Comparative Vagueness*

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1 Introduction

Can comparatives ‘a is ADJ-er than b’ be vague? A frequently expressed idea in certain linguistic circles is that they cannot. “[A]djectives in the comparative are uniformly non-vague” (Bochnak 2013: 42); “a core semantic difference between the positive [(i.e. unmodified)] and comparative forms”—e.g., between ‘tall’ and ‘taller’—“is that the latter lacks whatever semantic (or pragmatic) features give rise to the vagueness of the former” (Kennedy 2013: 270).¹ Broader work and more recent linguistic work has called this idea into question. Some have suggested that multidimensionality or uncertainty (indeterminacy, indecision, imprecision) about measurement procedures may lead comparatives to have borderline cases, a hallmark of vagueness (Williamson 1994: 156, Endicott 2000: 43–45, 140, Keefe 2000: 13–14; cf. Sassoon 2011: 102, 106–111, 2013: 172–178, 209). For instance, if individuals differ in “incommensurate dimensions” (Endicott 2000: 43) of niceness, there may be “no fact of the matter about who is nicer” (Keefe 2000: 13). Yet it has been maintained that “if we consider only one dimension of comparison, and suppose perfect accuracy in measurement, vagueness does affect the extension of the positive form of the adjective, but not that of the comparative” (Égré & Klinedinst 2011a: 10). Not so.

This paper provides new examples of vagueness phenomena with comparatives. I show that comparatives ‘a is ADJ-er than b’ can be vague due to a fuzziness in how much of some property determines a difference in ADJ-ness. The examples I provide cannot be assimilated to cases of fuzziness in what dimensions are relevant or measurement procedures.

¹First posted 2017. Please email me for permission before citing or quoting. Certain ideas in the paper draw on material from Silk 2016: chs. 6–7 and 2021a.

¹See also Cooper 1995: 246; Kennedy 2007b: 6, 2011: 74, 82–83, 93, 2013: 271, 269; McNally 2011: 164n.10; van Rooij 2011a: 65–69. I leave open to what extent these authors could allow for vague comparatives in cases such as those described below.
I focus on explicit comparatives of the form ‘a is ADJ-er than b’—synthetic, phrasal comparative constructions using a morphologically comparative form of a relative gradable adjective ‘ADJ’. I put aside “implicit” comparatives like (1a) in which a comparison is made using the positive form (Kennedy 2007a, 2011, van Rooij 2011a); and marked comparatives such as clausal comparatives like (1b) and (1c), and analytic comparatives like (1d) that have a synthetic counterpart (see Rett 2008: §§3.5, 2015: §§6.1.1, 6.2.2 and references therein). I also put aside examples with adjectives that generally imply positive-form predications, as with the “extreme” adjective in (1e) (see also Rett 2008: §3.7, 2015: §5.4, Morzycki 2012, Brasoveanu & Rett 2018).²

(1) a. a is strong compared to b.
   b. a is stronger than b is.
   c. a is stronger than b is fast.
   d. a is more strong than b.
   e. a is tinier than b. (→ b is tiny)

Restricting our attention in these ways will help abstract away from vagueness associated with a relation to a relevant standard, the sort of vagueness often observed with the positive form.³

The paper proceeds as follows. §2 presents the main examples. §3 examines their implications for traditional formal semantics for gradation. §4 develops a revised degree-based semantics with semiorders, a type of well studied threshold structure (Luce 1956, van Rooij 2011c). The semantics provides an improved interpretation of equatives in cases where certain key transitivity assumptions fail. §5 takes stock.

### 2 Comparative sorites

Suppose you are comparing edited versions of a dimly lit photo. The version with a 15% brightness increase, $x_{150}$, is prettier than the original, $x_0$. But it’s not as if every 0.1% change in brightness affects the prettiness of the photo. If you judge $x_i$ prettier than $x_0$, your judgment about $x_{i-1}$ — the otherwise identical version with a

²See Stassen 1985 on comparative constructions crosslinguistically.

³Terminology is fraught and varies widely. See e.g. linguistic work on “evaluativity” in the sense of Rett 2008, 2015. I also put aside possible connections with topics such as “subjectivity,” context-sensitivity, relativism, normativity (see also, e.g., Kennedy 2007b, 2013, Raffman 2014, Kamp & Sassoon 2016, Silk 2016, 2021a,b, Bylinina 2017, Solt 2018). I return to matters concerning standards of precision and multidimensionality briefly below.
brightness increase—should be no different. Consider (2), where \( \ldots x_i \ldots \) is the series of versions of \( x_0 \) differing only in brightness.

(2) (P1) \( x_{150} \) is prettier than \( x_0 \).

(P2) For all \( n \), if \( x_n \) is prettier than \( x_0 \), then \( x_{n-1} \) is prettier than \( x_0 \).

(C) \( \therefore \) For all \( n \), \( x_n \) is prettier than \( x_0 \).

