INDICATIVE CONDITIONALS ARE MATERIAL
EXPANDING THE SURVEY

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ABSTRACT

Adam Rieger (2013) has carried out a survey of arguments in favour of the material account of indicative conditionals. These arguments involve simple and direct demonstrations of the material account. I extend the survey with new arguments and clarify the logical connections among them. I also show that the main counter-examples against these arguments are not successful either because their premises are just as counter-intuitive as the conclusions, or because they depend on contextual fallacies. The conclusion is that the unpopularity of the material account is unjustified and that a more systematic approach in the analysis of arguments is long overdue in our attempts to understand the nature of conditionals.

1. INTRODUCTION

The material account of indicative conditionals states that indicative conditional sentences and the material implication have the same truth conditions. Recently, Adam Rieger (2013) has carried out a survey of arguments in favour of the material account. In this paper, I extend this survey by presenting yet more arguments for the material account. On top of presenting more arguments, I also want to argue that it is plausible to extend the material account to subjunctive conditionals. For that reason, the arguments here presented contain principles that hold for both indicative and subjunctive conditionals. Towards the end, I defend why there are good reasons to accept these new arguments and try to explain why similar arguments have been somewhat unpopular.

1.1 POSITIVE ARGUMENTS WITH (GCP)

One of the central arguments in Rieger's paper involves *General Conditional Proof* (GCP), the principle which states that if *A*, *B* entails *C*, it follows that *A* entails $B \rightarrow C$[1]. Rieger–correctly in my view–uses the following example to claim that this principle is intuitively valid: given that having eggs and olive oil entails that I can make mayonnaise, it follows that having eggs entails that if I have olive oil I can make mayonnaise[2].

[1] Here '→' stands for indicative conditionals, '⊃' stands for material conditional, and '⊨' stands for entailment. All argumentative forms and metalogical principles discussed will be initially named, and from then on will be referred by their respective abbreviations. Some of the known argumentative forms will be introduced only by their names and their logical form will not be introduced. For simplicity of exposition, I will use the same numeration (1,2,3…) for each positive argument and the capital letters *A*, *B*, *C*…. for both sentence letters and propositional variables—the context will make it clear which one is being used. I will not use quotes to highlight the use-mention distinction when there is no risk of confusion, and the symbols and variables quoted will be modified to ensure that the notation remains uniform.

[2] Rieger (2013: 3164).

In his paper, Rieger recognises that there are more arguments involving (GCP), but discusses just two arguments, one involving *Modus Ponens* (MP), and the other involving *Ex Contradictione Quodlibet* (ECQ), the principle that states that anything is entailed by a contradiction[3]. It is worth investigating if there are other versions of the same argument, for different versions tend to involve different logical assumptions. One can refuse a version of the argument with (GCP) because she denies the validity of one of the logical assumptions employed by it, but accepts a different version that involves logical assumptions that she endorses.

The principle that from $\neg(A\&\neg B)$ it follows $A \rightarrow B$ is only acceptable if indicative conditionals are material. Alternative accounts that involve more stringent truth conditions will block this inference, for instance, in possible world theories the fact that $\neg(A\&\neg B)$ is true is not enough to prevent $A \rightarrow B$ from being false if in the closest A-world(s), B is false. However, this principle follows from simple assumptions such as *Disjunctive Syllogism* (DS), *De Morgan*, (DM) and (GCP):

Prem	(1)	$\neg(A\&\neg B)$	
1	(2)	$\neg A \vee \neg\neg B$	1, (DM)
1	(3)	$\neg A \vee B$	2, (DN)
Sup	(4)	A	assumption
1, 4	(5)	B	3,4 (DS)
1	(6)	$A \rightarrow B$	4,5 (GCP)

The following argument proves that any conditional $A \rightarrow B$ can be inferred from $\neg A$, which is one of the infamous paradoxes of the material conditional implied by the material account of indicative conditionals. The argument assumes (TE), the truth conditions of the conjunction, '&', and *Contraposition* (CON)[4]:

Prem	(1)	$\neg A \vDash \neg(\neg B\&A)$	from the truth conditions of '&'
Prem	(2)	$\neg(\neg B\&A), \neg B \vDash \neg A$	from the truth conditions of '&'
Prem	(3)	$\neg B \rightarrow \neg A \vDash A \rightarrow B$	(CON)
2	(4)	$\neg(\neg B\&A) \vDash \neg B \rightarrow \neg A$	2, from (GCP)
1,2,3	(5)	$\neg A \vDash A \rightarrow B$	1–5, (TE)

It could be objected that this argument is not particularly strong since someone who denies the material account will promptly deny the validity of (CON). However, (CON) is accepted by many authors who do not admit the material account of indicative conditionals[5].

An even simpler version of (GCP) argument presented by Simons[6] involves the truth conditions of '&', (TE) and (DN):

Prem	(1)	$\neg A \vDash \neg(A\&B)$	from the truth conditions of '&'

[3] Rieger (2013: 1363; 1365).
[4] Simons (1965: 81).
[5] Lycan (2005: 34–35); Hunter (1993: 285); Austin (1961: 209); Anderson & Belnap (1975: 107–109).
[6] Simons (1965: 79–80).

1	(2)	$\neg A \vDash \neg(A\&\neg B)$	argumentative form similar to 1
1	(3)	$\neg(A\&B), A \vDash \neg B$	given the validity of 1
1	(4)	$\neg(A\&\neg B), A \vDash \neg\neg B$	given the validity of 2
1	(5)	$\neg(A\&\neg B), A \vDash B$	4, (DN)
1	(6)	$\neg(A\&\neg B) \vDash A \rightarrow B$	5, (GCP)
1	(7)	$\neg A \vDash A \rightarrow B$	2, 6 (TE)

Thus, the same conclusion is inferred, but without any need of (CON). Notice that argument can be criticised for using contradictories assumptions, since in the step (2) an argumentative form that involves $\neg A$ is used to draw conclusions about an argumentative form in the step (3), that involves A. But if both (2) and (3) are used in the same argument, then A and $\neg A$ are both true[7].

One way to reduce the fear of inconsistence is observing that the argument proves its conclusion without any premise generating a contradiction[8]. Besides, no assumption about the truth of A and $\neg A$ is needed in order to consider the validity of the steps (2) and (3). They are premises of argumentative forms that are intuitively valid, not assumptions.

Simons also proves the second paradox of material implication (SPM): $\neg A \vDash A \rightarrow B$[9]. The validity of this argumentative form is also implied by the material account. He uses the truth conditions of '&', (GCP), (DN), and (TE):

Prem	(1)	$\neg\neg B \vDash \neg(A\&\neg B)$	from the truth conditions of '&'
1	(2)	$\neg(A\&\neg B), A \vDash \neg\neg B$	given the validity of 1
1	(3)	$\neg(A\&\neg B), A \vDash B$	2, (DN)
1	(4)	$\neg(A\&\neg B) \vDash A \rightarrow B$	3, (GCP)
1	(5)	$\neg\neg B \vDash A \rightarrow B$	1,4 (TE)
1	(6)	$B \vDash A \rightarrow B$	5, (DN)

Finally, we can establish this paradox only with (E&) and (GCP):

Prem	(1)	$A\&B \vDash B$	(E&)
1	(2)	$B \vDash A \rightarrow B$	1, (GCP)

The argument is breathtakingly simple: it just involves the acceptance of (E&), an uncontroversial principle. Thus, what we need to prove that conditionals are material on top of (GCP) is (DN), (TE), and (E&). Each of these principles seem fundamental for our understanding of certain notions: (DN) is a consequence of negation[10], (TE) of the notion of

[7] Simons (1965: 81).
[8] Simons (1965: 81).
[9] Simons (1965: 80).
[10] With the exception of intuitionists, they don't need to apply.

entailment, and (E&) of the notion conjunction. Thus, the only way to block the argument is by refusing (GCP), which is a tall order.

