

# THE LOGICAL WEB

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## ABSTRACT

Different logic systems are motivated by attempts to fix the counter-intuitive instances of classical argumentative forms, e.g., strengthening of the antecedent, contraposition and conditional negation. These counter-examples are regarded as evidence that classical logic should be rejected in favour of a new logic system in which these argumentative forms are considered invalid. It is argued that these logical revisions are ad hoc, because those controversial argumentative forms are implied by other argumentative forms we want to keep. It is impossible to remove an argumentative form from a logical system without getting entangled in an intricate logical web, since these revisions imply the removal of other parts of a system we want to maintain. Consequently, these revisions are incoherent and unwarranted. At the very least, the usual approach in the analysis of counter-examples of argumentative forms must be seriously reconsidered.

**Keywords:** classical logic; material implication; conditionals; non-classical logic.

## 1. INTRODUCTION

The assumption that the use of classical logic is successful in the attempt to distinguish valid from invalid argumentative forms, and, consequently, valid from invalid arguments in natural language, assumes that there is a match between key components of classical logic and natural language operators such as ‘and’, ‘if, then’, ‘or’, and ‘not’. Inversely, any counter-intuitive argument that is authorised by classical logic suggests that system is incorrect and motivates the development of alternatives to classical logic. One recurrent criticism is that material implication, the truth-functional operator that is supposed to encapsulate the truth conditions of ‘if, then’ in natural language, is unsuitable for the task. If a conditional has the same truth conditions of a material implication it will always be true when the antecedent is false, no matter the consequent. For example, according to classical logic, the conditional ‘if the moon is made of cheese,  $2 + 2 = 4$ ’, will be vacuously true simply because the antecedent is false. Therefore, argue the critics, the instances in natural language of classical argumentative forms that involve the material implication are invalid because they violate conditionals truth conditions, and we should develop different logics that avoid these mistakes.

It will be argued that these criticisms ignore that those controversial argumentative forms are implied by other argumentative forms or meta-logical principles that are either less controversial or accepted by the very logicians that propose these revisions, thus making them unconvincing and incoherent. It is impossible for a valid argumentative form to have all its premises true and a false conclusion, which further implies that if the conclusion is false, at least one premise must be false. Thus, if less controversial argumentative forms, meta-logical principles, or a combination of both imply a controversial argumentative form, the negation of the latter implies the negation of the first. The article is divided as follows. Section 2 will be

divided in subsections, 2.1-2.6, that will discuss the logical relations between argumentative forms such as the first and the second paradox of material implication, conditional negation, hypothetical syllogism, strengthening of the antecedent, and or-to-if. Their logical dependence is explicated by direct proofs. Section 3 discuss which methodological principles should govern the comparison of logical systems. It is argued that attempts to present counter-examples to counter-intuitive argumentative forms need to be accompanied by counter-examples to the argumentative forms and metalogical principles that imply it in the first place. This is the *Disentanglement Requirement*. It is also argued that in order to provide a robust defence a given system of logic it will be necessary to explain away as illusions the counter-intuitive aspects of some valid argumentative forms and the intuitive aspects of some invalid argumentative forms. This is the *Pragmatic Task*. Section 4 concludes with observations about the need to analyse argumentative forms in groups and prevent costly theoretical enterprises.

## 2. THE FIRST PARADOX OF MATERIAL IMPLICATION

Since a material conditional  $A \supset B$ <sup>1</sup> is true when  $A$  is false, the following argumentative form is valid according to classical logic:  $\neg A \models A \supset B$ . This argumentative form has intuitive instances such as the following: ‘15 is not divisible by 9. However, if 15 is divisible by 9, then 15 is divisible by 3’<sup>2</sup>. But it also has counter-intuitive instances such as ‘I don’t drink sulphuric acid. Therefore, if I drink sulphuric acid, I will be healthy’. Let’s name this argumentative form *The First Paradox of Material Implication* (FPM). Despite these counter-intuitive aspects, any attempt to deny (FPM)’s validity must involve the refuse of one the argumentative forms employed in certain proofs. The first one involves *Ex Contradictione Quodlibet* (ECQ), the principle that states that anything is entailed by a contradiction, and *General Conditional Proof* (GCP), the principle that states that from  $A, B \models C$ , it follows that  $A \models B \rightarrow C$ . The proof of (FPM) is the following<sup>3</sup>:

Prem	(1)	$A \& \neg A \models B$	(ECQ)
	1	(2) $A \models \neg A \rightarrow B$	1, (GCP)

Another proof of (FPM) involves (ECQ), *Conditional Proof* (CP), the meta-logical principle that states that if  $A \models B$ , then  $A \rightarrow B$  is a tautology, and *Exportation* (EXP), the argumentative principle that allow us to infer  $A \rightarrow (B \rightarrow C)$  from  $(A \& B) \rightarrow C$ :

Prem	(1)	$A \& \neg A \models B$	(ECQ)
	1	(2) $(A \& \neg A) \rightarrow B$	1, (CP)
	1	(3) $\neg A \rightarrow (A \rightarrow B)$	2, (EXP)
	1	(4) $\neg A \models A \rightarrow B$	3, (CP)

It is also possible to prove (FPM) with (GCP), the *Transitivity of Entailment* (TE), the truth

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<sup>1</sup> Here ‘ $\rightarrow$ ’ stands for indicative conditionals, ‘ $\supset$ ’ stands for material conditional, and ‘ $\models$ ’ stands for entailment. All argumentative forms and metalogical principles discussed will be initially named, and from then on will be referred by their respective abbreviations. Some of the known argumentative forms will be introduced only by their names and their logical form will not be introduced. For simplicity of exposition, I will use the same numeration (1,2,3...) for each positive argument and the capital letters  $A, B, C, \dots$  for both sentence letters and propositional variables—the context will make it clear which one is being used. I will not use quotes to highlight the use-mention distinction when there is no risk of confusion, and the symbols and variables quoted will be modified to ensure that the notation remains uniform.

<sup>2</sup> Farrell ([24], 301).

<sup>3</sup> Rieger ([54], 3165).

conditions of the *Classical Conjunction*, ‘&’, and *Contraposition* (CON)<sup>4</sup>:

Prem	(1)	$\neg A \models \neg(\neg B \& A)$	from the truth conditions of ‘&’
Prem	(2)	$\neg(\neg B \& A), \neg B \models \neg A$	from the truth conditions of ‘&’
Prem	(3)	$\neg B \rightarrow \neg A \models A \rightarrow B$	(CON)
2	(4)	$\neg(\neg B \& A) \models \neg B \rightarrow \neg A$	2, from (GCP)
1, 2, 3	(5)	$\neg A \models A \rightarrow B$	1–5, (TE)

The following proof involves the truth conditions of ‘&’, *Double Negation* (DN), (TE), and (GCP)<sup>5</sup>:

Prem	(1)	$\neg A \models \neg(A \& B)$	from the truth conditions of ‘&’
1	(2)	$\neg A \models \neg(A \& \neg B)$	argumentative form similar to 1
1	(3)	$\neg(A \& B), A \models \neg B$	given the validity of 1
1	(4)	$\neg(A \& \neg B), A \models \neg\neg B$	given the validity of 2
1	(5)	$\neg(A \& \neg B), A \models B$	4, (DN)
1	(6)	$\neg(A \& \neg B) \models A \rightarrow B$	5, (GCP)
1	(7)	$\neg A \models A \rightarrow B$	2, 6 (TE)

It can also be argued that (FPM) follows from (IV), (OTF), (DN) and (TE)<sup>6</sup>:

Prem	(1)	$\neg A$	
1	(2)	$\neg A \vee B$	1, (IV)
1	(3)	$\neg\neg A \rightarrow B$	2, (OTF)
1	(4)	$A \rightarrow B$	3, (DN)
1	(4)	$\neg A \models A \rightarrow B$	1–3 (TE)

Thus, the denial of (FPM)’s validity implies the invalidity of (ECQ) or (GCP) in the first argument; the invalidity of (ECQ), (CP), or (EXP) in the second argument; the invalidity of (GCP), (TE) or (CON) in the third argument; the invalidity of (GCP), (DN) or (TE) in the fourth argument; and the invalidity of (IV), (OTF), (DN) or (TE) in the fifth argument.

### 2.1.1 REJECTING (FPM) VIA (ECQ)

None of these options are promising. One could argue that (ECQ) is the weakest link among the assumptions. After all, the idea that a contradiction entails anything goes against the common intuition that the premises must be relevant to the conclusion of a valid argumentative form. However, (ECQ) follows from basic argumentative principles such as *Conjunction Elimination* (E&), *Disjunction Introduction* (IV) and *Disjunctive Syllogism* (DS)<sup>7</sup>:

Prem	(1)	$A \& \neg A$	
1	(2)	$A$	1, (E&)
1	(3)	$\neg A$	1, (E&)
1	(4)	$A \vee B$	2, (IV)
1	(5)	$B$	3, 4 (DS)

<sup>4</sup> Simons ([63], 81).

<sup>5</sup> Simons ([63], 79–80).

<sup>6</sup> Gensler ([28], 370).

<sup>7</sup> Lewis & Langford ([39], 248–51).

