Comparative Vagueness*

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1 Introduction

Can comparatives '*a* is ADJ-er than *b*' be vague? A frequently expressed idea in certain linguistic circles is that they cannot. "[A]djectives in the comparative are uniformly non-vague" (Bochnak 2013: 42); "a core semantic difference between the positive [(i.e. unmodified)] and comparative forms"—e.g., between 'tall' and 'taller'—"is that the latter lacks whatever semantic (or pragmatic) features give rise to the vagueness of the former" (Kennedy 2013: 270).[1] Broader work and more recent linguistic work has called this idea into question. Some have suggested that multidimensionality or uncertainty (alternatively: indeterminacy, indecision) about measurement procedures may lead comparatives to have borderline cases, a hallmark of vagueness (Williamson 1994: 156, Endicott 2000: 43–45, 140, Keefe 2000: 13–14; cf. Sassoon 2011: 102, 106–111, 2013: 172–178, 209). For instance, if individuals differ in "incommensurate dimensions" of niceness (Endicott 2000: 43), there may be "no fact of the matter about who is nicer" (Keefe 2000: 13). Yet it has been maintained that "if we consider only one dimension of comparison, and suppose perfect accuracy in measurement, vagueness does affect the extension of the positive form of the adjective, but not that of the comparative" (Égré & Klinedinst 2011a: 10). Not so.

This paper provides new examples of vagueness phenomena with comparatives. I show that comparatives '*a* is ADJ-er than *b*' can be vague due to a fuzziness in how much of some property determines a difference in ADJ-ness. The examples I provide cannot be assimilated to cases of fuzziness in what dimensions are relevant or measurement procedures.

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[1]See also Cooper 1995: 246; Kennedy 2007b: 6, 2011: 74, 82–83, 93, 2013: 271, 269; McNally 2011: 164n.10; van Rooij 2011a: 65–69. I leave open to what extent these authors could allow for vague comparatives in cases such as those described below.

I focus on explicit comparatives of the form '*a* is ADJ-er than *b*'—synthetic, phrasal comparative constructions using a morphologically comparative form of a relative gradable adjective 'ADJ'. I put aside "implicit" comparatives like (1a) in which a comparison is made using the positive form (Kennedy 2007a, 2011, van Rooij 2011a); and marked comparatives such as clausal comparatives like (1b) and (1c) and analytic comparatives like (1d) that have a synthetic counterpart (see Rett 2008: §3.5, 2015: §§6.1.1, 6.2.2). I also put aside examples with adjectives that generally imply positive-form predications, as in (1e) with an "extreme" adjective (see also Rett 2008: §3.7, 2015: §5.4, Morzycki 2012, Brasoveanu & Rett 2018).

(1) a. *a* is strong compared to *b*.
 b. *a* is stronger than *b* is.
 c. *a* is stronger than *b* is fast.
 d. *a* is more strong than *b*.
 e. *a* is tinier than *b*. ($\rightarrow$ *b* is tiny)

Restricting our attention in these ways will help bracket vagueness associated with a relation to a relevant standard, often observed with the positive form.[2]

The paper proceeds as follows. §2 presents the main examples. §3 examines their implications for traditional formal semantics for gradation. §4 develops a revised degree semantics with semiorders, a type of well studied threshold structure (Luce 1956, van Rooij 2011c). The semantics provides an improved interpretation of equatives in cases where certain transitivity assumptions fail. §5 takes stock.

2 Comparative sorites cases

Suppose you are comparing edited versions of a dimly lit photo. The version with a 15% brightness increase, x_{150}, is prettier than the original, x_0. But it's not as if every 0.1% change in brightness affects the prettiness of the photo. If you judge x_i prettier than x_0, your judgment about x_{i-1}—the otherwise identical version with a $\frac{i-1}{10}$% brightness increase—should be no different. Consider (2), where $\ldots x_i \ldots$ is the series of versions of x_0 differing only in brightness.

(2) (P1) x_{150} is prettier than x_0.

[2] Terminology is fraught and varies widely. See e.g. linguistic work on "evaluativity" in the sense of Rett 2008, 2015. I also put aside possible connections with topics such as subjectivity, context-sensitivity, relativism, normativity (see also, e.g., Kennedy 2007b, 2013, Raffman 2014, Kamp & Sassoon 2016, Silk 2016, 2021a,b, Bylinina 2017, Solt 2018). I return to matters concerning standards of precision and multidimensionality briefly below.

(P2) For all n, if x_n is prettier than x_0, then x_{n-1} is prettier than x_0.
(C) So, for all n, x_n is prettier than x_0.

The premises seem true, and the argument seems valid. Yet the conclusion is false. x_0 is not prettier than itself.

