10

WEAK AND STRONG NECESSITY MODALS:

On Linguistic Means of Expressing "A Primitive Concept OUGHT"[1]

Alex Silk

'Ought' and 'Must'—they are contemptible auxiliaries.

George Eliot[2]

1. Introduction

A notion of 'ought' is central in many areas of philosophical discourse. "A primitive ought," Gibbard tells us, "is the basic conceptual atom that gives normative concepts their special character" (2006, 738). Yet, historically speaking, comparatively little attention has been paid among philosophers to the distinctive features of the meaning and use of 'ought'. 'Ought' is often treated as relevantly equivalent to a range of expressions of obligation and necessity. The eponym of Gibbard's Professorial Chair, Richard Brandt, observed as much half a century ago: "Philosophers often use the following expressions as approximate equivalents: 'It is X's duty to do *A*'; 'It is obligatory for X to do *A*'; 'It would be wrong for X not to do *A*'; and 'X ought to do *A*'" (1964, 374). Here is Åqvist:

2 Mary Garth, in *Middlemarch*, Bk. 2, Ch. 14. Shamelessly modified from the original.

[D]eontic logic . . . is the logical study of the normative use of language and . . . its subject matter is a variety of normative concepts, notably those of *obligation* (prescription), *prohibition* (forbiddance), *permission* and *commitment*. The first one among these concepts is often expressed by such words as 'shall', 'ought' and 'must', the second by 'shall not', 'ought not' and 'must not'.

<div align="right">(Åqvist 2002, 148)</div>

There has been extensive work in descriptive linguistics on discourse differences among Åqvist's "normative-concept-expressing" words. For instance, it's common to distinguish categories of so-called "weak" necessity modals such as 'ought' ('should', 'be supposed to') and "strong" necessity modals such as 'must' ('have to', '(have) got to', 'be required to').[3] Holding the reading of the modals fixed, (1a) is consistent in a way that (1b) is not. 'Ought ϕ' can be followed by 'Must ϕ', but not vice versa, as reflected in (2).

(1) a. I should help the poor, but I don't have to.
     b. #I must help the poor, but it's not as if I should.
(2) a. I ought to help the poor. In fact, I must.
     b. I must help the poor. #In fact, I ought to.

There are also conversational differences. Informally:

To say that one <u>ought</u> to take a certain option is merely to provide a nudge in that direction. Its typical uses are to offer guidance, a word to the wise . . . , to recommend, advise . . . In contrast, to say that one <u>must</u> take a certain option is to be quite forceful. Its typical uses are to command, decree, enact, exhort, entreat, require, regulate, legislate, delegate, or warn.

<div align="right">(McNamara 1990, 156)</div>

Gibbard hedges his bets on what natural language expression best approximates his favored normative notion:

Ewing's point is not that the English word 'ought' exactly captures the notion he has in mind. The word often suggests merely the weight of one set of considerations among others . . . The word 'must' might be better for his purposes and mine [?!?!], but it has its own flaws: it suggests greater urgency than Ewing's ought would have when factors in a decision nearly balance out.

<div align="right">(Gibbard 2012, 14–15; more on '?!?!' later)</div>

---

3 See, e.g., Sloman 1970, Leech 1971, Horn 1972, Wertheimer 1972, Harman 1975, Lyons 1977, Woisetschlaeger 1977, Williams 1981, Coates 1983, McNamara 1990, Palmer 1990, 2001, Bybee et al. 1994, Myhill 1995, Myhill and Smith 1995, Huddleston and Pullum 2002, von Fintel and Iatridou 2008, Rubinstein 2012; additional references to theoretical accounts will follow in due course. I focus on modal verbs; see Van Linden 2012 on weak/strong modal adjectives.

An interesting question for a paper—not this one (cf. Silk 2015b[4])—is to what extent insensitivity to linguistic differences among 'ought', 'should', 'must', 'have to', etc. may be a source of philosophical malaise. On the flip side, perhaps investigating the meaning and discourse function of such words can clarify the contours of Gibbard's basic normative "conceptual atom" and improve philosophical theorizing. In this paper I wish to set the stage for such a project by attending to the matters narrowly linguistic: what 'ought'—the eponymous expression of Gibbard's "primitive concept OUGHT" (2012, 204)—itself means. I focus on the distinction in "strength" between 'ought' (and its weak-necessity-modal kin) and strong necessity modals such as 'must'.

Although there has been growing interest in the semantics of 'ought', accounts are often developed in ways that bracket differences among necessity modals. Formal accounts of the weak/strong necessity modal distinction are typically developed piecemeal with an eye toward a narrow range of data. Adjudicating among theories can be difficult, if not premature. The aim of this paper is a more comprehensive theoretical investigation into weak and strong necessity modals. Building on previous work (Silk 2012, 2013), I develop an account of the meaning of 'ought' and the distinction between weak and strong necessity modals. The account systematizes a wide range of semantic and pragmatic phenomena: it generalizes across flavors of modality; it elucidates a special role that weak necessity modals play in discourse and planning; it captures contrasting logical, expressive, and illocutionary properties of weak and strong necessity modals; and it sheds light on how a notion of 'ought' is expressed in other languages. These phenomena have resisted systematic explanation.

Roadmap: §2 presents core data on the effects of standing contextual assumptions on the relative felicity of weak vs. strong necessity modals. The §2-examples highlight what I regard as the fundamental difference between the class of weak necessity modals and the class of strong necessity modals.

§3 presents the basic account of the weak/strong necessity modal distinction. No innovations are introduced in the semantics and pragmatics of strong necessity modals; uses of 'Must ϕ' predicate the (deontic, epistemic) necessity of the prejacent ϕ of the actual world.[5] The apparent "weakness" of weak necessity modals derives from their bracketing whether the necessity of the prejacent is verified in the actual world. 'Ought ϕ' can be accepted without accepting that ϕ is necessary (deontically, epistemically, etc.). Weak necessity modals afford a means of entertaining and planning for hypothetical extensions of the context in which certain considerations (norms, values, etc.) apply, without needing to commit that the considerations aren't defeated.

§4 examines how weak necessity is expressed crosslinguistically to motivate several formal implementations of the informal §3-account. For reasons that will become clear, I call the family of analyses a *modal-past approach* to 'ought' and the weak/strong necessity

---

modal distinction. I argue that the proposed treatment of the crosslinguistic data improves on the treatment by von Fintel and Iatridou (2008). The account gives precise expression to the informal idea that 'ought' is weaker than 'must', and captures ways in which 'ought', unlike 'must', patterns with past-marked modal forms.

§5 applies the formal semantics from §4 to several puzzles of entailingness and performativity with 'ought' and 'must'. Examining these puzzles highlights a second dimension along which modals differ, regarding their tendencies to be used in (what I call) an "endorsing" vs. "nonendorsing" way.

§6 recaps distinctive features of the account and contrasts it with several prominent alternatives, in particular the "collective commitment" analysis developed in Rubinstein 2012.

§7 concludes and raises directions for future research. Potential implications of the linguistic work on modals for broader philosophical theorizing are briefly considered.

For familiarity I follow the literature in labeling modals such as 'ought' and 'should' as "weak necessity" modals, and labeling modals such as 'must', 'have to', '(have) got to' as "strong necessity" modals. The terminology of "weak necessity" and "strong necessity" shouldn't mislead. I am not assuming that uses of the modals in the 'ought'-family invariably convey a weaker felt conversational force, that the modals express different "kinds" of necessity, or even that they comprise a scale of logical/quantificational strength. A claim such as that strong necessity modals truth-conditionally entail weak necessity modals, which truth-conditionally entail possibility modals, constitutes a substantive empirical hypothesis on my terminology. We will see reasons for questioning each of the above claims. (Hereafter I typically use 'ought' as my representative of the 'ought'/'should'/etc. family and 'must' as my representative of the 'must'/'have to'/etc. family.)

2. 'Ought' and 'Must' in Context

Descriptive and theoretical research on modals highlights various conversational differences among necessity modals. This section focuses on one such difference concerning the effects of contextual assumptions on the relative felicity of weak vs. strong necessity modals.[6]

---

6 See Woisetschlaeger 1977, ch. 5 and McNamara 1990, ch. 3 for prescient early discussion of contextual differences between 'ought' and 'must'. For extensive discussions in descriptive linguistics, see the references in nn. 3, 39, 41. See Rubinstein 2012 and Silk 2012 for recent theoretical emphasis. Rubinstein doesn't consider epistemic examples; I examine her alternative take on the data in §6. For reasons discussed in §§5–6, it's important not to substitute other strong necessity modals (e.g. 'have to') for 'must' in examples unless indicated otherwise; speakers who find 'should' more natural than 'ought' may substitute 'should' for 'ought' throughout. Judgments concerning some of the examples may be vague for some speakers and may vary given subtle changes in context. This is part of what needs to be explained. The positive account developed in the paper will crystalize the informal reactions described in this section. I use '?' to indicate that using the item is dispreferred; '?' marks a weaker infelicity than '#'.

A central purpose of conversation is to share and coordinate our expectations, values, and plans. Sometimes we assert propositions outright. We commit to settling on their truth for the remainder of the conversation. But sometimes we don't wish to impose such a strong restriction on the future course of the conversation. We may want to propose that someone is obligated to do something but be unsure about whether there might be competing norms that could outweigh or cancel her obligation. Or we may want to proceed as if some proposition is true while remaining open to the possibility that our apparent evidence for it is misleading. I suggest that the role of weak necessity modals is to afford a means of making such proposals and expressing such states of mind.

Start with an epistemic case. Suppose we're working on an art project and I ask you where the colored pencils are. Normally you put them in the drawer with the crayons but sometimes you accidentally put them on the shelf. In this scenario it's more appropriate for you to use 'ought' in responding to my question:

(3) *Me*: Do you know where the colored pencils are?
     You: They ought to (/should/?must/?have to) be in the drawer with the crayons.

Suppose, alternatively, that we're looking for the colored pencils together, and you saw something that leads you to conclude that they are in the drawer. Perhaps you noticed that they weren't on the shelf, and this is the only other place you think they could be. In this scenario it's more natural for you to use 'must':

(4) *Me*: Do you know where the colored pencils are?
     You: They must (/have to/?ought to/?should) be in the drawer with the crayons.

Its following from our evidence that the colored pencils are in the drawer depends on today not being one of the days when you accidentally put them on the shelf. Using 'must' is preferred if, and only if, you know that conditions are normal in this way. What is illuminating is that you can use 'ought' even if you aren't in a position to judge that they are. Accepting your 'ought'-claim doesn't require us to assume that your evidence is undefeated.

Consider a deontic case (n. 6). Suppose I am considering whether to fight in the Resistance or take care of my ailing mother. I mention that the value of family, which supports my helping my mother, is important, and you agree. But the issue is admittedly complex, and we haven't settled whether there might be more important competing values. Sensitive to this, you may find it more appropriate to express your advice that I help my mother by using 'ought' than by using 'must':

(5) *Me*: Family is very important.
     You: I agree. You ought to/should (/?must/?have to) tend to your mother.

But if we settle that family is of primary importance, it can become more natural to use 'must' and for us to accept that I have to help my mother:

(6) *Me*: Family is most important—more important than country.
     You: I agree. You must/have to (/?ought to/?should) tend to your mother.

My having an obligation to help my mother depends on the value of family being more important[7] in my situation than any competing value. Parallel to the epistemic case, what is illuminating is that you can felicitously use 'ought' to express your advice that I help my mother without assuming that this precondition for my having a genuine obligation is satisfied. Accepting your 'ought'-claim needn't require us to presuppose that the value of family is more important than other potentially competing values.

Cases such as (3)–(6) highlight what I regard as the fundamental difference between the class of weak necessity modals and the class of strong necessity modals. It's common to gloss epistemic notions of necessity as concerning what follows from a body of evidence (knowledge, information), and deontic notions of necessity as concerning what is obligatory.[8] Yet we can accept your epistemic 'ought'-claim in (3) without settling that conditions are relevantly normal and thus without settling that our evidence implies that the colored pencils are in the drawer; and we can accept your deontic 'ought'-claim in (5) without settling that family is the most important relevant value and thus without settling that I have a genuine obligation to help my mother. Accepting 'Ought ϕ' needn't commit one to accepting that ϕ is necessary (epistemically, deontically, etc.).

Whether 'ought' or 'must' is preferred depends on context in the sense of depending on whether certain preconditions for the prejacent to be necessary (in the above sense) are accepted. In (3)–(4), how you express your attitude toward the proposition that the colored pencils are in the drawer depends on your views about the (in)defeasibility of the relevant evidence; in (5)–(6), how you express your advice that I help my mother depends on the status in the context of the value of family vis-à-vis other potentially relevant values. This

---

7 Or at least not less important; I will bracket complications from incomparabilities and irresolvable dilemmas. For discussion of dilemmas and the 'ought'/'must' distinction, see Swanson 2011, Silk 2012, 2015b, and references therein.

8 See, e.g., Lyons 1977, Coates 1983, Palmer 1990, 2001, Sweetser 1990, Bybee et al. 1994, van der Auwera and Plungian 1998, Nuyts 2001, Huddleston and Pullum 2002. In saying that the uses of 'ought' and 'must' have the same type of reading, I am not assuming that the modals have the same interpretation or stand in entailment relations (§1). In calling uses "epistemic" I don't assume that they are factive/entailing, or even that they convey the same kind of doxastic attitude toward the prejacent (contrast Yalcin 2016). What is important about the uses in (3)–(4) is that they both directly address the question of where the colored pencils are and convey an assessment of the truth or likelihood of the prejacent given a body of information. In calling uses "deontic" I don't assume that they are performative or have the same directive force. What is important about the uses in (5)–(6) is that they both directly address the practical question of what I am to do.

effect of contextual assumptions on the relative felicity of 'ought' and 'must' has been generally underappreciated in theoretical accounts (see n. 6 for notable exceptions).[9]

Two clarificatory remarks: First, I said that 'ought' is preferred in contexts such as in (5) where it isn't settled that the precondition for me to have an obligation is satisfied; yet it is worth observing that 'must' may be appropriate in certain contexts. If you can be presumed a normative authority on the issue and use 'must', I may accommodate by accepting that the value of family takes precedence. This isn't an isolated phenomenon. Suppose that Alice, a young teen, is considering with her mother, Martha, whether to take the A-train or C-train to a concert. The A is quicker, but the C is safer. Martha regards Alice's safety as paramount. Although the primacy of safety isn't common ground, Martha can felicitously say:

(7)   You must take the C-train, not the A-train.

