A Fortiori Logic

Innovations, History and Assessments

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Dayeinu: al ahat kamah vekamah tovah kefulah umekhupelet laMakom aleinu…

It would have sufficed us [if He had done us any one or more of these many listed kindnesses, yet] how much more good, many times more and still more, did God [blessed be He] bring upon us!

A fortiori discourse in the PASSOVER HAGGADAH.
(My translation and brackets mine.)

Other works by Avi Sion (photo) include:

Future Logic: Categorical and Conditional Deduction and Induction of the Natural, Temporal, Extensional and Logical Modalities. Revised ed. Geneva: Author, 1996.[1]

Judaic Logic: A Formal Analysis of Biblical, Talmudic and Rabbinic Logic. Geneva: Author, 1995.[2]

Buddhist Illogic: A Critical Analysis of Nagarjuna's Arguments. Geneva: Author, 2002.

Phenomenology: Basing Knowledge on Appearance. Expanded ed. Geneva: Author, 2005.[3]

The Logic of Causation. Rev. & exp. ed. Geneva: Author, 2010.[4]

Volition and Allied Causal Concepts. Geneva: Author, 2004.

Ruminations: Sundry Notes and Essays on Logic. Expanded ed. Geneva: Author, 2005.[5]

Meditations: A Spiritual Logbook. Geneva: Author, 2006.

Logical and Spiritual Reflections. Rev. & exp. ed. Geneva: Author, 2009.[6]

All of these works may be consulted on the Internet, at:
www.TheLogician.net.

Also at: www.AviSionBooks.com.

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[1] First published by author in Vancouver, B.C., 1990.
[2] Published thereafter by Editions Slatkine in Geneva, 1997.
[3] First published by author in Geneva, 2003.
[4] First published by author in Geneva, 1999. The first edition comprised only Phase I (Macroanalysis); the second edition (2003) added Phase II (Microanalysis); the third edition added Phase III (Software Assisted Analysis).
[5] First published by author in Geneva, earlier 2005.
[6] First published by author in Geneva, 2008.

Abstract

A Fortiori Logic: Innovations, History and Assessments, by Avi Sion, is a wide-ranging and in-depth study of a fortiori reasoning, comprising a great many *new theoretical insights* into such argument, *a history* of its use and discussion from antiquity to the present day, and *critical analyses* of the main attempts at its elucidation. Its purpose is nothing less than to lay the foundations for a new branch of logic and greatly develop it; and thus to once and for all dispel the many fallacious ideas circulating regarding the nature of a fortiori reasoning.

The work is divided into three parts. The first part, **Formalities**, presents the author's largely original theory of a fortiori argument, in all its forms and varieties. Its four (or eight) principal moods are analyzed in great detail and formally validated, and secondary moods are derived from them. A crescendo argument is distinguished from purely a fortiori argument, and similarly analyzed and validated. These argument forms are clearly distinguished from the pro rata and analogical forms of argument. Moreover, we examine the wide range of a fortiori argument; the possibilities of quantifying it; the formal interrelationships of its various moods; and their relationships to syllogistic and analogical reasoning. Although a fortiori argument is shown to be deductive, inductive forms of it are acknowledged and explained. Although a fortiori argument is essentially ontical in character, more specifically logical-epistemic and ethical-legal variants of it are acknowledged.

The second part of the work, **Ancient and Medieval History**, looks into use and discussion of a fortiori argument in Greece and Rome, in the Talmud, among post-Talmudic rabbis, and in Christian, Moslem, Chinese and Indian sources. Aristotle's approach to a fortiori argument is described and evaluated. There is a thorough analysis of the Mishnaic *qal vachomer* argument, and a reassessment of the *dayo* principle relating to it, as well as of the Gemara's later take on these topics. The valuable contribution, much later, by Moshe Chaim Luzzatto is duly acknowledged. Lists are drawn up of the use of a fortiori argument in the Jewish Bible, the Mishna, the works of Plato and Aristotle, the Christian Bible and the Koran; and the specific moods used are identified. Moreover, there is a pilot study of the use of a fortiori argument in the Gemara, with reference to Rodkinson's partial edition of the Babylonian Talmud, setting detailed methodological guidelines for a fuller study. There is also a novel, detailed study of logic in general in the Torah.

The third part of the present work, **Modern and Contemporary Authors**, describes and evaluates the work of numerous (some thirty) recent contributors to a fortiori logic, as well as the articles on the subject in certain lexicons. Here, we discover that whereas a few authors in the last century or so made some significant contributions to the field, most of them shot woefully off-target in various ways. The work of each author, whether famous or unknown, is examined in detail in a dedicated chapter, or at least in a section; and his ideas on the subject are carefully weighed. The variety of theories that have been proposed is impressive, and stands witness to the complexity and elusiveness of the subject, and to the crying need for the present critical and integrative study. But whatever the intrinsic value of each work, it must be realized that even errors and lacunae are interesting because they teach us how *not* to proceed.

This book also contains, in a final appendix, some valuable contributions to general logic, including new analyses of symbolization and axiomatization, existential import, the tetralemma, the Liar paradox and the Russell paradox.

Contents in brief

Contents in detail

Please note that the Appendices are integral parts of the present work, to be studied in conjunction with the chapters in which they are mentioned. Also, the footnotes throughout this volume are intended to be read; they often contain important additional information or reflection.

As regards the spelling of foreign words, no great effort has been made here to harmonize it. The same word may have different spellings in different contexts. Very often, the spelling used depends on the spelling others prefer, who are discussed in the given context. The reader is asked to be indulgent in this matter.

Foreword

When I started writing the present work, in late 2010, I thought it would take a dozen pages and a couple of weeks at most to say what I felt the need to say. I had, I believed, said most of what needed to be said in my previous foray in the field of a fortiori logic, in my 1995 study of Judaic logic. But having noticed that some people were still writing on the subject without reference to my work, and to boot were making serious mistakes, I felt the need to show them the errors of their ways. However, as I proceeded in this set task, I found myself more and more involved in its intricacies.

For a start, to be fair the critiques had to be detailed, and show exactly what had been said and where lay the errors and lacunae. Secondly, I kept discovering more and more commentaries which needed to be similarly reviewed and evaluated. Thirdly, it became obvious that I needed to expand my theoretical investigations, to be able to answer various questions these commentaries brought up, consciously or unconsciously. Eventually, I realized that I had to aim for a history of the subject and a survey of more recent contributions to it, to be able to demonstrate precisely who said what first.

Thus, the work ended up taking me three years to complete. Three parts emerged. The first presented my new, much more detailed theory of a fortiori argument. The second part traced the early history of use and discussion of such argument, so far as I could make it out with the resources available to me. The third focused on modern commentaries on the subject. However, *these parts did not emerge separately, but repeatedly impinged on each other*, so that many chapters or sections had to be written more than once to be adapted to new findings. For this reason, it was impossible to publish any part of the work before it was all done.

It should be stressed that the work did not proceed in the order that the chapters are now set out. Whereas now all commentators are ordered chronologically, I did not comment on their work in their order of appearance in history. It was all a matter of chance encounter and personal mood. Moreover, my theoretical baggage at each stage was different. For this reason, some earlier chapters may appear more perspicacious or analytically cutting than some later ones. I tried, of course, to harmonize things as much as I could; but as the book grew in size, it became more and more unwieldy. No doubt my memory in these later years is not what it was once; so I may have missed some things.

1. Innovations

The present work is replete with valuable innovations in the field of a fortiori logic, and in other, related subjects. The present, wider ranging research confirms that my past work in this field, in my 1995 book *Judaic Logic*, was novel and important. But moreover, the present work corrects some inaccuracies in that past work, and greatly enlarges and sharpens our theory of a fortiori argument, so that it may be said to address almost every nook and cranny of the subject. There is not a single topic that I worked on here that did not yield some new insight or new theoretical development in a fortiori logic. This means that the research was certainly worthwhile and interesting; it is not a mere collection and rehashing of old material.

'Formalities', part one of the present volume, presents the author's largely original theory of a fortiori argument, in all its forms and varieties. Its four (or eight) principal moods are analyzed in great detail and formally validated, and secondary moods are derived from them. A crescendo argument is distinguished from purely a fortiori argument, and similarly analyzed and validated. These argument forms are clearly distinguished from the pro rata and analogical forms of argument. Moreover, we examine the wide range of a fortiori argument; the possibilities of quantifying it; the formal interrelationships of its various moods; and their relationships to syllogistic and analogical reasoning. Although a fortiori argument is shown to be deductive, inductive forms of it are acknowledged and explained. Although a fortiori argument is essentially ontical in character, more specifically logical-epistemic and ethical-legal variants of it are acknowledged.

The present work also contains, in a final appendix, valuable innovations relating to certain topics in general logic; namely, symbolization and axiomatization, existential import, the tetralemma, the Liar paradox and the Russell paradox.

2. History

Logic science, properly conceived, is not just a theoretical enterprise, but also an investigation into the historical roots of the forms of human discourse. The present work on a fortiori logic constitutes an excellent case study of how a particular form of thought is rooted deep in antiquity (in history), and probably much earlier, in language itself (in prehistory), and then gradually develops as awareness of it dawns, expands and intensifies. There is ample evidence that a fortiori discourse existed in very ancient times and in very diverse cultures. A fortiori reasoning was present in early Greek literature (Homer, Aesop), long before Aristotle first discussed it (in his *Rhetoric* and *Topics*); and it was present before that in Jewish literature (the Torah and other Biblical books). Aristotle did not invent the a fortiori argument, any more than he invented the syllogism; he 'merely' observed, described and explained them, as a botanist might notice and catalogue interesting plants.

'Ancient and Medieval History', part two of the present volume, looks into use and discussion of a fortiori argument in Greece and Rome, in the Talmud, among post-Talmudic rabbis, and in Christian, Moslem, Chinese and Indian sources. Aristotle's approach to a fortiori argument is described and evaluated. There is a thorough analysis of the Mishnaic *qal vachomer* argument, and a reassessment of the *dayo* principle relating to it, as well as of the Gemara's later take on these topics. The valuable contribution, much later, by Moshe Chaim Luzzatto is duly acknowledged. Lists are drawn up of the use of a fortiori argument in the Jewish Bible, the Mishna, the works of Plato and Aristotle, the Christian Bible and the Koran; and the specific moods used are identified. Moreover, there is a pilot study of the use of a fortiori argument in the Gemara, with reference to Rodkinson's partial edition of the Babylonian Talmud, setting detailed methodological guidelines for a fuller study. There is also a novel, detailed study of logic in general in the Torah.

3. Assessments

When I started to study a fortiori logic, I was little aware of the number of people who have since the late 19th century attempted to describe and explain this common form of reasoning. The field seemed nearly empty of contributors, a desert yet to be explored. Only little by little did I realize that many people have indeed tried their hand at solving the enigma of a fortiori argument – some, to be sure, more competently than others. It gradually became clear that a survey of existing contributors needed to be made, and their work had to be carefully studied and assessed. Such assessment depended, of course, on the theoretical and historiographical work undertaken earlier. It was interesting to see how many of the contributors studied past work very little before proposing their own ideas. Each apparently thought he was one of the first explorers.

'Modern and Contemporary Authors', part three of the present work, describes and evaluates the work of numerous (some thirty) recent contributors to a fortiori logic, as well as the articles on the subject in certain lexicons. Here, we discover that whereas a few authors in the last century or so made some significant contributions to the field, most of them shot woefully off-target in various ways. The work of each author, whether famous or unknown, is examined in detail in a dedicated chapter, or at least in a section; and his ideas on the subject are carefully weighed. The variety of theories that have been proposed is astonishing, and stands witness to the complexity and elusiveness of the subject, and to the crying need for the present critical and integrative study. But whatever the intrinsic value of each work, it must be realized that even errors and lacunae are interesting because they teach us how *not* to proceed.
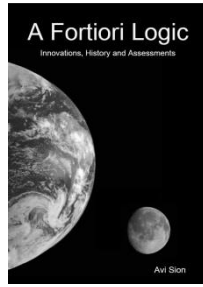
PART I - FORMALITIES

1. The standard forms

The present treatise on a fortiori logic has three purposes: (a) to present recent innovations I have made in the theory of a fortiori argument; (b) to retrace, as much as I can till now, the history of use and discussion of such argument; and (c) to review and evaluate (praise or criticize) ideas concerning such argument by other commentators or logicians. In comparison with the original theory of a fortiori argument presented in my book *Judaic Logic* over 15 years ago, the updated theory in the present work contains many significant improvements and enlargements. As this updated theory will, naturally, be the standard of judgment of all use and discussion of the argument throughout the present work, the reader is well advised to get acquainted with its main features before proceeding further[1].

1. Copulative a fortiori arguments

Based on close analysis of a large number of Biblical and Talmudic examples (some known to Jewish tradition and some newly identified by me), as well as examples from everyday discourse, I discovered and proposed in my book *Judaic Logic* the four valid moods of copulative a fortiori argument listed below.

An a fortiori argument consists of three propositions called the major premise, the minor premise and the conclusion. A copulative such argument is one involving terms. It comprises four terms, which are always symbolized in the same way. The four terms are called the major, the minor, the middle and the subsidiary; and the symbols for them are respectively P, Q, R and S[2]. Other terminology used will be clarified as we proceed.

a.   The **positive subjectal** {+s} mood:

> P is more R than (or as much R as) Q (is R),
> and Q is R enough to be S;
> therefore, all the more (or equally), P is R enough to be S.

Notice that the valid inference goes 'from minor to major'; that is, from the minor term (Q) to the major one (P); meaning: from the minor term as subject of 'R enough to be S' in the minor premise, to the major term as subject of same in the conclusion. Any attempt to go from major to minor in the same way (i.e. positively) would be invalid inference.

b.   The **negative subjectal** {−s} mood:

> P is more R than (or as much R as) Q (is R),
> yet P is R *not* enough to be S;
> therefore, all the more (or equally), Q is R *not* enough to be S.

Notice that the valid inference goes 'from major to minor'; that is, from the major term (P) to the minor one (Q); meaning: from the major term as subject of 'R not enough to be S' in the minor premise, to the minor term as subject of same in the conclusion. Any attempt to go from minor to major in the same way (i.e. negatively) would be invalid inference.

We can summarize all information about subjectal argument as follows: "Given that P is more R than or as much R as Q is R, it follows that: if Q is R enough to be S, then P is R enough to be S; and if P is R not enough to be S, then

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[1]   Those who have already read my *Judaic Logic* ought still to read the present treatise, because there are very many significant new insights and findings in it, and even some corrections.

[2]   Notice that the symbols R and S, respectively, happen to match the words "Range" (the middle item always refers to a range) and Subsidiary.

Q is R not enough to be S; on the other hand, if Q is R not enough to be S, it does not follow that P is R not enough to be S; and if P is R enough to be S, it does not follow that Q is R enough to be S." In this summary format, we resort to nesting: the major premise serves as primary antecedent, and the valid minor premises and conclusions appear as consequent conditions and outcomes, while the invalid moods are expressed as non-sequiturs.

For example: granted Jack (P) can run faster (R) than Jill (Q), it follows that: if Jill can run (at a speed of) one mile in under 15 minutes (S), then surely so can Jack; and if he can't, then neither can she. Needless to say, the conditions are presumed identical in both cases; we are talking of the same course, in the same weather, and so on. If different conditions are intended, the argument may not function correctly. The a fortiori argument is stated categorically only if there are no underlying conditions. Obviously, if there are conditions they ought to be specified, or at least we must ensure they are the same throughout the argument.

c.   The **positive predicatal** {+p} mood:

      More (or as much) R is required to be P than (as) to be Q,
      and S is R enough to be P;
      therefore, all the more (or equally), S is R enough to be Q.

Notice that the valid inference goes 'from major to minor'; that is, from the major term (P) to the minor one (Q); meaning: from the major term as predicate of 'S is R enough to be' in the minor premise, to the minor term as predicate of same in the conclusion. Any attempt to go from minor to major in the same way (i.e. positively) would be invalid inference.

d.   The **negative predicatal** {–p} mood:

      More (or as much) R is required to be P than (as) to be Q,
      yet S is R *not* enough to be Q;
      therefore, all the more (or equally), S is R *not* enough to be P.

Notice that the valid inference goes 'from minor to major'; that is, from the minor term (Q) to the major one (P); meaning: from the minor term as predicate of 'S is R not enough to be' in the minor premise, to the major term as predicate of same in the conclusion. Any attempt to go from major to minor in the same way (i.e. negatively) would be invalid inference.

We can summarize all information about predicatal argument as follows: "Given that more or as much R is required to be P than to be Q, it follows that: if S is R enough to be P, then S is R enough to be Q; and if S is R not enough to be Q, then S is R not enough to be P; on the other hand, if S is R not enough to be P, it does not follow that S is R not enough to be Q; and if S is R enough to be Q, it does not follow that S is R enough to be P." In this summary format, we resort to nesting: the major premise serves as primary antecedent, and the valid minor premises and conclusions appear as consequent conditions and outcomes, while the invalid moods are expressed as non-sequiturs.

For example: granted that it takes more strength (R) to lift 50 kilos (P) than 30 (Q): if someone (S) can lift 50 kilos, then surely he can lift 30; and if he can't lift 30, then he can't lift 50. Needless to say, the conditions are presumed identical in both cases; we are talking of the same handle, on the same day, and so on. If different conditions are intended, the argument may not function correctly. The a fortiori argument is stated categorically only if there are no underlying conditions. Obviously, if there are conditions they ought to be specified, or at least we must ensure they are the same throughout the argument.

Thus, to summarize, there are four valid moods of copulative a fortiori argument: two subjectal moods, in which the major and minor terms (P and Q) are the logical subjects of the three propositions concerned; and two predicatal moods, in which the major and minor terms (P and Q) are the logical predicates of the three propositions concerned. The major premise is always positive, though it differs in form in subjectal and predicatal arguments. In each of these types, there are two variants: in one, the minor premise and conclusion are positive; and in the other, they are negative. The positive and negative versions in each case are obviously closely related – the minor premise of the one is the negation of the conclusion of the other, and vice versa; that is, each can be used as a *reductio ad absurdum* for the other.

Note well the order in which the major and minor terms (P and Q) appear in the four moods: in the subjectal moods they are subjects; and in the predicatal ones they are predicates. It follows that in the two subjectal moods, the subsidiary term (S) is a predicate; and in the two predicatal moods, it (S) is a subject. The middle term (R), however, is a predicate in both premises and the conclusion of *all* the moods, note well. In subjectal moods it is a predicate of

the major and minor terms (P and Q); in the predicatal moods it is a predicate of unspecified subjects in the major premise and a predicate of the subsidiary term (S) in the minor premise and conclusion, the subsidiary term being one instance of the unspecified subject-matter of the major premise.

The difference between subjectal and predicatal moods is called a difference of structure. The difference between positive and negative moods is called a difference of polarity. The difference between moods that go "from minor to major" and those that go "from major to minor" is called a difference of orientation. Sometimes this difference of direction is stated in Latin, as "*a minori ad majus*" and "*a majori ad minus*"[3]. Note that the "from" term may be the minor or major and occurs in the minor premise; and the "to" term is accordingly the major or minor, respectively, and occurs in the conclusion. Notice the variations in orientation in accord with the structure and polarity involved.

In sum, these four valid moods are effectively four distinct figures (and not merely moods) of a fortiori argument, since the placement of their terms differs significantly in each case. This is clearly seen in the following table:

Figure/mood	+s	−s	+p	−p
major premise	PQR	PQR	RPQ	RPQ
minor premise	QRS	PRS	SRP	SRQ
conclusion	PRS	QRS	SRQ	SRP

Table 1.1

We shall deal with the validation of all these arguments further on. Meanwhile, the following clarifications should also be kept in mind:

- The expression "all the more," and others like it (such as "a fortiori," "how much more," and so on), are often used in practice to signal an intention of a fortiori argument. This is useful specifically when the argument is only partly explicit; but when the argument is fully explicit, as shown above, such expression is in fact redundant, and (as we shall see) can even be misleading (suggestive of 'proportionality'). When the argument is stated in full, it is sufficient to say "therefore" to signal the conclusion; nothing is added by saying "all the more."

  Incidentally, in practice people sometimes reserve "all the more" for argument that goes from minor to major and "all the less" for argument that goes from major to minor; but it is also true that the expression "all the more" (and others like it) is also often used indiscriminately, and this is the way we usually intend it here.

- The four arguments function just as well if the major term is greater (in respect of the middle term) than the minor term, or if they are equal. Whence, I have inserted in brackets in each mood: an "as much as" alternative clause to "more than" in the major premise, and an "equally" alternative to the traditional expression "all the more" in the conclusion. So though we have four figures, we may say that they contain two moods each, a 'superior' and an 'egalitarian' one, making a total of eight moods. Egalitarian a fortiori argument is also sometimes called '*a pari*'.

- Note that for subjectal moods, I have specified the major premise as "P is more R than Q (is R)" – this is done to avoid confusion with a proposition of the form "P is more R than (P is) Q." If we try using the latter with "P is Q enough to be S" to conclude "P is R enough to be S," we would have an argument vaguely resembling a fortiori but which is in fact invalid[4]. In the valid form, Rp > Rq; whereas in the fake form Rp > Qp. Watch out for occurrences of this fallacy in common discourse.

  The major premise of predicatal argument, i.e. "More R is required to be P than to be Q," does not have the same potential for ambiguity. Note, however, that it could alternatively be formulated as "To be P requires more R than to be Q (requires R)" – in which form it might be confused with the major premise of subjectal argument, viz. "What is P is more R than what is Q (is R)."[5]

- The major premise may occasionally in practice be converted – i.e. it may be stated, in subjectal argument, as "Q is less R than P" instead of as "P is more R than Q;" and in predicatal argument, as "Less R is required to be Q than to be P" instead of as "More R is required to be P than to be Q." The validity of the argument in such cases is not affected, *provided the minor premise and conclusion remain the same*. Note this proviso well. Very often,

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[3]   I notice that that the Soncino Talmud does not apparently use the term *a fortiori* as a general term, but distinguishes between *a minori* and *a fortiori* (instead of *a majori*). Maybe this was an error. In any case, in my opinion, such usage should be avoided as it would leave us with no general term. The term *a fortiori* is needed as a common label for all forms of the argument. Whereas *a majori* means from the major (term to the minor term), *a fortiori* means with stronger (reason); so these expressions are not equivalent.

[4]   For example, "This screw is longer than it is wide; and it is wide enough to fit into this hole; therefore it is long enough to do so." Clearly, this would be fallacious reasoning; the conclusion does not follow from the premises.

[5]   We might also put the major premise of subjectal argument in the form: "More R is found in P than in Q." However, the most natural form for the subjectal major premise is active and that for the predicatal major premise is passive.

such conversion of the major premise confuses people and they erroneously transpose the minor premise and conclusion[6]. Arguments involving such converted major premises, which may be labeled 'inferior', should not be counted as distinct moods.

- In practice, the major premise is very often simply left out. The proponent of a given argument may have it explicitly or tacitly in mind. But he may also be quite unaware of it, in which case it is only we logicians who tell him it is logically present in the background and playing an active role in the inference. This is not something peculiar to a fortiori argument, but is likewise often encountered in syllogism and other forms of argument. It is called enthymemic argument (a mere technical term); you can call it abridged or abbreviated argument, if you like.

- Concerning the minor premise and conclusion, the phrase "R enough to be" is often left out in practice. This may occur with the major premise absent, so that the middle term (R) is completely unstated (though of course still logically implicit); or it may occur with the major premise present, in which case the mention of the middle term in it is deemed sufficient for the whole argument. When the said phrase is left out, the minor premise and conclusion are usually stated in one if–then proposition: e.g. "If Q is S, then P is S," which (to repeat) may be combined with an explicit major premise or presented alone.

  The fact that often in practice the middle term R is left tacit should not blind us to the fact that it is a sine qua non for successful a fortiori argument. The proposition "P is more R than Q" combined with "Q is S" is logically quite compatible with "P is not S;" or combined with "P is not S" is logically quite compatible with "Q is S." Similarly, The proposition "More R is required to be P than to be Q" combined with "P is S" is logically quite compatible with "Q is not S;" or combined with "Q is not S" is logically quite compatible with "P is S." Note this well. Many commentators fail to realize this, or having learned it quickly forget it. Without the relation "R enough to be" in the minor premise, the a fortiori conclusion cannot be drawn and the argument is fallacious.

- Evidently, the clause "R enough to be" in positive moods, or "R not enough to be," in negative moods, even if it is not explicitly stated in the minor premise and conclusion, is absolutely essential to a fortiori argument. If there is no intended threshold of R to be attained or surpassed in order for S to be predicated of or to be subject to the major and minor terms, there is no operative a fortiori argument (though there might be some other thought-process, such as mere analogy). This is evident from the fact that, without this crucial clause, we simply cannot validate the argument. Keep that well in mind.

  Note that the expression "R not enough to be" can also be stated as "not enough R to be" or "not R enough to be," without change of meaning. The form "X is not R enough to be Y," which is used in the minor premise and conclusion of negative subjectal or predicatal arguments, is the most ambiguous, being used for cases where X *is not R at all*, as well as more obviously to cases where it *is R to some insufficient extent*. More will be said about this further on.

- Moreover, the middle term R must remain constant throughout the argument. That is, the middle term R specified in the minor premise must be identical with the one specified in the major premise. This can be seen by an example: although humans are more intelligent than horses, it does not follow that they can run faster than horses! Obviously, we can only speak of the superiority of humans over horses with respect to what was intended, viz. 'intelligence' in this case; this does not exclude the possibility that with respect to other attributes, such as leg muscles, horses are superior.

  On a formal level, what this means is that if we do not specify or keep in mind the middle term R intended in the major premise, we might easily intend *another* middle term, say R', in the minor premise and conclusion; in which case, our reasoning (whether unconsciously or deliberately done) would of course be faulty. This often happens in practice, and is one reason some people doubt the validity of a fortiori argument in general. But the problem here is not with the argument as such, but with the use of *two middle terms*. If we use, explicitly or implicitly, two middle terms, the argument is of course invalid, for it cannot be validated any longer. We could label such practice 'the fallacy of two middle terms' so as to remember to avoid it and not be taken in by it.

- Any or all of the four terms, P, Q, R, S, may be a compound, i.e. a conjunction of two or more terms. This of course happens in practice often enough.

It should be stressed that, albeit their various formal differences, the four principal forms of copulative a fortiori argument above enumerated truly deserve to be called by one and the same name; they constitute a family of arguments. The positive and negative moods of a given orientation (subjectal or predicatal) are obviously two facets of the same coin. But moreover, notice the similarity between the positive subjectal and negative predicatal moods, and also between the negative subjectal and positive predicatal moods. Note that the former two moods may be

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[6]   To avoid confusion always simply reflect on the question: which term 'is more R' or 'requires more R' than the other?

characterized as going "from minor to major," and the latter two as going "from major to minor." More will be said about this further on.

The positive subjectal mood may be viewed as the prototype of all a fortiori argument, because of its relative simplicity. Many accounts of a fortiori argument tend to mention only this mood; or rather, examples thereof. Nevertheless, this does not mean that the other three copulative moods, or indeed their implicational analogues, can be ignored. They are distinct movements of thought that merit separate attention.

I should also draw your attention to the possibility that the whole subjectal or predicatal a fortiori argument concerns only one subject, as shown next:

> When this thing (say, X) is P, it is more R than when it is Q,
> and when it is Q, it is enough R to be S;
> therefore, when it is P, it is enough R to be S.

> More R is required for this thing (say, X) to be P than for it to be Q,
> and when it is S, it is R enough to be P;
> therefore, when it is S, is R enough to be Q.

We can construct similar negative moods, of course. Notice that I have specified the subject as 'this thing' (or X) in both major premises, but these could equally be generalities, i.e. have 'something, anything' as their subject. Such single-subject a fortiori arguments are not mere theoretical possibilities, but often occur in practice. Note the conditional form the sentences take; these are really, therefore, cases of implicational argument (see next section). The conditioning may obviously be based on any type of modality – extensional, natural, temporal or spatial.

2. Implicational a fortiori arguments

In addition to the above four valid copulative moods, I identified in *Judaic Logic* four comparable 'implicational' moods. The first two I called antecedental (instead of subjectal) and the last two I called consequental (instead of predicatal). These four moods have the same figures as the preceding four; but they differ in involving the relation of implication instead of the copulative one, and therefore theses instead of terms as the items under consideration. I list them for you anyway, just to make sure there is no misunderstanding:

e.   The **positive antecedental** (+a) mood:

> P implies more R than (or as much R as) Q (implies R),
> and, Q implies enough R to imply S;
> therefore, all the more (or equally), P implies enough R to imply S.

f.   The **negative antecedental** (–a) mood:

> P implies more R than (or as much R as) Q (implies R),
> yet, P does *not* imply enough R to imply S;
> therefore, all the more (or equally), Q does *not* imply enough R to imply S.

g.   The **positive consequental** (+c) mood:

> More (or as much) R is required to imply P than to imply Q,
> and, S implies enough R to imply P;
> therefore, all the more (or equally), S implies enough R to imply Q.

h.   The **negative consequental** (–c) mood:

> More (as much) R is required to imply P than to imply Q,
> yet, S does *not* imply enough R to imply Q;
> therefore, all the more (or equally), S does *not* imply enough R to imply P.

Clearly, mostly similar comments can be made regarding the structures of these additional four valid moods (or eight, if we distinguish between superior and egalitarian moods) as for those preceding them.

In particular note well the fact that the middle thesis (R) is always a consequent (or non-consequent), whereas the other three theses (P, Q and S) have varied roles as antecedents (or non-antecedents) or consequents (or non-consequents) depending on the figure concerned. In antecedental argument, R is (or is not) a consequent of P and Q; while in consequental argument, R is (or is not) a consequent of S. Do not be misled by the fact that R is placed to the left of P and Q in the major premise of consequental a fortiori arguments. The thesis R does not there play the role of antecedent of P and Q (i.e. it does not imply them). The theses P, Q and R are there all consequents of some unstated antecedents; and thesis S is a specified instance of such unstated antecedent (in the positive case) or not so (in the negative case).

Variation of the middle thesis. Concerning the middle thesis R, the sense in which it is quantitatively variable (i.e. that more or less of it can be implied) needs to be clarified. *A proposition as such does not have degrees*; so it would be incorrect to imagine that the proposition R as a whole has degrees. A thesis (e.g. Rp) is not a quantity, and so cannot be "greater" than another thesis (e.g. Rq). Therefore, when in the major premises of implicational a fortiori argument we say that "more of thesis R" is implied or required, we must refer to a variation in the predicate and/or in the subject *within* thesis R. This insight can be better understood if we formulate an implicational a fortiori argument in such a way that the categorical propositions inherent in it are made explicit. This can be done with antecedental and consequental arguments of whatever polarity. Consider for instance the following case, which is doubtless the most frequent:

> P (= A is p) implies more R (= C is r) than Q (= B is q) does, and
> Q (= B is q) implies enough R (= C is r) to imply S (= D is s).
> So, P (= A is p) implies enough R (= C is r) to imply S (= D is s).

Here, I have shown each of the four categorical propositions as involving four different subjects (A, B, C, D) with four different predicates (p, q, r, s). The middle thesis R is here taken to mean that 'C is r'. The variation of R may in this light be understood in various ways. In the most frequent case, the subject C is constant and it is the predicate r within R that is variable, C being r_p in thesis Rp and C being r_q in thesis Rq ($r_p > r_q$). Comparatively rarely, the predicate r is constant and it is the subject C within R that is variable, Cp being r in thesis Rp and Cq being r in thesis Rq (Cp > Cq)[7]. In more complex cases, both the subject C and the predicate r might conceivably vary, Cp being r_p in thesis Rp and Cq being r_q in thesis Rq. The important point is that the resultant R theses can reasonably be said to satisfy the condition that Rp > Rq.

As regards language, the major and minor theses might in practice be stated in gerundive form, as 'A being p' and 'B being q', while the subsidiary term might more naturally be stated in the infinitive form, as 'D to be S'. For the middle thesis, we might say 'more r in C' to signify that it is the predicate that varies, or 'more C to be r' to signify that it is the subject that varies. Quite often in practice, people do not state the whole middle thesis, but only the most relevant term in it – i.e. the variable predicate (usually) or subject (rarely). Thus, instead of saying in the major premise "implies more R," they might say "implies more r" or "implies more C"; and likewise, instead of saying in the minor premise and conclusion "implies R enough," they might say "implies r enough" or "implies C enough."

Strictly speaking, of course, this is inaccurate, because *a lone term cannot be implied (or imply)*. The logical relation of implication concerns whole theses, never mere terms. But since this confusion occurs in everyday discourse, it is well to be aware of it and to take it into consideration. Thus, when in practice we encounter an a fortiori argument with whole theses as major and minor items, and a lone term as middle item, we should not think that this exemplifies a 'hybrid' type of argument which is partly copulative and partly implicational. Formally, such a construct is still implicational argument, except that the middle thesis is not entirely spoken out loud; i.e. either its subject or its predicate is left tacit. In the same way, the subsidiary thesis is sometimes incompletely stated. To validate such partly formulated arguments, we of course need to specify the intended unspoken term(s).

We could in fact say that *all a fortiori arguments are tacitly implicational*. The thin line between copulative and implicational argument becomes evident when we reword a typical copulative argument in implicational form, as follows:

> P (= something being p) implies more R (= r in it) than Q (= something being q) does, and

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[7]   That we have to acknowledge the possibility that the subject varies in magnitude will be evident further on, when we consider predicatal a crescendo (i.e. proportional a fortiori) argument. There it is manifest that this is logically possible and occurs in practice. As regards the assumption that Rp (Cp is r) > Rq (Cq is r) is implied when Cp > Cq (rather than when Cp < Cq) – this seems reasonable to me at this time, though some uncertainty persists.

Q (= something being q) implies enough R (= r in it) to imply S (= it to be s);
therefore, P (= something being p) implies enough R (= r in it) to imply S (= it to be s).

This argument is obviously a special case of the preceding one. Here, instead of four subjects (A, B, C, D), we only have two (or even just one). They are unspecific (i.e. not labeled A and B, as earlier done), in the sense that they each refer to 'something' (i.e. anything – the intent is general, not particular) that is solely defined by the predicate initially attached to it (viz. p, q, respectively). The 'something' that is intended in P and the 'something (else)' intended in Q are here distinct objects, note (although, as we have already seen, they could well in some cases be one and the same subject). Each of them is subject to a different measure or degree of the middle predicate 'r' (whence r is 'in it'). And each of them is or turns out to be subject to the subsidiary predicate 's'. The case shown (here again) is the positive antecedental mood; the same can obviously be done with the positive predicatal mood, and with the negative forms of both of these.

Looking back at the way I came upon these various argument forms when I wrote *Judaic Logic*, I remember first discovering the copulative forms and later, finding them insufficient to account for all examples of a fortiori argument I came across, I developed the implicational forms. In a sense, they were conceived as generalizations of the corresponding copulative forms. Indeed, I overgeneralized a bit, because I did not realize at the time that the notion that a thesis may "imply more" of another thesis is logically untenable. Much later, I started wondering whether 'hybrid' arguments signified additional types, besides the copulative and implicational. It is only recently that I better understood the relationships between the various forms of argument as above described. So the present account amends past errors and uncertainties.

I should also here mention the following special case, where the major premise "P implies more A to be B than Q does" means "P implies that a number x of A are B, and Q implies a that number y of A are B, and x > y." The change in magnitude involved in this case is not in the subject A or the predicate B inherent in the middle thesis, but in the quantifiers of A. So the middle thesis is not, as might be thought, about "how much A is B," or even "how much B A is," but about *the frequency* of occurrence of 'A being B'. In such case, the proposition could be stated less ambiguously as "P implies more instances of A to be B than Q does." The frequency involved may be extensional, as here; or it could have to do with another mode of modality, i.e. more often in time or place, or in more circumstances or contexts.

Moreover, though I have here presented the middle thesis R as a single categorical proposition, it should be kept in mind that R could contain a compound thesis, i.e. it could involve a complex set of variable factors.

In conclusion, when in formulating implicational a fortiori argument we refer to the middle thesis 'R', the intention is more precisely '***something in R***', meaning 'some term(s) in thesis R' or even 'some modal qualifier in thesis R'. That is, when we say: 'implies more R' or 'more R is required to imply' or 'implies enough R' – we must be understood to mean: 'implies more of something in R' or 'more of something in R is required to imply' or 'implies enough of something in R', respectively. Though I will continue to use the abridged formulae, these more elaborate formulae will be tacitly intended.

More will, of course, be said about implicational a fortiori argument as we proceed.

3. Validations

Validation of an argument means to demonstrate its validity. An argument is 'valid' if, given its premises, its conclusion logically follows. Otherwise, if the putative conclusion does not follow from the given premises or if its denial follows from them, the argument is 'invalid'. If the putative conclusion is merely not implied by the given premises, it is called a *non sequitur* (Latin for 'it does not follow'); in such case, the contradictory of the putative conclusion is logically as compatible with the given premises as the putative conclusion is. If a contrary or the contradictory of the putative conclusion is positively implied by the given premises, the putative conclusion is called an absurdity (lit. 'unsound') or more precisely an antinomy (lit. 'against the laws' of thought).

The validity of an argument does not guarantee that its conclusion is true, note well. An argument may be valid even if its premises and conclusion are in fact false. Likewise, the invalidity of an argument does not guarantee that its conclusion is false. An argument may be invalid even if its premises and conclusion are in fact true. The validity (or invalidity) of an argument refers to the logical *process*, i.e. to the claim that a set of premises of this kind formally implies (or does not imply) a conclusion of that kind.

A material a fortiori argument may be validated simply by showing that it can be credibly cast into any one of the valid moods listed above. If it cannot be fitted into one of these forms, it is invalid – or at least, it is not an a fortiori argument. The validations of the forms of a fortiori argument may be carried out as we will now expound. Invalid forms are forms that cannot be similarly validated. Obviously, material arguments can also be so validated; but the

quick way is as just stated to credibly cast them into one of the valid forms. Once the forms are validated by logical science, the material cases that fit into them are universally and forever thereafter also validated.

One way to prove the validity of a new form of deduction is through the intermediary of another, better known, form of deduction. Such derivation is called 'reduction'. 'Direct' reduction is achieved by means of conversions or similar immediate inferences. If the premises of the tested argument imply those of an argument already accepted as valid, and the conclusion of the latter implies that of the former, then the tested argument is shown to be equally valid. 'Indirect' reduction, also known as reduction *ad absurdum*, on the other hand, proceeds by demonstrating that denial of the tested conclusion is inconsistent with some already validated process of reasoning.

It works like this: Suppose A and B are the two (or more) premises of a proposed argument, and C is its putative conclusion. If the C conclusion is correct, this would mean that (A + B) implies C; which means that the conjunction (A + B + not-C) is logically impossible. Let us now hypothetically suppose that C is *not* a necessary implication in the context of A + B; i.e. that not-C is not impossible in it. In that case, we could combine not-C with one of the premises A or B, without denying the other. But we already know from previous research that, say, (A + not-C) implies not-B; which means that the conjunction (A + not-C + B) is logically impossible. Therefore, we must admit the validity of the newly proposed argument. Note that the two stated conjunctions of three items are identical except for the relative positions (which are logically irrelevant) of the items conjoined.

Analysis of constituents

The validation procedures[8] are accordingly uniform for copulative and implicational a fortiori arguments. They are based on analysis of the meanings of the propositions involved in such argument, i.e. on *reduction* of these more complex forms to simpler forms more studied and better understood by logicians.

The following are the two main reductions needed for validation of the earlier listed **copulative** arguments. The major premises (characterized as "*commensurative*" because they compare measures or degrees) of subjectal and predicatal arguments are always positive and have the following components:

The subjectal major premise, "P is more R than (or as much R as) Q is," means:

> P is R, i.e. P is to a certain measure or degree R (say, Rp);
>
> Q is R, i.e. Q is to a certain measure or degree R (say, Rq);
>
> and Rp is greater than (or equal to) Rq (whence: Rp implies Rq).

The predicatal major premise, "More (or as much) R is required to be P than to be Q," means:

> Only what is at least to a certain measure or degree R (say, Rp) is P;
>
> only what is at least to a certain measure or degree R (say, Rq) is Q;
>
> and Rp is greater than (or equal to) Rq (whence: Rp implies Rq).

We could more briefly write the first two components of the predicatal major premise as exclusive implications: 'If and only if something is Rp, then it is P' and 'If and only if something is Rq, then it is Q'; or more briefly still, as: 'Iff Rp, then P' and 'Iff Rq, then Q'[9]. Note that in my past treatment of the predicatal major premise, in my book *Judaic Logic*, I did not specify the exclusiveness of these two implications; but their exclusiveness is clearly implied by the word "required."

The positive minor premises and conclusions (labeled "*suffective*" because they concern sufficiency) of copulative arguments have the following four components in common. The symbols X and Y here stand for the symbols P or Q and S as appropriate in each mood; that is, we may have "P is R enough to be S," "Q is R enough to be S," "S is R enough to be P," or "S is R enough to be Q."

A proposition of the form "X is R enough to be Y" means:

> X is R, i.e. X is to a certain measure or degree R (say, Rx);
>
> whatever is at least to a certain measure or degree R (say, Ry), is Y, and
>
> whatever is *not* at least to that measure or degree R (i.e. is not Ry), is *not* Y;
>
> and Rx is greater than (or equal to) Ry[10] (whence: "Rx implies Ry"[11]).

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[8]   See my *Judaic Logic* chapter 3, section 2 – 'Validation Procedures' – for additional details on this topic. However, note well, there are significant changes in the present treatment.

[9]   Note that the form "If X, then Y" (or "X implies Y") can only strictly speaking be used if X and Y are *propositions*. If, as here, X and Y are *terms*, then we must strictly speaking say: "If something is X, then it is Y." However, it is all right to use the abridged form in practice if one is well aware of this caveat.

[10]   Note that in *Judaic Logic*, I here have "Ry includes Rx" – but it is evident that "Rx ≥ Ry" is a more meaningful and accurate statement, Ry being the threshold for Y and Rx being sufficient to pass that threshold and therefore equal to it or greater than it. The reason I complicated things in *Judaic Logic* is that I wanted to take into consideration all conceivable ranges (including discontinuous ones), whereas now I realize that the matter is simpler, because in relation to a fortiori argument specifically we need only consider continuous ranges.

[11]   This implication is intended in the sense that a larger number implies every smaller number. For example, if I have $5, then I obviously have $3.

All this implies that X is Y, of course. We could more briefly write the two middle components of a suffective proposition as: 'If something is Ry or more, then it is Y' and 'If something is not Ry or more, then it is not Y'; and these can be put together in a single proposition: 'If and only if something is Ry or more, then it is Y', which can be expressed still more briefly as: 'Iff ≥ Ry, then Y'.

Note that in my past treatment of suffective propositions, in my book *Judaic Logic*, I did not specify the third component, which is the inverse of the second component. I did not at the time realize the significance for a fortiori argument of this negative component, i.e. how essential it is to such argument; so this is an important new finding here. Note that since Ry implies Y and not-Ry implies not-Y, we may say that there is a causal relation – more precisely, a necessary and complete causation – between these two items.

It is this feature that gives meaning to the word "**enough**" (or "sufficiently") in such propositions. This tells us that X has whatever amount of R it takes to be Y; i.e. that X has at least the amount of R *required for* Y. It informs us that there is a ***threshold* of R** (viz. Ry) *as of and above which* X *is indeed* Y, and anywhere *before which* X is *not* Y; Rx is then specified as falling on the required side of the known threshold. In some cases, of course, Rx is exactly equal to Ry; in such cases, the proposition would be stated more precisely as: "X is R **just enough** to be Y." If it is known that Rx is (not equal to but) greater than Ry, we would say: "X is R **more than enough** to be Y." Thus, "enough" means "either just enough or more than enough." It is also clear from the above definition that another way to say "X is R enough to be Y" is: "X is **too much** R to be not-Y" (note the negation of the predicate in the latter form).

Although a proposition of the form 'X is R enough to be Y' implies that 'X is R' and 'X is Y' and 'Rx ≥ Ry', it does not follow that the latter propositions together imply the former, for it is not always true that there is a threshold value of R (Ry) as of which a subject (such as X) gains access to the predicate Y. Thus, we must know (or at least inductively assume) that 'Iff ≥ Ry, then Y' before we can construct a suffective proposition; without that threshold condition for predication, we do not have such a proposition.

The threshold (Ry), though in principle an exact quantity, need not be precisely specified in practice, but can be vaguely intended by saying "the minimum value of R corresponding to Y, whatever it happen to be." But in any case, note well, if there is a threshold, there has to be a negative as well as a positive side of it. We shall see the full significance of this insight further on, when we examine negative suffective propositions more closely. As regards the negative moods of copulative arguments (which involve such propositions), they can, as already mentioned, be validated by *reductio ad absurdum* to the corresponding positive moods, without pressing need to interpret their negative propositions.

It should be emphasized that the kind of thinking that makes a fortiori argument possible depends on there being a regular increase or decrease of the middle term, i.e. along the range R. If we came across a subject (X) whose predicate (Y) varies with respect to R in complex ways – unevenly rising and then falling and/or vice versa, or fluctuating from positive to negative and/or vice versa – we would just not use a fortiori argument. Such argument form is too simple to deal with more complex variables. We would normally only use it for continuous ranges; for discontinuous ones, we would resort to more detailed descriptions and perhaps to mathematical formulas.

Note also that 'X is R enough to be Y' implies 'X is Y' *provided* R is indeed *by itself* enough for Y. If R is in fact only *part of* a set of conditions necessary for Y, then factor R cannot be truthfully said to be 'enough' for Y – or, if it happens to be proposed as 'enough' for Y, the remaining required factors must at least be *tacitly* intended. This would mean, effectively, that the proposition 'X is R enough to be Y' is not as it appears categorical but in fact conditioned on the tacit factors, or alternatively that the outcome of R is not yet Y but some earlier stage of development than Y. To give an example of this important issue: suppose membership in an exclusive club depends on one's age, level of income and maybe other criteria. In that event, one might well say, "this man is old enough but not rich enough to be admitted" – and here, obviously, the man being old 'enough' does not imply he will be admitted, although he may be put on a waiting list till he gets rich 'enough' too. Thus, in common discourse, the word 'enough' may not signify full sufficiency but merely a tendency towards it. But in the present treatise, we intend the word 'enough' in its strict sense.

The above general form of suffective proposition will of course concretize in different ways according to the orientation of the copulative a fortiori argument under consideration:

In positive subjectal arguments (where P, Q are subjects), it will have the forms "P or Q is R enough to be S," which mean:

> P or Q (as the case may be) is to a certain measure or degree R (say, Rp or Rq, as appropriate);
> whatever is at least to a certain measure or degree R (say, Rs) is S and
> whatever is not at least to that measure or degree R (i.e. is not Rs) is not S;
> and Rp or Rq is greater than or equal to Rs.

In positive predicatal arguments (where P, Q are predicates), it will have the forms "S is R enough to be P or Q," which mean:

> S is to a certain measure or degree R (say, Rs);
> whatever is at least to a certain measure or degree R (say, Rp or Rq, as appropriate) is P or Q (as the case may be), and
> whatever is not at least to that measure or degree R (i.e. is not Rp or Rq) is not P or Q;
> and Rs is greater than or equal to Rp or Rq.

The formal difference between commensurative and suffective propositions ought to be clarified here, as I did not do this in my previous writings on this topic. Although their components are very similar in form, namely comparative and hypothetical propositions, what distinguishes them is that in commensurative forms the terms compared, viz. P and Q, are either both subjects or both predicates, whereas in suffective forms the terms compared, viz. X and Y, are one a subject and the other a predicate. For this reason, we cannot reduce commensuratives to suffectives or vice versa.

Even so, it is well to notice that the major premise of *predicatal* a fortiori argument, i.e. the commensurative proposition "More (or as much) R is required to be P than to be Q," is essentially about sufficiency. The word "required" tells us that there is an unstated quantity of R sufficient for P, whereas *lacking* that quantity, whatever it happen to be, being R does *not* entail being P; similarly with regard to Q, of course[12]. Thus, this major premise is a comparison between the thresholds for P and Q, telling us that amounts of R enough for Q are not all enough for P. On the other hand, the major premise of *subjectal* a fortiori arguments makes no mention of sufficiency, merely informing us that P is R and Q is R, and that these two quantities of R are one greater than (or equal to) the other.

All the above comments can be repeated with regard to the propositions involved in **implicational** a fortiori argument, *mutatis mutandis*. Briefly put, we can interpret the commensurative major premises of a fortiori arguments as follows.

The antecedental major premise "P implies more R than (or as much R as) Q does" means:

> P implies a certain measure or degree of R (say, Rp);
> Q implies a certain measure or degree of R (say, Rq);
> and Rp is greater than (or equal to) Rq (whence: Rp implies Rq).

The consequental major premise "More (or as much) R is required to imply P than to imply Q" means:

> Only what implies at least a certain measure or degree of R (say, Rp) implies P;
> only what implies at least a certain measure or degree of R (say, Rq) implies Q;
> and Rp is greater than (or equal to) Rq (whence: Rp implies Rq).

The suffective propositions which are used as minor premises and conclusions of a fortiori arguments can be interpreted as follows. Let us first look at the general positive form, "X implies R enough to imply Y;" this means:

> X implies to a certain measure or degree R (say, Rx);
> whatever implies at least to a certain measure or degree R (say, Ry) implies Y, and
> whatever does *not* imply at least to that measure or degree R (i.e. does not imply Ry) does *not* imply Y;
> and Rx is greater than or equal to Ry.

Notice that in the negative third clause of this definition, I have opted for the minimalist supposition. This choice seems sufficient to make the intended point, viz. that "without the power to imply at least Ry, Y does not follow." I could of course have opted for the more emphatic interpretation, viz. "whatever implies less than that measure or degree R (i.e. implies *not*-Ry), implies *not*-Y," but this would limit the application of the form considerably and unnecessarily. It could be that someone, or myself at a later date, considers the more emphatic option more appropriate; but until some specific reason is found to do so, we are wise to opt for the minimalist position. From the point of view of validation of a fortiori argument, both options are acceptable, because in both cases (as we shall presently see) the third and fourth clauses of the minor premise pass over intact into the conclusion.[13]

The above general form of suffective proposition will of course concretize in different ways according to the orientation of the implicational a fortiori argument under consideration:

In positive antecedental arguments (where P, Q are antecedents), it will have the forms "P or Q implies R enough to imply S," which mean:

> P or Q (as the case may be) implies to a certain measure or degree R (say, Rp or Rq, as appropriate);
> whatever implies at least to a certain measure or degree R (say, Rs) implies S and
> whatever does not imply at least to that measure or degree R (i.e. does not imply Rs) does not

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[12] A requirement is a *sine qua non*. On this basis, we may add two components to the above definition of the predicatal major premise, namely: "what is *not* to the required measure or degree R (i.e. Rp), is *not* P" and "what is *not* to the required measure or degree R (i.e. Rq), is *not* Q." I have done this simply by making the positive premise exclusive – i.e. adding "only" at the beginning of the clauses concerned.

[13] This question does not arise in the case of copulative suffectives, since "is" is negated solely by "is not;" in implicational suffectives, however, though the strict negation of "implies" is "does not imply," there is additionally a stronger form "implies not."

imply S;
and Rp or Rq is greater than or equal to Rs.

In positive consequental arguments (where P, Q are consequents), it will have the forms "S implies R enough to imply P or Q," which mean:

S implies to a certain measure or degree R (say, Rs);
whatever implies at least to a certain measure or degree R (say, Rp or Rq, as appropriate) implies P or Q (as the case may be), and
whatever does not imply at least to that measure or degree R (i.e. does not imply Rp or Rq) does not imply P or Q;
and Rs is greater than or equal to Rp or Rq.

For the rest, what was said earlier for copulatives may be adapted to implicationals.

As regards the **production** of commensurative and suffective propositions, the following should be said. How are they produced, one might ask? That is, how do we get to know them in the first place? The answer is very simple and obvious. The above stated components of commensurative or suffective propositions may be viewed as the premises of the productive arguments giving rise to them. That is to say, the simpler forms, which we have above identified as implied in and together defining these more complex forms, may be presented as premises of arguments whose conclusions are commensurative or suffective propositions. Note this well, for here we have numerous new arguments for formal logic to list as such. There is, to be sure, a bit of circularity in claiming such arguments. However, though that may be true at the most formal level, at more concrete levels such arguments are quite useful.

Validation procedures

We are now in a position to examine a fortiori argument for purposes of validation. What must be understood is that the middle term (R) of copulative argument is *its essential element*. Being the subject or predicate of the three other terms (the major term P, the minor term Q, and the subsidiary term S), the middle term underlies, is present in, all of them. Similarly, of course, implicational argument hinges on the middle thesis. We can say that a fortiori argument is principally about the middle item, and only incidentally about the other three items; it is the core or center of gravity of the whole argument; it is the common ground and intermediary of the three other items.

What a fortiori argument does is to relate together *three values of the middle item R* (here symbolized by Rp, Rq and Rs) found in relation to the other three items and thus representing them. The middle item of a fortiori argument is always something that varies quantitatively, in measure or degree – and the argument constitutes a comparison and hierarchical ordering of its different values (which are given in relation to the three other items). The truth of all this can be easily seen with reference to the following diagram, where quantities of R on the right are greater than quantities of R on the left.
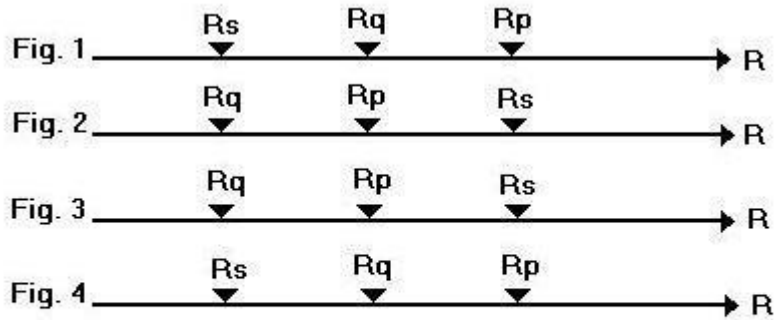
The a fortiori argument orders items P, Q, S, according to their position in a common continuum R

Diagram 1.1

That, then, is the essence of a fortiori argument: it is a comparison between the various quantities (measures or degrees) of the middle item (term or thesis) that are copulatively or implicationally involved in the other three items (as subjects or predicates, or antecedents or consequents, of it, as the case may be). We can thus present the

quantitative core of the validations very simply as follows, with reference to the comparative propositions implied in the premises and conclusions. Here, as always, ≥ means 'is greater than or equal to' and < means 'is less than'[14]:

Structure	Subjectal or antecedental		Predicatal or consequental	
Polarity	positive	negative	positive	negative
Major premise	Rp ≥ Rq	Rp ≥ Rq	Rp ≥ Rq	Rp ≥ Rq
Minor premise	Rq ≥ Rs	Rp < Rs	Rs ≥ Rp	Rs < Rq
Conclusion	So, Rp ≥ Rs	So, Rq < Rs	So, Rs ≥ Rq	So, Rs < Rp

Table 1.2

Note that the egalitarian positive subjectal (or antecedental) conclusion Rp = Rs can only be drawn from the premises Rp = Rq and Rq = Rs. Likewise, the egalitarian positive predicatal (or consequental) conclusion Rs = Rq can only be drawn from the premises Rs = Rp and Rp = Rq. In all other positive arguments, the conclusions would be Rp > Rs or Rs > Rq (as the case may be), even if one of the premises concerned involves an equation. It follows that the egalitarian negative argument of subjectal form has premises Rp ≥ Rq and Rp ≠ Rs and conclusion Rq ≠ Rs; while that of predicatal form has premises Rp ≥ Rq and Rs ≠ Rq and conclusion Rs ≠ Rp.

Another way to illustrate the quantitative aspect of a fortiori argument is by means of bar charts, as in the diagram below. Given that Rp is greater than (or equal to) Rq, there are three possible positions for Rs: in (a) Rs is greater than (or equal to) Rp and therefore than (or to) Rq; in (b) Rs is smaller than (or equal to) Rq and therefore than (or to) Rp; and in (c) Rs is in between Rp and Rq, in which case no conclusion can be drawn. Chart (a) can be used to illustrate the positive predicatal and negative subjectal moods, and chart (b) the positive subjectal and negative predicatal moods, while chart (c) can be used to explain invalid arguments.

| Rs   Rp   Rq | Rp   Rq   Rs | Rp   Rs   Rq |

| (a) | (b) | (c) |
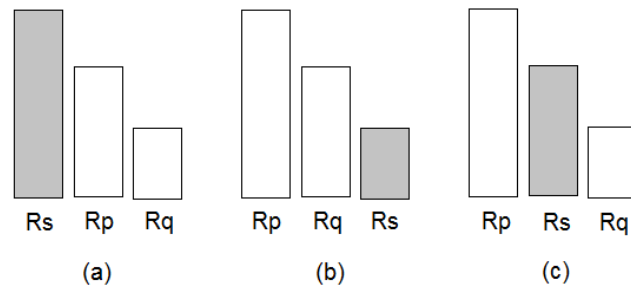
Diagram 1.2

In addition to the quantitative arguments above tabulated[15], we only need to select certain clauses from our premises to derive our conclusions, as follows (check and see for yourself):

• The conclusion of a positive subjectal argument, namely the positive suffective proposition "P is R enough to be S," is composed of four clauses:

P is to a certain measure or degree R (say, Rp);
whatever is at least to a certain measure or degree R (say, Rs), is S;
whatever is not at least to that measure or degree R (i.e. is not Rs), is not S;
and Rp is greater than (or equal to) Rs.

In this case, the four components are obtained as follows: *the first from the major premise, the second and third from the minor premise, and the fourth from the tabulated quantitative argument which is drawn from both premises.* Here, note well, the "enough R" condition of the conclusion (implied in its second and third components) comes from the minor premise, because it concerns the subsidiary term (S). Here, then, the crucial threshold value of R is Rs, i.e. the minimum value of R needed to be S; knowing that Rq equals or exceeds Rs, we can predict that Rp does so too.

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[14]   As regards use here of the 'is less than' relation in the negative moods, see the justification for this in the next section.
[15]   Looking at the tabulated quantitative arguments, we are tempted to say that there is 'something of syllogistic reasoning' in a fortiori argument, insofar as they all involve movement of thought from one item to another via an intermediary. But, note well, this is not really syllogistic inference from one class to another, but a more mathematical inference based on comparison of magnitudes. Clearly, we cannot say that a fortiori argument *is* syllogism; it is manifestly a distinct form of reasoning.

- The conclusion of a positive predicatal argument, namely the positive suffective proposition "S is R enough to be Q," is composed of four clauses:

    S is to a certain measure or degree R (say, Rs);
    whatever is at least to a certain measure or degree R (say, Rq), is Q;
    whatever is not at least to that measure or degree R (i.e. is not Rq), is not Q;
    and Rs is greater than (or equal to) Rq.

In this case, the four components are obtained as follows: *the first from the minor premise, the second and third from the major premise, and the fourth from the tabulated quantitative argument which is drawn from both premises*. Here, note well, the "enough R" condition of the conclusion (implied in its second and third components) comes from the major premise, because it concerns the minor term (Q). Here, then, the crucial threshold value of R is Rq, i.e. the minimum value of R needed to be Q; knowing that Rp equals or exceeds Rq, we can predict that Rs does so too.

Note that in both the above moods, the conclusion of the a fortiori argument comes solely and entirely from the two premises together (not separately). It is true that the premises contain more information than the conclusion does; but that only means that not all the information in them is used. This does not signify redundancies in the premises, because their form is essential to intuitive human understanding of the argument, whose conclusion has similar form to the minor premise.

The corresponding negative moods are most easily validated by *reductio ad absurdum*. We say: suppose the putative conclusion is denied, then combining such denial with the same major premise we would obtain a denial of the given minor premise; this being absurd, the putative conclusion must be valid.

More briefly put, the positive conclusions are composed of the following elements drawn from the respective premises: in subjectal argument, "P is Rp, what is Rs is S and what is not Rs is not S, and Rp ≥ Rs;" and in predicatal argument, "S is Rs, what is Rq is Q and what is not Rq is not Q, and Rs ≥ Rq." The corresponding negative conclusions imply that one or more of these four elements is denied.

It is worth here stressing the utility of the threshold condition, i.e. the implication of the minor premise that there is a threshold value of R (say, Rt), which has to be reached or surpassed before a subject X can accede to a predicate Y (i.e. Rx must be ≥ Rt which is ≥ Ry).

- In positive subjectal argument, the threshold of the minor premise and thence of the conclusion means that *not all* R are S (since some things are not Rs). Clearly, if all R were S, then we could from the major premise 'P is more R than Q' (which implies that 'P is R' and 'Q is R'), without recourse to the simplified minor premise 'Q is S', obtain the conclusion that 'P is S' (and even that 'Q is S')!

- In positive predicatal argument, one of the thresholds of the major premise and thence of the conclusion means that *not all* R are Q (since some things are not Rq). Clearly, if all R were Q, then we could from the major premise 'More R is required to be P than to be Q' (which implies that 'R is required to be P', and thence that 'all P are R'[16]), together with the simplified minor premise 'S is P', obtain (via the intermediate conclusion 'S is R') the conclusion that 'S is Q'!

In both these eventualities, the argument would be *merely syllogistic*, and not function like an a fortiori argument. Thus, the threshold condition is *essential* to the formation of a truly a fortiori argument; it is not something that can be ignored or discarded. Many people think that a fortiori argument can be formulated without this crucial condition, but that is a grave error on their part.

The same validation work can be easily done with implicational arguments, *mutatis mutandis*. We have thus formally and indubitably demonstrated all the said moods of a fortiori argument to be valid. As regards *invalid* a fortiori arguments, the following can be said. If the major item P is not identical in the major premise and in the minor premise or conclusion (so that there are effectively two major items), and/or if the minor item Q is not identical in the major premise and in the minor premise or conclusion (so that there are effectively two minor items), and/or if the middle item R is not identical in the major premise, the minor premise and the conclusion (so that there are effectively two or three middle items), and/or if the subsidiary item S is not identical in the minor premise and the conclusion (so that there are effectively two subsidiary items) – in any such cases, there is illicit process. Needless to say, "identical" here refers to identity *not only in the words used, but also in their intentions*; we are sometimes able

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[16]   Of course, 'more R is required to be P than to be Q' first implies that '*specifically* Rp is required to be P' (as well as '*specifically* Rq is required to be Q', and Rp > Rq), which means: 'if not Rp, then not P'. But here, 'Rp' refers to the value of R required for P, *whatever it happens to be*; so it is no different (except symbolically) than the mere, indefinite 'R'. In other words, 'all P are R' does not refer to just any or all values of R, but some appropriate value, whatever it happens to be. And clearly, this predicate R (meaning Rp) fits under the wider generality 'all R' in the syllogistic major premise 'all R are Q'. Thus, we can indeed infer, from 'S is P' and 'all things P are R' that 'S is R', and from the latter and the supposed 'all things R are Q' that 'S is Q'.

to formulate two terms in such a way as to make them seem the same superficially, although in fact they are not the same deeper down[17].

Likewise, if an item or a proposition is negative where it should be positive or vice versa – here again, we have fallacious reasoning. Although all such deviations from the established norms are obviously invalid, since we cannot formally validate them, they are often tried by people in practice, so it is worth keeping them in mind.

Identification in practice. We have so far theoretically described and validated a fortiori arguments. But the reader should also develop the ability *to recognize* such arguments when they occur in practice, in written text or oral discourse. The following are a few useful pointers. A fortiori argument is usually signaled by some distinctive word or phrase like "a fortiori" or "all the more/less," or "so much (the) more/less," or more rhetorically: "how much (the) more/less?!" Such signals are of course helpful, though they do not always occur (and moreover, they are sometimes used misleadingly, when there is no a fortiori argument in fact). Sometimes, we can guess that an a fortiori argument is involved, by noticing the use of an expression like "enough" or "sufficiently." But sometimes, there is no verbal indicator at all, and we can only determine the a fortiori form of the argument at hand by examining its content.

Very often, the major premise remains unstated, though it can be readily formulated in the light of the minor premise and conclusion. Very often, too, the middle term is left tacit, in the major premise or in the minor premise or in the conclusion, or even throughout the argument; in such cases, we have to guess at the underlying intent of the argument's author. All we are given, in very many cases, is an if–then statement with three terms; and often the 'if' and 'then' operators are missing too! There is nevertheless usually enough information for us to reconstruct the intended a fortiori argument, assuming some such argument is indeed intended (i.e. we must be careful not to artificially 'read in' the argument for our own purposes).

The following indices permit us to determine the exact mood of copulative argument. Find the term (S) common to both propositions (the premise and conclusion), and see whether it stands as subject or predicate. The positive subjectal form appears as: "Q is S; therefore, P is S;" and the negative subjectal form appears as: "P is not S; therefore, Q is not S." Notice here that S (the common term) is a predicate, and P and Q (the other two terms) are subjects. The positive predicatal form appears as: "S is P; therefore, S is Q;" and the negative predicatal form appears as: "S is not Q; therefore, S is not P." Notice that here S (the common term) is a subject, and P and Q (the other two terms) are predicates. Similarly for implicational arguments, except that "implies" appears instead of "is."

Of course, not even all the details given in the preceding paragraph may appear. For example, instead of "Q is S; therefore, P is S," the speaker may say "Q is S: all the more P!" But we can easily add the missing clause "is S" that makes the consequent (conclusion) a mirror image of the antecedent (minor premise). We must then look for a middle term R, such that "P is more R than Q" is true (or at least somewhat credible), and also such that "Q is R enough to be S" is true, and therefore "P is R enough to be S" is likewise true – and we have reconstructed the intended a fortiori argument.

Obviously, a proposition of the form "X is Y" does not, strictly speaking, imply one of the form "X is R enough to be Y" – that is, the mere fact that X is Y does not indicate that there is a threshold of R that needs to be crossed for X to be Y. Nevertheless, we often *inductively* infer the latter from the former by reasoning that if there indeed is an a fortiori argument there must indeed be such a threshold condition for the predication. Thus, we construct the more complex premise from the simpler given, thinking "well, if X is Y, it must have been R enough to be Y!" This concerns the minor premise; as regards the conclusion, we deduce the simpler proposition from the more complex.

It should be stressed that the term common to the two given propositions is in some cases the middle term (R), rather than the subsidiary term (S). An example of that would be the sentence: "Q is bad enough; imagine what P would be!" Here, the common term "bad" is of course the middle term (as the expression "enough" indicates); and no subsidiary term is mentioned, though one can guess what it might be. A fuller statement of the minor premise and conclusion would thus be: "if Q is bad (R) enough to be avoided (S), then all the more P is bad (R) enough to be avoided (S)."

Of course, though we may manage to fully reconstruct the intended a fortiori argument, it may yet be found invalid – e.g. if, as sometimes happens, the roles of P and Q are reversed; but this is another issue, of course. That is: first find out what form the author's intended argument has; then judge whether it is objectively valid or not. Also, do not confuse the issues of validity and truth: the argument may be well-formed, and yet be wrong due to its reliance on a false premise or other.

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[17]   See further on the discussion concerning 'Species and Genera'.

4. Ranging from zero or less

An observed practice

Often, in practice (e.g. in the Talmud), we find a fortiori arguments stated in the following discursive form:

> "*If Q, which is not R, is S, then (all the more) P, which is R, is S.*"

Many people get confused by this construction, and fail to understand the nature of a fortiori argument because of it. To put such an argument in standard positive subjectal form, and thus validate it, we must first realize that the antecedent proposition is the minor premise and the consequent one is the conclusion. Then we must see that the major premise is also present by the mention of Q being not R and P being R[18]. This tells us that R is the middle term, ranging from zero to some higher quantity. Whence we can formulate the major premise as "P (for which R > 0) is more R than Q (for which R = 0)." The minor premise can now be more precisely stated as "Q is R enough to be S;" and the conclusion likewise as "P is R enough to be S."

We can proceed in the same way to deal with a negative subjectal argument which is stated in the form:

> "*If P, which is R, is not S, then (all the more) Q, which is not R, is not S.*"

Similarly, we can readily standardize a positive predicatal argument that appears in the form:

> "*If S is P, even though P requires R, then (all the more) S is Q, since Q does not require R.*"

Or a negative predicatal argument that appears in the form:

> "*If S is not Q, even though Q does not require R, then (all the more) S is not P, since P requires R.*"

The clauses "P requires R" and "Q does not require R" used here should be understood to more precisely mean, respectively: "some amount of R is required, for something to be P" and "no amount of R is required, for something to be Q."

And similarly with the implicational equivalents of these four copulative arguments. In short, do not be confounded by the varying ways that a fortiori argument appears in practice in human discourse, but always be ready to reword it in standard form. Once we have mastered the formalities, no argument looks intractable.

It should be obvious that the four discursive forms we have just listed are merely special cases of another four, more broadly applicable and also often occurring in practice, namely, respectively:

> "*If Q, which is less R, is S, then P, which is more R, is S.*"
> "*If P, which is more R, is not S, then Q, which is less R, is not S.*"
> "*If S is P, even though P requires more R, it follows that S is Q, since Q requires less R.*"
> "*If S is not Q, even though Q requires less R, it follows that S is not P, since P requires more R.*"

These four statements are ways we often briefly articulate our a fortiori thoughts. The first two statements allude to subjectal argument. Their common major premise is "P is more R than Q is," and their minor premises and conclusion are: in the positive case, "if Q is (R enough to be) S, then P is (R enough to be) S;" and in the negative case, "if P is not (R enough to be) S, then Q is not (R enough to be) S." The second two statements allude to predicatal argument. Their common major premise is "More R is required to be P than to be Q," and their minor premises and conclusion are: in the positive case, "if S is (R enough to be) P, then S is (R enough to be) Q;" and in the negative case, "if S is not (R enough to be) Q, then S is not (R enough to be) P."

Clearly, "is R" and "is not R" are special cases of "is more R" and "is less R," respectively; and "requires R" and "does not require R" are special cases of "requires more R" and "requires less R," respectively. What the above observations mean is that we can, in theory as well as in practice, count the negation of the middle term R as a limiting or special case of R, i.e. as simply the value of R equal to zero in the range of possible values of R! Upon reflection, it occurs to me that the middle term R may even have *negative* values! For example, "Jack's financial situation is better than Jill's" may be true because Jack has a few dollars in the bank whereas Jill has debts; indeed, both of them may have debts, though his are less than hers. So R may in principle range anywhere from minus to plus infinity, without affecting the said forms of a fortiori argument.

What this insight implies is that, in the context of a fortiori logic, *if something* **is not R (i.e. is zero R or less than that), it is still formally counted as something that** **is R, with however the understanding that in its case R ≤ 0**. That is to say, for our purposes here, granting that R is broad enough to include not-R, it follows that everything is R and nothing is not-R!

How can this be? It must be understood that when we pass over from the logic of R *exclusive* of not-R, to that of R *inclusive* of not-R, the meaning of R is subtly changed. Instead of R meaning 'being R' (i.e. belonging to class R), it now means 'having to do with R' (i.e. merely pertaining to R). We can rightly say that not-R pertains to R, even

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[18]   Note that the middle term R may refer to a positive or negative characteristic, so long as indeed the major term P has (or requires) it, while the minor term Q lacks (or does not require) it, so that everything falls into place (i.e. so that the value of the middle for P be greater than that for Q, and not vice versa). In other words, when interpreting a given argument, we cannot automatically take what seems given as logically appropriate, but may have to judiciously choose as our effective middle term another (preferably derived from the given or suggested one) that fits the bill.

though not-R is not in a strict sense R. Another way to put it is to say that R as against not-R is denotative, whereas R including not-R is connotative[19]. More will be said on this issue further on, when we deal in a more general way with relative terms.

Although in principle an (inclusive) range R may have any value from minus infinity through zero to plus infinity, in practice it may be more limited. To be in accord with the law of the excluded middle, the range must include, as well as at least one positive value R > 0, the null value R = 0 or a negative value R < 0. An example of a limited range is that of temperature: although we can imagine temperatures to be infinitely cold, physicists have discovered through experiment that the minimum temperature in nature is -273°C (on this basis the Celsius scale was replaced by the Kelvin scale, in which this minimum is 0°K); similarly, we can expect there to be a maximum temperature in nature, even if we do not know its magnitude.

Note well that I did not invent this single-range artifice, but merely noticed its use in a fortiori practice and adopted it for theoretical purposes. This convention is, as we shall now see, a very important new finding and idea, which has important consequences for a fortiori logic, greatly simplifying it. It allows us, notably, to more precisely and positively interpret the negative forms of the commensurative and suffective propositions used in a fortiori argument. However, as we shall see, it is easier to apply to copulative reasoning than to implicational.

Implications of the commensurative forms

First, let us deal with *copulative* forms. The subjectal major premise "**P is more R than Q (is R)**" obviously implies both that "P is R (to some unspecified measure or degree)" and that "Q is R (to some unspecified measure or degree)," where P, Q are either designated individual things or they are classes (in which case the meaning is "all P are R" and "all Q are R"). As just pointed out, we can and often do use this form when the middle term R has a range of values from some negative lower limit (known or unknown, stated or unstated) or even from minus infinity, through zero, to some positive upper limit (known or unknown, stated or unstated) or even to plus infinity. That is to say, the possible values R may be a boundless range from negative infinity to positive infinity, or any more limited range in between (such as entirely positive or from zero upward).

Additionally, of course, the subjectal major premise implies that Rp (the value of R for P) is greater than Rq (the value of R for Q). It does not matter whether P is positive, zero or negative, and whether Q is positive, zero or negative, provided that the quantity Rp is superior to the quantity Rq. In the special case where Rq is zero, Rp must be positive; and in the special case where Rp is zero, Rq must be negative. In the special case where Rp and Rq are both zero, or both have the same positive or negative value of R, the proposition must be changed to the egalitarian form "P is as much R as Q (is R)." In cases where Rp is smaller than Rq, the form of course becomes "P is less R than Q (is R)."

Let us now consider the denial of the form "P is more R than Q (is R)." What is the meaning of the negative form "P is *not* more R than Q (is R)," in the light of the above insights? In the past, before I realized that the values of R for P and/or Q may be zero (or negative), I would have said that such zero (or negative) values are absent from the positive form and therefore implicit in the negative form. However, now that these values are perceived (or conceived, for the purposes of a fortiori logic) as possibilities within the positive form, the negative form acquires a much more specific meaning, namely the disjunction "P is R less than or as much as Q (is R)." Such disjunctive major premises often, of course, occur in practice.

That is to say, the three positive commensurative forms: "P is more R than Q (is R)," "P is less R than Q (is R)" and "P is as much R as Q (is R)" are the exhaustive repertoire of such propositions, so that *if any one of them is denied, one of the other two must be true*. In other words, in this context, negative propositions are redundant since their meanings can be fully represented by positive ones! This greatly simplifies our formal work in this field.

Similarly, of course, the predicatal major premise "**More R is required to be P than to be Q**" implies (in an extensional perspective) both that "some R are P," i.e. that some things that are R to a sufficient degree are also P; and that "some R are Q," i.e. that some things that are R to a sufficient degree are also Q. Moreover, it is implied that Rp is greater than Rq. Here again, note well, the form is to be taken in a wide sense, allowing in principle for any values of R, positive, zero or negative, though in practice a narrower range may be tacitly or specifically intended. And here again, the negative form, "More R is *not* required to be P than to be Q," is redundant, because it just means: "Less or as much R is required to be P than/as to be Q."

We can therefore define all copulative forms of commensurative proposition as follows, irrespective of the values of Rp and Rq (i.e. be they positive, zero or negative):

The subjectal forms:

P is to a certain measure or degree R (say, Rp);

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[19] We could note here that Oriental logic (the logic of Indian and Chinese cultures) is perhaps predominantly connotative, rather than denotative as in the West. This would explain why they regard terms as relative rather than absolute, and contradictions as possible.

Q is to a certain measure or degree R (say, Rq);
and Rp is greater than, equal to or lesser than Rq.

And the predicatal forms[20]:

What is to a certain measure or degree R (say, Rp), is P;
what is to a certain measure or degree R (say, Rq), is Q;
and Rp is greater than, equal to or lesser than Rq.

As can be seen from their above definitions, the subjectal commensurative propositions "P is more R than Q is" and "Q is less R than P is" are each other's *converse*; we can convert either to the other, without loss of information; similarly, "P is as R as Q is" and "Q is as R as P is" are equivalent, and so are compounds of the said forms. Likewise, the predicatal form "more R is required to be P than to be Q" is convertible to "less R is required to be Q than to be P," and vice versa; and similarly with the egalitarian and compound forms.

Similar interpretations can be made with regard to the commensurative major premises of *implicational* a fortiori arguments. Simply put: for the definitions, instead of saying "is" or "to be" in relation to the terms P, Q, and R, we would say "implies" or "to imply" in relation to the theses P, Q, and R.

Thus, the antecedental forms signify:

P implies to a certain measure or degree R (say, Rp);
Q implies to a certain measure or degree R (say, Rq);
and Rp is greater than, equal to or lesser than Rq.

The positive form refers to: "P implies more R than Q does," and the negation of that means: "P implies less R than or as much R as Q does."

And the consequental forms signify:

What implies to a certain measure or degree R (say, Rp), implies P;
what implies to a certain measure or degree R (say, Rq), implies Q;
and Rp is greater than, equal to or lesser than Rq.

The positive form refers to: "More R is required to imply P than to imply Q," and the negation of that means: "Less or as much R is required to imply P than/as to imply Q."

Implications of the suffective forms

First, let us deal with *copulative* forms. The positive form of suffective proposition, "**X is R enough to be Y**," used in the minor premise and conclusion of a fortiori argument, implies both that "X is R" and that "X is Y." In the subjectal form, X stands for P or Q (as the case may be) and Y for S; and in the predicatal form, X stands for S and Y for P or Q (as the case may be). Here again, the middle term R may conceivably be positive, zero or negative. The important thing to keep in mind is that there is a threshold value of R as of and above which X is Y, and below which X is not Y. This means that the negative form "X is *R not* enough to be Y"[21] implies both that "X is R" (whether R is greater than, equal to or less than zero) and that "X is not Y"[22].

As already said, the generic positive suffective form, "X is R enough to be Y" can be defined by means of four simpler propositions as follows:

X is to a certain measure or degree R (say, Rx);
whatever is to a certain measure or degree R (say, Ry), is Y, and
whatever is *not* to that measure or degree R (i.e. is not Ry), is *not* Y;
and Rx is greater than (or equal to) Ry.

What does *denial* of this collection of propositions mean, specifically? We can say that "X is Rx" remains true no matter what, because (as above explained) X like everything else is necessarily R if we define R broadly enough to include not R; and because Rx is by definition the value of R for X, whatever that happen to be. Similarly, Ry is by definition the quantity of R enough for Y, whatever that happen to be, so there is no sense in denying that "Ry is Y;" and likewise, "not-Ry is not Y" is not open to doubt. Therefore, the only way that the collection as a whole can be denied is by denying its last clause, viz. "Rx ≥ Ry," i.e. by saying that "Rx < Ry;" and this makes sense, because it is the same as saying that "X is not Y"[23]. Thus, the negative form, "X is R **not enough** to be Y" can also be defined in a positive manner, as follows:

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[20]   As earlier stated, we might add two components to this definition of the predicatal major premise, namely: "what is *not* to that measure or degree R (i.e. Rp), is *not* P" and "what is *not* to that measure or degree R (i.e. Rq), is *not* Q."
[21]   Or "X is not R enough to be Y" or "X is not enough R to be Y" – I take these three forms as equivalent.
[22]   Note well this implication, because one might intuitively, when looking at the negative form, erroneously think that maybe it *leaves open* whether X is or is not Y. To realize pictorially the truth of this implication, imagine a yardstick, and suppose that the threshold for some purpose is declared as 2 feet. If we measured a ribbon with it and found it 30 inches long, we would say: "it is long enough for our purpose;" whereas, if we found it 20 inches long, we would naturally say: "it is not long enough for our purpose."
[23]   It cannot be supposed that X might be Y *by virtue of something other than* R, say through Z. If X is Y, then Rx is necessarily greater than or equal to Ry; and if Rx is less than Ry, then X is necessarily not Y.

X is to a certain measure or degree R (say, Rx);

whatever is to a certain measure or degree R (say, Ry), is Y, and

whatever is *not* to that measure or degree R (i.e. is not Ry), is *not* Y;

and Rx is less than Ry.

That is to say, the difference between the positive and the negative forms is that the former has Rx greater than or equal to Ry, whereas the latter has Rx less than Ry. That's all. Note this well. Thus, other ways to say: "X is not enough R to be Y" would be: "X is **less than enough** R to be Y" or "X is **too little** R to be Y."

Note well again that the positive form "X is R enough to be Y" implies "X is Y," and its negation "X is not enough R to be Y" implies "X is not Y." It follows from this that "X is R enough to be Y" is equivalent to "X is not enough R to be not Y," and "X is R enough to be not Y" is equivalent to "X is not enough R to be Y." These are equations we will find good use for further on.

Note also that although "X is R enough to be Y" implies that "X is R" and "X is Y," it does not follow that given the latter two propositions we have enough information to construct the former one; we additionally need to know that "X is not Y" below some value of R, because this makes possible the statement that in the case of X, the amount of R is *enough* for it to be Y. Similarly, although "X is R *not* enough to be Y" implies that "X is R" and "X is *not* Y," it does not follow that given the latter two propositions we have enough information to construct the former one; we additionally need to know that "X is Y" as of and above some value of R, because this makes possible the statement that in the case of X, the amount of R is *not enough* for it to be Y.

We can therefore define all copulative forms of suffective proposition, irrespective of the values of Rx and Ry (i.e. be they positive, zero or negative), as follows:

The subjectal forms:

P or Q (as the case may be) is to a certain measure or degree R (say, Rp or Rq, as appropriate);

whatever is to a certain measure or degree R (say, Rs), is S and

whatever is *not* to that measure or degree R (i.e. is not Rs), is *not* S;

and Rp or Rq ≥ Rs (positive form), or Rp or Rq < Rs (negative form).

And the predicatal forms:

S is to a certain measure or degree R (say, Rs);

whatever is to a certain measure or degree R (say, Rp or Rq, as appropriate), is P or Q (as the case may be), and

whatever is *not* to that measure or degree R (i.e. is not Rp or Rq), is *not* P or Q;

and Rs ≥ Rp or Rq (positive form), or Rs < Rp or Rq (negative form).

Conversion of suffective propositions is not possible. This can be ascertained by examination of their defining implications. Try relating, for instance, a subjectal form "X is R enough to be Y" to a predicatal form "Y is R enough to be X." Since the underlying if–then components cannot be converted, nor can their suffective compounds.

Similar interpretations can be made with regard to the suffective minor premises and conclusions of *implicational* a fortiori arguments. Simply put: for the definitions, instead of saying "is" or "to be" in relation to the terms P, Q, and R, we would say "implies" or "to imply" in relation to the theses P, Q, and R.

5. Secondary moods

From the preceding interpretations we can derive a number of useful arguments. Such arguments may be considered as belonging to the family of 'a fortiori', although they are not among the four regular copulative moods. They can be labeled as 'secondary' a fortiori moods, as against the 'primary' moods that usually define the argument form for us. They are validable, note well, based on the understanding that the value of R may range from minus infinity through zero to plus infinity[24].

Producing a commensurative proposition. The first two arguments are distinctive in that their premises are both suffective and their conclusion is commensurative. These arguments describe for us how we might occasionally produce the major premises of primary arguments. Consider the following argument, for a start:

P is R enough to be S, and

Q is R *not* enough to be S.

Therefore, P is more R than Q (is R).

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[24]   This new understanding is the reason why my present treatment of the topic of secondary moods differs from the approach in my *Judaic Logic*.

This shows us how a *subjectal* commensurative proposition "P is more R than Q (is R)" can be constructed (i.e. deduced) from two suffective propositions. The positive premise tells us that the value of R for P is equal to or greater than that for S; and the negative one tells us that the value of R for Q is less than that for S. Since $Rp \geq Rs$ and $Rq < Rs$, it follows that $Rp > Rq$. We also know from the premises that what is P is Rp and what is Q is Rq. Whence the conclusion: P is more R than Q. Similarly, consider the following argument:

> S is R *not* enough to be P, and
> S is R enough to be Q.
> Therefore, more R is required to be P than to be Q.

This shows us how a *predicatal* commensurative proposition "More R is required to be P than to be Q" can be constructed (i.e. deduced) from two suffective propositions. The positive premise tells us that the value of R for S is equal to or greater than that for Q; and the negative one tells us that the value of R for S is less than that for P. Since $Rs \geq Rq$ and $Rs < Rp$, it follows that $Rp > Rq$. We also know from the premises that what is Rp is P and what is Rq is Q. Whence the conclusion: More R is required to be P than to be Q.

It should be emphasized that the above two arguments are not the only ways we can produce a subjectal or predicatal commensurative proposition. We can always produce such propositions with reference to their formal definitions. As we have seen, a subjectal commensurate, "P is more R than (or as much R as) Q is," is composed of the three elements: "P is R, i.e. P is to a certain measure or degree R (say, Rp); Q is R, i.e. Q is to a certain measure or degree R (say, Rq); and Rp is greater than (or equal to) Rq (whence: Rp implies Rq)." A predicatal commensurate, "More (or as much) R is required to be P than to be Q," is composed of the three elements: "Only what is at least to a certain measure or degree R (say, Rp) is P; only what is at least to a certain measure or degree R (say, Rq) is Q; and Rp is greater than (or equal to) Rq (whence: Rp implies Rq)."

That is, in general, to produce a commensurate proposition, we need only to supply its constituent parts. That is, of course, also logical argument: the said components are the premises and the composite commensurative proposition is the conclusion. The same, of course, can be said regarding the production of suffective from their constituent parts: that is also logical argument.

The above listed two secondary moods of a fortiori argument are only special cases of such production. Similar constructions are of course possible with regard to implicational propositions:

> P implies R enough to imply S, and
> Q implies R, but *not* enough to imply S.
> Therefore, P implies more R than Q does.

> S implies R, but *not* enough to imply P, and
> S implies R enough to imply Q.
> Therefore, more R is required to imply P than to imply Q.

As explained further on, we must here be extra careful, as the exact location of the negation in the proposition may affect its meaning and logic. That is to say, it is more difficult with implicationals, than it was with copulatives, to put positives, zeros and negatives in the same basket.

Using a negative commensurative major premise. From each of the above two secondary moods, we can derive two more by *reductio ad absurdum*. These four resemble primary moods, in that their major premises are commensurative and their minor premise and conclusion are suffective, but note well that they differ in that all have a negative major premise.

- Looking at the above subjectal argument: if the positive conclusion is denied and the positive major premise is retained, then the negative minor premise must be denied (so we have a negative and two positive propositions); the result is a mood apparently (since from P to Q) from major to minor (more on this in a moment).

> P is *not* more R than Q (is R),
> and P is R enough to be S.
> Therefore, Q is R enough to be S.

- Looking again at the above subjectal argument: if the positive conclusion is denied and the negative minor premise is retained, then the positive major premise must be denied (so we have three negative propositions); the result is a mood apparently (since from Q to P) from minor to major (more on this in a moment).

P is *not* more R than Q (is R),
and Q is R *not* enough to be S.
Therefore, P is R *not* enough to be S.

However, these two moods are not as new as they might seem. For their negative major premise, "P is not more R than Q," can be restated in positive terms as: "P is less R than or as much R as Q." That is, each negative mood refers to two positive moods, according as the major premise is in fact egalitarian or inferior. In the special case where the major premise is the egalitarian "P is as much R as Q," the two arguments would be, respectively, positive or negative subjectal *a pari* arguments. While in the special case where the major premise is the inferior "P is less R than Q," we could convert it to "Q is more R than P," and the two arguments would then be, respectively, positive or negative subjectal superior arguments. The difference between these two cases, note well, is that when the major premise is egalitarian, the terms P and Q can still be called the major and minor, whereas when the major premise is inferior, Q refers to the major term and P to the minor term (so the symbols would have to be switched). So really, the above arguments are not deeply new, but only superficially so.

- Looking at the above predicatal argument: if the positive conclusion is denied and the positive minor premise is retained, then the negative major premise must be denied (so we have a negative and two positive propositions); the result is a mood apparently (since from Q to P) from minor to major (more on this in a moment). Note that the conclusion would still be valid in the special case where the negative major premise means "As much R is required to be P as to be Q," since then this would be positive predicatal *a pari* argument (which can, as we have seen, go from minor to major).

    More R is *not* required to be P than to be Q,
    and S is R enough to be Q.
    Therefore, S is R enough to be P.

- Looking again at the above predicatal argument: if the positive conclusion is denied and the negative major premise is retained, then the positive minor premise must be denied (so we have three negative propositions); the result is a mood apparently (since from P to Q) from major to minor (more on this in a moment).

    More R is *not* required to be P than to be Q,
    and S is R *not* enough to be P.
    Therefore, S is R *not* enough to be Q.

However, here again, these two moods are not as new as they might seem. For their negative major premise, "More R is not required to be P than to be Q," can be restated in positive terms as: "Less R or as much R is required to be P than/as to be Q." That is, each negative mood refers to two positive moods, according as the major premise is in fact egalitarian or inferior. In the special case where the major premise is the egalitarian "As much R is required to be P as to be Q," the two arguments would be, respectively, positive or negative predicatal *a pari* arguments. While in the special case where the major premise is the inferior "Less R is required to be P than to be Q," we could convert it to "More R is required to be Q than to be P," and the two arguments would then be, respectively, positive or negative predicatal superior arguments. The difference between these two cases, note well, is that when the major premise is egalitarian, the terms P and Q can still be called the major and minor, whereas when the major premise is inferior, Q refers to the major term and P to the minor term (so the symbols would have to be switched). So really, the above arguments are not deeply new, but only superficially so.

Thus, the above listed four secondary moods of a fortiori argument with a negative commensurative major premise' can be directly reduced to primary forms of the argument. Nevertheless, even though they teach us nothing very new, they are still worth explicitly listing to draw attention to them. The moods shown are copulative arguments. Analogous moods can be formulated for implicational argument.

The above listings are obviously exhaustive, since all formal possibilities are accounted for.

Negative items

The valid primary and secondary moods are all formulated with positive terms or theses (P, Q, R, S). As regards moods involving the negations of some or all of these items, it is obvious that if we substitute not-P for P, and/or not-Q for Q, and/or not-R for R, and/or not-S for S, *throughout* a given primary or secondary argument, the validity of

the argument is in no way affected[25]. Every symbol (P, Q, R, S) is intended broadly enough to apply to any items, whether positive or negative, so switching its polarity throughout an argument has no effect on validity.

Difficulty arises only when such switching occurs in *only part* of an argument, or when two arguments are compared which have some item(s) of opposite polarity. In such cases, it is wise to tread very carefully, and not draw hasty conclusions. However, most such cases can be solved without too much trouble, as we shall discover in the next section. This is especially true as regards copulative arguments; implicational ones require additional reflection[26].

Arguments in tandem.

In an appendix to my *Judaic Logic*[27], I note that subjectal and predicatal (or antecedental and consequental) a-fortiori arguments are sometimes found in tandem, forming a sorites, so that the conclusion of one implicitly serves as minor premise in the other. For instances:

Positive subjectal followed by positive predicatal:
        A is more R than B,
        and B is R enough to be C;
        so, A is R enough to be C (this conclusion becomes the minor premise of the next argument).
        More R is required to be C than to be D,
        and A is R enough to be C (this premise being the conclusion of the preceding argument);
        therefore, A is R enough to be D.

Positive predicatal followed by positive subjectal:
        More R is required to be A than to be B,
        and C is R enough to be A;
        so, C is R enough to be B (this conclusion becomes the minor premise of the next argument).
        D is more R than C,
        and C is R enough to be B (this premise being the conclusion of the preceding argument);
        therefore, D is R enough to be B.

I wish to add here that, frankly, I do not remember if I ever saw a specific case. I may have just been assuming the occurrence of this phenomenon offhand. Rather my point was, I would say, that such conjunctions of related a fortiori arguments are conceivable. The thing to keep in mind is that the two a fortiori arguments *need not be contiguous,* in the discourse of a person or group, or in a document such as the Talmud. One argument may occur in one time or place, and the other in a completely different time or place. If you see things this way, you understand that there is some probability that the conclusion of one argument might be used as the premise of another. This happens in all knowledge all the time, but people pay little attention to it. It is bound to happen sooner or later, because no conclusion is ever left standing without being re-used in other arguments. Otherwise, why bother with it?

It also now occurs to me that the two arguments forming a sorites need not have the same middle term R. Let R1 be the first middle term, and R2 the second. We could equally well reason *with mixed middle terms*, as follows:

Positive subjectal followed by positive predicatal:
        A is more R1 than B,
        and B is R1 enough to be C;
        so, A is R1 enough to be C (this implies that A is C).
        More R2 is required to be C than to be D (this implies that what is C is R2),
        and A is R2 enough to be C (this follows from 'A is C' and 'C is R2', which together imply that A is R2);
        therefore, A is R2 enough to be D.

Positive predicatal followed by positive subjectal:
        More R1 is required to be A than to be B,
        and C is R1 enough to be A,
        so, C is R1 enough to be B (this implies that C is B).
        D is more R2 than C (this implies that C is R2),
        and C is R2 enough to be B (this follows from 'C is R2' and 'C is B');

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[25]   The ranking of arguments is also unaffected. Primary arguments remain primary; and secondary ones, secondary.
[26]   See on this topic my *Judaic Logic*, last section of chapter 3.3.
[27]   Chapter 16.1, first section.

therefore, D is R2 enough to be B.

Needless to say, we can predict other examples of sorites by involving other combinations of moods. For instance, the major premise "A is more R than B" might be converted to "B is less R than A," and so change the character of the first sorites, and so forth.

2. More formalities

1. Species and Genera

Since, as shown earlier, the propositions used in a fortiori argument can be reduced to simpler forms, it follows that we can formally combine a fortiori argument with syllogism in certain ways. Consider first the positive subjectal mood:

> P is more R than (or as much R as) Q (is R),
> and, Q is R enough to be S;
> therefore, all the more (or equally), P is R enough to be S.

Suppose that the given subsidiary term S refers to a species of a certain genus G. In that case, S is a subclass of G, i.e. 'All S are G' is true. The minor premise "Q is R enough to be S" of our a fortiori argument implies that "Q is S." By a first figure syllogism, we can infer that "Q is G." It follows that "Q is R enough to be G" is also true. If we now use this as our minor premise in lieu of the preceding, we can infer that "P is R enough to be G," since the major premise has remained unchanged. We can thus say, with reference to the subsidiary term: *if a positive subjectal a fortiori argument is true of a species (S), then it is equally true of any genus of it (G).***[1]**

It follows from this, by *reductio ad absurdum*, that: *if a negative subjectal a fortiori argument is true of a genus (G), then it is equally true of any species of it (S).* That is to say: given (with the same major premise) that "If P is R *not* enough to be G, then Q is R *not* enough to be G," and "All S are G," we can infer that "If P is R *not* enough to be S, then Q is R *not* enough to be S." (For otherwise, if Q were R enough to be S, then P would be R enough to be S and therefore G.)

The same can be done with positive predicatal a fortiori argument, as follows.

> More (or as much) R is required to be P than to be Q,
> and, G is R enough to be P;
> therefore, all the more (or equally), G is R enough to be Q.

Suppose that the given subsidiary term G refers to a genus of a certain species S. In that case, S is a subclass of G, i.e. 'All S are G' is true. The minor premise "G is R enough to be P" of our a fortiori argument implies that "G is P" (i.e. "all G are P"). By a first figure syllogism, we can infer that "S is P." It follows that "S is R enough to be P" is also true. If we now use this as our minor premise in lieu of the preceding, we can infer that "S is R enough to be Q," since the major premise has remained unchanged. We can thus say, with reference to the subsidiary term: *if a positive predicatal a fortiori argument is true of a genus (G), then it is equally true of any species of it (S).***[2]**

It follows from this, by *reductio ad absurdum*, that: *if a negative predicatal a fortiori argument is true of a species (S), then it is equally true of any genus of it (G).* That is to say: given (with the same major premise) that "If S is R *not* enough to be Q, then S is R *not* enough to be P," and "All S are G," we can infer that "If G is R *not* enough to be Q, then G is R *not* enough to be P." (For otherwise, if G were R enough to be P, then G and therefore S would be R enough to be Q.)

So much for copulative a fortiori argument. The same can be done with implicational a fortiori argument, except that we would here use hypothetical instead of categorical syllogism. That is, instead of "All S are G" (where S and G are

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[1]   We could, of course, equally well infer that "P is G" directly from the conclusion that "P is S." The end result is the same. But the point made here is that for any positive subjectal a fortiori argument in relation to a species S, there is a similar argument in relation to any genus G of S.
[2]   Here again, we could of course equally well infer that "S is P" directly from the conclusion that "G is P." The end result is the same. But the point made here is that for any positive predicatal a fortiori argument in relation to a genus G, there is a similar argument in relation to any species S of G.

terms), we would say "If S then, G" (where S and G are theses). Although such combinations of a fortiori argument and syllogism are simple and obvious, they are nevertheless sometimes useful and so well to keep in mind.

A case in point is the a fortiori argument suggested in Num. 12:14, viz. "If her father had but spit in her face, should she not hide in shame seven days? Let her be shut up without the camp seven days, and after that she shall be brought in again." If we look closely, we see that the subsidiary term in the minor premise "*hide in shame* seven days" is not identical (as it should be) to the one in the conclusion "*shut up without the camp* seven days;" in which case the inference is invalid. We can, however, draw a valid conclusion by using the common property (i.e. genus) of these two terms, i.e. what they both imply, viz. "*isolation* for seven days." To justify such syllogistic interference, we need the above formal treatment.

Now, it should be obvious that having in our example concluded with "isolation for seven days," we cannot take that generic conclusion as justification for a more specific conclusion such as "shut up without the camp seven days" which does not *exactly* correspond to the given species "hide in shame seven days." This would be fallacious reasoning. In other words, the new "shut up without the camp" specification cannot be claimed to be a *conclusion* of a fortiori reasoning, but must be regarded as a specification applied to the generic conclusion "isolation for seven days" *after* the a fortiori argument proper, through some *other* act of deductive or inductive reasoning or other justification. In this example, the justification would be that God[3] or the Torah has so decreed.

This can be stated in formal terms as follows. Consider a positive subjectal argument, to begin with. As we have just seen, if the minor premise affirms the minor term Q to have species S1 as predicate, then the conclusion must likewise affirm the major term P to have species S1 as predicate, or if we wish it may (via syllogism) affirm the major term P to have some genus G of S1 as predicate. We cannot, however, conclude with affirmation of predication for some separate species of G, say S2[4] – at least not through our a fortiori argument, though we might arrive at S2 by some other process of reasoning thereafter.

Regarding negative subjectal argument, this rule means (notice the changed positions of P and Q here) that if the minor premise denies the major term P to have genus G1 as predicate, then the conclusion must likewise deny the minor term Q to have genus G1 as predicate, or if we wish it may (via syllogism) deny the minor term Q to have some species S of G1 as predicate. We cannot, however, conclude with denial of predication for some separate genus of S, say G2[5] – at least not through our a fortiori argument, though we might arrive at G2 by some other process of reasoning thereafter.

In predicatal argument, the equivalent fallacy would consist in passing from a given genus to a not-given other genus. Thus, in the positive mood of predicatal argument, we might start from given genus G1, saying "G1 is R enough to be P;" from that, given that S is a known species of G1, we can legitimately infer (as above shown, through syllogism) that "S is R enough to be Q;" but then we swerve off and illicitly claim that "G2 is R enough to be Q," where G2 is another genus of S (though not a species of G1).

Similarly, in the negative mood of predicatal argument, it would be fallacious to pass from a given species to a not-given other species. That is, starting with "S1 is R not enough to be Q;" we can, given that G is a known genus of S1, legitimately infer (as above shown) that "G is R not enough to be P;" but we cannot likewise infer that "S2 is R not enough to be P," where S2 is another species of G (though not a genus of S1).

It should be added that the fallacious reasoning above described is not uncommon. To sum up, we have above, knowing that all S are G, established four rules of transmission of a fortiori argument from species to genera or vice versa:

1. If a positive subjectal argument is true of a certain subsidiary predicate (S), then it is also true of any genus of it (G).
2. If a negative subjectal argument is true of a certain subsidiary predicate (G), then it is also true of any species of it (S).
3. If a positive predicatal argument is true of a certain subsidiary subject (G), then it is also true of any species of it (S).
4. If a negative predicatal argument is true of a certain subsidiary subject (S), then it is also true of any genus of it (G).

And likewise, *mutatis mutandis*, with respect to implicational arguments.

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[3]   Though it is nowadays recommended in orthodox Jewish circles to write any name of God, even in languages other than Hebrew, in truncated fashion (e.g. as G-d), so as to avoid the eventual erasing or tearing of such a holy word – I have decided not to practice this restriction anymore, because this makes it difficult for people to search for the word in the Internet or in their computers. Of course, then, every time we change page with a computer or other reader, we effectively erase all the names of God that happen to be in it. This is very unfortunate – but I think inevitable in this day and age. In other words, the recommendation is in the last analysis impractical and impracticable.

[4]   By "separate species of G," I here mean that S2 is not a genus of S1.

[5]   By "separate genus of S," I here mean that G2 is not a species of G1.

We should also be aware that the middle term R may subtly differ in meaning in relation to the major and minor terms P and Q. In some cases, the meaning of R is identical; but in some cases, the R in relation to P (Rp) and the R in relation to Q (Rq) have the abstraction R in common, but they are each specifically relative to the term they concern. In effect, Rp and Rq are two species of the genus R. The argument, if properly constructed, remains nonetheless valid, because all that matters for its validity is that $Rp \geq Rq$ and this quantitative condition is here satisfied.

To give an example: "John loves his dog more than Jill does," could mean that Jill loves John's dog less than he does (in which case, the middle term is "loves John's dog"), or it could mean that Jill loves her own dog less than John loves his own dog (in which case, the middle term is the more abstract "loves his/her own dog," or even just "loves some dog"). In the latter case, the minor premise and conclusion would be: "Given that Jill loves her own dog enough to be classed as an animal lover, it follows that John loves his own dog enough to be classed as an animal lover." This example is positive subjectal. We may similarly construct examples for negative subjectal, predicatal and implicational arguments with this feature.

2. Proportionality

This section and the next are very important and should be read carefully.

Closely related to the issue of species and genera, is that of 'proportionality'. Often, rather than species and genera, what is involved are *different degrees or measures* of the same term. Thus, in positive subjectal or negative predicatal argument, the distinct species S1 and S2 (neither of which is included in the other, though they have a genus in common) would appear as different degrees or measures of the same genus G; similarly, in positive predicatal or negative subjectal argument, species S would appear as a single degree or measure of two distinct genera, G1 and G2 (neither of which includes the other, though they have a species in common).

For example, in the argument given in Num. 12:14 (see 2.4 below), though the minor premise specifies the quantity "seven days," we might be tempted (by considerations of proportionality, say) to conclude with another quantity like "fourteen days;" but such reasoning (without additional premises), as we have just shown, would be formally invalid. In purely a fortiori argument, the conclusion can never produce a different quality or quantity than the one given in the minor premise; this is a hard and fast rule based on strictly logical considerations.

Argument *a crescendo*. A fortiori argument with a 'proportional' conclusion is, in itself, by itself, fallacious. The copulative variant has, at least on the surface, the following four forms. The positive subjectal mood resembles that of regular a fortiori, except that, whereas the minor premise predicates a subsidiary term (S) of the minor term (Q), the conclusion predicates a *greater* subsidiary term (more than S) of the major term (P). It goes like this:

> P is more R than Q (is R),
> and Q is R enough to be S;
> therefore, P is R enough to be *more than* S.

To avoid confusion between the subsidiary term S in a general sense and its values in the minor premise and conclusion, think of 'S' in the former as 'Sq' and 'more than S' in the latter as 'more than Sq' or as 'Sp'. The corresponding negative subjectal mood has the same major premise, but as usual has the denial of the above conclusion as minor premise and the denial of the above minor premise as conclusion. That is, it argues: Since P is R *not* enough to be *more than* S, it follows that Q is R *not* enough to be S.

The positive predicatal mood resembles that of regular a fortiori, except that, whereas the minor premise predicates the major term (P) of a subsidiary term (S), the conclusion predicates the minor term of a *lesser* subsidiary term (less than S). It goes like this:

> More R is required to be P than to be Q,
> and S is R enough to be P;
> therefore, *less than* S is R enough to be Q.

To avoid confusion between the subsidiary term S in a general sense and its values in the minor premise and conclusion, think of 'S' in the former as 'Sp' and 'more than S' in the latter as 'less than Sp' or as 'Sq'. The corresponding negative predicatal mood has the same major premise, but as usual has the denial of the above conclusion as minor premise and the denial of the above minor premise as conclusion. That is, it argues: Since *less than* S is R not enough to be Q, it follows that S is R *not* enough to be P.

We could alternatively have, for the positive subjectal mood, 'less than S' in the minor premise and 'S' in the conclusion; and for the negative subjectal mood, 'S' in the minor premise and 'less than S' in the conclusion. And likewise, for the positive predicatal mood, 'more than S' in the minor premise and 'S' in the conclusion; and for the negative predicatal mood, 'S' in the minor premise and 'more than S' in the conclusion. What matters is that the *relative* magnitudes be as stated.

In practice, the subsidiary term in the minor premise would always be labeled 'S' and the subsidiary term in the conclusion would accordingly be labeled 'more than S' or 'less than S', as the case may be – for the simple reason that we normally know the minor premise before we get to know the conclusion. I have chosen the terminology above to stress that the negative moods are reducible *ad absurdum* to the positive ones.

We can likewise construct implicational forms. As these various forms show, 'proportional' a fortiori argument is based on the notion that if P is more R than Q is, or more R is required to be P than to be Q, then necessarily a larger amount of S will correspond to P than to Q. But in fact there is no such necessity; it may occasionally be true, but there is no logical reason why it should be. Such arguments, unlike regular a fortiori, simply cannot be validated as they stand.

Looking at the positive subjectal form of 'proportional' a fortiori argument, which is prototypical, it is evident that we can equally well refer to it as argument *a crescendo* (this being a name I invented in the course of my research, having found it useful). This name can be extended to all the other forms[6]. The advantage of such renaming is that it verbally completely distinguishes such argument from strict a fortiori.

Argument *pro rata*. Argument a crescendo (i.e. 'proportional' a fortiori) should not be confused with argument by proportion, which we can refer to as argument *pro rata* (this Latin name being already well established in the English language), this being understood to mean "at the same rate." Such argument concerns concomitant variations between two variables, and may be formulated as follows:

> Y varies in proportion to X. Therefore:
> given that: if X = x, then Y = y,
> it follows that: if X = more (or less) than x, then Y = more (or less) than y.

An example of it is Aristotle's statement: "Every good quality of the soul, the higher it is in degree, so much more useful it is" (*Politics* 7:1), which intends the argument: given that a certain quality of the soul is good, it is useful; if it is improved, it is still more useful. In practice, pro rata argument is often expressed in the form: "the more X, the more Y; and (by implication) the less X, the less Y." Note that two variants (which mutually imply each other) are possible: one with "more" and one with "less" – that is, the argument can go either way, increasing or decreasing the quantities involved.