The premises seem true, and the argument seems valid. Yet the conclusion is false. \( x_0 \) is not prettier than itself.

Or suppose you like sugar in your coffee. Yet you don’t care exactly how sweet it is. As far as your preferences go, one day’s spoonful of sugar is as good as the next. An extra, say, 0.1 gram of sugar doesn’t make a cup of coffee tastier. Consider (3)—where \( y_{100} \) is a sweetened cup of coffee with 10 grams of sugar, and \( \ldots y_i \ldots \) is a series of otherwise identical cups differing only in sugar content (cf. Luce 1956).

(3) (P1) \( y_1 \) is not tastier than \( y_{100} \).

(P2) For all \( n \), if \( y_n \) is not tastier than \( y_{100} \), then \( y_{n+1} \) is not tastier than \( y_{100} \).

(C) \( \therefore \) For all \( n \), \( y_n \) is not tastier than \( y_{100} \).

But not just any ordinary sweetened cup of coffee can be tastiest.

Sorites-susceptibility is one hallmark of vagueness. The comparatives in (2)–(3) exhibit “tolerance”: in (3), e.g., the added sweetness from 0.1 gram of sugar is “insufficient to affect the justice with which [the predicate ‘is (not) tastier than \( y_{100} \)’] applies” (Wright 1975: 349), given your preferences. Comparatives may also have “fuzzy” or “blurred boundaries of application”: “There is, for example, no sharp division between [cups] that are clearly [tastier than \( y_{100} \)] and [cups] that aren’t” (Raffman 1994: 41). The comparative may have borderline cases—cups in the “penumbra” (Russell 1923: 87) that are neither clearly tastier than \( y_{100} \) nor clearly not.

Such examples can be multiplied. Let \( F \) be a property relevant to how \( \text{ADJ} \) things are such that it is intuitively fuzzy in the context how much of a difference in \( F \)-ness makes for a difference in \( \text{ADJ} \)-ness (e.g., brightness and prettiness). Consider two items \( i, j \) in the domain of ‘\( \text{ADJ} \)’ that are significantly different in \( F \)-ness, where ‘\( i \) is (not) \( \text{ADJ} \)-er than \( j \)’ is clearly true. Continue from \( i \) to apply ‘is (not) \( \text{ADJ} \)-er than \( j \)’ to items incrementally different in \( F \)-ness. Find yourself at an item \( k \) such that ‘\( k \) is (not) \( \text{ADJ} \)-er than \( j \)’ is clearly false.

The examples in this section differ from previous types of examples of vague comparatives (§1). First, note that the force of (2)–(3) doesn’t turn on limitations in powers of discrimination. As Wright (1987: 239–243) shows, indiscriminability
between adjacent items in a sorites series isn’t necessary to generate the paradox. The incremental difference in brightness or sweetness in (2)–(3) are discriminable in the context. In (3), one simply doesn’t care exactly how sweet the coffee is. One cup is as tasty as the next given one’s preferences.

The force of (2)–(3) also doesn’t rely on multiple dimensions relevant for applying the comparative predicate. The examples proceed precisely by fixing a dimension (e.g., brightness). The interpretation with respect to a single dimension can be made linguistically explicit (e.g., ‘prettier with respect to brightness’). What is intuitively fuzzy isn’t what dimensions are relevant or their relative importance, but the relation between a particular dimension of comparison and the property associated with the adjective.

Preliminary upshot: Classic vagueness phenomena can arise with the comparative form independently of the sort of fuzziness in standards for counting as ADJ often linked with the positive form, and even if we “suppose perfect accuracy in measur[ing]” items along “only one dimension” (Égré & Klinedinst 2011a: 10).

3 Traditional semantics for gradation

This section examines the implications of examples such as (2)–(3) for traditional semantics for gradation.

It is common to locate the problem with sorites arguments with positive-form predicates such as (4)–(5) in the inductive premise. (Let $x_n$ be someone $4' + n$ millimeters tall.)

(4)   (P1) Someone who is $4'$ isn’t tall (for a pro basketball player).
      (P2) If someone who isn’t tall (for a pro basketball player) grows one nanometer, they still won’t be tall (for a pro basketball player).
      (C)  ∴ No one is tall (for a pro basketball player).

(5)   (P1) $x_0$ is not tall.
      (P2) For all $n$, if $x_n$ is not tall, then $x_{n+1}$ is not tall.
      (C)  ∴ For all $n$, $x_n$ is not tall.

For instance, even if we can’t point to any instance of (P2) in (5) that isn’t true, perhaps we can know that it isn’t true in any context (Soames 1999, Fara 2000), or

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4 Certain of the comparative sorites examples which I used in earlier work were problematic in failing to appreciate this point (Silk 2016: 198–199, 206). Thanks to Gunnar Björnsson for discussion.

5 If not, one’s discriminatory capacities or the difference between adjacent items could be adjusted accordingly.
no matter what formally precise language we might be speaking (Lewis 1970), or no matter how the conversation might evolve (Shapiro 2006), or on any competent way of applying ‘tall’ (Kamp 1981, Raffman 2014).