This shows that the common intuitions contrary to (ECQ) go against basic and intuitive argumentative principles<sup>8</sup>. It can also be shown that (ECQ) follows from (E&) and the *Principle of Antisyllogism* (PA), also known as *Contraposition Theorem*, which states that from  $A, B \models C$ , it follows that  $A, \neg C \models \neg B$ <sup>9</sup>:

- Prem (1)  $A \& B \models A$  (E&)  
 1 (2)  $A \& \neg A \models \neg B$  1, (PA)

We can also show that (ECQ) follows from the principle of *Trivial Validity* (TV) according to which any inference with a tautological conclusion is valid, using the *Truth Preservation* principle (TP) according to which if  $A \models B$ , then  $\neg B \models \neg A$ ; *Conditional Negation* (CN), which states that the negation of a conditional, i.e.,  $\neg(A \rightarrow B)$ , it's equivalent to a conjunction formed by its antecedent and negated consequent, i.e.,  $A \& \neg B$ ; and (DN). The demonstration is as follows:

- Prem (1)  $\neg B \models A \rightarrow A$  (TV)  
 1 (2)  $\neg(A \rightarrow A) \models \neg\neg B$  1, (TP)  
 1 (3)  $A \& \neg A \models \neg\neg B$  2, (CN)  
 1 (4)  $A \& \neg A \models B$  3, (DN)

Another argument for (ECQ) involves one of De Morgan's laws (DM), according to which the negation of a disjunction is the conjunction of the negations, (TV), (CP), (CON) and (DN):

- Prem (1)  $\neg B \models A \vee \neg A$  (TV)  
 1 (2)  $\neg B \rightarrow (A \vee \neg A)$  1, (CP)  
 1 (3)  $\neg(A \vee \neg A) \rightarrow \neg\neg B$  2, (CON)  
 1 (4)  $\neg(A \vee \neg A) \rightarrow B$  3, (DN)  
 1 (5)  $(\neg A \& \neg\neg A) \rightarrow B$  4, (DM)  
 1 (6)  $(\neg A \& A) \rightarrow B$  5, (DN)  
 1 (7)  $\neg A \& A \models B$  6, (CP), since 5 is a tautology derived from 2

So the admission that (ECQ) is invalid implies that at least one of the following argumentative forms must be invalid in each demonstration: (1) (E&), (IV), (DS); (2) (E&), (PA); (3) (TV), (TP), (CN), (DN); (4) (DM), (TV), (CP), (CON), (DN). The first two demonstrations stand out in their simplicity since they rely on relatively uncontroversial principles. Thus, (ECQ) reveals itself to be a surprising impenetrable stronghold. If there is a way to block (FPM) it should involve the denial of another principle.

### 2.1.2 REJECTING (FPM) VIA (CON)

The third proof of (FPM) involves (CON). This argumentative form has intuitive instances such as 'If this body is not being acted on by any external force, then it either remains at rest or moves in a straight line. Therefore, if this body is neither at rest nor moving in a straight line, then it is being acted on by some external force'<sup>10</sup>. However, it also has counter-intuitive instances such as 'If it rains tomorrow there will not be a terrific cloudburst. Therefore, if there

<sup>8</sup> Orayen ([51], 16).

<sup>9</sup> Nelson ([46], 268–69); Orayen ([51], 5).

<sup>10</sup> Hunter ([33], 288).

is a terrific cloudburst tomorrow it will not rain'<sup>11</sup>. That is a tough problem. It can be argued that the validity of (CON) is tied to the validity of *Modus Tollens* (MT), which is usually accepted. The example mentioned above has an analogous instance in (MT) form: 'if it rains tomorrow there will not be a terrific cloudburst. But there is a terrific cloudburst. Hence, tomorrow it will not rain'. In fact, it's possible to prove that contraposition is entailed by (MT) by (GCP) as follows<sup>12</sup>:

Prem (1)  $A \rightarrow B, \neg B \models \neg A$  (MT)  
 1 (2)  $A \rightarrow B, \models \neg B \rightarrow \neg A$  1, (GCP)

(CON) also follows from (GCP) and the *Sufficient Condition* principle (SC), according to which the assumption that the truth of the antecedent and the falsity of the consequent are sufficient for the falsity of the conditional. The proof is as follows:

Prem (1)  $A \rightarrow B$   
 Prem (2)  $\neg B$   
 1,2 (3)  $\neg A$  1, 2 (SC), for if  $A$  were true,  $A \rightarrow B$  would be false  
 1,2 (4)  $\neg B \rightarrow \neg A$  1, 2, 3 (GCP)

Another proof involves (TP) and (CP):

Prem (1) If  $A \models B$ , then  $\neg B \models \neg A$  (TP)  
 1 (2) If  $A \rightarrow B$ , then  $\neg B \rightarrow \neg A$  1, (CP)

It could even be argued that (CON) follows from the truth conditions of the biconditional ' $\Leftrightarrow$ '<sup>13</sup>:

Prem (1)  $A \Leftrightarrow B \equiv (A \rightarrow B) \& (B \rightarrow A)$  given the truth conditions of ' $\Leftrightarrow$ '  
 Prem (2)  $A \Leftrightarrow B \equiv (A \rightarrow B) \& (\neg A \rightarrow \neg B)$  given the truth conditions of ' $\Leftrightarrow$ '  
 1, 2 (3)  $B \rightarrow A \equiv \neg A \rightarrow \neg B$  1, 2 if  $A \& B \equiv A \& C$ , then  $B \equiv C$

To sum up: The first argument establishes that contraposition can only be invalid if either (GCP) or (MT) are invalid. Since (MT) follows from (SC), is necessary to refute (SC) or (GCP) in order to refute (CON). The second argument reinforces this conclusion for it establishes that in order to refute (CON), either (SC) or (GCP) must be invalid. The third argument establishes that in order to refute (CON) either (TP) or (CP) must be invalid, but both are impeccable. The last argument shows that (CON) follows from ' $\Leftrightarrow$ ', which is fairly standard.

It's also important to observe that (CON) is accepted by many authors who do not accept that conditionals are logically equivalent to the material implication<sup>14</sup>. In order to refute the third proof of (FPM), these authors will have to reject (GCP), since (TE) and the truth conditions of '&' are uncontroversial.

### 2.1.3 REJECTING (FPM) VIA (GCP) AND (EXP)

The attempts to refute (ECQ) and (CON) are not promising. Since (CP), (TE) and (DN) are

<sup>11</sup> Adams ([2], 15).

<sup>12</sup> Ortiz ([52], 41–42).

<sup>13</sup> Ortiz, H. (2009). Intuitive Arguments for Material Implication. Unpublished manuscript.

<sup>14</sup> For example, Lycan ([41], 34–35); Hunter ([33], 285); Austin ([7], 209); and Anderson & Belnap ([6], 107–109).

perfectively intuitive, it is arguable that the only eligible targets are (GCP) and (EXP). Let's consider (GCP) first. It is present in most proofs of (FPM). (GCP) sounds reasonable and apparently has intuitive instances in natural language, e.g., 'If having eggs and olive oil entails that I can make mayonnaise, it follows that having eggs entails that if I have olive oil, I can make mayonnaise'<sup>15</sup>.

Despite its prima facie plausibility, (GCP) faces the following counter-example. Let  $A$  be the disjunction 'Bob will retire next year or we will be invaded by Martians' and suppose that  $A$  is true only because the first disjunct is true. Now let  $B$  be 'Bob will not retire next year'. The conjunction of  $A$  and  $B$  entails 'We will be invaded by Martians'. From this it follows by (GCP) that 'Bob will retire next year or we will be invaded by Martians' entails 'If Bob does not retire next year, we will be invaded by Martians'. This is apparently a counter-example since we would be inclined to accept the first argument, but not the second<sup>16</sup>. But (GCP) follows from (CP), (EXP), *Conjunction Introduction* (I&) and *Modus Ponens* (MP). The proof is the following<sup>17</sup>:

Prem	(1)	$A, B \vDash C$	
	1	(2) $A \& B \vDash C$	1, (I&)
	1	(3) $\vDash (A \& B) \rightarrow C$	2, (CP)
	1	(4) $\vDash A \rightarrow (B \rightarrow C)$	3, (EXP)
	1	(5) $A \vDash B \rightarrow C$	4, (MP)

Since (CP) and (I&) are relatively uncontroversial, any attempt to refute (GCP) must involve the refusal of (EXP) or (MP), and since (MP) follows from (SC), is necessary to refute either (EXP) or (SC). (SC) is uncontroversial for it encapsulates a basic truth about conditionals: any conditional with a true antecedent and a false consequent is false. Thus, the only remaining target is (EXP). This argumentative form seems plausible and it has intuitive instances on natural language such as 'If he is a man and married, then he is a husband. Therefore, if he is a man, then if he is married, he is a husband'<sup>18</sup>. This argumentative form has counter-intuitive instances such as this 'If Harry runs fifteen miles this afternoon and he is killed in a swimming accident this morning, then he will run fifteen miles this afternoon. Therefore, if Harry runs fifteen miles this afternoon, then if he is killed in a swimming accident this morning, he will run fifteen miles this afternoon'<sup>19</sup>.

However, it's not clear how the second argument can be unacceptable if the first is allowed. If it is counter-intuitive to accept that 'if Bob will not retire next year, we will be invaded by Martians' follows from 'Bob will retire next year or we will be invaded by Martians', then it is also counter-intuitive to accept that 'we will be invaded by Martians' follows from 'Bob will retire next year or we will be invaded by Martians', and 'Bob will not retire next year'. The second argument is just as counter-intuitive as the first. Therefore, there is no counter-example. (EXP) is also preserved.

## 2.2 THE SECOND PARADOX OF MATERIAL IMPLICATION

Since a material implication,  $A \rightarrow B$ , is true when  $B$  is true, the it must be valid the following argumentative form,  $B \vDash A \rightarrow B$ . This argumentative form has intuitive instances such as 'The

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<sup>15</sup> Rieger ([54], 3165).

<sup>16</sup> Lycan ([41], 82).

<sup>17</sup> Rieger ([54], 3163–3164).

<sup>18</sup> Leavitt ([37], 10).

