THE INEXTRICABLE LINK BETWEEN CONDITIONALS AND LOGICAL CONSEQUENCE

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ABSTRACT

There is a profound, but frequently ignored relationship between logical consequence (formal implication) and material implication. The first repeats the patterns of the latter, but with a wider modal reach. It is argued that this kinship between formal and material implication simply means that they express the same kind of implication, but differ in scope. Formal implication is unrestricted material implication. This apparently innocuous observation has some significant corollaries: (1) conditionals are not connectives, but arguments; (2) the traditional examples of valid argumentative forms are metalogical principles that express the properties of logical consequence; (3) formal logic is not a useful guide to detect valid arguments in the real world; (4) it is incoherent to propose alternatives to the material implication while accepting the classical properties of formal implication; (5) some of the counter-examples to classical argumentative forms and known conditional puzzles are unsound.

1. INTRODUCTION

This paper aims to contribute to the traditional discussion about logical consequence from new and unexplored angles. It is almost universally accepted that conditional sentences should be interpreted as connectives alongside disjunctions and conjunctions. In this view, conditionals are statements about the world that can be true or false depending on the truth-values of their components. They can work as premises or conclusions in an argument, but they shouldn't be confused with arguments. One hypothesis about the truth-conditions of conditional sentences is the material account. This hypothesis states that natural language conditionals are logically equivalent to assertions of material implication, i.e., a conditional sentence is true if, and only if, it is not the case that both the antecedent is true and the consequent is false. This hypothesis is widely unpopular among conditional experts due to its many counter-intuitive features. For example, it implies that a conditional will be automatically true if the antecedent is false or if the consequent is true. These consequences motivate alternative systems of conditional logic.

On the other hand, we have arguments. They are attempts to defend a conclusion based on one or more premises. An argument is thought to be valid when the conclusion is a logical consequence of the premises. According to the classical account of logical consequence, an argument is valid if, and only if, it is not the case that both the premises are true and the conclusion is false in any possible world. This position also has its fair share of counter-intuitive features, such as that any argument will be trivially valid due to contradictory premises or a tautological conclusion. But unlike the counter-intuitive features of the material implication, these results are not widely perceived as being problematic, or at least are not considered to be strange enough to justify abandoning the classical account of logical consequence, which remains very popular to this day. So, to sum up, conditional sentences are perceived as statements that can be true or false, but not as valid or invalid; conditionals can be the premises or conclusions of arguments, but they are not arguments; the classical account of conditional sentences and arguments are both counter-intuitive, but the first is widely unpopular, while the latter is reasonably influent.

In this paper, it is argued that this current view is incoherent since there is a connection between the classical conception of logical consequence (formal implication) and material implication[1]. It is also argued that this connection suggests that material and formal implication are the same type of implication expressed in different modal ranges. Formal implication is simply material implication distributed across many worlds. The data that suggests that material and formal implication have many properties in common is presented in section 2. The idea that alternatives to the material implication face incoherencies is illustrated with an analysis of conditional-assertion theory and possible world theories in section 3. A unified view of implication in which conditional statements are interpreted as implication statements is presented in section 4. The counter-examples to classical argumentative forms are examined and, I hope, explained away in a principled manner in section 5. Section 6 tackles some known conditional puzzles such as conditional stand-offs and conditional embedding. Section 7 concludes with observations about the far-reaching consequences of this approach to our understanding of logic.

2. MATERIAL AND FORMAL IMPLICATION

The material implication $A \supset B$ amounts to a claim that a premise A materially implies a conclusion B if, and only if, it is not the case that both A is true and B is false in a given world that is assumed as a parameter. This reference to a parameter world is justified by the fact that we can evaluate an assertion of material implication in other worlds beyond the actual. This is evidenced in possible world theories that always redirect us to the closest A-world to evaluate a conditional statement, but in this A-world the relation of implication between A and B is also material. To make sense of the classical use of material implication and differentiate it from its use in possible world theories, we can observe that in the second, but not in the first, the parameter world is always the closest one where A is true. The mention of a premise and a conclusion instead of the usual notions of antecedent and consequent is also intentional: since we are talking about interpreting conditionals as assertions of material implication, and since this is an implication relation in some special sense, the antecedent and consequence of a conditional should be interpreted as a premise and a conclusion of an argument, respectively.

There are reasons to accept a connection between the classical conception of logical consequence and material implication occurs because the unrestricted relation of logical consequence (the formal implication[2]), repeats the patterns of a restricted relation of logical consequence (the material implication). The only difference between the two is that in the first the relation of implication has a wider modal reach, so to speak. If the relation of A materially implying B is valid when it is not the case that A is true and B is false, then the relation of A formally implying B is valid when it is not the case that A is true and B is false in any possible

[1] I will adopt the notation where '$\rightarrow$' stands for natural language conditionals, and '$\supset$' stands for material implication until the section 4, where we adopt '${}_{m}\vDash$' to denote an assertion of material implication. This modification in the choice of symbols will emphasise that material implication expresses the same relation of '$\vDash$' in a restricted form. I will use capital letters such as A, B, C for both propositional and formula variables. For simplicity of exposition, I will not use quotes to highlight the use-mention distinction when there is no risk of confusion. The context will make it clear which one is being used.

[2] For the sake of simplicity, I will use 'formal implication' as synonymous with 'the validity of an argumentative form', 'unrestricted relation of logical consequence' and 'arguments in natural language that carry an implicit claim to formal implication'. The context will make it clear in which sense the term is being used. This term is useful because it helps to emphasise the contrasts and similarities with material implication. It is also briefer than the long-winded 'unrestricted relation of logical consequence'.

world. In other words, the validity of formal implication will depend on how we interpret the validity of material implication and vice versa. See the table below.

Strengthening the Antecedent (SA)	**Left Weakening (Monotonicity) (LW)**
$A \supset B \vDash (A\&\mathrm{C}) \supset B$	if $A \vDash B$, then $A\&C \vDash B$
Exportation	**Deduction Theorem**
$(A\&\mathrm{B}) \supset C \vDash A \supset (B \supset C)$	if $A\&B \vDash C$, then $A \vDash B \supset C$
Hypothetical Syllogism (HS)	**Transitivity of Entailment (TE)**
$A \supset B, B \supset C \vDash A \supset C$	if $A \vDash B$ and $B \vDash C$, then $A \vDash C$
Conditional Negation	**Condition Negation Proof**
$A \supset B \leftrightarrows \neg(A\&\neg B)$	$A \vDash B \leftrightarrows \neg\Diamond(A\&\neg B)$
Conditional Antisyllogism	**Principle of Antisyllogism**
$(A\&B) \supset C \vDash (A\&\neg C) \supset \neg B$	if $A\&B \vDash C$, then $A\&\neg C \vDash \neg B$
Contraposition (CON)	**Contraposition Proof (CP)**
$A \supset B \leftrightarrows \neg B \supset \neg A$	$A \vDash B \leftrightarrows \neg B \vDash \neg A$
Or-to-If	**General Or-to-If**
$A \supset B \leftrightarrows \neg A \vee B$	$A \vDash B \leftrightarrows \Box(\neg A \vee B)$
Disjunction Principle	**General Disjunction Principle**
$(A\&B) \supset C \leftrightarrows A \supset (\neg B \vee C)$	$A\&\mathrm{B} \vDash C \leftrightarrows A \vDash \neg B \vee C$
Antecedent Disjunction Introduction	**Antecedent Disjunction Introduction Proof**
$((A \supset B)\&(C \supset B)) \vDash ((A \vee C) \supset B)$	if $A \vDash B$ and $C \vDash B$, then $(A \vee C) \vDash B$
Conjunctive Syllogism Conditional	**Conjunctive Syllogism**
$(\neg(A\&B) \,\&\, A) \supset \neg B$	$\neg(A\&B) \,\&\, A \vDash \neg B$
Limited Transitivity	**Limited Transitivity Proof**
$A \supset B, (A\&B) \supset C \vDash A \supset C$	if $A \vDash B$ and $(A\&B) \vDash C$, then $A \vDash C$

