

THE DEDUCTION PARADOX

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A deduction is an inference that aims for validity and can be either valid or invalid. An invalid deduction can never be valid, because if an inference is valid in one possible world, it must be valid in all. One possible world where an inference is valid implies that there are no worlds where the inference is invalid. Therefore, only valid inferences can truly be deductive.

The only plausible invalid deductive inferences are those mistakenly perceived as possibly valid. Examples include formal fallacies like affirming the consequent or denying the antecedent, which might be confused with valid forms like *modus ponens* or *modus tollens*. However, this implies that invalid deductive inferences are simply mistaken as deductive, which is contradictory. An erroneous intuition of validity is insufficient to label an inference deductive, since deductive inferences are, by definition, possibly valid.

This should give us pause: if the only genuine deductions are the valid ones, then our talk about deduction is an indirect and thoughtless way of referring to validity rather than to an inference type. There is simply no deduction to speak of—only validity. But then again, it's clear that validity is an attribute that some inferences possess, while others do not. This is a paradox.

One solution is to argue that the paradox arises from accepting S5's treatment of the accessibility relation, where all worlds are equally accessible. In S5, validity in one world implies validity in all. By adopting a weaker system like S4, where accessibility is not symmetric, an inference can be valid in some worlds without being valid in all. If not all worlds are fully accessible to each other, validity can be restricted to certain worlds, avoiding the paradox. This aligns with formal implication, which is transitive and reflexive, but not symmetric.

It could be objected that an inference valid in some worlds only because it lacks access to all worlds is not truly valid. It is this lack of access that creates the appearance of validity. If we accept that solution, we could declare an inference valid in one world simply by removing its access to the remaining worlds. This is implausible, as it generates validity too easily. Thus, such an inference would not only be invalid from the start but could never be valid, as required for a genuine deduction. Moreover, proponents of S5 would argue that weaker systems, such as S4, are too weak to capture our modal intuitions about the logic of necessity and possibility.