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Three Essays in Formal Ontology

Prefatory Note

When first reading Husserl's *Logical Investigations* it is very easy to pass by the third as a minor detour from the high road of Husserl's major concerns. In common with many other readers, I initially held this view: the many distinctions Husserl makes seemed to me to be, to use his own words about Twardowski, 'as subtle as they are queer'. To anyone accustomed only to the extensional whole-part theories of Leśniewski or Goodman this is a natural reaction. My change of view was influenced partly by Kevin Mulligan's insistence on the pivotal role of the third investigation in Husserl's work, and also by the increasing recognition of the themes of unity, dependence and self-sufficiency treated by Husserl, as concepts echoing loudly throughout the history of ontology. It was also Kevin Mulligan who unearthed Ginsberg's 1929 article on Husserl's six theorems, and discontent with her criticisms spurred me to attempt a formalised reconstruction of Husserl's ideas, which met with various difficulties on the way to the first of these essays.

At the same time I was attempting to use mereological considerations to offer an alternative to what I consider the unacceptable account of number put forward by Frege, using Schröderian and Husserlian ideas suggested to me by Barry Smith. My original view was that numbers are properties of what I then called manifolds, i.e. aggregates considered as composed in some determinate way. This is what I should now call a group or aggregate theory of number. In the second essay I present the considerations which forced me to abandon such a view and to recognise the distinctive nature of pluralities as against aggregated individuals. This in turn led me to reappraise the notions of reference and set, with the result seen in the third essay, where a formal theory of mani-

fold, now reconstrued as comprising both individuals and pluralities, is developed. Some manifolds are aggregable: to such aggregates mereological considerations still apply. These issues are dealt with in the second essay, where the opposition to Frege is also explicitly set out.

At each turn I found voices of encouragement from the past, some from unexpected quarters. Harkening to these has convinced me that the logical and philosophical harvest of the fecund years between Husserl's *Philosophie der Arithmetik* in 1891 and Russell's *Principles of Mathematics* in 1903 is yet far from being reaped in full.

I. The Formalisation of Husserl's Theory of Wholes and Parts¹

§ 1 Introduction

Husserl's third Logical Investigation is called "*On the theory of wholes and parts*". It has probably received less attention from commentators than any of the other investigations, including the shorter fourth, which Husserl himself saw as an application of the ideas of the third to questions of grammar. The ideas put forward in the third investigation play a crucial role in Husserl's subsequent philosophy, and he was able to recommend them, even much later in his life, as offering the best way into his philosophy.² Although they did not perhaps present such an attractive clarion-call to research, they might, had Husserl's advice been followed, have made a much greater contribution to philosophical work than in fact they did. I should like to suggest that it is not too late to learn from the third investigation, and that, in a tidier form than they there receive, the ideas could become indispensable weapons in the conceptual armoury of the philosopher interested in ontology. This paper has the more modest purpose of attempting to clarify and interpret what Husserl was trying to say, with a view to eventually offering a rigorous treatment of the most important notions, and I wish also briefly to suggest where such notions might prove important in ontology.

It is important to distinguish formalisation from mere symbolisation. Any expression may be symbolised: one simply introduces symbols for various words or other expressions: the difference is merely one of the graphic shape of the expression. However, symbols, unlike the natural language expressions they can conventionally replace, derive their sense from the specific convention setting up their use, whereas this freedom of interpretation is not available for the original natural language expressions. For this reason symbols are more easily detachable from their specific interpretation, and may be manipulated purely syntactically, without interpretation. It is this feature which makes symbolisation such a useful way of presenting a formal theory. A formal theory, in Husserl's sense,³ is one in which no mention is made of any particular things or kinds of things, but which deals with objects in complete ab-

straction from their specific natures. A formal theory need not even be expressed symbolically: a statement such as, 'If a thing bears a relation to another thing, then the second thing bears the converse relation to the first',⁴ contains no restriction to particular domains of application, but consists purely of logical constants and formal concepts, such as *thing*, *relation*, and concepts such as *converse* definable in terms of these. It is advantageous to present formal theories symbolically because we may use symbols which are not given any fixed interpretation, but belong to a grammatical class which corresponds to a formal concept; they are then free to vary in interpretation over any entities whatever falling under that formal concept. So, if we allow the usual sorts of formal grammar, the above formal statement, could be symbolised (If aRb then $b\check{R}a$). Symbolisation usually proceeds further, with symbolisation of the logical constants, which may indeed be necessary if they need some degree of regimentation for the specific purpose in hand. In this sense, a symbolised presentation of a purely formal theory in Husserl's sense fulfils the conditions suggested by Wittgenstein as marking an adequate *Begriffsschrift*.⁵ Each formal concept corresponds to a different type of variable, i. e. symbol with variable interpretation. Only the logical constants are fixed.⁶

Husserl thought that a purely formal theory of part and whole was possible, and regarded the second part of the third investigation as offering the beginnings of such a theory. But, for all its detail, the investigation remains only a sketch of what a fully developed formal theory would look like, and like all philosophical sketches, presents problems of interpretation, lacunae, and vagueness, as well as being highly suggestive of possible fruitful developments. Although Husserl makes a brief and somewhat half-hearted venture into a partial symbolisation of a few theorems, the investigation is largely couched in Husserl's semi-technical German, and he nowhere attempts to set up a formal language in the modern logistic sense, which means that his formal treatment falls well short of modern standards in terms of the rigour of its symbolisation. While Husserl was by no means unfamiliar with symbolic logic as such, he was less interested in symbolisation for its own sake than in the philosophical treatment of concepts, even those concepts where, as in logic and mathematics, symbolisation had become indispensable to progress. He never believed that problems could be resolved purely by recourse to symbolisation, and rejected strongly formalist tendencies in mathematics, which would have us believe that mathematics is simply a

game with symbols which do not themselves have any meaning.⁷ It might be suggested that a theory of whole and part cannot be formal in Husserl's sense, since where – as in the work of Leśniewski and Goodman – it has been formalised hitherto, it has proved to be a proper extension of logic in the normally accepted sense.⁸ Against this it must be pointed out that Husserl clearly states that *whole* and *part* are purely formal concepts.⁹ Whether Husserl is correct on this, depends on what is taken as the criterion for being a formal concept. I do not believe that enough has as yet been done in clarifying the idea of a formal concept to give a definitive answer on this point. To that extent, the title of this essay promises something which it is not clear can be given. However, to the extent that we can eliminate from the theory all *other* concepts which are clearly not formal, to that extent we have succeeded in outlining what Husserl would call a theory of the pure forms of whole and part.¹⁰

Although advertised as a theory of whole and part, Husserl's investigation spends as much time on the concepts of dependence and independence, which, while they bear crucially on Husserl's particular brand of whole-part theory, cannot be counted as purely mereological notions. However, Husserl lays great stress on the distinction between dependent and independent parts as being the chief distinction among parts, and since it is in this distinction that Husserl's theory is distinguished from later and symbolically more adequate whole-part theories, I shall also consider the question of dependence and independence in some detail.

Husserl draws a distinction in the investigation between two different kinds of part or constituent of a whole. Some parts, those normally so-called, could exist alone, detached from the whole of which they happen to be part. These Husserl calls 'pieces' or 'independent parts' of the whole. On the other hand there are parts or constituents of a whole which could not exist apart from the whole or sort of whole of which they are part. These Husserl calls 'moments' or 'dependent parts' of the whole. For example: the board which makes up the top of a table is a piece of the table, while the surface of the table, or its particular individual colour-aspect, are moments of it. This distinction amongst kinds of parts is certainly not new: indeed it may be claimed to go back to the *Categories* of Aristotle.¹¹ Husserl himself certainly derived the distinction from his teacher Stumpf, who used the terms 'partial content' and 'independent content', in his discussion of the distinction within the realm of phenomenological psychology.¹² Husserl first used the distinc-

tion himself in his 1894 article "Psychological studies in elementary logic", where many of the distinctions later made in the *Logical Investigations* are already to be found.¹³ The later exposition contains two major advances on the earlier version: firstly a recognition that the dependent/independent distinction has application outside the sphere of psychological contents to ontology generally, and secondly, connected with this, the idea of a formal theory of whole and part, which, as we have said, Husserl sketches but does not completely execute in the second half of the investigation.

In the hands of later whole-part theorists such as Leśniewski and Goodman, whole-part theory has become associated with nominalism and extensionalism, where its general applicability and algebraic similarities with set theory make it a substitute for set theory more acceptable to those who have ontological objections to sets as abstract objects.¹⁴ Part of the interest in examining Husserl's whole-part theory is that it is free from such nominalist scruples, being conceived within the richly Platonist ontology of pure species adopted by Husserl at the time. It is, furthermore, non-extensional, making indispensable use of the concepts of essence and necessity. The basic distinction Husserl makes between dependent and independent parts is not even expressible in an extensional language. However it seems to me that one need not buy Husserl's package of Platonism and non-extensional language as a whole in order to make use of his whole-part theory. It is usually taken for granted that a non-extensional language brings ontological commitment to Platonic entities of some kind, whether species, meanings, or something like possible worlds. But it is far from clear that we can even manage to make reasonable sense of the actual world in a purely extensional language. It may, further, be possible to use a whole-part theory of Husserl's type to buttress a more sophisticated nominalistic approach to universals *via* Husserlian moments, so the usual yoking together of Platonism and non-extensionalism is far from clearly established.

One of the problems with the interpretation of the third investigation is that not all traces of Husserl's earlier psychological approach and interests have been expunged. This affects both the language within which Husserl makes his points, and the range of examples to which he generally makes recourse. Thus the word 'content' is frequently used where the word 'object' is also appropriate, and where the latter ought to be used in preference. This is despite Husserl's acceptance that his remarks hold for all objects generally, and not just psychological contents. The

examples are drawn almost exclusively from the phenomenological psychology of perception; for instance, that in the visual perception of a coloured thing, the moment of colour and the moment of spatial extension are both dependent parts of the thing as a whole, and require each other's co-occurrence in the thing. When this observation is transposed from the phenomenological to the ontological mode, this yields the proposition that the moment of colour and moment of extension of the thing itself (rather than the thing as perceived) are dependent parts of it. In this case the transposition seems to go quite smoothly, and I believe that it was Husserl's opinion that this would be so quite generally: for 'content' substitute 'object' and the theory has been in principle extended. It seems to me questionable whether the extension of the theory to objects in general is in fact so easy. Particular attention must be paid to the fact that some objects at least may belong to more than one kind at once, and that its dependence relations vis-à-vis other objects may vary according to the kind. This consideration is lacking from the psychological case, and so may have been at work in moulding Husserl's thoughts about the general properties of the more important part-whole relations. It is often difficult to tell, at crucial junctures in the text, whether the unthematized background of examples was playing a part, and if so, what part.¹⁵

Arising from this is the fact that it is in general possible to give the concepts of dependence and independence a much wider application outside the theory of whole and part. Husserl may not have been unaware of this, but he does not embark on any such general development. I have therefore allowed myself to go beyond the range of Husserl's examples in order to open up the question of such a generalised theory of dependence. The attendant risks of distortion and misrepresentation of Husserl's own position are I believe worth running if we are to put his ideas to work quite generally.

§ 2 Problems of Formalisation

There is a wide range of formal languages among which to choose when we attempt to formalise Husserl's ideas. Choice among these must be motivated by considerations partly external to whole-part theory as such. But whichever language is chosen, it cannot, if it is to do justice to Husserl's ideas, be extensional. The whole-part theories of Leśniewski

on the one hand and Leonard and Goodman on the other are both extensional. So a minimalist solution to the choice problem would be to add to one of these a necessity operator and axioms for it. One could for instance take the axioms and rules of S4 and graft these on to the Leonard-Goodman calculus of individuals. This approach has all the merits of timidity: it causes least disturbance. But there are drawbacks as well. Since Husserl was writing before it was appreciated how modal logic would proliferate different systems, there is no chance of receiving a direct answer from his writings as to which of the many available would be the best to choose. In view however of formula (3) below, which tells us that whenever species stand in a relation of foundation they do so of necessity, it appears that any modal system used would have to contain the characteristic S4 axiom $\Box p \supset \Box \Box p$ as a theorem. One obvious candidate modal system is accordingly S4. However, since the applications of modal considerations in the present context do not seem to require that we decide among alternatives whose differences do not show up in the sorts of formula we shall be considering, I shall in fact shirk the choice, and suggest merely that the modal axioms be not weaker than S4.