The (teleological) necessity of Alice's taking the C-train depends on the goal of traveling safely taking priority over the goal of traveling quickly. Given Martha's authority in the context, she expects Alice to accommodate her assumption that this condition is satisfied. Such contexts notwithstanding—contexts in which the speaker doesn't have or doesn't wish to exercise the relevant authority—'ought' will be preferred.

Second, saying that accepting 'Ought ϕ' doesn't conventionally commit one to accepting that ϕ is necessary doesn't amount to the trivial claim that accepting 'Ought ϕ' needn't commit one to accepting 'Must ϕ'. One might analyze the examples by positing concepts of distinctive kinds of necessity, and explain accepting 'Ought ϕ' as accepting that the weaker kind of necessity holds of ϕ. For instance, one might posit and formalize a concept of weak epistemic necessity such that accepting that it's a weak epistemic necessity that d the colored pencils are in the drawer doesn't require accepting that today is relevantly normal and the evidence implies d; and one might posit and formalize a concept of weak deontic necessity (weak obligation) such that accepting that I have a weak obligation to help my mother doesn't require accepting that the value of family isn't defeated. Yet such a move isn't forced upon us. An alternative is to stick with the single familiar notions of necessity—e.g., understanding epistemic necessity as following from a body of evidence, and deontic necessity as being obligatory and following from a body of norms (n. 8)—and try saying that 'Ought ϕ' can be accepted without accepting that ϕ is necessary, period. The following sections investigate the prospects for this latter approach. (More on comparisons with the former approach in due course.)

---

9   The semantics in Finlay 2009, 2010, Lassiter 2011, Swanson 2011 have no obvious mechanism for capturing this effect of context on uses of 'ought' vs. 'must'. von Fintel and Iatridou (2008, 139–40) mention in passing a possibly relevant role for context, but the issue isn't investigated. See Rubinstein 2012, §2.2 for extensive critical discussion of previous comparative approaches and domain restriction approaches to weak necessity modals. (More on von Fintel and Iatridou's and Rubinstein's accounts in §§4, 6.)

3. The Analysis: Preliminary

The core of the approach to the weak/strong necessity modal distinction developed in the following sections is as follows. There is nothing specially "strong" about the necessity expressed by strong necessity modals. Strong necessity modals are given their usual semantics and pragmatics. 'Must ϕ' is true iff ϕ is necessary (in the relevant sense, i.e. epistemically, deontically, etc.; §2),[10] and uses of 'Must ϕ' predicate the necessity of ϕ of the actual world—just as 'May ϕ' is true iff ϕ is possible, and uses of 'May ϕ' predicate the possibility of ϕ of the actual world. The apparent weakness of weak necessity modals, we can try saying, derives from their bracketing the assumption that the necessity of ϕ need be verified in the actual world. Accepting 'Ought ϕ' needn't commit one to accepting that the actual circumstances verify the necessity of ϕ. Weak necessity modals afford a means of coordinating on the implications of our values, norms, etc. without having to settle precisely how they weigh against one another in particular circumstances, and while remaining open to new evidence about how they apply. This section begins developing these ideas within a standard premise-semantic framework for modals. The next section examines how the account may be implemented more precisely in the formal semantics and pragmatics.

As is common, I follow Kratzer (1977, 1981, 1991) in treating modals as semantically associated with a parameter determining a set of premises (propositions). Since modals can occur in intensional contexts, premise sets are indexed to a world of evaluation. What context supplies that determines the reading of a modal is the family of world-indexed premise sets $(P_w)_{w \in W}$, or *premise frame*: a function P from worlds w to premise sets $P(w)$.[11]

It is nontrivial how the considerations which seem intuitively relevant in interpreting modals are to be represented in the formal objects in the compositional semantics. First, the expressive, practical, and discourse-managing roles of modals such as 'ought' have been central in expressivist theories such as Gibbard's. However, the account of the weak/strong necessity modal distinction in what follows will be neutral on matters of expressivism (contextualism, relativism, invariantism)—e.g., on what type of psychological state of mind is conventionally expressed by certain uses of 'ought'/'must' sentences; whether notions of content or truth play a fundamental explanatory role in explaining semantic properties of sentences

---

10   I will often omit this parenthetical, but it should be understood.

11   See also van Fraassen 1973, Lewis 1973, 1981, Veltman 1976. Kratzer calls premise frames 'conversational backgrounds'. Kratzer's (1981, 1991) semantics uses two premise sets: a "modal base" $F(w)$ that represents a set of relevant background facts in w, and a (possibly inconsistent) "ordering source" $G(w)$ that represents the content of some ideal in w. Making the limit assumption (Lewis 1973, 19–20), Kratzer's semantics treats 'Must ϕ' as true at w iff ϕ follows from every maximally consistent subset of $F(w) \cup G(w)$ that includes $F(w)$ (equivalently (Lewis 1981), iff every $\leq_{G(w)}$-minimal world in $\bigcap F(w)$ is a ϕ-world).) These complications won't be relevant here; I simplify by treating modals as evaluated with respect to a single finite, consistent premise set. Nothing will turn on views about the limit assumption, or debates about so-called "weak vs. strong" semantics for epistemic 'must' (whether epistemic 'must' takes a nonempty ordering source).

or the dynamics of discourse; and whether particular premise frames figure in the compositional semantic value, and, if so, whether they are supplied by the context of utterance or a posited context of assessment. I will speak simply of "context," and I relativize parameters such as premise frames simply to worlds. The implementations to follow may be adapted along alternative contextualist/relativist/expressivist/invariantist lines.[12] (I revisit motivations for expressivism in §7. More on the practical roles of 'ought'/'must' below and in §5.)

Premise frames afford a natural way of encoding the contents of bodies of norms, preferences, etc.[13] Call a conditional norm, preference, expectation, etc. a *consideration*. A contextually supplied premise frame P encodes the content of a body of considerations. The premises in a premise set $P(w)$ represent what follows from a body of considerations given the relevant circumstances in w. For instance, suppose you want to go for a run given that it's sunny, you didn't just eat a burrito, and so on. The content of your preference can be encoded in a premise frame that assigns a premise set including the proposition that you go for a run to worlds where it's sunny, etc. Similarly the normative import of a value of charity might be encoded in a deontic premise frame that assigns a premise set including the proposition d that you donate to charity to worlds where you have means of supporting your family, etc.—as with P_d in (8), for relevant worlds characterized with respect to whether or not you have a J̲ob, there are R̲eputable charitable organizations, and there is a local S̲oup kitchen, and where u is the proposition that you undermine local aid organizations, and h is the proposition that you help at a local soup kitchen.

(8) $P_d(\text{JRS}) = \{d, h, \ldots\}$
     $P_d(\text{J}\overline{\text{R}}\text{S}) = \{u, h, \ldots\}$
     $P_d(\text{JR}\overline{\text{S}}) = \{d, \ldots\}$
     $P_d(\overline{\text{J}}\text{RS}) = \{h, \ldots\}$

The normative importance of helping those in need is reflected in P_d's assigning a premise set that includes h to worlds where there is a local soup kitchen. The normative importance of charitable giving is reflected in P_d's assigning a premise set that includes d to certain worlds where available aid organizations are reputable. However, the latter norm isn't unconditional; it applies only in worlds where you can support your family, and where the organizations will put the donations to good use. The relative importance of supporting your family over helping others is reflected in P_d's assigning a premise set that doesn't include d to worlds such as $\overline{\text{J}}$RS where the aid organizations are trustworthy but you don't have a job.

---

12 Cf., e.g., Gibbard 1990, 2003, Stephenson 2007, Dreier 2009, Yalcin 2012, MacFarlane 2014, Silk 2015a, 2016a, Swanson 2016a.

13 See Silk 2015a, 2016a, §5.6, 2017 for additional applications of the following approach to interpreting the formal premise-/ordering-semantic apparatus.

   With this way of understanding premise frames at hand, let's return to the semantics and diagnoses of the §2-examples. I give strong necessity modals their usual semantics of necessity. 'Must ϕ' is true at w, given a contextually supplied premise frame P, iff the prejacent proposition ϕ follows from $P(w)$, as in (9) (nn. 5, 11). The truth of 'Must ϕ' depends on the value of P at the evaluation world. Asserting 'Must ϕ' commits one to accepting that ϕ follows from what the relevant considerations enjoin given the facts, $P(w)$.[14]

   (9) $[\![\text{Must } \phi]\!]^{c,w} = 1$ iff $\bigcap P_c(w) \subseteq [\![\phi]\!]^c$ (*preliminary*)

   (10) A sentence S is *accepted* in c iff for every $w \in c$, $[\![S]\!]^{c,w} = 1$

What distinguishes weak necessity modals, I have said, is that they bracket whether the necessity claim is verified in the actual world. We can adopt the following constraint on a semantics and pragmatics for 'ought', for some relevant body of considerations P:

   (11) It's not the case that: 'Ought ϕ' is accepted in c only if for every $w \in c$,
         $\bigcap P(w) \subseteq [\![\phi]\!]^c$

In a manner to be made precise, uses of 'Ought ϕ' present the possibility that ϕ follows from the relevant considerations given certain circumstances, but without committing that such circumstances obtain or that the considerations actually apply. (We will examine what accepting 'Ought ϕ' does commit one to shortly.)

   Let's apply the preliminary analyses thus far to our §2-examples. Recall (3)–(4), reproduced in (12)–(13).

   (12) *Me*:   Where are the colored pencils?
         You: They ought to be in the drawer with the crayons.

   (13) *Me*:   Where are the colored pencils?
         You: They must be in the drawer with the crayons.

As with norms and preferences, one's expectations given a body of evidence can be conditional—e.g., conditional on things being normal in the relevant respects. Let w_N be a world in which your routine proceeds as normal, and let $w_{\bar{N}}$ be a world where something abnormal happens to disrupt your routine.[15] The conditional expectations concerning the colored pencils' location can be encoded in a premise frame P_e which (inter alia) assigns

---

to w_N a premise set $P_e(w_N)$ including the proposition d that the colored pencils are in the drawer, and which assigns to $w_{\bar{N}}$ a premise set $P_e(w_{\bar{N}})$ including $\neg d$. Given P_e, d is an epistemic necessity at w_N and not at $w_{\bar{N}}$. So in order for d to be accepted as epistemically necessary, per (9)–(10), the context set must be restricted to worlds like w_N—worlds where you didn't get distracted before putting the colored pencils away, no one played a trick on us and moved them, etc. If I'm unsure whether you are in a position to assume that nothing unusual led you to place the colored pencils somewhere else, I may challenge your assumption and raise a possibility that is incompatible with the epistemic necessity of d, as in (14).

(14) *You*: The colored pencils must be in the drawer with the crayons.
     Me: Really? I see that they aren't on the shelf. But don't you sometimes accidentally put them in the cabinet with the glue sticks?
     You: No, I never put them there. (/Oh, I forgot about that.)

But if you use 'ought', you aren't committing to conditions being normal, as reflected in (15); hence my mentioning such alternative possibilities may be beside the point, as in (16).

(15) *You*: The colored pencils ought to be in the drawer with the crayons.
     Me: I checked and they aren't there.
     You: Oh, then I'm not sure where they are. I would have expected them to be there.

(16) *You*: The colored pencils ought to be in the drawer with the crayons.
     Me: #Really? I see that they aren't on the shelf. But don't you sometimes accidentally put them in the cabinet with the glue sticks?
     You: I know; that's why I said *ought*!

Epistemic 'Ought ϕ' can be accepted even if it isn't settled that certain conditions relevant to the epistemic necessity of ϕ are satisfied.

Turn to our deontic modal examples in (5)–(6). My having an obligation to take care of my mother depends on the value of family being more important in my situation than other potentially competing values. Hence in order for the proposition m that I tend to my mother to follow from $P(w)$—what the normative considerations enjoin given the circumstances—it must be the case that the value of family takes precedence in my situation in w. In (5), unlike (6), after my assertion is accepted it still isn't settled whether this condition is satisfied. So, were you to use 'must' you would imply that you are foreclosing certain possibilities that I have left open. Unless you are in a position to do so (cf. (7)), your using 'must' is dispreferred. By accepting your claim with 'ought', we can provisionally proceed as if my helping my mother is required without needing to settle that the value of family is more important than other competing values we accept or may come to accept.

These examples highlight a critical role for weak necessity modals in discourse and deliberation. Take the deontic case. There are typically a range of interests, values, norms potentially relevant for determining what to do. How the relevant factors, whatever they are, interact is often highly complex. (One needs only a foray into deontic logic or normative ethics to convince oneself of this.) There may be uncertainty about the facts that would determine which considerations apply. For instance, in (8) one might not know the details about someone's financial or family situation. Other empirical factors—how donations are used, what the short- and long-term impacts are on those in need, etc.—can be even more difficult to assess. Hence one might not be in a position to commit to being in a world like JRS/JRS̄ where norms of charity aren't outweighed or defeated. Scenarios such as those in (5)–(6) compound such challenges for normative and empirical evaluation. Using deontic 'Must ϕ' may thus be inapt. One might not be in a position to commit to being in a world where ϕ follows from what the relevant norms enjoin given the facts. Deontic 'ought' affords a means of guiding our deliberations and plans—indeed our *conditional* plans, as emphasized throughout Gibbard's work on the psychology of normative judgment—while remaining open to new evidence about what values are at stake and how they interact with one another and the relevant facts.

I have proposed that what makes weak necessity modals "weak" is that they bracket whether the necessity of the prejacent is verified in the actual world. One can accept 'Ought ϕ' without presupposing that ϕ follows from what the relevant considerations P enjoin given the facts—e.g., without committing that all preconditions for ϕ to be a genuine obligation are satisfied, that one's evidence for ϕ isn't misleading, and so on. This feature of weak necessity modals isn't the only dimension along which necessity modals differ (more on which in §§5–6). However, I claim that it does distinguish the *class* of weak necessity modals from the *class* of strong necessity modals. Previous accounts of weak necessity modals have often been developed by considering a limited range of modal flavors in a limited range of contexts; extensions to other readings, to the extent that they are discussed at all, are often strained (e.g., Copley 2006, Swanson 2011, Rubinstein 2012, Charlow 2013, Ridge 2014, Portner and Rubinstein 2016, Yalcin 2016). The account in this paper generalizes across flavors of modality, and it captures a precise sense in which the relative felicity of 'ought' and 'must' depends on standing assumptions. Weak necessity modals afford a means of coordinating on the implications of our values, expectations, etc. without needing to settle precisely how they apply and weigh against one another in particular circumstances.

