IN DEFENSE OF HYPOTHETICAL SYLLOGISM

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ABSTRACT

Moti Mizrahi (2013) presented a putative counterexample to Hypothetical Syllogism (HS) for indicative conditionals aiming to succeed where previous attempts to refute HS have failed. Lee Walters (2014a) objected that Mizrahi's putative counterexample results from an inadequate analysis of conditionals with embedded modals, but advanced new putative counterexamples to HS for subjunctive conditionals that are supposed to bypass this issue (Walters, 2014a; 2014b). It is argued that Walter's analysis of embedded modals is unnecessary to prevent Mizrahi's putative counterexample, since the example can be disarmed either as a fallacy of equivocation or as a contextual fallacy. Walters's putative counterexamples to HS are also criticized as contextual fallacies. The conclusion is that neither Mizrahi nor Walters provided compelling reasons to reject HS.

Keywords: indicative conditionals; subjunctive conditionals; hypothetical syllogism; contextual fallacy; fallacy of equivocation.

1. MIZRAHI'S ASSAULT ON HS AND SOME MODAL COMPLICATIONS

A conditional is a sentence composed by a subordinate clause, usually accompanied by the term "if", that indicates the condition for the occurrence of the main clause. Two examples of conditionals are "If it rains there will be a flood in the city" and "If it had rained, there would be a flood in the city". The examples mentioned above are respectively of an indicative conditional and a subjunctive conditional. Subjunctive conditionals are also known as counterfactuals, since their antecedents seem contrary to fact. The terms "subjunctive" and "counterfactual" are slightly misleading (see Edgington, 1997: 99; von Fintel, 2012: 466–467), but they allow us to reliably identify a group sentences that exhibit a recognizable grammatical form, such as the use of "would" in the main clause (or consequent), and the past tense in the if-clause (or antecedent).

Now, suppose I accept that if it rains there will be a flood in the city. It turns out that is also true that if there will be a flood in the city, some people will lose their houses. It seems reasonable to conclude from these assumptions that if it rains there, some people will lose their houses. This argument is an instance of the argumentative form that logicians call "Hypothetical Syllogism" (HS). The example above was an instance of HS for indicatives, but HS is also intuitively valid for subjunctives. The acceptance of both that if it had rained, there would be a flood in the city, and that if there were a flood in the city, some people would lose their houses, plausibly implies that if it had rained, some people would lose their houses.

What is surprising is that despite its plausibility, HS was targeted with putative counterexamples. Consider the following putative counterexample to HS for indicatives advanced by Adams (1965: 166):

(1) If Brown wins the election, Smith will retire to private life.
(2) If Smith dies before the election, Brown will win it.
(3) Therefore, if Smith dies before the election, then he will retire to private life.

It seems that both (1) and (2) could be true, but (3) is obviously false. It is absurd to suppose that Smith could decide to retire after he died.

But this counterexample does not work since the premises and the conclusion are not evaluated in the same context. The context is illicitly shifted during the evaluation of the argument. Suppose that (3) is false, i.e., that it has a true antecedent and a false consequent. In this case, Smith will not be able to retire, because he will die before the election takes place. (1) has a false consequent and (2) has a true antecedent. It remains to be seen whether Brown will win the election in this context. If he does, (1) will have a true antecedent and a false consequent, and (2) must be true, since the antecedent is true and the consequent is true. Therefore, at least one of the premises will be false. Since there is no context where both premises are true and the conclusion is false, there is no counterexample. On the other hand, suppose that Brown will not win the election even if Smith dies—they are not the only candidates. In this case, the first premise will be vacuously true, since it will have a false antecedent; however, the second premise will be false, since it will have a true antecedent and a false consequent. Again, this is not a context in which the premises are true and the conclusion is false.

This discussion suggests that to properly refute HS is not as simple as it seems, since it also involves avoiding the quicksand of context shifting. Moti Mizrahi (2013, pp. 41–42) tried to meet this challenge with the following putative counterexample:

(4) If I am in Boston at time t, then I am in a city whose name starts with the letter B at time t.
(5) If I am in a city whose name starts with the letter B at time t, then I might be in Baltimore at time t.
(6) Therefore, if I am in Boston at time t, then I might be in Baltimore at time t.

The premises are intuitively true, but the conclusion seems false. I cannot be in Boston and Baltimore at the same time t.

Walters, however, argues that the apparent invalidity of HS in the example results form a failure to consider the way in which conditionals and modals interact. If we accept this naive treatment of conditionals with modals, argued Walters, we will have to accept the following counterexample against *Modus Ponens* (MP):

(7) If Wanchen does kill her father, she should do it gently.
(8) Wanchen does kill her father.
(9) Wanchen should kill her father gently.