The statement "Y varies in proportion to X" is not an argument but a mere proposition, reflecting some generalized empirical observations or a more theoretical finding. The above argument includes this proposition as its major premise, but requires an additional minor premise ("if X = x, then Y = y") to draw the conclusion ("if X = more/less than x, then Y = more/less than y"). The conclusion mirrors the minor premise in form, but its content is intentionally different. The quantities involved do not stay the same, but increase or decrease (as the case may be).

Notice that a pro rata argument has no middle term, unlike an a fortiori one. A pro rata argument is thus more akin to apodosis than to syllogism. Its major premise sets a broad principle, of which the minor premise and conclusion are two applications. The argument involved is thus simply inference of one quantity from another within the stated principle. If we found that contrary to expectations X and Y do not vary concomitantly as above implied, we would simply deny the major premise. In other words, this argument is essentially positive in form. A negative mood of it (with the same major premise and denials of the previous conclusion and minor premise) would not make much sense, since its minor premise and conclusion would be in conflict with its major premise.

The above formulas are at least true in cases of *direct* proportionality; in cases of *inverse* proportionality, the language would be: "the more X, the less Y; and (by implication) the less X, the more Y;" and the argument would have the following form:

> Y varies in inverse proportion to X. Therefore:
> given that: if X = x, then Y = y,
> it follows that: if X = more (or less) than x, then Y = less (or more) than y.

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[6]   'A crescendo', of course, refers to increase in magnitude – and so makes us think especially of positive subjectal argument, which goes from S to 'more than S'. We could by analogy name positive predicatal argument 'a diminuendo', since it involves decrease in magnitude, going from S to 'less than S'. Negative subjectal, going from not more than S to not S, would then count under the latter heading, and negative predicatal, going from not less than S to not S, under the former. But such multiplication of names would be silly pedantry.

And of course, in more scientific contexts, we may have access to a more or less complex mathematical formula – say $Y = f(X)$, where f refers to some function – that allows precise calculation of the proportion involved. In other words, the *validity* of pro rata argument is not always obvious and straightforward, but depends on our having a clear and reliable knowledge of the concomitant variation of the values of the terms X and Y. Given such knowledge, we can logically justify drawing the said conclusions from the said premises. Lacking it, we are in a quandary.

As its name implies, pro rata argument signifies that there is (if only approximately) some constant rate in the relative fluctuations in value of the variables concerned. The variables X and Y may be said to be proportional if $X/Y = a$ constant, or inversely proportional if $XY = a$ constant. In the exact sciences, of course, such a constant is a precisely measurable quantity; but in everyday pro rata discourse, the underlying 'constant' is usually a vague quantity, perhaps a rough range of possible values.

Proportionality or inverse proportionality as just defined, which can be represented by a straight line graph, and even when the graphical representation is more curved (e.g. exponential), may be characterized as simple. It becomes complex, when there are ups and downs in the relation of the two variables, i.e. when an increase in X may sometimes imply an increase in Y and sometimes a decrease in Y, it is obviously not appropriate to formulate the matter in the way of a standard pro rata argument. In such cases, we would just say: "the values of X and Y can be correlated in accord with such and such a formula," and then use the formula to calculate inferred quantities.

Proportionality may be continuous or not. Sometimes, there is proportionality of sorts, but it comes in slices: e.g. from X = 0 to 1, Y = k; from X = 1 to 2, Y = k +1; etc. That is, to each range of values for X, there corresponds a certain value of Y, and the two quantities go increasing (or decreasing, as the case may be). Such proportionality is compatible with pro rata argument. For this reason, it is wise to put the word 'proportionality' in inverted commas, so as to remember that it does not always imply continuity.

Note too that proportionality may be natural or conventional. An example of the latter would be a price list: bus fares for children under 16, $1; for adults 16+, $2. However, beware in such case of frequent exceptions or reversals: e.g. unemployed and pensioners, $1. In such cases, any pro rata argument must be stated conditionally: the bus fares are 'proportional' to age, provided the adults are not unemployed or pensioners.

It should also be reminded that proportionality (or its inverse), simple or complex, may or may not be indicative of a causal relation (in the various senses of that term). Two variables may vary concomitantly by virtue of being effects of common causes, in which case we refer to parallelism between them, or the one may cause or be caused by the other. Also, of course, such parallelism or causality may be unconditional or conditional. In such cases as it is unconditional, no more need be said. But in such cases as it is conditional, the condition(s) should ideally be clearly stated, although often they are not.

Pro rata argument may occur in discourse independently of a fortiori argument, or in conjunction with such argument. In any case, it should not be confused with a fortiori argument: they are clearly different forms of reasoning. Pro rata involves only two terms, or more precisely two values (or more) of two variables; whereas a fortiori involves four distinct terms, which play very different roles in the argument. Pro rata and a fortiori are both analogical arguments of sorts, but the former is much simpler than the latter.

3. A crescendo argument

Having thus clarified the differences between (regular) a fortiori argument, a crescendo argument (i.e. 'proportional' a fortiori) and pro rata argument, it is obvious that these three types of argument should not be confused, although many people tend to do so. Although these arguments are far from the same in form and in validity, such people wrongly identify pro rata argument and a crescendo argument with a fortiori argument. The reason for such confusion may be that argument a crescendo appears to be a combination of argument a fortiori and pro rata argument. This could be expressed as a formula:

A crescendo = a fortiori cum pro rata.

Though the latter two forms of argument may occur coincidentally (i.e. they may happen to be both true in certain material cases), it does not follow that they formally necessitate or imply each other. But, we may ask, do they together imply a crescendo argument? That is to say, is argument a crescendo valid in cases where both argument a fortiori and argument pro rata happen to be true? To answer this question, consider a regular *positive subjectal* a fortiori argument:

> P is more R than Q (is R),
> and Q is R enough to be S;
> therefore, P is R enough to be S.

It informs us that the quantity of R corresponding to P (Rp) is greater than the quantity of R corresponding to Q (Rq), and then argues: since the latter quantity (Rq) is big enough to imply Q to be S, then the former quantity (Rp) must be big enough to imply P to be S. It does not tell us that the subsidiary term S is a variable quantity; S is here clearly intended to have *one and the same value* in the minor premise and conclusion. And if S happens to be a variable quantity, we cannot automatically suppose that the variation of S is tied to that of R, so that S for P (Sp) is necessarily greater than S for Q (Sq), in concomitance with the variation of Rp with Rq.

Under what conditions, then, can we obtain the conceivable a crescendo conclusion "P is R enough to be *more than* S" from the above a fortiori premises? That is, what additional information do we need to transform the above valid a fortiori argument into a valid a crescendo argument? Or to put it another way again: having already come to the a fortiori conclusion "P is R enough to be S (i.e. Sq)," how can we proceed one step further and obtain the a crescendo conclusion "P is R enough to be *more than* S (i.e. Sp)"?

The answer is, of course, that we must obtain the additional information required to construct the following pro rata argument:

> If, moreover, (for things that are both R and S,) we find that:
> S varies in proportion to R, then:
> knowing from the above minor premise that: if R = Rq, then S = Sq,
> it follows in the conclusion that: if R = *more than* Rq = Rp, then S = *more than* Sq = Sp.

Note well the stipulation "for things that are both R and S." I have put this precondition in brackets, because it is in fact redundant, since as we saw earlier the minor premise of the a fortiori argument implies anyway that *not all* things that are R are S, but only those things that have a certain threshold value of R or more of it are S. We should not think of S varying with R as a general proposition applicable to all R (implying that all R are S[7]), but remain aware that this concomitant variation occurs *specifically* in the range of R where the threshold for S has indeed been attained or surpassed (i.e. where the "R enough to be S" condition is indeed satisfied).

If we know (by induction or deduction from other information) that the major premise of our above pro rata component is true (i.e. that S varies in proportion to R), we can infer its minor premise (viz. if R = Rq, then S = Sq) from the minor premise of the a fortiori component and thence draw the conclusion that "If the value of R for P is Rp (> Rq), then the value of S for P is Sp (> Sq)." This is assuming, as earlier specified, that the proportionality proposed in the major premise is direct and simple.

The desired a crescendo conclusion, viz. "P is R (Rp) enough to be *more than* Sq (Sp)" can then be confidently drawn. That is, the intermediate conclusions, i.e. the a fortiori conclusion (P, being more than Rq, is R enough to be Sq) and the pro rata conclusion (If R = Rp, then S = Sp) together imply the final, a crescendo conclusion (P, which is Rp, is R enough to be Sp). Note well that Sp, here, means nothing more precise than 'more than Sq'. Though given a distinct symbol, it is not an exact number, unless we are able to calculate it precisely through some sort of mathematical formula.

Note that we have here taken variations in the value of S to be concurrent with variations in the value of *the middle term* R. Very often, though, a crescendo thinking is based on the assumption that the values of S are proportional to those of the major and minor terms, P and Q. These two views are not necessarily in conflict, though the former (which we have adopted) is the more essential and more generally applicable. P and Q may be single values, or they may be variable over time – so long as their relative magnitudes or degrees are as specified in the major premise, i.e. that P is always more (more R, to be exact, though R is often left tacit) than Q. Thus, S may well be vaguely thought of as proportional to P or Q, although more precisely perceived the proportionality of S is in fact to R, the common factor of P and Q in relation to which P is greater than Q.

We have thus shown that, under certain circumstances, the formula "a fortiori + pro rata = a crescendo" is indeed true. That is to say, although the putative a crescendo conclusion is not per se valid, it can *in some cases* be valid if it so happen that the a fortiori conclusion from the same premises can be taken a step further by means of an appropriate argument pro rata. This has just been demonstrated for positive subjectal a crescendo.

A similar two-step argument can, of course, be formulated for *positive predicatal* a crescendo. In this case, we use the following combination of a fortiori and pro rata arguments:

> More R is required to be P than to be Q,
> and S is R enough to be P;

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[7]   Which is logically impossible, here.

therefore, S is R enough to be Q.

If, moreover, (for things that are both R and P or Q,) we find that:
R varies in proportion to S, then:
knowing from the above minor premise that: if S = Sp, then R = Rp,
it follows in the conclusion that: if S = *less than* Sp = Sq, then R = *less than* Rp = Rq.

Note well the stipulation "for things that are both R and P or Q." I have put this precondition in brackets, because it is in fact redundant, since as we saw earlier the major premise of the a fortiori argument implies anyway that *not all* things that are R are P and *not all* things that are R are Q, but only those things that have certain threshold values of R or more of it are P or Q. We should not think of R varying with S as a general proposition applicable to all S (implying that all S are R[8]), but remain aware that this concomitant variation occurs (at least) *specifically* in the range of R where the thresholds for P and Q have indeed been attained or surpassed (i.e. where the "R enough to be P" and "R enough to be Q" conditions are indeed satisfied).

If we know (by induction or deduction from other information) that the major premise of this pro rata component is true (i.e. that R varies in proportion to S), we can infer its minor premise (viz. if S = Sp, then R = Rp) from the minor premise of the a fortiori component and thence draw the conclusion that "If the value of S for Q is Sq (< Sp), then the value of R for Q is Rq (< Rp)." This is assuming, as earlier specified, that the proportionality proposed in the major premise is direct and simple.

The desired a crescendo conclusion, viz. "*Less than* Sp (Sq) is R (Rq) enough to be Q" can thence be confidently drawn. That is, the intermediate conclusions, i.e. the a fortiori conclusion (Sp, being more than Rq, is R enough to be Q) and the pro rata conclusion (If S=Sq, then R=Rq) together imply the final, a crescendo conclusion (Sq, which is Rq, is R enough to be Q). Note well that Sq, here, means nothing more precise than 'less than Sp'. Though given a distinct symbol, it is not an exact number, unless we are able to calculate it precisely through some sort of mathematical formula.

Thus, if we can provide an appropriate pro rata argument, we can credibly transform an a fortiori conclusion into an a crescendo conclusion, viz. in the case of positive subjectal argument, "P is R enough to be *more than* S," and in the case of positive predicatal argument, "*Less than* S is R enough to be Q."

As regards the corresponding subjectal or predicatal *negative a crescendo* arguments, they would consist of a negative a fortiori combined with the same positive argument pro rata. Referring to the above described positive arguments, keeping the major premise and additional premise about proportionality constant, if we deny the conclusion, we must deny the minor premise, as follows:

P is more R than Q (is R),
and P is R *not* enough to be *more than* S,
and S varies in proportion to R;
therefore, Q is R not enough to be S.

More R is required to be P than to be Q,
and *less than* S is R *not* enough to be Q,
and R varies in proportion to S;
therefore, S is R not enough to be P.

This is how we would derive the negative moods from the positive ones. But granting that the subsidiary term in the minor premise is thought of first, before the subsidiary term in the conclusion, it is more accurate to present these two arguments independently in the following revised forms:

P is more R than Q (is R),
and P is R not enough to be S,
and S varies in proportion to R;
therefore, Q is R not enough to be *less than* S.

More R is required to be P than to be Q,
and S is R not enough to be Q,

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and R varies in proportion to S;
therefore, *more than* S is R not enough to be P.

I hope the reader is not confused by this revision. Although the negative forms are validated by reduction ad absurdum to the positive ones, viewed as forms in their own right they would be rather worded as just shown. The comparisons between the subsidiary term in the minor premise and that in the conclusion remain the same – i.e. 'more than S > S' and 'S > less than S' mean the same, and 'less than S < S' and 'S < more than S' mean the same. Notice that the revised negative subjectal mood goes from S to less than S, like the earlier positive predicatal mood, while the revised negative predicatal mood goes from S to more than S, like the earlier positive subjectal mood. Similar developments to all those above for copulative argument are possible in relation to implicational argument.

Note well that *the relation between R and S changes direction*, according as our reasoning is subjectal (or antecedental) or predicatal (or consequental) in form. That is to say, whereas in subjectal argument S varies with R, in predicatal argument R varies with S. That is because, in positive subjectal argument, validation relies on the fact that Rq implies Rs; whereas, in positive predicatal argument, it relies on the fact that Rs implies Rp. In both cases, the R value of the subject precedes the R value of the predicate; but in the former, the subject is Q and the predicate is S, while in the latter, the subject is S while the predicate is P. It is because the subject has a certain value of R that it can be attributed the predicate, for which that value of R happens to be a precondition.

Thus, when reasoning a crescendo in the positive subjectal form, we reason from Rq to Sq, and from Rp to Sp; i.e. when R = Rq, then S = Sq, etc. Whereas, when reasoning a crescendo in the positive predicatal form, we reason from Sp to Rp, and from Sq to Rq, i.e. when S = Sp, then R = Rp, etc. Whence, we must reverse the order of dependence between the middle and subsidiary terms, according as we reason this way or that way, to make possible validation of the argument. Of course, if in a given case the required pro rata argument as above specified is not applicable, then the a fortiori argument (whether subjectal or predicatal, as the case may be) *cannot* be turned into a valid a crescendo one. All this is equally true of negative arguments, of course, as already made clear.

Validation. It is important to stress that the validity of a crescendo argument depends, as we have above clearly shown, on both its a fortiori constituent and its pro rata constituent. A crescendo is neither equivalent to the former nor equivalent to the latter, but emerges from the two *together*. The a fortiori element is only able to produce the conclusion "P is R enough to be S" (in the positive subjectal case) or "S is R enough to be Q" (in the positive predicatal case). The pro rata element is only able to produce the conclusion that the S is, parallel to R, greater for P than for Q, or lesser for Q than for P. The a crescendo conclusion is a merger of these two partial conclusions, namely (respectively) "P is R enough to be more than S" or "Less than S is R enough to be Q." Thus neither element is logically redundant; both are necessary to obtain the final conclusion.

Looking at the above descriptions of a crescendo argument, we see that, while the pro rata conclusion is partly based on information provided by the minor premise of the a fortiori argument, it could conceivably be built up without the latter, since it does not use all the information in it. However, although mere pro rata argument[9] no doubt exists, it remains true that the pro rata constituent could not by itself produce the stated final a crescendo conclusion, since the latter proposition is of suffective form; so the a fortiori constituent is also indispensable. Clearly, the final conclusion is made up of elements derived from both types of arguments; i.e. its semantic charge comes from all three premises. To repeat, then, the a crescendo argument cannot be identified with either constituent alone, but requires both to proceed successfully.

What we have done above is to formally demonstrate that, although drawing a 'proportional' conclusion from the premises of a valid a fortiori argument is not unconditionally valid, it is also not unconditionally invalid. Such a conclusion is in principle invalid, but it may exceptionally, under specifiable appropriate conditions, be valid. Formally, all depends on whether a pro rata argument can be truthfully proposed in addition to the purely a fortiori argument. In other words, to draw a valid a crescendo conclusion, the premises of a valid a fortiori argument do not suffice; but if they are combined with the fitting premises of a valid pro rata argument, as above detailed, such a conclusion can indeed be formally justified.

Of course, as with all deduction, even if in a given case the inferential process we propose is ideally of valid form, we must also make sure that the premises it involves are indeed true, i.e. that the content of the argument is credibly grounded in fact. Very often, in a crescendo argument, the process is convincing, but the major premise of the implicit pro rata argument is of doubtful truth; this is obviously something to be careful about. Merely declaring a certain proportionality to be true does not make it true – we have to justify all our premises, as well as their logical power to together produce the putative conclusion.

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[9]   That is, pro rata argument outside of a crescendo.

Another way to stress this is to remind that the concluding predicate Sp of positive subjectal a crescendo argument means nothing more than 'more than Sq', and the concluding subject Sq of positive predicatal a crescendo argument means nothing more than 'less than Sp'. These concluding subsidiary terms are not exact numbers, though in theory they might be exact if we happen to have a precise mathematical formula for their calculation – viz. $S = f(R)$ in the case of subjectal argument, or $R = f(S)$ in the case of predicatal argument. In most cases in practice, however, we do not have such a formula, and the terms we use are correspondingly vague and tentative. It is important to remember this.

Sometimes, unfortunately, rhetoric comes into play here, and albeit the lack of mathematical proof, the conclusion is made to seem more precise than deductive logic allows. We could at best refer to such conclusions as intuitively reasonable, or as inductive hypotheses, partly but not wholly sustained by the data in the premises; but we must realize and acknowledge that they are not deductive certainties. Otherwise, we would be engaged in misleading sophistry. Thus, it is important to keep in mind that, while we have shown that a crescendo argument is in principle, i.e. under ideal conditions, valid – it does follow that every a crescendo argument put forward in practice, i.e. in everyday discourse, is valid. It is potentially valid, but not necessarily actually valid. We have to carefully scrutinize each case.

Thus, to summarize: the expression 'proportional' a fortiori argument may be intended in a pejorative sense, as referring to argument that unjustifiably draws a proportional conclusion from only two a fortiori premises; or it may be intended to refer to valid a crescendo argument, consisting of three premises, viz. two a fortiori and one pro rata, yielding a justifiably proportional conclusion. A fortiori argument per se, as such, in itself, by itself, is not proportional; such argument may be verbally distinguished as *purely* a fortiori. However, when combined with pro rata argument, a proportional conclusion is justified, and we had best in such case speak distinctively of a crescendo argument, or as it is often called in a non-pejorative sense 'proportional' a fortiori argument.

Thus, a crescendo argument may be viewed as a special case of a fortiori argument; and it is fair to say that the field of a fortiori logic also deals with a crescendo argument. But strictly spoken, 'a fortiori' should be reserved for 'non-proportional' argument, and 'a crescendo' preferred for 'proportional' argument. The former is pure because the major and minor premise suffice for the conclusion; whereas the latter is a compound argument, comprising a pure a fortiori argument combined with a mere pro rata argument. Although people often appear to draw an a crescendo conclusion from a fortiori premises, such inference in fact relies on an unspoken additional pro rata premise, and so is not purely a fortiori.

In any case, it is useful to remember the formula: a crescendo equals a fortiori plus pro rata. This means that if you come across an a crescendo argument that looks valid, you can be sure that underlying it are a valid a fortiori argument and a valid pro rata argument. Inversely, if one and/or the other of the latter arguments cannot be upheld, then the former cannot either.

Alternative presentation. We have thus far considered a crescendo argument as a special case of a fortiori argument where purely a fortiori argument is combined with pro rata argument. Another way we might look upon the relationship between these arguments is to say that all a fortiori argument is a crescendo argument, while purely a fortiori argument is a special case where the pro rata argument involves a fixed quantity instead of a variable, i.e. where the 'proportionality' involved is a constant. That is to say, we could regard the general forms of a fortiori argument to be the following (for examples, with regard to positive subjectal and positive predicatal moods):

> P is more R than Q (is R),
> and Q is R enough to be *more than* S,
> and S is constant (pure) or varies in proportion to R (a crescendo);
> therefore, P is R enough to be S, or more than S (as the case may be).

> More R is required to be P than to be Q,
> and S is R enough to be P,
> and S is constant (pure) or R varies in proportion to S (a crescendo);
> therefore, S, or less than S (as the case may be), is R enough to be Q.

Note how the additional premise about proportionality is now so broadly stated that it includes both the cases of purely a fortiori argument (where S is constant) and those of a crescendo argument (where S varies in magnitude or degree). The advantage of this approach is that it goes to show that purely a fortiori argument and a crescendo argument are essentially two special cases of the same general form, and so that they can legitimately be referred collectively as forms of 'a fortiori argument' in the largest sense.

However, it must be emphasized that this joint formulation is just an ex post facto way of looking at things, a perspective. It remains true, as initially stated, that a fortiori argument is essentially pure, since it can be validated without reference to an additional premise about proportionality. In this more accurate perspective, a crescendo argument is an amplification of a fortiori argument, taking its 'equal' conclusion and enriching it by turning it into a 'proportional' one (by means of a pro rata argument). When we do not have any additional premise about proportionality, we may logically assume that the subsidiary item remains constant, since that is the minimum assumption of any a fortiori reasoning process. In effect, 'non-proportionality' is the default character of a fortiori argument.

4. Hermeneutics

We also need to deal with the important issue of hermeneutics. When someone formulates an argument a crescendo, that person presumably believes that its constituent a fortiori argument and pro rata argument both have true premises and are correctly combined to yield a valid and therefore true conclusion, even if all these factors are not explicitly laid out and confirmed. However, how can other persons know what he (or she) had in mind if some of these relevant factors are left tacit? This is the question of hermeneutics – how are we to interpret and judge an incompletely detailed a crescendo argument presented by someone else?

When judging a given concrete argument: (a) if it is formulated in such a way that only an a fortiori conclusion is claimed, we need only test whether the conclusion follows a fortiori from the premises (assuming them true); but (b) if it is formulated in such a way that an a crescendo (i.e. proportional) conclusion is claimed, we must needs test both whether the premises yield an a fortiori conclusion *and* whether an additional argument pro rata can be presented that makes possible the transformation of the latter conclusion into an a crescendo one, *and* whether the proportionality is vague or precisely calculable (and ultimately, of course, whether all these premises are true).

Without such additional information on proportionality, if the a fortiori premises are presented (explicitly or by implication) as *by themselves* yielding an a crescendo conclusion, the argument must obviously be declared invalid. Moreover, even if the need for an additional premise regarding proportionality is admitted, without a precise formula for the proportionality, an exact conclusion cannot legitimately be claimed. We have to assess from the author's words how well he understands the conditions for valid a fortiori or a crescendo argument, and whether the author considers that he has the required information at hand. If it seems likely that the author is well aware that a fortiori argument cannot logically by itself yield an a crescendo conclusion, and that a precise conclusion requires more information than a vague one, and is *tacitly intending* the required underlying pro rata argument or mathematical formula, his argument could be considered valid.

Of course, in interpreting a text or speech, we cannot estimate with certainty what its author's tacit intentions are or are not. We may be able to guess at the author's general logical knowhow from the context, and give him the benefit of the doubt in the case at hand. Or we may prefer to be strict, and demand explicit evidence that the author intends an argument pro rata in the case at hand, even if he knows the rule (since, after all, people do make mistakes). Thus, validation or invalidation often depends on the general credibility of the author of the text or speech, and on the severity of the interpreter and judge.

To put the problem in more concrete terms: when we read an ancient or modern text, by Aristotle, in the Tanakh, by a Talmudic sage, in the Christian Bible, or the Muslim Koran, or wherever, wherein the author seems to draw an a crescendo (i.e. 'proportional') conclusion from a fortiori premises, how should we react? We could possibly say: since the author of this argument has not justified his conclusion by *explicitly* proposing an appropriate accompanying pro rata argument, we must declare his reasoning fallacious. But this seems a bit rigid and lacking in subtlety, for logicians well know that discourse in practice is rarely if ever fully explicit. Our judgment in each case must clearly hinge on the wider context of the particular statement.

If we know from pronouncements elsewhere of that particular author that he has demonstrated clear understanding of the difference between a fortiori argument and a crescendo argument – i.e. that the former *per se* cannot yield a proportional conclusion unless it is backed up by an appropriate pro rata argument – then we can reasonably assume that in this particular case, though the author has not explicitly formulated the needed argument pro rata, he left out the missing pieces merely for brevity's sake. We can in such case give the author the benefit of the doubt and accept his a crescendo conclusion. Of course, even if he has in other contexts demonstrated his theoretical knowledge, or at least his intuitive rationality, it is still possible that in this particular case he unthinkingly made an error of form and/or content; so we can never be absolutely sure. Nevertheless, even if the hypothesis that he knowingly drew an a crescendo conclusion from a fortiori premises cannot definitely be proved, it has inductive support in his overall logical behavior patterns.

A contrario, if we find that there is no reliable evidence that this author has mentally grasped the difference between a fortiori argument and a crescendo argument, we should certainly consider all his a crescendo arguments as fallacious reasoning. This is true even if we find him sometimes drawing a valid a fortiori conclusion and sometimes drawing a doubtful a crescendo one, for he may be drawing different conclusions from similar premises as convenient to his discursive purposes, or without rhyme or reason, and not because of any awareness that there are precise rules to follow. We may also reasonably reject an a crescendo conclusion of his, if we find that the author has elsewhere in his works denied the truth of the proportionality (i.e. the major premise of the pro rata argument) which would be needed to justify this particular a crescendo conclusion.

Thus, judging a concrete a crescendo argument which is not entirely explicit as valid or invalid is not an easy matter. Of course, if the underlying purely a fortiori argument is formally invalid and/or one or both of its premises is/are untrue, or likewise if the required argument pro rata is obviously inappropriate in form and/or content, we can reject that particular a crescendo argument. But obviously such rejection does not always prove the author to be ignorant of the conditions under which an a fortiori argument may yield an a crescendo conclusion. The author may well have in all sincerity believed the implied proportionality to be true, even if we disagree with him and can prove him wrong. In such cases, it is not the inferential process we are attacking, but some premise(s).

In any case, what we must avoid doing is getting entangled in superficial verbal considerations. Usually, people who engage in a fortiori or a crescendo reasoning do so without explicitly labeling their argument as this or that in form. Sometimes, they call their argument 'a fortiori', or they use the words 'a fortiori' or some similar expression ('all the more', 'how much more', etc.) within the argument to signal its logical intent. But in the latter case, they make no verbal distinction between a fortiori and a crescendo: firstly, because the latter expression is new (my own invention) and they have no distinctive label for it; and secondly, because the issue of proportionality is vague and uncertain in most people's mind, if at all present.

What this implies is that we cannot reject an argument as invalid just because it has a fortiori premises and an a crescendo conclusion. Such an argument may indeed be fallacious, or it may merely be an incomplete statement of a valid argument (furthermore, in the latter case, the content may be true or false, of course). We cannot simply refer to the fact that it has not been labeled at all or that it has been labeled incorrectly. We must, as above explained, look into the matter more deeply and try to determine the actual intentions of the argument's author, even if they are tacit, and judge the matter in all fairness.

A classic illustration of a crescendo argument, and of the hermeneutic difficulties that surround such argument, is the Talmudic reading of Numbers 12:14 in Baba Qama 25a. Without here going into all the details of this example, which are dealt with in the appropriate chapter further on (7.4), I will here just describe the reasoning involved. The Torah passage reads: "*If her father had but spit in her face, should she not hide in shame seven days? Let her be shut up without the camp seven days, and after that she shall be brought in again.*" This may be construed as a valid positive subjectal a fortiori argument as follows:

> Causing Divine disapproval (P) is a greater offense (R) than causing paternal disapproval (Q).
> Causing paternal disapproval (Q) is offensive (R) enough to merit isolation for seven days (S).
> Therefore, causing Divine disapproval (P) is offensive (R) enough to merit isolation for seven days (S).

Note that the purely a fortiori conclusion is seven days, since this is the number of days given in the minor premise. The Gemara in BQ 25a (or more precisely, a *baraita* it quotes[10]), on the other hand, advocates an a crescendo conclusion, namely: "causing Divine disapproval (P) is offensive (R) enough to merit isolation for *fourteen* days (*more than* S)." This suggests that the author of this 'proportional' conclusion has in mind, consciously or not, the following pro rata argument:

> Granting the general principle that the punishment must vary in proportion to the offense, then:
> knowing from the above minor premise that: if the offense is paternal disapproval, then the punishment is seven days isolation,
> it follows with regard to the conclusion that: if the offense is Divine disapproval (a greater offense), then the punishment has to be fourteen days isolation (a greater punishment).

That is to say, in order to logically end up with the Gemara's a crescendo conclusion (fourteen days) we have to assume a general principle of proportionality between punishment and offense. Such a principle indeed exists in Jewish tradition – it is the principle of measure for measure (*midah keneged midah*). The hermeneutic issue here is

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[10]   A 'baraita' is a statement attributed to an author of Mishnaic times (a *Tanna*); whereas the Gemara is a collection of later (*Amoraic*) commentaries.

whether the author of the a crescendo conclusion (i.e. of the Gemara, or of the *baraita* it relies on) can be reasonably assumed to have reasoned thus (i.e. by means of an argument pro rata) – or whether he believed the a crescendo conclusion to proceed directly from the a fortiori premises, without need of the assistance of the principle of measure for measure.

Another issue in hermeneutics that needs underlining is the issue of the exactitude of the quantity specified in the a crescendo argument conclusion. As we have seen, the formal conclusion is essentially rather vague – that is, the concluding predicate Sp of positive subjectal a crescendo argument means nothing more than 'more than Sq', and the concluding subject Sq of positive predicatal a crescendo argument means nothing more than 'less than Sp'. In most discourse, the subsidiary term used in the conclusion of an a crescendo argument is accordingly vague. But in some cases, a rather precise quantity is proposed (for example, in the above Talmudic illustration, precisely 14 days are specified).

The questions then arise: on the basis of what precise information did the speaker arrive at this specific numerical result? Is he claiming to have a mathematical formula that makes possible its calculation, or at least a generally accepted conventional table? If so, what is it and how reliable is it (merely probable or sure)? Or is he making an unsubstantiated claim, giving the misleading impression that a vague a crescendo argument (or even purely a fortiori argument) can yield such a quantitatively precise conclusion? Is his discourse scientific or rhetorical? Here again, it is only by careful examination of the larger context that we can decide what the speaker consciously or subconsciously intended.

5. Relative middle terms

We cannot fully understand the practice of a fortiori argument without consideration of relative middle terms. Two middle terms R1 and R2 may be said to be *relative* (or antiparallel), if 'more R1' is equivalent to 'less R2', and vice-versa. Examples of such terms abound: much and little, long and short, big and small, far and near, hard and soft, heavy and light, stringent and lenient, good and bad, beautiful and ugly, hot and cold, rich and poor, and so forth. In principle, any term that varies quantitatively (in magnitude, in direction, in measure or degree of any sort) may give rise to a relative term, although we do not commonly construct relative terms without necessity.

Let us first consider **commensurative propositions with relative terms**. The two subjectal forms "A is more R1 than B is" and "B is more R2 than A is" may be taken to imply each other, i.e. are equivalent. Such propositions are said to be each other's *reverse* (note the reversion of roles of A and B in them). For example, if the relative terms are 'long' and 'short', then if A is longer than B, it follows that B is shorter than A, and vice versa. Similarly, the predicatal commensurative proposition "More R1 is required to be A than to be B" may be reverted to "More R2 is required to be B than to be A," and vice versa (again note the reversion of roles of A and B). For example, using the same relative terms: if more length is needed to be A than to be B, it follows that more shortness is needed to be B than to be A, and vice versa.

The formal difference between conversion and reversion is that, in conversion, the major term remains major (i.e. the more), and the minor remains minor (i.e. the less), and the middle term remains unchanged; whereas in reversion, the major term becomes the new minor, and the minor term becomes the new major, and the middle term is replaced by its relative. However, on closer scrutiny we realize that the converse and the reverse of a commensurative proposition are effectively the same. This is obvious, since they are both implicants of the same form. For instance, in the case of the subjectal form "A is more R1 than B," its converse "B is less R1 than A," and its reverse "B is more R2 than A," and indeed the converse of the latter "A is less R2 than B," are equivalent to it and to each other. Similarly for the corresponding predicatal forms.

Relative terms usually evolve from absolute terms. That is to say: initially, the terms R1 and R2 (e.g. much and little, or whatever) are intended absolutely, so that what is R1 is greater than what is R2. They are conceived as separated at some *conventional cut-off value* (say, *v*), such that what is *more than v* is R1 (e.g. much) and what is *less than v* is R2 (e.g. little). Then, when we realize that this dividing line *v* is rather conventional, and may in practice be fuzzy rather than precise[11], the terms are made relative, i.e. such that the whole range of values under consideration may be viewed as R1 in one direction and as R2 in the opposite direction. In one direction, the values of R1 increase and

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[11]   For example, we may at first divide women into beautiful and ugly ones, considering that all women fall under one or the other class. Then, we may perceive some cases as doubtful, so that the dividing line is difficult to pinpoint. Moreover, we may realize that individual men sometimes differ in opinion as to which women to class where, though they may agree in some cases. Also, individual men's viewpoints may vary over time. As a result, we relativize, and avoid laying down absolute rules. An example in physics is temperature: though the sensations of hot and cold are initially thought of as absolute, eventually the terms become relative; note that the location of the zero point and the size of the units varies in different scales (Fahrenheit, Celsius or Kelvin).

those of R2 decrease, and in the other direction the opposite occurs. Neither direction is formally preferable to the other. For this reason, such terms may be characterized as antiparallel.

The propositions "A is more R than B is" and "A is less R than B is" cannot both be true at once, but they can both be false. There is an alternative to them, viz. "A is as much R as B." Note that this third form is applicable to any equal quantity of R in A and B, just as the other two forms are applicable to any unequal quantities. Likewise, when dealing with relative middle terms R1 and R2, we must take into consideration the three alternatives: "A is more R1 (and less R2) than B is," and "A is less R1 (and more R2) than B is," and "A is as much R1 (and as much R2) as B is." These three forms are mutually exclusive, and usually but not always exhaustive.

This brings us to the issue of negative forms. In most cases, the three forms just mentioned are exhaustive, which means that the denial of any two of them implies the affirmation of the third. However, this is not always true. It is quite possible for A to be neither more R nor less R than B, nor as much R as B, for the simple reason that the whole concept of R is not applicable to A or to B. For example, though all objects extended in physical or mental space[12] may be said to be bigger or smaller or equal in size, such characterizations are inapplicable to spiritual and abstract objects; i.e. the latter must be admitted to be neither bigger nor smaller nor equal in size[13].

Thus, to determine the oppositional relations of given comparative forms, we must first ask whether the concept(s) used as middle term, viz. R (or R1 and R2), is (or are) universally applicable or applicable only within a given sphere. If it is (or they are) universally applicable the said three positive forms (more, less, or equally R – or ditto with R1 and R2) are exhaustive; but if they are applicable only within a circumscribed domain, they may be all three at once denied. Of course, in the latter case, it remains true that the three positive forms are exhaustive contextually, within the sphere of their relevance; so we may continue to think of them as exhaustive provided we keep in mind that this is true conditionally, granting the applicability of the middle term used.

All the above was said for subjectals. It can also be said, *mutatis mutandis*, for predicatals; and more broadly for implicationals.

Let us now consider the special case of **the relativity between a term R and its complement notR**. As we saw in the previous chapter (1.4), although these two terms are strictly speaking (by the law of non-contradiction) mutually exclusive, it is possible to conceive of a broader term with the same label 'R' which is *inclusive* of both R in the strict sense and notR, the negation of R in the strict sense. Such broader meaning of R has obviously *no negation* of its own, note well, since by definition it embraces all conceivable values of the original term R and its negation from plus infinity to minus infinity. However, just as we can construct a broader term R, we can also construct a broader term notR. The latter is *not a negation* of R in the broader sense, note well, but a term that like it by definition embraces all conceivable values of the original term R and its negation from plus infinity to minus infinity.

Thus, although in their strict senses the terms R and notR are absolutes, and clear contradictories, the broader or looser terms derived from them, also in everyday discourse labeled R and notR, may be viewed as relative terms, which mutually suggest each other, since they *both* embrace the full range of the strict terms R and notR, although they do so *in opposite directions*. That is, **what is *more* R is *less* notR, and vice versa**; thus, for instance, "A is more R than B" and "A is less notR than B" are equivalent. For examples: the subjectal forms: "A is more active (R) than B is" and "A is less inactive (notR) than A is;" and likewise, the predicatal forms: "More action (R) is required to be A than to be B" and "Less inaction (notR) is required to be B than to be A."

As regards eductions, we observed earlier that for any pair of relative terms (R1, R2) the converse and the reverse of a commensurative proposition are effectively the same. This is also true here, with regard to R and notR. For instance, in the case of "A is more R than B," its converse "B is less R than A," and its reverse "B is more notR than A," and indeed the converse of the latter "A is less notR than B," are four logically equivalent propositions. Similarly for the corresponding predicatal forms.

As regards oppositions, the three sets of propositions "A is more R (less notR) than B is," "A is less R (more notR) than B is" and "A is as much R (as much notR) as B is," are not only mutually exclusive but also exhaustive, since here the relative terms are contradictories (so that nothing can be said to lack both R and notR).

The egalitarian forms "A is as much R as B is" and "A is as much notR as B is" are quite compatible; indeed, they imply each other. This may seem odd at first sight, due to thinking in absolute terms. But it is clear that these two propositions do not imply that A and B are both R and notR in absolute terms. They just mean, respectively, that A and B have the same value of R and the same value of notR. And since that value, whatever its magnitude and polarity (positive, zero or negative) is one throughout, the two forms must imply each other.

So for subjectals, and similarly, *mutatis mutandis*, for predicatals; and more broadly for implicationals.

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[12]   That is, material bodies in physical space or mental images in the inner space of imagination.

[13]   Though analogies remain possible. Thus, we might say that the soul of God is greater than that of humans, or that a generic concept is larger than a specific one. But these analogous ideas would not be used interchangeably in different domains. We would not suggest, for instance, that a soul or a concept has physical size.

Some readers may find the above treatment a bit confusing, in view of the different senses of the terms R and notR, as absolute or as relative. For them, I propose **a more symbolic treatment**, as follows. In this context, let us use the following notation: given the *absolute* term R and its negation notR, we can conceive of the *relative* terms R and notR (symbolically distinguished by being underlined).

Whereas the terms R and notR are mutually *exclusive*, the terms R and notR are *inclusive*, in the sense that each of them includes both R and notR. Yet R and notR are not identical, because they differ in *direction*, each being the reverse of the other. That is, whereas R refers to R as a positive quantity and to notR as a zero or negative quantity of R, notR refers to notR as a positive quantity and R as a zero or negative quantity of notR. Thus, R signifies a direction from notR (negative or zero R) to R (positive R), while notR signifies a direction from R (negative or zero notR) to notR (positive notR). We can express these definitions as formulae (where 'iff' means 'if and only if'):

Iff X is R, then R > 0 (i.e. a positive quantity of R).
Iff X is not R, then R ≤ 0 (i.e. a zero or negative quantity of R).
Iff X is not R, then notR > 0 (i.e. a positive quantity of notR).
Iff X is R, then notR ≤ 0 (i.e. a zero or negative quantity of notR).

These formulae imply that 'R > 0' = 'notR ≤ 0' (since both imply 'X is R'), and that 'R ≤ 0' = 'notR > 0' (since both imply 'X is not R'). Note well that 'zero R' and 'zero notR' are *not* the same point, but contradictories, since the former means that R is absent whereas the latter means that R is present. This must be kept in mind to avoid inconsistency. However, the propositional forms involving the terms R and notR being comparative, this issue of 'zero' having a different meaning in each of the antiparallel continua has no impact. This will become evident when we consider oppositions and eductions, next.

By definition of 'more' and 'less', the propositions "A is more R than B" and "B is less R than A" are equivalent (if either is true, so is the other). Likewise, of course, "A is more notR than B" and "B is less notR than A."

By definition of 'more' and 'less', the propositions "A is more R than B" and "A is less R than B" are incompatible (only one may be true). Likewise, of course, "A is more notR than B" and "A is less notR than B."

By definition of R and notR, the propositions "A is more R than B" and "A is less notR than B" are equivalent. Likewise, "A is more notR than B" and "A is less R than B."

It follows that the propositions "A is more R than B" and "A is more notR than B" are incompatible. Likewise, of course, "A is less R than B" and "A is less notR than B."

We might define "A is as much R as B" in relation to the propositions "A is more R than B" and "A is less R than B," as either implying both (i.e. as their intersection) or as denying both (i.e. as an alternative to them). The latter definition seems best, since in accord with actual practice. Similarly, we may take it that "A is as much notR as B" denies both "A is more notR than B" and "A is less notR than B."

The propositions "A is as much R as B" and "A is as much notR as B" imply each other. For instance, if A and B are both at (say) R = 50, they are equally R (at +50) and equally notR (at –50). This is true even when R = 0 or when notR = 0, i.e. even though a zero quantity of R is a positive quantity of notR and a zero quantity of notR is a positive quantity of R, because each of the propositions "A is as much R as B" and "A is as much notR as B" refers to only one of the relative terms and anyway does not mention any actual quantity.

Whereas, as we have seen, some relative terms R1 and R2 might be both denied (if there exists things to which neither is applicable), in the case of relative complements R and notR, denial of both is impossible. Thus, here, the propositions "A is more R (= less notR) than B," "A is less R (= more notR) than B" and "A is as much R (or notR) as B," are always exhaustive (one of them must be true).

So for subjectals, and similarly, *mutatis mutandis*, for predicatals; and more broadly for implicationals.

Whereas the oppositional relation between the absolute terms R and notR is that they (when predicated of the same subject) are contradictory – i.e. they are incompatible (cannot both be true) and exhaustive (cannot both be false), the relative terms R and notR behave differently. They appear in comparative propositions only, and in that context may be affirmed together (one being more, the other less, or both as much). However, they cannot be both discarded. The peculiarity of such relative terms is that *neither of them has a true negation*, since both refer to the same full range of existential possibilities from minus infinity through zero to plus infinity (though in opposite directions). That is to say, each of them includes the whole world, as it were (but with a difference in perspective). Everything (not just some X) can be fitted in the continuum R, and simultaneously everything (not just some X) can be fitted in the continuum notR. For this reason, R and notR are not each other's negation, note well.

Although our introduction of the underlined symbols R and notR for relative complements, to distinguish them from the absolute terms R and notR, does clarify things somewhat, I will not make further use of them here, for the simple reason that I prefer a logic of ordinary language to symbolic language, and in ordinary language we would signify our intention when it is unclear simply by saying of a given term that it is intended as relative (or inclusive). It is just as easy to mentally or out loud say the word 'relative' as to say the word 'underlined'; and the disadvantage of the

latter over the former is that one must still add the thought (in words or wordlessly) 'and underlined means relative', so one's thinking is slowed down!

Thus far, we have compared commensurative propositions with relative terms. Let us now compare **suffective propositions with relative terms**. Two eductions from suffectives need to be investigated: movement from a positive to a negative form, or vice versa, and movement from a subjectal to a predicatal form, or vice versa. Concerning the said changes of polarity, we can do a good deal; but concerning changes of orientation, little can be done. We shall first deal with copulative forms, then with implicationals.

Copulative forms. First, let us interpret the negative forms. As already seen, the positive form "X is R enough to be Y" implies that "X is R" and "X is Y," as well as "Rx ≥ Ry," where R is understood as an inclusive middle term, which includes not only R > 0 but also R = 0 and possibly also R < 0. The *negation* of this form, i.e. "It is not true that X is R enough to be Y," may colloquially be loosely expressed as "X is R not enough to be Y" or "X is not enough R to be Y" or "X is not R enough to be Y" or "X is R enough not to be Y," putting the negation in various positions.

However, to avoid ambiguities, we might prefer to write more precisely "X is R [not-enough] to be Y" or "X is [not-enough] R to be Y" or "X is not [R-enough] to be Y" or "X is not [enough-R] to be Y," adding hyphens as shown. All these forms are equivalent in that they imply "X is R" and "X is not Y," as well as "Rx < Ry," note well. They all are contradictory to the said positive form, although they have in common with it that "X is R" (where R is inclusive of notR, remember), because they imply "Rx < Ry" (instead of "Rx ≥ Ry") and thence "X is not Y" (instead of "X is Y").

As regards the form "X is R enough to be [not-Y]," with the negation attached to the predicate, it is obviously incompatible with the form "X is R enough to be Y," since X cannot be R enough to be both Y and not-Y. But are these forms contrary or contradictory? We might think they could both be false, since they have in common that X is R, and this might be false. However, since R is here intended as an inclusive term, "X is not R" is implicitly included in to "X is R;" so it is useless to focus on this factor. We might alternatively compare the form "X is R enough to be [not-Y]" to the preceding three forms, and think that it is not equivalent to them, since it implies "Rx ≥ Rnot-y" whereas they imply "Rx < Ry." However, albeit the opposite directions of ≥ and < as well as the different suffixes involved, these forms have in common the implication that "X is not Y."

Wherever the dividing line along the continuum R for Y or for not-Y, once we know on which side of it a subject X falls, the matter is settled. Everything hinges on the result (Y or not-Y), however the condition for it is expressed in a given suffective proposition. The result is what matters, the rest is history. For this reason, whatever their apparent formal differences, i.e. their differences of wording, all suffective forms that imply "X is Y" are equivalent to each other, and all those that imply "X is not Y" are equivalent to each other.

Consider now the ambiguous form: "X is not R enough to be Y," when its intent is "X is [notR] enough to be Y" (note well the position of the hyphen). Albeit its negative middle term (notR), this is a positive form, which implies that "X is not R" and "X is Y." Since notR here is also an inclusive term, it does not contradict R but includes it as a possibility among others, and this form is really equivalent to "X is R enough to be Y." Therefore, we should be careful not to confuse the form: "X is [notR] enough to be Y" with any of the forms: "X is not R-enough to be Y" or "X is R not-enough to be Y" or "X is not-enough R to be Y" or "X is not enough-R to be Y." They are in fact contradictories.

These interpretations may well appear confusing, because we would not at first sight consider the forms "X is R enough to be Y" and "X is [notR] enough to be Y" to be equivalent. The reason we would not normally equate them is that the ranges R and notR go in opposite directions (the more R, the less notR, and vice versa), whereas the notion of sufficiency is originally in one direction only. Thus, given that "X is R enough to be Y," we would not naturally simultaneously think that "X is notR enough to be Y." We would rather wonder whether X is notR enough to be *not-*Y, associating the negation of R with the negation of Y. But the answer to that question would have to be that "X is *not* notR enough to be not-Y," which can also be stated as "X is notR enough to be Y."

Let me clarify this further. The expression "enough" originally signifies a minimum value, rather than a maximum. Normally, if we say that *as of* a certain value of R, there is "enough" for Y, we would not say that *below* that value there is "enough" for not-Y. However, if pressed to the wall, we are forced to say that since there are logically only two choices, viz. Y or not-Y, we must say that "X is R enough to be not-Y" is contradictory to "X is R enough to be Y." Similarly, as regards forms with notR. It is interesting to note that, although when dealing with commensuratives the difference in direction between R and notR is significant, because more R (less notR) and more notR (less R) are incompatible, when dealing with suffectives R and notR are effectively interchangeable.

In the case of suffectives, since the issue of direction is absent, the terms R and notR in their inclusive senses are just different labels for the same position on the scale. Whether the scale goes from here to there or from there to here, the

de facto intended position remains the same. For this reason, the terms are two sides of the same coin, merely differently labeled. In conclusion, though certain constructions are a bit artificial, from a formal point of view we have to accept them and evaluate them as above suggested, so as to be able to anticipate and deal with all possibilities. The same can be done with predicatal suffectives, and with implicational ones, of course.

This understanding for R and its complement notR can obviously be extended to any two terms, R1 and R2, known to be relative. Given, say, "X is R1 enough to be Y," or any form which like it implies that "X is R1" and "X is Y," we can readily educe that "X is R2 enough to be Y," or any form which like it implies that "X is R2" and "X is Y," because "X is R1" and "X is R2" are equivalent even though in opposite directions. The moment we introduce in either of these forms a negation that affects the predicate Y, or the relational element "enough," the proposition may be taken as contradictory to the preceding. If on the other hand we introduce a negation that affects the middle term only, making it notR1 or notR2, the relation of X to Y remains essentially unchanged.

Based on this reasoning, we can always interpret suffective propositions involving one or more negative elements. All we need to do is first decide or determine just what each negation is intended to negate. If a negation is aimed at the middle term, assuming it is inclusive, nothing is essentially changed. All other negations are significant; though of course pairs of them cancel each other out. Thus for examples, "X is notR1 not enough not to be Y" is equivalent to "X is R1 enough to be Y;" whereas "X is notR1 not enough to be Y" is equivalent to "X is R1 enough to be not-Y." The utility of this kind of inference will be seen when we deal with traductions, in the next chapter (3.5).

Second, let us consider the rewriting of subjectal suffectives as predicatals, or vice versa. If we look back at the definitions of these forms, we can see the difficulties that such pursuits present. Consider the components of a proposition of the copulative form "X is R enough to be Y":

>   X is to a certain measure or degree R (say, Rx);
>   whatever is to a certain measure or degree R (say, Ry), is Y, and
>   whatever is *not* to that measure or degree R (i.e. is not Ry), is *not* Y;
>   and Rx is greater than (or equal to) Ry (whence: "Rx implies Ry").

In order to obtain a suffective proposition with Y as subject and X (or even its negation) as predicate, we would need to contrapose the conditional propositions concerning X and Y. But though we can do that for Y, we do not have the necessary material for X. Moreover, the quantitative comparison would no longer be appropriate, anyway. So we cannot, as far as I can see, change the orientation of a suffective proposition.

Implicational forms. Let us first closely examine various negative forms. The positive form "X implies R enough to imply Y" implies that "X implies R" and "X implies Y" and "Rx ≥ Ry," where R is understood as an inclusive middle thesis, which includes not only R > 0 but also R = 0 and possibly also R < 0. The *negation* of this form, i.e. "It is not true that X implies R enough to imply Y," may colloquially be loosely expressed in various ways, but it is important here (more so than in the case of conjunctives) to avoid ambiguities. If we want a negative form to closely fit the positive form in all essential respects, we must ensure that it implies "X implies R" and "X does not imply Y" and "Rx < Ry." The form "X implies R, but not enough to imply Y" would best fit this bill, suggesting that there is a threshold value of R (say Ry) only as of which Y *is* implied, but that threshold is *not* reached and therefore Y is *not* implied; note that the negation here is primarily focused on the relational expression 'enough'. We should take this as the contradictory form.

As regards the form "X implies R, enough not to imply Y" – which suggests that there is a threshold value of R (say Ry) as of which Y is *not* implied, and that threshold *is* reached – although this form still in fact implies both "X implies R" and "X does not imply Y," it differs from the preceding in that here "Rx ≥ Ry" (though Ry has a different meaning here, note well); that is to say, this form is essentially positive as regards structure, though contrary to the above positive form. As regards the forms "X does not imply R enough to imply Y" or "X does not imply enough R to imply Y," which are hardly distinguishable, they are also contrary to the positive form, but in a different manner: neither of the propositions "X implies R" and "X does not imply Y" can be educed from them, although presumably they suggest that there is a threshold value of R (say Ry) as of which Y *is* implied, and that threshold is *not* reached, meaning that "Rx < Ry;" note that these forms do not clarify whether "X implies R" or "X does not imply R."

Consider now the form: "X implies notR enough to imply Y." We should be careful not to confuse this form with any of the preceding four forms: they are in fact antitheses. Albeit its negative middle thesis (notR), this is a positive form, which implies that "X implies notR" and "X implies Y." NotR here being presumably an inclusive thesis, it does not contradict R but includes it as a possibility among others; therefore, this form is really equivalent to "X implies R enough to imply Y" – from which it follows that their contradictory forms, "X implies notR, but not enough to imply Y" and "X implies R, but not enough to imply Y," are equivalent to each other.

As regards the rewriting of antecedental suffectives as consequentals, or vice versa, we can as we did for copulatives (and even more so) safely say that it is not formally feasible.

3. Still more formalities

1. Understanding the laws of thought

Many people regard Aristotle's three 'laws of thought' – the laws of identity, of non-contradiction and of the excluded middle[1] – as rigid prejudices. They think these are just conventions, that some moronic old fellow called Aristotle had the bad grace to impose on the rest of us, and that we can just chuck 'em out at will. In each of my past works, I have tried to explain why these are fundamental human insights that cannot under any pretext be discarded. I would like to add a few more explanations in the present work.

The laws of thought must not be thought of as mechanical rules, but as repeated *insights* of our *intelligence*. Every 'application' of these laws in a new context demands a smart new insight from us. We must in each new context reaffirm these laws, and use them creatively to deal with the complexities of the case at hand.

In a fortiori logic, where new forms are encountered, and new problems need solutions, we can expect our intelligence and creativity to be called upon. We have already come across many contexts where subtlety was required. The distinction between a proposition like 'X is Y' and 'X is R enough to be Y' was one such context. Another was our development of a distinction between absolute terms (R and notR) and relative terms (R and notR). The laws of thought are ever present in logical discourse, but they must always be understood and adapted in ways that are appropriate to the context at hand – so they are not mechanical laws, but 'smart laws'.

The laws of thought have to repeatedly be adapted to **the increasing complexity of discourse**. Originally, no doubt, Aristotle thought of the laws with reference to indefinite propositions, saying that 'A is B' and 'A is not B' were incompatible (law of non-contradiction) and exhaustive (law of the excluded middle). In this simplest of contexts, these laws implied only two alternatives. However, when Aristotle considered quantified propositions, 'All A are B' and 'Some A are B and some A are not B' and 'No A is B' – he realized that the application in this new context of the very same laws implied three alternatives. From this example, we see that the subtleties of each situation must be taken into consideration to properly 'apply' the laws. They are not really 'applied'; they are intelligently *formulated anew as befits* the propositional forms under consideration.

We could say that the disjunction "Either 'A is B' or 'A is not B'" refers to an individual subject A, whereas the disjunction "Either 'All A are B' or 'Some A are B and some A are not B' or 'No A is B'" refers to a set of things labeled A. But then the question arises: what do we mean when we say that an individual A 'is B'? Do we mean that A is 'entirely B', 'partly B and partly not B'? Obviously, the mutually exclusive and exhaustive alternatives here would be: "Either 'A is wholly B' or 'A is partly B and partly not B' or 'A is not at all B'." It seems obvious that in most cases 'A is B' only intends 'A is partly B and partly not B' – for if 'A is wholly B', i.e. 'A is *nothing but* B' were intended, why would we bother verbally distinguishing A from B? Well, such tautologies do occur in practice, since we may first think of something as A and then of it as B, and belatedly realize that the two names in fact refer to one and the same thing. But generally we consider that only B is 'wholly B', so that if something labeled A is said to have some property labeled B, A may be assumed to be intended as '*only partly* B'.

To give a concrete example: my teacup is white. This is true, even though my teacup is *not only* colored white, but also has such and such a shape and is made of such and such a material and is usually used to drink tea. Thus, though being this teacup intersects with being white, it does not follow that the identity of this individual teacup is entirely revealed by its white color (which, moreover, could be changed). With regard to classes, even though we may choose to define the class of all A by the attribute B, because B is constant, universal and exclusive to A, it does not follow that A is thenceforth limited to B. B remains one attribute *among* the many attributes that are observed to occur in things labeled A. Indeed, the class A may have other attributes that are constant, universal and exclusive to it (say C,

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[1]   Aristotle states the laws of non-contradiction and of the excluded middle in his *Metaphysics*, B, 2 (996^{b}26-30), Γ, 3 (1005^{b}19-23), Γ, 7 (1011^{b}23-24). *Metaph.* Γ, 7 (1011^{b}26-27) may be viewed as one statement by Aristotle of the law of identity: "It is false to say of that which is that it is not or of that which is not that it is, and it is true to say of that which is that it is or of that which is not that it is not." These references are found in the Kneales, p. 46 (although they interpret the latter statement as somewhat defining truth and falsehood, rather than as expressing the law of identity).

D, etc.), and yet B alone serves as the definition, perhaps because it intuitively seems most relevant. Thus, to define concept A by predicate B is not intended to limit A to B. If A were indeed limited to B, we would not name them differently.

These thoughts give rise to **the logical distinction between 'difference' and 'contradiction'**, which calls forth some further use of ad hoc intelligence. When we say that 'A and B are different', we mean that these labels refer to two distinct phenomena. We mean that to be A is not the same as to be B, i.e. that B-ness is different from A-ness. It does not follow from this that No A is B. That is to say, even though A is not the same thing as B, it is conceivable that some or all things that are A may yet be B in some way. To say the latter involves no contradiction, note well. Therefore, the laws of non-contradiction and of the excluded middle cannot in this issue be applied naïvely, but only with due regard for the subtleties involved. We must realize that 'difference' is not the same as 'contradiction'. Difference refers to a distinction, whereas contradiction refers to an opposition. Two propositions, say X and Y, may have different forms and yet imply each other. It is also possible, of course, that two propositions may be both different and contradictory.

Another subtlety in the application of the laws of thought is **the consideration of tense**. Just as 'A is B' and 'A is not-B' are compatible if they tacitly refer to different places, e.g. if they mean 'A is B here' and 'A is not-B there', so they are compatible if they tacitly refer to different times, e.g. if they mean 'A is B now' and 'A is not-B then'. Thus, if a proposition is in the past tense and its negation is in the present or future tense, there is no contradiction between them and no exclusion of further alternatives. Likewise, if the two propositions are true at different moments of the past or at different moments of the future, they are logically compatible and inexhaustive.

These matters are further complicated when we take into consideration the various modalities (necessity, actuality, possibility), and still further complicated when we take into consideration the various modes of modality (natural, extensional, logical). I have dealt with these issues in great detail in past works and need not repeat myself here. In the light of considerations of the categories and types of modality, we learn to distinguish factual propositions from epistemic propositions, which qualify our knowledge of fact. In this context, for instances, 'A is B' and 'A seems not provable to be B', or even 'A is B' and 'A seems provable not to be B', might be both true.

One of the questions Aristotle made a great effort to answer, and had some difficulty doing, was how to interpret the disjunction: "Either there will be a sea battle tomorrow or there will not be a sea battle tomorrow"[2]. But the solution to the problem is simple enough: if we can truly predict today what will (or will not) happen tomorrow, it implies that tomorrow is *already* determined at this earlier point in time and that we are able to know the fact; thus, in cases where the fact is *not* already determined (so that we cannot predict it no matter what), or in cases where it is already inevitable *but* we have no way to predict the fact, the disjunction obviously cannot be bipolar, and this in no way contravenes the laws of thought. Nothing in the laws of thought allows us to foretell whether or not indeterminism is possible in this world.

As a matter of fact, either now there will be the sea battle tomorrow or now there won't be one or the issue is still undetermined (three alternatives). As regards our knowledge of it, either now there will be the battle tomorrow and we know it, or now there won't be and we know it, or now there will be and we don't know it, or now there won't be and we don't know it, or it is still undetermined and so we cannot yet know which way it will go (five alternatives). We could partially formalize this matter by making a distinction between affirming that some event *definitely, inevitably* 'will' happen, and affirming only that it just *possibly or even very likely* 'will'; the former is intended in deterministic contexts, whereas the latter is meant when human volition is involved or eventually when natural spontaneity is involved. These alternatives can of course be further multiplied, e.g. by being more specific as regards the predicted time and place tomorrow.

What all this teaches us is that propositions like 'A is B' and 'A is not B' may contain many tacit elements, which when made explicit may render them compatible and inexhaustive. The existence of more than two alternatives is not evidence against the laws of thought. The laws of thought must always be *adapted* to the particulars of the case under consideration. Moreover, human insight is required to properly formalize material relations, in a way that keeps our reading in accord with the laws of thought. This is not a mechanical matter and not everyone has the necessary skill.

Another illustration of the need for intelligence and creativity when 'applying' the laws of thought is **the handling of double paradoxes**. A proposition that implies its contradictory is characterized as paradoxical. This is a logical possibility, in that there is a quick way out of such single paradox – we can say that the proposition that implies its contradictory is false, because it leads to a contradiction in knowledge, whereas the proposition that is implied by its contradictory is true, because it does not lead to a contradiction in knowledge. A double paradox, on the other hand, is a logical impossibility; it is something unacceptable to logic, because in such event the proposition and its contradictory both lead to contradiction, and there is no apparent way out of the difficulty. The known double

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[2]   See *De Interpretatione*, 9 (19ª30).

paradoxes are not immediately apparent, and not immediately resolvable. Insights are needed to realize each unsettling paradox, and further reflections and insights to put our minds at rest in relation to it. Such paradoxes are, of course, never real, but always illusory.

Double paradox is very often simply caused by *equivocation*, i.e. using the same word in two partly or wholly different senses. The way to avoid equivocation is to practice precision and clarity. Consider, for instance, the word "things." In its primary sense, it refers to objects of thought which are thought to exist; but in its expanded sense, it refers to any objects of thought, including those which are not thought to exist. We need both senses of the term, but clearly the first sense is a species and the second sense is a genus. Thus, when we say "non-things are things" we are not committing a contradiction, because the word "things" means one thing (the narrow sense) in the subject and something else (the wider sense) in the predicate. The narrow sense allows of a contradictory term "non-things;" but the wider sense is exceptional, in that it does not allow of a contradictory term – in this sense, everything is a "thing" and nothing is a "non-thing," i.e. there is no "non-thing."

The same can be said regarding the word "existents." In its primary sense, it refers to actually existing things, as against non-existing things; but in its enlarged sense, it includes non-existing things (i.e. things not existing in the primary sense, but only thought by someone to exist) and it has no contradictory. Such very large terms are, of course, exceptional; the problems they involve do not concern most other terms. Of famous double paradoxes, we can perhaps cite the Barber paradox as one due to equivocation[3]. Many of the famous double paradoxes have more complex causes. See for examples my latest analyses, in **Appendix 7.4** of the Liar paradox, and in **Appendix 7.5** of the Russell paradox. Such paradoxes often require a lot of ingenuity and logical skill to resolve.

2. Quantification

Many people, even trained logicians among them, find the field of a fortiori logic difficult because of the variety of issues of quantity in the terms. We have already dealt with the issue of proportionality, and the differences between purely a fortiori argument, pro rata argument and a crescendo argument. Here, we will deal with three distinctions between terms: that between constants and variables; that between individuals and classes; and that between distributive and collective terms.

Terms may be constants or variables

We have earlier presented and validated various moods of a fortiori argument, without specifying whether the terms they involve are constants or variables. The reason is simply that they may be either. Although initially all the terms involved might be thought of as constants, we eventually realize that some or all of them may equally well be variables, *provided the patterns of interrelationship between the variables continue to fit in to the formulae required for valid a fortiori argument.* That is, provided each value of one variable is related to the corresponding value of the other variable in accord with the said formulae; which simply means that the various values of variables are effectively alternative constants.

This should be obvious. But let us consider an example the following positive subjectal argument:

> P is more R than Q,
> and Q is R enough to be S;
> therefore, P is R enough to be S.

Here, P, Q, R, and S may all at first be assumed to have individual values, i.e. to be constants. But it is conceivable that P and Q may have different values of R at different times or places or in different circumstances or in different instances. That changes nothing in our argument, *provided* the relation 'Rp > Rq' remains unaffected for *every* pair of values of the variables Rp and Rq. If when variable Rp has value Rp1 and variable Rq has value Rq1, 'Rp1 > Rq1', and if the same is true for every other pair of values, like Rp2 and Rq2, Rp3 and Rq3, etc., then the a fortiori argument holds. It is also conceivable that Rp is constant while Rq is a variable, or vice versa, provided 'Rp > Rq' remains true. In all such cases, though the a fortiori argument seems like one statement, it may also be viewed as a summary of many similar statements[4].

Again, the predication 'Q is R' in the minor premise may intend the term R (which here more precisely means Rq) as a constant or as a variable. In the latter case, so long as every value of Rq – namely, Rq1, Rq2, Rq3, etc. – is sufficient to imply Q to be S (whether S is here a constant, or itself also a variable), the minor premise as a whole

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[3]   I deal with this one in my Future Logic, chapter 32.3.

[4]   This implies, incidentally, that generalization may have been used – i.e. there may be an element of prediction of future instances based on past instances.

remains true. The conclusion then (given also the major premise, of course) naturally follows, whether Rp is a constant or a variable (with values Rp1, Rp2, etc.). Note here again that it is possible for Rq to be a variable while Rp is a constant, or vice versa, or they might both be constants or both variables.

The important thing to note here is that whatever value S has in the minor premise, it must be repeated in the conclusion. Where S is variable in the former, it is *identically* variable in the latter. The a fortiori argument as such cannot change the value of S in the transition from the minor premise to the conclusion; only the very same value of S can be inferred a fortiori. This issue should not be confused with that of 'proportionality' of conclusion, which we dealt with in the preceding chapter (2.2-2.3). As we saw there, the value of S in the conclusion may differ from that in the minor premise, if and only if we have an additional premise capable of justifying such change.

Terms may be individuals or classes

We have so far treated the major and minor terms (P and Q) of **subjectal** a fortiori arguments as indivisible units. Obviously, P and Q may be named or pointed-to *individuals*; e.g. Tom is richer than Harry, or this man is richer than that man. Also, obviously, P and Q may be *whole classes*; e.g. gold is worth more than silver, meaning any amount of gold and any equal amount of silver; or again, humans are more intellectual than other animals, meaning all humans are more intellectual than all other animals. The latter universal major premise could conceivably give rise to an a fortiori argument in which the minor premise is particular (or even singular) and the conclusion is general, note well. This means the following positive and negative forms are valid:

> All P are more R than all Q,
> and, all or *some* Q are R enough to be S;
> therefore, *all* P are R enough to be S.

> All P are more R than all Q,
> yet, all or *some* P are R not enough to be S;
> therefore, *no* Q is R enough to be S.

That is, given the major premise "All P are more R than all Q," it follows that "if there is *any* Q that is R enough to be S, then *all* P are so too;" and inversely, "if there is *any* P that is R not enough to be S, then *no* Q can be so." Note well, "all P" here refers to each and every P, and "all Q" to any and every Q; consequently, the minor premises might be particular, i.e. refer to some Q or P, or even to just one specified or unspecified Q or P. This is obvious enough.

If the major premise is entirely particular, we can still draw a *particular* conclusion *provided* the minor premise is general. That is to say, the following positive and negative forms are also valid:

> Some P are more R than some Q,
> and, *all* Q are R enough to be S;
> therefore, *some* P are R enough to be S.

> Some P are more R than some Q,
> yet, *no* P is R enough to be S;
> therefore, *some* Q are R not enough to be S.

These moods are valid because there is still in them a guarantee of overlap between the instances referred to in the major and minor premises. Note that if we could pinpoint instances, we could treat particulars as generalities; i.e. the following would be valid moods:

> Certain P are more R than certain Q,
> and, all or some of *those* Q are R enough to be S;
> therefore, all of *these* P are R enough to be S.

> Certain P are more R than certain Q,
> yet, all or some of *these* P are R not enough to be S;
> therefore, none of *those* Q is R enough to be S.

When we have a mix of quantities in the major premise, we can for the same reason in certain cases draw a valid conclusion as follows:

Some P are more R than all Q,
and, all or *some* Q are R enough to be S;
therefore, *some* P are R enough to be S.

Some P are more R than all Q,
yet, *no* P is R enough to be S;
therefore, *no* Q is R enough to be S.

All P are more R than *some* Q,
and, *all* Q are R enough to be S;
therefore, *all* P are R enough to be S.

All P are more R than *some* Q,
yet, all or *some* P are R not enough to be S;
therefore, *some* Q are R not enough to be S.

This should cover all cases of subjectal argument. Moods not above mentioned as valid are intended as invalid. All this can be formally demonstrated in the usual manner. Note well that the quantities intended in the major premises are not intended as "one for one" correspondences between P and Q. That is, when we say "All (some) P are more R than all (or some) Q," we do not specifically mean that "for each instance of P referred to, there is *a corresponding instance* of Q such that this P is more R than that Q" (more on this topic presently); we are dealing with the classes in bulk (though "one for one" may be applicable in specific cases).

As regards **predicatal** a fortiori argument, the situation is very different since major and minor terms, P and Q, are predicates and not (as above) subjects, and therefore cannot properly be quantified. On the other hand, we can here quantify the subsidiary term, S, which is here a subject and not (as above) a predicate. The rule in this context is simple enough: the quantity in the conclusion is the same as that in the minor premise. If the minor premise (whether positive or negative) addresses some specified, some unspecified or all instances S, then so will the conclusion do. Again, this is obvious enough:

More R is required to be P than to be Q,
and, all (or some) S are R enough to be P;
therefore, all (or some) S are enough to be Q.

More R is required to be P than to be Q,
yet, all (or some) S are *R not* enough to be Q;
therefore, all (or some) S are *R not* enough to be P.

So much for copulative argumentation. Regarding implicational a fortiori argument, the theses involved may in principle contain any quantity or mix of quantities, without affecting the given implications; so there is nothing much to say about it. That is to say, though it is of course conceivable that in some cases the implications originally depend on the quantities involved, it remains true that once we are given the implications as premises the quantities in the theses concerned become irrelevant to the drawing of a conclusion from them. In such cases, quantities count as content, not as form, as regards the a fortiori process as such.[5]

Correspondences between terms. A special case of a fortiori argument that ought to be mentioned is when the terms involved are tied together by some correspondence. For example, the argument "all parents (P) are older (R) than their children (Q); therefore, if the children are old enough to vote, so are their parents" – it is clear that the major and minor terms refer to parents and *their respective* children, and not just to any children of any parents. With this in mind, we can construct the following valid moods in general terms:

Every P is more R than *its corresponding* Q,
and, a Q is R enough to be S;
therefore, *its corresponding* P is R enough to be S.

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[5]   We could perhaps, however, conceive of implicational equivalents of the above copulative quantifications, with reference to the modalities of implication. In this perspective, "if X, necessarily Y" is equivalent to a positive general proposition, and "if X, possibly Y" to a positive particular proposition. Similarly, "if X, necessarily not Y" is equivalent to a negative general proposition, and "if X, possibly not Y" to a negative particular proposition. This idea needs further consideration, but I will not do that here.

Every P is more R than its corresponding Q,
and, a P is R not enough to be S;
therefore, its corresponding Q is R not enough to be S.

In the above two subjectal moods, the correspondence is between P and Q. There could also be correspondence between P and Q and the subsidiary term (S), as in the following two predicatal moods.

More R is required (of something, e.g. S) to have *a corresponding* P than to have *a corresponding* Q,
and, S is R enough to have *its corresponding* P;
therefore, S is R enough to have *its corresponding* Q.

More R is required (of something, e.g. S) to have a corresponding P than to have a corresponding Q,
and, S is R not enough to have its corresponding Q;
therefore, S is R not enough to have its corresponding P.

Sometimes, additionally or alternatively, the middle term (R) is tied in this sense. For example, we may say that Noah was more righteous in *his* generation than many an absolutely more righteous man was righteous in *his own* respective generation. Valid moods can be constructed with such a variable middle term if this is done carefully – avoiding the fallacy of two middle terms by making sure the effective middle term throughout the argument is a generality. Clearly, in all such cases, the qualification "its corresponding" must be considered as part of the terms it qualifies, even though it is for emphasis stated outside them.
Needless to say, correspondences between terms are special cases; usually the terms involved are not tied together in this manner.

Terms may be distributive or collective
What has been said above about quantification concerns distributive terms – i.e. cases where the quantity all or some or whatever refers to the instances intended each one *singly*. The situation is very different where collective terms are intended, i.e. where the quantity is used to signify that a number of specified instances *together* make up a unit.
The latter situation can cause havoc in a fortiori argument, if misunderstood. Consider the following examples. With terms intended distributively, a fortiori argument is always possible: e.g. "If (any of) these five men are rich enough to pay for this object, then this one (of them) is rich enough." However, when the intent is collective, we cannot readily infer a fortiori. Given that five designated men together can lift a certain weight, it obviously does *not* follow that three of them are strong enough to do so; or: given that three designated men together can go through a certain door, it obviously does not follow that five of them are thin enough to do so. Thus, note, we cannot formulate an a fortiori argument in either direction – i.e. neither from five to three nor from three to five.
Clearly, what is involved here is an issue of upper and lower **limits**, i.e. of maxima and minima. A quantity can be enough, or too much or too little. We can formalize our above examples as follows. Suppose 'X' is a designated group of individuals, 'n' refers to the number of them involved, and 'Y' is a predicate concerning them collectively. Given the lower limit 'Less than nX together cannot be Y', it follows that 'nX together are Y' does not imply 'less than nX together are Y'. Similarly, given the upper limit 'More that nX together cannot be Y', it follows that 'nX together are Y' does not imply 'more than nX together are Y'. In some cases, both limits are set – neither more nor less than a stipulated quantity is allowed.
Moreover, note that collective terms are very specific in their intents: while *these* five strong men are able to jointly lift said weight, five (or even more) *other* (weaker) men may not be able to do so; or again, while these three thin men are able to jointly go through said door, three (or even less) other (fatter) men may not be able to do so.
More formally put, given a limit for 'nX1', i.e. a number n of designated individuals labeled X1, it does not follow that the same limit obtains for 'nX2', i.e. a number n of other designated individuals labeled X2. Collective terms can be very tricky in yet other ways. For instance, these five men may be able to lift a certain weight together, if they all have a handle to take hold of it, but not if they have a hard time grabbing it. Or these three men may be able to go through this door at once, if they are stacked one on top of the other, but not if they try to simply walk through it simultaneously. Thus, a collective term is not fully defined by numbers (like n) and designated individuals (like these men), but may involve more complex conditions. Note this well.

Also to keep in mind, a collective may have very different properties than its components; and conversely, its components may have very different properties than a collective. If we assume that what is true of some or all of its components is true of a collective, we commit the 'fallacy of composition'. If we assume that what is true of a collective is true of some or all of its components, we commit the 'fallacy of division'.

As regards application of the concept of **existential import** to the commensurative and suffective propositions constituting a fortiori argument, my position is simply that the existence of subjects is generally assumed. See **Appendix 7.2** for a fuller presentation of this position.

3. A fortiori through induction

As we have shown, a fortiori argument is essentially a deductive argument, one that can readily be formalized and validated. There is no denying that. Yet many commentators persist in erroneously regarding it as an essentially inductive argument, no doubt simply because they have not perceived its deductive form. We shall now examine the various ways induction may be involved in the formulation and justification of an a fortiori argument.

Induction in general. First, we must distinguish formal induction from material (or informal) induction. An inductive argument can be considered as part of formal logic if the relation between its premise(s) and conclusion can be expressed in formal terms, i.e. with reference to symbols acting as placeholders for any material terms or propositions. If we cannot treat the inductive argument in purely abstract ways, it constitutes a material (or informal) one. Many people think that only deduction can be treated formally, but the truth is that much inductive reasoning can be formulated and justified in formal terms. There are several types of formal induction.

a.   The most important type of formal induction is *factorial induction*. It is the most important type, because it is the most distinct from deduction and the most reliable. I have treated this topic in great detail in my work *Future Logic* (part VI).

Factorial induction may be described in the following general terms: given a premise p, or a set of premises p1 and p2 and…, then: if we have a single necessary implication c, then it is the deductive conclusion; whereas, if we have two or more logically possible implications c1 or c2 or…, and for some reason one of them (say, c1) is logically preferable to the other(s), then it is the adopted inductive conclusion. The latter conclusion may of course change as new data is discovered which changes the premises; or c1 may be found empirically untrue, in which case the next most probable inductive conclusion (say, c2) is selected. The conclusions c1, c2, c3, etc. are called the factors of the premise p, or of the set of premises p1, p2, p3, etc. The most probable conclusion is called the strongest factor.

To understand this process, let us look at the simplest example: *generalization* of a particular proposition. A deductive argument is distinguished by having a single conclusion regarding certain terms (say, X and Y) from the given premise(s); for example, that 'All X are Y' (symbol **A**) implies 'Some X are Y' (symbol **I**) is a deductive argument (an eduction). On the other hand, an inductive argument has in principle two or more conclusions; for example, 'Some X are Y' (**I**) implies either 'All X are Y' (**A**) or 'Some X are Y and some X are not Y' (**IO**) is (part of) an inductive argument[6].

But that is not all. The argument '**I** implies either **A** or **IO**' gives us a choice of inductive conclusions, but it does not give us any advice as to *which* of the two conclusions to opt for (or at best, it gives us a 50-50 probability for each conclusion, by virtue of symmetry). However, through the inductive act of generalization, we are able to prefer the conclusion **A** to the conclusion **IO**, by arguing that whereas **A** has the same positive polarity as given in the premise, **IO** involves an additional negative polarity *not at all found in the premise*. Thus, **A** requires *less assumption* than **IO**. For, though it is true that both **A** and **IO** claim quantitatively new information regarding the whole class, nevertheless **IO** introduces a claim to negative polarity not at all found in the premise **I**, while **A** remains entirely consistent in respect of polarity with the premise **I**. Thus, given **I**, alternative **A** is more probable than **IO**, because the latter is more speculative than the former.[7]

Thus, though our inductive argument starts by yielding a disjunction of conclusions, what makes it truly useful is that we are able to *formally* (before reference to any content) select one conclusion *in preference to* the other. It is the latter feature that allows us to refer to the argument as specifically inductive and distinguish it from deductive argument. An inductive argument allows deductively for two or more conclusions, but additionally (in most cases)

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[6]   Of course, **I** implies 'either **A** or **IO**' could be regarded as a deductive argument with a single disjunctive conclusion; so when we said that **A** implies **I** is a deductive argument because it has a single conclusion, we meant in this case that it has a single distinctively categorical conclusion. The inductive argument **I** implies either **A** or **IO**, by way of contrast, offers a choice of two possible categorical conclusions: either **I** implies **A**, or **I** implies **IO**. This is said in passing, being a minor issue.

[7]   Needless to say, we can similarly generalize from **O** to **E**, preferring the latter to **IO**.

provides us some formal reason(s) for preferring one of these conclusions over the others. Such induction is thus not a matter of guesswork or of purely material considerations, but is part of 'formal logic' in its own right.

What we have described above is called generalization from **I** to **A**. This process depends for its validity, note well, on two premises. First, that **I** is true; this might be determined empirically (i.e. by observation through the senses, as regards material phenomena, or by observation through the proverbial "mind's eye," as regards mental phenomena) or by deduction from previous inductions or deductions. Second, that there is no known evidence for claiming that **O** is true. Given these two premises *together*, and *not* just one of them alone (as some people erroneously think), we can inductively conclude that **A** is true.

It is obvious that if **I** concerns positive phenomena and is truly empirically established, it is very unlikely to ever turn out to be untrue. However, since **I** is usually made up of concepts and not just percepts, it is not inconceivable that it be later found inaccurate and should be abandoned; though, to repeat, that is relatively rare. On the other hand, the non-knowledge of **O** is not a definitive denial, and may more readily be conceived as being overturned. If upon further scrutiny we discover that in fact 'Some X are not Y', we must imperatively correct our previous judgment, and conclude with **IO** instead of **A**. This is called *particularization*. This is also a valid process within formal inductive logic. Particularization does not invalidate generalization, note well, but complements it by ensuring that all judgments are subject to correction if the need arise.

One more thing needs clarifying here: what constitutes inconclusiveness. If we lack, and cannot currently infer, any information regarding the relation of X and Y, so that the disjunction 'either **A** or **IO** or **E**' is true, we have effectively no conclusion from any known premises. If now, whether by observation or inference, we discover that **I** is true, this eliminates the **E** alternative of that disjunction, and we are left with the narrower disjunction 'either **A** or **IO**'. This is, as we have seen, a conclusion of sorts[8]. Thus, conclusiveness is distinguishable from inconclusiveness with reference to the range of alternatives implied. If the range is unlimited, there is no conclusion; if it is partly limited, we have a set of inductive conclusions; if it is limited to one conclusion, we have either a deduction or a preferred induction.

Another approach to factorial induction is with reference to individual cases. Starting with any particular X, we know from the laws of thought that it is either Y or not-Y. Having found that some other X are Y, and not having found any other X that are not-Y, we can predict with considerable certainty that this here X is Y. This is another way of presenting the above detailed generalization from **I** to **A**. Similarly, having found that some other X are not-Y, and not having found any other X that are Y, we can predict with considerable certainty that this here X is not-Y. This is generalization from **O** to **E**.

If on the other hand, from the very beginning we find that some X are Y and some X are not Y, we obviously cannot generalize either to **A** or to **E**. If we have already generalized to one of the latter universal forms, we must retract and admit **IO** instead. In that case, what can we predict concerning an individual case of X, about which we have no information as yet as to whether it is Y or not-Y? The way to resolve this issue is to compare *the frequencies* of cases of X that are known to be Y and cases of X that are known to be not-Y. If it looks as if the frequencies are about the same on both sides of the contingency, the probability that this here X is Y and the probability that it is not-Y are even, and no inductive conclusion can be drawn; since the factors have the same weight, they remain formally indistinguishable. But if one side is more frequent than the other, we can identify it as the strongest factor. That is, we can conclude: 'this X is most probably Y and less probably not-Y'. In some contexts, though not all, the probabilities involved can be precisely quantified.

The process just described may be described as *statistical induction*, because it relies on approximate or precise enumeration of known cases. If most (i.e. more than half of the) known instances of X are Y, then this unknown instance of X may be assumed to be Y. If most known instances of X are not-Y, then this unknown instance of X may be assumed to be not-Y. If the known instances of X are evenly distributed, with as many Y as not-Y, then this X being Y and this X being not-Y are equally probable, which means nothing definite can be said about the relation of X and Y or not-Y. Probabilities are of course subject to change as more instances of X are otherwise identified (empirically or by other arguments) to be Y or to be not-Y, so that our preferred conclusion may need updating.

In this light, generalization may be viewed as the special case where *all* known instances have the same polarity, so that we may assume the universal proposition **A** or (as the case may be) **E** to be true. We can also view the statistical treatment of contingencies, i.e. the movement from 'Most known instances' to 'Most instances' as a sort of generalization, in that we reiterate information available for known instances to unknown instances.

b.   The second most important type of formal induction is **adduction**, also known as 'the scientific method'. This form of reasoning is an offshoot of the logic of hypotheticals, in contrast to the preceding form, which is an offshoot of the logic of categoricals. The following are the two valid moods of apodosis, or hypothetical deduction:

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[8]   Which conclusion can eventually be further narrowed down to A (by generalization) or to IO (by particularization).

If P, then Q;   If P, then Q;
and P;   and not-Q;
therefore, Q   therefore, not-P

These moods are respectively called 'affirming the antecedent' (*modus ponens*) and 'denying the consequent' (*modus tollens*). These moods are used, in adduction, to connect new theories to given data or to make new predictions from old theories and, respectively, to exclude new theories unconnected to the data or reject old theories with false predictions. They are valid because the major premise, 'If P, then Q' (i.e. if the antecedent P is true, then the consequent Q is true), means that 'the theses P and not-Q cannot both be true'. From which it follows that if P is true, not-Q must be false; and if not-Q is true, P must be false. The following the two moods are, on the other hand, *in*valid:

If P, then Q;   If P, then Q;
and Q;   and not-P;
therefore, P   therefore, not-Q

Given the major premise, we cannot infer that P is true from the fact that Q is true, or that Q is false from the fact that P is false. These moods are invalid as apodoses, i.e. as deductive arguments. But they still have some utility as inductive arguments. We can say regarding the positive mood that the evident truth of Q in the context of 'If P, then Q' gives some probability to the hypothesis that P is true; for it could be that Q is true *because* P is true. Likewise, we can say regarding the negative mood that the evident falsehood of P in the context of 'If P, then Q' gives some *im*probability to the hypothesis that Q is true; for it could be that P is false *because* Q is false.

Such arguments are of course very tenuous taken individually. But if we have many, and still more, cases of evidence like Q confirming hypothesis P, thesis P becomes more and more credible. Likewise, if we have many, and still more, cases of evidence like not-P undermining hypothesis Q, thesis Q becomes more and more incredible. Thus, these forms of argument are used in adduction, respectively, to strengthen or weaken existing theories. Strengthening and weakening do not, of course, mean proving or disproving; they just refer to cumulative credibility.

All such cumulative credibility can fall apart instantly, if even a single instance is found that belies the thesis concerned. That is, if P is repeatedly confirmed through evidence like Q, and an instance of Q is found which implies not-P, the hypothesis that P is true must be rejected. Likewise, if Q is repeatedly undermined through evidence like not-P, and an instance of not-P is found which implies Q, the hypothesis that Q is false must be rejected. How definitive such theory 'rejection' is depends on how empirical the reason for it is. If the reason is truly empirical, the rejection is definitive; whereas, if the reason is itself merely theoretical, the rejection is more mooted.

Such thoughts can be compared to generalization and particularization. When we repeatedly find evidence that confirms our hypothesis, and find no evidence to the contrary, we generalize and assume all future evidence will go the same way. When and if we eventually encounter evidence to the contrary, we particularize that generality, and adopt a more contingent stance, if not the very opposite idea. Here again, given a contingency, we may still for statistical reasons prefer a thesis over its negation, or we may find the evidence evenly distributed.

Adduction also, like generalization-particularization, uses disjunctive argument. There may be, and usually there are, two or more hypotheses that could explain the accumulated evidence. In such case, we compare the probabilities for each of the alternative hypotheses, and we opt for the strongest. Other considerations may affect this judgment, such as the simplicity or complexity of a hypothesis; such considerations may be viewed as adding or subtracting credibility to the hypothesis, i.e. as increasing or diminishing its net probability. Thus, the disjunction of possible explanations of varying probability may be said to be a list of the factors of the data to be explained, ordered according to their relative strengths. So adduction may be viewed as a special case of factorial induction.

It is also true that we may regard generalization and particularization as special applications of theory confirmation and elimination, i.e. of adduction. When we generalize from **I** to **A**, say, we are effectively regarding the instances subsumed by **I** as confirmations of the hypothesis **A**. When we particularize, we reject hypothesis **A**, and opt for the more empirical thesis **IO**. We then look towards the relative frequencies of the two polarities, and formulate a new hypothesis that 'Most X are Y' or that 'Most X are not Y' or that 'As many X are Y as are not-Y', as the case may be. This in turn might be modified, as new data comes in. Thus, adduction and generalization-particularization are very closely related reasoning processes.

In this context, quite parenthetically, I would like to say a word or two about the term 'abduction', which many people confuse with and prefer to the term 'adduction'. The term 'abduction' refers to the assumption of a hypothesis

that explains available evidence[9]. From this definition it is clear that the term refers to a positive argument: If P, then Q; and Q; therefore, P. There is notably no mention here of the possible negative aspect, i.e. the possibility that hypothesis P might entail some false prediction, such as Q2, so that it is eliminated by the argument: If P, then Q2; and not-Q2; therefore, not-P. Nor is the competitive aspect brought out, i.e. that P may be supplanted by some other hypothesis P2 which is found more probable.

Abduction is thus very naïve guesswork. Adduction, on the other hand, refers to the full range of the scientific method: repeatedly testing a hypothesis by means of new evidence, which may turn out to be counter-evidence, and comparing competing hypothesis, to opt for the most likely, as above described. Thus, the two terms should not be confused.

c.   The third most important type of formal induction is ***induction based on deduction***. What this refers to is the following: suppose we have a duly validated deductive argument, say 'P and Q implies R'. Then we can logically rely on the arguments: 'If P and probably Q, then equally probably R', or 'If probably P and Q, then equally probably R', or again 'If probably P and probably Q, then probably R'. In these arguments, the degrees of probability of the premises are passed on to the conclusion, by multiplication. Thus, if one of the premises is 100% probable and the other 50%, the conclusion is 50% probable; whereas if both premises are 50% probable, the conclusion is only 25% probable. This is reasoning mathematically[10].

The important point to note is *the deductive underpinning of such induction*. The inductive form apes a deductive form. If the deductive form intended is valid, then the inductive form is a credible induction. But if the deductive form intended is *not* valid, then the inductive form is *not* a credible induction. The credibility of the induction is based on the credibility of the deduction it imitates. Thus, to give an example: given that 'All Y are Z and this X is Y, therefore this X is Z' is deductively valid, we can refer to a parallel inductive argument with, say, 'this X is probably Y' as the minor premise and 'this X is probably Z' as the conclusion. But, given the same premises, we cannot conclude 'this X is probably not Z' because such inductive conclusion has no deductive underpinning. It may well be true for other reasons that 'this X is probably not Z' – but this is not an inductive conclusion from the said premises.

Thus, an 'induction based on deduction' is very different from an unsupported statement of logical possibility. It is a type of formal induction, which relies for the credibility of its conclusion on the conclusion having a form that is validated by deduction. The relation between premises and conclusion is thus warranted. The implication involved is not merely probable but certain. The *process* remains certified, even if the premises and therefore the conclusion are only probable. In truth, most arguments we encounter in practice are of this sort, since most of our material knowledge is somewhat open to doubt. Our deductions naturally pass the doubtfulness of the premises onto the conclusion. But the process of deduction is unaffected: once shown logically necessary, it is reliable, no matter what the reliability of the propositions involved.

d.   The last and most unreliable type of 'induction' is precisely ***inductive inference that relies on a doubtful deductive process***. Here, the basis is only that 'P and Q *probably implies* R' (rather than 'P and Q implies R') – so that, whether the premises are certain or only probable, the conclusion is at best only probable, if at all credible. This may be called induction in only a very loose sense of the term, since by definition we have no formal justification for drawing the putative conclusion from the given premises. Does this happen? Yes, it does. And people indulge in it because it seems to them 'better than nothing'.

Someone may have information that suggests he might be able to formulate a certain deductive argument, but as yet he does not have enough information to actually formulate that argument – so he refers to a merely 'probable implication'. There is some truth in that since, indeed, the argument might eventually crystalize; but until it does, the expectation that it might must be regarded with the utmost skepticism. A 'probable implication' is, strictly speaking, no implication at all. Thus, though we have premises, we have no real process through which to elicit a conclusion from them. The conclusion's 'probability' is thus not much greater than the 'probability' of anything we imagine offhand.

This is comparable to having a few pieces of a jigsaw puzzle and trying to guess what the picture it comes from might be. It is in such contexts that the word 'abduction' is most appropriately used. And it is in such contexts that the skills of statisticians are called for, to find ways to select the most credible hypothesis from miniscule indices. The validity of such 'inference' is obviously very relative; in absolute terms, it is of doubtful worth.

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[9]   See for instance the definition given in *The Oxford Companion to Philosophy* (Ed. Ted Honderich, Oxford UP, 1995). The term was introduced by Charles S. Pierce (1839–1914).
[10]   If we toss two coins, what are the chances of getting, say, two 'heads' at once? Obviously, 50% * 50% = 25%. But it could alternatively be argued, with syllogistic logic in mind, that the probability of the conclusion is 100% minus the sum of the probabilities of the premises, for the conclusion is only sure to occur in contexts where the premises are sure to overlap. In that view, if the probabilities of the premises are both 50%, or otherwise add up to 100% or less, the premises might never overlap since their contexts of occurrence may be at odds, and the probability of the conclusion (from that conjunction of premises) must be taken as 0%. This issue needs to be reflected on.

It should be stated that the basis of all inference is ***analogy and disanalogy***, although of course there are more sophisticated and reliable inferences and more simplistic and unreliable ones. Our deductive and inductive capacities all depend on, among other cognitive processes, *the cognition of similarities and differences*. If appearances did not seem 'similar' in various respects and 'different' in others, we could not perceive or conceive any continuity in identity in any individual object across time, or be able to class it with some other objects and differentiate it from yet other objects.

Everything would seem completely different or completely the same. We could not claim any individual subjects to think or talk about, nor any abstractions to predicate of them. We would have no particular propositions to generalize, and all the more no general propositions. Nothing could be related in any way, positively or negatively, to anything else. There could be no analogy and no disanalogy. Abstract theories could not be formed or tested, since nothing could be claimed repetitive and distinguishable. In short, any attempt to deny human ability to cognize similarities and differences, and thence deductive and inductive processes, is a self-defeating proposition.

Induction in a fortiori logic. Now, let us illustrate the above four types of induction with reference to a fortiori argument. Consider an a fortiori argument, say the positive subjectal mood:

> P is more R than Q,
> and Q is R enough to be S;
> therefore, P is R enough to be S.

Put it this form, the argument takes for granted that the premises are true, so that the truth of the conclusion follows necessarily. However, in practice, more often than not, there is some degree of uncertainty in the premises, which is transmitted to the conclusion. Thus, insofar as the conclusion has some material uncertainty, it may be said to be somewhat 'inductive'. But this induction is quite significant and reliable insofar as the deductive process above described is certain, note well. The following is a general statement of the corresponding inductive argument:

> P is probably (x%) more R than Q,
> and Q is probably (y%) R enough to be S;
> therefore, P is probably (z%) R enough to be S.

The validity of this argument as induction is entirely based on the validity of the deductive argument it is modeled on. It is not an arbitrary construct designed to give an illusion of inference – it is inference. The probability of the conclusion (z%) is presumably a product of the probabilities of the premises (x% and y%), which may each range from > 0% to 100%; thus, we can say: $z/100 = x/100 * y/100$. When x% and y% both equal 100%, so does z. If x and/or y = 0% (i.e. if a premise is false), z = 0% (which means not that the conclusion is proved false, but that it cannot be determined from these premises).

We may be able to pinpoint more precisely where the uncertainties in the premises and conclusion lie, by looking at the components that make up each proposition. Thus, as we have seen, the major premise "P is more R than Q" is composed of three propositions, to wit: P is Rp, Q is Rq, and Rp > Rq. If P and Q are individuals, we may be assuming by generalization that they are *all* the time respectively Rp and Rq; or that the R value of P (Rp) is *always* greater than the R value of Q (Rq). If P and Q are classes, we may have generalized their stated relation from some instances to *all* instances. Similarly with regard to the minor premise: as we have seen, it is composed of four propositions, viz. Q is Rq, Whatever is at least Rs is S, Whatever is not at least Rs is not S, and Rq ≥ Rs. Here again, generalization may be involved in the claim that Q is Rq, whether Q is an individual or a class; furthermore, the two clauses relating Rs to S positively and negatively, and the clause comparing Rq and Rs all assume generalities.

Thus, beneath the surface discourse, we very probably rely on a number of inductions, which may have proceeded directly by generalization from experience, or indirectly by deduction from such generalities, or via adductive reasoning, or by means of induction based on deduction, or even by means of induction *not* based on deduction.

We can very well illustrate 'induction not based on deduction' with reference to a fortiori argument as follows. Instead of the above positive subjectal forms, someone might propose the following arguments as a deductive a fortiori (on the left) and the corresponding inductive a fortiori (on the right):

> P is more R than Q,   P is probably more R than Q,
> and Q is S;   and Q is probably S;
> therefore, P is S.   therefore, P is probably S.

Notice that the clause "R enough to be" is missing in both the minor premise and conclusion of these two arguments. This admittedly often occurs in practice, although the missing clause may usually be taken as tacitly intended. If it is not tacitly intended, but omitted out of ignorance of its being a requisite for validity, or as a sophistic attempt to mislead, the argument is inevitably deductively invalid – that is to say, there is no way for us to formally prove that the putative conclusion follows from the given premises. The process might superficially seem somewhat credible, because its minor premise and conclusion *partly resemble* the minor premise and conclusion of valid a fortiori argument. But this is of course an illusion; in fact, the conclusion is not implied by the given premises.

If the alleged deductive underpinning of the inductive argument is invalid, then the inductive argument is of course all the more so. Nevertheless, some people look upon such arguments as inchoate a fortiori, attempts at argument which might eventually crystalize into a fortiori form, at least inductive and then maybe even deductive. But we must keep in mind that the term 'inductive' here has its most disreputable sense. It does not refer to any actual inference, but only to an imaginary 'possible' inference.

The excuse often given in such circumstances is that it is 'material' or 'informal' inference. But what does this mean? The suggestion is that we are able to intuit logical relations *ad hoc*, without reference to logical principles. I agree we are able to do so; indeed, I believe all general logical principles are derived from particular logical intuitions. However, most appeals to 'material implication' are in fact admissions of the speaker's inability to elucidate the 'formal implication' underlying it – i.e. they are excuses for ignorance or laziness. For someone else, more skilled in logic and informed regarding the subject-matter, may very often be able to state the formal sources of the material intuition – or alternatively to demonstrate it to be invalid.

While a fortiori argument is often best characterized as inductive rather than deductive, for the above stated reasons, a crescendo argument is *very* often best so characterized, because of the additional problems involved in affirming proportionality, and all the more so in affirming some quantitatively specific proportionality. As we have seen, an a crescendo argument consists of an a fortiori argument combined with a pro rata argument, the two together yielding a 'proportional' conclusion. For instance, the positive subjectal form is:

> P is more R than Q (is R),
> and Q is R enough to be S,
> and S varies in proportion to R;
> therefore, P is R enough to be more than S.

As can be seen, the third premise, viz. "S varies in proportion to R" is rather vague, and can only justify the vague conclusion that P is not merely R enough to be S, but even R enough to *more than* S. Furthermore, the conclusion is at best probable, since the additional premise about proportionality is, of course, normally known by induction, and so is only probably true (say, to degree g%) rather than absolutely certain. Thus, here our formula for calculating the probability of the conclusion would be $z/100 = x/100 * y/100 * g/100$, where symbols x, y and z have the meanings already above assigned them.

The inductive status of a crescendo argument needs to be further emphasized when the additional premise is not merely that "S varies in proportion to R," but more specifically that S varies in proportion to R "in accord with such and such formula, say $S = f(R)$," when the formula concerned is natural (rather than conventional). The formula allows us to calculate a precise value for Sp (i.e. S in relation to P) given a precise value for Sq (S in relation to Q). Such a formula is of course conceivable, and we are often able to produce one. But of course, such a formula is not easy to establish with certainty (unless merely conventional, as happens – e.g. a price list for products of different sizes).

Very often, scientific experiments are necessary. We extrapolate from a number of correspondences between the values of the variables concerned. We propose a summary formula that allows us to predict untested values from the pattern suggested by past events. Thus, the formula constitutes a scientific hypothesis, which is relied on until and unless we come across some serious hitch in its application. Alternatively, the formula might be deduced from more general principles, which have earned our respect over time. But even then, it is still inductive, insofar as the general principles it is based on are themselves products of induction. Thus, in any event, the mathematical formula referred to must be admitted to be probable rather than certain.

Thus, we must remain aware that a crescendo argument is even more likely, than purely a fortiori argument, to deserve to be viewed as inductive. Needless to say, the same can be said for all forms of a crescendo argument, besides the example given above. Nevertheless, in practice, many a crescendo arguments may be viewed as effectively deductive, because the proportionality claimed seems pretty obvious and straightforward – all the more so if the putative conclusion is vague rather than precise. Their validity is so probable that we may take it for granted.

But very often, in everyday discourse, we come across a crescendo claims that are nowhere substantiated, and even that would be hard to substantiate if we tried. It is best to say in such cases that the speaker is proposing an inductive a crescendo argument, meaning that he is not so much deducing the conclusion by a crescendo from established premises but proposing his conclusion in the framework of a possible a crescendo development. That is, he is not so much inferring the putative conclusion as proposing it, although his proposition is not entirely arbitrary but comes embedded within a larger context.

4. Antithetical items

a.   We saw earlier that arguments with ***antithetical middle items***, such as: "If Q, which is not R, is S, then, all the more, P, which is R, is S," or "If S is P, though P requires R, then all the more S is Q, which does not require R," or their negative equivalents, or their implicational equivalents, all of which often occur in practice[11], can readily be assimilated into standard forms by viewing their middle items, R and notR, as inhabiting a common continuum, which is labeled R (or notR, as convenient). In this perspective, the absolute terms R and notR are both seamlessly included in a more expansive relative term R (or notR), and a standard form of a fortiori argument (or similarly, of a crescendo argument) can be formulated instead of the more awkward (i.e. difficult to formally validate) formula commonly given.

In this way, the standard forms are brought to bear to validate the said non-standard forms, which (to repeat) are often used in common discourse. However, it should be made clear that the standard forms we thus construct are based on the information provided in the said non-standard forms. That is to say, it is thanks to the information in the latter that we can put together the major premise of the former, which makes possible inference of the conclusion from the minor premise. In subjectal argument, given that P is R and Q is not R, we can say obviously that P (for which $R > 0$, i.e. a positive quantity of R) is more R than Q (for which $R \leq 0$, i.e. a zero or negative quantity of R) is. Again, in predicatal argument, given that R is required to be P and R is not required to be Q, we can say obviously that More R is required to be P (for which $R > 0$) than to be Q (for which $R \leq 0$).

Thus, there are forms of a fortiori argument (and similarly, a crescendo argument) which explicitly or implicitly involve a middle item R with contradictory or contrary values in relation to the major and minor terms. Such arguments can be formally validated through standardization, by replacing the absolute items R and notR with an all-encompassing but relative middle item R (or notR, as convenient), which ranges in value from any negative or zero absolute value (= notR) to any positive absolute value (= R). Note well, the validity of such arguments is not affected by the occurrence of antithetical middle items.

Let us now pursue this theme of antithetical terms or theses further, and investigate its application to major and minor items and/or to the subsidiary items.

b.   A fortiori argument (whether pure or a crescendo) is sometimes developed in relation to ***antithetical major and minor items***. Antithetical means contrary or contradictory. In copulative reasoning, the positive subjectal form of such argument is as follows:

> X (P) is more R than not-X (Q),
> and not-X (Q) is R enough to be S;
> therefore, X (P) is R enough to be S.

Clearly, X and not-X are here both R, with different values of R, the value of the first being superior to that of the second. We can obviously construct a similar negative subjectal argument. We can likewise formulate a positive predicatal argument, as follows:

> More R is required to be X (P) than to be not-X (Q),
> and S is R enough to be X (P);
> therefore, S is R enough to be not-X (Q).

And we can obviously construct a similar negative predicatal argument. Needless to say, in all these argument forms, X and not-X could just as well switch roles and be Q and P, respectively. The fact that the major and minor terms (whether X and not-X, or not-X and X) are contradictory does not affect the validity of the argument, since it has

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[11]   See for example Mishna Menahoth 8:5: "Just as the Menorah which is not to do with eating, requires 'pure olive oil', so meal offerings, which are to do with eating, is it not an inference that they should require 'pure olive oil'?"

standard form. It is just a special application of a fortiori, occasionally found in practice[12]. Similar comments can be made with respect to implicational reasoning.

The important thing to note here is that the *validity* of the standard forms of argument is not affected by their involving antithetical major and minor items, for the simple reason that in each mood the major premise (presumably given, explicitly or implicitly) formally allows us to draw the conclusion from the minor premise (and third premise, if any). We do not have to prove anything new in relation to the antitheses – all the information we need is already given in the premises.

c.   A crescendo argument (unlike purely a fortiori argument) sometimes concerns ***antithetical subsidiary items***. Antithetical means contrary or contradictory. In copulative reasoning, the positive subjectal form of such argument is as follows:

> P is more R than Q,
> and Q is R enough to be not-Y (S1),
> and S varies in proportion to R;
> therefore, P is R enough to be Y (S2).

Clearly, not-Y and Y are here two different values within a common continuum called S, one being labeled S1 and the other S2. Assuming direct proportionality of S to R, then S1 < S2, since the argument is from minor to major. Thus, the negative not-Y is the lesser value S1, corresponding to S ≤ 0, and the positive Y is the greater value S2, i.e. S > 0. In cases of inverse proportionality of S to R, the positive Y would be S1 and the negative not-Y would be S2. We could of course switch the roles of Y and not-Y (i.e. reason from Y to not-Y while S is directly proportional to R, or reason from not-Y to Y while S is inversely proportional to R) provided the meanings of these terms were such that S1 < S2. I have chosen the above order of presentation as more natural, taking Y as something more positively S and not-Y as something more negatively S. But these are just symbols, and the reverse order may be more accurate in some cases.

We can obviously construct similar negative subjectal arguments. We can likewise formulate a positive predicatal argument, as follows:

> More R is required to be P than to be Q,
> and Y (S1) is R enough to be P,
> and R varies in proportion to S;
> therefore, not-Y (S2) is R enough to be Q.

Here again, Y and not-Y are two different values within the continuum S, one being labeled S1 and the other S2. Assuming direct proportionality of R to S, then S1 > S2, since the argument is from major to minor. Thus, the positive Y is the greater value S1, i.e. S > 0. the negative not-Y is the lesser value S2, corresponding to S = 0. In cases of inverse proportionality of R to S, the negative not-Y would be S1 and the positive Y would be S2. The reverse order of Y and not-Y is here again conceivable, needless to say. And we can obviously construct similar negative predicatal arguments. Similar comments can be made with respect to implicational reasoning.

Now, the big question here is: are these arguments formally *valid*? That is to say, does the putative conclusion of each follow from the given premises? The answer is obviously: no – although it may be that with additional information these conclusions might well be logically justified. The answer is obviously 'no', because the third premise, which tells us about the proportionality of S to R or of R to S, only allows us a vague conclusion, namely that the subsidiary term in the conclusion is *more than* the subsidiary term in the minor premise, in the case of minor-to-major arguments (namely, the positive subjectal and negative predicatal moods), and that the subsidiary term in the conclusion is *less than* the subsidiary term in the minor premise, in the case of major-to-minor arguments (namely, the positive predicatal and negative subjectal moods).

If the given subsidiary term is the negative not-Y and the conclusion is supposed to be more than not-Y, it does not necessarily follow that the concluding subsidiary term is the positive Y. For if in a given case not-Y happens to refer to a specific negative value in the continuum S (i.e. S < 0) which happens to be well below zero, then 'more than not-Y' cannot be assumed to indeed refer to Y (i.e. S > 0), for it might conceivably still refer to another specific negative value in the continuum S (i.e. S ≤ 0), which is closer to zero but still not above zero. Likewise, if the given subsidiary term is the positive Y and the conclusion is supposed to be less than Y, it does not necessarily follow that the

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[12]   See for example Mishna Nedarim 10:7. "If he [a husband] can cancel vows [by his wife] which have already had [for a time, before he cancelled them] the force of a 'prohibition' [as any vow of his wife that he cancels], can he not also cancel vows which have not yet the force of a 'prohibition'?" This is positive predicatal.

concluding subsidiary term is the negative not-Y. For if in a given case Y happens to refer to a specific positive value in the continuum S (i.e. S > 0) which happens to be well above zero, then 'less than Y' cannot be assumed to indeed refer to not-Y (i.e. S ≤ 0), for it might conceivably still refer to another specific positive value in the continuum S (i.e. S > 0), which is closer to zero but still above zero.

It is only when Y and not-Y do not admit of degrees, i.e. when things are either precisely Y or precisely not-Y (S being binary, either = 1 or = 0), than we can take 'more than not-Y' to mean 'Y' and 'less than Y' to mean 'not-Y'. This happens, but is not a general truth. Therefore, though the moods above described with antithetical subsidiary terms are in exceptional cases indeed valid, they are not universally valid, so we must be careful when we reason in this way that we do not rush to judgment, but ponder the matter carefully.