<sup>19</sup> Lycan ([41], 82).

match was not cancelled. Therefore, if it rained, the match was not cancelled<sup>20</sup>. But it also has counter-intuitive instances in natural language such as ‘The match will not be cancelled. Therefore, if the players broke their legs, the match will not be cancelled’. Let’s call this argumentative form *The Second Paradox of Material Implication* (SPM). It can be shown that (SPM) follows from (E&) and (GCP):

Prem (1)  $A \& B \models B$  (E&)  
 1 (2)  $B \models A \rightarrow B$  1, (GCP)

Another proof involves (E&), (CP) and (EXP)<sup>21</sup>:

Prem (1)  $B \& A \models B$  (E&)  
 1 (2)  $(B \& A) \rightarrow B$  1, (CP)  
 1 (3)  $B \rightarrow (A \rightarrow B)$  3, (EXP)  
 1 (4)  $B \models A \rightarrow B$  4, (CP)

It’s also possible to prove (SPM) using the truth conditions of ‘&’, (GCP), (DN), and (TE)<sup>22</sup>:

Prem (1)  $B \models \neg\neg B$  (DN)  
 1 (2)  $\neg\neg B \models \neg(A \& \neg B)$  intuitively valid argumentative form  
 1 (3)  $\neg(A \& \neg B), A \models \neg\neg B$  given the validity of 2  
 1 (4)  $\neg\neg B \models B$  (DN)  
 1 (5)  $\neg(A \& \neg B), A \models B$  3,4 (TE)  
 1 (6)  $\neg(A \& \neg B) \models A \rightarrow B$  5, (GCP)  
 1 (7)  $\neg\neg B \models A \rightarrow B$  2,6 (TE)  
 1 (8)  $B \models A \rightarrow B$  7, (DN)

Another argument involves (IV), Commutativity of Disjunction (CD), (DN), (TE) and (OTF):

Prem (1)  $B$   
 1 (2)  $B \vee \neg A$  1, (IV)  
 1 (3)  $\neg A \vee B$  2, (CD)  
 1 (4)  $\neg\neg A \rightarrow B$  3, (OTF)  
 1 (5)  $A \rightarrow B$  4, (DN)  
 1 (6)  $B \models A \rightarrow B$  1–5 (TE)

It can be shown that the proponents of (HS) are committed to (SPM) with the principle known as *Necessary Consequent* (NC), which claims that  $A \rightarrow T$  is true, for any tautology,  $T$ ; and *Antecedent Disjunction Introduction* (ADI), which states that from  $((A \rightarrow B) \& (C \rightarrow B))$  it follows  $((A \vee C) \rightarrow B)$ . The demonstration is as follows<sup>23</sup>:

Prem (1)  $B$   
 1 (2)  $C \rightarrow B$   $C$  is irrelevant to whether or not  $B$  obtains  
 1 (3)  $\neg C \rightarrow B$   $\neg C$  is irrelevant to whether or not  $B$  obtains

<sup>20</sup> Clark ([12], 78).

<sup>21</sup> Leavitt ([37], 10).

<sup>22</sup> Simons ([63], 80–81).

<sup>23</sup> The argument is adapted from Walters ([69], 996), who uses subjunctives instead of indicatives and intends to demonstrate that (HS) is invalid.

- |     |     |                                 |     |       |
|-----|-----|---------------------------------|-----|-------|
| 1   | (4) | $(C \vee \neg C) \rightarrow B$ | 2,3 | (ADI) |
|     | (5) | $A \rightarrow (C \vee \neg C)$ |     | (NC)  |
| 1,5 | (6) | $A \rightarrow B$               | 4,5 | (HS)  |
| 1,5 | (7) | $B \vDash A \rightarrow B$      | 1,6 | (TE)  |

Thus, in order to deny (SPM) either (E&) or (GCP) must be denied due to the first argument; (E&), (CP) or (EXP) must be denied due to the second argument; the truth conditions of '&', (GCP), (DN) or (TE) must be denied in the third argument; and the validity of either (IV), (CD), (DN), (TE) or (OTF) in the fourth argument. But (E&), (CP), (DN) and (TE) are uncontroversial, and we already saw in the section 1.4 that (GCP) is basically implied by (EXP) and (SC), and that these argumentative forms are acceptable.

### 2.3 CONDITIONAL NEGATION

Classical logic asserts that from the negation of a conditional, i.e.,  $\neg(A \rightarrow B)$ , it's equivalent to a conjunction formed by its antecedent and negated consequent, i.e.,  $A \& \neg B$ . Let's call this logical equivalence *Conditional Negation* (CN). This logical equivalence has intuitive instances. Suppose that about a geometric figure someone is certain that is a polygon, but nothing more. In this circumstance, she could infer from 'is not the case that if this figure is a rectangle, it is a triangle' that 'this figure is a rectangle and it is not a triangle'<sup>24</sup>. But it also has counter-intuitive instances. From this equivalence, it follows that negating the conditional 'If God exists then the prayers of evil men will be answered' I must admit that, 'God exists and the prayers of evil men will not be answered'<sup>25</sup>. Thus, from the negation of a simple conditional, I can prove that God exists. This is implausible, because someone could refuse the conditional based on assumptions about the moral dispositions of God even if she doesn't believe in the existence of God. Thus, it seems natural that any logic system should remove (CN).

The problem with this approach is that (CN) follows from accepted principles such as *Reduction ad Absurdum* (RAA), (DM), (I&) (CP) and (DS):

- |      |     |  |     |            |
|------|-----|--|-----|------------|
| Prem | (1) | $\neg(A \rightarrow B)$                      |     |            |
| Sup  | (2) | $\neg(A \& \neg B)$                          |     | assumption |
| 2    | (3) | $\neg A \vee \neg \neg B$                    | 2,  | (DM)       |
| 2    | (4) | $\neg A \vee B$                              | 3,  | (DN)       |
| Sup  | (5) | $A$  |     | assumption |
| 2,5  | (6) | $B$  | 4,5 | (DS)       |
| 2    | (7) | $A \rightarrow B$                            | 5,6 | (CP)       |
| 1,2  | (8) | $(A \rightarrow B) \& \neg(A \rightarrow B)$ | 1,7 | (I&)       |
| 1    | (9) | $A \& \neg B$                                | 2-8 | (RAA)      |

We can also prove the validity of (CN) with (DS), (GCP), (TP), (DM) and (DN):

- |      |     |  |    |       |
|------|-----|--|----|-------|
| Prem | (1) | $(\neg A \vee B) \& A \vDash B$                      |    | (DS)  |
| 1    | (2) | $\neg A \vee B \vDash A \rightarrow B$               | 1, | (GCP) |
| 1    | (3) | $\neg(A \rightarrow B) \vDash \neg(\neg A \vee B)$   | 2, | (TP)  |
| 1    | (4) | $\neg(A \rightarrow B) \vDash \neg \neg A \& \neg B$ | 3, | (DM)  |
| 1    | (5) | $\neg(A \rightarrow B) \vDash A \& \neg B$           | 4, | (DN)  |

<sup>24</sup> Egré & Politzer ([17], 11).

<sup>25</sup> Stevenson ([68], 28).



Another proof involves (CP), (MT) and *Conjunctive Syllogism* (CS), the argumentative principle according to which  $\neg B$  follows from  $\neg(A \& B)$  and  $A$ <sup>26</sup>:

Prem	(1)	$\neg(A \rightarrow B)$	
Sup	(2)	$\neg(A \& \neg B)$	assumption
Sup	(3)	$A$	assumption
2,3	(4)	$B$	2,3 (CS)
2	(5)	$A \rightarrow B$	3,4 (CP)
2	(6)	$\neg(A \& \neg B) \rightarrow (A \rightarrow B)$	2,5 (CP)
1	(7)	$A \& \neg B$	1,6 (MT)

That  $A \& \neg B$  implies  $\neg(A \rightarrow B)$  is easily proved by (RAA) and (MP):

Prem	(1)	$A \& \neg B$	
Sup	(2)	$A \rightarrow B$	assumption
1	(3)	$A$	1, (E&)
1,2	(4)	$B$	2,3 (MP)
1	(5)	$\neg B$	1, (E&)
1,2	(6)	$B \& \neg B$	4,5 (I&)
1	(7)	$\neg(A \rightarrow B)$	2-6 (RAA)

Just so that there is no room for doubt about (CN) logical interdependence, the equivalence between  $\neg(A \& \neg B)$  and  $A \rightarrow B$  can also be proved. First, it can be proved that  $\neg(A \& \neg B)$  follows from  $A \rightarrow B$  by (RAA) and (MP)<sup>27</sup>:

Prem	(1)	$A \rightarrow B$	
Sup	(2)	$A \& \neg B$	assumption
1	(3)	$A$	2, (E&)
1,2	(4)	$B$	1,3 (MP)
1	(5)	$\neg B$	2, (E&)
1,2	(6)	$B \& \neg B$	4,5 (I&)
1	(7)	$\neg(A \& \neg B)$	2-6 (RAA)

Second, it can be proved that  $A \rightarrow B$  follows from  $\neg(A \& \neg B)$  by the same principles<sup>28</sup>:

Prem	(1)	$\neg(A \& \neg B)$	
Sup	(2)	$A$	assumption
Sup	(3)	$\neg B$	assumption
2,3	(4)	$A \& \neg B$	2,3 (I&)
1,2,3	(5)	$\neg(A \& \neg B) \& (A \& \neg B)$	1,4 (I&)
1,2	(6)	$B$	3,5 (RAA)
1	(7)	$A \rightarrow B$	2,6 (CP)

We can also show that  $A \rightarrow B$  follows from  $\neg(A \& \neg B)$  in a different way with (DS), (DM), (DN) and (GCP):

<sup>26</sup> Sherry ([62], 216).

<sup>27</sup> Hanson ([31], 54).

<sup>28</sup> Hanson ([31], 54).

Prem	(1)	$\neg(A \& \neg B)$	
1	(2)	$\neg A \vee \neg\neg B$	1, (DM)
1	(3)	$\neg A \vee B$	2, (DN)
sup	(5)	$A$	assumption
1,4	(6)	$B$	3,4 (DS)
1,2	(7)	$A \rightarrow B$	4,5 (GCP)

It is important to observe that (CN) is logically equivalent to (OTF). First, let's consider the demonstration that (CN) implies (OTF) below:

Prem	(1)	$A \rightarrow B$	
1	(2)	$\neg(A \& \neg B)$	1, (CN)
1	(3)	$\neg A \vee \neg\neg B$	2, (DM)
1	(4)	$\neg A \vee B$	3, (DN)

We can show that (OTF) implies (CN) in the following way:

Prem	(1)	$\neg A \vee B \models A \rightarrow B$	(OTF)
1	(2)	$\neg(A \rightarrow B) \models \neg(\neg A \vee B)$	1, (TP)
1	(3)	$\neg(A \rightarrow B) \models \neg\neg A \& \neg B$	2, (DM)
1	(4)	$\neg(A \rightarrow B) \models A \& \neg B$	3, (DN)

Thus, (OTF) and (CN) are intertwined. This shows that despite all its counter-intuitiveness, (CN) is still deeply cemented on other logical principles that are widely accepted. Thus, in order to refuse the validity of (CN), it is also necessary to refute either (RAA), (GCP), (DS), (CP), (MT), (OTF) or (CS). (RAA) is uncontroversial. (GCP) is ultimately implied by (SC) and (EXP), and both are undeniable argumentative principles. (MT) is implied by (SC). (DS) and (CS) both follow from the truth conditions of the classical disjunction, 'v'. (OTF) is intuitive.