It seems that (7) is true since Wanchen loves her father, so she would need to kill him, she would to it gently. Now, suppose that Wanchen does kill her father. It follows by MP that Wanchen should kill her father gently. The problem, however, is that Wanchen's father "is in good health, wants to live, and is a moral saint". Thus, it is not true that Wanchen should kill her father gently. But then we have a contradiction, since it is both true and false that Wanchen should kill her father gently. (7) and (8) are true, but (9) is false (Walters, 2014a: 91).

In the example above we deal with conditionals containing deontic modals, but the superficial analysis of conditionals with epistemic modals will also generate apparent counterexamples to MP. Consider the following example:

(10) Imran is either at work or in the park.
(11) If Imran is not at work, (then) he must be in the park.
(12) Imran is not at work.
(13) Imran must be in the park.

Suppose that I have enough information to attest that Imran is either at work or in the park, but no enough information to attest the truth of either disjunct. In this circumstance, I assume that Imran might be at work and he might be in the park. (10) is true. Now, from the disjunction it follows that "If Imran is not at work, (then) he must be in the park". (11) is true. Now, suppose that it turns out that Imran is not at work ((12) is true), we can then conclude by MP that Imran must be in the park. But the problem is that this would contradict our earlier assumption that Imran could be at work. Thus, (10)-(12) are true, but (13) is false (Walters, 2014a: 92).

Walters' point is that these contradictions generated by MP result from an incorrect treatment of conditionals with modals, and not the apparent invalidity of MP. The same thing could be said about Mizrahi's conclusion: it says more about his treatment of conditionals with modals, than about HS's supposed invalidity. In order to prevent the counterexamples to MP, Walters applies Dorr and Hawthorne's hypothesis about embedded modals, according to which modals in the consequents of conditionals should be interpreted as being constrained by the antecedent. In this analysis, the epistemic modals in the consequent of a conditional concern what is epistemically possible given the antecedent (Dorr and Hawthorne, 2013, pp. 867–914).

Thus, in the previous putative counterexamples to MP, (7) should be interpreted as (7*) "If Wanchen does kill her father, then in all epistemically possible worlds where she kills her father, she should do it gently". This together with (8) allow us to infer a perfectly acceptable conclusion: "In all epistemically possible worlds where Wanchen kills her father, she should do it gently". The other example can be explained in the same fashion. (11) should be analyzed as (11*) "if Imran is not at work, (then) in all epistemically possible worlds where he is not at work, he is in the park". This premise together with (12) will only allow us to infer "In all epistemically possible worlds where Imran is not at work, he is in the park", which is quite reasonable.

If we apply this analysis to Mizrahi's putative counterexample to HS we have:

(14) If I am in Boston at time t, then there is an epistemically possible world where I am in Boston at time t, and I am in a city whose name starts with the letter B at time t.

(15) If I am in a city whose name starts with the letter B at time t, then there is an epistemically possible world where I am in a city whose name starts with the letter B at time t and I am in Baltimore at time t.

(16) If I am in Boston at time t, then there is an epistemically possible world where I am in Boston at time t and I am in Baltimore at time t.

In this analysis, (4) and (5) are true because (14) and (15) are true, whereas (6) is false because (16) is false, since there is no epistemically possible world in which I am in Boston and Baltimore at the same time. But since the consequent of (15) and (16) are different, the example cannot be a counterexample to HS for the simple reason that it is not an instance of HS.

Of course, it is also possible to accept that the embedded modals should be taken in consideration, but dispute that they are constrained by the antecedent. In this alternative interpretation, the modals in the consequents of (5) and (6) have a wide scope over the conditional. But if that is the case, then Mizrahi's example is not an instance of HS. The same criticism holds if this alternative interpretation is adopted for the embedded modals of the putative counterexamples to MP. If the modals in (7) and (9), or (11) and (13), have wide scope, then these conditionals cannot be used in MP. There is no counterexample.

2. WHY A SUPERFICIAL ANALYSIS OF EMBEDDED MODALS IS NOT THE ISSUE

While it is true that there is much to say about the logical nuances of conditionals with embedded modals, there is very little to say about their role in the generation of these putative counterexamples. The first putative counterexample to MP can be explained away as a contextual fallacy, while the second putative counterexample to MP ignores how belief revision occurs during inferences, and Mizrahi's putative counterexample can be dismissed either as a fallacy of equivocation or a contextual fallacy.