To clarify this further: if we say that Q or P "is R enough to be" Y (or not-Y), do we mean *every* value of Y (or not-Y), or *some* value of Y (or not-Y)? Obviously, the latter. Therefore, even if our explicit reference to Y (or not-Y) is generic, it is tacitly specific – and we cannot tell from such a vague statement what the precise specification is. The only exception to this rule would be when we know for a fact that 'Y' has only one possible value (say, 1) and 'not-Y' has only one possible value (say, 0) – i.e. when they form a binary pair.

In cases where 'Y' and 'not-Y' refer to ranges of values of S, i.e. to S > 0 and S ≤ 0 respectively, we would need a precise mathematical formula, viz. S = f(R) or R = f(S), as the case may be, to draw the desired conclusion (i.e. one with an antithetical subsidiary term). That is, to know that 'more than not-Y' here means 'Y' we must be able to calculate *by how many degrees* of S the inferred 'more than not-Y' is more than the given 'not-Y'. Or, to know that 'less than Y' here means 'not-Y' we must be able to calculate *by how many degrees* of S the inferred 'less than Y' is less than the given 'Y'.

Note that the problem here is not specific to antithetical terms, but applies to any specific terms. Given only a vague premise about proportionality, i.e. "S varies in proportion to R" or "R varies in proportion to S" (as the case may be), we can only come to a vague conclusion. Even if the minor premise contains a very specific subsidiary term, the a crescendo conclusion cannot produce an equally specific subsidiary term, but only a term that is vaguely 'more than' or 'less than' the one given in the premise. An antithetical subsidiary term being a specific term, this rule applies equally to it.

Of course, if we can amplify our premise about proportionality with a formula that allows us to precisely calculate the concluding subsidiary term from the given subsidiary term, the problem dissolves and we can produce an exact conclusion – which means, in some cases, an antithetical subsidiary term. Though such a formula is conceivable and often available in scientific discourse, it is rarely available in everyday discourse. In the latter, we usually have no practical means of producing such a formula, and must rely more on intuitive understanding to decide whether the putative conclusion, involving an antithetical subsidiary term, seems reasonable or not. Thus, what shall we declare at the last? Shall we say such arguments are valid or invalid? I would suggest we declare them in principle *invalid,* while admitting that they are in practice often accepted as reasonable[13].

It is clear that to draw the desired conclusions we strictly need more precise information than the vague proportionality given in the above premises. However, being human, we function in practice in a more permissive mode, and grant many inferences on the basis of mere intuitions of proportionality. In such cases, if the inference of a positive from a negative or the inference of a negative from a positive seems reasonable to everyone, we might accept the argument as close enough to valid. Obviously, conclusions based on such relatively lenient standards are inductive rather than deductive. They are probable rather than certain, accepted as true until and unless some objection to them be found.

The logician's role is not to inhibit human knowledge if it does not meet strict ideal standards, but to help practitioners approach these standards as much as possible in each given context. We have to remain lucid at all times, and be aware of our limitations – but this should not stop us from proceeding apace with our pursuit of knowledge, which is a necessity for us as living beings. On this basis, while I have above demonstrated that deduction of an antithetical subsidiary term on the basis of a vague premise of proportionality is strictly speaking invalid, I would recommend that we in practice, in most cases, adopt a lenient attitude towards such inference.

Its result must, of course, be recognized as inductive rather than deductive. If the argument is based on public information, it is in principle subject to revision when and if new information arises. However, very often we have no means of scientific verification, so there is no likelihood that we shall change our mind that way. In such cases, we rely on a primary insight. A new such insight might conceivably arise in a wider context of knowledge and replace the former. In that event, of course, we would naturally revise our a crescendo argument. Thus, even an argument

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[13]   A possible illustration is 2 Samuel 12:18. "Behold: while the child was yet alive, we spoke unto him, and he hearkened not unto our voice; then how shall we tell him that the child is dead, so that he do himself some harm?" The relevant antithesis here is between 'being distracted' and 'harming himself'.

based on mere insight can in principle be reviewed. So it is fair to call such arguments inductive, and view them as more than just speculative pronouncements.

d.   Let us now consider a crescendo arguments (not purely a fortiori arguments, note) which involve both antithetical major and minor items and antithetical subsidiary items, in tandem. Such arguments may be characterized as **a contrario** a crescendo arguments, because the polarities of both the terms in minor premise are inverted in the conclusion, i.e. the subject and predicate in the former are negated in the latter.

Such arguments do occur in common discourse, although very rarely. For instance, R. Hananiah ben Gamaliel says, in Mishna Makkoth 3:15: "If in one transgression a transgressor forfeits his soul, how much more should one who performs one precept have his soul granted him!" Here, 'transgression' is replaced by 'performing precepts' and 'forfeiting of one's soul' is replaced by 'being granted one's soul'. If vice causes death, then virtue causes life. Sound reasonable. But is it? Does the first sentence logically *imply* the second?

The question posed is whether such arguments are valid. Consider for instances the following most typical forms (the first positive subjectal and the second positive predicatal):

> X (P) is more R than not-X (Q),
> and not-X (Q) is R enough to be not-Y (S1),
> and S varies in proportion to R;
> therefore, X (P) is R enough to be Y (S2).

> More R is required to be (X) P than to be not-X (Q),
> and Y (S1) is R enough to be X (P),
> and R varies in proportion to S;
> therefore, not-Y (S2) is R enough to be not-X (Q).

What distinguishes such arguments is that they compound the feature of antithetical major and minor terms and that of antithetical subsidiary terms. The expression *a contrario* is appropriately used in relation to such a crescendo arguments, because the movement of thought involved in them is obviously that of inversion. In our two samples, this means:

> If not-X, then not-Y; therefore, if X, then Y.
> If Y, then X; therefore, if not-Y, then not-X.

However, we know that inversion is not a valid process of eduction (i.e. immediate inference), even if many people wrongly imagine it to be. That is to say, the proposition 'if not-X, then not-Y' does *not* formally imply the proposition 'if X, then Y'; these propositions are admittedly sometimes true together, but just as often one is true and the other false. This can be proven very precisely[14]. Similarly, of course, for the other pair, 'if Y, then X' and 'if not-Y, then not-X'.

Admittedly, someone who formulates an argument like those above described is not relying on mere immediate inference, but on a complex a crescendo argument. Well, we have already in (b) above examined arguments with antithetical major and minor terms and found them formally valid, since the major premise guarantees the process. We have also in (c) above examined arguments with antithetical subsidiary terms and found them in principle invalid without more precise information regarding the proportionality, although often in practice accepted as effectively valid merely on the basis of intuitive understanding.

On the basis of these earlier findings, we should be able to readily determine the validity or invalidity of the more complicated a contrario forms. As regards the minor and major terms, they present no problem of validity here again, since the major premise explicitly refers to them. As regards the subsidiary terms, the movement of thought is strictly invalid if we lack a precise formula, although reasonable enough in most cases commonly encountered. However, we cannot resolve the issue of a contrario argument simply by this extrapolation, because here the problem of antithetical items is compounded.

That is, in a contrario a crescendo argument we are dealing with *two* changes of polarity in tandem, from negatives to positives (in our first sample, from not-X to X as well as from not-Y to Y), or from positives to negatives (in our second sample, from Y to not-Y as well as from X to not-X). Thus, here the difficulty is much greater: we need a formula that can express the parallelism between the X and Y values, not merely in a general way, but in a way so precise that it can pinpoint for us that the positives (X and Y) will occur together and the negatives (not-X and not-Y) will occur together[15].

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[14]   See my work *The Logic of Causation* in this regard.
[15]   Or similarly in cases of inverse proportionality, that X and not-Y and not-X and Y will be paired-off.

This is surgical precision that cannot simply be assumed offhand. Of course, it may happen that the terms behave in such an orderly manner, in lockstep, as tied couples. But how would we express the underlying determinism in a further premise? Perhaps in a statement like "Change from not-X to X and change from not-Y to Y are linked" (for the first mood) or "Change from Y to not-Y and change from X to not-X are linked" (for the second mood). But how would we know this? Presumably by observing the concomitant variation of the two terms. But then, if we already know that X is coupled with Y or that not-Y is coupled with not-X, what need have we of the proposed a crescendo argument? Does its conclusion teach us anything more?

Conversely, if we do feel the a crescendo argument to be useful, is it not because we do not have the said information on coupling, and seek to obtain it by deduction? I submit that the only way we could express the premise about coupling would be to adduce the conclusion! That is to say, the needed information is so precise that it cannot be specified indirectly. It can only be stated. But if it is stated, this is tantamount to granting the putative conclusion as a premise. We would be *begging the question*, engaging in circular argument.

I admit I could be wrong in this reasoning, but I think it is best to adopt it, until if ever someone proposes some credible additional premise that would formally guarantee the conclusion without repeating it. In sum, I prefer as a precaution to declare the proposed argument *invalid*. This does not mean that, in the above described a contrario arguments, the minor premise and conclusion cannot be true together, but it does mean that we cannot deduce the latter from the former, even granting the said major premise and third premise. There is a non-sequitur, but no antinomy.[16]

Thus, argument with antithetical minor and major items *and* antithetical subsidiary items is best avoided and regarded as rhetorical. Such argument appears reasonable to some people because they see it as mere inversion; but as we have seen, mere inversion is not valid inference, anyway. All the more is it not valid in the more complicated context of a crescendo argument.

5. Traductions

In the course of interpreting numerous Mishnaic and Talmudic a fortiori arguments, I noticed that there were often two or more ways a given argument could be interpreted to the same effect. I therefore resolved to try and justify such correspondences in formal terms, to facilitate such reinterpretation in the future. I have called the establishment of such rewriting of arguments in alternate forms – 'traductions' (which in French means 'translations'). Traduction should not be identified with reduction, the validation of arguments, since both the source and target arguments here are already known to be valid forms.

The purpose of traduction is more material – it is simply deriving from an argument, of one form, another argument, of a different form; that is, it is merely a different verbal expression of the same thought. The value of studying such changes of wording is that it helps one find the interpretation of a given material argument that is formally closest to the given speech or text, rather than imposing a more common form on it. In this way one demonstrates full understanding of the original movement of thought.

More specifically, the utility of traduction is that it allows us to pass from a positive to a negative form, or vice versa; or from a 'minor to major' to a 'major to minor' form, or vice versa; or from a subjectal (or antecedental) to a predicatal (or consequental) form, or vice versa – ideally, without loss of information. Correlations between copulative and implicational forms (some of which were dealt with earlier) can also of course be used for purposes of traduction. As we shall see, some traductions are of logical significance, others are more akin to linguistic manipulations.

We shall deal with copulative arguments, and assume offhand that the same can be done with implicational ones (the reader is invited to verify this, as an exercise). Regarding the former, knowing there are four moods of primary copulative a fortiori argument, we shall need to investigate the following possible traductions: the uniformly subjectal or predicatal, and mixtures of subjectal and predicatal. That is, from +s to –s, and vice versa; from +p to –p, and vice versa; from +p to +s and –s, and from –p to +s and –s; and finally from +s to +p and –p, and from –s to +p and –p.

Let us first consider the **uniform traductions**. The first type is the **purely subjectal** one: (1a) **from +s to –s**:

Positive subjectal (minor to major)	Negative subjectal (major to minor)
Given that P is more R1 than Q is, it follows that:	Given that Q is more R2 than P is, it follows that

[16] The best way to demonstrate the truth of my contention that a contrario a crescendo argument is not necessarily valid and is therefore invalid would be to produce an example where the premises are obviously correct and the conclusion is obviously wrong. For the time being, I have not managed to do that; but admittedly I have not tried very hard.

| if Q is R1 enough to be S, | if Q is R2 **not** enough to be **not**-S, |
| then P is R1 enough to be S. | then P is R2 not enough to be not-S. |

This is a logical process and one not hard to prove. The two major premises imply each other, given that their middle terms R1, R2 are relative. The minor premises likewise imply each other because they both imply that Q is S, and both the ranges R1 and R2 are fully inclusive, though in opposite directions. Similarly, the conclusions imply each other, and that P is S. Thus, albeit apparent differences in middle terms and in the polarities of their minor premises and conclusions and their subsidiary terms, these two a fortiori arguments are formally equivalent.

Compare for example the following two arguments: "given P is longer than Q, and Q is long enough to be S, then P is long enough to be S" and "given Q is shorter than P, and Q is not short enough to be not-S, then P is not short enough to be not-S." Clearly, if Q is not short enough to be not-S, then it must be long enough to be S; and if Q is long enough to be S, then it cannot be short enough to be not-S; and the conclusions follow.

Note well that both arguments are subjectal, and one is positive and goes from minor to major and the other is negative and goes from major to minor. Keep in mind, also, the special case where R1 is some concept R, and R2 its antithesis not-R; this sometimes occurs. Needless to say, if we substituted not-S for S in the first argument, and put S instead of not-S in the second, we would have another pair of equivalent arguments.

A likewise easily proved corollary of the above traduction is (1b) **from −s to +s**:

Negative subjectal (major to minor)	Positive subjectal (minor to major)
Given that P is more R1 than Q is, it follows that:	Given that Q is more R2 than P is, it follows that
if P is R1 **not** enough to be S,	if P is R2 enough to be **not**-S,
then Q is R1 not enough to be S.	then Q is R2 enough to be not-S.

The second sort of uniform traduction is the **purely predicatal** one: (2a) **from +p to −p**:

Positive predicatal (major to minor)	Negative predicatal (minor to major)
Given that more R1 is required to be P than to be Q, it follows that:	Given that more R2 is required to be Q than to be P, it follows that:
if S is R1 enough to be P,	if S is R2 **not** enough to be **not**-P,
then S is R1 enough to be Q.	then S is R2 not enough to be not-Q.

This is also a logical process and one easy enough to prove. The two major premises imply each other, given that their middle terms R1, R2 are relative. The minor premises likewise imply each other because they both imply that S is P, and both the ranges R1 and R2 are fully inclusive, though in opposite directions. Similarly, the conclusions imply each other, and that S is Q. Thus, albeit apparent differences in middle terms and in the polarities of their minor premises and conclusions and their major and minor terms, these two a fortiori arguments are formally equivalent.

Compare for example the following two arguments: "given more strength is required to be P than to be Q, and S is strong enough to be P, then S is strong enough to be Q" and "given more weakness is required to be Q than to be P, and S is weak not enough to be not-P, then S is weak not enough to be not-Q." Clearly, if S is not weak enough to be not-P, then it must be strong enough to be P; and if S is strong enough to be P, then it cannot be weak enough to be not-P; and the conclusions follow.

Note well that both arguments are predicatal, and one is positive and goes from major to minor and the other is negative and goes from minor to major. Needless to say, we could equally well have made the argument about R1 negative and that about R2 positive[17]. Keep in mind, also, the special case where R1 is some concept R, and R2 its antithesis not-R; this sometimes occurs.

A likewise easily proved corollary of the above traduction is (2b) **from −p to +p**:

Negative predicatal (minor to major)	Positive predicatal (major to minor)
Given that more R1 is required to be P than to be Q, it follows that:	Given that more R2 is required to be Q than to be P, it follows that:
if S is R1 **not** enough to be Q,	if S is R2 enough to be **not**-Q,
then S is R1 not enough to be P.	then S is R2 enough to be not-P.

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[17] But we cannot, of course, just change the polarities of the major and/or minor term(s) in the minor premise and/or conclusion without adjusting it/them in the major premise too. So this case differs from the preceding (subjectal argument), where we could just switch the polarity of the subsidiary term in the minor premises and conclusions without worrying about the major premise.

More complex are the **mixed traductions**, aimed at correlation of predicatal and subjectal forms of a fortiori argument. It is not easy, if not impossible, to effect such changes of form, because in subjectal arguments the minor premises and conclusions have P or Q as subjects and S as predicate, whereas in predicatal ones they have S as subject and P or Q as predicates, and we cannot simply convert one form to the other. However, such changes of form can be effected in a more convoluted manner, by constructing new items from the given elements of the argument.

Some such processes are of logical interest. But in many cases the change is rather artificial, in the sense that the underlying logical form of the argument is not really changed but only superficially made to appear to have been changed; that is, though the explicit wording looks different, the implicit thought is unchanged. Although such processing is thus a bit make believe from a logical point of view, it is still useful in that we can by this means verbally reproduce someone's actual thought process.[18]

The processes that go **from predicatal arguments to subjectal ones** are based on *a fusion of the given middle term R with the relational concept of its being (or not being) required* for some result, yielding either the new positive middle term "demanding of R" or the negative relative term "undemanding of R" as appropriate. We have in all four such traductions to consider.

(3a and 3b) **From +p to –s and +s**: starting with the following positive predicatal (major to minor) argument:

> Given that more R is required to be P than to be Q, it follows that:
> if S is R enough (for S) to be P,
> then S is R enough (for S) to be Q.

…we can construct the following two subjectal ones, whose equivalence is evidenced by their having the same net implications, viz. that S is P and Q:

Negative subjectal (major to minor)	Positive subjectal (minor to major)
Given that P is more [**demanding of** R] than Q is, it follows that:	Given that Q is more [**undemanding of** R] than P is, it follows that:
if P is [demanding of R] **not** enough to **prevent** [S (from being P)],	if P is [undemanding of R] enough **for** [S (to be P)],
then Q is [demanding of R] not enough to prevent [S (from being Q)].	then Q is [undemanding of R] enough for [S (to be Q)].

(3c and 3d) **From –p to +s and –s**: similarly, starting with the following negative predicatal (minor to major) argument:

> Given that more R is required to be P than to be Q, it follows that:
> if S is R **not** enough (for S) to be Q,
> then S is R not enough (for S) to be P.

…we can construct the following two subjectal ones, whose equivalence is evidenced by their having the same net implications, viz. that S is not-Q and not-P:

Positive subjectal (minor to major)	Negative subjectal (major to minor)
Given that P is more [**demanding of** R] than Q is, it follows that:	Given that Q is more [**undemanding of** R] than P is, it follows that:
if Q is [demanding of R] enough to **prevent** [S (from being Q)],	if Q is [undemanding of R] **not** enough **for** [S (to be Q)],
then P is [demanding of R] enough to prevent [S (from being P)].	then P is [undemanding of R] not enough for [S (to be P)].

Note that the move from +p to –s feels more natural than that from +p to +s, because in the former case, even though the minor premise and conclusion change polarity, the polarity of the middle term and the movement from major to

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[18]   These are the best solutions I have found so far to the problems posed, at any rate. It is not excluded that other, even perhaps better, traductions be found in the future, by me or others.

minor remain the same. For the same reasons, the move from –p to +s feels more natural than that from –p to –s. This is why I have listed the traductions in that order.

The reason I have characterized such traductions as artificial, i.e. as more verbal than logical, is partly of course due to their having recourse to a new middle term, viz. "demanding of R" (or "undemanding of R"), which has *a formal element* hidden in it, viz. the fact of requirement (or its lack) that is in fact indicative of predicatal argument. But there is a second, ultimately more important reason: it is that the subsidiary item is *not really the same* in minor premise and conclusion. Although the original subsidiary term S is still present and stands in the foreground at an explicit level in the derived arguments, at a more implicit level we have to specify its being the subject of the subject of the proposition. That is to say, though we say "for S" or "to prevent S," by the term "S" here we really mean the proposition "S is P" or "S is Q" (as appropriate).

As we all know by now, an a fortiori argument is formally invalid if the subsidiary item is not exactly the same in minor premise and conclusion. So we must regard such traductions as being, strictly speaking, misleading. But as is evident the derived arguments do carry some conviction! Why so? The reason they do so is that they are *only apparently* subjectal, at the surface level of their wording. In fact they are, at a deeper level, as regards their logical form, still very much predicatal, since the idea of requirement is inherent in their middle terms, viz. "demanding of R" (or "undemanding of R").

We could perhaps remedy the said fault, of the derived arguments having tacitly unequal subsidiary terms, by resorting to a more *abstract* subsidiary item. That is to say, instead of specifically saying (or even thinking) "S is P" or "S is Q" – we would state more vaguely "the subject (of the whole suffective proposition concerned, i.e. P or Q as the case may be) is predicated of S." This would indeed considerably reinforce the subjectal appearance of the derived arguments. But I maintain that these arguments would not be fully understood and believed if we did not have in mind the underlying predicatal discourse.

Consider, for instance, the first of the listed four traductions, viz. "from +p to –s." Note that both the arguments involved are from major to minor, and that both imply that S is P and Q. The middle term of the first is R, but the middle term of the second is "demanding of R." The major and minor terms P and Q remain the same, but the given subsidiary term S is amplified by a P or Q predicate (as appropriate) in the derived argument (though this may not be stated out loud, and only tacitly intended). Clearly, although the first argument is predicatal and positive, while the second is subjectal and negative, they tell us exactly the same thing. The latter argument is just as predicatal in essence as the former; such traduction is just a change of wording. But, to repeat, it is still useful sometimes.

For example, consider the positive predicatal argument: "given that more money (R) is needed to buy a car (P) than to buy a bicycle (Q), it follows that if $1000 (S) is enough money (R) for a car (P), then it (S) is enough for a bicycle (Q)." This can be restated in the following negative subjectal form: "given that a car purchase (P) calls for more funds (new R) than a bicycle purchase (Q) does, it follows that if a car purchase (P) calls for funds (new R) not large enough that $1000 cannot effect *it* (S, *in relation to P*), then a bicycle purchase calls for funds not large enough that $1000 cannot effect *it* (S, *in relation to Q*)." We can similarly explain and exemplify the other three traductions; the reader should perhaps do that as an exercise.

The processes that go **from subjectal arguments to predicatal ones** are based on making a very abstract subsidiary term, "the subject concerned," out of the subjects, P and Q; and fabricating two new major and minor items, "P is S" and "Q is S" (or their negations), out of the original major, minor and subsidiary terms. The middle term used here is R (or any relative of it, such as its negation[19]). We have in all four such traductions to consider.

From +s to –p and +p: starting with the following <u>positive subjectal</u> (minor to major) argument:

> Given that P is more R than Q is, it follows that:
> if Q is R enough (for Q) to be S,
> then P is R enough (for P) to be S.

…we can construct the following two predicatal ones, whose equivalence is evidenced by their having the same net implications, viz. that Q and P are S:

Negative predicatal (minor to major)	Positive predicatal (major to minor)
Given that more R is required for [P to be S] than for [Q to be S], it follows that:	Given that more **not**-R is required for [Q to be S] than for [P to be S], it follows that:
if [the subject concerned (i.e. Q)] is R **not** enough for	if [the subject concerned (i.e. Q)] is **not**-R enough for

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19   I take the pair R and its negation not-R here, but we could equally take two relatives R1 and R2.

[Q **not** to be S],	[Q to be S],
then [the subject concerned (i.e. P)] is R not enough for [P not to be S].	then [the subject concerned (i.e. P)] is not-R enough for [P to be S].

From –s to +p and –p: starting with the following <u>negative subjectal</u> (major to minor) argument:

> Given that P is more R than Q is, it follows that:
> if P is R **not** enough (for P) to be S,
> then Q is R not enough (for Q) to be S.

…we can construct the following two predicatal ones, whose equivalence is evidenced by their having the same net implications, viz. that P and Q are not-S:

<u>Positive predicatal</u> (major to minor)	<u>Negative predicatal</u> (minor to major)
Given that more **not**-R is required for [P **not** to be S] than for [Q not to be S], it follows that:	Given that more **not**-R is required for [Q to be S] than for [P to be S], it follows that:
if [the subject concerned (i.e. P)] is **not**-R enough for [P **not** to be S],	if [the subject concerned (i.e. P)] is **not**-R **not** enough for [P to be S],
then [the subject concerned (i.e. Q)] is not-R enough for [Q not to be S].	then [the subject concerned (i.e. Q)] is not-R not enough for [Q to be S].

Note that the move from +s to –p feels more natural than that from +s to +p, because in the former case, even though the minor premise and conclusion change polarity, the polarity of the middle term and the movement from major to minor remain the same. For the same reasons, the move from –s to +p feels more natural than that from –s to –p. This is why I have listed the traductions in that order.

We can strongly criticize this set of four traductions as we did the preceding set, and even more so. The major premises of the derived arguments are credible enough, merely regrouping information already present in the original arguments. The problem lies rather in the new minor premises and conclusions. What is problematic in the latter is not the middle term R (or its relative, not-R) or the predicated items, viz. "P is S" and "Q is S" (or their negations), which are clearly intended in the given arguments – the problem lies in the new subsidiary term.

This very abstract term, viz. "the subject concerned," is intended as a single substitute for the two original subjects, P and Q. We need a single term in this role, because an a fortiori argument cannot have more than one subsidiary. If there are two subsidiaries, the argument becomes invalid. So a vague term is introduced, "the subject concerned," which tacitly refers to a term (P or Q) which is present elsewhere in the same suffective proposition, namely within a new predicated item, viz. "P is S" and "Q is S" (or their negations), as the case may be.

This verbal artifice allows us to make predicatal arguments out of subjectal ones. But of course, it is a bit of a sleight of hand, because we cannot really understand the abstract term without mentally referring to the P or Q it stands for. Therefore, the tacitly intended P or Q remains the effective subject of the proposition concerned, even if we have hidden it away. So we must admit either that the argument is fallacious (having two subsidiary terms) or that it is not what it seems. That is to say, in the latter case, though we have reformulated the given subjectal argument in such a way that it now looks like a predicatal argument, the thought process involved is still really subjectal. Verbally, on the surface of thought, the argument may seem predicatal, but logically, in the depth of thought, it is quite subjectal. Although such rewording is theoretically a dead end, it can still as earlier indicated be useful for the practical purpose of interpretation.

To analyze one of the listed four traductions, consider for instance the first, viz. "from +s to –p." Note that both the arguments involved are from minor to major, and that both imply that P and Q are S. The middle term of the first is R, but the middle term of the second is "R in the subject." The new subsidiary term is the very abstract "there," and the new minor and major items are "Q is not S" and "P is not S," which contain the original minor and major terms (Q and P), respectively, to which the original subsidiary term (S) is negatively predicated. Since the new middle term is *not* enough to entail these negative items, it follows that Q and P are both S.

For example, consider the positive subjectal argument: "given that selling a car (P) generates more income (R) than selling a bicycle (Q) does, it follows that if selling a bicycle (Q) generates income (R) enough to buy a new suit (S), then selling a car (Q) generates income (R) enough to buy a new suit (S)." This can be restated in the following negative subjectal form: "given that more income generation is required for [a car sale to enable a suit purchase], than for [a bicycle sale to do so], it follows that if the subject concerned (here, sale of a bicycle) generated income not enough for [a bicycle sale not to enable purchase of a new suit], then the subject concerned (here, sale of a car)

generated income not enough for [a car sale not to enable purchase of a new suit]." We can similarly explain and exemplify the other three traductions; the reader should perhaps do that as an exercise.

To conclude this topic: uniform traductions (purely subjectal or purely predicatal ones) may be qualified as logical processes, whereas mixed traductions (from predicatal forms to subjectal ones, or vice versa) are rather verbal than truly logical.

Mongrel arguments. In this context, somewhat incidentally, I would like to draw attention to a mistake I have often found myself making when attempting to interpret examples of a fortiori argument in formal terms, i.e. when trying to fit them into some standard form. What happens is that we formulate an argument of mixed form; that is, mixing a subjectal major premise with a minor premise and a conclusion of predicatal form, or mixing a predicatal major premise with a minor premise and a conclusion of subjectal form. This produces arguments like the following positive copulative moods:

> P is more R than Q is,
> and S is R enough to be P;
> therefore, S is R enough to be Q.

> More R is required to be P than to be Q,
> and Q is R enough to be S;
> therefore, P is R enough to be S.

Such mixtures are best described as mongrels. The problem with them is not so much the order of the major and minor terms in the minor premise and conclusion, i.e. whether P is inferred from Q or Q is inferred from P; for the order could be changed. The problem is that the major and minor terms are subjects in the major premise and then predicates in the next two propositions, or predicates in the major premise and then subjects in the next two propositions. This is a problem, because it makes validation of these arguments impossible. Moreover, such arguments feel unnatural and unconvincing.

Note that the major premise of the first argument may also be stated as "More R is involved in being P than in being Q," which gives it a more predicatal air. Similarly, the major premise of the second argument may also be stated as "(to be) P requires more R than (to be) Q does," which gives it a more subjectal air. But such verbal reconstructions do not affect the essence of the matter. The conceptual difference between subjectal and predicatal argument is clear-cut, and the two forms should not be confused or mixed. As well, very often when one does this, the terms, though closely related, are not quite the same in the major premise on the one hand and in the minor premise and conclusion on the other hand. One should always make sure the terms are identical.

4. Apparently variant forms

The four copulative and four implicational moods of a fortiori argument described earlier should be viewed as representative of this form of argument, but obviously not as limiting its precise possible contents. They are theoretical models, by way of which we can test whether cases encountered in practice are 'true to form', i.e. valid, or not so. For in both types we are concerned with a broader range of propositional forms than may appear at first sight. We shall here describe in some detail some of the variations on the two theoretical themes that we may encounter in practice, and then we will enquire as to whether or when mixtures of them are conceivable[1].

1. Variations in form and content

In copulative arguments. I have called the first four moods 'copulative' because they involve categorical relations indicated by the copula 'is' (or 'to be'). But it should be clear that they could equally well involve other categorical relations; also, negative polarity may be involved and non-actual modalities (can, must, and different probabilities in between) of various modes (*de dicto* or various types of *de re*). To give an example: "If this man can run two miles so fast, he can surely run one mile just as fast" (positive predicatal) may be counted as a copulative argument; the effective copula (the relation between terms) here is 'run' and the modality (qualifying the relation) is 'can', the terms being 'this man' and 'a distance of one or two miles in a given lapse of time'. Moreover, past, present or future tenses may be involved, in various combinations, provided the major premise justifies it. For example: "If a man is that strong when old, he was surely as strong or stronger when younger."

The verb typically used to relate subjects and predicates in copulative a fortiori arguments is "is" or "to be." This can be taken very broadly to refer to any classification. But in practice, most verbs can be used here: to have (some quality or entity), to do (some action or go through some process), or whatever, with any object or complement, provided the statement can credibly be recast in the standard form (this process is called permutation[2]). This is true not only of a fortiori argument, but equally of syllogism and other forms of argument; so it requires no special dispensation. For example: "she sings Mozart well" can be recast as "she is a [good Mozart singer]." Sometimes, permutation is formally not possible, or at least not without careful consideration; for instances, the relations of 'becoming' and 'making' (or 'causing') cannot always be permuted.

One or more of the verbs involved may be of negative polarity. Be especially careful when one term is negated and another is posited, for this can confuse. In such cases, i.e. when in doubt, we can ensure that the argument is true to form (i.e. valid) by obverting the predicate(s) concerned. Obversion is permutation of the negation, passing it over from the copula to the predicate. For examples: instead of "is required not to be P" we would read "is required to be nonP;" or instead of "enough not to be S," read "enough to be nonS." The middle term (R) may likewise be negative in form, provided it is consistently so throughout the argument. However, if the major premise is negative, as in "P is not more R than Q" or "More R is not required to be P than to be Q," no such obversion of R is acceptable, although we may be able to convert the comparison involved from "more" to "less" (though in such case check carefully that the minor premise and conclusion are true to form).

Natural, temporal or spatial modality may be introduced in a fortiori argument; i.e. the predications involved may be modal and not merely actual. For example, in the major premise "More R is required to be able to be P than to be able to be Q" (and similarly in the minor premise and conclusion). In such case, I would say that the modality has to be looked on as part of the effective term. In our example, the effective major and minor terms are not really P and Q, but "able to be P" and "able to be Q." That is, here too permutation of sorts is used to verify that the inference is true to form. So the natural modality is operative here rather as in extensional conditional propositions, than as in categoricals[3].

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[1]   This is a question I have not (as I recall) previously asked myself.
[2]   See my *Future Logic*, chapter 18, on this topic.
[3]   That is, although permutation of modalities is not formally permissible, as the modality of a proposition concerns its copula rather than its predicate (i.e. 'can be [X]' cannot always be read as 'is [able to be X]' – to do so leads to errors of inference), we may nevertheless conceivably do so, provided the predication as a whole (i.e. copula plus predicate) is carried over, as occurs in extensional conditioning.

Logical and epistemic modalities, as well as ethical and legal modalities, are considered separately further on.

In implicational arguments. Similarly, though I have called the second set of four moods 'implicational', the relation 'implies' involved in them should not be taken in a limited sense, with reference only to logical implication. For it is obvious that, if the moods are valid for that mode, similar moods can be constructed and validated for other modes of conditioning, such as the extensional or the natural (to name two examples)[4]. Indeed, we can apply them more broadly still to a wide range causal propositions (concerning causation, volition or influence, notably). Thus, all sorts of relational expressions might appear in practice in lieu of 'implies' provided such a link is ultimately subsumed.

In implicational a fortiori argument, the items P, Q, R and S stand for theses instead of terms. It is clear that any categorical relation may be involved in these clauses – whether the copula 'is' or any other, whether positive or negative, whether actual or modal – and indeed ultimately any non-categorical relation. The proviso is that the claimed relations between the clauses and the middle thesis R be indeed applicable (which is not always the case, of course). The logical ('de dicto') relation of implication is the basic bond in such argument, but this may be replaced by any 'de re' relation that suggests it – such as natural, temporal, spatial or extensional modes of conditioning, or more broadly by the same various modes of causation (including logical causation, of course), and more broadly still (though in such cases the underlying bond becomes more tenuous, a probability rather than a certainty of sequence) by volition or influence.

The following is a sample of thoroughly causal a fortiori (positive antecedental). The important thing to realize here is that the a fortiori argument per se has nothing to do with causality. It takes the truth of the premises for granted and merely tells us the conclusion from them, on the basis of given quantitative relations to the middle term or thesis. It is an a fortiori argument, and not a causal argument.

> P causes more R than Q does,
> and, Q causes enough R to cause S;
> therefore, P causes enough R to cause S.

To give an example: "The car's good looks generate more sales than its technical features do; and, its technical features generate enough sales to keep the company afloat; therefore, its good looks generate enough sales to keep the company afloat." Here, the causal relations of generation and maintenance (keeping) replace the logical relation of implication.

Transformations. We may also note in this context that often (though not always) the same a fortiori argument can at will be credibly worded either in copulative form or in implicational form. If intelligently articulated, such transformations do not vitiate the argument. Consider for instance the following argument:

> A being C implies more E *in it* than B being D does,
> and, B being D implies enough E in it to imply it to be F;
> therefore, A being C implies enough E in it to imply it to be F.

Here, we have two subjects A and B (which may be the same subject, in some cases) with four different predicates C, D, E, F, brought together in truly implicational form. Notice that the middle term is "E in it" – i.e. it refers the predicate E to a *corresponding* subject, and not to just-any subject. In other words, it signifies the effective middle thesis to be variously "B is E" or "A is E," as the case may be. The argument can obviously be restated in truly copulative form, as follows:

> AC is more E than a BD is,
> and, BD is E enough to be F;
> therefore, AC is enough E to be F.

The terms AC and BD refer respectively to "A when it is C" and "B when it is D." This is valid transformation, provided the middle item E suggests a thesis in the implicational form (as above clarified) and a term in the copulative form. As we saw earlier, a middle *thesis* per se, being a proposition, cannot vary; so that when we say that more or less of it is implied, we always have in mind *something within it* that varies – usually a term (though not always). So, when we transform the implicational form into a copulative one, we have to identify precisely which

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[4]   Again see *Future Logic* part IV, concerning *de re* modes of conditioning.

content of the middle thesis to use as our middle term. We could similarly, of course, transform the copulative argument into the implicational one, if we proceed carefully.

In practice, it does not matter so much exactly how we word our argument, in implicational or copulative form, provided it ends up matching a valid form. The human brain is very clever and able to assimilate large variations in wording with little difficulty (though it can also, of course, be misled). So, we should not view the division too rigidly. However, such transformations are not always possible: we may have difficulty restructuring the middle item, or at least some information might be lost or might have to be added in the process. So we are justified in regarding copulative and implicational species of a fortiori argument as essentially distinct, even if in some special cases they can be transformed into each other.

2. Logical-epistemic a fortiori

We need to distinguish between purely 'ontical' (or *de re*) a fortiori argument and more 'logical-epistemic' (or *de dicto*) ones. The adjective 'ontical'[5] (from Gk. *ontos*, meaning 'existence') applies to the objects of ontology, the study of being, just as the adjective 'epistemic' (from Gk. *episteme*, meaning 'knowledge') applies to the objects of epistemology, the study of knowing. Ontical thus characterizes the things we allegedly know, whereas epistemic characterizes our alleged knowledge of them. Clearly, these terms are relative, in that something epistemic may be intended ontically inasmuch as it exists too.

A logical-epistemic a fortiori argument is one applying logical and/or epistemic qualifications to some relatively ontical information. A 'logical' qualification logically evaluates the proposed information in a given context of knowledge: it may logically evaluate *a term* as conceivable, significant, clear, precise, well-defined, and so forth, to various degrees, or the same in negative connotation; or it may evaluate a proposition through a modality with degrees[6] like probable, confirmed, evident, consistent, true, or their negations. An 'epistemic' qualification concerns the state of belief, opinion or knowledge of the speaker rather than the content spoken of or its purely logical evaluation; this refers to characterizations like credible, reliable, believable, understandable, to varying degrees, and their negative equivalents.

The distinction can be tested as follows: E.g. for 'credible' when we ask 'to whom?', we can answer 'to this person', or 'to most people', or 'to everyone', signifying that the issue is relatively subjective; whereas for 'probable', we would refer to a more objective issue, such as how often similar subjects have the same predicate.

A logical or epistemic thesis, then, is one which predicates such a logical or epistemic term to an ontical term or thesis. E.g. 'term X is vague', 'thesis X is probable' are logical propositions, 'term X is generally understood', 'thesis X is widely believed' are epistemic propositions. Of course, logical and epistemic propositions are in a sense themselves ontical; but they are always relative to information which is more ontical.

The following are examples of purely copulative logical-epistemic a fortiori argument, the first being subjectal and the second predicatal:

> Term P is 'better defined' (R) than term Q is,
> and, term Q is well defined (R) enough to be 'comprehensible' (S); therefore,
> all the more, term P is well defined (R) enough to be comprehensible (S).

> 'Better definition' (R) is required of a term to 'pinpoint its instances' (P) than to 'be comprehensible' (Q),
> and, term S is well defined (R) enough to pinpoint its instances (P); therefore,
> all the more, term S is well defined (R) enough to be comprehensible (Q).

Note that both samples involve only terms (i.e. they are not hybrid) and both have as their middle term R the logical qualification 'well defined'. In the subjectal example, R characterizes the terms P and Q, whereas in the predicatal example it characterizes the term S. In the subjectal example, the subsidiary term S is 'comprehensible', an epistemic qualification suitably related to R, and its major and minor terms P and Q are ontical (at least, relative to the two other terms). In the predicatal example, the major and minor terms P and Q are logical ('pinpoint its instances') or

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[5]   I personally prefer using the word 'ontal' (which I originally found used in the *Enc. Brit.*); but 'ontic' (used, e.g., by W. Windelband) and 'ontical' (commonly found in the Internet) seem more widely used; so, I have here opted for the latter as a compromise.

[6]   Note well this point – modalities like necessity or possibility do not strictly-speaking have degrees. When we assign them degrees we really refer to high or low probabilities. Necessity refers to maximum (100%) probability, while possibility refers to some unspecified (from >0 to100%) probability.

epistemic ('comprehensible') qualifications suitably related to R, while the subsidiary term S is (at least relatively) ontical.

The following are examples of purely implicational logical-epistemic a fortiori argument, the first being antecedental and the second consequental:

> Thesis P implies more 'correct predictions' (R) than thesis Q is,
> and, Q implies correct predictions (R) enough to imply that 'thesis A is probably true' (S);
> therefore,
> all the more, P implies correct predictions (R) enough to imply that thesis A is probably true (S).

> More 'correct predictions' (R) are required to imply 'thesis A probably true' (P) than to imply 'thesis B probably true' (Q),
> and, thesis S implies correct predictions (R) enough to imply that thesis A is probably true (P);
> therefore,
> all the more, S implies correct predictions (R) enough imply that thesis B is probably true (Q).

Note that both samples involve only theses (i.e. they are not hybrid) and both have as their middle thesis R the logical proposition that 'many of its predictions are correct'[7]. In the antecedental example, R characterizes the theses P and Q, whereas in the consequental example it characterizes the thesis S. In the antecedental example, the subsidiary thesis S is the logical proposition, suitably related to R, that 'thesis A is probably true', while the theses P and Q are (at least relatively) ontical. Here, the probability of Q due to correct prediction is declared in the minor premise high enough to imply A probable; therefore, given the major premise, the same can be concluded with regard to P and A. In the consequental example, the theses P and Q are the logical propositions, suitably related to R, that 'thesis A is probably true' and 'thesis B is probably true', respectively, while the subsidiary thesis S is (at least relatively) ontical. Here, the probability of S due to correct prediction is declared in the minor premise high enough to imply A probable; therefore, given the major premise, the same can be concluded with regard to S and B.

Though all the above examples are positive, we can easily construct similar arguments in negative form. In all of them, the logical-epistemic middle item (R) may be viewed as the basis of the deduction, and the suitably related logical-epistemic subsidiary item (S) or major and minor items (P and Q) may be viewed as the goal of the deduction; the remaining item(s) usually have ontical content, though they may in special cases (when that is what is discussed) be logical or epistemic too.[8]

These four samples make clear that logical-epistemic a fortiori arguments function like purely ontical a fortiori argument; there is nothing special about them, other than the logical-epistemic nature of some of the items involved. Nevertheless, such arguments seem rare; or at least, I find it difficult to formulate many examples of them. The matter gets more complicated when we, further on, look into 'hybrid' a fortiori arguments, which seem to involve mixtures of terms and theses.

3. Ethical-legal a fortiori

In my book *Judaic Logic*[9], I showed that, although the eight moods of a fortiori argument listed earlier are formulated very generically, they can be adapted to ethical or legal a fortiori argumentation. Generally, the middle item R may be any quantitative factor shared in some way by the other three items. In ethical or legal argument, this common thread will be specifically an ethical/legal characterization, or a proposition involving such characterization, by which I mean expressions like desirable, advantageous, useful, valuable, good, moral, ethical, legal, obligatory, demanding, important, stringent, and so on – and their negative versions – all of which, note well, have degrees. Coupled with that, either the subsidiary item or the major and minor items must refer to a physical, mental or spiritual *action or event* related to the ethical-legal qualification; for examples, as something desirable is *sought after*, or something good is *preferred*. The remaining item(s) are not ethical-legal in content.

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[7]   Though we state the middle item more briefly as a term, 'correct predictions', because this term is the essential variable in it, it is better to think of it as a proposition, 'many of its predictions are correct', so as to avoid making the a fortiori argument 'hybrid' in form. Notice also that the propositional form brings out more clearly that the predictions are made by a specific thesis (P or Q in the antecedental case and S in the consequental case).

[8]   Observe that logical-epistemic terms in this context come in 'suitably related' pairs, as e.g. 'better defined' comes with 'more comprehensible'. This does not mean, of course, that each term can only be paired off with one other term – it may well have many possible companions. But it does mean that not just any two such terms may be paired off.

[9]   In chapter 4.5.

A fortiori arguments involving such ethical or legal expressions must be examined and evaluated carefully, because these characterizations are rather vague and complex. One can easily err using them if one does not take pains to clarify just what they are intended to mean in each case. Consider, for instance, the following subjectal argument:

> P is more valuable (R) than Q,
> and, Q is valuable (R) enough to make A imperative (S);
> therefore, all the more, P is valuable (R) enough to make A imperative (S).

This argument can be interpreted and rewritten as follows:
- Major premise means: 'P does more to produce some value R than Q does', which in turn means:
'P produces R to degree R_P', and 'Q produces R to degree R_Q', and
'R_P is greater than R_Q' – whence, 'if R_P then R_Q'.
- Minor premise means: 'Q produces R to degree R_Q', and
'if R_Q then S (= the term 'makes A is imperative')'.
- Conclusion means: 'P produces R to degree R_P' (given), and
'if R_P then S' (since R_P implies R_Q, and R_Q implies S).

Alternatively, it might be read and rendered negatively, as follows:
- Major premise means: 'nonP does more to inhibit some value R than nonQ does', which in turn means:
'nonP inhibits R to degree $nonR_{nonP}$', and 'nonQ inhibits R to degree $nonR_{nonQ}$' , and
'$nonR_{nonP}$ is greater than $nonR_{nonQ}$' – whence, 'if $nonR_{nonP}$ then $nonR_{nonQ}$'.
- Minor premise means: 'nonQ inhibits R to degree $nonR_{nonQ}$', and
'if $nonR_{nonQ}$ then S (= the term 'makes A is imperative')'.
- Conclusion means: 'nonP inhibits R to degree $nonR_{nonP}$' (given), and
'if $nonR_{nonP}$ then S' (since $nonR_P$ implies $nonR_Q$, and $nonR_Q$ implies S).

Sometimes, both these interpretations are intended together. P and Q are two values; and S is some trait or behavior that is being recommended, say. The important factor here is of course the middle term R, which is implicit in the expression 'valuable'. What does it mean to be more or less valuable, or valuable enough? This has to refer to some causal concept – namely, the positive concept of production and/or the negative concept of inhibition. Where did R come from? It is implicit in the concept of value that something is valuable relative to some standard of value – call it R. So 'valuable' means valuable in the pursuit of (say) R.

What does 'makes A is imperative' (S) mean? It means that A is absolutely necessary for some unstated goal – or more probably for the ultimate goal here sought, namely R. However, note well, the necessity of A here referred to does not play any part in the actual a fortiori inference. The subsidiary item here is really not just A but the whole clause S (i.e. 'makes A is imperative'). Another such term like 'makes A allowed' or even 'makes A not imperative' or 'makes A forbidden' could equally well have occurred in that position without affecting the argument as a whole. Clearly, then, the conclusion can be formally inferred from the given premises, so the a fortiori argument as a whole is valid.

Of course, many questions can be asked about how we come to know the premises in the first place. The hierarchy of values P and Q proposed in the major premise has to be justified; and why the minor value Q implies the imperativeness (or whatever) of 'A' is not here explained (but taken for granted at the outset). The scale of values on which P and Q are measured could be a merely subjective scale, or one based on biological considerations, or again one based on spiritual ones. 'A' might for instance be a cause of Q, P and/or R, though need not be. But these issues stand outside the a fortiori reasoning as such. The a fortiori argument as such does not need more information than the said premises give to draw the said conclusion – provided that the message of each premise and of the conclusion are well understood.

Let's look at another sample, for instance the predicatal argument:

> More 'virtue' (R) is required to be (or have or do) P than to be (or have or do) Q,
> and, S is virtuous enough to be (or have or do) P;
> therefore, all the more, S is virtuous enough to be (or have or do) Q.

In this case, S refers to a person supposedly, and P and Q to character traits, or maybe behavior patterns, which require different degrees of 'virtue' (by S) to achieve. Here, the middle term 'virtue' has to be understood in a sufficiently uniform manner that the inference becomes possible. Obviously, if it means something different in each

proposition – say, courage in one and perseverance in another – we cannot logically draw the conclusion from the premises. Here again, then, caution is called for.

Apart from these words of warning, much the same can be said for ethical-legal a fortiori as was said regarding logical-epistemic a fortiori, so I won't repeat myself here.

4. There are no really hybrid forms

I have already shown that my inventory of copulative and implicational a fortiori arguments is in principle exhaustive[10], i.e. that 'hybrid' arguments are formally non-existent even if we often in everyday discourse seem to make use of them. The main reason given was that a standalone term cannot imply or be implied by a whole proposition. Terms can only be subjects or predicates; only theses can be antecedents or consequents.

This is true notwithstanding the fact, which we admitted, that since a thesis as such cannot have degrees like a term, the middle thesis of implicational arguments must be examined carefully, to determine *what it is in it* that is variable (i.e. more, equal or less, or sufficient or insufficient). The variable factor may be a subject or a predicate or a quantity or a modality, or a compound of such elements.

Thus, we can safely say that, formally speaking, there are no hybrid a fortiori argument. There are in principle no partly copulative and partly implicational a fortiori arguments. The four items P, Q, R, S of such arguments are necessarily either all terms (i.e. the main constituents of propositions) or all theses (i.e. propositions of whatever form, constituted by terms). Even if in everyday speech we often give the impression that terms and theses can be mixed indiscriminately, there is always some unspoken intent that explains the illusion. Some commentators have nevertheless tried, wittingly or unwittingly, to propose hybrid forms like the following:

> P is more R than Q is,
> and, Q is R enough to *imply* S;
> therefore, P is R enough to *imply* S.

In the above 'mostly subjectal' example, S seems to be a consequent of Q and P, although they seem to be subjects of predicate R. The solution may be that S is in fact a term, and what is thought of as implied is the thesis 'it (i.e. the subject Q or P, as appropriate) is S'. Alternatively, if S is in fact a thesis, it contains 'it' (which refers to Q or P, as appropriate) as subject and some additional term (here tacit) as predicate.

> More R is required to be P than to be Q,
> and, S *implies* R enough to be P;
> therefore, S *implies* R enough to be Q.

In the above 'mostly predicatal' example, S is both antecedent and subject, since it both implies R and is P and Q. Here, the solution may be that R is in fact a term, and by 'S implies R' is meant simply 'S is R'. Alternatively, if R is in fact a thesis, the thought may be that some proposition of which S is the subject (and whose predicate is here tacit) implies R.

> P implies more R than Q (implies R),
> and, Q implies R enough to *be* S;
> therefore, P implies R enough to *be* S.

In the above 'mostly antecedental' example, P and Q seem to be both antecedents and subjects, since they both imply R and are S. The solution here may be that P, Q and R are indeed theses, and 'to be S' is intended to mean 'to imply it (i.e. the subject, here tacit, of thesis Q or P, as appropriate) to be S'.

> More R is required to imply P than to imply Q,
> and, S *is* R enough to imply P;
> therefore, S *is* R enough to imply Q.

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[10]   This refers to my list of eight primary moods – ignoring here corresponding secondary moods, which may be viewed as mere derivatives of the primary ones.

In the above 'mostly consequental' example, S is both subject and antecedent, since it both is R and implies P and Q. Here, the solution may be that R is in fact a thesis, and by 'S is R' is meant 'S implies R'; or maybe, 'the subject (here tacit) of S has the predicate given (here tacitly) in R'. Alternatively, if R is in fact a term, 'S is R' might signify 'the subject (here tacit) of S is R'.

On the surface, the above four examples may seem conceivable, *because we are dealing in symbols*. But if we examine them more closely we find that appearance misleading. For it is a rule of logic that the same item cannot at once be a term and a thesis, as occurs in all of the above proposed moods. So these hybrids are not valid forms, strictly speaking. In each of them, some intent has been left tacit or some verbal or conceptual confusion occurred in the formulation. Nevertheless, it should be kept in mind that in practice we often do so word our sentences as to give the impression that we are mixing copulative and implicational clauses. This is occasionally confusing, but not always.

Let us analyze some more specific cases where confusion or doubt might occur in practice. These are mostly **logical-epistemic or ethical-legal** arguments that look partly implicational but are in fact wholly copulative. The reason such hybrid-looking arguments arise is that in them a thesis may actually function as (a) a subject-term or (b) a predicate-term.

(a) In the propositions "X is probable" or "X is desirable," where 'X' is a thesis, say 'that A is B', and 'probable' or 'desirable' is a predicate, thesis 'X' may be said to function effectively as a term (a subject), because it is taken as a unitary whole rather than as composed of parts.

For example, consider the a fortiori argument "Given that 'A is B' is more probable than that 'C is D', it follows that if 'C is D' is probable enough to be relied on, then 'A is B' is probable enough to be relied on." We might here think that since 'A is B' and 'C is D' are theses (the major and minor, respectively), the argument is implicational. On the other hand, since 'probable' and 'relied on' are terms (the middle and subsidiary, respectively), the argument seems copulative. The solution is not that the argument is hybrid, but that the major and minor theses are in this context intended as terms – i.e. they are the subjects for which the middle and subsidiary terms are predicates. Thus, the form of the argument is really subjectal, and not antecedental or hybrid.

The following is an example of predicatal form with similar effect. "More satisfaction of inductive criteria (R) is needed to adopt a thesis (P) than to merely conceive it possible (Q); and, thesis S satisfies inductive criteria (R) enough to be adopted (P); therefore, thesis S satisfies inductive criteria (R) enough to be conceivable (Q)." Here, although S is a thesis (say, 'that A is B'), it functions in the present context as a term (a subject), for which R, Q and P are indeed predicates. So, the form of the argument is really predicatal, and not consequental or hybrid.

(b) Again, looking the propositions "X makes Y probable" or "X makes Y desirable," where 'X' is a term, and 'Y' is a term or a thesis, say 'that A is B', and 'probable' or 'desirable' is a predicate, we are tempted to view the relation 'makes' as equivalent to an implication (which it indeed implies) and the combination 'Y is probable' or 'Y is desirable' as an implied thesis, in which case the given proposition as a whole seems to be implicational. However, because X is a subject-term (noun), we have to look upon 'makes' as a mere copula (verb) and upon the thesis made, i.e. 'Y is probable' or 'Y is desirable', as a predicate-term (object).

An example of this would be the following argument: "Term P is more well-defined (R) than term Q; and, term Q is well-defined (R) enough to 'make term or thesis A conceivable or credible' (S); therefore, term P is well-defined (R) enough to 'make term or thesis A conceivable or credible' (S)." This argument might be interpreted as partly copulative (since P, Q, and R are terms) and partly implicational (since S seems to refer to an implication, i.e. a thesis). But in fact it is wholly copulative, because S is a term, i.e. the clause 'makes term or thesis A conceivable or credible' must be taken as a unit and not be cut up. This example is thus subjectal.

A similar predicatal example would be the following: "More precision of definition (R) is required to 'make term or thesis A comprehensible' (P) than to 'make term or thesis B comprehensible' (Q); and, term S is precisely defined (R) enough to make A comprehensible (P); therefore, term S is precisely defined (R) enough to make B comprehensible (Q)." Here, the argument might be interpreted as partly copulative (since S and R are terms) and partly implicational (since P and Q seem to refer to implications, i.e. theses). But in fact it is wholly copulative, because P and Q are terms, i.e. the clauses 'make term or thesis A/B comprehensible' must be taken as units and not be cut up.

All the above examples involve logical-epistemic qualifications. We can similarly construct hybrid-looking arguments with ethical-legal qualifications. E.g. "That 'A be B' (P) is more desirable (R) than that 'C be D' (Q); and, Q is desirable (R) enough to be pursued regularly (S); therefore, P is desirable (R) enough to be pursued regularly (S)."

In conclusion, hybrid a fortiori argument do not really exist: when they do seem to occur, as they often enough do in logical-epistemic or ethical-legal contexts, it is due to some thesis being taken as a whole, i.e. as effectively a term.

5. Probable inferences

Very often in practice, though the given argument somehow seems to be an a fortiori, it is really not one at all. We may upon closer scrutiny decide that it is more precisely a hypothetical syllogism or an apodosis. Very often we are misled by expressions like 'all the more' indicative of a fortiori argument being inappropriately used in other forms of argument. Inversely, an argument may on the surface *not* look like an a fortiori at all, but really be one deeper down. Caution is always called for in interpreting arguments. We have to ask what form the underlying reasoning takes, irrespective of the wording used. In some cases, of course, no reasoning is at all intended; yet some people might assume an a fortiori argument to be intended, because a comparison or a threshold is mentioned. We have to always ask how the speaker intends his statement to be taken.

As just stated, some arguments do not immediately appear to be in standard a fortiori format, although one senses that there is an a fortiori 'flavor' to them. Consider the following arguments: Are these a fortiori in nature or something else? How are they to be validated?

Copulative form (X, Y, Z are terms):
      X more often occurs in Y than in Z; therefore:
      If X is found in Z, it is probably also in Y (positive mood), and
      If X is not found in Y, it is probably also not in Z (negative mood).

Implicational form (X, Y, Z are theses):
      X more often occurs in conjunction with Y than with Z; therefore:
      If X is found in conjunction with Z, it is probably also with Y (positive mood), and
      If X is not found in conjunction with Y, it is probably also not with Z (negative mood).

These closely resemble a fortiori argumentation. There are copulative and implicational forms (four in all), the former involving terms and the latter theses. In each case, the first proposition is the major premise, and the if–then propositions which follow it contain a minor premise (the antecedent) and a conclusion (the consequent). There is a positive and a negative mood, the positive one being minor to major and the negative one major to minor. However, these arguments as they stand are obviously not in standard form. They need to be reformulated to conform.

If such argument is to be viewed as a variant of a fortiori, the middle term has to be "the probability of occurrence," while the subsidiary term has to be "the actuality of occurrence." The major premise, which tells us that "X is in/with Y" occurs more frequently than "X is in/with Z," means that the former is more probable than the latter. The minor premise, which tells us that "X is in/with Z" has occurred, or that "X is in/with Y" has not occurred, refers to the actuality of occurrence or lack of it. And the conclusion *predicts* that "X is in/with Y" has probably also occurred, or respectively that "X is in/with Z" has probably also not occurred, again with reference to the actuality or inactuality of occurrence. We can thus reformulate the arguments as follows to bring out their 'a fortiori' aspect more clearly:

Positive mood (copulative [in] or implicational [with]):
      'X is in/with Y' is more probable than 'X is in/with Z', and
      'X is in/with Z' was probable enough to actually occur (at a certain time);
      therefore: 'X is in/with Y' is probable enough to actually have occurred or to later occur.

Negative mood (copulative [in] or implicational [with]):
      'X is in/with Y' is more probable than 'X is in/with Z', and
      'X is in/with Y' was not probable enough to actually occur (by a certain time);
      therefore: 'X is in/with Z' is not probable enough to actually have occurred or to later occur.

Note the introduction, in this improved formulation, of the crucial notion of sufficiency ("enough") or its absence, in accord with standard a fortiori format. So we can say that we here indeed have a fortiori arguments. The major and minor items P and Q are in this case the *theses* "X is in/with Y" and "X is in/with Z," respectively. The middle and subsidiary items R and S are the *terms* "probably" and "actually occurs." So the a fortiori argument involved, mixing theses and terms, is a hybrid-seeming one (although strictly-speaking it is wholly copulative, the theses in it being taken as terms). It is a logical a fortiori argument, probability and actuality being modalities.

Note well that the prediction of the conclusion should not be taken as a certainty. The argument makes no pretense to yield anything more than a probable conclusion, the degree of probability being that specified – clearly or vaguely – in the major premise. Though presented as a sort of deduction, the argument is essentially inductive. It could well be that the situation in fact, on the ground, is the opposite of what the argument predicts. Nevertheless, if *the only*

information we have at our disposal is that given in the argument, it is reasonable to adopt the conclusion's prediction as our 'best bet'. We have more rational basis for expecting the outcome that the conclusion predicts than we have for expecting the contradictory outcome.

Certainty from mere probability. I would like to draw attention in the present context to the fallacy inherent in certain probabilistic a fortiori arguments, namely those that seem to infer a certainty from a mere probability. The following two arguments, one positive and one negative, illustrate this pitfall:

> Thesis P is more probable (R) than thesis Q,
> and, thesis Q is probable (R) enough to imply thesis S;
> therefore, thesis P is probable (R) enough to imply thesis S.

> Thesis P is more probable (R) than thesis Q,
> and, thesis Q is probable (R) enough to deny thesis S;
> therefore, thesis P is probable (R) enough to deny thesis S.

In these hybrid-looking arguments, the items P, Q and S are theses and R is a logical-epistemic term (it is logical if 'probable' here means 'demonstrably likely to be true', but epistemic if it merely means 'believed by many people'). As we have seen, this apparent mix is not necessarily a problem, because theses may in such contexts be intended as (i.e. effectively function as) terms. However, in these two particular cases, the mix is a problem, because the subsidiary item (S) is definitely implied (or denied, i.e. its negation is implied). Since the implication (or denial) is quite intentional, it cannot be written-off as a badly-worded predication.

At first sight, the proposed argument may seem meaningful and credible; but upon closer scrutiny it is found fallacious. The main reason why it is fallacious is that in logic theory no propositions literally imply others when they (the implying ones) are more or less *probable*. In deductive logic, either a proposition X (Q or P in our example) implies another Y (S or notS, here) or it does not – there is no such thing as X implying Y if X is probable to a sufficient degree, and X not implying Y if X is not probable to that degree. Even in inductive logic, such a concept is unknown – there is only the concept of transmission of probability, i.e. if X implies Y, then increasing the probability of X being true increases that of Y being true.

As for degrees of implication, they are formally conceivable; but given that 'X probably implies Y', it does not follow that 'if X is probable to some high degree it implies Y to be *certain*'. Rather, probable implication is to be treated as a weakened form of implication, meaning that whereas the form 'X fully implies Y' transmits the high probability of X to Y (and in the limiting case, if X is certain, then Y is also certain), the form 'X only probably implies Y' transmits only a fraction of X's probability on to Y (i.e. here, if X is probable, then Y is 'probably probable'). This can be expressed quantitatively: if X implies Y with probability $m\%$, say; and X is itself only probable to degree $n\%$, say; then the resulting probability of Y is only $m\%$ of $n\%$.[11]

It should be added that it makes no difference whether the hybrid-seeming a fortiori argument involves an implication or a denial. It is fallacious either way. The principle of adduction that "no amount of right prediction ever definitely proves a hypothesis, but all it takes is a single wrong prediction to disprove it" has no relevance in the present context. Here, whether S is implied or denied the argument is invalid, because a probability cannot imply a certainty, whether positive or negative.

The lesson these examples teach us is that if we use a logical-epistemic middle term like 'probable', then we must also have a logical-epistemic term like 'reliable' contained in one or more of the other items of the a fortiori argument. Such terms occur together, not by chance but because their meanings have some rational relation (as probability rating is related to reliability). We cannot combine the middle term 'probable' with an assertion of the subsidiary item's implication or denial. There is in fact no logical discourse corresponding to that schema. It is artificial and conceptually faulty, for the reason already stated that a certainty cannot be implied by a mere probability.

6. Correlating ontical and probabilistic forms

Having examined the general forms of ontical a fortiori argument and various cases of more specifically logical-epistemic a fortiori argument, the question arises: can logical-epistemic arguments be constructed from ontical ones and/or vice versa? This question immediately comes to mind when we read Aristotle's descriptions of a fortiori argument, of which the following are some extracts:

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[11]   See my *Future Logic*, chapters 30.1 and 46.2.

Rhetoric, book II, chapter 23:

> "…if a quality does not in fact exist where it is *more* likely to exist, it clearly does not exist where it is *less* likely. Again, … if the less likely thing is true, the more likely thing is true also."

Topics, book II, chapter 10:

> "If one predicate be attributed to two subjects; then supposing it does not belong to the subject to which it is the more likely to belong, neither does it belong where it is less likely to belong; while if it does belong where it is less likely to belong, then it belongs as well where it is more likely."

Here, Aristotle's emphasis is clearly 'epistemological', since he repeatedly uses the word 'likely' as his middle term, yet judging by the examples he there gives the underlying subject-matter is arguably rather 'ontological'. This suggests that there are natural bridges between the ontical and logical-epistemic expressions of a fortiori argument. Let us look into the matter with reference to one of Aristotle's own examples, namely:

> A man is more likely to strike his neighbors than to strike his father:
> if a man strikes his father,
> then he is likely to strike his neighbors too.

This example is clearly intended as logical-epistemic, since it uses the relative *likelihood* of events to achieve its inference. But one senses that underlying it is another, more ontical argument, such as the following (others could of course be suggested):

> More antisocial attitude is required to strike one's father than to strike one's neighbor:
> if a man is antisocial enough to strike his father,
> then he is antisocial enough to strike his neighbor.

Aristotle's logical-epistemic wording does not reveal to us precisely why the concluding event (man striking neighbors) is more likely than the given event (man striking father), whereas my proposed ontical wording attempts to explain these events and their connection through some psychological attribute (being antisocial) of the subject (a man). Aristotle's effective middle term is a *vague, unexplained* 'likelihood' – whereas my ontical middle term (antisocial mentality) is more *specifically informative as to the causes* (different degrees of antisocial mentality) of the events (striking father or neighbor). One finds Aristotle's argument convincing especially because one (consciously or unconsciously) assumes that there are ontical reasons (such as those I propose) behind the probabilities he declares.

Thus, our first question arises: can we always, in formal terms, similarly infer an underlying ontical a fortiori argument from a given logical-epistemic (probabilistic) one? The answer, I would say, is that we cannot formally *infer* one, but we can hope to *construct* one that would seemingly fit the bill, i.e. explain the predicated likelihoods by means of some material property or properties. That is to say, with reference to the following forms, given the one on the left we may, using our knowledge and intelligence to propose an appropriate middle term R, construct the one on the right:

Given probabilistic argument	Constructed ontical a fortiori argument
'S is P' is more likely than 'S is Q':	More R is required to be P than to be Q;
if 'S is P' occurs,	and, S is R enough to be P;
then 'S is Q' is likely to occur too.	therefore, S is R enough to be Q.

This reconstruction seems reasonable, at least where the original middle term is the degree of 'likelihood'. But let us look into it more deeply. The given argument compares the likelihood of two events (theses) 'S is P' and 'S is Q' and tells us that if the more likely one indeed occurs then the less likely one is likely to occur too. Note well: it gives no guarantees as to this outcome, its conclusion being only probable though the minor premise is actual. Our proposed construct introduces a *new* term R that was not given in the original argument. R serves as middle term of our a fortiori, relative to which the predicates P and Q are compared in the major premise. R is a predicate of S. If the magnitude of R in S is large enough, then S is Q; and if its magnitude is even larger, then S is P. Whence, if S is P, it has to be Q.

Note that the conclusion 'S is Q' here is definite – it is not a mere probability as before. However, the proposed construct as a whole certainly cannot be inferred from the given argument. We can only posit our construct in the way of an inductive hypothesis that is hopefully fitting (if we have thought about it sufficiently), but which may turn out upon further experience and reflection to be inadequate (in which case it must be adapted or abandoned). So our

new conclusion is not as sure as it appears. Still, once we have a seemingly credible construct, we can claim it (on inductive, not deductive grounds – to repeat) to be the underlying ontical explanation of the given logical-epistemic argument.

Can such ontical explanation be provided for all logical-epistemic a fortiori arguments, or only for middle terms like 'likelihood'? I would offhand answer yes, arguing that we never use logical-epistemic characterizations entirely without reference to more ontical characteristics. That is, if we ask ourselves why we think a term or thesis deserves logical or epistemic evaluation X, we will argue the point ultimately with reference to some sort of more ontical information. Of course, we may have some such explanation in mind, but be unable to clearly put it in so many words, so this is difficult to prove in every case. Also of course, I am generalizing, since I cannot foresee all cases – so I may be found wrong eventually.

Now, let us turn the initial question around, and ask the reciprocal question: given an ontical a fortiori argument, can we formally derive from it a corresponding logical-epistemic argument (meaning, at least, a probabilistic argument similar to Aristotle's)? And if so, we might additionally ask, is there great utility in doing so – or is valuable information lost in the process?

If we reflect a moment, it is clear that behind my contention that underlying Aristotle's probabilistic argument there must be a more ontical argument that explains it – was the thought that Aristotle was really thinking in terms of the ontical argument even if he only verbalized a probabilistic one. So in fact the mental process we were looking for was in the reverse direction: not from logical-epistemic to ontical, but rather from ontical to logical-epistemic. We were not so much asking what ontical information can be drawn from Aristotle's probabilistic argument (not a lot, as we have just seen), but what ontical argument Aristotle had in mind even as he spoke in probabilistic terminology. We want to retrace his thought process from the pre-verbal ontical thought to its verbal probabilistic expression.

Consider therefore the following pair of arguments, this time the one on the left being a given ontical a fortiori argument and the one on the right a proposed probabilistic construct:

Given ontical a fortiori argument	Constructed probabilistic argument
More R is required to be P than to be Q;	'S is P' is more likely than 'S is Q':
and, S is R enough to be P;	if 'S is P' occurs,
therefore, S is R enough to be Q.	then 'S is Q' is likely to occur too.

If we examine these arguments carefully, we see that the latter cannot be inferred from the former. Of course, all information concerning R is lost in transition. But moreover, we have no basis for believing the major premise that 'S is P' is more likely than 'S is Q'; for, given that 'More R is required to be P than to be Q', it could still be true that 'S is P' is *less* likely (i.e. occurs less frequently) than 'S is Q'. The two minor premises are in agreement that 'S is P'; but, whereas the conclusion of the given a fortiori is definite that 'S is Q', the conclusion of the construct is that 'S is Q' is merely probable. Thus, not only does the proposed construct's major premise not follow from the given major premise, but the conclusion of the construct is less informative and sure than that of the original argument.

So, there is in fact no justification for supposing that an ontical a fortiori argument gives rise to a probabilistic argument as above proposed. The ontical argument does not formally tell us anything about the likelihood of the events it discusses. If such likelihood is asserted in an analogous probabilistic argument, it is *new* information (just as the middle term R was new information, in the opposite direction), which must be separately justified or admitted as a hypothesis to be assessed inductively (e.g. we would have to ask in Aristotle's above example whether it is empirically true that men strike their neighbors more often than they strike their fathers). Moreover, to repeat, the proposed new argument involves loss of information (about R) and has a less certain conclusion (about S being Q).

So, if we suppose that Aristotle really had an ontical argument in mind when he formulated his probabilistic one, we may say that such discourse on his part was inaccurate and wasteful. Conversely, granting that he meant only what he said, we could read more into it provided we realize that such interpretation on our part is not deductive inference but inductive hypothesis. In short, the relation between ontical and logical-epistemic a fortiori arguments can be described as hermeneutical rather than strictly logical. We often in practice do blithely hop from ontical to probabilistic form or vice versa – but we ought to be careful doing so, because formal analysis shows that it is not always licit. In logic, even the word 'likely' means something specific and cannot be used at will.

5. Comparisons and correlations

We need to be very clear about the differences between a fortiori argument and other forms of argument, with which a fortiori argument is often compared and even confused. Many commentators have wrongly characterized a fortiori argument as analogical argument[1], and many more as syllogism, and they need to be corrected once and for all. We shall begin our comparison and correlation of argument forms with regard to argument by analogy, and then deal with syllogism.

1. Analogical argument

Just what form does argument by analogy have, and how does it differ from a fortiori argument? Can either of these forms be reduced to the other?

The forms of analogy. Our first task is to formalize analogical argument and identify the conditions of its validity. Qualitative analogical argument, like pure a fortiori argument, consists of four terms, which we may label P, Q, R, S, and refer to as the major, minor, middle and subsidiary terms as before, although here without implying that the major term is greater in any way than the minor. The argument may then take the following four *copulative* forms:

a.   The **positive subjectal** mood. *Given that subject P is similar to subject Q with respect to predicate R, and that Q is S, it follows that P is S*. We may analyze this argument step by step as follows:

Major premise: P and Q are alike in that both of them have R.

              This implies both 'P is R' and 'Q is R', and is implied by them together.

Minor premise: Q is S.

              The term S may of course be any predicate; although in legalistic reasoning, it is usually a legal predicate, like 'imperative', 'forbidden', 'permitted', or 'exempted'.

Intermediate conclusion and further premise: All R are S.

              This proposition is obtained from the preceding two as follows. Given that Q is S and Q is R, it follows by a substitutive third figure syllogism that there is an R which is S, i.e. that 'some R are S'. This particular conclusion is then *generalized* to 'All R are S', provided of course we have no counter-evidence. If we can, from whatever source, adduce evidence that some R (other than Q) are *not* S, then of course we cannot logically claim that all R are S. Thus, this stage of the argument by analogy is partly deductive and partly inductive.

Final conclusion: P is S.

              This conclusion is derived syllogistically from All R are S and P is R.

If the middle term R is known and specified, the analogy between P and Q will be characterized as 'complex'; if R is unknown, or vaguely known but unspecified, the analogy between P and Q will be characterized as 'simple'. In *complex analogy*, the middle term R is clearly present; but in *simple analogy*, it is tacit. In complex analogy, the similarity between P and Q is indirectly established, being manifestly due to their having some known feature R in common; whereas in simple analogy, the similarity between them is effectively directly intuited, and R is merely some indefinite thing assumed to underlie it, so that in the absence of additional information we are content define it as 'whatever it is that P and Q have in common'.

Needless to say, the above argument would be equally valid going from P to Q. I have here presented it as going from Q to P to facilitate comparison and contrast to a fortiori argument, which topic will be dealt with further on.[2]

Quantification. Let us next consider the issue of quantity of the terms, which is not dealt with in the above prototype.

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[1]   Notably, Islamic commentators (such as al-Ghazali) seem to have tended in this direction, although this tendency has been implicit rather than explicit.

[2]   Note in passing that we could similarly validate an argument with a negative major premise. Given that 'P is *dissimilar* to Q with respect to R' (i.e. say, P is R but Q is not R), then since 'Q is S', there is an S which is not R, whence by generalization No S is R, and this together with P is R implies that 'P is not S'.

In the singular version of this argument, the major premise is 'This P is R and this Q is R', where 'this' refers to two different individuals. The minor premise is 'This Q is S', where 'this Q' refers to the same individual as 'this Q' in the major premise does. From the minor premise and part of the major premise we infer (by syllogism 3/RRI[3]) that there is an R which is S, i.e. that some R are S – and this is generalized to all R are S, assuming (unless or until evidence to the contrary is found) there is no R which is not S. From the generality thus obtained and the rest of the major premise, viz. this P is R, we infer (by syllogism 1/ARR) the conclusion 'This P is S', where 'this P' refers to the same individual as 'this P' in the major premise does.

In the corresponding general version of the argument, the major premise is 'All P are R and all Q are R' and the minor premise is 'All Q are S'. From the minor premise and part of the major premise we infer (by syllogism 3/AAI) that some R are S – and this is generalized to all R are S, assuming (unless or until evidence to the contrary is found) there is no R which is not S. From the generality thus obtained and the rest of the major premise, viz. All P are R, we infer (by syllogism 1/AAA) the conclusion 'All P are S'. Note that the minor premise *must* here be general, because if only some Q are S, i.e. if some Q are not S, then, if all Q are R, it follows that some R are not S (by 3/OAO), and we cannot generalize to all R are S; and if only some Q are R, we have no valid syllogism to infer even that some R are S.

As regards the quantity of P and Q, there is much leeway. It suffices for the major premise to specify only that some Q are R; because, even if some Q are not R, we can still with all Q are S infer that some R are S (3/AII), and proceed with the same generalization and conclusion. Likewise, the major premise may be particular with respect to P, provided the conclusion follows suit; for, even if some P are not R, we can still from some P are R and all R are S conclude with some P are S (1/AII). Needless to say, we can substitute negative terms (e.g. not-S for S) throughout the argument, without affecting its validity.

It is inductive argument. Thus, more briefly put, the said analogical argument has the following form: Given that P and Q are alike in having R, and that Q is S, it follows that P is S. The validation of this argument is given in our above analysis of it. What we see there is that the argument as a whole is *not entirely deductive, but partly inductive*, since the general proposition 'All R are S' that it depends on is obtained by generalization.

Thus, it may well happen that, given the same major premise, we find (empirically or through some other reasoning process) that Q is S but P is not S. This just tells us that the generalization to 'All R are S' was in this case not appropriate – it does not put analogical argument as such in doubt. Such cases might be characterized as 'denials of analogy' or 'non-analogies'. Note also that if 'All R are S' is already given, so that the said generalization is not needed, then the argument as a whole is not analogical, but entirely syllogistic; i.e. it is: All R are S and P is R, therefore P is S. Thus, analogy as such is inherently inductive. And obviously, simple analogy is more inductive than complex, since less is clearly known and sure in the former than in the latter.

It is interesting in passing to relate this argument form to the rabbinical hermeneutic principles. The second rule of R. Ishmael, the principle of *gezerah shavah*, which is based on the terms having some Biblical wording or intent in common, may be said to constitute simple analogy. This is because (evident) same wording, or (assumed) same 'intent' of different wordings, do not provide a sufficiently explicit predicate (R) in common to the subjects compared (P and Q). Words are explicit, but they are incidental to what they verbalize; therefore, the assumption that the Torah intends them as significant enough to justify an inference is open to debate[4].

The same can be said of the twelfth rule of R. Ishmael, which refers to contextual inferences (*meinyano*, *misofo*, and the like): such reasoning is simple analogy. However, the third rule of R. Ishmael, the principle of *binyan av*, falls squarely under the heading of complex analogy. In fact, our above description of complex analogy is an exact description of *binyan av* reasoning. When the rabbis want to extend the scope of a Torah law (S), they show that some new subject (P) has some feature (R) in common with the Torah-given subject (Q), and assuming that this feature is *the reason for* the law (this assumption constitutes a generalization, even if it superficially may seem to be a direct insight), they carry the law over from the given case to the unspecified case.

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[3]   Here, the symbol R refers to a singular affiRmative proposition, as against G for a singular neGative one. I introduced these symbols in my *Future Logic*, but singular syllogism is not something new. The Kneales (p. 67) point out that Aristotle gives an example of syllogism with a singular premise in his *Prior Analytics*, 2: 27. The example they mean is supposedly: "Pittacus is generous, since ambitious men are generous and Pittacus is ambitious" (1/ARR). Actually, there is another example in the same passage, viz.: "wise men [i.e. at least some of them] are good, since Pittacus is not only good but wise" (3/RRI). Note that the reason I did not choose the symbol F for aFfirmative was probably simply to avoid confusion with the symbol F for False. In any case, some symbols were clearly needed for singular propositions, since the traditional symbols A, E, I, O only concern plural propositions.

[4]   In other words, the traditional Judaic belief (or dogma) that names are part of the nature of the things they name, if not their very essence, is – as far as formal logic is concerned – only a theory. There is nothing obvious or axiomatic about it. It is a hypothesis that must remain open to scrutiny and testing like any other. Modern linguistics would deny this hypothesis in view of the demonstrable fact that all languages, including Hebrew, have evolved over time.

Other moods. The above, prototypical mood was positive subjectal. Let us now consider the other possible forms of analogical argument.

b.   The **negative subjectal** mood. *Given that subject P is similar to subject Q with respect to predicate R, and that P is not S, it follows that Q is not S*. This mood follows from the positive mood by reductio ad absurdum: given the major premise, if Q were S, then P would be S; but P is not S is a given; therefore, Q is not S. This argument is of course just as inductive as the one it is derived from; it is not deductive.

c.   The **positive predicatal** mood. *Given that predicate P is similar to predicate Q in relation to subject R, and that S is P, it follows that S is Q*. We may analyze this argument step by step as follows:

Major premise: P and Q are alike in that R has both of them.

This implies both 'R is P' and 'R is Q', and is implied by them together.

Minor premise: S is P.

Intermediate conclusion and further premise: S is R.

This proposition is obtained from the preceding two as follows. Given that R is P, it follows by conversion that there is a P which is R, i.e. that 'some P are R', which is then *generalized* to 'all P are R', provided of course we have no counter-evidence. If we can, from whatever source, adduce evidence that some P are *not* R, then of course we cannot logically claim that all P are R. Next, using this generality, i.e. 'all P are R', coupled with the minor premise 'S is P', we infer through first figure syllogism that 'S is R'. Clearly, here again, this stage of the argument by analogy is partly deductive and partly inductive.

Final conclusion: S is Q.

This conclusion is derived syllogistically from R is Q and S is R.

Note that the generalized proposition here concerns the major and middle terms, whereas in the preceding case it concerned the middle and subsidiary terms. Needless to say, this argument would be equally valid going from Q to P. I have here presented it as going from P to Q to facilitate comparison and contrast to a fortiori argument, which topic will be dealt with further on.

Let us now quantify the argument. In the singular version, the major premise is: this R is both P and Q, and in the general version it is: all R are both P and Q. The accompanying minor premise and conclusion are, in either case: and a certain S is P (or some or all S are P, for that matter); therefore, that S is Q (or some or all S are Q, as the case may be). We could also validate the argument if the major premise is: some R are P and all R are Q; but if only some R are Q, i.e. if some R are not Q, we cannot do so for then the final syllogistic inference would be made impossible[5]. Such argument is clearly inductive, since it relies on generalization. No need for us to further belabor this topic.

d.   The **negative predicatal** mood. *Given that predicate P is similar to predicate Q in relation to subject R, and that S is not Q, it follows that S is not P*. This mood follows from the positive mood by reductio ad absurdum: given the major premise, if S were P, then S would be Q; but S is not Q is a given; therefore, S is not P. This argument is of course just as inductive as the one it is derived from; it is not deductive.

We can similarly develop four *implicational* moods of analogical argument, where P, Q, R, S, symbolize theses instead of terms and they are related through implications rather than through the copula 'is'. The positive antecedental would read: *Given that antecedent P is similar to antecedent Q with respect to consequent R, and that Q implies S, it follows that P implies S*. The negative antecedental would read: *Given the same major premise, and that P does not imply S, it follows that Q does not imply S*. The positive predicatal mood would read: *Given that consequent P is similar to consequent Q in relation to antecedent R, and that S implies P, it follows that S implies Q*. The negative predicatal mood would read: *Given the same major premise, and that S does not imply Q, it follows that S does not imply P*. These are, of course, partly inductive arguments since they involve generalizations. Validation of these four moods should proceed in much the same way as that of the four copulative moods.

Quantitative analogy. Analogy may be qualitative or quantitative. The four (or eight) moods of analogical argument above described are the qualitative. In special cases, given the appropriate additional information, they become quantitative.

a.   The positive subjectal mood in such case would read: Given that subject P is greater than subject Q with respect to predicate R, and that Q is S (Sq), it follows that P is proportionately more S (Sp). Obviously, this reasoning depends on an additional (though often tacit) premise that *the ratio of Sp to Sq is the same as the ratio of P to Q (with respect to R)*.

Very often in practice the ratios are not exactly the same, but only roughly the same. Also, the reference to the ratio of P to Q (with respect to R) should perhaps be more precisely expressed as the ratio of Rp to Rq. Note that this

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[5]   However, if we know that some R are Q, and do *not* know that some R are not Q, we can generalize the positive particular to obtain the 'all R are Q' proposition needed to infer the final conclusion. In that case, the argument as a whole would be doubly inductive, since involving two generalizations.

argument effectively has five terms instead of only four (since term S splits off into two terms, Sp and Sq). Of course, the additional premise about proportionality is usually known by inductive means. It might initially be assumed, and thereafter found to be untrue or open to doubt. In such event, the argument would cease to be quantitative analogy and would revert to being merely qualitative analogy. Thus, quantitative analogy is inherently even more inductive than qualitative analogy.

Note that the argument here is, briefly put: 'just as P > Q, so Sp > Sq'. We can similarly argue 'just as P < Q, so Sp < Sq', or 'just as P = Q, so Sp = Sq'. In other words, positive subjectal quantitative analogy may as well be from the inferior to the superior (as in the initial case), from the superior to the inferior, or from equal to equal; it is not restrictive with regard to direction. In this respect, it differs radically from positive subjectal a fortiori argument, which only allows for inference from the inferior to the superior, or from equal to equal, and excludes inference from the superior to the inferior. All this seems obvious intuitively; having validated the qualitative analogy as already shown, all we have left to validate here is the idea of ratios, and that is a function of mathematics.[6]

Similar comments can be made with regard to the other three copulative moods of quantitative analogy, namely:

b.   The negative subjectal mood: Given that subject P is greater than subject Q with respect to predicate R, and that P is not S (Sp), and that the ratio of Sp to Sq is the same as the ratio of P to Q (with respect to R), it follows that Q is not proportionately less S (Sq).

This mood can be validated by reductio ad absurdum to the positive one. Both the major premise (viz. that P > Q, with respect to R) and the additional premise about proportionality (viz. that Sp:Sq = Rp:Rq) remain unchanged. What has 'changed' is that the minor premise of the negative mood is the denial of the conclusion of the positive mood, and the conclusion of the negative mood is the denial of the minor premise of the positive mood. Note that here instead of 'not more S (Sp)' and 'not S (Sq)', I have put 'not S (Sp)' and 'not less S (Sq)'; this is done only to preserve the normal order of thought – it does not affect the argument as such. Here again, needless to say, though the mood shown is based on P > Q, it can easily be reformulated with P < Q or P = Q; this only affects the conclusion's magnitude (making Sq mean 'more S' or 'equally S' as appropriate).

c.   The positive predicatal mood: given that predicate P is greater than predicate Q in relation to subject R, and that a certain amount of S (Sp) is P, and that the ratio of Sp to Sq is the same as the ratio of P to Q (in relation to R), it follows that a proportionately lesser amount of S (Sq) is Q.

Here, the argument is essentially that 'just as P > Q, so Sp > Sq', i.e. that the amounts of subject S (viz. Sp and Sq) in the minor premise and conclusion differ in accord with the amounts of predicates P and Q (in relation to R). Or maybe we should say that subject R differs in magnitude or degree when its predicate is P (Rp) and when its predicate is Q (Rq), and that subject S differs accordingly (i.e. Sp and Sq differ in the same ratio as Rp to Rq). This is again an inductive argument, and would be equally valid in the forms 'just as P < Q, so Sp < Sq', or 'just as P = Q, so Sp = Sq'.

d.   The negative predicatal mood: given that predicate P is greater than predicate Q in relation to subject R, and that a certain amount of S (Sq) is not Q, and that the ratio of Sp to Sq is the same as the ratio of P to Q (in relation to R), it follows that a proportionately greater amount of S (Sp) is not P.

This mood can be validated by reductio ad absurdum to the positive one. That is, given the same major premise and additional premise about proportionality, we would say: since the lesser amount of S (Sq) is not Q, it must be that the greater amount of S (Sp) is not P. Here again, if the major premise has P < Q or P = Q instead of P > Q, the conclusion follows suit (i.e. Sp < or = instead of > Sq).

We can similarly develop four implicational moods of quantitative analogy. Thus, all eight moods of qualitative analogical argument can be turned into quantitative ones, provided we add additional information attesting to 'proportionality'.

Face-off with a fortiori. Clearly, while qualitative analogy is somewhat comparable to purely a fortiori argument, quantitative analogy is somewhat comparable to a crescendo argument; but they are still far from the same. Let us first compare and contrast *qualitative* analogical argument to *pure* a fortiori argument. For this purpose, let us first focus on the positive subjectal mood, viz.:

P is more R than (or as much R as) Q,
and Q is R enough to be S;
therefore, P is R enough to be S.

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[6]   As regards negation of the major premise, here, we can deal with it very simply as follows. 'P is *not* greater than Q with respect to R' can be restated as 'P is either lesser than or equal to Q with respect to R'; therefore, given that Q is Sq and that Sp:Sq = P:Q (or Rp:Rq), it follows that P is Sp, where Sp < or = Sq. In other words, when the major premise is negative, we resort to two positive quantitative analogies in its stead.

Here, as in analogy, the major premise implies that both P and Q are R, but unlike in analogy, it additionally implies that $Rp \geq Rq$, i.e. that the quantity of R in P is greater than (or equal to) that in Q. Thus, though we can deduce the major premise of analogical argument from that of a fortiori argument, we cannot reconstruct the major premise of a fortiori argument only from that of analogical argument. Similarly, though the minor premise of a fortiori argument implies that Q is S, and therefore implies the minor premise of analogical argument, the reverse is not true. The difference between the two minor premises is that in a fortiori argument there is the element of *sufficiency of R to be* S, which is clearly lacking in argument by analogy. For the same reason, although the conclusion of a fortiori argument implies that of analogy, the latter does not by itself enable us to reconstruct the former.

Moreover, even though each of the propositions (the major and minor premises and the conclusion) involved in a fortiori argument implies the corresponding proposition of analogical argument, this does not mean that an a fortiori argument implies an analogical one. For, the a fortiori argument is deductive, i.e. its conclusion follow necessarily from its two premises; whereas, as we have just shown, the argument by analogy, even in its complex form, is inherently inductive, i.e. it requires a generalization of its minor premise to enable us to draw its conclusion. Therefore, even if both arguments may be said to yield a common conclusion, namely 'P is S', that conclusion has a very different logical status in the one and in the other.

It follows that we can neither reduce a fortiori argument to argument by analogy, since the latter's conclusion does not imply the former's (even though the premises of the former do imply those of the latter), nor can we do the reverse, since the premises of the latter do not imply those of the former (even though the conclusion of the former does imply that of the latter). It does happen that we know enough to form the major premise needed for a fortiori argument, but we do not know enough for its minor premise; or we know enough to form the minor premise needed for a fortiori argument, but we do not know enough for its major premise – in such cases we might have enough information to at least formulate an analogical argument. Thus, sometimes we have more information than we need for an analogy, but not enough for an a fortiori argument – in such cases we can only formulate an analogy.

Therefore, though we can say that a fortiori argument and argument by analogy have some features in common, we must admit that they are logically very distinct forms of argument. This is a formal and undeniable demonstration, once and for all. To repeat: neither argument can be *reduced* to the other. However, every valid a fortiori argument *implies* a corresponding argument by analogy involving less information and certainty. The premises of the latter, as we have just seen, lose the quantitative and/or sufficiency factors involved in the former; and the conclusion of the analogical argument is, as a result, both less informative and less sure (being now inductive instead of deductive). But of course, except for the present theoretical clarification, there is in practice no point in resorting to such implication, since the given a fortiori argument is better in all respects.

As regards the opposite direction, it cannot be said that every analogical argument implies a corresponding a fortiori argument. All we can say is that we can, sometimes, when the facts of the case permit it, construct an a fortiori argument which implies the given analogical argument. This is possible if the latter argument has a middle term (R), or an appropriate middle term can be found for it, which can both be used as a continuum of comparison (which, I think, is always possible in practice, although we *cannot* tell a priori which term is greater than the other) and at the same time serve as the sufficient condition for the subject (Q) to access the predicate (S) in the minor premise (and this is, of course, *not* always possible in practice). Thus, the construction of a corresponding a fortiori argument from a given analogical argument is not a mechanical matter and cannot always be performed. In effect, when it is found possible, it just means that we should in the first place have resorted to the stronger a fortiori argument yet foolishly opted for the weaker analogical argument.

All that we have said here applies equally well, *mutatis mutandis*, to the negative subjectal forms of these arguments, and to positive and negative predicatal forms, and again to the four implicational forms. These jobs are left to interested readers.

As regards comparison and contrast between *quantitative* analogy and *a crescendo* argument, i.e. 'proportional' a fortiori argument, the following need be said. The major premises are the same in both. But the minor premises and conclusions obviously differ, insofar as in quantitative analogy there is no idea of a threshold value of the middle term as there is in a fortiori argument. This explains why the 'proportionality' is essentially non-directional in quantitative analogical argument (inference is always possible both from minor to major and from major to minor); whereas it is clearly directional in a fortiori argument (inference is only possible from minor to major in positive subjectal and negative predicatal argument, and from major to minor in negative subjectal and positive predicatal argument).

Note in passing that although quantitative analogy and mere pro rata argument (i.e. used alone, outside of a crescendo argument) are not formally identical the two are effectively the same. Compare for example the following two formulas; clearly, the provisos in them are essentially the same (a concomitant variation between the values of S and the values of R) even if the terms are differently laid out.

Given that P is greater than Q with respect to R, and that Q is S (Sq), it follows that P is proportionately more S (Sp), *provided that* the ratio of Sp to Sq is the same as the ratio of P to Q (quantitative analogy).

Given that if R has value Rq then S has value Sq, it follows that if R has value more than Rq (say Rp), then S has value more than Sq (say Sp), *provided that* the values of S vary in proportion to the values of R (pro rata argument).

To conclude: there is, to be sure, an element of 'analogy' in all human thinking, including in syllogism and in a fortiori argument, since all abstraction is based on mental acts of comparison and contrast; but to say this loosely is not the same as equating syllogism or a fortiori argument to argument by analogy. When we look into the exact forms of these arguments, we clearly see their significant differences.

2. Is a fortiori argument syllogism?

The relationship(s) between a fortiori argument and syllogism have been a subject of debate for a long time, with some logicians and commentators equating the two or at least assimilating one to the other, and others denying such correlations. Ignoring for now the historical narrative, let us first focus on the formal issues involved and develop an independent judgment on them. I considered them very briefly in my *Judaic Logic*[7], saying:

> "It could be said that there is an a-fortiori movement of thought inherent in syllogism, inasmuch as we pass from a larger quantity (all) to a lesser (some). But in syllogism, the transition is made possible by means of the relatively incidental extension of the middle term, whereas, as we have seen, in a-fortiori proper, it is the range of values inherent to the middle term which make it possible."

Let us here look more deeply into the matter, without prejudice. First, it is well to realize that there are variant versions of the thesis of identification between these forms of argument. The most extreme position is of course that syllogism and a fortiori argument are one and the same thing. At the other extreme, all comparison and correlation between the two forms of reasoning might be rejected. But most logicians and commentators, myself included, adopt an median stance.

It should be made clear at the outset that we are here using the word 'syllogism' in its strict, Aristotelian sense, which is etymologically composed of 'syn' = together and 'logos' = thought, and which refers to an argument involving three items disposed in two premises and a conclusion in certain ways. We are not by this word referring more loosely (as some people do) to any form of argument in which an item serves as intermediary, i.e. to 'mediate inference'. The latter, more generic expression is equally applicable, for instance, to apodosis (i.e. *modus ponens* or *modus tollens*), and obviously equally to a fortiori argument.

Quite often, those who try to explain a fortiori argument do so by suggesting that a fortiori is a sort of syllogistic reasoning. As a fortiori has been less studied than syllogism, it is less widely known and understood; so it is natural for people to try to refer it to a more familiar form of reasoning. The way they proceed to do this, however, is (funnily enough) not very logical, since instead of showing that a fortiori argument is or can be reduced to syllogism, they do the opposite – they usually try to show that syllogism can be formulated as a fortiori argument.

Usually, this is done by means of an example (usually, a syllogism in 'Barbara' format, 1/AAA). Often, they try to buttress the demonstration by using the words 'a fortiori' instead of 'therefore' to introduce the syllogistic conclusion, effectively saying: since 'a fortiori' means 'perforce', and the conclusion of a syllogism follows perforce from its premises, syllogism is comparable to a fortiori argument; this is obviously silly. Rather, it seems to me, what they need to try and do is recast a fortiori argument into syllogistic form. But this is of course more difficult, as it requires a prior clear awareness of the formalities of a fortiori argument, which most people lack.

What are the proponents of the identification thesis claiming, exactly? Some seem to think that all a fortiori argument is syllogism and all syllogistic argument is a fortiori argument. Others seem only to claim that all syllogistic argument is a fortiori. Still others seem to claim the reverse, i.e. that all a fortiori argument is syllogistic. Yet others seem to regard that there are some convergences between the two forms of argument, without going so far as to claim that all cases of the one fall under the other or vice versa. They may speak of possible reduction or analogy, instead of outright equation.

An important issue here is what exactly is meant by equation between two forms distinct enough to be called two. It may be that these forms are logically equivalent, irrespective of superficial verbal differences; i.e. they are poetically different ways to express the exact same logical thought. Alternatively, perhaps, one form can be fully transformed

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[7]   In a footnote to chapter 4.

into or wholly reduced to the other, but not vice versa, so that the latter is logically prior to the former; or such transformation or reduction can occur in both directions. This may occur with or without loss of information, i.e. reversibly or not. Or again, the two forms may share some characteristics, i.e. be analogous in some respects, but differ sufficiently to require separate logical treatment.

That is to say, there is a big difference between saying that a fortiori argument 'is' or 'is a special case of' syllogism and saying that it 'can be expressed as' or 'is reducible to' syllogism; or vice versa; or both. Thus, these are different sorts and degrees of equation between two kinds of reasoning. To give a familiar example, even though all second and third figure syllogisms can be directly or indirectly reduced to first figure syllogisms, the former remain distinct forms of reasoning and significant in their own right. On the other hand, all cognoscenti agree, many fourth figure syllogisms have no real existence apart from first figure syllogisms, being distinguished only by the order of presentation of the premises and the order of terms in the conclusion.

Before proposing a theory of the precise correlation between the two forms of argument, let us first clarify what we mean by correlation in more formal terms and endow ourselves with the required vocabulary.

3.   Correlating arguments

Let us investigate the possible relations between any two arguments, whatever their forms. An argument consists of one or more premise(s), say p, and a conclusion, say q; if the argument is valid, then p implies q. The given premise(s) p may of course yield more than one conclusion, i.e. q need not be their only conclusion; however, if there are other conclusions, they together with the same premise(s) constitute and may be regarded as other arguments. We will symbolize the two arguments being correlated as p1q1 and p2q2. In all cases, to repeat, p1 implies q1 and p2 implies q2. (I recommend that the reader draw flow charts to illustrate what is described below.)

The two arguments may be said to be 'implicants' if their premises mutually imply each other or are identical, i.e. if p1 implies p2 and p2 implies p1; for it follows that p1 also has the conclusion q2, and that p2 also has the conclusion q1. In such case, either the two conclusions, q1 and q2, imply each other, or only one implies the other, or neither implies the other. If q1 and q2 do not mutually imply each other, then though the arguments are implicants they obviously remain logically distinct by virtue of this difference.

- If neither of q1, q2 implies the other, we have two effectively 'independent' arguments with identical or logically similar premises (i.e. premises that imply each other). Neither argument can be logically reduced to the other.
- If q1 implies q2 (but not vice versa), then the argument p2q2 may be said to be a 'subaltern' of p1q1; if the reverse is the case, the result is of course reversed. A subaltern argument is reducible to, i.e. can be validated by, the argument it is subaltern to. For example, the syllogism 1/AAI is a subaltern mood of 1/AAA, since the premises are the same and the conclusion I is a subaltern of the conclusion A.
- If q1 and q2 imply each other, we may call the two arguments 'intertwined'. Each is technically reducible to the other. For example, 1/AII and 3/AII are intertwined, being identical except that the minor premise of each is the converse of that of the other. If p1 is formally identical to p2 and q1 to q2, the two arguments can be characterized as 'same' or 'identical'. But if any premise and/or the conclusion is not formally identical in the two arguments, we may not thus fully equate them, even though they are indeed logically closely related. For in such case not only is there some formal difference between them, but that difference *may* together with some *other* common premise(s) result in some difference in conclusion.

Now, consider cases where p1 implies p2 but p2 does not imply p1. We may in such cases say that the argument p1q1 'implies' p2q2, but not vice versa. We can infer (via p2) that p1 also has the conclusion q2, but we cannot likewise infer (via p1) that p2 has the conclusion q1 (although that may happen in some cases). As regards the relationships between the conclusions, here again either they imply each other, or only one implies the other, or neither implies the other.

- If neither of q1, q2 implies the other, the two arguments are clearly independent, even though their premises are somewhat logically related. Neither argument is reducible to the other.
- If q2 implies q1 (but not vice versa), then the argument p1q1 may be said to be a subaltern of p2q2, because q1 may be viewed as following from p1 through the intermediary of p2 and q2, i.e. p1q1 is directly reducible to p2q2. For example, the syllogism 3/AAI is a subaltern mood of 3/AII or likewise of 3/IAI.
- If q1 implies q2 (but not vice versa), then the argument p2q2 may be said to be a 'corollary' of p1q1. Note well: this does not mean that p2q2 is a subaltern of p1q1 or vice versa; nor can the relation between the arguments be characterized as independent. We cannot here claim to logically reduce either argument to the other. We will encounter many examples of this relationship in the present context.

- If q1 and q2 imply each other, then the argument p1q1 is a subaltern of p2q2, whereas p2q2 is a corollary of p1q1. The relation between the arguments is not symmetrical, note well, because p1 implies p2 but p2 does not imply p1.

If p2 implies p1 but p1 does not imply p2, the same can be said of their possible relationships in reverse. If neither of p1 and p2 implies the other, they are 'unrelated' arguments; of course, they have to be compatible to occur together in a given context or body of knowledge. So much, then, for all the possible correlations between a pair of arguments, we now have a typology and terminology to work with. Note especially the applications of the term '*corollary*' here introduced.

4. Structural comparisons

Let us next compare and contrast the structures of syllogism and a fortiori argument. Consider the following typical samples:

a) A categorical syllogism	A copulative a fortiori argument
All Y are Z, and	P is more R than Q is,
All X are Y,	and, Q is R enough to be S;
therefore: All X are Z.	therefore, all the more, P is R enough to be S.

(Here, the items X, Y, Z and P, Q, R, S are terms, note.)

b) A hypothetical syllogism	An implicational a fortiori argument
Y implies Z,	P implies more R than Q does,
X implies Y,	and, Q implies enough R to imply S;
therefore: X implies Z	therefore, all the more, P implies enough R to imply S.

(Here, the items X, Y, Z and P, Q, R, S are theses, note.)

Syllogism refers to inference from one term or thesis (X) to a second (Z) via a third (Y). The third item, known as the middle item, is the intermediary through which the inference is made; and it is found in the two premises, but not in the conclusion. The other two items are found one in each premise and both in the conclusion; they are traditionally called the minor and major item, because valid positive moods of the first figure serve to include a narrower item in a larger (or equal) one – but this initial scenario does not apply to all valid moods (it does not apply to negative moods of the first figure or to moods of the second and third figures), so the words major and minor should not in all cases be taken literally, they are conventional labels. Do not be misled, either, by my adoption of similar terminology for a fortiori argument: the words here differ in meaning.

A fortiori argument also involves three propositions. But the first, called the major premise, *differs significantly in form* from the second, called the minor premise, and from the conclusion, while the latter two propositions are of the same form. Moreover, these three propositions involve, not just three terms or theses, but *four* of them. The middle item (R), here, is present (implicitly if not explicitly) in the conclusion as well as in the two premises; it is labeled 'middle' because it interrelates the three other items. The major and minor items (P and Q), here, are so named because they refer respectively to a larger and smaller quantity of the middle item; in the limiting cases of equality between them, their quantitative relations become interchangeable, of course. These two items are both present, together with the middle item, in the major premise.

Note well that the minor premise and conclusion do not always contain the same items (as they do in syllogism). In 'major to minor' moods the minor premise contains the major item and the conclusion contains the minor item, whereas in 'minor to major' moods the minor premise contains the minor item and the conclusion contains the major item. The middle item is, to repeat, found in both propositions. Finally, we here have a fourth item (S), called the subsidiary item, which is present in the minor premise and conclusion, but absent in the major premise.

Syllogism occurs through the inclusion or exclusion between the three classes or theses concerned; whereas a fortiori compares the measures or degrees of the quality or qualification signified by middle item in its relation to the three other items. Both forms of argument, it is true, involve quantity – but they do so with different emphasis and effect.

The quantities (like 'all' or 'some') involved in categorical syllogism are applicable to the classes concerned as conceptual groupings and not to the individual members they subsume; similar quantifications are involved in hypothetical syllogism with reference to the conditions underlying the theses. The quantities (like 'more' or 'enough') involved in copulative a fortiori argument relate to a common property of the individual instances of its other terms; or, in implicational a fortiori, of the conditions underlying its other theses. The focus of syllogism is not primarily on quantity, but on essentially qualitative information; in categorical syllogism, it relates to classification;

while in hypothetical syllogism, it relates to sequencing. The focus of a fortiori argument is primarily on quantity – it is quantitative ordering of thoughts, entities, qualities or events.

As just explained, the two forms of argument are structurally very different. As I will presently demonstrate, we can – though sometimes in a rather forced manner – derive a fortiori arguments from syllogisms, and vice versa. Nevertheless, these two species of reasoning cannot be considered the same, or one as a subspecies of the other, because, though some of the original meaning is retained when we attempt to transform syllogism into a fortiori argument or vice versa, some important information is lost.

5. From syllogism to a fortiori argument

Let us now investigate whether syllogism can be reworded as a fortiori argument (yes, it can) and whether such rewording entails any loss of information (yes, it does). We shall begin with a detailed analysis relative to the most typical valid syllogisms, namely the positive first figure moods 1/AAA and 1/AII of categorical syllogism (and their singular equivalent), and later extend our consideration to all other forms. The following translation of such syllogism into a fortiori is proposed. The major premise of the latter is derived from that of the former; the minor premise of the former becomes that of the latter; and the original conclusion is reworded as shown.

a) Categorical syllogism (given)	A fortiori argument (derived)
All Y are Z, and	The class Z is bigger than (or as big as) the class Y, and
All (or some) X are Y,	The class Y is big enough to include all (or certain) members of the class X,
therefore:	therefore: The class Z is big enough to include all (or certain)
All (or some) X are Z.	members of the class X.

We could view this transition from ordinary (Aristotelian) syllogism to a fortiori argument as made through the intermediary of a class-logic syllogism. That is, the major premise "All Y are Z" is first interpreted as "The class of Y is entirely subsumed by the class of Z," the minor premise likewise as "The class of X is entirely (or partly) subsumed by the class of Y," and the conclusion as "The class of X is entirely (or partly) subsumed by the class of Z." Then these three propositions are used to produce the three of the resultant a fortiori argument. This is said in passing.

The resulting a fortiori argument, note, is positive subjectal (minor to major). An example often given to illustrate and justify this operation is: "All men (Y) are mortal (Z) and Socrates (X) is a man, therefore Socrates is mortal" becomes "Since the class of mortals is more extensive than that of men (which is included in it), it follows that if the latter includes Socrates as a member, the former is bound to include him as a member as well." (People don't normally think like that, admittedly, but logicians have to sometimes!) But as we shall presently explain, this example is not fully accurate and so a bit misleading.

Note that, in this instance (though not always, as we shall later see), the major term (Z) of the syllogism gives rise to the major term (the class Z) of the a fortiori argument. However, the middle term (Y) of the syllogism gives rise to the minor term (the class Y) of the a fortiori, and the minor term of the syllogism (X) gives rise to the subsidiary term (the class X) of the a fortiori. Thus, do not confuse the appellations of the terms in the two arguments. The middle term of the a fortiori argument is "bigness;" although it is not explicitly given in the syllogism, it is read out of it. Here, "bigness" refers to *the extension* of each class concerned, i.e. *the number of members it includes*. We could as well have used the more technical term extensiveness.

Now, the major premise of the a fortiori argument tells us that the term Z is more extensive than the term Y (or only as extensive as Y, if it so happens that All Z are Y is also true). It does *not* and cannot tell us whether or not Z *includes* Y, note well. The minor premise and conclusion of the a fortiori argument likewise only tell us that the terms Y and Z respectively are more extensive than (or at least as extensive as) the term X. They do *not* and cannot tell us that Y and (therefore) Z *actually include* X; they only affirm in the infinitive that they are big enough *to* do so, i.e. their inclusion is a logical possibility but not an inferred certainty.

That is to say, the two premises of the derived a fortiori argument do not fully reproduce the information in the given syllogism – they only focus on the relative extensions implied by them, *but do not carry over the information about precise inclusions*. Consequently, the conclusion of the a fortiori – which likewise contains no actual (only 'potential') information about inclusions – cannot be used to infer the conclusion of the syllogism. This means that the given syllogism logically implies the proposed a fortiori argument – but the latter does not logically imply the former! So the derived a fortiori argument is a *corollary* of the given syllogism.

This correlation becomes more evident if we realize that the a fortiori argument shown here would remain true even if the classes Y and Z were mutually exclusive (!) – provided we knew somehow that their relative sizes (extensions) were as here stated (i.e. such that Z is greater or equal to Y). Even in such case (i.e. that of mutual exclusion), the said minor premise would imply the said conclusion, for neither of these two propositions formally means that the term X is (wholly or partly) actually included by the terms Y and Z – they only tell us that they could be.

