‘IF-THEN’ AS A VERSION OF ‘IMPLIES’

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Matheus Silva

ABSTRACT

Russell’s role in the controversy about the paradoxes of material implication is usually presented as a tale of how even the greatest minds can fall prey of basic conceptual confusions. Quine accused him of making a silly mistake in *Principia Mathematica.* He interpreted ‘if-then’ as a version of ‘implies’ and called it material implication. Quine’s accusation is that this decision involved a use-mention fallacy because the antecedent and consequent of ‘if-then’ are used instead of being mentioned as the premise and the conclusion of an implication relation. Quine’s interpretation on the topic became influential. This paper will present the following criticisms against this interpretation: (1) the notion of material implication does not involve a use-mention fallacy, since the components of ‘if-then’ are mentioned and not used; (2) Quine’s belief that the components of ‘if-then’ are used was motivated by a conditional-assertion theory of conditionals that is widely controversial and faces numerous difficulties; (3) the Philonian construction remains paradoxical even if it is called ‘material conditional’; (4) the Philonian construction is more plausible when it is interpreted as a material implication.

Keywords: material implication; conditionals; if-then; use-mention fallacy; conditional-assertion theories; *Principia Mathematica*.

1. INTRODUCTION

In the *Principia Mathematica*, Russell employed the notion of material implication to interpret ‘if-then’ constructions of natural language. According to Quine, this choice of terminology was fallacious, since it involves a confusion between the use and mention of words. It was his opinion that the criticisms and alternatives to the material implication presented by C. I. Lewis and others would never be made in the first place if Russell simply called the Philonian construction ‘material conditional’ instead of ‘material implication’. Quine’s interpretation on the topic became hugely influential, if not universally accepted. It will be argued that this widely accepted accusation is unfounded, for the antecedent and consequent of ‘if-then’ are mentioned, not used. It is also argued that interpreting conditionals as assertions of material implication can provide fruitful solutions to known puzzles in the literature.

It is important to notice, however, that while there’s an interesting proposal to be made and textual evidence that may justify Russell’s choice of terminology, a full-blown defence of material implication will require concepts and intuitions that were completely alien to Russell. For instance, a defence of ‘if-then’ as an assertion of material implication will require modal intuitions that he simply ignored. Russell himself refused modal intuitions in his posthumously published paper ‘Necessity And Possibility’ (1905). In this paper he argues that the modal operators of ‘necessity’ and ‘possibility’ have only an epistemological or psychological significance and should have no place in formal logic. Russell tried to deflect the criticisms with a different strategy in which he advanced a pragmatic defence of his choice of terminology. The position advanced in this paper couldn’t be more different even if it is inspired in Russell’s writings. Oddly enough, the notion of material implication that is currently perceived as an ancient artefact from the old days can only be reinvigorated into its full force with the use of contemporary ideas that weren’t popular in Russell’s time.

2. THE *PRINCIPIA* CONTROVERSY

Russell’s changed his ideas about logic constantly, but some core views remain the same throughout his lifework[1]. Russell firmly believed that symbolic logic captures the essence of deductive reasoning and that we should develop a symbolic logic capable of showing that mathematics is reducible to logic. More importantly, he endorsed the notion of implication as fundamental to our understanding of deduction and believed that there are two types of implication: material and formal. Material implication is a proposition which displays a relation between two propositions, let’s say, *p* and *q*. The statement ‘*p* materially implies *q*’ is symbolised as $p \supset q$ and is true unless *p* is true and *q* is false, i.e., whenever *p* is not true or *q* is true[2]. Russell interprets ‘if-then’ sentences as assertions of material implication, so ‘*p* materially implies *q*’ can also be read as ‘if *p*, then *q*’[3]. Formal implication is the implication we find today in first order predicate calculus in such formulas as (x) (Fx ⊃ Gx)[4].

From his discussion of material implication, Russell draws three curious inferences which would be known as the paradoxes of material implication: (1) for any two propositions, one of these propositions must imply the other; (2) false propositions imply all propositions; (3) true propositions are implied by all propositions. These counter-intuitive consequences were bombarded with criticisms. C. I. Lewis was its main detractor[5]. In *The Calculus of Strict Implication*, Lewis objected that material implication didn’t do justice to our intuitions about implication:

> If ‘*p* implies *q*’ means only ‘it is false that *p* is true and *q* false,’ then the implication relation is far too ubiquitous to be of any use[6].

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1 Russell’s views about logic are presented in works that are too numerous to mention. Some of the main references include ‘The Principles of Mathematics’ (1903), ‘The Theory of Implication’ (1906), ‘‘If’ And ‘Imply’, A Reply To Mr. MacColl’ (1908), ‘Principia Mathematica’ (1910), ‘Some Explanations in Reply to Mr. Bradley’ (1910), ‘The Philosophical Importance of Mathematical Logic’ (1913) and ‘Introduction to Mathematical Philosophy’ (1919). Two articles that were published posthumously, ‘Recent Italian Work on The Foundation Of Mathematics’ (1901) and ‘Necessity And Possibility’ (1905), repeat some of the main ideas of his other works.

2 Russell & Whitehead (1910: 7)

3 Russell & Whitehead (1910: 208).

4 Although Russell confusedly thought that formal implication also belongs in the propositional calculus.

5 For an overview of the clash between Russell and Lewis, see Barker (2006).

6 Lewis (1914: 246).

The idea that material implication ‘is far too ubiquitous to be of any use’ is motivated by Lewis’ view that *p* can only imply *q* when *q* is a logical consequence of *p*. In other words, the notion of implication is linked with the notion of logical consequence and its related cousins (‘logical inference’, ‘entailment’, ‘valid deduction’, etc.). In *Interesting theorems in symbolic logic*, Lewis drew the apocalyptic consequences from treating implication, and, therefore, logical consequence, as material implication. This meant that the *Principia* theorems were be under suspicion and symbolic logic would collapse:

> The consequences of this difference between the ‘implies’ of the algebra and the ‘implies’ of valid inference are most serious. Not only does the calculus of implication contain false theorems, but all its theorems are not proved. For the theorems of the system are implied by the postulates in the sense of ‘implies’ which the system uses. The postulates have not been shown to imply any of the theorems except in this arbitrary sense. Hence, it has not been demonstrated that the theorems can be inferred from the postulates, even if all the postulates are granted. The assumptions, e. g., of ‘Principia Mathematica,’ imply the theorems in the same sense that a false proposition implies anything, or the first half of any of the above theorems implies the last half[7].

