A QUANTIFICATIONAL ANALYSIS OF THE LIAR PARADOX

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Matheus Silva

A paradox is an unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises[1]. One of the simplest versions of the Liar Sentence is

L: This sentence is false.

Now, consider the following premises:

P1: Every declarative sentence with meaning is either true or false.
P2: L is a declarative sentence with meaning.

Yet, as surprising as it may seem, we cannot infer from those premises that L is either true or false. To understand why this happens, you need to consider the consequences of attributing truth values to the liar sentence. Let's suppose that L is false. If that is the case, L is true, since it claims it is false. But if L is true, what it says is the case, and thus it is false. Thus, we have to conclude that L is true if, and only if, it is false. This is the liar paradox.

In this note, I will defend that the reasoning that leads to the paradoxical conclusion is flawed, since the liar sentence is a quantified statement with a conjunction of different claims. Once the semantic structure of the liar sentence is unpacked by this analysis, L reveals itself as being merely false and without engendering any contradiction.

The quantificational analysis adopted here is inspired by Russell (1905, 1919), who claimed that we can interpret sentences with indefinite descriptions such as (1) as having the logical form of (1*),

(1) An *F* is *G*.
(1*) $\exists x(Fx \,\&\, Gx)$

Similarly, Russell argued, we can interpret a sentence with a definite description such as (2) as having the logical form of (2*),

(2) The *F* is *G*.
(2*) $\exists x(Fx \,\&\, \forall y(Fy \supset x = y) \,\&\, Gx)$

This analysis suggests that the semantics of definite descriptions involves a conjunction of three quantified statements:

(2a) There is an *F*.
(2b) At most one thing is *F*.

[1] I'm using the definition of paradox presented by Sainsbury (1988).

(2c) Something that is *F* is *G*.

This approach is surprising because it treats definite descriptions as quantified phrases instead of referring terms. The other puzzling aspect of Russell's approach is that existence and uniqueness claims are interpreted as a part of the content asserted instead of being understood as a presupposition of the sentence[2].

The next step is to extend this approach to complex demonstratives, which are expressions of the form 'That *F* is *G*', 'This *F* is *G*'. Thus, sentences like (3) can be interpreted as quantificational sentences such as (3*):

(3) This *F* is *G*.
(3*) $\exists x(Fx \,\&\, \forall y(Fy \supset x = y) \,\&\, Gx)$

The difference is minimal[3]. More importantly, this approach provides a way to access the semantic structure of the liar sentence, which is also a complex demonstrative. The key feature of this strategy is that the problematic aspect of the liar sentence that generates the paradox is interpreted as a part of its asserted content. This problematic aspect is, I believe, L's pretension to a lack of truthmakers. When we determine the truth value of a sentence we determine its connection to its truthmaker because we believe that its truth value should depend solely on how things stand in the world. But the liar paradox is an artificial sentence dissociated from the real world and thus any attribution of truth value seems arbitrary. This challenge can be overcome by incorporating the statement that L has no truthmaker into the asserted content of the liar sentence, thus creating a connection to the real world and a meaningful attribution of truth value. L can be interpreted as involving the following components:

La: There is a sentence L.
Lb: At most one thing is L.
Lc: Something that is L is false.
Ld: Something that is L requires no truthmaker.

Let '*Sx*' mean 'a sentence', '*Fx*' mean '*x* is false' and '*Nx*' mean '*x* requires no truthmaker'[4]. We can interpret L as follows:

L*: $\exists x(Sx \,\&\, \forall y(Sy \supset x = y) \,\&\, Fx \,\&\, Nx \,\&\, x = \mathrm{L})$

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[2] Of course, Russell's main motivation to defend this analysis of descriptions is that it would allow us to solve a puzzle involving definite descriptions with non-denoting expressions such as 'The present king of France is bald'. Intuitively, this sentence is false because currently France is not a monarchy. But its denial, 'The present king of France is not bald', also seems false. If a statement and its denial are both false the law of excluded middle is violated. Russell's solution attempts to dissolve this puzzle by interpreting the presuppositions of existence and uniqueness as a part of the asserted content. I will not discuss the details of Russell's analysis since this is common knowledge.

[3] For a defence that complex demonstratives are quantifiers see Barwise and Cooper (1981: 177, 184), Keenan and Stavi (1986), King (2000), Neale (1993, §9) and Taylor (1980).

[4] I will assume that the domain of quantification for this and the two other examples of quantified expressions that will be discussed in this article is everything that exists.

Thus, the semantics of the liar sentence involves not only an existence claim, a uniqueness claim, and a maximality claim, but also a non-truthmaking claim. In other words, the liar sentence is a general claim that the world contains a sentence that has a truth value but no truthmaker. But according to Truthmaker Maximalism (TM) all sentences with truth-values have truthmakers. Therefore, this sentence is false[5].

But wouldn't this conclusion create another paradox? If L is false, it is true, because it states that one of its properties is falsehood, but if it is true, it is false. It seems that we are back to square one. The reason why this is not the case is that L*'s truth requires the truth of each one of its conjuncts, but that conjunct that denies TM is false. Therefore, L* is simply false. This offer us a way to account for the meaningfulness of L in a way that does not commit us to a paradox[6].

There is still a need to explain where is the truthmaker responsible for the falsity of the liar sentence. The relation of truthmaking relates an entity in the world (e.g., a fact or state of affairs), to a representing entity (e.g., a statement). But if that is the case, TM itself cannot be the truthmaker of L* because the principle is representational in character whereas truthmakers should be non-representational worldly entities. The answer to this question is this: the truthmaker of L* is the truthmaking relation of every sentence that has a truth value. Those truthmaking relations are worldly entities in their own right. This implies that the truthmakers that are responsible for TM's truth are also responsible for L*'s falsity.