The preliminary account thus far raises many questions. Yet even at the present level of abstraction, we can see that the approach to the weak/strong necessity modal distinction in this paper differs crucially from other main approaches in the literature, such as probabilistic and comparative possibility approaches (Finlay 2009, 2010, 2014, Lassiter 2011) and domain restriction approaches (Copley 2006, von Fintel and Iatridou 2008, Swanson 2011, Rubinstein 2012, Charlow 2013). For instance, domain restriction accounts maintain that accepting 'Ought ϕ' requires accepting that ϕ is a necessity, and that the truth of 'Ought ϕ' at w requires that ϕ is a necessity at w; what distinguishes 'ought' from 'must' is the logical strength of the

necessity (roughly put, implication of ϕ by a superset of premises; equivalently, truth of ϕ throughout a subdomain of worlds). Weak necessity modals are treated as expressing a logically weaker *kind* of necessity. The present approach rejects these claims (§2).

4. Weak Necessity and the Modal Past

Our project is to develop an account to the weak/strong necessity modal distinction that systematizes a broader range of linguistic phenomena (§1). Taking on this more demanding goal requires sustained investigation into diverse domains. This section examines how notions of "ought" are expressed crosslinguistically in order to motivate several ways of formally implementing the proposed approach to the weak/strong necessity modal distinction and the semantics of 'ought'. §5 shows how the account helps explain various seemingly unrelated semantic and pragmatic properties of 'ought' and 'must'.

4.1. Data
Past forms of modals—in English, 'would' for 'will', 'could' for 'can', 'might' for 'may'—are often used not to indicate past time reference, but to express tentativeness or politeness and weaken the apparent force, as in (17)–(19). These forms are also the forms that appear in the consequents of subjunctive conditionals, as in (20). Palmer (2001) dubs such uses of past tense the "modal past."[16]

(17) a.   I will add one point to this discussion.
      b.   I would add one point to this discussion.

(18) a.   Alice will/may/can't be at home now.
      b.   Alice would/might/couldn't be at home now.

(19) a.   May/Can I comment on your proposal?
      b.   Might/Could I comment on your proposal?

(20) If you took the flight tomorrow, you would/could/might get there in time.

---

16 See also James 1982, Coates 1983, Fleischman 1989, Bybee 1995, Iatridou 2000, Huddleston and Pullum 2002. Terminology varies among authors. Weakness interpretations of past tense aren't limited to uses of modal verbs, as reflected in the present time readings of (i)–(iii).

(i)   I wanted to ask you a question.   (Bybee 1995, ex. 21)

(ii) I thought/was thinking about asking you to dinner.   (Fleischman 1989, 8)

(iii) *A*: How old is John?
       B: He'd be about sixty.   (cf. Huddleston and Pullum 2002, 200–201)

I don't assume that all modal-past forms necessarily convey weakness/tentativeness relative to their nonpast counterparts (cf. epistemic 'might' in contemporary English; Coates 1983, Palmer 1990, Huddleston and Pullum 2002, Collins 2007). My descriptive use of Palmer's label makes no theoretical assumptions about how the modal/weakness/tentativeness interpretations arise or are related to temporal interpretations of past morphology (n. 21).

Strikingly, 'ought' patterns with the past-marked modal forms. First, 'ought' weakens the apparent force of 'must'—hence the common label "weak necessity modal." Second, 'ought', unlike 'must', can appear in subjunctive conditionals:

  (21)   a.   If Alice came to the party tomorrow, Bert ought to leave.
          b.   #If Alice came to the party tomorrow, Bert must(ed) leave.

Third, 'ought' is nonentailing. For simple clauses 'ϕ', 'Ought ϕ' contrasts with 'Must ϕ' in being compatible with '$\neg\phi$':

  (22)   I could give to Oxfam, but I won't.

  (23)   a.   Alice ought to be here by now, but she isn't.
          b.   #Alice must be here by now, but she isn't.

Indeed, when used with the perfect, 'ought' implicates the negation of the prejacent.

  (24)   We could have given to Oxfam.   (*Implicates*: we didn't)

  (25)   a.   We ought to have given to Oxfam.   (*Implicates*: we didn't)
          b.   #We must have given to Oxfam (but we didn't).

'Must' cannot even receive a deontic reading when used with past time reference. 'Ought', unlike 'must', can be used to communicate that an obligation held in the past. That 'ought' can scope under the perfect in (25a) is a fourth respect in which 'ought' patterns with past-marked modal forms (Condoravdi 2002).

In sum, although 'must' doesn't have a past form, 'ought', we can try saying, functions notionally as its modal past (cf. Palmer 1990, 2001). This is surprising. But it becomes less surprising when we examine other languages. Let's use 'OUGHT' for the notion which in English is expressed with weak necessity modals such as 'ought' ('should', etc.), and let's use 'MUST' for the notion which in English is expressed with strong necessity modals such as 'must' ('have to', etc.). As emphasized in von Fintel and Iatridou's (2008) seminal discussion of weak necessity modals, it's crosslinguistically common to mark the semantic distinction between OUGHT and MUST morphologically rather than lexically (see also Palmer 2001, McGregor and Wagner 2006, Van Linden and Verstraete 2008, Matthewson 2010). A notion of OUGHT is often expressed not by using a different word—like 'ought' in English—but by using the modal-past form of a strong necessity modal, i.e. the form of a strong necessity modal that is used in counterfactuals.[17] von Fintel and Iatridou

---

17 As von Fintel and Iatridou note (2008, 126n.22), 'ought' fits the crosslinguistic pattern historically; it was formerly the past subjunctive of the verb 'owe'. Modal-past forms in other languages may be derived from various elements, not simply past tense (e.g., Iatridou 2000).

(2008) approach the crosslinguistic data in a domain restriction account of weak necessity modals. They treat weak necessity modals as quantifying over "the best of the best" worlds—the relevant $P(w)$-compatible worlds that are compatible with an additional premise set representing a secondary ideal (in Kratzer's terminology, a secondary ordering source).[18] In passing, von Fintel and Iatridou speculate that "the counterfactual marking is co-opted here in a somewhat meta-linguistic kind of way: 'if we were in a context in which the secondary ordering source was promoted [to primary status], then it would be a strong necessity that . . .'" (2008, 139). The tentativeness associated with counterfactual marking is attributed to the fact that the premises in the secondary premise set needn't apply: "The choice of whether to really promote the secondary ordering source is left open" (2008, 139).

I find these suggestions about the crosslinguistic data unsatisfying (see Rubinstein 2012 for further critiques). First, von Fintel and Iatridou briefly suggest treating the secondary ordering source for epistemic readings as representing what is normally the case, and the secondary ordering source for deontic readings as representing "less coercive sets of rules and principles" (2008, 119); yet no general story is given about what primary vs. secondary ordering sources represent per se, or how an ordering source is determined as primary or secondary across contexts. Absent an independent understanding of what makes it the case about a speaker that she is counterfactually promoting a secondary ordering source, the proposed story about the role of the counterfactual marking seems ad hoc. A worry is that the explanation of the tentativeness associated with counterfactual morphology redescribes what needs to be explained. The tentativeness is "explained" by introducing a parameter representing "shakier assumptions" (2008, 119n.9) that may not apply—uncharitably put, a parameter representing considerations one may only be tentatively committed to. Finally, it's unclear how von Fintel and Iatridou's explanation would generalize to tentativeness effects of counterfactual morphology on other lexical items—e.g., possibility modals like 'might', or desire verbs like 'wish' in other languages (as von Fintel and Iatridou note, many languages express a notion of WISH via counterfactual morphology on the word for 'want'; see also n. 16).[19] For instance, interpreting modal-past forms of possibility modals with respect

---

18   More precisely (see n. 11): For a set of worlds W and premise set S, let the S-minimal worlds in W be the worlds $u \in W$ such that no world $v \in W$ satisfies a proper superset of propositions $p \in S$. (For finite consistent premise sets the set of S-minimal worlds is $W \cap \bigcap S$.) Let s be a set of relevant worlds (e.g., as determined by a Kratzerian modal base), $G_1(w)$ be a premise set representing some primary ideal, and $G_2(w)$ be a premise set representing some secondary ideal. On von Fintel and Iatridou's semantics, whereas 'must' universally quantifies over the $G_1(w)$-minimal s-worlds, 'ought' universally quantifies over a subset of these worlds: the $G_2(w)$-minimal worlds among the $G_1(w)$-minimal s-worlds. 'Ought' quantifies over the $G_2(w)$-minimal worlds that 'must' quantifies over. See Sloman 1970, Williams 1981, McNamara 1990 for informal precedents. I return to general worries for domain restriction analyses below.

19   Interestingly, in various Italian dialects OUGHT can be expressed via counterfactual morphology on volitional verbs such as 'want', as in (i) below. Expressions of MUST with the indicative form are also possible, as in (ii). (Thanks to Federico Faroldi for bringing this to my attention.)

to a secondary ordering source incorrectly predicts a strengthening effect. The tentative use of 'might'/'could' in (19) cannot be derived by counterfactually "promoting" a secondary ordering source and then evaluating the possibility of the prejacent.[20]

4.2. Implementations. A Modal–Past Approach

This subsection examines how independent work on the semantics/pragmatics of counterfactual marking may be incorporated into our analyses of 'ought' and 'must'. I want to be clear that I am not assuming that lexicalized weak necessity modals are decomposed into a strong necessity modal and counterfactual features (schematically: *STRONG+CF*). The crosslinguistic data may provide insight into the semantics of lexicalized expressions of OUGHT; but we should be careful not to read off a semantics for 'ought' from a semantics for 'must' and counterfactual morphology. 'Ought' doesn't mean 'would have to'.

It's generally agreed that counterfactual marking signals that the worlds being talked about ("topic worlds") needn't be candidates for actuality. There are various ways of formalizing this signal and deriving it in the grammar. To fix ideas I assume that counterfactual marking cancels a presupposition that the set of topic worlds (e.g., a modal's domain of quantification) is a subset of the context set.[21]

I have said that no innovations are introduced into the semantics/pragmatics of strong necessity modals. 'Must ϕ' is given its familiar semantics of necessity: 'Must ϕ' is true at w iff ϕ follows from $P(w)$; and uses of 'Must ϕ' carry the usual indicative presupposition that the worlds being talked about are in the context set. One way of implementing the general indicative presupposition is as restricting the domain of the interpretation function to proper points of evaluation—contexts c and worlds w such that $w \in c$. Applying this to the case of a necessity modal yields:[22]

Definition 1. $[\![\text{Must}\,\phi]\!]^c = \lambda w : w \in c \,.\, \bigcap P(w) \subseteq [\![\phi]\!]^c$

(i) Su porceddu   e' tottu abruxau   it a d'essi boffiu   arrustiu   a fogu pracidu.
     pork.meat   get.burn.PPART   want.PAST.COND.3SG   roast.PPAST   with low fire
     'Pork meat got burned; it should have been roasted very slowly.'

(ii) Cussa   femina   bollit   ascurtada.
     DET.F.SG   woman.F.SG   want.PRES.3SG   listen.to.PPART.F.SG
     'That woman must be listened to.'   (Fanari 2007, 128; Campidanese, Sardinian)

20 For a set of relevant worlds s and primary and secondary ordering sources $G_1(w)$, $G_2(w)$, let i be the set of $G_1(w)$-minimal s-worlds, and j be the set of $G_2(w)$-minimal i-worlds. Since $j \subseteq i$, that there is a ϕ-world in j asymmetrically implies that there is a ϕ-world in i (n. 18).

21 An alternative is to treat counterfactual marking as positively presupposing that the set of topic worlds isn't included in the context set. For discussion and technical implementations, see Stalnaker 1975, von Fintel 1998, Iatridou 2000, Ippolito 2003, Schlenker 2005, Arregui 2009, Bittner 2011.

22 In a framework with object-language world-variables, the presupposition would be that the assignment function maps the given world-variable to a world in the context set (cf. Schlenker 2005).

Uttering 'Must ϕ' predicates the proposition that follows from $P(w)$ of every world w in the context set. The treatment of 'must' vis-à-vis necessity is parallel to the treatment of 'may', 'can', etc. vis-à-vis possibility:

Definition 2. $[\![\text{May}\,\phi]\!]^c = \lambda w : w \in c \;.\; \bigcap(P(w) \cup \{[\![\phi]\!]^c\}) \neq \varnothing$

Uttering 'Must ϕ' predicates the necessity of throughout the context set just as uttering 'May ϕ' predicates the possibility of ϕ throughout the context set or uttering 'ϕ' predicates ϕ throughout the context set. It's in this sense that there is nothing special or distinctively "strong" in the semantics/pragmatics of (so-called) strong necessity modals.

Turn to weak necessity modals. A natural idea is that the counterfactual marking in the relevant languages cancels an assumption that the relevant worlds at which the prejacent is necessary are in the context set[23]—hence the observation in §2 that accepting 'Ought ϕ' doesn't require accepting that all the preconditions for ϕ to be necessary are satisfied. The apparent weakness of uses of 'Ought ϕ', compared to uses of 'Must ϕ', derives from failing to presuppose that the topic worlds where ϕ is necessary are in the context set. Call this approach a *modal-past approach* to weak necessity modals and the weak/strong necessity modal distinction. The remainder of this section presents several ways of implementing a modal-past approach in the formal semantics and begins investigating their costs and benefits.

One straightforward way of treating 'ought' as the semantic modal past of 'must' would be to treat 'ought' as having an ordinary semantics of necessity, like 'must', but lacking the presupposition that the worlds at which ϕ is necessary are in the context set.

Definition 3. $[\![\text{Ought}\,\phi]\!]^c = \lambda w . \bigcap P(w) \subseteq [\![\phi]\!]^c$ (v1)

Informally put, uttering 'Ought' places the necessity claim on the "conversational table," but doesn't conventionally commit one to its truth (cf. Silk 2016b). Implementing a modal-past approach as in Definition 3 faces pressing challenges in the discourse dynamics and compositional semantics. On a standard Stalnakerian theory of conversation, assertions propose to restrict the context set to worlds where the asserted content is true (Stalnaker 1978). But for any $w \in c$, 'Ought ϕ' is true at w according to Definition 3 iff 'Must ϕ' is true at w according to Definition 1.[24] An alternative mechanism would be needed to distinguish how 'ought' and 'must' update context, e.g. allowing uses of 'ought' to distinguish among worlds outside

---

23 On this I disagree with Arregui 2010. Though Arregui associates 'should' with a past morphology feature, she denies that the feature is interpreted with 'should'. Arregui maintains that 'Should ϕ' presupposes that the modal's quantificational domain is included in the context set (for nonstative 'ϕ'). We have seen that this is incorrect. Nonentailing uses '(Should ϕ) $\wedge \neg\phi$' are consistent (more on which in §5).