Lewis’ point is that Russell identifies the deductibility of *q* from *p* with the material implication of *q* from *p*. This implies that in order for *q* being deducible from *p* it is enough that *p* is false or *q* is true. But this is unacceptable. Given that the proposition ‘Pigs fly’ is false, I’m not willing to admit that every proposition is inferable from ‘Pigs fly’. If any true proposition is implied by any proposition, and necessarily true propositions are implied by any proposition, it follows that every true proposition is necessarily true. If the proofs of *Principia* were made in this way they would not be truths, since a proof is based on premises that are assumed as true in order to arrive at the truth of a conclusion whose truth was not admitted.

One of Lewis criticisms is that the notion of material implication ignores modal distinctions that are intuitively tied to implication. To use our contemporary idiolect, a relation of material implication would only require certain combinations of truth values in the actual world, but logical inference requires a stronger connection:

> Material implication it will appear, applies to any world in which the all-possible is the real, and cannot apply to a world in which there is a difference between real and possible, between false and absurd. Strict implication has a wider range of application. Most importantly it admits of the distinction of true and necessary of false and meaningless[8].

In *Symbolic Logic*, Lewis and Langford proposed an alternative logical system based on a different notion of implication that would better represent Lewis’ intuitions about the subject:

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7 Lewis (1913: 242).
8 Lewis (1914: 241).

> It appears that the relation of strict implication expresses precisely that relation which holds when valid deduction is possible. It fails to hold when valid deduction is not possible[9].

The relation where *p* strictly implies *q* would be symbolised as $p \dashv q$, and it would only be true when *q* is logically inferable from *p*, and is logically equivalent to $\neg\Diamond(p\&\neg q)$. The possibility must be understood as a logical possibility, since $\Diamond p$ means '*p* is self-consistent'[10]. From this explanation, it follows that 'if *p*, then *q*' is only false if it is logically impossible that $p\&\neg q$. The strict implication can be hardly qualified as an improvement over material implication since it is also packed with paradoxical aspects. Suppose that a father tells his children: 'If it rains tomorrow, we will go to the cinema'. It is logically possible that the antecedent is true and the consequent false, since there is no logical inconsistence in admitting that it will rain, but we decided not to go to the cinema. It is also physically possible that it will rain tomorrow, but we will not go to the cinema. Actually, since the conditional represents an attempt from a father to please his children it is assumed that there is no logical necessity between the antecedent and the consequent[11]. Most examples of conditionals natural language are in disagreement with Lewis' explanation.

There are more problems. Since '*p* strictly implies *q*' can also be read as 'it is necessary that *p* materially implies *q*', it follows that that *p* strictly implies if *p* is a logical contradiction or *q* is a tautology. Thus, the proposition 'It is raining and it is not raining' strictly implies every proposition because it is a logical contradiction. The difference is that Lewis would feel more confident in accepting these counter-intuitive aspects since they are already present in the classical notion of logical consequence. But the notion of material implication was never intended as synonymous with the notion of logical consequence, even though it is an implication in its own right.

Russell's defence of the material implication is pragmatic in character. In a response to a letter of Bradley, which contained criticisms similar to the ones advanced by Lewis, Russell argued that the term 'implication' is used in a special technical sense that do not have the consequences claimed by the critics[12]:

> The essential property that we require of implication is this: "What is implied by a true proposition is true". It is in virtue of this property that implication yields proofs. But this property by no means determines whether anything, and if so what, is implied by a false proposition, or by something which is not a proposition at all. What it does determine is that, if *p* implies *q*, then it cannot be the case that *p* is true and *q* is not true[13].

This technical sense of implication would be justified since it would be enough to represent accurately 'if-then' mathematical propositions using the new logic. The strange nature of

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[9] Lewis & Langford (1932: 247).
[10] Lewis & Langford (1932: 123).
[11] Braine (1979: 155).
[12] Russell (1910: 350).
[13] Russell (1906: 161–62).

material implication is harmless because it doesn't allow you to infer false propositions from true propositions. In subsequent replies to Lewis, Russell insists on the same point:

> The essential point of difference between the theory which I advocate and the theory advocated by Professor Lewis is this: He maintains that, when one proposition *q* is "formally deducible" from another *p*, the relation which we perceive between them is one which he calls "strict implication," which is not the relation expressed by "not-*p* or *q*" but a narrower relation, holding only when there are certain formal connections between *p* and *q*[14].

In other words, the narrower relation of implication wasn't required to ensure the truth of conditional sentences. Russell's defence never pleased the critics, since the lingering intuition that implication is tied to logical consequence persisted. The only way to savage material implication requires modal distinctions, but Russell was against the use of modal intuitions in logic. In a sense, these distinctions would only become clear decades later with the development of modal logic.

Quine's reading of this controversy decades later is that that Russell committed a use-mention fallacy, which lead Clarence Lewis to fall in the same mistake with his strict implication proposal. The distinction between use and mention can be illustrated by the following example: the sentence 'China is a populated country' is a statement about an attribute of China. The word 'China' is being used in this context, not mentioned it. The sentence 'China has two syllables' is a statement about a word, the name of a country. In this context, China is being mentioned, not used to refer the county. According to Quine, Russell committed the use-mention fallacy because he interpreted the conditional connective as a statement of logical implication, in which the antecedent and consequent are mentioned as the premise and conclusion of an entailment relation; whereas genuine conditionals do not mention statements, but use them to express a relation between facts and objects in the world. If Russell had acknowledged this distinction, insisted Quine, he would never had equated 'if-then' with 'implies':

> Lewis founded modern modal logic, but Russell provoked him into it. For whereas there is much to be said for the material conditional as a version of 'if-then', there is nothing to be said for it as a version of 'implies'; and Russell called it implication, thus apparently leaving no place open for genuine deductive connections between sentences. Lewis moved to save the connections. But his way was not, as one could have wished, to sort out Russell's confusion of 'implies' with 'if-then'. Instead, preserving that confusion, he propounded a strict conditional and called it implication[15].