There is a problem about using a propositional necessity operator at all, in that traditionally the term 'essence' has related not to propositions but to properties, to *de re* rather than *de dicto* necessity. Husserl's writings show a willingness to accept both that individuals of certain kinds possess essential properties, and that there are general essences or *eide*, which are the abstract objects of imaginative variation among possibilities. For this reason I suggest that in addition to a necessity operator on propositions it is advantageous to consider a necessity operator on predicates, or property-abstracts. I shall use the expression 'nec' for this purpose. The operator was introduced by David Wiggins,¹⁶ who has given strong reasons, independent of Husserlian considerations, for believing that such an operator is indispensable to our ordinary conceptual scheme. It remains to be seen how 'nec' and ' \Box ' should be taken to interact, indeed whether a unified theory of them is possible at all. Because of these uncertainties, the account given in this paper must be regarded as only a tentative investigation into essentialistic whole-part theory.

There is yet a further reason for disquiet over simply grafting modal operators onto extensional mereology. For in extensional mereology (which I take to comprise both Leśniewski's mereology and the Leonard-Goodman calculus of individuals) a thing is identified with the

sum of its parts; indeed Goodman *defines* the identity of things as consisting in their having the same parts. But this rules out in advance the possibility of different things merely *coinciding* spatio-temporally. The case where such coincidence does not extend throughout the total life-span of both things is usually handled within extensional mereology by reconstruing things as four-dimensional space-time worms, and pointing out that temporary coincidence merely involves two such entities overlapping in a certain spatio-temporal region. However, there may also be cases in which we should wish to say that two things coincided over their total life-span, yet were not identical. This is connected with the fact that according to the everyday notion of a material thing, a thing can both gain and lose parts without prejudice to its identity, as can, most obviously, an organism.¹⁷ But a whole which conforms to the sum-principle of extensional mereology cannot lose any part. One way of avoiding recourse to four-dimensional objects, but which preserves the sum-principle, is Roderick Chisholm's theory of *entia successiva*.¹⁸ However, it seems somewhat drastic to abandon the paradigmatic role of organisms among material individuals for the sake of an abstract principle, when the normal three-dimensional thing-concept has not conclusively been shown to be beyond redemption. It would further be premature to abandon the normal conception in expounding Husserl's whole-part theory, if there is, as I believe, a chance that this very theory could provide assistance in explicating the normal conception of a thing.¹⁹

So I shall not be following a minimalist line: our mereology will not have the principle that coincident things are identical, and we shall use a *de re* necessity operator. It follows that the suggestions contained in this paper are largely exploratory: like Husserl's this is not a formal presentation with axioms and theorems, but an attempt to set out some of the possibilities and clarify some of the issues which need to be resolved before a formalisation of Husserl's ideas which is both intuitively and formally adequate can be presented.

One respect in which Husserl's whole-part theory is distinctive is its essential use of what Husserl calls *pure species*. I shall use lower-case Greek letters α , β etc. for such species, and lower-case Italic letters *a*, *b*, *c*, etc. for arbitrary members of α , β , γ respectively. Where we are treating an individual as such, in abstraction, as far as possible, from considerations of which species it belongs to, I shall use the letters *s*, *t*. Expressions of the form '*s* \in α ' will mean '*s* belongs to the species α '. But there

is here a problem of interpretation. What are such species? Do they indeed exist? If we follow Husserl in assuming that they do, we run the risk of building too many ontological presuppositions into the formalisation in advance. I shall accordingly give expressions of the form ' $s \in \alpha$ ' as far as possible a merely syntactic reading, allowing ' α ' to replace a common noun, and reading it as ' s is an α '. This leaves it open until later whether we should treat α as a proper name of a pure species, or of a set, or merely as a common name for s and maybe various other individuals. One thing to note, however, if we are to remain faithful to Husserl's way of construing species, is that we cannot allow contradictory species. Every species is, for Husserl, such that it *could* have members, even if it in fact does not. We shall accordingly make the informal stipulation that substituends for α , β etc. should be such that ' $\diamond (\exists x) (x \in \alpha)$ ' should be true.²⁰

Husserl explicitly warns the reader that he is using the term 'part' in a wider sense than it is usually given. He wishes it to comprise not only detachable pieces but also anything else discernible in an object, anything that is an actual constituent of it, apart from relational characteristics.²¹ In Aristotelian terminology, Husserl's parts would comprise parts normally so-called, accidents, and also boundaries.²² Doubts about the propriety of such a treatment are expressed by Findlay in the introduction to his English translation of the *Logical Investigations*. Findlay suggests that while there may be analogies between parts in the usual sense and individual accidents or moments, the two do not belong to the same category and it is therefore a mistake to treat them together ontologically. This does not recognise the expressly formal nature of Husserl's theory, for it is precisely the independence of restrictions to any particular category or region which mark what Husserl calls a formal theory. Husserl's account proceeds independently of doctrines concerning categories and category-mistakes.²³ The only way in which Husserl could be, in his own terms, mistaken, would be if he had confused either two formal concepts, or one formal concept and one material. Given only that Husserl does believe in individual accidents or property-instances, he cannot but treat them as falling within the formal concept of part. It is true that many philosophers have disputed whether there are such accidents. In answer it can be pointed out that not all the examples Husserl adduces as moments are property-instances; there are also boundaries, although he did not expressly include the latter until the later work *Experience and Judgment*.²⁴ It would be uncharitable to expect Husserl to pro-

duce a justification for treating of moments along with other parts in advance of judging how well the theory so produced managed to solve problems of unity and predication by comparison with other competing theories.

§ 3 Husserl's Basic Concepts: Whole and Foundation

The two most important concepts employed by Husserl in the third investigation are those of *whole* and *foundation*. Unfortunately, both these terms are ambiguous, and we must recognise their various senses before we can make clear sense of Husserl's theory. By contrast, Husserl does not make thematic the marks of the general concept *part* as such, but proceeds rather to make distinctions among the various kinds of part. It must be assumed that he considers the concept too primitive, being a formal concept, to allow of substantive elucidation.

Husserl distinguishes three different concepts of whole, a *narrow* concept, a *wide* concept, and a *pregnant* concept. The first two terms are mine; the last is Husserl's own. It is characteristic of Husserl's approach in the *Investigations* that he is reluctant to coin special terminology, even where he recognises ambiguities and is attempting to avoid them. This is in contrast to his later willingness to develop a specifically phenomenological vocabulary.

A narrow whole is one in which a number of entities are bound together into a unit by a further entity which Husserl calls a 'unifying moment' (*Einheitsmoment*). Narrow wholes are a rather special kind of whole, and cannot comprise all the wholes that there are. The supposition that all wholes are narrow in this sense leads, as Husserl points out in a passage reminiscent of Bradley,²⁵ to every complex being, appearances notwithstanding, infinitely complex. For if A and B are bound together by U, then A and U must be bound together by U₁, and so on *ad infinitum*. Husserl's own theory offers a way out of this regress of parts, by suggesting that some kinds of entity come together to form wholes just because they are the kinds of entity that they are, and thereby require partners, without requiring anything else which joins them together.²⁶

The wide concept of whole seems to me to be very like Goodman's concept of an individual; no restrictions are placed on how tightly or loosely connected the various parts of the whole are, whether they are

scattered or not, so long as we can still regard the whole as a single thing. It is indeed in the possibility of being regarded as a single thing that Husserl considers that bare unity consists.²⁷ This does not mean however that there are only individuals. Husserl expressly contrasts *unity* and *plurality* as formal concepts.²⁸ But any plurality may be taken together as something unitary, thereby founding a new higher unity, whose unity is, however, extrinsic to it, in the collective act.²⁹ So I shall allow as individuals anything which can possess a (singular) proper name. This will include even arbitrary *collectiva*. This liberality is reflected in extensional mereologies by allowing that arbitrary sums of individuals are themselves individuals. The reason for this is not that we wish to take most of these arbitrary *collectiva* seriously, but rather that it is not clear in advance where to draw the line between things which are wholes in this widest and weakest sense, and those which have some more intrinsic unity.³⁰

The third or pregnant concept of whole is defined by Husserl in terms of the concept of foundation. A pregnant whole is one each of whose parts is foundationally connected, directly or indirectly, with every other, and no part of the whole so formed is founded on anything else outside the whole. This of course presupposes Husserl's own concept of foundation, which means that Husserl attempts to define one sense of *whole* in terms of foundation, which in fact itself presupposes another concept of whole, the wide concept, which is, as Husserl points out, not a real or determining predicate.³¹ The unity of a pregnant whole is intrinsic to it, by contrast with the extrinsic unity of a mere sum or aggregate.

When we turn to foundation, matters are not so clear. It is most important to clarify Husserl's meaning here, since the concept of foundation turns out to be the most basic one of the whole investigation. I believe that we must distinguish two very different types of relation, both of which Husserl calls 'foundation'. There is a *generic* concept, which relates species, and there is an *individual* concept, which relates individuals which belong to species related according to the generic relation. It would in fact be more correct to speak of generic and individual concepts in the plural, since Husserl offers several formulations which do not exactly coincide, and it is possible to discern further definitional possibilities not considered by Husserl. It is chiefly in connection with the generic relations that one can speak, as Husserl does, of laws of essence. Husserl is mainly interested in the essential relations, and so does not offer an account of individual relations as such. But if one is to be

able to discuss the foundational relations of determinate individuals, such an account is needed, and there are crucial places in the investigation where Husserl is clearly talking about relations between determinate individuals, albeit individuals considered as belonging to a certain species. In his official introduction of the concept of foundation, Husserl, in addition to speaking of the case where the species α and β are foundationally related, also mentions the case where we should say that two members a and b of these respective species are themselves foundationally related.³² The definition of relative dependence and independence offered earlier speaks clearly of one thing's being dependent or independent relative to another.³³ In each case it is clear from either the context or the notation that the schematic letters used by Husserl are to be taken as singular terms.³⁴ Finally the notion of a pregnant whole requires that we talk about the foundational connectedness of the individual parts making up the whole. For these reasons an account of the foundational relatedness of individuals is necessary. However, Husserl was of the opinion that it is possible to move back and forth between talk about individuals and talk about species without difficulty, and so does not enlarge upon the difference.³⁵ It is however this difference which constitutes the major difficulty in developing a Husserlian whole-part theory.

Husserl defines foundation in the first instance as a relation holding between two pure species. The verbal rendering of the definition goes thus:³⁶

an α as such requires	foundation by a β	:=	there is an essential law to the effect that an α cannot exist as such except in a more comprehensive unity which associates it with a β .
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Later Husserl contends that the concept of whole or unity here employed is dispensable, and reformulates what he takes to be the same idea thus:³⁷ in virtue of the essential nature of an α , an α cannot exist as such unless a β also exists.

In this second version reference to a more comprehensive whole is missing. But this suffices to make the two concepts of foundation not equivalent. For according to the second definition, every species is self-founding. This means that, according to a statement Husserl elsewhere makes about absolute dependence,³⁸ everything is dependent absolutely. This is clearly not what Husserl wanted since it obliterates the distinc-

tion between dependent and independent objects. So the concept of foundation used in defining absolute and relative dependence cannot be the weaker second concept. One solution to this problem which readily suggests itself is that the species α and β have to be *different*. We could then say that something is dependent only if it is dependent on something belonging to another species.³⁹ This will not do, however, since it turns out that there are species which are non-trivially self-founding, which the suggestion does not allow.