Or suppose you like sugar in your coffee. Yet you don't care exactly how sweet it is. As far as your preferences go, one day's spoonful of sugar is as good as the next; an extra, say, 0.1 gram of sugar doesn't make a cup of coffee tastier. Consider (3), where y_{100} is a sweetened cup of coffee with 10 grams of sugar, and $\ldots y_i \ldots$ is a series of otherwise identical cups differing only in sugar content (cf. Luce 1956).

(3) (P1) y_1 is not tastier than y_{100}.
(P2) For all n, if y_n is not tastier than y_{100}, then y_{n+1} is not tastier than y_{100}.
(C) So, for all n, y_n is not tastier than y_{100}.

But not just any sweetened cup of coffee can be tastiest.

Sorites-susceptibility is one hallmark of vagueness. The comparatives in (2)–(3) exhibit "tolerance": in (3), e.g., the added sweetness from 0.1 gram of sugar is "insufficient to affect the justice with which [the predicate 'is (not) tastier than y_{100}'] applies" (Wright 1975: 349). Comparatives may also have "fuzzy" or "blurred boundaries of application": "There is, for example, no sharp division between [cups] that are clearly [tastier than y_{100}] and [cups] that aren't" (Raffman 1994: 41). The comparative may have borderline cases — cups in the "penumbra" (Russell 1923: 87) that are neither clearly tastier than y_{100} nor clearly not.

Such examples can be multiplied. Let F be a property relevant to how ADJ things are such that it is intuitively fuzzy in the context how much of a difference in F-ness makes for a difference in ADJ-ness (e.g., how much of a difference in brightness makes for a difference in prettiness of the photo). Consider two items i, j in the domain of 'ADJ' that are significantly different in F-ness, where 'i is (not) ADJ-er than j' is clearly true. Continue from i to apply 'is (not) ADJ-er than j' to items incrementally different in F-ness. Find yourself at an item k such that 'k is (not) ADJ-er than j' is clearly false.

The examples in this section differ from previous types of examples of vague comparatives (§1). First, note that the force of (2)–(3) doesn't turn on limitations in powers of discrimination. As Wright (1987: 239–243) shows, indiscriminability between adjacent items in a sorites series isn't necessary to generate the paradox.[3] The

[3] Certain of the examples which I used in earlier work were problematic in failing to appreciate this point (Silk 2016: 198–199, 206). Thanks to Gunnar Björnsson for discussion.

incremental differences in brightness and sweetness in (2)–(3) are discriminable in the context.[4] In (3), one simply doesn't care exactly how sweet the coffee is. One cup is as tasty as the next, given one's preferences.

The force of (2)–(3) also doesn't rely on multiple dimensions relevant for applying the comparative predicate. The examples proceed precisely by fixing a dimension (e.g., brightness). The interpretation with respect to a single dimension can be made linguistically explicit (e.g., 'prettier with respect to brightness'). What is intuitively fuzzy isn't what dimensions are relevant or their relative importance, but the relation between a particular dimension of comparison and the property associated with the adjective.

Preliminary upshot: Vagueness phenomena can arise with comparatives '*a* is ADJ-er than *b*' independently of the fuzziness in standards for counting as ADJ observed with the positive form, and even if we "suppose perfect accuracy in measur[ing]" items along "only one dimension" (Égré & Klinedinst 2011a: 10).

3 Traditional semantics for gradation

This section examines the implications of examples such as (2)–(3) for traditional semantics for gradation.

It is common to locate the problem with sorites arguments such as (4)–(5) with positive-form predicates in the inductive premise. (Let x_n be someone $4' + n$ millimeters tall.)

(4) (P1) Someone who is 4′ isn't tall (for a pro basketball player).
(P2) If someone who isn't tall (for a pro basketball player) grows one millimeter, they still won't be tall (for a pro basketball player).
(C) So, no one is tall (for a pro basketball player).

(5) (P1) x_0 is not tall.
(P2) For all n, if x_n is not tall, then x_{n+1} is not tall.
(C) So, for all n, x_n is not tall.

For instance, even if we can't point to any instance of (P2) in (5) that isn't true, perhaps we can know that it isn't true in any context (Soames 1999, Fara 2000), or no matter what formally precise language we might be speaking (Lewis 1970), or no matter how the conversation might evolve (Shapiro 2006), or on any competent way of applying 'tall' (Kamp 1981, Raffman 2014).

[4] If not, one's discriminatory capacities or the difference between adjacent items could be adjusted accordingly.

There is an important difference between the inductive premises (P2) in (2)–(3) vs. (5). To help illustrate, consider the three-premise variant of (2) in (6). The premise (P3) is an instance of what is sometimes called *IP-transitivity*, which is a weakening of transitivity — i.e., if a relation $\geq$ satisfies transitivity, IP-transitivity in (7) is also satisfied, where $>$ and $\sim$ are the strict (asymmetric) and non-strict (symmetric) subsets, respectively, of $\geq$.[5]

(6) (P1) x_{150} is prettier than x_0.
(P2′) For all n, x_{n-1} is as pretty as x_n.
(P3) For all a, b, c, if a is as pretty as b, and b is prettier than c, then a is prettier than c.
(C) So, for all n, x_n is prettier than x_0.