What is most surprising is that (GCP) can be used with another metalogical principle to devastating effect. Each rule of inference has a corresponding rule of proof. (GCP) is a rule of proof corresponding to *Exportation* (EXP): $(B\&A) \rightarrow B \vDash A \rightarrow (B \rightarrow C)$. There is also a metalogical principle corresponding to *Importation* (IMP): $A \rightarrow (B \rightarrow C) \vDash (A\&B) \rightarrow C$. We can call this rule of proof *General Importation Proof* (GIMP):

(GIMP): Give that $A \vDash B \rightarrow C$, it follows that $A\&B \vDash C$

This rule is intuitively valid. If having eggs entails that I can make mayonnaise if I have olive oil, it follows that having eggs and olive oil entails that I can make mayonnaise. Now, in his paper, Rieger shows that from (GCP) and (ECQ), it follows what we could call the *First Paradox of Material Implication* (FPM): $\neg A \vDash A \rightarrow B$[11]. We can use this information together with (GIMP) to show that (ECQ) is equivalent to (FPM) as follows:

Prem	(1)	$\neg A\&A \vDash B$	(ECQ)
1	(2)	$\neg A \vDash A \rightarrow B$	1, (GCP)
1	(2)	If $\neg A \vDash A \rightarrow B$, then $\neg A\&A \vDash B$	1,2 (GIMP)
1	(3)	$\neg A \vDash A \rightarrow B \equiv \neg A\&A \vDash B$	1,2

Since (SPM) follows from (E&) and (GCP), we can use this information together with (GIMP) to show that (SPM) is equivalent to (E&):

Prem	(1)	$B\&A \vDash B$	(E&)
1	(2)	$B \vDash A \rightarrow B$	1, (GCP)
1	(2)	If $B \vDash A \rightarrow B$, then $B\&A \vDash B$	1,2 (GIMP)
1	(3)	$B\&A \vDash B \equiv B \vDash A \rightarrow B$	1,2

These results are surprising. Someone who denies the validity of (SPM) would not think of himself as denying the validity of an uncontroversial argumentative form such as (E&), but this is the consequence of accepting (GCP) and (GIMP). The relation between (FPM) and (ECQ) is even more perplexing, because (FPM) is the most counter-intuitive aspect of the material account and (ECQ) is a consequence of the classical conception of validity. Thus, denying (FPM) amounts to deny (ECQ) and, consequently, the classical conception of validity.

There are other ways of making this connection more intuitive. The connection between (FPM) and (ECQ) can be explained as follows: when $A \rightarrow B$ is asserted under the assumption of $\neg A$, what is actually being asserted is $(\neg A\&A) \rightarrow B$, which should be interpreted as saying that any consequent is materially implied by the conjunction of two inconsistent

[11] Rieger (2013: 5).

antecedents[12]. The connection between (SPM) and (E&) can also be made plausible if we consider that a material implication with a true consequent has the truth of this consequent implicit in its antecedent. The assertion of $A \rightarrow B$ under the assumption of B should be interpreted as $B \rightarrow (A \rightarrow B)$, which by (IMP) leads us to $(B\&A) \rightarrow B$, which means that a consequent is materially implied by its combination with its antecedent (Ceniza, 1988: 511).

Fitelson argued that (EXP) and (IMP) can also be used to prove that conditionals are intuitionist[13]. While this is true, it does not represent a threat to the material account for one simple reason: it has too many similarities with the material account to be perceived as convincing alternative by its opponents. The intuitionist conditional validates paradoxes such as (FPM) and (SPM)[14], which can hardly be considered an improvement.

1.2 POSITIVE ARGUMENTS WITH (CP)

Rieger presented arguments involving *Conditional Proof* (CP)[15]:

(CP): From $A \vDash B$, it follows that $\vDash A \rightarrow B$

There are some principles with similar logical forms that are also intuitively valid. They can be considered different versions of conditional proof. I will use a number and a connective to identify each version of conditional proof:

($CP^1\rightarrow$) From $A \vDash B$, it follows that $\vDash A \rightarrow B$
($CP^2\rightarrow$) From $\vDash A \rightarrow B$, it follows that $A \vDash B$
($CP^1\supset$) From $A \vDash B$, it follows that $\vDash A \supset B$
($CP^2\supset$) From $\vDash A \supset B$, it follows that $A \vDash B$

One of the most striking examples involving (CP) is the proof that all tautological conditionals are material. The proof that $A \supset B$ is logically equivalent to $A \rightarrow B$ is made in two steps. First, we need to show that if $A \supset B$ is a logical truth, so is $A \rightarrow B$. Here is the demonstration:

Prem	(1)	$\vDash A \supset B$	
1	(2)	$A \vDash B$	1, by ($CP^2\supset$)
1	(3)	$\vDash A \rightarrow B$	2, by ($CP^1\rightarrow$)
1	(4)	$\vDash A \supset B$ entails $\vDash A \rightarrow B$	from 1–3

Now we need to show that if $A \rightarrow B$ is a logical truth, so is $A \supset B$[16]:

[12] Russell (1970: 136).
[13] Fitelson (2013).
[14] For an exposition of the paradoxes see Priest (2008: 113).
[15] Rieger (2013: 3164).
[16] Fitelson (2008: 2).

Prem	(1)	$\vDash A \rightarrow B$	
1	(2)	$A \vDash B$	1, by ($CP_2\rightarrow$)
1	(3)	$\vDash A \supset B$	2, by ($CP_1\supset$)
1	(4)	$\vDash A \rightarrow B$ entails $\vDash A \supset B$	from 1–3

Another argument for the conclusion that tautological conditionals are material involves the principle of *Trivial Validity* (TV) according to which any inference with a tautological conclusion is valid:

Prem	(1)	$\vDash A \rightarrow B$	tautology
1	(2)	From $\neg A$ it follows that $\vDash A \rightarrow B$	1, by (TV)
1	(3)	From B it follows that $\vDash A \rightarrow B$	1, by (TV)
1	(4)	$\vDash A \rightarrow B \equiv \vDash A \supset B$	from 2,3

This conclusion that tautological conditionals are material reinforces the view that true conditionals in mathematics are material. The connection lies in the fact that mathematical truths are necessary. The proposition 'If 3 is a prime, it is only divisible by 1 and itself' is not just true, but necessarily true; it is a logical consequence of the axioms of arithmetic. Now, if a conditional expressing a mathematical truth is a tautology, it must be material, since any tautological conditional is material.

If tautological conditionals are material, alternatives to the material account of indicative conditionals are forced to assume that the logic of conditionals in mathematics is distinct from the one used in natural language. This concession puts some pressure on them, for any acceptable theory of indicative conditionals must be simplified to the material account to mathematical contexts[17]. After all, it cannot be ignored that the language of mathematics is in continuity with the ordinary language, since the mathematical activity occurs in natural language. Thus, since mathematical conditionals are conditionals of natural language, a proper explanation of conditionals must be a proper explanation of mathematical conditionals[18].

In order to avoid the material account, it would be needed to argue that mathematical conditionals are only material because mathematical contexts have special features, but this is implausible since we understand the use of conditionals in mathematics by following our ordinary use, and since mathematical conditionality is expressed by using standard conditional forms of a given natural language[19]. Perhaps there is little doubt that mathematical conditionals are material since the counter-intuitive aspects usually associated with the material implication such as temporal flexion and causal connections are not present in mathematics[20]. But if that is the only difference between mathematical conditionals and other conditionals, perhaps these counter-intuitive aspects have no logical significance. In any

17 Burgess (2004: 568).
18 Barwise (1986: 21).
19 Rumfitt (2013: 183).
20 Orayen (1985: 235–236).

case, the fact that the material account must not assume that indicative conditionals have a different semantics when they are tautological is an advantage over its alternatives.

