“IF-THEN” AS A VERSION OF “IMPLIES”

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ABSTRACT

Russell’s role in the controversy about the paradoxes of material implication is usually presented as a tale of how even the greatest minds can fall prey of basic conceptual confusions. Quine accused him of making a silly mistake in *Principia Mathematica.* He interpreted “if-then” as a version of “implies” and called it material implication. Quine’s accusation is that this decision involved a use-mention fallacy because the antecedent and consequent of “if-then” are used instead of being mentioned as the premise and the conclusion of an implication relation. It was his opinion that the criticisms and alternatives to the material implication presented by C. I. Lewis and others would never be made in the first place if Russell simply called the Philonian construction “material conditional” instead of “material implication”. Quine’s interpretation on the topic became hugely influential, if not universally accepted. This paper will present the following criticisms against this interpretation: (1) the notion of material implication does not involve a use-mention fallacy, since the components of “if-then” are mentioned and not used; (2) Quine’s belief that the components of “if-then” are used was motivated by a conditional-assertion view of conditionals that is widely controversial and faces numerous difficulties; (3) if anything, it was Quine who could be accused of fallacious reasoning: he ignored that in the assertion of a conditional is the whole proposition that is asserted and not its constituents; (4) the Philonian construction remains counter-intuitive even if it is called “material conditional”; (5) the Philonian construction is more plausible when it is interpreted as a material implication.

Keywords: material implication; conditionals; if-then; use-mention fallacy; conditional-assertion theories; *Principia Mathematica*.

1. INTRODUCTION

> “much confusion has been produced in logic by the attempt to identify conditional statements with expressions of entailment.” – William & Martha Kneale, *The Development of Logic*
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> “whereas there is much to be said for the material conditional as a version of “if-then”, there is nothing to be said for it as a version of ‘implies’.” – W. V. O. Quine, *Word and Object*

In the *Principia Mathematica*, Russell employed the notion of material implication in order to interpret “if-then” constructions in formal logic. According to Quine, this choice of terminology was fallacious, since it involves a confusion between the use and mention of words. Quine’s accusation became influential. This paper will argue that this widely accepted accusation is unfounded, for the antecedent and consequent of “if-then” are mentioned, not used. It is also argued that interpreting conditionals as assertions of material implication can provide fruitful solutions to known puzzles in the literature.

It is important to notice, however, that while there's an interesting proposal to be made and textual evidence that may justify Russell's choice of terminology, a full-blown defense of material implication will require concepts and intuitions that were completely alien to Russell. Not only Russell never intended to use material implication to interpret "if-then" constructions in natural language, as the present proposal will require modal intuitions that he openly refused in his posthumously published paper "Necessity And Possibility" (1905). According to Russell, the modal operators of "necessity" and "possibility" have only an epistemological or psychological significance and should have no place in formal logic. Instead, Russell tried to deflect the criticisms against material implication with a pragmatic defense of his choice of terminology. The position advanced in this paper couldn't be more different even if it is inspired in Russell's writings. Oddly enough, the notion of material implication that is currently perceived as an ancient artefact from the old days can only be reinvigorated into its full force with the use of contemporary ideas that weren't popular in Russell's time.

The paper will be divided as follows. Section 2 presents the Principia controversy over material implication in detail. Bertrand Russell choice of terminology and his defence of the material implication interpretation are discussed, followed by C.I. Lewis's criticism and Quine's accusation that the notion of material implication is a use-mention fallacy. In section 3, the attribution of use-mention fallacy to Russell's interpretation is questioned. Not only Russell was perfectly aware of the distinction between use and mention, as it is arguable that the components of a conditional are mentioned and not used. Quine's belief that the components of "if-then" are used was motivated by a conditional-assertion view of conditionals that is widely controversial and faces numerous difficulties. If anything, it was Quine who could be accused of fallacious reasoning: he ignored that in the assertion of a conditional is the whole proposition that is asserted and not its constituents. Section 4 presents a revamped interpretation of material implication. This approach allows us to solve a series of puzzles in the current literature about conditionals. Section 5 concludes.

2. THE PRINCIPIA CONTROVERSY

Russell's changed his ideas about logic constantly, but some core views remained the same throughout his lifework[1]. Russell firmly believed that symbolic logic captures the essence of deductive reasoning and that we should develop a symbolic logic capable of showing that mathematics is reducible to logic. More importantly, he endorsed the notion of implication as fundamental to our understanding of deduction and believed that there are two types of implication: material and formal. Material implication is a proposition which displays a relation between two propositions, let's say, p and q. The statement "p materially implies q" is symbolized as $p \supset q$ and is true unless p is true and q is false, i.e., whenever p is not true or q is true[2]. Russell interprets "if-then" sentences as assertions of material implication, so "p