(7) *IP-transitivity:* $\forall u, v, w : (u \sim v \wedge v > w) \rightarrow u > w$

Traditional semantics for gradation validate IP-transitivity premises such as (P3). Consider, first, a degree-based semantics which treats gradable adjectives as associating items with degrees on a scale (Bartsch & Vennemann 1973, von Stechow 1984, Kennedy 1999, 2007b, Heim 2001, Morzycki 2015). For instance, on a Kennedy-style implementation, 'tall' denotes a function *tall* from an individual to a degree representing the individual's maximal height. Though some theories assume that degrees are isomorphic to rational numbers, a minimal constraint is that the relation $\geq$ on the set of degrees D have the structure of a partial order, i.e. that $\geq$ be a reflexive, transitive, antisymmetric relation on D (Kennedy 1999, 2007b, Barker 2002, Lassiter 2015). Compositional details aside, the comparative (8) is true iff the maximal degree to which Alice is tall, $tall(Alice)$, is greater than the maximal degree to which Bert is tall, $tall(Bert)$.

(8) Alice is taller than Bert.

The interpretation of (P3) from (6) is as in (9),[6] where P, the denotation of 'pretty', is a function from items to a degree representing how pretty they are.[7]

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[5]See, e.g., Sen 1970: 10–11. See Luce 1956 for earlier use of 'P' and 'I' in discussion of transitivity and binary preference and indifference relations.

[6]I assume an "equally" reading of the equative (cf. Bhatt & Pancheva 2007, Rett 2008). In theories positing a basic "at least" meaning, '=' can be substituted with '≥'.

[7]What is important about degrees is that they represent how pretty, tall, etc. things are, and that they can be associated with qualitative orderings on items in adjectives' domains. Nothing of metaphysical significance is implied by things having "degrees" of prettiness, tastiness, etc.

(9) $\forall a,b,c : \big(P(a) = P(b) \wedge P(b) > P(c)\big) \rightarrow P(a) > P(c)$

(P3) follows from the transitivity of the relation $\geq$ on the domain of degrees D.

Analagous points hold with the other main approach to gradation in formal semantics: delineation semantics ("partial predicate," "inherent vagueness" semantics). Gradable adjectives are analyzed here as predicates whose denotation partitions a *comparison class CC* of relevant individuals into a positive extension, a negative extension, and an extension gap ("borderline cases") (Klein 1980, Burnett 2012).[8] Following Klein 1980, a comparative such as (8) is true iff there is some comparison class in which Alice is tall and Bert is not tall. To avoid problematic entailments, delineation theories impose qualitative restrictions on comparisons among individuals across comparison classes (Fine 1975, Klein 1980, Fara 2000). For instance, if a counts as tall in some comparison class and a's height is greater than b's height, then there is no comparison class in which b counts as tall and a does not. Delineation theorists prove that the qualitative restrictions derive a preorder (reflexive, transitive relation) $\succcurlyeq_A$ "at least as ADJ as" on the set of individuals in the domain of 'ADJ', for any adjective 'ADJ' (van Benthem 1982, Klein 1991, van Rooij 2011a).[9] The interpretation of any adjective thus relies on a preorder $\succcurlyeq_A$ on the set of individuals. (P3) follows from the transitivity of $\succcurlyeq_A$.

So, traditional semantics validate premises such as (P3) in (6) — due to the structure of scales $\langle D, \geq \rangle$ in degree-based semantics, or the qualitative ordering $\succcurlyeq_A$ on the set of individuals in delineation semantics. That leaves premises such as (P2′) as the culprit for theories of vagueness seeking to deny the inductive premise.

(P2′) For all n, x_{n-1} is as pretty as x_n.

Yet there are costs to locating the problem with the argument in (P2′). (P2′) is, on the face of it, true in the context. Saying this isn't simply to say there is a paradox; *something* plausible must be denied. Denying (P2′) requires denying the aesthetic possibility that a 0.1% difference in brightness might fail to make a difference in how

[8] Some theories also invoke a parameter for relevant standards (cf. Barker 2002), e.g. where the positive extension of 'tall' is the set of individuals in CC whose height is at least the standard of tallness.

[9] Degrees and scales may be derived from these qualitative orderings (Cresswell 1977, Bale 2008). The set of degrees D is the set of equivalence classes under $\succcurlyeq_A$; and the relation $\geq_A$ on D is defined accordingly where $[a]_A \geq_A [b]_A := a \succcurlyeq_A b$ (with $[u]_A$ being the equivalence class $\{v : v \succcurlyeq_A u \wedge u \succcurlyeq_A v\}$).

pretty a photo is. It is hard to see why that might be so. Or consider an analogous version of (P2′) for our example with 'tasty' (cf. (3)):[10]

(10) For all n, y_n is as tasty (to you) as y_{n+1}.