So we shall revert to Husserl's first formulation, with its reference to a more comprehensive unity. This suggests that every α should be found together with a β in something of which the α in question is a *proper* part. In what follows, we shall use Goodman's symbols ' \ll ' for 'is a proper part of' and '<' for 'is a part of', where the latter allows, while the former excludes, coincidence. Hence our suggestion for a rendering of the definition is:

$$(1) \quad \Box(\forall x)(x \in \alpha \supset (\exists yz)(y \in \beta \ \& \ x \ll z \ \& \ y < z))$$

This condition appears still not to be strong enough. Whilst it captures the letter of Husserl's formulation it misses something of the spirit, in that in line with this definition the more comprehensive whole could be simply the β itself. This appears implausible as capturing the idea that we want: while we might say that the species *husband* is founded on that of *wife* (and vice versa of course) we should not want to say that because the existence of husbands required that of married couples that husbands are founded as such on married couples. This appears to have got marital carts before horses. Similar remarks would apply, *mutatis mutandis*, to foundation relations cited by Husserl, such as the mutual foundedness of colour-moments and moments of extension. Husserl takes foundation to be a relation of necessary *association*,⁴⁰ and the connotations of this word preclude either the α or the β in question from exhausting the more comprehensive whole of which each is a part. Indeed Husserl's formulation is itself ambiguous in that it could be read as implying that the whole is more comprehensive just than the α or as more comprehensive than both the α and the β , which is our second and preferred reading. Although Husserl does talk of wholes in the pregnant sense being founded upon the range of their parts,⁴¹ this is a regrettable equivocation, and probably stems from the etymology and previous use by others as well as Husserl of the word. In the sense we have formulat-

ed, everything which is founded on something is thereby dependent, whereas in this other sense we could describe even independent wholes as founded upon their parts. It would be better to describe such wholes as *constituted by* their parts, reserving the word 'foundation', despite its misleading etymology, for the associative relationship. However, we cannot merely strengthen the last conjunct ' $y < z$ ' of (1) to ' $y \ll z$ ', for it would follow from this that any species whose instances had to exist as part of some greater whole would thereby be a self-founding species. But while a lake cannot exist as such unless surrounded by land, and a child cannot exist as such unless it has parents, this cannot be regarded as making the species *lake* and *child* self-founding, whereas the species *sibling* clearly *is* self-founding, since a sibling cannot as such exist unless *another* sibling exists. So, using ' $\alpha \sqsupset \beta$ ' for ' α s are founded on β s' we arrive at the following definition of generic foundation:

$$(2) \quad \alpha \sqsupset \beta := \Box (\forall x) (x \in \alpha \supset (\exists y) (y \in \beta \ \& \ x \not\prec y \ \& \ y \not\prec x))$$

where ' $x \not\prec y$ ' abbreviates ' $\sim(x < y)$ '.

The essential nature of the foundation relation is expressed by the prefixed necessity operator. Since we have assumed the availability of the S4 principle $\Box p \supset \Box \Box p$, we have as a consequence that all generic foundation relationships hold of necessity:

$$(3) \quad (\alpha \sqsupset \beta) \supset \Box (\alpha \sqsupset \beta)$$

a result which would meet with Husserl's approval. It might be questioned, however, whether strict implication adequately fits the bill for expressing the relationship between α s and β s that we are aiming for. Should it perhaps involve some relationship of logical relevance, connecting the two species? For instance, would it be better to adopt the following as a definition of foundation:

$$(4) \quad (\forall x) (x \in \alpha \rightarrow (\exists y) (y \in \beta \ \& \ x \not\prec y \ \& \ y \not\prec x))$$

where the arrow represents the *entailment* connective?⁴² This would appear to be in harmony with Husserl's view of the relationship as arising out of the very nature of α s as such. It would preserve the theorem (3) above, since the logical system E of entailment has an S4 modal structure. And suppose that it is necessarily false that there be an α : would

not definition (2) make it trivially true that $\alpha \supset \beta$ for all species β ? This would also appear to favour an approach via entailment. But we have stipulated informally that no such case can arise, because it would violate the requirement that every species be such as to be capable of having instances, so this problem cannot arise as long as we remain within the limits imposed by this stipulation. While it would seem both possible and perhaps in the long run desirable to develop a foundation theory in terms of entailment or some other relevant connective, this course places additional difficulties in the path of interpreting Husserl, and so will not be followed here.

While the definition of foundation given above as (2) includes many important essential part-whole relationships, it does not include them all, so it is worth noting that the wider sense of foundation given by Husserl can be captured as follows:

$$(5) \quad \alpha \supset \beta := \square (\forall x) (x \in \alpha \supset (\exists y) (y \in \beta))$$

It is in this sense, rather than that of (2), that a whole which needs a part of a certain kind may be said to be founded on that part. For instance, it is essential to men that they possess brains, or tables that they possess tops. Such essential parts cannot however be described as being *associated* with the wholes which include them, since associated parts are co-ordinated, neither being the whole itself. It may be because Husserl was not quite clear which of the various possible essentialistic relations he wished to describe as foundation that we get from him more than one, non-equivalent definition. It is more to the point, however, simply to note the differences, remarking that both concepts of foundation have their uses. We shall in what follows concentrate predominantly on ' \supset ', since this appears to carry the greater weight for Husserl. However, as we shall see, some of the results which Husserl takes to hold for foundation in general hold for ' \supset ' but not for ' \supset '.

As an application of our definitions let us consider one of the propositions put forward by Husserl in § 14 of the Investigation, and for which he offers informal proofs. This is Husserl's Theorem I⁴³:

If an α as such requires to be founded on a β , every whole having an α , but not a β , as a part, requires a similar foundation.

To represent this we introduce by definition a complex general term ' $\alpha)\beta$ ', to be read 'object which contains an α but not a β as part'. Defini-

tions of general terms take the form of showing what condition an individual must satisfy to fall under the term, and accordingly have the form ' $t \varepsilon \alpha := (\dots t \dots)$ ', where ' t ' is an arbitrary singular term and ' $(\dots t \dots)$ ' stands for a sentential context containing occurrences of ' t ' but not of ' α ' or any other term defined in terms of ' α '. Thus we give a definition of ' α) β ' as follows:

$$(6) \quad t \varepsilon \alpha) \beta := (\exists x) (x \varepsilon \alpha \ \& \ x < t) \ \& \ \sim (\exists x) (x \varepsilon \beta \ \& \ x < t)$$

It is understood that if an open sentence of the form ' $x \varepsilon \alpha$) β ' occurs in a proof, that when replaced by an open sentence corresponding to the right-hand side of the definition (6), we reletter bound variables if necessary so as to ensure that scope problems do not arise; otherwise the use of open sentences containing defined general terms is the same as that when there are no bound variables present.

Given (2) and (6) it is a simple matter of modal predicate logic, using only the transitivity property of '<' to prove

$$(7) \quad \alpha \sqsupset \beta \supset \alpha) \beta \sqsupset \beta$$

and its necessity, which is the obvious way of representing Husserl's Theorem I.⁴⁴ Thus what Husserl confidently calls its axiomatic self-evidence is seen to stand up in the present formalisation.

While the relation ' \sqsupset ' is trivially reflexive, the relation ' \sqsupset ' is not. Only certain species are self-founding in the stronger sense. The most obvious examples are those using derelativised nouns. These do not figure as such in Husserl's examples, although his exposition uses such nouns a good deal. We can offer the following as examples: sibling, spouse, partner, colleague, cousin, accomplice, companion, fellow, enemy, peer, associate. The last example uses the very idea Husserl employs to characterise the foundational tie as such. We might offer examples of non-self-founding species such as house, mountain, planet.

A crude grammatical test for whether a noun corresponds to a founded species or not is to see whether it is natural to describe an α as, say, an α *of* something or someone. So every colour is the colour of something, every spouse is the spouse of someone, every planet is the planet of some star, every monarch is the monarch of some realm, and so on. The test is only crude, however, in that some founded species are not so spoken of, e.g. we do not call a lake or an island a lake *of* the sur-

rounding land, or island *of* the surrounding sea, and the word 'of' can mean many other things. Nevertheless the test is a useful rough guide. For self-founding species, for instance, it often makes sense to say that every α is an (or the) α *of* another α .

Some foundation relations between species are symmetric; Husserl calls such relations *two-sided* or *mutual* foundation.⁴⁵ For example the species husband and wife, or colour and extension, are mutually founding.⁴⁶ On the other hand, some foundation relations are not symmetric; these are *one-sided*. Thus in Brentano's psychology judgments are one-sidedly founded on presentations or ideas, while feelings of love and hate are founded on judgments, again one-sidedly, and hence indirectly on ideas. To take our geographical example again, a lake is as such one-sidedly founded on dry land. Several terms from physical geography show such one-sided foundation, e.g. mountain, plateau, cwm, island, peninsula, and so on.

Whereas ' \neg ' is transitive, ' \lrcorner ' is not. The definition (2) has to be examined to see why not. The conditions $\alpha \lrcorner \beta$ and $\beta \lrcorner \gamma$ do not suffice to show that $\alpha \lrcorner \gamma$, because if we have $a \in \alpha$, $b \in \beta$ and $c \in \gamma$ satisfying the conditions for (2), the fact that neither a and b nor b and c are part of one another does not suffice to show that a cannot have c as part or vice versa. Examples of this are hard to come by, but the following suggests itself: there cannot be a person conducting the defence at a trial unless there is a trial, and there cannot be a trial unless there is a defendant or defendants. But there is nothing to stop the or a defendant conducting the defence at the trial. Another consideration which reinforces the position that ' \lrcorner ' be not transitive is that if it were, all species two-sidedly founded on some other species, would by transitivity and symmetry be self-founding, in the strong sense, and this is surely not intended.

It is possible to define certain more general concepts relating to foundation if we allow ourselves to quantify over species, introducing bound general term variables. We may say that a species is *founded* or is *founding* according as there is a species it is founded upon or, respectively, founds:

$$(8) \quad \alpha \lrcorner := (\exists \xi)(\alpha \lrcorner \xi)$$

$$(9) \quad \lrcorner \alpha := (\exists \xi)(\xi \lrcorner \alpha)$$

Here we use a notational device which we find convenient elsewhere also: to represent existential generalisation by omission. We can also de-

fine an important concept of essential independence for species: α s as such are *essentially independent* when they are not founded:

$$(10) \quad I(\alpha) := \sim(\alpha \supset)$$

The other important concept of foundation concerns not the relations between species but those between individuals. Husserl, as we mentioned, brushes very lightly over the distinction, and in the 1929 commentary on the formal work in the third investigation by Eugenie Ginsberg, whose later article appears in this volume in translation, the distinction goes quite unnoticed.⁴⁷ If we are to be able to speak of foundational relationships between individuals at all, we cannot rest content with defining such individual foundation in terms of generic foundation, after a fashion such as this: a , as an α , is founded on b as a β := $a \in \alpha \ \& \ b \in \beta \ \& \ \alpha \supset \beta$. The first and most obvious reason is that just because $a \in \alpha$ and $b \in \beta$ and $\alpha \supset \beta$ it does not follow that a is founded on b . For b must be not just any β but *the right one*. If Alice, as a wife, is founded on Bob, as a husband, it is not sufficient that Alice be a wife and Bob a husband: they must be married *to one another*. Similarly, although, to use Husserl's example again, a moment of colouredness requires a moment of extension and vice versa, merely taking the colour-moment of one thing and the extension-moment of another does not yield the more independent colour-extension whole required.

A definition of individual in terms of generic foundation would be forthcoming were we able to specify a condition $F(a,b,\alpha,\beta)$ to be added as a conjunct to the right-hand side of the attempted definition above. I have been unable to find such a general condition, and indeed have come to believe, somewhat reluctantly, that there is none to be found. We can certainly find formulae for many particular cases: for example in the marital case we simply need the relation ' a is married to b '. But it is clear that in this case the species terms are derived from the relative by derelativisation. Furthermore, if Husserl is correct in saying that the foundational relations between species rest on the essential natures of the species in question, and not on formal considerations, we *ought not* to be able to find such a general formula.

One initially promising way of trying to define individual foundation from generic is to introduce different concepts of whole. For the whole formed by the colour of this + the extension of that is a mere whole, in the widest sense mentioned above, whereas the whole formed by the co-

lour of this + the extension of this is much more coherent. This coherence cannot consist in total independence however, since both the colour and extension may (and in this case do) require completion by something beyond them. Nor can it mean simply completability, since the colour of this + the extension of that can also be completed into a self-sufficient whole, namely this + that. Similarly, while Mr. Smith and Mrs. Jones do not form a maritally self-sufficient whole, together with Mrs. Smith and Mr. Jones they do, namely a pair of married couples. This sort of completion results in a whole which is in a certain sense too large, giving us the sum of two of the sort of whole we were looking for. So the whole resulting from completion must be specified more closely. It would seem that the best way to do this is to invoke the concept of a *pregnant* whole. The colour of this + the extension of this is part of a self-sufficient visual datum, say, which is not merely summed aggregatively with another. Similarly a single married couple is the smallest maritally independent whole; every member of the collection is maritally connected to every other, whereas in a pair of couples each member of a couple is maritally unconnected with the members of the other couple. The pregnant whole for the foundation relation in question offers the promise of being neither too large nor too small. But this concept is itself defined in terms of the relation of individual foundation, as we shall see below, so it cannot be invoked without circularity. I do not believe that Husserl saw the threat of circularity here, so it is not to be expected that we could find from his account any indication as to how it might be avoided.

