THE TRIVIALITY RESULT IS NOT COUNTER-INTUITIVE

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ABSTRACT

The Equation (TE) states that the probability of $A \rightarrow B$ is the probability of B given A. Lewis (1976) has shown that the acceptance of TE implies that the probability of $A \rightarrow B$ is the probability of B, which is implausible: the probability of a conditional cannot plausibly be the same as the probability of its consequent, e.g., the probability that the match will light given that is struck is not intuitively the same as the probability that it will light. Here I want to counter Lewis' claim. My aim is to argue that: (1) TE express the coherence requirements implicit in the probability distributions of a *modus ponens* inference (MP); (2) the triviality result is not implausible because it results from these requirements; (3) these coherence requirements measure MP employability, so TE significance is tied to it; (4) MP employability doesn't provide either the acceptability or the truth conditions of conditionals, since MP employability depends on previous independent reasons to accept the conditional and some acceptable conditionals are not MP friendly. Consequently, TE doesn't have the logical significance that is usually attributed to it.

Keywords: The Equation, triviality result, Dutchman conditionals.

1. INTRODUCTION

The Equation (TE) states that the probability of $A \rightarrow B$ is the probability of B given A[1]. Lewis (1976) has shown that the acceptance of TE implies that the probability of $A \rightarrow B$ is the probability of B, which is implausible: the probability of a conditional cannot plausibly be the same as the probability of its consequent, e.g., the probability that the match will light given that is struck is not intuitively the same as the probability that it will light. This counter-intuitive consequence is the triviality result. Here I want to counter Lewis' claim and argue that the triviality result is plausible when it is properly understood. This paper will be divided as follows. In section 2, it is argued that TE express the coherence requirements implicit in the probability distributions of a *modus ponens* inference (MP). Thus, the triviality result is not implausible, since it is also a coherence requirement implicit in MP. Section 3 makes the case that TE measure our willingness to employ a conditional in MP, but MP employability does not provide either the acceptability or the truth conditions of conditionals. The reasons are that that MP employability depends on previous independent reasons to accept the conditional and some acceptable conditionals are incompatible with MP. Thus, TE does not have the logical significance that is usually attributed to it. Section 4 concludes.

2. THE REAL MEANING OF THE EQUATION

Our inferential disposition to employ $A \rightarrow B$ in a MP is measured by $\Pr(A \supset B|A)$, which is equal to $\Pr(B|A)$. The proof is as follows:

[1] Jeffrey (1964: 702–703).

1	$\Pr((\neg A \vee B)\|A) = \Pr(B\|A)$	since $\Pr(\neg A\|A) + \Pr(B\|A) = \Pr(B\|A)$
2	$\Pr(A \supset B\|A) = \Pr(B\|A)$	from 1, given that ¬A ∨ B is logically equivalent to $A \supset B$

This is plausible since the probability that 'if the match is struck, it will light' given that 'the match is struck' is intuitively the same as the probability that the match will light given that is struck. In other words, our willingness to accept a material conditional given that its antecedent is true is the same as the probability of its consequent given its antecedent.

Now, the fact that $\Pr(A \rightarrow B) = \Pr(B|A)$ implies that $\Pr(A \rightarrow B) = \Pr(B)$ is perfectly intuitive if TE tracks our inferential disposition to employ $A \rightarrow B$ in a MP. To support this, I propose the following proof:

1	$\Pr(A \rightarrow B) = \Pr(B\|A)$	TE
2	$\Pr(B\|A) = \Pr((\neg A \vee B)\|A)$	since $\Pr(\neg A\|A) + \Pr(B\|A) = \Pr(B\|A)$
3	$\Pr((\neg A \vee B)\|A) = \Pr((A \supset B)\|A)$	given that $A \supset B$ is logically equivalent to $\neg A \vee B$
4	$\Pr(A \rightarrow B) = \Pr((A \supset B)\|A)$	from 1 and 3
5	$(A \supset B) \,\&\, A \vDash B$	given the validity of MP
6	$\Pr((A \supset B)\|A) \leq \Pr(B)$	from 5, for it is irrational to be more confident of the premises than of the conclusion
7	$\Pr(A \rightarrow B) \leq \Pr(B)$	from 4 and 6

So $\Pr(A \rightarrow B)$ is tantamount to $\Pr(A \supset B|A)$, which is less or equal to $\Pr(B)$. The point of this argument is that if $\Pr(A \rightarrow B) = \Pr(B)$ is counter-intuitive, the conclusion that $\Pr(A \rightarrow B) \leq \Pr(B|A)$ in the proof above should be equally counter-intuitive, but it isn't. To understand why this is the case, imagine if one would attribute a bigger probability to a conditional, 'if the match is struck, it will light', given its antecedent, 'the match is struck', than to its consequent, 'the match will light'. This reasoner could be correctly accused of being irrational, since he has more confidence in the premises than in the conclusion of a MP. In other words, in order to preserve the validity of MP, $\Pr(A \rightarrow B)$ should be less or equal than $\Pr(B|A)$, given that $\Pr(A \rightarrow B)$ is interpreted as equivalent to $\Pr(A \supset B|A)$. This is perfectly acceptable. Therefore, $\Pr(A \rightarrow B) = \Pr(B)$ shouldn't be considered counter-intuitive given the acceptance of TE.