> It is doubtful that Lewis would have even started this [modal logic] if Whitehead and Russell, who followed Frege in defending Philo of Megara's version of 'If *p* then *q*' as

[14] Russell (1919: 154).
[15] Quine (1961: 323).

> ‘Not(*p* and not *q*)’, had not made the mistake of calling the Philonian construction “material implication” instead of the material conditional[16].

It is difficult to overestimate the influence of Quine’s opinions about this subject. It is due to his criticisms alone that the name of material implication fell into disuse and was replaced by material conditional. But being influential is not synonymous with being truthful and there are plenty of reasons to question Quine’s conclusions.

3. A FALLACY THAT NEVER WAS

The accusation that the material interpretation involves a use-mention fallacy is predicated on the idea that the antecedent and the consequent in a conditional are used, not mentioned. But the antecedent and the consequent are not used. This becomes clearer when we consider the interpretation and formalisation of arguments in natural language. An argument such as ‘*p*, therefore *q*’ is not interpreted as using, in the sense of asserting, either *p* or *q*, but instead is interpreted as a more complex statement, namely, ‘*p* deductively implies *q*’. The same could be said about ‘if *p*, then *q*’, where *p* and *q* are not being used in the sense of being asserted, but are mentioned. This conditional should be interpreted as a more complex statement, namely, ‘*p* materially implies *q*’.

Quine was misled into thinking that the antecedent and consequent of a conditional are used because he was influenced by a conditional-assertion view of conditionals, according to which ‘if *p*, then *q*’ should be interpreted as a conditional assertion of *q* given the assumption of *p*. This becomes clear in the following passage of *Methods of Logic*:

> An affirmation of the form ‘if *p* then *q*’ is commonly felt less an affirmation of a condition than as a conditional affirmation of the consequent. If after we have made such an affirmation, the antecedent turns out true, then we consider ourselves committed to the consequent, and are ready to acknowledge error if it proves false. If on the other hand the antecedent turns out to have been false, our conditional affirmation is as if it had never been made[17].

But this type of conditional-assertion theory of conditionals is widely controversial and counter-intuitive. Suppose one says ‘If it rains, I won’t go to the supermarket’. Now suppose that it didn’t rain. According to the conditional-assertion theory, the speaker never said anything because the antecedent turned out to be false. This violates our intuitions about conditionals. In fact, given that many of conditionals in natural language probably have false antecedents, this would imply that speakers are not saying anything with conditionals a significant part of the time! Of course, one could try to defend the conditional-assertion theory and dismiss these criticisms, but it is doubtful that everyone who endorses Quine’s criticisms as an uncontroversial truism in logic would gladly accept this theoretical burden.

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[16] Quine (1964: 196).
[17] Quine (1950:12).

In fact, it can be argued that if there is one who is guilty of committing a fallacy here is Quine, who erroneously assumed that the components of a complex proposition are asserted like usual propositions. But when a conditional is asserted, it's the whole proposition that it is asserted, and not its antecedent and consequent. The assertion of a conditional then is as a statement about a relation between the propositions expressed by the antecedent and consequent. In other words, the antecedent and consequent *are mentioned*, not used. In order for them to be used, they would need to be asserted, but they cannot be asserted, for what is asserted is the conditional, not its propositional constituents. In the assertion of the conditional 'If it rained on Thursday, the match was cancelled', the antecedent 'it rained on Thursday', and the consequent 'the match was cancelled' are mentioned, not used. The speaker is stating that the consequent follows from the antecedent. It is a statement of a logical implication of some sort. The same could be said about disjunctions, conjunctions, and biconditionals. Russell understood this distinction and observed that 'the proposition "*p* implies *q*" *asserts* an implication, though it does not *assert p* or *q*'[18].

What is worse, there is textual evidence to suggest that Russell was perfectly aware of the distinction between use and mention, as is illustrated in the following passage:

> Any proposition may be either asserted or merely considered. If I say "Caesar died", I assert the proposition "Caesar died" if I say "Caesar died is a proposition", I make a different assertion, and "Caesar died" is no longer asserted, but merely considered. Similarly in a hypothetical proposition, e. g. "if $a = b$, then $b = a$", we have two unasserted propositions, namely "$a = b$" and "$b = a$" while what is asserted is that the first of these implies the second[19].

Now, notice that Russell uses the term 'considered' as synonymous of 'mentioned'. A proposition is merely considered when it is mentioned, instead of being asserted.

The other point that needs to be considered is whether Russell was aware of the distinction between material implication and logical implication understood in the sense of entailment or deducibility. Even though Russell ignored the modal distinctions that would settle the discussion on this issue in his favour, he presented independent arguments that made it clear that he didn't make this confusion. While explaining Lewis Carroll's puzzle about the tortoise in *Principles of Mathematics*, Russell observes that 'We need, in fact, the notion of therefore, which is quite different from the notion of implies, and holds between different entities'[20]. In other words, the notion that *p* materially implies *q* is quite different from the notion that *q* deductively follows from *p*.

That Russell made a clear distinction between material implication and deductibility is evidenced by his argument that the notion of implication is more primitive than deductibility. Russell claims that '*q* is implied by *p*' cannot mean the same as '*q* is deducible from *p*' since it would mean that there exists a set of principles of deduction from which it can be demonstrated that *p* implies *q*. The notion of 'deducible from' is defined in terms of the principles of deduction that employ the notion of implication. Thus, it would not be permissible to substitute 'implied by' for 'deducible from' due to the charge of circularity.

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[18] Russell (1903: 35).
[19] Russell (1906: 161).
[20] Russell (1903: 35).