24 The right-to-left direction is obvious. Left-to-right: if $[\![\text{Ought}\,\phi]\!]^c(w) = 1$ then $\bigcap P(w) \subseteq [\![\phi]\!]^c$ but then $[\![\text{Must}\,\phi]\!]^c(w) = 1$ since, by hypothesis, $w \in c$.

the context set or have some non-eliminative effect. Second, it isn't obvious how the relative weakness of 'ought' vis-à-vis 'must' would carry over in embedded environments that shift the evaluation world, such as indicative conditionals or attitude ascriptions.

One way of avoiding these challenges is to build a counterfactual element into the semantics of 'ought'. Consider Definition 4, where h is a contextually supplied selection function.

Definition 4. $[\![\text{Ought } \phi]\!]^{c,w} = 1 \text{ iff } \forall w' \in h_c(w) : \bigcap P_c(w') \subseteq [\![\phi]\!]^c$ (v2)

To a first approximation, one can think of h as picking out a set of relevant worlds that are minimal/"preferred" in some contextually relevant sense—most normal, expected, desirable, etc., depending on the context (cf. Grosz 2012; Starr 2010, 167).[25] In (12) $h(w)$ might be the maximally w-normal worlds where you don't get distracted before putting the colored pencils away, no one hides them, etc.—worlds like w_N. In (26) $h(w)$ might be the maximally w-similar worlds where you have a job, the available charities are trustworthy, etc.—worlds like JRS from (8).

(26)   You ought to donate to charity.

'Ought ϕ' is true iff *these* worlds verify the necessity of ϕ, i.e. iff ϕ follows from the relevant considerations P at every $w' \in h(w)$. The point from (11) that accepting 'Ought ϕ' doesn't require settling that ϕ is actually necessary is captured via h. The set of worlds $h(w)$ at which the necessity of ϕ is evaluated might include the evaluation world w, but it might not.

Definition 4 uses a simple selection function to determine the worlds at which the necessity of the prejacent is evaluated. The semantics could be complicated by deriving the set of selected worlds from more basic elements, such as independently represented orderings or premise sets of the relevant types (normality, desirability, etc.). The selection of worlds could also be treated as explicitly depending on the prejacent ϕ or a contextually relevant set of circumstances C, i.e. $h(w, \phi, C)$. This would reflect the idea that in interpreting 'Ought ϕ' one evaluates what follows from P at worlds satisfying certain conditions plausibly relevant to whether ϕ is a necessity.[26] In (12) one looks at worlds u satisfying what is normally the case in matters concerning where the colored pencils are, and one checks whether, conditional on such facts, the relevant information implies that the colored pencils are in the drawer, i.e. whether $\bigcap P_e(u) \subseteq drawer$; in (26) one looks at worlds v satisfying what is normally or preferably the case in matters concerning whether to donate, and one checks whether, conditional

---

25 I often use double quotes around 'preferred' as a reminder that what is intended is the generalized notion of minimality, rather than a specifically bouletic or deontic notion.

26 In Silk 2012 I implemented these ideas by analyzing weak necessity modals as expressing a kind of conditional necessity (cf. Wertheimer 1972). I no longer endorse this way of capturing the points in §§2–3. Conditional necessity analyses make pressing the challenge of distinguishing uses of 'ought' from (implicit or explicit) conditional necessity claims (see below and §7.1).

on such facts, the relevant norms enjoin you to donate, i.e. whether $\bigcap P_d(v) \subseteq donate$. Alternatively, the semantics might use a simple world-indexed function h, and such additional factors might be invoked in an extra-semantic account of how h is determined in concrete discourse contexts. For present purposes I assume the latter option.

The approach in Definition 4 raises the question of what distinguishes 'Ought ϕ' from counterfactual necessity sentences 'If χ, it would have to be that ϕ' (more on which shortly). A comparative semantics such as Definition 5 avoids this issue—where $\prec_w$ is a partial order on propositions along a contextually relevant dimension (likelihood, normality, desirability, etc.), and $s(w, p)$ is the set of closest p-worlds to w, understanding the relevant closeness relation as the relation that would figure in interpreting a counterfactual.

Definition 5.

$\quad [\![\text{Ought } \phi]\!]^{c,w} = 1$ iff $\{u : u \in s(w, \bigcap P_c(u) \subseteq [\![\phi]\!]^c)\} \; \prec_w \{v : v \in s(w, \bigcap P_c(v) \not\subseteq [\![\phi]\!]^c)\}$ (v3)

This treats 'Ought ϕ' as saying that it would be better (in a relevant sense) if ϕ was necessary (in a relevant sense).[27] Like Definition 4, Definition 5 avoids treating the truth of 'Ought ϕ' at w as requiring that ϕ be a necessity at w. The closest worlds u at which ϕ is necessary needn't be in the context set. Yet we can still see how uses of 'ought' may bear on interlocutors' views about what is necessary. 'Ought ϕ' introduces the possibility that ϕ is necessary and comments on it. The attitudinal comment is that the (closest) worlds in which ϕ is necessary are $\prec_w$-better—more desirable, normal, expected, etc., depending on the context. A critical issue is why the semantics for 'ought' and 'must' should be so dissimilar. One must provide a precise sense in which 'ought' is weaker than 'must' (logically, conversationally). Further, it isn't obvious how a comparative semantics like Definition 5 might shed light on how OUGHT-interpretations of *STRONG+CF* arise in other languages.

Definitions 3–5 provide several avenues for developing a modal-past implementation of the core ideas from §2. The semantics avoid analyzing accepting 'Ought ϕ' in terms of ϕ being a necessity (in any sense) at every w in the context set, and they offer precise representations of 'ought' as the notional modal-past of 'must'. 'Ought' is interpreted with respect to the same body of considerations P_c as 'must', and "introduces" the proposition $[\lambda w \,.\, \bigcap P_c(w) \subseteq [\![\phi]\!]^c]$ that the prejacent is a necessity relative to P_c; what distinguishes 'ought' is that the attitude taken toward that proposition needn't be acceptance—in Definition 3, by allowing improper points of evaluation; in Definition 4, by including an additional

---

27 Definition 5 could be refined depending on one's views on the semantics of comparative possibility (Lassiter 2011, Kratzer 2012), but the basic idea should be clear enough. Contrast the comparative probability semantics for 'ought' from Finlay (2009, 2010, 2014), which treats 'Ought ϕ' as saying (roughly) that ϕ is more likely than any relevant alternative to ϕ. In contrast, Definition 5 treats 'Ought ϕ' as making a comparative claim about the necessity of ϕ, rather than a comparative claim about ϕ, and it generalizes the relevant comparative notion via the context-dependent parameter $\prec$. (See Rubinstein 2014, Portner and Rubinstein 2016) for additional discussion of connections between weak necessity modals and comparatives/gradability.)

layer of modality and evaluating the truth of $[\lambda w . \bigcap P_c(w) \subseteq [\![\phi]\!]^c]$ throughout a set of possibly counterfactual worlds; and in Definition 5 by commenting on the preferredness (in a relevant sense) of $[\lambda w . \bigcap P_c(w) \subseteq [\![\phi]\!]^c]$. It will be useful in what follows to have a particular analysis at hand. To fix ideas I will generally assume the semantics in Definition 4. It's less clear how a comparative meaning such as Definition 5 might come to be associated with certain uses of *STRONG+CF* in other languages, and I think that the ideas motivating Definition 3 are more perspicuously developed in a dynamic setting (Silk 2016b). I leave more thorough comparisons for future work.

4.3 Entailments?

An initially plausible thought is that strong necessity modals entail weak necessity modals, which entail possibility modals. However, a worry for the modal-past analyses is that they seem to predict the consistency of sentences 'Ought ϕ and must $\neg\phi$' and sentences 'Ought ϕ and $\neg$may ϕ', as in (27).

(27)   #I mustn't/can't/may not lie to Alice, but I ought to.

The 'must'/'may' conjunct is true at w iff the necessity of $\neg\phi$ is verified at w; the 'ought' conjunct is true at w, according to Definition 4, iff the necessity of ϕ is verified at every "preferred" (desirable, expected, etc.) world $w' \in h(w)$. Accepting (27) requires restricting the context set to worlds w such that $P(w)$ implies $\neg\phi$ and, for every $w' \in h(w)$, $P(w')$ implies ϕ. Absent further constraints on h, such conditions are consistent.

I think this prediction is actually a feature, not a bug. 'Must', 'ought', and 'may' cannot be ordered by logical strength simply in virtue of their conventional meanings. 'Must ϕ' $\vDash$ 'May ϕ', no doubt about that; but 'Must ϕ' $\nvDash$ 'Ought ϕ', and 'Ought ϕ' $\nvDash$ 'May ϕ'.

Epistemic 'Must ϕ' commits the speaker to high credence in ϕ and epistemic 'May ϕ' commits the speaker to some credence in ϕ. Epistemic 'must' entails epistemic 'may'. But epistemic 'Ought ϕ' doesn't commit the speaker to any credence in ϕ. This invalidates the entailments between epistemic 'ought' and epistemic 'must'/'may' (cf. Copley 2006, Silk 2016b, Swanson 2016a, Yalcin 2016; see n. 8).[28]

(28)   #Alice can't be home yet; she hasn't called, and she always calls right away to let us know she got back safely. She must be home already. I hope there wasn't an accident.

$$(\#\neg Can\ \phi \wedge Must\ \phi)$$

---

[28] I use 'can't' for the external negation ($\neg < can$), since with epistemic 'may not' the negation is internal. With 'mustn't' the negation is internal ($must < \neg$). With deontic 'may not' the negation is external ($\neg < can/may$).

(29)   Alice can't/must not be home yet; she hasn't called, and she always calls right away to let us know she got back safely. She ought to be home already. I hope there wasn't an accident.

$$(Must \ \neg\phi \land Ought \ \phi)$$
$$(\neg Can \ \phi \land Ought \ \phi)$$

Although epistemic 'Ought ϕ' doesn't commit the speaker to any unconditional credence in ϕ, it still conveys a doxastic attitude. Informally, it conveys a conditional attitude about the necessity of ϕ given a body of evidence, on the assumption that conditions are relevantly normal (§§2–4; cf. Groefsema 1995, 72–73, Wedgwood 2007, 118–19, Yalcin 2016). One may infer from the denial of 'ϕ' that conditions aren't normal. Yet since 'ought' can be used to talk about the modal status of ϕ at nonactual possibilities, epistemic 'Ought ϕ' can still be true and accepted.

It's hard to come up with coherent deontic examples analogous to (29). Hard, but perhaps not impossible:

(30)   I must tell my wife about the affair. I know I shouldn't; it'll only hurt her. But I must.

(31)   I know I shouldn't tell my wife about the affair; it'll only hurt her. But I can't lie to her.

(32)   #I must tell my wife about the affair. I know I can't; it'll only hurt her. But I must.

It isn't immediately obvious what to say about these examples. It's interesting that entailments from 'must' to 'ought' and from 'ought' to 'may' seem more compelling with nonepistemic readings. Yet insofar as we want unified semantics for modal verbs that generalize across readings, we should prefer an account that avoids treating the entailments as semantically valid.

5. "Weakness" in Weak Necessity Modals

§4 developed the §3-account by treating 'ought' as the notional modal past of 'must' and incorporating general insights about the semantics/pragmatics of counterfactual marking. This section shows how a modal-past account of the "weakness" of weak necessity modals systematizes several seemingly unrelated puzzles of entailingness and performativity with 'ought' and 'must'.

5.1. Diagnosing Weakness

A modal-past analysis gives precise expression to the informal intuition that 'ought' is weaker and more tentative than 'must'. In uttering 'Ought ϕ' the speaker fails to mark the necessity claim as being about worlds that are candidates for actuality. Yet, as Stalnaker notes,

"normally a speaker is concerned only with possible worlds within the context set, since this set is defined as the set of possible worlds among which the speaker wishes to distinguish" (1975, 69). So, uttering 'Ought ϕ' implicates that one isn't in a position to commit to ϕ being a necessity throughout the set of live possibilities. Grice's first quantity maxim—"Make your contribution as informative as is required" (Grice 1989, 26)—can then be exploited to generate a familiar upper-bounding implicature (Horn 1972, Gazdar 1979; n. 29). Using 'ought' implicates that for all one knows—better, for all one is willing to presuppose in the conversation—'Must ϕ' is false. This implicature has the usual properties of implicatures; it is reinforceable, cancellable, and suspendable:

(1a) I ought to help the poor, but I don't have to.
(2a) I ought to help the poor. In fact, I must.
(33) I ought to help the poor. Maybe I have to.

In (2a) the speaker first conveys that the worlds in which my helping the poor is deontically necessary needn't be live possibilities, and then commits that what holds in the former worlds also holds in the actual world. The implicature data with 'ought' can be treated analogously to implicature data with subjunctive conditionals.

(34) a. If you had the flu, you would have exactly the symptoms you have now.
                                                                (cf. Anderson 1951, 53)
      b. If you had the flu, you would have very different symptoms from the symptoms you have now.
      c. If you had the flu, you would be sick. Maybe you do have the flu; you *are* pretty congested.

Likewise we can assimilate the tentativeness of 'ought' to the tentativeness of modal-past forms generally, as in non-counterfactual subjunctive conditionals ("future-less-vivid" conditionals) such as (20) above and (35).

(35) If you came to our party tomorrow—and I'm not saying that you will—you would have a great time.

Using the past form highlights the possibility that the marked clause might not ultimately be accepted. The basis of the scale between 'ought' and 'must' isn't fundamentally logical but epistemic strength (§4). 'Ought' and 'must' are ordered, not in terms of subset/superset relations in their domains of quantification, as per domain restriction accounts (§§2–3), but in terms of epistemic attitude toward the proposition that ϕ is necessary.[29]

---

29 Cf. Verstraete 2005b, 2006, Van Linden and Verstraete 2008.

5.2. *Entailingness and Directive Force*

Though many authors have claimed that epistemic 'Ought ϕ' expresses that ϕ is probable,[30] we have seen that this isn't quite right (§4). Epistemic 'Ought ϕ' doesn't commit the speaker to any unconditional credence in ϕ, as reflected in (23), reproduced in (36).

(36) a. Alice ought to be here by now, but she isn't.
      b. #Alice must be (/may be, /is probably) here by now, but she isn't.