Someone could object that proof has a weak link in the second line. It assumes without argument that $\neg A$ and B are independent events. If they are not, the second line should be different: $\Pr((\neg A \vee B)|A) = \Pr(\neg A|A) + \Pr(B|A) - \Pr((\neg A \,\&\, B)|A)$. But $\Pr((\neg A \,\&\, B)|A)$ is the same as zero, since the conjunction will be false when one of its conjuncts is false. So the result is the same[2]. Another objection is that it assumes that $\Pr(A \supset B|A)$ is the same as $\Pr(A \rightarrow B)$, but the relationship between the material conditional and natural language conditionals is widely controversial. In fact, the majoritarian position is that conditionals cannot be material since they can't be vacuously true when their antecedent is false, e.g., we don't think that 'If the moon is made of cheese, today is Thursday' is true just because the moon is not made of

[2] I owe this objection to Adam Olszewski.

cheese. But this is the conclusion we should endorse if conditionals were material. This objection can be answered by observing that while the equivalence between $A \rightarrow B$ and $A \supset B$ is controversial, it is widely accepted that $A \supset B$ behaves like $A \rightarrow B$ when A is true, because in these cases the conditional cannot be vacuously true due to the falsity of the antecedent. Since we are considering a circumstance in which A is assumed as true, the scepticism about the use of the material conditional is neutralised.

Another strategy is to resort to a similar result that doesn't involve the material conditional[3]. We can prove that the probability of any conditional given its antecedent equals the conditional probability of the consequent given the antecedent. All that is needed is assuming that $(A \rightarrow B)$ & $A \equiv (A$ & $B)$. This assumption is minimal; it amounts to the generally accepted assumption that if A and B are both true, $A \rightarrow B$ is true. From this assumption we can prove that $\Pr(A \rightarrow B) = \Pr(B|A)$. The proof is as follows: For any conditional that satisfies the robustness requirement, i.e., the principle that $A \rightarrow B$ is acceptable when B is robust with respect to A, i.e., when $\Pr(B)$ is high and would remain high after learning that A[4], $\Pr(A \rightarrow B|A)$ and $\Pr(A \rightarrow B)$ need to be both high. $\Pr(A \rightarrow B|A)$ equals $\Pr((A \rightarrow B)$ & $A))|\Pr(A)$. Since the probability is considered given the assumption that A is the case, the fact that A is assumed in conjunction with $A \rightarrow B$ does not affect its probability. From $(A \rightarrow B)$ & $A \equiv (A$ & $B)$ it follows that $(A \rightarrow B)$ & $A \equiv A$ & B. Thus, $\Pr(A \rightarrow B|A)$ equals $\Pr(A$ & $B)|\Pr(A)$, which according to the definition of conditional probability equates to $\Pr(B|A)$[5]. Since the assumption that $\Pr(A \rightarrow B|A) = \Pr(B|A)$ leads to the conclusion that $\Pr(A \rightarrow B|A)$ equals $\Pr(B)$, the same conclusion follows: the triviality result is not counter-intuitive.

The triviality result was interpreted in multiple ways and adapted for many purposes. Lewis (1976) himself interpreted the result as a proof that no conditional connective will have its truth conditions measured by the conditional probability of the consequent given the antecedent. Edgington (1986) argued that since TE encompass our intuitions about conditionals truth conditions, and there is no connective that can satisfy TE, we should conclude that conditionals have no truth conditions. Both interpretations miss the target. TE doesn't express our semantic intuitions about conditionals truth conditions, but coherence requirements about what should be our probability distributions in a MP. Consequently, the triviality result is a direct result from having coherence requirements in probability distributions of a MP inference. Not only it is not implausible, as it represents a rational conclusion that prevents incoherence.

3. THE EQUATION AND MODUS PONENS

The question is whether TE should have any real significance above and beyond this coherence requirement. A different angle from which we can approach the problem is that TE is just a different way to measure our willingness to employ a conditional in a MP. This provides us with another perspective to approach TE. If MP employability provides acceptability or truth conditions for conditionals, so is TE. Inversely, if doesn't provide such conditions, neither does TE.

It could be argued that any attempt to treat MP employability as an acceptability test results in many false negatives, since some conditionals are accepted only when we are willing to employ $A \rightarrow B$ in *modus tollens.* Take, for instance, 'If John's speaking the truth, I'm a Dutchman'. This conditional can be perfectly reasonable when John is lying, but it fails MP as

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[3] Ellis (1984: 58).

[4] Jackson (1987: 26–31).

[5] Ellis (1984: 58, note 11).

an acceptability test. If it turns out that John is speaking the truth, I won’t infer that I’m a Dutchman. I would rather abandon the conditional. Another false negative concerns the deceiving wife’s case. The conditional ‘If my wife is deceiving me, I will never know’ is acceptable, because my wife is too smart to get caught. However, if I discover that she is deceiving me, I would not infer that I would never know. I would abandon the conditional.

From a quick glance at the examples above, two things catch one’s eye. First, we are only able to decide whether or not we are willing to employ a conditional in a MP if we already have independent reasons to accept the conditional in the first place. If we didn’t have any reasons to accept $A \rightarrow B$, how would we know if B should be inferable from A? So MP as a test must be parasitic of our previous epistemic commitments, and not the other way around. Secondly, some of the many reasons why we accept a conditional can be incompatible with MP. So conditionals can be acceptable and fail in the MP test. This shows that MP employability doesn’t provide acceptability conditions for conditionals and, consequently, doesn’t provide truth conditions for conditionals. But if MP employability shouldn’t have the centrality it has in our understanding of conditionals, neither should TE.

4. CONCLUDING REMARKS

It was argued that TE and its implications, such as the triviality result, represent coherence requirements in probability distributions of a MP inference. So TE is not unacceptable when it is properly understood. In an attempt to go beyond this simple observation, it was argued that TE tracks MP employability, but there are reasons to think that this requirement doesn’t provide acceptability or truth conditions for conditionals. Consequently, TE doesn’t have the logical significance that is usually attributed to it, even if it is true.

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