Another argument for (CON) involves the biconditional. It can be said that 'A if and only if B' is intuitively equivalent to 'If A then B and if B then A'. A conjunction is true when both of its conjuncts are true. A biconditional is true when both of its members A and B have the same truth value, i.e., when both are true or both are false. Thus, when both are false, 'A if and only if B' is true, but in this case the conjunction can only be true if each of the conjuncts are true. Thus, if 'A if and only if B' is true when A and B are false, then 'If A then B and if B then A' is true when A and B are false<sup>29</sup>.

Someone could attempt to deny (CP) on the grounds that if it was valid, it would imply (SPM), since from the mere acceptance of B and the assumption of A,  $A \rightarrow B$  follows from B alone by (CP). This happens because (CP) allows us to reason with assumptions instead of accepted premises, as it is evidenced by the argumentative strategy used by Hanson above. The assumption of A is introduced in the step 2 only to be later used in a *reductio* in order to obtain the desired conclusion in the step 6; then is finally discharged from the assumption dependence column as the antecedent of the conclusion in the step 7<sup>30</sup>.

However, in order to deny (CP) we need to accept that  $A \rightarrow B$  can be false when A entails B, which is patently absurd<sup>31</sup>. Moreover, denying the validity of (CP) would require a complete reformulation of mathematics as it is known since mathematical proofs rely heavily on applications of (CP). When a mathematician deduces B from a hypothesis A and axioms X, she

<sup>29</sup> Ortiz ([52], 87).

<sup>30</sup> Adams ([2], 24).

<sup>31</sup> Hanson ([31], 54).

asserts  $A \rightarrow B$  on the strength of  $X$  alone<sup>32</sup>. If its validity on mathematics is not to be abandoned, at the very least, it would be needed to explain why (CP) is invalid in our nonmathematical deductions, and no explanation of this sort seem promising. (CN) is secured.

## 2.4 HYPOTHETICAL SYLLOGISM

Consider now *Hypothetical Syllogism* (HS),  $A \rightarrow B, B \rightarrow C \vDash A \rightarrow C$ . This argumentative form has intuitive instances such as the following: ‘If Eclipse wins the 2.30, I will win £400. If I win £400, I will settle my debts. Therefore, if Eclipse wins the 2.30, I will settle my debts’<sup>33</sup>. But it also has counter-intuitive instances such as the following: ‘If it is seven o’clock you can hear the news report; and if you can hear the news report you have ears. Therefore, if it is seven o’clock you have ears’<sup>34</sup>. But it is easy to prove that (HS) follows from standard argumentative principles such as (MP) and (CP)<sup>35</sup>:

Prem	(1)	$A \rightarrow B$	
Prem	(2)	$B \rightarrow C$	
Sup	(3)	$A$	assumption
1,3	(4)	$B$	1, 3 (MP)
1,2,3	(5)	$C$	2, 4 (MP)
1	(6)	$A \rightarrow C$	3, 4 (CP)

Suppose that (1)  $A \rightarrow B$ , and (2)  $B \rightarrow C$ . Now assume that  $A$ . From (1) and  $A$  it follows that  $B$  by (MP). From (2) and  $B$  it follows that  $C$ , again by (MP). Thus, by the assumption of  $A$  it follows that  $C$ , i.e.,  $A \rightarrow C$ <sup>36</sup>. The fact that inferences with (MP) and (CP) imply that  $A \rightarrow B, B \rightarrow C, A \vDash C$  is valid, also lead to (HS) with (GCP)<sup>37</sup>:

Prem	(1)	$A \rightarrow B, B \rightarrow C, A \vDash C$	
1	(2)	$A \rightarrow B, B \rightarrow C \vDash A \rightarrow C$	1, (GCP)

It can also be shown that (HS) follows from (TE) and (CP):

Prem	(1)	If $A \vDash B$ , and $B \vDash C$ , then $A \vDash C$	(TE)
Prem	(2)	If $A \rightarrow B$ , and $B \rightarrow C$ , then $A \rightarrow C$	1, (CP)

(CN) is also enough to ensure the validity of (HS). We can demonstrate this from (CN): if  $A \rightarrow C$  is false, then  $A$  is true and  $C$  is false, by (CN); but then for any truth value of  $B$ , if  $A \rightarrow B$  is false, then  $B \rightarrow C$  is false. This, it’s impossible that the premises are false and the conclusion is true<sup>38</sup>.

The first argument establishes that (HS) is implied by (MP), and, therefore, (SC) and (EXP). The second argument establishes that (HS) is implied by (GCP). The third argument establishes that (HS) is implied by (TE) and (CP), and the last that it is implied by (CN), and consequently, by (RAA), (SC), (EXP) or (DS).

<sup>32</sup> Rumfitt ([59], 183).

<sup>33</sup> Newton-Smith ([47], 25).

<sup>34</sup> Stevenson ([68], 28).

<sup>35</sup> Braine ([9], 36).

<sup>36</sup> Braine ([9], 36).

<sup>37</sup> Ortiz ([52], 42).

<sup>38</sup> Dale ([15], 91–95).

## 2.5 STRENGTHENING OF THE ANTECEDENT

*Strengthening of the antecedent* (SA) is the principle that allow us to infer  $(A \& C) \rightarrow B$  from  $A \rightarrow B$ . It has intuitive instances such as ‘If this switch is pressed down, the light comes on. Therefore, if this switch is pressed down and I stand on one leg, the light comes on’<sup>39</sup>. But it also has the following counter-intuitive instance in natural language: ‘If Brown wins the election, Smith will retire to private life. Therefore, if Smith dies before the election and Brown wins it, Smith will retire to private life’<sup>40</sup>.

First, one could argue that (SA) follows from *Left Weakening* (LW), i.e., if  $A \vDash B$ , then  $A \& C \vDash B$ , (CP) and (TE):

Prem	(1)	$A \rightarrow A$	tautology
	1	(2) $A \vDash A$	1, (CP)
	1	(3) $A \& C \vDash A$	2, (LW)
	1,3	(4) $(A \& C) \rightarrow A$	3, (CP)
	1,2,3	(5) $A \rightarrow A \vDash (A \& C) \rightarrow A$	1–4 (TE)

The following demonstration of (SA) involves *Or-to-If* (OTF), i.e., the argumentative form according to which  $A \rightarrow B$  and  $\neg A \vee B$  are logically equivalent, (IV), (DM), (CD), *Associativity of Disjunction* (AD), and *Commutativity of Conjunction* (CC)<sup>41</sup>:

Prem	(1)	$A \rightarrow B$	
	1	(2) $\neg A \vee B$	1, (OTF)
	1	(3) $(\neg A \vee B) \vee \neg C$	2, (IV)
	1	(4) $\neg C \vee (\neg A \vee B)$	3, (CD)
	1	(5) $(\neg C \vee \neg A) \vee B$	4, (AD)
	1	(6) $\neg(C \& A) \vee B$	5, (DM)
	1	(7) $\neg(A \& C) \vee B$	6, (CC)
	1	(8) $(A \& C) \rightarrow B$	7, (OTF)

(SA) is also entailed by (HS). Consider the following hypothetical syllogism:  $(A \& B) \rightarrow A$ ,  $A \rightarrow C \vDash (A \& B) \rightarrow C$ . The first premise,  $(A \& B) \rightarrow A$ , is necessarily true. If this argumentative form preserves the truth,  $A \rightarrow C \vDash (A \& B) \rightarrow C$  will also preserve the truth. Ergo, (SA) will preserve the truth<sup>42</sup>. If (SA) is entailed by (HS), which is entailed by (MP) and (CP), implies that (SA) is also entailed by (MP) and (CP).

That there is a relation between the two argumentative forms can be shown by the fact that the invalidity of (MP) follows from exportation (EXP) and the invalidity of (SA):

Prem	(1)	$A \rightarrow B$	
	1	(2) $\neg((A \& C) \rightarrow B)$	1, invalidity of (SA)
	1	(3) $(A \& C) \rightarrow \neg B$	2, intuitive negation of ‘ $\rightarrow$ ’
	1	(4) $C \rightarrow (A \rightarrow \neg B)$	3, (EXP)
	1	(5) $C$	assumption
	1	(6) $A \rightarrow \neg B$	

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<sup>39</sup> Allott & Uchida ([4], 2).

<sup>40</sup> Adams ([1], 166).

<sup>41</sup> Fulda & Ortiz ([26], 329).

<sup>42</sup> Jackson ([34], 84).

The point is that we can't infer 6 from 4 and 5 by (MP) because we denied  $(A \& C) \rightarrow B$  in step 2 under the assumption that SA is invalid<sup>43</sup>. Thus, (SA) and (MP) are also linked.

There are no obvious candidates for exclusion. The first argument uses (LW), which can be plausibly interpreted as a consequence of monotonicity in formal logic, in addition to (CP) and (TE), which are undeniable logic platitudes. The second argument uses (OTF), which is intuitive—see section 1.6 of this article—and (IV), (DM), (CD) and (AD), which all follow from the truth conditions of 'v', and (CC), which follows from the truth conditions of '&'. It is hard to imagine anyone doubting any of those assumptions. The third proof uses (HS), which is entailed by (CP) and (MP), which in turn is entailed by (EXP) and (SC).

One could object that (SA) is invalid since it implies that  $(A \& \neg B) \rightarrow B$  follows from  $A \rightarrow B$ . It is argued that this cannot be correct, since  $(A \& \neg B) \rightarrow B$  is intolerable, for its antecedent is inconsistent with its consequent<sup>44</sup>. This happens because while  $\neg A \supset A$  is a contingent truth when  $A$  is false,  $\neg A \rightarrow A$  is intuitively a contradiction in natural language<sup>45</sup>. This intuition is known as *Aristotle's Thesis* (AT) and it is so common that sometimes it is used by logicians as an example of contradiction<sup>46</sup>. Thus, it can be argued that (SA) is inconsistent with (AT).