Material and formal implication also have in common the fact that they are antisymmetric: from $A \supset B$ it doesn't follow $B \supset A$, and from $A \vDash B$ it doesn't follow $B \vDash A$, and they are both reflexive since $A \supset A$ and $A \vDash A$ are both valid. There are two degrees of implication here. In the first degree, we have the relation of material implication, which preserves the truth of the premise in a given world taken as a parameter. In the second degree, we have a relation of formal implication that ensures that the truth of the premises is preserved in every possible world. The important thing is that the same pattern of implication presented in the first degree is repeated in the second degree.

Take for instance the relationship between (TE) and (HS). The first states that if *A* formally implies *B*, and *B* formally implies *C*, then *A* formally implies *C*; while the latter states that if *A* materially implies *B*, and *B* materially implies *C*, then *A* materially implies *C*. This means that (HS) can be considered a restricted form of transitivity. Another example is the relationship between (SA) and (LW). The first states that if *A* materially implies *B*, then the conjunction of *A* and *C* materially implies *B*; whereas the second states that if *A* formally implies *B*, then the conjunction of *A* and *C* formally implies *B*. Now consider the examples presented in the table below.

First Paradox of Material Implication (FPM)	**Ex Contradictione Quodlibet (ECQ)**
$\neg A \vDash A \supset B$	If $\neg\Diamond A$, then $A \vDash B$
Second Paradox of Material Implication (SPM)	**Trivial Validity (TV)**
$B \vDash A \supset B$	If $\Box B$, then $A \vDash B$

In the case of (FPM), a material implication trivially holds because the antecedent is false in the world parameter, and in (ECQ), a formal implication trivially holds because the antecedent is false in every possible world. In (SPM), the material implication trivially holds because the consequent is true in the parameter world, whereas in (TV) the formal implication trivially holds. After all, the conclusion is true in every possible world. In both examples the relations of formal implication mirror the behaviour of the relations of material implication, with the only difference being that the first covers all possible worlds while the latter is bounded to a given parameter world.

The connection between (FPM) and (ECQ) can also be explained as follows: the conditional ‘If *A*, then *B*’ is synonymous with ‘if *A* is true, then *B* is true’. Now suppose that *A* is false, and the truth of *A* is incompatible with the truth of *B*. Because of the incompatibility, I decide to abandon the conditional and ignore that *A* is false. But this would amount to accepting that the following conditional is false: ‘if *A* is true and false, then *B* is true’. So, the denial of (FPM) would amount to the denial of (ECQ). The connection between (FPM) and (ECQ) can also be explained as follows: when $A \rightarrow B$ is asserted under the assumption of $\neg A$, what is being asserted is $(\neg A \& A) \rightarrow B$, which should be interpreted as saying that any consequent is materially implied by the conjunction of two inconsistent antecedents (Russell, 1970, p. 136). We can also explain this as follows: from (FPM) we have $\neg A \rightarrow (A \rightarrow B)$, which by importation amounts to $(\neg A \& A) \rightarrow B$.

Thinking along similar lines shows that (SPM) is also connected with conjunction elimination. If a material implication is motivated by a true consequent, then it has the truth of this consequent implicit in its antecedent. The assertion of $A \rightarrow B$ under the assumption of *B* should be interpreted as $B \rightarrow (A \rightarrow B)$, which by importation leads us to $(B \& A) \rightarrow B$, which means that a consequent is materially implied by its combination with its antecedent (Ceniza, 1988, p. 511). So now we can’t deny (SPM) without denying conjunction elimination either. The relationship between (SPM) and (TV) can be established as follows:

1. $B \to (A \to B)$ inferential justification
2. $(A\&B) \to B$ 1, importation
3. $\neg(A\&B) \lor B$ 2, or-to-if
4. $(\neg A \lor \neg B) \lor B$ 3, &
5. $\neg A \lor (\neg B \lor B)$ 4, associativity
6. $A \to (\neg B \lor B)$ 5, or-to-if

This means that denying that 'if *A* and *B* are true, then *B* is true' leads to the negation of 'if *A* is true, then *B* is true or false'.

Another relevant argument was advanced by Lee Archie (1979). He defended that if any truth values are consistently assigned to a natural language conditional to which modus ponens (MP) and modus tollens (MT) are valid argumentative forms, and affirming the consequent is an invalid argumentative form, this conditional would have the same truth conditions of a material implication. His argument relies on circumstance surveyors and some relatively uncontroversial assumptions: when the antecedent of a conditional is true and the consequent is false, the conditional is false; (MP) and (MT) have at least one substitution instance with true premises and a true conclusion; affirming the consequent have at least one substitution instance with true premises and a false conclusion; a conditional is truth-functional without any suppositions concerning the assignment of its truth values. This conclusion is in agreement with our view. There is no consistent attribution of truth values to a conditional different from material implication that can satisfy (MP), (MT), and invalidate affirming the consequent, for the simple reason that material and formal implication express the same properties in different modal ranges[3].

Formal implication and material implication are tied in such a fundamental manner that our intuitions, arguments and hypotheses about the former should be translated into intuitions, arguments and hypotheses about the latter. This relation represents a bridge between formal implication and conditionals. This connection is of the most importance because conditional logic experts tend to be very critical of the material implication while simultaneously accepting the classical notion of formal implication. They will be inclined to accept argumentative forms such as (ECQ) and (TV) at the same time they reject (FPM) and (SPM). This differential treatment probably occurs because the classical notion of formal implication is fairly simple and intuitive, whereas the material implication is still poorly understood given its close ties to natural language and it is susceptible to a wide variety of biases that muddle our perception of the issue and clouds our understanding[4].