There is an important difference between the inductive premises (P2) in (2)–(3) vs. (5). To help illustrate, consider the three-premise variant of (2) in (6).

(6) (P1) $x_{150}$ is prettier than $x_0$.
    (P2’) For all $n$, $x_{n-1}$ is as pretty as $x_n$.
    (P3) For all $a, b, c$, if $a$ is as pretty as $b$, and $b$ is prettier than $c$, then $a$ is prettier than $c$.
    (C) $\therefore$ For all $n$, $x_n$ is prettier than $x_0$.

(P3) is an instance of what is sometimes called IP-transitivity, which is a weakening of transitivity — i.e., if a relation $\succeq$ satisfies transitivity, IP-transitivity in (7) is also satisfied, where $>$ and $\sim$ are the strict (asymmetric) and non-strict (symmetric) subsets, respectively, of $\succeq$.

(7) IP-transitivity: $\forall u, v, w : (u \sim v \land v > w) \rightarrow u > w$

Traditional semantic frameworks for gradation validate IP-transitivity premises such as (P3). Consider, first, a degree-based framework which treats gradable adjectives — adjectives that can take degree morphemes and modifiers (e.g., ‘-er’, ‘very’) — as associating items with degrees on a scale (Bartsch & Vennemann 1973, von Stechow 1984, Kennedy 1999, 2007b, Heim 2001, Morzycki 2015). For instance, on a Kennedy-style implementation, ‘tall’ denotes a function $\text{tall}$ from an individual to a degree representing the individual’s maximal height. Though some theories assume that degrees are isomorphic to rational numbers, a minimal constraint is that the relation $\geq$ on the set of degrees $D$ have the structure of a partial order, i.e. that $\geq$ be a reflexive, transitive, antisymmetric relation on $D$ (Kennedy 1999, 2007b, Barker 2002, Lassiter 2015). Compositional details aside, a comparative such as (8) is true iff the maximal degree to which Alice is tall, $\text{tall}(\text{Alice})$, is greater than the maximal degree to which Bert is tall, $\text{tall}(\text{Bert})$.

(8) Alice is taller than Bert.

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See, e.g., Sen 1970: 10–11. See Luce 1956 for earlier use of ‘P’ and ‘I’ in the context of transitivity and binary preference and indifference relations, respectively.
The interpretation of (P3) in (6) is in (9), where \( P \) — the denotation of ‘pretty’ — is a function from items to a degree representing how pretty they are.

\[
(9) \quad \forall a, b, c: (P(a) = P(b) \land P(b) > P(c)) \rightarrow P(a) > P(c)
\]

(P3) follows from the transitivity of the relation \( \geq \) on the domain of degrees \( D \).

Analogous points can be made for the other main approach to gradation in formal semantics: delineation semantics (“partial predicate,” “inherent vagueness” semantics). Gradable adjectives, on delineation theories, are analyzed as predicates whose denotation partitions a comparison class \( CC \) of relevant individuals into a positive extension, a negative extension, and an extension gap (the “borderline cases”) (cf. Klein 1980, von Stechow 1984, Burnett 2012). Following Klein 1980, a comparative such as (8) is true iff there is some comparison class in which Alice is tall and Bert is not tall. To avoid problematic entailments, delineation theories impose qualitative restrictions on comparisons among individuals across comparison classes (Fine 1975, Klein 1980, Fara 2000). For instance, if \( a \) counts as tall in some \( CC \) and \( a \)'s height is greater than \( b \)'s height, then there is no \( CC' \) in which \( b \) counts as tall and \( a \) does not. Delineation theorists prove that the qualitative restrictions derive a preorder (reflexive, transitive relation) \( \geq_A \) “at least as \( \text{ADJ} \) as” on the set of individuals in the domain of ‘\( \text{ADJ} \)’, for any adjective ‘\( \text{ADJ} \)’ (van Benthem 1982, Klein 1991, van Rooij 2011a). The result is that the interpretation of any adjective relies on a preorder \( \geq_A \) on the set of individuals. (P3) follows from the transitivity of \( \geq_A \).

Traditional semantics thus validate premises such as (P3) in (6) — due to the basic structure of scales \( (D, \geq) \), in degree-based semantics, or the qualitative ordering \( \geq_A \) on the set of individuals, in delineation semantics. That leaves premises such as (P2') as the culprit for theories of vagueness seeking to deny the inductive premise.

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7I assume an “equally” reading of the equative (cf. Bhatt & Pancheva 2007, Rett 2008). In theories assuming a basic “at least” meaning, ‘=’ can be substituted with ‘\( \geq \)’.

8What is important about degrees is that they represent how pretty, tall, etc. things are, and that they can be associated with qualitative orderings on items in adjectives’ domains. Nothing of metaphysical significance is implied by things having “degrees” of prettiness, tastiness, etc.