It could be argued that (AT) is only plausible because  $\neg A \rightarrow A$  is conceived in a context where  $\neg A$  is true. But no one denies that  $\neg A \rightarrow A$  is false in these circumstances. The disagreement with (AT) is in cases in which  $\neg A$  is false. We can show with the material account why (AT) must be false in these cases at the same time we do justice to the intuitive aspects about (AT). The thesis claims that  $A \rightarrow \neg A$  is always false, i.e.,  $\neg(A \rightarrow \neg A)$  is always true. But if the conditional is material, this negation cannot be always true since it will be false when  $A$  is false. This may seem absurd but it is justified by a marriage between the material account and our intuitions. If conditionals are material,  $\neg(A \rightarrow \neg A)$  is logically equivalent to  $A$ . This is plausible and captures the rationale behind (AT), since it represents the belief that no truth can imply its own negation. Thus, the belief that  $A$  is true cannot imply its own negation, i.e., it is logically equivalent to the belief that  $\neg(A \rightarrow \neg A)$  is true; while the negation of  $\neg(A \rightarrow \neg A)$  is logically equivalent to the belief that  $\neg A$ .

The equivalence between  $\neg A$  and  $A \rightarrow \neg A$  can be shown with some principles. First,  $A \rightarrow \neg A$  follows from  $\neg A$  with (HS), (NC), and *Even-if* (EF):  $A \rightarrow B$  is true when  $B$  is true, because  $A$  is irrelevant to whether or not  $B$  obtains<sup>47</sup>:

Prem	(1)	$\neg A$	
Prem	(2)	$A \rightarrow (B \vee \neg B)$	(NC)
1	(3)	$B \rightarrow \neg A$	1, (EF)
1	(4)	$\neg B \rightarrow \neg A$	1, (EF)
1	(5)	$(B \vee \neg B) \rightarrow \neg A$	3,4 (EF)
1,2	(6)	$A \rightarrow \neg A$	2,5 (HS)

Of course,  $A \rightarrow \neg A$  could also be derived from  $\neg A$  by (SPM). Now, the inference of  $\neg A$  from  $A \rightarrow \neg A$  involves only (SC) and (DM):

Prem	(1)	$A \rightarrow \neg A$	
1	(2)	$\neg(A \& \neg A)$	1, (SC)
1	(3)	$\neg A \vee \neg A$	2, (DM)
1	(4)	$\neg A$	3, tautology

Thus, there are good reasons to think that  $A \rightarrow \neg A$  and  $\neg A$  are equivalent. It is also easy to show

<sup>43</sup> Mizumoto ([45], 13–14).

<sup>44</sup> Stalnaker ([67], 123–124).

<sup>45</sup> Mitchell ([44], 64).

<sup>46</sup> Cooper ([14], 194–195).

<sup>47</sup> The argument is adapted from Walters [69, 90].

that the violation of (AT) follows from (HS) and two irrelevant true conditionals<sup>48</sup>:

Prem	(1)	$\neg A \& \neg B$	
1	(2)	$\neg A$	from 1, (E&)
1	(3)	$\neg B \rightarrow \neg A$	from 2, $\neg B$ is irrelevant to whether or not $\neg A$ obtains
1	(4)	$\neg B$	from 1, (E&)
1	(5)	$A \rightarrow \neg B$	from 4, $A$ is irrelevant to whether or not $\neg B$ obtains
1	(6)	$A \rightarrow \neg A$	3,5 (HS)

Thus, anyone who endorses (HS) will have to deny (AT). (AT) can also be criticised for its connection with what Rieger called *Global Principle of Conditional Non-contradiction* (GCNC):  $\neg[(A \rightarrow B) \& (A \rightarrow \neg B)]$ <sup>49</sup>. (GCNC) says that conditionals that have the same antecedents but contradictory consequents are contradictory. The connection between (GCNC) and (AT) is the following: if  $\neg A \rightarrow \neg A$  is necessarily true then its contradictory  $\neg A \rightarrow A$  is a contradiction<sup>50</sup>. Thus, accepting (SA) implies the logical costs of refusing both (AT) and (GCNC).

(GCNC) could be criticised in many fronts. Notice that if  $A \rightarrow B$  and  $A \rightarrow \neg B$  are contradictories, we could never employ them in reductio arguments. In reductios we assume that  $A \rightarrow B$  and  $A \rightarrow \neg B$  are both true and infer from this that  $A$  is false. This can be exemplified in an informal proof that there are infinite prime numbers: If there is a  $N$  which is the biggest prime number, there is a prime number bigger than  $N$ . If there is a  $N$  which is the biggest prime number, there is no prime number bigger than  $N$ . Therefore, there is no  $N$  which is the biggest prime number<sup>51</sup>. This reasoning is justified by the thought that the conjoint acceptance of  $A \rightarrow B$  and  $A \rightarrow \neg B$  it is equivalent to a conditional with the form  $A \rightarrow (B \& \neg B)$ , which on its turn is logically equivalent to  $\neg A$ <sup>52</sup>.

(GCNC) also implies that (CON) is invalid. The argument is as follows:  $A \rightarrow B$  and  $\neg A \rightarrow B$  are consistent, but we can infer from them by (CON) that  $\neg B \rightarrow \neg A$  and  $\neg B \rightarrow A$ , which are contradictory propositions accordingly to (GCNC)<sup>53</sup>. Thus, the acceptance of (GCNC) implies the abandonment of (CON), with all that this entails.

Another argument against (GCNC) involves *U-to-if* (UTF): Every  $F$  is  $G \models Fa \rightarrow Ga$ . Thus, if 'Every  $F$  is  $G$ ' entails 'If  $a$  is  $F$ , then  $a$  is a  $G$ ', then 'Every  $F$  is not  $G$ ' entails 'If  $a$  is  $F$ , then  $a$  is not a  $G$ '. Now, if the conditionals 'If  $a$  is  $F$ , then  $a$  is a  $G$ ' and 'If  $a$  is  $F$ , then  $a$  is not a  $G$ ' are contradictories, then the universally quantified statements 'Every  $F$  is  $G$ ' and 'Every  $F$  is not  $G$ ' would be also contradictories. But they are not, since they can be both false in some circumstances, e.g., if there are some  $F$ s that are  $G$ s and some  $F$ s that are not  $G$ s; and they can be both true in some circumstances, e.g., if there are no  $F$ s. They are only inconsistent if there are some  $F$ s that are  $G$ s, or some  $F$ s that are not  $G$ s, but not both. A similar conclusion applies to the conditionals 'If  $a$  is  $F$ , then  $a$  is a  $G$ ' and 'If  $a$  is  $F$ , then  $a$  is not a  $G$ '. They are both true if there is no  $a$  that is  $F$ , and are only inconsistent if there are some  $F$ s that are  $G$ s, or some  $F$ s that are not  $G$ s, but not both. Thus, the acceptance of (UTF) and the truth conditions of universally quantified statements imply that (GCNC) is false.

There is also an indirect way of showing that (GCNC) is misleading. Grice presented a case in which two people erroneously think of themselves to be disagreeing even if their assertions are consistent. Suppose that two people are debating about the potential outcomes of the British election in 1966. One of them says, 'It will be either Wilson or Heath', but the second denies this by saying, 'No, it will be either Wilson or Thorpe'. Grice uses this example to show that

<sup>48</sup> The argument is adapted from Walters [70, 992].

<sup>49</sup> Rieger ([54], 3167).

<sup>50</sup> Young ([72], 61).

<sup>51</sup> Jackson ([34], 53).

<sup>52</sup> Ceniza ([10], 511).

<sup>53</sup> Mackie ([42], 109).

both of them can be correct<sup>54</sup>. Now, since both disjunctions can be true if Wilson wins the election, their disagreement is about the second disjunct of each disjunction. If Wilson loses the election, then just one of the disjunctions can be true. The corresponding conditionals of those disjunctions by (OTF) are, respectively, ‘If it is not Wilson, is Heath’ and ‘If it is not Wilson, is Thorpe’. The supporter of each conditional seems to be in disagreement with each other, but only if the antecedent is true, i.e., if Wilson does not win. Again, their disagreement is only about the consequent (the second disjunct of each disjunction), and it presupposes the truth of the antecedent (the falsity of the first disjunct). Just as two people erroneously think of themselves as disagreeing with both disjunctions, they would also mistakenly think of themselves as disagreeing with both conditionals.

## 2.6 OR-TO-IF

*Or-to-if* (OTF) is the principle that allow us to infer  $\neg A \rightarrow B$  from  $A \vee B$ . It has intuitive instances such as ‘Either the butler did it or the footman did it. Thus, if the butler did not do it, the footman did it’<sup>55</sup>. However, it also has a counter-intuitive instance in the following context: suppose that there are two balls in a bag, labelled as  $x$  and  $y$ . We know that ball  $x$  comes from a collection in which 99% of the balls are red. But I don’t have any reason to think that ball  $y$  is red. Maybe ball  $y$  comes from a collection in which only 1% of the balls are red. My confidence that  $x$  is red, justifies my belief that either  $x$  is red, or  $y$  is red, but doesn’t justify the conclusion that if  $x$  is not red,  $y$  is red<sup>56</sup>.

However, (OTF) it’s entailed by (SA), and *Limited Transitivity* (LT), the principle according to which  $A \rightarrow C$  follows from  $A \rightarrow B$  and  $(A \& B) \rightarrow C$ . Let  $T$  be a tautology; the argument can be presented as follows<sup>57</sup>:

Prem	(1)	$A \vee B$	
1	(2)	$T \rightarrow (A \vee B)$	from 1
1	(3)	$(T \& \neg A) \rightarrow (A \vee B)$	2, (SA)
1	(4)	$((T \& \neg A) \& (A \vee B)) \rightarrow B$	3, logical truth since $T \& \neg A \vDash B$
1	(5)	$(T \& \neg A) \rightarrow B$	3,4 (LT)
1	(6)	$\neg A \rightarrow B$	it is equivalent to 5

We can also show that (OTF) follows from simple inferences and (CD)<sup>58</sup>:

Prem	(1)	$A \rightarrow B$	
Prem	(2)	$\neg A \rightarrow \neg A$	tautology
Prem	(3)	$A \vee \neg A$	tautology
1,2	(4)	$B \vee \neg A$	1–3, given the possible inferences with $A$ and $\neg A$ in 1 and 2
1,2	(5)	$\neg A \vee B$	4, (CD)

It will be really difficult to avoid the conclusion in the argument above. There is no denying that (2) and (3) are tautologies, or that (4) follows from the previous steps, or that (CD) is valid.