It is also possible to show that conditionals are material if they obey conditional negation (CN), i.e., the principle that $A \rightarrow B$ is logically equivalent to $\neg(A\&\neg B)$. The following argument involves (E&), (MP), (I&), (CP^{1}→) and reduction to absurdity (I¬)[21]:

Prem	(1)	$A \rightarrow B$	
Sup	(2)	$A\&\neg B$	assumption
2	(3)	A	2, (E&)
1,2	(4)	B	1,3 (MP)
2	(5)	$\neg B$	2, (E&)
1,2	(6)	$B\&\neg B$	4,5 (I&)
1	(7)	$\neg(A\&\neg B)$	2–6, (I¬)

Prem	(1)	$\neg(A\&\neg B)$	
Sup	(2)	A	assumption
Sup	(3)	$\neg B$	assumption
2,3	(4)	$A\&\neg B$	2,3 (I&)
1,2,3	(5)	$\neg(A\&\neg B)\ \&\ (A\&\neg B)$	1,4 (I&)
1,2	(6)	B	3,5 (I¬)
1	(7)	$A \rightarrow B$	2,6 (CP^{1}→)

One could object that (CP^{1}→) implies that any conditional of the form $A \rightarrow B$ is entailed by B, since from the mere acceptance of B and the assumption of A, it follows that $A \rightarrow B$ from B alone by (CP^{1}→). This happens because (CP^{1}→) allows us to reason with assumptions instead of accepted premises, as it is evidenced by the argumentative strategy used above. The assumption that A is introduced in the step 2 only to be later used in a *reductio* in order to obtain the desired conclusion in the step 6; then is finally discharged from the assumption dependence column as the antecedent of the conclusion in the step 7[22].

The use of (CP^{1}→) can be unsettling in many ways. It allows us, for instance, to infer conditionals by making vacuous discharge of assumptions. Take the following example:

Prem	(1)	B	
1	(2)	$A \rightarrow B$	1, (CP^{1}→)
2	(3)	$B \rightarrow (A \rightarrow B)$	1,2 (CP^{1}→)

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[21] Hanson (1991: 54).

[22] Adams (1975: 24).

The conditional derived in line 3 is the usual case in which the line number corresponding to the antecedent of $B \rightarrow (A \rightarrow B)$ is discharged from the assumption dependence column. But in line 2 there is vacuous discharge of assumptions. Since the antecedent of 2 has no prior occurrence, there is no assumption to discharge from the dependence column[23]. And as if this result was not unnatural enough, notice that the use of ($CP^1\rightarrow$) in the step 3 would imply that B entails $A \rightarrow B$, which is one of the paradoxes of the material conditional.

However, in order to deny ($CP^1\rightarrow$) we need to accept that $A \rightarrow B$ can be false when A entails B, which is patently absurd[24]. Moreover, denying the validity of ($CP^1\rightarrow$) would require a complete reformulation of mathematics as it is known since mathematical proofs rely heavily on applications of ($CP^1\rightarrow$). When a mathematician deduces B from a hypothesis A and axioms X, she asserts $A \rightarrow B$ on the strength of X alone[25]. If its validity on mathematics is not to be abandoned, at the very least, it would be needed to explain why ($CP^1\rightarrow$) is invalid in our nonmathematical deductions. The fact of the matter is that our uneasiness with the unforeseen aspects of ($CP^1\rightarrow$) involve extreme examples that are not supported by our inferential practices. If these intuitions go against a basic and intuitive principle, maybe they should not be trusted[26]. Rather, we should rely on the basic assumptions that are grounded on inferential practices.

Another distinctive argumentative form implied by the material account is strengthening of the antecedent (SA): $A \rightarrow B \vDash (A\&C) \rightarrow B$. We can show that (SA) follows from Left Weakening (LW), i.e., if $A \vDash B$, then $A\&C \vDash B$, ($CP^2\rightarrow$) and ($CP^1\rightarrow$):

Prem	(1)	$A \rightarrow A$	Tautology
1	(2)	$A \vDash A$	1, ($CP^2\rightarrow$)
1	(3)	$A\&C \vDash A$	2, (LW)
1	(4)	$(A\&C) \rightarrow A$	3, ($CP^1\rightarrow$)
1	(5)	$A \rightarrow A \vDash (A\&C) \rightarrow A$	1–4, (TE)

Since ($CP^1\rightarrow$), ($CP^2\rightarrow$) and (TE) are fundamental features of entailment, any attempt to bar the argument will focus on (LW). However, (LW) should be interpreted as a consequence of monotonicity. Thus, in order to refuse (SA), we should refuse monotonicity for deductive logic. That is a high price to pay to avoid the material account.

Another argument for the conclusion that any conditional $A \rightarrow B$ can be entailed by B involves ($CP^1\rightarrow$), ($CP^2\rightarrow$), simplification (E&), and exportation (EXP)[27]:

Prem	(1)	$B\&A \vDash B$	(E&)
1	(2)	$(B\&A) \rightarrow B$	1, ($CP^1\rightarrow$)

[23] Sherry (2006: 205).
[24] Hanson (1991: 54).
[25] Rumfitt (2013: 183).
[26] Orayen (1983: 16).
[27] Leavitt (1972: 10). Leavitt attributes the authorship of the argument to Bertrand Russell. Its original formulation is presented in natural language. Rieger (2013: 3163–4) uses a similar argument to prove that ($CP^1\rightarrow$), (EXP) and (E&) entail (GCP). The only difference is that the premise is $A, B \vDash C$, which leads to the conclusion that $A \vDash B \rightarrow C$.

1	(3)	$B \rightarrow (A \rightarrow B)$	2, (EXP)
1	(4)	$B \vDash (A \rightarrow B)$	3, ($CP^2\rightarrow$)

There are many arguments for the material account involving Or-to-If (OTF): $A \vee B \vDash \neg A \rightarrow B$. The following argument proves (OTF) with syllogism disjunctive (SD), ($CP^1\rightarrow$), ($CP^2\rightarrow$) and (EXP)[28]:

Prem	(1)	$(A \vee B)\&\neg A \vDash B$	(SD)
1	(2)	$((A \vee B)\&\neg A) \rightarrow B$	1, ($CP^1\rightarrow$)
1	(3)	$(A \vee B) \rightarrow (\neg A \rightarrow B)$	2, (EXP)
1	(4)	$A \vee B \vDash \neg A \rightarrow B$	from 3, by ($CP^2\rightarrow$)

The logical assumptions of this argument are beyond suspicion. (SD) is universally accepted, while ($CP^1\rightarrow$) and ($CP^2\rightarrow$) seem to be a triviality about the relation between tautological conditionals and entailment. (EXP) would also be regarded as irreproachable weren't for its association with other positive arguments for the material account. I will discuss some attempts to refute (EXP) and other fundamental principles in section 3.

Rumfitt, however, observes that (CP) implies the material account only if the negation assumed is classic. In an intuitionist logic, (CP) is valid, but the conditional is not material for $\neg A \vee B$ entails $A \rightarrow B$, but the converse entailment is invalid[29]. However, since the main alternatives to the material account, such as suppositional theory and possible world theories endorse the classic negation, the assumption of (CP) is still strong.

1. 3 POSITIVE ARGUMENTS WITH (OTF)

The argumentative form (OTF) is intuitively plausible. Consider the following reasoning: The disjunction 'Either *A* or *B*' is true if at least one of its disjuncts is true, i.e., if either *A* is true or *B* is true. Thus, if not-*A* or *B* is true then either not-*A* is true or *B* is true. But if *A* is true not-*A* can't be true, and since either not-*A* or *B* must be true then *B* must be true, i.e., 'If *A* then *B*' is true. Therefore, it was shown that if either not-*A* or *B* is true then 'If *A* then *B*' is true[30]. Thus, if (OTF) is valid, conditionals are material, for if the disjunction is material, so it is the conditional that is implied by it[31]. This point is reinforced by the fact that (FPM) follows from (I∨), (OTF), (DN) and (TE)[32]:

Prem	(1)	$\neg A$	
1	(2)	$\neg A \vee B$	1, (I∨)
1	(3)	$\neg\neg A \rightarrow B$	2, (OTF)

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[28] Katz (1999: 411).
[29] Rumfitt (2013: 166).
[30] Pollock (1969: 19).
[31] For a similar argument see also Rieger (2013: 3165–3166).
[32] Gensler (2010: 370).