As far as the a fortiori argument is concerned, all information about inclusion found in the two premises of the given syllogism is redundant – it is simply not carried over in it; it is effectively lost. The propositional forms used in the a fortiori argument are simply incapable of containing this significant classificatory information, and they have no need of it to deduce the conclusion they yield. Similarly, the latter conclusion (of the a fortiori) has nothing definite to tell us about the inclusion (i.e. of X in Z) that the original syllogistic conclusion refers to, and therefore is unable to reconstruct it.

We could of course have formulated the major premise as "The class Z is bigger than (or as big as) *its subclass* Y," and similarly the minor premise as "The class Y is big enough to include all (or certain) members of *its subclass* X" – but the additional information so tagged on would still *play no role* in the inference made possible by the a fortiori reasoning as such. It would of course suggest to us that X is indeed included in Y and therefore in Z – but that suggestion would be emerging not from the a fortiori argument itself, *but from the syllogism* mentally underlying it. The a fortiori argument as such could logically still only yield the said conclusion "The class Z is big enough to include all (or certain) members of the class X," without specifying that the Xs are indeed included in Z.

It follows that we cannot say that the derived a fortiori argument "is" or "is identical to" the given syllogism. The most we can say is that the former is implicit in the latter; i.e. that it is a part of it or an aspect of it. The derived format cannot logically replace the given format in all respects. Important information is lost in the transition. To return to the above example about Socrates, we now see that the clause about the class of men being included in that of mortals, and the suggestion that Socrates is included in both these classes, are not in fact constituents of the a fortiori argument as such.

As already mentioned, many people affirm that syllogisms can be reworded in a fortiori form, and they give an example or two to prove it, usually a positive first figure mood. But I asked myself two original questions: (a) can *all* valid moods of the syllogism be likewise recast in a fortiori format, and (b) are *all* valid forms of a fortiori argument generated by such translations?

To my surprise, the answers to both these questions were found to be: yes, even if some of the processes do seem rather artificial. At the risk of boring the reader stiff (skip it all if you take my word for it), I will now show step by step that all valid moods of syllogism can be recast in a fortiori form; and I will also show that such translations or reinterpretations generate all the varieties of a fortiori argument. Though most of this work was easy enough, some of it was a bit difficult – so it was well worth doing.

We have already dealt with wholly positive first figure syllogism (moods 1/AAA, 1/AII). We can similarly propose a translation of first figure syllogism with a negative major premise and conclusion (moods 1/EAE, 1/EIO), by simply using the term nonZ in place of Z. as follows:

b) Syllogistic format	A fortiori format
No Y are Z (= All Y are nonZ), and	The class nonZ is bigger than the class Y, and
All (or some) X are Y,	The class Y is big enough to include all (or certain) members of the class X,
therefore: All (or some) X are not Z (= are nonZ).	therefore: The class nonZ is big enough to include all (or certain) members of the class X.

Clearly, however, this is still positive a fortiori argument (albeit with a negative term nonZ), and does not correspond to negative a fortiori argument. The question arises, can we generate a negative a fortiori argument from some other syllogism? Yes, but to do so we need to move over to the second figure of syllogism, as follows:

c) Syllogistic format	A fortiori format
All Z are Y, and	The class Y is bigger than the class Z, and
All (or some) X are not Y,	The class Y is *not* big enough to include all (or certain) members of the class X,
therefore:	therefore: The class Z is *not* big enough to include all (or certain)
All (or some) X are not Z.	members of the class X.

The derivation might be considered roughly adequate, if we accept that X is not Z (or Y) means that Z (or Y) is not "big enough to include" X. But even if we do so, it is very reluctantly, as we are clearly indulging in an unnatural way of thinking. The above translation concerns the syllogistic moods 2/AEE, 2/AOO; we can do the same for the moods 2/EAE, 2/EIO simply by replacing Z with nonZ, as shown next:

d) Syllogistic format	A fortiori format
No Z are Y (= All Z are nonY), and	The class nonY is bigger than the class Z, and
All (or some) X are not Y (= are nonY),	The class nonY is *not* big enough to include all (or certain) members of the class X,
therefore:	therefore: The class Z is *not* big enough to include all (or certain)
All (or some) X are not Z.	members of the class X.

Of course, such a fortiori argument is even more awkward. Still, let us say we have now managed to translate all first and second figure syllogism to positive and negative a fortiori arguments, respectively. What of third figure syllogism – to what would we translate them? Moreover, all the a fortiori forms encountered so far have been subjectal – what of predicatal a fortiori? The answers to these two questions are the same. We can propose the translations of positive and negative moods of third figure syllogism into positive and negative predicatal a fortiori arguments, as follows:

e) Syllogistic format	A fortiori format
All Y are Z, and	A bigger class is required to subsume the class Z than to subsume the class Y, and
Some (or all) Y are X,	The class X is big enough to include some (or even all) members of the class Y,
therefore:	therefore: The class X is big enough to include some members of
Some X are Z.	the class Z.

The above is appropriate for the translation of the mood 3/AII (and likewise its subaltern 3/AAI). For the mood 3/IAI (and its same subaltern 3/AAI), we would have to transpose the premises and convert the particular conclusion.

f) Syllogistic format	A fortiori format
Some (or all) Y are Z, and	A bigger class is required to subsume the class X than to subsume the class Y, and
All Y are X,	The class Z is big enough to include some (or even all) members of the class Y,
therefore:	therefore: The class Z is big enough to include some members of
Some X are Z.	the class X.

The mood 3/OAO (and its subaltern 3/EAO) can be translated as shown next; note the transposition of premises here too.

g) Syllogistic format	A fortiori format
Some (or all) Y are not Z, and	A bigger class is required to subsume the class X than to subsume the class Y, and
All Y are X,	The class Z is *not* big enough to include some (or even all) members of the class Y,
therefore:	therefore: The class Z is *not* big enough to include some members
Some X are not Z.	of the class X.

And similarly for the mood 3/EIO (and its subaltern 3/EAO) shown next.

h) Syllogistic format	A fortiori format
No Y are Z (= All Y are nonZ), and	A bigger class is required to subsume the class nonZ than to subsume the class Y, and
Some (or all) Y are X (= are not nonX),	The class nonX is *not* big enough to include some (or even all) members of the class Y,

| therefore: Some X are not Z (= Some nonZ are not nonX). | therefore: The class nonX is *not* big enough to include some members of the class nonZ. |

Note in passing that we could similarly process modal syllogism, the modalities simply passing over from the syllogism to the derived a fortiori argument. We have thus shown that all valid moods of the syllogism can be rephrased (however awkwardly) as a fortiori argument of various sorts. The first figure moods yielded positive subjectal a fortiori; the second figure moods yielded negative subjectal a fortiori; and the third figure moods yielded positive and negative predicatal a fortiori. We have thus also shown that all four forms of a fortiori argument are produced by these processes.

Needless to say (but I will add it anyway, to be foolproof), the middle term of these a fortiori arguments, viz. "big," is just one possible middle term – most a fortiori arguments we encounter in practice do not use this particular middle term but may use any of countless middle terms. That is to say, although we have generated instances of all sorts of a fortiori, we have certainly not generated all particular instances of a fortiori! Thus, 'a fortiori argument in general' must be admitted to be a larger class than 'a fortiori argument generated from syllogism' is. Keeping that in mind, we will not overestimate the import of the preceding demonstration.

Our demonstration was made with reference to categorical syllogism and copulative a fortiori – but obviously the same can be done with reference to hypothetical (and likewise, 'de re' conditional) syllogism. We can I think take that for granted without worry, so I won't bore you further with a repetition of all the above work to prove the point. However, it is worth our while looking at just one sample of translation of hypothetical syllogism into a fortiori argument, to highlight and examine the different concepts and language involved in the latter:

Hypothetical syllogism (given)	A fortiori argument (derived)
If Y, then Z, and	Thesis Z is bigger than thesis Y, and
If X, then Y	Thesis Y is big enough to include all (or certain) conditions of thesis X,
(or if X, not-then not Y),	
therefore: If X, then Z	therefore: Thesis Z is big enough to include all (or certain) conditions of thesis X.
(or if X, not-then not Z).	

The given propositions can also be written/read respectively as "Y implies Z," "X implies Y" (or in the weaker case, "X does not imply not Y"), and "X implies Z" (or in the weaker case, "X does not imply not Z"), note. The derived a fortiori argument here is still copulative, note, and not implicational. Its language is clearly similar here to that we used in translating categorical syllogism, except that we say "thesis Z" instead of "class Z," etc., and we speak of "conditions" instead of "members."

This change of wording reflects the nature of implication: necessary implication (the stronger variant) means that the consequent occurs under *all* the conditions applicable to the antecedent, whereas possible implication (the weaker variant) refers to *some* of these conditions. In logical conditioning, the conditions referred to are internal contexts of knowledge; in 'de re' modes of conditioning – i.e. the natural, temporal and spatial modes – they are external circumstances, times or places.

Note well that when we say that "thesis Z is bigger than thesis Y," we do not mean that the subject of Z is bigger than the subject of Y (taking Z and Y as single categorical propositions here, for the sake of argument). We are not referring to the extensions of the terms involved *within* the theses, but to the conditions *underlying* the theses. The proposition "Y implies Z" does not formally exclude the possibility that Y may be singular and Z universal, or vice versa, provided that under all the conditions concerned Y is accompanied by Z. Similarly with the other forms mentioned in the above arguments.

To conclude, it is now formally proved true that syllogism can always be recast in a fortiori form, although it was also evident in the course of our demonstration that such translations are, to varying degrees, very contrived. Maybe logicians occasionally have such convoluted thoughts; but we do not ordinarily think in such awkward ways. Syllogism is quite thinkable and commonly thought by itself, i.e. without need to refer to a fortiori argument as above done. Indeed, syllogism is a simpler and more primitive movement of thought; more easily and widely comprehensible. So we would not in practice rephrase a syllogism as an a fortiori argument. Still, we have answered half the question initially asked.

6. From a fortiori argument to syllogism

We need now turn to the second part of our initial question: can a fortiori argument be said to be syllogism or at least be expressed as or reduced to syllogism? We can put it more technically, and ask: how can a four-term argument (a fortiori) be reworded as a three-term one (syllogism), with minimal loss of meaning and conviction?

One possible answer is: by ignoring or concealing one of the terms – namely the middle term. We very commonly do leave tacit the middle term in our a fortiori arguments. An argument used by Aristotle that does this is: *if even the gods are not omniscient, certainly human beings are not*. Here, the middle term is tacitly present in the words 'even' and 'certainly'; often, it is signaled by expressions like 'enough' and 'all the more'. Put in formal terms, this would read: if Q is S enough, then all the more P is so – the a fortiori middle term R being left out entirely. This is a three-term argument – but is it syllogism? Clearly not, since we have no operative syllogistic middle term (S cannot be said to play that role here).

We can in this context, incidentally, pinpoint the usual error of the logicians and commentators who seek to explain a fortiori argument as a sort of syllogism – they wrongly assume that the three terms of such derived syllogism would be the minor, major and subsidiary (P, Q, S), whereas in fact they are the different quantities of the middle term (R) occurring relative to these three terms. This is a subtle distinction that they could not see, because they had not sufficiently analyzed a fortiori argument as such as gravitating around a fourth term, which is often in practice left unstated or so intertwined with the other three that it is almost imperceptible.

To understand how we can transmute a fortiori argument into syllogism, we need to go back to the way a fortiori argument is formally validated – i.e. we need to decorticate its propositions and find out whether the constituents of the given premises justify the constituents of the putative conclusion. To do this we need to look again at Table 1.1 and Diagram 1.1, given in the first chapter.

Once the centrality of the middle term of a fortiori argument is grasped, it is easy for us to formulate its various moods in syllogistic form. We shall now do this with reference to copulative a fortiori argument. The major premise of all four a fortiori arguments can be expressed in the hypothetical form "Rp implies Rq," which means "the quantity of R corresponding to P implies the quantity of R corresponding to Q." Rp may be said to imply Rq because a larger quantity implies all lesser quantities; as for example 5 equally implies, 4, 3, 2 or 1 – i.e. you cannot reach the larger amount without passing through the smaller amounts. If you have $5, it is also true that you have $4, $3, etc.

The minor premises and conclusions can similarly, by consideration of their quantitative significances, be expressed as if–then propositions or negations of such, as shown below. Note well that the middle items of the syllogisms produced vary: in cases (a) and (d) the middle theses is Rq, while in cases (b) and (c) it is Rp. Note too that in cases (a) and (b) the premises must be switched to get the stated conclusion; i.e. the major premise becomes the minor and vice versa. Notice, finally, that we obtain valid moods in all three figures of the syllogism, viz. 1/AAA (twice), 2/AOO and 3/OAO, although (predictably, in view of the numbers) we do not produce all the valid moods of syllogism out of those of a fortiori.

a) Positive subjectal copulative a fortiori	Syllogism (1/AAA)
P is more R than (or as much R as) Q is,	Rp implies Rq
and, Q is R enough to be S; therefore,	Rq implies Rs
all the more (or equally), P is R enough to be S.	So, Rp implies Rs

b) Negative subjectal copulative a fortiori	Syllogism (3/OAO)
P is more R than (or as much R as) Q is,	Rp implies Rq
yet, P is *R not* enough to be S; therefore,	Rp does *not* imply Rs
all the more (or equally), Q is *R not* enough to be S.	So, Rq does *not* imply Rs

c) Positive predicatal copulative a fortiori	Syllogism (1/AAA)
More (or as much) R is required to be P than to be Q,	Rp implies Rq
and, S is R enough to be P; therefore,	Rs implies Rp
all the more (or equally), S is R enough to be Q.	So, Rs implies Rq

d) Negative predicatal copulative a fortiori	Syllogism (2/AOO)
More (or as much) R is required to be P than to be Q,	Rp implies Rq
yet, S is *R not* enough to be Q; therefore,	Rs does *not* imply Rq
all the more (or equally), S is *R not* enough to be P.	So, Rs does *not* imply Rp

To give an example[8]: "It is psychologically more difficult (R) for a man to strike his father (P) than to strike his neighbors (Q); so, if John (S) was able to strike his father, he is all the more capable of striking his neighbors" (major to minor, positive predicatal) becomes "John's psychological state is such that he could strike his father, and striking one's father is more difficult than striking one's neighbors, therefore John's psychological state is such that he could strike his neighbors."

Now, just as we found that in the attempted transformation of syllogism into a fortiori argument there was a loss of information (about inclusions between the classes concerned), so it is with regard to the attempted transformation of a fortiori argument into syllogism some information is inevitably lost in the process. What information is lost? I will now explain, with reference to our earlier analysis of the propositional forms involved in a fortiori argument. Consider for instance process (a), the translation of positive subjectal a fortiori argument into syllogism.

The form "P is more R than Q" tells us more than just "Rp implies Rq" – it also tells us that "P implies Rp" and "Q implies Rq" and "Rp is greater than Rq." Likewise, the form "Q is R enough to be S" tells us more than just "Rq implies Rs" – it also tells us that "Rs implies S" and "Q implies Rq" and "Rs includes Rq." Thus, the two premises of the derived syllogism do not carry over all the information that was in the given a fortiori argument, but only parts of it. The conclusion of the derived syllogism, viz. "Rp implies Rs," is similarly less informative than the conclusion of the given a fortiori argument, which tells us that "Rs implies S" and "P implies Rp" and "Rs includes Rp." From "Rp implies Rs," we can infer the element "Rs includes Rp," but not the elements "Rs implies S" and "P implies Rp."

So we are unable to logically reconstitute the conclusion of the original a fortiori argument from the conclusion of the derived syllogism – we only have part of its discourse leftover, some of it having been left out on the way. It follows that, though a significant syllogism is formally discernible within the a fortiori argument, that derived syllogism does not store enough information to in turn produce the original a fortiori argument.

It follows that the a fortiori argument logically implies but is not implied by the syllogism shown. That is to say, we cannot claim that the given a fortiori argument "is" or "is identical to" the derived syllogism. The most we can say is that the latter is a *corollary* of the former: it is implicit in it, a part of it or an aspect of it. But evidently, the latter cannot logically replace the former in all respects. Important information is lost in the transition.

The same can be said with regard to the translation of other forms of a fortiori argument to syllogism, i.e. the processes labeled (b), (c) and (d) above. Though these processes are obviously valid, they do not produce a syllogistic replica of a fortiori argument – only at best a corollary. We could do and say the same for implicational a fortiori argument, but there is no need to repeat ourselves. Suffice it to show and comment on one sample:

Implicational a fortiori argument	Syllogism
P implies more R than (or as much R as) Q does,	Rp implies Rq
and, Q implies enough R to imply S; therefore,	Rq implies Rs
all the more (or equally), P implies enough R to imply S.	So, Rp implies Rs

Note that the language of the derived syllogism here is exactly the same as that for copulative a fortiori, since we are still concerned with various degrees of R implying each other. The syllogism derived here – as that derived from copulative a fortiori – is formally hypothetical, not categorical. The items involved in all such derived syllogisms, viz. Rp, Rq and Rs, are the terms or theses "the value of R that P is or implies," "the value of R that Q is or implies," and "the value of R that S is or implies," respectively (using "is" for copulative sources and "implies" for implicational ones). No more need be said.

We have thus demonstrated using formal means that the essence of a fortiori argument can be expressed through syllogism. This is equally true of positive or negative, subjectal or predicatal, copulative or implicational a fortiori. This does not mean that a fortiori argument is the same as syllogism, but it does mean that syllogistic movements of thought are involved in a fortiori reasoning, or (in other words) that a fortiori argument can be *partly* reduced to and validated by means of syllogism. Even so, as just explained, some information is invariably lost on the way.

7. Reiterating translations

To complete our analysis of the relationships between a fortiori argument and syllogism we need to examine one more issue. Having found that we can generate a fortiori arguments from syllogisms, and syllogisms from a fortiori arguments – we need to ask the question of reiteration. This does not mean reversibility, since we have already shown that such translations are not reversible – i.e. the arguments generated by such translations are always mere corollaries because some information is always lost in the process. Rather, the question to ask is this: having

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[8]   This is one of Aristotle's examples reworked a bit.

extracted an a fortiori argument from a syllogism, what syllogism can we in turn extract from that derived a fortiori argument? And conversely, having extracted a syllogism from an a fortiori argument, what a fortiori argument can we in turn extract from that derived syllogism?

Every syllogism implies a corresponding a fortiori argument which does not in turn imply it back – but it does go on to imply *some* syllogism. Similarly, every a fortiori argument implies a corresponding syllogism which does not in turn imply it back – but it does go on to imply *some* a fortiori argument. It should be obvious that these pairs of statements are not in contradiction, but I will explain why anyway. Consider the following samples:

a) Syllogism	A fortiori argument
All Y are Z, and	The class Z is bigger than (or as big as) the class Y, and
All (or some) X are Y;	The class Y is big enough to include all (or certain) members of the class X,
therefore:	therefore: The class Z is big enough to include all (or certain)
All (or some) X are Z.	members of the class X.

b) A fortiori argument	Syllogism
P is more R than (or as much R as) Q is, and,	Rp implies Rq, and
Q is R enough to be S; therefore,	Rq implies Rs;
all the more (or equally), P is R enough to be S.	So, Rp implies Rs.

Consider first process (a): if we wanted to draw a syllogism in accord with process (b) out of the a fortiori argument derived from the given syllogism, we would obtain the syllogism shown next, which is considerably different from the original one:

> The size of class Z implies the size of class Y,
> the size of class Y implies the size of class X;
> therefore, the size of class Z implies the size of class X.

I use the word "size" here to avoid using the more barbaric "bigness;" we are of course concerned with extensions, meaning that a bigger size implies a smaller one, as already explained. Note that this new syllogism is not categorical (with "is" relating terms) like the original one, but hypothetical (with "implies" relating theses[9]). If now we compare the information in this derived syllogism to that in the original syllogism, we note that valuable information was lost in the transition: the *inclusions* of Y in Z and X is Y (and therefore of X in Z) are long gone; instead, all we now discuss are the *relative sizes* of these classes (irrespective of any subsumption between them). Thus, though we have indeed obtained a new syllogism, it is certainly not the same as the original syllogism but a watered-down corollary of it.

Consider now process (b): if we wanted to draw an a fortiori argument in accord with process (a) out of the syllogism derived from the given a fortiori argument, we would obtain the a fortiori argument shown next, which is considerably different from the original one:

> Thesis Rp is bigger than (or as big as) thesis Rq, and
> Thesis Rq is big enough to include all the conditions of thesis Rs; therefore,
> all the more (or equally), Thesis Rp is big enough to include all the conditions of thesis Rs.

Here, as already explained, sizes (as in "bigger," "big enough") refer to numbers of conditions underlying theses, not to numbers of members within classes. Note that the new a fortiori argument is copulative, since it uses "is" rather than "implies" to relate items. Here again, if we compare the information in this derived a fortiori argument to that in the original a fortiori argument, we note that valuable information was lost in the transition: for instance, information we had initially on Q actually being (R enough to be) S is long gone – all we still know about them now is that the value of R corresponding to Q occurs in numerically more conditions than the value of R corresponding to S does. Thus, though we have indeed obtained a new a fortiori argument, it is certainly not the same as the original a fortiori argument but a watered down corollary of it.

Clearly, such reiterations do not produce very interesting results. No doubt if we tried reiterating further, i.e. translating the syllogism and a fortiori argument produced by the above reiterations into new derivatives, we would

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[9]   The propositions involved, written more fully, would be: 'If a class has the number of members in class Z, then it has the number of members in class Y' (meaning that the first number is greater or equal to the second number, as e.g. 5 implies 3), and so forth.

likewise not produce anything very interesting. But the main point we wished to make – viz. that reiteration should not be confused with reversing – has been convincingly established.

8. Lessons learned

We can here conclude our research into the relationships between a fortiori argument and syllogism. We looked into this topic because it is one often and hotly debated in literature on a fortiori logic. We decided to carry out a more formal and systematic investigation than past logicians and commentators have done, so as to be able to judge the matter more objectively and definitively. We first looked into the conceivable correlations between any two arguments, and in particular formally defined what we mean by a corollary. Then we compared the structures of syllogism and a fortiori argument, showing the various respects in which they differ significantly.

We then demonstrated that every syllogism contains an implicit a fortiori argument – but we also found that the latter is a mere corollary of the former, information being lost in transition, so that the process of derivation cannot be reversed. We then demonstrated the converse, i.e. that every a fortiori argument contains an implicit syllogism – but we also found that the latter is a mere corollary of the former, information being lost in transition, so that the process of derivation cannot be reversed. Finally, we showed that reiteration of these processes, though logically possible, is not very interesting.

We are now in a position to formulate the following overall lessons learned. Syllogism and a fortiori argument are very different movements of thought. They are structurally different, and each serves a different rational purpose; so they are not equivalent or interchangeable. Although they can be formally interrelated in various ways, they remain logically distinct and irreplaceable in important ways. Syllogism orders terms or theses by reference to the inclusions (or exclusions) between them, while a fortiori argument orders them by reference to the measures or degrees of some property they have (or do not have) in common. Neither function can be substituted for the other. We can (however awkwardly) reword one form of argument into the other; but such translations are not exactly transformations, because significant information cannot be passed on from one form to the other; therefore, neither form of argument can be fully reduced to the other. Thus, both forms are needed by reason to pursue its business; they are complementary instruments of reasoning.

In spite of all that, it should be remembered that a fortiori argument, whether copulative or implicational, is inextricably dependent on hypothetical (rather than categorical) syllogism, as we have shown in detail in the first chapter of the present volume, in the section dealing with validation (1.3). Hypothetical syllogisms (arguments such as if a, then b and if b then c, therefore if a then c) are not by themselves sufficient for validation of a fortiori arguments, since we also need comparative arguments (such as a > b and b > c, therefore a > c); but without them the validations would be impossible. Thus, a fortiori argument is a sort of composite argument, whose components are brought to the surface during the process of validation. Nevertheless, it is a form of argument in its own right, insofar as we are able to reason through it directly, without always having to resort to validation – i.e. it is intuitively credible anyway.
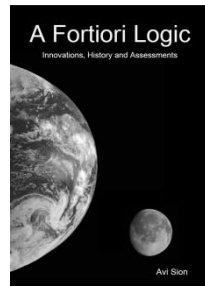
PART II – ANCIENT AND MEDIEVAL HISTORY

6. A fortiori in Greece and Rome

1. Aristotle's observations

Looking at the sayings or writings of ancient Greek philosophers – Thales, Anaximander, Anaximenes, Heraclitus, Pythogoras, Philolaus, Xenophanes, Parmenides, Zeno, Empedocles, Leucippus, Democritus, Anaxagoras, Socrates, Plato, and Aristotle, and their successors – one cannot but be awed by the extraordinary breadth and profundity of their thinking, and their anticipation of many ideas considered important today. For example, I recently realized that Empedocles[1] could be regarded as the precursor of the phenomenological approach, on the basis of his statement: "Think on each thing in the way in which it is manifest."

It is not surprising, therefore, to find some discussion of a fortiori argument in the works of Aristotle (Greece, 384-322 BCE)[2]. The following quotations from his works (dated *c*. 350 BCE) seem relevant to our research.[3]

In his *Rhetoric* 2:23 (i.e. book II, chapter 23), in §4, Aristotle writes:

> "Another line of proof is the *a fortiori*[4]. Thus it may be argued that if even the gods are not omniscient, certainly human beings are not. The principle here is that, if a quality does not in fact exist where it is more likely to exist, it clearly does not exist where it is less likely. Again, the argument that a man who strikes his father also strikes his neighbors follows from the principle that, if the less likely thing is true, the more likely thing is true also; for a man is less likely to strike his father than to strike his neighbors. The argument, then, may run thus. Or it may be urged that, if a thing is not true where it is more likely, it is not true where it is less likely; or that, if it is true where it is less likely, it is true where it is more likely: according as we have to show that a thing is or is not true."

In this passage, Aristotle shows he considers a fortiori argument as a "line of proof" – by which he presumably means that it is a deductive argument. He marks his understanding of a fortiori argument as going from denial of the 'more' to denial of the 'less', or from affirmation of the 'less' to affirmation of the 'more'. On this basis, we can say that Aristotle was aware of at least two valid moods: positive argument "from minor to major," and negative argument "from major to minor," though he does not use such terminology, but only says: "according as we have to show that a thing is or is not true"[5]. Clearly, therefore, what he has in mind here are positive and negative subjectal arguments. His arguments can be reworded as follows to clarify their standard formats (with the symbols P, Q, R, and S, denoting the major, minor, middle and subsidiary terms, respectively):

His first example is negative subjectal: that the gods are omniscient (P) is more credible (R) than that human beings are so (Q); therefore if the gods' omniscience is not credible enough to be assumed (S), the omniscience of human beings is not credible enough to be assumed. This illustrates the principle: if a quality in a certain place (P) is more likely to be found (R) than the same quality in another place (Q) is, then if the quality in the first place is not

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[1]   Of Acragas, a Greek colony in Sicily, ca. 482 – ca. 432 BCE. Cited by Freely, p. 18.

[2]   When I researched a fortiori argument, back in 1991-92, although my main interest was Judaic logic, I wondered – as a big fan of Aristotle, the undoubted founder of formal logic – whether he had noticed and discussed a fortiori argument. But I lacked the research tools and free time to find out (the Internet did not exist, for a start, and I had little access to reference books). Just recently, looking at Allen Wiseman's new study of the subject, I was pleased to see that he had found use and mention of a fortiori argument in Aristotle and other ancients (p. 7). Apparently, he did so at least in part thanks to the Kneales' historical study, to which he refers at length (p. 25). I have since then done some research in the works of Aristotle, and his predecessor Plato, and determined more accurately the extent of use of a fortiori discourse in these authors. The results are given here.

[3]   The full texts of Aristotle's *Rhetoric* and *Topics* are available online at the Internet Classics Archive: classics.mit.edu/Aristotle/rhetoric.html and classics.mit.edu/Aristotle/topics.html.

[4]   'A fortiori' is of course a Latin expression meaning 'all the more strongly'. Aristotle's words in Greek are "ἄλλος ἐκ τοῦ μᾶλλον καὶ ἧττον" – meaning, literally: "Another topic is derived from the more and less." www.perseus.tufts.edu/hopper/text?doc=Perseus%3Atext%3A1999.01.0059%3Abook%3D2%3Achapter%3D23%3Asection%3D4.

[5]   Or, in a more literal translation: "according as it is necessary to prove either that a predicate is affirmable or that it is not." (See Perseus Digital Library reference mentioned earlier.)

sufficiently likely to be found to be considered as existing in fact (S), it follows that the quality in the second place is not sufficiently likely to be found to be considered as existing in fact (S).

His second example is positive subjectal: a man striking his neighbors (P) is a more likely event (R) than the man striking his father (Q); therefore, if a man striking his father is likely enough to be expected (S), then the man striking his neighbors is likely enough to be expected. This illustrates the principle: if something somewhere (P) is more likely (R) than the same thing elsewhere (Q), then if the latter is likely enough to be declared true (S), it follows that the former is likely enough to be declared true. (To which he adds the negative mood: if the former is not likely enough to be declared true, it follows that the latter is not likely enough to be declared true.[6])

Noteworthy here is Aristotle's formulation of these a fortiori arguments in logical-epistemic terms, i.e. using a logical middle term (such as 'likely') and an epistemic subsidiary term (such as 'believed')[7]. His above two examples could of course have been formulated in purely ontical terms, as follows. The gods (P) are more well-endowed (R) than human beings (Q) are; therefore, if the gods are not well-endowed enough to be omniscient (S), then human beings are not well-endowed enough to be omniscient. Or again: striking one's neighbors (P) generally seems more natural (R) than striking one's father (Q); therefore, if striking his father seems natural enough to a certain man for him to actually do it (S), then striking his neighbors seems natural enough to him for him to actually do it.[8]

Still in *Rhetoric* 2:23, Aristotle adds a number of examples of allegedly *a pari* a fortiori argument. I say allegedly, because the proposed arguments are not complete enough to judge the matter. Note that five of the examples have negative form, while two have positive form. In any case, this serves to show us his awareness of such argument:

> "This argument might also be used in a case of parity, as in the lines: Thou hast pity for thy sire, who has lost his sons: Hast none for Oeneus, whose brave son is dead? And, again, 'if Theseus did no wrong, neither did Paris'; or 'the sons of Tyndareus did no wrong, neither did Paris'; or 'if Hector did well to slay Patroclus, Paris did well to slay Achilles'. And 'if other followers of an art are not bad men, neither are philosophers'. And 'if generals are not bad men because it often happens that they are condemned to death, neither are sophists'. And the remark that 'if each individual among you ought to think of his own city's reputation, you ought all to think of the reputation of Greece as a whole'."

In his *Topics* 2:10 (book II, chapter 10), where Aristotle begins with: "Moreover, argue from greater and less degrees…," I will divide what he thereafter says in three parts for purposes of analysis:

> "See whether a greater degree of the predicate follows a greater degree of the subject: e.g. if pleasure be good, see whether also a greater pleasure be a greater good: and if to do a wrong be evil, see whether also to do a greater wrong is a greater evil. Now this rule is of use for both purposes: for if an increase of the accident follows an increase of the subject, as we have said, clearly the accident belongs; while if it does not follow, the accident does not belong. You should establish this by induction."

This first paragraph, if it is at all related to a fortiori argument, makes clear by implication that Aristotle does not universally approve of *a crescendo* argument, i.e. of argument resembling a fortiori but having a 'proportional' conclusion. He is clearly *not* saying, for instance, that if pleasure is good *it follows deductively that* more pleasure is better – he is only saying that the question should be asked and that the answer is to be sought *by induction*; he explicitly conceives the possibility that it may *not* follow. This is an important finding concerning Aristotle, considering that (as we shall see) many people who historically came after him did not likewise realize the invalidity of 'proportional' a fortiori argument. He goes on:

> "If one predicate be attributed to two subjects; then supposing it does not belong to the subject to which it is the more likely to belong, neither does it belong where it is less likely to belong; while if it does belong where it is less likely to belong, then it belongs as well where it is more likely. Again: If two predicates be attributed to one subject, then if the one which is more generally thought to belong does not belong, neither does the one that is less generally thought to belong; or, if the one that is less generally thought to belong does belong, so also does the other. Moreover: If two predicates be attributed to two subjects, then if the one which is more usually thought to belong to the one subject does not belong, neither does the remaining predicate belong to the remaining subject; or, if the one which is less usually thought to belong to the one subject does belong, so too does the remaining predicate to the remaining subject."

Aristotle here details the positive and negative moods of three seemingly distinct a fortiori arguments. The first concerns two subjects (A, B) with a common predicate (C), and its major premise is: 'A is C' (P) is more likely (R)

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[6]   However, note that this further remark is not found in all extant versions of the text. (See Perseus Digital Library reference mentioned earlier.)

[7]   Actually, judging by another, more literal translation, it is not sure that Aristotle intended the logical-epistemic interpretation in the second example (concerning a man striking his father): "And to say that a man who beats his father also beats his neighbors, is an instance of the rule that, if the less exists, the more also exists." Compare the wording here "if the less *exists*, the more also *exists*" to the wording above "if the less *likely* thing is true, the more *likely* thing is true also." (See Perseus Digital Library reference mentioned earlier.)

[8]   Needless to say, I am only here discussing the formal aspect of these arguments; I am not endorsing their content.

than 'B is C' (Q). The second concerns one subject (A) with two predicates (B, C), and its major premise is: 'A is B' (P) is more generally thought (R) than 'A is C' (Q). The third concerns two subjects (A, B) with two predicates (C, D), and its major premise is: 'A is B' (P) is more usually thought (R) than 'C is D' (Q). Although the middle term (R) is differently worded in each case, no great significance should be attached to this variation: all three may be taken to mean about the same, say 'likely'. The subsidiary term (S) may in all cases be regarded as 'believed' (or 'adopted' or any similarly convenient qualification). In each case, the said major premise is followed by the minor premises and conclusions in the standard forms below:

> Given something (P) is more likely (R) than another thing (Q) is, it follows that:
> if Q is R enough to be believed (S), then P is R enough to be S;
> and if P is R not enough to be S, then Q is R not enough to be S.

Clearly, the three sets of argument of positive and negative forms are effectively one and the same set. They illustrate subjectal a fortiori argument with a logical middle term (e.g. 'likely') and an epistemic subsidiary term (e.g. 'believed'). Note well, though these arguments concern whole propositions (labeled P and Q by me), they are not to be regarded as antecedental since no implication between propositions is suggested. Though the major and minor terms, P and Q, are propositions, each stands in this context as a unitary term, a subject of which may be predicated the said logical and epistemic qualifications. The four terms of the a fortiori argument as such are the two effective subjects P and Q, and the two predicates 'likely' (R) and 'believed' (S). Aristotle does not seem aware of all that.

The three or four terms mentioned by Aristotle as subjects and predicates (labeled A, B, C and D by me) are terms within the propositions P and Q, and *not* the terms of the a fortiori argument as such, note well. These terms (A, B, C, D) are thus quite incidental to the argument, which are used to illustrate possible uses of such argument. Aristotle could well have mentioned only one such illustration if his intention was to abstractly describe a fortiori argument as such. It appears, then, that it was not his primary intention to do that here. His primary intention was probably to concretely describe different ways a predicate may be found to belong or not belong to a subject, by *using* a fortiori argument.

Nonetheless, judging by this second paragraph, which clearly concerns a fortiori argument, we can again say that Aristotle was well aware of the positive and negative subjectal moods. Thus far, however, there is still no evidence of his being aware of predicatal arguments. We might, on a superficial reading, have thought that Aristotle here marks the difference between subjectal and predicatal a fortiori, when he speaks of one predicate for two subjects or two predicates for one subject. He might have been referring, in the first case to the subsidiary term being predicated of the major and minor terms (the subjectal mood), and in the second case to the major and minor terms being predicated of the subsidiary term (the predicatal mood). But when we actually set out his arguments in standard forms, we see clearly that they are all subjectal.

I should also stress that though Aristotle's arguments in the above paragraph of *Topics*, as well as in the Rhetoric passage earlier considered, can be cast in standard forms using qualifications like 'likely' as middle term and 'believed' as subsidiary term, it is obvious that Aristotle himself does not formulate his arguments as clearly. He is not sharply aware of the distinct functions of these two terms (R and S) in his arguments. In fact, he tends to lump them together, i.e. treat them as one and the same. This observation will be further confirmed further on, when we analyze *Topics* 3:6. Still in *Topics* 2:10, he goes on:

> "Moreover, you can argue from the fact that an attribute belongs, or is generally supposed to belong, in a like degree, in three ways, viz. those described in the last three rules given in regard to a greater degree. For supposing that one predicate belongs, or is supposed to belong, to two subjects in a like degree, then if it does not belong to the one, neither does it belong to the other; while if it belongs to the one, it belongs to the remaining one as well. Or, supposing two predicates to belong in a like degree to the same subject, then, if the one does not belong, neither does the remaining one; while if the one does belong, the remaining one belongs as well. The case is the same also if two predicates belong in a like degree to two subjects; for if the one predicate does not belong to the one subject, neither does the remaining predicate belong to the remaining subject, while if the one predicate does belong to the one subject, the remaining predicate belongs to the remaining subject as well."

Looking at this third paragraph, we can also say that Aristotle realized that a fortiori inference is also possible between equals, not just from the more to the less or vice versa. And as he points out, in such case the argument can function either way, i.e. from minor to major or from major to minor, whether it is positive or negative. I have in my *Judaic Logic* account called such argument, in which the major premise is a statement of equality, egalitarian a fortiori; another name for it is *a pari*.

Apart from that, there is nothing new in this paragraph – it still concerns only subjectal moods. There is still no mention of equivalent predicatal moods (which involve quite different arrangements of terms). Even so, this insight of his has some importance. He also says further on in the *Rhetoric* chapter above quoted: "This argument [i.e. a fortiori] might also be used in a case of parity." He again implies as much in *Topics* 2:11: "You can argue, then, from greater or less *or like* degrees of truth in the aforesaid number of ways" (italics mine) and elsewhere.

It should be noted that, though Aristotle, as we have seen in *Rhetoric* 2:23 and in the second passage of *Topics* 2:10, formulates a fortiori argument primarily in logical-epistemic terms, looking at the third passage of *Topics* 2:10 it appears that he also conceives of purely ontical a fortiori argument, since he speaks repeatedly of a predicate *belonging* to a subject, as against being *supposed* to belong. This is confirmed by some of his a fortiori pronouncements in other contexts; for example in *Topics* 3:6 (see below), or again in the *History of Animals* 5:14, where he says: "If a sow be highly fed, it is all the more eager for sexual commerce, whether old or young," implying that being well-fed physically causes (and not merely implies) a sow to want sex.

It is reasonable to suppose that, though Aristotle only mentions the distinction between a property and a supposed property in the said third passage, which deals with terms of "like degree," he does not consider this distinction as exclusive to egalitarian a fortiori arguments. For a start, he makes no mention of such exclusiveness; and besides, examples like the one just cited from the *History of Animals* show that he does not intend it. Thus, this distinction between a property and a supposed property can be fairly applied to non-egalitarian a fortiori arguments too.

In other words, we may say that Aristotle is somewhat aware of purely ontical argument, and does not limit on principle a fortiori to the logical-epistemic variety, even if he appeals to the latter more often (so far). In my theory of a fortiori argument, note, the emphasis is rather on the ontological variety. This does not of course exclude the epistemological variety, which Aristotle seemingly emphasized, nor for that matter the ethical and legalistic variety, which the Rabbis and others have emphasized; I view (and have from the start viewed) all these other varieties as special cases of the primary, ontological variety.

A further thing to notice is the uncharacteristic lack of formalization in Aristotle's treatment of a fortiori argument. This is no doubt because he mentions such argument in passing, without focusing on it particularly or very deeply. Although he does discuss the argument in relatively abstract terms, as when he says in the *Rhetoric* passage: "if a thing is not true where it is more likely, it is not true where it is less likely; or … if it is true where it is less likely, it is true where it is more likely," and not merely through concrete examples (like the man striking his father or neighbors), he does not go one step further as he did with syllogistic reasoning and use symbols (A, B, Γ, Δ) in lieu of terms to list all possible moods of the argument and, most importantly, to formally validate or invalidate them. My theory of a fortiori argument does this crucial job.

In this regard, we should note too that Aristotle does not here (or elsewhere, to my knowledge) formulate any rule of reasoning comparable to the rabbinical *dayo* principle (which appears on the stage of documented history perhaps some four and a half centuries later[9]) – or more precisely, to the principle of deduction as it applies specifically to a fortiori argument, namely the rule that the subsidiary term (which is a predicate in subjectal argument) must be identical in the minor premise (where it concerns the minor term) and in the conclusion (where it is applied to the major term). Aristotle may well in practice reason correctly in accord with this principle, but he does not explicitly express theoretical awareness of it – unless we count the already mentioned passage: "See whether a greater degree of the predicate follows a greater degree of the subject," which we interpreted as an effective rejection of a crescendo argument, as intended by him to be an admonishment by him not to always reason proportionately.

Let us now move on and examine a passage of his *Topics* 3:6 (book III, chapter 6), which again I split up as convenient:

> "Moreover you should judge by means of greater or smaller or like degrees: for if some member of another genus exhibit such and such a character in a more marked degree than your object, while no member of that genus exhibits that character at all, then you may take it that neither does the object in question exhibit it; e.g. if some form of knowledge be good in a greater degree than pleasure, while no form of knowledge is good, then you may take it that pleasure is not good either."

In this first paragraph, Aristotle shows stronger awareness of the middle term of a fortiori argument, namely the "such and such a character" (R) which the "other genus" (i.e. the major term, P) exhibits in a more marked degree than "your object" (i.e. the minor term, Q). Notice, too, that this middle term (R) is definitely ontological, rather than as before epistemological. However, his argument is not very well formulated, in that his major premise states that "some members" of P "exhibit this character," whereas his minor premise states contradictorily that "no members" of P "exhibit this character." This confusion is not due to his insertion of quantification issues into the equation, but to

his conflation between the middle term (in the major premise) and the subsidiary term (in the minor premise and conclusion). The latter is a not uncommon error of formulation[10]. He goes on:

> "Also, you should judge by a smaller or like degree in the same way: for so you will find it possible both to demolish and to establish a view, except that whereas both are possible by means of like degrees, by means of a smaller degree it is possible only to establish, not to overthrow. For if a certain form of capacity be good in a like degree to knowledge, and a certain form of capacity be good, then so also is knowledge; while if no form of capacity be good, then neither is knowledge. If, too, a certain form of capacity be good in a less degree than knowledge, and a certain form of capacity be good, then so also is knowledge; but if no form of capacity be good, there is no necessity that no form of knowledge either should be good. Clearly, then, it is only possible to establish a view by means of a less degree."

This second paragraph serves to show (only by means of example, but clearly enough) that Aristotle is aware that, even though one may argue positively, from predication of the subsidiary term to the minor term to predication of the subsidiary term to the major term, it does *not* follow that one may argue negatively, from denial of predication of the subsidiary term to the minor term to denial of predication of the subsidiary term to the major term – except, of course, where the argument is *a pari*. He here obviously refers specifically to subjectal argument, since in fact (although he makes no remark to that effect) the opposite rule would hold for predicatal argument. His statement of this rule is significant, since he thereby declares a mood *in*valid, whereas previously he only declared moods valid.

Note however that he does not similarly point out that, though (in subjectal argument) one may argue negatively, from denial of predication of the subsidiary term to the major term to denial of predication of the subsidiary term to the minor term, it does *not* follow that one may likewise argue positively, from predication of the subsidiary term to the major term to predication of the subsidiary term to the minor term – except, of course, where the argument is *a pari*. That is, even though he has previously mentioned both positive and negative moods for validation purposes, in the present remark he only mentions a negative mood for invalidation purposes and omits to mention the corresponding positive mood for invalidation purposes.

Moreover, Aristotle's "invalidation" of a mood of a fortiori argument here is merely intuitive, i.e. a raw rational insight – he does not explain or formally prove the invalidity of the mood in question. He tells us that it is wrong reasoning, but he does not tell us why it is so.

Furthermore, in the example he gives, the major term is "knowledge" and the minor term is an unspecified "capacity," while the middle and subsidiary terms are "good." In this passage, then, he again confuses the issue somewhat by contradicting elements of his major premise, viz. "if a certain form of capacity be good [middle term, R] in a like degree to knowledge," in his minor premise and conclusion, viz. "if no form of capacity be good [subsidiary term, S], then neither is knowledge."

The error here, as already pointed out, is to use one and the same term (viz. "good," in this example) both as middle and as subsidiary. For the argument to be consistent and valid, these two must be distinct (the middle term might, say, be "valuable" and the subsidiary term "pursued," so that the argument reads: if a certain capacity is as valuable as knowledge, it follows that if no capacity is valuable enough to be pursued, then knowledge is not valuable enough to be pursued). Aristotle, then, is apparently not aware of this important rule, i.e. of the need to distinguish the middle and subsidiary terms.

We might more generously see, in Aristotle's affirmation of something in one premise and negation of it in the other, as a recognition by him of the possibility of using a term so abstractly that both its position (e.g. "good") and its negation ("not good") are included in it, as different degrees of it (above zero and zero or less, respectively). Looking at a term R in this way, we can both claim that P is more R than Q, and claim that P and Q are not R at all, without self-contradiction. This seems to be the thought in Aristotle's head, though he does not (here at least) make any explicit remark to that effect. To be sure, knowing that Aristotle is not prone to self-contradiction, this is a credible hypothesis.

It is worth noting too in this context that, although Aristotle associates a fortiori argument with the idea of greater, lesser or equal degrees, there is no evidence in the above cited passages of any notion of "sufficiency," i.e. of there being a threshold as of which predication occurs and before which it does not occur. This is an important deficiency in his treatment (if indeed, as I presume, he nowhere else mentions this feature of a fortiori argument). Had he been aware of the "sufficiency" issue (i.e. the need to have *enough* of the middle term for predication) in a fortiori inference, he would have quickly realized that the middle term mentioned in the major premise cannot reasonably be identical with the predicate inferred from the minor premise to the conclusion.

As we shall see further on, all but one of Aristotle's many a fortiori arguments in practice are formulated without the crucial feature of "sufficiency" of the middle term for predication. The one exception shows that Aristotle was

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[10]   See for instance a similar fault in the sample argument given by Alexander of Aphrodisias, further on.

slightly aware of this feature, but not enough to make it explicit in all his a fortiori discourse, and not enough to take it into consideration in his theorizing.

2. The Kneales' list

In their historical opus, *The Development of Logic*[11], William and Martha Kneale give seven references in Aristotle's *Topics* concerning a fortiori argument, namely: "ii. 10 (114^{b}37); iii. 6 (119 ^{b}17); iv. 5 (127 ^{b}18); v. 8 (137 ^{b}14); vi. 7 (145 ^{b}34); vii. 1 (152 ^{b}6); vii. 3 (154 ^{b}4)"[12]. I have above dealt in detail with the first two of these passages (namely, 2:10 and 3:6), which are the most interesting, in that Aristotle is in them effectively teaching us something about a fortiori argument. The remaining passages are less interesting: Aristotle uses rather than discusses a fortiori argument in two of them (namely, 4:6 and 7:3), while the rest (namely, 5:8, 6:7 and 7:1) have nothing to do with such argument but were only apparently listed because they contain a reference to degrees. Only the following two remaining passages, then, concern a fortiori argument:

Topics 4:6 – This chapter contains an a fortiori argument of positive subjectal form:

> "On the other hand, the comparison of the genera and of the species one with another is of use: e.g. supposing A and B to have a like claim to be genus, then if one be a genus, so also is the other. Likewise, also, if what has less claim be a genus, so also is what has more claim: e.g. if 'capacity' have more claim than 'virtue' to be the genus of self-control, and virtue be the genus, so also is capacity. The same observations will apply also in the case of the species. For instance, supposing A and B to have a like claim to be a species of the genus in question, then if the one be a species, so also is the other: and if that which is less generally thought to be so be a species, so also is that which is more generally thought to be so."

The reasoning here is: Given that A seems more fitting to be a genus (or a species) than B is, it follows that: if B seems so fitting that it may be declared a genus (or a species), then A must also be fitting enough for that; if A and B are equally fitting (parity), then the inference goes both ways. We can distinguish two moods (from minor to major, and *a pari*), each with two alternative middle terms (one for genus and one for species); but all four arguments have really one and the same thrust.

Topics 7:3 – This chapter contains a very similar a fortiori argument:

> "Moreover, look at it from the point of [*sic*][13] and like degrees, in all the ways in which it is possible to establish a result by comparing two and two together. Thus if A defines a better than B defines [b?] and B is a definition of [b?] so too is A of a. Further, if A's claim to define a is like B's to define b, and B defines b, then A too defines a. This examination from the point of view of greater degrees is of no use when a single definition is compared with two things, or two definitions with one thing; for there cannot possibly be one definition of two things or two of the same thing."

The reasoning here is: Given that A defines 'a' more fittingly than B does 'b', it follows that if B defines 'b' so fittingly that it may be declared the definition, then A defines 'a' must also be fitting enough for that; if Aa and Bb are equally fitting (parity), then the inference from Bb to Aa is also valid (more significantly, the reverse inference is also possible now: though Aristotle does not say so, he probably intended it). Here again, note, there is only really one argument, though it is worded in two ways.

We do not learn anything new about a fortiori argument from these two passages; they each give an example of a fortiori argument, rather than a discussion of it. I should perhaps, after all, say a bit more about the three passages listed by the Kneales that do not contain a fortiori arguments. Aristotle seems there and elsewhere[14] to have some beliefs about the degrees of things that I do not entirely agree with.

Consider for instance the following comment drawn from *Topics* 5:8:

> "Next look from the point of view of greater and less degrees… See, for destructive purposes, if P simply fails to be a property of S simply; for then neither will more-P be a property of more-S, nor less-P of less-S, nor most-P of most-S, nor least-P of least-S. … For constructive purposes, on the other hand, see if P simply is a property of S simply: for then more-P also will be a property of more-S, and less-P of less-S, and least-P of least-S, and most-P of most-S."

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[11]   Oxford, London: Clarendon, 1962. This is available (in part) online at Google Books: books.google.com/books?id=FtXAwgy1w9cC&printsec=frontcover&dq=Kneale&hl=en&ei=RV7ZTOONOZCbOpnd_fAI&sa=X&oi=book_result&ct=result&resnum=1&ved=0CCUQ6AEwAA#v=onepage&q&f=false. This is a great piece of work. Pity, though, that it contains so much material in the Greek or Latin original without English translation. Someone should remedy this and prepare a new edition.

[12]   On p. 42, fn. 4. Note that I assume that there was a typing error with regard to "iv. 5," and that the intent was really "iv. 6," since the former chapter has nothing of relevance in it, whereas the latter does. The text here reproduced is drawn from the Internet Classics Archive; translation by W. A. Pickard-Cambridge.

[13]   Presumably the original text said "from the point of view of greater, lesser and like degrees."

[14]   Just search for all occurrences in the *Rhetoric* and *Topics* text of the words "degree" or "greater," and you will find many cases.

What this, and more of the same (which I have left out, for brevity's sake), suggests is that Aristotle considers concomitant variation to be a universal law. According to him, if S is P, then to every degree of S there corresponds a comparable degree of P, and if such parallel increase and decrease in magnitude does not occur, then S is not P. This is highly to be doubted, in my view. In some cases, the same value of a predicate P is applicable to all values of a subject S. In some cases, a constant subject S has (over time) different degrees of a predicate P. The variations may be inverted, with increase on one side and decrease on the other, or vice versa. Many other complications are conceivable and occur in practice.

As an example of such inference that Aristotle gives us is: "Thus, inasmuch as a higher degree of sensation is a property of a higher degree of life, a lower degree of sensation also would be a property of a lower degree of life, and the highest of the highest and the lowest of the lowest degree, and sensation simply of life simply." Well, it may be true that degrees of sensation are proportional to degrees of life (whatever that means: presumably complexity of organization?), but I very much doubt that we can universally *infer* a concordance of lesser degrees from one of a higher degrees, and so on, as he apparently recommends. Perhaps he only means that such concomitant variation is a good working hypothesis, a probability to be verified empirically.

Again, consider the following comment drawn from *Topics* 4:6:

> "Moreover, judge by means of greater and less degrees: in overthrowing a view, see whether the genus admits of a greater degree, whereas neither the species itself does so, nor any term that is called after it… If, therefore, the genus rendered admits of a greater degree, whereas neither the species does so itself nor yet any term called after it, then what has been rendered could not be the genus."

Let G be a genus and S be a species, or a species of a species. The question here posed is whether G is or is not indeed a genus of S; or conversely, whether S is or is not indeed a species of G. The answer is sought through comparison of changes in magnitude; actually, only increase in magnitude is mentioned, not decrease (no explanation is given for this unreasonable stipulation). It is not clarified what is here increased – it seems to be the degree of G or S itself, rather than of some property thereof. The changes in degree seem to refer to comparisons of instances (extensional mode), rather than to changes over time (temporal mode).

Aristotle reasons syllogistically that if the genus is variable then the species must be variable too. But to my mind this is an error of logic. Surely a variable is a set of constants, in which case a genus may be variable and yet composed of species some or all of which are (different) constants. The error is to treat the predicate 'variable' as distributive, whereas it is here intended as collective – it applies to the class as a whole, not necessarily to any of its parts.

Such comments by Aristotle, though not directly relevant to a fortiori argument, have indirect relevance, since belief in the universality of concomitant variation would lead us to automatically draw an a crescendo conclusion from a fortiori premises, whereas in fact an appropriate pro rata argument is a formally required intermediary for such deduction. But as we have earlier seen (in the previous section, in the first passage of *Topics* 2:10), Aristotle explicitly (though without naming it) presents argument pro rata as inductive rather than deductive. It follows that he cannot (without self-contradiction) have here intended to suggest that pro rata argument always possible, i.e. formally universal for any terms. Thus, a fortiori argument must be distinguished from a crescendo.

About *a contrario*. In this context, I could additionally point to some of Aristotle's remarks in his *Rhetoric*, which give the impression that he advocates a contrario argument, which has some resemblance to a fortiori argument but is really very different. The following passage, drawn from the already mentioned chapter of *Rhetoric* will illustrate what I mean:

> "One line of positive proof is based upon consideration of the opposite of the thing in question. Observe whether that opposite has the opposite quality. If it has not, you refute the original proposition; if it has, you establish it. E.g. 'Temperance is beneficial; for licentiousness is hurtful'. Or…: 'If war is the cause of our present troubles, peace is what we need to put things right again'."

If we read this literally, we would suppose that 'If all X are Y, then *no* not-X is Y'. But such inversion, as Aristotle surely well knew, is not universally valid. We can only educe from 'all X are Y' (via: 'all not-Y are not-X') that '*some* not-X are not Y'; it remains possible that 'some not-X are Y'. So, we should view his remarks on such arguments as mere observations. They are presented as forms of rhetoric, rather than of logic, so as to point out noncommittally that people do use them, without intent to imply them to be necessarily valid.

The examples he gives seem credible enough, being particular causative arguments. Since licentiousness hurts, we should try temperance to diminish if not remove our pain. Since war causes troubles, we should try peace to diminish if not stop our malaise. These are only probable arguments, however, which do not guarantee that the desired change will occur. They are not, of course, a fortiori arguments, although they have some resemblance.

Compare the commonly used formulation of a fortiori argument: 'If Q, which is not R, is S, then, all the more, P, which is R, is S', with the following a contrario statement (which for the sake our present demonstration involves the

vaguer term 'something' in the places of P and Q): '**If something which is not-R is S, then something which is R is not-S**' – and it is easy to see the difference. In the former case (i.e. a fortiori), the predicate is S in both the antecedent and consequent, whereas in the latter case (i.e. a contrario), the predicate is S in the premise and not-S in the conclusion. The resemblance is thus quite superficial.

A contrario argument, like a fortiori, can be copulative or implicational. In the former case, it has the form: 'If X is Y, then not-X is not-Y'; and in the latter case, it has the form: 'If X implies Y, then not-X implies not-Y'. While such reasoning is sometimes applicable, it is not – to repeat – universally valid.

Finally, let me quote the Kneales' sole remark about Aristotle in relation to a fortiori argument:

> "…the theory of arguments a fortiori, or, as Aristotle says, 'from the more and the less'. This is a topic to which he refers many times and always in a way which suggests that he thinks of it as a well-recognized theme. It was natural, therefore, that he should wish to incorporate his views on the subject into his later work on logic, and it seems probable that this is what he had in mind when he spoke later of his intention to write on arguments 'according to quality' (*κατὰ ποιότητα*)." (Pp. 42-43.)

This comment suggests that Aristotle was rather interested in a fortiori argument and seemingly intended to treat the subject in more detail eventually. The Kneales do not specifically cite the passages in Aristotle's works they base these remarks on. As already mentioned, they do give a number of references in the *Topics*, but I do not see that these passages justify the above claims. Not that it matters greatly, but I would have liked to know what the Kneales meant more precisely. Because, judging by the texts analyzed above, Aristotle's involvement in theoretical a fortiori logic was not very intense.

3. Aristotle in practice

Let us now take a closer look at Aristotle's practice of a fortiori argument, which differs considerably from his theoretical treatment. For this purpose, I looked into all instances I could find of Aristotle's use of the argument[15]. See **Appendix 4** for a detailed list of citations[16]. These included 40 occurrences of the fifteen key phrases most often used to signal a fortiori discourse, namely: a fortiori (12), all the more (22), how much more (2), how much less (0), so much more (1), so much less (0), much more (2), much less (1), (how/so) much the more (0), (how/so) much the less (0). Plus 3 occurrences of more widely used character strings, namely: more so (1), less so (0), even more (2), even less (0). Additionally, I referred to the passages in Aristotle's *Rhetoric* and *Topics* found by the Kneales (see previous two sections), which contain numerous a fortiori arguments without use of the key phrases (except once), and found another 37 occurrences.

Altogether, I found in Aristotle's works, 80 cases of a fortiori argument, of which at least 11 were a pari (i.e. involved a major premise with equal major and minor terms). As could be expected, most cases, 48 to be exact, were positive subjectal in form; and indeed, of these 8 could be said to be a crescendo. Without surprise, another 22 cases were found to be negative subjectal. The interesting findings were that 5 cases were positive predicatal and 3 cases were negative predicatal; and moreover that 2 cases were antecedental. What these findings teach us is that, although Aristotle reasoned often enough in subjectal formats, which he mentions in his more theoretical exposés, he also occasionally reasoned in other formats, which he does not consciously distinguish in theoretical contexts.

Aristotle, as everyone knows, was Plato's star student. Examining the latter's main works, I found at least 15 instances of a fortiori discourse, 9 of them spoken (if we are to believe Plato) by Socrates, and the rest by others. Of these instances, 9 are positive subjectal in form (and of those, 4 seem to have an a crescendo intent), 1 is negative subjectal, 4 are negative predicatal, and 1 is negative consequental in form. These findings are based on computer searches for specific strings; more cases, involving other wording, may conceivably yet be found. These figures on Plato are also significant, assuming that Aristotle read these works (a fair assumption), since they are additional evidence that Aristotle did not closely examine all the data he had on hand when analyzing a fortiori argument. The corresponding findings for Aristotle are as follows:

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[15]   In a pdf copy of *The Works of Aristotle*. (Ed. William David Ross. Chicago: Encyclopædia Britannica, 1952.) Presumably, this contains all his extant works; as for works which may have been lost, nothing can be said, obviously.

[16]   It is interesting to note that I did not find (using the main key phrases) use, mention or discussion of a fortiori argument in the *Prior Analytics*.

Mood of a fortiori argument	Orientation	Number found	Of which a pari	Of which crescendo
Copulative				
Positive subjectal {+s}	from minor to major (Q-P)	48	7	8
Negative subjectal (–s)	from major to minor (P-Q)	22	4	
Positive predicatal {+p}	from major to minor (P-Q)	5		
Negative predicatal (–p)	from minor to major (Q-P)	3		
Implicational				
Positive antecedental (+a)	from minor to major (Q-P)	2		
Negative antecedental (–a)	from major to minor (P-Q)	0		
Positive consequental (+c)	from major to minor (P-Q)	0		
Negative consequental (–c)	from minor to major (Q-P)	0		
Totals		80	11	8

Table 6.1

Needless to say, the arguments are here classified on the basis of their apparent forms, without regard to the truth or falsehood of their contents.

As regards Aristotle's own use of predicatal argument, 1 case occurs in *On the Soul*, 1 case in *Parva Naturalia*, 1 case in *History of Animals*, 1 case in *Metaphysics*, 2 cases in the *Posterior Analytics*, and 2 cases in *Rhetoric*. For example: "But if the Soul does not, in the way suggested [i.e. with different parts of itself acting simultaneously], perceive in one and the same individual time sensibles of the same sense, *a fortiori* it is not thus that it perceives sensibles of different senses" (*Parva Naturalia*, 7). This has to be read as a predicatal argument[17], since the subjects of the minor premise and conclusion are one and the same (viz. "the soul") and their predicates are different (viz. "it perceives sensibles of the same sense" and "it perceives sensibles of different senses").

Aristotle's two uses of implicational argument (both positive antecedental) occur in *History of Animals*; notice that there is no use of negative antecedental or of positive or negative consequental argument. An example is: "Now, as the nature of blood and the nature of the veins have all the appearance of being primitive, we must discuss their properties first of all, and *all the more* as some previous writers have treated them very unsatisfactorily" (3:2). This has to be read as an implicational argument[18], because in the minor premise and conclusion, the antecedents and consequents contain different subjects and predicates, so that these propositions consist of theses implying theses.

Thus, judging by his extant works, Aristotle did not pay close attention to his own uses, or his teacher's uses, of a fortiori argument, when discussing this form of reasoning. Had he done so, he would have discovered predicatal argument and implicational argument.

Furthermore, as regards his 8 uses of a crescendo argument (all positive subjectal), it may be supposed that Aristotle uttered them in good faith, i.e. that he believed that in these specific cases proportionality was justified. But he apparently nowhere remarks on the important difference between purely a fortiori argument and the more elaborate a crescendo argument, even though he uses both these types of reasoning. That is to say, he does not formulate a rule comparable to the much later rabbinical "sufficiency (*dayo*) principle," according to which (in the simplest reading of it[19]) the conclusion of an a fortiori argument should exactly mirror its minor premise, and not indulge in proportionality (to which we should add: unless, of course, an appropriate pro rata argument can be additionally put forward to justify such proportionality).

It is noteworthy that, in all the instances of a fortiori argument I found in Plato and Aristotle works, only one instance contains the word 'enough' or 'sufficient'. The instance is found in Aristotle's work and reads: "But since even water

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[17]   I read the argument as: If the soul (S) is not versatile (R) enough to perceive simultaneously sensibles of the same sense (Q), then the soul (S) is not versatile (R) enough to perceive simultaneously sensibles of different senses (P). The required major premise is obviously: More versatility (R) is required for P than for Q.

[18]   I read the argument as: If the primitiveness of the properties of blood and veins (Q) implies urgency (R) enough for us to discuss them first (S), then their having been unsatisfactorily treated by past writers (P) implies urgency (R) enough for us to discuss them first (S). The required major premise is obviously: P implies more urgency (R) than Q.

[19]   In truth, as discussed elsewhere, the *dayo* principle is more complex and more specifically religious than here suggested, and we should rather refer to a larger 'principle of deduction'.

by itself alone, that is, when unmixed, will not *suffice* for food – for anything which is to form a consistency must be corporeal – , it is still much less conceivable that air should be so corporealized [and thus fitted to be food]" (*On Sense and the Sensible*, 5). This shows that Plato was unaware of this crucial feature of a fortiori argument, and Aristotle was a bit more but still barely aware of it.

Finally, it is interesting to note the following statistics: of the a fortiori arguments used by Aristotle, only 16 are logical-epistemic[20], the remaining 57 being ontical. What this tells us is that the impression given by *Rhetoric* 2:23 and *Topics* 2:10 that he regards a fortiori argument as essentially logical-epistemic is belied by his actual practice.

4. Relation to syllogism

One more important question to ask regarding Aristotle's theoretical treatment of a fortiori is whether he regarded such argument as capable of identification with syllogism. Wiseman[21] suggests that Aristotle did not make such an equation, saying:

> "Interestingly, Aristotle did not consider the a fortiori to be the same as his categorical syllogism; rather, he understands it as an analogic[al] device, unlike what we have encountered in some definitions so far that meant to show it as deductively valid. Perhaps Aristotle was the first to view the a fortiori as an inductive analogy."

As regards Wiseman's claim that Aristotle viewed a fortiori as a mere analogical device, I tend not to agree. Wiseman is basing this assumption, I take it, on the first of the above quoted paragraphs in *Topics* 2:10– which, as already pointed out, is not clearly about a fortiori argument (even though the next paragraph indeed is about it). Aristotle is here neither proposing a necessary deduction (a fortiori or other) nor suggesting a weaker argument by analogy – on the contrary, he is saying one cannot predict which way things will go ("See whether a greater degree of the predicate follows a greater degree of the subject…") and must resort to induction for the answer. Moreover, if we look at the earlier *Rhetoric* quotation, a different picture emerges.

As regards the suggestion that the two forms of argument are different, note that Wiseman does not *quote* Aristotle as saying so; he only theorizes it is so, based on the information available to him. I would certainly lean towards the same assumption, however. It would seem (given his extant works) that Aristotle did not ask himself or try to answer that specific question, about whether a fortiori argument is or is not a sort of syllogistic argument; had he done so, he would surely have stressed the fact explicitly, one way or the other. On the other hand, it could be argued that Aristotle tended to consider syllogism as the essential form of all argument (certainly many people after him seem to have thought he did so) – in which case he would not necessarily think he needed to specifically subsume a fortiori for us.

Consider now an example of a fortiori argument given by Aristotle in *Rhetoric* 2:23: *a man is less likely to strike his father than to strike his neighbors; therefore, if a man strikes his father, he is likely to strike his neighbors too.* We see here that Aristotle is aware of the major premise[22], as well as of the minor premise and conclusion. However, he does not discuss the real middle term, which tacitly underlies and would explain and justify the apparent middle term 'likely' that he takes for granted. *Why* is a man more likely to strike his neighbors than his own father? Because it is generally easier*, psychologically, socially and ethically* to strike one's neighbors than one's father. The apparent middle term 'likely' is based on an emotional and cultural fact (or at least, the assumption of such a fact).

A fortiori argument usually appears as essentially deductive – in the sense that given the premises we can confidently infer the conclusion – yet in the present example there is clearly a sense that the conclusion is at best probable. Why is that? Because it so happens that the example under scrutiny is about human volition, i.e. something that by nature cannot be predicted with certainty. A man may well generally find it easier to hit neighbors than his own father; but in truth, a man may consider the latter action as more legally permissible, being a private as against public matter, or again, he may out of cowardice hit on his weak old father more readily than he assaults his strong young neighbors.

Such actions are based on personal perceptions or belief systems, and depend on personal inclinations and conscience, and they are ultimately produced by freewill. For this reason, Aristotle indeed had to qualify things as only "likely" throughout his example. But such approximation is not inherent to a fortiori, but a function of the content in this particular sample. If we look at the other example Aristotle gives in the same passage of *Rhetoric* – *if*

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[20]   These are distributed as follows: 4 in *Rhetoric* (2:23), 8 in *Topics* (2:10, 7:3), 3 in *Posterior Analytics* (1:1, 1:3, 1:10), and 1 in *Metaphysics* (3:4). Note in passing that none of the a fortiori arguments used by Plato are logical-epistemic.

[21]   *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions*, p. 25.

[22]   Even though the other example here given, about the non-omniscience of gods and humans, does not likewise mention a major premise (namely that gods are more qualified to be omniscient than humans) at all. Needless to say, not verbalizing the major premise does not mean it is not mentally present in the background. This is true not only in a fortiori argument but in all reasoning (and is called abridged argument, or enthymeme). Much of our thought remains tacit, even if it has a logical impact on what we do verbalize.

even the gods are not omniscient, certainly human beings are not – it is clear that he sees the conclusion as certain[23], and not as a mere rough analogy[24].

We can thus, to conclude, say that since – as far as we know – Aristotle did not fully analyze a fortiori argument, he is not likely to have made a pronouncement as to whether it was the essentially same as syllogism or not; or, for that matter, as to whether it is deductive or merely analogical. The truth is, Aristotle was a genius who ranged far and wide in logic, philosophy and the special sciences, and touched upon a great many subjects, some of which he took time to look into more deeply and systematically, and some of which he only briefly considered in passing. Regarding a fortiori, the latter seems to be applicable. Moreover, of course, Aristotle was human, and however authoritative his viewpoints on many issues, he was not omniscient (as he readily admits in one of the said examples).

Whatever Aristotle may have or not have privately thought on the issue, my own formalization of a fortiori, presented in the preceding chapters, justifies our henceforth definitively adopting the position that Aristotle's categorical syllogism (and also for that matter hypothetical syllogism, which is very similar in overall form) is very different from copulative (or implicational, as the case may be) a fortiori argument, though the latter is also a form of deduction. Moreover, although we can correlate these two forms of argument in various ways, we cannot formally reduce either of them to the other; they are distinct and relatively independent movements of thought.

5. Cicero

Marcus Tullius Cicero (Rome, 106-43 BCE), who was an influential philosopher and jurist among many other things, left us some interesting reflections on a fortiori argument in his *Topics*[25]. Cicero there tells us (this was a year before his death) he composed the book as a commentary to Aristotle's work with the same name, from memory; but his treatment is distinctive. It seems to have been equally influenced by Aristotle's *Rhetoric* (II, 23) and by some later, Stoic texts[26]. Concerning argumentation in general, Cicero has this to say:

> "6. Every systematic treatment of argumentation has two branches, one is concerned with invention of arguments and the other with judgment of their validity; Aristotle was the founder of both in my opinion."

By "invention of arguments" he apparently means formulation of arguments. From his mention here of validation, we see that Cicero's interest was in logic, and not merely in rhetoric. He discusses in some detail all the arguments he lists, giving examples from Roman law practices. Arguments by comparison (i.e. a fortiori) are classified as arguments "from the things which are in some way closely connected with the subject," which in turn fall under the heading of arguments "inherent in the nature of the subject." This teaches us that Cicero looked upon a fortiori argument as essentially ontical, rather than as logical-epistemic. He introduces a fortiori argument in §23 as follows:

> "23. All arguments from comparison are valid if they are of the following character: what is valid in the greater should be valid in the less (*Quod in re maiore valet, valeat in minori*), as for example… Likewise the reverse: what is valid in the less should be valid in the greater (*Quod in minori valet, valeat in maiore*); the same example may be used if reversed. Likewise, what is valid in one of two equal cases should be valid in the other (*Quod in re pari valet valeat in hac quae par est*); for example… Equity should prevail, which requires equal laws in equal cases."

Cicero here apparently lists three varieties of the argument: from major to minor; from minor to major; and from equal to equal. Let us look at the examples here proposes for them. The first example concerns reasoning from major to minor: "since there is no action for regulating boundaries, there should be no action for excluding water in the city." This argument seems to be a negative subjectal; we can formalize it as follows:

> Regulating boundaries (P) is more serious a matter (R) than excluding water in the city (Q) is,
> yet, regulating boundaries (P) is *not* a serious matter (R) enough to justify an action (S);
> therefore, excluding water in the city (Q) is not a serious matter (R) enough to justify an action (S).[27]

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[23]   Though of course we might contend that, since gods do not exist and are figments of the imagination, his certainty was in fact unjustified. But our concern here is with inference – given the truth of the premises, does the conclusion's truth follow or not? This issue applies to all inference, not just to a fortiori.

[24]   Even if Aristotle goes on to abstract from this example a principle stated in terms of likelihood, the fact remains that the example itself is distinctively stated in terms of certainty.

[25]   *Topica*. Trans. H. M. Hubble. Cambridge, Mass. Harvard UP, 1949. The full text of this book in Latin, with an English translation, may be read online at: www.scribd.com/doc/45159491/Cicero-Topica.

[26]   See the Introduction, presumably written by the translator, H. M. Hubbell.

[27]   This matter is a bit obscure to us; a footnote explains that "boundaries" refers to five foot strips no man's land between estates, and "excluding water" refers to water diverted by one neighbor into another's property.

For reasoning from minor to major, Cicero unfortunately gives no example here, but only says "the same example may be used if reversed." It is not clear what "reversed" (*convertere*) here means. It surely does not mean simple conversion, for such argument would obviously be logically invalid[28]. That is, we can reasonably assume he is not suggesting that "since there is no action for excluding water in the city, there should be no action for regulating boundaries" follows from the preceding case. Therefore, he presumably intends a hypothetical contraposition of it: "if there was a possibility of action for excluding water in the city, there would be a possibility of action for regulating boundaries," which signifies: positive subjectal argument.

The example Cicero adduces for *a pari* argument is: "since use and warranty run for two years in the case of a farm, the same should be true of a (city) house. But a (city) house is not mentioned in the law, and is included with the other things use of which runs for one year"[29]. It is not clear to me what the intended conclusion is, here. The first sentence seems to conclude with equality; but the second sentence denies the equality. I think that the solution to that problem is simply that Cicero here proposes two *a pari* arguments, one positive and one negative. The first says hypothetically: "if farm and city house were equal, the law of the former would apply to the latter." The second says factually: "but since they are not equal, the law of the former does not apply to the latter."

Thus, to summarize, Cicero seems to have pointed to positive and negative subjectal a fortiori argument, including their *a pari* versions. What about the positive and negative predicatal moods? I do not think that we can judge on the basis of the examples he gives that Cicero consciously limited a fortiori to the subjectal moods, to the exclusion of the predicatal ones; or for that matter, that he intended to limit it to copulative forms, to the exclusion of implicational ones. He obviously simply stated three directions "from major to minor," "from minor to major," and "from equal to equal" – *unaware of* the distinctions between positive and negative, subjectal and predicatal, or copulative and implicational. In other words, let us not misinterpret his vagueness as an exclusive (or even inclusive) intent.

It seems that some of this ambiguity was corrected by later writers, judging by a maxim claimed by Mielziner to have been in use in 19th century jurisprudence[30]: "*Quod in minor valet, valebit in majori; et quod in majori non valet, nec valet in minori*" – meaning: "what avails in the less, will avail in the greater; and what will *not* avail in the greater, will not avail in the less." The similarity of this statement to Cicero's is striking, but so is the difference. Here, the minor to major case is consciously positive, since the major to minor case is explicitly negative. The trouble with this more precise later statement, however, is that (if it was intended as exhaustive) it effectively limits a fortiori reasoning to the subjectal mode, to the exclusion of the predicatal mode. But such exclusiveness may have been, and probably was, unintentional.

In fact, Cicero further expounds "the topic of comparison" in §68-71.

> "68… a definition and example were given above. Now, I must explain more fully how it is used. To begin with, comparison is made between things which are greater, or less or equal. And in this connexion, the following points are considered: quantity, quality, value, and also a particular relation to certain things."

He then goes on to clarify each of these considerations with many examples. I will reproduce here one example for each. For "quantity": "more 'goods' are preferred to fewer;" for "quality": "we prefer… the easy task to the difficult;" for "value": "we prefer… the stable to the uncertain;" for "relation to other things": "the interests of leading citizens are of more importance than those of the rest." Clearly, these considerations refer to possible contents of a fortiori argument: the examples he proposes are sample major premises.

The uniform 'X is preferred to Y,' format of his proposed major premises suggests to me that Cicero was only consciously aware of subjectal a fortiori argument; he did not consciously notice (though he might have in practice used) predicatal a fortiori argument. Granting this, it follows that when earlier Cicero referred to inference from major to minor, he did have in mind negative subjectal argument; and therefore for him inference from minor to major meant positive subjectal. Note also that the format is also always copulative, never implicational and the middle term is always 'preference' – one thing is preferable to another. This is a limitation which we might excuse by saying that Cicero had in mind disputes between people in front of a court.

We can thus guess the forms of argument Cicero had in mind to have been: given 'X is better than Y,' it follows that 'if Y is good enough for Z, then so is X' and 'if X is not good enough for Z, then neither is Y.' He also says: "70… And just as these are the things which in a comparison are regarded as the better, so the opposites of these are

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[28]   I found a commentary on this issue:
books.google.ch/books?id=p6vy4KUNfUEC&pg=PA124&lpg=PA124&dq=cicero+topica+23&source=bl&ots=0wxi2LvYti&sig=EFKrSHtZUhi 3P8klVoxkty9ZIHM&hl=en&ei=mxJ5TfaZJI32sga5mvXzBw&sa=X&oi=book_result&ct=result&resnum=3&ved=0CCMQ6AEwAg#v=onepage &q=fortiori&f=false.

[29]   It is explained in a footnote that a farm owner would sell the warranted use of his land for two years, after which the purchaser would acquire title by "adverse possession".

[30]   On p. 131, footnote 1. Mielziner gives as reference: "quoted by Coke on Littleton, 260."

regarded as worse." What he had in mind here is: since 'X is better than Y' is convertible to 'Y is worse than X,' it also follows that 'if X is bad enough for Z, then so is Y' and 'if Y is not bad enough for Z, then neither is X.' Cicero does not say this explicitly, but that is evidently what he means. Note that these alternate arguments are formally the same, i.e. just as subjectal.

Regarding *a pari* argument, he adds: "71. When equals are compared, there is no superiority or inferiority; everything is on the same plane." He gives a new example of it: "If helping one's fellow-citizens with advice and giving them active assistance are to be regarded as equally praiseworthy, then those who give advice and those who defend ought to receive equal glory. But the first statement is true, therefore the conclusion is also." Now, my impression here is that Cicero is having trouble formulating a sample *a pari* argument! What he has just put forward is not a fortiori argument, but simply apodosis: 'If A, then B; but A, therefore B.'

The correct formulation of an *a pari* argument would be, according to me: 'X is as good as Y, therefore: if X is good enough for Z, so is Y; and if Y is good enough for Z, so is X; and if either is not good enough for Z, neither is the other.' Or, to use Cicero's sample terms: 'Giving advice and actively assisting are equally praiseworthy, therefore: if either is praiseworthy enough to deserve glory, so is the other; and if either is not praiseworthy enough to deserve glory, neither is the other.' It seems that Cicero did not fully grasp this form.

Finally, we should note that Cicero does not mention anywhere the principle of deduction for purely a fortiori argument, according to which the subsidiary term should be identical in the conclusion to what it is in the minor premise, and not made 'proportional' (in an attempt to reflect the proportion between the major and minor terms). There is accordingly no mention by him of the a crescendo argument, where a 'proportional' conclusion is indeed allowed, being made possible by means of an additional premise about concomitant variation.

The rabbinical *dayo* (sufficiency) principle, being first mentioned in the Mishna Baba Qama 2:5, may be said to have appeared in Jewish legal discourse sometime in 70-135 CE at the latest, this being the period when R. Tarfon (who is mentioned in the said Mishna) was active. This principle, as we shall see, prohibits lawmakers from inferring a greater penalty for a greater crime from a lesser penalty for a lesser crime given in the Torah. I have not found evidence of a similar restriction in Cicero's *Topics*. However, Roman law does seem to have generated an apparently similar principle, which reads: "*In poenis bensignior est interpretatio facienda,*" meaning: in penalties, the more benign interpretation is to be applied[31].

I do not know when this principle first appeared in Roman law. If it was developed before or during Cicero's time, he would surely have mentioned it somewhere (in his *Topics* or elsewhere), being an expert in Roman law. If it emerged later, it might still have done so before it made its appearance in Jewish jurisprudence – or it may have come after. This historical question must be resolved by competent historians. In any case, it cannot be said with certainty that the law system where the principle appeared first influenced the law system where it appeared second. There could have been a common inspiration, or an inspiration from one to the other, or the two cultures could have arrived at the same idea independently[32].

To summarize, what is evident is that though Cicero had some knowledge of a fortiori argument, he was not conscious of all its forms (namely, predicatal and implicational forms); also, some of the forms he was conscious of (namely the *a pari*) he did not quite master. Moreover, the issue of 'proportionality' apparently eluded him. Another important observation we must make is there is no evidence of formalization or validation in Cicero's treatment of the subject, though he mentions the issue of "validity" at the beginning of his book. Thus, we must say that on the whole Cicero did not go much further than Aristotle as regards a fortiori logic. Still, he enriches the field a bit through his more conscious distinction between three variants of a fortiori argument (viz. major to minor, minor to major, and *a pari*) and his listing of various possible contents (quantity, quality, value and importance).

All this is certainly interesting historically, in that it gives us an idea of the state of knowledge and skill regarding a fortiori argument in Cicero's lifetime in Rome. Because Cicero was one of the foremost legal thinkers, lawyers and orators of his generation, we can reasonably consider his level as the 'state of the art' for his time and place, that is about three centuries after Aristotle in the Greco-Roman world. Needless to say, this is said on the basis of a spot check, and not on the basis of a thorough study of all the relevant literature in that region and period. There may well have been other logicians or rhetoricians who said more on a fortiori argument than we have discovered thus far.

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[31]   This is cited by Wiseman (p. 165). The reference he gives is: Digest of Justinian, no 49, in Albert Gautier, *Introduction to Roman Law for Studies in Canon Law*, (Rome: Faculty of Canon Law, St. Thomas University, 1994), page 154. I cannot compare and contrast this principle more precisely to the *dayo* principle, because I have not so far seen examples of just how it was used in practice.

[32]   Thus, Maccoby's suggestion, in his essays on the subject, that the *dayo* principle was an independent rabbinical production may turn out to be true, or false – it is not possible to tell which without more thorough research.

6. Alexander of Aphrodisias

The Kneales' account makes no mention of any discussion of a fortiori argument in the Hellenistic world in the centuries between Aristotle and Alexander of Aphrodisias, who was a 3rd century CE Peripatetic philosopher and commentator of Aristotle's works. In particular, they do not mention Cicero's contribution to the subject, which we presented in the previous section, even though they do examine his work on other topics. Obviously, then, their silence regarding a fortiori argument should not be interpreted to mean that there was no discussion of the subject; it could well just mean that they did not consider it important enough to mention. Anyway, as regards the said Alexander, the Kneales tell us the following, further to their earlier comments regarding the treatment of a fortiori argument by Aristotle:

> "From Alexander's explanation it appears that an argument of type (5), i.e. *κατὰ ποιότητα*, is an *a fortiori* argument with a general conditional premiss[33]. His example is:
>
>> If that which appears to be more sufficient for happiness is not in fact sufficient, neither is that which appears to be less sufficient.
>> Health appears to be more sufficient for happiness than wealth and yet is not sufficient.
>> Therefore wealth is not sufficient for happiness.
>
> The theory of arguments *κατὰ ποιότητα* was probably an attempt to systematize what Aristotle says of *a fortiori* arguments in various passages of his *Topics*" (p. 111).

I do not see that this remark tells us much more about Aristotle or about a fortiori argument, but I quote it to be exhaustive. As regards Alexander's example, I would rephrase it in standard format as follows:

> Health (P) is apparently more conducive to happiness (R) than wealth (Q) is.
> Health (P) is not conducive to happiness (R) sufficiently to actually produce happiness (S).
> Therefore, wealth (Q) is not conducive for happiness (R) sufficiently to actually produce happiness (S).

In this format, it is seen to be a valid negative subjectal (major to minor). Let us analyze Alexander's statement in detail, now. The Kneales' remark about this being "an *a fortiori* argument with a general conditional premiss" refers to the first proposition: "If that which appears to be more sufficient for happiness is not in fact sufficient, neither is that which appears to be less sufficient." If we look at this proposition, we see that it is only general regarding the major and minor terms P and Q (respectively, "more sufficient for happiness" and "less so"), but not general as regards the middle term R (which is specified as "sufficient for happiness"). Thus, it is only partly general. To be fully general, i.e. effectively a formal statement, the middle term should have been "something." That is to say, the proposition should have read: "If that which appears to be more sufficient for *something* is not in fact sufficient, neither is that which appears to be less sufficient."

In fact, therefore, since it is not "general" enough to be formal, this first proposition is redundant. Alexander's second and third propositions contain, without need of the initial not-quite-abstract statement, the whole concrete a fortiori argument. The second proposition, "Health appears to be more sufficient for happiness than wealth and yet is not sufficient," lists both the operative major premise ("Health appears to be more sufficient for happiness than wealth") and minor premise ("and yet [health] is not sufficient [for happiness]"); and the third proposition ("Therefore wealth is not sufficient for happiness") concludes the argument. Now, this is a well-constructed a fortiori argument, because it has an explicit middle term ("sufficient for happiness" – meaning, rather, conducive to happiness), relative to which the major and minor terms are compared, and it has two premises and a conclusion, and its minor premise and conclusion contain the idea of sufficiency (in negative form) for a certain result (actual happiness, in this case).

So this is on the whole a good effort by Alexander, although not perfect. The imperfections are (a) the first proposition, which is not general enough to count as a formal statement and therefore redundant (since the next proposition does the job just as well without it); (b) the lumping together of the operative major and minor premises into an apparently single statement (so that the different roles of the conjuncts in it are blurred); and (c) the use of the term "sufficient" in two senses: as 'conducive ' and 'enough (to actualize)'. The latter equivocation causes some confusion in the reading of Alexander's a fortiori argument, and is indicative of some confusion within him. It is indicative of a commonplace error, which we have already spotted in Aristotle's treatment – namely, the conflation of the middle and subsidiary terms, the failure to clearly distinguish them in view of their quite distinct roles in the argument.

Thus, all things considered, Alexander's statement is a well-constructed example of (subjectal) a fortiori argument, showing considerable implicit understanding of the form of inference – but it is not a successful explicit formalization, showing complete understanding. And of course, so far as we can tell from the Kneales' account, there

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[33]   *In Aristotelis An. Pr. Lib. I Commentarium*, ed. Wallies, *C.I.A.G.* ii (i), p. 265. (Footnote by the Kneales.)

is no effort at validation. This is all a bit surprising, since Alexander was an Aristotelian, and so presumably well acquainted with Aristotle's formal methods. We could regard Alexander's first, "general" proposition as his attempt at validation. He perhaps viewed this statement as justifying the inference from the second proposition to the third (much like in syllogism the general major premise justifies the inference from the minor premise to the conclusion). But though such application of a wider generality gives an impression of validation, it does not in fact constitute validation, since the wider generality remains unproved.

Still, Alexander's work is an improvement. He places more emphasis than Aristotle seems to have done on ontical a fortiori. He is also more advanced in his clear focus on sufficiency in the example quoted, whereas Aristotle does not use the word in the present context. Of course, several centuries separate the two. Note in passing that in Alexander's case we are already in Talmudic times (not that I suggest a causal relation between his thought and that of the rabbis – but the parallelism is interesting).

It is (according to Ventura[34]), be it said in passing, to this Alexander that we owe the Greek word *logika* in the sense of the modern term 'logic'. Previously, the word had rather the sense of 'dialectic' (e.g. as used by Cicero). Aristotle's word for what we call logic was 'analytic'; whence the titles of two of his works: *Prior Analytics* and *Posterior Analytics*. Alexander also inaugurated the term *Organon* to refer to a collection of Aristotle's logical works[35].

As for the Kneales, their failure to analyze the "general conditional premiss" sufficiently to realize its relative informality shows that they did not have an entirely clear idea of what constitutes formalization. For this reason, and because I have in the past found errors in their analyses in other contexts, I do not take for granted their following statement: "The theory of arguments κατά ποιότητα was probably an attempt to systematize what Aristotle says of *a fortiori* arguments in various passages of his *Topics*." They do not specify which passages. I would want to see these passages for myself before accepting that there is significant "systematization" in them. All we are shown here is a negative subjectal argument; there is no positive subjectal and there are no predicatal forms on display, to convince us that Alexander indeed achieved a systematic understanding. He made a valuable contribution, but I reserve judgment as to its full scope.

7. Historical questions

What is the precise history of a fortiori argument in ancient Greek, Roman and Hellenistic literature, whether philosophical, religious or secular? This question is always answered briefly and rather vaguely by historians of logic, if at all, because no one has apparently ever systematically researched the answers to it. In fact, this question should be asked for every type of argument, in every culture, if we want to be able to eventually trace the development of reasoning by human beings. But historical research into the a fortiori argument would be a good start, a good model, as it is a rather distinctive form of argument which is used and discussed in the said ancient Western civilizations though not so frequently as to be overwhelming. This is a scientific task, akin to biological research into a particular species of life in a particular environment, and it should be carried out with appropriate rigor and exhaustiveness.

The first step in such research would be collection of all relevant data. This means identifying the precise locations in various extant texts where such argument appears (in full or in part) to be *used*, and of course registering the argument made there in a data base so that it becomes henceforth readily available for future discussions. The literature[36] to be looked into dates from about the 8th century BCE to about the 5th century CE, in the Greco-Roman world, mainly in the Greek and Latin languages. Apart from actual occurrences of a fortiori argument, abstract *discussions* relating to the use of such argument must be identified and collected. Discussion of a form of argument signifies a higher degree of logical awareness than mere usage; and any attempts at theory, i.e. to formalize it, to find its varieties and to validate it, signify a higher level still. All these stages in logical awareness should obviously be distinguished, assuming instances of all of them are found.

Once the said raw data is collected, logicians can begin to sift through it and analyze its full significance. We can find out when and where the argument first and subsequently appeared within the period and region studied, and

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[34]   In his Introduction to Maimonides' *Terminologie Logique* (p. 14).

[35]   Ventura, p. 12, footnote 17. The term was later extended to include not only the said purely logical works, but related works like the *Categories*, *On Interpretation*, the *Topics* and *On Sophistical Refutations*. At one time, the *Rhetoric* and the *Poetics* were also (with some justification) included in the *Organon*, but later dropped out. Parts of the *Metaphysics* could have been included but were not.

[36]   Literature in whatever form, of course – including archaeological fragments, epigraphy and the like. Obviously, too, when dealing with second-hand information, distinction must be made between the date of a report and the alleged date of what is reported. Remember, too, that a lot of the early literature was oral for a long time before it was put in writing. Also, even written material changes a bit over time, during transcription or by deliberate editing or amplification. All such factors must of course be taken into consideration and specified when estimating historical dates.

what form it took in each case. We can follow the flowering of varieties of the argument over time and in different places, as practice becomes more sophisticated. We can distinguish the different contexts of usage: poetic, business, legal, philosophical, scientific. We can compare the frequencies of use of such argument in different cultures[37]. We can perhaps trace the travels of the argument from one culture or subculture to another, as it is passed on from one people or social group to another, along trade routes or through various kinds of intellectual influence (for examples, through a philosophical author or a religious holy book). We can hopefully perceive the dawning self-awareness of those using the argument, as they begin to marvel at it, discern its parts and try to understand how it functions.

Clearly, we have here a sketch of a very interesting and enriching research project that someone or some people could and should take up. Similar research should of course also be carried out for other periods of history and regions of the world.

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[37]   This certainly exists. There is no doubt that a fortiori argument plays a larger role in Jewish law deliberations than in those of any other culture, for instance. I would also suggest, as another example, comparison between colloquial use of a fortiori discourse in French and English; the French seem to me to use it much more often.

7. A fortiori in the Talmud

1. Brief history of a fortiori

There is credible written evidence that a fortiori argument was *in use* in very early times thanks to the Jewish Bible. Five instances are apparent in the Torah proper (the Five Books of Moses, or Pentateuch) and about forty more are scattered throughout the Nakh (the other books of the Bible). According to Jewish tradition, the Torah dates from about 1300 BCE (the time of the Exodus from Egypt and wanderings in the Sinai desert)[1], and subsequent Biblical books range in age from that time to about the 4th century BCE (the period of the return from Babylon of some of the captives after the destruction of the first Temple). The oldest apparent a fortiori (actually, a crescendo) argument in the Torah is the one formulated in Gen. 4:24 by Lamekh (before the deluge); while the oldest purely a fortiori argument is the one formulated in Gen. 44:8 by Joseph's brothers (patriarchal era). A fortiori arguments are also found in some of the latest books of the Bible (first exile period).

Of the 46 or so instances of a fortiori argument in the Tanakh (see **Appendix 1**), at least 10 were known to (i.e. were consciously *identified as such* by) the rabbis of the Talmud – so it is not surprising that this form of argument came to play such an important role in the development of Jewish law. The *qal vachomer* argument, as it is called in Hebrew, is mentioned in several lists of Talmudic hermeneutic principles. It is the first rule in the list of 7 attributed to **Hillel** (the Elder, Babylonia and Eretz Israel, c. 110 BCE-10 CE) and the first rule in the list of 13 attributed to **R. Ishmael** (ben Elisha, Eretz Israel, 90-135 CE), both of which are given at the beginning of the *Sifra* (a halakhic midrash, attributed by many to Rab, i.e. Abba Arika, 175–247 CE). It is also found (as rules 5 and 6) in the slightly later list of 32 rules of **R. Eliezer b. Jose ha-Gelili** (Eretz Israel, ca. 2nd cent. CE)[2], and among the much later 613 rules of the Malbim (Meïr Leibush ben Yechiel Michel Weiser, Ukraine, 1809-1879) in his work *Ayelet haShachar,* the introduction to his commentary on the Sifra.[3]

As regards historical source, there can be little doubt that the rabbis learned a fortiori argument from its use in the Tanakh – and not (as some commentators have suggested) from surrounding cultures (Greek, Roman, or whatever). We can be sure of that knowing that the Talmudic rabbis' attention was wholly turned towards Jewish Scriptures and oral tradition; and a fortiori arguments were clearly in use in these sources; and moreover, everyone agrees that the Torah, at least, antedates by several centuries the historical appearance of a fortiori argument in other cultures. This does not, of course, imply that the Greeks and other early users of a fortiori argument learned this form of reasoning from the Torah or other Jewish sources. There is no doubt that a fortiori argument arose independently in different cultures at different times, simply due to its being a natural form of human reasoning[4].

In the lists of Hillel and R. Ishmael, all that is offered is a title or heading: "*qal vachomer*," which is variously translated as light and heavy, easy and difficult, lenient and stringent, or minor and major. It should be said that the language of a fortiori argument in the Tanakh, though very varied (but not always distinctive, i.e. not always specifically reserved for such argument), does not include the words *qal vachomer*. This expression is presumably therefore of rabbinical origin. Two other expressions indicative of a fortiori discourse are also found in rabbinic

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[1]   Some historians, on the basis of debatable evidence or lack of evidence, claim the Torah to date from as late as the 8th cent. BCE. Even if this were true, it would signify a very early date for the a fortiori arguments present in it.
[2]   R. Eliezer's list is known indirectly from later texts. Jacobs characterizes it as "a post-Talmudic work" (in his *Rabbinic Thought in the Talmud*, p. 78, fn. 9). See en.wikipedia.org/wiki/R._Eliezer_ben_Jose_ha-Gelili. It is however a very significant work in that it includes the hermeneutic principles of **R. Akiva**, which rivaled those of R. Ishmael and yet were not (to my knowledge) collected in a list bearing his name. You can find all three lists in the Appendix to A. Schumann's Introduction to the *Judaic Logic* collection he edited.
[3]   You can easily find additional information on the various lists in a number of Wikipedia articles. Note that Hillel, R. Ishmael and R. Eliezer b. Jose ha-Gelili were all three Tannaim, i.e. Mishnaic rabbis. (More accurately, Hillel is classified as pre-Tannaic, forming together with Shammai the last of the *zugot*, i.e. "pairs.")
[4]   It would perhaps be more accurate to postulate that a fortiori argument was first formulated far in prehistory, soon after language and logic first formed in the cognitive apparatus of the human species; but it stood out as a recognizable meme at different times in different cultures during the historic period.

literature: *kol she ken* (which seems to be the Hebrew equivalent of 'all the more so') and *al achat kama vekama* (which seems to be the Hebrew equivalent of 'how much the more').[5]

The term *qal vachomer* is somewhat descriptive, in the way of a hint – but note well that it is certainly *not a description of a fortiori argument in formal terms*, and it *does not validate* or even discuss the validity of the argument (but, obviously, takes it for granted). The list of R. Eliezer b. Jose ha-Gelili is not much more informative in that respect than those of its predecessors, since it only adds that *qal vachomer* may be *meforash* (i.e. explicit) or *satum* (i.e. implicit)[6]. Other early rabbinic literature does not go much further in elucidating the definition and more theoretical aspects of *qal vachomer*; it is all taken for granted.

Rather, the form and operation of a fortiori argument are taught through concrete examples. Ten Biblical examples of the argument, four in the Torah and six elsewhere[7], are listed in *Genesis Rabbah* (in Heb. *Bereshith Rabbah*), a midrashic work (closed ca. 400-450 CE) attributed by tradition to R. Oshia Rabba (d. ca. 350 CE). This just says: "R. Ishmael taught: [There are] ten a fortiori arguments recorded in the Torah" (92:7), and lists the ten cases without further comment. But of course, the main teaching of such argumentation is through the practice of the rabbis. There are a great many concrete examples of a fortiori reasoning in the Talmud and other rabbinic literature[8], which incidentally serve to clarify the form for future generations.

There is, however, one passage of the Talmud which is very instructive as to how the rabbis theoretically understood the *qal vachomer* (a fortiori type) argument and the *dayo* (sufficiency) principle related to it – and that is pp. 24b-25a and further on pp. 25b-26a of the tractate Baba Qama (meaning: 'first gate'), which is part of the order of *Neziqin* ('damages'). For the time being we shall concentrate on this important passage. We shall have occasion further on in the present volume to consider and explore some other significant Talmudic a fortiori arguments.

However, this book makes no claim to constituting an exhaustive study of this subject. Nonetheless, while I must confess being largely ignorant of the 'Sea of the Talmud', I believe the present contribution will be found very valuable due to the considerable extent and depth of new logical insight it contains. We shall in the present chapter, further on, describe in detail just what the said passage of the Talmud reveals. But first permit me to prepare you, the reader, with some background information and analysis, so that you come properly armed to the crux of the matter.

The Talmud (meaning: the teaching) in general consists of a series of rabbinical discussions on various legal and other topics stretching over centuries, roughly from about the 1st century BCE to about the 5th century CE. It has two essential components: the first and historically earliest stratum (ca. 200 CE) is the **Mishna** (meaning: repetition) and the second and later stratum is the **Gemara** (meaning: completion)[9]. The Gemara is a commentary (in Aramaic) on the Mishna (which is in Hebrew), clarifying, explaining and amplifying it[10].

The compiling and editing of the Mishna (whose participants are known as Tannaim, teachers) is traditionally attributed to R. Yehudah HaNassi (d. 219 CE), while the redaction of the Gemara (whose participants are known as Amoraim, expounders) took more time and was the work of many (until ca. 500 CE). This refers to the main, Babylonian (*Bavli*) Talmud, with which we are here concerned; there is an earlier, less authoritative compilation

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[5]   Feigenbaum, in his *Understanding the Talmud* (pp. 88-90), explains the terminology more precisely as follows. The expression *qal vachomer* is Tannaic. The premise is introduced by them saying *mah* or *umah* and the conclusion is signaled by *eino din she*, or *al achat kama vekama*, or *lo kol she ken*. Amoraim on the other hand, use *tashta* before the premise and *mibaya* or *tserikha lemeimar* before the conclusion. R. Nosson Dovid Rabinovich, in his *M. Mielziner's Talmudic Terminology* (Jerusalem: Ahavath Torah, 1988 – pp. 69-70), presents the matter slightly differently.

[6]   This is of course an important distinction to note, because it indicates that rabbis were already aware quite early that a fortiori argument is in practice not always as fully verbalized as it could and ought to be. Indeed, the Biblical examples of such argument are typically not fully verbalized (to various degrees), so they did not need to look far to realize the fact.

[7]   I list and analyze these ten examples in detail in my *Judaic Logic* (chapters 4, 5 and 6). I show there that one of the cases listed, viz. Esther 9:12, is doubtfully a fortiori. More important, I show there that there are at least another twenty cases of a fortiori in the Bible, one of which is in the Torah, Genesis 4:24. See summary of these and more recent findings in **Appendix 1** to the present volume.

[8]   Precisely how many concrete cases of *qal vachomer* argument there are in the Talmud and related documents has never, to my knowledge, been researched. This gigantic task should imperatively be done by someone – not just anyone, but someone with the needed logical knowhow. Indeed, the precise location and form of all rabbinic use of all explicit and implicit hermeneutic principles needs to be researched, so that a fully scientific assessment of Talmudic logic can be effected. The Babylonian and Jerusalem Talmuds should also be compared in this respect, though the latter contains much less commentary than the former. Although I unfortunately have never learned Hebrew and Aramaic well enough to take up the task in the original languages, I hope one day to at least try and draw up a rough list in English based on perusal of the Soncino Talmud.

[9]   In between Mishna and Gemara is the **Tosefta** (ca. 300 CE), a later supplement to the Mishna that the Gemara sometimes refers to for additional information.

[10]   The term Talmud is often taken as equivalent to the term Gemara, for whereas the Mishna is published separately, the Gemara is always published in conjunction with the Mishna since the Gemara's purpose is to comment on the Mishna. But I think the correct use is to say Talmud when referring to the conjunction, and Gemara when referring specifically to the commentary, as one says Mishna when referring to the older material.

known as the Jerusalem (*Yerushalmi*) – or more precisely put, the Land of Israel[11] – Talmud (closed ca. 350-400 CE).[12]

The genesis of these various documents is an interesting historical issue, which has received much attention over time and more critical attention in modern times. Their redactors are thought to have been numerous and stretched out over centuries[13]. Some of the individuals involved are known by tradition, others remain anonymous. They should not, of course, be viewed as standing outside looking in on the collective discursive process they describe. Some of them were without doubt active or passive contemporary participants in some of the Talmudic discussions they report. But even those who do not fall in the category of eye-witnesses must be considered as effectively participants, albeit sometimes centuries after the fact, since by their selection, ordering and slanting of scattered material, their paraphrases and explanations, not to mention their outright interpolations, they necessarily affect our perceptions of the presumed original discussions. It would be a grave error to regard such redactors as entirely self-effacing, perfectly objective and impartial, contemporary observers and stenographers.

The Mishna and the Gemara[14] were conceived as written records of past and present oral legal (halakhic) and to a lesser extent, non-legal (haggadic) traditions. The rabbis (as we shall here indifferently call all participants) mentioned or implied in them did not all live at the same time and in the same place, note well. Their discussions were rarely face to face, but were brought together in one continuous document by the redactors, who were therefore perforce (albeit often invisibly) themselves important participants in the discussions, by virtue of their work of selection, structuring and commentary. Keep in mind this scattering in time and place of participants, and also the constant presence of the redactors in the background of all discourse[15]. Too often, traditional students of the Talmud approach it naïvely and idealistically as an essentially indivisible unit, somehow transcending time and space, perfectly harmonious.

There were perforce long periods of time when the traditions that were eventually put down in writing were transmitted by word of mouth. It must be considered whether such transmission was always perfect, or whether some elements were lost, transformed or added along the way. While it is true that people in those days were more used to memorizing things than we are today, and that they used various mnemonic devices to do so, one may still reasonably assume that some change in the information occurred over time if only unwittingly. Also, as Louis Jacobs has pointed out[16], in the name of I. H. Weiss, with reference to modern day scholars who are able to recite the whole of the Talmud by heart, it is surely easier to memorize a document one has read than to memorize information never seen in written form. It should also be considered that people naturally vary in intelligence, and students often do not understand all that their teachers do, and indeed sometimes students understand more than their teachers do. In short, the oral tradition should never be looked upon as some static solid phenomenon, but rather as a living mass subject to some change over time.

2. A brief course in the relevant logic

Before we examine any Talmudic text in detail, we need to briefly clarify the logical point of view on a fortiori argument. This clarification is a necessary propaedeutic, because many of the Talmudists and students of the Talmud who may choose to read this essay are probably not acquainted with any objective analysis of the underlying logic, having only been trained in rabbinical ways, which are rarely very formal. The treatment proposed in the present

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[11]   Some call it the Palestinian Talmud, because the Land of Israel was at the time of its formation under Roman rule and the Romans chose to rename Judea "Palestine" (more precisely, the Roman emperor Hadrian so decreed after the Bar Kochba rebellion). But it is wise to stop using this name, because it has nowadays, after intense propaganda efforts by anti-Israeli journalists and revisionist "historians," become associated with current Arab inhabitants of the Jewish homeland, to make them seem like natives (or even aborigines).

[12]   See Neusner for a more detailed exposition of these various documents and their interrelationships. I cannot here, of course, get into discussions about dating that emerge from the different modern theories of Talmudic formation, including those of Abraham Weiss and David Weiss-Halivni. This is not my field, though truly a fascinating one.

[13]   According to some commentators, the Talmud, though mainly the work the Tannaim and the Amoraim, may have received some further editing by the hand of some Savoraim (ca. 500-600 CE) and perhaps even some Geonim (ca. 600-1000 CE). Abraham Weiss considers that some editing was done in almost every generation, while David Weiss-Halivni attributes most of this work to those he calls the Stammaim (ca. 427 to 501 or 520 CE). See the interesting essays on these subjects in *Essential Papers on the Talmud*.

[14]   Individual sentences or topics in the Mishna are called *mishna* in the sing., *mishnayot* in the pl. Likewise for the Gemara: *gemara*, *gemarot*.

[15]   Note that when in the coming pages I refer to the Gemara's "author," I intend this singular term as very vague. It could be taken to refer to some anonymous Amora(s) whose ideas the Gemara just reports, or it could refer to the later redactor(s) injecting his/their own ideas. It is by no means clear in either case whether one person was involved or many; and if they were many, it is not clear whether they cooperated as a team, or they simply succeeded each other, each modifying or adding to the work of his predecessor. Moreover, keep in mind that the author(s) of one *sugya* may be different from that/those of other *sugyas*, for all we know.

[16]   In his *Studies*, in a footnote on p. 60. Moreover, note that Maimonides considers, in the introduction to his *Commentary on the Mishnah*, that "it is not possible for any person to remember the entire Talmud by heart" (p. 110).

section is of course minimal – much more can be learned about the a fortiori argument in other chapters of the present volume and in my past work called *Judaic Logic*.

Formal validation of a fortiori argument. The paradigm of a fortiori argument, the simplest and most commonly used form of it, is the positive subjectal mood[17], in which the major and minor terms (here always labeled P and Q, respectively) are subjects and the middle and subsidiary terms (here always labeled R and S, respectively) are predicates. It proceeds as follows[18]:

> P is R more than Q is R (major premise).
> Q is R enough to be S (minor premise).
> Therefore, P is R enough to be S (conclusion).

An example of such argument would be: "*If her father had but spit in her face, should she not hide in shame seven days? Let her be shut up without the camp seven days, and after that she shall be brought in again.*" (Num. 12:14). This can be read as: if offending one's father (Q) is bad (R) enough to deserve seven days isolation (S), then surely offending God (P) is bad (R) enough to deserve seven days isolation (S); the tacit major premise being: offending God (P) is worse (R) than offending one's father (Q).

This form of argument can be logically validated (briefly put) as follows. The major premise tells us that P and Q are both R, though to different measures or degrees. Let us suppose the measure or degree of R in P is Rp and that of R in Q is Rq – then the major premise tells us that: if P then Rp, and if Q then Rq, and Rp is greater than Rq (which in turn implies: if something is Rp then it is also Rq, since a larger number includes all numbers below it[19]). Similarly, the minor premise tells us that nothing can be S unless it has at least a certain measure or degree of R, call it Rs; this can be stated more formally as: if Rs then S and if not Rs then not S. Obviously, since Q is R, Q has the quantity Rq of R, i.e. if Q, then Rq; but here we learn additionally (from the "enough" clause) that Rq is greater than or equal to Rs, so that if Rq then Rs; whence, the minor premise tells us that if Q then S. The putative conclusion simply brings some of the preceding elements together in a new compound proposition, namely: if P then Rp (from the major premise) and if Rs then S and if not Rs then not S (from the minor premise), and Rp is greater than Rs (since Rp > Rq in the major premise and Rq ≥ Rs in the minor premise), so that if Rp then Rs; whence, if P then S. The conclusion is thus proved by the two premises (together, not separately, as you can see). So the argument as a whole is valid – i.e. *it cannot logically be contested*.