9Some theories also invoke a parameter for relevant standards (Lewis 1970, Barker 2002), e.g. where the positive extension of ‘tall’ is the set of individuals in \( CC \) whose height is at least the standard of tallness.

10Degrees and scales may be derived from these qualitative orderings (Cresswell 1977, Bale 2008): the set of degrees \( D \) is the set of equivalence classes under \( \geq_A \); and the relation \( \geq_A \) on \( D \) is defined accordingly such that \([a]_A \geq_A [b]_A := a \geq_A b\) (where \([u]_A\) is an equivalence class \( \{ v : v \geq_A u \land u \geq_A v \} \)).
(P2′) For all \(n\), \(x_{n-1}\) is as pretty as \(x_n\).

Yet there are costs to going this route. (P2′) is, on the face of it, true in the context. Saying this isn’t simply to say that there is a paradox; something plausible must be denied. Denying (P2′) requires denying the aesthetic possibility that a 0.1% difference in brightness might fail to make a difference in how pretty a photo is. It is hard to see why that might be so. Or consider an analogous version of (P2′) for our example with ‘tasty’ (cf. (3)).:11

(10) (P2′) For all \(n\), \(y_n\) is as tasty (to you) as \(y_{n+1}\).

(P2′) describes your non-obsessiveness about sugar in your coffee. The extra sweetness from (say) 0.1 gram of sugar doesn’t make a difference to you in how tasty the coffee is. Such tastes don’t seem impossible or incoherent, so as to be ignorable by a semantic theory. You simply don’t care exactly how sweet the coffee is.

Note that premises such as (P2′) in our examples cannot be denied by denying that adjacent items are discriminable. As discussed above, the adjacent versions of the photo are distinguishable in brightness, and the adjacent cups of coffee are distinguishable in sweetness. What is at issue is whether such differences need make one prettier or tastier than the other.

The previous points are perhaps not decisive. There may be other resources for traditional theories to help explain away the intuition that (P2′) is true, say in terms of “loose use” or “granularity” (Lasersohn 1999, Bittner & Smith 2001, Krifka 2007, Sauerland & Stateva 2011). A challenge for replies along these lines is again the observation that the intuition that (P2′) is true can persist in contexts of maximal discriminability. One cup of coffee in the series is as tasty to you as the next, not just “loosely speaking” but, on the face of it, speaking precisely, and even if you happen to be a supertaster. Saying otherwise would mischaracterize your state of mind. I won’t continue to press these issues here. Instead, I would like to use the remainder of the paper to begin investigating the prospects for an alternative framework, which avoids validating generalizations such as (P3) in (6) as a matter of conventional

11 The version of the argument in (i) uses an instance of PI-transitivity ((ii)) for (P3). As with IP-transitivity, PI-transitivity is entailed by transitivity of a relation \(\succeq\), and (P3) is validated in traditional frameworks.

(i) (P1) \(y_{100}\) is tastier than \(y_1\).
   (P2′) For all \(n\), \(y_n\) is as tasty as \(y_{n+1}\).
   (P3) For all \(a, b, c\), if \(a\) is tastier than \(b\), and \(b\) is as tasty as \(c\), then \(a\) is tastier than \(c\).
   (C) \(\therefore\) For all \(n\), \(y_{100}\) is tastier than \(y_n\).

(ii) PI-transitivity: \(\forall u, v, w:\ (u \succ v \land v \sim w) \rightarrow u > w\)
meaning. The semantics in §4 allows for the truth of \((P2')\) in (6) and (10), while circumscribing a class of cases in which the premises \((P2)\) in three-step comparative sorites arguments are false. I hope these preliminary developments may provide a useful basis for future work and eventual theory comparison.

4 Semiorders in a degree semantics

Our aim is a revised semantics for gradation that allows for the truth of equatives such as \((P2')\) and the falsity of transitivity assumptions such as \((P3)\) in certain comparative sorites cases.

\[(P2')\] For all \(n, x_{n-1}\) is as pretty as \(x_n\).

\[(P3)\] For all \(a, b, c\), if \(a\) is as pretty as \(b\), and \(b\) is prettier than \(c\), then \(a\) is prettier than \(c\).

A natural approach is to move from thinking of the relation on the set of degrees or individuals as at least a partial order (§3) to thinking of the relation as a semiorder, instead. Semiorders have been used fruitfully in measurement theory, choice theory, and mathematical psychology for representing intransitive indifferences.\(^{12}\) There are precedents for using semiorders in accounts of vagueness phenomena with predicative uses and the positive form as well (cf. Luce 1956, van Rooij 2011a,b,c, Cobreros et al. 2012).\(^{13}\) The broader research on semiorders provides an independently motivated resource to incorporate into the semantics of gradation.

