

IS CLASSICAL LOGIC MONOTONIC?

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It is usually accepted that one of the properties of classical logic is monotonicity, the property that ensures that the validity of implication is not affected by the addition of new premises. In this piece, I will argue that this common notion is unjustified since it is motivated by a category mistake. The notion of monotonicity is primarily epistemic in character and can't be meaningfully attributed to a system. This is acutely clear in the contrast of monotonicity with non-monotonicity, which we tend to associate with defeasible inferences, where reasoners can abandon a previous inference based on new information. So non-monotonicity is actually the reasoners' willingness to abandon a previous inference based on new information and can only be a property of a system in a manner of speaking. It follows that monotonicity should be understood in a similar epistemic fashion as the reasoners' willingness to *maintain* a previous inference after newly discovered information. By analysing the problem from this perspective, a classical inference can be considered non-monotonic if the reasoner is willing to retract her previous inference based on new information.

Let's say that in classical logic if a given premise A implies B , then A will still imply B given the addition of any other premise C . This property is integral to the notion of implication in a classical system. Now, imagine that I assumed that A implied B , but later on realised that another proposition, C , is also true, but it is incompatible with the truth of B in conjunction with A . In this case, not only would I not accept that the conjunction of A and C implies B , as I would withdraw the initial statement that A implies B . This is how things should be: if a premise implies a conclusion, then the negation of the conclusion *must imply* the negation of the premise. Otherwise it would not be valid in the first place. The fact that a claim to implication can freely incorporate new information is consistent with the abandonment of a claim to implication precisely due to the addition of the newly found information.

One way to make this more intuitive is with antecedent strengthening. This inferential rule states that $A \supset B$ implies $(A \& C) \supset B$. This rule also encapsulates the supposed monotonicity of classical logic with material implication, but faces apparent counterexamples such as the following: 'If the match is struck it will light. Therefore, if the match is struck and it is held under water, it will light.' This apparent counterexample is also intended to show that the inferential rule fails because it doesn't do justice to the defeasible nature of our reasoning in some matters. But this counterexample does not work, since the premise is only true when there are normal background conditions that ensure that the match will light when stroke, i.e., it is dry, there is the presence of oxygen in the atmosphere, etc. However, the truth of one the conjunctives of the conclusion's antecedent is inconsistent with one these conditions, namely, that the match is dry. Thus, in the only context where the premise is true, the background conditions necessary for the causal relation are maintained, but in this context the conclusion is vacuously true due to the falsity of the antecedent. Or, to put in other words, if the conclusion is false, its antecedent is true, but in that case the premise is also false. If I find out that the match is also held under the water, I will not conclude that it will light. Instead, I will withdraw the initial premise. This is exactly how things should be.

This example is also important for a different reason. It is common to assume that a material implication is a conditional connective and not a claim to an implication relation. This is motivated by the popular notion that to interpret conditionals as claims to implication is to commit a use-mention fallacy in which the antecedent and consequent are mentioned as the premise and conclusion of an implication relation; whereas genuine conditionals do not mention statements, but use them to express a relation between facts and objects in the world. This popular view is baseless though. When a conditional is asserted, it's the whole proposition that it is asserted, and not its antecedent and consequent. The assertion of a conditional then can be understood as a statement about a relation between the propositions expressed by the antecedent and consequent. In other words, the antecedent and consequent are mentioned, not used.

So there is nothing that prevent us from interpreting conditionals as claims to implication. The important part is that by interpreting conditional sentences as claims to material implication in the example mentioned above we have a clear demonstration of how material implication is consistent with reasoners' non-monotonic dispositions. In an antecedent strengthening we accept that if A materially implies B , then A and C materially imply B . If the reasoner realises that the last claim to implication is invalid she will abandon the former claim as well. This is just a repetition of the same behaviour expressed by formal implication in a more restricted range. The reason is that the reasoner can use classical logic in a way that is compatible with her disposition to change her mind when confronted with new information.

I believe that classical logic is wrongly perceived as monotonic due to the questionable notion that monotonicity can be expressed as a formal relationship between implication statements. This erroneous perception is motivated by the excessive importance attributed to the formal aspects of valid implication to the detriment of the way flesh and blood people may reason using these propositional forms.