Another suggestion would be that two foundationally related items can only be found together in one substance. This would mean restricting the examples of foundation relations unduly, since a planet would normally be regarded as a substance, yet planets as such cannot exist unless stars exist, for example. The suggestion is quite foreign to the spirit of Husserl's enterprise, for Husserl never speaks of substances. It would I think have been much more to his liking to work towards a definition or definitions of substance through his theory rather than the other way around. This is not to say that we cannot use the notion of substance to guide our investigation in various directions, merely that the general problem of individual foundation is not to be resolved by recourse to the notion. It accordingly seems best that we treat the concept of individual foundation as primitive.

That a particular α cannot exist without a β may be true: that it cannot

exist without the particular β which satisfies this requirement need not also be true: for Bob to be a husband he must be married, but he need not be married to Alice; he might have married Carol instead. So at the level of individual foundation a measure of unavoidable factuality enters in, though not to all relations of individual foundation.

One of the general problems facing a theory of individual foundation is the question as to how far we may be taking Husserl to be working within assumptions about logical form which are implicitly Aristotelian, and whether such assumptions must be rejected. This concerns in particular the question whether an individual may be an instance of different species such that its foundational relations to other individuals vary according to the species in question. For instance, Jupiter falls into the species *planet* and also *heavenly body*. *Qua* planet, Jupiter is foundationally related to the Sun, the substantive relation in this case being gravitational. But *qua* heavenly body, Jupiter is not founded on the Sun. The problem is that if we are to say, as Husserl appears to want to, that a given individual either is or is not founded on another individual, we have to either deny that an individual can belong to two co-ordinate species, or else insist that there is some one privileged species with respect to which all talk about foundational relatedness of an individual is to be carried on. This sort of supposition can be roughly characterised as Aristotelian, and there are indications of such a position in Husserl.⁴⁸ An obvious candidate for such a privileged species is an individual's *infima species*, the product species of all those to which it belongs, which would, on Husserl's view, have only that individual as extension. It would be what he calls an *eidetic singularity*.⁴⁹ The problem with this is that in order for such a species to guarantee individuation of the object in question it would, *pace* Leibniz, have to comprise relational characteristics. This is not in itself objectionable, since many of the clearest cases of foundation rest on relations. But again the contingency of many of the relationships into which a thing enters means that we should have to find a way to distinguish essential from accidental attributes of something in order to arrive at a stable and useful conception of individuals' relative dependence and independence. Also an *infima species* will almost certainly have an infinite intension, so it could not be a working tool for the investigation of individual foundation. To take this problem into account, we shall have to mark explicitly the species under which we are considering an individual's foundedness.

The individual foundational relations hold between individuals not

merely as such, then, but considered as belonging to given species. The only way in which this consideration can be excluded from explicit mention is either by generalising, or by assuming that for certain individuals there are species to which they could not but belong in order for them to exist at all, in other words to assume essentialism for individuals. We shall explore both possibilities.

The basic relation of individual foundation we can accordingly gloss, in full dress, as '*s, qua* α , is founded on *t, qua* β '. We shall symbolise this as ' $s_{\alpha} \sqsupset_{\beta} t$ '. The similarity of basic symbol is intentional, but note that it is flanked by singular rather than general terms, and is indexed by a pair of general terms. We must take care to distinguish this formulation from the similar sounding '*s, which is an* α , is founded on *t, which is a* β '. The latter, while mentioning the species to which *s* and *t* belong, does not, like the former, say that it is in virtue of belonging to these species that they are so related. If we take '*s* is founded on *t*' as merely meaning that *s* and *t* belong to *some* species whereby they are so related (cf. (20) below) then the latter form may be true while the former is false: e. g., it is true that Jupiter is a heavenly body, and is founded on the Sun, which is also a heavenly body, but it is not in virtue of Jupiter's being a heavenly body that it is founded on the Sun, but rather in virtue of its being a planet of the Sun.

Expressions like 'as such', '*qua*', 'in virtue of being', and others repeatedly used by Husserl and by ourselves in discussing foundation, are logically peculiar in that they do not form unrestrictive relative clauses as 'which', 'that' etc. do, but create an intensional context. To see this, let us take a pair of examples. Suppose the owner of the Casa Negra nightclub is also the husband of Dolores, its principal singer. Then while the following are true:

- (a) The owner of the Casa Negra cannot exist as such unless the Casa Negra exists.
- (b) The husband of Dolores cannot exist as such unless Dolores exists.

The sentences obtained by interchanging subjects of (a) and (b) are false. Similarly, supposing that all and only rational animals are featherless bipeds, it does not follow that

- (c) A rational animal as such (by nature) has two legs.
or that
- (d) Jones, *qua* rational animal, has two legs.

It follows that there is no such entity as Jones *qua* rational animal, which is to be distinguished from Jones *qua* loving father, for instance. Expressions like 'Jones *qua* loving father' are not genuine singular terms, but sentential fragments having the force e. g. of 'Jones is a loving father, and as such, he . . .' where the 'as such' creates the intensional context.⁵⁰ This property of 'as such' and related expressions throws into relief the difficulties about the connection between generic and individual foundation. To the extent that we have either to mention or otherwise assume, with expressions like '*qua*' or 'as such', a general kind or species, Husserl is right in taking individuals to stand in foundational relations in virtue of their belonging to species which stand in generic foundational relations.⁵¹

We can give a specification of the connection between generic and individual foundation by the following axioms:

- (11) $\Box (s \overset{\neg}{\alpha} \beta t \supset (s \varepsilon \alpha \ \& \ t \varepsilon \beta \ \& \ \alpha \supset \beta \ \& \ s \prec t \ \& \ t \prec s))$
 (12) $\Box (\alpha \supset \beta \supset (\forall x) (x \varepsilon \alpha \supset (\exists y) (x \overset{\neg}{\alpha} \beta y)))$

The converse implication to that given in (12) follows from (11) together with the definition (2) of generic foundation. This gives us the desirable result that α s as such are founded on β s if and only if any α is as such founded on some β as such. The appearance of triviality of this result disappears when it is remarked that 'founded on' does not mean the same in both occurrences. The indefinability of the individual relation in terms of the generic amounts to the lack of a general formula $F(s, t, \alpha, \beta)$ which could be added as a conjunct on the right of the implication in (11) so as to turn it into an equivalence.

§ 4 Dependence

Having dealt at length with the problems of foundation, we should now turn to the more general concepts of dependence and independence, which will of course vary according to the conception of foundation by means of which they are defined. Given the definition of foundedness (8) and essential independence (10) for species, we can define related notions of dependence and independence for individuals: an individual is *partly dependent*, written 'dep', when some species it belongs to is founded:

$$(13) \quad \text{dep}(s) := (\exists \xi)(s \varepsilon \xi \ \& \ \xi \neg)$$

while an individual is *totally independent*, written 'ind', when it is not partly dependent, that is:

$$(14) \quad \text{ind}(s) := (\forall \xi)(s \varepsilon \xi \supset I(\xi))$$

One could similarly define partial independence and total dependence. It follows from (11), (12) and (14) that an individual is totally independent if and only if it is not founded in any way on any other individual. It should be noted that by definition no individual can be self-founding, since every individual is a part (albeit improper) of itself. By contrast, some species are, as we have seen, self-founding.

The condition of total independence is extraordinarily strong, because of the universal quantification. It might be wondered what, if anything, could satisfy it. Since, according to orthodox cosmology, God falls under the term 'creator', and there can be no creator without creatures, even God would not, according to this view, be totally independent, being reciprocally founded on his works.

Because of the strength and uncertainty of application of such conditions, it would appear advantageous to develop more readily applicable conditions. One way to do this is to attempt to distinguish in individuals those species to which they belong of necessity from those to which they belong adventitiously. It is here that we shall use Wiggins' *de re* operator 'nec'. This will be used to offer a faithful formal rendering of such expressions as '*s* must be an α ', '*s* is essentially/necessarily/by its very nature an α '. This would normally be written, using property-abstraction, as

$$[\text{nec } (\lambda x)(x \varepsilon \alpha)](s);$$

however, to avoid unnecessary symbolic complication, I shall adopt the abbreviation

$$s! \varepsilon \alpha$$

and in general, for any *simple* predicate, where there is no risk of confusion, *de re* necessity will be marked by an exclamation mark after the occurrences of terms of which the predicate holds of necessity; so '*s*! < *t*!' will be short for $[\text{nec } (\lambda x)(\lambda y)(x < y)](s, t)$ and so on.

We may now define an individual as being *essentially independent*, written ‘essind’, when every species to which it belongs of necessity is independent:

$$(15) \quad \text{essind}(s) := (\forall \xi)(s! \varepsilon \xi \supset I(\xi))$$

while an individual is *essentially dependent* when it is not essentially independent:

$$(16) \quad \text{essdep}(s) := (\exists \xi)(s! \varepsilon \xi \ \& \ \xi \neg I)$$

Armed with these new concepts we may resolve the theological problem about God’s dependence on the world, in a manner suggested by Aquinas,⁵² by noting that since God need not have created the world, his being a creator is not essential to him, so he can be secured essential independence. The world, on the other hand, is essentially dependent, at least according to the traditional cosmology. The only possible candidate for total independence on such a view would be the totality comprising both God and the world.

According to traditional theology, the world is dependent on God both because he created it and because he continuously sustains it. The Husserlian concept of dependence covers both kinds of dependence, because it makes no reference to time. So something which needs to be produced by something else, but which can thereafter survive without this, is dependent on it in a different way from that in which something is dependent on something which it requires to exist at every time at which it exists itself. It is worthwhile contrasting the views of Husserl on dependence with those of his student Ingarden. In his chief work, *Der Streit um die Existenz der Welt*, Ingarden distinguishes four basic senses of dependence/independence.⁵³ Since these are given in opposing pairs, we need only characterise one of each pair. They may be set out in a table as follows:

- | | |
|----------------------|-----------------------------|
| (1) Autonomy | – Heteronomy |
| (2) Originality | – Derivation |
| (3) Self-sufficiency | – Non-self-sufficiency |
| (4) Independence | – Dependence. ⁵⁴ |

An object is autonomous or self-existent if it has its existential foundation in itself, is immanently determined.⁵⁵ An object is original if, in its

essence, it cannot be produced by any other object.⁵⁶ An object is self-sufficient if it does not need, by virtue of its essence, to coexist with something else within a single whole.⁵⁷ Finally, an object which is self-sufficient is independent if it does not require, by virtue of its essence, the existence of any other object which is also self-sufficient.⁵⁸ Ingarden draws attention to Husserl's examination of dependence and independence in the third investigation, but regards the eight concepts he sets out as belonging to a kind of theory which Husserl did not recognise, which Ingarden calls existential ontology, and which he contrasts with both formal and material ontology. There are considerable differences of background between Husserl's and Ingarden's respective treatments of dependence and independence, which we cannot enter into here. It is clear however that Ingarden's distinctions (2)–(4) could be variously interpreted within Husserl's theory of foundation. Ingarden in particular models his (3) on Husserl's definition of foundation. The difference between (3) and (4) is not highlighted by Husserl, and in making it Ingarden must have in mind some concept of whole stronger than the wide concept employed by Husserl. The only one of Ingarden's pairs which does not obviously fall within the general Husserlian account of foundation is (1).⁵⁹

If an object t is essentially independent, it follows that it is possible that t has no supplement, i. e. that t could constitute all there is, there being no whole (in the wide sense) of which t were a part. This possibility, which shows the self-sufficiency of the object in a perspicuous light, coincides with the conception of an object which is something for itself (*Etwas-für-sich*) in the late ontology of Brentano, as formulated by Chisholm:⁶⁰

$$(17) \quad t \text{ is Etwas-für-sich} := \Diamond \sim (\exists x)(t \ll x)$$

This coincidence of notions is an interesting sidelight on the otherwise very different worlds of Husserl's ontology of the *Logical Investigations* and the ontology of Brentano in the *Kategorienlehre*. It suggests that Husserl's concepts of dependence and independence could contribute valuable insights to the problem of substance, which looms much larger for Brentano.