4.1 Semiorders

Formally, a semiorder \(\succeq\) on a set \(S\) is an interval order—a reflexive, Ferrers binary relation—that satisfies semitransitivity ((11)); equivalently, \(\succeq\) is a semiorder iff there is a real-valued function \(f\) and fixed positive number \(\epsilon\) such that \(u \succeq v\) iff \(f(u) \geq f(v) - \epsilon\), for all \(u, v \in S\).\(^{14}\)

\[(11)\] Reflexive: \(\forall u : u \succeq u\)

Ferrers: \(\forall u, v, w, z : (u \succeq v \land w \succeq z) \rightarrow (u \succeq v \land w \succeq z)\)

Semitransitive: \(\forall u, v, w, z : (u \succeq v \land v \succeq w) \rightarrow (u \succeq z \lor z \succeq w)\)

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\(^{13}\)I return to this briefly in §4.4.3.

\(^{14}\)See the references in n. 12. For present purposes we can assume that the sets are finite. For results generalizing to arbitrary sets, see Bouyssou & Pirlot 2021a,b. See Riguet 1951 on applying Ferrers graphs to representations of certain relations.
Intuitively speaking, semiorders generalize weak orders by comparing items under a “range of fuzziness,” representable by uniform-length intervals. The Ferrers property ensures the interval representation, whereby \( u \succsim v \) iff \( u \) and \( v \) can be associated with intervals \( U, V \), respectively, such that \( V \) doesn’t wholly follow \( U \) (i.e., \( u_i \geq v_i \), for some \( u_i \in U, v_i \in V \)). Semitransitivity implies that all the intervals can be made the same length; the “fuzziness” is the same for each item (cf. Fishburn 1970: 212, 1973: 93). Strict \( \succ \) and non-strict \( \sim \) subsets of \( \succsim \) can be defined in the usual way:

\[
\begin{align*}
\text{Intuitively, the semiorder } & \succsim \text{ in (12) weakens an order in which } u > v > w > z \text{ by comparing items under a “fuzziness” that fails to distinguish } u \text{ and } v \text{ and fails to distinguish } v \text{ and } w. \text{ Notably, the non-strict part } \sim \text{ is no longer an equivalence relation. We have a case where } u \sim v \sim w; u \sim v \text{ and } v \sim w \text{ but } u \not\sim w, \text{ indeed } u > w. \text{ Such intransitivities of } \sim \text{ will be important in what follows.}
\end{align*}
\]

4.2 Degree semantics

Let’s turn to the semantics. To fix ideas I assume a Kennedy-style degree-based framework (see §3). The denotations of adjectives ‘ADJ’ may still be understood
as associating items with degrees, conceived as points on a scale; yet a scale is now \((D, \succsim_A)\), with \(\succsim_A\) a semiorder on the set of degrees \(D\). Truth conditions for the comparative and equative are in (13)–(14) (n. 7), where \(\text{adj}\) is the function denoted by ‘ADJ’ from items to degrees, and \(f_A\) and \(\epsilon_A\) are a real-valued function and positive number, respectively, such that \(u \succsim_A v\) iff \(f_A(u) \geq f_A(v) - \epsilon_A\), for all \(u, v \in D\).

\[(13)\] ‘\(a\) is ADJ-er than \(b\)’ is true
\[
\text{iff } \text{adj}(a) >_A \text{adj}(b)
\]
\[
\text{iff } f_A(\text{adj}(a)) - f_A(\text{adj}(b)) > \epsilon_A
\]
\[(14)\] ‘\(a\) is as ADJ as \(b\)’ is true
\[
\text{iff } \text{adj}(a) \sim_A \text{adj}(b)
\]
\[
\text{iff } |f_A(\text{adj}(a)) - f_A(\text{adj}(b))| \leq \epsilon_A
\]

One shouldn’t be misled by the numerical values in the formalism. A relation \(\succsim\) is a semiorder only if there is a function \(f\) and positive number \(\epsilon\) such that \(u \succsim v\) iff \(f(u) \geq f(v) - \epsilon\), for all \(u, v\). As noted in §3, degrees needn’t be isomorphic to numbers, and properties of ADJ-ness needn’t be quantifiable. Talk of the numerical relation between \(f_A(\text{adj}(a))\) and \(f_A(\text{adj}(b))\) is compatible with \(a\) and \(b\) being as ADJ as one another, imperceptibly different in ADJ-ness, or even incomparable.

The value \(\epsilon_A\) can be understood as representing a threshold of distinguishability in matters of ADJ-ness. The relations \(>_A\), \(\sim_A\) relate items that do/don’t meet the threshold and count as relevantly distinguishable. The operative notion of distinguishability is specific to matters of ADJ-ness. Being discriminable doesn’t imply being “distinguishable,” in the sense of being related by \(>_A\). Conversely, the fact that \(\text{adj}(a) \sim_A \text{adj}(b)\) doesn’t imply that \(a\) and \(b\) are indistinguishable, in general or in properties relevant to determining how ADJ they are. Per §§2–3, the adjacent versions of the photo may be discriminable in brightness, and the adjacent cups of coffee maybe discriminable in sweetness. To say that items are related by \(\sim_A\) is to imply that any such differences fail to distinguish them in ADJ-ness in the context.