1	(4)	$A \to B$	3, (DN)
1	(4)	$\neg A \vDash A \to B$	1–3 (TE)

A similar argument could be used to show (SPM):

Prem	(1)	B	
1	(2)	$B \vee \neg A$	1, (I∨)
1	(3)	$\neg A \vee B$	2, *Commutativity of Disjunction* (CD)
1	(4)	$\neg\neg A \to B$	3, (OTF)
1	(5)	$A \to B$	4, (DN)
1	(6)	$B \vDash A \to B$	1–5 (TE)

Notice that while (I∨) and (OTF) allow us to infer (FPM), the validity of (FPM) and (OTF) enable us to deduce the same conclusion allowed by (I∨)[33]:

Prem	(1)	A	
1	(2)	$\neg A \to B$	1, (FPM)
1	(3)	$A \vee B$	2, (OTF)

The same can be said about (I∨) and (OTF) entailing (SPM), since (SPM) and (OTF) are enough to infer the same conclusion allowed by (I∨):

Prem	(1)	B	
1	(2)	$\neg A \to B$	4, (SPM)
1	(3)	$B \vee A$	5, (OTF)
1	(4)	$A \vee B$	6, (CD)

Thus, (I∨), (OTF), (FPM) and (SPM) are linked. One who denies the material account could block either argument by refusing (I∨) instead of attacking (OTF). That's what Anderson and Belnap do when they argue that *A* only entails $A \vee B$ if the disjunction is extensional (Anderson & Belnap, 1975: 165–167). One reply is that a disjunction 'Either *A* or *B*' is denied by 'Neither *A*, nor *B*', and that from the last proposition we can infer 'not-*A*'. Thus, if we could not infer 'Either *A* or *B*' from *A*, we should not be able to infer 'not-*A*' from 'Neither *A*, nor *B*'[34].

Since (OTF) implies the material account, any argument that shows the validity of (OTF) is an argument for the material account. One powerful argument for (OTF) involves the principle that if two propositional forms imply the same propositional form by means of the same propositional form they must be equivalent. Given the argumentative forms $\neg A \vee B, A \vDash B$ and $A \to B, A \vDash B$, together with the propositional *A*, both $\neg A \vee B$ and $A \to B$ entail the same propositional form *B*. $\neg A \vee B, \neg B \vDash \neg A$ and $A \to B, \neg B \vDash \neg A$ when combined with the

[33] The argument is adapted from Johnson (1996).
[34] Slater (1988: 124).

same propositional form $\neg B$, both $\neg A \vee B$ and $A \rightarrow B$ entail the same conclusion $\neg A$[35]. What is important about this argument is that it relies on a basic principle that establishes the plausibility of (OTF) by means of the validity of (DS), (MP) and (MT).

Another powerful argument for (OTF) involves nothing more than two tautologies and commutativity of disjunction (CD)[36]:

Prem	(1)	$A \rightarrow B$	
Prem	(2)	$\neg A \rightarrow \neg A$	tautology
1	(3)	$A \vee \neg A$	tautology
1	(4)	$B \vee \neg A$	1–3, given the possible inferences with A and $\neg A$ in 1 and 2
1	(5)	$\neg A \vee B$	4, (CD)

It is not obvious how someone could prevent the conclusion above given its logical assumptions. Thus, a case can be made that (OTF) is valid and therefore the material account is true.

1.4 POSITIVE ARGUMENTS WITH (CN)

In these discussions, there is too much focus on certain argumentative forms that have conditionals in the premise or in the conclusion. Other connectives are either ignored or have a marginal role–for instance, disjunctions are only considered in the discussion about (OTF). However, the truth conditions of conjunctions and conditional negation (CN), $A \rightarrow B \equiv \neg(A \& \neg B)$, are central to this discussion. (CN) is a mark of material account since it implies that any conditional is false if, and only if, its antecedent is true and its consequent is false. Otherwise it is true. A conditional that requires a stronger truth conditions, e.g., that could be false when the antecedent is false, would violate (CN) and thus would not be material.

(CN) is intuitive. Consider the following reasoning: if $A \rightarrow B$ is true, then if A is true, B must be true. Since is not the case that A is true and B is false, $A \& \neg B$ is false, and consequently $\neg(A \& \neg B)$ is true. If $\neg(A \& \neg B)$ is true, and A is true, then B is not false, since $A \& \neg B$ is not true. Therefore, B must be true. Thus, if A is true, so is B, i.e., $A \rightarrow B$ is true. Thus, (CN) is valid[37].

If we accept (CN) and consider the circumstances under which the conditional and disjunction are false, we can show that (OTF) is valid, and, therefore, that the material account is true. If $\neg A \vee B$ is false, then A is true and B is false, which is exactly the circumstance in which $A \rightarrow B$ is predicted to be false according to (CN). If $A \rightarrow B$ is false, $\neg A \vee B$ will be false since $A \& \neg B$ will be true. Thus, the validity of (CN) implies the validity of (OTF) and, consequently, the truth of the material account. (CN) can also be used to infer either (FPM) in the following demonstration:

Prem	(1)	$\neg A$	
1	(2)	$\neg(A \& \neg B)$	1, given the truth conditions of '&'

[35] Sen (1961: 46).
[36] Russell (1970: 136).
[37] Restall (2006: 92–93).

1	(3)	$A \rightarrow B$	2, (CN)
1	(4)	$\neg A \vDash A \rightarrow B$	1,3 (TE)

The importance of (CN) lies in the fact that it provides a bridge between a conditional and a disjunction via the negation of a conjunction. A similar argument works for (SPM):

Prem	(1)	B	
1	(2)	$\neg(A\&\neg B)$	1, given the truth conditions of '&'
1	(3)	$A \rightarrow B$	2, (CN)
1	(4)	$B \vDash A \rightarrow B$	1,3 (TE)

It is important to observe that (CN) is logically equivalent to (OTF). First, let's consider the demonstration that (CN) implies (OTF) above:

Prem	(1)	$A \rightarrow B$	
1	(2)	$\neg(A\&\neg B)$	1, (CN)
1	(3)	$\neg A \vee \neg\neg B$	2, (DM)
1	(4)	$\neg A \vee B$	3, (DN)

We can show that (OTF) implies (CN) using (DM), (DN), and the *Truth Preservation* principle (TP), according to witch if $A \vDash B$, then $\neg B \vDash \neg A$:

Prem	(1)	$\neg A \vee B \vDash A \rightarrow B$	(OTF)
1	(2)	$\neg(A \rightarrow B) \vDash \neg(\neg A \vee B)$	1, (TP)
1	(3)	$\neg(A \rightarrow B) \vDash \neg\neg A\&\neg B$	2, (DM)
1	(4)	$\neg(A \rightarrow B) \vDash A\&\neg B$	3, (DN)

This proves that (OTF) and (CN) are logically equivalent. One indirect way to criticise (CN) is by attacking (MP), since it is implied by it. (MP) follows from (CN)[38] and the transitivity of entailment (TE). The demonstration is as follows[39]:

Prem	(1)	$A \rightarrow \neg B \vDash \neg(A\&B)$	(CN)
1	(2)	$\neg(A\&B)\ \&\ A \vDash \neg B$	1, given the truth conditions of '&' and '¬'
1	(3)	$(A \rightarrow \neg B)\ \&\ A \vDash \neg B$	1, 2 (CN) and (TE)
1	(4)	$(A \rightarrow B)\ \&\ A \vDash B$	1,3 (TE) given '¬', every proposition is equivalent to a negation, ergo, (CN) entails (MP)

However, even though the refusal of (MP) is a path to block (CN), it is not a promising one. Everyone who rejects (MP) will most likely already reject (CN)[40] for independent reasons.

38 Rumfitt refers this rule as 'a particular case of importation'.
39 Rumfitt (2013: 176).
40 For further discussions involving (CN) see Wiredu (1972: 253–254) and Neidorf (1967: 66–67).