(10) describes your non-obsessiveness about sugar in your coffee. The extra sweetness from (say) 0.1 gram of sugar doesn't make a difference to you in how tasty the coffee is. Such tastes don't seem impossible or incoherent, so as to be ignorable by a semantic theory. You simply don't care exactly how sweet the coffee is.

Note that the premises (P2′) in our examples cannot be denied on the ground that adjacent items are not discriminable. As discussed above, the adjacent versions of the photo are discriminable in brightness, and the adjacent cups of coffee are discriminable in sweetness in the given contexts. What is at issue is whether such differences need make one item prettier or tastier than the other.

The previous points are perhaps not decisive. There may be other resources for traditional theories to help explain away the intuition that (P2′) is true, say in terms of "loose use" or "granularity" (Lasersohn 1999, Bittner & Smith 2001, Krifka 2007, Sauerland & Stateva 2011). A challenge for replies along these lines is again the observation that the intuition that (P2′) is true can persist in contexts of maximal discriminability. One cup of coffee in the series is as tasty to you as the next, not just "loosely speaking" but, on the face of it, speaking precisely, and even if you happen to be a supertaster. Saying otherwise would mischaracterize your state of mind.

I won't continue to press these issues here. Instead I would like to use the remainder of the paper to begin investigating an alternative framework which avoids validating generalizations such as (P3) in (6) as a matter of conventional meaning. The semantics in §4 allows for the truth of (P2′) while circumscribing a class of cases in which the premises (P2) in two-premise comparative sorites arguments are false. I hope these preliminary developments may provide a fruitful basis for future work and theory comparison.

[10] The version of the argument in (i) below uses an instance of PI-transitivity ((ii)) for (P3). As with IP-transitivity, PI-transitivity is entailed by transitivity of a relation $\succeq$, and (P3) is validated in traditional frameworks.

(i) (P1) y_{100} is tastier than y_1.
(P2′) For all n, y_n is as tasty as y_{n+1}.
(P3) For all a, b, c, if a is tastier than b, and b is as tasty as c, then a is tastier than c.
(C) So, for all n, y_{100} is tastier than y_n.

(ii) *PI-transitivity:* $\forall u, v, w : (u \succ v \wedge v \sim w) \rightarrow u \succ w$

4 Semiorders in a degree semantics

Our aim is a semantics that allows for the truth of equatives such as (P2′) and the falsity of transitivity assumptions such as (P3) (reproduced below from (6)).

(P2′) For all n, x_{n-1} is as pretty as x_n.
(P3) For all a, b, c, if a is as pretty as b, and b is prettier than c, then a is prettier than c.

One natural approach is to move from thinking of the relation on the set of degrees or individuals as at least a partial order or preorder (§3) to thinking of the relation as a *semiorder*, instead. Semiorders have been used fruitfully in measurement theory, choice theory, and mathematical psychology for representing intransitive indifferences.[11] There are precedents for using semiorders in accounts of vagueness phenomena with predicative uses and the positive form as well (cf. Luce 1956, van Rooij 2011a,b,c, Cobreros et al. 2012).[12] Semiorders afford an independently motivated resource for semantics for gradation.

4.1 Semiorders

Formally, a semiorder $\gtrsim$ on a set S is an interval order — a reflexive, Ferrers binary relation — that satisfies semitransitivity ((11)); equivalently, $\gtrsim$ is a semiorder iff there is a real-valued function f and fixed positive number ϵ such that $u \gtrsim v$ iff $f(u) \geq f(v) - \epsilon$, for all $u, v \in S$ (n. 11).[13]

(11) *Reflexive:* $\forall u : u \gtrsim u$
Ferrers: $\forall u, v, w, z : (u \gtrsim v \wedge w \gtrsim z) \rightarrow (u \gtrsim z \vee w \gtrsim v)$
Semitransitive: $\forall u, v, w, z : (u \gtrsim v \wedge v \gtrsim w) \rightarrow (u \gtrsim z \vee z \gtrsim w)$

Intuitively speaking, semiorders generalize weak orders by comparing items under a "range of fuzziness," representable by uniform-length intervals. The Ferrers property ensures the interval representation, whereby $u \gtrsim v$ iff u and v can be associated with intervals U, V, respectively, such that V doesn't wholly follow U (i.e., $u_i \geq v_i$, for some $u_i \in U$, $v_i \in V$). Semitransitivity implies that all the intervals can be made the same length; the "fuzziness" is the same for each item (cf. Fishburn 1970: 212,

[11] For classic discussion see Luce 1956, Scott & Suppes 1958, Fishburn 1985; see also Suppes et al. 1989: ch. 16, Pirlot & Vincke 1997, Aleskerov et al. 2007.