Epistemic '(Must ϕ) ∧ ¬ϕ' is anomalous in a way that epistemic '(Ought ϕ) ∧ ¬ϕ' is not. Surprisingly, there is robust evidence that this is the case with deontic readings as well.[31] When one wishes to convey that one thinks a given obligation won't be satisfied, one typically uses 'ought'/'should' rather than 'must'.

(37) a. He should/ought to come tomorrow, but he won't.
      b. *He must come tomorrow, but he won't.

                                        (Palmer 1990, 123; judgment Palmer's)

(38) a. You ought to help your mother, but you won't (/I know you won't).
      b. ??You must help your mother, but you won't (/I know you won't).

As Eric Campbell insightfully puts it, "The idea that one *must* (not) do something . . . does not generally indicate that one option is simply better than another option, but that the other is out of bounds," "off the table," "closed off"; alternative possibilities become "*unthinkable*" (2014, 463–65). Of course obligations can go unfulfilled. What is interesting is that speakers appear to assume otherwise, at least for purposes of conversation, when expressing obligations with 'must'.[32]

Call data concerning sentences of the form '(*MODAL* ϕ) ∧ ¬ϕ' *entailingness data* (though see n. 33). Our discussion in §4 suggests a natural way of capturing entailingness data such as (36)–(38). Evaluating 'Ought ϕ' can take us to worlds outside the context set when assessing the necessity of ϕ. There is no requirement that the value of the premise frame P at worlds outside the context set be compatible with the common ground. So, 'Ought ϕ' can be true at a world w in the context set even if every world in the context set is a ¬ϕ-world; hence the

---

30 E.g., Sloman 1970, Horn 1972, Wertheimer 1972, Thomson 2008, Finlay 2010, 2014, Lassiter 2011, Wheeler 2013, Ridge 2014.

31 For theoretical discussion, see esp. Werner 2003, 124–37; Ninan 2005; Portner 2009, 103, 189–96; Silk 2016b. See also, e.g., Leech 1971, Wertheimer 1972, Harman 1973, Lyons 1977, Woisetschlaeger 1977, Williams 1981, Coates 1983, Palmer 1990, Sweetser 1990, Myhill 1996, Huddleston and Pullum 2002, Swanson 2008, 2016a, Close and Aarts 2010, Campbell 2014, Goddard 2014.

32 Cf.: "The basic strong obligation component common to ['got to', 'have to', and 'must'] is 'I can't think: X will not V'; note that this does not mean that the obligated event will inevitably take place but rather that the speaker is operating on this assumption" (Myhill 1996, 348–49).

coherence of '(Ought ϕ) $\wedge$ ¬ϕ'. Since 'Must ϕ' doesn't have a broadly counterfactual element to its meaning, the context set must include $\bigcap P(w)$, for all worlds w in the context set. So, if 'Must ϕ' is accepted, ¬ϕ cannot be satisfied throughout the context set; hence the incoherence of '(Must ϕ) $\wedge$ ¬ϕ' on any reading (qualifications shortly).[33,34]

How to derive these points about the relation between the modals' premise sets and the context set depends on general issues of presupposition and verbal mood (cf. §4). For instance, with modals that lack a counterfactual meaning component, there might be a context-set presupposition on the embedded clause that it denotes a proposition defined only at worlds in the context set (e.g., due to an indicative presupposition transmitted to the embedded clause, or a presupposition in the modal's lexical semantics). So, for 'must ϕ' to be true at w, $\bigcap P(w)$ must be a subset of $\{u: [\lambda v: v \in c . [\![\phi]\!]^{c,v} = 1](u) = 1\}$, hence $\bigcap P(w)$ must be a subset of the context set. Alternatively, there might be a general presupposition on the values of quantifier domain variables that they be compatible with the common ground at worlds in the context set. What is important here is that non-counterfactual modal constructions (modal sentences, indicative conditionals, etc.) presuppose that the domain of quantification is included in the context set. 'Ought ϕ' lacks this presupposition. Parallel to our points in §3, there is nothing distinctively "strong" posited about 'must' vis-à-vis entailingness. The incoherence of accepting '(Must ϕ) $\wedge$ ¬ϕ' follows from the modal's ordinary semantics of necessity and general context-set presuppositions associated with indicative.

---

33 This explanation leaves open whether '¬ϕ' may be true at some worlds in the context set, and thus seems to predict that accepting deontic 'Must ϕ' is compatible with accepting the epistemic possibility of ¬ϕ. This prediction appears to be borne out by corpus data. Verstraete (2007) cites the naturally occurring example in (i) with an imperative, adapted with 'must' in (ii).

  (i)   You've got to take a stand Tom. You've got to do it mate. [. . .] Don't stand for it, because if you
         do you'll just get trampled on. (CB ukspok)   (Verstraete 2007, 242)

  (ii)   You mustn't stand for it, because if you do you'll just get trampled on.

  (iii) ?You must go to confession, but maybe you won't (/you might not).

  Though (iii) strikes me as somewhat anomalous, this is arguably due to a general norm of cooperative conversation that interlocutors do what they can to make the actual world be among the preferred/best worlds (cf. Portner 2007, 358). On this diagnosis, (iii) would be anomalous to the extent that it's anomalous to commit someone to help see to it that ϕ while expressly admitting the possibility of ¬ϕ.

34 Alternatively, one might attempt to explain the entailingness data by positing additional features in the conventional meaning of 'must'. For instance, first, one might say that 'must' is only interpreted with respect to a modal base (or the union of several modal bases), not an ordering source (n. 11). (As far as the interpretation of 'must' goes, "all laws are natural laws" (cf. Piaget 1962, 340).) Since modal bases consist of propositions true at the evaluation world, 'Must ϕ' would entail 'ϕ'. Second, following Swanson 2016a one might include in the semantic entry for 'must' but not 'ought' a constraint requiring high credence in the prejacent. Third, one might appeal to more basic performative properties of 'ought' and 'must' (n. 35). However, it would be theoretically preferable if we could explain the data in terms of independent features of the semantics of 'ought' and 'must', as pursued in the main text. I argue below that the analysis in the main text derives the modals' contrasting performative properties without needing to take them as basic. We can explain the entailingness data without ad hoc stipulations about 'must'.

It's the *non*-entailingness of 'ought', and the consistency of '(Ought ϕ) $\wedge$ $\neg\phi$', that is given special explanation.

Deontic 'ought' and 'must' are often thought to differ in illocutionary force. Paul McNamara characterizes the phenomena well:

> To say that one <u>ought</u> to take a certain option is merely to provide a nudge in that direction. Its typical uses are to offer guidance, a word to the wise ("counsel of wisdom"), to recommend, advise or prescribe a course of action . . . In contrast, to say that one <u>must</u> take a certain option is to be quite forceful. Its typical uses are to command, decree, enact, exhort, entreat, require, regulate, legislate, delegate, or warn. Its directive force is quite strong.   (McNamara 1990, 156)

Many previous accounts capture this contrast by stipulating an ad hoc performative element in the lexical semantics of 'must'.[35] Our account of entailingness data suggests a strategy for deriving the contrasting speech-act properties of deontic 'ought' and 'must' from their static semantics and general pragmatic considerations. Accepting 'Must ϕ' is incompatible with denying 'ϕ'. So, if the truth of 'ϕ' is assumed to depend on the actions of the addressee, updating with 'Must ϕ' will commit her to seeing to it that ϕ (or, in the general case, commit the interlocutors to presupposing that the subject of the obligation is committed to seeing to it that ϕ) (cf. Bybee et al. 1994). So, it's no surprise that 'must' should often be thought to be conventionally directive. By contrast, since accepting 'Ought ϕ' is compatible with denying 'ϕ', updating with 'Ought ϕ' needn't commit anyone to seeing to it that ϕ. Even if deontic 'ought' can be used to perform a directive speech act in certain contexts, it doesn't do so as a matter of conventional meaning. Yet given our discussion of the conversational role of 'ought' (§§2–3), it's unsurprising that utterances of deontic 'Ought ϕ' should often perform more moderate speech acts of recommending or advising. Uttering 'Ought ϕ' can convey one's preference that 'ϕ' be accepted, but without imposing the truth of 'ϕ' on the common ground. As Gibbard, following Stevenson, writes, the speaker "is making a conversational demand. He is demanding that the audience accept what he says, that it share the state of mind he expresses" (Gibbard 1990, 172), though in a "more subtle, less fully conscious way" than by issuing "an imperative" (Stevenson 1937, 25)—or, we might say, than by using 'must'. Deontic 'ought' generally provides a less face-threatening alternative to deontic 'must', in particular in contexts where the speaker might be construed as imposing on the addressee or relevant subject.[36]

---

35 See, e.g., Ninan 2005; Portner 2007, 363–65, 2009, 103–5, 189–96; Swanson 2008, 1203–4 (cf. Finlay 2014, 172–74; Ridge 2014, 27–36). See Van Linden and Verstraete 2011, 153 and Van Linden 2012, 69 on differences in force among weak and strong deontic adjectives.

36 Cf. "Bradshaw said, he must be taught to rest. Bradshaw said they must be separated. 'Must,' 'must,' why 'must'? What power had Bradshaw over him? 'What right has Bradshaw to say "must" to me?' he demanded" (*Mrs Dalloway*, Virginia Woolf). See also n. 39.

Our treatments of differences in "strength" between 'ought' and 'must' are compatible with the observation that uses of 'must' may carry an intuitively weaker conversational force in certain contexts. Consider (39).

(39) [Context: You're hosting a party. You wish to offer the guests a cake that you baked yourself. You say:]
    a. You must have some of this cake.
    b. You should/ought to have some of this cake.
    c. You may have some of this cake.   (cf. Lakoff 1972a, 910)

As Lakoff observes, using 'must' would be most polite, conveying an offer, while using 'should' would be less polite and using 'may' would be flat-out rude. It's an interesting question what conversational factors are responsible for such apparent reversals of the modals' felt forces. Given our focus on the weak/strong necessity modal distinction I put the issue aside, since the phenomenon generalizes to intuitively "weak" uses of imperatives, as in permission/invitation uses such as (40) (von Fintel and Iatridou 2015), and intuitively "strong" uses of possibility modals, as in (39c) and command uses such as (41).

(40) Here, have some of this cake!

(41) [Context: Celebrity to entourage:]
    You may/can leave now.

5.3 Qualification: Endorsing and Nonendorsing Use

I have said that '(Must ϕ) $\wedge \neg\phi$' cannot be coherently accepted on any reading. This claim needs to be qualified. Though there is robust data attesting to the anomalousness of deontic '(Must ϕ) $\wedge \neg\phi$' (n. 31), some speakers report being able to hear sincere uses as consistent in certain contexts.[37] However, a key observation is that even speakers who can hear examples such as (42) as consistent agree that it would be more natural to use a strong necessity modal such as 'have to' or 'be required to', as in (43).

(42) I must go to confession; I'm a Catholic. But I'm not going to. I haven't practiced for years.

(43) I have to (/I'm required to) go to confession; I'm a Catholic. But I'm not going to. I haven't practiced for years.

---

37 I haven't seen this judgment expressed in published work, though I've heard it voiced in personal conversation. Thanks to Jan Dowell for discussion.

In (43) it's consistent for the speaker to dismiss going to confession because she isn't endorsing the norms which imply that she is obligated to do so. She is simply reporting what these norms require.

This observation suggests that one can hear examples such as (42) as felicitous to the extent that one accepts "objective" uses of 'must' in (roughly) the sense of Lyons 1977, 1995. Adapting Lyons's terminology, say that a modal is used *endorsingly* in an utterance of '*MODAL ϕ*' if the utterance presents the speaker as endorsing/accepting the considerations with respect to which the modal is interpreted; and say that the modal is used *nonendorsingly* if it doesn't. Among strong necessity modals, 'be required to' is typically used nonendorsingly; 'have to' and '(have) got to' are more flexible, with 'have to' tending more toward the nonendorsing side of the spectrum and '(have) got to' toward the endorsing side; and 'must' is typically used endorsingly.[38] It's easier to hear sincere utterances with (e.g.) 'have to'/'be required to' as compatible with the speaker's rejecting or being indifferent about the considerations that would verify the modal claim:

(44) [Context: Some friends are deciding whether to go home or stay out late for a party.]
    a. #You must get home by 11, but I don't care whether you do.
    b. #Bert must get home by 11. Aren't his parents stupid? I would stay out if I were him.

(45) a. You have to (/are required to) get home by 11, but I don't care whether you do.
    b. Bert has to (/is required to) get home by 11. Aren't his parents stupid? I would stay out if I were him.

When one wishes to describe relevant norms without necessarily expressing endorsement of them, one typically uses (e.g.) 'have to' rather than 'must'.

So, the promised qualification is this: endorsing uses of '*STRONG ϕ*' are incompatible with a denial of '*ϕ*'. It's only with endorsing uses of strong necessity modals that $\bigcap P(w)$ must be included in the context set. What makes the claims about entailingness and performativity particularly compelling in the case of 'must' (versus e.g. 'have to') is that 'must' is typically

---

38 These generalizations are supported by robust data in descriptive linguistics. In addition to Lyons 1977, 1995, see Leech 1971, 2003, Lakoff 1972a,b, Coates 1983, Perkins 1983, Palmer 1990, 2001, Sweetser 1990, Myhill 1995, 1996, Myhill and Smith 1995, Verstraete 2001, Huddleston and Pullum 2002, Smith 2003, Leech et al. 2009, Close and Aarts 2010, Goddard 2014. For critical discussion see Swanson 2012, 2016b, Silk 2015b, 2016a. The distinction between endorsing/nonendorsing uses has been noted under various labels in diverse disciplines (e.g., Hare 1952, von Wright 1963, Verstraete 2007, Silk 2016a). I adapt Lyons's terminology since use of 'subjective' and 'objective' can be fraught. I use 'endorsement' as a cover term for acceptance attitudes of various kinds.

used endorsingly.[39] But to the extent to which one finds nonendorsing uses of 'must' acceptable, to that same extent one is predicted to find uses of deontic 'Must ϕ' to be nonentailing and lack directive force.

What is distinctive about weak necessity modals is that even when they are used endorsingly, they are nonentailing and may lack imperative force. 'Ought' and 'should' are like 'must' in typically being used endorsingly (n. 38). Parallel to (44)–(45), the claims in (46) with 'ought' would be more naturally expressed with a modal such as 'supposed to', as in (47).

(46)   a. #You ought to get home by 11, but I don't care whether you do.
        b. #Bert ought to get home by 11. Aren't his parents stupid? I'd stay out if I were him.

(47)   a. You're supposed to get home by 11, but I don't care whether you do.
        b. Bert is supposed to get home by 11. Aren't his parents stupid? I'd stay out if I were him.