In the first putative counterexample, (7) derives its plausibility from the fact that Wanchen finds herself in a circumstance where she has to kill her father. Since she would have to kill him she would opt for the lesser of evils, i.e., she would kill him gently. On the other hand, (9) is evaluated in an entirely different context where the only relevant facts are his good health, his will to live, and the fact that he is a moral saint. However, the contradictory conclusion cannot be inferred if the context is kept constant during the evaluation of (7) and (9). If (7) and (9) are evaluated in a context where Wanchen has to kill her father, she would kill her father, notwithstanding his will to live, etc. On the other hand, if (7) and (9) are evaluated in a context where the only background facts are Wanchen's father good health, will to live, and the fact that he is a moral saint, then (7) and (9) are both false, because she should not kill her father. The role of background facts in the context also explains why Walters thinks that the modal in (7) does not represent what Wanchen should do simpliciter, but only what she should do given the antecedent. These are the background facts that are responsible for the truth of (7), i.e., she must kill her father gently, but *only if* this tragic circumstance imposes itself.

In the second putative counterexample to MP, we assume that Imran Imran might be at work and he might be in the park, only to discover that Imran was not at work. Walters asserts that we must conclude by MP from the conditional that Imran must be in the park. This conclusion is in contradiction with the initial assumption but only if we disregard that the falsity of this assumption in the second premise of the MP: we conclude that he must be in the park because we find out that he was not at work. There is no contradiction in the second example, but just a lack of attention to the way beliefs are updated during inferences. In the inferential passage from the disjunction to the conditional, the reasons to accept both propositions involve merely the assumption that Imran must be in one of two locations. But in the employment of the conditional in a MP, the realization that the antecedent is true engenders a belief revision where one of the locations is discarded.

Notice that the role of the conditional in the inference of the supposed contradiction is negligible. After all, all we need is (10) "Imran is either at work or in the park" and (12) "Imran is not at work", to infer (13) "Imran must be in the park". When the problem is formulated in this way, it becomes clear that there that there is no contradiction in the example, but just a lack of attention to the way beliefs are updated during inferences. Otherwise, we would have to admit that there is a contradiction every time one accepts a disjunction without knowing the truth values of its disjuncts, and later on end up using the disjunction in a *Disjunctive Syllogism*.

That these counterexamples have nothing to do with the modals involved is also evidenced by the fact that from (10) we should not be able to infer a conditional with a modal such as (11), but a different conditional such as (11*) If Imran is not at work, (then) he is in the park. Moreover, each counterexample can be formulated without modals to the same effect. The first counterexample could be reinterpreted with the fist premise as "If Wanchen does kill her father, she will kill him gently". The acceptance of this premise together with the realization that Wanchen killed her father, allow us to infer by MP that she killed him gently. But since Wanchen's father "is in good health, wants to live, and is a moral saint", we can also infer that Wanchen will not kill her father, gently or not. Since we used two different context sets, the contradiction ensues: Wanchen will kill and not kill her father. The second counterexample can also be interpreted without modals. We know that there are only two excluding alternatives about Imran's location: he is either at work or in the park. From this disjunction it follows that "If Imran is not at work, (then) he is in the park". The realization that he is not at work will allow us to infer by MP that he is in the park. But this conclusion is in contradiction with the initial assumption that being at the work was one of his alternative locations.

Now, let us consider again Mizrahi's putative counterexample to HS. Just like the apparent counterexamples to MP, Mizrahi's example is not generated by the presence of embedded modals. In fact, the example either involves a fallacy of equivocation or a contextual fallacy. Consider (5). Unless the city whose name starts with a letter B in the antecedent is Baltimore, the conditional will be false. The reason is the same one that lead to the rejection of the conclusion: it is not possible to be in two cities at the same time t. On the other hand, in order to make (5) true, we must admit that the city in the antecedent is Baltimore. If we incorporate this information in the antecedent, we will have "If I am in the city of Baltimore, whose name starts with the letter B at time t, then I might be in Baltimore at time t". The problem now is that the HS does not apply, since the antecedent of (5) is not

equivalent to the consequent of (4). Thus, the counterexample can be accused of being a fallacy of equivocation.

One possible reply is that the belief that the city mentioned in the antecedent of (5) is Baltimore is an assumption accepted in a tacit manner. This solution, however, will turn the counterexample in a contextual fallacy. In (4), the consequent is accepted under the assumption that the city whose name starts with B is Boston, but in (5) the antecedent is accepted under the assumption that the city whose name starts with the same letter is Baltimore. Therefore, we can accept (4) and (5) in different contexts, but never in the same context. This throws out of the window the idea that both premises are true; an idea which is necessary to demonstrate that both premises can be true when the conclusion is false. Thus, there is no need to resort to an analysis involving embedded modals to prevent Mizrahi's putative counterexample to work, since it fails for independent reasons. The validity of HS for indicatives is sheltered from Mizrahi's attack.