[12] I return to this briefly in §4.4.3.

[13] For present purposes we can assume that the sets are finite. For results generalizing to arbitrary sets, see Bouyssou & Pirlot 2021a,b. See Riguet 1951 on applying Ferrers graphs to representations of certain relations.

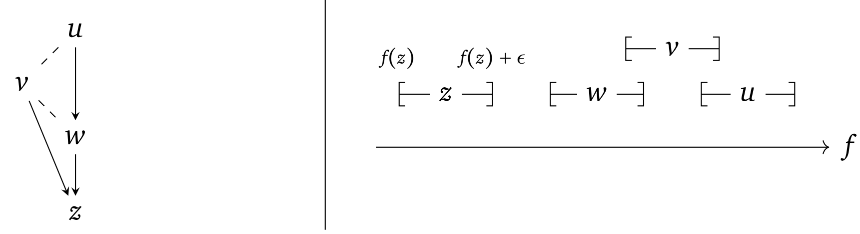
1973: 93). Strict $\succ$ and non-strict $\sim$ subsets of $\succsim$ can be defined in the usual way: $u \succ v$ iff $u \succsim v \wedge v \not\succsim u$ iff $f(u) - f(v) > \epsilon$; and $u \sim v$ iff $u \succsim v \wedge v \succsim u$ iff $|f(u) - f(v)| \leq \epsilon$ (again for all $u, v \in S$ and some real-valued function f and positive number ϵ).

For an example, consider the semiorder $\succsim$ and alternative graph and interval representations in (12). A dashed line between items x and y in the graph represents that $x \sim y$, and an arrow from x to y represents that $x \succ y$ (reflexive $\sim$-loops and transitive $\succ$-arcs are omitted). $x \succ y$ just in case the "fuzziness" around x, represented by $[f(x),\ f(x) + \epsilon]$ in the interval representation, strictly follows the fuzziness around y, represented by $[f(y),\ f(y) + \epsilon]$.

(12) $S = \{u, v, w, z\}$

$$\begin{aligned}\succsim = \{&\langle u,u\rangle, \langle u,v\rangle, \langle u,w\rangle, \langle u,z\rangle,\\ &\langle v,u\rangle, \langle v,v\rangle, \langle v,w\rangle, \langle v,z\rangle,\\ &\langle w,v\rangle, \langle w,w\rangle, \langle w,z\rangle,\\ &\langle z,z\rangle\}\end{aligned}$$

Graph representation: | Interval representation:



Intuitively, the semiorder $\succsim$ in (12) weakens an ordering in which $u > v > w > z$ by comparing items under a fuzziness that fails to distinguish u and v and fails to distinguish v and w. Notably, the non-strict part $\sim$ is now not an equivalence relation. We have a case where $u \sim v \sim w$; $u \sim v$ and $v \sim w$ but $u \not\sim w$, indeed $u \succ w$. Such intransitivities of $\sim$ will be important in what follows.

4.2 Degree semantics

Let's turn to the semantics. To fix ideas I assume a Kennedy-style degree-based framework (§3). As previously, the denotations of adjectives can be understood as associating items with degrees, conceived as points on a scale; yet a scale is now $\langle D, \succsim_A\rangle$, with $\succsim_A$ a semiorder on the set of degrees D. Truth conditions for the comparative and equative are in (13)–(14) (n. 6), where *adj* is the function denoted by 'ADJ' from items to degrees, and f_A and ϵ_A are a real-valued function and positive number, respectively, such that $u \succsim_A v$ iff $f_A(u) \geq f_A(v) - \epsilon_A$, for all $u, v \in D$.

(13) 'a is ADJ-er than b' is true
iff $adj(a) \succ_A adj(b)$
iff $f_A(adj(a)) - f_A(adj(b)) > \epsilon_A$

(14) 'a is as ADJ as b' is true
iff $adj(a) \sim_A adj(b)$
iff $|f_A(adj(a)) - f_A(adj(b))| \leq \epsilon_A$

One shouldn't be misled by the numerical values in the formalism. A relation $\succsim$ is a semiorder only if *there is* a real-valued function f and positive number ϵ such that $u \succsim v$ iff $f(u) \geq f(v) - \epsilon$, for all u, v. As noted in §3, degrees needn't be isomorphic to numbers, and ADJ-ness needn't be quantifiable. Talk of the numerical relation between $f_A(adj(a))$ and $f_A(adj(b))$ is compatible with a and b being as ADJ as one another, imperceptibly different in ADJ-ness, or even incomparable.