Another argument is that (OTF) can be proved by the principle that if two propositional forms imply the same propositional form by means of the same propositional form they must be equivalent. Given the argumentative forms  $\neg A \vee B$ ,  $A \vDash B$  and  $A \rightarrow B$ ,  $A \vDash B$ , together with

<sup>54</sup> Grice ([30], 64).

<sup>55</sup> Jackson ([34], 5).

<sup>56</sup> Edgington ([16], 55–56).

<sup>57</sup> Bennett ([8], 139–140).

<sup>58</sup> Russell ([60], 136).

the propositional form  $A$ , both  $\neg A \vee B$  and  $A \rightarrow B$  enable us to infer the same propositional form  $B$ .  $\neg A \vee B$ ,  $\neg B \models \neg A$  and  $A \rightarrow B$ ,  $\neg B \models \neg A$  when combined with the same propositional form  $\neg B$  led to the same conclusion  $\neg A$ <sup>59</sup>. What is important about this argument is that it relies on a basic principle that establishes the plausibility of (OTF) by means of the validity of (DS), (MP) and (MT).

It can be also argued that (OTF) is logically equivalent to (GCP) since we can obtain the same inference allowed by (GCP) using (OTF) and we can obtain the same inference allowed by (OTF) using (GCP). First, let us show that (OTF) can substitute (GCP) using what we could call *General Or-to-IF* (GOTF) that states that if  $A, B \models C$  then  $A \models \neg B \vee C$ , in addition to (DN) and (TE)<sup>60</sup>:

Prem	(1)	$A, B \models C$	
Prem	(2)	$A \models \neg B \vee C$	1, (GOTF)
1	(3)	$\neg B \vee C \models \neg \neg B \rightarrow C$	(OTF)
1	(4)	$\neg B \vee C \models B \rightarrow C$	3, (DN)
1	(5)	$A \models B \rightarrow C$	2,4 (TE)

We can use (GCP) and (DS) to obtain the same conclusion allowed by (OTF):

Prem	(1)	$A \vee B, \neg A \models B$	(DS)
1	(2)	$A \vee B \models \neg A \rightarrow B$	1, (GCP)

Thus, in order to deny the validity of (OTF) is necessary to deny the validity of either (SA) or (LT) in the first argument; the validity of (CM) in the second argument; the validity of (DS), (MP) or (MT) in the third argument; or the validity of (GCP), since it is equivalent to (OTF) as demonstrated in the fourth argument.

### 3. SOME METHODOLOGICAL CONSIDERATIONS

Which methodological principles should govern the comparison of logical systems? It is usually accepted that we need to consider whether the sentences presented in the counter-examples have any ambiguities, hidden variables or are poorly formulated<sup>61</sup>, whether the logical system includes closely related phenomena governed by the same fundamental principles<sup>62</sup>, is accountable to some kind of evidential basis (logical intuitions, etc.)<sup>63</sup>; and reflectively balances this evidential basis with our theoretical principles<sup>64</sup>. The present discussion suggests that we also need to take in account the entangled behaviour of logical systems in a consistent manner. Any attempt to remove an argumentative form will generate logical ripples that reverberate across the logical system and even the slightest alteration has significant consequences. It is impossible to remove a counter-intuitive argumentative form without getting entangled in an intricate logical web. This implies that attempts to present counter-examples to counter-intuitive argumentative forms need to be accompanied by counter-examples to the argumentative forms and metalogical principles that imply it in the first place. Let's call this principle *Disentanglement Requirement*.

This principle should be treated as a logical platitude, since its violation leads to incoherence, but if the arguments presented in this article are to be believed, many authors

<sup>59</sup> Sen ([61], 46).

<sup>60</sup> Hong Tang ([32], 27).

<sup>61</sup> Cooper ([14], 181).

<sup>62</sup> Ellis ([18], 50–51).

<sup>63</sup> Ellis ([18], 51).

<sup>64</sup> Lowe ([40], 46); Sherry ([62], 215).



violate this requirement. For instance, Lowe states that we cannot abandon (MP)'s validity, while maintaining that (MT)'s validity is open to debate<sup>65</sup>; but both (MP) and (MT) are implied by (SC): from  $A \rightarrow B$  and  $\neg B$  it follows that  $\neg A$  by (SC), for if  $A$  were true,  $A \rightarrow B$  would be false; and from  $A \rightarrow B$  and  $A$  it follows that  $B$  by (SC), for if  $B$  were false,  $A \rightarrow B$  would be false. Moreover, (MP) entails (MT), since if  $A \rightarrow B$ ,  $A \vDash B$  by (MP), then  $A \rightarrow B$ ,  $\neg B \vDash \neg A$  by (PA). Hunter commits a similar slip when he stipulates that (MP) and (CON) are both valid, but denies the validity of (HS)<sup>66</sup>. But (HS) is implied by (MP) and (CP), and it is also implied by (TE) and (CP), which goes against the rule III of his own system<sup>67</sup>. Goldstein presents a theory of conditional assertion which requires the validity of (MP) and (FPM), but he also insists that the 'conditional's truth is unsettled when the antecedent and consequent are both true'<sup>68</sup>. But (MP) is predicated on the notion that given the truth of a conditional,  $A \rightarrow B$ , and the truth of the antecedent  $A$ , the consequent,  $B$ , must be true as well, otherwise the conditional would be false. The conditional's truth cannot be unsettled when the antecedent and consequent are both true, otherwise (MP) wouldn't work. Next is Gauker<sup>69</sup> and his semantic theory of conditionals which validates (FPM) and (SPM), but invalidates (MT), (CON), (OTF). However, (OTF) implies (FPM) and (SPM) with (IV), (CD), (DN) and (TE). The violations of the Disentanglement Requirement are too numerous to catalogue, because every attempt to depart from classical logic tends to ignore the cobweb-like character of logical systems. The main point is that 'proofs' of consistency of a new system motivated by individual intuitions are not nearly enough to ensure coherence.

It is interesting to note that in section 2.1.3, we tried to explain why two allegedly counter-examples to (GCP) and (EXP) failed with the argument that the conclusions were just as counter-intuitive as the premises. This means that at a certain point will be necessary to explain why a given argumentative form seems invalid if it is to be maintained in the logical system. More than that, we need to explain why some valid argumentative forms seem invalid and why some invalid argumentative forms seem valid. More specifically, in order to provide a robust defence a given system of logic it will be necessary to explain away as illusions the counter-intuitive aspects of some valid argumentative forms and the intuitive aspects of some invalid argumentative forms. Let's call this requirement *The Pragmatic Task*.

There have been some attempts to perform this task, and most of them are presented as a support of classical logic, with a special focus on the counter-intuitive aspects of the material implication<sup>70</sup>. Grice postulated that counter-intuitive conditionals seem counter-intuitive only because they are conversationally inappropriate. If the only reason to assert, 'If Bob does not retire next year, we will be invaded by Martians' is that Bob will retire next year, then the speaker should have just said so. It is misleading. Therefore, insisted Grice, we can explain the counter-intuitive conclusions of these arguments as conversationally inappropriate. Jackson argued that the conditional seems false because we wouldn't be willing to accept the conditional if the antecedent turned out to be true, i.e., the conditional is not robust in relation to the truth of its antecedent<sup>71</sup>.

These authors tried to explain in a principled way why some valid argumentative forms seem invalid, but they don't explain why some classically invalid argumentative forms seem valid. For instance, the argument 'It will not both rain and shine. Therefore, it is wrong to say both that if the barometer drops it will rain, and also that if the barometer drops it will shine'<sup>72</sup> or the argument 'If this is gold it is insoluble in water; so it's not true that if this is gold it is

<sup>65</sup> Lowe ([40], 45).

<sup>66</sup> Hunter ([33], 279).

<sup>67</sup> Hunter ([33], 293).

<sup>68</sup> Goldstein ([29], 296).

<sup>69</sup> Gauker [27].

<sup>70</sup> See, for example, Ajdukiewicz [3], Allott & Uchida [4, 5], Clark [11, 12], Fulda [25], Grice [30], Hanson [31], Lewis [38], Jackson [34, 35], Mellor [43], Noh [48]; Rieger [53; 55]; Smith [64]; Smith & Smith [65].

<sup>71</sup> Jackson [34].

<sup>72</sup> Cooper ([13], 299).

soluble in water'<sup>73</sup> are both intuitively valid, but invalid in classical logic. It could be argued that these theories could explain these examples as cases in which there is a preservation of conversation appropriateness or robustness instead of preservation of truth, but a detailed explanation of these cases would be necessary.

The only opponent of classical logic that tried to explain why some classical argumentative forms are intuitive is Stalnaker. Stalnaker argued that an instance of (OTF) such as 'Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did' seems valid because it is a reasonable inference, in the sense that in every context in which the premise is assertable and it is accepted, is a context that entails the proposition expressed by the corresponding conclusion. But to ensure that anyone who accepted the premise would be in position accept the conclusion is not a guarantee that it is impossible that the premise is true and the conclusion is false. In other words, it preserves reasonability, not truth<sup>74</sup>.

However, Stalnaker does not attempt to explain away the counter-intuitive aspects of his own system. One problem is his endorsement of conditional excluded middle, the principle that states that  $(A \rightarrow B) \vee (A \rightarrow \neg B)$  is always true. This has counter-intuitive instances when  $A$  and  $B$  have nothing to do with each other, e.g., 'If you say the magic words, it will rain tomorrow, or if you say the magic words, it will not rain tomorrow'<sup>75</sup>. Another problem is that his system implies that Dutchman conditionals are always false. These conditionals are used to express the speaker's scepticism about the antecedent, e.g., 'If John's speaking the truth, I'm a Dutchman'. This type of conditional will always be false in his logical system since in the closest world in which John is speaking the truth, I'm not a Dutchman.