### 4.3 Comparative sorites revisited

Let’s apply the semantics to the comparative sorites arguments from §§2–3. To fix ideas I focus on the alternative versions of the example with ‘pretty’. Start with (6), reproduced below, where \(x_0\) is the original dimly lit photo and \(x_i\) is an otherwise identical version with a \(\frac{1}{10}\%\) brightness increase. Truth conditions for the non-distinguishability premise (P2′) and the IP-transitivity premise (P3) are in (15)–(16), again letting \(P\) be the function denoted by ‘pretty’ from items to degrees representing how pretty they are.
(6) (P1) \( x_{150} \) is prettier than \( x_0 \).
(P2') For all \( n \), \( x_{n-1} \) is as pretty as \( x_n \).
(P3) For all \( a, b, c \), if \( a \) is as pretty as \( b \), and \( b \) is prettier than \( c \), then \( a \) is prettier than \( c \).
(C) \( \therefore \) For all \( n \), \( x_n \) is prettier than \( x_0 \).

(15) (P2') is true iff \( \forall n : P(x_{n-1}) \sim_p P(x_n) \)

(16) (P3) is true iff \( \forall a, b, c : \left( P(a) \sim_p P(b) \land P(b) \succ_p P(c) \right) \rightarrow P(a) \succ_p P(c) \)

The non-distinguishability premise (P2') is true according to (15) in the given scenario. Any difference in prettiness between adjacent versions of the photo fails to exceed the distinguishability threshold \( \epsilon_p \). Discriminable though they might be, one version is as pretty as the next. However, (P3) is no longer semantically validated. Semiorders don't in general satisfy IP-transitivity. The subset \( \langle u \sim v \sim w \rangle \) from (12) is a simple countermodel: \( v \sim u \) and \( u \succ w \) but \( v \nsucc w \); hence \( (\sim \cdot \succ) \notin \succ \). The falsity of the conclusion (C) is compatible with the truth of (P1)–(P2').

Consider the simplified example in (17), letting \( o \) be the original photo (=\( x_0 \)) and \( a, b, c, d \) be edited versions differing in brightness in increments of, say, 1%. Suppose \( f_p \) maps each version's degree of prettiness to its percent brightness increase compared to the original, e.g. \( f_p(P(b)) = 2 \), with \( \epsilon_p = 1.5 \). Visually:

\begin{align*}
\text{(17) Graph representation:} & \quad \text{Interval representation:} & \quad \text{Numerical representation:} \\
\begin{array}{c}
P(d) \quad P(c) \quad P(b) \quad P(a) \quad P(o) \\
\end{array} & \quad \begin{array}{c}
P(a) \quad P(c) \quad P(b) \quad P(d) \quad P(o) \\
\end{array} & \quad \begin{array}{c}
f \quad f \\
0 \quad 1 \quad 2 \quad 3 \quad 4 \\
\end{array} \\
\end{align*}

\begin{align*}
\sim_p \quad \succ_p \\
\end{align*}

The (P1)-style premise is true: \( P(d) \succ_p P(o) \); \( d \) is prettier than \( o \). The non-distinguishability premise is also true: \( P(o) \sim_p P(a) \land P(a) \sim_p P(b) \land \cdots \); adjacent photos aren’t distinguished in prettiness. Yet, as discussed above, the intransitivity of \( \sim_p \) invalidates (P3), and the conclusion (C) is false: \( o \) isn’t prettier than itself (nor is \( a \) in the given scenario); \( P(o) \nsucc_p P(o) \).
The semantics also avoids validating the inductive premises from the three-step versions of the arguments in (2)–(3). Truth conditions for (P2) in (2), reproduced below, are in (18).

\[(\text{P1}) \quad x_{150} \text{ is prettier than } x_0.\]
\[(\text{P2}) \quad \text{For all } n, \text{ if } x_n \text{ is prettier than } x_0, \text{ then } x_{n-1} \text{ is prettier than } x_0.\]
\[(\text{C}) \quad \therefore \text{ For all } n, x_n \text{ is prettier than } x_0.\]

(18) (P2) is true iff \(\forall n: P(x_n) \succ_p P(x_0) \rightarrow P(x_{n-1}) \succ_p P(x_0)\)

The counterinstance to (P2) occurs at the photo \(x_i\), \(i = \min\{k: P(x_k) \succ_p P(x_0)\}\). In the toy example in (17), photo version \(b\) is prettier than \(o (=x_0)\), i.e. \(P(b) \succ_p P(o)\). Not so for \(a\); the difference in brightness is insufficient to make one prettier than the other, i.e. \(P(a) \sim_p P(o)\). (P2) is false, and (C) doesn’t follow.