[1] Russell's views about logic are presented in works that are too numerous to mention. Some of the main references include "The Principles of Mathematics" (1903), "The Theory of Implication" (1906), ''If' And 'Imply', A Reply To Mr. MacColl" (1908), "Principia Mathematica" (1910), "Some Explanations in Reply to Mr. Bradley" (1910), "The Philosophical Importance of Mathematical Logic" (1913) and "Introduction to Mathematical Philosophy" (1919). Two articles that were published posthumously, "Recent Italian Work on The Foundation of Mathematics" (1901) and "Necessity and Possibility" (1905), repeat some of the main ideas of his other works.
[2] Russell & Whitehead (1910: 7).

materially implies *q*" can also be read as "if *p*, then *q*"[3]. Formal implication is the implication we find today in first order predicate calculus in such formulas as (x) (Fx ⊃ Gx)[4].

From his discussion of material implication, Russell draws three curious inferences which would be known as the paradoxes of material implication: (1) for any two propositions, one of these propositions must imply the other; (2) false propositions imply all propositions; (3) true propositions are implied by all propositions. These counter-intuitive consequences were bombarded with criticisms. C. I. Lewis was its main detractor[5]. In *The Calculus of Strict Implication*, Lewis objected that material implication didn't do justice to our intuitions about implication:

> If '*p* implies *q*' means only 'it is false that *p* is true and *q* false,' then the implication relation is far too ubiquitous to be of any use[6].

The idea that material implication "is far too ubiquitous to be of any use" is motivated by Lewis' view that *p* can only imply *q* when *q* is a logical consequence of *p*. In other words, the notion of implication is linked with the notion of logical consequence and its related cousins ("logical inference", "entailment", "valid deduction", etc.). In *Interesting theorems in symbolic logic*, Lewis drew the apocalyptic consequences from treating implication, and, therefore, logical consequence, as material implication. This meant that the *Principia* theorems were be under suspicion and formal logic would collapse:

> The consequences of this difference between the 'implies' of the algebra and the 'implies' of valid inference are most serious. Not only does the calculus of implication contain false theorems, but all its theorems are not proved. For the theorems of the system are implied by the postulates in the sense of 'implies' which the system uses. The postulates have not been shown to imply any of the theorems except in this arbitrary sense. Hence, it has not been demonstrated that the theorems can be inferred from the postulates, even if all the postulates are granted. The assumptions, e. g., of 'Principia Mathematica,' imply the theorems in the same sense that a false proposition implies anything, or the first half of any of the above theorems implies the last half[7].

Lewis' point is that Russell identifies the deductibility of *q* from *p* with the material implication of *q* from *p*. This implies that in order for *q* being deducible from *p* is enough that *p* is false or *q* is true. Lewis objected to the implausibility of this consequence. Given that the proposition "Pigs fly" is false, I'm not willing to admit that every proposition is inferable from "Pigs fly". If any true proposition is implied by any proposition, and necessarily true propositions are implied by any proposition, it follows that every true proposition is necessarily true. If the proofs in the *Principia* were made in this way, they would not be truths, since a proof is based on premises that are assumed as true in order to arrive at the truth of a conclusion whose truth was not admitted.

One of Lewis criticisms is that the notion of material implication ignores modal distinctions that are intuitively tied to implication. To use our contemporary idiolect, a relation of material implication would only require certain combinations of truth values in the actual world, but logical inference requires a stronger connection:

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3 Russell & Whitehead (1910: 208).
4 Although Russell confusedly thought that formal implication belongs in the propositional calculus.
5 For an overview of the clash between Russell and Lewis, see Barker (2006).
6 Lewis (1914: 246).
7 Lewis (1913: 242).

> Material implication it will appear, applies to any world in which the all-possible is the real, and cannot apply to a world in which there is a difference between real and possible, between false and absurd. Strict implication has a wider range of application. Most importantly it admits of the distinction of true and necessary, of false and meaningless[8].

In *Symbolic Logic*, Lewis and Langford proposed an alternative logical system based on a different notion of implication that would better represent Lewis' intuitions about the subject:

> It appears that the relation of strict implication expresses precisely that relation which holds when valid deduction is possible. It fails to hold when valid deduction is not possible[9].

The relation where p strictly implies q is symbolized as $p \prec q$, and it is only be true when q is logically inferable from p, and it is logically equivalent to $\neg\Diamond(p\&\neg q)$. The possibility must be understood as a logical possibility, since $\Diamond p$ means "p is self-consistent"[10]. From this explanation, it follows that "if p, then q" is only false if it is logically impossible that $p\&\neg q$. But the strict implication can be hardly qualified as an improvement over material implication since it is also packed with paradoxical aspects. Suppose that a father tells his children: "If it rains tomorrow, we will go to the cinema". It is logically possible that the antecedent is true and the consequent false, since there is no logical inconsistence in admitting that it will rain, but they decided not to go to the cinema. It is also physically possible that it will rain tomorrow, but they will not go to the cinema. Actually, since the conditional represents an attempt from a father to please his children it is assumed that there is no logical necessity between the antecedent and the consequent[11]. This conditional, like most examples of natural language conditionals, are in disagreement with Lewis' explanation.