Yet a characteristic use of 'Ought ϕ' is with an explicit or implicated denial of 'ϕ' (§4).

6. Negotiability and Collective Commitment

Along the way we have noted various ways in which the proposed approach to the weak/strong necessity modal distinction differs from other approaches in the literature. For instance, alternative approaches generally agree in treating uses of 'Ought ϕ' as predicating a distinctive kind of necessity, namely weak necessity, of the prejacent ϕ at the actual (evaluation) world. Very roughly: On domain restriction accounts (Copley 2006, von Fintel and Iatridou 2008, Swanson 2011, Rubinstein 2012, Charlow 2013), ϕ is a weak necessity if ϕ is true throughout a certain set S of worlds, where S is a subdomain of the set of worlds quantified over by 'must'. On probabilistic/comparative possibility accounts (Finlay 2009, 2010, 2014, Lassiter 2011), ϕ is a weak necessity if ϕ is sufficiently likely/desirable, or more likely/desirable than any relevant alternative to ϕ. Although Yalcin's (2016) normality-based semantics denies that 'ought' and 'must' are logically related, 'Ought ϕ' is still interpreted by evaluating the truth of ϕ throughout a set of minimal worlds relative to the actual world.

The approach in this paper rejects treating acceptance of 'Ought ϕ' in terms of ϕ being a necessity, in any posited sense of necessity, at every candidate for the actual world. The apparent "weakness" of (so-called) weak necessity modals is diagnosed instead in terms of a failure to presuppose that the relevant worlds at which the prejacent is a necessity are in the context set (§4). For example, the semantics in Definition 4 adds a layer of modality,

---

39   It isn't implausible that the drastic decline in frequency of deontic 'must' is due in no small part to the above features of its meaning and use (cf. Myhill 1995, 1996, Krug 2000, Smith 2003, Tagliamonte 2004, Leech et al. 2009, Close and Aarts 2010, Goddard 2014).

predicating the ordinary necessity claim $[\lambda w \,.\, \bigcap P_c(w) \subseteq [\![\phi]\!]^c]$ of every world in a certain set of possibly counterfactual worlds. The proposed modal-past approach (even if not implemented in precisely this way) captures contrasting discourse properties of 'ought' and 'must' such as differences in conversational force and relations to standing contextual assumptions; it captures logical properties such as differences in entailingness and the lack of entailments between 'ought' and 'must'/'may'; it captures relations between weak necessity modals and broader modal-past phenomena; and it generalizes across flavors of modality.

There are insights from previous accounts that are preserved in the account developed here. The semantics in §4 allow for connections between weak necessity modals and common ground assumptions (n. 6); mood, counterfactuality, and conditionality (nn. 17, 23, 26); and notions of comparison (n. 27), normality (Makinson 1993, Frank 1996, Yalcin 2016), and probability (though not unconditional probability; n. 30). Previous accounts are often developed with an eye toward one or several of these issues to the exclusion of others. Although we haven't examined each of the connections in equal depth, I hope the discussion has illustrated the fruitfulness of a modal-past approach and its potential for systematizing diverse linguistic phenomena.

Before concluding I would like to compare in more detail the account of the weak/strong necessity modal distinction in this paper with the domain restriction account in Rubinstein 2012, which constitutes the most extensively developed alternative from the literature (see also Portner and Rubinstein 2012, 2016, Rubinstein 2014). We have already observed general points of disagreement between the modal-past approach and domain restriction accounts (see also §§3, 5). Here I focus on the distinctive feature of Rubinstein's account: the appeal to *collective commitment* to a body of priorities (norms, goals, ideals).[40]

Rubinstein's main innovation is to supplement von Fintel and Iatridou's domain restriction semantics with a substantive account of the distinction between primary vs. secondary ordering sources (§4): what makes a primary ordering source "primary" is that it includes premises which are presupposed to be collectively committed to by the interlocutors; what makes a secondary ordering source "secondary" is that it includes premises which are presupposed *not* to be collectively committed to by the interlocutors. 'Must' is only interpreted with respect to primary premises; 'ought' is logically weaker in also being interpreted with respect to secondary premises, which are presupposed to be not collectively endorsed:

Strong necessity modals are only sensitive to prioritizing premises that the conversational participants are presupposed to be collectively committed to . . . [I]f any participant in the

---

40 Rubinstein doesn't examine epistemic modals. I leave open how the account would be extended to epistemic readings. As discussed previously, there will be general worries regarding the lack of entailments between epistemic 'ought' and 'may/'must', and the fact that epistemic 'ought' doesn't in general quantify over a set of worlds that are regarded as epistemically possible (§§4–5). See Silk 2018 for critical discussion of Portner and Rubinstein's (2012) appeal to collective commitment in an account of mood selection.

conversation were given the chance to defend these priorities, it is assumed in the context of the conversation that they would do so. Weak necessity modals take into account all these premises plus some more. For these additional premises, lack of collective commitment is presupposed . . . [A] speaker uses a weak modal when he or she believes (perhaps mistakenly) that the secondary priorities it depends on are still up for discussion.

(Rubinstein 2012, 51–52)

In (48) although the speaker is committed to a priority favoring cost-effectiveness, it isn't presupposed that Alice is committed to it (cf. Rubinstein 2012, 55–60). This lack of collective commitment is what calls for using 'ought', on Rubinstein's view.

(48) [Context: Alice is considering whether to take the subway or a cab to a concert. The subway is cheaper; the cab is quicker. You say:]
You ought to (/should, /?have to, /?must) take the subway.

For clarity, use 'commitment$_R$' for the notion of commitment in Rubinstein's analyses. Rubinstein explicitly identifies commitment$_R$ to a priority *p* with *endorsing that p is desirable* (e.g., 2012, 78; also Portner and Rubinstein 2012, 471, 475, 477–81). What determines whether *p* is a "primary" or "secondary" priority, according to Rubinstein, is the interlocutors' mutual presuppositions about the desirability of *p*.

§5 delineated two dimensions along which modals differ: strength and tendencies for endorsing/nonendorsing use, i.e. the extent to which uses of '*MODAL ϕ*' convey the speaker's endorsement of the considerations that would verify the modal claim. Delineating these dimensions brings out problems with diagnosing the weak/strong necessity modal distinction in terms of collective commitment$_R$. As we have seen, the weak/strong necessity modal distinction crosscuts the distinction between necessity modals that express endorsement in the above sense and those that don't, and hence crosscuts the distinction between necessity modals that express collective commitment$_R$ and those that express a lack of commitment$_R$.

First, there are uses of weak necessity modals that express collective endorsement of the relevant priorities. In (49) we are both publicly committed$_R$ to the value of family; my helping my mother is "publicly endorse[d] . . . as desirable" (Rubinstein 2012, 78) by every conversational participant. Yet 'ought' is felicitous, indeed preferred.

(49) *Me*:   Family is very important. I think I would rather stay here.
       You: I agree. You ought to tend to your mother.

Second, there are uses of strong necessity modals where the interlocutors expressly deny commitment$_R$ to the relevant priorities, as in (43) and (45). The lack of endorsement can even be common ground:

(50) [Context: Our parents are asleep. We've settled on staying out to go to a party.]
    You: When is curfew, again? We need to make sure that we tell Mom we got back before then if she asks.
    Me: We have to be home by 11. Aren't her rules stupid? This party is going to be great.

Counterexamples such as these aren't atypical. Corpus studies attest to variations among weak necessity modals, and variations among strong necessity modals, vis-à-vis tendencies to express collective commitment$_R$. Collective commitment$_R$ has been invoked as a basis for distinguishing among the modals in each class.[41] Summarizing his corpus analyses of 'ought' and 'should' in contemporary American English, Myhill concludes, "Using *ought* suggests that people have the same feelings about the specific obligation in question and there is agreement about it, while *should* does not suggest the same feelings or agreement" (1997, 8). Far from being exceptional, expressing collective commitment$_R$ with 'ought' is the norm, as in the naturally occurring discourse in (51) (slightly abbreviated).

(51)   *A*:   I won't tell anyone . . . but the Dean, of course.
     B: And Mrs. Reynolds.
     A: Yes. She ought to know.   (cf. Myhill 1997, 10)

Conversely, though 'must' tends to express collective commitment$_R$, other strong necessity modals such as '(have) got to' do not. '(Have) got to' differs from 'must' in typically being "associated with *conflicts* between the speaker and the listener" (Myhill 1996, 365), as in the naturally occurring example in (52).

(52) Edie, you've got to stop bothering me when I'm working.   (Myhill 1996, 369)

Collective commitment$_R$ to the contents of premise sets may affect the distribution of modals in discourse, but it isn't what explains the distinction between weak and strong necessity modals.

    A modal-past approach captures intuitions about negotiability and collective commitment which may be motivating Rubinstein's account. Uses of strong necessity modals "presuppose (collective) commitment" in the same ordinary sense as uses of any context-sensitive expression: one commits that the value of the context-dependent item is as one's utterance assumes, and that the world is as one's utterance says it is, given this assumed value. Such commitments are compatible with not endorsing-as-desirable the premises $p \in P(w)$, for any $w \in c$. In (50) we can accept that the house rules require us to be home by 11 while denying that those rules are desirable or that they are to guide our plans. Conversely, uses

---

41 See, e.g., Joos 1964, Myhill and Smith 1995, Myhill 1996, 1997, Verstraete 2001, Facchinetti et al. 2003, Leech et al. 2009, Close and Aarts 2010, Goddard 2014.

of weak necessity modals express "negotiability" in the sense that they don't conventionally commit one to being in a world where the relevant considerations verify the necessity of the prejacent. Failing to express commitment to the prejacent's being necessary is compatible with (collectively) endorsing the considerations encoded in the given premise frame or the premises from which the prejacent would follow. In (5) we can endorse the value of family even if we prefer not to settle how it interacts with other potentially competing values.

7. Conclusion

This paper developed a *modal-past approach* to 'ought' and the distinction between so-called weak necessity modals ('ought', 'should') and strong necessity modals ('must', 'have to'). There is nothing specially "strong" about the necessity expressed by strong necessity modals. Strong necessity modals are given their ordinary semantics/pragmatics of necessity; uses of 'Must ϕ' predicate the (deontic/epistemic/etc.) necessity of the prejacent ϕ of every candidate for the actual world. The apparent "weakness" of weak necessity modals derives from their bracketing whether the prejacent is necessary in the actual world. Uses of 'Ought ϕ' fail to presuppose that the topic worlds in which ϕ is necessary are included in the context set. 'Ought ϕ' can be accepted without needing to settle that the relevant considerations (norms, goals, etc.) which actually apply verify the necessity of ϕ. This analysis carves out important roles for weak necessity modals in conversation and deliberation. As emphasized throughout Gibbard's (1990, 2003, 2012) developments of expressivism, 'ought' affords a means of coordinating our *conditional* attitudes and plans. Weak necessity modals allow us to entertain and plan for hypothetical continuations or minimal revisions of the current context; they afford conventional devices for coordinating our norms, values, expectations, without having to settle precisely how the relevant considerations apply and compare. The proposed account systematizes a spectrum of semantic and pragmatic data—e.g., concerning the relative felicity of weak and strong necessity modals, relations between uses of weak and strong necessity modals and standing contextual assumptions, morphosyntactic properties of expressions of ought crosslinguistically, and contrasting logical and illocutionary properties of weak and strong necessity modals. The range of linguistic phenomena that are unified under and explained by the account lend it a robust base of support.

The data considered here aren't the only data to be explained by an overall theory of weak and strong necessity modals. For instance, there are additional linguistic contrasts between the classes of weak and strong necessity modals, such as in data with incomparabilities, comparatives, quantifiers, conditionals, modifiers, and neg-raising.[42] Second, we briefly examined one additional dimension of difference among modals in tendencies for endorsing/non-endorsing use. It's worth investigating interactions between weak/strong necessity modals and other such

---

42 See, e.g., Lassiter 2011, 2012, Swanson 2011, Rubinstein 2012, 2014, Charlow 2013, Iatridou and Zeijlstra 2013, Silk 2013, 2015b, 2016b, Portner and Rubinstein 2016.

differences in modal meanings, e.g. with implicatures (cf. Verstraete 2005a). Third, although I argued against treating 'ought' and 'must'/'may' as being ordered by logical or quantificational strength, more thorough comparisons of inference patterns with different types of readings are needed. Fourth, our discussion highlighted interactions between weak and strong necessity modals and general issues such as context-sensitivity, counterfactuality, attitude expression, and performativity. These interactions afford rich avenues for future research. In closing I would like to raise two potential avenues for future work, one more narrowly linguistic, one more philosophical.

7.1. Counterfactuality?

In §4 we noted that many languages express a notion of OUGHT by using the form of a strong necessity modal used in counterfactuals. Such languages use the same string to express OUGHT and counterfactual necessity (WOULD HAVE TO). Yet as von Fintel and Iatridou (2008, 128–31) observe, 'ought' cannot generally be replaced by 'would have to':

> (53) a. I ought to help the poor, but I don't have to.
>   b. ??I would have to help the poor, but I don't have to.

Open questions include how non-lexicalized expressions of OUGHT are diachronically related to the dedicated lexical items; how certain uses of *STRONG+CF* might come to be conventionally interpreted as expressions of OUGHT; how counterfactual interpretations of *STRONG+CF* are related to meanings of grammaticalized forms like 'ought'; and how meanings of the grammaticalized forms compare across languages. The modal-past analysis in Definition 4 avoids conflating 'ought' with 'would have to' and giving 'Ought ϕ' the semantics of an implicit counterfactual necessity claim. The necessity of ϕ isn't evaluated at the closest ψ-worlds, for any implicit condition ψ, but at the relevant minimal/"preferred" (desirable, normal, etc.) worlds determined by h_c. Although the focus of this paper has been on lexicalized English weak necessity modals, it's worth considering how such an interpretation might become associated with certain uses of *STRONG+CF* in other languages. Channeling my inner Gibbard, "What I can suggest will have to be quite speculative, but the speculations, I hope, may prompt more solid investigation" (1990, 61).

Using a clause that lacks the usual indicative presupposition places a burden on the interpreter's task of inferring which possibilities are being talked about. A condition specifying which possibilities are relevant typically must be salient, either implicitly or in the linguistic context, as in (54). (cf. Schueler 2011).

> (54) [Context: We're trying out guitars in a music store. Looking at an expensive vintage Les Paul, I say:]
>   a. (If I bought it,) my partner would kill me.
>   b. (If I wanted to buy it,) I would have to check with my partner first.

So, if no condition is retrievable, using 'would have to' is anomalous, as in (55a). Uses of 'ought' lack this salience/retrievability requirement (cf. (55b)).