If $s \sqsupset_{\alpha} t$ then, as we have pointed out, it may be quite accidental to t as such, even t as a β , that it should *satisfy* the requirement for s for a β . One fact which shows this clearly is the possibility in certain cases of *dis-*

junctive satisfaction. Suppose, for instance, that Brown is a cat-owner. Then, as such, he must possess some cat. But he may possess more than one, each of which would, on its own, be sufficient to render him a cat-owner. At any time at which he owned more than one cat, the loss of one would not affect his status as a cat-owner. Indeed, provided he replaced cats as they died or he lost them etc., he could, barring catastrophe, remain a cat-owner for a time-span far longer than the life of any of his cats. In a similar way, a man is biologically dependent for continued life upon a regular supply of oxygen, water and nutrients, but the particular consignment of such material which actually sustains him will vary widely over time.⁶¹ Similar considerations apply to those parts of a thing which are essential to its being the sort of thing it is, but which can suffer replacement without the thing's ceasing to exist, either because it has more than one, and can acquire more as need be, or if it can temporarily survive without one. The replacement of cells in organisms gives an example of the first kind, while the repairing of machines gives one of the second.⁶²

Having defined the dependence and independence, whether essential or not, of individuals, we should now define *relative* dependence and independence, concepts of which Husserl makes much use in the investigation. We first define some more general concepts of individual foundation, following the practice established earlier of marking generalisation by omission of symbols.

- (18) $s_\alpha \sqsupset t := (\exists \xi)(s_\alpha \sqsupset_\xi t)$
 (19) $s \sqsupset_\beta t := (\exists \xi)(s_\xi \sqsupset_\beta t)$
 (20) $s \sqsupset t := (\exists \xi \eta)(s_\xi \sqsupset_\eta t)$

The general concept of foundation given by (20) does not make it explicit why s is founded on t . This more general concept frequently occurs in Husserl's exposition.

Husserl defines relative dependence as follows:⁶³

A content α is relatively dependent with regard to a content β (or in regard to the total range of contents determined by β and all its parts), if a pure law, rooted in the peculiar character of the kinds of content in question, ensures that a content of the pure genus α has an *a priori* incapacity to exist except in, or as associated with, other contents from the total ranges of the pure genera of contents determined by β .

I have quoted this in full because it illustrates vividly the sorts of problem of interpretation we face in the investigation. In the middle of what purports to be a definition, which should therefore be totally unambiguous, one finds inserted hedges and adjustments, which make a significant difference to the sense. It also displays Husserl's indifference to the possible problems of an individual's belonging to various species, since the same schematic letters are used for species and for members of these.

Three possible concepts of relative dependence suggest themselves to me on the basis of this passage. The first is that relative dependence is nothing other than individual foundation. This is arrived at by simply ignoring the bracketed adjustments in the passage and the clause 'or as associated with'. This identification may not be exact, because of the ambiguity of the phrase 'determined by β ', which might refer to parts of β , or essential parts of β , or simply some species to which β belongs (it must be remembered that here we are following Husserl's ambiguous lettering). It may be that the concept of individual dependence here suggested is not quite the same as that given by our (11)–(12).

By taking account of the adjustments beginning 'or . . .' we may arrive at the reading that s , say, is not directly founded on t but on something 'in its range', i. e. something which is, in the widest sense, a part of t . So we have the following alternative concept of individual relative dependence:

$$(21) \quad \text{dep}_1(s,t) := (\exists x)(x < t \ \& \ s \sqsupset x)$$

According to (21) anything which is founded on something else is thereby dependent₁ with respect to it, a result which is quite in the spirit of Husserl's exposition. The converse to this is not true: an object may be dependent₁ on another without being founded on it. To take an example from Eugenie Ginsberg's discussion of the Investigation, the shape of a particular brick is founded upon other aspects of the brick, and so this individual shape is dependent₁ upon the wall of which the brick happens to be a part, yet the shape could hardly be said to be founded upon the wall.⁶⁴ Ginsberg does not however distinguish between foundation and relative dependence, and so some of her attempts to show that Husserl's theorems are not all valid are vitiated. The wide concept of dependence₁ here canvassed is perhaps somewhat unnatural, and we should perhaps take closer cognisance of the phrase 'the total range of contents determined by β and all its parts'. There is, I think, no telling exactly

what this phrase is intended to mean, but the Ginsberg example suggests that we choose not merely an adventitious part of the whole t as something upon which s is founded, but rather take a part which t could not but have, i. e. something u such that $u < t!$, using our abbreviated device for showing essential predicates. This then suggests a third possible concept of relative dependence:

$$(22) \quad \text{dep}_2(s,t): = (\exists x)(x < t! \ \& \ s \sqsupset x)$$

According to this sense, whenever an individual is founded on another, it is also dependent₂ upon it, since for any individual t it is true that $t < t!$.

Dependence₂ does not reduce to foundation however. For one thing, s may be dependent₂ on t and at the same time a part of it, which means that, according to (11), s cannot be founded on t . In a case such as this we may say that, in one sense at least, s is a *dependent part* of t :

$$(23) \quad \text{deppt}_2(s,t): = \text{dep}_2(s,t) \ \& \ s < t$$

while of course it is similarly possible to define another sense of 'dependent part' through dep_1 :

$$(24) \quad \text{deppt}_1(s,t): = \text{dep}_1(s,t) \ \& \ s < t$$

It is clear of course that if $\text{deppt}_2(s,t)$ then $\text{deppt}_1(s,t)$: the second sense is stronger than the first. The close connection between foundation and dependent parts may be seen by the following theorem:

$$(25) \quad s \sqsupset t \supset \text{deppt}_2(s,s+t)$$

where $s+t$ is the aggregate or sum of s and t : the theorem follows from definition (23) together with the result that $t < (s+t)!$; clearly the very sum $s+t$ could not but have had t as part.⁶⁵ This shows that anything which is founded on something else is thereby a dependent part of a whole which is more comprehensive than either the founded or the founding part. It is because of this that the three notions of foundation, dependence, and being a dependent part, are so readily confused. It may be that Husserl himself did not make the distinctions so clearly as we have drawn them, but there is, as has been shown, sufficient evidence

from the investigation to show that such fineness of distinction can, and was perhaps intended to be, read from the text.

Recalling the supposition that entities may essentially belong to certain species, being the individuals they are, we can introduce various notions of essential foundation and dependence which are stronger than those we have used hitherto. We can for instance describe an individual s as *essentially founded* on an individual t when s is founded on t in virtue of some species to which s belongs essentially:

$$(26) \quad \text{essfd}(s,t) := (\exists \xi)(s! \varepsilon \xi \ \& \ s \sqsupset_{\xi} t)$$

while t essentially founds s when s is founded on t through a species to which t must belong:

$$(27) \quad \text{essfg}(s,t) := (\exists \xi)(t! \varepsilon \xi \ \& \ s \sqsupset_{\xi} t)$$

and a yet stronger relation can be obtained either by conjoining these, or, stronger yet, by insisting that the species α, β such that $s \sqsupset_{\alpha} t$ are such that $s! \varepsilon \alpha$ and $t! \varepsilon \beta$.

We have already mentioned the possibility of disjunctive or generic satisfaction of an individual's need by other individuals. For though s may be in some sense essentially founded on t , this may not mean that s could not have been essentially founded on something other than t satisfying the same requirement. To take a biological example, an organism as such is, let us suppose, essentially founded at any time on some consignment of water, but any other consignment would have done equally well. Similarly an internal combustion engine is essentially founded on a supply of lubricant (here it is obvious that we mean a functioning engine, not a museum-piece), but again which particular mass of lubricant does the job is not important. A ship-launching ceremony might be thought to be essentially founded upon a bottle of champagne, but it need not have been just the one which was used. In other cases, however, an individual s is not only essentially founded on some other thing t , but it could only have been t upon which it was so founded. In such cases we may introduce definitions based on formulas such as $s \sqsupset t!$, $s! \varepsilon \alpha \ \& \ s \sqsupset_{\alpha} t!$, and $t! \varepsilon \beta \ \& \ s \sqsupset_{\beta} t!$; which can themselves be used to further define notions of dependent parts. So we might be then equipped to say in what sense it is essential to a man that he has not just any brain, but this very brain, whereas it is not essential to him that he have this very

heart, or in what sense it is essential to a person that he or she should have the very parents he or she did have.⁶⁶ There is perhaps at present little point in doing more than indicating that there is here a wide range of questions and issues, some of them bearing on regularly-debated issues such as personal identity, together with a rich fund of possible concepts of dependence, all developed out of Husserl's ideas, and requiring further refinement.

We can however indicate a possible formulation of Husserl's attempt to define the pregnant concept of whole in terms of foundation. We need here individual foundation, as was argued earlier. Firstly we define *direct foundational relatedness*: two things are directly foundationally related when one is founded on the other:

$$(28) \quad \text{dfr}(s,t) := s \supset t \vee t \supset s$$

Then we define *foundational relatedness* as the (proper) ancestral of the relation of direct foundational relatedness:⁶⁷

$$(29) \quad \text{fr}(s,t) := \text{dfr}^{>0}(s,t)$$

Thus two entities are foundationally related if one founds the other, or both found or are founded on some third thing, or one founds and the other is founded on some third thing, etc. Then an entity is a *pregnant whole* when all its parts, in this case its proper parts, are foundationally related to one another, and no part is foundationally related to anything else outside this entity:

$$(30) \quad \text{Prwh}(s) := (\forall xy)(x \ll s \supset ((y \ll s \ \& \ x \neq y) \equiv \text{fr}(x,y)))$$

While it is thus not too difficult to express Husserl's idea symbolically it is much harder to see what it amounts to in practice. In theory the world should partition itself neatly into discrete entities, each of which is a pregnant whole. (Entities are discrete when they have no common part.) It might however be the case that every entity is foundationally related to every other, in which case there would be no partition, and only one pregnant whole, the world itself. This result would certainly be counted as in some sense monistic. It is possible that the sort of whole which Husserl had in mind when discussing pregnant wholes would be lesser in extent; to capture such wholes we might need to take a tighter foundation

relation as the basis for the definition of foundational connectedness. In general, the stronger such a relation, the tighter the organisation of the resulting wholes, the smaller in extent they are, and the more there are of them. So it seems that rather than there being a single concept of pregnant whole, there are several, having in common a recipe for generation from a concept of individual foundation. This is a characteristic outcome of studying the third investigation: ideas which at first sight seem sharp show themselves to hide various possible interpretations.

§ 5 Husserl's Six Theorems

An illustration of the difficulty is the attempt to interpret the six theorems of § 14: one has to use these as a guide to what Husserl meant at the same time as attempting to see whether they are valid or not. It is instructive to examine these and Husserl's proofs for them. We already saw above how Theorem I, interpreted as (7), is valid. Here is Theorem II:⁶⁸

A whole which includes a non-independent moment without including, as its part the supplement which that moment demands, is likewise non-independent, and is so relatively to every superordinate independent whole in which that non-independent moment is contained.

Husserl states that this follows from Theorem I as a corollary, given a definition of relative dependence. But he is wrong in this. Theorem I is stated in terms of species, whereas Theorem II relates to individuals. Here is a place where the transition between these two levels is not so simple as Husserl believes. We can give an example of things satisfying the intuitions represented by Theorem II which do not in any obvious way satisfy those of Theorem I. Let us call any expression which requires completion by only names or other singular terms to yield a sentence a *predicate*. Then the English verb 'loves' is a predicate, requiring completion by two names to obtain a sentence. In the sentence 'John loves Mary' the names 'John' and 'Mary' satisfy this double requirement. Now the predicate 'loves Mary' also has a requirement for supplementation by a name, and in the given sentence this requirement is met by the name 'John'. We might say that in the given sentence the predicate 'loves Mary' *inherits* from the predicate 'loves' that requirement which is met by the name

'John'.⁶⁹ This is in conformity with the way in which the first part of Theorem II is phrased: the predicate 'loves Mary' does not contain *all* the supplements demanded by its part 'loves', and so inherits from the latter the demand satisfied by 'John'. But the most obvious way of expressing this in the terms of Theorem I is to substitute the term 'predicate' for ' α ' and 'name' for ' β '. But in that case we should render ' $(\alpha)\beta$ ' as 'predicate which does not contain a name as part': but precisely 'loves John' *is* a predicate which contains a name as part. It may be that this particular kind of multiple satisfaction was not considered by Husserl in his phrasing of Theorem I. To show that Theorem II does indeed follow from Theorem I we should have to be assured that *whenever* we have things satisfying the premisses in Theorem II we can always find a pair of species α and β such that Theorem I is satisfied with respect to the supplement which the larger whole inherits from the smaller moment. It seems to me dubious that we should be able to establish this in full generality, so it may be that Husserl's theorems require another axiom to support them, such as the following:

$$(31) \quad (s \supset_{\beta} u \ \& \ s < t \ \& \ u \not< t) \supset t \supset_{\beta} u$$

Some such principle does indeed seem to be taken as self-evident by Husserl, but it cannot be directly proved from the proof of Theorem I, because it is compatible with the principles of this theorem that a is an α , b is a β such that $a \supset_{\beta} b$, and that c is an $(\alpha)\beta$, and so itself requires a β for completion, but rather than inheriting a 's requirement satisfied by b , its requirement is satisfied by some further β , say b' . It is hard to find a convincing example of this state of affairs, which leads me to concur with Husserl. The nearest to a counterexample that I have managed is this: let ' α ' be replaced by 'represented district' and ' β ' by 'representative': the relevant whole being a district together with its representative. Now a council ward may be part of a parliamentary constituency, but the constituency, even if it does not contain the councillor who represents the ward, does not inherit the requirement for him, but has its own requirement met by its Member of Parliament. However, the force of this purported counterexample is somewhat blunted by the possible ambiguity in the notion of 'district', which might, one may say, have a bare geographical meaning and a more sophisticated administrative one. It might be argued that it is only in the administrative sense that a district's representation requirements arise, whereas it is only in the geographical

sense that the ward is part of the constituency. In administrative terms the ward is not part of the constituency, but a completely different entity entering into quite different governmental arrangements. It is here that we face the problem of whether it is one and the same thing which is both a council ward and part of the parliamentary constituency, or rather whether these two *coincide*.