There are other problems. Since "p strictly implies q" can also be read as "it is necessary that p materially implies q", it follows that p strictly implies if p is a logical contradiction or q is a tautology. Thus, the proposition "It is raining and it is not raining" strictly implies every proposition because it is a logical contradiction. The difference is that Lewis would feel more confident in accepting these counter-intuitive aspects since they are already present in the classical notion of logical consequence. But the notion of material implication was never intended as synonymous with the notion of logical consequence, even though it is an implication in its own right.

Russell's defense of the material implication is pragmatic in character. In a response to a letter of Bradley, which contained criticisms similar to the ones advanced by Lewis, Russell argued that the term "implication" is used in a special technical sense that does not have the consequences claimed by the critics[12]:

> The essential property that we require of implication is this: "What is implied by a true proposition is true". It is in virtue of this property that implication yields proofs. But this property by no means determines whether anything, and if so what, is implied by a false proposition, or by something which is not a proposition at all. What it does determine is that, if p implies q, then it cannot be the case that p is true and q is not true[13].

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[8] Lewis (1914: 241).
[9] Lewis & Langford (1932: 247).
[10] Lewis & Langford (1932: 123).
[11] Braine (1979: 155).
[12] Russell (1910: 350).
[13] Russell (1906: 161–62).

This technical sense of implication would be justified since it would be enough to represent accurately "if-then" mathematical propositions using the new logic. The strange nature of material implication is harmless because it doesn't allows us to infer false propositions from true propositions. In subsequent replies to Lewis, Russell insists on the same point:

> The essential point of difference between the theory which I advocate and the theory advocated by Professor Lewis is this: He maintains that, when one proposition *q* is "formally deducible" from another *p*, the relation which we perceive between them is one which he calls "strict implication," which is not the relation expressed by "not-*p* or *q*" but a narrower relation, holding only when there are certain formal connections between *p* and *q*[14].

In other words, the narrower relation of implication wasn't required to ensure the truth of conditional sentences. Russell's defense never pleased the critics, since the lingering intuition that implication is tied to logical consequence persisted. The only way to savage material implication requires modal distinctions, but Russell was against the use of modal intuitions in logic. In a sense, these distinctions would only become clear decades later with the development of modal logic.

Quine's reading of this controversy decades later is that Russell committed a use-mention fallacy, which lead Clarence Lewis to fall in the same mistake with his strict implication proposal. The distinction between use and mention can be illustrated by the following example: the sentence "China is a populated country" is a statement about an attribute of China. The word "China" is being used in this context, not mentioned it. The sentence "China has two syllables" is a statement about a word, the name of a country. In this context, China is being mentioned and not used to refer to the country. According to Quine, Russell committed the use-mention fallacy because he interpreted the conditional connective as a statement of logical implication, in which the antecedent and consequent are mentioned as the premise and conclusion of an entailment relation; whereas genuine conditionals do not mention statements, but use them to express a relation between facts and objects in the world. If Russell had acknowledged this distinction, insisted Quine, he would never have equated "if-then" with "implies":

> Lewis founded modern modal logic, but Russell provoked him into it. For whereas there is much to be said for the material conditional as a version of 'if-then', there is nothing to be said for it as a version of 'implies'; and Russell called it implication, thus apparently leaving no place open for genuine deductive connections between sentences. Lewis moved to save the connections. But his way was not, as one could have wished, to sort out Russell's confusion of 'implies' with 'if-then'. Instead, preserving that confusion, he propounded a strict conditional and called it implication[15].

> It is doubtful that Lewis would have even started this [modal logic] if Whitehead and Russell, who followed Frege in defending Philo of Megara's version of 'If *p* then *q*' as 'Not(*p* and not *q*)', had not made the mistake of calling the Philonian construction "material implication" instead of the material conditional[16].

[14] Russell (1919: 154).
[15] Quine (1961: 323).
[16] Quine (1964: 196). The notion that Philonean implication is equivalent to the material implication is also doubtful since the stoics viewed propositions as tensed. For Filo, a conditional $p \rightarrow q$ is true if, and only if, it is not the case that *p* is true and *q* is false *at the present moment*. But it is arguable that the notion of material implication ignores this temporality restriction (Rescher, 2007: 48). I will ignore this anachronism for the sake of exposition, since Quine himself ignored this distinction.

It is difficult to overestimate the influence of Quine's opinions about this subject. It is due to his criticisms alone that the term "material implication" fell into disuse and was replaced by "material conditional". But being influential is not synonymous with being truthful and there are plenty of reasons to question Quine's conclusions.