The value ϵ_A can be understood as representing a threshold of distinguishability in matters of ADJ-ness. The relations $\succ_A$, $\sim_A$ relate items that do/don't meet the threshold and count as relevantly distinguishable. The operative notion of distinguishability is specific to matters of ADJ-ness. Being discriminable doesn't imply being "distinguishable," in the sense of being related by $\succ_A$. Conversely, the fact that $adj(a) \sim_A adj(b)$ doesn't imply that a and b are indiscriminable, in general or in properties relevant to determining how ADJ they are. Per §§2–3, adjacent versions of the photo may be discriminable in brightness, and adjacent cups of coffee may be discriminable in sweetness. Saying that items are related by $\sim_A$ implies that such differences fail to distinguish them in ADJ-ness in the context.

4.3 The comparative sorites revisited

Let's apply the semantics to the comparative sorites arguments from §§2–3. To fix ideas I focus on the alternative versions of the example with 'pretty'. Start with (6), reproduced below, where x_0 is the original dimly lit photo and x_i is an otherwise identical version with a $\frac{i}{10}\%$ brightness increase. Truth conditions for the non-distinguishability premise (P2′) and the IP-transitivity premise (P3) are in (15)–(16), again where P is the function denoted by 'pretty' from items to degrees representing how pretty they are.

(6) (P1) x_{150} is prettier than x_0.
(P2′) For all n, x_{n-1} is as pretty as x_n.
(P3) For all a, b, c, if a is as pretty as b, and b is prettier than c, then a is prettier than c.
(C) So, for all n, x_n is prettier than x_0.

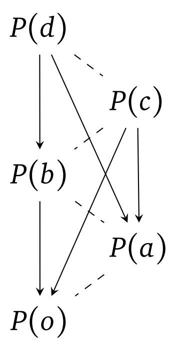
(15) (P2′) is true iff $\forall n\colon P(x_{n-1}) \sim_P P(x_n)$

(16) (P3) is true iff $\forall a,b,c\colon \big(P(a) \sim_P P(b) \wedge P(b) \succ_P P(c)\big) \rightarrow P(a) \succ_P P(c)$

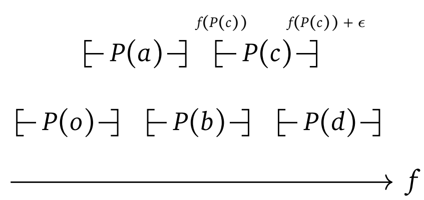
(P2′) is true according to (15) in the given scenario. Any differences between adjacent versions of the photo fail to distinguish them in prettiness. Discriminable though they might be, one version is as pretty as the next. However, (P3) is no longer semantically validated. Semiorders don't in general satisfy IP-transitivity. The subset $u \sim v \sim w$ from (12) is a simple countermodel: $v \sim u$ and $u \succ w$ but $v \nsucc w$, indeed $v \sim w$; hence $(\sim \cdot \succ) \nsubseteq \succ$. The falsity of the conclusion (C) is compatible with the truth of (P1)–(P2′).

A simplified version of the example is illustrated in (17), letting o be the original photo ($=x_0$) and a, b, c, d be edited versions differing in brightness in increments of, say, 1%; and where f_P maps each photo's degree of prettiness to its percent brightness increase compared to the original, e.g. $f_P(P(b)) = 2$, with $\epsilon_P = 1.5$.

(17) Graph representation:



Interval representation:



Numerical representation:

	f
$P(o)$	0
$P(a)$	1
$P(b)$	2
$P(c)$	3
$P(d)$	4
ϵ = 1.5	

The (P1)-style claim is true: $P(d) \succ_P P(o)$; d is prettier than o. The non-distinguishability claim is also true: $P(o) \sim_P P(a) \wedge P(a) \sim_P P(b) \wedge \cdots$; adjacent photos aren't distinguished in prettiness. Yet the intransitivity of $\sim_P$ invalidates the IP-transitivity claim, as discussed above, and the paradoxical conclusion is false: o isn't prettier than o; $P(o) \nsucc_P P(o)$.

The semantics also avoids validating the inductive premises from the two-premise versions of the arguments in scenarios where the non-distinguishability claim is true. Truth conditions for (P2) in (2), reproduced below, are in (18).

(2) (P1) x_{150} is prettier than x_0.

(P2) For all n, if x_n is prettier than x_0, then x_{n-1} is prettier than x_0.

(C) So, for all n, x_n is prettier than x_0.

(18) (P2) is true iff $\forall n{:}\, P(x_n) \succ_P P(x_0) \rightarrow P(x_{n-1}) \succ_P P(x_0)$

The counterinstance to (P2) occurs at the photo x_i, $i = \min\{k : P(x_k) \succ_P P(x_0)\}$. In the toy example in (17), photo b is prettier than o, i.e. $P(b) \succ_P P(o)$. Not so for a; the difference in brightness between them is insufficient to make one prettier than the other, i.e. $P(a) \sim_P P(o)$. (P2) is false, and (C) doesn't follow.

