THE DEDUCTION PARADOX

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A deduction is an inference that aims for validity and can be either valid or invalid. An invalid deduction can never be valid, because if an inference is valid in one possible world, it must be valid in all. One possible world where an inference is valid implies that there are no worlds where the inference is invalid. Therefore, only valid inferences can truly be deductive.

The only plausible invalid deductive inferences are those mistakenly perceived as possibly valid. Examples include formal fallacies like affirming the consequent or denying the antecedent, which might be confused with valid forms like *modus ponens* or *modus tollens*. However, this implies that invalid deductive inferences are simply mistaken as deductive, which is contradictory. An erroneous intuition of validity is insufficient to label an inference deductive, since deductive inferences are, by definition, possibly valid.

This should give us pause: if the only genuine deductions are the valid ones, then our talk about deduction is an indirect and thoughtless way of referring to validity rather than to an inference type. There is simply no deduction to speak off—only validity. But then again, it's clear that validity is an attribute that some inferences possess, while others do not. This is a paradox.

One solution is to argue that the paradox arises from accepting S5's treatment of the accessibility relation, where all worlds are equally accessible. In S5, validity in one world implies validity in all. By adopting a weaker system like S4, where accessibility is not symmetric, an inference can be valid in some worlds without being valid in all. If not all worlds are fully accessible to each other, validity can be restricted to certain worlds, avoiding the paradox. This aligns with formal implication, which is transitive and reflexive, but not symmetric.

It could be objected that an inference valid in some worlds only because it lacks access to all worlds is not truly valid. It is this lack of access that creates the appearance of validity. If we accept that solution, we could declare an inference valid in one world simply by removing its access to the remaining worlds. This is implausible, as it generates validity too easily. Thus, such an inference would not only be invalid from the start but could never be valid, as required for a genuine deduction. Moreover, proponents of S5 would argue that weaker systems, such as S4, are too weak to capture our modal intuitions about the logic of necessity and possibility.

Perhaps a way out of this paradox is to maintain that validity should not be viewed as the ideal end goal of an inferential process, but rather as constitutive of the very notion of inference. Any inference that could be valid is valid. Conversely, there are no invalid inferences, because they would have to be merely possibly valid, which is impossible. Inference consists in truth preservation. *Inference is validiy*. Inferences are truth-conducting processes in the same way that causation transfers a conserved quantity (such as energy or charge) from cause to effect. To suggest that inferences aim at truth-transference is as implausible as suggesting that causality aims at the transference of a physical quantity. Both are simply transferences, through and through.

We can also adopt concepts from epistemology[1] to support the thesis that inference is co-extensive with validity. If we accept the view that evidence is equivalent to knowledge, it follows that every inference is always valid, as the premise of an inference serves as evidence for the conclusion. Since one cannot have evidence for a proposition without knowing it, one cannot have a premise for a conclusion without implying it.

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1   Williamson, Timothy. *Knowledge and its Limits*. Oxford: Oxford University Press, 2000.