Having thus validated the positive subjectal mood of a fortiori argument, it is easy to validate the negative subjectal mood by *reductio ad absurdum* to the former. That is, keeping the former's major premise: "P is R more than Q is R," and denying its putative conclusion, i.e. saying: "P is R *not* enough to be S," we must now conclude with a denial of its minor premise, i.e. with: "Q is R *not* enough to be S." For, if we did not so conclude the negative argument, we would be denying the validity of the positive argument.

We can similarly demonstrate the validity of the positive, and then the negative, predicatal moods of a fortiori argument. In this form, the major, minor and middle terms (P, Q and R) are predicates and the subsidiary term (S) is a subject.

> More R is required to be P than to be Q (major premise).
> S is R enough to be P (minor premise).
> Therefore, S is R enough to be Q (conclusion).

An example of such argument would be: "*Behold, the money, which we found in our sacks' mouths, we brought back unto thee out of the land of Canaan; how then should we steal out of thy lord's house silver or gold?*" (Gen. 44:8). This can be read as: if we (S) are honest (R) enough to return found valuables (P), then surely we (S) are honest (R) enough to not-steal (Q); the tacit major premise being: more honesty (R) is required to return found valuables (P) than to refrain from stealing (Q).

Here the validation proceeds (again briefly put) as follows. The major premise tells us that iff (i.e. if only if) Rp then P, and iff Rq then Q, and Rp is greater than Rq (whence if Rp then Rq). The minor premise tells us additionally that

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if S then Rs, and (since it is "enough") Rs is greater than or equal to Rp (whence if Rs then Rp), from which it follows that if S then Rp; and since iff Rp then P, it follows that if S then P. From the preceding givens, we can construct the putative conclusion, using if S then Rs (from the minor premise), and Rs is greater than Rq (from both premises, whence if Rs then Rq); these together imply if S then Rq, and this together with iff Rq then Q (from the major premise) imply if S then Q. The conclusion is thus here again incontrovertibly proved by the two premises jointly. The negative predicatal mood can in turn be validated, using as before the method of *reductio ad absurdum*. That is, if the major premise remains unchanged and the putative conclusion is denied, then the minor premise will necessarily be denied; but since the minor premise is given and so cannot be denied, it follows that the conclusion cannot be denied.

Notice that the reasoning proceeds from minor to major (i.e. from the minor term (Q) in the minor premise, to the major term (P) in the conclusion) in the positive subjectal mood; from major to minor in the negative subjectal mood; from major to minor in the positive predicatal mood; and from minor to major in the negative predicatal mood. These are valid forms of reasoning. If, on the other hand, we proceeded from major to minor in the positive subjectal mood, from minor to major in the negative subjectal mood; from minor to major in the positive predicatal mood; or from major to minor in the negative predicatal mood – we would be engaged in fallacious reasoning. That is, in the latter four cases, the arguments cannot be validated and their putative conclusions do not logically follow from their given premises. To reason fallaciously is to invite immediate or eventual contradiction.

Note well that each of the four arguments we have just validated contains only four terms, here labeled P, Q, R, and S. Each of these terms appears two or more times in the argument. P and Q appear in the major premise, and in either the minor premise or the conclusion. R appears in both premises and in the conclusion. And S appears in the minor premise and in the conclusion. The argument as a whole may be said to be properly constructed if it has one of these four validated forms and it contains *only four terms*. Obviously, if any one (or more) of the terms has even slightly different meanings in its various appearances in the argument, the argument cannot truly be said to be properly constructed. It may give the illusion of being a valid a fortiori, but it is not really one. It is fallacious reasoning.

The above described a fortiori arguments, labeled subjectal or predicatal, relate to terms, and may thus be called 'copulative'. There are similar 'implicational' arguments, which relate to theses instead of terms, and so are labeled antecedental or consequental. To give one example of the latter, a positive antecedental argument might look like this:

> Ap (A being p) implies Cr (r in C) more than Bq (B being q) does,
> and Bq implies Cr enough for Ds (for D to be s);
> therefore, Ap implies Cr enough for Ds.

Notice the use of 'implies' instead of 'is' to correlate the items concerned. I have here presented the theses as explicit propositions 'A is p', 'B is q', 'C is r' and 'D is s', although they could equally well be symbolized simply as P, Q, R, and S, respectively. The rules of inference are essentially the same in implicational argument as in copulative argument.

The principle of deduction. This forewarning concerning *the uniformity throughout an argument of the terms used* may be expressed as a law of logic. It is true not just of a fortiori argument, but of all deductive argument (for instances, syllogism or apodosis). We can call this fundamental rule 'the principle of deduction', and state it as: *no information may be claimed as a deductive conclusion which is not already given, explicitly or implicitly, in the premise(s)*. This is a very important principle, which helps us avoid fallacious reasoning. It may be viewed as an aspect of the law of identity, since it enjoins us to acknowledge the information we have, as it is, without fanciful additions. It may also be considered as *the fifth law of thought*, to underscore the contrast between it and the principle of induction[20], which is the fourth law of thought.

Deduction must never be confused with induction. In inductive reasoning, the conclusion can indeed contain more information than the premises make available; for instance, when we generalize from some cases to all cases, the conclusion is inductively valid *provided and so long as* no cases are found that belie it. In deductive reasoning, on the other hand, the conclusion must be formally implied by the given premise(s), and no extrapolation from the given data is logically permitted. In induction, the conclusion is tentative, subject to change if additional information is

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[20]   In its most general form, this principle may be stated as: what in a given context of information appears to be true, may be taken to be effectively true, unless or until new information is found that puts in doubt the initial appearance. In the latter event, the changed context of information may generate a new appearance as to what is true; or it may result in some uncertainty until additional data comes into play.

found, *even if* such new data does not contradict the initial premise(s)[21]. In deduction, on the other hand, the conclusion is sure and immutable, so long as no new data contradicts the initial premise(s).

As regards the terms, if a term used in the conclusion of a deductive argument (such as a fortiori) differs *however slightly* in meaning or in scope from its meaning or scope in a premise, the conclusion is invalid. No equivocation or ambiguity is allowed. No creativity or extrapolation is allowed. If the terms are not exactly identical throughout the argument, it might still have some inductive value, but as regards its deductive value it has none. This rule of logic, then, we shall here refer to as 'the principle of deduction'.

The error of 'proportional' a fortiori argument. An error many people make when attempting to reason a fortiori is to suppose that the subsidiary term (S) is *generally* changed in magnitude in proportion (roughly) to the comparison between the major and minor terms (P and Q). The error of such **'**proportional' a fortiori argument, as we shall henceforth call it, can be formally demonstrated as follows.

Consider the positive subjectal mood we have described above. Suppose instead of arguing as we just did above, we now argue as do the proponents of such fallacious reasoning that: *just as 'P is more R than S' (major premise), so S in the conclusion (which is about P) should be greater than it is in the minor premise (which is about Q)*. If we adhered to this 'reasoning', we would have *two different subsidiary terms*, say S1 for the minor premise and S2 for the conclusion, with S2 > S1, perhaps in the same proportion as P is to Q, or more precisely as the R value for P (Rp) is to the R value for Q (Rq), so that S1 and S2 could be referred to more specifically as Sq and Sp. In that case, our argument would read as follows:

> P is R more than Q is R (major premise).
> Q is R enough to be S1 (minor premise).
> Therefore, P is R enough to be S2 (conclusion).

The problem now is that this argument would be difficult to validate, since it contains *five terms* instead of only four as before. Previously, the value of R sufficient to qualify as S was *the same* (viz. R ≥ Rs) in the conclusion (for P) as in the minor premise (for Q). Now, we have two threshold values of R for S, say Rs1 (in the minor premise, for Q) and Rs2 (in the conclusion, for P). Clearly, if Rs2 is assumed to be greater than Rs1 (just as Rp is greater than Rq), we cannot conclude that Rp > Rs2, for although we still know that Rp > Rq and Rq ≥ Rs1, we now have: Rp > Rs1 < Rs2, so that the relative sizes of Rp and Rs2 remain undecidable. Furthermore, although previously we inferred the "If Rs then S" component of the conclusion from the minor premise, now we have no basis for the "If Rs2 then S2" component of the conclusion, since our minor premise has a different component "If Rs1 then S1" (and the latter proposition certainly does not formally imply the former).[22]

It follows that the desired conclusion "P is R enough to be S2" of the proposed 'proportional' version of a fortiori argument is simply invalid[23]. That is to say, *its putative conclusion does not logically follow from its premises*. The reason, to repeat, is that we have effectively a *new* term (S2) in the conclusion that is not explicitly or implicitly given in the premises (where only S1 appears, in the minor premise). Yet deduction can never produce *new* information of any sort, as we have already emphasized. Many people find this result unpalatable. They refuse to accept that the subsidiary term S has to remain unchanged in the conclusion. They insist on seeing in a fortiori argument a profitable argument, where the value of S (and the underlying Rs) is greater for P than it is for Q. They want to 'quantify' the argument more thoroughly than the standard version allows.

We can similarly show that 'proportionality' cannot be inferred by positive predicatal a fortiori argument. In such case, the subsidiary term (S) is the subject (instead of the predicate) of the minor premise and conclusion. If that term is different (as S1 and S2) in these two propositions, we again obviously do not have a valid a fortiori argument, since our argument effectively involves five terms instead of four as required. We might have reason to believe or just imagine that the subject (S) is diminished in some sense in proportion to its predicates (greater with P, lesser with Q), but such change real or imagined has nothing to do with the a fortiori argument as such. S may well vary in meaning or scope, but if it does so it is not *due to* a fortiori argument as such. Formal logic teaches generalities, but

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[21]   For example, having generalized from "some X are Y" to "all X are Y" – if it is thereafter discovered that "some X are not Y," the premise "some X are Y" is not contradicted, but the conclusion "all X are Y" is indeed contradicted and must be abandoned.

[22]   Of course, if Rs1 was assumed as greater than Rs2, we would be able to infer that Rp > Rs2. But this is not the thrust of those who try to "quantify" a fortiori argument, since the proportion between P and Q would be inversed between Rs1 and Rs2. Moreover, the next objection, viz. that "If Rs2 then S2" cannot be deduced from "If Rs1 then S1," would still be pertinent.

[23]   I put the adjective 'proportional' in inverted commas because the proportion of S2 to S1 is usually not exactly equal to that of P to Q. But whether this expression is intended literally or roughly makes no difference to the invalidity of the argument, note well. If it is invalid when exact, as here demonstrated, then it is all the more so when approximate!

this does not mean that it teaches uniformity; it allows for variations in particular cases, even as it identifies properties common to all cases.

People who believe in 'proportional' a fortiori argument do not grasp the difference between knowledge by a specific deductive means and knowledge by other means. *By purely a fortiori deduction*, we can only conclude that P relates to precisely S, just as Q relates to S in the minor premise. But this does not exclude the possibility that *by other means*, such as observation or induction, or even a subsequent deductive act, we may find out and prove that the value of S relative to Q (S1) and the value of S relative to P (S2) are different. If it so happens that we *separately* know for a fact that S varies in proportion to the comparison of P and Q through R, we can *after* the a fortiori deduction *further process* its conclusion in accord with such additional knowledge[24]. But we cannot claim such further process as part and parcel of the a fortiori argument as such – it simply is not, as already demonstrated in quite formal terms.

Formal logic cuts up our long chains of reasoning into distinguishable units – called arguments – each of which has a particular logic, particular rules it has to abide by. Syllogism has certain rules, a fortiori argument has certain rules, generalization has certain rules, adduction has certain rules, and so on. When such arguments, whether deductive or inductive, and of whatever diverse forms, are joined together to constitute a chain of reasoning (the technical term for which is *enthymeme*), it may look like the final conclusion is the product of all preceding stages, but in fact it is the product of only the last stage. Each stage has its own conclusion, which then becomes a premise in the next stage. The stages never blend, but remain logically distinct. In this way, we can clearly distinguish the conclusion of a purely a fortiori argument from that of any other argument that may be constructed subsequently using the a fortiori conclusion as a premise.

Some of the people who believe that a fortiori argument yields a 'proportional' conclusion are misled by the wording of such conclusion. We say: "since so and so, therefore, *all the more*, this and that." The expression "all the more" seems to imply that the conclusion (if it concerns the major term) is *quantitatively more* than the minor premise (concerning the minor term). Otherwise, what is "more" about it? But the fact is, we use that expression in cases of major to minor, as well as minor to major. Although we can say "how much more" and "how much less," we rarely use the expression "all the less"[25] to balance "all the more" – the latter is usually used in both contexts. Thus, "all the more" is rather perhaps to be viewed as a statement that the conclusion is *more certain* than the minor premise[26]. But even though this is often our intention, it is not logically correct. In truth, the conclusion is always (if valid) *as* certain as the minor premise, neither more nor less. Therefore, we should not take this expression "all the more" too literally – it in fact adds nothing to the usual signals of conclusion like "therefore" or "so." It is just rhetorical emphasis, or a signal that the form of reasoning is 'a fortiori'.

The argument *a crescendo*. Although 'proportional' a fortiori argument is not formally valid, it is in truth *sometimes* valid. It is valid under certain conditions, which we will now proceed to specify. When these conditions are indeed satisfied, we should (I suggest) name the argument differently, and rather speak of 'a crescendo' argument'[27], so as to distinguish it from strict 'a fortiori' argument. We could also say (based on the common form of the conclusions of both arguments) that 'a crescendo' argument is a particular type of a fortiori argument, to be contrasted to the 'purely a fortiori' species of a fortiori argument. More precisely, a crescendo argument is *a compound* of strictly a fortiori argument and 'pro rata' argument. It combines premises of both arguments, to yield a special, 'proportional' conclusion.

The **positive subjectal** mood of a crescendo argument has three premises and five terms:

> P is more R than Q is R (major premise);
> and Q is R enough to be Sq (minor premise);
> and S varies in proportion to R (additional premise).
> Therefore, P is R enough to be Sp (a crescendo conclusion).

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[24]   A neutral example would be: suppose we know that product A is more expensive than product B; knowing a certain quantity of product B to cost $1000, we could only predict by purely a fortiori argument that the same quantity of product A will cost 'at least $1000'. But this would not prevent us from looking at a price list and finding the actual price of that quantity of product A to be $1250. However, such price adjustment would be an *after the fact* calculation based on the price list rates, and not an inference based on the a fortiori argument. In fact, once we obtained the price list we would not need the a fortiori argument at all.

[25]   Not to be confused with "none the less".

[26]   This is evident in the Latin expression *a fortiori ratione*, meaning 'with stronger reason'.

[27]   The term is of Italian origin, and used in musicology to denote gradual increase in volume.

The 'additional premise' tells us there is proportionality between S and R. Note that the subsidiary term (Sp) in the conclusion differs from that (Sq) given in the minor premise, although they are two measures or degrees of one thing (S). This mood can be validated as follows:

The purely a fortiori element is:

> P is more R than Q is R,
> and Q is R enough to be Sq.
> (Therefore, P is R enough to be Sq.)

To this must be added on the pro rata element:

> Moreover, if we are given that S varies in direct proportion to R, then:
> since the above minor premise implies that: if R = Rq, then S = Sq,
> it follows that: if R = more than Rq = Rp, then S = more than Sq = Sp.

Whence the a crescendo conclusion is:

> Therefore, P is R enough to be Sp.

If the proportion of S to R is direct, then Sp > Sq; but if S is inversely proportional to R, then Sp < Sq. The **negative** subjectal mood is similar, having the same major and additional premise, except that it has as minor premise "P is R not enough to be Sp" and as a crescendo conclusion "Q is R not enough to be Sq."

The **positive predicatal** mood of a crescendo argument has three premises and five terms:

> More R is required to be P than to be Q (major premise);
> and Sp is R enough to be P (minor premise);
> and S varies in proportion to R (additional premise).
> Therefore, Sq is R enough to be Q (a crescendo conclusion).

As before, the 'additional premise' tells us there is proportionality between S and R. Note that the subsidiary term (Sq) in the conclusion differs from that (Sp) given in the minor premise, although they are two measures or degrees of one thing (S). This mood can be validated as follows:

The purely a fortiori element is:

> More R is required to be P than to be Q,
> and Sp is R enough to be P.
> (Therefore, Sp is R enough to be Q.)

To this must be added on the pro rata element:

> Moreover, if we are given that R varies in direct proportion to S, then:
> since the above minor premise implies that: if S = Sp, then R = Rp,
> it follows that: if S = less than Sp = Sq, then R = less than Rp = Rq.

Whence the a crescendo conclusion is:

> therefore, Sq is R enough to be Q.

If the proportion of R to S is direct, then Rq < Rp; but if R inversely proportional to S, then Rq > Rp. The **negative** predicatal mood is similar, having the same major and additional premise, except that it has as minor premise "Sq is R not enough to be Q" and as a crescendo conclusion "Sp is R not enough to be P."

In practice, we are more likely to encounter subjectal than predicatal a crescendo arguments, since the subsidiary terms in the former are predicates, whereas those in the latter are subjects, and subjects are difficult to quantify. We can similarly construct four implicational moods of a crescendo argument, although things get more complicated in such cases, because it is not really the middle and subsidiary theses which are being compared but terms within them. These matters are dealt with more thoroughly in earlier chapters, and therefore will not be treated here.

From this formal presentation, we see that purely a fortiori argument and a crescendo argument are quite distinct forms of reasoning. The latter has the same premises as the former, *plus* an additional premise about proportion, which makes possible the 'proportional' conclusion. Without the said 'additional premise', i.e. with only the two premises (the major and the minor) of a fortiori argument, we cannot legitimately draw the a crescendo conclusion.

Thus, people who claim to draw a 'proportional' conclusion from merely a fortiori premises are engaged in fallacy. They are of course justified to do so, if they explicitly acknowledge, or at least tacitly have in mind, the required additional premise about proportion. But if they are unaware of the need for such additional information, they are definitely reasoning incorrectly. The issue here is not one of names, i.e. whether an argument is called a fortiori or a crescendo or whatever, but one of information on which the inference is based.

To summarize: Formal logic can indubitably validate properly constructed a fortiori argument. The concluding predication (more precisely, the subsidiary item, S) in such cases is identical to that given in the minor premise. It is not some larger or lesser quantity, reflecting the direct or inverse proportion between the major and minor items.

Such 'proportional' conclusion is formally invalid, if all it is based on are the *two* premises of a fortiori argument. To draw an a crescendo conclusion, it is necessary to have *an additional* premise regarding proportionality between the subsidiary and middle items.

Regarding the rabbis' *dayo* (sufficiency) principle. It is evident from what we have just seen and said that there is no formal need for a "*dayo* (sufficiency) principle" to justify a fortiori argument as distinct from a crescendo argument. It is incorrect to conceive, as some commentators do (notably the Gemara, as we shall see), a fortiori argument as a crescendo argument artificially circumvented by the *dayo* principle; for this would imply that the natural conclusion from the two premises of a fortiori is a crescendo, whereas the truth is that a fortiori premises can only logically yield an a fortiori conclusion. The rule to adopt is that to draw an a crescendo conclusion an additional (i.e. third) premise about proportionality is needed – it is *not* that proportionality may be assumed (from two premises) unless the proportionality is specifically denied by a *dayo* objection.

In fact, the *dayo* principle can conceivably 'artificially' (i.e. by Divine fiat or rabbinic convention) restrain only a crescendo argument. In such case, the additional premise about proportion is disregarded, and the conclusion is limited to its a fortiori dimension (where the subsidiary term is identical in the minor premise and conclusion) and denied its a crescendo dimension (where the subsidiary term is greater or lesser in the minor premise than in the conclusion). Obviously, if the premise about proportionality is a natural fact, it cannot logically ever be disregarded; but if that premise is already 'artificial' (i.e. a Divine fiat or rabbinic convention), then it can indeed conceivably be disregarded in selected cases. For example, though reward and punishment are usually subject to the principle of 'measure for measure', the strict justice of that law might conceivably be discarded in exceptional circumstances in the interest of mercy, and the reward might be greater than it anticipates or the punishment less than it anticipates.

Some commentators (for instance, Maccoby) have equated the *dayo* principle to the principle of deduction. However, this is inaccurate, for several reasons. For a start, according to logic, as we have seen, an a fortiori argument whose conclusion can be formally validated is necessarily in accord with the principle of deduction. In truth, there is no need to refer to the principle of deduction in order to validate the conclusion – the conclusion is validated by formal means, and the principle of deduction is just an ex post facto observation, a statement of something found in common to all valid arguments. Although useful as a philosophical abstraction and as a teaching tool, it is not necessary for validation purposes.

Nevertheless, if a conclusion was found not to be in accord with the principle of deduction, it could of course be forthwith declared invalid. For the principle of deduction is also reasonable by itself: we obviously cannot produce new information by purely rational means; we must needs get that information from somewhere else, either by deduction from some already established premise(s) or by induction from some empirical data or, perhaps, by more mystical means like revelation, prophecy or meditative insight. So obvious is this caveat that we do not really need to express it as a maxim, though there is no harm in doing so.

For the science of logic, and more broadly for epistemology and ontology, then, a fortiori argument and the 'limitation' set upon it by the principle of deduction are (abstract) natural phenomena. The emphasis here is on the word natural. They are neither Divinely-ordained (except insofar as all natural phenomena may be considered by believers to be Divine creations), nor imposed by individual or collective authority, whether religious or secular, rabbinical or academic, nor commonly agreed artificial constructs or arbitrary choices. They are universal rational insights, apodictic tools of pure reason, in accord with the 'laws of thought' which serve to optimize our knowledge.

The first three of these laws are that we ***admit facts as they are*** (the law of identity), ***in a consistent manner*** (the law of non-contradiction) ***and without leaving out relevant data pro or con*** (the law of the excluded middle); the fourth is the principle of induction and the fifth is that of deduction.

To repeat: for logic as an independent and impartial scientific enterprise, there is no ambiguity or doubt that an a fortiori argument that is indeed properly constructed, with a conclusion that exactly mirrors the minor premise, is valid reasoning. Given its two premises, its (non-'proportional') conclusion follows of necessity; that is to say, if the two premises are admitted as true, the said conclusion must also be admitted as true. Moreover, to obtain an a crescendo conclusion additional information is required; without such information a 'proportional' conclusion would be fallacious. A principle of deduction can be formulated to remind people that such new information is not producible *ex nihilo*; but such a principle is not really needed by the cognoscenti.

This may all seem obvious to many people, but Talmudists or students of the Talmud trained exclusively in the traditional manner may not be aware of it. That is why it was necessary for us here to first clarify the purely logical issues, before we take a look at what the Talmud says. To understand the full significance of what it says and to be able to evaluate its claims, the reader has to have a certain baggage of logical knowledge.

The understanding of *qal vachomer* as a natural phenomenon of logic seems, explicitly or implicitly, accepted by most commentators. Rabbi Adin Steinsaltz, for instance, in his lexicon of Talmudic hermeneutic principles, describes

qal vachomer as "essentially logical reasoning"[28]. Rabbi J. Immanuel Schochet says it more forcefully: "*Qal vachomer* is a self-evident logical argument"[29]. The equation of the *dayo* principle to the principle of deduction is also adopted by many commentators, especially logicians. For instance, after quoting the rabbinical statement "it is sufficient if the law in respect of the thing inferred be equivalent to that from which it is derived," Ventura writes very explicitly: "We are resting here within the limits of formal logic, according to which the conclusion of a syllogism must not be more extensive than its premises"[30].

However, as we shall discover further on, the main reason the proposed equation of the *dayo* principle to the principle of deduction is ill-advised is that it incorrect. There are indeed applications where the *dayo* imperative happens to correspond to the principle of deduction; but there are also applications where the two diverge in meaning. Commentators who thought of them as equal only had the former cases in mind when they did so; when we consider the latter cases, we must admit that the two principles are very different.

3.   A fresh analysis of the Mishna Baba Qama 2:5

In the Mishna Baba Qama 2:5, there is a debate between **the Sages** and **R. Tarfon** about the concrete issue of the financial liability of the owner of an ox which causes damages by goring on private property. This debate has logical importance, in that it reveals to a considerable extent skills and views of Talmudic rabbis with regard to the a fortiori argument. The Sages consider that he must pay for half the damages, whereas R. Tarfon advocates payment for all the damages[31].

The Sages (*hachakhamim*) are unnamed rabbis of Mishnaic times (Tannaim) and R. Tarfon is one of their colleagues (of the 3rd generation), who lived in Eretz Israel roughly in the late 1st – early 2nd century CE. We are not told how many were the Sages referred to in this Mishna (presumably there were at least two), nor who they were. The contemporaries of R. Tarfon include R. Eleazar b. Azariah, R. Ishmael b. Elisha, R. Akiva, and R. Jose haGelili; it is conceivable that these are the Sages involved in this debate. They are all big names, note; the latter three, as we have seen, produced hermeneutic principles. R. Tarfon, too, was an important and respected figure. So the debate between them should be viewed as one between equals.[32]

The Mishna (BQ 2:5) is as follows[33]:

> "What is meant by 'ox doing damage on the plaintiff's premises'? In case of goring, pushing, biting, lying down or kicking, if on public ground the payment is half, but if on the plaintiff's premises R. Tarfon orders payment in full whereas the Sages order only half damages.
>
> R. Tarfon there upon said to them: seeing that, while the law was lenient to tooth and foot in the case of public ground allowing total exemption, it was nevertheless strict with them regarding [damage done on] the plaintiff's premises where it imposed payment in full, in the case of horn, where the law was strict regarding [damage done on] public ground imposing at least the payment of half damages, does it not stand to reason that we should make it equally strict with reference to the plaintiffs premises so as to require compensation in full?
>
> Their answer was: it is quite sufficient that the law in respect of the thing inferred should be equivalent to that from which it is derived: just as for damage done on public ground the compensation [in the case of horn] is half, so also for damage done on the plaintiff's premises the compensation should not be more than half.
>
> R. Tarfon, however, rejoined: but neither do I infer horn [doing damage on the plaintiff's premises] from horn [doing damage on public ground]; I infer horn from foot: seeing that in the case of public ground the

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law, though lenient with reference to tooth and foot, is nevertheless strict regarding horn, in the case of the plaintiff's premises, where the law is strict with reference to tooth and foot, does it not stand to reason that we should apply the same strictness to horn?

They, however, still argued: it is quite sufficient if the law in respect of the thing inferred is equivalent to that from which it is derived. Just as for damage done on public ground the compensation [in the case of horn] is half, so also for damage done on the plaintiff's premises, the compensation should not be more than half."

This discussion may be paraphrased as follows. Note that only three amounts of compensation for damages are considered as relevant in the present context: nil, half or full; there are no amounts in between or beyond these three, because the Torah never mentions any such other amounts.

(a) R. Tarfon argues that in the case of damages caused by "tooth and foot," the (Torah based) law was lenient (requiring no payment) if they occurred on public ground and strict (requiring full payment) if they occurred on private ground – "*does it not stand to reason that*" in the case of damages caused by "horn," since the (Torah based) law is median (requiring half payment) if they occurred on public ground, then the law (i.e. the rabbis' ruling in this case) ought to likewise be strict (requiring full payment) if they occurred on private ground? Presented more briefly, and in a nested manner, this *first argument* reads as follows:

> If tooth & foot, then:
> > if public then lenient, and
> > if private then strict.
>
> If horn, then:
> > if public then median, and
> > if private then strict (R. Tarfon's putative conclusion).

R. Tarfon thus advocates full payment for damage on private property. The Sages disagree with him, advocating half payment only, saying "*dayo*—it is enough."

(b) R. Tarfon then tries another tack, using the same data in a different order, this time starting from the laws relating to public ground, where that concerning "tooth and foot" is lenient (requiring no payment) and that concerning "horn" is median (requiring half payment), and continuing: "*does it not stand to reason that*" with regard to private ground, since the law for "tooth and foot" damage is strict (requiring full payment), the law (i.e. the rabbis' ruling in this case) for "horn" damage ought to likewise be strict (requiring full payment)? Presented more briefly and in a nested manner, this *second argument* reads as follows:

> If public, then:
> > if tooth & foot then lenient, and
> > if horn then median.
>
> If private, then:
> > if tooth & foot then strict, and
> > if horn then strict (R. Tarfon's putative conclusion).

R. Tarfon thus advocates full payment for damage on private property. The Sages disagree with him again, advocating half payment only, saying "*dayo*—it is enough."

More precisely, they reply to him both times: "*it is quite sufficient that the law in respect of the thing inferred should be equivalent to that from which it is derived*" – meaning that only half payment should be required in the case under consideration (viz. damages by "horn" on private grounds). In Hebrew, their words are: דיו לבא מן הדין להיות כנדון (*dayo lavo min hadin lihiot kenidon*) – whence the name *dayo* principle[34].

Now, the first thing to notice is that these two arguments of R. Tarfon's contain the exact same given premises and aim at the exact same conclusion, so that to present them both might seem like mere rhetoric (either to mislead or out of incomprehension). The two sets of four propositions derived from the above two arguments (by removing the nesting) are obviously identical. All he has done is to switch the positions of the terms in the antecedents and transpose premises (ii) and (iii). The logical outcome seems bound to be the same:

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[34]   A comparable statement of the *dayo* principle is found in *Pesachim* 18b, whence we can say that it is intended as a statement of principle and not just as an *ad hoc* position.

(a) If tooth & foot and public, then lenient (i).
If tooth & foot and private, then strict (ii).
If horn and public, then median (iii).
If horn and private, then strict (R. Tarfon's putative conclusion).

(b) If public and tooth & foot, then lenient (same as (i)).
If public and horn, then median (same as (iii)).
If private and tooth & foot, then strict (same as (ii)).
If private and horn, then strict (same putative conclusion).

However, as we shall soon realize, *the ordering of the terms and propositions does make a significant difference*. And we shall see precisely why that is so.

(a) What is R. Tarfon's logic in **the first argument**? Well, it seems obvious that he is making some sort of argument *by analogy*; he is saying (note the identity of the two sentences in italics):

> Just as, in one case (that of tooth & foot), *damage in the private domain implies more legal liability than damage in the public domain* (since strict is more stringent than lenient).
> So, in the other case (viz. horn), we can likewise say that *damage in the private domain implies more legal liability than damage in the public domain* (i.e. given median in the latter, conclude with strict, i.e. full payment, in the former, since strict is more stringent than median).

Just as in one case we pass from lenient to strict, *so* in the other case we may well pass from median to strict[35]. Of course, as with all analogy, a generalization is involved here from the first case (tooth & foot being more stringent for private than for public) up to "all cases" (i.e. the generality in italics), and then an application of that generality to the second case (horn, thusly concluded to be more stringent for private than for public). But of course, this is an inductive act, since it is not inconceivable that there might be specific reasons why the two cases should behave differently. Nevertheless, if no such specific reasons are found, we might well reason that way. That is to say, R. Tarfon does have a point, because his proposed reasoning can well be upheld as an ordinary analogical argument. This might even be classified under the heading of *gezerah shavah* or maybe *binyan av* (the second or third rule in R. Ishmael's list of thirteen)[36].

The above is a rather intuitive representation of R. Tarfon's first argument by analogy. Upon reflection, this argument should be classified more precisely as a quantitative analogy or *pro rata* argument:

> The degree of legal liability for damage is 'proportional' to the status of the property the damage is made on, with *damage in the private domain implying more legal liability than damage in the public domain*.
> This is true of tooth and foot damage, for which liability is known to be nil (lenient) in the public domain and full (strict) in the private domain.
> Therefore, with regard to horn damage, for which liability is known to be half (median) in the public domain, liability may be inferred to be full (strict) in the private domain.

This argument, as can be seen, consists of three propositions: a general major premise, a particular (to tooth and foot) minor premise and a particular (to horn) conclusion. The major premise is, in fact, known by induction – a generalization of the minor premise, for all damage in relation to property status. But once obtained, it serves to justify drawing the conclusion from the minor premise. The pro rata argument as such is essentially deductive, note, even though its major premise is based on an inductive act. But its conclusion is nevertheless a mere rough estimate,

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[35]   Indeed, R. Tarfon could buttress his argument by pointing out that the latter transition is only half the distance, as it were, compared to the former. Alternatively, we could insist on 'proportionality' and say: from lenient (zero) to strict (full) the change is 100%, therefore from moderate (half) we should infer not just strict (full), which is only 50%, but 'stricter than strict', i.e. 150% payment! This is just pointed out by me to show that R. Tarfon's argument by analogy was more restrained than it could have been. Evidently, 100% is considered the maximum penalty by both parties; no punitive charges are anticipated.

[36]   I am here just suggesting a possibility, without any intent to make a big issue out of it. The advantage of this suggestion is that it legitimates R. Tarfon's line of reasoning as an application of *another* rabbinic hermeneutic principle. The format would be: 'just as private is stricter in the known case, so private should be stricter in the case to be determined'.

since the 'proportionality' it is based on is very loosely formulated. Notice how the minor premise goes from zero to 100%, whereas the conclusion goes from 50% to 100%[37].

The Sages, on the other hand, seem to have in mind, instead of this ordinary argument by analogy or pro rata argument, a more elaborate and subtle *a fortiori* argument of positive subjectal form. They do not explicitly present this argument, note well; but it is suggested in their reactions to their colleague's challenge. Their thinking can be construed as follows:

> Private domain damage (P) implies more legal liability (R) than public domain damage (Q) [as we know by extrapolation from the case of tooth & foot].
> For horn, public domain damage (Q) implies legal liability (R) enough to make the payment half (median) (S).
> Therefore, for horn, private domain damage (P) implies legal liability (R) enough to make the payment half (median) (S).

We see that the subsidiary term (S) is the same (viz. 'median', i.e. half payment) in the Sages' minor premise and conclusion, in accord with a fortiori logic; and they stress that conclusion in reply to R. Tarfon's counterarguments by formulating their *dayo* principle, viz. "it is quite sufficient that the law in respect of the thing inferred should be equivalent to that from which it is derived," to which they add: "just as for damage [by horn] done on public ground the compensation is half, so also for damage [by horn] done on the plaintiff's premises the compensation should not be more than half."[38]

We see also that the major premise of the Sages' *qal vachomer* is identical to the statements in italics of R. Tarfon's argument by analogy, i.e. to the major premise of his pro rata argument. In both R. Tarfon and the Sages' arguments, this sentence "private damage implies more legal liability than public damage" is based on the same generalization (from tooth & foot, in original premises (i) and (ii), as already seen) and thence applicable to the case under scrutiny (horn, for which proposition (iii) is already given)[39]. So both their arguments are equally based on induction (they disagreeing only as to whether to draw the conclusion (iv) or its contrary).

But the most important thing to note here is that the *same* premises (viz. (i), (ii) and (iii)) can be used to draw *contrary* conclusions (viz. full payment vs. half payment, respectively, for damage by horn on private grounds), according as we use a mere analogical or pro rata argument, like R. Tarfon, or a more sophisticated strictly a fortiori argument, like the Sages. This discrepancy obviously requires explanation. Since both arguments are built on the same major premise, produced by the same inductive act of generalization, we cannot explain the difference by referring to the inductive preliminaries.

The way to rationalize the difference is rather to say that the argument by analogy or pro rata is more approximate, being a mere projection of the *likely* conclusion; whereas the a fortiori argument is more accurate, distilling the *precise* conclusion inherent in the premises. That is to say, though both arguments use the same preliminary induction, the argument of R. Tarfon is in itself *effectively a further act of induction*, whereas the argument of the Sages is in itself *an act of pure deduction*. Thus, the Sages' conclusion is to be logically preferred to the conclusion proposed by R. Tarfon.

Note well that we have here assumed that R. Tarfon's first argument was merely analogical/pro rata, and that the Sages proposed a purely a fortiori argument in response to it. It is also possible to imagine that R. Tarfon *intended* a purely a fortiori argument, but erroneously drew a 'proportional' conclusion from it; in which case, the Sages' *dayo* objection would have been to reprove him for not knowing or forgetting (or even maybe deliberately ignoring) the principle of deduction, i.e. that such argument can only yield a conclusion of the same magnitude as the minor premise. However, I would not support this alternative hypothesis, which supposes R. Tarfon to have made a serious error of reasoning (or even intentionally engaged in fallacy), because it is too far-fetched. For a start, R. Tarfon is an important player throughout the Mishna, someone with in general proven logical skills; moreover, more favorable readings of this particular argument are available, so we have no reason to assume the worst.

Another possible reading is that R. Tarfon's first argument was not merely analogical/pro rata but was *intended as a crescendo*, i.e. as a combination of a fortiori argument with pro rata argument, which can be briefly presented as follows:

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[37]   Because, to repeat, judging by Torah practice, it can go no further – i.e. there is no "150%" penalty.
[38]   The words "by horn" in square brackets added by me; but they are in accord with the interpolation in the Soncino edition.
[39]   Note that the general major premise of the Sages' *qal vachomer* can be stated more specifically as "for horn" – in which case, since the minor premise and conclusion are both specified as "for horn," the whole a fortiori argument can be considered as conditioned by "for horn" and this condition need not be specified as here done for each proposition in it.

Private domain damage (P) implies more legal liability (R) than public domain damage (Q) [as we know by extrapolation from the case of tooth & foot].

For horn, public domain damage (Q) implies legal liability (Rq) enough to make the payment half (median) (Sq).

The payment due (S) is 'proportional' to the degree of legal liability (R).

Therefore, for horn, private domain damage (P) implies legal liability (Rp) enough to make the payment full (strict) (Sp = more than Sq).

In that case, the *dayo* statement by the Sages may be viewed as *a rejection of the additional premise about 'proportionality'* between S (the subsidiary term) and R (the middle term) in the case at hand. That would represent them as saying: while proportionality might seem reasonable in other contexts, in the present situation it ought not to be appealed to, and we must rest content with a purely a fortiori argument. The advantage of this reading is that it conceives R. Tarfon as from the start of the debate resorting to the more sophisticated a fortiori type of argument, even though he conceives it as specifically a crescendo (i.e. as combined with a pro rata premise). The Sages prefer a purely a fortiori conclusion to his more ambitious a crescendo one, perhaps because it is easier to defend (i.e. relies on less assumptions), but more probably for some other motive (as we shall see).

(b) So much for the first argument; now let us examine **the second argument**. This, as many later commentators noticed, and as we shall now demonstrate, differs significantly from the preceding. The most important difference is that, here, the mere argument by analogy (or argument pro rata, to be more precise), the purely a fortiori argument and the a crescendo argument (i.e. a fortiori and pro rata combo), *all three yield the same conclusion*. Note this well – it is crucial. The second analogical argument proceeds as follows:

Just as, in one case (that of the public domain), *damage by horn implies more legal liability than damage by tooth & foot* (since median is more stringent than lenient).

So, in the other case (viz. the private domain), we can likewise say that *damage by horn implies more legal liability than damage by tooth & foot* (i.e. given strict in the latter, conclude with strict, i.e. full payment, in the former, since strict is 'more stringent than' [here, as stringent as[40]] strict).

This argument is, as before, more accurately represented as a pro rata argument:

The degree of legal liability for damage is 'proportional' to the intentionality of the cause of damage, with *damage by horn implying more legal liability than damage by tooth & foot.*

This is true of the public domain, for which liability is known to be nil (lenient) for damage by tooth and foot and half (median) for damage by horn.

Therefore, with regard to the private domain, for which liability is known to be full (strict) for damage by tooth and foot, liability may be inferred to be full (strict) for damage by horn.

This argument visibly consists of three propositions: a general major premise, a particular (to the public domain) minor premise and a particular (to the private domain) conclusion. The major premise is, in fact, inductive – a generalization of the minor premise, for all damage in relation to intentionality (in horn damage the ox intends to hurt or destroy, whereas in tooth and foot damage the negative consequences are incidental or accidental). But once obtained, the major premise serves to justify drawing the conclusion from the minor premise. Here again, the 'proportionality' is only rough; but in a different way. Notice how the minor premise goes from 0% to 50%, whereas the conclusion goes from 100% to 100%.

The purely a fortiori reading of this second argument would be as follows:

Horn damage (P) implies more legal liability (R) than tooth & foot damage (Q) [as we know by extrapolation from the case of public domain].

For private domain, tooth & foot damage (Q) implies legal liability (R) enough to make the payment full (strict) (S).

Therefore, for private domain, horn damage (P) implies legal liability (R) enough to make the payment full (strict) (S).

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[40]   Note that whereas in the first argument by analogy the movement is 'from median to strict', in the second argument by analogy the movement is 'from strict to strict'. Assuming here again that 100% payment is the maximum allowed. Otherwise, if we insisted on 'proportionality', arguing that just as the increase from lenient (zero) to median (half) is 50%, so the increase from strict (full) ought to be 50%, we would have to conclude an 'even stricter' penalty of 150%!

Note that *the conclusion would be the same if this argument was constructed as a more elaborate a crescendo argument*, i.e. with the additional pro rata premise "The payment due (S) is 'proportional' to the degree of legal liability (R)." The latter specification makes no difference here (unlike in the previous case), because (as we are told in the minor premise) the minimum payment is full and (as regards the conclusion) no payment greater than full is admitted (by the Torah or rabbis) as in the realm of possibility anyway. Thus, whether we conceive R. Tarfon's second argument as purely a fortiori or as a crescendo, its conclusion is the same. Which means that the argument, if it is not analogical/pro rata, is essentially a fortiori rather than a crescendo.

Observe here the great logical skill of R. Tarfon. His initial proposal, as we have seen, was an argument by analogy or pro rata, which the Sages managed to neutralize by means of a logically more powerful a fortiori argument; or alternatively, it was an a crescendo argument that the Sages (for reasons to be determined) limited to purely a fortiori. This time, R. Tarfon takes no chances, as it were, and after judicious reshuffling of the given premises offers an argument which yields the same strict conclusion whether it is read as an argument by analogy (pro rata) or a more elaborate a crescendo – or as a purely a fortiori argument. A brilliant move! It looks like he has now won the debate; but, surprisingly, the Sages again reject his conclusion and insist on a lighter sentence.

Note well *why* R. Tarfon tried a second argument. Here, the stringency of the target law (viz. horn in the private domain) is *equal to* (and not, as in his first argument, greater than) the stringency of the source law (viz. tooth & foot in the private domain); i.e. both are here 'strict'. This makes R. Tarfon's second argument consistent with a fortiori logic and with the *dayo* principle that the Sages previously appealed to, since now "the law in respect of the thing inferred" is apparently "equivalent to that from which it is derived." Yet, the Sages reiterate the *dayo* principle and thus reject his second try. How can they do so?

What is odd, moreover, is that the Sages answer both of R. Tarfon arguments *in exactly the same words*, as if they did not notice or grasp the evident differences in his arguments. The following is their identical full reply in *both* cases:

> "It is quite sufficient that the law in respect of the thing inferred should be equivalent to that from which it is derived: just as for damage done on public ground the compensation is half, so also for damage done on the plaintiff's premises the compensation should not be more than half."

<div dir="rtl">(אמרו לו דיו לבא מן הדין להיות כנדון מה ברה"ר חצי נזק אף ברשות הניזק חצי נזק)</div>

One might well initially wonder if the Sages did not perchance fail to hear or to understand R. Tarfon's second argument; or maybe some error occurred during the redaction of the Mishna or some later copying (this sure does look like a 'copy and paste' job!). For if the Sages were imputing a failure of *dayo* to R. Tarfon's second argument, in the same sense as for the first argument, they would not have again mentioned the previous terms "public ground" for the minor premise and "the plaintiff's premises" for the conclusion, but instead referred to the new terms "tooth and foot" and "horn." But of course, we have no reason to distrust the Sages and must therefore assume that they know what they are talking about and mean what they say.

Whence, we must infer that *the Sages' second dayo remark does not mean exactly the same as their first one*. In the first instance, their objection to R. Tarfon was apparently that if the argument is construed as strictly a fortiori, the conclusion's predicate must not surpass the minor premise's predicate; in this sense, the *dayo* principle simply corresponds to the principle of deduction, as it naturally applies to purely a fortiori argument. Alternatively, if R. Tarfon's first argument is construed as pro rata or as a crescendo, the Sages' first *dayo* objection can be viewed as rejecting the presumption of 'proportionality'. However, such readings are obviously inappropriate for the Sages' *dayo* objection to R. Tarfon's second argument, since the latter however construed is fully consistent with the *dayo* principle in either of these senses.

How the second *dayo* differs from the first. An explanation we can propose, which seems to correspond to a post-Talmudic traditional explanation[41], is that the Sages are focusing on *the generalization* that precedes R. Tarfon's second argument. The major premise of that argument, viz. "Horn damage implies more legal liability than tooth & foot damage" was derived from two propositions, remember, one of which was "In the public domain, *horn* damage entails *half* payment" (and the other was "In the public domain, tooth & foot damage entails no payment"). R. Tarfon's putative conclusion after generalization of this comparison (from the public domain to all domains), and a further deduction (from "In the private domain, tooth & foot damage entails full payment"), was "In the private domain, *horn* damage entails *full* payment." Clearly, in this case, the Sages cannot reject the proposed deduction, since it is faultless however conceived (as analogy/pro rata/a crescendo or even purely a fortiori). What they are

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[41]   In the notes in the Artscroll Mishnah Series, Seder Nezikin Vol. I(a), Tractate Bava Kamma (New York: Mesorah, 1986), the following comment is made regarding 2:5 in the name of Rav: "Even in this [second] *kal vachomer*, we must resort to the fact that keren [i.e.horn] is liable in a public domain; otherwise, we would have no *kal vachomer*." Other commentators mentioned in this context are: Tos. Yom Tov, Nemmukei Yosef, Rosh and Rambam.

saying, rather, is that *the predicate of its conclusion cannot exceed the predicate* (viz. half payment) *of the given premise involving the same subject* (viz. horn) *on which its major premise was based.*

We can test this idea by applying it to R. Tarfon's first argument. There, the major premise was "Private domain damage implies more legal liability than public domain damage," and this was based on two propositions, one of which was "For tooth & foot, *private* domain damage entails *full* payment" (and the other was "For tooth & foot, public domain damage entails no payment"). R. Tarfon's putative conclusion after generalization of this comparison (from tooth & foot to all causes), and a further deduction (from "For horn, public domain damage entails half payment"), was "For horn, *private* domain damage entails *full* payment." Clearly, in this case, the Sages cannot object that the predicate of its conclusion exceeds the predicate of the given premise *involving the same subject* (viz. private domain, though more specifically for tooth & foot) on which its major premise was based, since they are the same (viz. full payment). Their only possible objection is that, conceiving the argument as purely a fortiori, the predicate of the conclusion cannot exceed the predicate (viz. half payment) *of the minor premise* (i.e. "For horn, public domain damage entails half payment"). Alternatively, conceiving the argument as pro rata or a crescendo, they for some external reason (which we shall look into) reject the implied proportionality.

Thus, the Sages' second objection may be regarded as introducing an extension of the *dayo* principle they initially decreed or appealed to, applicable to any generalization preceding purely a fortiori argument (or possibly, pro rata or a crescendo arguments, which as we have seen are preceded by the same generalization). The use and significance of generalization before a fortiori argument (or eventually, other forms of argument) are thereby taken into consideration and emphasized by the Sages. This does not directly concern the a fortiori deduction (or the two other possible arguments), note well, but only concerns an inductive *preliminary* to such inference. However, without an appropriate major premise, no such argument can be formed; in other words, the argument is effectively blocked from taking shape.

The question arises: how is it possible that by merely reshuffling the given premises we could obtain two different, indeed conflicting, a fortiori (or other) conclusions? The answer is that the two major premises were constructed on the basis of *different directions of* generalization[42]. In the first argument, the major premise is based *entirely* on tooth & foot data, and we learn something about horn *only* in the minor premise. In the second argument, the major premise relies *in part* on horn data, and the minor premise tells us *nothing* about horn. Thus, the two preliminary generalizations in fact cover quite different ground. This explains why the two a fortiori processes diverge significantly, even though the original data they were based on was the same.

The first *dayo* objection by the Sages effectively states that, if R. Tarfon's first argument is construed as purely a fortiori, the conclusion must logically (i.e. by the principle of deduction) mirror the minor premise; alternatively, construing it as pro rata or a crescendo, the needed 'proportionality' is decreed to be forbidden (for some reason yet to be dug up). For the second argument, which has one and the same conclusion however construed (whether a fortiori or other in form), the Sages' *dayo* objection cannot in the same manner refer to the minor or additional premise, but must instead refer to the inductive antecedents of the major premise, and constitute a rule that the conclusion cannot exceed in magnitude such antecedents. This explains the Sages' repetition of the exact same sentence in relation to both of R. Tarfon's arguments.

A problem and its solution. There is yet one difficulty in our above presentation of the Sages' second *dayo* objection that we need to deal with.

As you may recall, the first dialogue between R. Tarfon and the Sages could be described as follows: R. Tarfon proposes an a crescendo argument concluding with full payment for damage by horn on private property, whereas the Sages conclude with half payment through the purely a fortiori argument leftover after his tacit premise of 'proportionality' is rejected by their *dayo*. That is, they effectively say: "The payment due (S) is *not* 'proportional' to the degree of legal liability (R)." Thus, the first exchange remains entirely within the sphere of a fortiori logic, despite the *dayo* application.

But the second dialogue between these parties cannot likewise be entirely included in the sphere of a fortiori logic, because the final conclusion of the Sages here is not obtained by a fortiori argument. Since the effect of their second *dayo* objection is to block the formation by generalization of the major premise of R. Tarfon's second a fortiori argument, it follows that once this objection is admitted his argument cannot proceed at all; for without a general major premise such argument cannot yield, regarding horn damage on private property, a conclusion of half compensation any more than a conclusion of full compensation. Yet the Sages do wish to conclude with half compensation. How can they do so?

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[42]   To give a simpler example, for the reader's assistance: suppose we are given that 'Some X are Y'; this is equivalent to 'Some Y are X'. In such case we have two possible directions of generalization: to 'All X are Y', or to 'All Y are X'. Clearly, while the sources of these two results are logically identical, the two results are quite different.

The answer to the question is, traditionally, to refer back to the Torah passage on which the argument is based, namely Exodus 21:35: "And if one man's ox hurt another's, so that it dieth; then they shall sell the live ox, and divide the price of it; and the dead also they shall divide". This signifies half compensation for horn damage without specifying the domain (public or private) in which such damage may occur – thus suggesting that the compensation may be the same for both domains. In the above two a fortiori arguments, it has been assumed that the half compensation for horn damage applies to the public domain, and as regards the private domain the compensation is unknown – indeed, the two a fortiori arguments and the objections to them were intended to settle the private domain issue.

This assumption is logically that of R. Tarfon. Although the said Torah passage seems to make no distinction between domains with regard to damage by horn, R. Tarfon suspects that there is a distinction between domains by analogy to the distinction implied by Exodus 22:4 with regard to damage by tooth and foot (since in that context, only the private domain is mentioned[43]). His thinking seems to be that the owner of an ox has additional responsibility if he failed to preempt his animal from trespassing on private property and hurting other animals in there. So he tries to prove this idea using two arguments.

The Sages, for their part, read Exodus 21:35 concerning horn damage as a general statement, which does not distinguish between the public and private domains; and so they resist their colleague's attempt to particularize it. For them, effectively, what matters is that two oxen belonging to two owners have fought, and one happened to kill the other; it does not matter who started the fight, or where it occurred or which ox killed which – the result is the same: equal division of the remaining assets between the owners, as the Torah prescribes. Effectively, they treat the matter as an accident, where both parties are equally faultless, and the only thing that can be done for them is to divide the leftovers between them.

Clearly, if compensation for horn damage on public grounds could be more than half (i.e. if half meant at least half), R. Tarfon could still (and with more force) obtain his two 'full compensation' conclusions (by two purely a fortiori arguments), but the Sages' two *dayo* objections would become irrelevant. In that event, the conclusion regarding horn damage would be full compensation on *both* the public and private domains. But if so, why did the Torah specify *half* compensation ("division" in two)? Therefore, the compensation must at the outset be only half *in at least one domain*. That this would be the public domain rather than the private may be supposed by analogy from the case of tooth and foot[44]. This is a role played by the major premise of the first argument. This means that the first argument (or at least, its major premise) is needed *before* the formulation of the second. They are therefore not independent arguments, but form (in part) a chain of reasoning (a sorites) – and their order of appearance is not as accidental as we might initially have thought.

It should be realized that the assumption that the liability for horn damage on private property is equal to or greater than same on public grounds is not an *a priori* truth. It is *not unthinkable* that the liability might be less (i.e. zero) in the former case than in the latter. Someone might, say, have argued that the owner of the private property, whose animal was gored there, was responsible to prevent other people's oxen from entering his property (e.g. by fencing it off), and therefore does not deserve any compensation! In that case, it would be argued that on public grounds he deserves half compensation because he has no control over the presence of other people's oxen thereon. In this perspective, the onus would be on the property owner, rather than on the owner of the trespassing ox.

Given this very theoretical scenario, it would no longer be logically acceptable to generalize from the liability for damage by tooth & foot, which is less (zero) on public ground and more (full) on private ground, and to say that liability for damage of any sort (including by horn) is greater in a private domain than in the public domain. However, this scenario is not admitted by the rabbis (I do not know if they even discuss it; probably they do not because it does not look very equitable[45]). Therefore, the said generalization is accepted, and serves to determine the compensation for damage by horn on private property in both arguments. In the first argument, this generalization

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[43]   "If a man cause a field or vineyard to be eaten, and shall let his beast loose, and it feed in another man's field; of the best of his own field, and of the best of his own vineyard, shall he make restitution."
[44]   Note that although Ex. 22:4 only mentions the private domain, it is taken to imply the opposite penalty for the public domain. That is to say, if we take it to mean that damage by tooth & foot in the private domain must be compensated in full, then we can infer *from the non-mention* here or elsewhere of the public domain that this level of compensation does not apply. This is called a *davka* (literal) reading of the text. Although strictly speaking the denial of 'full' may mean either 'only half' or 'zero' compensation, the rabbis here opt for an extreme inversion, i.e. for zero compensation for tooth & foot damage in the public domain. Presumably, their thinking is that if half compensation was intended in this case, the Torah would have said so explicitly, since there is no way to arrive at that precise figure by inference.
[45]   Another very theoretical possibility is that the compensation, which as we have argued must be only half in at least one domain (since the Torah specifies equal division of remains), is half in the private domain and either nil or full in public domain. It could be argued that it is nil in the public domain because the owner of the killed ox should have watched over his animal, or that it is full in the public domain because the owner of the killing ox should have watched over his animal. These logical possibilities are also ignored no doubt because they do not look equitable: they make one party seem more responsible than the other.

(from tooth & foot damage to all damage) produces the major premise. In the second argument, it serves only to eliminate in advance the possibility of zero compensation in such circumstance.

Thus, we can interpret the Torah as teaching that compensation for horn damage is generally *at least* half – and more specifically, *no more than* half on public grounds and *no less than* half on private property. Thereafter, the issue debated in the Mishna is whether the latter quantity is, in the last analysis, 'only half' or 'more than half (i.e. full)' compensation. Both parties in the Mishna take it for granted that the half minimum is a maximum as regards public grounds; but they leave the matter open to debate as regards its value on private property. R. Tarfon tries, in his second argument, to prove that the compensation in such circumstance ought to be full, by comparison to the law relating to tooth & foot damage in the same circumstance. But the Sages, interdict his major premise by saying *dayo*, in view of the textual data that premise was based on, and thus opt for only half compensation.

Following this *dayo*, note well, the Sages' conclusion is not obtained by a modified a fortiori argument, since (as already mentioned) such an argument cannot be formulated without an appropriate major premise, but is obtained by mere *elimination*. Their form of reasoning here is negative disjunctive apodosis (*modus tollens*):

> The appropriate compensation for horn damage on private property is, according to the Torah, at least (*lav davka*) half, i.e. *either* only half *or* full.
> But it cannot be proved to be full (since the major premise of R. Tarfon's attempt to do so by a fortiori cannot be sustained due to a *dayo* objection).
> Therefore, it must be assumed to be only (*davka*) half (as the Sages conclude).

It should be said that this reasoning is not purely deductive, but contains an inductive movement of thought – namely, the generalization *from* the failure to prove full compensation specifically through R. Tarfon's a fortiori argument in the light of the Sages' renewed *dayo* objection *to* the impossibility henceforth to prove full compensation by any means whatever. This is a reasonable assumption, since we cannot perceive any way that the *dayo* might be avoided (i.e. a way not based on the given of half compensation for damage by horn on public grounds[46]); but it is still a generalization. Therefore, the apodosis is somewhat inductive; this means that further support for the Sages' conclusion of only half compensation for damage by horn on private property would be welcome.

Thus, strictly speaking, in the last analysis, although a fortiori argument is attempted in the second dialogue, it is not finally used, but what is instead used and what provides us with the final conclusion is a disjunctive argument.

The essence of the *dayo* principle. We can thenceforth propose a more inclusive formulation of the Sages' *dayo* principle, which merges together the said two different cases, as follows. ***Whenever (as in the present debate) the same original propositions can, via different directions of preparatory induction and/or via different forms of deduction, construct two or more alternative, equally cogent arguments, the chain of reasoning with the less stringent final result should be preferred***. This, I submit, is to date the most accurate, all-inclusive statement of the *dayo* principle formulated on the basis of this Mishnaic *sugya*.

In the light of this broader statement of the *dayo* principle, we can read the two applications given in the present debate as follows. In the first argument, where there was a choice between a pro rata or a crescendo argument with a stringent conclusion, and a purely a fortiori argument with a median conclusion, the Sages chose the latter argument, with the less stringent conclusion, as operative. In the second argument, where all three forms of argument yielded the same stringent conclusion, the Sages referred instead to the preliminary generalization; in this case they found that, since the terms of one of the original propositions generalized into the major premise corresponded to the terms of the putative final conclusion, and the former proposition was less stringent than the latter, one could not, in fact, perform the generalization, but had to rest content with the original proposition's degree of stringency in the final one.

In the first instance, the *dayo* principle cannot refer to the inductive antecedent of the argument, because that original proposition does not have the same terms as the final conclusion, however obtained; so we must look at the form of the deductive argument. In the second instance, the *dayo* principle cannot refer to the deductive argument, since whatever its form it results in the same the final conclusion; so we must look at the preliminary generalization preceding such argument. Thus, one and the same *dayo* principle guides both of the Sages' *dayo* objections. Their teaching can thus be formulated as follows: 'Given, in a certain context, an array of equally cogent alternative arguments, the one with the less stringent conclusion should be adopted'.

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[46]   Actually, I believe I have found such a way. We could use the *kol zeh assim* argument proposed by Tosafot to put the Sages' *dayo* principle in doubt, at least in the present context. See my analysis of this possibility in a later chapter (9.7). Even though I do there decide that the *dayo* principle trumps the *kol zeh assim* argument, it remains true that this at least proves the Sages' conclusion to be inductive rather than deductive.

In other words, the *dayo* principle is a *general* guideline to opt for the less stringent option whenever inference leaves us a choice. It is a principle of *prudence*, the underlying motive of which seems to be moral – *to avoid any risk of injustice in ethical or legal or religious pronouncements based on inference*. We could view this as a guideline of inductive logic, insofar as it is a safeguard against possible human errors of judgment. It is a reasonable injunction, which could be argued (somewhat, though not strictly) to have universal value. But in practice it is probably specific to Judaic logic; it is doubtful that in other religions, let alone in secular ethical or legal contexts, the same restraint on inference is practiced.

An alternative translation of the Sages' *dayo* principle that I have seen, "*It is sufficient that the derivative equal the source of its derivation*," is to my mind very well put, because it highlights and leaves open the variety of ways that the "derivation" may occur in practice. The *dayo* principle, as we have seen, does not have one single expression, but is expressed differently in different contexts. The common denominator being apparently an imperative of caution, preventing too ready extrapolation from given Scriptural data. In the last analysis, then, the *dayo* principle is essentially not a logical principle, but rather a moral one. It is a Torah or rabbinical decree, rather than a law of logic. As such, it may conceivably have other expressions than those here uncovered. For the same reason, it could also be found to have exceptions that do not breach any laws of logic. Traditionally, it is deemed as applicable in particular to *qal vachomer* argument; but upon reflection, in view of its above stated essential underlying motive or purpose, it is evident that it could equally well in principle apply to other forms of argument. Such issues can only be definitely settled empirically, with reference to the whole Talmudic enterprise and subsequent developments in Jewish law.

Alternative scenarios. Our proposed scenario for the Mishna debate is thus as follows. R. Tarfon starts the discussion by proposing a first argument, whose form may be analogical/pro rata or a crescendo, which concludes with the imperative of full payment in the case of horn damage in the private domain. The Sages, appealing to a *dayo* principle, interdict the attempted 'proportionality' in his argument, thus effectively trumping it with a purely a fortiori argument, which concludes with a ruling of half payment. In response, R. Tarfon proposes a second argument, based on the very same data, which, whether conceived as analogical/pro rata or a crescendo, *or as purely a fortiori*, yields the very same conclusion, viz. full payment. This time, however, the Sages cannot rebut him by blocking an attempt at 'proportionality', since (to repeat) a non-'proportional' argument yields the very same conclusion as 'proportional' ones. So the Sages are obliged to propose an extension or enlargement of the initial *dayo* principle that focuses instead on the generalization before deduction. In this way, they again rule half payment.

This scenario is obvious, provided we assume the Sages' two *dayo* objections are expressions of a *dayo* principle. It is also conceivable, however, that they have no such general *principle* in mind, but merely intend these objections to be *ad hoc decisions* in the two cases at hand. In that case the *dayo* principle is a "principle," not in the strict sense of a universal principle that must be applied in *every* case of the sort, but in the looser sense of a guiding principle that may *on occasion*, for a variety of unspecified motives, be applied[47]. In fact, if we look at the Mishna passage in question, we see that nowhere is there any mention of a *dayo* "principle." There is just statement "It is quite sufficient that the law in respect of the thing inferred should be equivalent to that from which it is derived," which was presumably labeled "the *dayo* principle" by later commentators. This statement could be interpreted equally well as having a general or particular intent.

If we adopt the latter assumption, the scenario for the Mishna debate would be as follows: when R. Tarfon proposes his first argument, whether it is construed as pro rata or a crescendo, the Sages merely refuse his inherent 'proportional' premise *in this particular case*, without implying that they would automatically refuse it in other eventual cases. Similarly, when he proposes his second argument, whether it is construed as pro rata, a crescendo, or purely a fortiori, they merely refuse his preparatory generalization *in this particular case*, without implying that they would automatically refuse it in other eventual cases. Thus, the Sages might be said to making 'ad hoc' *dayo* objections, rather than appealing to a *dayo* 'principle' in the strict sense. Why would the Sages raise a *dayo* objection in this particular case, and not raise it in other cases? Conceivably, they perceive some unspecified danger in the present case that may be absent in other cases.

Granting this alternative view of the *dayo* principle, be it said in passing, there is conceivably no need to mention *qal vachomer* argument at all in this Mishna debate! In this view, it is possible that neither R. Tarfon nor the Sages intended any genuine a fortiori type of reasoning, but were entirely focused on mere analogy. As we shall see, although the Gemara probably does intend an a crescendo interpretation of the two arguments of R. Tarfon, it is not inconceivable that its author simply had in mind analogical/pro rata argument. Although the expression *qal vachomer* does appear in the Gemara, it does not necessarily have to be taken as referring to a fortiori or a crescendo argument,

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[47]   Thus, for instance, we speak in philosophy of the uniformity principle, not meaning that everything is uniform, but that there is considerable uniformity in the universe. Or again, in physics there is the uncertainty principle, which is applicable not in all systems but only in the subatomic domain.

but could be read as referring to pro rata. It is anyhow worthwhile stating that another viewpoint is possible, because this allows us to conceptually uncouple the *dayo* principle from *qal vachomer*.

But the main value of our proposing alternative scenarios is that these provide us with different explanations of the disagreement between R. Tarfon and the Sages. Where, precisely, did they disagree? Given the primary scenario, where the *dayo* principle is *a hard and fast principle* in the eyes of the Sages, the question arises: how come R. Tarfon forgot or did not know or chose to ignore this principle? If the Sages claim it as a Divine decree, i.e. an ancient tradition dating "from the Sinai revelation," whether inferred from Scripture or orally transmitted, it is unthinkable that a man of R. Tarfon's caliber would be ignorant of it or refuse to accept it. Thus, the primary scenario contains a difficulty, a *kushia*.

One possible resolution of this difficulty is to say that the Sages were here legislating, i.e. the *dayo* principle was here *in the process of* being decided by the rabbis collectively, there being one dissenting voice, viz. that of R. Tarfon, at least temporarily till the decision was declared law. In that event, the conflict between the two parties dissolves in time. Another possible resolution is to say that the Sages did not intend their *dayo* statement as a hard and fast principle, but as *a loose guideline* that they considered ought to be applied in the present context, whereas their colleague R. Tarfon considered it ought not to be applied in the present context. In that event, the two parties agree that the *dayo* principle is not universal, but merely conditional, and their conflict here is only as to whether or not its actual application is appropriate in the case at hand.

This would explain why R. Tarfon can put forward his first and second arguments failing each time to anticipate that the Sages would disagree with him. He could not offhand be expected to predict what their collective judgment would be, and so proposed his opinion in good faith. That they disagreed with him is not a reflection on his knowledge of Torah or his logical powers; there was place for legitimate dissent. Thus, while the hypothesis that the Sages' *dayo* objections signify a hard and fast rule of Sinaitic origin is problematic, there are two viable alternative hypotheses: namely, that the Sages' *dayo* objections constituted a general rabbinical ruling in the making; or that they were intended as ad hoc, particular and conditional statements, rather than as reflections of a general unbreakable rule. The problem with the former hypothesis is explaining away R. Tarfon's implied ignorance or disagreement; this problem is solved satisfactorily with either of the latter two hypotheses.

The Gemara commentary revolves around this issue, since its first and main query is: "Does R. Tarfon really ignore the principle of *dayo*? Is not *dayo* of Biblical origin?" The Gemara's thesis thus seems to be that *dayo* is a principle of Biblical origin and that therefore R. Tarfon knew about it and essentially agreed with it. We shall presently see where it takes this assumption.

About method. An issue arising from this Mishnaic discussion is whether it is based on revelation or on reason. If we examine R. Tarfon's discourse, we see that he repeatedly appeals to reason. Twice he says: "does it not stand to reason?" (*eino din*) and twice he claims to "infer" (*edon*)[48]. This language (the translations are those in the Soncino edition) suggests he is not appealing to Divine revelation, but to ordinary human reason. And, significantly, the Sages do not oppose him by *explicitly* claiming that their *dayo* principle is Divinely-ordained (as the Gemara later claims) and thus overrides his merely rational argument – no, they just affirm and reaffirm it as something intuitively self-evident, on moral if not logical grounds. Thus, from such positive and negative evidence, it is possible to suppose that both R. Tarfon and the Sages regard their methodological means as essentially rational.

Concerning the logical skills of R. Tarfon and the Sages, neither party to the debate commits any error of logic, even though their approaches and opinions differ. All arguments used by them are formally valid. At no stage do the Sages deny R. Tarfon's reasoning powers or vice versa. The two parties understand each other well and react appropriately. There is no rhetorical manipulation, but logic is used throughout. Nevertheless, a pertinent question to ask is: why did R. Tarfon and the Sages not clarify all the logical issues involved, and leave their successors with unanswered questions? Why, if these people were fully conscious of what they were doing, did they not spell their intentions out clearly to prevent all possible error? The most likely answer is that they functioned 'intuitively' (in a pejorative sense of the term), without awareness of all the formalities involved. They were skillful practitioners of logic, but evidently not theoreticians of it. They did not even realize the importance of theory.

4. A logician's reading of Numbers 12:14-15

We have thus far analyzed the Mishnaic part of Baba Qama 24b-25a. Before we turn to the corresponding Gemara, it is wise for us – in the way of a preparatory study – to look at a Torah passage which plays an important role in that

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[48]   See the sentences: "does it not *stand to reason* that we should make it equally strict with reference to the plaintiffs premises?" and "does it not *stand to reason* that we should apply the same strictness to horn?" Also: "R. Tarfon, however, rejoined: but neither do I *infer* horn from horn; I *infer* horn from foot." (My italics throughout.)

Gemara, as an illustration of the rabbinical hermeneutic rule of *qal vachomer* (a fortiori argument) and as a justification of its attendant *dayo* (sufficiency) principle.

The Torah passage in question is Numbers 12:14-15. The reason why this passage was specifically focused on by the Gemara should be obvious. This is *the only* a fortiori argument in the whole Tanakh that is both spoken by God and has to do with inferring a penalty for a specific crime. None of the other four a fortiori arguments in the Torah are spoken by God[49]. And of the nine other a fortiori arguments in the Tanakh spoken by God, two do concern punishment for sins but not specifically enough to guide legal judgment[50]. Clearly, the Mishna BQ 2:5 could only be grounded in the Torah through Num. 12:14-15.

Num. 12:14-15 reads: "14. *If her father had but spit in her face, should she not hide in shame seven days? Let her be shut up without the camp seven days, and after that she shall be brought in again.* 15. *And Miriam was shut up without the camp seven days; and the people journeyed not till she was brought in again.*" Verse 14 may be construed as a *qal vachomer* as follows:

> Causing Divine disapproval (P) is a greater offense (R) than causing paternal disapproval (Q). (Major premise.)
> Causing paternal disapproval (Q) is offensive (R) enough to merit isolation for seven days (S). (Minor premise.)
> Therefore, causing Divine disapproval (P) is offensive (R) enough to merit isolation for seven days (S). (Conclusion.)

This argument, as I have here rephrased it a bit, is a valid purely a fortiori of the positive subjectal type (minor to major)[51]. Some interpretation on my part was necessary to formulate it in this standard format[52]. I took the image of her father spitting in her face (12:14) as indicative of "paternal disapproval" caused presumably, by analogy to the context, by some hypothetical misbehavior on her part[53]. Nothing is said here about "Divine disapproval;" this too is inferred by me from the context, viz. Miriam being suddenly afflicted with "leprosy" (12:10) by God, visibly angered (12:9) by her speaking ill of Moses (12:1). The latter is her "offense" in the present situation, this term (or another like it) being needed as middle term of the argument.

The major premise, about causing Divine disapproval being a "more serious" offense than causing paternal disapproval, is an interpolation – it is obviously not given in the text. It is constructed in accord with available materials with the express purpose of making possible the inference of the conclusion from the minor premise. The sentence in the minor premise of "isolation" for seven days due to causing paternal disapproval may be inferred from the phrase "should she not hide in shame seven days?" The corresponding sentence in the putative conclusion of "isolation" for seven days due to causing Divine disapproval may be viewed as an inference made possible by a fortiori reasoning.

With regard to the term "isolation," the reason I have chosen it is because it is *the conceptual common ground* between "hiding in shame" and "being shut up without the camp." But a more critical approach would question this term, because "hiding in shame" is a voluntary act that can be done within the camp, whereas "being shut up without the camp" seems to refer to involuntary imprisonment by the authorities outside the camp. If, however, we stick to the significant distinctions between those two consequences, we cannot claim the alleged purely a fortiori argument to be valid. For, according to strict logic, we cannot have more information in the conclusion of a deductive argument (be it a fortiori, syllogistic or whatever) than was already given in its premise(s).

That is to say, although we can, logically, from "hiding in shame" infer "isolation" (since the former is a species the latter), we cannot thereafter from "isolation" infer "being shut up without the camp" (since the former is a genus of the latter). To do so would be illicit process according to the rules of syllogistic reasoning, i.e. it would be fallacious.

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[49]   One is by Lemekh (Gen. 4:24), one is by Joseph's brothers (Gen. 44:8), and two are by Moses (Ex. 6:12 and Deut. 31:27). The argument by Lemekh could be construed as concerning a penalty, but the speaker is morally reprehensible and his statement is more of a hopeful boast than a reliable legal dictum.

[50]   The two arguments are in Jeremiah 25:29 and 49:12. The tenor of both is: if the relatively innocent are bad enough to be punished, then the relatively guilty are bad enough to be punished. The other seven a fortiori arguments in the Nakh spoken by God are: Isaiah 66:1, Jer. 12:5 (2 inst.) and 45:4-5, Ezek. 14:13-21 and 15:5, Jonah 4:10-11. Note that, though Ezek. 33:24 is also spoken by God, the (fallacious) argument He describes is not His own – He is merely quoting certain people.

[51]   Actually, it would be more accurate to classify this argument as positive antecedental, since the predicate S (meriting isolation for seven days) is not applied to Q or P (causing disapproval), but to the subject of the latter (i.e. the person who caused disapproval). That is, causing disapproval *implies* meriting isolation. But I leave things as they are here for simplicity's sake.

[52]   I say 'on my part' to acknowledge responsibility – but of course, much of the present reading is not very original.

[53]   The Hebrew text reads 'and her father, etc.'; the translation to 'if her father, etc.' is, apparently, due to Rashi's interpretation "to indicate that the spitting never actually occurred, but is purely hypothetical" (Metsudah Chumash w/Rashi at: www.tachash.org/metsudah/m03n.html#fn342).

It follows that the strictly *correct* purely a fortiori conclusion is either specifically "she shall hide in shame seven days" or more generically put "she shall suffer isolation seven days." In any case, then, the sentence "she shall be shut up without the camp seven days" cannot logically be claimed as an a fortiori conclusion, but must be regarded as a *separate and additional* Divine decree that even if she does not voluntarily hide away, she should be made to do so against her will (i.e. imprisoned).

We might of course alternatively claim that the argument is intended as a crescendo rather than purely a fortiori. That is to say, it may be that the conclusion of "she should be shut up without the camp seven days" is indeed *inferred* from the minor premise "she would hide in shame seven days" – in 'proportion' to the severity of the wrongdoing, comparing that against a father and that against God. For this to be admitted, we must assume a tacit *additional premise* that enjoins a pro rata relationship between the importance of the victim of wrongdoing (a father, God) and the ensuing punishment on the culprit (voluntary isolation, forced banishment and incarceration).

Another point worth highlighting is the punishment of leprosy. Everyone focuses on Miriam's punishment of expulsion from the community for a week, but that is surely not her only punishment. She is in the meantime afflicted by God with a frightening disease, whereas the hypothetical daughter who has angered her father does not have an analogous affliction. So the two punishments are not as close to identical as they may seem judging only with reference to the seven days of isolation. Here again, we may doubt the validity of the strictly a fortiori argument. This objection could be countered by pointing out that the father's spit is the required analogue of leprosy. But of course the two afflictions are of different orders of magnitude; so a doubt remains.

We must therefore here again admit that this difference of punishment between the two cases is not established by the purely a fortiori argument, but by a separate and additional Divine decree. Or, alternatively, by an appropriate a crescendo argument, to which no *dayo* is thereafter applied. We may also deal with this difficulty by saying that the punishment of leprosy was *already* a fact, produced by God's hand, before the a fortiori argument is formulated; whereas the latter only concerns the punishment that is *yet to be* applied, by human intervention – namely, the seven days' isolation. Thus, the argument *intentionally* concerns only the later part of Miriam's punishment, and cannot be faulted for ignoring the earlier part.

It is perhaps possible to deny that an a fortiori argument of any sort is intended here. We could equally well view the sentence "Let her be shut up without the camp seven days" as an independent decree. But, if so, of what use is the rhetorical exclamation "If her father had but spit in her face, should she not hide in shame seven days?" and moreover how to explain to coincidence of "seven days" isolation in both cases? Some sort of analogy between those two clauses is clearly intended, and the a fortiori or a crescendo argument serves to bind them together convincingly. Thus, although various objections can be raised regarding the a fortiori format or validity of the Torah argument, we can say that all things considered the traditional reading of the text as a *qal vachomer* is reasonable. This reading can be further justified if it is taken as in some respects a crescendo, and not purely a fortiori.

What, then, is the utility of the clause: "And after that she shall be brought in again"? Notice that it is not mentioned in my above a fortiori construct. Should we simply read it as making explicit something implied in the words "Let her be shut up without the camp seven days"? Well, these words do not strictly imply that after seven days she should be brought back into the camp; it could be that after seven days she is to be released from prison (where she has been "shut up"), but not necessarily brought back from "without the camp." So the clause in question adds information. At the end of seven days, Miriam is to be both released from jail *and* from banishment from the tribal camp.

Another possible interpretation of these clauses is to read "Let her be shut up without the camp seven days" as signifying a sentence of *at least* seven days, while "And after that she shall be brought in again" means that the sentence should *not exceed* seven days (i.e. "after that" is taken to mean "immediately after that"). They respectively set a minimum and a maximum, so that *exactly* seven days is imposed. What is clear in any case is that "seven days isolation" is stated and implied in both the proposed minor premise and conclusion; **no other quantity, such as fourteen days, is at all mentioned**, note well. This is a positive indication that we are indeed dealing essentially with a purely a fortiori argument, since the logical rule of the continuity between the given and inferred information is (to that extent) obeyed.

As we shall see when we turn to the Gemara's treatment, although there is no explicit mention of fourteen days in the Torah conclusion, it is not unthinkable that fourteen days were implicitly intended (implying an a crescendo argument from seven to fourteen days) but that this harsher sentence was subsequently mitigated (brought back to seven days) by means of an additional Divine decree (the *dayo* principle, to be exact) which is also left tacit in the Torah. In other words, while the Torah apparently concludes with a seven-day sentence, this could well be a final conclusion (with unreported things happening in between) rather than an immediate one. Nothing stated in the Torah implies this a crescendo reading, but nothing denies it either. So much for our analysis of verse 14.

Let us now briefly look at verse 15: "And Miriam was shut up without the camp seven days; and the people journeyed not till she was brought in again." The obvious reading of this verse is that it tells us that the sentence in verse 14 was duly executed – Miriam was indeed shut away outside the camp for exactly seven days, after which she was released and returned to the camp, as prescribed. We can also view it as a confirmation of the reasoning in the previous verse – i.e. as a way to tell us that the apparent conclusion was the conclusion Moses' court adopted and carried out. We shall presently move on, and see how the Gemara variously interpreted or used all this material.

But first let us summarize our findings. Num. 12:14-15 may, with some interpolation and manipulation, be construed as an a fortiori argument of some sort. If this passage of the Torah is indeed a *qal vachomer*, it is not an entirely explicit (*meforash*) one, but partly implicit (*satum*). In some respects, it would be more appropriate to take it as a crescendo, rather than purely a fortiori. It could even be read as not a *qal vachomer* at all; but some elements of the text would then be difficult to explain.

It is therefore reasonable to read an a fortiori argument into the text, as we have done above and as traditionally done in Judaism. It must however still be stressed that this reading is somewhat forced if taken too strictly, because there are asymmetrical elements in the minor premise and conclusion. We cannot produce a valid purely a fortiori inference without glossing over these technical difficulties. Nevertheless, there is enough underlying symmetry between these elements to suggest a significant overriding a fortiori argument that accords with the logical requirement of continuity (i.e. with the principle of deduction). The elements not explained by a fortiori argument can and must be regarded as separate and additional decrees. Alternatively, they can be explained by means of a crescendo arguments.

In the present section, we have engaged in a frank and free textual analysis of Num. 12:14-15. This was intentionally done from a secular logician's perspective. We sought to determine objectively (irrespective of its religious charge) just what the text under scrutiny is saying, what its parts are and how they relate to each other, what role they play in the whole statement. Moreover, most importantly, the purpose of this analysis was to find out what relation this passage of the Torah might have to a fortiori argument and the principle of *dayo*: does the text clearly and indubitably contain that form of argument and its attendant principle, or are we reading them into it? Is the proposed reasoning valid, or is it somewhat forced?

We answered the questions as truthfully as we could, without prejudice pro or con, concluding that, albeit various difficulties, a case could reasonably be made for reading a valid a fortiori argument into the text. These questions all had to be asked and answered before we consider and discuss the Gemara's exegesis of Num. 12:14-15, because the latter is in some respects surprisingly different from the simple reading. We cannot appreciate the full implications of what it says if we do not have a more impartial, scientific viewpoint to compare it to. What we have been doing so far, then, is just preparing the ground, so as to facilitate and deepen our understanding of the Gemara approach to the *qal vachomer* argument and the *dayo* principle when we get to it.

One more point needs to be made here. As earlier said, the reason why the Gemara drew attention in particular to Num. 12:14-15 is simply that this passage is the only one that could possibly be used to ground the Mishna BQ 2:5 in the Torah. However, though as we have been showing Num. 12:14-15 can indeed be used for this purpose, the analogy is not perfect. For whereas the Mishnaic *dayo* principle concerns inference by a rabbinical court from a law (a penalty for a crime, to be precise) explicit in the Torah to a law *not* explicit in the Torah (sticking to the same penalty, rather than deciding a proportional penalty), the *dayo* principle implied (according to most readings) in Num. 12:14-15 relates to an argument whose premises and conclusion are all in the Torah, and moreover it infers the penalty (for Miriam's *lèse-majesté*) for the court to execute by derivation from a penalty (for a daughter offending her father) which may be characterized as intuitively-obvious morality or more sociologically as a pre-Torah cultural tradition.

For if we regard (as we could) both penalties (for a daughter and for Miriam) mentioned in Num. 12:14-15 as Divinely decreed, we could not credibly also say that the latter (for Miriam) is *inferred* a fortiori from the former (for a daughter). So the premise in the Miriam case is not as inherently authoritative as it would need to be to serve as a perfect analogy for the Torah premise in the Mishnaic case. For the essence of the Mishnaic sufficiency principle is that the court must be content with condemning a greater culprit with the same penalty as the Torah condemns a lesser culprit, rather than a proportionately greater penalty, on the grounds that the only penalty explicitly justified in the Torah and thus *inferable* with certainty is the same penalty. That is, the point of the Mishnaic *dayo* is that the premise is *more authoritative* than the conclusion, whereas in the Num. 12:14-15 example this is not exactly the case. What this means is that although the Mishnaic *dayo* can be somewhat grounded on Num. 12:14-15, such grounding depends on our reading certain aspects of the Mishna *into* the Torah example. That is to say, the conceptual dependence of the two is mutual rather than unidirectional.

5. A critique of the Gemara in Baba Qama 25a

As regards the Gemara of the Jerusalem Talmud, all it contains relative to the Mishna Baba Qama 2:5 is a brief comment in the name of R. Yochanan[54] that R. Tarfon advocates full payment for damages in the private domain, whereas the Sages advocate half payment[55]. This is typical of this Talmud, which rarely indulges in discussion[56]. On the other hand, the Gemara of the Babylonian Talmud has quite a bit to say on this topic (see p. 25a there), though perhaps less than could be expected. When exactly that commentary on our Mishna was formulated, and by whom, is not there specified; but keep in mind that the Gemara as a whole was redacted in Babylonia ca. 500 CE, i.e. some three centuries after the Mishna was closed, so these two texts are far from contemporaneous[57]. It begins as follows:

> "Does R. Tarfon really ignore the principle of *dayo*? Is not *dayo* of Biblical origin? As taught: How does the rule of *qal vachomer* work? And the Lord said unto Moses: 'If her father had but spit in her face, should she not be ashamed seven days?' How much the more so then in the case of divine [reproof] should she be ashamed fourteen days? Yet the number of days remains seven, for it is sufficient if the law in respect of the thing inferred be equivalent to that from which it is derived!"

The a crescendo reading. Reading this passage, it would appear that the Gemara conceives *qal vachomer* as a crescendo rather than purely a fortiori argument; and the *dayo* principle as a limitation externally imposed on it. It takes the story of Miriam (i.e. Numbers 12:14-15) as an illustration and justification of its view, claiming that the punishment due to Miriam would be fourteen days by *qal vachomer* were it not restricted to seven days by the *dayo* principle. The *dayo* principle is here formulated exactly as in the Mishna (as "It is sufficient, etc."); but the rest of the Gemara's above statement is not found there.

In fact, the Gemara claims that the thesis here presented is a *baraita* – i.e. a tradition of more authoritative, Tannaic origin, even though it is not part of the Mishna[58]. This is conventionally signaled in the Gemara by the expression 'as taught': דתניא (*detania*)[59]. The *baraita* may be taken as the Hebrew portion following this, i.e. stretching from "How does the rule of *qal vachomer* work?" to "…from which it is derived." Note well that *baraita* thesis is clearly delimited: the preceding questions posed by the Gemara – viz. "Does R. Tarfon really ignore the principle of *dayo*? Is not *dayo* of Biblical origin?" – are *not* part of it; we shall return to these two questions further on.

As we have shown in our earlier analysis, Num. 12:14-15 could be read as devoid of any argument; but then we would be hard put to explain the function of the first sentence: "If her father had but spit in her face, etc.," and its relation to the second: "Let her be shut up without the camp, etc.". It is therefore a reasonable assumption that an argument is indeed intended. This argument can be construed as purely a fortiori; in that event, its conclusion is simply seven days isolation, the same number of days as mentioned in the minor premise; and if the *dayo* principle have any role to play here it is simply that of the principle of deduction, i.e. a reminder that the conclusion must reflect the minor premise. It is also possible to interpret the argument as a crescendo, as the Gemara proposes to do; in that event, its conclusion is a greater number of days of isolation (say, fourteen days); and the *dayo* principle plays the crucial role of resetting the number of days to seven.

The latter is a conceivable hypothesis, but by no means a certainty, note well. There is clearly no mention of "fourteen days" in the Torah passage referred to, i.e. no concrete evidence of an a crescendo argument, let alone of a *dayo* principle which cuts back the fourteen days to seven. The proposed scenario is entirely read into the Biblical text, rather than drawn from it, by the *baraita* and then the Gemara; it is an interpolation on their part. They are saying: though the Torah does not explicitly mention fourteen days, etc., it tacitly intends them. This is not inconceivable; but it must be admitted to be speculative, since other readings are equally possible.

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[54]   I presume offhand this refers to R. Yochanan bar Nafcha, d. ca. 279 CE.

[55]   See page 11b, chapter 2, law 7.

[56]   This Talmud (closed in Eretz Israel, ca. 400 CE) may of course contain significant comments about *qal vachomer* and the *dayo* principle elsewhere; I have not looked into the matter further.

[57]   Since R. Tarfon flourished in 70-135 CE, and the Mishna was redacted about 220 CE, the Gemara under examination here must have been developed somewhere in between, i.e. in the interval from c. 220 CE to c. 500 CE. The thesis upheld in this particular anonymous Gemara may have existed some time before the final redaction, or may have been composed at the final redaction (or possibly even later, if some modern scholars are to be believed).

[58]   According to a note in *Talmud Bavli*, this *baraita* first "appears at the beginning of *Toras Kohanim*," by which they presumably mean the introduction to *Sifra* listing the thirteen hermeneutic principles of R. Ishmael and some Biblical illustrations of them.

[59]   According to the *Introduction to the Talmud* of R. Shmuel Ha-Nagid (Spain, 993-1060 – or maybe Egypt, mid-12th cent.), a *tosefta* (addition) is a form of *baraita* (outside material) "usually introduced by the word *tanya*;" so, the use of this word here could be indicative of a *tosefta*. Further on in the same work, it is said that "an anonymous statement in the Tosefta is according to R. Nechemia;" so, the statement here cited by the Gemara might have been made by the Tanna R. Nechemia (Israel, fl. c. 150 CE). This is just speculation on my part, note well. An English translation of the book by R. Shmuel Ha-Nagid can be found in Aryeh Carmell's *Aiding Talmud Study* (5th ed. Jerusalem: Feldheim, 1991); see there pp. 70, 74.

The *baraita* apparently proposes to read, not only the particular *qal vachomer* about Miriam, but *qal vachomer* in general as a crescendo argument, since it says "How does the rule of *qal vachomer* work?" rather than "how does the following example of *qal vachomer* work?" Thus, the Tanna responsible for it may be assumed to believe unconditionally in the 'proportionality' of a fortiori argument. Likewise, the Gemara – since it accepts this view without objection or explanation. If it is true that this Gemara (and the *baraita* it is based on – but I won't keep mentioning that) regards a fortiori argument to always be a crescendo argument, it is way off course, of course.

As we have seen, as far as formal logic is concerned a fortiori argument is essentially not a crescendo, even though its premises can with the help of an additional premise about proportionality be made to yield an a crescendo conclusion. It is conceivable that the particular argument concerning Miriam is in fact not only a fortiori but a crescendo (assuming the premise of proportionality is tacitly intended, which is a reasonable assumption); but it is certainly *not* conceivable that *all* a fortiori arguments are a crescendo. The Gemara's identification of a fortiori argument with a crescendo is nowhere justified by it. The Gemara has not analyzed a fortiori argument in general and found its logical conclusion to be a crescendo (i.e. 'proportional'); it merely asserts this to be so in the case at hand and, apparently, in general.

While it is true that, empirically, within the Talmud as well as outside it, convincing examples of seemingly a fortiori argument yielding a (roughly or exactly) proportional conclusion can be adduced, it is also true that examples of a fortiori argument yielding a *non*-proportional conclusion can be adduced. This needs to be explained – i.e. commentators are duty-bound to account for this variation in behavior, by specifying under what logical conditions a 'proportional' conclusion is justified and when it is not justified. The answer to that is (to repeat) that a fortiori argument as such does not have a 'proportional' conclusion and that such a conclusion is only logically permissible if an additional premise is put forward that justifies the 'proportionality'. The Gemara does not demonstrate its awareness of these theoretical conditions, but functions 'intuitively'. Its thesis is thus essential dogmatic – an argument by authority, rather than through logical justification.

Thus, for the Gemara, or at least this here Gemara, the words "*qal vachomer*," or their English equivalent "a fortiori argument," refer to what we have called a crescendo argument, rather than to purely a fortiori argument. There is nothing wrong with that – except that the Gemara does not demonstrate awareness of alternative hypotheses.

A surprising lacuna. Furthermore, it should imperatively be remarked that the Gemara's above explanation of the Mishna debate, by means of the Miriam story, is only relevant to the first exchange between R. Tarfon and the Sages; *it does not address* the issues raised by the second exchange between them.

For in the first exchange, as we have seen, R. Tarfon tries by means of a possible pro rata argument, or alternatively an a crescendo argument (as the Gemara apparently proposes), to justify a 'proportional' conclusion (i.e. a conclusion whose predicate is greater than the predicate of the minor premise, in proportion to the relative magnitudes implied in the major premise); and here the Sages' *dayo* objection limits the predicate of conclusion to that of the minor premise; so the analogy to the Miriam case is possible. But in the second exchange, the situation is *quite* different! Here, as we earlier demonstrated, the *dayo* objection refers, not to the information in the minor premise, but to the information that was generalized into the major premise. That is to say, whereas the first objection is aimed at the attempted pro rata or a crescendo deduction, the second one concerns the inductive preliminary to the attempted pro rata or a fortiori or a crescendo deduction.

The Gemara makes no mention of this crucial distinction between the two cases. It does not anywhere explicitly show that it has noticed that R. Tarfon's *second* argument draws the same conclusion whether it is considered as pro rata, a crescendo, or even purely a fortiori, so that it formally does *not* contravene the Sages' first objection. The Gemara does not, either, marvel at the fact that the Sages' second objection is made in *exactly the same terms*, instead of referring to the actual terms of the new argument of R. Tarfon. It does not remark that the Miriam story (as the Gemara interprets it) is therefore *irrelevant* to the second case, since it does not resemble it, and some other explanation must be sought for it. This lacuna is of course a serious weakness in the Gemara's whole hypothesis, since it does not fit in with all the data at hand.

To be sure, the distinction between the two cases does appear in rabbinic literature. This distinction is solidified by means of the labels *dayo aresh dina* and *dayo assof dina* given to the two versions of the *dayo* principle. But I do not think the distinction is Talmudic (certainly, it is absent here, where it is most needed). Rather, it seems to date from much later on (probably to the time of Tosafot). These expressions mean, respectively, applying the *dayo* "to the first term (or law)" and applying it "to the last term (or law)." In my opinion, *assof dina* must refer to the *dayo* used on the first *qal vachomer*, while *aresh dina* refers to the *dayo* used on the second *qal vachomer*[60].

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[60]   The reason I say "in my opinion," is that the text where I found this distinction, namely *La mishna* (Tome 8, Baba Kama. Tr. Robert Weill. Paris: Keren hasefer ve-halimoud, 1973), posits the reverse, i.e. *aresh dina* for the first argument and *assof dina* for the second. But that would not make sense in my view. Either there was a typing error, or (less likely) whoever originally formulated this distinction did not really understand how the two *dayo* applications differ. For it is clear from the analysis presented in the present volume that, in the first argument *dayo* is

Be that as it may, what concerns us here is the Gemara, which evidently makes no such distinction (even if later commentators try to ex post facto give the impression that everything they say was tacitly intended in the Gemara). What this inattentiveness of the Gemara means is that even if it manages to prove whatever it is trying to prove (we shall presently see just what) – it will not succeed, since it has not taken into account all the relevant information. Its theory will be too simple, insufficiently broad – inadequate to the task. The Gemara's failure of observation is of course also not very reassuring.

The claim that *dayo* is of Biblical origin. Let us now return to the initial questions posed by the Gemara, viz. "Does R. Tarfon really ignore the principle of *dayo*? Is not *dayo* of Biblical origin?" (ור"ט לית ליה דיו והא דיו דאורייתא הוא). As already remarked, it is important to notice that these questions are *not* part of the *baraita*. They are therefore the Gemara's own thesis (or an anonymous thesis it defends as its own) – indeed, as we shall see, they are the crux of its commentary. The *baraita* with the a crescendo reading is relatively a side-issue. What the Gemara is out to prove is that R. Tarfon "does *not* ignore" the *dayo* principle, because "it *is* of Biblical origin." What is not of Biblical origin may conceivably be unknown to a rabbi of Tarfon's level; but what is of Biblical origin must be assumed as known by him.

The question of course arises what does "of Biblical origin" (*deoraita*) here mean exactly? It cannot literally mean that the principle of *dayo* is *explicitly* promulgated and explicated in the Torah. Certainly it is nowhere to be found in the Torah passage here referred to, or anywhere else in that document. Thus, this expression can only truly refer to an *implicit* presence in the Torah. And indeed the Torah passage about Miriam, brought to bear by the Gemara, seems to be indicated by it as the needed *source and justification* of the principle, rather than as a mere illustration of it. However, as we shall see further on, there is considerable circularity in such a claim. So claiming the *dayo* principle to have "Biblical origin" is in the final analysis just *say-so*, i.e. a hypothesis – it does not solidly ground the principle and make it immune to all challenge, as the Gemara is suggesting.

It could well be thought, reading the Mishna, that R. Tarfon was *not* previously aware of the Sages' alleged *dayo* principle, since he did not preempt their two *dayo* objections. Had he known their thinking beforehand, he would surely not have wasted his time trying out his two arguments, since he would expect them to be summarily rejected by the Sages. Since he did try, and try again, the Sages must have been, in his view, either unearthing some ancient principle unknown to him, or deciding a new principle, or proposing ad hoc decisions. It is this overall reasonable conclusion from the Mishna that the Gemara seeks to combat, with its claim that the *dayo* principle was of Biblical origin and therefore R. Tarfon must have known it. Note this well.

I do not know why the Gemara is not content with the perfectly legal possibilities that the *dayo* principle might be either a tradition not known to R. Tarfon, or a new general or particular decision by the Sages (*derabbanan*). For some reason, it seeks to impose a more fundamentalist agenda, even though the alternative approaches are considered acceptable in other Talmudic contexts. The Gemara does not say why it is here unacceptable for the Sages to have referred to a relatively esoteric tradition or made a collegial ruling (by majority, *rov*)[61]. It seems that the Gemara is driven by a desire to establish that R. Tarfon and the Sages are more in harmony than they at first seem; but it is not clear why it has chosen the path it has, which is fraught with difficulties.

The claim that *dayo* is conditional. The Gemara shifts the debate between R. Tarfon and the Sages from one as to *if* the *dayo* principle is applicable to one as to *when* it is applicable. The two parties, according to the Gemara, agree that the *dayo* principle is "of Biblical origin," and thus that there is a *dayo* principle; but they disagree on whether or not it is applicable unconditionally. In this view, whereas the Sages consider the *dayo* principle as universally applicable, R. Tarfon considers it as only conditionally applicable. Thus, the parties agree in principle, and their disagreement is only in a matter of detail. The Gemara then proceeds to clarify R. Tarfon's alleged conditions[62]:

> "The principle of *dayo* is ignored by him [R. Tarfon] only when it would defeat the purpose of the a fortiori, but where it does not defeat the purpose of the a fortiori, even he maintains the principle of *dayo*. In the instance quoted there is no mention made at all of seven days in the case of divine reproof; nevertheless, by the working of the a fortiori, fourteen days may be suggested: there follows, however, the principle of *dayo* so that the additional seven days are excluded, whilst the original seven are retained. Whereas in the case before us the payment of not less than half damages has been explicitly ordained [in all kinds of

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applied to the premise about proportionality (which is relatively downstream, whence "at the end"), while in the second argument it is applied before the formation of the major premise (thus, well upstream, i.e. "at the beginning"). Moreover, my view seems to be confirmed by the following comment in the Artscroll Mishnah mentioned in an earlier footnote: "it is easier to apply the principle of *dayyo* to the first *kal vachomer*, because in that instance it applies to the end of the *kal vachomer*." It also seems to be confirmed by the article on the *dayo* principle in Encyclopedia Talmudit (reviewed in a later chapter, viz. 31.3).

[61]   And I have found no explanation by later commentators.

[62]   In truth, the Gemara's explanations are not entirely clear; it is only by referring to later commentaries (paraphrased in *Talmud Bavli* ad loc) that I was personally able to fathom them.

grounds]. When therefore an a fortiori is employed, another half-payment is added [for damage on the plaintiff's premises], making thus the compensation complete. If [however] you apply the principle of *dayo*, the sole purpose of the a fortiori would thereby be defeated."

Let us try and understand what the Gemara is saying here. It is proposing a distinction (allegedly by R. Tarfon) between two obscure conditions: when applying the *dayo* principle "would defeat the purpose of the *qal vachomer*," it is *not* applied; whereas where applying the *dayo* principle "would *not* defeat the purpose of the *qal vachomer*," it is applied. What does this "defeating the purpose of the a fortiori argument" condition refer to? The Gemara clarifies it by comparing R. Tarfon's (alleged) different reactions to two cases: that concerning Miriam and the (first) argument in the Mishna (the Gemara has apparently not noticed the second argument at all, remember).

The Gemara here reaffirms its theory that, although the Torah ("the instance quoted" – i.e. Num. 12:14-15) does not mention an initial or an additional seven days[63], "nevertheless, by the working of the a fortiori" (as conceived by the Gemara, meaning a crescendo) fourteen days in all (i.e. seven plus seven) are intended, and the *dayo* principle serves after that to "exclude" the additional seven days, admitting only the "original" seven days. In this case, then, the *dayo* principle *is* to be applied. The Gemara then turns to R. Tarfon's (first) argument, claiming that in its case the *dayo* principle is *not* to be applied. Why? Because "the payment of not less than half damages has been explicitly ordained [in all kinds of grounds]." This is taken by commentators (Rashi is mentioned) to mean that since the Torah does not make a distinction between public and private property when it specifies half liability for damage by horn[64], it may be considered as intending this penalty to be (the minimum[65]) applicable to both locations.

The Gemara goes on to tell us that through "a fortiori" inference "another half-payment is added, making thus the compensation complete." The implication is that, whereas the Sages would at this stage apply the *dayo* principle and conclude with *only* half payment, R. Tarfon (according to the Gemara) considered that doing so would "defeat the purpose of the a fortiori" and he concluded instead with full payment. In the Miriam case, we go from *no* information to fourteen days and back to seven; so we still end up with new information (seven) after the *dayo* application to the *qal vachomer* increase. Whereas in the Mishna case, we go from half to full payment and back to half; so that *dayo* application here would altogether cancel out the *qal vachomer* increase. Thus, R. Tarfon is presented by the Gemara as knowing and accepting the *dayo* principle, but applying it *more conditionally* than the Sages do[66].

But I would certainly challenge the underlying claim that the a fortiori argument used by R. Tarfon (which concludes with full payment for damage by horn on private property) is "nullified" by the Sages' objection to it (which limits the payment to half). What is given in the Torah is that such damage (on whatever domain) is liable to half payment. This "half" is indefinite, and must be interpreted as at least half (i.e. a minimum of half, *no less than* half), which leaves open whether only half (i.e. a maximum of half, *no more than* half) or full (i.e. *more than* half) is intended. R. Tarfon's argues (through a crescendo, i.e. 'proportional' a fortiori argument) in favor of the conclusion "full," whereas the Sages argue (through *dayo*, or purely a fortiori argument) in favor of the alternative conclusion "only half." R. Tarfon's argument is certainly *not made logically useless* by the Sages' dismissal of it, but constitutes a needed acknowledgment of one of the two possible interpretations of "half," just as the Sages' *dayo* duly

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[63]   It is not clear which seven days the Gemara intends to refer to, when it says "there is no mention made at all of seven days in the case of divine reproof." It could be referring to the initial seven days (the minor premise of the a fortiori argument), which as we shall later see the Gemara considers as tacit. Or it could be referring to "the additional seven days" mentioned a bit further on in the same paragraph, i.e. the seven days added on to the presumed initial seven to make a total of fourteen (the a crescendo conclusion of the argument), which the Gemara also takes for granted though absent in the text. In any case, the Gemara's explicit admission that information is lacking is worth underlining.

[64]   Here reference is made to Ex. 21:35, which concerns an ox killing (by goring or other such means) another's ox, in which case the live ox is sold and the price of it divided between the two owners. And this situation is contrasted to Ex. 22:4, which does specify private property with regard to tooth & foot damage. However, this comparison seems a bit forced to me, because though it is true that there is no mention of where the ox was killed, that is because the damage done has nothing to do with location; whereas in the case of someone's beast feeding in another's field, it is the field that has been damaged. In any event, the rabbis are evidently making a generalization, from the case of an ox goring another ox (i.e. Ex. 21:35), to an ox goring or similarly damaging *anything* found on public or private property. Just as in the first case, the oxen are split between the owners, so the minimum for any *other* such damage by an ox is half liability. This is at least true for damage on public property, and the question asked is whether more than that can be charged for damage on private property.

[65]   If we did not say "the minimum," and instead interpreted the "half damages" on private property as *davka*, we would be suggesting that this penalty is Torah-given, and therefore no greater penalty can be inferred. If the latter were assumed, the Sages' *dayo* objections would only be *ad hoc* Scriptural stipulations and not expressions of a broad principle. In that event, R. Tarfon's two arguments were not rejected by the Sages because of any technical fault in them, but simply because the conclusion was *already settled* by Scriptural decree, so that there was no sense in his trying to *infer* anything else. But this does not seem to be the intent of the Mishna or the Gemara.

[66]   Obviously, this more specific difference of opinion between the parties does not disturb the Gemara authorship. The implication is that the viewpoint attributed to R. Tarfon (about the conditionality of *dayo*) is not "of Biblical origin" – or, of course, it would be known to and agreed by the Sages! What credence does it have, then? Why hang on to it, if it is just one man's opinion? One senses a double standard in the Gemara's approach.

acknowledges the other possibility. If the Mishna had directly interpreted "half" as "only half," without regard to the possibility of "full," the interpretation would have seemed unjustified.[67]

An argument *ex machina*. But let us dig deeper into the alleged conditionality of *dayo* application. Why, more precisely, does the Gemara's R. Tarfon consider that applying the *dayo* principle in the case of the Miriam argument does not "defeat the sole purpose of the a fortiori," yet would do so in the case of his formally similar (first) argument? What is the significant difference between these two cases? And what sense are we to make of the Gemara's further explanations, viz.:

> "And the Rabbis? — They argue that also in the case of divine [reproof] the minimum of seven days has been decreed in the words: Let her be shut out from the camp seven days. And R. Tarfon? — He maintains that the ruling in the words, 'Let her be shut out etc.', is but the result of the application of the principle of *dayo* [decreasing the number of days to seven]. And the Rabbis? — They argue that this is expressed in the further verse: And Miriam was shut out from the camp. And R. Tarfon? — He maintains that the additional statement was intended to introduce the principle of *dayo* for general application so that you should not suggest limiting its working only to that case where the dignity of Moses was involved, excluding thus its acceptance for general application: it has therefore been made known to us [by the additional statement] that this is not the case."[68]

It seems[69] that R. Tarfon's thought (still according to the Gemara, note well) is that, with regard to Miriam, *no part of the penalty for offence against God is explicitly mentioned in the Torah* (Num. 12:14-15), so that all fourteen days must be inferred by "a fortiori" (i.e. a crescendo); after which the *dayo* principle is used to revoke seven of those days, leaving seven. Whereas, in the case of horn damage on private property, the minimum liability of half payment is *already explicitly given in the Torah* (Ex. 21:35), so that the "a fortiori" (i.e. a crescendo) argument only serves to add on half payment; in which case, applying the *dayo* principle here would completely nullify the effect of the *qal vachomer*.

Thus, it is implied, the *dayo* principle is applicable in the Miriam case, but inappropriate in the case of a goring ox. The Sages (allegedly) then object that the initial seven days are indeed given in the Torah, in the sentence "Let her be shut out from the camp seven days." To which R. Tarfon (allegedly) retorts that this sentence refers to the *dayo* principle's "decreasing the number of days to seven." The Sages reply that that function is fulfilled by the sentence "And Miriam was shut out from the camp." To which R. Tarfon retorts that the latter rather has a generalizing function from the present case to all others. As far as I am concerned, most of this explanation by the Gemara is artificial construct and beside the point. It is chicanery, *pilpul* (in the most pejorative sense of that term).

The claim it makes (on R. Tarfon's behalf) that *all fourteen days* for offence against God must be inferred is untrue – for the fourteen days are not inferred *from nothing*, as it suggests; they are inferred from the seven days for offence *against a father*. The inference of the conclusion, whether it is a crescendo or purely a fortiori, depends on this minor premise. The seven days for a father are indeed a given minimum, *also* applicable to God; *otherwise, there would be no a crescendo or a fortiori inference at all*. The Gemara is claiming an "a fortiori" (i.e. a crescendo) argument to be present in the text, and yet denying the relevance of the textual indicators for such an assumption. Its alleged "a fortiori" argument is therefore injected into the discussion *ex machina*, out of the blue, without any textual justification whatsoever. This is not logic, but rhetoric.

The situation in the argument about Miriam is thus in fact technically exactly identical to the (first) argument relating to liability for damages by horn in the Mishna. Both arguments do, in fact, have the minor premise needed to draw the conclusion. Whence the Gemara's concept of "defeating the sole purpose of the a fortiori" is a red herring; it is just a convenient verbal artifice, to give the impression that there is a difference where there is none. The Gemara has evidently tried to entangle us in an imaginary argument. For, always remember, it is the Gemara's reading which is at stake here, and not R. Tarfon's actual position as it appears in the Mishna, which is something quite distinct.

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[67]   Thus, the comment in *Talmud Bavli* that "applying *dayyo* in this case would leave the *kal vachomer* teaching us absolutely nothing" is not correct. The Mishna does *not* go from 'half' to 'full' and *back to* 'half' – it goes from 'at least half' to 'full' and thence to '*only* half'. We could similarly interpret the Miriam argument as going from 'at least 7 days' to '14 days' to 'only 7 days', and thus show the two cases are logically quite similar, contrary to the Gemara's claim.