This analysis also provides us with a promising strategy to solve a recent paradox involving truthmaking. Consider the following sentence:

M: This sentence has no truthmaker.

Peter Milne (2005) argued that this sentence represents a counterexample to TM. His reasoning is as follows: suppose that M has a truthmaker and it is true. But if it is true, it has no truthmaker since it claims that it has no truthmaker. This implies that M has a truthmaker if it has no truthmaker. By *reductio* we conclude that M has no truthmaker. But this implies that M was true all along even without a truthmaker. Therefore, TM is false.

The quantificational treatment of this sentence offers us a different perspective. M can be analysed as follows:

Ma: There is a sentence M.
Mb: At most one thing is M.
Mc: Something that is M has no truthmaker.
Md: Something that is M is true.

Let '*Tx*' mean '*x* is true'. The complete semantic structure of M can be interpreted as M*:

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[5] It is controversial what is the most convincing formulation of TM. One could argue, for instance, that it is implausible that analytic truths, essential predications or negative existential truths have entities as truthmakers. This motivated some authors, e.g., Gonzalo Rodriguez-Pereyra (2005), to defend more restrictive truthmaking principles. In any case, it is reasonable to assume that any plausible version of TM or of a more restrictive truthmaking principle will imply that that a sentence that claims it has no truthmaker is false.

[6] This solution also holds for different formulations of the paradox such as the strengthened liar and liar cycles. I will leave this as an exercise for the reader.

M*: $\exists x(Sx \,\&\, \forall y(Sy \supset x = y) \,\&\, Tx \,\&\, Nx \,\&\, x = \mathrm{M})$

In other words, M* asserts the existence of a true sentence without a truthmaker, but that is precisely what a proponent of TM will deny. Thus, the counterexample can be accused of being motivated by circular reasoning[7].

Another interesting case is the truth teller sentence:

TL: This sentence is true.

This sentence does not involve any obvious contradiction, but it is counter-intuitive since we have no obvious way to determine the truth value of TL. Roy Sorensen (2001) went so far as to suggest that the truth value of TL is unknowable and that it represents a deep truthmaker gap. Sorensen's reasoning is that in order to ascertain the truth value of a sentence we determine its connection to its truthmaker, but since the truthteller has no truthmaker we will never know its truth value. The quantificational treatment suggests a different conclusion. TL can be interpreted as involving the following claims:

TLa: There is a sentence TL.
TLb: At most one thing is TL.
TLc: Something that is TL is true.
TLd: Something that is TL requires no truthmaker.

The logical structure of TL is as follows:

TL*: $\exists x(S\mathrm{x} \,\&\, \forall y(S\mathrm{y} \supset y = x) \,\&\, Tx \,\&\, Nx \,\&\, x = \mathrm{TL})$

But this shows that just like M*, TL* cannot be used as a counterexample to TM without begging the question. In fact, it is false if TM is true. Therefore, we can determine the truth value of the truthteller. It is simply false. This diagnosis is reinforced by the fact that TL refuses an interpretation in which it has no truth values. If TL is neither true nor false, it is not true. But TL says that is true, so it is false. Thus, the initial assumption that TL is neither true nor false is incorrect.

It is no wonder that Dan López de Sa and Elia Zardini (2006) have recently argued that Milne's argument allows one to prove the negation of anything. TM is motivated by the belief that a sentence is true because of what exists in the world and not the other way around. If we accept the truth of M or similar paradoxical sentences, the world is the way it is because of which sentences we take as truths.

One potential objection to the present solution is that it's not obvious that L* really expresses L, since L is self-referential, but L* is not. If we change the target sentence when we moved from L to L*, the solution does not work. But that's like arguing that it is not obvious that (2*) really express (2), because the later is referential and the first is not. If it is

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[7] Gonzalo Rodriguez-Pereyra (2006) made a similar accusation that Milne's counterexample begs the question against TM. I agree with him.

plausible to interpret (2*) as a quantified phrase, it is also plausible to interpret L* as a quantified phrase. Moreover, there is nothing in the quantificational analysis that prevents us from interpreting L* as claiming that the sentence that has a truth value without a truthmaker is L* itself. In fact, this is indeed false since L* does have a truthmaker.

Following Donnellan (1966), one could also object that definite descriptions may have both attributive and referential uses, but only an attributive use admits a quantificational analysis. If that is the case, it is not obvious that L has an attributive use instead of referential one. Consequently, it is not a given that L can be interpreted as L*. My answer to that is that is that unlike (2), there is no clear referential use for L. If I point at L and say, 'This sentence is false', I'm not communicating the same thought expressed by L. The difference here is that if L is to be interpreted as a referential expression at all, it must be interpreted as a self-referential one, but self-referential expressions are artificial by nature and have no obvious communicative use. In fact, even the argument employed by Kripke (1977) in his reply to Donnellan's argument does not seem appropriate here since there is no obvious speaker's reference of L in addition to the actual semantic reference of L.

Positing the need for truthmakers enable us to strike at the heart of the liar paradox: it is a sentence that states that it has a truth value that would depend solely on whether it has a truth value, instead of being dependent on how things stand in the world. This means that the fact that L states that its own truth value is false is a negligible aspect that distract us from the real issue, which is the pretension of having no truthmakers.

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