Now, let's consider Quine's statement that the Philonian construction wouldn't generate any controversy if it was called 'material conditional'. It wouldn't be an exaggeration to say that nine out of ten logic textbooks adopted his choice of terminology. In retrospect, this change of nomenclature made no difference whatsoever. Logic textbooks always highlight the paradoxes of material conditional and the connective is widely unpopular among conditional experts. Let's take an intuitively false conditional such as 'If John drank acid, he improved his health'. Now suppose that John didn't drink acid. If we interpret this conditional as logically equivalent to the material conditional, it will be true simply because the antecedent is false. This goes against our intuitions. Another example is the conditional 'If today is hot, the Earth is round'. This conditional is true due to the truth of the consequent alone, but this is weird since the antecedent and consequent have no mutual relevance. Now, if the decision to call the Philonian construction 'material conditional' didn't make the connective less paradoxical, then the original diagnosis that blamed the 'material implication' moniker for its paradoxical aspects was certainly flawed. Worse, this change of names moves us far away from the crux of the matter. There is much to be gained by interpreting conditionals as material implications when the notion is properly understood.

It could be objected that is not obvious that Quine was an enthusiast of the conditional-assertion theory of conditionals. The evidence is the passage that immediately follows the one quoted above:

> Departing from this usual attitude, however, let us think of conditionals simply as compound statements which, like conjunctions and alternations, admit as wholes of truth and falsity[21].

By 'usual attitude', Quine is referring to the intuition that supports the conditional-assertion theory. My reply to this objection is two-fold. First, it seems that the reason he departs from the usual attitude is the lack of a developed theory that supports this intuition more than anything else. It is natural to suppose that he would endorse a full-fledged account of the theory[22]. Second, the conditional-assertion theory is the only option in which it can be argued that the components of 'if-then' are used instead of being mentioned.

Another objection is that is not clear whether by 'used' Quine meant the same as 'asserted'. Since the present criticism against his position takes this equivalence for granted, there is some additional argumentation required. My reply to this criticism is that the only reasonable interpretation is one where the antecedent and the consequent of a conditional are used in the sense of assertion. Perhaps, there is one additional criticism to take in consideration. In the conditional-assertion theory, the consequent is asserted under the assumption of the antecedent. Thus, the antecedent and the consequent are used, but in two different ways. But conceptual details aside, the criticism advanced against Quine remain uncorrected. The only meaningful sense where the antecedent and the consequent are used involves a conditional-assertion theory that is widely controversial.

21 Quine (1950: 12–13).

22 Like the work of Edgington (1995), to take one of many examples.

4. MATERIAL IMPLICATION 2.0

Intuitively, conditional statements express some sort of deductive reasoning, but the precise nature of this relation is controversial. It seems obvious that if *p* entails *q*, $p \rightarrow q$[23] is necessarily true, and inversely, if $p \rightarrow q$ is necessarily true, *p* entails *q*. This relation, however, doesn't hold in most cases, since most true conditionals are not necessarily true. Is there some other connection between the two though? Mackie suggested that conditionals are condensed arguments. Thus, to accept 'if *p* then *q*' is to be willing to infer that *q* while discovering that *p*. In this sense, the conditional 'If it rains, the street is wet' would express an inference we would be willing to perform given the assumption that it rains, and not a belief on a proposition[24]. Ryle defended a similar view by suggesting that conditional sentences are like inferential tickets. To accept 'if *p* then *q*' is to find out that one is entitled to argue that '*p*, therefore *q*', given the condition that the premise *p* is obtained. The reasoner does not actually need to make the inference she is entitled to, in the same way that an owner of a railway ticket does not need to use it to travel, even though she would be entitled to[25].

Other philosophers also highlighted conditionals' relationship with arguments, but were cagier about its precise nature. For instance, Strawson proposed that 'if *p*, then *q*' conventionally implies the existence of a ground-consequence relation between the two propositions and means the same as '*p*, so *q*'[26]. The hypothesis is that if '*p*, so *q*' is a conventional argument-form, 'if *p*, then *q*' would be the conventional quasi-argument-form, and that the only difference between the two is that the premises of a quasi-argument-form are 'entertained rather than asserted'. Strawson thinks that this would explain why we may hesitate to call conditional statements true, and prefer to call them 'reasonable or well-founded'[27].

One attempt to establish this relation between conditionals and arguments is to emphasise its relationship with *modus ponens*. Hare hinted at this idea when he said that 'to understand the 'If … , then' form of sentence is to understand the place that it has in logic (to understand its logical properties). It is, in fact, to understand the operation of modus ponens and related inferences'[28]. Jackson endorsed a similar view according to which the acceptance of $p \rightarrow q$ is measured by our willingness to employ it on a *modus ponens.* He argued for the importance of *modus ponens* as condition for the assertibility of conditionals using the concept of robustness: $p \rightarrow q$ is acceptable when *q* is robust with respect to *p*, i.e., when $\Pr(q)$ is high and would remain high after learning that *p*. In this sense, $p \rightarrow q$ would only be acceptable when it can be employed on a *modus ponens* inference[29].

It is fair to say then that despite the prevalent Quinean view that rigidly extricates conditionals from arguments, the association between the two was perceived as natural by multiple authors. In fact, the supposed differences between conditionals and arguments are usually exaggerated. Sometimes it is said that a conditional 'if *p*, then *q*' does not involve an assertion of *p* and *q*, while an argument '*p*, therefore *q*', involves both an assertion of *p* and *q*,

[23] I will adopt the notation where '→' stands for natural language conditionals.
[24] Mackie (1973: 81).
[25] Ryle (1950: 312).
[26] Strawson (1952: 35).
[27] Strawson (1952: 83).
[28] Hare (1970: 16).
[29] Jackson (1987: 26–31).

and an additional assertion that *p* implies *q*. But this interpretation has some problems. First, it ignores that a commitment to the truth-values of *p* and *q* can be expressed on the terms employed even if neither *p* nor *q* are asserted, e.g., '*q* because *p*', 'given *p*, *q*', etc. Second, it would mean that expressions such as '*p*, therefore *q*' contain three assertions, instead of one. In fact, it would imply that the word 'therefore' alone should be read as '*p* strictly implies *q*', which is absurd. We could instead interpret '*p*, therefore *q*' as meaning only '*p* strictly implies *q*', where *p* and *q* are not asserted, just mentioned. The commitment to *p* and *q* is expressed, but not stated.