This pragmatic task needs to be supported by our intuitions in an elegant way, instead of being just an *ad hoc* device tailored made to save the proposed logical system from criticism, but it shouldn't be also discarded the possibility that some of these logical biases admit different causes. In this case, a comprehensive taxonomy of all the causes would be required, since the usual list of fallacies is far from exhaustive. The conceptual analyses of these mistakes should also consider the experiments involved in conditional reasoning<sup>76</sup>, and all the data should be properly integrated in a general corpus. Nothing should be left to chance.

#### 4. THE NEED TO AVOID COSTLY THEORETICAL ENTERPRISES

The offer of an alternative conditional logic is usually guided by a piece-meal approach, where each attempt to fix an individual counterexample motivates a different logic system. This methodological practice leads to a fragmented understanding of conditionals' role in logic. Rather, the discussion about the merits of alternative conditional logics should rely on broader, systemic considerations that take in consideration the logic relations between argumentative forms. But once this requirement is observed, the problems of alternative systems become apparent. The figures below present a mental map and a network graph containing the examples presented in this paper. This is the type of diagram we should consider when evaluating an argumentative form. The different argumentative forms gather in clusters that highlight their logical relationships. The data set that captures the systematic nature of logic systems is too complex to be analysed by the traditional cherry picking of individual counterexamples. Instead, it requires a different conceptual approach that will lead to a better understanding of their properties and new insights. This new way of looking at an old problem is more promising than new systems that bypass this obvious, but usually ignored, rationale.

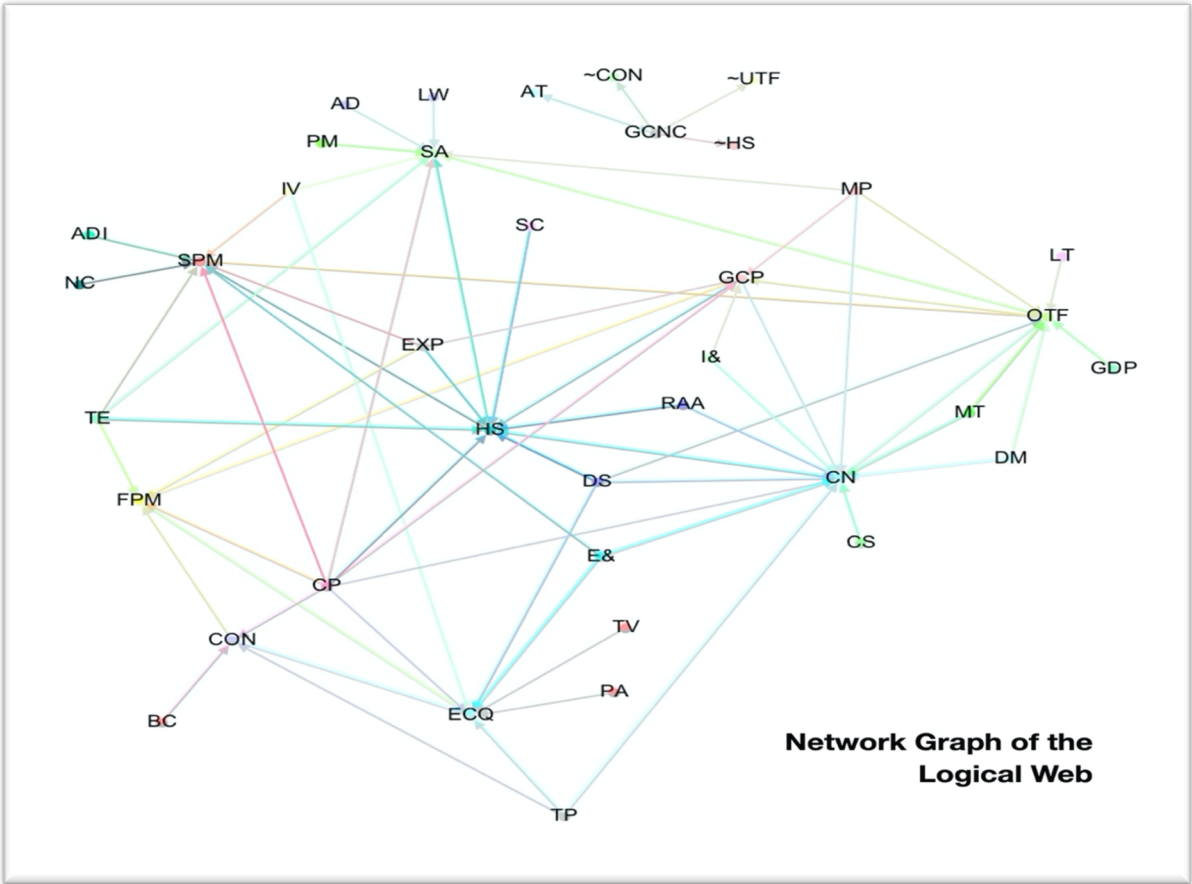
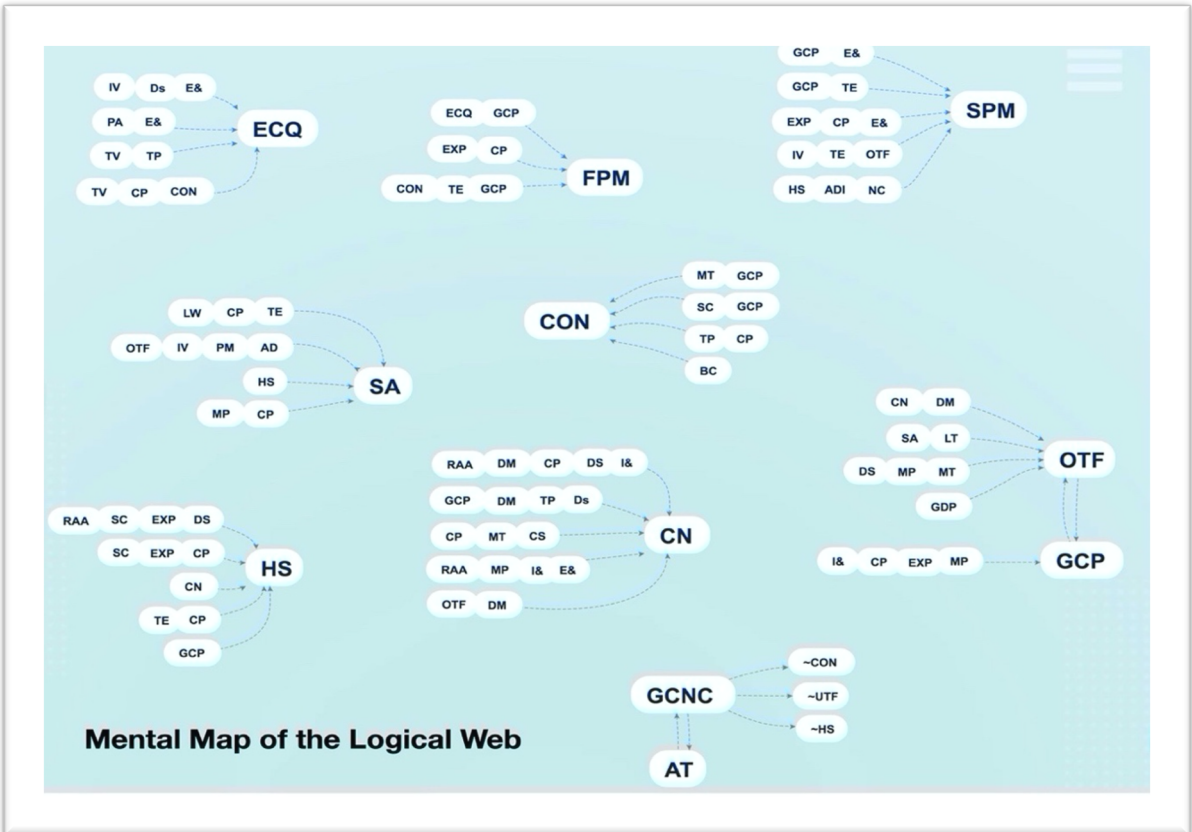
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<sup>73</sup> Stevenson ([68], 28).

<sup>74</sup> Stalnaker ([66], 270).

<sup>75</sup> Hunter ([33], 284).

<sup>76</sup> See, for example, Evans [19]; Evans et al; [23]; Evans et al. [22]; Evans & Beck (1981); Evans & Newstead [21]; Johnson-Laird, P; Byrne, R. [36]; Oberauer & Wilhelm ([49], 1710); Oberauer et al. ([50], 1237); Rips & Marcus [56]; Roberge [57]; Romain et al. [58]; Wason & Johnson-Laird [71].



It was mentioned that we are tied to a logical web. Another useful metaphor to describe logical systems is the logical wall. Logicians can't just remove bricks from the logic wall without additional revisions, and these revisions must be understood for what they are: costly theoretical enterprises. There is no reason to think that all this work will pay off in the end since

these revisions would need to be much more drastic and theoretically costly in order to be consistent across the board. Therefore, alternative systems will inevitably lose their intuitive and elegance appeal, which ultimately defeats the initial purpose of developing an alternative that is more intuitive than classical logic. Rushing into the development of alternative systems based on a superficial analysis of argumentative instances in natural language will only lead to additional logical mistakes down the line.