3. ALTERNATIVE NOTIONS OF LOGICAL CONSEQUENCE

The relationship between material and formal implication suggests that there should be also a connection between the notion of logical consequence and the truth conditions of conditionals

[3] Notice that Archie s argument can be extended to many-valued systems. I will leave this exercise to the reader. This suggests that even if we would not believe in bivalence and the classical negation operator, we would still have good reasons to accept that natural language conditionals and the material implication share truth conditions.
[4] For different explanations of the supposed biases see Ajdukiewicz (1956), Allott & Uchida (2009a; 2009b), Clark (1971), Hanson (1991), Lewis (1976), Grice (1989), Jackson (1987, 2006), Mellor (1993), Noh (1998); Rieger (2006; 2013); Smith (1983), Smith & Smith (1988), and Silva (2017).

in any given logical system. After all, conditional sentences simply express the properties of logical consequence in a narrower modal scope. This implies that we need to posit different conceptions of logical consequence that are consistent with different theories of conditionals to evaluate their relative merits. As a pilot study of this research program, we evaluate two new notions of logical consequence motivated by conditional-assertion theory and possible world theories. Those alternatives can be compared unfavourably with the classical conception of logical consequence.

According to possible world theories, $A \rightarrow B$ is true iff either B is true in the closest A-world, or A is impossible. If B is false in the closest A-world, the conditional is false (Stalnaker, 1975). David Lewis (1973) offers a distinct version inspired by the same idea. The difference is that $A \rightarrow B$ is true if, and only if, in every possible A-world that is as closest to the actual world as it is allowed by the truth of A, B is true. There are many other possible worlds semantics inspired by the same idea[5]. These details and their subtle differences don't impact my argumentation, since they all face the same difficulties associated with their pre-theoretic intuitions. From here on I will take Stalnaker's semantics as default and refer to it as vanilla possible world semantics or simply possible world account. The possible world account denies the following argumentative forms:

- **(SA)**, for B can be true in the closest A-world, but false in the closest A&C-world;

- **(HS)**, for even if B is true in the closest A-world, and C is true in the closest B-world, C can still be false in the closest A-world;

- **(CON)**, because B can be true in the closest A-world, while $\neg A$ is false in the closest $\neg B$-world;

- **(FPM)**, since B could be false in the closest A-world even if $\neg A$ is true in the actual world;

- **(SPM)**, given that B could be false in the closest A-world even if B is true in the actual world.

The consequence of this departure from material implication is the denial of the following metalogical principles:

- **(LW)**: if a relation of material implication cannot be monotonic, neither should be a relation of formal implication;

- **(TE)**: if the relations of material implication in the hypothetical syllogism are not transitive, neither are the relations of formal implication in an argumentative form;

- **(CP)**: if the falsity of the conclusion (i.e., the consequent) in a material implication relation does not imply the falsity of the premise (i.e., the antecedent), then the falsity of a conclusion in a formal implication does not imply the falsity of the premise;

[5] See Davis (1979).

- **(ECQ)**: if a material implication does not follow trivially from a false premise (i.e., the antecedent), then a formal implication will not follow trivially from a contradictory premise;

- **(TV)**: if a material implication does not follow trivially from a true conclusion (i.e., the consequent), then a formal implication will not follow trivially from a necessary conclusion.

I'm not sure that most people would be willing to adopt such revisionist consequences based on criticisms of the material implication, but that's the price they will have to pay for the sake of coherence. There is no doubt that metalogical principles such as (LW), (TE) and (CP) represent a fundamental aspect of any believable notion of logical consequence. (ECQ) and (TV) are more controversial and the proponent of the possible world account could argue that denying (ECQ) is a consequence that should be seen as a bonus and not a hindrance. After all, we already have paraconsistent systems specifically designed to deal with contradictions in a way that prevents them from implying anything. But the reason why this answer is unconvincing in this case is that the possible world account also has its device of triviality that is reminiscent of (ECQ), namely, that $A \rightarrow B$ is true when A is impossible. This is not consistent with the type of thinking we would expect from the denial of (ECQ). Besides, it would not be a stretch to suggest that most people will accept them.

One obvious reply to my criticism is that possible world theories can be modified to accommodate counterpossible conditionals and impossible worlds[6]. Some proposals along these lines would allow us to refuse both (FPM) and (ECQ). The reason why I don't consider these proposals promising is that they are too timid and half-hearted to be presented as a proper full-fledged alternative to the material implication in their own right. For example, if (FPM) and (ECQ) are abandoned, (SPM) and (TV) should be abandoned too. After all, they are also examples of trivial validity in material and formal guises. But that is not the case, which suggests that the intuitions that motivate these reconstructions involve theoretical purposes that are alien to the whole enterprise and probably involve some basic misconception about the nature of conditionals and arguments in general. Never mind the fact that these proposals would still acknowledge the denials of (LW), (TE) and (CP), which simply defy belief.

One could argue that metalogical principles such as (SA), (HS) and (CON) are not inconsistent with the possible world account after all. They only seem inconsistent with the theory if we make an illicit context shift in the evaluation of the principles. If the context is maintained fixed, those principles will turn out true in the possible world account (Brogaard & Salerno, 2008). While it is undeniable that avoiding contextual fallacies remains a fundamental tenet of semantics, this solution doesn't have its intended effect in relation to the possible world account for two reasons. First, if we follow this stricture through, it will imply that (FPM) and (SPM) are true principles as well (Silva, 2017). This result would undermine the whole reason for a possible world account in the first place. Second, as Cross (2011) so eloquently put it, there is no such thing as a contextual fallacy as far as the possible world system is concerned. The contextual fallacy is embedded in the very truth conditions of the logic system, which was motivated by modal intuitions that rely on context shifting. So, there is no way to correct it by simply adding further restrictions in the evaluation of conditionals.

[6] See, for example, Sorensen (1996), Nolan (1997), and Zalta (1997).

Now, let's move on to conditional-assertion theories. They state that if B is an assertive act, $A \rightarrow B$ is used to conditionally assert that B given A[7]. One of the most surprising features of conditional-assertion theory is its non-propositional requirement. The theory states that $A \rightarrow B$ is just a conditional act of B given A. Thus, it is not a proposition with truth values, much less a connective that combines two propositions to produce an additional proposition whose truth values are determined by its propositional constituents (DeRose & Grandy, 1999, p. 407). This puts conditionals in an entirely new light. Instead of being seen as static truth-functions, conditionals are now portrayed as action movements in natural language. Conditional-assertion theories will deny the following metalogical principles:

- **(SA)**: one could be willing to assert B given the assumption of A, but not willing to assert B under the assumption of $A\&C$;

- **(HS)**: one could be willing to assert B under the assumption of A, or willing to assert C under the assumption of B, but not willing to assert C under the assumption of A;

- **(CON)**: one could be willing to assert B given the assumption of A, but still reject the assertion of $\neg A$ given the assumption of $\neg B$;

- **(FPM)**: from $\neg A$ nothing follows about whether one would be willing to assert B given the hypothetical acceptance of A;

- **(SPM)**: from B nothing follows about whether one would be willing to assert B given the hypothetical acceptance of A.