1.5 POSITIVE ARGUMENTS WITH CIRCUMSTANCE SURVEYORS

Lee Archie presented an argument for the material account using circumstance surveyors[41]. His assumptions are the following:

1. (MP) and (MT) have solid instances in natural language
2. The fallacy of affirming the consequent can have true premises and a false conclusion
3. $A \to B$ is false when A is true and B is false
4. The other truth values can be attributed in an arbitrary manner, but consistently: [T, T = 1] [F, T = 3] [F, F = 4]

First, let's examine the circumstance surveyor of (MP):

A	B	$A \to B$	A	$\vDash$	B
T	T	**1**	**T**		**T**
T	F	F	T		F
F	T	3	F		T
F	F	4	F		F

The only line where (MP) has a solid instance is the first one. Thus, $A \to B$ must be true when A and B are true. Now, consider the circumstance surveyor of (MT):

A	B	$A \to B$	$\neg B$	$\vDash$	$\neg A$
T	T	1	F		F
T	F	F	T		F
F	T	3	F		T
F	F	**4**	**T**		**T**

The only line where modus tollens has a solid instance is the fourth. Thus, $A \to B$ must be true when A and B are false. The circumstance surveyor of the fallacy of affirming the consequent contains the last piece of the puzzle:

A	B	$A \to B$	B	$\vDash$	A
T	T	1	T		T
T	F	F	F		T
F	T	**3**	**T**		**F**
F	F	4	F		F

The only instance with true premises and a false conclusion is the third line. Thus, $A \to B$ must be true when A is false and B is true. With a few uncontroversial assumptions, Archie made a compelling case that conditionals must have the same truth conditions of the material implication truth table.

[41] Lee Archie (1979).

Archie, Hurdle and Thomblison made another argument with the following assumptions[42]:

5. $A \rightarrow B$ is false when A is true and B is false.
6. $(A \rightarrow B)\&(\neg A \rightarrow B)$ is logically equivalent to ‘B, whether or not A’.
7. $(A \rightarrow B)\&\neg(\neg A \rightarrow B)$ is not self-contradictory.
8. $(\neg A \rightarrow B)\&\neg(A \rightarrow B)$ is not self-contradictory.

The first assumption is uncontroversial. The second assumption is intuitively plausible, since asserting both $A \rightarrow B$ and $\neg A \rightarrow B$ amounts to assert that ‘B, whether or not A’, e.g., asserting ‘If John works harder, he will be fired, and if John does not work harder, he will be fired’ amounts to accept that ‘John will be fired whether he works harder or not’. It is just a different way of saying that B is true, and its truth is independent on A. The third assumption can be justified by someone who accepts that A implies B, but $\neg A$ doesn't imply B. The fourth assumption is justified by someone who accepts that $\neg A$ implies B, but A doesn't imply B. These assumptions can be used to build the following truth table:

	I	II	III	IV	V	VI	VII	VIII
	A	*B*	Not-*A*	If *A* then *B*	If not-*A* then *B*	*B*, whether or not *A*	If A then B, but not if not-A then B	If not-A then B, but not if A then B
1	T	T	F	?	?	T	?	?
2	T	F	F	F	?	F	F	?
3	F	T	T	?	?	T	?	?
4	F	F	T	?	F	F	?	F

The truth values in columns IV and V are determined the first assumption. The truth values on column VI are determined by the truth values of column II. The truth values in columns VII and VIII are determined by the falsity of one of their conjuncts in columns IV and V. Now, we can use these truth values to make new inferences about the remains truth values. Since the column VI is true on lines 1 and 3, the columns VII and VIII must be true on the same lines, since the conjunct assertion of $A \rightarrow B$ and $\neg A \rightarrow B$ amounts to assert that ‘B, whether or not A’. Since columns IV and V are the conjuncts of VII and VIII, they must be true on lines 1 and 3. And it is clear that ‘Not if not-*A* then *B*’ (second conjunct of column VII) is inconsistent with ‘if not-*A* then *B*’ (column V). Thus, column VII must be false when column V is true, i.e., on lines 1 and 3. A similar reasoning applies to ‘not if *A* then *B*’ (second conjunct of column VIII), which is the negation of ‘if *A* then *B*’ (column IV). Therefore, VIII must be false on lines 1 and 3, since column IV is true on these lines. The following table can be constructed with these conclusions:

	IV	V	VI	VII	VIII
	If *A* then *B*	If not-*A* then *B*	B, whether or not A	If A then B, but not if not-A then B	If not-A then B, but not if A then B
1	T	T	T	F	F

[42] Archie, Hurdle and Thomblison (1977).

2	F	?	F	F	?
3	T	T	T	F	F
4	?	F	F	?	F

From the assumptions 3 and 4, we can infer that columns VII is true on line 4 and column VIII is true on line 2, since they are not self-contradictory. Now, if the column VII is true on line 4, both of its conjuncts must be true on line 4, which means that column IV is true. If the column VIII is true on line 2, column V must be true on line 2, since it is one of its conjuncts. Thus, all the truth values have been properly assigned in a way that is in accordance with the material account.

1.6 MISCELLANEOUS ARGUMENTS

In other to show that $A \supset B$ and $A \rightarrow B$ are logically equivalent, we need to show that the entailment relation between them goes both ways. First, we need to show that '→' entails '⊃'. This conclusion follows from the assumption that (MP) is valid for '→'. If the entailment of $A \rightarrow B$ and A to B were to fail, there would be an A and a B for which $A \rightarrow B$ is true, but $A \supset B$ is false. But if A were true and B false, we would be able to infer by (MP) that for '→' B is true, which is a contradiction[43].

Now, we need to show that '⊃' entails '→'. This can be shown with hypothetical syllogism (HS). Given T, a tautology, consider the following proof[44]:

Prem	(1)	$A \supset B$	
Prem	(2)	$A\&B$	
1	(3)	Even if A, B	from 2
	(4)	$A \rightarrow T$	tautology
1	(5)	$T \rightarrow (A \supset B)$	from 1
1	(6)	$A \rightarrow (A \supset B)$	4, 5 (HS)
1	(7)	$A \rightarrow (A\&(A \supset B))$	since $A \rightarrow A$ is also a tautology, we can combine their similar antecedents to make this inference
	(8)	$A \rightarrow B$	from 7, if we weaken the consequent

The only way to block the argument is by refusing the validity of (HS).

Another argument involves the fact that it is relatively uncontroversial that conditionals satisfy the first line of the truth table (FL). Based on this information and the validity of (CON), we can easily show that conditionals must satisfy the fourth line of the truth table. The demonstration is as follows:

Prem	(1)	$A\&B$	
1	(2)	$A \rightarrow B$	1, (FL)
1	(3)	$\neg B \rightarrow \neg A$	2, (CON)

43 Rieger (2013: 3163).

44 Morreau (2009: 448–449).

1	(6)	$A\&B \vDash \neg B \rightarrow \neg A$	1–3 (TE)

This shows that any conditional with a false antecedent and a false consequent must be true[45]. Given that (FL) is widely accepted, any criticism should be directed to the validity of (CON), but it is arguable that (CON) follows from the truth conditions of '$\leftrightarrows$'[46]:

Prem	(1)	$A \leftrightarrows B \equiv (A \rightarrow B)\&(B \rightarrow A)$	given the truth conditions of '$\leftrightarrows$'
Prem	(2)	$A \leftrightarrows B \equiv (A \rightarrow B)\&(\neg A \rightarrow \neg B)$	given the truth conditions of '$\leftrightarrows$'
1, 2	(3)	$B \rightarrow A \equiv \neg A \rightarrow \neg B$	1, 2 if $A\&B \equiv A\&C$, then $B \equiv C$

Another argument for (CON) involving the biconditional is that '*A* if and only if *B*' is intuitively equivalent to 'if *A* then *B* and if *B* then *A*'. A conjunction is true when both of its conjuncts are true. A biconditional is true when both of its members *A* and *B* have the same truth value, i.e., when both are true or both are false. Thus, when both are false, '*A* if and only if *B*' is true, but in this case the conjunction can only be true if each of the conjuncts are true. Thus, if '*A* if and only if *B*' is true when *A* and *B* are false, then 'If *A* then *B* and if *B* then *A*' is true when *A* and *B* are false[47]. Now if (CON) and (FL) entails the material account, and (CON) follows from '$\leftrightarrows$', it is necessary to refuse either '$\leftrightarrows$' or (FL) in order to refute the material account.

