TWO DEGREES OF IMPLICATION

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One premise *p* materially implies a conclusion *q* if, and only if, it is not the case that both *p* is true and *q* is false in a given world that is assumed as a parameter. This is a relation of material implication. This reference to a parameter world is justified by the fact that when we evaluate arguments that contain a material implication in the premise, we consider all the possible worlds in which the premise is true. The set of these possible worlds might include the actual world, but don't need to be restricted by it. The relation of formal implication is slightly different. One premise *p* formally implies a conclusion *q* if, and only if, it is not the case that *p* is true and *q* is false in any possible world.

We can say then that in a material implication the relation of logical consequence is restricted to a parameter world, whereas in a formal implication the relation of logical consequence is unrestricted and extends over many worlds. One could infer from these similarities that the material implication should be reduced to formal implication with the argument that it is just a restricted version of it, or, inversely, that formal implication should be reduced to material implication with the argument that it is an unrestricted version of it. But both reductionist claims would betray a superficial understanding of this connection. The obvious kinship between material and formal implication just means that they express the same type of implication. If they seem any different is because they differ in scope.

One way to talk about this difference is to maintain that formal and material implication are the same type of implication presented in two degrees. In the first degree, we have what is usually referred to as the relation of material implication, which is restricted by a parameter world. In the second degree, we have what is known as a relation of formal implication, which ensures that in every possible world in which their premises are true, their truth is preserved. The important thing is that the same pattern of implication presented in first degree is repeated in the second degree.

Let's use '$^{1\bullet}$' and '$^{2\bullet}$' to represent first and second degrees. We can put this degree distinction into practice by evaluating a simple argumentative form such as *modus ponens*: $p \supset q, p \vDash q$. This argumentative form can be interpreted as claiming that that in every possible world in which *p* $^{1\bullet}$implies *q* and *p* is true, *q* is also true. Or, to put it more simply, the truth of $(p \supset q)\&p$ $^{2\bullet}$implies *q*. The $^{2\bullet}$implication is a $^{1\bullet}$implication repeated across different worlds. It is the same process, only with a different modal reach.

Conditional sentences are claims to a deductive inference, which means that the assertion of a conditional contains the implicit claim that the antecedent (or premise) necessitates the consequent (or conclusion) relatively to the parameter world. The fact that the claim to a necessitation relation is restricted to a given world does not alter the dynamic. Conditionals are arguments, not connectives. If they happen to be used in arguments, it's because we make arguments that involve arguments either as a premise or a conclusion, or both. The argument that uses an argument as a premise or a conclusion is a $^{2\bullet}$implication, but the argument that is used either as a premise or a conclusion is a $^{1\bullet}$implication.

Let's consider the first paradox of material implication, i.e., the argumentative form $\neg p \vDash p \supset q$. One apparent counterexample to this argumentative form is 'Some John did not drink poison. Therefore, if John drinks poison, it will be good for his health'. Intuitively, the conclusion is false. But let's analyse this argumentative form using the present heuristic. The first paradox can be interpreted as claiming that that in every possible world in which $\neg p$ is true, p $^{1\bullet}$implies q. Or, to use a different description, the truth of $\neg p$ $^{2\bullet}$implies $p \supset q$. The conclusion of the argument seems false if we conceive a world where p is true, but this modal intuition is motivated by poor reasoning. This way of thinking ignores that $^{1\bullet}$implications are always restricted to a world parameter, and the world parameters in this case are worlds where p is false, not true. It would be as if we would determine whether a premise $p\&\neg p$ $^{2\bullet}$implied q by considering worlds where $p\&\neg p$ is true knowing that this premise is necessarily false.

Notice that the present distinction is also important because it allows what is usually referred to as a material implication to travel between different worlds. I say this because for some strange reason people assume that material implications are confined in the actual world and that only possible world theories are allowed to make use of modal intuitions, and these are the only correct ones. The limits of this point of view become clear when we consider the evaluation of a simples *modus tollens* argument. In these cases there are no possible worlds where both $p \rightarrow q$ and $\neg q$ are true, but $\neg p$ is false. But all the possible worlds in this evaluation are worlds where $p \rightarrow q$ and $\neg q$ are both true, are also worlds where $\neg p$ is also true. Otherwise, the first conditional would contain a true premise and a false conclusion. Thus, the only meaningful way to make sense of a simple *modus tollens* argument is to abandon possible world theories. This shows that the conventional wisdom on the subject needs revision: it's the material implication, or first degree implication, that is flexible, and not the conditionals in possible world theories.

The main lesson we should learn from the present criticism is that the usual presentation in logic textbooks of the material implication (or, if we decide to use the more usual term, 'material conditional') as a form of connective is completely misguided. This probably will help us explain why the counter-intuitive aspects of the material implication and the perplexities surrounding conditionals in general seem much deeper and resilient than the weird features of truth-functional connectives such as disjunction and conjunction. The material implication is not a connective, but a deductive argument with a restriction to a parameter. Consequently, the current terminology that relies on a distinction between material implication and formal implication should be replaced by a distinction between first and second degree implications.