Given such uncertainties, it is far from apparent that Theorem II is, as Husserl takes it to be, a mere corollary of Theorem I. For this reason I shall confine myself to discussing the *consequences* of (31) taken as axiomatic, together with our other assumptions, rather than attempt to establish (31) or something like it. It can be seen that Theorem II follows very readily from (31), in its two parts, if interpreted as follows:

- (32) $(s \supset u \ \& \ s < t \ \& \ u \not< t) \supset \text{dep}_1(t, u)$
 (33) $(s \supset u \ \& \ s < t \ \& \ u \not< t \ \& \ u < v) \supset \text{dep}_1(t, v)$

In fact we can show not just (32), but the stronger formula obtained by replacing the consequent of (32) by ' $t \supset u$ '. Further, there does not appear to be any need for Husserl to restrict the superordinate wholes v merely to those which are independent. With these minor reservations, we can endorse Husserl's Theorem II provided we are prepared (a) to gloss 'dependent', as 'dependent₁' and provided (b) we accept (31).

Husserl's Theorem III is given in two versions: these both in effect amount to the transitivity of the relation 'is an independent part of'. We shall use therefore a simple version:

If s is an independent part of t and t is an independent part of u then s is an independent part of u .

To clarify this we must first give a definition of 'independent part'. The obvious one will do:

- (34) $\text{indpt}_1(s, t) := s < t \ \& \ \sim \text{dep}_1(s, t)$

One could also define similarly a relation indpt_2 based on the relation dep_2 but the one we have given here fits the bill more closely. For in the presence of (31–3) it becomes easy to prove that

- (35) $(\text{indpt}_1(s, t) \ \& \ \text{indpt}_1(t, u)) \supset \text{indpt}_1(s, u)$

by much the method Husserl uses in his informal proof of Theorem III, except that Husserl appeals both to Theorems I and II, whereas, because of the difficulties we have alluded to, we appeal only to (31) and its consequences.

Irrespective of the merits of (31), Husserl's fourth theorem is valid. His formulation is:⁷⁰

If s is a dependent part of a whole t , it is also a dependent part of every other whole of which t is a part.

We can represent this as

$$(36) \quad (\text{deppt}_1(s,t) \ \& \ t < u) \supset \text{deppt}_1(s,u)$$

and it follows immediately from the definition of deppt_1 and the transitivity of the part-whole relation ' $<$ '. In fact it is a more general thesis that

$$(37) \quad (\text{dep}_1(s,t) \ \& \ t < u) \supset \text{dep}_1(s,u)$$

It should be noticed that this was the assumption questioned by Ginsberg in her brick example, and the principle is harmless once the difference between individual foundation and the more general relation of relative dependence, in the sense of dep_1 , is made clear. One particular restriction of (36) yields the transitivity of deppt_1 . It must be noted that both deppt_1 and indpt_1 are transitive, but that the former is in many ways the more obvious notion. For as Husserl defines relative independence, it does not entail independence *tout court*, whereas this is true for relative dependence. The reason can be seen in the notion of independent part. That a is an independent part of b means only that a is not founded on anything within the range of b ; it does not mean that there is not something else outside b upon which a is founded. Husserl states this explicitly as his Theorem V: to represent this we must give some derelativised notions of dependence and independence derived from the relative notions we have been using. It is for instance possible to define ' s is founded' as meaning simply ' s is founded on something', and similarly for ' s is dependent'. But because of the interrelation between dep_1 and \neg these amount to the same thing, so we shall simply say

$$(38) \quad \text{dep}_1(s) := (\exists x)(\text{dep}_1(s,x))$$

and define something as independent₁ when it is not dependent₁:

$$(39) \quad \text{ind}_1(s) := \sim \text{dep}_1(s)$$

The nice thing about this definition is that we can link now the notion of independence and dependence of an individual previously given as (13–14) in terms of its membership of a species, with the new derelativised notions stated in terms of individuals; by virtue of the principles (11–12) the following is a theorem:

$$(40) \quad \square (\forall x)(\text{ind}(x) \equiv \text{ind}_1(x))$$

so naturally the two contraries, dep and dep_1 , are necessarily equivalent also. This shows that the detour through relative dependence and independence brings us back to the same position as we started from when considering the generic concept of foundation.

Husserl's Theorem V simply says

A relatively dependent object is also absolutely dependent, whereas a relatively independent object may be dependent in an absolute sense.

and we can see how, in our interpretation, this is unproblematically correct.

The final Theorem VI reads

If a and b are independent parts of some whole c , they are also independent relative to one another.

If we render this as

$$(41) \quad (\text{indpt}_1(a,c) \ \& \ \text{indpt}_1(b,c)) \supset \sim (\text{dep}_1(a,b) \vee \text{dep}_1(b,a))$$

then brief consideration shows that it is true, for were either a or b dependent on the other, since each is a part of c , the dependent one would by definition be dependent on c , contrary to the assumption; this is precisely the form of reasoning followed by Husserl in his proof.

We can thus see a way through the six theorems of § 14. Given the axioms and definitions hitherto suggested, the principle (31), which Husserl took to be self-evident, and the selection of dep_1 and not dep_2 as the relevant notion of dependence, all six follow. It is suggested then that

this constitutes an acceptable interpretation of what Husserl meant, which has the merit of making the theorems all valid if the axioms (11–12,31) are valid. This verdict on the semi-formal work of § 14 may be contrasted with that of Ginsberg,⁷¹ whom we suggested did not separate individual foundation from relative dependence, and whose criticisms of Husserl cannot therefore be accepted.

If, as suggested earlier, there are various possible concepts of dependence and independence which we could formulate without being unfaithful to Husserl's intentions, then it would be necessary to test these against the six theorems of § 14 in much the same way as we have done for the concepts connected with dep_1 . But the tests would be more complex, because of the essentialistic nature of many of the stronger definitions. After § 14 Husserl moves on to discuss various other whole-part notions which can be defined in his terms, such as mediate and immediate parts, abstractum and concretum, etc. These will obviously inherit any ambiguities possessed by the basic notions. Rather than follow up all the various possible interpretations, I shall instead turn to possible applications of Husserl's concepts within ontology. Applications in grammar, in particular the question of the dependence-status of different sentence-parts, and the structure of sentences, I hope to deal with elsewhere. For a summary of other applications which have been made, the reader should consult the essay by Smith and Mulligan earlier in this volume.

§ 6 Applications

One problem which was very much a live issue in Husserl's day, but which subsequently became buried, is the question of a distinction between ordinary or genuine objects and objects of higher order.⁷² Such a distinction was fundamental to Meinong's theory of objects, and suggests a kind of logical or ontological atomism whereby the basic objects are those of lowest order, there being aggregates, classes and complexes constituted on the basis of these. Husserl's account of categorial objects, or objects of the understanding, is very much in the same vein,⁷³ and Ingarden too, defends the difference between his concepts of self-sufficiency and independence by invoking this distinction.⁷⁴ Findlay has suggested, in commentary on Meinong, that the implied atomism is untenable.⁷⁵ We have, in Husserl's concepts of the third investigation, the

wherewithal for re-examining the issues. It may be simply misleading to regard objects with other objects as their pieces as somehow less self-sufficient than the pieces. The organs of an organism, while pieces of the organism in the sense that they are both separately presentable and physically separable,⁷⁶ considered as living tissue they are dependent for their continued existence on that of the organism of which they are part; in this sense they are moments rather than pieces of it. For the most vital organs, this dependence is reciprocal. The way is quite open to allow that some larger objects are in fact more self-sufficient than their smaller parts. One example which is mentioned by Husserl,⁷⁷ and which Findlay also cites as militating against the atomistic view of objects,⁷⁸ involves time. Temporal durations, considered not merely as abstractly extended parts of an abstract extended whole, but as concretely occupied by events and processes in the natural world, can no longer be seen as mere pieces, but must be regarded as dependent parts or moments of the whole. This suggests that the ontology which conceives of the world as made up of four-dimensional entities, of which the familiar three-dimensional objects of everyday experience constitute merely temporal cross-sections, is mistaken in supposing that temporally determined objects are sliceable in time in just the same way as a thing is sliceable in space. The theory of four-dimensional space-time objects can be accused of failing to distinguish between things and processes.

A similar consideration might help to dampen somewhat that perennially appealing aspect of all forms of atomism, micro-reductionism. If an entity can be shown to be complex, to consist of parts in a determinate relation to one another, it is the assumption of micro-reductionism that everything which could be meaningfully said about the complex could be expressed mentioning only its parts and their properties and relations. There is no doubt that in many areas of empirical investigation our understanding of entities is furthered by seeing how they are put together. The gains in understanding achieved fuel the drive to find ever more fundamental particles or constituents of matter in physics. It is sometimes suggested that there is no end to how far such reductions can be carried. But the assumption need not go unchallenged. At some stage of our knowledge of the physical world it might be reasonable for the philosopher to suggest that the bunch-of-grapes model of complexity is not the appropriate one. This might occur when the known fundamental particles fall into families by their characteristics, but there has been a prolonged inability to isolate the supposed constituents of these. Rather

than seeing the particles as consisting of more fundamental ones held together by a particularly strong natural glue, it might be hypothesised that the more fundamental parts of the isolable particles are not pieces but moments, which are mutually founding. As Husserl pointed out, such parts need not have any other part or constituent whose job was to hold them together, but require each other by their very nature. Such moments might be compared with the distinctive features of phonological theory,⁷⁹ which cannot be isolated but which explain the resemblances of phonemes, which can.

A rather similar but less universally appealing kind of unifying reduction of explanation is reduction *upwards*, macro-reduction, which seeks explanation of phenomena in terms of the objects in question belonging to some more inclusive totality with its own properties, a whole of which they can be seen to be mere moments. The supreme macro-reductionist was Hegel. Like the micro-reductionist, the macro-reductionist claims that nothing gets lost in his reductive explanation. An intermediate position might contend that micro- and macro-reductionism make opposite but cognate mistakes, the micro-reductionist taking all part-whole relations as relations of piece to whole, while the macro-reductionist takes all such relations as relations of moment to whole. The benefit of the observations drawn from Husserl is not just that it gives us a way to draw the parallels between the atomist and the holist, but that because there are various possible senses of dependence and cognate concepts, it can be made clear that there is not just one possible atomism or holism, but several, so that atomism of one kind might be quite compatible with holism of another. The atomist who sees a man as an aggregate of particles, and the holist who sees him as a mere mode or moment of some greater whole, may simply have different criteria for what it is to be an independent whole.