The material account also follows from the use of the conditional 'If Oswald did not kill Kennedy, someone else did'. There is nothing strange in saying that this conditional depends on whether Kennedy was killed, and thus on whether Kennedy was killed by someone. If the logical form of the proposition 'Someone killed Kennedy' is represented as (∃x)Fx, the logical form of the proposition 'Oswald did not kill Kennedy' can be represented as ¬F*a*. If we apply the existential instantiation rule to the first propositional form, we have F*b*, and this together with ¬F*a* give us ($a \neq b$) by indiscernibility of identicals. The conjunction then gives us F*b* & ($a \neq b$) and by applying the existential generalisation we have (∃x)Fx & ($a \neq$ x), which is the logical form of the consequent of the conditional. Thus, the conditional is entailed by its consequent. Now suppose that the antecedent of the conditional is false. Thus, it is true that Oswald killed Kennedy, and, therefore, that someone killed Kennedy. Therefore, the conditional will again be true[48].

Thus, from a simple conditional and an impeccable line of reasoning we are forced to admit that the conditional is material. The simplicity of the argument is also its weakness since it could be said that this only shows that *that* specific indicative conditional is material, which hardly works as an argument for the materiality of all indicative conditionals.

One could object that the assumption that someone killed Kennedy is only a ground to accept the indicative conditional, but does not entail it. Suppose that Kennedy died because of a blow to the head. Either Kennedy accidentally tripped and fell on a rock or Oswald

45 Ortiz (2007: 87).
46 Ortiz (2009: 3).
47 Ortiz (2007: 87).
48 Mellor (1993: 238–239). In fact, it could be said that the premise 'Someone killed Kennedy' not only entails, but is logically equivalent to the conclusion, 'If Oswald did not kill Kennedy, someone else did'; since there are no circumstances in which the conditional is true and the negation of the premise, namely, 'No one killed Kennedy', is true. See Lowe (1979: 139–140) and Johnston (1996: 99–100).

clobbered Kennedy with the rock and made it look like he tripped. No one other than Oswald was within a fifty-mile radius of Kennedy at the time of his death. In these circumstances, it is true that someone killed Kennedy, but the conditional is false[49].

Now, this conditional is only false in this circumstance if we assume that the truth conditions of the conditional involves a possible-worlds approach. Given the imagined circumstances, in the world in the world that is most similar to the actual world overall and where Oswald did not kill Kennedy, nobody else killed him because he accidentally tripped and fell on a rock. The problem is that this begs the question against the material account. Someone who defends the material account could just as easily dismiss this argument by arguing that if someone killed Kennedy and the only possible killer is Oswald, both the antecedent and consequent of the conditional are false, and the conditional is vacuously true.

One could also argue that this particular conditional is a different type from the ones presented in the usual counter-examples against the material account. For one thing, this conditional involves an attribution of responsibility for an action that happened and not a causal relation between the antecedent and the consequent. However, this line of reasoning is not entirely successful for two reasons. First, it concedes too much to the material account since it admits that a whole class of conditionals is material. Secondly, it is highly implausible and suggests that something is wrong with the usual counter-examples presented against the material account. The alternatives have the burden of proof of explaining what makes the other types of conditional non-material.

In his paper, Rieger presents an argument involving universally quantified statements. He uses *U-to-if*: (UTF) Every F is $G \vDash Fa \rightarrow Ga$. Given the statement 'Everyone studying French is studying German' and the assumption that Anna is one of the students, we have to conclude by (UTF) that 'If Anna is studying French, then she is studying German'. But this conditional must be material, for it can only be false if Anna is studying French but is not studying German, since that is the only way of falsifying (UTF)[50].

Now, there is an underlying counter-intuitive aspect in the truth conditions of universally quantified statements that makes the connection between (UTF) and the material account even more compelling. The underlying fact assumed in the argument above is that general statements such as 'Every *F* is *G*' are only true if every x that is a *F* is a *G*, or there are no *F*s. Thus, a proposition such as 'All unicorns are sad' is vacuously true, since there are no unicorns. The reason for this vacuous truth value is that the negation of 'Every *F* is *G*' is 'Some *F*s are not *G*s', which reads as 'There is an x, such that x is *F*, but not *G*'. This explains why 'All unicorns are sad' must be true, since its negation is 'There is a x, such that this x is a unicorn and is not sad'. But this is false, since there are no unicorns. If this proposition is false, its contradictory must be true, thus 'All unicorns are sad' is true. Thus, when there are no *F*s, 'Every *F* is a *G*' is vacuously true and imply vacuously true conditionals such as 'if *a* is an *F*, *a* is a *G*'. Thus, the only way to block (UTF) is by arguing that universally quantified statements are not true when there are no *F*s, because they have qualitative existential import[51], but this seems a desperate solution to avoid the material account.

[49] Davis (1980: 184).
[50] Rieger (2013: 3166–7).
[51] Anjum (2008: 111).

Faris designed an argument for the material account that tries to do justice to our intuitions that the connection between A and B is a necessary condition for the truth of $A \rightarrow B$[52]. This connection can be expressed by the *Condition E* (CE):

> (CE): There is a set S of true propositions such that B is inferable from A together with S.

Thus, a conditional such as 'If Smith is taller than Jones, Smith is taller than Robinson' will be true if there is at least one set S that makes B inferable from A. This set could be the statement *Jones is at least as taller as Robinson*. The truth of this statement satisfies (CE). Now for any propositions A and B, (CE) is a condition necessary and sufficient for the truth of $A \rightarrow B$. It is sufficient since in any case in which we believe that a set S existed as specified we must be prepared to assert $A \rightarrow B$ and necessary for without it we are prepared to deny the truth of $A \rightarrow B$. In the cases in which B is inferable directly from A, (CE) is satisfied since S can be considered any set of true propositions. About the conditional 'If no French was saved then nobody that was saved was French' we can say that (CE) is satisfied since the consequent is inferable from the antecedent together with any set of true propositions.

Now, B can be inferred from A and $A \supset B$. Therefore, if $A \supset B$ is true there is a set S of true propositions, i.e., the set that consists solely of the proposition $A \supset B$, such that B is inferable from A together with S. It follows that if $A \supset B$ is true, (CE) is satisfied. But if (CE) is satisfied, $A \rightarrow B$ is true. Thus, if the proposition $A \supset B$ is true, $A \rightarrow B$ is true. Since it is accepted that $A \supset B$ is inferable from $A \rightarrow B$, it follows that $A \supset B$ and $A \rightarrow B$ are logically equivalent[53].

There are two interesting aspects of this argument. First, it derives the conclusion that conditionals are material from what is supposed to be its nemesis, namely, the relevantist intuition that there should be a connection between the components of a conditional. Secondly, it could be argued that (CE) is logically equivalent to (GCP). Notice that in the demonstration that uses (CE) the reasoning is as follows: if B is inferable from A and $A \supset B$, if $A \supset B$ is true, (CE) is satisfied and $A \rightarrow B$ is true; but this is just a different way of saying that from $A, A \supset B \vDash B$ it follows that $A \supset B \vDash A \rightarrow B$, which is the formulation of (GCP). Consequently, (CE) is logically equivalent to (GCP).

2. ARE THE POSITIVE ARGUMENTS COUNTER-EXAMPLES?

Despite the intuitive appeal of the positive arguments, some philosophers will use these logical connections as arguments *against* the material account. First, let's consider the counter-examples presented against (EXP) and (IMP). For instance, we can use $(A\&B) \rightarrow A$ as a logical truth to infer by (EXP) and (IMP) that $B \rightarrow (A \rightarrow B)$ is also a logical truth. If $B \rightarrow (A \rightarrow B)$ is a logical truth, then B entails $A \rightarrow B$ by ($CP^2\rightarrow$). Now, if $B \rightarrow (A \rightarrow B)$ is a logical truth, both conditionals 'if I am going to be run over by a car tomorrow, then even if I take precautions, I will still be run over', and 'if I am not going to be run over by a car tomorrow,

[52] Faris (1962: 209–210).
[53] Faris (1962: 209–210).

then even if I fail to take precautions, I won't be run over' will be true. But both conditionals seem false since the can be used in an argument for fatalism: if both conditionals are true, I must conclude that any precautions are useless[54].