As Fara (2000) emphasizes, an overall account of the sorites must do more than predict that the inductive premise is not true. For instance, if the inductive premise is not true, why do we find it plausible? What should we say about the seemingly predicted “sharp boundary” between (e.g.) photo versions that are prettier than the original \(x_0\) and versions that are not? Several directions for approaching such questions in the present semiorder-based framework are as follows.

First, unlike the traditional semantics from §3, the semantics in this section avoids conflating premises such as (P2) and (P2'). As we have seen, the non-distinguishability claim expressed by (P2') in (19) doesn’t imply the claim expressed by (P2) in (20).

\[(\text{19}) \quad \forall n: P(x_{n-1}) \sim_p P(x_n) \quad \text{(true)}\]
\[(\text{20}) \quad \forall n: P(x_n) \succ_p P(x_0) \rightarrow P(x_{n-1}) \succ_p P(x_0) \quad \text{(false)}\]

Key is the possible intransitivity of the non-distinguishability relation \(\sim\) (cf. Cobreros et al. 2012). Scenarios in which \(\hat{u} \sim \check{v} \sim \check{w}\) invalidate IP-transitivity and falsify (P2). Paradoxical conclusions needn’t follow from truisms such as (P1) and the non-distinguishability in ADJ-ness in the context of adjacent items in the series.

Second, the formal semantics is compatible with alternative philosophical theories of vagueness (e.g., epistemicism, contextualism, supervaluationism). Consider, for example, the treatment of the distinguishability threshold. On an epistemicist theory (Sorensen 1988, Williamson 1994), facts about competent use across contexts would determine a specific set of items that are (non)distinguishable in ADJ-ness from \(x_i\), for any \(x_i\) in the series. There would be a context-invariant counterinstance to inductive premises such as (P2) in (2)–(3). The apparent fuzziness in
the boundary between (e.g.) photo versions that are prettier than \( x_0 \) and those that are not may be diagnosed as uncertainty about the metasemantic facts determining what precise language is being spoken. Alternatively, on a broadly contextualist line, the “size” of the distinguishability threshold can be treated as a contextual parameter, with different contexts \( c \) determining different semiorders \( \succsim_{A,c} \) and thresholds for counting as distinguishable in ADJ-ness.\(^{15}\) Even if the compositional semantics assumes a particular representation of context \( c \), there may be a range of scales and thresholds compatible with speakers’ interests (cf. Fara 2000), psychological states or verbal dispositions (cf. Raffman 1994, 1996), or discourse moves (cf. Kamp 1981, Soames 1999, Shapiro 2006, Silk 2016, 2021a). We may not be able to point to any instance of (P2) we reject, or any instance of the sharp boundaries claim we accept. In a supervaluationist theory, supervaluating over such live representations of context could be encoded in the truth conditions and formal semantics (cf. also Sassoon 2013 and references therein).

If comparatives can be vague, whence the traditional idea that comparatives such as (8) with “prototypical relative adjectives” (McNally 2011: 163) are not?

(8) Alice is taller than Bert.

The vagueness of comparatives such as those in §2 turns on a fuzziness in how much of a certain property makes for a difference in ADJ-ness — how much brightness makes for a difference in how pretty the photo is, how much sweetness makes for a difference in how tasty the coffee is, and so on. It is hard to imagine a situation in which not every difference in height affects how tall something is. Uses of ‘ADJ’ in contexts of maximal distinguishability are a limiting case in which \( \sim_A \) is an equivalence relation. Every difference in properties determining ADJ-ness affects how ADJ things are. The non-distinguishability claim is false, and the comparative sorites doesn’t arise.

4.4 Remarks

4.4.1 Threshold structures

The semantics developed in this section uses a traditionally defined semiorder, a relation that is “crisp” and complete.\(^{16}\) Both features are inessential to the framework.

\(^{15}\) The size of the distinguishability threshold \( \delta_A \) for an adjective ADJ may be understood as the cardinality of the set of \( \succsim_{A} \)-minimal degrees that are \( \succ_A \)-better than \( u \), for any degree \( u \) — formally, \( \delta_A = \left| \{ v : v \succ_A u \land \sim \exists w : w \succ_A u \land v \succ_A w \} \right| \). Depending on one’s theory, the threshold may be treated as a contextual parameter. (I continue to ignore intensionality and indexing to a world of evaluation.)

\(^{16}\) Reflexivity and the Ferrers property (§4.1) entail completeness, i.e. \( u \succ v \lor v \succ u \), for all \( u, v \in S \).
Cases with incomparability (“incommensurability”) between items, such as from incomparable dimensions relevant for applying the adjective, may call for partial semiorders. The semantics could also be developed with a variant threshold structure. One could use a “pseudo-order,” which introduces an intermediate “hesitation” zone between non-distinguishability ∼ and distinguishability > (compare higher-order vagueness). Or, for proponents of many-valued approaches to vagueness, a “fuzzy” (“valued”) semiorder could be substituted. Formal properties of such structures have been extensively studied (see Moretti et al. 2016: §§5–6).