[68]   The Gemara goes on and on, the next sentence being "R. Papa said to Abaye: Behold, there is a Tanna who does not employ the principle of dayo even when the a fortiori would thereby not be defeated…" (note the two negations, implying there may be yet *other* exceptions to *dayo* application). But this much later comment (dating from the late 3rd cent. CE) goes somewhat against the theory the Gemara attributes to R. Tarfon. So it is safe to stop where we have. Incidentally, if the sequence of events was really as implied in the Gemara, then the anonymous thesis that R. Tarfon "did not ignore" that the *dayo* principle "is of Biblical origin" would be dated roughly somewhere in the 3rd cent. CE – that is, one or two centuries after the fact, rather than three or more. But it is also possible that the said anonymous thesis was composed after the "R. Papa said to Abaye" part, the latter being adapted by the redactors to "fit in" – as modern scholars say often happens in the Talmud.

[69]   I base this interpretation on explanations given in *Talmud Bavli* ad loc.

The roles of the verses in Num. 12:14-15. What is evident is that neither of the readings of the said Torah portion that the Gemara attributes to R. Tarfon and the Sages fully corresponds to the simple reading (*peshat*). They are both awkward inventions[70] designed to justify the Gemara's own strange thesis. The Gemara's thesis is not something necessary, without which the Mishna is incomprehensible; on the contrary, it clouds the issues and misleads. Whatever the author's authority, it is unconvincing.

The simple reading of Num. 12:14-15 is, as we saw earlier[71], that the sentence "If her father had but spit in her face, should she not hide in shame seven days?" (first part of v. 14, call it 14a) provides the minor premise of a possible a fortiori argument (whether strict or a crescendo), while the sentence "Let her be shut up without the camp seven days, and after that she shall be brought in again" (second part of v. 14, call it 14b) provides its immediate conclusion. Note well that it is from these two sentences (i.e. v. 14a & 14b) that we in the first place surmise that there is an a fortiori argument in the text; *to speak of an a fortiori argument without referring to both these indices would be concept stealing*. The further sentence "And Miriam was shut up without the camp seven days; and the people journeyed not till she was brought in again" (v. 15) plays no part in the a fortiori argument as such, but serves to confirm that the sentence was carried out by Moses' court as prescribed by God.

The Gemara's R. Tarfon makes no mention of the role of v. 14a in building a *qal vachomer*, and regards v. 14b as the final conclusion of the argument, *after* the operation of an *entirely tacit* a crescendo inference to fourteen days and an *also tacit* application of *dayo* back to seven days; as regards v. 15, it effectively plays no role within the argument in his view, having only the function of confirming that the *dayo* application is a general principle and not an exceptional favor[72]. The Gemara's Sages, on the other hand, regard v. 14b (not 14a, note well) as the minor premise of the *qal vachomer*, and v. 15 its final conclusion, after the operation of an a crescendo inference to fourteen days and an application of *dayo* back to seven days.

Both parties make serious errors. The first of these is that neither of them accounts for v. 14a – why is it mentioned here if as both parties suppose it plays no role? No a fortiori argument can at all be claimed without reference to this information. The R. Tarfon thesis here is largely imaginary, since he ignores the role of v. 14a in justifying a *qal vachomer*; there is no trace in the Torah text of the a crescendo argument he claims, other than v. 14b. On the basis of only the latter textual given of seven days, he *projects into the text* a minor premise of seven days, an intermediate a crescendo conclusion of fourteen days and a *dayo* principle application, yielding a final conclusion of seven days (v. 14b). But if all the textual evidence we rely on is v. 14b, on what basis can we claim any a crescendo reasoning has at all occurred before it, let alone a *dayo* application, with this verse as the final conclusion? The whole process becomes a patent fabrication.

Nowhere in the proof text, note well, are the words *qal vachomer* or *dayo* used, or any verbal signal to the same effect. And this being so, what credence can be assigned to the Gemara's central claim, viz. that the *dayo* principle is "of Biblical origin?" It is surely paradoxical that it is able to support this ambitious claim only by means of a very debatable mental projection of information into the Torah, like a magician pulling a rabbit out of a hat after showing us it was empty. This means that the Gemara's proposed argument in favor of this claim is circular: it assumes X in order to prove X. This is of course made possible through the use of complicated discourse; but the bottom line is still the same.

The Sages' thesis is a bit more credible in that, even if they also grant no role to v. 14 a, they at least do propose a minor premise (v. 14b), as well as a final conclusion (v. 15). However, it is hard to see how "Let her be shut up without the camp seven days" (v. 14b) could be the minor premise of *qal vachomer* yielding the conclusion "And Miriam was shut up without the camp seven days" (v. 15)! These two propositions have the same subject (as well as the same explicit predicates), so where is the *qal vachomer*? Moreover, the Sages thereby subscribe to R. Tarfon's strange misconception regarding a fortiori argument.

A fortiori argument with a single subject. I am referring here to the bizarre notion that (in the *qal vachomer* argument under consideration, which is positive subjectal) the subject of the minor premise must be repeated in the conclusion, while the subsidiary terms (i.e. the predicates of these propositions) go from less to more (implicitly). In fact, positive subjectal argument, whether a fortiori or a crescendo, *formally* has different subjects (the minor and the major terms, respectively) in the minor premise and conclusion (as for the predicate, i.e. the subsidiary term, it

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[70]   I call this 'pegging' – this sort of arbitrary association of rabbinical claims with Torah passages irrespective of content. When meaningful reasons are not available, the rabbis sometimes unfortunately engage in such lame excuses to give the impression that they have some Scriptural basis. The conclusions of such arguments are foregone – there is no process of logical inference. Such interpretations would supposedly be classed as **asmakhta** by the rabbis.

[71]   See the earlier section on Num. 12:14-15 for a fuller exposé.

[72]   If Miriam was spared the extra seven days incarceration due to the exceptional circumstance that Moses prayed for her, then it was not due to application of a *dayo* principle but to an ad hoc special favor. Note that there is nothing in v. 15 that suggests either interpretation – all it says is that Miriam was indeed shut up for seven days.

remains constant in pure a fortiori, while it increases in a crescendo). *There has to be* two subjects for the argument to logically function. The bizarre notion in the Gemara of a single subject argument is *the reason why* both parties in it ignore v. 14a and look for some other proposition to use as minor premise.

It should be stressed that there is *no allusion whatsoever* to such an idea in the Mishna. The Mishna's R. Tarfon and Sages manifestly have an entirely different dialogue than the one the Gemara attributes to them. The discussion in the Mishna is much more credible than that in the Gemara. The Gemara makes up this notion solely in order to create a distinction between the Miriam case and the Mishna's (first) argument. It needs to do this, remember, in order to justify its theory that R. Tarfon and the Sages agree on the *dayo* principle, although R. Tarfon applies it conditionally whereas the Sages apply it universally. But as we shall demonstrate formally, this notion is logically untenable. Buying the Gemara's scenario is like buying Brooklyn Bridge from someone who doesn't own it.

The thesis of R. Tarfon in the Gemara is that, in the Miriam case, we *must have* a minor premise that offending *God* (rather than merely one's father) justifies a minimum of seven days of punishment, in order to be able to infer *qal vachomer* (i.e. a crescendo) that offending God justifies fourteen days of punishment – just as with regard to an ox, we (allegedly) reason from half liability for damage done on *private* (rather than public) property to full liability on private property. The Sages do not object to this claim. But this claim is simply not true – there is no such technical requirement for positive subjectal a crescendo (or a fortiori) inference. We can very well, and normally do, reason with a change of subject, i.e. from the penalty for offence to one's father to that for offence to God, or from the liability for damage on public grounds to that on private grounds. This is precisely the power and utility of a fortiori (and a crescendo) inference.

Moreover, we in fact can, by purely a fortiori argument, infer the needed minor premise about seven days penalty for offending God (from the same penalty for offending one's father), and likewise the half liability on private property (from the same liability on public property)[73]. One cannot claim an a crescendo argument to be valid without admitting the validity of the purely a fortiori argument (and pro rata argument) underlying it. Obtaining the minor premise demanded by the Gemara's R. Tarfon is thus not the issue, in either case. The issue is whether such a minor premise will allow us to draw the desired 'proportional' conclusion. And the answer to that, as we show further on, is: No!

Furthermore, if we carefully compare the Gemara's argument here to the first argument laid out in the Mishna, we notice a significant difference. As we just saw, the Gemara concludes with full liability for horn damage on private property on the basis of half liability for horn damage on *private* property. As earlier explained, it bases this minor premise on the fact that Ex. 21:35 does not make a distinction between public and private property when it prescribes half liability for damage by horn, so that this may be taken as a minimum in either case. Thus, for the Gemara, half liability for horn damage on private property is a Torah given, which does not need to be deduced. On the other hand, in the Mishna, the minor premise of the first argument refers to the public domain rather than to private property.

In his first argument, R. Tarfon argues thus (italics mine): "…in the case of horn, where the law was strict regarding [damage done on] *public* ground imposing at least the payment of half damages, does it not stand to reason that we should make it equally strict with reference to the plaintiffs premises so as to require compensation in full?" And to justify his second argument he argues thus: "but neither do I infer horn [doing damage on the plaintiff's premises] from horn [doing damage on *public* ground]; I infer horn from foot, etc."[74] Thus, his first argument is clearly intended as an inference from the penalty for horn damage in the *public* domain (half) to that in the private domain (full). The Gemara's construct is thus quite different from the Mishna's, and cannot be rightly said to represent it.

As regards the rule here apparently proposed by the Gemara (which it attributes to R. Tarfon), viz. that the subject must be the same in minor premise and conclusion, as already stated there is no such rule in formal logic for positive subjectal argument[75]. Such argument generally has the minor and major terms as subjects of the minor premise and conclusion respectively, even if the subsidiary term sometimes (as is the case in a crescendo argument) varies in magnitude 'proportionately'. In the case of a crescendo argument, where the predicate (subsidiary term) changes, *there absolutely must be a change of subject, since otherwise we would have no explanation for the change of predicate*. That is, we would have no logical argument, but only a very doubtful 'if–then' statement. The proposed rule is therefore fanciful nonsense, a dishonest pretext.

We can examine this issue in more formal terms. A positive subjectal a fortiori argument generally has the form: "P is more R than Q is; and Q is R enough to be S; therefore, P is R enough to be S" (two premises, four terms). If the

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[73]   These two a fortiori arguments are given in full in previous sections of the present chapter.
[74]   The explanations in square brackets are given in the Soncino edition.
[75]   Perhaps, then, the Gemara's authorship rather has in mind predicatal argument? For in the latter, the subject is normally constant while the predicates vary. But the difference is that in predicatal argument, the subject of the minor premise and conclusion is the subsidiary term, while the predicates are the major and minor terms; and the major premise differs in form, too. However, this schema does not accord with the form of the Miriam argument, so it is unlikely to be intended by the Gemara for R. Tarfon's first argument, which it considers formally analogous to the Miriam argument.

argument is construed as a crescendo, it has the form: "P is more R than Q is; and Q is R enough to be Sq; and S is 'proportional' to R; therefore, P is R enough to be Sp" (three premises, five terms). The argument form attributed by the Gemara to R. Tarfon simply has the form: "If X is S1, then X is S2" (where X is the sole subject, and S1 and S2 the subsidiary terms, S2 being greater than S1); that is, in the Miriam sample: "if offending God merits seven days penalty, then offending Him merits fourteen days penalty," and again in the Mishna's first dialogue: "If liability for horn damage on private property is half payment, then liability for same on private property is full payment." This is manifestly not a fortiori or a crescendo argument, but mere if–then assertion; it could conceivably happen to be true, but it is not a valid inference.

It is clear that the latter inference, proposed by the Gemara in the name of R. Tarfon, has no logical leg to stand on. It has *no major premise* comparing the subjects (P and Q); and no need or possibility of one, since there is only one subject (X). Having no major premise, it has no middle term (R); and therefore *no additional premise* in which the subsidiary term (S) is presented as 'proportional' to it. Thus, *no justification or explanation* is given why S should go from Sq in the minor premise to Sp in the conclusion. It is therefore not an a fortiori or a crescendo argument in form, even if it is arbitrarily so *labeled* by the Gemara. You cannot credibly reason a fortiori or a crescendo, or any other way, if you cannot produce the requisite premises. There is no such animal as "argument" *ex nihilo*.

The Gemara's proposed if–then statement is certainly not universal, since that would mean that if any subject X has any predicate Y then it has a greater predicate Y+, and if Y+ then Y++, and so forth ad infinitum – which would be an utter absurdity[76]. From this we see that not only has the Gemara's argument no textual bases (as we saw earlier), but it has no logical standing. There is in fact no "argument," just arbitrary assertion on the Gemara's part. For both the Miriam sample and the (first) Mishna sample, the Gemara starts with the convenient premise that "there is a *qal vachomer* here," which it considers as given (since it is traditionally assumed present, on the basis of *other* readings of these texts), and then draws its desired conclusion without recourse to any other proposition, i.e. *without premises*![77]

If this requirement for a single subject is not a rule of logic, is it perhaps a hermeneutic principle, i.e. a rule prescribed by religion? If so, where (else) is it mentioned in the oral tradition or what proof-text is it drawn from? Is it practiced in other contexts, or only in the present one, where it happens to be oh-so-convenient for the Gemara's interpretative hypothesis? If it is an established rule, how come the Sages do not agree to it? The answers to these questions are pretty obvious: there is no such hermeneutic rule and no basis for it. It was unconsciously fabricated by the Gemara author in the process of developing the foolish scenario just discussed. It is not a general necessity (or even a possibility, really), but just an ad hoc palliative.

Unfortunately, when people use complex arguments (such as the a fortiori or the a crescendo) without prior theoretical reflection about them, they are more or less bound to eventually try to arbitrarily tailor them to their discursive needs.

To sum up. We have seen that the Gemara introduces a number of innovations relative to the Mishna it comments on. The first we noted was that the Gemara, in the name of an anonymous Tanna, reads the *qal vachomer* in Num. 12:14-15, and apparently all a fortiori argument in general, as a crescendo argument. Next we noted a surprising lacuna in the Gemara's treatment, which was that while it dealt with R. Tarfon's first argument, it completely ignored his second, and failed to notice the curious verbatim repetition in the Sages' two *dayo* objections. Third, we showed that the thesis that *dayo* is "of Biblical origin," so that R. Tarfon must have been aware of it, was the Gemara's main goal in the present *sugya*. In the attempt to flesh out this viewpoint, the Gemara proceeds to portray R. Tarfon as regarding the *dayo* principle as being applicable only conditionally, in contrast to the universal *dayo* principle seemingly advocated by the Sages.

To buttress this thesis, the Gemara is forced to resort to an argument *ex machina* – that is, although vehemently denying the role of both parts of Num. 12:14 in the formation of a *qal vachomer*, the Gemara's R. Tarfon nevertheless assumes one (i.e. a phantom a fortiori argument) to be somehow manifest between the lines of the proof-text. Moreover, in order to make a distinction between the Miriam example and the (first) Mishna argument, so as to present the *dayo* principle as applicable to the former and inapplicable to the latter, the Gemara's R. Tarfon invents a preposterous rule of inference for *qal vachomer*, according to which the subject must be the same in the minor premise and the conclusion. In the Miriam example, the absence of a minor premise with the required subject (offending God) means that *dayo* is applicable, for applying it would not "defeat the purpose of the *qal vachomer*;"

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[76]   It is of course possible that in a specific case of Y, "all Y1 are Y2" is true; so that predicating the value Y1 entails predicating the value Y2. But this cannot be proposed as a general truth without absurd infinite reiteration.
[77]   This is very much the mentality of a conventional mind – what Ayn Rand has called a "second-hander" in her novel *The Fountainhead*. Such a person takes the say-so of 'authorities' for granted, and makes no effort at independent verification. It builds buildings without foundations. It disregards the natural order of things.

whereas in the (first) Mishna argument, the presence of a minor premise with the required subject (damage by ox on private property) means that *dayo* is inapplicable, for applying it would "defeat the purpose of the *qal vachomer*."

This all looks well and good, if you happen to be sound asleep as the Gemara dishes it out. For the truth is that at this stage the whole structure proposed by the Gemara comes crashing down.

The trouble is, there is no such thing as an a fortiori argument (or a crescendo argument) that takes you *from no information to a conclusion*, whether maximal or minimal. If the proposed *qal vachomer* "argument" has no minor premise (since v. 14a is explicitly not admitted as one) and no major premise (since the subject of the conclusion must, according to this theory, be the same in the minor premise as in the conclusion), then there is *no* argument. You cannot just declare, arbitrarily, that there is an argument, while cheerfully denying that it has any premises. And if you have no argument with a maximum conclusion, then you have no occasion to apply the *dayo* principle, anyway.

Moreover, there is no such one-subject rule in a fortiori logic; indeed, if such a rule were instituted, the argument would not function, since it would have no major premise, and no major, minor or middle term; consequently, if it was intended as 'proportional' (as the Gemara claims), it would imply an inexplicable and absurd increase in magnitude of the subsidiary term. Thus, even if the Gemara's textually absent argument about Miriam were generously granted as being at least 'imaginable' (in the sense that one might today imagine, without any concrete evidence, Mars to be inhabited by little green men), the subsequent demand that a *qal vachomer* have only one subject would make the proposed solution formally impossible anyway.

The Gemara's explanation is thus so much smoke in our eyes, a mere charade; it has no substance. We need not, of course, think of the Gemara as engaging in these shenanigans cynically; we can well just assume that the author of this particular commentary was unconscious. In fine, the Gemara's scenario, in support of its claim that the *dayo* principle is "of Biblical origin" and so R. Tarfon did not ignore it—is logically unsustainable.

6. A slightly different reading of the Gemara

As we saw previously, the two arguments featured in Mishna BQ 2:5 may objectively be variously interpreted. R. Tarfon's first argument may be read as pro rata or as a crescendo, though not as purely a fortiori (since his conclusion is 'proportional'), while his second argument may be read in all three ways. As regards the Sages' first *dayo* objection, if R. Tarfon's first argument is supposed to be intended as a pure a fortiori, the objection to it would simply be that such argument cannot logically yield a 'proportional' conclusion; this reading is very unlikely. Rather, the first *dayo* objection may be taken as a refusal of the 'proportionality' of the pro rata or a crescendo arguments, and possibly the proposal of a purely a fortiori counterargument, i.e. one without a 'proportional' conclusion. The Sages' second *dayo* objection, on the other hand, cannot have the same intent, since in this case all three forms of argument yield the very same 'proportional' conclusion; so it must be aimed at the inductive processes preceding these arguments.

In our above analysis of the corresponding Gemara, we have mostly represented it as conceiving of one possible scenario for both[78] arguments of the Mishna, that of a crescendo argument moderated by a *dayo* principle. This is the traditional and most probable interpretation, but it should be said that an alternative reading is quite possible. Certainly, the Gemara here does not accept, *or even consider*, the alternative hypothesis that purely a fortiori argument may be involved in the second argument of R. Tarfon, since it clearly assumes that the conclusion's predicate is bound to be greater than the minor premise's predicate. However, it would be quite consistent to suppose that the Gemara is in fact not talking of two a crescendo arguments, but of two analogical/pro rata arguments. There is some uncertainty as to the Gemara's real intent, since it does not explicitly acknowledge the various alternative hypotheses and eliminate all but one of them for whatever reasons.

Looking at the Mishna and Gemara discourses throughout the Talmud, it is obvious that the people involved use purely a fortiori argument, a crescendo argument, and argument pro rata in various locations. But it is not obvious that there is a clear distinction in their minds between these three forms of argument. It is therefore not impossible that when they say "*qal vachomer*," they might indiscriminately mean any of these three forms of argument. It should be clear to the reader that the issue I am raising here is not a verbal one. I am not reproaching the Talmud for using the words "*qal vachomer*" in a generic or vague sense. I certainly cannot reproach it for not using the expressions 'a crescendo' or 'pro rata', as against 'a fortiori', since these names were not in its vocabulary.

What I am drawing attention to is the Talmud's failure to demonstrate its theoretical awareness of the difference between the three forms of argument, whatever they are called. How could such awareness be demonstrated? It would have sufficed to state (if only by means of concrete examples, without abstract explanations) that the two

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[78]   Although, as already remarked, the Gemara does not in fact pay any heed to the second argument or at all take it into consideration in its theorizing.

premises of a fortiori per se do not allow a 'proportional' conclusion to be drawn, but must be combined with a third, pro rata premise for such a conclusion (i.e. a crescendo) to be justified; and that it is also possible to arrive at a 'proportional' conclusion without a fortiori reasoning, through merely analogical (i.e. pro rata) reasoning.

That is to say, for instance in the positive subjectal mood, the major premise "P is more R than Q is" and the minor premise "Q is R enough to be S" do not suffice to draw the conclusion "P is R enough to be *more than* S." To deduce the latter a crescendo conclusion, an additional premise must be given, which says that "S is proportional to R." Given all three said premises, we can legitimately conclude that "P is R enough to be (proportionately) *more than* S;" but without the third one, we can only conclude "P is R enough to be S." Alternatively, we might infer from "S is in general proportional to R," combined with "a given value of S is proportional to a given value of R," that "a greater value of S is proportional to a greater value of R" (this is pro rata without a fortiori).

Thus, although we have taken for granted in our above analysis the traditional view that when the Gemara of Baba Qama 25a speaks of *qal vachomer*, it is referring to a fortiori argument, i.e. more precisely put to a crescendo argument (since it advocates 'proportional' conclusions), it is quite conceivable that it was unconsciously referring to mere pro rata argument. The *dayo* principle is not something conceptually, even if halakhically, tied to a fortiori (or a crescendo) argument, but could equally well concern pro rata argument (or even other forms of reasoning). And what I have above called the "bizarre notion," which the Gemara credits to R. Tarfon, that the minor premise and conclusion of a positive subjectal argument must have the same subject for the argument to work, could equally be applied to pro rata argument as to a crescendo, since it is an arbitrary rule of Judaic logic without formal support in generic logic. Therefore, our above analysis of the Gemara would not be greatly affected if we assume it to refer to pro rata instead of to a crescendo argument. This is not a very important issue, but said in passing.

8. In the Talmud, continued

The present chapter is a continuation of the preceding, aimed at further clarifying some details.

1. Natural, conventional or revealed?

Our above critique of the Gemara was based to some extent on the assumption that it considers *dayo* as a principle, which the Sages regard as a hard and fast rule and R. Tarfon views as a conditional rule, depending on whether or not its application "defeats the purpose of the *qal vachomer*." But in truth, the idea of *dayo* as a "principle" may be an interpolation, because the original Aramaic text (viz. "ור"ט לית ליה דיו והא דיו דאורייתא הוא") does not use the word "principle" in conjunction with the word "*dayo*."

The translation given in the *Soncino Babylonian Talmud* (viz. "Does R. Tarfon really ignore the principle of *dayo*? Is not *dayo* of Biblical origin?") does of course use this word. But if we look at the *Talmud Bavli* translation (with their running commentary here put in square brackets), viz. "And does R' Tarfon not subscribe to [the principle of] 'It is sufficient…' – Why, [the principle of] 'It is sufficient…' is contained in the [Written] Torah, [and R' Tarfon must therefore certainly accept it!]" – it becomes evident that the word "principle" is an add-on. This of course does not mean that it is unjustified, but it opens possibilities.

If we do accept the translations, it is clear that the word "principle" is here equivocal, anyway – granting that for the Sages it means a universal proposition whereas for R. Tarfon it means a merely conditional one. This equivocation implies that the positions of the two parties are not as harmonious as the Gemara tries to suggest. They do not agree on principle and merely differ on matters of detail, as it were. On one side, there is a hard and fast rule; and on the other, one that is subject to adaptation in different situations. This is a radical difference, which is hardly diminished by assuming the "principle" to be of Biblical origin.

In view of this, it is difficult to guess what might be the Gemara's purpose in positing that the *dayo* principle is *deoraita* (of Biblical origin – as against *derabbanan*, of rabbinic origin) and is known and essentially accepted by R. Tarfon. Moreover, as we have exposed, the Gemara's scenario for R. Tarfon's thesis is forced and untenable, being based on doubtful readings of the Torah and Mishna texts it refers to and, worst of all, on a parody of logic. Certainly, the Gemara's scenario does not prove the claim of Biblical origin. If anything, that claim is weakened by virtue of having been supported by such rhetoric. But is the claim now disproved, or can it be supported by other means?

The Gemara is, of course, correct is in linking the issue of Biblical origin with that of R. Tarfon's knowledge and acceptance. If the principle is of Biblical origin – i.e. is given in the Written Torah, or (since it is not manifest in the Pentateuch) at least the Oral Torah – it must be assumed to be known and accepted by him, as well as by the Sages. If he did not know and accept it, but only the Sages did, it cannot be of Biblical origin. However, I do not see how the Gemara can claim a different understanding of the *dayo* principle of Biblical origin for R. Tarfon than for the Sages. What would be the common factor between their views, which would be a "principle" of Biblical origin? The difference between universal and only-conditional applicability is too radical; these two theses are logically contrary. Their only possible intersection is that valid *dayo* objections may occur. This is hardly enough to constitute a "principle," although we might in the limit grant it such status.

On the other hand, it would be quite consistent to say that the Sages and R. Tarfon *both* believe in a *dayo* principle of Biblical origin that is *only conditionally* applicable, but only differ with regard to the precise conditions of its application. Thus, the Biblical origin hypothesis remains conceivable, provided the word "principle" is understood in its softer sense, in such a way that debate is logically possible in particular cases, so that R. Tarfon might win in some cases and the Sages in other cases. The *dayo* principle would then consist in the bare fact that "some *dayo* objections are justifiable, though some are not;" and its being of Biblical origin would mean that this vague, contingent prediction was given at Sinai. Such conceivability does not of course prove that this much-reduced *dayo* principle was indeed of Biblical origin. Nor does it explain why the Gemara tried so hard to establish it as such. But it at least leaves the hypothesis in the running, so long as no other plausible reasons are found to discard it.

As mentioned at the end of our analysis of the Mishna, there are yet other equally viable hypotheses. We can still uphold the conflict between the Sages and R. Tarfon to be one between a hard and fast view of the *dayo* principle and an only-conditional view of it, *provided* we do not claim this principle to be of Biblical origin, but only of rabbinic origin (*derabbanan*). In the latter case, the Sages are collectively in the process of legislating the *dayo* principle in our Mishna, and though R. Tarfon initially tries to argue against this innovation by means of his two arguments, at the end he is forced to accept the majority decision. This scenario is equally consistent, and to my knowledge the Gemara offers no reason for dismissing it.

In this context, we could suggest that the *dayo* principle being "of Biblical origin" means, not that is was explicitly mentioned in or logically deduced from the Torah, but simply that something to be found in the Torah *inspired* the rabbis to formulate and adopt this principle. We might even propose (this is pure speculation on my part) the inspiration to have come specifically from Deuteronomy 4:2[1], which reads: "Ye shall not add unto the word which I command you, neither shall ye diminish from it." It could well be that the rabbis, consciously or otherwise, saw in this warning of the Torah a justification for the cautiousness called for by their *dayo* principle. In that event, both R. Tarfon and the Sages obviously agreed regarding the truth of the inspiring Torah passage, but they differed as to how far the inspiration should be allowed to go. The *dayo* principle is not, in either case, precisely deducible from the said Torah passage, but a relation of sorts between the two can be claimed. The rabbinical principle, however broadly understood, is not in 'the letter of the law', but it is surely in 'the spirit of the law'.

Another possibility is that there is no *dayo* principle, whether universal or conditional, at all, but each recorded *dayo* objection stands on its own as an *individual* rabbinical decree, for whatever reason the rabbis consider fit. This too can be used to explain the disagreements between R. Tarfon and the Sages in a consistent manner. This hypothesis logically differs very little from the above mentioned one of a conditional *dayo* principle, except in that the conditional *dayo* principle scenario implies an explicit Divine prediction at Sinai, whereas the no *dayo* principle scenario assumes no specific Sinaitic transmission on this topic (even if the general authority of the rabbis to judge and maybe innovate may have there been explicitly established). Here again, then, we have a consistent alternative hypothesis that the Gemara did not take into consideration and eliminate, before affirming its own thesis.

The methodology of the Talmud is of course essentially dogmatic. It engages in discussions and arguments, usually genuinely logical; but it does not go all the way with logic, systematically applying its techniques and referring to its results. It accepts some arbitrary ideas. This here seems to be a case in point, where the Gemara seeks to prove some preconceived notion and does everything it can to give the impression that it has. But we must always consider alternatives and evaluate them fairly.

The issue we will explore now is whether the *dayo* principle is to be regarded as natural, conventional or revealed. By 'natural' I mean that it is a law of nature, i.e. more specifically of logic or perhaps of natural ethics. By 'conventional' I mean that it is a collective decision of the rabbis, or more generally of human authorities, for whatever motive. And by 'revealed' I mean here that it is Divinely-decreed, handed down to us through prophecy or other supernatural means; i.e. more specifically, primarily at the Sinai revelation through Moses, and then written in the Torah or passed on orally through an unbroken tradition.

We have, I believe, definitely established in our above treatment that the *dayo* principle is not a law of logic. Many people have thought of it – and for a long time, I must confess, I too did so – as signifying that the (predicate of the) conclusion of (purely) a fortiori argument cannot quantitatively surpass the (predicate of the) minor premise. The *dayo* principle, in that view, corresponds to the principle of deduction, i.e. to a reminder that you cannot get more out of it than you put into it. In that perspective, I used to think the rabbis collectively instituted the *dayo* principle in order to prevent *other people* from erroneously drawing a 'proportional' conclusion from purely a fortiori premises. I was misled into this belief, perhaps, by the fact that rabbinical a fortiori reasoning is *in practice* usually correct, and also by the fact that the mentions of *qal vachomer* in the lists of Hillel and R. Ishmael do not mention the *dayo* principle as a separate hermeneutic rule, and therefore apparently consider the latter as an integral part of the former's structure, which though it can be distinguished from it cannot correctly be dissociated from it.[2]

But as we have demonstrated in the present study the *dayo* principle is something much more complex than that. However, although this principle is not a natural principle in the sense of a law of logic, it might still be considered as a natural principle in the sense of a truth of ethics in a secular perspective. If we were to consider it as such, we would have to say that when the rabbis apply it, they are merely expressing their moral sensibilities as ordinary human beings. In that event, we would have to say that the *dayo* principle is applicable not only in legal contexts peculiar to the Jewish religion, but in all legal contexts, whether Jewish or non-Jewish, religious or secular. But the

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[1] Likewise, Deut. 13:1.
[2] To tell the truth, I had inexcusably, at the time I wrote *Judaic Logic,* not actually studied this Talmudic *sugya*, but instead took for accurate what other commentators said about it. I was at the time much more naïvely trusting than I am today!

latter does not seem true – certainly, if we look at legal rulings in other traditions, the idea of *dayo* hardly if at all arises. So this idea seems to be a particularly Jewish (indeed, rabbinical) sensibility.

Thus, the *dayo* principle should rather be viewed as either conventional or revealed. As we have seen, contrary to what the Gemara insists, there is no incontrovertible proof that it is revealed. It may be "of Torah origin" in a broad sense, in the sense of "of Sinaitic origin." But it is clearly (for any honest observer) not explicitly stated in the Written Torah; so it must be assumed to be part of the Oral Torah. Of course, the Gemara does seem to be claiming this principle to be logically derived from Num. 12:14-15 – but as we have seen, this 'proof' is unfortunately circular: it is read into the text rather than out of it. This means that the only way we know that the principle is "of Torah origin" is because the rabbis (led by the Gemara) *tell* us that it is. Such assertion is considered by the rabbis as sufficient proof that the alleged tradition is indeed Sinaitic. But scientifically it is surely not sufficient, as all sorts of things could have happened in the millennia in between.

Thus, while in the first instance (*lehatchila*) the rabbis would affirm the principle as derived from the Written Torah, if they are pressed hard enough they would probably as a last resort (*bedieved*) opt instead for the Oral Torah explanation. But, to my mind at least, this is logically equivalent to saying that the rabbis are the effective source of the principle. That is, it is *derabbanan*, and not at all *deoraita*. For we only have their say-so as proof of their assertion. Of course, it is still conceivable that the principle was indeed handed down at Sinai – we have not disproved that, and have no way to do so. But, as there is no way (short of a new revelation) to prove it, either, this conceivable scenario remains a mere speculation. So that the logical status of the principle is pretty much exactly the same as if the rabbis had simply conventionally decided to adopt it. This is the conclusion I adopt as a result of the present study: the *dayo* principle is of rabbinical origin.

To conclude, it is not clear why the Gemara makes such a big thing about the "Biblical origin" of the *dayo* principle, even going so far as to construct fictitious inference rules and arguments to prove its point. Did the Gemara have some halakhic purposes in mind, or was it just engaging in idle chatter (*pilpul*)? As we have seen, the Mishna can well be understood – indeed, in a number of ways – without pressing need to resolve the issue of the origin of the *dayo* principle. Why then is the Gemara's commentary so focused on this specific issue, ignoring all other aspects? Perhaps it needs the proposition that the *dayo* principle is "of Biblical origin" for some other purpose(s), elsewhere. Not being a Talmudic scholar, I cannot answer this question. But in any event, to my mind, whatever the Gemara's motives may have been, it failed miserably in this particular discourse.

Moreover – let us not forget this fact – when the Gemara refers to the *dayo* principle, it means just the first expression of that principle, as it is applicable to R. Tarfon's first argument. The Gemara has not shown any awareness of the existence and significance of R. Tarfon's second argument, and therefore of the difference in the Sages' *dayo* objection to it. Thus, even if it had succeeded to prove somehow that the Sages' first *dayo* objection was "of Biblical origin," it would not have proven that their second objection was of equally elevated origin. This, too, is a disappointment concerning the Gemara: its powers of observation and analytic powers were here also less acute than they ought to have been.

We have thus far considered the issue of the origin of the *dayo* principle, but now let us look into that of *qal vachomer*. It is worth noting for a start that *qal vachomer* and the *dayo* principle are viewed by the Gemara as two distinct thought processes. The *dayo* principle is applied ex post facto, to the conclusion of a preexisting *qal vachomer*. The *dayo* principle (presumably) cannot be invoked until and unless a *qal vachomer* is formulated. If the *dayo* principle is not applied (as is possible in R. Tarfon's view, according to the Gemara), the *qal vachomer* stands on its own. Thus, *qal vachomer* inference is independent of the *dayo* principle, even if the latter process is not independent of the former. Therefore, claiming that the *dayo* principle is "of Biblical origin" does not necessarily imply a claim that *qal vachomer* inference is also so justified. It may thus well be a natural process, if not a rabbinical convention.

In this context it is interesting to note that, in the lists of hermeneutic principles of Hillel and R. Ishmael, the *dayo* principle is nowhere mentioned, but only *qal vachomer* is mentioned. Since *qal vachomer* can occur, according to the Gemara, without the *dayo* principle, why is the latter not mentioned also as a separate hermeneutic principle? And if the *dayo* principle is "of Biblical origin," as the Gemara has it, should it not all the more be mentioned in such lists? Conversely, if *qal vachomer* is a natural thought process, why does it need to be mentioned is such lists? Perhaps the answer to these questions is simply that the term "*qal vachomer*" in these lists is intended as an all-inclusive title, meaning "anything to do with *qal vachomer*, including on occasion application of the *dayo* principle." Since, whatever the source of *qal vachomer*, whenever it is mentioned the question arises as to whether or not the *dayo* principle is applicable to it, the former always brings to mind the latter. Moreover, the traditional view seems to be that the *dayo* principle is only applicable to *qal vachomer*, so this question will not arise in other contexts.

In the Mishna, there is no explicit reference to the issue of the origin of the inference processes used. No explicit claim is made by anyone there that the *dayo* principle is "of Biblical origin" or any other origin; and nothing of this sort is said of *qal vachomer*. If we look at R. Tarfon's wording, we are tempted to say that he regards his reasoning as natural. When he says: "I infer horn from foot" and "does it not stand to reason that we should apply the same strictness to horn?" – he seems to be appealing to logic rather than to some dogmatic given; and furthermore, by saying "I" and "we," he seems to suggest that the decision process is in human hands. The Sages do not in their replies reprove him for this naturalistic approach; but they merely, it seems, say what they for their part consider to be a wiser ruling.

For the Gemara (i.e. the particular Gemara commentary that concerns us here, and not necessarily the Gemara in general), as we have seen, "*qal vachomer*" is understood as referring specifically to a crescendo argument, i.e. to a fortiori argument with a 'proportional' conclusion. The Gemara bases this understanding on the *baraita* it quotes. It does not mention purely a fortiori argument, which suggests that it is not aware of such form of argument. This is of course an important error on its part, because without awareness of the difference between purely a fortiori argument and a crescendo argument it cannot realize the logical skill of R. Tarfon's second argument and the challenge it posed to the Sages' first formulation of the *dayo* principle. The Gemara's blindness to purely a fortiori argument explains its blindness to R. Tarfon's second argument.

Even so, it is safe to say that the Gemara considers *qal vachomer* as natural in origin. Certainly, it does not explicitly state it to be "of Biblical origin," as it does for the *dayo* principle. Although the Gemara's assumption that Num. 12:14-15 contains an example of *qal vachomer* is reasonable, this Torah passage certainly does not use any verbal expression indicative of it, like "*qal vachomer*" or "all the more;" so, human insight is needed to see the implicit *qal vachomer*. The Gemara cannot be said to regard *qal vachomer* as a conventional construct by the rabbis, since the argument is in its view already found in the Torah. Since the Gemara does not even raise the issue (though it could and should have), it may be supposed to regard *qal vachomer* as ordinary human reasoning.

We might, however, suppose that the Gemara considers that the Miriam example is also given in the Torah to teach us that the correct conclusion of *qal vachomer* is 'proportional' – i.e. that this rule of inference was Divinely-ordained together with the *dayo* principle. But such a supposition is objectively nonsensical, since a fortiori argument is in fact *not* universally 'proportional'. It would suggest that God, well after the Creation, may tell us to disregard logic and judge contrary to its laws. Yet, the laws of logic are not arbitrary dictates that can be discarded at will – even at Divine will – they are inextricably tied to the world as it is and our rational cognition of it. Therefore, to attribute such opinion to the Gemara would be to its discredit.

If we look at the three other a fortiori arguments in the Pentateuch listed in *Genesis Rabbah*, there is as in the Miriam instance no explicit 'proportionality', but we could in two of them at least similarly assume implicit 'proportionality', namely Ex. 6:12 and Deut. 31:27. Moreover, there is one passage in the Pentateuch that is explicitly 'proportional', namely Gen. 4.24: "If Cain shall be avenged sevenfold, truly Lamekh seventy and seven-fold"[3] — but the speaker of this statement being Lamekh, someone apparently not regarded as exemplary, it can hardly be considered as halakhically authoritative. There are also many passages in the rest of the Bible that seem either explicitly or implicitly 'proportional', and so could be brought to bear in the present context. But the Gemara does not (at least, not here) find it necessary to mention any of them.

Thus, it is reasonable to suppose that the Gemara views *qal vachomer* (or at least its 'proportional' version) as natural argumentation – i.e. as not needing a special Divine dispensation to be credible. In other words, it is purely logical. In Talmudic terminology, this would qualify *qal vachomer* as a sort of *svara*, an inference naturally obvious to human reason. This seems to be the way most rabbis throughout history would characterize the argument. Certainly, most of the exceptional rules and dispensations they have enacted in relation to this argument form suggest it; although the fact that some have tried to interdict its free use suggests a doubt in their mind in this regard.

But even though *svara* refers to natural and universal logical insight, *qal vachomer* is always counted as one of the "*midot*," i.e. of the rabbinical hermeneutic principles. There is a difficulty in this fact, because a hermeneutic principle is thought of as a discursive tool (ordained directly by God or indirectly by rabbinical decision) for use specifically in Torah interpretation. Such principles being essentially non-natural, they may well be *not* rationally evident or even perhaps *contrary to* logic. Not so in the case of *qal vachomer*. So there is a problem with its inclusion in the lists of *midot*. The solution of this paradox, I would say, is simply that the rabbis themselves did not make such fine distinctions between natural and conventional logic. Or equally well: they could lump *qal vachomer* with more uncommon forms of reasoning, because in their minds all are "logical." This is indeed suggested in many rabbinical texts in English, where the word "*midot*" is translated as "principles of logic."

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[3]   This passage is not included in the *Genesis Rabbah* listing of ten cases of *qal vachomer*, but is mentioned in Rashi's commentary. According to Jacobs in his *Rabbinic Thought in the Talmud* (p. 116) this instance is mentioned in much earlier rabbinic texts: "*Avot de-Rabbi Nathan* (version B) 44; Gen. Rabbah 4:24 (ed. Theodor-Albeck, p. 225) and the Jerusalem Talmud *Sank*. 10:1 (27d)."

2. Measure for measure

The Gemara perhaps sought to justify the *dayo* principle by claiming it to be "of Biblical origin" – but there was no pressing need for it to do so, since other explanations were readily available and perhaps less problematic. It seems that the Gemara, not having previously analyzed *qal vachomer* reasoning in formal terms, was unable to precisely perceive its constituent premises, and under what conditions they resulted in this or that conclusion; and thence, how such an argument could be rebutted. In the Gemara author's mind, therefore, apparently, the status of a Divine decree ("Biblical origin") was necessary for the *dayo* principle to have the power to rebut the *qal vachomer* argument (as he saw it).

As we have shown, the two arguments proposed by R. Tarfon and the *dayo* objections to them put forward by the Sages can be interpreted in a number of ways. R. Tarfon's two arguments could have been (1) intended as two mere arguments by analogy (more precisely, pro rata); or (2) the first one may have been pro rata, while the second was (purely) a fortiori; or (3) they could (as the Gemara did) both be construed as having been a crescendo. The Sages' *dayo* statements, could be viewed as (a) particular ad hoc objections, decided by the rabbis collegially; or (b) as general objections, either (i) clearly given in the Written Torah or deduced from it (as the Gemara wrongly claims); or (ii) inductively or rhetorically derived from it (as the Gemara actually attempted); or (iii) known from the Oral Torah (i.e. by unbroken tradition since the Sinai revelation); or again (iv) decided by the rabbis.

If we said that R. Tarfon's first argument was purely a fortiori, we would thereby imply that he did not know how to reason correctly in the a fortiori mode; nevertheless, if he did so reason incorrectly, the Sages' *dayo* objection to his argument would in that event be equivalent to the principle of deduction, interdicting a 'proportional' conclusion from the given premises. Many commentators have so interpreted the debate, but in truth they did so without paying attention to R. Tarfon's second argument, which could also be considered as purely a fortiori and yet be free of the Sages' same objection. So this hypothesis is farfetched and unconvincing, and best brushed aside.

More probably, R. Tarfon put forward his first argument in pro rata or a crescendo form; and the Sages objected "*dayo*" to it in particular or in general, as already said. The purpose of this objection was to annul the premise of 'proportionality' inherent in R. Tarfon first argument. R. Tarfon, being an intelligent man, got the message and proposed instead a neat second argument, which was not subject to the same rebuttal, for the simple reason that whatever its form (pro rata, a crescendo *or purely a fortiori*) it yielded one and the same seemingly 'proportional' conclusion. Nevertheless, the Sages again objected "*dayo*" to it, in particular or in general, in exactly the same terms. By so doing, the Sages enlarged the meaning of their *dayo* objection, since it could here only refer to the generalization process preceding the deduction, since annulling the premise of 'proportionality' was useless.

As earlier explained, the principle of deduction is that the putative conclusion of any deductive argument whatsoever must in its entirety follow necessarily from (i.e. be logically implied by) the given premise(s), and therefore cannot contain any information not found explicitly or implicitly in the said premise(s). If a putative conclusion contains *additional* information and *yet seems true*, that information must be proved or corroborated from some *other* deductive or inductive source(s). This principle is true not only of valid a fortiori argument, but of all other valid forms of deductive argument, such as for instances syllogism or dilemma. Inference in accord with this principle is truly deductive. Inference not in accord with this principle may still be inductively valid, but is certainly not deductively valid.

It seems evident that when the Gemara says "a fortiori" (*qal vachomer*) it means a crescendo. Yet the Gemara does not clearly acknowledge the implications of such an assumption (at least not in the *sugya* under scrutiny). To be fully credible, the Gemara should have demonstrated its understanding that the arguments it characterized as a fortiori were not purely so, but involved an additional premise, one which establishes a pro rata relationship between the subsidiary and middle items. The issue is not merely verbal, note well, but depends on acknowledging a logical precondition for validity. Unfortunately, (to my knowledge) the Gemara nowhere explicitly acknowledges this crucial precondition. Nevertheless, we can generously suppose that the Gemara unconsciously or tacitly intends it, and move on. Our inquiry must now turn to the question: What is the required additional premise, in more concrete terms?

The tacit premise. It is a principle of justice (perhaps even the essence of it) that: *on the positive side, the reward ought to fit the good deed and be commensurate with it; and on the negative side, the punishment ought to fit the wrongdoing and be commensurate with it*. If these conditions are not fulfilled, justice has not been entirely served. This principle is in accord with our natural human 'sense of justice'. It is an insight which cannot be proved, but which expresses (at least in part, if not wholly) what we commonly mean by 'justice'. It is the basis of many laws legislated by mankind and guides many courts of law (namely, those that are characterized as 'just') in their deliberations and their rulings. For examples, a greater penalty is incurred by armed bank robbery than by

shoplifting; or by premeditated murder than by murder in a moment of passion. In this negative guise, the principle of justice is known (in Latin) as the *lex talionis*, or law of retaliation.

Of course, the 'sense of justice' is not something literally 'sensory', but rather something 'intuitive', an insight of sorts. We know from within ourselves what is just and what is not. Of course, such knowledge is mere opinion that has to be confirmed over time using inductive techniques. We individually may see things differently at different times; and different people may see things differently. The sense of justice may be honed by use or blunted by disuse. It may be influenced by surrounding culture, whether incidentally or by deliberate propaganda. All the same, even though this faculty can be put to sleep or smothered, swayed or manipulated, each of us (as a being capable of personally suffering in a similar situation) does have an underlying sense of justice.

Of course, it is not always easy to intuit, much less demonstrate indubitably, what is 'fitting' and 'commensurate' reward or punishment. Justice is not an exact science. In Judaism, where this principle is known as ***midah keneged midah*** (meaning: measure for measure), the right measure is determined either by Divine fiat or by rabbinical decision; in the latter case the wisdom of the rabbis being assumed to be above average. I have not seriously researched the issue as to when this principle began to play an explicit role in rabbinical decision making, but I assume it was very early in view of its implicit presence in many stories and commandments of the Jewish Bible (Torah and Nakh).

The story of Miriam's punishment for criticizing Moses, which the Gemara focuses on so insistently, is a case in point. In the Mishna debate, it is obvious that R. Tarfon's two arguments are motivated by the measure for measure principle, even though not in so many words, but in the background, pre-verbally. Some commentators see the statement by God in Gen. 9:6, "Whoso sheddeth man's blood, by man shall his blood be shed," as the Biblical precursor of the measure for measure principle, even though it is more specific, in view of its symmetrical format (shed blood justifies blood shedding). The value and importance of justice in Judaism may be seen, for instance, in the Deut. 16:20 injunction: "Justice, justice shalt thou pursue."

As regards stories, an illustration often appealed to, of God's *practice* of 'measure for measure', is the correspondence between the crimes of the Egyptians against the Israelites and the punishments that later befell them; for example: they wanted to drown the babies (Ex. 1:22) – their army was drowned in the sea (Ex. 14:28). In Joshua 7:25, "Why hast thou troubled us? Hashem shall trouble thee this day," a 'tit for tat' is clearly implied. The principle is well-nigh explicit in 2 Samuel 22:24-28; for instance in v. 26, David says: "With the merciful Thou dost show Thyself merciful, with the upright man Thou dost show Thyself upright." Or compare Proverbs 1:11 and 1:18. Many examples of such reciprocity can also be found in the Talmud; see for instance *Sotah* 8b-11b. The concept is certainly older than the name attached to it.

I have not to date managed to find out when and where the exact Hebrew phrase "*midah keneged midah*" first appears. But I found a Mishna (*Sotah* 1:7) with very similar words: "By the measure that a man measures, so is he measured (במדה שאדם מודד בה מודדין לו, *bemidah sheadam moded bah, modedin lo*)"[4]. The meaning is admittedly not literally identical, since 'measure for measure' is understood to mean more broadly that the way a man behaves determines his recompense. However, if we understand "the measure that a man measures" as signifying the thoughts which determine his behavior, and "so is he measured" as referring to the Divine judgment in consequence of his actions, which determines his recompense, the two ideas may be pretty well equated.[5]

On the basis of this equity principle, it appears reasonable to us (for instance) that someone who has offended God deserves *more* punishment than someone who has merely offended a human being even if the latter be one's own father. On this basis, then, it appears reasonable to us that, in the episode narrated in Num. 12:14-15, Miriam should indeed, as the Gemara suggests, theoretically *deserve* a penalty of (say) fourteen days isolation instead of just seven days. The fourteen is perhaps just an illustrative number, because surely offending God deserves more than double the punishment due for offending one's father. Indeed, even the seven days penalty in the latter case is an arbitrary number – in this case, a Divine decree – so the fourteen days penalty is bound to be so too.[6]

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[4]   This concerns a suspected adulteress. The Mishna goes on, giving examples: "She adorned herself for a transgression; the Holy One, blessed be He, made her repulsive;" etc. And the corresponding Gemara starts with: "R. Joseph said: although the measure has ceased, [the principle] in the measure has not ceased." I found this passage thanks to Jacobs, who quotes it in his *Rabbinic Thought in the Talmud* (p. 78, fn. 1). I had previously by chance found this maxim in the *Mekhilta de Rabbi Ishmael* (*Beshallach,* 1); but this Midrash is a later document, thought to date from the 3rd century.

[5]   In Oriental religions, of course, the 'measure for measure' principle is expressed as the 'law of karma'. This is a more mechanical version than the Judaic principle, which clearly involves Divine intervention and thus distinctively allows for eventual exceptions, i.e. reduced punishment or increased reward out of Divine love.

[6]   Actually, there are explanations of these specific numbers in later commentaries, but I won't go into them here, so as not to complicate matters unduly. (E.g. one explanation refers to the fact that 7 days is the minimum period of quarantine in the event of leprosy, so that another 7 days is the least possible additional period of quarantine.)

Clearly, the Sages' *dayo* principle is *not* a redundant restatement of the principle of deduction for a fortiori argument, as it might sometimes appear to be; nor does it have any other purely logical purpose. Rather, it serves an important additional, more *moral* purpose. We could imagine that the Gemara tacitly agrees that, in the Miriam example, the *qal vachomer* by itself (*per se*) can only logically yield the conclusion of seven days. But in the present case, even though this is not explicitly said anywhere, the *qal vachomer* is not 'by itself': it happens (*per accidens*) to be accompanied by an expectation of fourteen days based, *not* on formal grounds relating to purely a fortiori inference, but on the principle of justice that we have just now enunciated.

The *dayo* principle then comes to teach us: ***even in a case like this, where a greater penalty is expected due to implications of the principle of justice, the rabbinical conclusion (i.e. the law, the halakha) should not diverge from the quantity given in the Torah-based premises, whether such premises are used to draw a conclusion by mere analogy or by a fortiori argument or any other inductive or deductive means.*** The use of inference should not end up concealing and exceeding the penalty amounts mentioned in the premises given by Scripture. Such quantities should be understood as *davka* (as is), and not used for extrapolations however just those might seem based on human reasoning. The *dayo* principle is then, as the Gemara suggests, "Biblical," if only in the sense that it advocates strict adherence to Biblical givens whenever penalties are to be inferred, whether by deduction or by induction.

The motive of the Sages seems obvious enough: the *dayo* principle is essentially a precautionary measure, enacted to avoid human errors of judgment in processes of inference in legal contexts. When a human court condemns an accused to some penalty, it is taking on a very serious responsibility. If that penalty is Divinely-ordained, i.e. explicitly written in the Torah, the responsibility of the human judges is limited to whether or not they correctly subsumed the case at hand to a given set of laws. Whereas, if the judges add something to the given penalty, on the basis of some 'proportional' reasoning, they are taking an additional risk of committing an injustice. So, it is best for them to stick to the Torah-given penalty.

It is interesting to note the comment by R. Obadiah Sforno (Italy, 1475-1550), regarding the principle of "an eye for eye" in Exodus 21:23-25, that "strict justice demanded the principle of measure for measure, but Jewish tradition mitigated it to [monetary] compensation to avoid the possibility of exceeding the exact measure."[7] This suggests that the idea of compensation was instituted in that context to prevent eventual *excess* in the application of physical retribution – which, of course, would not be justice, but injustice[8]. We may refer to this idea to perhaps better understand and justify the *dayo* principle. In instituting this principle, the rabbis were not merely "tempering justice with mercy," but also making sure that there would not be occasional occurrences of injustice, by mistake or due to excessive zeal. It was, at least in part, a precautionary measure.[9]

Viewed as a restraint on 'proportional' inference, the Sages' *dayo* principle is not a principle of logic, but a merely hermeneutic principle inclining rabbinical judgment to mercy. It is not intended to regulate the *qal vachomer* inference as such, but rather to restrict a parallel application of the principle of justice – or perhaps more accurately put, a parallel intuition from our 'sense of justice'. The Sages are telling us: although our human sense of justice produces in us an expectation that (to take the Gemara's example) Miriam deserves (say) a fourteen days penalty, nevertheless God mercifully decreed (in the Torah) only seven days penalty for her. On the basis of this exemplary decree in the Biblical story of Miriam, Jewish legislators and law courts must henceforth *always* judge with the same restraint and limit the concluding penalty to the penalty given in the premise, *even when* the principle of justice would suggest a more severe punishment.

This is surely the real sense of the Sages' *dayo* principle: they were not reiterating any law of logic, but setting a limitation on the principle of justice. And now, having perceived this, we can understand many things in this Talmudic *sugya*. We can understand why the Gemara would wish to establish that the *dayo* principle is Divinely-decreed. For it might seem unjust to restrict application of the principle of justice; it might be argued that the conclusion of a strict deduction is as reliable as its premises. Moreover, we can see how it is conceivable that, as the Gemara has it, R. Tarfon can differ from the Sages' view and ignore the *dayo* principle in some situations. For no law

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[7]   I am here quoting the paraphrase of Sforno's comment given in *The Soncino Chumash* (ad loc.), not Sforno directly.

[8]   It is interesting that, in Shakespeare's *The Merchant of Venice* (Act IV, Scene 1), the Italian Jewish protagonist, Shylock, is refused the "pound of flesh" he had contracted for on the basis that he might inadvertently take more than that (namely, some blood with the flesh). So it seems, ironically, that the legal principle Shakespeare appealed to might have been formulated a few decades before him by… an Italian Jew, i.e. Sforno! (Indeed, according to a Wikipedia article, Shakespeare's play was written in 1596-98, and this and other elements of it are based on a tale by Giovanni Fiorentino called *Il Pecorone,* published in Milan in 1558. Sforno died in 1550.)

[9]   There is of course some tension between what I said a bit higher up, about the *dayo* principle being "Biblical," if only in the sense that it advocates strict adherence to Biblical givens etc., and Sforno's suggestion that "eye for eye" was mitigated to monetary compensation. But, in the latter case, the literal reading of the Biblical law is looked upon as metaphorical and is replaced by a less harsh reading; whereas, in cases of *dayo*, the literal reading is not discarded, but proportional inferences from it are disallowed, so as to prevent harsher practices. These are two clearly very different treatments of Biblical text by the rabbis.

of logic is being ignored or breached thereby, but only a moral principle; and a moral principle is logically more flexible, i.e. it may apply differently to different situations.[10]

Other angles. The *dayo* principle as above presented is designed to prevent the rabbis from ruling too severely. What of rulings that are too lenient, we might ask? Surely, ruling too leniently can conceivably be a problem. Justice is not served if criminals are not punished as they deserve (as indeed unfortunately often happens in practice in present day society). Too much leniency can be a bad thing for society, just as too much severity often is. So the *dayo* principle ought conceivably to forbid excessive mercy, as well as excessive justice.

If we think about it, measure for measure is essentially a principle of justice rather than one of mercy. By definition, mercy is intended to temper strict justice. It is not measure for measure, but beyond measurement. Justice is logical, while mercy is humane. Logically, the judgment should be so and so; mercy mitigates the conclusion. Mercy is surely desirable; but excessive mercy would obviously constitute injustice. Overdoing it would be negation of measure for measure! Thus, the right balance is needed. Arguing thus, we might easily advocate that the *dayo* principle is applicable to inferences that increase leniency, as well as to those that increase severity.

But my impression from rabbinic discourse generally is that the *dayo* principle is always intended as a principle of justice, and not occasionally as a principle of mercy. The rabbis are not so worried about irrational bursts of magnanimity; they are worried about inflicting undeserved punishment.

There is another objection that can be raised to our moral interpretation of the *dayo* principle. It seems reasonable enough in the present *negative* legal context, where the *qal vachomer* has as its conclusion a punishment for a wrongdoing. But what of equivalent *positive* legal contexts, where the *qal vachomer* has as its conclusion a reward for a good deed? Surely, the rabbis cannot here say that it is merciful to diminish the reward's proportionality. Also, what of non-legal contexts, when the *qal vachomer* is constructed in pursuit of a factual conclusion – do the rabbis simply ignore the *dayo* principle in such cases? The question is, then: how general is the Sages' *dayo* principle, or rather: what are the limits of its application?

The answers to these questions are, I think, broadly speaking, as follows. Jewish law, like most law systems, is essentially concerned with sanctions for wrongdoing rather than with rewarding good deeds. For this reason, only the negative side of the measure for measure principle is relevant to the rabbinical legislative process, and applications of the *dayo* principle occur only in relation to penalties. I doubt that any legalistic a fortiori argument with a conclusion of reward occurs in Jewish law; but if any indeed does, and the principle of measure for measure seems applicable, I very much doubt that the rabbis would block, on the basis of the *dayo* principle, the inference of increased or decreased rewards.

As regards a fortiori arguments in homiletic and other non-legal contexts, I do believe the *dayo* principle is indeed ignored in practice. It is admittedly sometimes apparently used – but such use is rhetorical. In other contexts, maintaining the a crescendo conclusion may be preferred. Since the principle has no binding legal impact either way, the decision to use or not-use it depends entirely on what the speaker wishes to communicate.

All the above comments circumscribing use of the *dayo* principle are of course mere personal impressions and educated guesses; they are open to discussion. They would have to be justified empirically, by thorough systematic research through the whole Talmud and indeed all Jewish law literature. Until such data is gathered by scholars, and fully analyzed by competent logicians, we cannot answer the said questions with much greater precision and certainty than just done. Nevertheless, by asking questions and proposing answers, we have at least raised issues and sketched possible results. It would, of course, be interesting and valuable to find rabbinical statements that clearly justify what has been said.

3. The dayo principle in formal terms

We shall here review our new interpretation of the *dayo* principle in more formal terms. This is done with reference to Mishna Baba Qama 2:5, where the principle is traditionally given pride of place, first dealing with the Sages' objection to R. Tarfon's first argument, and then with their objection to his second argument. As already seen, these are two distinct expressions of the *dayo* principle, although they have a common motive. The corresponding Gemara in Baba Qama 25a, as we saw, only seems to have noticed the first version of the *dayo* principle; but later

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[10]   For my part, I must confess that I originally believed the *dayo* principle to be a rabbinical statement of the principle of deduction, proposed specifically for *qal vachomer* only because such argument was for the rabbis the very essence of deductive reasoning. This is essentially the position I took in my *Judaic Logic*, although I also there considered that proportionality was still possible though a separate act of reasoning (whether deductive or inductive). But now, having realized this more accurate interpretation of the *dayo* principle, as applicable to *any* extrapolation attempted on the basis of *midah keneged midah* on the products or preliminaries of *qal vachomer* (or any type of reasoning with similar effect), I definitely opt for this latter hypothesis.

commentators (notably, it seems, Rashi and Tosafot) did notice the second[11]. We shall show here more precisely why the Gemara's view is inadequate.

A further reason why we wish to now investigate the *dayo* principle in more formal terms is because both formulations in the Mishna relate specifically to the positive subjectal form of a crescendo argument. Nothing is there said of eventual applications to the negative subjectal form, or to the positive or negative predicatal forms. Our purpose here is to consider theoretically what such other applications would look like. Whether such other applications actually occur or not in the Talmud (or other rabbinic literature) is not the main issue, here; but it is abstractly conceivable that they might occur. In any case, we are sure to clarify our concept of the *dayo* principle by this enlarged research.

Let us to begin with deal with **the Sages' *dayo* objection to the first argument of R. Tarfon**. Here, R. Tarfon tried to infer a liability of full payment for damage by horn on private property (conclusion), from a liability of half payment for damage by horn on public property (minor premise). He was thus presumably using a crescendo argument, of positive subjectal form, as follows:

> Action P is a more serious breach of a certain law (R) than another action Q is.
> Action Q is a breach of that law (R) enough to merit a certain penalty (S).
> The magnitude of penalty S is 'proportional' to the seriousness of the breach of law R.
> Therefore, action P is a breach of that law (R) enough to merit a *greater* penalty (S+).

The Sages' *dayo* objection to this attempt can be stated as: if the minor premise predicates a certain penalty (S) for a certain action (Q), then the conclusion cannot predicate a *greater* penalty (S+) for a more illegal action (P). This objection can be perceived as neutralizing the additional premise concerning 'proportionality'. The Sages are saying: although by commonsense such 'proportionality' seems just, by Jewish law it is not to be applied, and we can only predicate the same penalty (S) in the conclusion as was previously given (in the minor premise).

What the *dayo* objection does here is to block, or *switch off*, as it were, the operation of the additional premise regarding 'proportionality': though that moral premise might usually be granted credibility, it is rendered inoperative in the present context, to avoid any possible excess of penalization (as earlier explained). This means that *the a crescendo argument is effectively abolished and replaced with a purely a fortiori argument*. Evidently, then, the Gemara's view, according to which the a crescendo argument is allowed to proceed, and then the *dayo* principle reverses its action[12], is technically incorrect. The action of *dayo* is preventive, rather than curative; it takes place before the 'proportional' conclusion is drawn, and not after.

We can easily, by formal analogy, extend this principle to other forms of a crescendo argument, if only out of theoretical curiosity. The analogous positive predicatal argument would have the following form:

> A more serious breach of a certain law (R) is required to merit penalty P than to merit another
> penalty Q.
> Action S is a breach of that law (R) enough to merit penalty P.
> The seriousness of the breach of law R is 'proportional' to the magnitude of action S.
> Therefore, a *lesser* action (S–) is a breach of that law (R) enough to merit penalty Q.

Notice that the additional premise about 'proportionality' is different in subjectal and predicatal arguments. The order is reversed. In the former, the subsidiary term S, being a predicate, is proportional to the middle term R; whereas in the latter, it is the middle term R that is proportional to the subsidiary term S, which is a subject. This is due to the order of things in the minor premise, which the conclusion naturally reflects, where predication is made possible only if the value of R for the subject matches or exceeds the minimum value of R necessary for the predicate.

In this context, the Sages' *dayo* objection would be stated as: if the minor premise predicates a certain penalty (P) for a certain action (S), then the conclusion cannot predicate a lesser penalty (Q) for a *less* illegal action (S–). This objection can be perceived as a denial of the additional premise concerning 'proportionality'. Here, the Sages might say: although by commonsense such 'proportionality' seems just, by Jewish law it is not to be applied, and we can only address the same action S in the conclusion as was given (in the minor premise). This statement, to repeat, is formulated by analogy, merely for theoretical purposes; it is not given in the original Mishna debate.

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[11]   These later commentators generously project their insights onto the Gemara; but this is of course anachronism, motivated by their wish to claim a continuity of tradition.
[12]   Notice the sequence of events in the following sentence in the Gemara: "nevertheless, by the working of the a fortiori, fourteen days may be suggested: there follows, however, the principle of *dayo* so that the additional seven days are excluded." This means that: first, fourteen days are inferred using *qal vachomer*; and *after that* ("there follows"), the number of days is reduced by *dayo* to seven.

There is admittedly a difficulty in the latter extension of the *dayo* principle. For whereas applying *dayo* to a positive subjectal argument results in preventing potentially excessive justice, by mechanically attributing a greater penalty to a more serious breach of law, the application of *dayo* to a positive predicatal argument results in the prevention of increasing leniency, which is what attributing a lesser penalty to a less serious breach of law would constitute. We shall return to this issue further on.

As regards the corresponding negative arguments, they can easily be determined using the method of *ad absurdum*. In each case, the major premise and the additional premise about 'proportionality' remain the same, while the negation of the conclusion becomes the new minor premise and the negation of the minor premise becomes the new conclusion. Application of the (first) *dayo* principle to them would have the effect of inhibiting the deduction of the putative negative a crescendo conclusion from the given negative minor premise, through rejection of the additional premise.

As for implicational arguments, they can be dealt with in comparable ways.

Let us now deal with the **Sages' *dayo* objection to the second argument of R. Tarfon**. Here, R. Tarfon tried to infer a liability of full payment for damage by horn on private property (conclusion), from a liability of full payment for damage by tooth & foot on private property (minor premise). He was thus using an argument, again of positive subjectal form, that yields the same conclusion whether construed as a crescendo argument or as purely a fortiori. This means that the first version of the Sages' *dayo* principle would be useless in this second case, for the minor premise and conclusion naturally have the exact same predicate (full payment). Therefore, since the Sages nevertheless declared *dayo* applicable, they must have been referring to some *other* feature of the argument.

The only other logical operation they could have been referring to is the inductive formation of the major premise, by generalization from the liability of half payment for damage by horn on public property and the liability of no payment for damage by tooth & foot on public property. That is, the major premise that 'liability for damage by horn is *generally* greater than liability for damage by tooth & foot' was derived from the same given concerning horn as before, namely that 'liability for damage by horn on public property is half payment'. Here, then, the *dayo* principle must be stated in such a way as to interdict this preliminary generalization.

The Sages apparently hint at this solution to the problem by restating their second objection in exactly the same terms as the first. There is no other explanation for their using the exact same words. In this context, then, the Sages' *dayo* objection would be stated as: if the major premise is inductively based on information about a certain action (P) meriting a certain penalty (S), in one set of circumstances, then the conclusion drawn from it cannot be that the same action (P) in another set of circumstances merits a greater penalty (S+). That is, under the *dayo* principle, we can only conclude that 'P is S', not that 'P is S+'. Note well how this second version of the *dayo* principle is very different from the previous.

It is important to realize that, unlike the preceding one, this *dayo* objection cannot be perceived as neutralizing the additional premise concerning 'proportionality'. For here, a crescendo and purely a fortiori argument have *the exact same* conclusion; so that whether or not we 'switch off' this third premise *makes no difference whatever* to the result. This means that, in the present case, the argument is necessarily purely a fortiori, i.e. devoid of an additional premise. No a crescendo argument can usefully be proposed here, since the conclusion is already maximal through purely a fortiori argument. Therefore, in such case, we must prevent the unwanted conclusion *further upstream* in the reasoning process; that is, at the stage where the major premise is getting formed by means of a generalization.

We can easily, by formal analogy, formulate a similar principle with regard to positive predicatal argument. In this context, the Sages' *dayo* objection would be stated as: if the major premise is inductively based on information about a certain action (S) meriting a certain penalty (Q), in certain circumstances, then the conclusion drawn from it cannot be that a lesser action (S–) in whatever other circumstances merits the same penalty (Q). That is, under the *dayo* principle, we can only conclude that 'S is Q', not that 'S– is Q'. This statement, to repeat, is formulated by analogy, merely for theoretical purposes; it is not given in the original Mishna debate.

Admittedly, our formal extension of the second *dayo* principle from positive subjectal argument to positive predicatal argument is open to debate. For whereas in the former case *dayo* serves to prevent increased severity, in the latter case it seems to have the opposite effect of preventing increased leniency. This issue will have to be addressed, further on.

Returning now to the Gemara, we can see from the above formal treatment, that it was wrong in considering the *dayo* principle as concerned essentially with a crescendo argument. In the first case, which the Gemara did try to analyze, the Sages' *dayo* objection effectively advocated a purely a fortiori argument instead of R. Tarfon's apparent attempt at a crescendo argument. But in the second case, which was unfortunately ignored by the Gemara, the Sages' *dayo* objection couldn't function in a like manner, by blocking the usual velleity of 'proportionality', since this would be

without effect on the conclusion. It had to apply to a presupposition of R. Tarfon's argument, however construed – namely the generalization earlier used to construct its major premise.[13]

Let us now return to the issue glimpsed above, as to whether or not the *dayo* principle is only meaningful in relation to positive subjectal a crescendo argument, which proceeds from a lesser penalty for a lesser infraction to a greater penalty for a greater infraction. We have seen that we can formally enlarge the idea of preventing proportionality implied in *dayo* application to positive subjectal argument, to negative subjectal, and to positive and negative predicatal arguments – but is such analogy meaningful when more concretely examined? We shall here try to answer this question.

Remember our earlier determination that the *dayo* principle is not a logical principle, but a "moral" one, i.e. it has to do with ethics or law in the context of the Jewish religion. It is not logically necessitated by the principle of deduction or by the use of a fortiori argument or any other purely logical consideration; no contradiction would arise if we simply ignored it. It is, rather, something Divinely or rabbinically prescribed, to lawmakers and courts of law, for cases where a *qal vachomer* is being attempted in order *to infer a greater penalty for some wrongdoing*. It is an artificial injection into the Jewish legislative process apparently motivated by mercy, i.e. to temper justice. There is no reason to apply it in contexts other than the sort just specified, or for that matter in other religions or outside religion.

We could eventually expect the same idea to be extended from penalties to *duties*. Such conceptual extrapolation might well be found exemplified in the Talmud or other Jewish literature (I have not looked for examples). That is conceivable if we think of penalties and duties as having in common the character of *burdens* on the individual or community subjected to them. If we look on increased duties (*mitzvoth*) as positive rewards, in the way that a servant might rejoice at receiving increased responsibilities, the analogy of course fails. But if we look on duties as burdens, an analogy is possible. It that case, the *dayo* principle could be taken to mean more broadly that burdens in general must not be increased on the basis of a *qal vachomer* argument from the Torah.

Granting the above clarifications of the *dayo* principle, the first question to ask is: is its function limited to contexts of positive subjectal *qal vachomer* – or can this definition be extended to other a fortiori argument formats? The format focused on by the rabbis is, to repeat, positive subjectal, which means that it is minor to major (*miqal lechomer*), whence the appropriateness of the name *qal vachomer*. Let us now consider what *dayo* application to the negative subjectal format would mean. Such argument is, of course, major to minor (*michomer leqal*) in orientation. It would look as follows:

> Action P is a more serious breach of a certain law (R) than another action Q is.
> Action P is a breach of that law (R) *not* enough to merit a certain penalty (S).
> The magnitude of penalty S is 'proportional' to the seriousness of the breach of law R.
> Whence, action Q is a breach of that law (R) *not* enough to merit a *lesser* penalty (S–).

The major premise and the additional premise about 'proportionality', which (as we saw earlier) is in practice derived from the principle of *midah keneged midah* (measure for measure), both remain the same, here. What changes is that the minor premise and conclusion are now negative propositions and the major term (P) appears in the former and the minor term (Q) appears in the latter. It remains true that the value of S associated with P is greater than that associated with Q; however, note that here the greater value appears in the minor premise and the lesser in the conclusion.

Our question is: what would be the significance of the *dayo* principle, in either of its senses, in such negative subjectal context? Note that above argument is formally valid. The question is thus not whether its conclusion follows from its said premises. The question is whether to reject its additional premise (first type of *dayo* application) or its major premise (second type of *dayo* application).

At first sight the answer is that the *dayo* principle would not be called for – because there is no velleity in such a context to use the principle of measure for measure, and *dayo* is intended as a restraint on such velleities. Since the minor premise and conclusion are negative, we can say that no actual penalty, small or large, is claimed in either of these propositions; in that case, we are not naturally inclined to engage in measure-for-measure reasoning, and therefore no *dayo* principle is needed to block such reasoning. It would appear, then, that the *dayo* principle is not useable in such negative context.

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[13]   It should be said that R. Tarfon's first argument could conceivably be inhibited by the second type of *dayo* objection (viz. blocking formation of the major premise by generalization), as well as by the first type (viz. blocking operation of the third premise about proportionality). But this does not seem to be the thrust of the Sages' rebuttal of the first argument; they seem rather to adopt a purely a fortiori stance in opposition to their colleague's a crescendo approach.

However, we could also look upon such negative argument as tacitly positive. *Assuming that all law-breaking merits some penalty*, we could argue that where an illegal action is not sufficiently illegal to merit a certain penalty we may infer it to positively merit a *lesser* penalty, though we cannot predict *how much* less. In that case, the negative subjectal argument would be interpreted as saying that P is illegal enough to positively merit a penalty of magnitude 'somewhat less than S', and therefore Q is illegal enough to positively merit a penalty of magnitude even smaller than 'somewhat less than S'. This thought clearly involves measure-for-measure reasoning; so the *dayo* principle ought to now be applicable.

But of course it is not in fact applicable, because this new argument infers a decrease in penalty, whereas the *dayo* principle is essentially aimed at preventing inferences of increase in penalty. It is intended as a principle of mercy, pushing towards leniency rather severity of judgment; therefore, its application here would be inappropriate. In other words, we would not normally try to interdict the conclusion of a negative subjectal argument (even one recast in more positive form), whether by denial of the additional premise or of the major premise, for the simple reason that such reaction would not be in accord with the spirit and intent of the *dayo* principle.

We can argue in much the same way with respect to positive predicatal a crescendo argument:

> A more serious breach of a certain law (R) is required to merit penalty P than to merit another penalty Q.
> Action S is a breach of that law (R) enough to merit penalty P.
> The seriousness of the breach of law R is 'proportional' to the magnitude of action S.
> Therefore, a *lesser* action (S–) is a breach of that law (R) enough to merit penalty Q.

Here again, we have reasoning from major to minor – specifically, from a more illegal action (S) with a greater penalty (P) to a less illegal action (S–) with a smaller penalty (Q) – so, there would be no sense in applying (in either way) the *dayo* principle to it. Such an argument would, if our analysis of the moral motives of this principle has been correct, be allowed to proceed unhindered.

However, things get more complicated when we turn to negative predicatal argument, since the orientation is again from minor to major, while the minor premise and conclusion are negative in polarity:

> A more serious breach of a certain law (R) is required to merit penalty P than to merit another penalty Q.
> Action S is a breach of that law (R) not enough to merit penalty Q.
> The seriousness of the breach of law R is 'proportional' to the magnitude of action S.
> Therefore, a *greater* action (S+) is a breach of that law (R) not enough to merit penalty P.

In view of the negative polarities involved, we are tempted to say that there is no call for the *dayo* principle since no actual penalties are claimed. However, if we recast the argument in more positive form, following the idea that *all law-breaking merits some penalty*, we could say that the minor premise concerns some positive penalty of magnitude 'somewhat less than Q' (for action S) and likewise the conclusion concerns some positive penalty of magnitude 'somewhat less than P' (for action S+). Assuming that 'somewhat less than P' is greater than 'somewhat less than Q', which seems reasonable granting the additional premise, we can say that this argument is indeed from minor to major in a positive sense. In that case, the *dayo* principle ought to be applied to it, to prevent justification of the increased penalty advocated by the conclusion. Thus, either the additional premise about 'proportionality' or the generalization leading to the major premise will be interdicted.

Thus, to sum up, whereas when we think in bare formalities the four forms of a crescendo argument might seem liable to *dayo* principle interference, upon reflection it is only the positive subjectal and negative predicatal forms which are concerned, because they go from minor to major. The other two forms, the negative subjectal and the positive predicatal, are not concerned, because they go from major to minor. So the issue is not so much the polarity of the argument as its orientation. All the above can be repeated regarding implicational arguments, of course.

What we have said here, of course, refers to arguments that predicate penalties[14]. Arguments that predicate rewards are not to be treated in an analogous manner, because (as we have seen earlier) the *dayo* principle is only aimed at preventing increased punishment, not increased reward. But, one might ask, what of decreased rewards? Is not a decrease in reward comparable to an increase in punishment? The answer to that I would suggest is again practical rather than formal: Jewish law is not concerned with rewarding good deeds, but in penalizing bad ones. Furthermore, it does not address all bad deeds, but only some of them – namely, those subject to judgment by rabbinical courts.

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[14]   Or eventually, maybe, duties – viewed as burdens, as earlier explained.

The purpose of Jewish law, as indeed most law systems, is to ensure at least social peace; it is not to control everything. Accordingly, the *dayo* principle is not intended to deal with changes in magnitude relating to rewards. It will simply not be invoked in such contexts; and indeed, such contexts are not expected to arise.

This is all assuming, of course, that my understanding of the matter is correct. It is not unthinkable that the empirical truth is a bit different from what I have assumed; and for instance, there are in fact occasional applications of the *dayo* principle in situations where I have just said it is logically inapplicable. In that event, needless to say, the above account would have to be modified in accord with actual facts. This should not be too difficult, since the formal issues are already transparent. It is not unthinkable that over time the original intent of the Sages' *dayo* (given in Mishna Baba Qama 2:5) has been misunderstood, forgotten or intentionally ignored, and the concept of *dayo* was eventually used more broadly. This is in fact suggested by the broad or vague way that the *dayo* principle is usually presented in rabbinical literature.

Judging by the study of Mishnaic *qal vachomer* presented in **Appendix 2**, we cannot resolve the empirical issue with reference to the Mishna. For, surprisingly, of the 46 arguments found there, only the famous two in Mishna Baba Qama 2:5 involve the *dayo* principle! This is an important finding. There are nine other arguments which are possibly a crescendo, and therefore could be subject to *dayo*; but there is no mention of *dayo* in relation to them – either because they are not really a crescendo or because they do not serve to infer a penalty from the Torah.

Therefore, we must look to the Gemara (and indeed, later rabbinic literature), to find out whether the *dayo* principle is consistently applied in practice as here postulated. Only after all a fortiori arguments in the whole rabbinic corpus have been identified and properly analyzed will this question be scientifically answered. In **Appendix 3**, I try to at least partly answer the question, using the Rodkinson English edition of the Talmud. My finding in this pilot study is that there are only six Talmudic contexts where the *dayo* principle is explicitly appealed to! In five of these cases, the *dayo* principle may be said to be used as I have predicted, i.e. to prevent increase in legal responsibility through a fortiori argument. In the remaining case, this is partly true (see fuller explanation there).

Considering the prime position given to *qal vachomer* in the rabbinic lists of *middot* (hermeneutic principles), and the great attention accorded by rabbinical commentators to the Mishna Baba Qama 2:5 which introduces the *dayo* principle, one would expect the Tannaim (the rabbis of the Mishnaic period) to resort to *dayo* objections quite often. That this is statistically not the case is, to repeat, quite surprising. It may well be that more instances of *dayo* use by Tannaim will be found in some *baraitot* (statements attributed by Tannaim not included in the Mishna), many (maybe most) of which are quoted by Amoraim (the rabbis of the Gemara period) in different passages of the Talmud. This matter deserves systematic research, if we want to get a realistic idea of the quantity of *dayo* use by the Tannaim[15].

Besides that, we of course need to further research independent *dayo* use by the later rabbis, i.e. the Amoraim and their successors, respectively. Its use also in the early and late Midrashic literature deserves close study too. As regards the Amoraim, it is also quite surprising how little they appeal to the principle, at least explicitly, at least in the Rodkinson edition. However, my expectation is that, though some more use of the *dayo* principle by the Tannaim and the Amoraim may well be found, it will not be significantly much more.

I would like now to deal with a couple of further details, before closing this topic.

To begin with, let us reflect on the fact that rabbinical formulations (apparently of more recent vintage historically) usually describe a fortiori argument as an instrument of legal reasoning that can proceed *in both directions*, i.e. both from minor to major and from major to minor. For instance, consider the following formulation by R. Feigenbaum:

> "Any stringent ruling with regard to the lenient issue must be true of the stringent issue as well; [and] any lenient ruling regarding the stringent issue must be true with regard to the lenient matter as well."[16]

According to this statement, given that a stringent ruling (S) applies to the lenient issue (Q), it must also apply to the stringent issue (P); and given that a lenient ruling (S) applies to the stringent issue (P), it must also apply to the lenient issue (Q). The first part of that statement matches positive subjectal a fortiori (minor to major). The second part of it presumably refers to the negative subjectal form, since it is major to minor (and obviously not predicatal).

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[15]   I have read that there are separate collections of *baraitot*. These would, of course, have to be consulted too to resolve the issue once and for all.
[16]   *Understanding the Talmud*, p. 88-90. Feigenbaum rightly characterizes *qal vachomer* as "a particular logical structure," but he introduces the above formula by saying: "it is logical to assume that…." This is a sort of contradiction: if the structure is truly logical, the argument is not a mere assumption, but a thought process that can be validated. Feigenbaum evidently has not attempted to logically validate his formula. He does, however, describe two ways in which the Gemara may "refute" such argument – either by showing that the proposed ruling is found inapplicable in relation to another relatively stringent (or, respectively, relatively lenient) issue, or by showing that the lenient issue is in some respects more stringent (or, respectively, that the stringent issue is in some respects more lenient). But the latter "refutations" are, of course, material rather than formal: they effectively deny the truth of the minor or major premise in a given case, not the validity of the argument properly formulated.

Indeed, that is how I interpreted it in my *Judaic Logic*[17]. My thinking there was that: Given that there has been some breach of law (R), then some penalty is deserved; in that event, "not-deserving a stringent penalty" implies "deserving a lenient penalty"! The terms stringent and lenient being understood as relative to each other, not as absolute.

Thus, a formulation such as R. Feigenbaum's tacitly assumes that "all law-breaking merits some penalty." It is only on this basis that we can indeed logically transfer a lenient ruling from a stringent issue to a lenient matter, as he and others postulate. Although his above formula is stated entirely in positive terms, it in fact refers to both positive and negative arguments. Note in passing that the *dayo* principle is not mentioned in that writer's formula. That is because he is here thinking in purely a fortiori terms, and not a crescendo like the Gemara. He is not saying that the inferred ruling is to be *more* stringent or *more* lenient, but only *as much* so. The same stringency or leniency is passed on.

Not having R. Feigenbaum's book in my possession any longer, I do not know what, if anything, he said in it about the *dayo* principle. I doubt offhand that he distinguished between purely a fortiori and a crescendo argument, and that he related that principle exclusively to the latter form and limited *dayo* use to increased stringencies. But, using at the language of his above statement, I would say it ought to be amplified as follows. In cases where purely a fortiori inference is appropriate, the same degree of stringency or leniency is concluded, and the *dayo* principle is irrelevant. But in cases where a crescendo inference is appropriate, the natural conclusion would be more stringency or more leniency. In such cases, if the conclusion is a more stringent penalty than the one proposed in the Torah, *dayo* should be applied; whereas if it is more lenient it need not be.

Another point I would like to clarify is the idea emitted above that in predicatal a crescendo argument the subsidiary term (the subject of the minor premise and conclusion) is decreased (in the positive mood) or increased (in the negative mood). What does it mean to say, as we did, that *an action* is lesser or greater? This is best clarified by giving an example. We might, for instance, conceive two kinds of killing: intentional killing and unintentional killing, and argue thus: More badness (middle term, R) is required to merit a more severe penalty (major term, P) than to merit a less severe penalty (minor term, Q); so if, under the law relating to killing, intentional killing (S1) is bad enough to merit a more severe penalty, then unintentional killing (S2) is bad enough to merit a less severe penalty. This is a positive predicatal a crescendo argument.

Formal application of the *dayo* principle to this reasoning would mean that it is forbidden to here follow the principle of measure for measure and infer a lesser penalty for the less serious crime. Intuitively, such interdiction is obviously contrary to reason: we would rather let the 'proportional' conclusion stand since it is more indulgent. Neither justice nor mercy would be well served by applying the *dayo* principle to such cases. To punish a less serious crime the same way as a more serious one would be contrary to both justice and mercy. To punish a less serious crime less severely than a more serious one is in accord with both our sense of justice and our sense of mercy.

Clearly, then, the *dayo* principle should remain inoperative in cases of positive predicatal a crescendo argument concerning retribution for crime. Similar reasoning, as we have seen, applies to negative subjectal a crescendo argument. It is only with regard to positive subjectal or negative predicatal a crescendo arguments that the *dayo* principle makes sense and has relevance, for only in their case may there be an over-enthusiastic upsurge of justice, so that mercy requires a more cautious and temperate approach. In other words, *dayo* is potentially relevant only to a crescendo arguments that go from minor to major; it plays no role in such arguments that go from major to minor. *Dayo* is also, of course, irrelevant to purely a fortiori arguments (whether *a minori* or *a majori*), since the subsidiary term (whether it is a subject or a predicate) remains unchanged in them.[18]

This is spoken entirely from a theoretical perspective. It does not mean that the rabbis have all always been as conscious as that of the various possibilities. But I suspect they at least subconsciously have indeed reasoned in this way and limited *dayo* in the ways above described. Exceptions might conceivably be found in the mass of Talmudic and other rabbinic literature. This is an empirical question that must be answered empirically. If examples of upside down application of *dayo* are found, they would need to be rationalized somehow ad hoc – or, alternatively, they could be viewed as occasional errors of reasoning.

To conclude our formal exposition, we can say that the *dayo* principle is much leaner than what we may have originally imagined. It is not a formal law of a fortiori logic, but a very specific religiously-inspired rule for Jewish legislators and judges. Moreover, it is not a rule to be applied indiscriminately, but specifically with regard to attempts at increasing penalties on the basis of proportional *qal vachomer* reasoning. I should add: since a crescendo argument as such, i.e. as distinct from the *dayo* principle used to freeze its conclusions as just explained, is purely logical – it is inaccurate to call *qal vachomer* a hermeneutic rule! The first hermeneutic rule in Hillel's list or in R. Ishmael's list is, strictly speaking, not the *qal vachomer* argument, but the *dayo* principle applied in the context of

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[17]   See chapter 4.5 there.
[18]   The same can of course be said of the implicational equivalents of those various arguments. *Dayo* will only apply to positive antecedental or negative consequental a crescendo arguments concerning punishment for illegal acts.

such argument. We may nevertheless maintain the use of "*qal vachomer*" as the title of the first rule on the basis that the *dayo* principle is called for solely in that specific context, *because it is only in such context that a quantitative increase (in penalty) might be inferred*.

One might unthinkingly assume that the *dayo* principle might equally well be used in conjunction with other forms of analogical reasoning (e.g. *gezerah shavah* or *binyan av*). Indeed, one might argue that if *dayo* is applicable in such a maximally deductive context as *qal vachomer*, then it should all the more be applicable in more inductive contexts like *gezerah shavah* or *binyan av*. But further reflection should convince that what distinguishes *qal vachomer* is that it deals with quantities and the *dayo* principle is a restriction of increase in quantity (of the subsidiary term, to be exact) when inferring a penalty from the Torah. Since *gezerah shavah*, *binyan av* and other hermeneutic principles do not prescribe quantitative changes, the *dayo* principle does not concern them.

It remains conceivable, however, that yet other forms of reasoning could result in quantitative changes that would call for application of *dayo*. Come to think of it, it does seem like the rabbis "temper justice with mercy" even in situations that do not involve *qal vachomer* or any other hermeneutic principle. But of course such judgments might not be characterized as based on the *dayo* principle, since they are made more directly. What I am referring to here is the rabbinical interpretation of the *lex talionis* (the law of retaliation) found in Exodus 21:23–25 and Leviticus 24:19–21 – the famous "an eye for an eye, a tooth for a tooth" principle. The rabbis do not read this Torah law literally, but as a call for monetary compensation in cases of injury; this is shown using various arguments, including a *qal vachomer*.[19]

4. The human element

Looking at rabbinical practices and principles, we can safely say that the rabbis were very careful to acknowledge the human element in reasoning a fortiori, or by means of any other of the listed hermeneutic principles (and by extension, even unlisted thought processes).

This is evident, first of all, in their practice of *teshuvah* (Heb.) or *pirka* (Aram.) – usually rendered in English as 'objection' or 'challenge' – consisting in retorting to or rebutting an argument, and in particular an a fortiori argument, by showing or at least pointing out that one (or more) of its premises is (wholly or partly) open to doubt or false, or that the putative conclusion cannot in fact be drawn from the given premises. This demonstrated their awareness, if only pre-verbally in some instances, of the *inductive* sources of many of the propositions used in their reasoning. In some cases, as well, such practice on their part demonstrated awareness of the relative artificiality of certain forms of argumentation they used and thence the tenuousness of their conclusions.

Such awareness of the human element in apparently deductive inference is also made evident in their setting a number of explicit restrictions on the use of a fortiori argument. Such argument could only be used for inferring laws by qualified rabbis involved with their peers in the development of Jewish law (meaning in principle members of the Sanhedrin, though in practice some participants were probably not officially members). Inferences made had to be accepted unanimously or by ruling of a majority. Inferences could be made only from written Torah laws, and not from oral Torah traditions, even if they were reputed to go all the way back to Moses, and all the more so if they were considered to be of more recent vintage. One could not infer a new ruling from a previously inferred ruling, i.e. use the conclusion of one a fortiori argument as a premise in the next.

I would additionally suggest, *an a fortiori inference from a Torah law would be considered questionable if it was found to conflict with another Torah law*. This seems reasonable on the general understanding that written Torah law carries more weight in Judaism than any human inference. An example is apparently given by Louis Jacobs in his *The Jewish Religion: A Companion*[20] with reference to a responsum of the Radbaz (Spain-Israel, R. David ben Zimra, 1479-1573) to the question why the Torah does not forbid a man's marriage to his own grandmother, and yet forbids him his wife's grandmother (who is a more remote relative), although we would expect by a fortiori argument from the prohibition in the latter case that the former case would also be prohibited. Jacobs explains: "Typical of Radbaz's attitude to the limited role of human reasoning in Judaism is his reply that the *a fortiori* argument is based on human reasoning, whereas the forbidden degrees of marriage are a divine decree, so that human reasoning is inoperative there. All we can say is that God has so ordained. One degree of relationship is forbidden, the other permitted."

The a fortiori argument here is: a man's own grandmother (P) is more closely related (R) to him than his wife's grandmother (Q); if his wife's grandmother (Q) is closely related (R) enough to be forbidden in marriage to him (S), then a man's own grandmother (P) is closely related (R) enough to be forbidden in marriage to him (S). The

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[19]   See in the present volume, in the chapter on Moses Mielziner, the section called 'Concerning the *jus talionis*' (13.3). Also see *Baba Qama*, 83b-84a.
[20]   Oxford: OUP, 1995.

difficulty is that, although the former is forbidden, the latter is *not* forbidden. However, I do not see why the rabbis do not accept this a fortiori argument, as they do many others, and simply prohibit marriage to one's own grandma, since there is no written permission to contend with. The answer given by the Radbaz, and before him by Menahem Meiri (France, 1249-1316), is that there is no need for the inferred prohibition as no one would be likely to do such a thing anyway in view of age differences. That is, more precisely put, while a man might be attracted to his wife's grandmother (e.g. if his wife is thirteen years old, and her mother twenty-six and her grandmother thirty-nine, and he is forty), he is unlikely to be attracted to his own grandmother (who would be in her mid-sixties at least). But this argument may seem a bit weak, as some men are attracted by much older women, even if rarely.[21]

Another restriction was that a ruling based on a fortiori argument could not take precedence over a Torah law from which it was inferred, if the two happened to come into conflict. For example, it is inferable from the Torah law (Ex. 23:4) that one should return one's enemy's lost ox or ass that one should likewise, a fortiori, return one's friend's lost ox or ass. One might think that, having thus made a deductive inference, it would follow that when simultaneously encountering two lost animals, one from each of these people, one could legally prefer to return that belonging to one's friend rather than (or at least before) returning that belonging to one's enemy. But no: the premise remains more binding than the conclusion, and one must therefore give precedence to the enemy's animal[22]. Yet another important restriction was that a rabbinical law court could not sentence someone to corporeal punishment on the basis of a legal ruling derived by a fortiori argument. Meaning that, however reliable the justifying deduction might well have been, there was still a drop of doubt in it sufficient to preclude such drastic penalties.

Some of these restrictions were perhaps more theoretical than practical, because if we look at Talmudic discussions (Mishna, Gemara and later commentaries and super-commentaries all included) one is struck by the ease and frequency with which the rabbis engaged in a fortiori argument if only rhetorically. One would have to examine all rabbinic literature in great detail to determine whether these theoretical restrictions have all in fact been consistently adhered to in practice (this is certainly a worthwhile research project for someone). Nevertheless, on the whole, these restrictions show the rabbis' acute awareness of the natural limits of the human powers of experience and reason.[23]

The *dayo* principle as I have above described it falls right into this pattern of restricting excessive reliance on logical means. A ruling based on *qal vachomer* argumentation remains somewhat doubtful, even though the conclusion (if correct) follows the premises with absolute certainty, because there is inevitably some human element in the induction of the premises. These premises may be in part or even largely Torah-based, but still some part(s) of them were inevitably based on human insight or convention, so it is wise to remain a bit open-minded concerning their conclusion[24]. But this is nothing to do with the *dayo* principle, as we have latterly discovered. This principle is not designed to throw doubt on *qal vachomer* argumentation as such, but to prevent extrapolation from Torah-based premises by means of the principle of justice.

A question we could ask is: why is the *dayo* (sufficiency) principle not *directly and always* applied to the *midah keneged midah* (measure for measure) principle? In my above treatment of these principles, I have identified the latter as inserting an additional premise of 'proportionality' between the minor premise and conclusion, and the former as either blocking the operation of this additional premise or preventing the formation of the major premise through generalization. Thus, we may view the measure for measure principle as tending to turn a purely a fortiori conclusion into an a crescendo one, and the sufficiency principle as on the contrary tending to restrain (in one way or another) such proportionality. The two balance each other out, and the result is that the purely a fortiori conclusion stands unchanged.

The question is: could we not say, more generally: *whenever* we encounter a *midah keneged midah*, we must apply *dayo*? Why does the *qal vachomer* need to be mentioned at all? Obviously, if such a general rule was promulgated,

[21]   In any case, this is not a very good example of the above stated restriction on a fortiori inference, because the conflict here is between an inferred prohibition and a Torah 'permission' (presumed merely due to *absence of* written prohibition, note well), and not between an inferred permission (or exemption) and a written Torah prohibition (or imperative). But, even though I cannot here adduce a fully appropriate example, I think the said restriction does exist and is quite reasonable. Even if I turn out to be wrong, the issue is worth investigating.

[22]   This example and its explication are given by R. Schochet in the already cited online video. However, I have not found the Talmudic reference for it (though it is one of the five examples given by Saadia Gaon in his commentary on the 13 *midot*). Moreover, elsewhere, namely here: www.come-and-hear.com/supplement/so-daat-emet/en_gentiles3.html, it is pointed out that returning a lost animal to a brother is based on Deut. 22:2 – in which case, I do not see the need for a *qal vachomer* from Ex. 23:4 (unless a 'friend' and a 'brother' mean different things). Nevertheless, I will not get into a discussion of this concrete issue, nor look for a less controversial example – an illustration of the rabbinic restriction was all that was needed here and this perhaps hypothetical one will suffice.

[23]   This is the general point I want to make here. In fact, rabbinic restrictions on use of *qal vachomer* (and/or the *dayo* principle) and other hermeneutic principles are far more numerous and intricate than here suggested (indeed, sometimes they seem to me *ad hoc*, i.e. tailored for the convenience of a particular discussion only). But I do not want to get bogged down in this special field of study. You can find some further details and clarifications in Steinsaltz or Mielziner, for instances.

[24]   Francis Bacon, in his *The Advancement of Learning*, expresses a similar thought: "As in nature, the more you remove yourself from particulars, the greater peril of error you do incur; so much more in divinity, the more you recede from the Scriptures by inferences and consequences, the more weak and dilute are your positions" (2:25:12).

the two said principles would effectively cancel each other out and cease to exist! Obviously, too, this is not the intent of the *dayo* principle; i.e. it is not meant to altogether neutralize the *midah keneged midah* principle. So it is reasonable to suppose the *dayo* principle to be intended for a specific context; namely, for when a *qal vachomer* is formulated and we are tempted to extrapolate its conclusion by a thought of measure for measure. And more specifically still, for when the speaker (like R. Tarfon in mBQ 2:5) attempts to infer a larger penalty from a lesser penalty prescribed in the Torah.

If there were no *qal vachomer*, or other deductive inference, the measure for measure principle might conceivably have been applied without restriction. Why then, we might well ask, was the *dayo* principle needed in the context of *qal vachomer*? Perhaps the answer to that important question is that if the measure for measure extrapolation occurs in a non-deductive context, we naturally remain aware of the human element in it and maintain a healthy measure of skepticism. Whereas *in a deductive context*, especially where the powerful logic of *qal vachomer* is used, since we have already *proved* part of the quantity, we are more likely to view its measure for measure extrapolation as also 'proved'. The *dayo* principle comes to remind us that the proposed extrapolation does *not* have the same degree of reliability as the more limited conclusion of the *qal vachomer* has. Indeed, the *dayo* principle precludes any temptation to extrapolate rather than let us run the risk wrongful extrapolation.

This may conceivably have been the justification of the *dayo* principle in the rabbis' minds. Even if they did not fully realize that it concerned a thought of *midah keneged midah* accompanying a *qal vachomer*, rather than the latter argument *per se*, they would have sensed the danger of unbridled extrapolation. And according to the Gemara, as we have seen, the preemptive measure against such extrapolation (viz. the *dayo* principle) was not a mere rabbinical ruling (by the Sages), but a Divine decree (through Num. 12:14-15). It perhaps had to be a Torah-based hermeneutic rule, so that it could not in turn be open to doubt as a human construct. Even so, as we have seen, R. Tarfon and others did (according to the Gemara) claim the *dayo* principle could in some situations be bypassed or even ignored. But, for the most part, the Sages' posture has prevailed.

It is worth noting lastly that, according to later authorities (at least some of them), *qal vachomer* argument (or more precisely the *dayo* principle associated with it) could only be used in the *Talmudic* law making process. After the closure of this process, it was considered illegal to use this hermeneutic principle, or any other of the thirteen rules of R. Ishmael for that matter, to interpret the written Torah for legislative purposes. The references for this sweeping ruling are given by R. Bergman[25] as: "*Maharik Shoresh* 139; *Ra'ah* to *Ketubos* cited in *Yad Malachi* 144." This limitation in time is additional evidence that Judaism does not view the *dayo* principle as a law of logic but as a revealed *ad hoc* religious law. Laws of logic cannot be abrogated; decrees can. Similarly for the other hermeneutic principles.

Why this limitation in time? Because, I presume, the hermeneutic rules were a prerogative of the Sanhedrin, the Jewish Supreme Court; when its deliberations were interrupted due to foreign conquest and rule, rabbis were no longer empowered to use these interpretative principles. An implication of this explanation is that if – or when – the Sanhedrin is reinstituted (presumably by the Messiah) the *dayo* principle and other such guidelines will again be useable by its members. This is a neat answer to the question, except that most of the Babylonian Talmud's deliberations took place in Babylon, far from the traditional seat of the Sanhedrin in the Land of Israel. Presumably, the Babylonian rabbis involved were considered to be worthy successors to the Sanhedrin. The reason for the time limitation would then simply be that the Talmud was 'closed' in about 500 CE (say), and subsequent rabbis were considered as at a lower spiritual level than their teachers.

5. Qal vachomer without dayo

It should be pointed out that Talmudic use of *qal vachomer* does not always require application of the *dayo* principle, for the simple reason that the conclusion sometimes naturally lacks the required quantitative aspect, i.e. there is no propensity to 'proportionality' that needs to be interdicted. In other words, the argument is purely a fortiori rather than a crescendo. Consider the following argument:

> "All these things they [the rabbis] prescribed [as culpable] on a Festival, how much more [are they culpable] on Sabbath. The Festival differs from the Sabbath only in respect of the preparation of food." (Mishna *Beitzah*, 5:2.)[26]

There is, surprisingly, no remark in the corresponding Gemara (*Yom Tov*, 37a) on this significantly different use of a fortiori reasoning. Here, unlike in the Miriam example and cognate cases, there is no appeal to the *dayo* principle.

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[25]   See his chapter 13.
[26]   See www.halakhah.com/pdf/moed/Beitzah.pdf.

Does the Talmud notice and discuss this difference anywhere else? I do not know. In any case, this example is very interesting and worth analyzing further.

The Mishna here clearly teaches that: *what is **forbidden** (assur) on a Festival is, a fortiori, also forbidden on the Sabbath*. We can express this in a standard form of a fortiori argument (namely, the positive subjectal, from minor to major) as follows:

> The Sabbath (P) is more religiously important (R) than any Festival (Q); whence:
> if a certain action on a Festival (Q) is important (R) enough to be forbidden (S),
> it follows that the same action on the Sabbath (P) is important (R) enough to be forbidden (S).

This is a passable representation of the argument. However, if we ask what we mean here by more "religiously important," we might reply that the Sabbath is more "demanding" (or strictly regulated) than any Festival. In that perspective, the argument would seem to be, though still 'minor to major', more precisely negative predicatal in form, and we should preferably formulate it as follows:

> More holiness (R) is required to observe the Sabbath (P) than to observe any Festival (Q).
> If some action[27] (S) is *not* sufficiently holy (R) to be *compatible with* observance of a Festival (and thus must[28] be forbidden on it) (Q),
> then that action (S) is not sufficiently holy (R) to be compatible with observance of the Sabbath (and thus must be forbidden on it) (P).

Note that I have inserted "holiness" (of an action) as this argument's operative middle term (R) on the basis of rabbinical explanatory statements in the present context that the holiness of the Sabbath is greater than that of any Festival day. The way I have used this word is a bit awkward, I'll admit; but it does the job anyway.

More fully expressed the argument has three components: (a) Given that (in the minor premise) S implies not-Q, it follows by contraposition that if Q is prescribed, S must be forbidden. (b) And given that S implies not-Q, it follows by a fortiori that S implies not-P. Finally, (c) since (in the conclusion) S implies not-P, it follows by contraposition that if P is prescribed, S must be forbidden. The two 'contrapositions' used are simple ethical logic: anything that interferes with achievement of a set goal is obviously to be prohibited; the means must be compatible with the ends.

We can present the corresponding positive predicatal (major to minor) as follows:

> More holiness (R) is required to observe the Sabbath (P) than to observe any Festival (Q).
> If some action (S) is sufficiently holy (R) to be compatible with observance of the Sabbath (and thus may be permitted on it) (P).
> then that action (S) is sufficiently holy (R) to be compatible with observance of a Festival (and thus may be permitted on it) (Q),

This follows from the negative form by *reductio ad absurdum*, of course. The meaning of this new argument is: *what is **permitted** (i.e. not forbidden) (*mutar*) on the Sabbath is, a fortiori, also permitted on a Festival*. That is, the argument could as well be put in negative subjectal form, as follows:

> The Sabbath (P) is more religiously important (R) than any Festival (Q); whence:
> if a certain action on the Sabbath (P) is important (R) *not* enough to be forbidden (S),
> it follows that the same action on a Festival (Q) is important (R) *not* enough to be forbidden (S).

The expression "not enough to be forbidden" may be taken to imply that the action in in fact "permitted."

Obviously, we cannot reverse these two statements, viz. that what is forbidden on a Festival must be forbidden on the Sabbath, and what is permitted on the latter must be permitted on the former. Obviously, something forbidden on the Sabbath (e.g. cooking food) is not necessarily also forbidden on a Festival. Something permitted on a Festival (e.g. cooking food) is not necessarily also permitted on the Sabbath. Reasoning of the latter sort would be fallacious by the ordinary rules of a fortiori logic.

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[27]   I call the subsidiary term S an "action" to stress that it is something that the people towards whom the law is addressed have a choice to do or not do. No law is possible or meaningful if not addressed to humans with freewill; and no law can be made about something which it is outside their control.

[28]   The injunction "must be forbidden" is addressed to the judges who will legislate and implement the law, whereas the law which says that "S is forbidden, etc." is addressed to the people.

Note also: although I have above classified the two arguments as predicatal (i.e. copulative), it might be more accurate to call them consequental (i.e. implicational). For, what the negative form tells us is that a certain action (S) *by a Jew* causes some deficiency of, let us say, holiness (R) *in him* and thus causes *him* to fail to observe a Festival (Q) or the Sabbath (P); similarly for the positive form, *mutatis mutandis*. In other words, while it is true that P, Q, R, S are terms, there is an unstated underlying subject (a Jewish man, or woman) in relation to which they are all predicates, so that theses (rather than terms) are in fact tacitly intended here.

Furthermore, according to formal logic, if the above two arguments are true, the following two (in which the negative term not-S replaces the positive term S) must also be true.

> More holiness (R) is required to observe the Sabbath (P) than to observe any Festival (Q).
> If some inaction[29] (not-S) is *not* sufficiently holy (R) to be *compatible with* observance of a Festival (and thus must be forbidden on it) (Q),
> then that inaction (not-S) is not sufficiently holy (R) to be compatible with observance of the Sabbath (and thus must be forbidden on it) (P).

This is a negative predicatal (minor to major) argument. The meaning of this new argument is, clearly: *what is imperative (chayav) on a Festival is, a fortiori, also imperative on the Sabbath*. In this form, it is positive subjectal.

More fully expressed the argument has three components: (a) Given that (in the minor premise) not-S implies not-Q, it follows by contraposition that if Q is prescribed, S must be prescribed. (b) And given that not-S implies not-Q, it follows by a fortiori that not-S implies not-P. (c) Finally, since (in the conclusion) not-S implies not-P, it follows by contraposition that if P is prescribed, S must be prescribed. The two 'contrapositions' used are simple ethical logic: anything without which a set goal cannot be achieved is obviously to be prescribed; the means necessary for an end are indispensable.

We can present the corresponding positive predicatal (major to minor) as follows:

> More holiness (R) is required to observe the Sabbath (P) than to observe any Festival (Q).
> If some inaction (not-S) is sufficiently holy (R) to be compatible with observance of the Sabbath (and thus may be permitted on it) (P).
> then that inaction (not-S) is sufficiently holy (R) to be compatible with observance of a Festival (and thus may be permitted on it) (Q),

This follows from the negative form by *reductio ad absurdum*, of course. The meaning of this new argument is: *what is exempted (i.e. not prescribed) (patur) on the Sabbath is, a fortiori, also exempted on a Festival*. In this form, it is negative subjectal.

Obviously, here again, we cannot reverse these two statements, viz. that what is imperative on a Festival must be imperative on the Sabbath, and what is exempted on the latter must be exempted on the former. Something imperative on the Sabbath (e.g. the additional sacrifices on it) is not necessarily also imperative on a Festival. Something exempted on a Festival (e.g. the said additional sacrifices) is not necessarily also exempted on the Sabbath. Reasoning of the latter sort would be fallacious by the ordinary rules of a fortiori logic.

Clearly, the Sabbath and the Festivals involve some distinctive practices; and Festivals are not all identical. The Festivals are not merely lighter forms of Sabbath, and the Sabbath is not merely a heavier form of Festival; and the various Festivals involve different rituals. We cannot deductively predict *all* features of one holy day from the other, or vice versa, but must refer to Biblical injunctions or hints for the special features of each. The above a fortiori arguments do not provide a complete set of relationships, which mechanically exclude innovations from the Biblical proof-text.

What can be inferred from the Sabbath to Festivals or vice versa is a product of two forces: (a) the major premise, which relates these two kinds of holy day *through a middle term* that we took to be 'holiness'; and (b) the minor premise, which links one of these holy days to a certain subsidiary term *through the same middle term*. This limits the possibilities of inference, insofar as the middle term does not have unlimited scope. For a start, 'holiness' is a vague abstraction, difficult to establish objectively; moreover, it does not provide links to any and all subsidiary terms, but only at best to a specified few.