3. QUINE'S UNFAIR ACCUSATION

The accusation that the material interpretation involves a use-mention fallacy is predicated on the idea that the antecedent and the consequent in a conditional are used, not mentioned. But the antecedent and the consequent are not used. This becomes clearer when we consider the interpretation and formalization of arguments in natural language. An argument such as "*p*, therefore *q*" is not interpreted as using, in the sense of asserting, either *p* or *q*, but instead is interpreted as a more complex statement, namely, "*p* deductively implies *q*". The same could be said about "if *p*, then *q*", where *p* and *q* are not being used in the sense of being asserted, but are mentioned. This conditional should be interpreted as a more complex statement, namely, "*p* materially implies *q*".

It is arguable that Quine was misled into thinking that the antecedent and consequent of a conditional are used because he was influenced by a conditional-assertion view of conditionals, according to which "if *p*, then *q*" should be interpreted as a conditional assertion of *q* given the assumption of *p*. This becomes clear in the following passage of *Methods of Logic*:

> An affirmation of the form 'if *p* then *q*' is commonly felt less an affirmation of a condition than as a conditional affirmation of the consequent. If after we have made such an affirmation, the antecedent turns out true, then we consider ourselves committed to the consequent, and are ready to acknowledge error if it proves false. If on the other hand the antecedent turns out to have been false, our conditional affirmation is as if it had never been made[17].

But this type of conditional-assertion theory of conditionals is widely controversial and counter-intuitive. Suppose one says "If it rains, I won't go to the supermarket". Now suppose that it didn't rain. According to the conditional-assertion theory, the speaker never said anything because the antecedent turned out to be false. This violates our intuitions about conditionals. In fact, given that many of conditionals in natural language probably have false antecedents, this would imply that speakers are not saying anything with conditionals a significant part of the time! Of course, one could try to defend the conditional-assertion theory and rebut these criticisms, but it is doubtful that everyone who endorses Quine's criticisms as an uncontroversial truism in logic would gladly accept this theoretical burden.

In fact, it can be argued that if there is one who is guilty of committing a fallacy here is Quine, who erroneously assumed that the components of a compound proposition are asserted like usual propositions. But perhaps with the exception of conjunctions, when a compound proposition such as a conditional, biconditional or disjunction are asserted, it's the whole proposition that is asserted and not its propositional constituents. The assertion of a conditional then is as a statement about a relation between the propositions expressed by the antecedent and consequent. In other words, the antecedent and consequent *are mentioned*, not used. In order for them to be used, they would need to be asserted, but they cannot be asserted, for what is asserted is the conditional, not its propositional constituents. In the assertion of the conditional "If it rained on Thursday, the match was cancelled", the antecedent "it rained on Thursday", and the consequent "the match was cancelled", are mentioned, not used. The speaker is stating that the consequent follows from the antecedent. It is a statement of logical

[17] Quine (1950: 12).

implication of some sort. Russell understood this distinction and observed that "the proposition '*p* implies *q*' *asserts* an implication, though it does not *assert p* or *q*"[18].

In fact, there is textual evidence to suggest that Russell was perfectly aware of the distinction between use and mention, as is illustrated in the following passage:

> Any proposition may be either asserted or merely considered. If I say "Caesar died", I assert the proposition "Caesar died" if I say ''Caesar died is a proposition", I make a different assertion, and "Caesar died" is no longer asserted, but merely considered. Similarly in a hypothetical proposition, e. g. "if $a = b$, then $b = a$", we have two unasserted propositions, namely "$a = b$" and "$b = a$" while what is asserted is that the first of these implies the second[19].

Now, notice that Russell uses the term "considered" as synonymous of "mentioned". A proposition is merely considered when it is mentioned, instead of being asserted.

The other point that needs to be considered is whether Russell was aware of the distinction between material implication and logical implication in the sense of entailment or deducibility. Even though Russell ignored the modal distinctions that would settle the discussion on this issue in his favour, he presented independent arguments that made it clear that he didn't make this confusion. While explaining Lewis Carroll's puzzle about the tortoise in *Principles of Mathematics*, Russell observes that "We need, in fact, the notion of therefore, which is quite different from the notion of implies, and holds between different entities"[20]. In other words, the notion that *p* materially implies *q* is quite different from the notion that *q* deductively follows from *p*.

That Russell made a clear distinction between material implication and deductibility is also evidenced by his argument that the notion of implication is more primitive than deductibility. Russell claims that "*q* is implied by *p*" cannot mean the same as "*q* is deducible from *p*" since it would mean that there exists a set of principles of deduction from which it can be demonstrated that *p* implies *q*. The notion of "deducible from" is defined in terms of the principles of deduction that employ the notion of implication. Thus, it would not be permissible to substitute "implied by" for "deducible from" due to the charge of circularity.

