knowledge is understood in the way of recursive reliabilism, including simple process reliabilism, the explanation above no longer holds. Suppose that $a \land b \land c \land d \land e \land f \land g$ is true in Example 1 and that there are no circumstances that make this example Gettiered. In this case, S knows that $a \land b \land c \land d \land e \land f \land g$. However, in a future situation similar to Example 1, it is not highly probable that S could acquire true beliefs even if the same complex process is used. Therefore, S's knowledge in Example 1 can no longer be said to be of extra value. This shows that knowledge does not necessarily signal a larger number of true beliefs in the future, and therefore recursive reliabilism causes a standard reliabilist account of the value of knowledge to fail.

3.3 Comparison with the preface paradox and the discursive dilemma

The above puzzle is partially similar to, but different from, several existing problems. The first is the preface paradox discovered by Makinson (Makinson, 1965). In the story of the paradox, an author claims s_1, s_2, s_3..., s_n in his new book and is thought to believe these statements to be true. In other words, if we express the author's belief in x as B(x), then B(s_1), B(s_2), B(s_3)…, B(s_n) are true and thus B($s_1 \land s_2 \land s_3 \land ... \land s_n$) is also true. Here, each of these beliefs is rationally acquired. However, in the preface to the book, he also states, as many authors do, that 'there will be some mistakes', which also reflects his beliefs. That is, ¬B($s_1 \land s_2 \land s_3 \land ... \land s_n$) is true. This belief also stems from his awareness of himself as fallible from past experience and thus is rationally held. Therefore, paradoxically, this author has contradictory beliefs, both of which are rationally acquired.

The preface paradox resembles the puzzle I have posed in that it concerns the gap between beliefs about individual propositions and beliefs about their conjunction. On the other hand, as Sorensen points out, the paradox does not rely on a probabilistic conception of belief formation or of justification assessment criteria (Sorensen 2006: Sec. 4). The author does not derive the probability that all of s_1, s_2, s_3..., s_n are true from the probability that each belief is true. He rationally believes in the negation of the conjunction, not by quantitative considerations of the probability that individual claims are correct, but by qualitative considerations based on his experience of some of his mistakes being pointed out after the publication of his previous works. In contrast, in Example 1, the justification for the belief in the conjunction is questionable because the probability of the conjunction itself being true is not sufficiently high when the probability of each part being true is considered.

Secondly, some may point out the similarities with the discursive dilemma raised by Pettit (Pettit 2001: 272–273). In this dilemma, the subject in question is not an individual but a group. As a premise, it is assumed that at least some groups can be the subjects of judgements, and the judgements of the groups are given by applying some aggregation procedure, e.g., a majority vote, to the judgements of its members. The dilemma is as follows. Group G consists of three members, A, B, and C. Each of them makes consistent judgements about the truth or falsity of propositions p, q, and $p \wedge q$, as shown in Table 1. At this point, even if only the majority rule is considered, the group's decision will depend on the decision aggregation procedure adopted. First, if the procedure is to aggregate each member's judgement on $p \wedge q$ directly, G will also judge $p \wedge q$ to be false because the majority of members judge it to be false. In contrast, if we take a procedure that aggregates each member's judgements about p and q separately, and then derive logical consequences from G's judgements about the truth or falsehood of p and q, then G will judge both p and q to be true and therefore $p \wedge q$ to be true. The reason why this is called a dilemma is that it causes problems whichever procedure is adopted. First, if the former procedure is adopted, then G will have an inconsistent set of beliefs, p, q, $\neg(p \wedge q)$, as long as G's beliefs about p and q are also determined by majority voting on each proposition. Second, if the latter procedure is adopted, then G will believe $p \wedge q$ even though there is only one member, C, who rationally believes $p \wedge q$ to be true.

Unlike my puzzle, the discursive dilemma is unrelated to justification understood in terms of reliability, and is only related to rationality as having a consistent set of beliefs. However, there is a notable similarity. On the one hand, in the discursive dilemma, the different order in which the majority voting procedure and the inference rule of conjunction introduction are applied produces different group beliefs. On the other hand, in the case of my puzzle, the extension by recursion is at odds with the evaluation of probability. In other words, the combination of the consideration of continuous matters, such as probabilities or proportions, and the consideration of discontinuous matters, such as the application of inference rules that classical logic allows or extensions by logical operations, which do not allow steps, causes difficulties.

Table 1
The discursive dilemma.

	p	q	$p \wedge q$
A	True	False	False
B	False	True	False
C	True	True	True
	↓	↓	
G	True	True	?

3.4 Reliable in every part or reliable as a whole

Even if, as a comparison with the above two problems suggests, consideration of probability causes problems when combined with a recursive structure, it seems inevitable for reliabilists to appeal to probability or proportion. When this point is considered together with the gap between the part and the whole suggested by the comparison with the preface paradox, the question of which of the following divergent paths to choose emerges as a point of contention, that is, whether to focus on the probability that each of the constituent processes produces a true belief or on the probability that the whole series of processes that led to the belief in question produces a true belief.

