THE TRIVIALITY RESULT IS NOT COUNTER-INTUITIVE

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ABSTRACT

The Equation (TE) states that the probability of $A \rightarrow B$ is the probability of B given A (Jeffrey, 1964: 702–703). Lewis has shown that the acceptance of TE implies that the probability of $A \rightarrow B$ is the probability of B, which is implausible: the probability of a conditional cannot plausibly be the same as the probability of its consequent, e.g., the probability that the match will light given that is struck is not intuitively the same as the probability that it will light (Lewis, 1976: 299–300). Here I want to counter Lewis' claim. My aim is to argue that: (1) TE doesn't track the probability of $A \rightarrow B$, but instead our willingness to employ it on a *modus ponens*; (2) the triviality result doesn't strike us as implausible if our willingness to employ $A \rightarrow B$ on a *modus ponens* implies a similar result; (3) TE is still inadequate in this limited role given that some conditionals are only employable on a *modus tollens* or can't be employed on a *modus ponens*; (4) TE does not have the logical significance that is usually attributed to it, since inferential disposition is a pragmatic phenomenon.

Keywords: The Equation, triviality result, Dutchman conditionals.

1. INTRODUCTION

This paper will be divided in two sections. In the first section, I will argue that TE doesn't track the probability of $A \rightarrow B$, but instead our willingness to employ it on a *modus ponens*; and that the triviality result doesn't strike us as implausible if our willingness to employ $A \rightarrow B$ on a *modus ponens* implies the triviality result. In the second section, I will argue that TE is still inadequate in this limited role given that some conditionals are employable only on a *modus tollens* or can't be employed on a *modus ponens*; and that TE does not have the logical significance that is usually attributed to it, since inferential disposition is a pragmatic phenomenon.

2. THE RELATION BETWEEN THE EQUATION AND MODUS PONENS

Our inferential disposition to employ $A \rightarrow B$ on a *modus ponens* is measured by $\Pr(A \supset B/A)$, which is equal to $\Pr(B/A)$. The proof is as follows:

1	$\Pr((\neg A \vee B)/A) = \Pr(B/A)$	since $\Pr(\neg A/A) + \Pr(B/A) = \Pr(B/A)$
2	$\Pr(A \supset B/A) = \Pr(B/A)$	From 1, given that ¬A ∨ B is logically equivalent to $A \supset B$

This is plausible since the probability that 'if the match is struck, it will light' given that 'the match is struck' is intuitively the same as the probability that the match will light given that is struck. In other words, our willingness to accept a material conditional given that its antecedent is true is the same as the probability of its consequent given its antecedent.

Now, the fact that $\Pr(A \rightarrow B) = \Pr(B/A)$ implies that $\Pr(A \rightarrow B) = \Pr(B)$ is perfectly intuitive if TE tracks our inferential disposition to employ $A \rightarrow B$ on a *modus ponens*. To support this, I propose the following proof:

1	$\Pr(A \rightarrow B) = \Pr(B/A)$	TE
2	$\Pr(B/A) = \Pr((\neg A \vee B)/A)$	since $\Pr(\neg A/A) + \Pr(B/A) = \Pr(B/A)$
3	$\Pr((\neg A \vee B)/A) = \Pr((A \supset B)/A)$	given that $A \supset B$ is logically equivalent to $\neg A \vee B$
4	$\Pr(A \rightarrow B) = \Pr((A \supset B)/A)$	from 1 and 3
5	$(A \supset B) \,\&\, A \vDash B$	given the validity of m*odus ponens*
6	$\Pr((A \supset B)/A) \leq \Pr(B)$	from 5, for it is irrational to be more confident of the premises than of the conclusion
7	$\Pr(A \rightarrow B) \leq \Pr(B)$	from 4 and 6

From the proof above it follows that $\Pr(A \rightarrow B)$ is tantamount to $\Pr(A \rightarrow B/A)$, which is less or equal to $\Pr(B)$. The point of this argument is that if $\Pr(A \rightarrow B) = \Pr(B)$ is counter-intuitive, $\Pr(A \rightarrow B) \leq \Pr(B/A)$ should be equally counter-intuitive, but it isn't. To see why $\Pr(A \rightarrow B) \leq \Pr(B)$ is not counter-intuitive, we only need to consider that $\Pr(A \rightarrow B)$ is tantamount to $\Pr((A \supset B)/A)$ given the acceptance of TE, which is less or equal to $\Pr(B)$. The probability of 'if the match is struck, it will light' given that 'the match is struck' is less or equal to the probability that 'the match will light'. This is perfectly acceptable. Therefore, $\Pr(A \rightarrow B) = \Pr(B)$ shouldn't be considered counter-intuitive given the acceptance of TE.