It could be objected that it is not obvious that Quine was an enthusiast of the conditional-assertion theory of conditionals. The evidence is the passage that immediately follows the one quoted above:

> Departing from this usual attitude, however, let us think of conditionals simply as compound statements which, like conjunctions and alternations, admit as wholes of truth and falsity[21].

By "usual attitude", Quine is referring to the intuition that supports the conditional-assertion theory. My reply to this objection is two-fold. First, it seems that the reason he departs from the usual attitude is the lack of a developed theory that supports this intuition more than anything else. It is natural to suppose that he would endorse a full-fledged account of the theory[22]. Second, the conditional-assertion theory is the only option in which it can be argued that the components of "if-then" are used instead of being mentioned.

Another objection is that it is not clear whether by "used" Quine meant the same as "asserted". Since the present criticism against his position takes this equivalence for granted, there is some additional argumentation required. My reply to this criticism is that the only

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[18] Russell (1903: 35).
[19] Russell (1906: 161).
[20] Russell (1903: 35).
[21] Quine (1950: 12–13).
[22] Like the work of Edgington (1995), to take one of many examples.

reasonable interpretation is one where the antecedent and the consequent of a conditional are used in the sense of assertion. Perhaps, there is one additional criticism to take in consideration. In the conditional-assertion theory, the consequent is asserted under the assumption of the antecedent. Thus, the antecedent and the consequent are used, but in two different ways. But conceptual details aside, the criticism advanced against Quine remain uncorrected. The only meaningful sense where the antecedent and the consequent are used involves a conditional-assertion theory that is widely controversial.

4. MATERIAL IMPLICATION UPDATED

Now, let's consider Quine's statement that the Philonian construction wouldn't generate any controversy if it was called "material conditional". It wouldn't be an exaggeration to say that nine out of ten logic textbooks adopted his choice of terminology. In retrospect, this change of nomenclature made no difference whatsoever. Logic textbooks always highlight the paradoxes of material conditional and the connective is widely unpopular among conditional experts. Let's take an intuitively false conditional such as "If John drank acid, he improved his health". Now suppose that John didn't drink acid. If we interpret this conditional as logically equivalent to the material conditional, it will be true simply because the antecedent is false. This goes against our intuitions. Another example is the conditional "If today is hot, the Earth is round". This conditional is true due to the truth of the consequent alone, but this is weird since the antecedent and consequent have no mutual relevance. Now, if the decision to call the Philonian construction "material conditional" didn't make the connective less paradoxical, then the original diagnosis that blamed the "material implication" moniker for its paradoxical aspects was certainly flawed. Worse, this change of names moves us far away from the crux of the matter. There is much to be gained by interpreting conditionals as material implications when the notion is properly understood.

Intuitively, conditional statements express some sort of deductive reasoning, but the precise nature of this relation is controversial. It seems obvious that if p entails q, $p \rightarrow q$[23] is necessarily true, and inversely, if $p \rightarrow q$ is necessarily true, p entails q. This relation, however, doesn't hold in most cases, since most true conditionals are not necessarily true. Is there some other connection between the two? Mackie suggested that conditionals are condensed arguments. Thus, to accept "if *p* then *q*" is to be willing to infer *q* while discovering *p*. In this sense, the conditional "If it rains, the street is wet" would express an inference we would be willing to perform given the assumption that it rains, and not a belief on a proposition[24]. Ryle defended a similar view by suggesting that conditional sentences are like inferential tickets. To accept "if *p* then *q*" is to find out that one is entitled to argue that "*p*, therefore *q*", given the condition that the premise *p* is obtained. The reasoner does not actually need to make the inference she is entitled to, in the same way that an owner of a railway ticket does not need to use it to travel, even though she would be entitled to[25].

Other philosophers also highlighted the relationship of conditionals with arguments, but were cagier about its precise nature. For instance, Strawson proposed that "if *p*, then *q*" conventionally implies the existence of a ground-consequence relation between the two propositions and means the same as "*p*, so *q*"[26]. The hypothesis is that if "*p*, so *q*" is a conventional argument-form, "if *p*, then *q*" would be the conventional quasi-argument-form,

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[23] I will adopt the notation where "$\rightarrow$" stands for natural language conditionals, '$\supset$' stands for material implication, and '$\vDash$' stands for entailment. I will not use quotes to highlight the use-mention distinction when there is no risk of confusion.

[24] Mackie (1973: 81).

[25] Ryle (1950: 312).

[26] Strawson (1952: 35).

and that the only difference between the two is that the premises of a quasi-argument-form are "entertained rather than asserted". Strawson thinks that this would explain why we may hesitate to call conditional statements true, and prefer to call them "reasonable or well-founded"[27].