This means that just as the possible world account, conditional assertion theories are also inconsistent with (LW), (TE), (CP), (ECQ) and (TV), and would have to endorse a notion of logical consequence that has no resemblance with anything we have in mind as far as the subject goes. It is important to notice that not every proponent of the theory will accept this interpretation. Edgington (1995, p. 254), who is known for being the main proponent of this theory, objects that the invalidity of (HS) only seems plausible due to an illicit context shift. In the context where both premises $A \rightarrow B$ and $B \rightarrow C$ reflected acts of conditional assertion; it would also be a context where $A \rightarrow C$ would be an act of conditional assertion. But this type of answer is inadequate because even if one would be willing to make an act of conditional assertion, it does not follow that she did make an act of conditional assertion. This concession is also inadequate because a speaker may not anticipate the conclusions of her previous assertion commitments. The other problem is that even if we could demand consistency for arguers with a fixed context requirement, this would put in doubt the whole conditional-assertion theory enterprise since the counter-examples to the material implication will also be disarmed with a fixed context (Silva, 2017, p. 4).

[7] Some of the main proponents of the theory are Appiah (1985), Edgington (1986, 1995), Barker (1995), Woods (1997); DeRose (1999) and DeRose & Grandy (1999). One could object that I'm ignoring conditional-assertion theories in their propositional version. These theories state that $A \rightarrow B$ is true when A and B are both true; false when A is true and B is false; and has no truth value when A is false, regardless of B's truth value. In other words, if A is false, $A \rightarrow$ B express no proposition. See Jeffrey (1963); Manor (1974) and McDermott (1996). The reason why I don't consider these views as versions of conditional-assertion theory is that this line of reasoning doesn't interpret conditionals as conditional speech acts, but as categorical assertions that are null when the antecedent is false.

The relationship between the classical notion of logical consequence and the material implication only occurs if the latter is a form of implication in its own right. This seemingly innocuous observation presents another challenge for conditional assertion-theories, since in their analysis of conditional sentences they are not analysed as a sort of implication, but as a conditional assertion act.

What is curious is that despite their many alleged differences, possible world and conditional-assertion theories end up facing similar difficulties. The reason lies in the shared intuition that motivated each theory: Ramsey's test. The test states that we accept $A \rightarrow B$ if, and only if, after the hypothetical addition of A to our belief system, and after making the required adjustments to maintain consistency without modifying the hypothetical belief in A, we would be willing to accept B (Stalnaker, 1968, p. 102)[8]. Conditional assertion theories are analogous to Ramsey's test as follows: the theory predicts that if B is an assertive act, $A \rightarrow B$ is synonymous with the willingness to assert B after the hypothetical addition of A to our belief system, and corresponding adjustments to maintain consistency without modifying the hypothetical belief in A (Edgington, 2014, sec. 3.1). It is also common knowledge that possible world accounts were initially designed as an ontological analogue of Ramsey's test: the closest A-world is equivalent to the addition of A to our belief system after making the required adjustments to maintain consistency without abandoning the belief in A (Stalnaker, 1968, p. 102).

It could be objected that any relation between material and formal implication in a classical logic system shouldn't be indicative of any dependence between the truth conditions of conditionals and the explanation of logical consequence in alternative systems. After all, this relationship just reflects the patterns of one system among many. To insist on this pattern as a universal restriction would ensue accusations of logical provincialism if not outright dogmatism. The reason why this objection is wrong-headed is that alternative logic systems all start as a departure from classical logic. This is especially the case for conditional logics and theories of conditionals, which always target the material implication with criticisms and offer tailor-made alternatives to fix these counter-intuitive features. But if conditionals were assertions of implication to begin with, most of these alternatives are poorly motivated from the start.

4. THE NATURE OF LOGICAL CONSEQUENCE

Suppose one asserts that a premise A implies a conclusion B. This corresponds to the assertion of a conditional, 'If A, then B'. Let us assume you believe that A implies B only in relation to one world w_1. This means that you believe it is not the case that A is true and B is false in w_1. This description encapsulates the modal range assumed in most conditionals used in daily life. If I say, 'If John left, he went to the supermarket', I assume that it is not the case that John left but went somewhere else. Otherwise, I remain uncorrected. If this conditional is true, the premise implies the conclusion. This is a material implication.

The 'if' part in the conditional is also known as its antecedent, and what is expressed after 'then 'is the consequent. According to a popular view influenced by Quine (1961, p. 323; 1964, p. 196), to interpret conditionals as claims to implication is to commit the use-mention fallacy in which the antecedent and consequent are mentioned as the premise and conclusion of an implication relation. Instead, genuine conditionals do not mention statements, but use them to express a relation between facts and objects in the world. This popular view is baseless though.

[8] This is the modified and more widely discussed formulation of the test. The original idea and formulation can be found in Ramsey (1929, p. 143).

When a conditional is asserted, it's the whole proposition that is asserted, and not its antecedent and consequent. The assertion of a conditional, then, is made as a statement about a relation between the propositions expressed by the antecedent and consequent. In other words, the antecedent and consequent are mentioned, but not used. The speaker is stating that the consequent follows from the antecedent. So, the interpretation of conditionals as claims to implication is still the correct one.

In the example above you believe that *A* implies *B* only relatively to one world w_1. But, depending on the subject at hand, you could assume that *A* implies *B* in all possible worlds. In this interpretation, it is not the case that *A* is true and *B* is false in any possible world. In this case, we say that, in this interpretation, *A* formally implies *B*. Other similar expressions that can be used here are '*A* strictly implies *B*', '*A* entails *B*' or '*B* is a logical consequence of *A*'. The reference to 'formally implies' is not an accident. When *A* implies *B* in all possible worlds, it is because it involves some necessity that can be recognised or reinterpreted in terms that are exclusively due to formal terms.

Formal implication is an unrestricted material implication, i.e., a material implication under the scope of a necessity operator. When *A* formally implies *B*, *A* also materially implies *B* in any possible world. What we have here is a relation of implication that can admit different degrees of modal reach. When it is extended to all possible worlds, we say the relation of implication is formal, but when is restricted to a given world, we call it material (see figure 1 below).

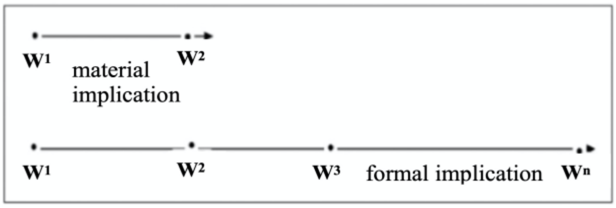

Figure 1. The paths of implication. Material and formal implication are the same type of relation expressed in different modal ranges.

The material implication is satisfied in at least one possible world, while the formal implication extends over all possible worlds. How about the implication relations in between? Their range will vary according to the arguer's beliefs about the subject matter. If we do not expect physical truths to be metaphysically necessary, we will place them in the intermediary range between material and formal implication (see figure 2 below). This is not written in stone and it does not have a pre-existent nature. The range of the implication will be dependent on the arguer's beliefs. The only objective thing is that no implication is valid if it fails in its range. If it is formal, it will fail if there is a possible world where the premise is true and the conclusion is false. If it is material, it will fail in case the premise is true and the conclusion is false in the world that is taken as a parameter.