The question as to what constitutes a *natural* whole is probably not one which could receive a single answer. Which entities constitute natural wholes is something which cannot be settled *a priori*, but must be the concern of the empirical sciences. The sorts of object which we consider as having a tightness of organisation making it fitting to call them wholes in a natural sense seem to have a greater degree of causal coherence, and relative causal isolation from outside phenomena, than those which we should be less inclined to describe as natural wholes. The necessity to speak in terms of *degrees* of isolation and coherence suggests that there can be a spectrum of natural wholes of which some are more clearly

units than others. The paradigmatic examples of natural wholes would appear to be organisms, although these too can be from certain points of view taken as mere moments of some greater whole, involving say a species or an eco-system, while from other points of view they are aggregates of other wholes, such as cells, molecules etc., which have an integrity of their own. Other natural unities are not dissimilar from organisms e. g. in the manner in which they are able to utilise energy. Thunderstorms and river-systems have been suggested as examples.⁸⁰ Aristotle considered stars were not only natural but living unities, an opinion which is by no means so implausible as it appears at first sight.⁸¹ Such a readiness to see analogies between living or organic unities and other natural wholes need be neither anthropomorphic nor need it deny the ubiquity of causal explanation, since it is precisely the causal integrity of a natural whole or system which binds it together. This is not something imposed on reality from outside by our mode of cognition, but represents organisation which is intrinsic and which we discover.

According to this way of considering the multiplicity of ways in which things are connected in the physical world, the distinction between lower- and higher-order objects need not be an absolute one, with a single bedrock layer of natural units, but an object may be from one point of view a natural unit, from another it may coincide with an aggregate of differently organised units, or again be a moment of a greater whole. The fact that objects are naturally organised in many ways ensures that this relativity is not the mere imposition of a conceptual scheme on an otherwise unstructured world, but cuts along natural seams in reality.

When we move from considerations of units in nature to units in other spheres, such as social, legal and economic wholes, causal considerations are no longer so predominant, although they still apply. The unity of many man-machine wholes, such as a manned vehicle, is still predominantly one of relative causal self-containedness, while that of social wholes such as clubs, families, societies, or the various differently-sized units in an army or a business enterprise, require further considerations relating also, e. g., to functions and lines of control or authority. Such considerations may cut across those of causal or spatio-temporal proximity. It is, again, the merit of the vocabulary developed by Husserl that such matters can be discussed without an undue reliance on metaphor, and in full recognition that there will be very many different kinds of relation constituting the various kinds of whole brought into consideration.

One strand in the skein going to make up the traditional notion of substance is that a substance is what exists by itself, without needing the existence of anything beyond itself. In Husserl's terms, such an object is absolutely independent. Given the many different possible senses of 'independent' we could envisage various different senses of 'substance'. It might indeed be the case that some of the historic disputes over substance could be clarified by showing how different philosophers were operating with different concepts of independence. It is noteworthy that Husserl nowhere speaks in the third investigation of substance. His account is furthermore purely formal, and proceeds without assumptions as to which sorts of object are the most basic or paradigmatic independent wholes.

§ 7 Relations and Foundation

We have mentioned in several places the importance of relations between parts of a whole in constituting it as the whole it is. Many of our examples used nouns with a clearly derelativised sense, such as 'husband', 'sibling' and so on. We can very often generate one or more such nouns from a relative term, sometimes artificially. Sometimes the derelativised nouns are common enough to be etymologically unconnected with the relative term in question, as e. g. 'husband' and 'wife' have no etymological connection with the relative 'is married to', and may indeed be far more familiar than the relative notion which defines them. The term 'lake' for instance corresponds to no cognate verb expressing the relation of being land surrounding an expanse of water. Generally speaking, the more closely related things are affected in their properties by their particular relation, the more likely we are to have derelativised nouns to describe the relata as such. This is a partial explanation for the richness of the vocabulary of derelativised nouns dealing with human social and kinship relations, for the relations human beings have to one another mark and are marked by characteristic forms of behaviour of the people concerned.

It might be thought that we always can generate a foundation relation whenever we can obtain a pair of derelativised nouns from a relative term. Suppose for instance that given any binary relation R we define a pair of nouns by derelativisation as follows:

- (42) $t \in R_1 := (\exists x)(tRx)$
 (43) $t \in R_2 := (\exists x)(xRt)$

Does it follow automatically that $R_1 \supset R_2$ and $R_2 \supset R_1$? The answer is no: while we automatically get that $R_1 \supset R_2$ and vice versa, the stronger condition imposed by (2) means that R gives rise to foundation relations in the strong sense under these conditions:

- (44) $R_1 \supset R_2$ iff $\square(\forall x)(x \in R_1 \supset (\exists y)(xRy \& x \not\prec y \& y \not\prec x))$
 (45) $R_2 \supset R_1$ iff $\square(\forall x)(x \in R_2 \supset (\exists y)(yRx \& x \not\prec y \& y \not\prec x))$

Clearly any relation which is symmetric and for which (44–5) held, would give rise to a derelativised *self-founding* species term: for example from ‘possesses the same parents as and is different from’ we get ‘sibling’ while ‘is working together with’ gives ‘collaborator’.

Certain relative terms which possess etymologically related derelativised nouns fail this test, perhaps rather surprisingly. For example ‘employs’, ‘loves’, ‘shaves’, with their nouns ‘employer’ / ‘employee’, ‘lover’ / ‘loved’ etc. have neither of the cognate pair of nouns founding the other. The reason is that it is possible that all employers, lovers, shavers, etc. employ, love and shave only themselves. In general, so long as a relation *could* be reflexive, even by accident, i. e.

- (46) $\diamond(\forall xy)(xRy \supset x = y)$

then there is no reason why either of R_1 , R_2 should be founded on the other. Of course, in the weaker sense of foundation given by ‘ \supset ’, there is always reciprocal foundation: there can be no employer without an employee, no lover without a loved one etc. But where general reflexivity is possible, this sort of requirement is not a requirement for an *associated* entity as such. It follows that any relative term possessing the logical property of reflexivity, including all equivalence relations, all partial orderings and especially identity, fails to give rise to foundation relations in the strong sense.

One obviously germane relation is the whole-part relation. In fact, if we consider the relation of being a proper part, symbolised ‘ \ll ’, we shall see that this gives rise to one self-founding derelativised term. For ‘ \ll_1 ’ is self-founding, whereas it is not true that $\ll_1 \supset \ll_2$, or that $\ll_2 \supset \ll_1$, or that $\ll_2 \supset \ll_2$. The reason that none of the last three is true is that we can

envisage the situation where the world consists of precisely two atoms, i. e. is a whole with only two proper parts. Again, it is certainly true that $\ll_1 \not\sqsupset \ll_2$ and vice versa, i. e. that there cannot be a proper part unless there is a proper whole or container, and vice versa, but the stronger relation of founding is ruled out by the restrictions of (2) as manifested in (44–5). The obvious noun-phrase corresponding to ‘ \ll_1 ’ is simply ‘proper part’. Because of the mereological law that to every proper part of a whole there must correspond a complementary proper part of that whole, i. e. an object disjoint from it (sharing no parts) which together with it makes up the whole, or, symbolically:

$$(47) \quad \Box(\forall xy)(x \ll y \equiv (\exists z)(x \sqcap z \ \& \ y = x + z))$$

it follows that \ll_1 or ‘proper part’ stands for a self-founding species. It turns out then that even the terms ‘whole’ and ‘part’ are derelativised from one or other of the relations ‘is part of’ or ‘is a proper part of’; this fact leads Husserl into local difficulties in expounding the idea of an independent part, since while it is natural to say that a (proper) part as such cannot exist apart from its whole, for independent parts we also want to say that the object which is here in fact a part *could* exist outside this particular whole.⁸² The difficulty is only one of expression, however, not of substance.

Having seen how foundation relations may arise of relative terms, we might turn the issue round and ask whether all foundation relations point back to some underlying and more basic relative term. The question must first be made more precise however, since for any pair of species α, β such that $\alpha \sqcap \beta$, we always have the relative term ‘ $\alpha \sqcap \beta$ ’. We are trying to get beyond this however and ask whether an α which is founded on a β is so because of some relation which is not defined in terms of α and β , but which may indeed be used in definition of these terms, as in the case of derelativisation already mentioned. If we follow Husserl’s opinion on this, we should have to deny it. For Husserl claims that although *colour* and *extension* are mutually founding, there is nothing in the concepts colour and extension which points to any such underlying relation.⁸³ It is Husserl contends, precisely in this lack of a means to render the law of mutual dependence for colour and extension as an instance of a *logical* or formal principle that there consists the synthetic *a priori* status of the statement that colour is impossible without extension and vice versa. Were it possible to treat ‘colour’ and ‘extension’ as nouns

definable by derelativisation from some antecedently given relative term, the dependence in question would be analytic rather than synthetic. While it seems to me that Husserl's distinction between analytic and synthetic is not so sharp as he thought it was,⁸⁴ the mere possibility that there should be acceptable cases of foundation where the necessity is not obviously logical leaves in doubt the possibility of always finding an underlying relation.

It is worth considering a way of making a distinction among relations which can be found at its clearest perhaps in Meinong, who also brings this distinction into play when discussing the difference between genuine and higher-order objects.⁸⁵ Some relations, such as difference, similarity, being the same height, and the like, do not bring their terms into any real connection, but rather leave them quite unaffected by being thus related. Standing in such relations makes no difference to the properties of the terms; it is indeed often the case that they stand in such a relation in virtue of the separate properties that they possess. Such relations are themselves built or founded on their terms. We may call these *ideal* relations. Other relations, such as acting upon, magnetically attracting, playing tennis against, bring their terms into connection in that, had the relation not obtained, the properties of one or both of the terms would have been different. We may call these *real* relations. The most obvious examples of real relations involve some causal link. Now some foundation relations have underlying relative terms corresponding only to ideal relations, which means that the unity engendered by the foundation is in a sense extrinsic to the objects related. Many ideal relations are equivalence relations, and since these are reflexive they are in any case, by the result above, powerless to engender genuine foundation relations. But where there is some real connection between the terms of a relation, these terms, described in a way which implies the properties induced by the relation, will, if the relation in question satisfies one of (44–5), be foundationally related. We could then describe the relation as a moment of the whole uniting the parts. While these remarks are only schematic, it does seem to me that a theory of the unity of wholes can only be developed in conjunction with an adequate theory of relations: the two enterprises must proceed together. It is perhaps not accidental that the importance of the interconnection between relations and wholes only arises as a serious issue once the Leibnizian dogma that whatever exists is one is called into question.

One of the considerations we derive from examining the role of rela-

tions in engendering foundation is the impoverished role of reflexive relations, including especially equivalence relations. The role of the latter in modern theories of abstraction is well-known. But from the ontological point of view reflexive relations are as such highly dubious. While we may be perfectly prepared to allow a relative *term* to be flanked by a pair of names for the same thing, and yield a true sentence, it is a different matter again if we ask what *relation* corresponds to the term. The whole notion of a relation which holds between a thing and itself is suspect,⁸⁶ and the more especially when, in the case of identity, it can only hold between a thing and itself. This difficulty can be found for instance in Hume and Wittgenstein.⁸⁷ It is usual these days to dismiss their problem as a pseudo-problem resulting from the confusion of a sign with the thing signified.⁸⁸ But the objection is not that there is a certain kind of relative term which can generate true sentences. It is rather that nothing intrinsically relational is represented by this sign, if indeed anything at all is represented. Nor is this to deny the cognitive value of such relative terms. It is to object that they are ontologically sterile. Where a reflexive relation may also hold between different things, as e.g. 'is the same height as', it can always be traded in for the anti-reflexive variant, e.g. 'is the same height as and different from'. Such terms may now generate foundation relations between their derelativisations. Indeed those perplexing derelativisations like 'employer' / 'employee', etc. are most happily applied when reflexivity is not envisaged: it does sound wrong to describe a self-employed person as either an employer or an employee, or a narcissist as a lover, and it is because of such anti-reflexive uses that we have the derelativised nouns at all. It may be of more than etymological interest that many of the terms for equivalence relations are in fact derived from their associated adjectives or nouns, even, it should be noted, *identity*.

Notes

References in these notes are to works listed in the bibliography at the end of these three essays. Works are cited under the name and year in which they appear there.

References to Husserl's *Logische Untersuchungen* (Husserl, 1900–01) will be to the volume and page of the 5th edition of 1968, which will be abbreviated *LU*, and to the page of the English translation of Findlay 1970, abbreviated *LI*. Section numbers, unless otherwise specified, are to the third investigation.