One attempt to establish this relation between conditionals and arguments is to emphasize its relationship with *modus ponens*. Hare hinted at this idea when he said that "to understand the 'If … , then' form of sentence is to understand the place that it has in logic (to understand its logical properties). It is, in fact, to understand the operation of modus ponens and related inferences"[28]. Jackson endorsed a similar view according to which the acceptance of $p \rightarrow q$ is measured by our willingness to employ it on a *modus ponens.* He argued for the importance of *modus ponens* as condition for the assertibility of conditionals using the concept of robustness: $p \rightarrow q$ is acceptable when q is robust with respect to p, i.e., when $\Pr(q)$ is high and would remain high after learning that p. In this sense, $p \rightarrow q$ would only be acceptable when it can be employed on a *modus ponens* inference[29].

The aforementioned examples show that the association between conditionals and arguments is natural. In fact, the supposed differences between conditionals and arguments are usually exaggerated. For example, one can argue that a conditional "if *p*, then *q*" does not involve an assertion of *p* and *q*, while an argument "*p*, therefore *q*", involves both an assertion of *p* and *q*, and an additional assertion that *p* implies *q*. But this interpretation has some problems. First, it ignores that a commitment to the truth-values of *p* and *q* can be expressed on the terms employed even if neither *p* nor *q* are asserted, e.g., "*q* because *p*", "given *p*, *q*", etc. Second, it would mean that expressions such as "*p*, therefore *q*" contain three assertions, instead of one. In fact, it would imply that the word "therefore" alone should be read as "*p* strictly implies *q*", which is absurd. We could instead interpret "*p*, therefore *q*" as meaning only "*p* strictly implies *q*", where *p* and *q* are not asserted, just mentioned. The commitment to *p* and *q* is expressed, but not stated.

It seems undeniable that it is part of the meaning of a conditional that the consequent follows from the antecedent in some sense to be specified. This intuition is reinforced by the fact that the terms that are usually associated with the protasis ("if", "given that", "when", "antecedent", etc.) or the apodosis ("then", "consequent") should be interpreted as indicatives of premise(s) and conclusion, respectively. The strict implication view advanced by Clarence Lewis states that the consequent follows from the antecedent in the same sense that a conclusion deductively follows from the premise of an argument. It is an understandable mistake, since it tries to emulate the notion of entailment into the meaning of conditionals in order to do justice to the intuition that they involve some sort of implication, but it is a mistake nonetheless. Lewis' view is unsatisfactory and somewhat ad hoc because it leaves no room for the specific role of conditionals in deductive arguments. In this proposal, conditionals will exhibit the same entailment relations of the deductive arguments to which they take part, but this is implausible since conditionals are not deductive arguments.

The notion of material implication advanced by Russell is more promising in that regard. It offers a notion of implication that is somewhat associated with our intuitions about entailment, but it also manages to have its own distinct characteristics. The only aspects in which Russell's characterization was lacking are the modal distinctions that highlight both the similarities and differences between material implication and entailment. As it happens, these distinctions will also provide a compelling strategy to explain away the counter-intuitive aspects of material implication.

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[27] Strawson (1952: 83).
[28] Hare (1970: 16).
[29] Jackson (1987: 26–31).

The Russelian analysis can be modified to accommodate the relevant intuitions if conditionals are interpreted as elliptical for *local* material inferences. Thus, $p \rightarrow q$ should be interpreted as an elliptical for "p ensures the truth of q in *this world*" or "it is not the case both that p is true and that q is false in *this world*". The use of qualifications such as "this world" and "local" are a reference to the fact that the material implication relation is restricted to the truths grounded on a given world that is assumed as a parameter. This qualification is necessary to allow us to make sense of entailment and its dependence on possible worlds. For instance, the argumentative form $p \supset q$, $p \vDash q$ should read as "In every possible world in which p materially implies q, and p is true, q is true". The material implication can be satisfied in other worlds beyond the actual world. The parameter-world that is relevant to the discussion of the paradoxes of material implication is usually the actual one.

It could be objected that the material implication is too artificial to count as an explanation of conditionals. Rescher, for example, argues that "material implication is a technical concept that has a life of its own, detached from any propositional relationships that have their natural home in ordinary language"[30]. The problem with this line of reasoning is that it ignores that the phenomena of natural language usually have a complex nature that requires "artificial" and "technical" solutions. The classical notion of validity is also assumed in our daily argumentation, but its precise behavior is unnatural. Ordinary speakers don't have the notion that their arguments will only work if it is impossible that their premises are true and the conclusion is false. However, nobody would dismiss the traditional notion of validity with the observation that is "a technical concept that has a life of its own, detached from any propositional relationships that have their natural home in ordinary language"[31].