### 4.4.2 Intransitivities

A key feature of the account of the comparative sorites in this section is the possible intransitivity of the relation ∼A figuring in the interpretation of equatives ‘a is as ADJ as b’. This allows us to capture the truth of the non-distinguishability premise (P2’), that adjacent items are as ADJ as one another, while invalidating the IP/PI-transitivity premise (P3). One might worry, though, that this feature is a bug. Sentences such as (21) are predicted to be consistent.¹⁷

(21) ??Alice is as strong as Bert, who is as strong as Chloe. But Alice isn’t as strong as Chloe.

I agree that such sentences would generally be odd in an “out of the blue” context. What we see from comparative sorites cases, however, on the present account, is that they shouldn’t be treated as inconsistent as a matter of linguistic meaning. Recall the illustration in (17): we have a case in which P(a) ∼ P(b) ∼ P(c) ∼ P(a), indeed P(c) > P(a).

There are further reasons not to treat sequences of the form in (21) as semantic contradictions. Consider a variant on our coffee cup case. Suppose you like both cream and sugar in your coffee. As with sweetness, it’s not as if you care exactly how creamy it is. An extra sprinkle of sugar won’t make a cup of coffee tastier (to you), and an extra drop of cream won’t either. That combination of extra sweetness and creaminess, however, can sometimes hit the spot. That is, where o is an ordinary cup of coffee, and o₁,ᵢ is an otherwise identical cup with an extra i drops of cream and j sprinkles of sugar:

(22) o is as tasty (to you) as o₁,₀, which is as tasty (to you) as o₁,₁. But o isn’t as tasty (to you) as o₁,₁.

¹⁷Thanks to an anonymous referee for raising such examples.
Or consider a case involving incomparabilities. Suppose you are considering two job opportunities, one on the East Coast (E) and one on the West Coast (W). Neither is preferred to the other; they’re just so different! Likewise if job E “sweetened the deal” with, say, a gift basket from a famous local deli (E+). Yet getting the gift basket is better than not, other things equal. That is:

(23) Job E is as preferable (to you) as job W, which is as preferable (to you) as job E+. But it’s not the case that job E is as preferable (to you) as job E+.

One might wonder whether such examples are true. That should hopefully suffice for present purposes. We shouldn’t rule out a semantics on grounds of allowing for intransitivities with equatives.

4.4.3 Comparisons
Our discussion has focused on comparatives of the form ‘a is ADJ-er than b’. Proceeding in this way has enabled us to bracket vagueness potentially associated with the positive form and implications arising from marked constructions (see §1). Indeed, a precedent for invoking semiorders in semantics for comparatives is van Rooij’s (2011a) delineation semantics for “implicit” comparatives, such as (1a) or (24), in which a comparison is made using the positive form. The truth of an implicit comparative ‘a is ADJ compared to b’ requires that the difference between the items be significant, as reflected in (25) (adapted from van Rooij 2011a: 65–66).

(24) Alice is tall compared to Bert.

(25)

| Alice | Bert |

| a. Alice is taller than Bert. (true) |
| b. Alice is tall compared to Bert. (false) |

van Rooij uses semiorders to capture this “significantly ADJ-er than” relation in the interpretation of (25b) with the positive form. “[S]tandard explicit comparatives like” (25a) with ‘taller’ are interpreted in terms of weak orders, as in traditional

18Cases of putative intransitive indifferences are familiar from multidisciplinary work in choice theory (e.g., Armstrong 1939, Fishburn 1970).
19See also van Rooij’s 2011b, 2011c for appeals to semiorders in accounts of vagueness associated with positive-form predications.
frameworks (2011a: 65; cf. §§3, 4.3). As van Rooij doesn’t consider other adjectives, I leave open how he might extend his account. The semantics in this paper invokes semiorders in the scale structure and allows for vagueness phenomena with both positive and comparative forms. (I won’t take a stand here on how the positive form should be analyzed.)

5 Recap

Vagueness phenomena with comparatives have received comparatively little attention in literatures on linguistic vagueness. Consideration of adjectives such as ‘tall’ has led some theorists to assume that the comparative form cannot be vague. This paper builds on more recent work which has questioned this assumption. First, I show that vagueness phenomena with explicit comparatives ‘a is ADJ-er than b’ can arise in situations of fuzziness regarding how much of some property makes for a difference in ADJ-ness. Examples such as those from §§2–3 cannot be assimilated to cases of indiscriminability or fuzziness in relevant standards, dimensions, or measurement procedures. Second, I develop a revised semantics for gradation with semiorders that provides an improved treatment of such examples. The treatment of equatives captures the truth of claims expressing that adjacent items in a sorites series are not relevantly distinguishable in ADJ-ness. And it avoids validating transitivities that would then yield paradoxical conclusions. The formal semantics can be adapted according to one’s preferred theory of vagueness.

References