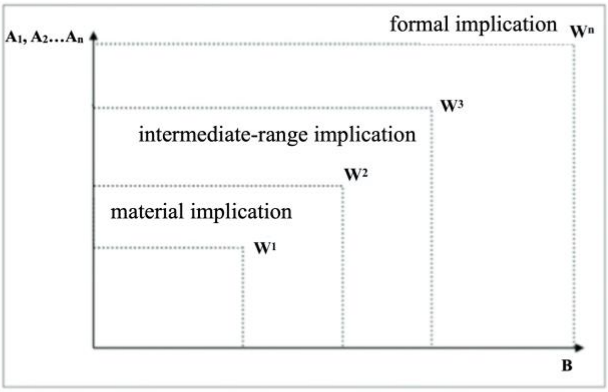

Figure 2. Different claims to implication. They are all classical.

One could insist on a third option, namely, a variably strict implication whose range varies according to the closest worlds in which the premise is true. The variable strict implication would then be 'valid' if the conclusion is true in the closest worlds (in the sense of most similar to the actual world) where the premise is true. You could in principle debate what should be our criteria of similarity. There is a distinct possibility that we will not be able to tell in certain cases if the claim to variable implication is true or not. There are a couple of reasons to avoid this approach. First, this idea makes the relation of implication dependent on a check involving hypothetical assumptions about the truth of the premise. This incorporates an element that is completely foreign to the notion of implication, which intuitively will hold even if the premise is false. Second, it gives the wrong answer in some obvious examples, e.g., when I say, 'If John is telling the truth, I'm more intelligent than Barcan Marcus', I'm making a claim to implication that can be satisfied in the material range, but will be always invalid in the variable one. The conclusion is false in all the closest worlds where the premise is true, but that's because it's a claim to a material implication made under the assumption that the premise is false to demonstrate that the premise is false by a reductio of sorts. This type of blunder occurs with a notion of variable implication because it is motivated by intuitions associated with the use of (MP). They are tilted in the direction of the premise being true because that is what occurs when you are employing a conditional in a (MP). Thus, if we want proper implication variation, it will be different ranges of necessitation. The closest worlds where the premises are true do not need to factor in it.

So, we have claims to implication. These claims will be true if the corresponding truth-value combinations about the premise(s) and conclusion hold. To determine if these combinations are satisfied, we need to consider the worlds in which they are supposed to hold. This modal range is determined by the proponent of the implication and her beliefs on the subject. We can give different names to implication relations according to their modal range. The intrusion of epistemic elements is allowed when they involve the demarcation of modal ranges, but what happens in each range is the necessitation of the truth of the conclusion given the truth of the premises. The implication is valid when the truth of the premises necessitates the truth of the conclusion relatively to a certain range. In a material implication, the truth of the premises necessitates the truth of the conclusion in a parameter world, while in a formal implication, the truth of the premises necessitates the truth of the conclusion tout court.

Conditional statements express implication statements, so they can also be interpreted as arguments in their own right. Notice that once we have decided to interpret conditionals as implication statements, our conceptual habits will change significantly and we will need to revise our baby logic conventions. For instance, the talk about (HS) as an argumentative form commits a category mistake. The reason is that it does not make sense to talk about an argumentative form that contains two implication statements as premises and another one as a conclusion, since real people do not make arguments where the premises are arguments themselves. Rather, (HS) should be interpreted as a metalogical principle about the properties of implication involving three implication statements. The received view of logic is incorrect since the supposed argumentative forms are metalogical statements about the properties of implication. They are nothing more than expressions of these properties.

Logic offers us an explanation of the nature of implication. This insight allows us to make a classification of metalogical principles, and each will express a different property of implication. (MP) express the property of reflexivity, which is an uninteresting property. This can be shown with general conditional proof (GCP), the principle that states that if two premises formally imply a conclusion, then one of these premises will formally imply that the other premise materially implies the conclusion. The demonstration is as follows:

Prem.	(1)	$A \,_{m}\!\vDash B, A \vDash B$	(MP)
1	(2)	$(A \,_{m}\!\vDash B) \vDash (A \,_{m}\!\vDash B)$	1, (GCP)

(MP) is then the claim that $A \,_{m}\!\vDash B$ implies itself, i.e., is the property of reflexivity. It's symptomatic that some authors tried to refute (MP) because this would amount to denying that $A \,_{m}\!\vDash B$ formally implies itself. Now let's consider (MT), which can be expressed as $A \,_{m}\!\vDash B$, $\neg B \vDash \neg A$. This amounts to accepting that from a material implication and the denial of its conclusion it follows the denial of its premise. This is the same meaning conveyed by (CON). Consider the following inference using (MT) and (GCP):

Prem.	(1)	$A \,_{m}\!\vDash B, \neg B \vDash \neg A$	(MT)
1	(2)	$(A \,_{m}\!\vDash B) \vDash (\neg B \,_{m}\!\vDash \neg A)$	1, (GCP)

The acceptance of (MT) leads us to (CON), which is nothing more than the claim that if a premise materially implies the conclusion, then the denial of the conclusion leads to the denial of the premise. It's a simple property of material implication.

(SA) represents the monotonic aspect of material implication. If a premise materially implies a conclusion, the inclusion of another premise will not defeat this claim. So, we have $(A_1 \,_{m}\!\vDash B) \vDash (A_1 \& A_2 \,_{m}\!\vDash B)$. The other metalogical principles will follow through in a seamless fashion. (HS) can be expressed as $(A \,_{m}\!\vDash B_1), (B_1 \,_{m}\!\vDash B_2) \vDash (A \,_{m}\!\vDash B_2)$, and represents the transitivity of material implication. Conditional negation (CN) can be expressed as $\neg(A \,_{m}\!\vDash B) \vDash A \& \neg B$ and express what follows from the invalidity of a material implication, and so on.

The same patterns repeat themselves with formal implication. (SA) has an unrestricted cousin in (LW), namely, that if $A_1 \vDash B$ then $A_1 \& A_2 \vDash B$. (HS) has an unrestricted parallel in transitivity of entailment, which is the same logical form now with formal implication instead of material implication, $(A \vDash B_1), (B_1 \vDash B_2) \vDash (A \vDash B_2)$. The unrestricted version of (CN) can be expressed as $\neg(A \vDash B) \vDash \Diamond(A \& \neg B)$, and so on.

The supposed tautological character of valid deductive arguments is another perplexing notion that we created due to the excessive importance we attribute to pedagogical tools like (MP). Indeed, in a (MP) we are supposed to start with a claim to an implication statement, proceed to state the premise of this implication statement and then infer the conclusion already contained in the initial statement. But if arguments aim to convince the audience, the premises need to be more plausible than the conclusion, and an implication statement cannot be more plausible than the conclusion it already contains. This goes against the very aim of argumentation.