- ¹ An earlier version of this essay was read at the Colloquium 'Whole-Part Theory and the History of Logic' held by the Seminar for Austro-German Philosophy at the University of Sheffield in May 1978. My thanks go especially to David Bell, Kevin Mulligan, Herman Philipse and Barry Smith for their help and constructive criticism.
- ² When William Kneale visited Husserl in Freiburg in January 1928, he relates that Husserl "told me that his essay *Zur Lehre von den Ganzen und Teilen* in his *Log. Unt.* was the best starting point for a study." (From part of a letter to Herbert Spiegelberg quoted in Spiegelberg, 1971, n. 25, p. 78.)
- ³ For Husserl on formal theories see *LU I* §§ 67–72. The ideas are expanded considerably in Husserl, 1929.
- ⁴ The example is Husserl's: *LU II/1* 254, *LI* 457. A purely formal proposition which is true is free of all existential assumptions, § 12 *ibid.*
- ⁵ Wittgenstein, 1961, 4.1272 tells us that words like 'object', 'concept', 'complex' and 'fact' signify formal concepts, and are represented in a *Begriffsschrift* by variables. Cf. also 3.325.
- ⁶ This concept of variability of all propositional constituents except the logical constants can be found already in Bolzano. Cf. his definition of logical analyticity and universal satisfaction in Bolzano, 1837, §§ 147–8, a work which influenced Husserl profoundly.
- ⁷ For explicit repudiations of formalism in mathematics cf. e.g. Husserl, 1929, § 39.
- ⁸ For an introduction to Leśniewski's work see Luschei, 1962 or Lejewski, 1958. For Leśniewski the division between logical and non-logical theories comes between his Ontology and his Mereology, so for him whole-part theory contains non-logical constants. Leonard and Goodman, 1940 or Goodman, 1977 blur such a distinction by defining identity mereologically. For an axiomatisation of the whole-part theory in Goodman, 1977 see Breitkopf, 1978.
- ⁹ *LU II/1* 252, *LI* 455.
- ¹⁰ Such a theory is explicitly canvassed at § 24.
- ¹¹ Aristotle, 1928, Ch. 2, where Aristotle contrasts being part of a subject with being in a subject in such a way as to be incapable of existence apart from it.
- ¹² Stumpf, 1873.
- ¹³ Husserl, 1894.
- ¹⁴ But compare my third essay below, where a nominalistically acceptable conception of set is described.
- ¹⁵ Cf. the discussion in § 18, where Husserl is not altogether clear whether it is possible to give examples of proper parts of a whole which are not proper parts of proper parts of this whole.
- ¹⁶ Wiggins, 1976. Cf. the mereological applications in Wiggins, 1980.
- ¹⁷ Cf. Locke, 1975, Book II, Ch. 27: "In the state of living Creatures, their Identity depends not on a Mass of the same Particles; but on something else. For in them the variation of great parcels of Matter alters not the Identity"; p. 330.
- ¹⁸ Chisholm, 1976, Ch. 3 and appendices A–B.
- ¹⁹ Wiggins, 1980 uses a whole-part theory strengthened with the operator *nec* to argue for this conception and against Chisholm's *entia successiva*.
- ²⁰ Cf. § 12 of the fourth investigation, where it is declared that 'round square' cannot correspond to any object: *LU II/1* 326, *LI* 517. Later, in Husserl, 1948, § 91 the extension of a pure species is said to comprise pure *possibilities*.
- ²¹ § 2, *LU II/1* 252, *LI* 455.
- ²² In commentary on Aristotle, Anscombe in fact replaces Aristotle's accident example by a boundary example: Anscombe and Geach, 1961, pp. 7–8.
- ²³ It is indicative of Husserl's low reliance on categories that he is very reluctant, by comparison with later philosophers, to brand sentences as nonsensical. Cf. his distinction in Investigation IV, § 12, between nonsense and absurdity.

- ²⁴ Husserl, 1948, § 32a. In § 32b Husserl adds *connections* as yet a further distinct kind of dependent part to accidents (which he calls 'qualities') and boundaries.
- ²⁵ § 22, *LUII/1* 280, *LI* 478–9.
- ²⁶ *Ibid.*, *LUII/1* 279, *LI* 477.
- ²⁷ As Husserl says, *LUII/1* 280, *LI* 478, "Unity is... a categorial predicate."
- ²⁸ § 11, *LUII/1* 252, *LI* 455.
- ²⁹ Perhaps the clearest statement of this is in § 148 of Husserl, 1913. Cf. Beilage 74 of the *Husserliana* edition (p. 625), where Husserl clarifies the statement in the text. It will become clear in my third essay below that I do not share Husserl's view that a nominalisation is necessary to constitute a set as a new object on the basis of plural reference.
- ³⁰ Cf. Husserl on *collectiva* at Investigation VI, § 51, *LUII/2* 159, *LI* 798. Though the section title also mentions *disjunctiva* the section has strangely nothing to say about such things.
- ³¹ § 22, *LUII/1* 279–80, *LI* 478.
- ³² § 14, *LUII/1* 261, *LI* 463.
- ³³ § 13, *LUII/1* 258, *LI* 460.
- ³⁴ In § 14 this is seen by the *ad hoc* use of a suffix, in § 13 by the lack of articles. Husserl's usage of symbols is sloppy by modern standards.
- ³⁵ Cf. the remark that we can use the same expressions for individuals and species as a 'harmless equivocation'. § 14, *LUII/1* 261, *LI* 463.
- ³⁶ *Ibid.*
- ³⁷ § 21, *LUII/1* 275, *LI* 475.
- ³⁸ Cf. Proposition 5 of § 14, *LUII/1* 263, *LI* 465.
- ³⁹ This suggestion was made to me by Barry Smith, as an improvement on an earlier formulation of mine which required that α s and β s be such that no α ever be part of a β or vice versa. Both these ideas are inadequate, as the case of self-founding species shows.
- ⁴⁰ § 14, *LUII/1* 261, *LI* 463.
- ⁴¹ § 21, *LUII/1* 276, *LI* 475.
- ⁴² On entailment see above all Anderson and Belnap 1975.
- ⁴³ § 14, *LUII/1* 262, *LI* 463.
- ⁴⁴ This is one place where the definition of foundation using entailment suggested at (4) is too strong for our purposes, because the definition of ' α ' β ' is extensional while that of ' \neg ' in (4) is relevant and intensional, and the restrictions in E on e. g. importation make it impossible to prove (7) as it stands. However by strengthening the definition (6) a relevant version of (7) could be proved.
- ⁴⁵ § 16.
- ⁴⁶ Herman Philipse has objected that the husband/wife type examples are analytic, whereas Husserl is clearly interested in synthetic connections such as the colour/extension example. Two things may be said in reply. Firstly, the distinction Husserl draws between analytic and synthetic is not as sharp as he thought it was. This is an issue which I hope to take up elsewhere, though note the remarks in n. 77 of the opening essay by Smith and Mulligan. Secondly, as Husserl is really interested in an *a priori* theory (§ 24) no harm at all can be done by including analytic as well as synthetic examples.
- ⁴⁷ Ginsberg, 1929. Cf. my note to the translation of her later paper in this volume.
- ⁴⁸ Cf. Husserl, 1913, § 14.
- ⁴⁹ *Ibid.*, § 12.
- ⁵⁰ For an implicit recognition of this in Leibniz, cf. Leibniz 1903, 261. Cf. more explicitly Ishiguro, 1972, 16.
- ⁵¹ I had previously thought that it made some sense to talk of individuals simply as such, without mention or assumption of any kind to which they might belong. Many sources have dissuaded me of this view, but David Bell and Herman Philipse have done so most directly.

- ⁵² Aquinas, 1964–76, Ia, 44, 4; Vol. 8, p. 21, where Aquinas argues that in creating God does not act from need but ‘simply to give of his goodness’. At the same time the idea that God’s nature could have been other than it is not particularly congenial for Aquinas, so his problem is not completely cleared.
- ⁵³ Ingarden, 1964–5, §§ 12–15.
- ⁵⁴ For a note on these translations cf. my introduction to Ginsberg’s paper.
- ⁵⁵ *Ibid.* § 12.
- ⁵⁶ *Ibid.* § 13.
- ⁵⁷ *Ibid.* § 14.
- ⁵⁸ *Ibid.* § 15.
- ⁵⁹ One of Ingarden’s examples of a heteronomous object would be any entity which is purely noematic, a correlate of consciousness, so Ingarden could fairly claim that the material for such a distinction exists already in Husserl.
- ⁶⁰ Chisholm, 1976, 208. This paper in general furnishes abundant evidence that late Brentano was working using whole-part theoretic considerations akin to those we find in Husserl, which is not surprising, given his influences on his pupils, in particular Husserl’s former teacher Stumpf. Unfortunately we have not here the space to compare Husserl and Brentano at length.
- ⁶¹ The example, though not the application of it, is drawn from Ingarden, 1964/5, § 15. The idea of disjunctive satisfaction of requirements clearly has applications in biology. The importance of such biological considerations is urged in the Preface to Wiggins, 1967.
- ⁶² Because Chisholm denies that a genuine entity may lose parts, he must construe organisms and machines as less than genuine, with an identity which is a simulacrum of true identity. Cf. Chisholm, 1976. This is a thought which can be found *inter alia* in Hume and Leibniz, and in a modified form pervades extensional mereology. It is certainly attractive, and more tractable than the Aristotelian alternative, but I am convinced it is wrong.
- ⁶³ § 13. *LU* II/1 258, *LI* 460.
- ⁶⁴ Ginsberg 1929, 112. Cf. also my prefatory note to her paper in this volume.
- ⁶⁵ Wiggins uses this consideration in his 1980 to discredit the idea that a cat can be identical with the mereological sum of its body + its tail, for the cat, but not the sum, could lose the tail.
- ⁶⁶ As suggested in Kripke, 1972, 312 f.
- ⁶⁷ This notation for the proper ancestral is due to Carnap. Cf. his 1954, § 36.
- ⁶⁸ § 14, *LU* II/1 262, *LI* 464.
- ⁶⁹ On Frege’s use of whole-part terminology to describe the phenomenon which he calls ‘unsaturatedness’ of predicates and concepts see his late essays “Die Verneinung” and “Gedankengefüge”, Frege, 1976a. I have expanded elsewhere on the appropriateness of using Husserlian ideas in this connection. Frege’s use of terms like ‘ergänzungsbedürftig’, unlike that of Husserl, is not backed by a theory of dependent and independent parts. Indeed, if we are to believe his remarks in Frege, 1895, Frege had a rather low opinion of whole-part theory in general.
- ⁷⁰ § 14, *LU* II/1 263, *LI* 464. We have adjusted the symbolism to our convention.
- ⁷¹ Ginsberg, 1929.
- ⁷² Cf. Meinong, 1899.
- ⁷³ On categorial unities see § 23, for instance. The notion can be found throughout Husserl’s writings.
- ⁷⁴ Ingarden, 1964/5, § 15, n.
- ⁷⁵ Findlay, 1963, 148f.
- ⁷⁶ The idea of separate presentation here derives from Stumpf, 1873. Cf. § 3 for Husserl’s comments, and the historical remarks in Ginsberg’s paper in this volume.

⁷⁷ §25, *LU* II/1 292, *LI* 488.

⁷⁸ Findlay, 1963, 150.

⁷⁹ The Prague and Moscow schools of linguistics were in fact influenced by the third investigation. Cf. Jakobson, 1973, 13–4, Holenstein, 1975.

⁸⁰ Cf. Pantin, 1968, 37f.

⁸¹ Aristotle, 1930, 292^a 19f.

⁸² § 11, *LU* II/1 253, *LI* 456.

⁸³ *Ibid.*

⁸⁴ As mentioned in n. 46 above.

⁸⁵ Cf. Findlay, 1963, 141f. Husserl in fact makes a very similar distinction between two sorts of relation in Husserl, 1887 and 1891a. His terminology is however more unfortunate, since he calls the relations 'physical' and 'psychical' rather than 'real' and 'ideal'. The term 'psychical' indicates not that the relation is mental, but that it is *of a sort with* the relation between object and content of an idea. Cf. Findlay, 1963, 35, where it is made clear that the mental relation is ideal for Meinong. The terminology of Husserl readily misled Frege into criticising Husserl's theory of number as psychologistic, which it was not. For a clear refutation of the myth of Husserl's early psychologism see Willard, 1974. We have, for obvious reasons, adopted the less misleading terminology of Meinong.

⁸⁶ Cf. the arguments against reflexive relatedness in Vol. 2 of Armstrong, 1978, 91f.

⁸⁷ Hume, 1978, Book I, Pt. IV, § II, p. 200f. Wittgenstein, 1961, 5.53f.

⁸⁸ Cf. Quine, 1960, § 24.