Following a similar reasoning, some philosophers argued that the use of (EXP) and (IMP) coerces us to abandon the validity of (MP). If $(A\&B) \to A$ is a logical truth, given (EXP) and (IMP), $B \to (A \to B)$ will also be a logical truth. But if (MP) was valid, we should always infer $A \to B$ from B and $B \to (A \to B)$. However, it is assumed that the inference of $A \to B$ from B is not valid. Thus, either we should maintain (EXP) and (IMP), or abandon the validity of (MP)[55]. This criticism will resonate later on with Vann McGee, who claims that if (EXP) and (IMP) are valid, either (MP) is invalid or the material account is true[56].

One could also deny the validity of (EXP) with the following counter-intuitive: 'If Harry runs fifteen miles this afternoon and he is killed in a swimming accident this morning, then he will run fifteen miles this afternoon. Therefore, if Harry runs fifteen miles this afternoon, then if he is killed in a swimming accident this morning, he will run fifteen miles this afternoon'[57].

Notice that the counter-intuitive instances of $B \to (A \to B)$ reflect a certain disbelief with the counter-intuitive aspects of (SPM), which follows from this propositional form by (CP), since it is a logical truth. Thus, these counter-examples can also be interpreted as an attack against (SPM), since what it is being argued is that (EXP) and (IMP) cannot be valid since this would imply the validity of (SPM).

Other attacks were made against (OTF). Stalnaker uses the counter-intuitive consequences of the material account as an indication that (OTF) should not be trusted. First, let's consider an instance of (OTF): 'Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did'. This is intuitively valid, but now consider the following example: 'The butler did it; therefore, if he didn't, the gardener did.' The premise of this counter-intuitive argument entails the premise of the (OTF) instance–for if the butler did it, that implies that either the butler or the gardener did it by (I∨)–and the conclusion of both arguments are the same. The problem is that if the first intuitive argument is valid, then the second counter-intuitive argument must be valid. But since it isn't, (OTF) must be invalid after all[58].

Another counter-intuitive instance of (OTF) is presented in the following context: suppose that there are two balls in a bag, labelled as x and y. We know that ball x comes from a collection in which 99% of the balls are red. But I don't have any reason to think that ball y is red. Maybe ball y comes from a collection in which only 1% of the balls are red. My confidence that x is red, justifies my belief that either x is red, or y is red, but doesn't justify the conclusion that if x is not red, y is red[59]. What these counter-examples have in common is the assumption that (OTF) is invalid since it implies (FPM) given that the premise of (OTF) must be obtained by (I∨). Since (OTF) implies (FPM), and (FPM) is invalid, so it is (OTF).

No attack against the material account would be complete without counter-examples aimed specifically at (GCP). Let A be the disjunction 'Bob will retire next year or we will be

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[54] Kremer (1987: 212). See also Lycan (2005: 82) for a similar criticism. Notice that a similar objection could be directed against the use of (E&) and (GCP) to derive (SPM).
[55] Adams (1975: 33).
[56] McGee (1985).
[57] Lycan (2005: 82).
[58] Stalnaker (1975: 269).
[59] Edgington (1987: 55–56).

invaded by Martians'[60] and suppose that *A* is true only because the first disjunct is true. Now let *B* be 'Bob will not retire next year'. The conjunction of *A* and *B* entails 'We will be invaded by Martians'. From this it follows by (GCP) that 'Bob will retire next year or we will be invaded by Martians' entails 'If Bob does not retire next year, we will be invaded by Martians'. This is apparently a counter-example since we would be inclined to accept the first argument, but not the second[61]. The use of (GCP) when the premises are contradictory can also be questioned. The accusation is that it is reasonable to doubt that $A\&\neg A \vDash B$ entails $A \vDash \neg A \rightarrow B$ because maybe one should be able to derive anything from a contradiction even if the conditional of the second argumentative form is false[62].

Notice that what is being assumed in both counter-examples is that (GCP) must be invalid since it entails (FPM). In the first counter-example, the use of (GCP) involves the inference of an instance of (OTF), which is assumed to be intuitively invalid, from another instance of (DS), which is perceived as intuitively valid. The instance of (OTF) is intuitively invalid for the same reason the previous counter-examples against (OTF) are counter-intuitive: it implies (FPM) since the conclusion has a false antecedent given that the premise of (OTF) is obtained by (I∨). Thus, in order to deflect the attack to (GCP), we also need to save (OTF) from the counter-aspects of (FPM). The connection between (FPM) and (GCP) in the second counter-example is more direct, since it is directly entailed by (GCP) from (ECQ).

An exhaustive list of every counter-example would be beyond the scope of this paper, but this list is representative of the material account's main forces, since it includes the most plausible argumentative forms associated with it. If the apparent invalidity of these counter-intuitive instances could not be explained away, this would be a decisive blow against the material account.

3. RESISTING THE COUNTER-EXAMPLES

The fact that philosophers can draw the opposite conclusion based on the same data supports the dictum that one philosopher's *modus ponens* is another philosopher's *modus tollens*. However, in order to interpret the data in the opposite direction, a principled rationale must be provided, otherwise there is a risk of begging the question. The line of reasoning of the objections could be summarised in the following way: some intuitive argumentative forms that imply the material account of indicative conditionals also imply some counter-intuitive argumentative forms. But since these counter-intuitive argumentative forms are invalid, these intuitive argumentative forms must be invalid after all. This line of reasoning distorts the dialectics of discussion. Instead, what we should say about the role of positive arguments is that some intuitive argumentative forms imply the material account of indicative conditionals, and, consequently, indirectly imply all of its argumentative forms, including the counter-intuitive ones. This is exactly what we should expect if the data was strong enough to confirm a theory. It would also confirm its theoretical implications, no matter how counter-intuitive

[60] I'm slightly simplifying the original counter-example proposed by Lycan (2005: 82). The only differences is that in the original the left disjunct is 'My friend Bob will retire next year' and the right disjunct is 'in 2004 the planet Ynool will spontaneously explode, causing a rain of blood over Fairbanks, Alaska'. This will not affect the argumentation.
[61] Lycan (2005: 82).
[62] Gibbins (1979: 451).

they may be. To claim that positive arguments should not be trusted because they force us to accept the paradoxes of the material account amounts to claim that arguments should be ignored if they force us to accept the material account. This is circular reasoning, because it is assumed that only the counter-intuitive aspects of classical argumentative forms have an evidentiary role in our understanding of conditionals. It makes the defence of the material account an impossible task.

This line of reasoning assumes that the counter-intuitive instances of material account are the last word about the subject because they cannot be explained away. But these counter-intuitive instances can be explained away. They are not genuine counter-examples since in each case the premise is just as counter-intuitive as is the conclusion. Genuine counter-examples have premises that are intuitively true and a conclusion that is intuitively false since they are supposed to exemplify an instance of an argumentative form that has a true premise and a false conclusion.

The first counter-example is that conditionals with the propositional form $(A \& B) \rightarrow A$ cannot be equivalent to conditionals with the propositional form $B \rightarrow (A \rightarrow B)$, since this implies that they are logical truths and that the following pair of conditionals is true:

> (C1) If I am going to be run over by a car tomorrow, then even if I take precautions, I will still be run over.
>
> (C2) If I am not going to be run over by a car tomorrow, then even if I fail to take precautions, I will still be run over.

But notice how in the counter-example above no consideration is given to the premise that implies each conclusion. They are assumed as intuitively true given that they remain uninterpreted under the logical form of $(A \& B) \rightarrow A$, but a closer look reveals that they are just as counter-intuitive as their corresponding conclusions are:

> (P1) If I am going to be run over by a car tomorrow and take precautions, I will still be run over.
>
> (P2) If I am not going to be run over by a car tomorrow and fail to take precautions, I will still be run over.

What is counter-intuitive about these premises is that it is assumed in the antecedent of each that one person, maybe a speaker that asserts the conditional, will be run over by a car tomorrow. That is what give both the premises and their corresponding conclusions their fatalist character. (EXP) and (IMP) do not allow us to draw any conclusions that are not already assumed in the premises. Thus, there is no genuine counter-example in this case. If this example is the reason that would force us to choose between the validity of (EXP)-(IMP) and (MP), as Adams would want us to believe, then there is no real dilemma. We can choose both of them, since they face no real threat.