But while the admission that argumentative forms with conditionals are metalogical principles can reduce the logical toolbox, one could object that it still lets unscathed numerous valid argumentative forms, e.g., De Morgan's laws, conjunction elimination, conjunctive syllogism, double negation and disjunction introduction. But since conditionals are logically equivalent to disjunction statements, and conditionals are ultimately claims to implication, then claims to implication are logically equivalent to disjunction statements. Therefore, the supposed argumentative forms involving disjunction statements are also metalogical principles about the properties of implication. Thus, a schema such as disjunction introduction is 'valid' because it is a metalogical principle that expresses a property of inclusive disjunctions. This has nothing to do with real-world argumentation and shouldn't be taught as if it was a useful guide to it. If formal logic were a useful guide to argumentation, the only thing that would be needed to demonstrate a conclusion is to retroactively infer a known, but trivial premise that has the same truth-conditions as the conclusion.

Perhaps the most important consequence of the previous statements is that we can't have a formal demonstration of validity in general. Let's consider an obvious example of validity, such as (MP). It is undeniable that if the argument presented as a premise is valid, and the second premise that states the premise of this argument is true, then the conclusion of the argument presented in the premise *must be true*. But this assurance will confirm nothing of substance about whether the initial argument, the 'premise', was valid or not. The conclusion is merely conditional: if the argument is valid and the 'second premise' is true, the conclusion is true. But this is just a reinstatement of the nature of validity. It doesn't guide us to the validity of real-world arguments.

The implication premise in an (MP) is not circular because it states that it is not the case that A is true and B is false in a given modal range, and that these truth values can be ascertained independently. But if the purposed implication claim is substantial, the truth values of A and B will be controversial and we will have to consider independent reasons and possibly other implication statements to decide whether A implies B or not. Let's say that the implication statement is 'If the Taniyama–Shimura conjecture is true, then Fermat's last theorem is true'. The justification for such a statement involves hundreds of additional assumptions and at least a few dozen additional theorems. The truth of this conjecture was unknown until Andrew Wiles came along. It is certainly not something whose validity can be ascertained by a circumstance surveyor. What can be shown by using such textbook techniques, however, is that if this statement is valid, you can infer the truth of its conclusion from its premise, but this result is uninteresting and doesn't help us in our attempts to gain more knowledge.

The circularity of metalogical principles is also evidenced in the use of (CON), which allows us to move from a premise that states that A materially implies B to a conclusion that states that $\neg B$ materially implies $\neg A$. Where is the justification to accept that A materially implies B in the first place? The fact that is not the case that A is true and B is false. How do I know that? I need to check in the real world to determine whether this truth value combination is satisfied or not. But real-life implication statements are not so obvious that you can tell the truth-value of their propositional components in advance. So, you will have to rely on indirect evidence and other implication statements to get to this conclusion. This is not something that can be achieved by the calculus of truth values alone. The same can be said about (MT), (HS) and (SA). Their logical form will assure you that the conclusion is valid if the premise is valid, but that doesn't provide any formal guarantees that the premise was valid in the first place. It is assumptions in, assumptions out. The conclusion is just as good as the information you inserted on the premises in the first place.

One potential misunderstanding here is assuming that logic can be useful as an epistemic guide to finding valid implication in the real world since it encapsulates the nature of valid implication. It is a non-sequitur and it is similar to the trap of confusing truth conditions with the criteria of truth. Truth conditions have logical significance for they determine the conditions in which a proposition is true or false, but criteria of truth only have epistemic significance because they are standards used in contexts of imperfect information to distinguish whether a given proposition is true or false, i.e., in contexts where the only evidence available to assess the relevant proposition is intensional. Neither truth conditions provide criteria of truth, nor a concept of implication provides criteria of validity.

The circularity of metalogical statements explains why the conclusion of a valid deductive argument appears to be contained in its premises in some trivial sense. If your notion of implication is determined by an innocuous metalogical observation about the properties of implication, then valid deductive arguments will be seen as merely tautological. Things don't work that way in the real world. To ascertain whether a particular deductive argument is valid or not, we need to consider the reasons that support this statement. That occurs because the elements that allow you to assess the validity of a deductive argument are the same devices that allow you to build a deductive argument. That is, you know that a deductive argument is valid in a non-trivial way by considering whether the claim to a conceptual connection between the premises and the conclusion is true.

That also explains why experts in other areas will be able to carry on doing their cognitive business without the hassle of reading logic textbooks. The ability to judge the merits of a claim to implication is specific in nature and requires an understanding of the subject matter. A body of knowledge and conceptual understanding cannot be moved around and automatically inserted in different domains like a pile of books is inserted on an empty shelf. We are not able to determine the validity or invalidity of a deductive argument in any domain of our choice, irrespectively of whether we have any knowledge about the subject or not. That would require a single method that could be applied to any domain and all it would require of us is taking a mere logic class and doing some exercises on a piece of paper. It would trivialize all deductive enterprises so that they could fit into our preconceived notions.

Naturally, one can learn many interesting things about the nature of implication itself. The metalogical statements express properties that are interdependent and assure us that any alternative logical system is severely limited in scope. If an alternative notion of implication violates (HS), it must also violate (TE) and (MP); the refusal of (CN) implies a denial of (MT) and (CON), and the abandonment of (SA) violates monotonicity and (HS), and so on. But beyond that, these properties will either presuppose the validity of the target implication used as a source, or they will be trivial. Consequently, these principles should not be confused with real-life argumentative forms or an epistemic guide to real-life argumentation, which is too complex and dependent on content to be identified exclusively by formal means. Logic is the science of logical consequence, but paradoxically this means that it cannot help us find meaningful logical consequence.

5. PUTTING SOME COUNTEREXAMPLES TO REST

The perspective defended in this article allows us to explain the counter-examples against classical metalogical principles in a consequential manner. Let's consider the (FPM). One apparent counterexample to this principle is 'Some John did not drink poison. Therefore, if John drinks poison, it will be good for his health'. Intuitively, the premise can be true, while the subsequent implication statement is false. But the first paradox claims that in every possible world in which $\neg A$ is true, A implies B in that parameter world. The implication statement seems false if we conceive a world where *A* is true, but this intuition ignores that the world parameter in this case is a world where *A* is false, not true.

The (SPM) also has counter-intuitive instances such as 'The match will not be cancelled. Therefore, if the players broke their legs, the match will not be cancelled.' This principle claims that in every possible world in which *B* is true, *A* implies *B* in that parameter world. The apparent counter-example only seems plausible if we ignore that this claim to implication is restricted to parameter worlds in which *B* is true. Since in those worlds the match is not cancelled, the assumption that the players broke their legs is also false. Once again, the contrary intuition is motivated by a modal illusion and poor reasoning: what seems obvious results from an illicit shift in the parameter world.

It was observed that (SA) encapsulates the monotonic aspect of material implication. It is a claim that any accepted implication statement should be robust given new information. This goes against the prevalent notion that most implication statements are non-monotonic since many of the past arguments were abandoned by their proponents after new findings. But it must be observed that no one advances an argument predicated on the notion that is going to be abandoned. Instead, one argues for an argument given the expectation that it will be resilient to new information. If the argument doesn't meet this requirement, it is going to be abandoned.