As Fara (2000) emphasizes, an overall account of the sorites must do more than predict that the inductive premise is not true. For instance, if the inductive premise is not true, why do we find it plausible? What should we say about the seemingly predicted "sharp boundary" between (e.g.) photo versions that are prettier than the original x_0 and versions that are not? Several directions for approaching such questions in the present semiorder-based framework are as follows.

First, unlike the traditional semantics from §3, the semantics in this section avoids conflating premises such as (P2) and (P2′) from (2) and (6). As we have seen, the non-distinguishability claim in (19) expressed by (P2′) doesn't imply the claim in (20) expressed by (P2).

(19) $\forall n{:}\, P(x_{n-1}) \sim_P P(x_n)$ *(true)*

(20) $\forall n{:}\, P(x_n) \succ_P P(x_0) \rightarrow P(x_{n-1}) \succ_P P(x_0)$ *(false)*

Key is the possible intransitivity of the non-distinguishability relation ~ (cf. Cobreros et al. 2012). Scenarios in which $u \sim v \sim w$ invalidate IP-transitivity. Paradoxical conclusions needn't follow from truisms such as (P1) and the non-distinguishability in ADJ-ness of adjacent items in the context .

Second, the formal semantics is compatible with different philosophical theories of vagueness (e.g., epistemicism, contextualism, supervaluationism). Take the treatment of the distinguishability threshold.[14] On an epistemicist theory (Sorensen 1988, Williamson 1994), facts about competent use would determine a specific set of items that are (non)distinguishable in ADJ-ness from i, for any i in the domain of 'ADJ'. There would be a context-invariant counterinstance to inductive premises such as (P2) in (2)–(3). The apparent fuzziness in the boundary between (e.g.) photo versions that are prettier than x_0 and those that are not may be diagnosed

[14]One way of formalizing the size of the distinguishability threshold δ_A for an adjective is as the cardinality of the set of degrees that are minimally $\succ_A$-better than u, for any degree u (assuming a constant threshold) — i.e., $\delta_A = |\{v : v \succ_A u \wedge \neg\exists w{:}\, w \succ_A u \wedge v \succ_A w\}|$. Depending on one's theory the threshold could be treated as a contextual parameter, as discussed below. (I continue to ignore intensionality and indexing to a world of evaluation.)

as uncertainty about the metasemantic facts determining what precise language is being spoken. Alternatively, on a broadly contextualist line, the distinguishability threshold could be treated as a contextual parameter, with different contexts c determining different semiorders $\gtrsim_{A,c}$ and thresholds for counting as distinguishable in ADJ-ness. Even if the compositional semantics assumes a particular representation of context c, there may be a range of scales compatible with speakers' interests (cf. Fara 2000), psychological states or verbal dispositions (cf. Raffman 1994, 1996), or discourse moves (cf. Kamp 1981, Soames 1999, Shapiro 2006, Silk 2016, 2021a). We may not be able to point to any instance of (P2) we reject, or any instance of the sharp boundaries claim we accept. In a supervaluationist theory, quantification over the live representations of context could be encoded in the truth conditions and formal semantics (cf. also Sassoon 2013).

If comparatives can be vague, whence the traditional idea that comparatives such as (8) with "prototypical relative adjectives" (McNally 2011: 163) are not?

(8) Alice is taller than Bert.

The vagueness of the comparatives in the examples from §2 arises from the fuzziness in how much of a certain property yields a difference in ADJ-ness — how much brightness makes for a difference in how pretty the photo is, how much sweetness makes for a difference in how tasty the coffee is, and so on. It is hard to imagine a situation in which not every difference in height affects how tall something is. Uses of 'ADJ' in contexts of maximal distinguishability are a limiting case in which $\sim_A$ is an equivalence relation. Every difference in properties determining ADJ-ness affects how ADJ things are. The non-distinguishability claim is false, and comparative sorites arguments are not compelling.

4.4 Remarks

4.4.1 Threshold structures

The semantics developed in this section uses a traditionally defined semiorder, a relation that is "crisp" and complete.[15] Both features are inessential to the framework. Cases involving incomparability ("incommensurability") between items — e.g., due to incomparable dimensions relevant for applying the adjective — may call for partial semiorders. The semantics could also be developed with a variant threshold structure. One could use a "pseudo-order," which introduces an intermediate "hesitation" zone between non-distinguishability $\sim$ and distinguishability $>$ (compare higher-order vagueness). Or, for proponents of many-valued approaches

[15] Reflexivity and the Ferrers property (§4.1) entail completeness, i.e. $u \gtrsim v \vee v \gtrsim u$, for all u, v.

to vagueness, a "fuzzy" ("valued") semiorder could be used. Formal properties of such structures have been extensively studied (see Moretti et al. 2016: §§5–6).