Whatever way you look at it; it is part of the meaning of a conditional that the consequent follows from the antecedent in some sense to be specified. This intuition is reinforced by the fact that the terms that are usually associated with the protasis ('if', 'given that', 'when', 'antecedent', etc.) or the apodosis ('then', 'consequent') should be interpreted as indicative of premise(s) and conclusion, respectively. The strict implication view advanced by Clarence Lewis states that the consequent follows from the antecedent in the same sense that a conclusion deductively follows from the premise of an argument. It is an understandable mistake, since it tries to emulate the notion of entailment into the meaning of conditionals in order to do justice to the intuition that they involve some sort of implication, but it is a mistake nonetheless. Lewis view is unsatisfactory and somewhat ad hoc because it leaves no room for the specific role of conditionals in deductive arguments. In this proposal, conditionals will exhibit the same entailment relations of the deductive arguments to which they take part, but this is implausible since conditionals are not deductive arguments.

The notion of material implication advanced by Russell is more promising in that regard. It offers a notion of material implication that is somewhat associated with our intuitions about entailment, but in which the implication also manages to have its own distinct characteristics. The only aspects in which Russell's characterisation was lacking are the modal distinctions that highlight both the similarities and differences between material implication and entailment. As it happens, these distinctions will also provide a compelling strategy to explain away the counter-intuitive aspects of material implication.

The Russelian analysis can be slightly modified to accommodate the relevant intuitions if conditionals are interpreted as elliptical for *local* material inferences. Thus, $p \rightarrow q$ should be interpreted as an elliptical for '*p* ensures the truth of *q* in *this world*' or 'it is not the case both that *p* is true and that *q* is false in *this world*'. The use of qualifications such as 'this world' and 'local' are a reference to the fact that the material implication relation is restricted to the truths grounded on a given world that is assumed as a parameter. This qualification is necessary to allows us to make sense of entailment and its dependence on possible worlds. For instance, the argumentative form $p \supset q, p \vDash q$ should read as 'In every possible world in which it *p* materially implies *q*, and *p* is true, *q* is true'. The material implication can be satisfied in other worlds beyond the actual world. In an impossible world where entailment did not involve possible worlds, $p \vDash q$ would collapse on $p \rightarrow q$. The parameter-world that is relevant to the discussion of the paradoxes of material implication is usually the actual one.

It is important to acknowledge that material implication is the close cousin of the classical conception of validity. Therefore, they stand or fall together. If $p \rightarrow q$ is only true when there is a connection between *p* and *q* that is stronger than material implication, then $p \vDash q$ is only true when there is a connection between *p* and *q* that is stronger than classical validity. If $p \rightarrow q$ is true, the *material argument* is valid since *q* is materially implied by, or it is a material consequence of p. If $p \vDash q$ is true, the *deductive argument* is valid since q is deductively implied by, or it is a logical consequence of p. The material implication $p \rightarrow q$ can be vacuously true due to the mere falsity of p or the mere truth of q, since it is not the case that p is true and q is false. Similarly, a deductive argument $p\&\neg p \vDash q\vee\neg q$ is vacuously valid due to either the contradiction of $p\&\neg p$ or the tautology of $q\vee\neg q$, because there is no possible world in which $p\&\neg p$ is true and $q\vee\neg q$ is false.

It could be objected that the material implication is too artificial to count as an explanation of conditionals. Rescher argues that 'material implication is a technical concept that has a life of its own, detached from any propositional relationships that have their natural home in ordinary language'[30]. The problem with this line of reasoning is that it ignores that phenomenon of ordinary language usually have a complex nature that requires 'artificial' and 'technical' solutions'. The classical notion of validity is also assumed in our daily argumentation, but its precise nature is unnatural. Ordinary speakers don't have the notion that their arguments will only work if it is impossible that their premises are true and the conclusion is false. However, nobody would dismiss the traditional notion of validity with the observation that is 'a technical concept that has a life of its own, detached from any propositional relationships that have their natural home in ordinary language'[31].

It is a curious state of affairs that a widely unpopular material implication is supported by intuitions that are similar to the ones supporting the widely accepted classical conception of validity[32]. The only reason for this discrepancy is Quine's influence on the terminology.

This is unfortunately because the use of 'material implication' make it possible the use of intuitions and questions that are smothered by the use of 'material conditional'. Indeed, the rehabilitation of material implication allows us to explain puzzles that seem intractable otherwise. Take for instance the problem of understanding how conditionals correspond to reality:

> Truths correspond to reality. Falsehoods don't. 'The cat is on the mat' is true if and only if the cat is on the mat. There is no apparent problem in understanding what state of affairs must actually obtain for 'The cat is on the mat' to be true so long as it is obvious in the situation which cat and which mat are being referred to. … Consider now the conditional 'Someone let in the cat if the cat is on the mat'. What sort of situation or state of affairs makes it true? We know how to draw a picture of a cat on a mat, or a cat not on a mat, and of a mat with no cat on it. How can we draw a picture of a conditional state of affairs: if the cat is on the mat, then such-and-such? Given an event description, an event so described either occurs in a certain vicinity during a certain period, or it does not occur. There is no such thing as the conditional occurrence of an event. Declarative conditional sentences about occurrences are therefore not about conditional occurrences. What are they about? What in the world makes a declarative conditional sentence true?[33]

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[30] Rescher (2007: 45).

[31] Nobody except the relevant logicians. But it still could be argued that their concept of relevant implication would be too complicated and technical for an ordinary speaker. Thus, I remain uncorrected.

[32] There are a few exceptions, of course. See as an example of this type of work Anderson & Belnap (1975), which tried to develop a logical system with a notion of logical consequence which is stronger than the classical notion.

[33] Sanford (2003: 5–6).

The present interpretation provides an easy answer for this problem: conditional statements are declarative statements about a relation of material implication between two propositions, the antecedent and the consequent. In other words, they are statements about how one proposition ensures the truth of another in a given world. There is no need to resort to conditional state of affairs in our explanation, for we only need facts and regular truthmaking. They are categorical or declarative statements about facts associated with implication. A conditional corresponds to reality if the antecedent materially implies the consequent, i.e., if certain state of affairs or facts obtain. A conditional does not correspond to reality if the antecedent does not materially imply the consequent.