It is a curious state of affairs that a widely unpopular material implication is supported by intuitions that are similar to the ones supporting the widely accepted classical conception of validity. The only reason for this discrepancy is Quine's influence on the terminology. This is unfortunately because the use of "material implication" make it possible the use of intuitions and questions that are smothered by the use of "material conditional". Indeed, the rehabilitation of material implication allows us to explain puzzles that seem intractable otherwise. Take, for instance, the problem of understanding how conditionals correspond to reality:

> Truths correspond to reality. Falsehoods don't. 'The cat is on the mat' is true if and only if the cat is on the mat. There is no apparent problem in understanding what state of affairs must actually obtain for 'The cat is on the mat' to be true so long as it is obvious in the situation which cat and which mat are being referred to. … Consider now the conditional 'Someone let in the cat if the cat is on the mat'. What sort of situation or state of affairs makes it true? We know how to draw a picture of a cat on a mat, or a cat not on a mat, and of a mat with no cat on it. How can we draw a picture of a conditional state of affairs: if the cat is on the mat, then such-and-such? Given an event description, an event so described either occurs in a certain vicinity during a certain period, or it does not occur. There is no such thing as the conditional occurrence of an event. Declarative conditional sentences about occurrences are therefore not about conditional occurrences. What are they about? What in the world makes a declarative conditional sentence true?[32]

The present interpretation provides a natural solution for this problem: conditional statements are declarative statements about a relation of material implication between two propositions,

[30] Rescher (2007: 45).
[31] Nobody except the relevant logicians. For example, Anderson & Belnap (1975) tried to develop a logical system with a notion of logical consequence which is stronger than the classical notion. But their concept of relevant implication would be also too complicated and technical for an ordinary speaker.
[32] Sanford (2003: 5–6).

the antecedent and the consequent. In other words, they are statements about how one proposition ensures the truth of another in a given world. There is no need to resort to conditional state of affairs in our explanation, for we only need facts and regular truthmaking. They are categorical or declarative statements about facts associated with implication. A conditional corresponds to reality if the antecedent materially implies the consequent, i.e., if certain state of affairs or facts obtain. A conditional does not correspond to reality if the antecedent does not materially imply the consequent.

Now consider the phenomenon of embedded conditionals. These examples are known for being tricky and resistant to analysis. Suppose that in relation to a piece of glass that had been held a foot above the floor, you say: "If it broke if it was dropped, it was fragile" (Gibbard, 1981, pp. 235–6). The present interpretation offers a way to unpack this sentence as follows: "the premise 'the glass was dropped' materially implies that the premise 'the glass broke' materially implies 'the glass was fragile'". Thus, a conditional such as $p \rightarrow (q \rightarrow r)$ can be interpreted as "p materially implies that q materially implies r". This provides us with a clear rationale to interpret successive reiterations of embedding in conditionals, with increasing orders of complexity. We can explain conditionals in embedding contexts as composed assertions of material implication. Just as we may have one or more premises in an argument, we may have one more proposition in an antecedent. This is another puzzle that was laid to rest.

This reasoning also allows us to explain some of the counter-examples against classical argumentative forms in a principled manner. Consider *antecedent strengthening*: $p \rightarrow q \vDash (p\&\mathrm{r}) \rightarrow q$. This argumentative form faces the following counter-example: "If the match is struck it will light. Therefore, if the match is struck and it is held under water, it will light". In order to understand what is wrong with this counter-example, let's take a step back and consider one feature of deductive validity, namely, monotonicity. If $p \rightarrow q$ and p deductively entail q, this implication will persist notwithstanding additional information, including information that may render one of the premises false. Thus, the following instance of *modus ponens* will preserve the truth of the premise, "If the match is struck, it will light. The match is struck. Therefore, it will light". Now, if we add an additional premise that makes the conclusion false, the argument will still be valid. Thus, the following instance of *modus ponens* is valid, "If the match is struck, it will light. The match is struck. The match is held under water. Therefore, it will light". This argument is somewhat counter-intuitive because the truth of the additional premise is incompatible with a background condition required for the conclusion, i.e., that the match is dry. But then again, if this premise is true, the conclusion is false, but so is the first premise. So, there is no conceivable circumstance where all premises are true and the conclusion is false. Therefore, the counter-example is merely apparent.

The same reasoning holds for the material implication. If $p \rightarrow q$ is true, p materially implies q and the addition of another premise will not make this implication invalid. Thus, "If the match is struck it will light" is materially valid, it will remain valid given the addition of the premise that the match is held under water. Thus, "if the match is struck and it is held under water, it will light" will remain materially valid. This is somewhat counter-intuitive, because we know that under typical background conditions, the strengthened conditional will not have a true antecedent and a true consequent. However, this is not a counter-example, since the strengthened conditional will only be false with a true antecedent and a false consequent, and in this circumstance the premise is also false. Or to put in other words, in the only circumstance where the attempt of material implication exhibited by the strengthened conditional is invalid is also a circumstance where the attempt of material implication exhibited by the premise is also invalid. The validity of *antecedent strengthening* can be explained as a form of monotonicity related to the relations of material implication in the premise and in the conclusion. The reason why *antecedent strengthening* is perceived as invalid is that the material implication is monotonic, while the evidential support between the antecedent and the consequent is not. If the evidential support may well be undone by additional findings, the implication still holds. But we can't approach deductive logic as nonmonotonic logic.