4.4.2 Intransitivities

The possible intransitivity of the relation $\sim_A$ in the interpretation of equatives has played a prominent role in the account in this section. It enables us to capture the truth of the non-distinguishability premise (P2′) expressing that adjacent items are as ADJ as one another, without validating the IP-transitivity premise (P3). One might worry, though, that this feature is a bug. Examples such as (21) are predicted to be consistent.[16]

(21) ??Alice is as strong as Bert, who is as strong as Chloe. But Alice isn't as strong as Chloe.

I agree that such discourses would generally be odd in an "out of the blue" context. What we see from comparative sorites cases, however, is that they shouldn't be treated as inconsistent as a matter of linguistic meaning. Recall the simplified version of the case in (17): $P(a) \sim_P P(b) \sim_P P(c) \not\sim_P P(a)$, indeed $P(c) \succ_P P(a)$.

There are further reasons not to treat sequences of the form in (21) as semantic contradictions. Consider a variant on our coffee cup case. Suppose you like both cream and sugar in your coffee. As with sweetness, it's not as if you care exactly how creamy it is. An extra sprinkle of sugar won't make a cup of coffee tastier (to you), and an extra drop of cream won't either. That combination of extra sweetness and creaminess, however, can sometimes hit the spot — i.e., where o is an ordinary cup of coffee with cream and sugar, and $o_{i,j}$ is an otherwise identical cup with an extra i drops of cream and j sprinkles of sugar:

(22) o is as tasty (to you) as $o_{1,0}$, which is as tasty (to you) as $o_{1,1}$. But o isn't as tasty (to you) as $o_{1,1}$.

Or consider a case involving incomparabilities. Suppose you are considering two job opportunities, one on the East Coast (job E) and one on the West Coast (job W). Neither is preferred to the other — they're just so different! And still, even if job E "sweetened the deal" with, say, a gift basket from a famous local deli (job E$^+$). Yet getting the gift basket is better than not, other things equal. That is:

(23) Job E is as preferable (to you) as job W, which is as preferable (to you) as job E$^+$. But it's not the case that job E is as preferable (to you) as job E$^+$.

[16] Thanks to an anonymous referee for raising such examples.

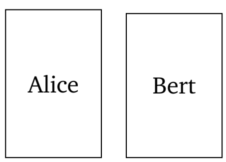
One might wonder whether (22)–(23) are true.[17] That should suffice for present purposes. We shouldn't rule out a semantics on grounds of allowing for intransitivities with equatives.

4.4.3 Comparisons

Our discussion has focused on comparatives of the form '*a* is ADJ-er than *b*'. Proceeding in this way has enabled us to bracket vagueness potentially associated with the positive form and implications of marked constructions (§1). Indeed, a precedent for invoking semiorders in semantics for comparatives is van Rooij's (2011a) delineation semantics for "implicit" comparatives, such as (1a) or (24), in which a comparison is made using the positive form.[18] The truth of the implicit comparative requires that the difference in ADJ-ness between the items be significant, as reflected in (25) (adapted from van Rooij 2011a: 65–66).

(24) Alice is tall compared to Bert.

(25)



a. Alice is taller than Bert. (*true*)

b. Alice is tall compared to Bert. (*false*)

van Rooij uses semiorders to capture the "significantly ADJ-er than" relation in the interpretation of (25b) with the positive form. "[S]tandard explicit comparatives like" (25a) with 'taller' are interpreted in terms of weak orders, as in traditional frameworks (2011a: 65; cf. §§3, 4.3). As van Rooij doesn't consider other adjectives, I leave open how he might extend his account. The semantics in this paper invokes semiorders in the scale structure and allows for vagueness phenomena with both positive and comparative forms. (I won't take a stand here on how the positive form should be analyzed.)

[17] Putative cases of intransitive indifferences have a rich history in multidisciplinary work in choice theory (e.g., Armstrong 1939, Fishburn 1970).

[18] See van Rooij 2011b,c for further appeals to semiorders in accounts of vagueness associated with positive-form predications.

5 Recap

Vagueness phenomena with comparatives have received comparatively little attention in literatures on linguistic vagueness. This paper builds on recent work which questions common assumptions that the comparative form cannot be vague. First, I show that vagueness phenomena with explicit comparatives '*a* is ADJ-er than *b*' can arise in situations of fuzziness regarding how much of some property makes for a difference in ADJ-ness. Examples such as those from §§2–3 cannot be assimilated to cases of indiscriminability or fuzziness in relevant standards, dimensions, or measurement procedures. Second, I develop a revised semantics for gradation with semiorders that provides an improved treatment of such examples. The treatment of equatives captures the truth of the claim that adjacent items in the sorites series are not relevantly distinguishable in ADJ-ness; and it avoids validating transitivities that would yield paradoxical conclusions. The formal semantics can be adapted according to one's preferred theory of vagueness.

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