Another important issue is the endless discussion about whether indicative and subjunctive conditionals have the same truth conditions or not. The intuition that supports the *Apartheid* thesis can be explained away in the following manner: since an indicative 'if *p* is the case, then *q* is the case' should be interpreted as saying '*p* materially implies *q*', is also natural to think that a subjunctive 'if *p* were the case, then *q* would be the case' should be interpreted as saying 'if *p* were true, it would materially imply *q*'. But one can accept that *p* materially implies *q,* but deny that if *p* were true, it would materially imply *q.* The error is in assuming that the fact that the antecedent is knowingly false makes any difference to the type of statement involving in a material implication.

The material implication view nullifies approaches that give much importance to the grammatical aspects of different conditionals since they are all removed from the expanded propositional content. The propositional content of complete conditionals does not admit the subjunctive mode of the propositions involved in the material implication. For instance, we cannot say 'The proposition 'Kennedy were not killed by Oswald' entails 'Someone else would have killed Kennedy', since this is ungrammatical. But if the full propositional content removes the subjunctive mode, then all theoretical intuitions motivated by the subjunctive mode are eliminated as a consequence.

Now consider the phenomenon of embedded conditionals. The conditional $p \rightarrow (\mathrm{q} \rightarrow r)$ can be interpreted as '*p* materially implies that *q* materially implies *r*'. This provide us with a clear rationale to interpret successive reiterations of embedding in conditionals, with increasing orders of complexity. We can explain conditionals in embedding contexts as composed assertions of material implication. Thus, the propositional form of $p \rightarrow (q \rightarrow r)$ should be interpreted as *p* materially implies that *q* materially implies *r*. Just as we may have one or more premises in an argument, we may have one more proposition in an antecedent. This is another puzzle that was laid to rest.

This reasoning also allows us to explain the counter-examples against classical argumentative forms in a principled manner. Consider *antecedent strengthening*: $p \rightarrow q \vDash (p\&\mathrm{r}) \rightarrow q$. This argumentative form faces the following counter-example: 'If the match is struck it will light. Therefore, if the match is struck and it is held under water, it will light'. In order to understand what is wrong with this counter-example, let's take a step back and consider one feature of deductive validity, namely, monotonicity. If $p \rightarrow q$ and *p* deductively entails *q*, this implication will persist notwithstanding additional information, including information that may render one of the premises false. Thus, the following instance of *modus ponens* will preserve the truth of the premise, 'If the match is struck, it will light. The match is struck. Therefore, it will light'. Now, if we add an additional premise that makes the conclusion false, the argument will still be valid. Thus, the following instance of *modus* *ponens* is valid, 'If the match is struck, it will light. The match is struck. The match is held under water. Therefore, it will light'. This argument is somewhat counter-intuitive because the truth of the additional premise is incompatible with a background condition required for the conclusion, i.e., that the match is dry. But then again, if this premise is true, the conclusion is false, but so is the first premise. So there is no conceivable circumstance where all premises are true and the conclusion is false. Therefore, the counter-example is merely apparent.

The same reasoning holds for the material implication. If $p \rightarrow q$ is true, p materially implies q and the addition of another premise will not make this implication invalid. Thus, 'If the match is struck it will light' is materially valid, it will remain valid given the addition of the premise that the match is held under water. Thus, 'if the match is struck and it is held under water, it will light' will remain materially valid. This is somewhat counter-intuitive, because we know that under typical background conditions, the strengthened conditional will not have a true antecedent and a true consequent. However, this not a counter-example, since the strengthened conditional will only be false with a true antecedent and a false consequent, and in this circumstance the premise is also false. Or to put in other words, in the only circumstance where the attempt of material implication exhibited by the strengthened conditional is invalid is also a circumstance where the attempt of material implication exhibited by the premise is also invalid. The validity of *antecedent strengthening* can be explained as a form of monotonicity related to the relations of material implication in the premise and in the conclusion. The reason why *antecedent strengthening* is perceived as invalid is that the material implication is monotonic, while the evidential support between the antecedent and the consequent is not. If the evidential support may well be undone by additional findings, the implication still holds. But we can't approach deductive logic as nonmonotonic logic.

There are plenty of similarities that show the natural kinship between deductive validity and material implication. To take just one example, if transitivity of entailment is the principle that if $A \vDash B$, $B \vDash C$, then $A \vDash C$, then hypothetical syllogism, which states that if $p \rightarrow q, q \rightarrow r \vDash p \rightarrow r$, can be considered a form of transitivity of material implication.

The relationship between the classical conception of validity and material implication is also evidenced by the fact that the negation of both may seem counter-intuitive from an epistemic point of view. If indicative conditionals are material, from the negation of $\neg(p \rightarrow q)$ it follows $p \& \neg q$. This assumption faces counter-intuitive instances when someone accepts the premise due to intensional evidence, but the conclusion is a conjunction he ignores. For example, if I deny the conditional 'If God exists then the prayers of evil men will be answered' I must admit that, 'God exists and the prayers of evil men will not be answered'[34]. Thus, from the negation of a simple conditional, I can prove that God exists. This is implausible, because someone could refuse the conditional based on assumptions about the moral dispositions of God even if she does not believe in the existence of God.

The root of the counter-example result from the tension between the material account and our common epistemic practices. The material account rests on extensional calculus, which works under an assumption of omniscience logic, i.e., that the evaluator of the conditional knows the truth-values of its propositional constituents, but in practice the evidence that is usually available when we evaluate a conditional is intensional. When we evaluate $p \rightarrow q$, we

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34 Stevenson (1970: 28).

usually do not know if p and q are true or not. If I want to establish whether John wasn't late to work if he left his home late, we need to consider how the traffic was today, etc. However, our eventual ignorance about the truth-values of p and q are completely ignored by the extensional calculus. This explains why it is intuitive to think that p and $\neg q$ entails $\neg(p \rightarrow q)$, but the converse is not intuitively true: the extensional evidence is sufficient to discard the conditional, but the refusal of the conditional can be motivated by the intensional evidence available, even though the later is defeatable.

The negation of a deductive argument may also look incompatible with classical logic due to epistemic intuitions. Suppose that a mathematician denies the validity of an attempt of proof. Let's say, the attempt to prove that p entails q. The negation of this inference in classical terms implies that it is possible that p is true and q is false. However, the mathematician who denies this inference may well believe that q is not only true, but necessarily true, since all mathematical truths are necessary. Thus, he would deny the inference as a failed attempt of a proof, but would also deny its logical implications from a classical point of view. This refusal to accept classical negation is similar to the one we see with the negation of conditionals. The heart of the problem is that both the inference and the conditional are being evaluated with intensional evidence that are negligible from a logical point of view.