## REFERENCES

1. Adams, E. (1965). The logic of conditionals. *Inquiry: An Interdisciplinary Journal of Philosophy*, 8(1-4), 166–197.
2. Adams, E. (1975). *The Logic of Conditionals - An Application of Probability to Deductive Logic*, (Ed.) D. Reidel Publishing Company, Dordrecht-Holland, Boston-U.S.A.
3. Ajdukiewicz, K. (1956). Conditional sentence and material implication. *Studia Logica*, 4(1), 135–153.
4. Allott, N; Uchida, H. (2009a). Natural language indicative conditionals are classical. *UCL Working Papers in Linguistics*. 21: 1–17.
5. Allott, N; Uchida, H. (2009b). *Natural language indicative conditionals are classical*. UCL Working Papers in Linguistics, 1–17.
6. Anderson, A; Belnap, N. (1975). *Entailment - The Logic of Relevance and Necessity* (Vol. 1). Princeton: Princeton University Press.
7. Austin, J. (1956). Ifs and Cans. *Proceedings of the Aristotelian Society* 42, 109–132. Reprinted in 1961, James O. Urmson and Geoffrey J. Warnock (eds.), *Philosophical Papers* (205–232). Oxford: Clarendon Press.
8. Bennett, J. (2003). *A Philosophical Guide to Conditionals*. Oxford: Clarendon Press.
9. Braine, M. (1979). On Some Claims about “If-Then”. *Linguistics and Philosophy*, 3(1), 35–47.
10. Ceniza, C. (1988). Material Implication and Entailment. *Notre Dame Journal of Formal Logic*. 29(4), 510–519.
11. Clark, M. (1971). Ifs and Hooks. *Analysis*, 32(2), 33–39.
12. Clark, M. (1974). Ifs and Hooks: A Rejoinder. *Analysis*, 34(3), 77–83.
13. Cooper, W. (1968). The Propositional Logic of Ordinary Discourse, *Inquiry*, 11, 295–320.
14. Cooper, W. (1978) *Foundations of Logico-Linguistics*. Reidel, Dordrecht.
15. Dale, A. (1974), A Defence of Material Implication. *Analysis*, 34(3), 91–95.
16. Edgington, D. (1987). Un argumento de Orayen en favor del condicional material, *Revista Latinoamericana de Filosofía*, 13(1), 54–58.
17. Egré, P., Politzer, G. (2013). *On the negation of indicative conditionals*. Proceedings of the 19th Amsterdam Colloquium. Maria Aloni, Michael Franke and Floris Roelofsen (eds.), 10–18.
18. Ellis, B. (1984). Two Theories of Indicative Conditionals. *Australasian Journal of Philosophy*, 62(1).
19. Evans, J. (1977). Linguistic factors in reasoning. *Quarterly Journal of Experimental Psychology*, 29, 297–306.
20. Evans, J., Beck, M. (1981). Directionality and Temporal Factors in Conditional Reasoning. *Current Psychological Research*, 1(2), 111–120.
21. Evans, J., Newstead, S. (1977). Language and reasoning: A study of temporal factors. *Cognition*, 5(3), 265–83.

22. Evans, J., Newstead, S. & Byrne, R. (1993). *Human reasoning: The psychology of deduction*. Hove: Erlbaum: Psychology Press.
23. Evans, J. St. B. T; Handley, S; Over, D. (2005). Suppositions, Extensionality, and Conditionals: A Critique of the Mental Model Theory of Johnson-Laird and Byrne (2002). *Psychological Review*, 112(4), 1040–1052.
24. Farrell, R. J. (1975). A Note on the Truth-Table for P then Q. *Notre Dame Journal of Formal Logic*, 16(2), 301-304.
25. Fulda, J. (1989). Material Implication Revisited, *The American Mathematical Monthly*, 96(3), 247–250.
26. Fulda, J., Ortiz, H. (2012). Strengthening the antecedent, concessive conditionals, conditional rhetorical questions and the theory of conditional elements. *Journal of Pragmatics*, 44: 328–331.
27. Gauker, C. (2005). *Conditionals in Context*, MIT Press.
28. Gensler, H. (2010). *Introduction to Logic*. Routledge.
29. Goldstein, S. (2019). A Theory of Conditional Assertion. *The Journal of Philosophy*, 116(6), 293—318.
30. Grice, P. (1989). Indicative Conditionals. In: *Studies in the way of words*. Cambridge: Harvard University Press.
31. Hanson, W. (1991). Indicative Conditionals Are Truth-Functional. *Mind*, 100(1), 53–72.
32. Hong Tang, W. (2006). *If and hook*. Master's thesis, National University of Singapore, Singapore. URL <http://scholarbank.nus.edu.sg/handle/10635/15332>.
33. Hunter, G. (1993). The Meaning of “If” in Conditional Propositions. *Philosophical Quarterly*, 43(172), 279–97.
34. Jackson, F. (1987). *Conditionals*. Oxford: Basil Blackwell.
35. Jackson, F. (2006). *Indicative Conditionals Revisited*. Seminar at The Chinese University of Hong Kong, 27, 1–16.
36. Johnson-Laird, P. N; Byrne, R. M. J. (2002). Conditionals: A theory of meaning, pragmatics, and inference. *Psychological Review*, 109, 646–678.
37. Leavitt, F. (1972). An Unpublished Remark of Russell's on “If ... Then”. *Russell: the Journal of Bertrand Russell Studies*, 92 (2), 10.
38. Lewis, D. (1976). Probabilities of Conditionals and Conditional Probabilities. *Philosophical Review*, 85(3), 297–315; reprinted in Harper et al. (eds.) (1981) *Ifs*, Dordrecht: D. Reidel.
39. Lewis, C; Langford, C. H. (1932) *Symbolic Logic*, 2<sup>o</sup> ed., New York, Dover, 1959.
40. Lowe, E. (1995). The Truth About Counterfactuals. *The Philosophical Quarterly*, 45(178), 41–59.
41. Lycan, W. (2005). *Real Conditionals*. Oxford: Oxford University Press.
42. Mackie, J. (1973). *Truth, probability and paradox: studies in philosophical logic*. USA: Oxford University Press.
43. Mellor, D. (1993). How to Believe a Conditional. *The Journal of Philosophy*, 90(5), 233–248.
44. Mitchell, D. (1962). *An Introduction to Logic*. London: Hutchison.
45. Mizumoto, M. (2009). Bennett on Modus Ponens. *Annals of the Japan Association for Philosophy of Science*, 18, 11–28.
46. Nelson, E. (1933). On Three Logical Principles in Intension. *The Monist*, 43, 268-284.
47. Newton-Smith, W. (2005). *H. Logic - An introductory course*. London: Routledge.
48. Noh, E.-J. (1998). A relevance-theoretic account of metarepresentative uses in conditionals. In Rouchota, V. and A. H. Jucker, eds., *Current Issues in Relevance Theory*, Amsterdam:

- John Benjamins, pp. 271-304.
49. Oberauer, K., & Wilhelm, O. (2000). Effects of directionality in deductive reasoning: I. The comprehension of single relational premises. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 26 (6), 1702–1712.
  50. Oberauer, K., Hörnig, R., Weidenfeld, A. (2005). Effects of directionality in deductive reasoning: II. Premise integration and conclusion evaluation. *The Quarterly Journal of Experimental Psychology* 58 (7), 1225–1247.
  51. Orayen, R. (1983) Deducibility Implies Relevance? A Negative Answer (II) (On the Philosophical Status of Relevant Logic). *Crítica: Revista Hispanoamericana de Filosofía*, 15 (44), 3–25.
  52. Ortiz, H. (2007). *Una nueva defensa de la explicación veritativo funcional de los condicionales indicativos*. [Master Thesis] Universidad Nacional Autónoma de México. Unam Repository. [https://repositorio.unam.mx/contenidos/una-nueva-defensa-de-la-explicacion-veritativo-funcional-de-los-condicionales-indicativos-79634?c=4Mw8w6&d=false&q=\\*.:\\*&i=1&v=1&t=search\\_0&as=0](https://repositorio.unam.mx/contenidos/una-nueva-defensa-de-la-explicacion-veritativo-funcional-de-los-condicionales-indicativos-79634?c=4Mw8w6&d=false&q=*.:*&i=1&v=1&t=search_0&as=0).
  53. Rieger, A. (2006). A simple theory of conditionals. *Analysis*, 66(3), 233–40.
  54. Rieger, A. (2013). Conditionals are material: the positive arguments. *Synthese*. 190 (15), 3161–3174.
  55. Rieger, A. (2015). Defending a Simple Theory of Conditionals. *American Philosophical Quarterly*, 52(3), 253–260.
  56. Rips, L; Marcus, S. (1977). Suppositions and the analysis of conditional sentences. In M.A. Just & P.A. Carpenter, (Eds.), *Comprehension processes in comprehension*. Hillsdale, NJ: Lawrence Erlbaum Associates Inc.
  57. Roberge, J. (1982). Linguistic Factors in Conditional Reasoning. *Quarterly Journal of Experimental Psychology*, 34(2), 275–284.
  58. Romain, B., Connell, I., & Braine, M.D.S. (1983). Conversational comprehension processes are responsible for reasoning fallacies in children as well as adults. *If is not the biconditional. Developmental Psychology*, 19, 471–481.
  59. Rumfitt, I. (2013). Old Adams Buried. *Analytic Philosophy*, 54 (2), 157–188.
  60. Russell, L. (1970). ‘If’ and ‘ $\supset$ ’, *Mind*, 79.
  61. Sen, P. (1961). Entailment, Implication and Material Implication. *Journal of the Indian Academy of Philosophy*, 61, 109–128.
  62. Sherry, D. (2006). Formal Logic for Informal Logicians. *Informal Logic*, 26(1), 199–220.
  63. Simons, L. (1965). Intuition and Implication. *Mind, New Series*, 74(293), 79–83.
  64. Smith, N. (1983). On Interpreting Conditionals. *Australian Journal of Linguistics*, 3, 1–24.
  65. Smith, N; A. Smith. (1988). A Relevance-theoretic Account of Conditionals. In Larry M. Hyman; Charles N. Li, eds., *Language, Speech and Mind: Essays in Honour of Victoria A. Fromkin*, pp. 322–352. London: Routledge.
  66. Stalnaker, R. (1975). Indicative Conditionals. *Philosophia*, 5(3), 269–286.
  67. Stalnaker, R. (1984). *Inquiry*. The MIT Press. Cambridge, Massachusetts.
  68. Stevenson, C. (1970). If-Iculties. *Philosophy of Science*, 37(1), 27–49.
  69. Walters, L. 2014a. Conditionals, Modals, and Hypothetical Syllogism. *Thought* 3: 90–97.
  70. Walters, L. 2014b. Against Hypothetical Syllogism. *Journal of Philosophical Logic* 43: 979–997.
  71. Wason, P., Johnson-Laird, P. (1972). *Psychology of reasoning: Structure and content*. Cambridge, Mass. Harvard University Press.
  72. Young, J. (1972). Ifs and Hooks: A Defence of the Orthodox View. *Analysis*, 33(2), 56–63.