Now (SA) faces the following counter-example: 'If the match is struck it will light. Therefore, if the match is struck and it is held under water, it will light'. To understand what is wrong with this counter-example, remember that what (SA) means is that if an implication statement is true, it will remain true given the addition of new information. Conversely, if a strengthened implication statement turns out to be false, is because the original statement was never true in the first place.

This is precisely what happens in the supposed counterexample. There are no circumstances where 'If the match is struck it will light' is valid given the addition of the premise that the match is held underwater. The temptation is to assume that the initial statement can be accepted, while the strengthened claim to material implication must be discarded. But this can only occur if we evaluate them in different contexts, thus committing a contextual fallacy.

Another classical principle that has counter-intuitive instances is (CON). Take for instance 'If it rains tomorrow there will not be a terrific cloudburst. Therefore, if there is a terrific cloudburst tomorrow it will not rain' (Adams, 1975, p. 15). Let's call the first and second implication statements S1 and S2, respectively. S2 only seems false if we consider a parameter world where its premise is true, but S1 is only true in a parameter world where this premise of S2 is false. Thus, we can't have an evaluation in which S1 is true and S2 false if they involve the same parameter worlds.

Now consider the following counter-example to (HS): 'If Brown wins the election, Smith will retire to private life. If Smith dies before the election, Brown will win it. Therefore, if Smith dies before the election, then he will retire to private life.' Let's call each implication statement S1*, S2* and S3*, respectively. Intuitively, one could plausibly accept both S1* and S2*, but reject S3*. It is absurd to suppose that Smith could decide to retire after he died (Adams, 1965, p. 166).

The counterexample does not work, however, since the implication statements are not evaluated in the same parameter world. Suppose that S3* is false, i.e., that it has a true premise and a false conclusion. In this case, Smith will not be able to retire, because he will die before the election takes place. S1* has a false conclusion and S2* has a true premise. It remains to be seen whether Brown will win the election in this context. If he does, S1* will have a true premise and a false conclusion, and S2* must be true, since the premise and the conclusion are true. Therefore, at least one of S1* and S2* will be false. There is no counterexample.

Now let's tackle (CN). This principle faces counter-intuitive instances such as the following: 'It is not the case that if God exists then the prayers of evil men will be answered. Therefore, God exists and the prayers of evil men will not be answered' (Stevenson, 1970, p. 28). Thus, from the negation of a simple conditional, I can prove that God exists. This is implausible because someone could refuse the conditional based on assumptions about the moral dispositions of God even if she does not believe in the existence of God.

Let's that a person believes that God's moral dispositions are essentially inconsistent with answering the prayers of evil men. The conditional then is interpreted as 'There is no possible world in which God exists but the prayers of evil men are not answered', which can be accepted without any commitment to the truth-values of either its premise or conclusion in the actual world. In other words, the counter-example is motivated by a simple mistake. It confuses a material implication statement, which is restricted to a parameter world, with a formal implication statement, which is an unrestricted material implication that extends over many worlds. This is perfectly normal if we understand that conditionals are implication statements that can admit different modal ranges.

6. SOLVING CONDITIONAL PUZZLES

There is a long list of conditional puzzles presented in the literature. It includes conditional stand-offs, the triviality results, conditional embedding and the Apartheid thesis, to name just a few. These enigmas have defied philosophers for decades and stirred dozens of ingenious solutions as a result. In this section, I offer a new way to address these conditional puzzles. Let's start with conditional standoffs. In very loose terms, a conditional stand-off occurs when one individual has grounds to accept a conditional, while another has equally compelling grounds to accept what seems to be the same conditional with a negated consequent. If conditionals have truth conditions, they cannot both be true, because they seem contradictory. The reasoning then is that for one of the conditionals to be false, someone would have to make a mistake about the facts of the case. However, both individuals have perfectly good reasons to accept each conditional. If none of them is making a mistake, none of them is saying something false. Therefore, conditionals have no truth conditions. This puzzle was presented by Gibbard (1981, pp. 226–32) in the following example:

> Sly Pete and Mr. Stone are playing poker on a Mississippi riverboat. It is now up to Pete to call or fold. My henchman Zack sees Stone's hand, which is quite good, and signals its content to Pete. My henchman Jack sees both hands, and sees that Pete's hand is rather low, so that Stone's is the winning hand. At this point, the room is cleared. (…) Zack knows that Pete knew Stone's hand. He can thus appropriately assert 'If Pete called, he won.' Jack knows that Pete held the losing hand, and thus can appropriately assert 'If Pete called, he lost.' From this, we can see that neither is asserting anything false.

There is a caveat with this example though. Arguably, the example is not symmetric because Jack has better reasons to justify his belief than Zack. This led to attempts to offer new stand-off examples which ensured perfect symmetry:

> In a game, (1) all red square cards are worth 10 points, and (2) all large square cards are worth nothing. X caught a glimpse as Z picked a card and saw that it was red. Knowing (1), he believes 'If Z picked a square card, it's worth 10 points'. Y, seeing it bulging

under Z's jacket, where Z is keeping it out of view, knows it's large. Knowing (2), he believes 'If Z picked a square card, it's worth nothing' (Edgington, 1995, p. 294).

The obvious response is that one can have epistemic bad luck even if she hadn't done anything. You can 'do your job' as an epistemic agent and still be wrong if the context is somewhat averse to your belief justification process. Moreover, if both statements are interpreted as implication statements, they can both be correct if their common premises turn out to be false, i.e., if Z didn't pick a square card. Otherwise, notwithstanding X and Y's good epistemic practices, one of the conditionals will have a true premise and a false conclusion, and one of them will be mistaken.

The Apartheid thesis is another perplexing issue that defied conditional experts for decades. It states that indicative and subjunctive conditionals have different truth conditions. One of the main arguments that have been presented to support this thesis is the Adam pairs. Consider the following pair of conditionals:

(1) If Oswald did not kill Kennedy, someone else did.

(2) If Oswald had not killed Kennedy, someone else would have.

Intuitively, these conditionals have different truth conditions. After all, to accept (1) is enough to know that Kennedy was killed by someone, but to accept (2) is necessary to assume a conspiracy theory regarding its murder (Lewis, 1973, p. 3)[9].

The intuition that supports the *Apartheid* thesis can be explained away in the following manner: since an indicative 'if *A* is the case, then *B* is the case' should be interpreted as saying '*A* implies *B* in a parameter world', it is also natural to think that a subjunctive 'if *A* were the case, then *B* would be the case' should be interpreted as saying 'if *A* were true, *A* would imply *B* in a parameter world'. But one can accept that *A* implies *B* in a parameter world, at the same she denies that if *A* were true, *A* would imply *B* in a parameter world. The error is in assuming that the fact that the antecedent is knowingly false makes any difference to the type of claim involved in an implication.