3. WALTER'S COUNTEREXAMPLES TO HS

Walter's interpretation of Mizrahi's putative counterexamples implies that if HS is indeed invalid, this must be shown by other counterexamples. And this is exactly what Walters tried to do by proposing another putative counterexample to HS for subjunctives that is intended to be immune to appeals to context. The first premise of the counterexample is accepted under the assumption that it is plausible to think, at least in deterministic contexts, that a subjunctive conditional is true when the consequent is true and the antecedent does no determine the truth of the consequent. Thus, we can say that the following subjunctive is true: "it would not rain if he did not do a rain dance, but it would not rain if he did either". If we represent subjunctive conditionals with ">", the conditional has the following form: $(A \lor \neg A) > C$. It is also plausible to think that a subjunctive conditional with a possible antecedent and a necessary consequent is true. Thus, any subjunctive conditional with the form $B > (A \lor \neg A)$ satisfies this requirement. We can then infer from the two conditionals that $B > C$, for any given B, by HS. But since subjunctive conditionals are not generally true simply because the consequent is true, HS for subjunctives is invalid (Walters, 2014a, pp. 95–96).

One obvious problem of this counterexample is that the rationale behind the acceptance of both premises implies the acceptance of the conclusion. The first premise, $(A \lor \neg A) > C$, is accepted as true because C is true and independent of $A \lor \neg A$. The second premise, $B > (A \lor \neg A)$, is accepted as true because $A \lor \neg A$ is necessary and B is possible. In both premises the antecedent is irrelevant to the consequent. But then we can say that the conclusion inferred from these premises, $B > C$, will be true simply because C is true. After all, due to its inferential path, B has no relevance to the truth of C. Thus, there is a context shift during the evaluation of the argument in the sense that the rationales used to accept both premises are abandoned in the conclusion.

Walters also advanced another counterexample to HS for subjunctives that is supposed to be immune to appeals to context:

(17) If I had lived on the moon, I would have drank freeze-dried tea.
(18) But if I had drank freeze-dried tea, I would not have lived on the moon.
(19) Therefore, If I had lived on the moon, I would not have lived on the moon.

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The first premise is a conditional assumed as true, but where the antecedent is completely irrelevant to the consequent. The second premise is true under certain circumstances. The conclusion, however, is false. Thus, HS must be invalid (Walters, 2014b: 994).

Notice that there is no tension in asserting both premises and rejecting the conclusion within the same context because Walters assumes that (17) is an irrelevant conditional. But what does he mean by an irrelevant conditional? An irrelevant conditional is one whose "consequent is true and the antecedent is irrelevant to whether or not the consequent obtains" (Walters, 2014b: 987). Thus, we can say that the acceptance of (17) implies that the speaker is committed to drink freeze-dried tea, whether she is living on the moon or not. But if living on the moon has no bearing on the speaker's willingness to drink freeze-dried tea, (18) must be false, since it asserts that the speaker's willingness to drink freeze-dried tea is relevant to whether or not she would have lived on the moon.

It seems that we are onto something important, since Walters asserts that (18) is true under certain circumstances without specifying what they are. In order for the counterexample to work, however, both (17) and (18) need to be accepted within the same context, but there is no common context where living on the moon is irrelevant to drinking freeze-dried tea, whereas drinking freeze-dried tea is relevant to living on the moon. It is a contextual fallacy.

The only way to maintain the context fixed is to assume that both the premises and the conclusion are irrelevant conditionals of some sort. Since (17) is accepted because the consequent is assumed as true, (18) has a true antecedent. In that case, we have to posit that the consequent is true to ensure that (18) is true, while stipulating that (17) has a false antecedent to ensure context consistency. The conclusion will then have a false antecedent and a true consequent, but since "If I had lived on the moon, I would not have lived on the moon" only seem false in a context where the antecedent is true, there is no counterexample anymore.

It could be objected that the conclusion is intuitively false even if we recognize that the antecedent is false, but the only reason to think that a conditional with the form $A > \neg A$ is false is that if A were true, $\neg A$ would be false. But this is irrelevant since the counterexample already assumes that $\neg A$ is the case. This can be interpreted as a contextual fallacy during the evaluation of a single proposition. The proposition, in this case, the conclusion of the argument, is accepted in a context where the antecedent is taken as false, but the conclusion is evaluated in a different context, where the antecedent is taken as true. This can also be interpreted as an argumentative contextual shift in the sense that $\neg A$ implies $A > \neg A$ in the same contexts, but this argument is refused due to a contextual shift in the passage of the premise to the conclusion. Thus, Walters' putative counterexamples to HS cannot be considered an improvement over previous attempts to refute HS, since they did not break the curse of contextual fallacy. The validity of HS for subjunctives is secured.

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