Recursive reliabilism, including simple process reliabilism, evaluates the reliability of each part of the path to a belief separately. This is why recursive reliabilism leads to the above conclusions in Examples 1 and 2. To capture this feature, I would like to call recursive reliabilism a kind of 'reliable-in-every-part reliabilism', which concludes that justification holds if and only if every process that contributes to producing the belief is reliable.

Here is a fork in the road where reliabilists can steer themselves in another direction: 'reliable-as-a-whole reliabilism'. As I have shown in 2.5, reliable-in-every-part reliabilism formulates a reliable complex process as a series of unconditionally reliable or conditionally reliable processes. Contrarily, we can understand the reliability of the complex process in different ways. In an alternative way, reliabilists can regard the reliability of a complex process as the tendency of that process as a whole to produce true beliefs. Then, they can set a condition for justification that the complex process which produced the belief must be reliable in that sense, namely, reliable as a whole, instead of demanding all constituent processes to be reliable. Although reliability as a whole partly depends on the rates at which ingredient processes produce true beliefs, it can hold as long as the complex process tends to produce true beliefs as a whole, without every part of it being reliable. At the same time, even when all the partial processes are reliable, the reliability as a whole sometimes does not hold. This branch of reliabilism is what I call 'reliable-as-a-whole reliabilism'.

Reliable-as-a-whole reliabilism is immune to the puzzle. Contrary to reliable-in-every-part reliabilism, the reliable-as-a-whole theory judges that S's belief in $a \wedge b \wedge c \wedge d \wedge e \wedge f \wedge g$ in Example 1 is not justified, and S's belief in $a \vee b \vee c \vee d \vee e \vee f \vee g$ in Example 2 is justified. This is because the complex process which produced the latter belief is reliable as a whole, but the process of the former is not. At the same time, reliable-as-a-whole reliabilism does not acknowledge the justification and hence knowledge in the value of knowledge case, a detailed version of Example 1. Therefore, it does not hinder the reliabilist account of extra value.

Because two variations of theories are competing, reliabilists should choose reliable-as-a-whole reliabilism as long as they take the puzzle seriously.

3.5 Insufficient motivation for recursive reliabilism

Finally, it should be considered whether there is enough motivation to adopt recursive or reliable-in-every-part reliabilism going so far as biting the bullet. In the first place, why did Goldman attempt an explanation by recursion? He does not explicitly state the reason, but looking at competing theories with a recursive clause which he exemplifies may provide a clue. For example, he formulates the infallibility analysis of justification as below (Goldman 1979: 6). In the following, a recursive clause is supplemented because he only presents the base clause.

> BC_{inf}:
> S's belief in *p* is justified if *p* is an infallible proposition.
> Proposition *p* is infallible iff, for any S, if S believes *p*, then *p* is true.
>
> RC_{inf}:
> S's belief in *p* is justified if S's believing *p* results from S's valid reasoning and all beliefs on which the reasoning is based are justified.

As Goldman himself points out, there are substantial problems with BC_{inf} (Goldman 1979: 7). Furthermore, the added recursive clause, RC_{inf}, reflects only the application of deductive reasoning, and thus other recursive clauses would need to be added or RC_{inf} strengthened to constitute the complete set of justified beliefs. However, we will not dwell on such flaws and focus here on the assumptions behind this analysis. It is assumed here that there are first some justified beliefs that are certainly true, and that by adding truth-preserving operations to them, one can extend the justified beliefs that are certainly true. However, such extensions seem allowable because the foundational justified beliefs defined by the base clause are always true. This is because it is only truth that valid inferences preserve, not some other property, such as a tendency to be true or justification.

However, according to the base clause of the simple process reliabilism, foundational justified beliefs are not necessarily true. This is because beliefs that are unfortunately false, despite arising from unconditionally reliable processes, are also justified. Extending the set thus defined by RC, a strengthened version of RC_{inf}, does not preserve any properties and does not constitute the desired set. It can be imagined that Goldman adopted the recursive structure because he thought he could transfer it from existing theories, including infallibility analysis, to his own process reliabilism. Contrary to his expectation, however, he had to part company with the recursive structure when he introduced probabilistic considerations into the theory as well as externalising the justification criterion.

4. Conclusions

Fumerton has pointed out that Goldman's recursive reliabilism falls into an infinite regress. This problem arises because Goldman incorporates his no-defeater condition into the base case in the recursive explanation. The regress can be avoided by providing a recursive explanation of prima facie justification before imposing the no-defeater condition on the whole of the recursion. Therefore, the regress puzzle is not a basis for abandoning reliabilism after the manner of Goldman. However, recursive reliabilism has a more difficult problem concerning the gap between the whole and the part. This difficulty arises from a mismatch between the consideration of the gradual matter of probability and the binary consideration of extension through logical operations. Faced with this puzzle, reliabilists can choose reliable-as-a-whole reliabilism instead of reliable-in-every-part reliabilism, including recursive reliabilism, because the latter leads to less plausible judgements in some cases and causes a reliabilist explanation of the extra value of knowledge to fail. In addition, it seems that reliabilists do not have enough motivation to adopt reliable-in-every-part reliabilism by accepting these pains.

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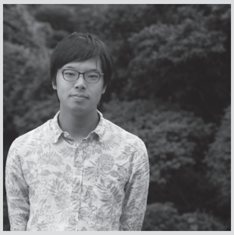

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