Someone could object that proof has a weak link. It assumes that $\Pr(A \supset B/A) = \Pr(A \rightarrow B)$, but the relationship between the material conditional and natural language conditionals is widely controversial. In fact, the majoritarian position is that conditionals cannot be material since they can't be vacuously true when their antecedent is false, e.g., we don't think that 'If the moon is made of cheese, today is Thursday' is true just because the moon is not made of cheese. But this is the conclusion we should endorse if conditionals were material.

This objection can be answered by observing that while the equivalence between $A \rightarrow B$ and $A \supset B$ is controversial, it is widely accepted that $A \supset B$ behaves like $A \rightarrow B$ when A is true, because in these cases the conditional cannot be vacuously true due to the falsity of the antecedent. Since we are considering a circumstance in which A is assumed as true, the scepticism about the use of the material conditional is neutralised

Another strategy is to resort to a similar result that doesn't involve the material conditional[1]. We can prove that the probability of any conditional given its antecedent equals the conditional probability of the consequent given the antecedent. All that is needed is assuming that $(A \rightarrow B) \,\&\, A \equiv (A \,\&\, B)$. This assumption is minimal; it amounts to the

[1] Ellis (1984: 58).

generally accepted assumption that if A and B are both true, $A \rightarrow B$ is true. From this assumption we can prove that $\Pr(A \rightarrow B) = \Pr(B/A)$. The proof is as follows: For any conditional that satisfies the robustness requirement, i.e., the principle that $A \rightarrow B$ is acceptable when B is robust with respect to A, i.e., when $\Pr(B)$ is high and would remain high after learning that A[2], $\Pr(A \rightarrow B/A)$ and $\Pr(A \rightarrow B)$ need to be high. $\Pr(A \rightarrow B/A)$ equals $\Pr((A \rightarrow B) \& A))/\Pr(A)$. Since the probability is considered given the assumption that A is the case, the fact that A is assumed in conjunction with $A \rightarrow B$ does not affect its probability. From $(A \rightarrow B) \& A \equiv (A \& B)$ it follows that $(A \rightarrow B) \& A \equiv A \& B$. Thus, $\Pr(A \rightarrow B/A)$ equals $\Pr(A \& B)/\Pr(A)$, which by the definition of conditional probability equates to $\Pr(B/A)$[3]. Since the assumption that $\Pr(A \rightarrow B/A) = \Pr(B/A)$ leads to the conclusion that $\Pr(A \rightarrow B/A)$ equals $\Pr(B)$, the same conclusion follows: the triviality result is not counter-intuitive.

3. THE REAL PROBLEM OF THE EQUATION

The triviality result was interpreted for many purposes. Lewis himself interpreted the result as a proof that no conditional connective will have its truth conditions measured by the conditional probability of the consequent given the antecedent[4]. Edgington argued that since TE encompass our intuitions about conditionals truth-conditions, and there is no connective that can satisfy TE, we should conclude that conditionals have no truth-conditions[5]. Both interpretations miss the target. TE doesn't express our semantic intuitions about conditionals' truth-conditionals, but our willingness to employ a conditional on a *modus ponens*. The assumption that $\Pr(A \rightarrow B) = \Pr(B/A)$ is just a different ways to express the assumption that $A \rightarrow B$ is true if we are willing to infer B from A, i.e., if we are willing to employ $A \rightarrow B$ in a *modus ponens* (EM). This provides us with another reason to reject TE. If EM is false, so is TE.

It could be argued that EM results in many false negatives, since some conditionals are accepted only when we are willing to employ $A \rightarrow B$ in *modus tollens*: e.g., 'If John's speaking the truth, I'm a Dutchman'. This conditional can be perfectly reasonable when John is lying, but it is not employable in a *modus ponens*. If it turns out that John is speaking the truth, I won't infer that I'm a Dutchman. I would rather abandon the conditional. Another false negative concerns the deceiving wife's case. The conditional 'If my wife is deceiving me, I will never know' is acceptable, because my wife is too smart to get caught. However, if I discover that she is deceiving me, I would not infer that I would never know; I would abandon the conditional.

Thus, TE is acceptable only if is interpreted as follows: $A \rightarrow B$ is acceptable if, and only if, our disposition to employ it in an inference is compatible with the reasons that lead us to accept it in the first place. TE is not particularly impressive in this new form. It is trivially true that in most cases our inferential dispositions are determined by our reasons to accept a conditional, but that is not a universal principle, as the deceiving wife case makes clear.

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2 Jackson (1987: 26–31).
3 Ellis (1984: 58, note 11).
4 Lewis (1976).
5 Edgington (1986: 3).

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