The other similarity between classical deduction implication and material implication involves their almost non-existent role as epistemic guides. Briskman summarises the problem of classical deduction implication as follows:

> If we start from the assumption, unquestioned by almost all writers on entailment, that mathematical truths are necessarily true, then an identification of entailment with strict implication leads to the seemingly absurd consequence that all mathematical truths entail each other, and so are logically equivalent, and thus, assuming our derivation rules are complete, inter-deducible. But this makes a nonsense of mathematical (and logical) practice; for in mathematics (and logic) we usually prove theorems from axioms and can only rarely reverse the procedure and prove the axioms from theorem[35].

That every theorem follows from every theorem vacuously is known, but mathematicians are not interested in vacuous validity since they have an epistemic requirement in their work. They want to know whether some conjecture follows from accepted truths non-vacuously. This is not a problem, however, since the notion of logical consequence was never intended be an epistemic guide to truth. Validity does not increase knowledge. It preserves it. In a typical attempt of proof, a mathematician does not know yet the truth values of the conjecture they are trying to prove or which premises will be required to prove it. At least not until they have actually done it. If this attempt of proof is successful, it will be due to the conceptual connections between the premises and the conclusion (an epistemic requirement), not because of the truth values of either the premises and conclusion alone. The reasons to accept the validity of a proof are independent of the truth-values of the relevant propositions (we don't know whether the conclusion is true or not, since we want to establish its true), and involve the conceptual connections between the premises and the conclusion. If the required

[35] Briskman (1975: 118-19).

conceptual connection between the premises and the conclusion is established, then its validity is also established.

Anderson and Belnap made a similar accusation against material implication in *Entailment: The Logic of Relevance and Necessity*:

> Imagine, if you can, a situation as follows. A mathematician writes a paper on Banach spaces, and after proving a couple of theorems he concludes with a conjecture. As a footnote to the conjecture, he writes: 'In addition to its intrinsic interest, this conjecture has connections with other parts of mathematics. … For example, if the conjecture is true, then the first order functional calculus is complete; whereas if it is false, then it implies that Fermat's last conjecture is correct'[36].

Fermat's last conjecture is now recognised as a theorem proven by Andrew Wiles in 1994, but we can ignore this detail for the sake of argumentation. The point of the counter-example is that you weren't supposed to imply a conjecture whose truth-value is still unknown based on a falsity. No serious mathematician would ever allow such a thing. But then again no proponent of the material implication ever defended that a conditional that is vacuously true due to the falsity of its antecedent can work as a proof of its consequent. The only thing that a vacuous material implication proves is that is valid in a mere vacuous sense. You can't extract with a *modus ponens* the consequent you intended to prove by this artificial mean because the antecedent was false and the only reason to accept the material implication was the falsity of this antecedent. In other words, you can't derive falsities from truth with a material implication relation that is established on vacuous grounds.

In *The Calculus of Strict Implication*, Lewis presented a similar criticism and accused the material implication of being impractical:

> As has already been said, the few theorems in which the present calculus of propositions clearly reveals its meaning of 'implies' … are not capable of any proper use as rules of reasoning. In order so to use them one would need to know the truth or falsity which the reasoning is supposed to discover. Does p (materially) imply q? Tell us first whether p is false or q true, and we can answer.
>
> Even if we take it to be the case that every truth implies every other, the process of *reasoning* about their relations must follow entirely different paths. It must proceed to ask *how* the one fact implies the other, and this inquiry always turns upon possibilities, which will seem to be wider than the mere facts. Would it be a successful piece of deduction if the instructor enlightened the student in accordance with such rules as the system of material implication affords: "The theorem is true; I've proved it. A true proposition is implied by anything. Therefore your postulates imply your theorem. *Q.E.D*"[37].

The knowledge that there is a connection between p and q is sufficient to decide whether $p \rightarrow q$ is true, but knowing that $p \rightarrow q$ is true is not sufficient to decide whether there is a connection between p and q because the material implication is a matter of truth-functional

[36] Anderson & Belnap (1975: 17).
[37] Lewis (1914: 246-7).

combination of truth values. The temptation is to assume that logic should be suited to establish relations between matters of fact because we use conditionals to express relations between matters of fact, but logic isn't in the business of providing knowledge about the world. Instead, its task is to ascertain which deductive argumentative forms ensure truth preservation. Similarly, it is erroneous to assume that a system of logic should correspond to assumptions about real world connections because a connective behaviour has little to do with its content. The truth conditions of a conditional should not encapsulate the reasons to accept the connection between the antecedent and the consequent.

Take for instance the conditionals 'If there were 10 people in this room, we would be short of chairs' and 'If this were a fish, it would have gills'. The connection between the antecedent and consequent in the first conditional depends mainly on a principle of arithmetic, while the connection in the second conditional relies on a non-causal generalisation. The notion that the truth conditions of a connective in logic should guide us in the process of finding the facts that will help to establish the connection in conditionals so different, never mind the connections of every single type of conditional in natural language, defies belief.

One could object that $p \rightarrow q$ cannot be an argument since the main inferences associated with the conditional can go both ways. The reasoner can infer q from the truth of $p \rightarrow q$ and p, just as he can infer $\neg p$ from the truth of $p \rightarrow q$ and $\neg q$. But in an argument we have only one direction, which is from the premise(s) to the conclusion. This reply, however, is naive. Let's take a look at a *modus ponens*, $p \rightarrow q, p \vDash q$. From the truth of $p \rightarrow q$ and p we can infer the truth of q, while from the truth of $\neg q$ we can infer the truth of $\neg((p \rightarrow q) \,\&\, p)$. Once again, the inference can go both ways.

One additional reason to the present interpretation is that conditionals cannot be complete propositions without being interpreted as an indirect assertion of implication its propositional constituents. It is not just the antecedent and consequent need to be expanded to be interpreted as full propositions in their own right, but the conditional as a whole. Thus, conditionals can only be interpreted as complete propositions if they are assertions of implication.