The other counter-example to (EXP) can be explained in a similar fashion. The conclusion of the argument, 'if Harry runs fifteen miles this afternoon, then if he is killed in a swimming accident this morning, he will run fifteen miles this afternoon', is just as counter-intuitive as its premise, 'if Harry runs fifteen miles this afternoon and he is killed in a swimming accident this morning, then he will run fifteen miles this afternoon'. The reason why the premise is counter-intuitive is that the antecedent of the premise can only be true if Harry run fifteen miles after dying in an accident, i.e., the antecedent of the premise is false. The inference with (EXP) only transfers this counter-intuitive aspect.

Now consider the counter-examples against (OTF). The first argument is that (OTF)'s validity implies the validity of the following instance of (FPM): 'The butler did it; therefore, if he didn't, the gardener did.' However, the instance of (OTF) is just as counter-intuitive as this conclusion, since we accept the argument 'Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did', under the assumption that the butler did it. A similar reasoning explains what is wrong with the other counter-intuitive instance of (OTF). The disjunction in the premise is also counter-intuitive since it involves the consideration of two alternatives when in fact it is accepted under the assumption of just one of them, i.e., the premise that 'either *x* is red, or *y* is red' is accepted only because it is accepted that *x* is red. Thus, the counter-intuitiveness of concluding that 'if *x* is not red, *y* is red' does not matter, since it was already present in the premise.

One could object that the disjunction that is accepted under the assumption of the truth of one of its disjuncts is less counter-intuitive than a conditional that is accepted under the assumption that its antecedent is false, but that already involves the admission that the premise is counter-intuitive, which defeats the whole purpose of presenting a clear example in which the premises are true and the conclusion is false. At the very least, it would be necessary to admit that only the counter-intuitive aspects of the disjunction can be properly explained away, which is far from obvious.

Finally, let's consider the counter-example against (GCP). The example involves a counter-intuitive instance of (OTF) that is entailed by an instance of (DS). But this is not a proper counter-example since the instance of (DS) that is the basis of the inference is just as counter-intuitive as the instance of (OTF). One of the premises is the disjunction 'Bob will retire next year or we will be invaded by Martians', which is accepted only because Bob will retire next year, and the other premise is the claim 'Bob will not retire next year'. The conjunction of the disjunction and the claim entail, 'We will be invaded by Martians'. Now notice how counter-intuitive is the disjunction 'Bob will retire next year or we will be invaded by Martians', which includes two completely unrelated facts, and how counter-intuitive is the second premise given that the disjunction was accepted mainly due to the assumption of its negation. Thus, the instance of (DS) has both counter-intuitive premises and a counter-intuitive conclusion, being the second premise and the conclusion assumed as false by the arguer. Of course, it is important to observe that each counter-intuitive instance of (OTF) and (DS) is not invalid, since in each case the premise is just as counter-intuitive as the conclusion.

However, not every counter-intuitive instance of argumentative form can be explained away as cases in which the conclusion preserves the counter-intuitiveness of the premise. This solution does not work in all cases, e.g., the argument 'The butler did it; therefore, if he didn't, the gardener did', has an intuitive premise and a counter-intuitive conclusion. It does not work also in the following counter-example to strengthening of the antecedent: 'If Brown wins the election, Smith will retire to private life. Therefore, if Smith dies before the election and Brown wins it, Smith will retire to private life'[63]. Thus, if the validity of these

[63] Adams (1965: 166).

argumentative forms are to be rescued from the contrary intuitions, a different explanation is required[64].

I think that we can explain these counter-intuitive aspects[65] as the result of an illicit alteration of the context in the evaluation of the argument. They seem invalid because we commit a contextual fallacy[66]. The conclusion 'if the butler didn't, the gardener did' is counter-intuitive if the only reason to accept it is the assumption that 'the butler did', because it is evaluated in a context that ignores this contextual assumption made in the premise. But if we retain the contextual assumptions fixed, the conclusion will lose its counter-intuitive aspect. The same explanation holds for the second example. The conclusion seems false because it is evaluated in a context where is assumed that Smith dies before the election, but the premise that is the basis of this conclusion discards this possibility. Once we recognise that this possibility must be discarded in the evaluation of the conclusion, its seeming falsity is neutralised.

The principle that the context should be kept constant is not an *ad hoc* solution, but a basic tenet of semantics[67]. The violation of this basic tenet implies that all classical inference rules would be invalid[68]. Conversely, its observance implies that all classical inference rules are valid. Thus, in order to make their case, the critics of the material account must refute this basic principle, but it is far from obvious how this could be successful.

Moreover, if we maintain the context constant, the paradoxes of material conditional turn out to be valid for indicative conditionals. From the premise 'John will not drink sulphuric acid' it is legitimate to conclude that 'If John drinks sulphuric acid, he will gain super powers'. The conclusion only seems false if we consider a context where the antecedent is true, but the conclusion only follows from the premise because the antecedent is false. The perception that the conditional is false when the antecedent is true is irrelevant because the antecedent is false in the context of evaluation.

The importance of using a constant context also explains why it is so plausible to think that conditionals in mathematics are material. In mathematics, an argument is evaluated using a single context-set, but in evaluating arguments in general, especially the counter-intuitive instances presented as counter-examples, logicians tend to change the background facts in the passage of the premises to the conclusion.

The absence of contextual fallacy is also behind examples that support the material account such as 'If Oswald did not kill Kennedy, someone else did'. This conditional is

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[64] It also does not explain the counter-intuitive aspect of trivially valid argumentative forms in which the premises are contradictory or the conclusion is tautological, e.g., the argumentative form, $\neg A \& A \vDash B$, is counter-intuitive, but while the premise is counter-intuitive for being a contradiction, B can still be intuitive; and the the argumentative form, $A \vDash B \vee \neg B$, can have an intuitive conclusion even if the premise is intuitive. What make these argumentative forms counter-intuitive is the fact that they are valid despite the irrelevance of the premise for the conclusion. But while these counter-intuitive aspects need to be explained, they are not urgent as the other counter-examples contrary to the material account. The reason is that they assume a strong relevance requirement that is inconsistent with widely accepted principles such the first line of the truth table of material implication. These principles are accepted even by most critics of the material account. However, a proper rebuttal of these relevantist intuitions are beyond the scope of this paper.

[65] Traditionally, the principled defence of logical systems in face of counter-intuitive instances has being the bread and butter of the proponents of the material account. See Ajdukiewicz (1956); Allott & Uchida (2009a; 2009b); Grice (1989); Jackson (1987; 2006); Noh (1998); Rieger (2006; 2015); Smith (1983) and Smith & Smith (1988). However, an analysis and comparison of the strengths of the different approaches would go beyond the scope of this paper.

[66] Allott & Uchida (2009a; 2009b).

[67] Allott & Uchida (2009a; 2009b; Brogaard and Salerno (2008); Gauker (2005: 94); Kaplan (1989).

[68] Brogaard and Salerno (2008: 40–41).

obviously entailed by the falsity of the antecedent or the truth of the consequent because it is a known fact that someone killed Kennedy and Oswald is the main suspect. Thus, either Oswald did not kill Kennedy or someone else did. An illicit contextual change is not even considered in this case because it would only be plausible if these assumptions were not known facts.

4. CONCLUDING REMARKS

This systematic approach is long overdue and we should not waste the knowledge we acquired about the logical connections between these principles and meta-principles. The use of surveys of positive arguments not only provides invaluable data in our attempt to understand conditionals, but also represent a change of paradigm in the way the evidence is examined. Instead of thinking in terms of individual argumentative forms, and what our individual intuitions about their intuitive instances in natural language are, we should think in terms of clusters of argumentative forms that gravitate together due to their logical dependence. This change of mindset will represent a significant improvement over the prevailing approach to counter-examples.

Perhaps even more importantly, the counter-examples against the material account fail either because they have counter-intuitive premises or because they commit contextual fallacies. Therefore, it would be prudent to scrutinise with a fine-tooth comb every argumentative principle and theoretical assumption in order to determine whether they fall in the same traps or not. It would be hard to overestimate the impact of this simple methodological observance on the prevailing views about conditionals.

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