Thus, much in these arguments depends on traditional understanding of the terms involved. That is to say, the arguments are descriptive propositions as much as deductive processes. They give verbal expression to pre-existing

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[29]   Note that I here call S an action and not-S an inaction merely for convenience – it may be that S is an inaction and not-S is an action. The important thing is that they be contradictories.

traditions or traditions taking shape, as well as assist in the inference of information. They are formulas designed to enshrine traditional principles and facilitate logical access to them.

It is perhaps historically in this way, by development from the *Beitzah* 5:2 example of a fortiori argument, that **the more general rabbinic definition of *qal vachomer*** emerged (presumably later)[30]. To take a modern statement, R. Chavel defines the argument as follows:

> "A form of reasoning by which a certain stricture applying to a minor matter is established as applying all the more to a major matter. Conversely, if a certain leniency applies to a major matter, it must apply all the more to the minor matter."[31]

This seems to refer primarily to the first two of our above examples, where the "minor matter" is a Festival day and the "major matter" is the Sabbath, and the "stricture" is the proscribing of some action and the "leniency" is its permission. Stricture, of course, suggests restriction, a negative; but it can here be taken to mean *more broadly* strictness or stringency and thus also refer to a prescription, just as leniency can also refer to an exemption. This is evident in the similar but more accurately worded description of a fortiori reasoning by R. Feigenbaum:

> "Any stringent ruling with regard to the lenient issue must be true of the stringent issue as well; [and] any lenient ruling regarding the stringent issue must be true with regard to the lenient matter as well."[32]

A similar description may also be found in Steinsaltz's *Reference Guide* and many other books. What this tells us is that although the examples traditionally drawn from *Beitzah* 5:2 initially refer to *qal vachomer* inferences from prohibition to prohibition and from permission to permission, the rabbis also eventually admit the inferences from imperative to imperative and from exemption to exemption that we have just logically demonstrated.

Mielziner, by the way, shows explicit awareness of all four moods, to the extent that where the conclusion is "*assur*" (forbidden) he adds in brackets the alternative of "*chayav*" (imperative), and where the conclusion "*eino din sheassur*" (permitted) he adds in brackets the alternative of "*[eino din] shechayav*" (exempt). That is, he makes allowance for both the negative and the positive interpretations. He additionally gives us Talmudic examples of an imperative implying an imperative by such *qal vachomer*: in *Baba Metzia* 95a, it is inferred that the borrower must restore what was stolen (from him the borrower by some third party) to the lender; or again, in *Baba Metzia* 94b, that the borrower must restore what he (the borrower) lost to the lender.[33]

However, I am not sure exactly when, in documented history, the transition occurred from the principle specifically concerning Festivals and Sabbaths given in Mishna *Beitzah* 5:2, and perhaps other passages of the Mishna with a similar thrust, to the general formulations that authors like Mielziner, Chavel, Feigenbaum or Steinsaltz, give nowadays. I suspect the general formulations are not that modern, and may be found in the Talmud or other early literature. It would be very interesting to discover exactly how the progression from material principle to formal principle occurred, i.e. thanks to whom and on what dates.

To conclude this section, what we need to note well is that *no application of the dayo principle is needed or even possible* in cases of the sort here considered, since obviously an action is either forbidden or permitted, either imperative or exempted, and there are no degrees in between. Admittedly, as regards permitted actions, some may be more 'desirable' or 'to be preferred' or 'recommended' than others, but these are not degrees of permission as such. Observe that we have no inclination, in the above inference from permission on the Sabbath to permission on a Festival, to regard the latter permission as of a lesser (or greater) degree than the former. Similarly with regard to exemption: it has in itself no degrees. Very often, the conclusion of a fortiori argument is like that – *without degree*. This is clearly purely a fortiori inference, and not to be confused with a crescendo inference.

I do not know if the rabbis explicitly made this distinction, between *qal vachomer* use with appeal to *dayo* principle and *qal vachomer* without relevance of *dayo*. As I have explained, the *dayo* principle is needed to block reasoning through the *midah keneged midah* (measure for measure) principle or similar 'proportional' propositions. It is not directly related to a fortiori argument as such; it is only indirectly related, to prevent a common penchant for 'proportionality' in special cases. In many cases, if not in most, there is no such propensity, because there is no parallel principle like *midah keneged midah* pressing us towards 'proportionality', and therefore the issue of *dayo* does not even arise. In truth, a fortiori reasoning is always the same, irrespective of whether there is 'proportionality' or not and whether *dayo* is thereafter used or not.

In view of all this, it is hard to understand why the Gemara commentary in Baba Qama 25a is so categorical in its treatment, giving the impression that a fortiori argument is necessarily a crescendo, and failing to explicitly note that the *dayo* principle, whether it is applied to all a crescendo arguments (as the Sages apparently hold, in the Gemara's

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[30]   This is of course a historical question worth investigating empirically.
[31]   *Encyclopedia of Torah Thoughts*, p. 27, n. 106.
[32]   *Understanding the Talmud*, p. 88.
[33]   *Introduction to the Talmud*, pp. 132-4. "Must restore" is, of course, an imperative, a positive instruction.

view) or only to some (as R. Tarfon holds, according to the Gemara), is not applicable to purely a fortiori arguments, i.e. those which do not involve (explicitly or implicitly) an additional premise about 'proportionality'. Surely, if the author of this Gemara was aware of the full sweep of Talmudic discourse, he would have noticed these distinctions and taken them into consideration in his commentary.[34]

6. Three additional Gemara arguments

Further on in tractate Baba Qama, on pp. 25b-26a, the Gemara proposes three a fortiori arguments in which the previously used propositions, about damage by horn and by tooth & foot on public and private grounds, are recycled and reshuffled in various ways, and the resulting conclusions are tested. For this reason, I have dubbed them "experimental" arguments. It is not immediately clear what the purpose(s) of these additional arguments might be. At first sight, their insertion here looks like a process of consistency checking. Possibly, the Gemara is using them to settle some legal matter specified in the larger context. Alternatively, it is merely exploring theoretical possibilities, trying different permutations and seeing where they lead. Or again, perhaps the Gemara is simply engaged in intellectual exercise for its own sake. In any case, we shall here try to throw some light on these arguments by means of logical analysis.

Before we do so, however, let us briefly recall here **the original Mishna (BQ 2:5) arguments** to which they refer, for this will facilitate our work. *The first* Mishna argument can be presented in several ways. Its premises and conclusion can be laid out as a set of if-then propositions spelling out the legal liability for damage by different causes in different domains, as follows:

> If tooth & foot and public, then no liability (by extreme inversion of Ex. 22:4).
> If tooth & foot and private, then full liability (Ex. 22:4).
> If horn and public, then half liability (Ex. 21:35).
> If horn and private, then full liability (R. Tarfon's putative conclusion).
> If horn and private, then half liability (the Sages' conclusion, after application of *dayo* type I).

As we saw in our earlier detailed treatment, this basic argument can be recast in analogical, pro rata, a crescendo or purely a fortiori forms, as follows:

> Analogy
> Just as, in the case of tooth & foot, damage in the private domain implies *more* legal liability than damage in the public domain (since the former implies full liability and the latter none).
> Likewise, in the case of horn, damage in the private domain implies more legal liability than damage in the public domain (i.e. given half liability in the latter, conclude with full in the former).
>
> Pro rata
> The degree of legal liability for damage is 'proportional' to the status of the property the damage is made on, with damage in the private domain implying more legal liability than damage in the public domain.
> This is true of tooth and foot damage, for which liability is known to be nil in the public domain and full in the private domain.
> Therefore, with regard to horn damage, for which liability is known to be half in the public domain, liability may be inferred to be full in the private domain.
>
> A crescendo
> Private domain damage (P) is more important (R) than public domain damage (Q) [as we infer by extrapolation from tooth & foot damage (where liability is respectively full and half in the two domains) to all causes of damage, including horn].
> Horn damage in the public domain (Q) is important (Rq) enough to make the payment half (Sq).
> The payment due (S) is 'proportional' to the degree of legal liability (R).
> Therefore, horn damage in the private domain (P) is important (Rp) enough to make the payment full (Sp = more than Sq).

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[34]   I have not here resolved the question as to whether in the Talmud (Mishna and Gemara) the language of purely a fortiori argument is different from that of a crescendo argument. Probably not, but it is worth looking into the matter empirically. I do so with regard to the Mishna in Appendix 2, but only in English translation (not in the original Hebrew).

Pure a fortiori
Private domain damage (P) is more important (R) than public domain damage (Q) [as we infer by extrapolation from tooth & foot damage, to repeat].
Horn damage in the public domain (Q) is important (R) enough to make the payment half (S).
Therefore, horn damage in the private domain (P) is important (R) enough to make the payment half (S).

As we learned previously, the above analogical, pro rata or a crescendo arguments correspond to R. Tarfon's reasoning. The Mishna Sages reject his reasoning by means of a *dayo* objection of the first type, i.e. which denies the 'proportionality' assumed by their colleague. Effectively, then, the Sages advocate the purely a fortiori argument exclusively. *The second* Mishna argument can likewise be presented in several ways. As a set of if-then propositions, it looks as follows:

If tooth & foot and public, then no liability (by extreme inversion of Ex. 22:4).
If horn and public, then half liability (Ex. 21:35).
If tooth & foot and private, then full liability (Ex. 22:4).
If horn and private, then full liability (R. Tarfon's same putative conclusion).
If horn and private, then half liability (the Sages' conclusion, after application of *dayo* type II).

And here again, the basic argument can be recast in analogical, pro rata, a crescendo or purely a fortiori forms, as follows:

Analogy
Just as, in the public domain, damage by horn implies *more* legal liability than damage by tooth & foot (since the former implies half liability and the latter none).
Likewise, in the private domain, damage by horn implies more legal liability than damage by tooth & foot (i.e. given full liability in the latter, conclude with full in the former).

Pro rata
The degree of legal liability for damage is 'proportional' to the intentionality of the cause of damage, with damage by horn implying more legal liability than damage by tooth & foot.
This is true of the public domain, for which liability is known to be nil for damage by tooth and foot and half for damage by horn.
Therefore, with regard to the private domain, for which liability is known to be full for damage by tooth and foot, liability may be inferred to be full for damage by horn.

A crescendo
Horn damage (P) is more important (R) than tooth & foot damage (Q) [as we infer by extrapolation from the public domain (where liability is respectively half and nil in the two cases) to all domains, including the private].
Tooth & foot damage in the private domain, (Q) is important (R) enough to make the payment full (S).
The payment due (S) is 'proportional' to the degree of legal liability (R).
Therefore, horn damage in the private domain (P) is important (R) enough to make the payment full (S).

Pure a fortiori
Horn damage (P) is more important (R) than tooth & foot damage (Q) [as we infer by extrapolation from the public domain, to repeat].
Tooth & foot damage in the private domain, (Q) is important (R) enough to make the payment full (S).
Therefore, horn damage in the private domain (P) is important (R) enough to make the payment full (S).

As we found out previously, this time *all* of the above argument forms, including the purely a fortiori one, match R. Tarfon's reasoning. So, the Mishna Sages cannot reject his reasoning by means of a *dayo* objection of the first type, since 'proportionality' is not essential to its stringent conclusion of full liability. Nevertheless, they maintain their *dayo* objection, and again advocate a moderate conclusion of only half liability. Therefore, the latter *dayo* objection must be of a second type. It is indeed, interdicting the inductive process of generalization through which the major premise of such argument is produced. We need not say more than that here, having already dealt with the issues involved at length.

Now, what is interesting is the way the Gemara takes the final conclusion of the Mishna Sages, namely that horn damage in the private domain implies half liability, and uses it as a constant premise in each of its three experimental arguments. This proposition is of course implied by Ex. 21:35, which specifies half liability for horn damage, without specifying a domain; but the Sages have effectively ruled that it is not a minimum but a maximum[35], i.e. it is to be read as *davka* half. Nevertheless, the Gemara here additionally uses a watered down version of Ex. 21:35 in two of its arguments (the first two).

Another proposition relevant to all three Gemara arguments is Ex. 22:4, which specifies full liability for tooth & foot damage in the private domain[36]. This proposition is repeated in two of the Gemara arguments (the first and last). In the Mishna, the liability for tooth & foot damage in the public domain is taken to be the extreme inverse of Ex. 22:4, i.e. no liability. And this is also assumed in two Gemara arguments (the last two); however, at the end of one Gemara argument (the first one), a moderate inversion is attempted, i.e. "not full" is taken to mean "half" rather than "nil."

Let us now examine **the three new arguments in the Gemara** more closely.

First experiment. The Gemara states: "*But should we not let Tooth and Foot involve liability for damage done [even] on public ground because of the following a fortiori:*
If in the case of Horn, where [even] for damage done on the plaintiff's premises only half payment is involved, there is yet liability to pay for damage done on public ground,
does it not necessarily follow that in the case of Tooth and Foot, where for damage done on the plaintiff's premises the payment is in full, there should be liability for damage done on public ground?
— Scripture, however, says: And it shall feed in another man's field, excluding thus [damage done on] public ground. But have we ever suggested payment in full? It was only half payment that we were arguing for!"[37]

Note at the outset the sources of the premises in the Gemara's argument. One is the earlier conclusion of the Mishna Sages (via their *dayo* objections to R. Tarfon's claims) that for damage by horn on private property the ox owner's liability is half. The other two premises are more directly derived from the Torah (Ex. 22:4 and Ex. 21:35). The conclusion concerns damage by tooth & foot on public property.

Expressed as a set of brief if-then statements, this Gemara argument looks as follows. Note that the first two have in common the factor of private property.

> If horn and private, then half liability (ruling of the Mishna Sages).
> If tooth & foot and private, then full liability (Ex. 22:4).
> If horn and public, then *some* liability (from Ex. 21:35).
> If tooth & foot and public, then *some* liability (putative conclusion).

Or in analogical format, as follows:

> Just as, in the private domain, damage by tooth & foot implies more legal liability than damage by horn, since the former implies full and the latter half.
> Likewise, in the public domain, damage by tooth & foot implies more legal liability than damage by horn; whence given that the latter implies *some* liability (note that although Ex. 21:35 implies a specific amount, the Gemara here deliberately avoids mentioning it in its premise), then the former implies *some* liability.

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[35]   The Sages opinion is obviously accepted as henceforth binding.
[36]   Which is taken to mean to the exclusion of the public domain. Such exclusion is based on *davka* interpretation of Scripture. That is, what is specified as applicable to private property is taken to include *only* private property, thus excluding public property. The thinking here is: 'Otherwise, why specifically mention private property?' In general, "If A and B, then C" does not formally exclude "If A and not B, then C"; taken together they imply "If A, then C." However, in the exclusive reading, "If A and B, then C" is taken to imply "If A and not B, then not C."
[37]   Note that I have left out a sentence here, because I do not understand it and do not see its logical significance. This says: "*Scripture further says, And they shall divide the money of it [to indicate that this is confined to] 'the money of it' [i.e. the goring ox] but does not extend to compensation [for damage caused] by another ox.*" What has "another ox" got to do with it?

Or again, in purely a fortiori format, of positive antecedental form (minor to major), as follows[38]:

> Tooth & foot damage (P) is more important (R) than horn damage (Q) [as we infer by extrapolation from their liabilities for damage in the private domain, respectively full and half, to all domains, including the public].
> Horn damage in the public domain (Q) is important (R) enough to imply *some* liability (S).
> Therefore, tooth & foot damage in the public domain (P) is important (R) enough to imply *some* liability (S).

The Gemara is thus justified in describing its argument here as *qal vachomer* (מקל וחומר), although this must be taken to refer to purely a fortiori argument and not a crescendo. We see clearly from the a fortiori formulation that the major premise is produced by a generalization, from the particular case of private property to all property, and its application to the particular case of public property. On this basis, the minor premise about unspecified liability for horn leads to the conclusion about unspecified liability for tooth & foot.

Now, the main question to ask here is: *why is the Gemara opting for such vague language?* There are actually two separate questions, here: (a) Why is its premise is deliberately vague, saying "there is yet liability" (חייבת), i.e. *some* liability, without specifying just how much liability even though the amount is already known from Ex. 21:35 to be precisely half? And (b) Why is its conclusion also vague, saying "there should be liability" (חייב), i.e. *some* liability, although the amount of this liability may be assumed by partial instead of full denial of Ex. 22:4 to be half? We shall now propose our answers.

The way to answer our question about the vagueness of the minor premise is to consider what would happen if more explicit language were to be used. To start with, had the Gemara used half liability as the consequent of the minor premise, and argued a crescendo instead of purely a fortiori, its conclusion would have been full liability for tooth & foot damage in the public domain, and thus contrary to Ex. 22:4, according to which full liability is reserved for tooth & foot damage in the private domain. This is evident in the following lines:

> Tooth & foot damage (P) is more important (R) than horn damage (Q) [as we infer by extrapolation, as before].
> Horn damage in the public domain (Q) is important (R) enough to imply half liability (S) (as specified in Ex. 21:35).
> The payment due (S) is 'proportional' to the degree of legal liability (R).
> Therefore, tooth & foot damage in the public domain (P) is important (R) enough to imply *full* liability (S) (contrary to the *davka* reading of Ex. 22:4).

Thus, the Gemara's thinking (consciously or otherwise) in this respect was effectively as follows. Since the full liability conclusion is contrary to a Scriptural given (namely Ex. 22:4, which specifies full liability to be applicable only to private property) the argument must be rejected somehow. Since the major and minor premises are already accepted, and the inference process is clearly valid, the only way to reject the argument is by denying the additional premise about 'proportionality' – or, in other words, by applying a *dayo* objection of type I. That is to say, the a crescendo argument is to be discarded, leaving only the underlying purely a fortiori argument. This leftover argument is similar to the Gemara's (previously mentioned), except that it infers half liability from half liability[39], instead of some liability from some liability.

Another route the Gemara may have tried is the following. As we learned from R. Tarfon, we can by judicious reshuffling of the premises obtain an alternative a fortiori argument. In the present case, this would be done as shown next. In terms of if-then statements, our competing argument would be as follows. Note that the first two statements, which we use to form our major premise, are both about horn damage.

> If horn and private, then half liability (ruling of the Mishna Sages).
> If horn and public, then half liability (Ex. 21:35).
> If tooth & foot and private, then full liability (Ex. 22:4).
> If tooth & foot and public, then full liability (putative conclusion, contrary to Ex. 22:4).

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[38]   Note that, to simplify, I here use "is more important" as equivalent to "implies more liability."

[39]   It is perhaps to this implicit a fortiori argument that the Soncino edition refers, when it explains (in a footnote) the Gemara's conclusion of half liability for tooth & foot damage in the public domain by saying: "On the analogy to Horn where the liability is only for half damages in the case of Tam. The Scriptural text may have been intended to exclude only full compensation."

This can be recast in analogical form thusly:

> Just as, in the case of horn, damage in the public domain implies *as much* legal liability as in the private domain (since both imply half liability).
> Likewise, in the case of tooth & foot, damage in the public domain implies *as much* legal liability as in the private domain; whence given that the latter implies full liability, then the former implies full liability (contrary to Ex. 22:4, which specifies full liability to be applicable only to private property).

More to the point, we can formulate it in purely a fortiori format as follows. Note that this argument is positive antecedental and *a pari* (i.e. egalitarian).

> Public domain damage (P) is as important (R) as private domain damage (Q) [as we infer by extrapolation from horn damage (where liability is half in both domains) to all causes of damage, including tooth & foot].
> Tooth & foot damage in the private domain (Q) is important (R) enough to imply full liability (S).
> Therefore, tooth & foot damage in the public domain (P) is important (R) enough to imply *full* liability (S) (contrary to Ex. 22:4).

Now, observe why this argument seems more secure than the preceding a crescendo. It also goes from minor to major; but since the minor premise predicates what is *already the maximum* amount allowable (namely, full liability), the conclusion has to predicate *the same maximum* amount (i.e. full liability). Yet here again the conclusion is contrary to a Scriptural given (Ex. 22:4, which specifies full liability to be applicable only to private property). Therefore, it must be rejected. The only way to do this is through a *dayo* objection of type II, i.e. by preventing the generalization that gave rise to its major premise from proceeding. The final conclusion will then again be half liability.

What the above suggests, then, is that the Gemara opted for vague language in the minor premise, speaking of liability indefinitely, because it knew or at least sensed that specifying half liability would in any event lead to a conclusion of full liability, contrary to Scripture; which conclusion would have to be prevented by application of *dayo* objections of both types. In the Judaic frame of reference, a conclusion contrary to what the Torah teaches is a conclusion contrary to 'fact', which must be prevented to avoid inconsistency. Apparently, then, rather than get involved in that long discussion, or *pilpul*, it opted for a vaguer statement of the minor premise, to arrive at its desired conclusion more directly.

As regards its vague conclusion, a minimum of reflection shows that the liability implied, though stated indefinitely, can only be half liability. This is evident already in the above two arguments from the minor premise of half liability, since their conclusion of full liability is unacceptable because contrary to Scripture. However, we could arrive at the same result by working on the vague conclusion of the Gemara's own purely a fortiori argument (from some to some liability). Given the conclusion that tooth & foot damage on public property implies *some* liability, i.e. denies no liability, this *can only* mean half liability, since full liability is excluded by Ex. 22:4. This seemed so obvious to the Gemara that it did not even see any necessity to say it out loud.

As we have seen, according to the rabbis, based on Biblical practice, the variable "liability" allows in the present context for only three possible values; namely, no liability, half liability and full liability. Therefore, an indefinite amount of liability, i.e. *some* liability, which is the negation of no liability, means "half or full" liability. Therefore, to say "there is liability," meaning some liability, is not as open a statement as it might seem – it allows for only two possibilities, viz. half or full liability. So, if one of these is known to be false (in this case, with reference to the Torah), the other must be true. The latter argument is a disjunctive apodosis: "either this or that, but not this, therefore that."

Note well that the Gemara here proposes an alternative judgment on damage by tooth & foot on public property to that previously accepted (in the debate between R. Tarfon and the Sages). Previously, the Mishna and the Gemara interpreted Ex. 22:4 ("If a man… shall let his beast loose, and it feed in another man's field, etc."), which imposes full liability for tooth & foot damage on specifically private grounds, as implying that there is no liability for tooth & foot damage on public grounds. Here, the Gemara (logically enough) proposes an alternative reading for the latter case, such that "not full" is taken to mean "half" instead of the more extreme "nil," and it backs up this moderate reading by reasoning that so concludes.

Thus, the Gemara's use of vague language in its first argument was not some subterfuge relying on half-truths; it was just intended as a shortcut to a result that was in any case logically inevitable. The Gemara achieved its objective here, which was to establish that Ex. 22:4, which imposes full liability for tooth & foot damage on private grounds,

need not be taken to imply (as it was in the Mishna) that there is no liability for tooth & foot damage on public grounds; for the alternative of half liability is logically equally cogent. That the Gemara was consciously doing this is evident from its statement: "It was only half payment that we were arguing for!" At worst, the Gemara can be criticized for being too laconic; but its reasoning is sound.

Second experiment. The Gemara states: "*But should we not let Tooth and Foot doing damage on the plaintiff's premises involve the liability for half damages only because of the following a fortiori:*
If in the case of Horn, where there is liability for damage done even on public ground, there is yet no more than half payment for damage done on the plaintiff's premises,
does it not follow that in the case of Tooth and Foot, where there is exemption for damage done on public ground, the liability regarding damage done on the plaintiff's premises should be for half compensation [only]?[40]
— Scripture says: He shall make restitution, meaning full compensation."
We should here again at the outset note that the Gemara's argument uses as a premise the earlier conclusion of the Mishna Sages (via their *dayo* objections to R. Tarfon's claims) that for damage by horn on private property the ox owner's liability is half. The other two premises are derived from the Torah as follows: one directly, from Ex. 21:35; and the other indirectly, by extreme inversion of Ex. 22:4 (by which I mean that "not full" is here taken to mean "nil" as in the Mishna, instead of "half" as proposed in the preceding experimental argument of the Gemara). The conclusion concerns damage by tooth & foot on private property. The Gemara demonstrates that a conclusion of half liability, contrary to the full liability given in Ex. 22:4, would follow from the said premises.
Expressed as a set of brief if-then statements, this Gemara argument looks as follows. Note that the first two have in common the factor of public property.

> If horn and public, then *some* liability (from Ex. 21:35).
> If tooth & foot and public, then no liability (by extreme inversion of Ex. 22:4).
> If horn and private, then only half liability (ruling of the Mishna Sages).
> If tooth & foot and private, then [only] half liability (putative conclusion, contrary to Ex. 22:4).

This can be expressed in analogical form, as follows. Note that I here use the term "exemption" in the sense of "freedom of liability," allowing for degrees of zero, half and total exemption; the term is thus intended as the reverse of the range of "liability."

> Just as, in the public domain, damage by tooth & foot implies more legal *exemption* than damage by horn, since the former implies no liability and the latter *some* liability (note that although we can infer from Ex. 21:35 the amount to be half, the Gemara here deliberately avoids specifying it in its premise).
> Likewise, in the private domain, damage by tooth & foot implies more legal exemption than damage by horn; whence given that the latter implies only half liability, then the former implies only half liability (contrary to Ex. 22:4, which imposes full liability for this).

We can represent the same argument in purely a fortiori form, as follows. Note the negative polarity of the middle term (R) used; this is necessary to ensure that tooth & foot damage emerge as the major term (P) and horn damage as the minor term (Q). The resulting argument is thus minor to major, positive antecedental.

> Tooth & foot damage (P) is more *un*important (R) than horn damage (Q) [as we infer by extrapolation from their liabilities for damage in the public domain (respectively none and *some*) to all domains, including the private].
> Horn damage in the private domain (Q) is unimportant (R) enough to imply only half liability (S).
> Therefore, tooth & foot damage in the private domain (P) is unimportant (R) enough to imply only half liability (S) (contrary to Ex. 22:4, which imposes full liability for this).

The Gemara is thus justified in describing its argument here as *qal vachomer* (מק"ו), although again this should be understood to refer to purely a fortiori argument rather than a crescendo. We see clearly from the a fortiori formulation that the major premise is produced by a generalization, from the particular case of public property to all

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[40]   I have added the square brackets around this last "only," because it is not found in the original and therefore seems to be an interpolation by the Soncino edition translators.

property, and its application to the particular case of private property. On this basis, the minor premise about half liability for horn leads to the conclusion about half liability for tooth & foot.

Thus, whether we reason analogically or purely a fortiori, we obtain a conclusion contrary to Scripture. Since the processes used are faultless, what this means is that one or more of the premises must be wrong. In order to try and understand where the problem lies, let us look again at the Gemara's formulation. The first question to ask (in view of what we learned in the previous case) is why does the Gemara say vaguely "there is liability" (חייבת) for damage by horn in the public domain, when it is known from Ex. 21:35 that the amount of liability is precisely half? Looking at the major premise of the above a fortiori argument, which is generalized from this information, it is clear that it would have made no difference to it if the Gemara had specified half liability. The argument by analogy would similarly be unaffected. So there seems to be no reason for the Gemara not to have said half[41].

Another question is why does the Gemara find it necessary to say "no more than" (אלא) half regarding the liability for damage by horn on private property? Until now, "half" has always meant precisely half, without need to specify that *only* half is intended. If more than half liability was possibly included in the term half, the meaning of it would have been "half or full," and this could be stated as before as indefinite "liability." Perhaps the answer is that if the liability for damage by horn on private property had been full, as R. Tarfon advocated, then the conclusion here would be full liability for damage by tooth and foot on private property. So the Gemara is specifying "no more than half" merely to indicate that it is abiding by the ruling of the Mishna Sages, and not adopting the contrary opinion of R. Tarfon.

In fact, we could represent almost the same argument in a crescendo form, as follows. Note the similarities to the preceding purely a fortiori formulation, but also the totally different conclusion. Instead of half liability, the conclusion here is no liability. But the effect is the same, in that this is contrary to Ex. 22:4.

> Tooth & foot damage (P) is more *un*important (R) than horn damage (Q) [as we infer by extrapolation from their liabilities for damage in the public domain (respectively none and *some* (or more precisely half)) to all domains, including the private].
> Horn damage in the private domain (Q) is unimportant (R) enough to imply half liability (S).
> The payment due (S) is 'proportional' to the degree of legal liability (R).
> Therefore, tooth & foot damage in the private domain (P) is unimportant (R) enough to imply no liability (S) (contrary to Ex. 22:4, which imposes full liability for this).

If, in view of the conflict of this conclusion with Ex. 22:4, we interdicted the premise about 'proportionality' by means of a *dayo* objection of type I, we would obtain the same conclusion as the pure a fortiori argument above; namely, half liability. This would of course still leave us with a conclusion contrary to the Scriptural given of Ex. 22:4. Although the Gemara originally does not express this conclusion, however obtained, as "only half," it is interesting to note that the translator does add on the qualification of exclusion. This is no doubt to exclude "full" liability (rather than to exclude "no" liability), because this is the crux of the issue in this Gemara argument.

Now, this conclusion of half (i.e. not full) liability is especially troubling because the premises that give rise to it were previously regarded as quite acceptable. The major premise is based on Ex. 21:35 (whether we read it as half liability or more vaguely as some) and on the extreme inversion of Ex. 22:4 (i.e. reading not-full as nil, to the exclusion of half) taken for granted by all participants in the Mishna. And the minor premise is the ruling of the Sages in the Mishna, which is in any case implied in Ex. 21:35 (since this verse does not make an explicit distinction between public and private property). How then can these givens result in a conclusion contrary to Scripture, i.e. to Ex. 22:4? This is the difficulty.

Obviously, the problem must lie with the major premise of the a fortiori argument (whether non-proportional or proportional). The extrapolation of "Tooth & foot damage is more unimportant than horn damage" from public property to private property has to be interdicted by a *dayo* objection of type II, so as to avoid the antinomic conclusion. This could be considered as the intent of the final statement "Scripture says: He shall make restitution, meaning full compensation," although there is no explicit mention of *dayo* here. The Gemara is effectively saying: the conclusion cannot be right, therefore block it from happening. This is regular *reductio ad absurdum* reasoning.

We could also, by the way, obtain the conclusion of no liability by purely a fortiori argument (instead of a crescendo, as just shown), by imitating the Mishna's R. Tarfon and using another direction of generalization, as shown next. First, let us reshuffle the initial if-then statements, so that the ones we use to form our major premise are both about horn damage, as follows:

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[41]   Possibly it used vague language here simply to harmonize the language in this experiment with that in the preceding one.

If horn and public, then half liability (Ex. 21:35).
If horn and private, then half liability (ruling of the Mishna Sages).
If tooth & foot and public, then no liability (by extreme inversion of Ex. 22:4).
If tooth & foot and private, then no liability (putative conclusion, contrary to Ex. 22:4).

Next, let us formulate the argument in analogical form, keeping to the language of exemption for symmetry with the previous formulation, as follows:

Just as, in the case of horn, damage in the private domain implies *as much* legal exemption as in the public domain (since both imply half liability):
So, in the case of tooth & foot, damage in the private domain implies as much legal exemption as damage in the public domain; whence given the latter implies no liability, then the former implies no liability (contrary to Ex. 22:4, which imposes full liability for this).

Lastly, we formulate the argument as a purely a fortiori one, of positive antecedental form (minor to major), as follows:

Damage in the private domain (P) is as *un*important (R) as damage in the public domain (Q) [as we infer by extrapolation from horn damage (where liability is half in both domains) to all causes of damage, including tooth & foot].
Tooth & foot damage in the public domain (Q) is unimportant (R) enough to imply no liability (S).
Therefore, tooth & foot damage in the private domain (P) is unimportant (R) enough to imply no liability (S) (contrary to Ex. 22:4, which imposes full liability for this).

This argument seems more solid than the preceding a crescendo argument because it argues from no liability to no liability, rather than from half to none. So it cannot be prevented by means of a *dayo* objection of type I. And yet its conclusion is the same, viz. no liability. Which poses a problem, since it is inconsistent with the Scriptural imposition of full liability (in Ex. 22:4). Here, then, we must resort to a *dayo* objection of type II, interdicting the generalization that led to the major premise. We might then be tempted to accept the next amount of half liability as the final result – but no, this is still contrary to Ex. 22:4, and so must be avoided too.

To sum up, the initial premises used in different ways in the various arguments we considered representing the Gemara's second experiment cannot readily be rejected, yet they lead to a conclusion contrary to Scripture. To prevent such paradoxical result, we had to again resort to *dayo* objections of both types. This means that the initial premises are together viable provided we do not indulge in proportional thinking or in generalizations in relation to them. Our room for maneuver with them is severely limited; we must proceed with caution.

Third experiment. The Gemara states: "*But should we not [on the other hand] let Horn doing damage on public ground involve no liability at all, because of the following a fortiori:*
If in the case of Tooth and Foot, where the payment for damage done on the plaintiff's premises is in full there is exemption for damage done on public ground.
does it not follow that, in the case of Horn, where the payment for damage done on the plaintiff's premises is [only][42] *half, there should be exemption for damage done on public ground?*
— Said R. Johanan: Scripture says. [And the dead also] they shall divide, to emphasise that in respect of half payment there is no distinction between public ground and private premises."

We can here again at the outset note that the Gemara's argument uses as a premise the earlier conclusion of the Mishna Sages (via their *dayo* objections to R. Tarfon's claims) that for damage by horn on private property the ox owner's liability is half. The other two premises are derived from the Torah as follows: one directly, from Ex. 22:4; and the other indirectly, by extreme inversion of Ex. 22:4 (by which I mean that "not full" is here taken to mean "nil" as in the Mishna, instead of "half" as proposed in the first experimental argument of the Gemara). The conclusion concerns damage by horn on public property. The Gemara demonstrates that a conclusion of no liability, contrary to the half liability given in Ex. 21:35, would follow from the said premises.

Expressed as a set of brief if-then statements, this Gemara argument looks as follows. Note that the first two have in common the factor of private property.

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[42]   I have added the square brackets around this last "only," because it is not found in the original and therefore seems to be an interpolation by the Soncino edition translators.

If tooth & foot and private, then full liability (Ex. 22:4).
If horn and private, then [only] half liability (ruling of the Mishna Sages).
If tooth & foot and public, then no liability (by extreme inversion of Ex. 22:4).
If horn and public, then no liability (putative conclusion, contrary to Ex. 21:35).

This can be expressed in analogical form, as follows. Note that I here use the term "exemption" in the sense of "freedom of liability," allowing for degrees of zero, half and total exemption; the term is thus intended as the reverse of the range of "liability."

Just as, in the private domain, damage by horn implies more legal *exemption* than damage by tooth
& foot, since the former implies [only] half liability and the latter full liability.
Likewise, in the public domain, damage by horn implies more legal exemption than damage by tooth
& foot; whence given that the latter implies no liability, then the former implies no liability (contrary
to Ex. 21:35, which imposes half liability).

We can represent the same argument in purely a fortiori form, as follows. Note the negative polarity of the middle term (R) used; this is necessary to ensure that horn damage emerge as the major term (P) and tooth & foot damage as the minor term (Q). The resulting argument is thus minor to major, positive antecedental.

Horn damage (P) is more *un*important (R) than tooth & foot damage (Q) [as we infer by
extrapolation from private domain damage (for which the liabilities are half and full respectively) to
all domains, including the public].
Tooth & foot damage in the public domain (Q) is unimportant (R) enough to imply no liability (S).
Therefore, horn damage in the public domain (P) is unimportant (R) enough to imply no liability (S)
(contrary to Ex. 21:35, which imposes half liability).

The Gemara is thus justified in describing its argument here as *qal vachomer* (מק"ו), although again this should be understood to refer to purely a fortiori argument rather than a crescendo. We see clearly from the a fortiori formulation that the major premise is produced by a generalization, from the particular case of private property to all property, and its application to the particular case of public property. On this basis, the minor premise about no liability for tooth & foot leads to the conclusion about no liability for horn.
No 'proportionality' can be presumed here, for the simple reason that the minor premise and conclusion are already an extreme value (namely, no liability). Thus, an a crescendo argument with the same terms would be identical with the above purely a fortiori argument.
Manifestly, whether we reason analogically or purely a fortiori, we obtain a conclusion contrary to Scripture. Since the processes used are faultless, what this means is that one or more of the premises must be wrong. Examining the Gemara's formulation, we see that in the present case, unlike the preceding two, there is no ambiguous language. The word exemption (פטורה) is clearly intended here, in both its occurrences, in the sense of full exemption, i.e. zero liability.
It is noteworthy that, although the Gemara originally does not express the liability for damage by horn as "only half," the translator adds on the qualification of exclusion. But this is no doubt simply to exclude the "full" liability here due according to the dissenting opinion of R. Tarfon; and it does not seriously affect the argument, since if full were adopted instead of half, the major premise would become egalitarian, but the minor premise and conclusion would remain the same.
Now, this conclusion of no liability (instead of half) is obviously problematic, since the premises that give rise to it were previously regarded as quite acceptable. The major premise is based on Ex. 22:4 and on the ruling of the Sages in the Mishna, which is in any case implied in Ex. 21:35 (since this verse does not make an explicit distinction between public and private property, as R. Johanan reminds us[43]). And the minor premise is based on the extreme inversion of Ex. 22:4 (i.e. reading not-full as nil, to the exclusion of half) taken for granted by all participants in the Mishna. How then can these givens result in a conclusion contrary to Scripture, i.e. to Ex. 21:35? This is the difficulty.

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[43]   More precisely, R. Johanan, an early authority, interprets the Scriptural verse "[And the dead also] they shall divide," which is the last sentence of Ex. 21:35, to mean that half liability applies to the public domain as well as to the private domain. Taken literally, of course, this verse does not have exactly that meaning (i.e. another reading is conceivable); but it is reasonable to suppose that Ex. 21:35 as a whole applies to both domains, since neither is explicitly specified or excluded.

Obviously, the problem must lie with the major premise of the a fortiori argument. The extrapolation of "Horn damage is more unimportant than tooth & foot damage" from private property to public property has to be interdicted by a *dayo* objection of type II, so as to avoid the antinomic conclusion. This could be considered as the intent of the final statement concerning damage by horn that "in respect of half payment there is no distinction between public ground and private premises," although there is no explicit mention of *dayo* here. The Gemara is effectively saying: the conclusion cannot be right, therefore block it from happening. This is regular *reductio ad absurdum* reasoning.

Our next obvious move would be to investigate if a conclusion consistent with Scripture would be obtained by imitating the Mishna's R. Tarfon, and judiciously reshuffling the given information so as to attempt another direction of generalization. This would proceed as follows. First, we reshuffle the initial if-then statements, so that the ones we use to form our major premise are both about tooth & foot damage, as follows:

> If tooth & foot and private, then full liability (Ex. 22:4).
> If tooth & foot and public, then no liability (by extreme inversion of Ex. 22:4).
> If horn and private, then [only] half liability (ruling of the Mishna Sages).
> If horn and public, then [only] half liability (conclusion in accord with Ex. 21:35).

Next, we formulate the argument in analogical form, keeping to the language of exemption for symmetry with the previous formulation, as follows:

> Just as, in the case of tooth & foot, damage in the public domain implies more legal exemption than in the private domain (since these respectively imply no and full liability):
> So, in the case of horn, damage in the public domain implies more legal exemption than damage in the private domain; whence given the latter implies only half liability, then the former implies only half liability (in accord with Ex. 21:35).

Lastly, we formulate the argument as a purely a fortiori one, of positive antecedental form (minor to major), as follows:

> Damage in the public domain (P) is more *un*important (R) than damage in the private domain (Q) [as we infer by extrapolation from tooth & foot damage (for which liability is respectively nil and full) to all causes of damage, including horn].
> Horn damage in the private domain (Q) is unimportant (R) enough to imply only half liability (S).
> Therefore, horn damage in the public domain (P) is unimportant (R) enough to imply only half liability (S) (in accord with Ex. 21:35).

However, before we can adopt this purely a fortiori argument we must look into the corresponding a crescendo argument. The latter is as follows:

> Damage in the public domain (P) is more *un*important (R) than damage in the private domain (Q) [as we infer by extrapolation from tooth & foot damage (for which liability is respectively nil and full) to all causes of damage, including horn].
> Horn damage in the private domain (Q) is unimportant (R) enough to imply half liability (S).
> The payment due (S) is 'proportional' to the degree of legal liability (R).
> Therefore, horn damage in the public domain (P) is unimportant (R) enough to imply no liability (S) (contrary to Ex. 21:35, which imposes half liability).

Evidently, arguing a crescendo with these premise results in the undesirable conclusion of no liability for horn damage in the public domain, which is contrary to Scripture (Ex. 21:35). This being the case, such a crescendo argument has to be interdicted by means of a *dayo* objection of type I. So doing, we return to the purely a fortiori argument formulated just before, which yields the conclusion of half liability. Since the latter conclusion is consistent with Scripture (Ex. 31.35), we have no need to interdict it by means of a *dayo* objection of type II. We can therefore adopt the said a fortiori argument as a viable alternative to the third one proposed by the Gemara, which yielded an unacceptable conclusion.

From this we see that, while the Gemara's third experiment is in many ways similar to its second, they are ultimately quite different, in that while the second experiment leaves us without a viable a fortiori counter-argument, the third one does have a viable a fortiori counter-argument. It is surprising that the Gemara did not remark on this significant

difference, but remained content with simply listing two arguments with conclusions inconsistent with Scriptural givens.

To sum up. The Gemara's three experimental arguments have in common as a premise the conclusion of the Sages in the Mishna that damage by horn in the private domain implies half liability. The arguments then seek to determine what conclusion can be drawn from that constant premise about the other situations, viz. tooth & foot damage in the public and private domains, and horn damage in the public domain, respectively. The purpose of the exercise is apparently to compare such conclusions to, respectively, an assumption in the Mishna (viz. that tooth & foot damage on public property implies no liability, based on extreme inversion of Ex. 22:4) and to certain Scriptural givens (viz. Ex. 22:4, which imposes full liability for tooth & foot damage on private property, and Ex. 21:35, which imposes half liability for horn damage on public property).

The Gemara's logical virtuosity in proposing these three arguments is rather impressive, considering its lack of formal tools. Although the above proposed explicit logical analyses of the three arguments are absent in the Gemara, similar analyses may be reasonably be supposed to have consciously or subconsciously colored the Gemara's thinking, for otherwise it would be difficult to explain its intent in presenting these arguments. Note in particular that though the *dayo* principle is nowhere here mentioned by the Gemara, both versions of it are very present in the background of its discourse.[44]

7. Assessment of the Talmud's logic

We have in the preceding pages examined in great detail, using up to date methods of formal logic, the a fortiori reasoning of both the Mishna and the Gemara, or at least their reasoning in the immediate vicinity of the present *sugya*[45] (i.e. mBQ 2:5 and bBQ 25a). We judged these texts on their own merits, note well, and not through the prism of later commentaries. Our general conclusion may well be that both the earlier and later Talmudic sages, the Tannaim and the Amoraim, were amazingly powerful logic practitioners, even if they were not great theoreticians. Judging by the Talmudic material we have looked at here, their reasoning seems on the whole sound, even if too often much is left unstated.

What is amazing is precisely that, albeit the brevity of their statements, the people involved were able to reason with such accuracy. I am amazed because, with my pedestrian mind, without reference to formal methods and without full exposition of all implicit discourse, I would be unable to arrive at similar results with equal aplomb. Nevertheless, it must be said and admitted that self-assurance, however esthetically impressive, is not enough. Logic is not just an art; it is first of all a science. To reason correctly is good; but to know just why one's reasoning is correct is much better. To reason correctly based only on intuition, i.e. on immediate logical insight, is not as convincing as to do so based on broad theoretical understanding, i.e. on abstract study of the exact conditions for correct reasoning (even if, to be sure, such study is also based on the same faculty of logical insight). In the former case, there is some reliance on luck; in the latter, nothing is left to chance.

Comparing now the logic in the Mishna to that in the Gemara, certain trends are evident. The Mishna's thinking is more straightforward; the Gemara's thinking is more tortuous. In the Mishna, R. Tarfon puts forward an argument in support of his contention that the legal liability for damage by an ox on private property ought to be full compensation. This argument is not accepted by his colleagues, the Sages, apparently because it relies on proportionality. R. Tarfon then very skillfully proposes an alternative argument, which is not open to such objection. The Sages nevertheless reject the latter argument, apparently by resorting to another kind of objection.

R. Tarfon's two arguments are traditionally presumed to be *qal vachomer*, i.e. a fortiori arguments, although just what that means (besides the descriptive name) is nowhere defined. In fact, looking at these arguments very objectively, they could be interpreted as arguments by analogy or more precisely as arguments pro rata, or as arguments a crescendo (i.e. proportional a fortiori) or as purely a fortiori arguments. Moreover, there is no attempt to theoretically validate these arguments. But in any event, they are intuitively quite reasonable; and it seems from the text that it is on this logical basis that R. Tarfon advocates them.

The Sages' objections, labeled *dayo* (from their opening word, which means "it is enough") are not likewise justified by any theoretical discussion. What is clear after our detailed analysis is that they are not essentially logical objections; they are not indicative of breaches of deductive logic, though they might be postulated to signify some inductive restraint. They should rather be viewed as arbitrary decisions (I here use the term 'arbitrary' non-pejoratively, in the sense of 'resorting to arbitration') by the Sages themselves, based on certain ethical

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[44]   It should be noted in passing that all the a fortiori arguments explicitly formulated by the Gemara in the present context are pure; none are a crescendo. This implies that the Gemara does (unconsciously if not consciously) admit that some a fortiori arguments are not a crescendo (unlike the *baraita* it quotes earlier on, which seems to suggest that a fortiori argument is always a crescendo).

[45]   A *sugya* is a portion of the Talmud dealing with a specific topic.

considerations. It can reasonably be doubted that the Sages are here evoking some ancient tradition, perhaps a teaching dating back from Sinai, because R. Tarfon, their colleague and equal, evidently does not preemptively take it into consideration in his two arguments.

Turning now to the Gemara, i.e. the later Talmudic commentary on this passage of the Mishna, we find a very different frame of mind. One would expect the Gemara to initiate a thorough theoretical reflection on R. Tarfon's two lines of reasoning and the difference in the Sages' *dayo* objections to them. But no; the Gemara ignores these burning issues and goes off on a tangent, focusing on the relatively not very relevant issue of the distance between R. Tarfon's and the Sages' positions. Apparently, the Gemara's only concern here is whether R. Tarfon knew and agreed with the Sages' *dayo* considerations. Obviously, he could not have fully agreed with them, since his conclusions differ from theirs; so the question is how far their views on the *dayo* principle differ.

In pursuit of the answer to that question, the Gemara engages is a very complicated scenario of its own, according to which R. Tarfon advocated a more conditional *dayo* principle than the Sages did. Briefly put, it proposes a distinction (which it attributes to R. Tarfon ex post facto) between applications of the *dayo* principle that "would defeat the purpose of" the *qal vachomer* and those that "would *not* defeat" it. In the former case, the 'proportional' gain made possible by an a fortiori argument (taken by the Gemara, on the authority of a *baraita*, to mean a crescendo argument) would be wiped out by *dayo*, so it should not be applied; whereas in the latter case, it would not by wiped out by *dayo*, so it may be applied.

In defense of this fanciful scenario, the Gemara proposes different readings of a Torah text, viz. Num. 12:14-15, by R. Tarfon and the Sages. However, both these readings are far removed from the plain meaning of the text, in that they do not take all of it into consideration. Most important, the view attributed by the Gemara to R. Tarfon assumes an a fortiori argument to be intended in the text while discarding the verses that would justify such assumption! It thus mendaciously infers an a crescendo conclusion of fourteen days ex nihilo, instead of with reference to the textual given of seven days. This means that the Gemara's whole idea, of a distinction between applications of the *dayo* principle that "would defeat the purpose of" the *qal vachomer* and those that "would *not* defeat" it, is an outright deception. The bottom line is that the Gemara in fact fails to achieve its stated goal of harmonizing the opinions of the Mishna contestants.

Now, this is a bit of a shock, but not too astonishing. Anyone who has studied the Gemara to any extent can see for himself that its thinking, though based on the Mishna to some extent, is often more convoluted and open to doubt. Of course, more fundamentalist readers would never agree with such an assessment, but instead insist that in such cases the Gemara has intellectual intentions and ways too sublime for us ornery folk to grasp. But we, while making no claim to infallibility or omniscience, do claim to be honest and lucid, and stand by what is evident to the senses and to reason. In the present case, the Gemara's ideas must obviously not be confused with the discourses found in the Mishna. With regard to this, the following general comment of Louis Jacobs in *Rabbinic Thought in the Talmud* (pp. 17-18) is apropos:

> "A much discussed question is whether the interpretations of the Mishnah found in the Gemara are really a reading of ideas into the Mishnah or whether they are authentic accounts of what the Mishnah itself intended. Now students of the Mishnah in the Middle Ages noted that some, at least, of the Gemara's interpretations of the Mishnah are so far-fetched and artificial that they cannot possibly be accepted as real interpretations of what the Mishnah intends to say, which is why Maimonides and other early commentators were prepared to disregard the Gemara to interpret the Mishnah on its own terms. To conclude from this that the Gemara has, at times, 'misunderstood' the Mishnah is precarious. It is possible that the Gemara, at times, consciously departs from the plain meaning of the Mishnah in order to produce its own original work…."

There is no harm, in our view, in producing original work, provided it is openly acknowledged as such. Unfortunately, the traditionalist's way of thinking is that what he reads into a text must have been intended in the original; to him, interpretation is a sort of deduction. This is applicable at all levels – from the Mishna reading meanings into the Torah, to the Gemara reading them into the Mishna, to later commentators reading them into the Talmud[46]. And this applies to both halakhic and haggadic material. What is sorely needed to cure this serious intellectual malady is to understand *the inductive nature of interpretation*. An interpretation is a theory designed to fit the 'facts' that the given text constitutes. Its logical status is that of a hypothesis, which may and probably does have competing hypotheses. Rarely is an explanation the only conceivable hypothesis, though this happens occasionally. Therefore, a reading should always be acknowledged to be *one possible* interpretation, even if it fits the given data.

But in the case under consideration, as we have definitely shown, the Gemara's proposed interpretation of the Mishna simply does not convince. It is not a credible theory, because it is built on illusion, on make-believe.

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[46]   Not to mention the Bible writer(s) interpreting empirical givens.

Furthermore, the Gemara does not demonstrate its having noticed and understood the differences between R. Tarfon's two arguments and between the Sages' two *dayo* objections to them. That later commentators have projected such understanding into the Gemara does not prove that the Gemara in fact had it, only at best that it might have. Such ex post facto attribution of knowledge to the Gemara is only evidence of the faith later commentators had in it. The Gemara itself does not explicitly remark on these crucial issues, nor even implicitly suggest them.

It is interesting to note that, whereas the Mishna participants are involved in a purely legal debate, without stepping aside and reflecting on the methodological issues it implies, the Gemara does, in an attempt to clarify the primary, legal discussion, initiate a secondary, more methodological reflection. The latter discourse is intended as an accessory to the former, in that the legal conclusions that might be drawn depend on the methodological lessons learned. Thus, we can say that there is in the Gemara a *sugya* within a *sugya*, or there are two intertwined layers of discussion – the main one being legalistic in content, whereas the accessory one is methodological in content. The problem is that, although the Gemara could have used this opportunity to develop a deep reflection on the methodological issues involved, it disappointingly engaged in a very tangential and artificial discourse, driven by quite ideological considerations.

The present work being a treatise on logic, with an emphasis on a fortiori logic, our concern is naturally with the methodological topic of conversation; we are not really interested in the legal topic except possibly as an example. I personally have no legal axe to grind; I am not out to modify or overturn any halakhah. I certainly have no desire to put down anyone, either. Our interest in this research is relatively abstract, and certainly impartial. Our present study is aimed at logic theory and history; it is not essentially Talmudic in orientation, in contradistinction to the rabbis, whose main interest is always legal rather than logical. Nevertheless, we had to consider the legal debate in some detail, since it houses information we needed for historical purposes and to empirically judge the level of rabbinical understanding of a fortiori reasoning.

The author of the Gemara commentary we have studied is obviously someone with an intelligent, imaginative and logically sharp mind. But it is not an entirely scientific mind, which frankly considers all alternatives, lays out all the pros and cons, and judges the matter fairly in accord with objective standards. It is an authoritarian mind, which therefore functions to some extent manipulatively. The Gemara's author does not derive a conclusion from given premises in an unbiased manner; he starts with a desired conclusion and proceeds to give the impression of having proven it by intricate argument. He is satisfied with the result, even though he in fact did not prove it, either because he fools himself or because he assumes no one would notice the logical trickery involved in his argument and dare cry foul. In the latter event, he relies on the psychology denounced in H. C. Anderson's *The Emperor's New Clothes*.

What is evident looking at Baba Qama 25a is that the Mishna's narrative and that of the Gemara are quite distinct. The Gemara presents itself as a mere conduit, authoritatively clarifying and explaining the Mishna – but it speaks for itself alone. There is no evidence that it truly represents the views of R. Tarfon and the Sages. When the Gemara speaks in their names, it is just telling us what it thinks they said or meant. The thesis the Gemara presents must be treated as just a hypothesis, even as a mere speculation, since there is no way to establish its historicity. The dialogues it puts forward are imaginary. Its argument is rhetorical and not logical.

The Gemara's doctrinal goal seems to be to reconcile the seemingly antagonistic positions of the participants in the Mishna, i.e. R. Tarfon and the Sages. This is in accord, we may remark in passing, with the general rabbinical dogma that everything a Talmudic rabbi (or indeed an important later rabbi) says is essentially right, even if it seems to conflict with what others say. This doctrine that "the Torah has seventy facets" presumably arose ex post facto, first implicitly and then explicitly, perhaps somewhere mid-course in the Talmud, maybe only in the Gemara (when exactly, I do not know[47]). In any case, it clearly plays an active role in the present portion of the Gemara, and this is important to keep in mind.

To repeat, the Gemara's treatment of the Mishna is quite superficial, failing to spot and take into consideration important details in the proof-text. The Gemara cheerfully refers to the Miriam story as its model for understanding the Mishna, failing to notice that though this passage of the Torah can be used to throw light on the first argument of the Mishna, it is useless with regard to the second. Moreover, the Gemara not having even tried to make a preparatory theoretical analysis of *qal vachomer*, fails to realize the different possibilities of interpretation inherent in the Mishna. It takes for granted without reflection that the *qal vachomer* inferences in the Mishna are all 'proportional', and does not see the possibility in it of purely a fortiori arguments or even *non* a fortiori arguments.

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[47]   The explicit sentence is found in *Bamidbar (Numbers) Rabbah* 13:15, an 11th or 12th century CE midrash. But the implicit concept is no doubt much earlier. I would guess that it is a viewpoint of the Amoraim with regard to the Tannaim. It is essentially an expression of absolute faith in the tradition as handed down. The philosophic (logical, epistemological, ontological) significance of such a doctrine needs to be reflected on. Can conflicting viewpoints all be true?

The Gemara then embarks on a quite abstruse theory of *qal vachomer*, which it attributes to R. Tarfon, according to which (in positive subjectal a fortiori argument) a conclusion can only be drawn from a minor premise with the same subject. The Gemara does not notice that this imagined narrative is not in accord with what is explicitly given in the Mishna, let alone realize that it has no basis in formal logic. It does this to justify making a distinction between the argument implied in the Miriam story and the argument (it only perceives one, the first) given in the Mishna, so as to explain the difference of opinion there between R. Tarfon and the Sages in relation to a presumed *dayo* principle. Furthermore, in attempting to depict this theory, the Gemara has R. Tarfon drawing an alleged *qal vachomer* conclusion from no premises at all when he applies it to the Miriam story.

The Gemara's general idea that the Miriam story contains an a crescendo conclusion of fourteen days restricted by a *dayo* principle to seven days (rather than a straight a fortiori argument with a conclusion of seven days) is still *not unthinkable*, note well. Even though the Gemara does not admit v. 14a (about offending one's father) as a premise of this argument, and takes v. 14b (about seven days quarantine) as its final conclusion, the argument can be *imagined* as occurring in between these verses. It must however be stressed that, contrary to what the Gemara claims, there is no actual concrete hint of this scenario in the Biblical text. Even a purely (i.e. non-proportional) a fortiori reading is open to debate; all the more so an a crescendo one. Consequently, any claim that the passage points to a *dayo* principle is also open to debate.

The a fortiori reading is not inevitable; but it is a reasonable assumption, provided it is made to explain the connection between the first and second part of v. 14. Note well that the a fortiori argument is not just used to infer a number of days, but especially the punishment of isolation away from the community. The seven days prescribed are only a qualification of this predicate, serving to quantify the penalty; they are not the main issue of the argument. Thus, the final exchange in the Gemara between R. Tarfon and the Sages, regarding where in the Biblical text the alleged two sets of "seven days" come into play focuses on a side issue, diverting attention from the main one. The Gemara gives the impression that the *qal vachomer* is all about numbers of days; this is misleading.

Our wisest course is to blame the Gemara alone for these various rationalizations. The Gemara is plainly indulging in sophistry, masquerading as rational discourse. Its narrative is an obvious and absurd invention, which has little to do with the Mishna's. If we accept the scenario the Gemara advocates, we would be unfairly imputing the errors of reasoning it commits to R. Tarfon and the Sages. We cannot justify lumping together the players in the Mishna with those perhaps two or three centuries later (and some five hundred miles away) in the Gemara, just out of some ideological desire to make them appear to all speak with one voice. It is better to blame the author of this Gemara in particular for them than to insist they are true and embarrass everyone else! These very critical remarks of mine are sure to revolt traditional Talmudists, but they are unavoidable.[48]

I am, of course, well aware that such statements undermine rabbinical authority. We can say, having found such casuistry, that the rabbis are *not always* right, i.e. that their logic is not infallible. But I knew this already, having uncovered much problematic reasoning by them in the course of my earlier research on rabbinic hermeneutics, as detailed in my *Judaic Logic*[49]. Many more instances are uncovered in the present work; see for examples the fallacies discussed in chapters 3.4, 9.7, and especially 18.2.

The issue is *how often* do they reason incorrectly? This question cannot be answered offhand but requires systematic and thorough research throughout Talmudic literature – by competent researchers, I might add (for someone who does not know logic much better than them cannot judge theirs). If errors are only occasional, that is surely not too serious, since we are all human beings with limitations; if they are very frequent, that is certainly quite serious, since some inexcusable negligence is involved. It might be possible to lay the blame for all or most errors found on some specific rabbis. This would somewhat improve the logical credibility of the rabbis collectively, although we could still wonder why the errors were not spotted and corrected by other rabbis.

In any case, our approach as logicians must be objective and impartial, and not swayed by any imagined or actual threat of hostility and rejection. From a metaphysical point of view, if God is the ultimate reality of the world we experience, and the meaning of human life is to tend towards Him, then truth is a paramount value and honesty is an indispensable virtue. There is no rational excuse for evading or stonewalling, let alone opposing and denigrating, just criticism. It would be unrealistic to expect utter perfection from any human being, even if he is an important rabbi. When we come across logical faults, we should not deny them, but humbly admit them and try to correct them.

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[48]   It is not unthinkable that the Mishna and Gemara under scrutiny have the same views, but the data at hand phenomenologically does not justify such an extreme judgment. We are duty bound to look at the matter more subtly, and keep track of who said what and when, so that we can pinpoint more precisely who is in error and where, and judge with greater accuracy. This is the scientifically sound approach, and it is also more favorable to the honor of the rabbis. It is preferable by far, and is the policy I have adopted.

[49]   See especially chapter 11, on harmonization processes. But in chapter 10, too, there are relevant findings. See for instance my doubts regarding the 'freedom' of terms (*mufneh*) doctrine relative to *gezerah shavah*; this issue arises in the present Gemara on p. 25b, by the way.

While some might consider criticism of rabbinical arguments as cause for condemnation, we should rather view such events as welcome opportunities for improvement.

By this I mean that once we realize and admit that Talmudic and more broadly rabbinic logic is not inerrant – but sometimes debatable, contrived or erroneous – we open a safe door to halakhic review and revision. This of course cannot be taken as a blanket license for general change in Judaism as convenient; but there may be circumscribed opportunities for evolution based on ad hoc logical analysis. For the law must surely be in accord not only with empirical scientific knowledge of nature and history, but also with logic. Just as ignorance of the former is bound to lead to error in law, so is faulty logic also bound to lead to such error.

One of the major rabbinical authorities of modern times, R. Moses Sofer (Germany, 1762-1839) wrote this about logic (*higayon*): "whoever mixes words of logic with matters of Torah offends against the law of: 'Thou shalt not plough with an ox and an ass together' [Deut. 22:10]."[50] But logic is not, as this farfetched statement suggests, something arbitrary that we have a choice about using or not. Mentally, we are of course able disregard it; but intellectually, if we are honest, we cannot do that, because logic is our main means for verifying and certifying the truthfulness and consistency of our judgments. If any verse of the Torah is to be brought to bear in this matter, it is rather this: "Thou shalt not have in thy bag diverse weights" (Deut. 25:13). But there is no need for that; it is obvious. Apologists for religion reproach secular scientific knowledge of nature and history for varying in time. They suggest that such variation is proof of its unreliability. But this is of course a spurious argument. Scientific knowledge varies because it is essentially inductive, freely and dynamically adapting to new empirical discovery and rational review. This is not a fault or weakness – it is the very virtue of science. The truly scientific view[51] at any point in time is comparatively *the best* hypothesis human beings as a group have to offer. That it may later change does not make it any the less 'the best' at the time concerned. Certainly, it is always better than a static hypothesis based on religious dogma that is out of touch with empirical fact and rational scrutiny.

Browbeating is not a form of proof. Religion must learn to humbly adapt to scientific change. This would certainly not be the end of religion, because religion is a necessary expression and instrument of human spirituality. See how those who lost it suffer, from the lack of direction in their lives. Just as science makes possible the accumulation and transmission of human knowledge of nature and history, so religion makes possible the accumulation and transmission of human knowledge of spirituality. Of course, the latter tends to be more plural than the former, because spirituality allows for many paths. But in any case, whatever the chosen path, empirical science and logic must be taken into consideration to ensure its full truth.

8. The syllogistic Midot

As regards syllogism, it is also naturally found in rabbinic thinking and even within many of their hermeneutic techniques (*midot*). This is said to contradict the claim of many commentators that none of the rabbinical hermeneutic techniques are syllogistic.

This position, for instance, is to some extent found in Louis Jacobs' treatment, insofar as the only rabbinic argument he sees as syllogistic is the one referred to as *ha-kol*. In his *Studies* (1961), he says: "There is a form of Talmudic reasoning which has no connection with the *qal wa-chomer* but bears a remarkable affinity to the Syllogism;" and he goes further in a footnote, saying: "the '*ha-kol*' formula… is identical with the Syllogism," and giving as example the following argument implied in *Avot* 6:3:

>   "He who learns from his fellow has to pay him honour;
>   I have learned from my fellow;
>   Therefore I am obliged to pay him honour."

Michael Avraham, for his part, asserts categorically (in the English abstract of a 1992 Hebrew paper) that none of the 13 principles of R. Ishmael are syllogistic; as he puts it: "the Kal Vachomer – *like the rest of the 13 'Middot'* – is not a syllogism" (my italics). This opinion is apparently not new, judging by a statement made by Aviram Ravitsky (my italics):

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[50]   In *Hatam Sofer*, *OH*, No. 51. Quoted by Louis Jacobs, in *A Tree of Life: Diversity, Flexibility and Creativity in Jewish Law* (2nd ed. London: Littman, 2000), p. 8. It is true that earlier authorities like Maimonides or Nachmanides defended the use of logic in Torah related contexts – but they lived in a period when the dangers to religion inherent in strict logical scrutiny were not yet known. R. Sofer was contending with the *Haskalah*, the modern Jewish Enlightenment critics.

[51]   Of course, by scientific, I mean strictly scientific – and not pseudo-scientific pronouncements by scientists (in physics, biology or psychology) about having 'proved' the non-existence of God or of soul or of volition or even of consciousness.

> "Maimonides viewed most of the halakhic world as conventional, and this view enabled him to treat the halakhic arguments as dialectical ones, although *he did not think that halakhic arguments could be reduced to syllogistic figures.*" [52]

But in my *Judaic Logic* (1995), I show that many of the thirteen *midot* of R. Ishmael involve syllogistic thought processes. For a start, a fortiori argument is in part based on hypothetical syllogism. Syllogistic reasoning is implicit in the *midot* dealing with the scope of terms, collectively called *klalim uphratim* (rules 4-7), insofar as these have to do with subsumption and exclusion of cases in classes[53]. But more to the point, most of the *midot* dealing with harmonization (specifically the rules 8-11) are clearly syllogistic, so much so that they can be represented and resolved diagrammatically. While my work on a fortiori argument has attracted some attention, my work on these harmonization *midot* has apparently not been noticed. For this reason, I think it useful to reiterate some of these findings in the present context, to show how a lot of rabbinic thinking is syllogistic.

The first three (actually, four) of the principles of R. Ishmael concerned with harmonization begin with the phrase ***kol davar shehayah bikhlal veyatsa...***, meaning literally "anything which was in a generality and came out...". Broadly put, in formal terms, these rules are concerned with the following exegetic situation:

Given the three premises (#s 1, 2, 3), common to the four harmonization rules 8a, 8b, 9, 10:

All S1 are P1   (common **major** premise, #1),

and All S2 are P2   (common **minor** premise, #2),

where All S2 are S1, but not all S1 are S2   (common **subjectal** premise, #3),

and the fourth premise (#4), as applicable in each of these rules:

P1 and P2 are in some relation $f\{P1, P2\}$ (d)   (distinctive **predicatal** premise, #4):

- *In rule No. 8a*, P2 implies but is not implied by P1; that is:

All P2 are P1, but not all P1 are P2.

- *In rule No. 8b*, P1 implies P2 (and P2 may or not imply P1); that is:

All P1 are P2 (whether All P2 are P1 or some P2 are not P1).

- *In rule No. 9*, P1 and P2 are otherwise compatible; that is:

Some P1 are P2 and some P1 are not P2; some P2 are P1 and some P2 are not P1.[54]

- *In rule No. 10*, P1 and P2 are incompatible; that is:

No P1 is P2 and No P2 is P1.[55]

What, other than the above given, are resulting relations (conclusions)?

Between S1 and P2 (this is the primary issue, #5);

and (secondarily) between S2 and P1, and between S1 and P1, and between S2 and P2.

We can for a start, by means of syllogism, draw the following conclusions, common to all four rules, from the first three premises, without reference to the fourth premise:

- From the minor and subjectal premises, Some S1 are P2   (mood 3/AAI).
- From the major and subjectal premises, All S2 are P1   (mood 1/AAA).
- From the major and subjectal premises, Some P1 are not S2   (mood 3/OAO).

What this means is that, no matter which predicatal premise is used, it cannot logically yield a conclusion incompatible with 'Some S1 are P2'. The following specifies what can additionally be said in each of the four rules under scrutiny (the sources and discussion of the examples here proposed are given ad loc. in my *Judaic Logic*):

In rule No. 8a, nothing further about S1 and P2 can be deductively inferred; yet R. Ishmael apparently claims 'All S1 are P2' (which is too much). *For example*: A sorceress (or by extension, a sorcerer) is liable to the death penalty (#1); a male or female medium or necromancer is liable to death by stoning (#2); a male or female medium or necromancer is a sorcerer or sorceress (#3); death by stoning is a species of death penalty (#4); therefore, all sorts of sorcerers or sorceresses are liable to be stoned (#5).

In rule No. 8b, we can syllogistically infer (mood 1/AAA) that 'All S1 are P2'; yet R. Ishmael apparently claims 'Some S1 are not P2' (which is inconsistent). *For example*: whoever approaches holy offerings while impure is liable to the penalty of excision (#1); anyone who eats peace-offerings while impure is liable to the penalty of excision (#2); peace-offerings are holy offerings (#3); the penalty is the same in both cases, viz. excision (#4); therefore, the consumption of offerings of lesser holiness than peace-offerings is *not* subject to the penalty of excision (#5).

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[52]   See his essay: "Halakhic Arguments as Dialectical Arguments and Exegetical Principles as Aristotelian Topoi in Maimonides' Philosophy." In *Tarbits*, 73 (2004), p. 219.

[53]   R. Akiva's competing principles of *ribbui umiut* could also be argued to suggest syllogistic reasoning. For interesting examples, see Nissan Mendel, pp. 100-102.

[54]   Of course, 'some P1 are P2' and 'some P2 are P1' imply each other.

[55]   Of course,' no P1 is P2' and 'no P2 is P1' imply each other.

In rule No. 9, we can syllogistically infer (mood 2/AOO) that 'Some P2 are not S1'; it is not clear how R. Ishmael's proposed conclusion here should be presented in formal terms (such lack of clarity being of course a deficiency). I have not found a sufficiently informative example of application of this rule[56].

In rule No. 10, the predicatal premise is logically incompatible with the other three premises, so no syllogistic inference is possible; R. Ishmael apparently resolved the conflict by modifying the major premise to read 'Some, but not all, S1 are P1' (which is logically acceptable, though not the only option open to us). *For example*: the release of a Hebrew slave is subject to a certain set of laws (#1); the release of a daughter sold as maid-servant is subject to another set of laws (#2); a daughter sold as maid-servant is nominally a subcategory of Hebrew slave (#3); yet, the laws for the maid-servant and those for the Hebrew slave in general are very different (#4); therefore, the category of Hebrew slave intended here is in fact not so broad as to subsume such maid-servants (#5).

From these reflections, we learn that at least four of the rules of R. Ishmael (as I have tentatively interpreted them, *based on a small number of examples*) are syllogistic in form. These four all include at least the syllogism: 'All S2 are S1 and All S2 are P2, therefore Some S1 are P2' (3/AAI). Two of the four involve an additional syllogism (of form 1/AAA in rule 8b, and of form 2/AOO in rule 9); one rule involves no additional syllogism (rule 8a); and the fourth rule involves inconsistent premises. It is interesting to note that R. Ishmael's apparent solutions to these four syllogistic problems are in some way or other deficient. Nevertheless, it does not change the fact that these four rules are essentially of a syllogistic nature.[57]

I have also demonstrated, in an earlier chapter of the present volume, in the section on analogical argument (5.1), the presence of syllogistic thinking in rabbinic analogical arguments, namely in rule 2 (*gezerah shavah*), rule 3 (*binyan av*) and rule 12 (*meinyano* and *misofo*) in R. Ishmael's list. These arguments are not solely syllogistic – they involve inductive processes too – but they definitely do include syllogism. These findings are indubitable, and they put to rest once and for all the rather widespread notion in some quarters that the rabbinic hermeneutic principles do not depend on syllogistic reasoning.

Syllogism can, I suspect, be discerned in yet other *midot*, if we examine them closely enough. I would go much further than that, and assert that these examples drawn from the 13 *midot* are only the tip of the iceberg. The *midot* are by far not a full listing of the reasoning processes actually used by the rabbis; it is certain many of their actual reasoning processes are not included in their listings. The listings only bring together certain forms of thought which the rabbis considered worthy of notice and emphasis for some reason. But like all human beings, they used many thought processes unconsciously – including the process of syllogism. It is impossible for anyone to reason without certain basic thought forms; and the syllogism is definitely one of these unavoidable thought forms, since it is required for all mental acts of inclusion or exclusion.

9.   Historical questions

There is, I would say, a significant difference between a fortiori use in Talmudic contexts its use in other ancient literature, such as in Platonic or Aristotelian texts[58]. In the latter, it is probably more accurate to speak of a fortiori *discourse* rather than a fortiori *argument*, because it is used more as a rhetorical device than as a form of reasoning. The author in such cases could well have rephrased his text in such a way as to pass the same message without using a fortiori language. Whereas, in Talmudic contexts, the use of a fortiori is definitely argumentative; it is necessary to prove something that has a legal impact and that could not be arrived at by other means. So when we speak of a fortiori use in the Talmud, we are referring to something much more serious.

When we speak of Talmudic and rabbinic logic, we must have in mind and look for both *explicit* theories and *implicit* practices by people concerned. Theorizing has different levels: just being aware that one is engaged in an argument is one level; the next higher level is awareness that the argument is of a certain peculiar type, and a name is assigned to it (such as 'a fortiori argument'); the third level consists in attempting to give form to such argument, using symbols in the place of terms; and the highest level is wondering at the argument's validity and seeking to establish it once and for all. Study of the history of an argument is also theory, though of a more intellectual-cultural sort. As regards practice, it may be far ahead of theory. Theory can improve further practice, but is generally based on prior 'intuitive'

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[56]   An example is given in *Sifra* on Lev. 13, but I have not so far found a way to formalize it. See Mendel Nissan, pp. 104-105. There may also be useful examples in Mielziner's book.

[57]   The fifth rule which begins with the phrase *kol davar shehayah bikhlal veyatsa...* (rule 11) is somewhat different but also syllogistic. See my analysis of it in my *Judaic Logic*.

[58]   For example, in one of Plato's Dialogues, Socrates says: "I am not a match for one of you; and *a fortiori* I must run away from two." In truth, Aristotle does often use a fortiori argument with a more scientific intent; but even then his argument as a whole does not depend so heavily on such argument as it does in Talmudic literature. And of course, Aristotle must be acknowledged for his early reflections on a fortiori argument in the *Topics* (2:10) and the *Rhetoric* (2:23), even if such reflections were scant.

practice. Therefore any investigation that aims to understand the logic of some group of people or humanity in general must focus strongly on actual practices.

Even if much conscious research has been carried out on Talmudic and rabbinic logic (including hermeneutics), I wager that there is still a lot to discover in this field. We shall never arrive at an accurate, scientific history – or indeed, theory – of Talmudic logic, and in particular of Talmudic a fortiori logic, without a thorough, systematic listing and competent analysis of all the arguments in the Talmud and related texts. Someone has to do this major work some day; or else we shall always be dealing in rough hypotheses based on limited samples.

Take for instance a fortiori argument, which is our object of study here. What we need, for a start, is a table listing all the apparent cases of a fortiori argument. In each case, we should note its location in the Talmud, and who (named or unnamed) is apparently formulating it, so that the best estimate of its date can later to be put forward. We must distinguish the person(s) formulating it from the person(s) commenting upon it in subsequent developments of the Talmud (by which I mean here, the two Talmuds, that of the Land of Israel and the Babylonian, and related contemporary literature).

Each argument must be analyzed, first by classifying it, i.e. identifying which of the eight standard moods it fits into. Is it copulative or implicational? Label the major term (or thesis) P, the minor term (or thesis) Q, the middle term (or thesis) R and the subsidiary term (or thesis) S. Is the argument subjectal (or antecedental) or predicatal (or consequental)? Is it positive or negative? Moreover, is the argument purely a fortiori or a crescendo? Was *dayo* applied? Very often, some creative work will be necessary, insofar as the a fortiori argument is not entirely explicit. It may, for instance, be necessary to construct an appropriate major premise, and the operative middle term (or thesis) may have to be suggested by the researcher. Obviously, any such contributions to the argument made by the researcher must be noted as such and not confused with the raw data. That is to say, the fact that the original argument is in some way incompletely formulated is a significant detail of the analysis.

Once we have such an exhaustive database of the a fortiori arguments in the Talmud, we can begin to develop a truly scientific account of this argument form in that document. We can say with certainty what moods (if not all) of the argument were known to the named participants and anonymous redactors, and how well they understood them. We can compare the logical skills in this domain of the different players involved. We can find out more precisely what their theoretical understanding of a fortiori forms were and what terminology they used for them. We must not forget that the Talmud is a document built-up over centuries, by hundreds of people. The Talmud is not a monolith, but has many temporal and geographical layers[59]. Therefore, research must also try to trace the development of skills and understanding of a fortiori and other argumentation across time and place.

We can also more accurately compare Talmudic use and knowledge of the a fortiori argument to use and knowledge in surrounding cultures – notably the Greek and Roman as regards the Mishna and the Jerusalem Talmud, and possibly further afield as regards the Gemara of the Babylonian Talmud, since it was developed in Babylon where perhaps some Indian influences might have occurred. This too, of course, has a long timeline. Take for example the distinction between *miqal le chomer* (from minor to major) and *michomer leqal* (from major to minor). This distinction is taken for granted today – but it surely has a rich history. Does it appear anywhere in the Talmud, or is it a later discovery? It is found in later rabbinic literature – but the question is when and where did it first appear? Was this an independent Jewish discovery, or can Greek or Roman or other ancient influence, or later on Christian or Moslem influence, be traced? If the distinction is made in the Talmud, just when and by whom, in what context(s)?

This distinction, note well, signifies some level of conceptual analysis of a fortiori reasoning. But it is still relatively vague or equivocal, insofar as 'minor to major' can signify either positive subjectal or negative predicatal argument, while 'major to minor' can signify either negative subjectal or positive predicatal argument[60]. A question to ask is, therefore: what was the original intent of this distinction – was it meant as a distinction between positive and negative moods of subjectal a fortiori argument, or was it a distinction between positive subjectal and positive predicatal arguments, or was there awareness of all four possibilities, or did it remain vague? Indeed, granting that positive subjectal argument is the most obvious and widespread form, when were negative subjectal and positive and negative predicatal argument forms first realized in Judaic logic (or elsewhere, for that matter)?

And so on. There are evidently many questions worth asking and the answers cannot be settled till we have a thorough database, as already said. It should be noted that today, with the digitalization of most ancient texts well on the way if not already completed, the job is immensely facilitated, since exhaustive searches of different verbal strings are possible in a jiffy and information can be cut and pasted without difficulty! The historical work and the logical analysis involved may or may not be done by the same person(s). The ideal scholar would be a good wide-

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[59]   I must draw attention, here, to the different theories of Talmudic formation, including those of Abraham Weiss and David Weiss-Halivni. They are of course very relevant to attempts at dating specific passages of the Talmud.
[60]   And similarly with implicational moods. I doubt however that the distinction between copulative and implicational moods is to be found anywhere outside my book *Judaic Logic* – so it is no use asking that question.

ranging historical researcher, knowledgeable and at ease in the Talmud and other significant texts in the original languages, and a good logician to boot. These qualities are not necessarily all found in the same person, but a multi-disciplinary team might be constituted by a university. I do hope some people someday realize the need and value of such research and organize a determined effort in that direction.

9. Post-Talmudic rabbis

1. Logic and history issues

In the present chapter, our object shall be to discuss and to some extent trace some of the developments in rabbinic and more broadly Jewish thought concerning the a fortiori argument, and to a lesser extent more broadly the hermeneutic principles. This is of course a massive task that we cannot remotely hope to carry out exhaustively in the present study; we can however hopefully reflect on some of the issues involved and give scattered examples of the kind of research and evaluation that are needed in this context.

The first thing to make clear is the distinction between hermeneutic and logical principles. Although the rabbis to some extent regarded their hermeneutic principles as logical principles, the truth is that logic was not a prime interest for them: their primary interest was in justifying the traditional legal system enshrined in the Mishna and expanded on in the Gemara and subsequently. I will not here even try to roughly trace the development of Jewish law from its Biblical beginnings, through the formative period from Ezra to the Mishna, followed by the Gemara and later rabbinic work. I can only recommend to the reader who has not already done so to read works (preferably critical) on the subject, such as Mielziner's *Introduction to the Talmud*. The important thing, in the present context, is to take to heart what Mielziner writes regarding the "circumstances that necessitated artificial interpretation":

> "As long as the validity of this oral law had not been questioned, there was no need of founding it on a Scriptural basis. It stood on its own footing, and was shielded by the authority of tradition. From the time however when the Sadducean ideas began to spread, which tended to undermine the authority of the traditional law and reject everything not founded on the Scriptures, the effort was made by the teachers to place the traditions under the shield of the word to the Thora. To accomplish this task, the plain and natural interpretation did not always suffice. More artificial methods had to be devised by which the sphere of the written law could be extended so as to offer a basis and support for every traditional law, and, at the same time, to enrich the substance of this law with new provisions for cases not yet provided for. This artificial interpretation which originated in the urgent desire to ingraft the traditions on the stem of Scripture or harmonize the oral with the written law, could, of course, in many instances not be effected without strained constructions and the exercise of some violence on the biblical text…" (pp. 120-121).[1]

Two ideas should be emphasized in this context. The first is that *the hermeneutic principles have a history*. They did not come out of the blue all of a sudden, whether at Sinai or later, but were gradually developed in response to specific needs by specific persons, and often against conflicting opinions by other persons. Changes evidently occurred over time. This development can be precisely traced to some extent, even though traditional commentators make every effort to deny significance to the known history. The second idea is that *the hermeneutic principles are not necessarily logical*. Mielziner rightly refers to "artificial" as against "natural" methods, using exactly the same terms as I did independently fifteen years ago when I wrote my *Judaic Logic*. In that work, I showed, clearly and by formal means, to what extent the hermeneutic principles could be regarded as logical and to what extent they could not. Mielziner, in his reference to "strained constructions and the exercise of some violence on the biblical text," had the honesty and courage to admit the limits of rabbinic logic.

In the present work, following detailed logical analyses mainly of the Mishna Baba Qama 2:5 and the related Gemara Baba Qama 25a, I have developed a more precise assessment of Talmudic logic. It appeared from this exploration that the a fortiori logic found in the Mishna is more natural, less artificial, than that found in the Gemara. Judging from the Talmudic passage we examined, the understanding of a fortiori argument by the earlier rabbis was simpler and more straightforward, while that of the later rabbis was more complicated and tortuous. The two groups should not be lumped together. This is as regards their practice; neither group engages in much theoretical reflection (if any)

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[1]   Further on, Mielziner remarks that there were "some legal traditions… for which the Rabbis were unable to find a biblical support or even a mere hint" (and informs us that 55 such cases have been enumerated). These were suggestively labeled as *halakhot leMoshe miSinai* – laws handed down to Moshe from Sinai.

on the subject. So the artificiality that Mielziner speaks of is more centered in the Gemara than in the Mishna (at least as regards a fortiori argument).

What is clear from our research is that it is misleading and futile to try to interpret and justify the rabbinical hermeneutic principles entirely through logic. They undoubtedly have some logical character, and are often thought of and intended as logic, but they are not purely and entirely logical. They are, as Mielziner well described them, ad hoc responses to the problem of anchoring the oral law so-called, i.e. the Jewish legal tradition existing at a certain period of history, in the more authoritative written Torah. Sometimes that anchoring is possible by quite natural (i.e. purely logical) means; but sometimes some intellectual artifices are necessary to achieve the desired end. With this frank admission in mind, we can more clearly trace the history of commentaries on the hermeneutic principles and practices in general, and on the a fortiori argument in particular, from two points of view.

The first viewpoint is that of the *uncritical traditionalists*. Their writings or lectures on the a fortiori argument or on hermeneutics are simply designed to pass on as clearly as possible the information received from tradition. This teaching is presumed true and valid without question, and the only role of the teacher is to clarify it and give examples of it. The second viewpoint is that of the *critical logicians*, among which I count myself. Their written or oral reflections on the subject are aimed at scientific evaluation, and are therefore perforce more formal and not necessarily in agreement with tradition. Truth and validity are not automatically granted, simply because the argument in question is claimed to be, directly or indirectly, of Divine or prophetic origin, or to have the stamp of approval by rabbinical or whatever authorities. These two viewpoints are pretty well bound to be at odds in some cases, though not in all.

Many indices can be used as litmus tests for the classification of a commentator in one camp or the other. We must look and see where each commentator stands in relation to the debate between R. Tarfon and the Sages in the Mishna; how he perceives the argument(s) of the former and the objection(s) of the latter. We must also pay attention to his eventual reactions to the Gemara: to its general equation (on the basis of a *baraita*) of a crescendo argument with a fortiori; to its readings of the argument about the isolation of Miriam in Num. 12:14-15; to its claims about R. Tarfon's ideas about when the *dayo* principle may or may not be applied to an a fortiori argument. In short, we must look out for the depth and breadth of a commentator's awareness of the issues involved. Certain authors will judge such matters dogmatically: they are the traditionalists. Others will be more circumspect: to the extent they are so, they belong to the critical school.

That is our theoretical stance; but in practice, as we shall presently discover, there is rarely need to get that fancy, because most commentators on the a fortiori argument treat the issues relatively superficially.

2. Philo of Alexandria

Philo of Alexandria, aka Philo Judaeus or the Jew (20 BCE – 50 CE), has some importance in the history of philosophy, because he embodies an early confluence between Jewish religion (and philosophy) and Greek (religion and) philosophy, and also of course because he later had considerable influence on Christian religion (and philosophy). He interpreted the Jewish Bible (the Septuagint version, in Greek) allegorically, by adaptation of Greek and Hellenistic ideas (derived from works of Plato, Aristotle, and the Stoics) as well as reference to Jewish traditions. I include him here because he lived, though outside of the Holy Land, in the Mishnaic period. According to Encyclopedia Judaica article on him, there is no mention of Philo is any ancient Jewish source other than Josephus (*Antiquities* 18:8). This could mean that mentions of him by rabbis of the Mishnaic and/or Talmudic periods have been lost; or that they did not know of his work; or that they knew of his work but disapproved of it. Philo has occasionally anticipated positions that rabbis in the Talmud later adopted, presumably independently, since they did not acknowledge his authorship. He was subsequently (much later) not considered by the rabbis as particularly kosher, despite his evidently intense faith in God and the Torah, and his services to the Jewish community of Alexandria[2], due no doubt partly to the Greek influences evident in his views and partly because Christians eventually adopted many of them.

Philo did not, to my knowledge, discuss a fortiori argument, but he did use it. I found one instance of a fortiori argument in a work by Philo, namely *De Fuga et Inventione*[3] (§84, p. 163), translated from the Greek original into French by Esther Starobinski-Safran: "No blasphemer has a right to forgiveness. For if those that cursed their mortal parents are chastised, what punishment ought to be meted to those who dare blaspheme the Father and Creator of the

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[2]   He was part of an embassy sent in 40 CE by the Alexandrian Jewish community to Rome, to plead their cause before the emperor Gaius Caligula.

[3]   Paris: Cerf, 1970. The title means "escape and discovery."

universe?" The minor premise of this argument comes from the Biblical passage: "May he who curses his father and his mother perish" (Exodus 21:17).[4]

The reference to blasphemy in the conclusion is derived from the earlier Biblical passage: "And if a man come presumptuously upon his neighbor, to slay him with guile; thou shalt take him from Mine altar, that he may die" (Exodus 21:14). According to Philo, an intentional murderer seeking refuge at the altar is effectively guilty of "blasphemy," since by this gesture he blames God for his sins. The "blasphemy" perceived by Philo is, note well, not explicit, but implicit in the blame of God, which is in turn implicit in the act of seeking refuge with Him albeit guilty of intentional murder (as explained in §79-80)[5].

The proposed argument is thus: if someone who curses his parents deserves punishment, then someone who blasphemes God deserves punishment – possibly greater punishment[6]. Evidently, the argument is positive subjectal; and possibly a crescendo (note that Philo only asks "what punishment?" – he does not explicitly give an answer, whether equal or greater). Notice its similarity to the argument in the Torah concerning Miriam (Num. 12:14-15), which also proceeds from parents to God. I have not looked for use of a fortiori argument by Philo in other books.

I do not know why Philo finds it necessary to infer from the penalty for cursing parents an equal (or maybe greater) penalty for blaspheming God, when the Torah already states, in Leviticus 24:15-16: "Whoever curseth his God shall bear his sin. And he that blasphemeth the name of the Lord, he shall surely be put to death; all the congregation shall certainly stone him...." Perhaps he differentiated these two verses, since v. 15, which refers to cursing (Heb. *yeqalel*), does not mention a death penalty, whereas v. 16, which refers to blasphemy (Heb. *noqev*), does mention a death penalty[7]. But in that case, his purpose would be to prove (by a fortiori argument from parents) that the death penalty for cursing parents also applies to *cursing* God. But if so, why use the word "blaspheme" instead of "curse" in his stated argument? So there is some confusion or redundancy, here. In any case, note that he does not mention the death penalty at all in his argument, but only chastisement or punishment[8].

I should say that I noticed Philo's a fortiori argument thanks to Starobinski-Safran, who in her introductory commentary (p. 38) reads the argument as: "If the outrage inflicted on parents is punished by [bodily] death (Ex. 21:15-16[9]), then the outrage to God, which is infinitely more grave, calls for death of the soul, an even heavier penalty" (again, my translation from the French). She does not specify 'bodily' death (my addition in square brackets), but we may assume this to be her intent from the explicit contrast to death "of the soul." Note that the penalty for cursing parents is implicitly specified in Leviticus 20:9 as death by stoning (according to the rabbis, who point out that it there says: "his blood shall be upon him"), the same as the penalty explicitly specified in Lev. 21:16 for blasphemy. So as far as this-worldly punishment goes, the two cases are identical; but nothing is said (or denied) concerning other-worldly punishments.

The distinction here made by Starobinski-Safran between death (of the body) and death of the soul, and the proportionality of punishment she assumes, are not (as far as I can discern) explicitly given in Philo's wording. The commentator is reading these details into the text. Perhaps she mentions "death of the soul" because no actual verbal blasphemy (meriting bodily death under Jewish law) has occurred in the case under consideration, but only blasphemy by implication (from the attempt to take refuge at the altar illicitly). She does not say what she precisely means by "death of the soul;" maybe she means the penalty of *karet* (usually translated as 'excision')? Certainly, even if Philo did have similar intentions in mind, the matter is open to debate, just as the intent of the argument concerning Miriam is debatable.

The trouble with allegory, as we see in this example, is that while it may provide a neat explanation and integration of the text at hand, there is a risk that it ends up being taken literally by people and thus have untoward legal consequences. The plain meaning of the Torah text referred to here is that an intentional killer who seeks refuge at the altar may be forcibly taken from there and executed anyway, because he committed murder and this crime is

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[4]   My translation from the French of both Philo's words and the Biblical passage.

[5]   It is interesting to note, here, that the words 'blasphemy' and 'blame' are etymologically very close. Starobinski-Safran informs us in a footnote that the Greek words Philo uses in this context have many possible meanings, including blaspheming, cursing, slandering, or insulting. She further remarks that "to seek refuge at the altar without right is offensive towards the Divinity; in effect, it is discharging onto [the Divinity] the faults one is responsible for" (my translation).

[6]   Note that Philo does not prove, by means of his a fortiori argument, his initial claim that "No blasphemer has a right to forgiveness;" he only proves that they deserve punishment of some sort, possibly death.

[7]   Blasphemy, according to Ibn Ezra, could signify reviling or, more likely, use of the Tetragrammaton. Rashi states that only if the Tetragrammaton is uttered is blasphemy liable to the death penalty. (The latter comments are drawn from *The Soncino Chumash*, pp. 759-60.)

[8]   Of course, if the death penalty is specified for cursing parents, then (a fortiori) the death penalty (at least) must be applied for cursing God. If we say that the death penalty is the maximal sentence, then the argument is purely a fortiori; it cannot be a crescendo. But if a more severe penalty is conceivable, as Starobinski-Safran suggests, then the argument might well still be a crescendo.

[9]   Starobinski-Safran writes verse 16, but that must be an error of inattention, for v. 16 concerns kidnapping; it is v. 17 that concerns cursing of parents. Verse 15 concerns the smiting of parents; but this seems to me irrelevant in the present context, since no one can physically smite God.

liable to the death penalty (Ex. 21:14). On the other hand, Philo allegorically explains the killer's seeking refuge at the altar as effectively an act of blaming God for his sin, and thence accuses the killer of "blasphemy;" this gives the impression that when the killer is pulled away from the altar to be executed, he is being executed for "blasphemy;" but what he has done is not strictly-speaking blasphemy according to the law (Lev. 24:16).

An additional note. In today's political context, when fundamentalist Moslems are trying to impose worldwide restrictions on the freedom of speech, by pressuring governments to outlaw what they call "blasphemy," even by non-Moslems, and when many Western politicians, journalists, academics and sundry institutions, seem about ready to abjectly comply, it is important to add the following disclaimer. Whereas we frequently read in news bulletins about Moslems savagely lynching people they accuse of "blasphemy" (meaning, any doubt or criticism of Islamic beliefs), it must be stressed that Judaism – like Christianity – nowadays no longer applies past laws against blasphemy (meaning, speaking ill of God). Jews today believe in and are happy with the rights of thought, speech and conscience they have in the West, including Israel; and moreover they do not approve of capital punishment for blasphemy or other religious transgressions anywhere in the world. I am not aware of any segment of the Jewish population who would say otherwise.

The outlook of today's Jews on Torah passages like those imposing the death penalty for blasphemy or for cursing parents is *not literal, but symbolic*. Their practical purpose is simply to make people aware of the great immorality of such acts. The more severe the penalty theoretically prescribed by the Torah for a certain act, the greater the implied immorality of that act. But the prescribed penalty is *merely potential, not actual*. That is, retribution for such immoral deeds is today de facto and de jure no longer a rabbinic responsibility; it is left to God[10]. Thus, the aim of such passages is no more than moral education – to raise awareness of the full weight of thoughts, words and deeds. Each person remains free to choose – and, of course, to pay the natural consequences for his or her choice (including Divine retribution, if God so chooses). And that is a good thing. We certainly do not want the development of a Taliban mentality in our community, and most of us would surely combat it as regressive nonsense if it ever arose.

In this context, it is worth citing the Mishna: "A Sanhedrin that effects an execution once in seven years, is branded a destructive tribunal. R. Eliezer b. Azariah says: once in seventy years. R. Tarfon and R. Akiba say: were we members of a Sanhedrin, no person would ever be put to death. [Thereupon] Rabban Simeon b. Gamaliel remarked, [yea] and they would also multiply shedders of blood in Israel!" (Makkoth 1:10)[11]. For most crimes deserving capital punishment, with the possible exception of murder, the penalty was never in fact applied past the Biblical period. Moreover, capital punishment was effectively abandoned altogether during the Mishnaic period, although of course it has remained a topic of theoretical discussions. Jewish law is clearly very different in spirit and practice from Islamic law.

3. Sifra

The *Sifra* is a halakhic midrash to Leviticus, which is occasionally called *Torat Kohanim* like the Torah book (Leviticus) that it is an exegesis of (JE[12]). According to Jacob Neusner (USA, b. 1932), in *Rabbinic Literature: An Essential Guide*[13], it is considered as dated ca. 300 CE (p. 3). I nevertheless include it in the present chapter – as an extra-Talmudic document, rather than as a post-Talmudic one, for lack of a better place. Indeed, though later than the Mishna, it is often referred to in the Talmud (JE). Neusner describes it as an effort to more thoroughly anchor the 'oral Torah' – meaning the Mishna (and the Tosefta) – in the 'written Torah' – i.e. essentially the Pentateuch (pp. 56-57). Neusner does not mention the work's author, but JE discusses the matter[14].

My interest here is in certain features of Sifra's logic that are mentioned by Neusner. I have not personally read Sifra, but take Neusner's description of these features for granted. As Neusner puts it, Sifra's purpose is to show that "the Mishnah is subordinated to Scripture and validated only through Scripture;" and it does so by means of a "critique of the Mishnah" which too often seems to rely on its own logic rather than explicitly refer to the Pentateuch (p. 58ff). This critique, as we shall see, focuses both on syllogistic and a fortiori logic. Sifra reportedly makes (or seems to make) assertions concerning these fields that simply, as I will definitively show in formal terms, cannot be upheld.

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[10]   We might here refer to the statement "vengeance is Mine" (Deut. 32:35).
[11]   Soncino Talmud, Tractate Makkoth 7a. See also on this topic: Mishna Sanhedrin 7:4, 9:1, 11:1 and Tractate Sanhedrin 41a.
[12]   The   Jewish   Encyclopedia   (JE)   article   referred   to   here   may   be   consulted   online   at: www.jewishencyclopedia.com/view.jsp?artid=697&letter=S.
[13]   Nashville, Tenn.: Abingdon, 2005.
[14]   Maimonides (in the introduction to his *Yad haḤazaḳah*), among others, considers its author to be Rab, aka Abba Arika (Babylonia, 175–247 CE), in view of the book's other title, *Sifra debe Rab*. Another theory, proposed by Malbim (in the introduction to his Sifra edition), is that R. Ḥiyya b. Abba (ca. 180-230 CE), a late Tanna or early Amora who lived in Eretz Israel, was the book's redactor. The latter is also credited with compilation of the *Tosefta*. A lot more is said on this topic, which need not concern us here.

Syllogism. First, Sifra disputes "that we can classify things on our own by appeal to the traits or indicative characteristics, that is, utterly without reference to Scripture." According to Sifra (or according to Neusner's reading of it), "on our own, we cannot classify species into genera. Everything is different from everything else in some way. But Scripture tells us what thing are like what other things for what purposes, hence Scripture imposes on things the definitive classifications, not traits we discern in the things themselves." And again:

> "The thrust of *Sifra*'s authorship's attack on taxonomic logic is easily discerned… things have so many and such diverse and contradictory indicative traits that, comparing one thing to something else, we can always distinguish one species from another. Even though we find something in common, we can also discern some other trait characteristic of one thing but not the other."

If I understand such statements correctly, what Sifra is saying (or more probably, just implying through its many particular discursive acts, since rabbinic literature is rarely if ever so abstract in its approach) is that antithetical syllogisms can consistently be constructed. This would mean the following in formal terms:

All S1 are G;	No S2 are G;
and X is S1.	and X is S2.
Therefore, X is G.	Therefore, X is not G.

On the surface, such a situation might seem conceivable. The individual or class called X might be classified under species S1 in some respects and under species S2 in other respects; and S1 might fall under genus G, while S2 does not fall under genus G. The two major premises do not seem incompatible, since they concern different subjects, S1 and S2; and the two minor premises do not seem incompatible, since a term may well have different predicates, S1 and S2. Yet the two conclusions are clearly incompatible!

However, logic is quite able to show where in the said premises the contradiction lies, by constructing a 2nd figure syllogism using the two initial major premises:

No S2 are G
All S1 are G
Therefore, No S1 is S2.

Using the latter conclusion as our new major premise, it follows by syllogism that if X is S1, it cannot be S2, and vice versa. That is, despite surface appearances, the two species, S1 and S2, are in fact mutually exclusive, by virtue of being related in contrary ways to the genus G. Thus, in fact, the two minor premises 'X is S1' and 'X is S2' cannot both be true at once. Therefore, the contradiction between 'X is G' and 'X is not G' will *in fact* never arise.

That is to say, the apparent argument of Sifra that contradictions are possible if we rely only on logic, so that appeal to Scripture is necessary to help us choose one side or the other, is not credible. It only seems credible due to superficial appeal to syllogistic reasoning; but in fact such quandaries cannot occur in practice for someone who truly knows logic. It should be said that the supposition that such quandaries are conceivable is not peculiar to Sifra; Greek and Roman sophists have also often imagined them possible.

Of course, Sifra may not be saying what I have here assumed it to say. It may just be saying that without Scripture's guidance we cannot know whether X is S1 or S2; or perhaps (more likely) we cannot know whether species S1 or S2 falls under genus G or not, where G is some law or legal ruling. Such arguments would be logically acceptable. But what is sure, anyway, is that no one can legitimately argue that the initially listed two 1st figure syllogisms are compatible. This is not open to discussion.

The same of course can be said with regard to rival hypothetical syllogisms:

If B1 then C;	If B2 then not C;
and if A then B1.	and if A then B2.
Therefore, if A then C.	Therefore, if A then not C.

The if–then premises of such arguments may offhand seem compatible, but their conflicting conclusions (assuming thesis A is not a paradoxical proposition) show them to be in fact incompatible. However, it should be obvious that this restriction is only applicable in cases of strict implication; if some of the implications involved are less firm, a situation of rivalry might conceivably occur. If A deductively implies B and B deductively implies C, then the conclusion is that A deductively implies C. But if A merely inductively implies B and/or B merely inductively implies C, then the conclusion is that A merely inductively implies C. Whereas 'deductive implication' signifies a

100% certainty, what I call 'inductive implication' refers to a looser relationship where the antecedent *probably* (with less than 100% certainty) implies the consequent. In such cases, the conclusions 'if A, maybe then C' and 'if A, maybe then not C' may both be justified, even though there is some degree of tension between them.

We can similarly admit that potential (though not actual) conflicts might occur in categorical syllogism. If for instance the rival syllogisms have as major premises that Most S1 are G and Most S2 are not G, and as minor premises that X is S1 and X is S2, then the conclusions will be respectively that X is probably G and X is probably not G. Though these two conclusions are in tension, they are not strictly speaking incompatible, and therefore they might conceivably occur together (especially if their probabilities are expressed so vaguely). It is probably the possibility of such *tendencies* to conflict that the author of Sifra had in mind. Another possibility is that there are unstated conditions to the premises of the categorical or hypothetical syllogisms, which make the rival arguments compatible although they superficially seem incompatible.

A fortiori argument. Second, concerning the argument a fortiori or *qal vachomer*, Neusner tells us in the name of Sifra that it "will not serve" – for "if on the basis of one set of traits that yield a given classification we place into hierarchical order two or more items on the basis of a different set of traits, we have either a different classification altogether or, much more commonly, simply a different hierarchy." This is intended as a critique of "the Mishnah's… logic of hierarchical classification." To wit: "Things are not merely like or unlike, therefore following one rule or its opposite, Things are also weightier or less weighty."

Here, the suggestion is that we can construct compatible a fortiori arguments, with reference to different middle terms (R1, R2), which yield contrary conclusions. This is a very similar suggestion to the previous one, but one specifically centered on a fortiori argument. It should again be stated that Sifra is not alone in this error (if it indeed makes it) – many people seem to think that such a situation is logically possible. Such people do not truly understand the logic involved, as I will now formally show. Consider the following two arguments:

P is more R1 than Q is;	P is more R2 than Q is;
and Q is R1 enough to be S.	and Q is R2 enough *not* to be S.
Therefore, P is R1 enough to be S.	Therefore, P is R2 enough *not* to be S.

On the surface, looking at the premises superficially, such a situation may seem possible. After all, such major premises certainly occur in practice. P and Q may be in a certain relation within the hierarchy R1 and in a very different (even opposite) relation in another hierarchy R2. But such differences could not give rise to contrary conclusions, one implying that 'P is S' and the other that 'P is not S' – for the simple reason that the minor premises are incompatible, one implying that 'Q is S' and the other that 'Q is not S'. Thus, in fact, such a situation is logically inconceivable.

Thus, contrary to what Sifra seems to be (according to Neusner's analysis) suggesting, we do not need to appeal to Scripture to choose between this hierarchy and that one so as to avoid contradiction. Two hierarchies that lead to contrary conclusions will never be true together. This is logically obvious and demonstrable. Of course, here again, we might defend Sifra by saying that it perhaps does not claim such antithetical a fortiori arguments, but merely says that Scripture is required to establish the major and/or minor premises. This would present no problem. But if the claim is indeed one to viable antitheses, it is untenable.

We could also defend Sifra by pointing out that many rival arguments that seem to adhere to the formal conflict presented above are in fact not intended so strictly. The major premises, which tell us that P is more R than Q, may be tacitly intended to mean that P is *usually* (though not always) more R than Q; and/or the minor premises may really have the form 'Q is *usually* R enough to be S'; in which cases, the conclusions will also be probabilistic at best. Thus, there may be an appearance of conflict, when in fact there is only some logical tension. This, I believe, often occurs in practice, and may well be what the author of Sifra had in mind when he raised this issue. Or again, there may be unstated conditions to the premises of the rival a fortiori arguments, which make them compatible although they superficially seem incompatible.

To sum up and conclude. If, as Neusner seems to be implying, Sifra criticizes the Mishna on the ground that it relies on *logic* independently of Scripture, and that by doing so it opens itself to irresolvable contradictions, Sifra can and must be opposed on purely formal grounds. Logic does not lead to contradictions, but on the contrary deflects them, or uncovers and resolves them. If, however, Sifra is only saying that the Mishna has to refer to Scripture for its major and/or minor premises, i.e. for the content of its propositions – that is another matter entirely: it is then an issue not of logic, but of *fact* or even of moral and legal *evaluation* of fact.

But upon reflection, even in the latter cases we must distinguish between deductive and inductive logic. It is true that *deductive* logic cannot prescribe facts and even less so their evaluations (though it can be used to ensure that such prescriptions are kept internally consistent). But *inductive* logic certainly can strongly impinge on issues of fact or

even of moral and legal evaluation of facts. Through experience and scientific method we can, for instances, contest that hare are to be classified with ruminants, or that there are no fish that have scales but lack fins. And moreover, from purely factual material, we can put in doubt the credibility of evaluations; for example, how we conceive sunrise and sunset to occur directly affects the times prescribed for beginning and ending the Sabbath.**15**

4. The Korach arguments

The Midrash called *Bemidbar* (Numbers) *Rabbah*, which is closely related to the Midrash called *Tanhuma* (named after a rabbi), is (or at least its earliest portions are) thought to date from the 5ᵗʰ century CE, apparently before the completion of the Babylonian Talmud**16**. What interests us in it here is its commentary regarding Numbers 16:1, which reads:

> "'Now Korach... took'. What is written in the preceding passage (Num. 15:38)? 'Bid them that they make them... fringes (Heb. *tzitzith*)... and that they put with the fringe of each corner a thread of blue (Heb. *techeleth*)'. Korach jumped up and asked Moses: 'If a cloak is entirely of blue, what is the law as regards its being exempted from the obligation of fringes?' Moses answered him: 'It is subject to the obligation of fringes'. Korach retorted: 'A cloak that is entirely composed of blue cannot free itself from the obligation, yet the four blue threads do free it?!'
>
> He [Korach] asked again: 'If a house is full of Scriptural books, what is the law as regards its being exempt from the obligation of *mezuzah* [a small scroll with a selection of Torah verses, which is affixed to the doorposts of Jewish gates and homes]?' He [Moses] answered him: 'It is [still] under the obligation of having a mezuzah.' He [Korach] argued: 'The whole Torah, which contains two hundred and seventy-five sections, cannot exempt the house, yet the one section in the mezuzah exempts it?! These are things which you have not been commanded, but you are inventing them out of your own mind!'"

There is, note well, no evidence of this discourse in the Torah itself; it only appears much later in history, in the Midrash. These two arguments attributed to Korach are traditionally regarded as samples of *qal vachomer*, although (I presume) most commentators view them as *qal vachomer* of a fallacious sort. For that reason, they are especially interesting, in that they illustrate a possibility of erroneous reasoning in the a fortiori mode. We may paraphrase the two arguments briefly as follows:

a) If mere threads of blue wool (on each of the four corners) are sufficient to make a garment lawful to wear, then surely if the whole garment is made of such blue wool (even if without the corner threads) it is likewise lawful.

b) If a few passages of the Torah (in a mezuzah affixed to the doorposts) are sufficient to make a house lawful to live in, then surely if the whole Torah is stored in a house (even if without a mezuzah) it is likewise lawful.

These two arguments have the following form in common: If a *small quantity* of something (Q) is sufficiently in accord with the norm (R) to make so-and-so be declared lawful (S), then surely a *large quantity* of that thing (P) is sufficiently in accord with the norm (R) to make so-and-so be declared lawful (S). The preceding hypothetical proposition comprises the minor premise and conclusion of the a fortiori argument. Its tacit major premise is therefore: a large quantity of something (P) is more in accord with the norm (R) than a small quantity of same (Q). The argument is clearly positive subjectal, from minor to major.