The interpretation of conditionals as arguments is the only explanation of Dutchman conditionals, which are explicit reductios. The conditional 'If John is telling the truth, I'm a dutchman' is an argument that intends to show the ridiculous consequences of the premise. Since the conclusion of the argument is false, the speaker who asserted the conditional would want us to infer by *modus tollens* that the premise is false. Another reason to interpret conditionals as arguments is that conditionals cannot be complete propositions without being interpreted as an indirect assertion of implication of its propositional constituents. It is not just the antecedent and consequent need to be expanded to be interpreted as full propositions in their own right, but the conditional as a whole. Take the following conditional "If I strike the match, it will light". This sentence should be expanded as "It is not the case that the premise 'x strikes the match at the time *t*' is true and the conclusion 'the match will light at the time *t*' is false". Thus, conditionals can only be interpreted as complete propositions if they are assertions of material implication.

Here one might object that the acceptance and assertion of the conditional assumes a causal relation that extends over different worlds, say, worlds that are very similar to ours in terms of laws, background facts, etc. Therefore, the conditional should be reinterpreted as "There are no closest worlds where the premise 'x strikes the match at the time *t*' is true and the conclusion 'the match will light at the time *t*' is false". But this reading seems too strong. For one thing, the conditional refers to a specific match, in a specific moment in time, and in a specific world (the actual one). This will still be true even if the stronger assumption is also an assumption of the speaker. Worse, it doesn't seem that stronger readings can be made to work. One example of stronger conditional that would fit this different reading is as follows: "Every match that is struck in standard conditions, will light". But this seems false, because some matches will fail to light even in standard conditions. So we can adopt a weaker version of the original conditional as follows: "Most matches that are struck in standard conditions, will light". But now we face a different problem. Suppose I struck the match at the time *t*, but it fails to light. The proposition "Most matches that are struck in standard conditions and closer worlds, will light" is still true, because in most worlds that are similar to ours, the conditional will light at the time *t*. However, the conditional "If I strike this match at the time *t*, it will light" is still false, because in the actual world the match failed to light at the time *t*.

It can be objected that it is widely accepted that, in natural language, if a speaker asserts '*p*, therefore *q*', they use *p* and *q* in the argument. Now, since a conditional 'if *p* then *q*' is interpreted as an argument of sorts (an assertion of material implication), we would have to conclude that *p* and *q* are used in a conditional as well. Now, this objection misses the target. Our intuitions on the subject can get murky, because the premises and the conclusion of '*p*, therefore *q*' construction are complete propositions that can be analysed individually, so they seem to be asserted when an argument is used. Of course, the premise and conclusion of argument can also be independently asserted, but when they are used qua premise and qua conclusion, they are only mentioned in an argument as the former ensuring the truth of the latter. Besides, the premises and the conclusion of an 'if *p* then *q*' construction need to be expanded to be interpreted as full propositions in their own right, so the suggestion that they are also individually asserted seems even more far-fetched.

Perhaps more importantly, not only $p \rightarrow q$ is a statement of implication, as also its logical equivalents such as $\neg(p\&\neg q)$ and $\neg p \lor q$. If $\neg p \lor q$ is an implication statement, so are $p \lor q$, and $\neg(p \lor q)$. Few people would look at a disjunction as a version of implies, but this is exactly what follows from its logical equivalence with material implication.

5. SOME REVISIONS ARE IN ORDER

If conditionals are disguised assertions of material implication, the contemporary literature on the subject is headed on the wrong direction. The questions that conditional experts should be asking themselves couldn't possibly be more different from the questions they are asking now. Take for instance the discussion involving conditional probability and principles such as Adam's thesis. This principle states that the assertability of $p \rightarrow q$ is measured by the probability of q given p. If the probability of the consequent given the antecedent is high, the conditional is assertable. Otherwise, it is unassertable. But since conditionals truth-conditions are dependent on relations of material implication, the probability attributions of the consequent given the antecedent have no bearings on its assertability. A similar criticism can be extended to intuitions and questions that gravitate around principles such as the equation, the Ramsey's thesis and possible world theories, just to name a few. Moreover, this is also a call for coherence, since the intuitions that support the unpopular material implication are very similar to the intuitions that support that widely accepted classical conception of validity. Russell's terminology is not only appropriate, but it is also more insightful than the terminology proposed by Quine and can represent an interesting hypothesis among the theoretical alternatives proposed by conditional experts.

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