(55)   [Context: We're strangers in a hotel lobby. I notice you fumbling with your bags. I say:]
   a. ??Here, I would have to help you.
   b. Here, I ought to help you.

A hypothesis, then, is that distinctive discourse effects of weak necessity modals are products of using an expression that is neither marked as being about the context set (as via indicative mood) nor indicated as being about some other topical possibility (via an implicit or explicit condition).

Suppose we are inquiring about the necessity of ϕ. Uttering '$STRONG\ \phi$' requires settling which considerations apply and aren't defeated. We might not be prepared to restrict the future course of the conversation in this way. Nevertheless each of us takes some ways of extending the conversation and addressing the question under discussion to be better or more likely than others. Using the modal-past form allows us to consider the necessity of ϕ as holding not necessarily in the current context, as with a strong necessity modal, but in a preferred (likely/normal/desirable) continuation or minimal revision of the current context, whatever that might turn out to be (§§2–3). One's judgments about replacing 'would have to' with 'ought' should thus improve to the extent that the antecedent for 'would have to' describes what one regards as "preferred" conditions relevant for evaluating questions about the necessity of ϕ. Indeed the discourse effects of 'would have to' and 'ought' are strikingly closer in (57) than in (56).

(56) a. (If I was a mobster, which I'm not,) I would have to kill you.
   b. ??I ought to kill you.
(57) [Context: Alice is considering with her mother whether to take the A- or C-train. The A is quicker; the C is safer. Her mother says:]
   a. If safety was most important, you would have to take the C-train. In fact, safety is more important, as we can agree. So you must/have to take the C.
   b. You ought to take the C-train. In fact, safety is most important, as we can agree. So you must/have to take the C.

As often occurs in grammaticalization and pragmaticalization,[43] the original (counterfactual) meaning may be semantically bleached, and the reinterpreted (weakness) meaning becomes more abstract; the attitude expressed about the necessity claim can thus appear

---

43 See, e.g., Horn 1984, Heine et al. 1991, Lehmann 1995, Traugott 1995, Diewald 2011, Narrog and Heine 2011, Davis and Gutzmann 2015.

vague or nonspecific. Such an interpretation is essentially what is delivered by the lexically unspecified generalized notion of minimality associated with *h* in Definition 4.[44] This grammaticalization path predicts the observed variations in how ᴏᴜɢʜᴛ is expressed crosslinguistically—i.e., with some languages using only a conventionalized weakness interpretation of *STRONG+CF* (French); some languages using only a grammaticalized form (English); and some languages using both (German).

To my knowledge, no general account has been given of how weakness interpretations of past forms are derived, or how weakness vs. implicit-counterfactual interpretations of past forms are determined across contexts (cf. (17)–(19), n. 16). The above hypothesis may provide a basis for weakness (tentativeness, politeness) effects associated with modal-past forms generally. Thorough synchronic/diachronic crosslinguistic work on the interpretation of modal-past forms, and how ᴏᴜɢʜᴛ-interpretations of *STRONG+CF* are conventionalized and in some cases grammaticalized, is called for.

7.2. Philosophical Therapy?

I would be remiss if I concluded without asking how, if at all, our linguistic inquiry might inform theorizing about a "primitive concept ᴏᴜɢʜᴛ" (Gibbard 2012, 204). How might investigating a word 'ought' teach us something about "concepts fraught with" a "primitive ought" (Gibbard 2003, x, 21, 179; 2012, 14)?[45] Recall our discussions of the practicality of deontic 'ought' and 'must' judgments. It's common in discourse and deliberation to investigate what we are actually required to do. We wish to guide our planning and influence one

---

44 The hypothesis in the main text considers weakness uses of *STRONG+CF* on the model of a conditional consequent (i.e., lacking a particular retrievable antecedent). An alternative is to understand the uses on the model of the antecedent. Interestingly, there is a precedent for associating an interpretation like the one given to *h* in Definition 4 with a certain type of 'if'-clause. Grosz (2012) argues that optatives and exclamatives should be treated on the model of nonlogical 'if'-clauses under a covert exclamation operator *EX* ("nonlogical" in the sense that the embedded clause provides the subject matter of the attitude), as reflected in (i)–(ii).

   (i) Oh, that it would snow!
        a. *EX* [it snows]
        b. ≈ "It would be good if it snowed"

   (ii) Oh, that it snowed!
        a. *EX* [it snowed]
        b. ≈ "It's surprising that it snowed"

   Grosz's *EX* operator is essentially a scalar analogue of *h*, also generalized to express a lexically unspecified attitude toward the embedded proposition. A Grosz-style scalar version of Definition 4—that 'Ought ϕ' is true iff the proposition that ϕ is necessary is at least as *h*-"preferred" (desirable/normal/etc.) as a contextually relevant standard—might help capture connections between weak necessity modals and gradability/comparatives (Finlay 2014, Rubinstein 2014, Portner and Rubinstein 2016). Crosslinguistic comparisons among morphological and lexical expressions of ᴏᴜɢʜᴛ, nonlogical 'if'-clauses, and optatives and exclamatives may provide fruitful avenues to explore.

45 See Silk 2015b (esp. §§6–7) for additional discussion and applications.

another's behavior in light of the norms we accept. An endorsing strong necessity modal like 'must' is well suited to the task. However, using 'must' is often awkward. We may want to talk about obligations which held in the past, or which may go unfulfilled or be overridden in the future. Or we may want to communicate information about a body of norms without necessarily registering commitment to them or enjoining others to share in such commitment. A modal like 'ought' or 'be required to' may thus be more suitable. There is a range of expressive resources at our disposal for coordinating our actions and attitudes. This is for the better given the variety of our purposes. But it also raises a philosophical risk. Bracketing differences among necessity modals might turn out to be harmless for the purposes of (meta)normative inquiry. But it might not.

A central innovation of Gibbard's expressivism is the thesis that normative concepts are essentially *plan-laden*:

(58) "To believe that one ought to do *X* is to plan to do *X*."
      "*Thinking what I ought to do is thinking what to do.*"
      "*Ought* claims . . . are claims about what to do."

                                                      (Gibbard 2003, ix–x, 10; 2012, 204)

Generalizing: "The clear distinctive feature of normative concepts, I now think, lies in their conceptual ties. Oughts of action tie in conceptually with acting" (Gibbard 2011, 36). Many take it as obvious that some form of judgment internalism is true. After all, normative judgments are constitutive of deliberation, and deliberation is essentially practical; its aim is action. But many find clear counterexamples. What about the psychopath, or someone who is tired or depressed (such as perhaps the poor reader who has made it thus far)?

Attending to various dimensions of difference among necessity modals may shed light on conflicting intuitions about the ostensible "plan-ladenness" of "concepts fraught with ought." Gibbard hedges: the concepts "fraught with ought" are so-fraught "not for every sense of the term ['ought']," but for the "crucial sense" explained by the pattern in (58) (2003, x). It is revealing that although Gibbard couches the thesis about the practicality of the normative using 'ought', he pumps the intuition using 'must':

> Oughts of action tie in conceptually with acting. Take, for example, the belief that the building is on fire and the one and only way to keep from being burned to a crisp is to leave forthwith. If that's the case, we'd better leave forthwith, but it isn't strictly incoherent, conceptually, to have this belief and not to leave. Contrast this with the normative belief that one *must* leave forthwith. It is, I maintain, conceptually incoherent to hold this belief and not leave, if one can.
>
>   (Gibbard 2011, 36)

As we have seen, it's hard to hear a sincere utterance of 'Must ϕ' as consistent with the speaker's being indifferent about whether ϕ. Such judgments aren't nearly as anomalous

when expressed with weak necessity modals or modals that are more naturally used non-endorsingly (§§4–6).

(59) #I *must* (/I've *got* to) get home by 10, but forget that; I'm not going to.

(60) I {ought to, should, am supposed to, have to} get home by 10, but forget that; I'm not going to.

We should be wary of general claims about normative language and judgment. A thesis such as (58) may seem compelling when considering examples using terms that are paradigmatically entailing and endorsing; but when we consider cases using other terms, counterexamples can appear in the offing. If only "fraught with MUST" were catchier.

The skeptically inclined might be apt to wonder what is at-issue in debates about judgment internalism—i.e., if not linguistic issues about directive/entailing/endorsing uses of language, or empirical psychological issues about conditions under which different attitudes and judgments guide and motivate us. One might wonder how probative it is to couch metaethical inquiry in terms of a class of "normative" language/concepts/judgments at all. More fruitful, perhaps, to leave talk of the "normative" to the side, and ask directly about motivational states of mind and directive uses of language, and what type of person to be and how to live.

References

Anderson, Alan Ross (1951). "A Note on Subjunctive and Counterfactual Conditionals." *Analysis* 11: 35–38.

Åqvist, Lennart (2002). "Deontic Logic." In *Handbook of Philosophical Logic, Vol. 8*, 2nd ed., edited by Dov M. Gabbay and Franz Guenthner, 147–64. Dordrecht: Kluwer Academic Publishers.

Arregui, Ana (2009). "On Similarity in Counterfactuals." *Linguistics and Philosophy* 32: 245–78.

——— (2010). "Detaching *If*-Clauses from *Should*." *Natural Language Semantics* 18: 241–93.

van der Auwera, Johan, and Vladimir A. Plungian (1998). "Modality's Semantic Map." *Linguistic Typology* 2: 79–124.

Bittner, Maria (2011). "Time and Modality without Tenses or Modals." In *Tense across Languages*, edited by Monika Rathert and Renate Musan, 147–88. Tübingen: Niemeyer.

Brandt, R. B. (1964). "The Concepts of Obligation and Duty." *Mind* 73: 374–93.

Bybee, Joan (1995). "The Semantic Development of Past Tense Modals in English." In *Modality in Grammar and Discourse*, edited by Joan Bybee and Suzanne Fleischman, 503–17. Amsterdam: Benjamins.

Bybee, Joan, Revere Perkins, and William Pagliuca (1994). *The Evolution of Grammar: Tense, Aspect, and Modality in the Languages of the World*. Chicago: University of Chicago Press.

Campbell, Eric (2014). "Breakdown of Moral Judgment." *Ethics* 124: 447–80.

Charlow, Nate (2013). "What We Know and What to Do." *Synthese* 190: 2291–323.

Close, Joanne, and Bas Aarts (2010). "Current Change in the Modal System of English: A Case Study of *Must*, *Have To* and *Have Got To*." In *The History of English Verbal and Nominal Constructions*, edited by Ursula Lenker, Judith Huber, and Robert Mailhammer, 165–81. Amsterdam: John Benjamins.

Coates, Jennifer (1983). *The Semantics of the Modal Auxiliaries*. London: Croom Helm.

Collins, Peter (2007). "*Can*/*Could* and *May*/*Might* in British, American and Australian English: A Corpus-Based Account." *World Englishes* 26: 474–91.

Condoravdi, Cleo (2002). "Temporal Interpretation of Modals: Modals for the Present and for the Past." In *The Construction of Meaning*, edited by David I. Beaver, Luis D. Casillas Martínez, Brady Z. Clark, and Stefan Kaufmann, 59–88. Stanford: CSLI Publications.

Copley, Bridget (2006). "What Should *Should* Mean?" MS, CNRS/Université Paris 8.

Davis, Christopher, and Daniel Gutzmann (2015). "Use-Conditional Meaning and the Semantics of Pragmaticalization." In *Proceedings of Sinn und Bedeutung 19*, edited by Eva Csipak and Hedde Zeijlstra, 197–213. Göttingen: University of Göttingen.

Diewald, Gabriele (2011). "Pragmaticalization (Defined) as Grammaticalization of Discourse Functions." *Linguistics* 49: 365–90.

Dreier, James (2009). "Relativism (and Expressivism) and the Problem of Disagreement." *Philosophical Perspectives* 23: 79–110.

Fanari, Martina (2007). *Expressing Deontic Modality with Volitional Verbs: The Case of Sardinian*. PhD thesis, University of Pavia.

Finlay, Stephen (2009). "Oughts and Ends." *Philosophical Studies* 143: 315–40.

——— (2010). "What *Ought* Probably Means, and Why You Can't Detach It." *Synthese* 177: 67–89.

——— (2014). *Confusion of Tongues: A Theory of Normative Language*. New York: Oxford University Press.

von Fintel, Kai (1998). "The Presupposition of Subjunctive Conditionals." In *MIT Working Papers in Linguistics 25: The Interpretive Tract*, edited by Uli Sauerland and Orin Percus, 29–44. Cambridge: MIT Press.

von Fintel, Kai, and Sabine Iatridou (2008). "How to Say *Ought* in Foreign: The Composition of Weak Necessity Modals." In *Time and Modality*, edited by Jacqueline Guéron and Jacqueline Lecarme, 115–41. Springer.

von Fintel, Kai, and Sabine Iatridou (2015). "A Modest Proposal for the Meaning of Imperatives." MS, MIT.

Fleischman, Suzanne (1989). "Temporal Distance: A Basic Linguistic Metaphor." *Studies in Language* 13: 1–50.

van Fraassen, Bas C. (1973). "Values and the Heart's Command." *Journal of Philosophy* 70: 5–19.

Frank, Anette (1996). *Context Dependence in Modal Constructions*. PhD thesis, University of Stuttgart.

Gazdar, Gerald (1979). *Pragmatics: Implicatures, Presupposition, and Logical Form*. New York: Academic Press.

Gibbard, Allan (1990). *Wise Choices, Apt Feelings: A Theory of Normative Judgment*. Cambridge: Harvard University Press.

——— (2003). *Thinking How to Live*. Cambridge: Harvard University Press.

——— (2006). "Reply to Critics." *Philosophy and Phenomenological Research* 72: 729–44.

——— (2011). "How Much Realism? Evolved Thinkers and Normative Concepts." In *Oxford Studies in Metaethics*, vol. 6, edited by Russ Shafer-Landau, 33–51. New York: Oxford University Press.

——— (2012). *Meaning and Normativity*. New York: Oxford University Press.

Goddard, Cliff (2014). "*Have To*, *Have Got To*, and *Must*: NSM Analyses of English Modal Verbs of 'Necessity.'" In *Nonveridicality and Evaluation: Theoretical, Computational and Corpus Approaches*, edited by Maite Taboada and Radoslava Trnavac, 50–74. Leiden: Brill.

Grice, Paul (1989). *Studies in the Ways of Words*. Cambridge: Harvard University Press.

Groefsema, Marjolein (1995). "*Can*, *May*, *Must*, and *Should*: A Relevance Theoretic Account." *Journal of Linguistics* 31: 53–79.

