

THE DEDUCTION PARADOX

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What is a deduction? It is a type of inference that aims for validity and, consequently, can be valid. A deduction is either valid, if it is successful, or invalid otherwise. Now, notice that an invalid deduction could never be valid even in principle, because a single possible world where a deductive inference is valid would “spread”, so to speak, its validity across all remaining possible worlds. One possible world where an inference is valid implies that there are no worlds where the inference is invalid; thus, only valid inferences can be deductive.

The only plausible candidates for invalid deductive inferences are those mistakenly perceived to be possibly valid. Paradigmatic examples of invalid deductive inferences include formal fallacies such as affirming the consequent or denying the antecedent, which an incompetent reasoner might confuse with valid inferential forms like *modus ponens* and *modus tollens*, respectively. But this is tantamount to saying that invalid deductive inferences are inferences mistakenly perceived to be deductive, which is a contradiction. The mere erroneous intuition of possible validity is not enough to identify an inference as deductive because, by definition, a deductive inference is possibly valid.

This should give us pause: if the only genuine deductions are the valid ones, then our talk about deduction is an indirect and thoughtless way of referring to validity rather than to an inference type. There is simply no deduction to speak of—only validity. But then again, it’s clear that validity is an attribute that some inferences possess, while others do not. This is a paradox.

One solution is to argue that validity is nothing more than a coherence requirement for inferences. In fact, under this interpretation, validity should not be viewed as the ideal end goal of a specific inferential process, but as the starting point of any inference. *An incorrect deduction is not a well-intentioned inference that fails to reach its goal, but rather an illogical process.* It is constitutive of the very notion of inference that it preserves the truth of the premises. However, this solution is not really an improvement. If we follow this line of reasoning, the real distinction is not between valid and invalid inferences but between genuine and merely apparent ones. But how can we conceive of an apparent inference as being genuine without implying its validity across all possible worlds? We are back to square one.

Another solution is to argue that the paradox results from accepting the S5 treatment of the accessibility relation between worlds. The accessibility relation in S5 has three key properties: reflexivity (every world can access itself); symmetry (if one world can access another, then that world can access the first); and transitivity (if one world can access a second, and the second can access a third, then the first can access the third). It is because all possible worlds are considered equally accessible from any other possible world in S5 that validity in one world implies validity in all of them.

However, if we adopt a weaker system such as S4, where the accessibility relation is not symmetric, an inference could be valid in some worlds without being valid in every possible world. If not every world is fully accessible to every other, conceivable validity could be restricted to certain worlds without “contaminating” the rest. This should not be surprising, since formal implication is transitive—if $A \models B$ and $B \models C$, then $A \models C$ —reflexive, as $A \models A$ is always valid, but not symmetric, because from $A \models B$, it does not follow that $B \models A$. In other words, accessibility relations should reflect the same properties as formal implication to allow for invalid deductions. Of course, proponents of S5 would argue that weaker systems, where axiom 5 is not accepted, are too weak to capture our modal intuitions about the logic of necessity and possibility.