The implication heuristic nullifies approaches that give too much importance to the grammatical aspects of different conditionals since they are all removed from the expanded propositional content. The propositional content of complete conditionals does not admit the subjunctive mode of the propositions involved in the implication. For instance, we cannot say "The proposition 'Kennedy were not killed by Oswald' entails 'Someone else would have killed Kennedy'" because this sentence is ungrammatical. But if the full propositional content removes the subjunctive mode, then all theoretical intuitions motivated by the subjunctive mode are eliminated as a consequence.

Let's move on to the next puzzle. The thesis known as the Equation states that the probability of $A \rightarrow B$ is the probability of *B* given *A* (Jeffrey, 1963, pp. 702-703). Lewis (1976, pp. 299-300) has shown that the acceptance of the equation implies that the probability of $A \rightarrow B$ is the probability of *B*, which is implausible: the probability of a conditional cannot plausibly be the same as the probability of its consequent, e.g., the probability that the match will light given that is struck is not intuitively the same as the probability that it will light.

However, we can show that a similar result is not only expected but intuitively satisfactory if we interpret conditionals as implications restricted to a parameter world. To realize this task,

[9] This example is a modification of the original example presented by Adams (1970, p. 90). Hence the name 'Adam pairs'.

we need to make some assumptions. First, let's assume that conditional probability is primitive, i.e., that $\Pr(B|A)$ can't be defined as $\Pr(A\&B)|\Pr(A)$. Intuitively, I can attribute a high probability to B given the assumption of A even if I don't know the probability of A. For instance, I can attribute a high probability to the ceremony being cancelled tomorrow given the assumption that there will be heavy rainfall, even if I don't know the probability that there will be heavy rainfall tomorrow. Thus, saying that we are considering the probability of B given A amounts to saying that we are evaluating the probability of B in a context where A is taken as true. Now, if we accept that $A \rightarrow B$ is an assertion of material implication, i.e., equivalent to $\neg A \vee B$, and assume the equation, it follows that the probability of $\neg A \vee B$ equals the probability of B given A. But the probability of $\neg A \vee B$ is the same as the probability of B in a context where A is taken as true. In other words, if A is assumed as true, the probability of $\neg A \vee B$ is the same as the probability of B. Now, we already agreed that $A \rightarrow B$ is equivalent to $\neg A \vee B$. Consequently, if A is assumed as true, the probability of $A \rightarrow B$ is the same as the probability of B.

Another way to see the harmless of the triviality results is to observe that the acceptance of $A \,{}_{\mathrm{m}}\!\vDash B$ given the belief in A is tracked by the probability attributed to B. If the probability attributed to B is low, so is the acceptance of the assertion of material implication in a context where A is taken as true. Properly understood, this result is required to preserve coherence. Otherwise, one would be able to accept both an implication statement and its premise but deny its conclusion.

Now let's consider the phenomenon of embedded conditionals. Some will argue that if conditionals were regular truth-valuable sentences, conditional embedding would not be so rare and obscure[10]. But if conditionals are interpreted as claims to implication this rarity becomes understandable. If arguments that contain an argument in either its premises or conclusion are pretty unusual in theorisation, imagine in natural language. But they can be made intelligible with the implication heuristic, even if they happen to be convoluted. For instance, the conditional $A \rightarrow (B \rightarrow C)$ can be interpreted as 'A implies that B implies C in a parameter world'. This provides us with a clear rationale to interpret successive reiterations of embedding in conditionals, with increasing orders of complexity. We can explain conditionals in embedding contexts as composed assertions of implication restricted to parameter worlds. Just as we may have one or more premises in an argument, we may have one more proposition in an antecedent or consequent. This is another puzzle that was laid to rest.

How can we verify a conditional when the antecedent is contrary-to-fact?[11] This is the problem of counterfactuals. Consider the following example: suppose that my friend almost touched a live wire. I say, with a sign of relief: 'If you had touched that wire, you would get an electric shock'. How are we supposed to confirm the conditional if you did not touch the wire? There is the intuition that what interests us is knowing whether she would get an electric shock in a hypothetical circumstance where she touched the wire. This intuition can be interpreted as a demand for knowing whether the premise of the conditional would imply the conclusion in a world where the premise is true. But this is simply a matter of knowing which world is taken as a parameter. If the arguer wants to know whether the implication is valid in a parameter world where the antecedent is true, and the antecedent is false in the actual world, then the actual world is irrelevant; otherwise, it is not irrelevant and the implication is vacuously valid. The notion that the only implication conveyed is one where the premise is true is misguided and confuses logic with epistemological considerations. The important thing is that while

[10] See, for example, Appiah (1985, pp. 205–10), Gibbard (1981, pp. 234–8), Edgington (1995, pp. 280–4), and Woods (1997, pp. 58–68).
[11] See Chisholm (1946), Goodman (1947), Will (1947), Watling (1957), Walters (1961), and Tredwell (1965).

evaluating an argument, the premise and the conclusion are analysed in the same parameter world.

One could insist that only possible world theories are adequate to capture this intuition, but that would be an overly hasty conclusion. For some strange reason people assume that only possible world theories are allowed to make use of modal intuitions, and these are the only correct modal intuitions. The limits of this point of view become clear when we consider the evaluation of a simple (MT). In these cases, there are no possible worlds where both $A \rightarrow B$ and $\neg B$ are true, but $\neg A$ is false; but all the possible worlds where $A \rightarrow B$ and $\neg B$ are true, are worlds where $\neg A$ is true. Otherwise, the first premise would contain a true antecedent and a false consequent. Thus, the only meaningful way to make sense of (MT) is to abandon possible world theories.

Conditionals seem to have a dual nature. On one hand, they are used to represent reality, so they have categorical-like features; but on the other hand, they are also inferential in nature, so they can be also interpreted as arguments. So, are conditionals statements or arguments? Maybe both? We have a tried and tested metaphysical vocabulary that allows us to make sense of the truth-value distinctions of categorical sentences and their connection to reality. But when we try to extend this vocabulary to conditional sentences, it falls apart. We can't make heads or tails of it. The present interpretation provides an easy answer for this problem: conditional statements are claims to a relation of implication between two propositions, the premise (antecedent) and the conclusion (consequent). In other words, they are statements about how one proposition ensures the truth of another in a parameter world. There is no need to resort to a dual nature, for they are categorical statements about facts associated with implications. A conditional corresponds to reality if the premise implies the consequent, otherwise, they do not.

7. CONCLUDING REMARKS

Here, we defended that conditional sentences should be interpreted as assertions of logical consequence that can vary in modal range. More specifically, it was also argued that these assertions can be plausibly interpreted as having the same truth conditionals of classical implication. Interpreting conditionals as claims to implication restricted to a parameter world highlights the flexible nature of conditionals, for we can ascertain their truth values in different worlds. It also brings formal implication closer to conditionals, since they are simply the same type of implication without being restricted to a single world. And, on top of it all, it allows us to explain away counter-examples and puzzles that have plagued conditional experts for decades. There is no telling how much this hypothesis can affect the recent work in conditional logic. This impact includes but is not limited to possible-world semantics, probabilistic semantics, conditional assertion theory, works involving belief revision, etc.

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