Grosz, Patrick Georg (2012). *On the Grammar of Optative Constructions*. Amsterdam: Benjamins.

Hare, R. M. (1952). *The Language of Morals*. Oxford: Oxford University Press.

Harman, Gilbert (1973). "Review of Roger Wertheimer, *The Significance of Sense: Meaning, Modality, and Morality*." *The Philosophical Review* 82: 235–39.

——— (1975). "Reasons." *Crítica: Revista Hispanoamericana de Filosofía* 7: 3–17.

Heine, Bernd, Ulrike Claudi, and Friederike Hünnemeyer (1991). *Grammaticalization: A Conceptual Framework*. Chicago: University of Chicago Press.

Horn, Laurence R. (1972). *On the Semantic Properties of Logical Operators in English*. PhD thesis, University of California, Los Angeles.

——— (1984). "Toward a New Taxonomy for Pragmatic Inference: Q-Based and R-Based Implicature." In *Meaning, Form, and Use in Context: Linguistic Applications*, edited by Deborah Schiffrin, 11–42. Washington, DC: Georgetown University Press.

Huddleston, Rodney, and Geoffrey K. Pullum, eds. (2002). *The Cambridge Grammar of the English Language*. Cambridge: Cambridge University Press.

Iatridou, Sabine (2000). "The Grammatical Ingredients of Counterfactuality." *Linguistic Inquiry* 31: 231–70.

Iatridou, Sabine, and Hedde Zeijlstra (2013). "Negation, Polarity, and Deontic Modals." *Linguistic Inquiry* 44: 529–68.

Ippolito, Michela (2003). "Presuppositions and Implicatures in Counterfactuals." *Natural Language Semantics* 11: 145–86.

James, Deborah (1982). "Past Tense and the Hypothetical: A Cross-Linguistic Study." *Studies in Language* 6: 375–403.

Joos, Martin (1964). *The English Verb: Form and Meanings*. Madison: University of Wisconsin Press.

Kratzer, Angelika (1977). "What 'Must' and 'Can' Must and Can Mean." *Linguistics and Philosophy* 1: 337–55.

——— (1981). "The Notional Category of Modality." In *Words, Worlds, and Contexts: New Approaches in Word Semantics*, edited by Hans-Jürgen Eikmeyer and Hannes Rieser, 38–74. Berlin: de Gruyter.

——— (1991). "Modality/Conditionals." In *Semantics: An International Handbook of Contemporary Research*, edited by Arnim von Stechow and Dieter Wunderlich, 639–56. New York: de Gruyter.

——— (2012). *Modals and Conditionals: New and Revised Perspectives*. New York: Oxford University Press.

Krug, Manfred (2000). *Emerging English Modals: A Corpus-Based Study of Grammaticalization*. Berlin: Mouton de Gruyter.

Lakoff, Robin (1972a). "Language in Context." *Language* 48: 907–27.

——— (1972b). "The Pragmatics of Modality." In *Proceedings of the Chicago Linguistic Society 8*, 229–246. Chicago: CLS.

Lassiter, Daniel (2011). *Measurement and Modality: The Scalar Basis of Modal Semantics*. PhD thesis, New York University.

——— (2012). "Quantificational and Modal Interveners in Degree Constructions." In *Proceedings of SALT 22*, edited by Anca Chereches, 565–83. Ithaca, NY: CLC Publications.

Leech, Geoffrey (1971). *Meaning and the English Verb*. London: Longman.

——— (2003). "Modality on the Move: The English Modal Auxiliaries 1961–1992." In *Modality in Contemporary English*, edited by Roberta Facchinetti, Manfred Kung, and Frank Palmer, 223–40. New York: Mouton de Gruyter.

Leech, Geoffrey, Marianne Hundt, Christian Mair, and Nicholas Smith (2009). *Change in Contemporary English: A Grammatical Study*. Cambridge: Cambridge University Press.

Lehmann, Christian (1995). *Thoughts on Grammaticalization*. München: Lincom.

Lewis, David (1973). *Counterfactuals*. Cambridge: Harvard University Press.

——— (1981). "Ordering Semantics and Premise Semantics for Counterfactuals." *Journal of Philosophical Logic* 10: 217–34.

Lyons, John (1977). *Semantics*, vol. 2. Cambridge: Cambridge University Press.

——— (1995). *Linguistic Semantics: An Introduction*. Cambridge: Cambridge University Press.

MacFarlane, John (2014). *Assessment Sensitivity: Relative Truth and Its Applications*. Oxford: Clarendon Press.

Makinson, David (1993). "Five Faces of Minimality." *Studia Logica* 52: 339–79.

Matthewson, Lisa (2010). "Cross-Linguistic Variation in Modality Systems: The Role of Mood." *Semantics and Pragmatics* 3: 1–74.

McGregor, William B., and Tamsin Wagner (2006). "The Semantics and Pragmatics of Irrealis Mood in Nyulnyulan Languages." *Oceanic Linguistics* 45: 339–79.

McNamara, Paul (1990). *The Deontic Quadecagon*. PhD thesis, University of Massachusetts.

Myhill, John (1995). "Change and Continuity in the Functions of the American English Modals." *Linguistics* 33: 157–211.

——— (1996). "The Development of the Strong Obligation System in American English." *American Speech* 71: 339–88.

——— (1997). "*Should* and *Ought*: The Rise of Individually-Oriented Modality in American English." *English Language and Linguistics* 1: 3–23.

Myhill, John, and Lauren A. Smith (1995). "The Discourse and Interactive Functions of Obligation Expressions." In *Modality in Grammar and Discourse*, edited by Joan Bybee and Suzanne Fleischman, 239–92. Amsterdam: Benjamins.

Narrog, Heiko, and Bernd Heine, eds. (2011). *The Oxford Handbook of Grammaticalization*. Oxford: Oxford University Press.

Ninan, Dilip (2005). "Two Puzzles about Deontic Necessity." In *New Work on Modality*, edited by Jon Gajewski, Valentine Hacquard, Bernard Nickel, and Seth Yalcin, 149–78. Cambridge: MITWPL.

Nuyts, Jan (2001). *Epistemic Modality, Language, and Conceptualization*. Amsterdam: John Benjamins.

Palmer, F. R. (1990). *Modality and the English Modals*, 2nd ed. New York: Longman.

—— (2001). *Mood and Modality*, 2nd ed. Cambridge: Cambridge University Press.

Perkins, Michael R. (1983). *Modal Expressions in English*. Norwood: Ablex.

Piaget, Jean (1962). *The Moral Judgment of the Child*. New York: Collier Books.

Portner, Paul (2007). Imperatives and Modals. *Natural Language Semantics* 15: 351–83.

—— (2009). *Modality*. Oxford: Oxford University Press.

Portner, Paul, and Aynat Rubinstein (2012). "Mood and Contextual Commitment." In *Proceedings of SALT 22*, edited by Anca Chereches, 461–87. Ithaca, NY: CLC Publications.

—— (2016). "Extreme and Non-Extreme Deontic Modals." In *Deontic Modality*, edited by Nate Charlow and Matthew Chrisman, 256–82. New York: Oxford University Press.

Ridge, Michael (2014). *Impassioned Belief*. Oxford: Oxford University Press.

Rubinstein, Aynat (2012). *Roots of Modality*. PhD thesis, University of Massachusetts Amherst.

—— (2014). "On Necessity and Comparison." *Pacific Philosophical Quarterly* 95: 512–54.

Schlenker, Philippe (2005). "The Lazy Frenchman's Approach to the Subjunctive (Speculations on Reference to Worlds and Semantic Defaults in the Analysis of Mood)." In *Romance Languages and Linguistic Theory 2003*, edited by Twan Geerts, Ivo van Ginneken, and Haike Jacobs, 269–309. Amsterdam: John Benjamins.

Schueler, David (2011). *The Syntax and Semantics of Implicit Conditionals: Filling in the Antecedent*. PhD thesis, University of California, Los Angeles.

Silk, Alex (2012). "Modality, Weights, and Inconsistent Premise Sets." In *Proceedings of SALT 22*, edited by Anca Chereches, 43–64. Ithaca, NY: CLC Publications.

—— (2013). *What Normative Terms Mean and Why It Matters for Ethical Theory*. PhD thesis, University of Michigan, Ann Arbor.

—— (2015a). "How to Be an Ethical Expressivist." *Philosophy and Phenomenological Research* 91: 47–81.

—— (2015b). "What Normative Terms Mean and Why It Matters for Ethical Theory." In *Oxford Studies in Normative Ethics*, vol. 5, edited by Mark Timmons, 296–325. Oxford: Oxford University Press.

—— (2016a). *Discourse Contextualism: A Framework for Contextualist Semantics and Pragmatics*. Oxford: Oxford University Press.

—— (2016b). "Update Semantics for Weak Necessity Modals." In *Deontic Logic and Normative Systems*, edited by Olivier Roy, Allard Tamminga, and Malte Willer, 237–55. Milton Keynes: College Publications.

—— (2017). "Modality, Weights, and Inconsistent Premise Sets." *Journal of Semantics* 34: 683–707.

——— (2018). "Commitment and States of Mind with Mood and Modality." *Natural Language Semantics* 26: 125–66.

Sloman, Aaron (1970). "'Ought' and 'Better.'" *Mind* 79: 385–94.

Smith, Nicholas (2003). "Changes in Modals and Semi-Modals of Strong Obligation and Epistemic Necessity in Recent British English." In *Modality in Contemporary English*, edited by Roberta Facchinetti, Manfred Kung, and Frank Palmer, 241–66. New York: Mouton de Gruyter.

Stalnaker, Robert (1975). "Indicative Conditionals." In Stalnaker (1999), 63–77.

——— (1978). "Assertion." In Stalnaker (1999), 78–95.

——— (1999). *Context and Content: Essays on Intentionality in Speech and Thought*. Oxford: Oxford University Press.

Starr, William (2010). *Conditionals, Meaning, and Mood*. PhD thesis, Rutgers University.

Stephenson, Tamina (2007). "Judge Dependence, Epistemic Modals, and Predicates of Personal Taste." *Linguistics and Philosophy* 30: 487–525.

Stevenson, Charles L. (1937). "The Emotive Meaning of Ethical Terms." *Mind* 46: 14–31.

Swanson, Eric (2008). "Modality in Language." *Philosophy Compass* 3: 1193–207.

——— (2011). "On the Treatment of Incomparability in Ordering Semantics and Premise Semantics." *Journal of Philosophical Logic* 40: 693–713.

——— (2012). "Imperative Force in the English Modal System." Handout, Philosophy-Linguistics Workshop, University of Michigan.

——— (2016a). "The Application of Constraint Semantics to the Language of Subjective Uncertainty." *Journal of Philosophical Logic* 45: 121–46.

——— (2016b). "Metaethics without Semantics." MS, University of Michigan.

Sweetser, Eve (1990). *From Etymology to Pragmatics: Metaphorical and Cultural Aspects of Semantic Structure*. Cambridge: Cambridge University Press.

Tagliamonte, Sal (2004). "*Have To*, *Gotta*, *Must*: Grammaticalisation, Variation and Specialization in English Deontic Modality." In *Corpus Approaches to Grammaticalization in English*, edited by Hans Lindquist and Christian Mair, 33–55. Philadelphia: John Benjamins.

Thomson, Judith Jarvis (2008). *Normativity*. Chicago: Open Court.

Traugott, Elizabeth Closs (1995). "Subjectification in Grammaticalisation." In *Subjectivity and Subjectivisation. Linguistic Perspectives*, edited by Dieter Stein and Susan Wright, 31–54. Cambridge: Cambridge University Press.

Van Linden, An (2012). *Modal Adjectives: English Deontic and Evaluative Constructions in Synchrony and Diachrony*. Berlin: Mouton de Gruyter.

Van Linden, An, and Jean-Christophe Verstraete (2008). "The Nature and Origins of Counterfactuality in Simple Clauses: Cross-Linguistic Evidence." *Journal of Pragmatics* 40: 1865–95.

——— (2011). "Revisiting Deontic Modality and Related Categories: A Conceptual Map Based on the Study of English Modal Adjectives." *Journal of Pragmatics* 43: 150–63.

Veltman, Frank (1976). "Prejudices, Presuppositions, and the Theory of Conditionals." In *Amsterdam Papers in Formal Grammar*, vol. 1, edited by Jeroen Groenendijk and Martin Stokhof, 248–81. Central Interfaculteit, University of Amsterdam.

Verstraete, Jean-Christophe (2001). "Subjective and Objective Modality: Interpersonal and Ideational Functions in the English Modal Auxiliary System." *Journal of Pragmatics* 33: 1505–28.

——— (2005a). "Scalar Quantity Implicatures and the Interpretation of Modality: Problems in the Deontic Domain." *Journal of Pragmatics* 37: 1401–18.

——— (2005b). "The Semantics and Pragmatics of Composite Mood Marking: The Non-Pama-Nyungan Languages of Northern Australia." *Linguistic Typology* 9: 223–68.

——— (2006). "The Nature of Irreality in the Past Domain: Evidence from Past Intentional Constructions in Australian Languages." *Australian Journal of Linguistics* 26: 59–79.

——— (2007). *Rethinking the Coordinate-Subordinate Dichotomy: Interpersonal Grammar and the Analysis of Adverbial Clauses in English*. Berlin: Mouton de Gruyter.

Wedgwood, Ralph (2007). *The Nature of Normativity*. Oxford: Oxford University Press.

Werner, Tom (2003). *Deducing the Future and Distinguishing the Past: Temporal Interpretation in Modal Sentences in English*. PhD thesis, Rutgers University.

Wertheimer, Roger (1972). *The Significance of Sense: Meaning, Modality, and Morality*. Ithaca, NY: Cornell University Press.

Wheeler, Samuel C. III (2013). *Neo-Davidsonian Metaphysics: From the True to the Good*. New York: Routledge.

Williams, Bernard (1981). "Practical Necessity." In *Moral Luck*, 124–31. Cambridge: Cambridge University Press.

Woisetschlaeger, Erich Friedrich (1977). *A Semantic Theory of the English Auxiliary System*. PhD thesis, MIT.

von Wright, Georg Henrik (1963). *Norm and Action: A Logical Inquiry*. London: Routledge and Kegan Paul.

Yalcin, Seth (2012). "Bayesian Expressivism." *Proceedings of the Aristotelian Society* 112: 123–60.

——— (2016). "Modalities of Normality." In *Deontic Modality*, edited by Nate Charlow and Matthew Chrisman, 230–55. New York: Oxford University Press.