Perhaps more importantly, not only $p \rightarrow q$ is a statement of implication, as also its logical equivalents such as $\neg(p \& \neg q)$ and $\neg p \vee q$. Few people would look at a disjunction as a version of implies, but this is exactly what follows from its logical equivalence with material implication. Statements of implication in deductive argumentative forms admit a similar treatment. Not only $p \vDash q$ is a statement of implication, as also its logical equivalents such as $\neg\Diamond(p \& \neg q)$. More than that, if $\neg p \vee q$ is an implication statement, so are $p \vee q$, $\neg(p \vee q)$, $p \& \neg q$ and any other compound proposition for that matter.

The similarities between conditionals and arguments should give us pause for thought. A logical connective connects propositions such that the value of the compound proposition produced depends only on the truth value of the original propositions and on the truth conditions of the connective. If 'if-then' is both a connective and an argument, what stop us from thinking that that 'therefore' constructions are also connectives? The only difference is that in the case of an argument the truth value of the sentence that says that the premises entail the conclusion is dependent of the truth values of the premises in all possible worlds in which they are true. A statement such as 'p and q, therefore r' means 'The premises p and q entail r', and this statement will only be true if in every possible world where p and q are true, r is true. This shows us that the distinction between connectives and arguments is only a matter of modal breadth. Connectives involve entailment relations restricted to the actual world whereas arguments are about entailment relations that include all the possible worlds in which the premises are true.

CONCLUDING REMARKS

If conditionals are disguised assertions of material implication, the contemporary literature on the subject is headed on the wrong direction. The questions that conditional experts should be asking themselves couldn't possibly be more different from the questions they are asking now. Take for instance the discussion involving conditional probability and principles such as Adam's thesis. This principle states that the assertability of $p \rightarrow q$ is measured by the probability of q given p. If the probability of the consequent given the antecedent is high, the conditional is assertable. Otherwise, it is unassertable. But since conditionals truth-conditions are dependent on material implication, the probability attributions of the consequent given the antecedent have no bearings on its assertability. A similar criticism can be extended to intuitions and questions that gravitate around principles such as the equation, the Ramsey's thesis and possible world theories, just to name a few. Moreover, this also a call for coherence, since the intuitions that support the unpopular material implication are very similar to the intuitions that support that widely accepted classical conception of validity. Russell's terminology is not only appropriate, but it is also more insightful than both the terminology proposed by Quine and the theoretical alternatives proposed by conditional experts.

REFERENCES

Anderson, A., & Belnap, N. 1975. *Entailment: The Logic of Relevance and Necessity*, *Vol. I*. Princeton University Press.

Barker, S. 2006. *Lewis on Implication*. Transactions of the Charles S. Peirce Society: *A Quarterly Journal in American Philosophy*, 42(1), 10–16.

Briskman, L. 1975. Classical Semantics and Entailment. *Analysis*, 35(4), 118–126.

Edgington, D. 1995. On Conditionals, *Mind* 104: 235–329.

Hare, R. 1970. Meaning and speech acts, *Philosophical Review*, 79(1): 3–24.

Jackson, F. 1987. *Conditionals*. Oxford: Basil Blackwell.

Lewis, C. I. 1913. Interesting theorems in symbolic logic. *The Journal of Philosophy*. 10(9), 239–242.

Lewis, C. I. 1914. The Calculus of Strict Implication. *Mind, New Series*, 23(90): 240–247.

Lewis, C. I. 1917. The Issues Concerning Material Implication. *The Journal of Philosophy, Psychology and Scientific Methods*, 14(13), 350–356.

Lewis, C. I. 1928. Review of Principia Mathematica. *The American Mathematical Monthly*, 35(4): 200–205.

Lewis, C. I., Langford, C. 1932. *Symbolic Logic*, second edition. New York, Dover, 1959.

Mackie, J. 1973. *Truth, probability and paradox: studies in philosophical logic*. USA: Oxford University Press.

Moore, G. (ed.). 1993. *Collected Papers, vol. 3: Towards the 'Principles of Mathematics' 1900-1902*. London/New York: Routledge.

Quine, W. 1950. *Methods of Logic,* New York, Holt, Rheinhart and Winston.

Quine, W. 1961. Reply to Professor Marcus. *Synthese*, 13(4), 323–330.

Quine, W. 1964. *Word and Object*. Cambridge, Massachusetts: The MIT Press.

Rescher, N. 2007. *Conditionals*. Cambridge, Massachusetts: The MIT Press.

Ryle, G. 1950. 'If', 'So', and 'Because'. In: *Collected Papers*, Vol. 2: Collected Essays 1929-1968. Routledge, 2009.

Russell, B. 1901. *Recent Italian Work on the Foundations of Mathematics*. In: Moore, G. (ed.). 1993: 350–62.

Russell, B. 1903. *The Principles of Mathematics*. W. W. Norton & Company, 2 ed.

Russell, B. 1905. *Necessity and Possibility*. In: Urquhart, A. (ed.). 1994: 508–20.

Russell, B. 1906. The Theory of Implication. *American Journal of Mathematics*, 28(2), 159–202.

Russell, B. 1908. 'If' and 'Imply', A Reply to Mr. MacColl. *Mind, New Series*, 17(66): 300–301.

Russell, B. 1910. *Some Explanations in Reply to Mr. Bradley*. *Mind, New Series*, 19(75): 373–378.

Russell, B. 1913. The Philosophical Importance of Mathematical Logic. *The Monist*, 23(4): 481–49.

Russell, B. 1919. *Introduction to Mathematical Philosophy*, London: Allen & Unwin.

Russell, B., Whitehead, A. 1910. *Principia Mathematica - Volume One*. Merchant Books, 2009.

Sanford, D. 2003. *If P, Then Q: Conditionals and the Foundations of Reasoning.* London: Routledge.

Stevenson, C. 1970. If-Iculties. *Philosophy of Science*, 37(1), 27–49.

Strawson, F. 1952. *Introduction to Logic Theory*. London: Metheuen & CO. LTD.

Urquhart, A. (ed.). 1994. *The Collected Papers of Bertrand Russell, Volume 4: Foundations of Logic, 1903-05*. London/New York: Routledge.