What is wrong with this argument? The answer is obvious: its major premise does not have the logical necessity it is implied to have. While on the surface it might seem like a large quantity is preferable to a small one, this is not necessarily the case, because the two quantities may present *significant qualitative differences*. That is, the terms of the proposed major premise are incompletely specified, and therein lies the fallacy. The minor premise, regarding the sufficiency of a small quantity (Q) to satisfy the norm (R) for a certain result (S), may be true only provided that this quantity fulfills certain qualitative criteria (which may have additional quantitative aspects). If the larger quantity (P) does not fulfill these same qualitative criteria, it may well *not* be able to satisfy the norm (R) for a certain result (S). Therefore, the major premise should, to be truly universal, more precisely read: a large quantity of something *precisely specified* (P) is more in accord with the norm (R) than a small quantity of the *exact* same thing (Q).

Returning now to the two Korach arguments for the purpose of illustration, we can say the following. In both cases, the sophistry consisted in occulting the details given in brackets. In (a), what makes the garment kosher is not merely that it contains blue threads, but that it contains them *on the four corners*. In (b), what makes the house kosher is not merely that it contains Torah words, but that it contains them *on the doorposts*. The details do matter – they are not expendable. Therefore, in effect, Korach's two arguments may be said to commit the fallacy of having more than

four terms. The major and minor terms in the major premise are made to appear the same as the subjects in the minor premise and conclusion, but they are in fact different from them.

The two arguments might have been a bit more credible, had they respectively advocated an inference from a garment not made of blue wool yet having kosher tzitzit, to a garment entirely made of blue wool as well as having kosher tzitzit; or from a small mezuzah affixed to the doorposts, to a giant mezuzah affixed to doorposts. But even then, such inference would not be *necessarily* true, because there is no formal reason why the law might not interdict garments made entirely of blue wool (even with kosher tzitzit) or giant mezuzot (even affixed to doorposts). The major premise in use in any argument must be in fact true, for a true conclusion to be drawn from it. Very rarely is the major premise logically necessary; it is only so if its contradictory is self-contradictory. In most cases, the major premise has to be determined empirically – or, in such a religious context, be given in the proof text.

In my opinion, the two arguments attributed to Korach are not factual reports, but post facto fabrications with an educational purpose. Because: either Korach had some brains and could see for himself the fallacy of his reasoning, but he cynically proposed these arguments anyway, thinking no one else would notice; or he was not intelligent enough to realize his own errors of logic. But in either case, surely Moses had the intelligence needed to see through the fallacy, and would have publicly reproved Korach for his dishonesty or his intellectual deficiency, so as to stop the rebellion in its tracks by discrediting its leader. However, since according to the Torah account Divine intervention was used, we can infer that Moses did not use this logical means. I think the arguments were imagined by the author(s) of the Midrash for three reasons. One was to flesh out the story of Korach with some (most likely anachronistic) Talmudic-style legal debate, showing up the perversity and stupidity of the rebel. Another was perhaps to intimidate eventual readers, saying in effect: if you behave like Korach, expressing doubts in the law of Moses, you will be punished like Korach. The third was perhaps to teach people some a fortiori logic, to make sure they do not make similar errors of reasoning.

However, this is not how some later commentators have understood the purpose of this Midrash. They have taken it to mean, not that Korach was arguing fallaciously, but that Korach was being too logical, so that we ought to learn from this story to suspend our rational judgment now and then. For instance, R. Ephraim Buchwald, in an essay called "The Excesses of Rationality" (2007)[17], explains the matter as follows:

> "According to Korach, human logic always prevails. Korach is certain that the rational processes are the ultimate determinant of right and wrong. Since the laws handed down from Moses and Sinai have no internal logic, they must be summarily rejected. It is for that very reason that parashat Chukat follows parashat Korach. The Torah, in Numbers 19:2, declares: '*Zoat chukat HaTorah*': This is the statute of the Torah! There is no logic to the laws of the Red Heifer. Reason is of little value when it comes to this irrational ritual. The Red Heifer comes to confirm to Korach and all his fellow rationalists, that the ultimate authority is the law of Moses and Sinai, not mortal logic! … While Judaism in general is a most rational and logical faith, true believers must eventually conclude that there are certain aspects of the religion that one can not rationally fathom or master. It is that leap of faith that a believer must make, and this doubt that we all must overcome, and for which we are ultimately rewarded."

This is obviously, in view of what our analysis above has demonstrated, an erroneous interpretation of the Midrash. The commentator evidently does not have great logical knowhow, since he seems to think that the two Korach arguments are valid. He is therefore not qualified to discuss the limits of human logic. Korach cannot be presented on the basis of the two arguments attributed to him as a "rationalist," or proponent of reason, since they are in fact not in accord with logic. If he was not an idiot, he was a sophist who cynically faked logical argument. In the Midrashic story, Moses does not answer Korach by sullenly saying: "your arguments are sound, but I will stick dogmatically to my positions," as our commentator implies. Rather, I'd say, Moses refutes Korach, as often done in Talmudic debate, by denying his conclusion, thereby tacitly implying that at least one of his premises is incorrect; and since the minor premise is in accord with the law of Moses, it must be the major premise that is mistaken. In other words, the correct interpretation is that Moses does not concede Korach's reasoning powers, but rather challenges them.

R. Buchwald is, of course, relying on the traditional commentaries regarding the statute of the red heifer (Numbers 19). They find it odd that the ritually clean people involved in preparing the ashes of the red heifer should be made unclean (v. 7, 8, 10), while those ashes are used to ritually clean people who are unclean due to having come in contact with a dead person (v. 12). Rashi comments, citing Yoma 67b: "Because Satan and the nations of the world taunt Israel, saying, 'What is this commandment, and what purpose does it have?' Therefore, the Torah uses the term "statute." I have decreed it; You have no right to challenge it."

But in truth, what has this to do with logic? It is not logically inconceivable that the same substance (the ashes of the red heifer) might have one effect (ritual uncleanliness) on one set of people (the people producing or handling it) and

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another, opposite effect (ritual cleaning) on another set of people (the people it is sprinkled on). Such complex relations can readily be found in nature – e.g. a chemical substance might be harmful to one kind of organism and beneficial to another. Or consider, to take an extreme example, the particle-wave duality in quantum mechanics, where the same phenomenon seems different viewed from different perspectives.

The red heifer ritual is no more 'illogical' than the ritual of sacrificing animals to purify people of their sins, or the rituals of tzitzit or mezuzah, or that of matza, or those of shofar, lulav and succah, or any other religious ritual. When dealing with the supernatural, everything is equally artificial, i.e. inexplicable by natural means. Rituals are not given in nature, or rationally inferred from it. Such truths (if they are indeed true) can only be known through revelation or similar (alleged) extraordinary means. Belief in them – at least in the case of people without prophetic powers of their own, and maybe even for prophets – depends on faith. Even prayer, the most natural expression of belief in God, depends on faith.[18]

Moreover, the inexplicability of alleged spiritual practices is not a reflection on human logic. Human logic does not promise omniscience. There are many things we do not, and perhaps can never, understand, even in the natural world; all the more so, in the (presumed) spiritual world. The fact that there are limits (whether short or long term) to the power of logic can never be used as an argument against the power of logic within its natural limits. There is no logical argument by which logic might be invalidated, because such argument would be claiming to have some logic, and thus be self-defeating. Even if logic admittedly cannot predict all truth, it can certainly eliminate quite a bit of falsehood. For this reason, we should not hasten to ditch it just because it does not deliver everything we wish for.

R. Buchwald's attempt to compare the Korach argumentation to the red heifer statute is, anyway, ingenuous. He regards the Korach arguments as perplexing because though sound (in his view), they lead to conclusions that are contrary-to-fact (i.e. to Biblical fact); and he regards the red heifer ashes as perplexing, because (I presume, though he does not say so) they have contrary behavior patterns in relation to different subjects. But even supposing these two perplexities are justified, they are certainly logically very different and cannot be lumped together. If they are, as he supposes, both 'illogical', they are 'illogical' in significantly different ways.

In any case, there is one kind of illogic that no amount of faith can ignore or cure – and that is any breach of the laws of thought. Faith is acceptable where there is some gap or uncertainty in knowledge; but if a claim – however 'authoritative' – goes against these fundamental laws, we can be absolutely sure it is incorrect. This applies equally well to other-worldly claims as to this-worldly ones. Our reaction in such case should not be blind faith, but to demand a credible resolution of the paradox. This is the adult, mentally-healthy reaction to such conundrums. In this sense, logic has much to say even about spiritual claims. Logic is mankind's main protection against falsehood of any kind.

5. Saadia Gaon

When I found out that Saadia Gaon, ben Yosef (Egypt, ca. 882 – Iraq, 942), had written a short book, entitled in Hebrew *Perush Shelosh Esre Midot* (Explanation of the Thirteen Hermeneutic Principles), and actually found a copy of it on the Internet[19], I was overjoyed, hoping to find in it some interesting original insights into *qal vachomer*. However, upon reading it (with the help of a friend), I was rather disappointed. Saadia Gaon there in fact says nothing theoretical about *qal vachomer*, other than to say that it may be used for non-legal as well as legal purposes.

He does not analyze the argument in any way, but is content to present five rabbinical examples of it – without, by the way, explaining why he chose those particular ones. If a man is obligated to take good care of his second wife, all the more so his first wife. Since, if one finds one's enemy's strayed animal, one is obligated to return it to him, it follows a fortiori that one must do that for a friend. And so forth. All these examples are in fact legal in content; he does not actually give any with non-legal content, but simply repeats (somewhat lamely, as if he could not think of any offhand) that non-legal content is possible. That's it. Of course, examples have their importance; but they are certainly not enough.

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[18]   To form objective judgments on such matters, one must take into consideration not only one's own religious beliefs but also those of other people. There are many religions in the world, each with its own rituals and its own rationales for these. They cannot *all* be absolute truths. Most, if not all, must be human inventions. Of course, each of us conveniently believes it is *the others'* belief systems that are imaginary. But try proving that! Therefore, all of us should have a measure of modesty and tolerance in his beliefs. This is not relativism, but honesty.

[19]   Originally written in Arabic; translated into Hebrew by Nahum ha-Maarabi in the 13th century. Full Hebrew text is given in the *Œuvres Complètes*. According to the introduction to this volume, the authenticity of the text has been demonstrated. Some comments are included in footnotes. You can also read it online at the Internet Archive: www.archive.org/stream/oeuvrescomplete01josegoog#page/n130/mode/2up. The Wikipedia article on Saadia Gaon informs us that, according to Azulai, Saadia has also written (again, in Arabic) a methodology of the Talmud entitled *Kelale ha-Talmud*.

According to the introduction (in French) to the *Œuvres Complètes*[20], Saadia does not always thus limit his commentary on the *midot* to examples, but in some cases gives explanations, even if his explanations are sometimes obscure (e.g. as to what distinguishes the 7th and 8th rules). In any case, he does not go into the details concerning the rules. Moreover, we are told, Saadia considers that anyone has a right to put forward new applications of the thirteen rules, which liberty is far from admitted by other commentators. Nevertheless, I should add, Saadia is known to have defended the rabbinic tradition that the thirteen *midot* were Divinely revealed to Moses at Mt. Sinai[21]. He no doubt did so in the context of his polemics with the Karaites, who of course rejected rabbinic interpretation[22].

So I was taught, anyhow; but I have not offhand found an explicit statement to that effect. Perhaps he merely implied it. We might, for example, so interpret his citation of *Sanhedrin* 88b, "With the increase in numbers of the disciples of Shammai and Hillel, who did not advance far enough in their studies, the controversies increased" (*The Book of Doctrines and Beliefs*, pp. 32-33), to explain the existence of disagreements between rabbis. The implication is that originally, when the Torah was first given, there were no doubts; these developed over time, when levels of learning diminished. This matter could be further pursued, but I will leave it at that for now and move on.

I would like, rather, to take this opportunity to quote Saadia Gaon on the value of empiricism and rationalism:

> "Furthermore [authentic tradition] verifies for us the validity of the intuition of reason. It enjoins us, namely, to speak the truth and not to lie. Thus it says: *For my mouth shall utter truth….* Besides that it confirms for us the validity of knowledge inferred by logical necessity, [that is to say] that whatever leads to the rejection of the perception of the senses or rational intuition is false…. Next [tradition] informs us that all sciences are [ultimately] based on what we grasp with our aforementioned senses, from which they are deduced and derived." (*The Book of Beliefs and Opinions*, Pp. 18-19.)

It is also interesting to note here certain rules for inference set by Saadia Gaon:

> "In endeavoring to establish the truth of inferential knowledge, we shall henceforth be on guard against these five possible forms of mistakes, namely: (1) that it does not conflict with knowledge established by sense-perception; (2) that is does not conflict with knowledge established by Reason; (3) that it should not conflict with some other truths; (4) that it should not be self-contradictory; still more, that it should not (5) involve a difficulty more serious than the one intended to avoid." (*The Book of Doctrines and Beliefs*, p. 42.)[23]

Taking Saadia at his word, we can predict that were he placed squarely before new facts and shown the validity of certain logical inferences, he would have the intellectual and moral integrity to admit them, and would not dogmatically insist on contrary, more traditional 'facts' or 'inferences'. Unfortunately, there are still today some people who think they do religion a service by refusing to face facts and logic. Just yesterday, I had the hilarious experience of watching an online video showing an Islamic apologist claiming in 2007 on Iraqi TV that the earth is flat and much larger than the sun, which is also flat![24] Fortunately, apologists for Judaism never go so far; but they also sometimes show considerable resistance to change.

I say this here because readers of the present volume must obviously be prepared to adapt to new discoveries and insights, and not cling at all costs to traditional views. I want to emphasize in passing that to be critical does not signify to be hostile and willfully negative. Though critical, I have personally no desire to contradict or denigrate our religious tradition. Not all critical commentators are so moderate in their views or intentions; some are very eager to find fault with the rabbis or the Torah. For my part, I would prefer to always justify the rabbis and the Torah, and confirm their wisdom, and it is only reluctantly that I criticize some of their claims. Nevertheless, I try to be scrupulously fair and honest – i.e. to be scientific – and to admit that there is a problem when there indeed appears to be one. This is the golden mean – neither dishonestly attacking nor dishonestly defending, but sincerely looking for the truth.

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[20]   *Œuvres Complètes de R. Saadia ben Iosef al-Fayyoumi* (vol. 9). Ed. Joseph Derenbourg, Hartwig Derenbourg and Mayer Lambert. Paris : Leroux, 1897.

[21]   It is stated (apparently in the *Sifra*) that Exodus 21:1, "Now these are the ordinances which thou shalt set before them," was said by R. Ishmael to refer to the thirteen rules for interpretation of the Bible revealed to Moses on Sinai. This equation may be convenient, but it is not based on a literal reading.

[22]   His book, *Emunot veDeot* (Constantinople, 1562; in Hebrew), can be downloaded free at:
www.seforimonline.org/seforimdb/index.php?table_name=seforim_database&function=details&where_field=id&where_value=2.

[23]   Needless to say, the two books quoted here, *Of Doctrines and Beliefs* (Abridged ed. Trans. Alexander Altmann. Oxford: Phaidon, date not specified.) and *Of Beliefs and Opinions* (vol. I. Trans. Samuel Rosenblatt. New Haven: Yale, 1948.) are two translations of Saadia Gaon's *Emunot veDeot*. Incidentally, I am amazed how different they are; so much so that I had to quote them both because I could not find the same material in both!

[24]   See: www.memritv.org/clip/en/1684.htm.

6. Rashi and Tosafot

Concerning the contribution of Rashi, i.e. R. Shlomo ben Yitzhak (France, 1040-1105), to the understanding of the hermeneutic principles, Mielziner tells us that he "occasionally explained, in his lucid way, the single rules where they are applied in the Talmudic discussions." There is, he adds, "a separate treatise on the hermeneutic rules ascribed to this commentator and published under the title of *Perush Rashi al Hamidot*," which however "seems to be spurious." This is found "in Kobak's *Jeschurun*, vi, Hebrew part, pp. 38-44, 201-204; the remaining commentaries on the thirteen rules are enumerated by [Adolf] Jellinek in *Ḳonṭres ha-Kelalim*, Nos. 163-175."

I have no access to these various sources, so must make do with a more ad hoc treatment. The question that interests me here is: firstly, what does Rashi say about the *qal vachomer* in Numbers 12:14-15 (and eventually, the other cases found in the Torah, and maybe also those in the Nakh)? And secondly, what does he say about the discussion concerning the *dayo* principle in Baba Qama 25a-b? I shall also try and determine the viewpoints on these topics of Rashi successors, the Tosafot. The basic issue to my mind is: do these post-Talmudic commentators accept the idea seemingly advocated in the Gemara (based on a *baraita*) that *qal vachomer* is naturally 'proportional' and the *dayo* principle is designed to reign in such velleity in it? The answer to expect is, obviously: yes, they do.

First, let me mention in passing Rashi's comments on other a fortiori arguments appearing in the Torah. Concerning Genesis 44:8, all Rashi says is: "This is one of the ten a fortiori inferences that are found in Scripture, which are all listed in *Bereishit Rabbah* (92:7)." For Exodus 6:12: he is likewise content to say: "This is one of the ten a fortiori inferences in the [Tanakh]," although he additionally explains Moses' speech defect as an "obstruction of the lips." He has no comment regarding Deuteronomy 31:27. Evidently, Rashi does not question the Midrashic statistic of just ten *qal vachomer* in the Tanakh.

As regards Numbers 12:14, Rashi's comment is: "*If her [Miriam's] father were to display, to her, an angry face, would she not be humiliated for seven days? Certainly, then, in the case of the Divine Presence, [she should be humiliated] for fourteen days. However, it is sufficient that the derivative equal the source of its derivation. Therefore, even with My rebuke, let her be confined for seven days.*" As can be seen, this is just a repetition of the thesis given in a *baraita* transmitted in the said Gemara. If we look for Rashi's comment opposite that *baraita* in the Gemara, we find that he has none. That means he considers the matter sufficiently clear as it is and sees no point in adding anything to it. There you have it. Rashi does not ask or answer any theoretical questions concerning *qal vachomer* reasoning or the *dayo* principle, but takes them for granted.

Rashi comments somewhat more extensively on another *qal vachomer* and *dayo* principle application, namely in Tractate *Zevachim* 69b (*Seder Kodashim*)[25]. There, the Mishna explicitly refers to both the argument (by R. Meir) and the application of the principle (by R. Jose), and the Gemara expounds almost exactly in the same words as in Baba Qama 25a, saying: "Does not R. Meir accept the principle of *dayo* [it is sufficient]? Surely the principle of *dayo* is Biblical, for it was taught: How is a *qal vachomer* applied? And the Lord said unto Moses: If her father had but spit in her face, should she not hide in shame seven days? How much more should a divine reproof necessitate [shame for] fourteen days; but it is sufficient for that which is inferred by an argument to be like the premise!" But Rashi does not add much, other than to (rightly) point out that the *qal vachomer* in the Miriam story is implicit rather than explicit.

That Rashi uncritically accepts the common notion that a fortiori argument is 'proportional' is evident not only in his acceptance without comment of the "fourteen days" given in the Gemara of Baba Qama 25a, but also in his comment to Genesis 4:24 (which, it should be noted, is not included in the traditional list of ten a fortiori arguments in the Tanakh). There, based on *Tanchuma Bereshit* 11, Rashi elucidates Lamekh's statement "If Cain shall be avenged sevenfold, truly Lamekh seventy and seven-fold" as a *qal vachomer*, as follows: "If Cain killed intentionally, [and yet] his punishment was delayed for seven generations, [then] I, who killed unintentionally, surely will have my punishment deferred for many periods of seven generations."

Note, however, that though in the case of Miriam Rashi acknowledges the *dayo* principle, he does not mention its application in the case of Lamekh; nor does he tell us why he doesn't. I suggest that the reason why it seems reasonable in one case and not the other is the following. In the example of Miriam, the conclusion (14 days penalty) is more stringent than the minor premise (7 days penalty), in accord with the principle of *midah keneged midah* (measure for measure), so the *dayo* principle is required to mitigate the punishment; whereas, in the example of Lamekh, the conclusion, though likewise quantitatively superior (77 instead of 7 generations), is more lenient as regards the sanction than the minor premise (i.e. signifies longer deferral of punishment), and so is not subject to the *dayo* principle (which if applied would speed up the punishment).

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[25]   This can be read in English in the Soncino Talmud, at: www.halakhah.com/pdf/kodoshim/Zevachim.pdf.

Successors of Rashi, known as Tosafot[26], comment on the Mishna and Gemara in more detail. Essentially, they subscribe to the scenario apparently advocated by the Gemara when interpreting the Mishna of Baba Qama 25a. That is to say, they accept uncritically that R. Tarfon's two arguments are a crescendo. Nevertheless, to their credit, they consider that his first and second try are logically (and not merely rhetorically) different, due to reshuffling. They also agree with R. Tarfon that his second argument is able to avoid the *dayo* restriction as set by the Sages against his first argument, because while the first argues from half to full damages, the second argues from full to full damages. As a result of which, they intelligently explain the Sages' continued insistence on *dayo* application with reference to premises antecedent to the *qal vachomer* itself.[27]

However, since Tosafot accept the Gemara reading of both arguments as a crescendo, they also accept the "fourteen days" notion proposed by the Gemara (following a *baraita*), i.e. the claim that *qal vachomer* naturally yields a 'proportional' conclusion. Without questioning this claim, they only focus on trying to explain this number (rather than any other large number[28]). An explanation they give is to refer to seven days as the minimum period of quarantine in the event of leprosy (Leviticus 13:4); a more severe confinement must be at least another seven day period.[29] Tosafot also consequently make efforts to defend the obscure notion, ascribed by the Gemara to R. Tarfon, that the *dayo* principle can on occasion be ignored, specifically where it would "defeat the purpose of" the *qal vachomer*.[30]

But such glosses are superficial in their concerns; they gloss over the more serious underlying issues. I have shown in my detailed analysis of this *sugya* in the two preceding chapters (7-8) that we cannot countenance some of the commonplace interpretations of this Mishna and Gemara without getting ensnared in a multitude of logical errors, which make at least some of the rabbis involved look very foolish. Once the logical errors are understood, it is seen that many of the explanations proposed in the Gemara, and later by others, including Tosafot, are vain attempts to uphold a very wobbly structure. If we want to redeem the rabbis involved, we must approach the whole matter much more lucidly, and consider a moral instead of logical explanation of the *dayo* principle.

I do not want to seem to be dismissing Tosafot in a debonair manner, being fully aware of their importance, but simply see no point in repeating here what I demonstrated earlier. So I invite the reader to go there.

7. Kol zeh assim

A thorough study of the logic in Tosafot, and even just of its a fortiori logic, would doubtless result in a thick and interesting book. Not having the necessary language skills, I cannot myself undertake such a study; but I would certainly recommend that someone duly qualified in both logic (especially as taught in the present volume) and the Talmud do the job. But we can here get an idea of the logical resourcefulness of Tosafot through one example, which has to do Baba Qama 25a. This is thanks to Yisrael Ury, who in his book *Charting the Sea of Talmud*[31] provides an English translation of a commentary by Tosafot and some useful clarifications as to its intents[32]. This passage of Tosafot is only incidentally concerned with Baba Qama 25a, using it to illustrate a certain form of argument; so we shall not here cite all of it, but only quote or paraphrase the parts of it relevant to our narrower purpose.

The Tosafot commentary, whose precise author is not named, proceeds in three stages, we might say. In a **first stage**, it refers to one of the arguments originally given in the Mishna Baba Qama 2:5, which it paraphrases as follows:

> "Whereas tooth and foot, for which damages are not paid for damage done in the public domain, yet are liable for full damages for damages done in the domain of the damaged party, then horn, for which half damages are paid for damage done in the public domain, certainly should pay full damages for damage done in the damaged party's domain."

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[26]   The Tosafot were medieval rabbis, active between the 12th and mid-15th centuries mainly in France and Germany, who elucidated and explicated many passages of the Talmud. Their commentaries (*tosafot* means additions) were very important to subsequent development of Jewish law. Many grandsons of Rashi are counted among them, by the way. I here refer to them collectively, because I do not know precisely which one(s) commented on the issue here concerning us; perhaps his or their names is/are known to experts.

[27]   For a summary in English of the comments of Tosafot and others, see the Art Scrolls' *Talmud Bavli*.

[28]   Even thousands considering God's exaltedness.

[29]   Another commentator has suggested: "Why particularly fourteen? The Rabbis (*Nidah* 31a) remark that each parent provides a child with five essential parts (the father with bones, sinews, etc., the mother with skin, flesh, etc.), whereas God provides him with ten (spirit, soul, etc.). Since God doubles the father's portion, the humiliation for his rebuke is also double, fourteen days to the father's seven." From the Metsudah Chumash w/Rashi at: www.tachash.org/metsudah/m03n.html#fn343.

[30]   As I have shown, this notion is based on rhetoric; it has no basis in logic. But note that since I regard the *dayo* principle in its broadest sense as a moral rather than logical principle, I do not deny that it might have exceptions, as R. Tarfon claims in the Gemara's scenario.

[31]   Jerusalem: Mosaica, 2012.

[32]   Although Ury does not clearly state where this Tosafot is found, it seems from the context to be opposite *Kiddushin* 4b. The relevant pages in Ury's book are 113-118.

Looking at this argument, we easily recognize the first argument of R. Tarfon, since it proceeds by mentioning first tooth & foot damage in the public and private domains and then horn damage in the same domains. As I have shown previously, this argument can be put in standard a fortiori form as follows:

> Private property damage (P) implies more legal liability (R) than public domain damage (Q) [as we know by extrapolation from the case of tooth & foot[33]].
> Public domain damage (Q) implies legal liability (Rq) enough to necessitate *half* payment for damage by horn (Sq) [this is derived from the Torah[34]].
> The payment due (S) is 'proportional' to the degree of legal liability (R).
> Therefore, private property damage (P) implies legal liability (Rp) enough to necessitate *full* payment for damage by horn (Sp = more than Sq).

We shall here label this argument as argument (1a). Notice that it is positive antecedental. The major premise is obtained by generalization from the givens regarding damage by tooth & foot. The major and minor terms are 'damage on private property' (P) and 'damage on public domain' (Q). The middle term is 'legal liability' (R); and the subsidiary term is 'to make the payment for damage by horn have a certain magnitude' (S). In fact, note well, the argument is not purely a fortiori but a crescendo, since the magnitude of S in the conclusion is greater than that in the minor premise. This means there is a tacit premise to take into consideration, about the proportionality of 'payment due' (S) to 'legal liability' (R).

Although not directly mentioned by Tosafot, the second argument of R. Tarfon is, as we shall see, also (if not more) relevant to the present discussion; so we shall restate it here, in standard form:

> Horn damage (P) implies more legal liability (R) than tooth & foot damage (Q) [as we know by extrapolation from the case of public domain].
> Tooth & foot damage (Q) implies legal liability (R) enough to necessitate *full* payment for damage on private property (S).
> Therefore, horn damage (P) implies legal liability (R) enough to necessitate *full* payment for damage on private property (S).

We shall here label this argument as argument (1b)[35]. Notice that it is also positive antecedental. The major premise is, here, obtained by generalization from the givens regarding damage on public grounds. However, the major and minor terms are 'damage by horn' (P) and 'damage by tooth & foot' (Q). The middle term is again 'legal liability' (R); but the subsidiary term is 'to make the payment for damage on private property full' (S). Note that this argument is purely a fortiori, and not a crescendo. But it is clear that it could also be stated in a crescendo form, and that if it were would yield the same conclusion (viz. full payment for horn damage on private property), since no payment greater than full is admitted by the Torah or the rabbis. For this reason, it suffices to state it in pure form.

The **second stage** of our Tosafot commentary concerns an objection, and the reply to it, put forward in the past by a commentator called the Ri (presumably this refers to R. Isaac ben Samuel, a 12th century French Tosafist). The Ri's objection is described as follows:

> "But consider that the damages of tooth and foot are common!"

To which objection the Ri himself replies:

> "Paying full damages in the damaged party's domain is not a severity (*chumra*) to be used in an objection (*pirka*), for it does not at all cause tooth and foot to lead to the requirement of half damages for damage done in the public domain as does horn."

I have to say that I only understood the Ri's objection thanks to the clarifications given by Ury, which I presume are traditional. He explains it as follows: because damage caused by tooth & foot is "commonplace," the ox's owner is obligated to take extra care "that his animal not cause damage when it comes in proximity to the property of others;" so that if such damage does indeed occur, he is more open to blame. As regards damage by horn, since the goring of another animal by an ox is "a rare event," it is unexpected by the ox's owner and he is justified in not taking special precautions against it; so that if such damage does indeed occur, he is not as liable.

Thus, the Ri's objection means that, whereas *on public grounds* tooth & foot damage implies less liability than horn damage (no liability against half liability), as the Mishna teaches (based on the Torah), it may well be that *on private*

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[33]   Based on Ex. 22:4, and its extreme inversion, as explained in an earlier chapter (8.6).
[34]   Ex. 21:35 – "And if one man's ox hurt another's, so that it dieth; then they shall sell the live ox, and divide the price of it; and the dead also they shall divide."
[35]   Its premises are based on the same Biblical information as the first argument.

property tooth & foot damage implies more liability than horn damage (full liability against, say, only half). This reasoning thus constitutes an objection to the original Mishna argument – i.e. it is designed to show that the conclusion that seems inevitable in the latter (namely, full liability for damage by horn) is perhaps not so inevitable. Putting this reasoning in standard form, we obtain the following:

> Tooth & foot damage (P) implies more legal liability (R) than horn damage (Q) [since the former is common and the latter is uncommon].
> Tooth & foot damage (P) implies legal liability (R) enough to necessitate full payment for damage on private property (S).
> *From which it does not follow that* horn damage (Q) implies legal liability (R) enough to necessitate full payment for damage on private property (S).

We shall label this as argument (2a). This argument should be compared to the second argument of R. Tarfon, which we labeled (1b). Notice that they are very similar, except that the major premise has been reversed so that the putative conclusion no longer follows. In (2a), tooth & foot damage is the major term, while the horn damage is the minor term. The middle term is unchanged. The subject of the minor premise is unchanged (still tooth & foot damage), but now this subject is the major term. The subject of the putative conclusion is unchanged (still horn damage), but now this subject is the minor term. Since the format of the attempted a fortiori argument is still positive antecedental, inference from major to minor is illicit. Thus, we can no longer draw the conclusion of (1b) that 'horn damage on private property necessitates full payment'.

Such conclusion is now a non sequitur – it is not excluded by the new premises (it does not contradict them), but it is not justified by them, either. This argument is not itself an a fortiori argument, note well, but merely serves to put in doubt R. Tarfon's second a fortiori argument. It obstructs his conclusion, without needing to actually contradict it. It rejects his argument by reversing its major premise[36]. If, as R. Tarfon takes it, the owner of an ox is more responsible for horn damage than for tooth & foot damage, then the inference from full liability in the latter to full liability in the former is perfectly logical. But if, as the Ri contends with reference to 'frequencies of occurrence', the owner of an ox is more responsible for tooth & foot damage than for horn damage, then the inference from full liability in the former to full liability in the latter is debatable.

Another way to look at the objection (2a) is to say that the major premise of R. Tarfon's first a fortiori argument (1a) – which take note is the one that Tosafot mentions – is no longer granted. This premise, viz. "private property damage (P) implies more legal liability (R) than public domain damage (Q)," was obtained *by generalization* from the given that damage *by tooth & foot* implies no liability in the public domain and full liability on private property. However, now the objection makes us aware that this generalization is open to question, since *the conditions for* legal liability are not the same in the case of damage by horn, due to there being different frequencies of occurrence. Thus, analogy is blocked. What applies to tooth & foot does not necessarily apply to horn.

This, then, is the objection conceived of as possible by the Ri, stated in more formal terms. Let us now try to understand the way he himself neutralized the objection. Remember that we are *given* by the Mishna (based on certain Torah verses) that tooth & foot damage on public grounds does not necessitate any payment for damages, while horn damage on public grounds necessitates payment of half damages. On this basis, the Ri replies to the objection by saying: (i) that "paying full damages in the damaged party's domain" ought to "cause tooth and foot to lead to the requirement of half damages for damage done in the public domain as does horn;" and (ii) that since this consequence does not in fact occur, "paying full damages in the damaged party's domain is not a severity to be used in an objection."

The first part of his remark (i) refers to an a fortiori argument with the same major premise as (2a) combined with the given information about horn damage on public grounds necessitating half payment; these premises would conclude that tooth & foot damage on public grounds necessitates half payment (at least – more than half, i.e. full, if proportionality is applied). We may label this argument (2b), and put it in standard a fortiori form (positive antecedental, from minor to major) as follows:

> Tooth & foot damage (P) implies more legal liability (R) than horn damage (Q) [since the former is common and the latter is uncommon].
> Horn damage (Q) implies legal liability (R) enough to necessitate half payment for damage on public grounds (S).
> Therefore, tooth & foot damage (P) implies legal liability (R) enough to necessitate *half* payment for

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[36]   Such reversal of course means that the two major premises are in conflict, and therefore that the two arguments cannot be both upheld. It is not surprising, then, that they yield conflicting results.

damage on public grounds (S).

But, the Ri tells us in the second part of his remark (ii), *this conclusion cannot be true*, since the Torah tells us that tooth and foot damage on public grounds is exempt from any payment! Therefore, he concludes, this last a fortiori argument must be rejected. This last argument, which is a *reductio ad absurdum*, can be labeled argument (2c). It says: since the minor premise of argument (2b) is Torah given, and the process is valid, the only way to reject it is by abandoning its major premise[37]. That is to say, tooth & foot damage *cannot* be taken to imply more legal liability than horn damage on the basis of the former being more common and the latter being less common, as the objection (2a) initially attempts. Thus, the Ri shows that the objection, although reasonable sounding in itself, leads to absurdity and must be dropped.

The **third stage** of the Tosafot commentary we are analyzing is introduced by the statement in Hebrew: "*vekhol zeh assim bakal vachomer*," meaning in English: "and all this I will put into the a fortiori argument"[38]. The unnamed Tosafist then argues as follows[39]:

"Whereas tooth and foot, even though their damages are common, they are exempt from payment for damage done in the public domain, but necessitate a full payment for damage done on the property of the injured party – then horn, even though its damage is not common, and it necessitates payment of half damages for damage done in the public domain, does it not follow (lit. *eino din*) that it necessitates payment of full damages for damage done on the property of the injured party?"

The question posed is of course rhetorical – the author's intention is clearly that the proposed conclusion does follow. Where the author says "even though" (lit. *af al pi*) – as in even though the damage is common or even though the damage is uncommon – he is obviously referring back to the objection of the Ri, which suggests an inverse proportionality between frequency of occurrence and legal liability, i.e. that the more common a certain kind of damage is, the less the liability for it, and conversely that the less common a certain kind of damage is, the more the liability for it.

The purpose of this Tosafot commentary is, as its introduction ("all this I will put into the a fortiori argument") implies, to somehow *merge together* the original argument of the Mishna and the Ri's objection and his retort to the objection. Obviously, "all this" refers to the two preceding stages. Our job now is to judge whether the argument here proposed by Tosafot does indeed perform what it is designed to do. We can, for a start, put the proposed argument in standard form, as follows:

Tooth & foot damage (P) is more *common* (R) than horn damage (Q) [since the former is common and the latter not so].

Yet, tooth & foot damage (P) is common (R) *not* enough to make the ox's owner exempt from full payment for damages on private property (S) [since tooth & foot damage on private property necessitates full payment[40]].

Therefore, horn damage (Q) is common (R) *not* enough to make the ox's owner exempt from full payment for damages on private property (S) [whence, horn damage on private property does necessitate full payment].

We shall label this argument as argument (3), or refer to it more familiarly and briefly as "*kol zeh assim.*" As can be seen, it is *negative subjectal* in form (it goes major to minor). Its major and minor terms are respectively 'damage by tooth & foot' (P) and 'damage by horn' (Q). Its middle term is 'frequency of occurrence' (R), and its subsidiary term is 'to make the ox's owner exempt from full payment for damages on private property' (S). The major premise is known to us by generalization from the frequencies of occurrence observed in the public domain, where P and Q are characterized as common and uncommon, respectively. The minor premise is based on Torah information. The argument *has to be* put in negative subjectal form to be validated, note well, because it has as its subjects the two causes of damage and it goes from major to minor. Note that the subsidiary term is identical in minor premise and

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[37]   What the Ri actually says is: "paying full damages in the damaged party's domain is not a severity to be used in an objection," which could be taken to mean that he advocates denial of the minor premise of the objection (2a); but obviously, he cannot be intending that, since he knows that the minor premise is given in the Torah; therefore, it must be the major premise of the objection, which institutes the greater severity for tooth & foot damage compared to horn damage, that he intends to abandon.

[38]   Note that Ury has it as "*kol zeh achnis*," which he (or whoever) translates as "all this I will fold." But the Hebrew portion he quotes clearly has "*assim*," so I have preferred that word. Maybe there are different versions of the same Tosafot text. It is not an important issue.

[39]   I have referred to the translation given by Ury, but modified it considerably so as to make it both more literal and more readable.

[40]   Although strictly speaking "tooth & foot damage on private property necessitates full payment" does not imply "tooth & foot damage is common (R) not enough to make the ox's owner exempt from full payment for damages on private property," we can inductively assume this implication granting that there is a threshold value of the middle term (R) that allows access to the predicate.

conclusion; this means that the argument is purely a fortiori. The negative conclusion can finally be restated in the more familiar positive form (this being a simple eduction).

Alternatively, we could formulate the argument in *positive subjectal* form (going from minor to major) as follows. Note the change of polarity in the middle and subsidiary terms, and the change in the order of the terms tooth & foot and horn. The net result is the same:

> Horn damage (P) is more *uncommon* (R) than tooth & foot damage (Q).
> Yet, tooth & foot damage (Q) is uncommon (R) enough to make the ox's owner have to pay in full for damage on private property (S).
> Therefore, horn damage (P) is uncommon (R) enough to make the ox's owner have to pay in full for damage on private property (S).

The question we must ask here is: what does Tosafot mean by "all this"? In other words, what features of the preceding arguments (1a), (1b), (2a), (2b) and (2c), is argument (3) really referring to? "All this" is rather vague and needs to be specified more precisely. The two essential features that the Tosafot *kol zeh assim* argument shares with the discourse preceding it are the following: first, it has the same final conclusion as R. Tarfon's two arguments, viz. that damage by horn on private property entails full payment; second, it takes into consideration the Ri's objection, in that it is built around the observed fact of tooth & foot damage being more common than horn damage, and at the same time, it takes into consideration the Ri's reply to the objection, in that the *kol zeh assim* argument abstains from inferring greater liability for tooth & foot damage than for horn damage from their different frequencies of occurrence.

Thus, it can be said that the unnamed Tosafist's *kol zeh assim* argument does indeed, in a certain sense, conflate all the preceding arguments. Nevertheless, it does not annul and replace the preceding discourse. Especially note that we cannot formally derive the *kol zeh assim* argument from either or both of R. Tarfon's arguments, or derive them from it. However, unlike the Ri's objection (2a), this argument (3) is compatible with R. Tarfon's (1a) and (1b), since the major premise here has a different middle term. Thus, Tosafot's argument is a new, additional argument – not a substitute for the others. Its major premise comes from the Ri's commentary, taking both the objection (2a) and the retort to it (2b) and (2c) into consideration; its minor premise comes from the Mishna, and before that the Torah; and its conclusion agrees with that of R. Tarfon. Therefore, the *kol zeh assim* argument is a clever artifice, a way to allude to a number of issues in one shot.

Nevertheless, it should be stressed that the Tosafot argument is logically quite redundant, once we have become aware of the Ri's objection and his own retort to it. For the Ri's objection to R. Tarfon's original argument is that 'frequency of occurrence' may have an impact on 'legal liability', while his own retort to the objection is that if this impact were admitted a contradiction to Torah law would ensue; whence it follows that such objection is inadmissible. Once this inadmissibility is realized, there is no utility whatsoever in at all mentioning 'frequency of occurrence' as this Tosafot commentary so glibly does, since all connotation of 'legal liability' has been permanently removed from it.

Moreover, note well, Tosafot's argument does not constitute decisive proof of anything, just as R. Tarfon's arguments do not. We have to admit that these arguments are not decisive, anyway, if we wish to leave room in the discussion for the Sages' *dayo* principle. For the conclusion that damage by horn on private property entails full payment is eventually denied by the Sages (the colleagues of R. Tarfon in the Mishna), when they say and insist: "*dayo*—it is enough!" Their preferred conclusion is that damage by horn on private property entails only half payment. The Tosafot commentary (at least that part of it translated for us by Ury, which is all I have on hand) does not deal with this important issue here, its purpose being only to illustrate *kol zeh assim* argument (i.e. it refers to Baba Qama 25a only incidentally, here, so as to clarify another issue entirely).

Upon reflection. After writing the above, it occurred to me that Tosafot's argument (3), *unlike* R. Tarfon's arguments (1a) and (1b), is *immune* to both of the Sages' *dayo* objections. R. Tarfon's two arguments, you may recall, were neutralized by the Sages' two *dayo* objections, because they both relied in some way on the information that damage by horn on public grounds obligates the ox's owner to half compensation, in order to arrive at the conclusion that damage by horn on private grounds entails full compensation. The *kol zeh assim* argument differs radically from those in that it does not rely on the said information to arrive at the same conclusion. This means that Tosafot's argument is not logically affected by the Sages' *dayo* rebuttals, and conversely that they are logically unable to neutralize it.

As far as I know, Tosafot did not realize the collateral damage his *kol zeh assim* argument was capable of causing in this *sugya*. His argument, as we have seen, was only intended to save some of the insight of the Ri on 'frequency of occurrence' (the leftover, as it were, after the Ri's objection was neutralized by his retort) and to reaffirm R. Tarfon's

conclusion. But actually, Tosafot's argument does not merely buttress R. Tarfon's – it definitely proves it, since it is not subject to reproof by *dayo*. Does this then mean that the Sages' *dayo* objections, and therefore (at least in this particular case) the *dayo* principle, are null and void? Hopefully not – but then, under what conditions, exactly, could we still sustain them?

Since Tosafot's argument (3) is formally clearly valid, we can only find fault with its content – i.e. by denying its major premise and/or minor premise. The major premise, "tooth & foot damage is more common than horn damage," does not seem easily deniable assuming it is based on empirical observation; if it is not based on empirical observation, however, it could be denied as factually inaccurate. The minor premise, "tooth & foot damage is common not enough to make the ox's owner exempt from full payment for damages on private property," was, you may recall, based on the information given in the Torah (Exodus 22:4[41]) that tooth & foot damage on private property necessitates full payment.

It could be argued that this Torah passage does not actually specify *full* compensation, but rather refers to the quality of the feed or food restituted, leaving open the issue of quantity. But the rabbis, to my knowledge, do not accept this interpretation, and probably would not do so. We could still, however, deny Tosafot's minor premise by denying that there is a threshold of the middle term, i.e. a frequency of occurrence *as of which* the ox's owner is exempt from full payment for damages on private property and before which he is not. This is a more subtle yet technically possible approach, aimed at still more thoroughly detaching the concept of legal liability from that of commonness.

That is, if we say that no matter how common or uncommon tooth & foot damage is, this statistical feature has no effect whatever on the legal liability for the owner of an ox to pay (in full or whatever) for depredations on private property – then the minor premise of the *kol zeh assim* argument is dissolved, and the conclusion of that argument (concerning horn) no longer logically follows. Indeed, if we reflect on the meaning of the minor premise, we see that it does not make sense, anyway. It seems to suggest that if tooth & foot damage was more common than it is, it might at some point be common enough to exempt from full compensation on private property, whereas the Torah seems to unconditionally impose full payment.

Thus, it is possible in various ways to attack Tosafot's argument, and the probably best way to do so is by totally disconnecting the issue of legal liability from that of frequency of occurrence. In that event, nothing of the Ri's initial objection would be left over, and the "all this" of the Tosafot *all this I will put* claim ceases to be credible. This would be, I daresay, an acceptable price to pay if we wish to continue to uphold the Sages' *dayo* principle – at least in the present context, and more likely in all contexts, since the present context is in fact the root context for that principle, on which all subsequent appeals to that principle in the Talmud historically depend.

One thing is sure, we cannot cling to both the Sages' *dayo* principle and Tosafot's present *kol zeh assim* a fortiori argument – they are logically incompatible. We have to choose between them. It is obvious that we ought to choose to hang on to the *dayo* principle, which is more ancient (about late 1st – early 2nd century CE) and seems more important in Talmudic discourse, rather than on to the *kol zeh assim* argument, which appears much later in Jewish history (about the 12th cent. CE, say) and whose loss has less impact on Jewish jurisprudence. Therefore, the *kol zeh assim* argument seems condemned – at least in the present context (i.e. with reference to Mishna Baba Qama 2:5), even if a similar form of argument might be attempted in some other context(s) without unpleasant consequences.

8. Maimonides

Rabbi Moshe ben Maimon (Spain, 1135 – Egypt, 1204), known in Jewish literature by the acronym "Rambam," and more widely as Moses Maimonides[42], wrote at about the age of sixteen[43] a treatise on logic, called *Maqala Fi-Sana'at Al-Mantiq* (in Arabic), first translated into Hebrew by Moses Ibn-Tibbon (France, ca. 1240-1283)[44], and thence into other languages, including Latin (Basel, 1527), German (19th century), French and English (20th century). The edition I have in hand is a 1982 reprint of a 1935 critical edition with the text in Hebrew and in French

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[41] "If a man cause a field or vineyard to be eaten, and shall let his beast loose, and it feed in another man's field; of the best of his own field, and of the best of his own vineyard, shall he make restitution."
[42] The suffix '-ides' means 'son of'.
[43] This is according to Ventura's Introduction to his *Terminologie Logique* (p. 7). The Wikipedia article claims he was "in his twenties."
[44] The Tibbonides were a famous family of translators in the 12th-13th century, in the south of France. According to Ventura (pp. 14-17), it is they who translated the Arabic word *mantik*, which means much the same as the Greek word *logos*, into the Hebrew word *higayon*. A better choice would have been *dibbur*, but they avoided it because of certain philosophical connotations (association with the Arab dialectics of Kalam). Until then, the word *higayon* did not have the precise sense of 'logic'. Opponents of Maimonides used this word (which the translators chose, not him) against him, quoting the Bab. Talmud (*Berachot* 28a): "Prevent your children from using *higayon*." But other commentators, namely R. Joseph b. Caspi and Jacob Anatolio (a relative of the Tibbon family), objected to this reading, the former arguing that by *higayon* the rabbis meant pseudo-logical babble, while the latter pointed out that this statement referred specifically to children and not to adults.

(translation by Moise Ventura); its title is *Milot haHigayon* in Hebrew and *Terminologie Logique* in French (meaning, in English, *Terms of Logic*)[45]. It also contains the original Arabic-language version (extant parts) written in Hebrew letters. The scope of this work is considerable; it is not a mere lexicon, as its title suggests. It is an earnest teaching of formal logic and many of the more philosophical concepts surrounding it.

> Briefly put, the contents of Maimonides' study are as follows, chapter by chapter. 1) proposition: subject, predicate, affirmation, negation; 2) quantity: universal, particular, indeterminate, singular; 3) terms, copula, tenses, modalities; 4) oppositions, modalities; 5) immediate inferences, conversion, inversion; 6) syllogism, premise, conclusion, major, middle and minor terms; 7) figures and moods of the syllogism, conclusive-inconclusive, hypothetical and disjunctive arguments, direct reduction and reduction *ad absurdum*, induction, analogy, juridical reasoning; 8) sensory experience, axioms of reason and their derivatives, widespread opinions, traditional assertions, true propositions, demonstrative syllogism, dialectical syllogism, rhetorical syllogism, sophistical syllogism, poetic syllogism, enthymeme; 9) the four causes, material, formal, efficient and final, proximate and remote causes, the four elements, the material substratum; 10) species, genus, difference, attributes *per se* and *per accidens*, substance, definition, description, the ten categories[46]; 11) essential and accidental, potentiality, actuality, contraries with or without an intermediary, property, privation, relative, opposite; 12) anteriority in time, in nature, in rank, in merit, in cause; 13) names of various sorts, synonyms, homonyms, amphibologies, metaphors; 14) logos as rational faculty, thought and verbal discourse, logic as art and science, divisions of theoretical and practical philosophy, logic as instrument of all other sciences.

Effectively, Maimonides was importing into Jewish culture some very powerful tools developed by Aristotle and his successors[47]. But, while this book contains an interesting exposé of the main elements of Aristotelian logic, what is surprising is that it does *not* mention a fortiori argument[48]. One might understand such silence regarding most of the other of the hermeneutic rules, since they are principles of interpretation used specifically in Talmudic contexts. But the a fortiori argument is, as well as one of the means used in the Talmud for exegetic purposes, a universal method of reasoning. One would therefore have expected Maimonides to have included this form of argument in his treatise on logic[49]. However, two excuses can be put forward on his behalf. The first is that Aristotle himself, and subsequent logicians to the time of Maimonides, hardly mentioned this form of argument and never treated it in any significant detail. The second is that Maimonides wrote this book at a very young age, and perhaps was not then fully aware of the great significance of a fortiori argument in Talmudic discourse.

However, it does not appear that Maimonides subsequently dealt with a fortiori argument, or for that matter the other hermeneutic rules. Maimonides of course freely used a fortiori discourse in his works. In his *Guide for the Perplexed*, for instance, I found 23 instances. But, such use is practice, not theory (I have checked them all). As far as I know, he did not anywhere specifically stop and reflect on the reason why this argument works, even though he was exceptionally conscious of logical issues. The following are some examples of use of a fortiori argument by Maimonides, which I found in the said work:

- "If the firmament, with that which is over it, be supposed to be above the heavens, it would *a fortiori* seem to be unreal and incomprehensible." (Part 2, chapter 30.)
- "We thus learn that his prophetic perception was different from that of the Patriarchs, and excelled it; *a fortiori* it must have excelled that of other prophets before Moses." (Part 2, chapter 35.)

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[45]   Paris: Librairie Philosophique, 1935. Reprinted, 1982.

[46]   Here Maimonides lists the *Organon* of Aristotle as including eight works. Today, only six are included therein, the *Rhetoric* and the *Poetics* being excluded. But such inclusion, as Ventura points out (p. 13), is reasonable from Maimonides' point of view, since for him rhetorical syllogism is based on traditional assertions and poetical syllogism is based on fictions or imitations. Incidentally, the latter concept was introduced by Al-Farabi.

[47]   Maimonides apparently learned logic at least partly through his readings of Muslim commentators, especially Abu Nasr Al-Farabi (Central Asia, 872-950); he certainly mentions the latter (see e.g. p. 106). The impact of such early study of logic on the rest of his work is evident; such studies explain his orderly mind, rationalism and conceptual powers. It cannot be said that the rabbis after Maimonides all studied this work and took it to heart. Unfortunately, many still today carefully avoid studying logic, even as they refer to Maimonides' halakhic works.

[48]   Ventura expresses the same surprise, I think, when he remarks, in his Appendix to Chapter VIII (pp. 76-77): "Maimonides did not in his treatise mention a fortiori reasoning." Notwithstanding, he then suggests that Maimonides' mention of enthymeme (p. 72) might be construed as a tacit reference to a fortiori argument, since M. Lalande, in his *Vocabulaire technique et critique de la philosophie*, defines a fortiori as "an enthymeme that assumes a premise like the following: 'Who can do the more can do the less.'" But this argument of Ventura's is clearly spurious, since 'enthymeme' may refer to abridged argument of any sort. He goes on, describing how a fortiori is understood in the Talmud. Here, he adheres to the idea that "Miriam should have been sequestered fourteen days instead of seven," suggesting that a fortiori argument is naturally 'proportional' and requires the *dayo* principle to restrict its excesses. This belief of Ventura's is indicative of a lack of reflection on his part on the logic of a fortiori argument, no doubt under the influence of the Gemara.

[49]   Ventura (p. 23) lists some modern authors who have made an effort to relate Aristotelian logic and Talmudic hermeneutics, namely A. Schwarz and M. Mielziner, and M. Ostrovski. He also mentions (p. 18) a commentary by Moses Mendelssohn on Maimonides's treatise on logic (I have not read it).

- "The best test is the rejection, abstention, and contempt of bodily pleasures: for this is the first condition of men, and *a fortiori* of prophets." (Part 2, chapter 40.)
- "Man is superior to everything formed of earthy matter, but not to other beings; he is found exceedingly inferior when his existence is compared with that of the spheres, and *a fortiori* when compared with that of the Intelligences." (Part 3, chapter 13.)
- "But I agree with Aristotle as regards all other living beings, and *a fortiori* as regards plants and all the rest of earthly creatures." (Part 3, chapter 16.)
- "The law forbids us to imitate the heathen in any of these deeds, and *a fortiori* to adopt them entirely." (Part 3, chapter 29.)

Although Maimonides does not mention or discuss a fortiori argument in his treatise on logic, *Terms of Logic*, he does (in chapter 7) mention inductive reasoning and argument by analogy, both of which are involved in the background of a fortiori thinking. He describes induction as follows: "it proceeds from some particular assertions, admitted as true due to experience, to arrive at a general proposition that can be made into a premise of syllogism." Regarding analogy he says: "when one of two objects that resemble each other by a certain trait has some attribute that is not apparent in the other [object], we affirm of the latter [object] the same attribute." Also noteworthy is the great credence Maimonides gives to sense data and to the axioms of reason and deductive inferences from them (in chapter 8), saying: "all that is perceived by a healthy organ is indubitably true. And the same can be said of rational data… and their derivatives."[50]

These remarks and similar ones of his strike me as very 'modern', because in the past (and in very many cases still today) people thought of logic as essentially a deductive enterprise. Maimonides here devotes some space to the more inductive and analogical aspects of reasoning (which are also found in Aristotle, of course[51]). Yet his outlook is not quite modern, in that he does not mention the all-important proviso that a generalization is always tentative, i.e. subject to rejection or particularization if subsequent experience reveals instances to the contrary. That is to say, he makes the common error of focusing on the positive side and ignoring the negative side. The same is true, of course, of analogy – it is an inductive act, which may later be repudiated; i.e. its credibility remains dependent on further experience. Moreover, Maimonides does not mention that induction and analogy are closely related logical acts. First, as already said, in that analogy is inductive. And second, in that every generality is a statement that the individuals constituting it have some attribute in common, i.e. are analogous in some respect; this was also known to Aristotle[52].

Still, Maimonides' outlook is considerably different from that of Talmudic scholars who preceded him[53]. This is true not only in his frank acknowledgment of experience, axiom, induction and analogy, as important elements of human judgment, as already stated, but even with regard to deduction. Having been influenced by Aristotelian logic and philosophy, Maimonides' understanding of deductive reasoning was no doubt a lot more structured and rigorous. I would speculate that, even if he did not openly criticize any of the Talmudic inferential processes, he was personally aware of the tenuousness of some of the arguments used. This may perhaps explain, at least in part, his viewpoint concerning the hermeneutic rules in his *Sefer Hamitzvot* (Book of Commandments). In this important halakhic work, he adopts a more sweeping and severe position than the Talmud itself regarding the legislative effectiveness of the hermeneutic principles, including a fortiori argument (presumably, since he does not explicitly except it):

> "And now I will begin to discuss the Principles (*shorashim*, roots), totaling fourteen, to be relied upon in enumerating the *mitzvot*… The Second Principle: Do not include laws which are derived from one of the 13 principles of Torah interpretation [of Rabbi Yishmael] or from a *ribui* [an extra word, letter, etc. in a Scriptural verse]."[54]

Mielziner comments on this ruling as follows: "Maimonides holds that laws derived from the Mosaic law by means of the hermeneutic rules are, in general, not to be regarded as *biblical laws* (*min hatorah*) except when expressly characterized as such in the Talmud. But this somewhat rational view is strongly criticized by Nachmanides (in his annotations to that book) who shows that from the Talmudical standpoint every law which the Rabbis derived by the

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[50]   Pp. 64 and 69; translations from the French my own. Compare a similar statement by Saadia Gaon quoted earlier.

[51]   See for instance *Topics* 1:12: "Induction is a passage from individuals to universals."

[52]   For instance: "For there is no name common to all the objects that I mean, but, for all that, these things are all in the same class by analogy." (*Meteorology*, 4:9)

[53]   This does not mean that the Talmudic rabbis did not engage in induction and analogy. As Ventura points out (pp. 77-78), they engaged in analogy (e.g. *mah matsinu*, *hequesh*, *gezerah shavah*); and in induction (e.g. *binyan av*), more or less consciously. But in either case they do not seem to have reflected on the issue from a logician's viewpoint. This is of relevance to a fortiori argument, since it depends on induction for its premises and involves a sophisticated form of analogy. See also Saadia Gaon.

[54]   See www.shiur.org/daily/rambam-mitzvos.pdf.

authoritative interpretation from sacred Scripture, has the character and sanctity of a Mosaic Law"[55]. To quote Nachmanides: "all elucidated in the Talmud through one of the thirteen methods are words of Torah and they are the interpretation of the Torah which was told to Moshe"[56]. Be that as it may, the fact remains that for Maimonides, even if an inference a fortiori is highly deductive, it does not pass the Biblical status of its premises onto its conclusion.

Other, more recent authors concur with this assessment. Halbertal writes: "Maimonides also defines *mitsvot* of a Scriptural status in terms of their traditional pedigree and their non-controversial nature. As a result of this definition, all laws derived from the application of legal hermeneutical principles, such as *a fortiori* and analogy, are relegated to Rabbinic status, not that of Scripture. This definition is a direct result of Maimonides' theory of *mitsvot*, according to which direct linkage with Sinaitic Revelation is incompatible – at least at the level of Scripture – with controversial laws. This definition brought Maimonides into conflict with Nahmanides, who strongly criticized the Maimonidean position on this issue. Nahmanides' critique is based upon both the corpus of Talmudic law and considerations of an ideological nature." Sinclair likewise: "According to Maimonides, the status of Scriptural law (*de'oraita*) is conferred by tradition alone upon laws which are free of controversy. Any controversial law is *ipso facto* Rabbinical in nature, including a law which is derived from the Scriptural text by means of hermeneutic principles such as *a fortiori* and analogy."[57]

We can dig more deeply into Maimonides' thinking on this issue in his commentary on the Mishna, found in his introduction to *Seder Zeraim*[58]. He does believe that "all the commandments were stated with their generalities, specifics and fine details at Sinai.... [Moshe was given] the 613 precepts with their explanations; the commandments in writing, and the explanations by oral transmission." He makes a similar statement in the introduction to his *Mishneh Torah*, citing Exodus 24:12. This is the doctrine that the revelation consisted of two components, viz. a written Torah and an oral Torah, which is of course relevant to any discussion of the hermeneutic principles.

Moreover, the Rambam explains, for those, like Joshua and the Elders, who received the Torah entirely and directly from Moshe Rabbeinu, there were no doubts or disputes (*machlokot*); it is only those who came after them that had to resort to inferences (*svara*), i.e. to the thirteen *midot*. Regarding the latter, some of the inferences made convinced everyone; but in other cases, there were disagreements concerning the inferences to be made: in such cases the sages resorted to majority vote (*rov*). Thus, apparently, he believed the hermeneutic rules were given at Sinai, even while acknowledging that some disagreements arose over time concerning them too.

I have not anywhere found more specific comments by Maimonides on the individual hermeneutic principles, and in particular on *qal vachomer* and the *dayo* principle. It may, however, be that he has scattered significant remarks in his halakhic works: I do not know. So I will stop here.

9. More on medieval authors

In this section, we will examine bits and pieces of additional information drawn from various sources regarding a fortiori and other reasoning found in Jewish medieval literature.

Moise Ventura, in his very fine 1935 critical edition of Maimonides' *Terms of Logic*, which was based on thorough comparative research in numerous past editions and manuscripts, as well as various commentaries, wrote somewhat wonderingly:

> "When one browses through the Hebrew manuscripts in the great libraries, one is struck to see the considerable number of works written in the Middle Ages to abridge or comment on Aristotle's Logic.

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[55] Nachmanides, also known as Rabbi Moses ben Nachman, acronym Ramban (Spain, 1194 – Israel, 1270). Note that the Ramban was born ten years before the Rambam passed away.

[56] Cited in: www.daatemet.org.il/articles/article.cfm?article_id=8. However, the Ramban is also there quoted as saying: "know that though they [the Sages] said a man does not rule via analogies by himself, they did not mean to say that all analogies were explicated to them from Sinai and given to them from the mouth of Moshe [etc.]; this is not true, since we have found them always disagreeing in many places [etc.], and were this received tradition from Sinai [etc.], there would be no occasion for these questions and for the answers that were said in the Gemara [etc.]. But the intent of an analogy which is from Sinai is that they had received a tradition that a certain ruling is learned from an analogy, but from where exactly it is derived was not a part of the received tradition." In other words, the Ramban's opinion is not as sweeping as it first seems, at least as regards analogical reasoning.

[57] M. Halbertal, *Maimonides'* Sefer Hamizvot *and the Structure of the* Halakhah. (Heb.) Tarbiz 59 (1990), 457-480. D.B. Sinclair, *Legal Reasoning in Maimonidean Jurisprudence*. L'Eylah 29 (1990), 32-35. Both quotations found on the Internet at: www.mucjs.org/JLAS/reasoning.htm.

[58] The full Hebrew version can be read at: www.daat.ac.il/daat/mahshevt/hakdama/1-2.htm. I have found a large portion of this text in English online at: rambam.merkaz.com/Class%204%20-%20Intro%20to%20Mishnah.pdf. See also a short extract in English at:books.google.ch/books?id=OKL4bGl-S80C&pg=PA377&lpg=PA377&dq=Maimonides,+Introduction+to+Seder+Zera%27im&source=bl&ots=LgkwH1ljFv&sig=j3Za_U2oGiiUUZ6TRxIDROGruWw&hl=en&ei=vMxWTdP5FoiAOuT8zKAF&sa=X&oi=book_result&ct=result&resnum=12&ved=0CGEQ6AEwCw#v=onepage&q&f=false.

Among these writings, some are due to Moslem authors, whose works were subsequently translated from Arabic to Hebrew, and the others to Jewish authors who wrote in Hebrew on this subject. Almost all these works have remained unpublished.…" (p. 18, my translation from French).

He goes on, asking why Maimonides' work received such special attention, that it was so often translated, published and commented on. Was it his authority or the literary qualities of the work that earned it such exceptional popularity? His explanation is that Maimonides' book (written while yet in his teens) was not intended to vulgarize Aristotle's *Organon*, but to prepare the ground for his own philosophical system in the framework of Judaism, which came to maturity decades later (when he was fifty-five) in his *Guide for the Perplexed*[59]. It would, of course, be very interesting to examine all the above mentioned manuscripts and to see what is said in them, if anything, concerning a fortiori argument, and to evaluate the level of understanding of such argument exhibited in them.

In a recent but unfortunately too brief article by Aviram Ravitsky, entitled "Aristotelian Logic and Talmudic Methodology: The Commentaries On The 13 Hermeneutic Principles And Their Application Of Logic," included in Schumann's collection *Judaic Logic*[60], we are informed that there are "probably… dozens of treatises" on this subject:

"In 1917 Aaron Freimann published a bibliographic list[61] of commentaries on the thirteen principles, in which he counted over fifty different commentaries. Today, some sixty manuscripts are known to consist of commentaries on the principles (though some of them overlap)" (p. 120).

Ravitsky rightly distinguishes between "material" and "formal" commentaries. The former class, which most commentaries fall into, make use of examples drawn from the Talmud and related literature to illustrate and explain hermeneutic principles. The latter refer to Aristotelian logic and philosophy to elucidate them (the qualification of 'Aristotelian' being here broadly understood to include later developments). Ravitsky informs us, based on his careful examination of some thirty documents, that "a recognizable trend of [such more 'formal'] commentaries… began in the 14th century" (p. 117)[62].

This article is of considerable interest to us here, since a few of these commentaries (hopefully their most significant elements) are actually quoted. This gives us a chance to discover and evaluate the thinking of their authors, especially regarding a fortiori argument. The first quoted is **R. Avraham Elijah Cohen** (late 14th – early 15th centuries); referring to the argument of *qal vachomer*, he writes:

"And I contend that this would be… explained by the art of logic. […] A bull is robust compared to a donkey, and nonetheless it is not robust compared to a man; a cat is not as robust as a donkey, all the more it is not robust compared to a man." (P. 122.)

Ravitsky regards this as an "instance of formalistic commentaries" because it uses non-halakhic concepts (in this case, features of animals) instead of the legal or rabbinical content usually found in Talmudic examples. I would not however call this statement an example of formal analysis, even if it does refer abstractly to "the art of logic." But I do agree that its use of a secular illustration is significant (although, to be precise, such illustrations also do occasionally occur in the Talmud and related literature[63]), since it is indicative of recognition that the argument can be used in any context. In any event, let us examine this example in formal terms:

Donkeys (D) are less robust (R) than bulls (B).
Even so, bulls (B) are less robust (R) than men (A).
Cats (C) are less robust (R) than donkeys (D).
Therefore, cats (C) are less robust (R) than men (A).

Although some sort of a fortiori argument is explicitly *intended* here, if we label the five terms involved as shown above we see that what we are actually given is a chain of three quantitative comparisons (relative to R) resulting in a fourth: "A > B and B > D and D > C; therefore, A > C." But this cannot be considered as a fortiori argument, for the simple reason that there is no predication involved – i.e. A, which seems to play the role of subsidiary term, is not a

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[59] Ventura's idea that Maimonides' *Terms of Logic* was effectively a propaedeutic to his *Guide* is perhaps mirrored in Joseph A. Buijs essay *Maimonides' Use of Logic in the Guide to the Perplexed* (in Schumann's *Judaic Logic* collection), where the earlier logical and epistemological work is said to "infuse the development of issues in his later philosophical work." Buijs does not, however, mention Ventura's commentary.

[60] Piscataway, N.J.: Gorgias, 2010.

[61] "Die Hebräischen Kommentare zu den 13 Middot des Rabbi Ismail" in *Festschrift Adolf Schwarz*, ed. S. Krauss (Berlin and Vienna, 1917).

[62] See also, by the same author, "Talmudic Methodology and Aristotelian Logic: David ibn Bilia's Commentary on the Thirteen Hermeneutic Principles" in *Jewish Quarterly Review* - Volume 99, Number 2 (Spring 2009), pp. 184-199.

[63] E.g. *Chullin*, 60a. Not to mention the Bible, where most of the a fortiori discourse has a non-legal content. Note too that one of the first medieval commentators, Saadia Gaon (in his *Commentary on the Thirteen Midot*), explicitly teaches that *qal vachomer* may be legal or non-legal in content.

predicate of B or C (or even D). In other words, the author of this example did not (at least, not in this instance) understand a fortiori argument! (Nor, incidentally, does Ravitsky show understanding, since he does not raise the issue!)

The next author quoted is **R. Isaac Aboab of Castile** (1433-1493), a disciple of R. Isaac Canpanton (whose school, Ravitsky tells us (p. 139), was distinguished in that its interest in Aristotelian logic was not merely philosophical, but had a potential impact on halakha). R. Isaac Aboab describes "the essence of the argument" as follows:

> "A fortiori is a principle that teaches the scale of astringency from lenient to strict, and the scale of extenuation from strict to lenient." (P. 123.)

He then gives the following illustration of this argument: Even though Reuven received a scholarship, he was not given a place; therefore, Shimon, who did not get a scholarship, would "all the more" not be given a place. This author demonstrates some understanding of a fortiori argument, both in his abstract description of it and in the example he proposes for it. We can show this sample argument valid by casting it in standard (negative subjectal) form:

> Reuven (P) was given greater regard (R) than Shimon (Q) was given, since the former received a scholarship whereas the latter did not.
> Yet, Reuven (P) was not given enough regard (R) to be given a place (S).
> Therefore, all the more, Shimon (Q) will not be given enough regard (R) to be given a place (S).

I wonder whether R. Isaac Aboab was the first to express a fortiori argument in this terminology of "strict" and "lenient," which has remained the rabbinical norm to this day? This is a historical question that is worth investigating. If the answer is yes, that would make him a significant figure in the development of a fortiori logic[64]. Be that as it may, we have to note that his above quoted description of the argument is a bit vague. What does he mean by "the scale of astringency from lenient to strict" and "the scale of extenuation from strict to lenient"? All it tells us is that stringency increases as we go from lenient to strict and decreases as we go from strict to lenient. We have to refer to his example to get a better grip on what he is trying to say.

As for his example, it only illustrates the negative subjectal mood of a fortiori argument. He does not (at least, not in the segment of his discourse that Ravitsky has quoted for us) give examples of the other three (or seven) valid moods. On the other hand, Ravitsky mentions that this author noticed "the discrepancy … between the single Hebrew term of *qal vachomer* and the two forms of the application of this principle" (p. 132), as the earlier quotation ("from lenient to strict" and "from strict to lenient") makes clear. We can wonder whether Isaac Aboab might not be the first Jew to have noticed this difference of direction. Nevertheless, it is not clear whether he identified it as one between positive and negative subjectal moods, or as one between positive subjectal and positive predicatal moods; I do not suppose he did either.

Moreover, although Ravitsky classifies this effort as "formal" analysis, and there is indeed some formalism in it insofar as abstract terms like "strict" and "lenient" are used, it is strictly-speaking not very formal. Aristotle's theory of syllogism may be characterized as formal in that he used abstract symbols like A, B, Γ (or labels like "the minor," "the middle," and "the major," or ordinal numbers) instead of concrete terms, and because he systematically developed all possible figures and moods and determined with reference to the laws of thought which are valid or invalid. R. Isaac Aboab, on the other hand, is still apparently stuck in the realm of sample concrete labels like "Reuven" and "Shimon," and makes no attempt at systematization or validation. This is, admittedly, closer to formal than the earlier Talmudic total absorption in concrete cases; but it is not yet quite formal.

Regarding the issue of validity, Ravitsky quotes the unknown author of *Sharei Tsedek* (apparently in Spain, ca. late 14th – early 15th centuries)[65]:

> "The reason [R. Ishmael] began with this principle [i.e. *qal vachomer*] is that it features self-explanatory truth more than the other [hermeneutic] principles, alike the first figure of logical syllogism that is more self-evident than the rest of the figures." (P. 124.)

What this author seems to be saying is that a fortiori argument is (at least, comparatively to the other hermeneutic principles) self-evident, just as first figure syllogism is (compared to the other figures) an irreducible primary. But the analogy in fact stopped there. He was certainly not claiming, as Ravitsky seems to suggest, that by placing it in first position in his list of thirteen principles R. Ishmael was implying that the other hermeneutic principles can be reduced to *qal vachomer*, just as the other figures of syllogism can be verified by means of the first. In truth, as I have shown in my book *Judaic Logic*, a fortiori argument is not an irreducible primary, but is reducible to simpler

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[64]   Of course, this is assuming these English words reflect similar ones in Hebrew, and are not mere interpolations by the translator.
[65]   Whose identity is uncertain according to Ravitsky, though some have identified him with Gersonides. He refers us to his essay "On the Date of Sha'are Sedek, attributed to Gersonides" (in Hebrew), *Daat*, 63 (2008), pp. 87-102.

forms of argument, including hypothetical syllogisms and quantity comparisons; as for the other hermeneutic principles, see my comments there.

Ravitsky goes on to quote other medieval authors regarding other hermeneutic principles: Moses of Narbonne, who equated *gezerah shavah* to analogical inference; R. Avraham Elijah Cohen, who analyzed *mah matsinu* in terms of the distinctive properties of subjects; R. David Ibn Bilia, who analyzed *klal uphrat* using the terminology of genus and species. We need not in the present context discuss these issues. I only wish to remark in passing that I agree with Ravitsky that the influence of Aristotelian logic (and more broadly, philosophy) is evident in all these cases.

Two other authors are quoted by Ravitsky on the subject of a fortiori argument. One is **R. Immanuel ben Isaac Aboab** (ca. 1555-1628), a great-grandson of the earlier quoted Isaac Aboab of Castille. He explained as it follows: "The initial principle is *qal vachomer*. Meaning, the Torah is expounded by the element and manner that lead from lenient to strict, and is what the logicians refer to as: Argumentum a minori ad maius, vel a fortiori" (pp. 131-2). The other is the much latter **Isaac Samuel Reggio** (1784-1855), who says essentially the same thing, viz. that the rabbinical hermeneutic principles are mostly "based on the rules of the art of logic. E.g. the first principle, named *qal vachomer* is extremely fluent amongst the scholars of the art of logic under the title of 'Argumentatio a minori ad majus'…" (p. 131). Note that both these definitions focus solely on minor to major reasoning, ignoring major to minor, and making no distinction between positive and negative, subjectal and predicatal, forms.

These two authors are quoted in support of the notion that the hermeneutic principles and the art of logic are, on the whole, in agreement. Another, much earlier author, R. Hillel ben Samuel of Verona (ca. 1220 – ca. 1295), went so far in this optimistic vein as to declare sweepingly: "the Sages of the Talmud established all of their scrutinies (*sic*) on the methods of syllogism and demonstration" (p. 127)[66]. Naturally, some rabbinical authorities expressed their disagreement with such naïve statements. For instance, Isaac ben Joseph Ibn Polgar (ca. 14th century), who argued: "when they begin to study logic, foolery and error enter their minds, for they think that the conditions of syllogism are necessary in legal-religious matters [… whereas] our sacred Torah is expounded by the thirteen principles alone" (p. 129).

Having studied the issues involved in great detail in my earlier work *Judaic Logic*, I would place myself somewhere in between these various opinions; I will not go into detail here, but merely repeat some conclusions. The rabbinical hermeneutic principles are variously logical: some are quite logical (notably *qal vachomer*), some are more or less so, some are not logical (*non sequiturs*), and some are antithetical to logic (antinomies). Thus, it is inaccurate to regard them as either all logical or all illogical. In my view, they all ought to have been logical; logic is not something one can discard at will.

The claim that the hermeneutic principles were originally a secret code applicable only to Torah interpretation may seem conceivable *prima facie*; but once one considers it seriously, it is seen to be difficult to uphold. Briefly put: for a start, since this code is not given in the written Torah, to claim it as given in the oral Torah as a tool for the justification of the oral Torah is a circular argument. Secondly, one can imagine a secret code as being necessary, assuming that God wanted only some people (namely the Jewish people, or perhaps more specifically the rabbis) to truly understand the Torah; but once this code is no longer secret, the past argument in its favor falls apart.

Mielziner, who I quoted on this topic in an earlier section (9.1), rightly identifies the hermeneutic principles as developed ad hoc by the rabbis over time, as means by which traditional laws existing and developed from pre-Mishnaic times to post-Talmudic times, could be anchored – by hook or by crook, if I may so put it – to the written Torah. I suggested much the same in my own study, *Judaic Logic*. Ravitsky, I think, shows the same awareness when he defines them as "basic and fundamental rules by which the oral tradition is related to the Scriptures" (note the guardedly vague term 'related' he uses, p. 117).

It is of course not possible with so limited a sample to describe and evaluate medieval attempts to relate the rabbinical hermeneutic principles, and in particular the first of these, viz. *qal vachomer*, to Aristotelian logic. We cannot even be sure that Ravitsky, on whose brief study we have heavily relied in the present section, selected and quoted the most significant authors and works. We have seen that this commentator did not notice certain weaknesses in logic in his selections, notably R. Avraham Elijah Cohen's confusion between simple quantitative comparisons and a fortiori argument. Moreover, we saw that Ravitsky was too quick to acknowledge as 'formal' arguments that were still, strictly speaking, material. Furthermore, his uncritical acceptance of Saul Lieberman's claims in "Rabbinic Interpretation of Scripture" makes me doubt his judgment. So we cannot take for granted that he acquitted his set task in a fully representative manner.[67]

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[66]   Similarly optimistic statements by R. Avraham Shalom (15th century) and R. Elijah Galipapa (18th century) are quoted by Ravitsky.

[67]   Regarding Saul Lieberman, see my comments in a later chapter (15). Another index that makes me wary of Ravitsky's reliability is his failure to take account of my work *Judaic Logic* when discussing 'modern research.' He mentions this book in passing twice, on issues of minor import; the first time, regarding the expression '*qal vachomer*,' and the second, only to imply that, like Jacobs, I perceived in *binyan av* reasoning a type of induction close to J.S. Mill's Method of Agreement. All the original work in *Judaic Logic*, such as the formalization of *qal vachomer* and

Much more detailed studies would be needed to arrive at some solid historical conclusions. Nevertheless, based on the data we have at hand, we can tentatively propose the following conclusions. Medieval Jewish commentators wished to correlate (at least some of) the rabbinical hermeneutic principles with Aristotelian logic, out of a desire to reconcile the philosophy and science of their day with the worldview and claims of the Torah and subsequent Judaic tradition. Some of these commentators were themselves rabbis, some were lay philosophers. They were on the whole not critical, in the modern sense; rather, they had faith that the two fields of human endeavor could indeed be reconciled. They remained in the mainstream of Rabbinism, although presumably some passed over to Karaism.

Logic was regarded by many as a neutral discipline, without conceivable negative impact on religion. R. Jedaiah ben Abraham Bedersi Ha-Penini (ca. 1270 – ca. 1340), for instance, wrote "this art [i.e. logic] is comprised of knowledge or views that would result in neither harm nor benefit to faith" (p. 135)[68]. Some rabbis, on the contrary, realized the dangers posed by logic for the Judaic viewpoint. As Ravitsky points out, "they cast restrictions on the study of logic or even opposed it;" some of them could well see that logic is "a discipline that educates for rational criticism, or even animadversion, of the type that would make it difficult to accept religious truths" (p. 134). Nowadays, no one can contest that the study of logic has both a positive and a negative impact on religious belief; mostly, perhaps, the latter. Ravitsky clearly agrees when he concludes: "Attempts to reconcile [the two are] farfetched and artificial."

From a logician's perspective (as far as I can see so far), these various medieval commentators cannot be claimed to have entirely succeeded in their endeavor to correlate hermeneutics and logic, because: (a) their approach was not formal enough; (b) they were not sufficiently systematic; and (c) they did not make the required efforts of validation. Aristotle and his successors had given them examples of formalism, systematic treatment and validation, in relation to the syllogism and other forms of argument; but they had not done the same job in relation to a fortiori argument or the logic of causation. So the later commentators were not able to draw on such past work. Of course, many of the hermeneutic principles could be explicated somewhat in non-formal terms. For instance, rules like *gezerah shavah* or *klal uphrat* could be adequately discussed informally. But some, such as *qal vachomer* and *binyan av*, to name but two, could only be dealt with credibly by formal means. These means were, in fact, largely available in the epoch under study; but apparently none of the medieval commentators surveyed had the logical competence needed to apply them.

10.   Moshe Chaim Luzzatto

Although the Ramchal deserves in many ways to be classed as a modern author, I have put him here so as to count him among the post-Talmudic Jewish logicians[69]. Surprisingly, this important author is not even mentioned in many standard studies of Talmudic logic, such as Mielziner's; somehow, and quite unfairly, he has passed unnoticed. Ravitsky, likewise, does not mention him.

Formulation. R. Moshe Chaim Luzzatto (Italy-Netherlands-Israel, 1707-1746), also known in Jewish literature by his acronym "Ramchal," wrote two books on logic, namely *Sepher haHigayon* (The Book of Logic, 1741) and *Derech Tevunot*[70] (The Way of Understanding, 1742); he also wrote a couple of books on grammar which may have some logical significance, though I have not read them. Concerning *Derech Tevunot*, which is more intended as a teaching of Talmudic reasoning than of logic in general, I wrote the following in my *Judaic Logic* review of it (or more precisely, of a 1989 translation of it, called *The Ways of Reason*[71]):

> As well, he mentions *a-fortiori* argument, in the form: X1 is greater than X2, and X2 is Y, therefore X1 is Y; we may notice, however … that the middle term which explains and justifies the process, being *the respect* in which X1 and X2 are compared, is lacking, and also that he is not apparently aware of the formal varieties of the argument (but the form of his argument is correct, as a positive subjectal).[72]

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of the *midot* used for harmonization, is not even mentioned, let alone taken into consideration. This suggests to me that he did not take the trouble to study this important book, but only mentioned it to 'pad' his references. No wonder he can say in his abstract: "To date, the application of logic to the realm of the 13 principles has not received proper attention in the research literature." This is of course factually inaccurate and only stated so as to amplify the importance of his paper. Nevertheless, his paper is informative and thoughtful.

[68]   Ravitsky also quotes R. Joseph Ibn Caspi and the Moslem philosopher Al-Ghazali to the same effect. He also mentions Maimonides, R. Abraham Ibn Izra, and others.

[69]   In this regard, it is interesting to quote Louis Jacobs in his *Religion and the Individual* (Cambridge: UP, 1992), p. 101: "Although Luzzatto lived in the eighteenth century, the historian Zunz rightly remarked that the Jewish middle ages lasted until the end of the eighteenth century."

[70]   An image in pdf of this book in Hebrew may be viewed/downloaded at: hebrewbooks.org/19760.

[71]   Trans. Rabbis D. Sackton and Ch. Tscholkowski. Jerusalem: Feldheim, 1989.

[72]   Note that all symbols introduced here [viz. X1, X2, and Y] are my own. N.B. I do not have my copy of the book on hand, and therefore cannot quote exactly what is said in it, as I would have preferred to today. I assume my past summary was accurate, although it is possible that today I would see things differently.

But at the time I wrote that comment, I had not seen *Sepher haHigayon* (i.e. the English translation of it, called *The Book of Logic*)[73], for the simple reason that it was first published in 1995, the same year my said book was first published. About this work by Ramchal much needs be said, but what will be said here is only what it says about the a fortiori argument (in chapter 14). I have to admit that R. Luzzatto's understanding of a fortiori argument is surprisingly original and advanced[74]. On second thoughts, we should perhaps not be surprised; the mid-18th century is after all not so long ago, and writers of that period are normally counted as 'early modern'.

> "**Quantified commensurates** [are terms that] **share a common quality, but not in the same degree**. One exhibits a Greater degree and the other a Lesser degree of the same quality. Rules of Greater Degree: 1. To whichever subject the greater is predicated, the lesser will also be predicated… 2. What cannot be predicated to the greater term cannot be predicated be predicated to the lesser… Rules of Lesser Degree: 1. To whichever subject the lesser is not predicated, the greater will not be predicated either. 2. Whatever is affirmed about the lesser will surely be affirmed about the greater." (Pp. 89-90.)[75]

Let us examine these four "rules," and see to which of the standard models of a fortiori argument they respectively correspond. The middle term (R) of each argument is left tacit in these rules, but may be identified with the "common quality shared in different degrees" referred to in the definition. The major (greater), minor (lesser) and subsidiary terms (P, Q, S) are noted symbolically (as P, Q, and S, respectively) by me in each rule. I give the Hebrew original so everyone can verify the accuracy of the translation:

- מי שיפל בו היתר, יפל בו הפחות. "**To whichever subject** (S) **the greater** (P) **is predicated, the lesser** (Q) **will also be predicated**." This, being major to minor and positive, refers to positive predicatal argument; note that P and Q are predicates.
- מה שלא יפל ביתר, לא יפל בפחות. "**What** (S) **cannot be predicated to the greater term** (P) **cannot be predicated to the lesser** (Q)[76]." This, being major to minor and negative, refers to negative subjectal argument; note that P and Q are subjects.
- מי שלא יפל בו פחות, לא יפל בו יתר. "**To whichever subject** (S) **the lesser** (Q) **is not predicated, the greater** (P) **will not be predicated either**." This, being minor to major and negative, refers to negative predicatal argument; note that P and Q are predicates.
- מי שמחיב בפחות, כל שכן ביתר. "**Whatever** (S) **is affirmed about the lesser** (Q) **will surely be affirmed about the greater** (P)." This, being minor to major and positive, refers to positive subjectal argument; note that P and Q are subjects.

Amazing! This is the first time I see *all four* moods of (copulative) a fortiori argument listed by anyone before me. They are classed in the following order: first the two major-to-minor moods, the positive and the negative; then the two minor-to-major moods, the positive and the negative. For this reason, their order of presentation seems odd by my standards: positive predicatal, negative subjectal, negative predicatal, positive subjectal. But that, of course, is an unimportant issue – the fact remains all four forms are clearly there.

Since these definitions are explicitly built around a "common quality shared with varying degrees," we can say that[77] they do include the middle term (R). However, what is manifestly lacking in them is the notion of a threshold of R that any subject must cross before it gets the predicate. Yet, this is an essential feature of a fortiori that anyone must acknowledge who claims to understand the argument. We can therefore say without any exaggeration that R. Luzzatto correctly formulated the four moods of a fortiori argument, in the sense of perceiving their two possible orientations (subjectal and predicatal) and two possible polarities (positive and negative) some 250 years before I did. As far as I know, he was the first to do this important work (in or before 1741, presumably while a resident of Amsterdam). However, although his formulation does mention the middle term, it is still incomplete since it does not mention the crucial issue of the sufficiency (or insufficiency) of that term, without which the argument cannot be validated. Therefore, while Ramchal should be regarded as an important contributor to a fortiori logic, he cannot fairly be said to have been the first to formalize the argument correctly.

Assessment. That these four moods are not listed by him using symbols instead of terms (like my P, Q, R, S) is not important. Nor is it important that he does not devise descriptive names for the arguments (positive/negative,

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[73]   Trans. Rabbis D. Sackton and Ch. Tscholkowski. Jerusalem: Feldheim, 1995.

[74]   I wondered at first if the translators had, perhaps unwittingly, infused their own relatively modern ideas into the original text – because it seems so modern! But the original Hebrew is shown and it is evident that the translation is correct. With regard to their translation of *Derech Tevunot*, I had in my *Judaic Logic* expressed strong disappointment – not because I doubted that they rendered the Ramchal's words accurately, but because I felt that the English wording they used for various items and processes was not in accord with more familiar works and therefore could be misleading.

[75]   Bold fonts used by the translators omitted by me.

[76]   בשמירת היחס "…provided that the relationship of greater and lesser is maintained in regard to that predicate."

[77]   Contrary to what I say in my *Judaic Logic*, with reference to his treatment in *Derech Tevunot* (above quoted).

subjectal/predicatal). What we have here is still a considerable measure of formalization in the strict sense of the term, since abstract concepts are used instead of concrete examples. Expressions like "what," "whatever," "whichever subject," "predicated," "affirmed," "the greater (term)," "the lesser" are all equivalent to use of symbols – they serve the same function of theoretical generalities allowing for any specific values that may occur in practice.

Note, too, that his definitions are not made in narrowly legal terms, but in terms adaptable to any subject matter. We can also say that he was clearly aware of the middle term underlying the major and minor terms, which puts him ahead of many past and present logicians. So, R. Luzzatto may *almost* be said to have been the first to formalize a fortiori argument, or at least its four forms (the primary copulatives). I say 'almost' – because some criticism of his presentation is possible and necessary. There are a number of significant deficiencies in it, from a formal logician's point of view.

Firstly, the preamble, "Quantified commensurates share a common quality, but not in the same degree" (הם שחלוקים בכמות איכותם), is actually the unstated *major premise* of all four "rules," telling us that the major and minor terms P and Q share a common quality R to different degrees. This premise should not be detached from the four arguments, because it is an integral part of each of them, making the inference possible. It should be repeated every time it is relied on.

Secondly, the *middle term* R should also be explicitly mentioned in the minor premises and conclusions, whereas it is left tacit in them. It is not enough to state there that P and Q are greater or lesser, with implicit reference to the major premise. The term R mentioned in the major premise must be repeated in the minor premise and conclusion, to ensure that it is with respect to that exact same term that they are intended; otherwise, we risk committing the fallacy of two middle terms. If the middle term is not mentioned in all three propositions, it is not fulfilling its role of intermediary, which explains why the putative conclusion follows the given premises.

Thirdly, and most importantly, note again the absence of the explanatory concept of *sufficiency* (or insufficiency) in the minor premises and conclusions, i.e. the awareness that there is in each case a threshold value of R as of which S is applicable (or not). Without this subtle feature, we have no explanation for the link between the subject and predicate in the minor premise, and therefore no explanation for our claiming the same link in the conclusion. The middle term, and its being present enough or not enough, are essential details to succeed in validating the argument. The minor premise *must* specify these details, even if the conclusion is stated without them.

Even so, it would not be fair to say that Ramchal confused a fortiori argument with argument by analogy. If he had done so, he would have allowed for inference in positive subjectal form from major to minor and in positive predicatal form from minor to major. The fact that he did *not* count such reasoning (and its negative corollaries) as valid shows that he was referring to a fortiori reasoning rather than to qualitative[78] analogy. For whereas the analogical argument is non-directional, able to function indifferently in either direction, a fortiori argument is distinctively directional.

In sum, although we can rightly attribute the formulation and listing of all four moods of (primary copulative) a fortiori argument to R. Luzzatto – assuming no one preceded him in this feat that I do not know about – we cannot say that he succeeded in *fully formalizing* these arguments. A little bit more work was needed to thoroughly define each unit of reasoning he listed, in a way that made their validations possible. The deficient way he has formulated the arguments (i.e. without mention of sufficiency or insufficiency of the middle term in the minor premise) makes their putative conclusions invalid – i.e. the contradictories of these conclusions *remain logically possible*.

We should also note that he does not actually analyze the four moods he has listed, and distinguish between those in which P and Q are subjects and those in which they are predicates. We can suppose that he was aware of the differences between major-to-minor and minor-to-major moods, and between positive and negative moods, because of the way he has ordered the material. But he does not seem to have clearly noticed the also important structural difference between subjectal and predicatal. Still, what he did achieve should not be belittled. It deserves high praise. This historical finding is a quite unexpected and somewhat humbling for me. Although I independently formulated these four principal moods back in 1995 (no doubt some time before), I must now admit they were already roughly known. But I can still claim as original, their more precise formulation and analysis, as above detailed. I can also claim the discovery of the corresponding implicational forms and various derivative secondary forms.

Moreover, fourthly, as far as I know, R. Luzzatto made no effort of *validation*, but accepted the reasoning involved in his four "rules" as self-evident. But of course, that won't do – logicians have to justify all arguments they acknowledge in appropriately detailed and convincing ways[79]. So, I can claim this important achievement, having

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[78] The analogy would be qualitative rather than quantitative (pro rata), since in each of Ramchal's four moods the subsidiary term S remains constant, i.e. the same in the minor premise and conclusion. As pointed out further on, Ramchal does not show awareness of a crescendo argument, or even (to my knowledge) of quantitative (pro rata) analogy.

[79] In their Foreword (p. xxi), the translators state: "He goes beyond the logical investigation of validity and nonvalidity to find the means of evaluating what is true and what is false. In his treatment of syllogisms, the Ramchal passes over how given premises make a conclusion

formally demonstrated that a fortiori arguments can be reduced to more familiar and proven arguments. And that was of course made possible by my more precise formulations and analyses.

Needless to say, the said four deficiencies in R. Luzzatto's treatment of a fortiori argument are not a reflection on his intellectual capacities. It is evident that he could easily have further developed his study of the subject in the stated directions had he wished to. Obviously, he was a logician more concerned with teaching practical logic than in researching theoretical issues. Of course, deeper theoretical analysis does improve practice, but it is a fact that most people do not feel the need to go that far.

An insight by R. Luzzatto also worth noting is the following:

> "In addition, it is necessary to distinguish between quantified commensurate terms which are greater or lesser on the one hand, and more or less likely, on the other. For when a certain quality is exhibited to a greater degree, it is not, therefore, more likely to occur; in fact, it is often less likely." (P. 90.)

Here again, we see the lucidity of the man. Many people, from Aristotle's time to the present day, have made the mistake, when discussing a fortiori argument, of confusing *ontical* differences in degree between the major and minor terms in relation to an underlying middle term with *epistemic* differences in degree, i.e. with degrees of likelihood. R. Luzzatto is evidently aware of the alternative possibilities involved, since he stresses that the major term may in fact (in some cases) be less likely than the minor term. Clearly, this is an author whose work on logic deserves careful reading or rereading.

Nevertheless, it must be pointed out that the Ramchal's above listed four arguments are all purely a fortiori; he does not like many logicians before and after him attempt to draw 'proportional' conclusions. This is in one sense to his credit, in that purely a fortiori argument is the essence of a fortiori. But in another sense this is a deficiency, in that a crescendo argument is also valuable, provided we understand that it involves an additional premise about 'proportionality'.

The Ramchal's non-mention of a crescendo argument – and for that matter of the *dayo* principle – is surprising, considering the large role such argument plays in the Talmud, and in particular in Baba Qama 25a. I do not know whether he has anywhere written any comments regarding Talmudic a fortiori argument. If he did, it would be very interesting to know what he said, in view of his above-average clarity of insight and logical skill.

11.   More research is needed

What we have found so far in the preceding pages concerning the views of early Jewish commentators on a fortiori argument may look a bit slim. The truth is that my linguistic skills are insufficient to do a much more thorough job than that on them. Someone with better Hebrew and Aramaic than mine will have to look into this matter more fully, carefully examining any relevant comments in the literature, and publish a new work on the subject.

This of course means, for a start, examination of the later commentaries placed all around the Mishna and Gemara in current editions of the Talmud; but all other possible sources must also be investigated. A thorough, chronologically-ordered *bibliography* on Talmudic logic, hermeneutics and methodology needs, perhaps, to be drawn up for this purpose. This can be done using material from various sources. Possible starting points: the Jewish Encyclopedia article on Talmudic hermeneutics[80]; the 'Logic and methodology' section of the Wikipedia article on Talmud[81]; M. Mielziner's *Introduction to the Talmud*, pp. 83, 96, 128-9; and others. These various lists are doubtless far from exhaustive[82]. The works they include seem offhand to relate to logic and the rabbinical hermeneutic rules in general, and thence to the subject-matter of *qal vachomer* in particular. But some or even many of them might have no distinctive additional information or even no relevant information at all.

The reason I mention these lists, and the potentially relevant authors and works in them, is in order to stress that the present work is far from complete: it is probably at best sketchy. The work of history and evaluation that needs to be systematically done is yet to be done. We need to collect all the relevant information in those and similar works before we can hope to write a thorough history of the subject and to more precisely trace the evolution in understanding of *qal vachomer* and other arguments among Jewish logicians and commentators. I of course should

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logically necessary." But of course, there is no exemption from the obligation to demonstrate validity – the "beyond" they claim for Ramchal is a cop-out.

[80]   Online at www.jewishencyclopedia.com/view.jsp?artid=34&letter=T&search=Talmudical%20Hermeneutics. The list there is reproduced in the corresponding Wikipedia article: en.wikipedia.org/wiki/Talmudical_hermeneutics.

[81]   See: en.wikipedia.org/wiki/Talmud.

[82]   My passing on of this bibliographic material should not be construed as an attempt on my part to appear more "learned" than I really am. The material I list here is, for the most part, material I am ignorant of. Had I consulted it, I would have treated in within my book. The reason I include it here is merely to give other researchers a bit of a starting point for further investigation.

have – and would dearly have loved to – study all the works listed there, but I have so far not found many of them in English (or French) translation: most are, of course, in Hebrew.

I could only here, for now at least, do some of the work – whatever was within my linguistic purview. Ideally, the books listed in the eventual bibliography should all be translated into English by someone so as to be available for scrutiny by international scholars like me who do not necessarily master Hebrew. At least the segments relevant to our study should be translated; that is, those to do with the hermeneutic principles and practices of the rabbis, and in particular those to do with the a fortiori argument. The ideal would be to create a freely accessible '.org' website in which such works would be collected and their translations posted for all to see and study.

In truth, the probability is high that most of the books listed – especially those by medieval rabbis – simply repeat the same old platitudes about the *qal vachomer* argument, the *dayo* principle, and other hermeneutic principles. The reason for this is simple – the obligation of rabbis to conform to orthodox standards in order to be accepted by their peers. There are, to be sure, sometimes disagreements and even very passionate disputes among them. But these are all probably within certain bounds, for otherwise they would not be considered as kosher and perpetuated and read.

We must, however, remain open to the possibility of the unexpected: the Ramchal's seemingly original discovery of the four moods of a fortiori argument is a felicitous case in point.

10. A fortiori in the Christian Bible

In this chapter, I am called upon for the sake of comprehensiveness to comment on some of the a fortiori discourse found in Christian literature, especially the Gospels. I must stress that I do not intend the following treatment to be exhaustive. I am merely breaking ground for a more extensive treatment by others. Being personally not very interested in the Christian religion, I am not sufficiently motivated to do a thorough job on the subject. I do hope someone else will take up the challenge and do the necessary research.

Needless to say, although I am a Jew, I have no desire to engage here in religious polemics against Christianity. Jews do not normally try to convert non-Jews to their views. My interest here is entirely logical. The proof is that I am not always critical. When I am critical, it is with an impartial, scientific spirit – the same spirit I apply to assessment of a fortiori and other forms of reasoning found in Jewish texts or texts of other traditions.

1. In the Christian Bible

Using a Kindle edition of the Christian Bible[1], the *Revised King James New Testament* edited by Brad Haugaard (2008), I searched mechanically for the main key words and phrases of a fortiori argument and found 28 instances[2]. These were: *How much more* (17) in Matthew 7:11, 10:25, 12:11-12; Luke 11:13, 12:24, 12:28; Romans 5:8-9, 5:10, 5:15, 11:12 (2 instances), 11:24; 1 Corinthians 6:3; 2 Corinthians 3:11; Philemon 1:15-17; Hebrews 9:13-14, 10:28-29. Other *much more* (4) in Matthew 6:26, 6:30; Romans 5:17; Hebrews 12:9. *Much less* (1) in Hebrews 12:25. *Even more* (1) in 2 Corinthians 3:7-8. No distinctive wording (5) in Luke 13:15-16, 16:11, Romans 11:15, 2 Corinthians 3:9, 1 John 4:20 (the connectives used in these five cases were, respectively: *ought not*, *if not – who*, *if – what, but, far more* and *how*).

This list may, of course, not be exhaustive, since a fortiori argument is not always distinctively worded. Key words or phrases for which no hits were registered are not here mentioned, for brevity's sake. Sometimes, words or phrases that usually signal a fortiori argument turn out not to do so, their intent being merely to express increasing magnitude; this occurred, for example, with the 2 instances of *so much the more* and all 10 instances of *all the more* (their intent here is simply *a lot more*). I did not bother to look at residual hits of *all the*, *more*, *less*, and *therefore*, although there were over a hundred hits for each of these search strings; so I may have missed a few cases.

As can be seen, the 28 instances of a fortiori argument were found in the following 8 books: Matthew (5), Luke (5), Romans (8), 1 Corinthians (1), 2 Corinthians (3), Philemon (1), Hebrews (4) and 1 John (1). Note that the epistles to the Romans, 1 & 2 Corinthians and Philemon (13 inst.) were written by Paul; the author of the epistles to the Hebrews is unknown[3]. No cases were found in the other 19 books, namely: Mark, John, Acts, Galatians, Ephesians, Philippians, Colossians, 1 Thessalonians, 2 Thessalonians, 1 Timothy, 2 Timothy, Titus, James, 1 Peter, 2 Peter, 2 John, 3 John, Jude, and Revelation.

No one has to my knowledge spotted all 28 a fortiori arguments in NT here listed. As we shall see further on, Neusner discusses one example in some detail, namely Matthew 12:11-12. Jacobs mentions three examples, namely: Matthew 12:11-12, Luke 13:15 and Romans 5:10[4]. As we shall see further on, Maccoby cites four examples, all from Romans, namely: 5:10, 5:17, 11:15 and 11:24. Abraham, Gabbay and Schild give the same three examples as Jacobs. I did not discover Luke 13:15 and Romans 11:15 through mechanical search, but only thanks to Jacobs and Maccoby

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[1]   The books constituting the Christian Bible are referred to by Christians as the "New Testament" (abbr. NT), while those constituting the Jewish Bible as the "Old Testament" (abbr. OT).

[2]   Please note that I have more recently come across 8 additional instances. These I report in a separate section of the present chapter, further on. The total is now therefore 36.

[3]   Paul was in the past assumed by some to have also authored the epistles to the Hebrews, but this assumption was not universally accepted; nowadays, most experts reject it.

[4]   In *Rabbinic Thought in the Talmud*, pp. 113-114. Note that Jacobs also cites three examples from the Biblical Apocrypha: Ecclesiasticus 10:31, 14:5, and Wisdom of Solomon 13:3. I have not searched through the Apocrypha, or for that matter the Pseudepigrapha, for a fortiori arguments, but clearly this is a job worth doing and likely to reap a rich and interesting harvest. Even if such literature is non-canonical, it is historically significant.

mentioning them. I discovered 1 John 4:20 thanks to H.W.B Joseph[5]. I came across Luke 16:10 and 2 Corinthians 3:9 more or less by chance, due to their proximity to other cases found mechanically.

Out of the 28 instances of a fortiori discourse found, 18 are positive subjectal and 6 are positive antecedental in form; 2 are positive predicatal and 2 are negative predicatal; the other standard forms are not used. Thus, we can safely say that the overwhelming majority of a fortiori arguments in NT are of the most obvious type (positive subjectal or antecedental); however, it is interesting that four of them are more complicated (predicatal). Of the arguments, 17 are purely a fortiori and 9 are clearly or probably intended as a crescendo (i.e. proportional)[6], while the remaining two can safely be classed as logically invalid. One of the a crescendo arguments[7] may be said to breach the rabbinic rule of *dayo* against inferring a stronger penalty from a lesser penalty given in the Torah.

Needless to say, to acknowledge an argument as 'valid' in form is not necessarily to agree with its content. An argument is valid if its premises, be they true or false, do indeed logically imply its conclusion, be it true or false; and argument is invalid it its premises and conclusion are not related in this way. A false premise may, in a valid argument, imply a true conclusion, i.e. a proposition whose truth can be established by other means; and true premises may, in an invalid argument, wrongly seem to imply a false conclusion. Thus, we may well accept an argument as valid without accepting its premises and conclusion, or as invalid without rejecting its premises and conclusion.

Moreover, it should be said that the NT arguments listed above are all in rather abridged form. They do not consciously lay out all the premises involved and all their terms, including all material necessary to draw the putative conclusion. In all cases, the major premise is left tacit, though the middle term is sometimes explicitly stated. This is excusable in most cases, insofar as the minor and major terms are readily apparent, so that given or injecting a middle term, the major premise can in fact be reconstructed; however, in certain cases, some effort and ingenuity may be needed for such reconstruction. In all cases, most of the minor premise and conclusion are explicit; but usually, if not always, the minor premise lacks the necessary information that the subject has *enough of* the middle term to gain access to the predicate. Strictly speaking, without this crucial factor of 'sufficiency', the conclusion cannot logically be drawn.[8]

Furthermore, in cases of a crescendo argument, i.e. where the subsidiary term is not identical in the minor premise and conclusion, but varies 'proportionately' to the values of the middle term in relation to the major and minor terms, the NT does not explicitly specify the third premise, the premise about proportionality (pro rata variation), which is logically needed to justify the conclusion. We must take it for granted that the speakers subconsciously had the necessary information in mind when they formulated their conclusions. That these various details are missing from their discourse of course indicates that they did not have full awareness of the formal conditions of a fortiori argument or a crescendo argument. Nevertheless, we are not here to test their abstract knowledge of logic, but are satisfied with making reasonable demands on their actual thinking processes.

Thus, we can be generous and say: if the a fortiori or a crescendo intent of an argument is obvious enough, and the argument can, even though some required elements of it are missing, potentially be fitted into a credible template, i.e. one of the standard forms validated by logic theory, then we may accept it as a 'valid' argument for our purposes here. This remains true, even if due to rhetorical flourishes the terms used in an argument vary somewhat (but of course, not if they vary too wildly). We are not interested in a nitpicking evaluation of abstract knowledge, but in a fair assessment of practical knowhow. Of course, we must remain attentive to detail and make sure the argument is not invalid. With these considerations in mind, 26 of the arguments found in NT have been assessed as formally valid, although 2 were declared invalid.

The 10 arguments in Matthew and Luke are attributed to Jesus, the initiator of Christianity, who is regarded by Christians as the son of God. Of these arguments, 9 are positive subjectal in form, including 1 a crescendo; and 1 is negative predicatal. The language used by Jesus (in the translation here used, at least) is: *how much more* (6), simply *much more* (2), and non-distinctive (2). All 10 arguments can be considered as formally valid[9].

It should be noted that 4 of the arguments in Luke correspond to 4 in Matthew, so that the net number by Jesus is 6 (rather than 10). That is, Luke 11:13, 12:24, 12:28 and 13:15-16 correspond to Matthew 7:11, 6:26, 6:30 and 12:11-12, respectively. It is interesting to note that the wording in these pairs of equivalents is not identical, implying one or

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[5]   See the section devoted to this author later in the present volume.

[6]   Namely: Matthew 12:11-12; Luke 13:15-16; Romans 5:10, 11:15, 2 Corinthians 3:7-8, 3:9, 3:10-11; Hebrews 9:13-14, 10:28-29. For example, in Luke 13:15-16 the increase is from 'untying an animal from its stall so as to allow it to drink water' to 'cutting loose a woman from a demonic bond so as to heal her'; these are obviously two degrees of 'setting free and relieving'.

[7]   Namely, Hebrews 10:28-29.

[8]   Though all these remarks are made with regard to 'copulative' a fortiori arguments, similar ones can be made with regard to 'implicational' arguments.

[9]   However, before surmising that Jesus is infallible, see the section on Additional findings (10.5), where it is shown that Jesus commits one error of reasoning (in Luke 16:10).

both of them not to be verbatim accounts. The variations may be indicative of poetic license or of interpolation of interpretations, but they are anyway relevant to historical studies. The gospel of Matthew is considered as earlier than that of Luke. Matthew was a Jew living in Israel in Jesus' day, and may have been an eyewitness to many or most events he recounts; whereas Luke (ca. 1-84 CE) was a Greco-Syrian from Antioch, who did not witness what he reports. Modern scholars suggest both writers based their books partly on the book of Mark and partly on a hypothetical Q document that no longer exists.

The argument in 1 John 4:20 is by John himself; i.e. he is not quoting anyone else. Concerning Paul, as already said 13 arguments are attributed to him, as the author of Romans, 1 & 2 Corinthians and Philemon. Of these arguments, 6 are positive subjectal in form, including 4 a crescendo; 5 are positive antecedental, including 1 a crescendo; and 2 are positive predicatal. The language used by Paul (in the translation used, at least) is: *how much more* (9), simply *much more* (1), and non-distinctive (3). Only 11 of these arguments can be considered as formally valid; and 2 (being a contrario in form) must be classed as invalid.

Some of the arguments by Paul are very similar, repeating the same idea in perhaps slightly different words. Compare: Romans 5:8-9 and 5:10; Romans 5:15 and 5:17; Romans 11:12 (which itself contains two similar minor premises with a common conclusion) and 11:15; 2 Corinthians 3:7-8 and 3:9. Here, the same author is rewording his thoughts in different ways, to get his points across. Paul (Tarsus, ca. 5 CE – Rome, ca. 47 CE) was apparently a diaspora Jew who lived for some time in Israel. Initially actively anti-Christian, he later converted to Christianity, and became one of its foremost leaders and missionaries. Paul may be said to have turned the Christian movement into a distinct religion, or at least given the evolving religion a new impetus.

Finally, let us mention the 4 a fortiori arguments in Hebrews. Of these, 3 are positive subjectal in form, including 2 a crescendo; and 1 is positive antecedental. The language used in them (in the translation here used, at least) is: *how much more* (2), simply *much more* (1), and *much less* (1). All 4 arguments can be considered formally valid. However, one argument (namely, Hebrews 10:28-29) goes against the rabbinic principle of *dayo* which forbids using an a crescendo argument to infer a stronger penalty from a lesser penalty given in the Torah. Although the Epistle to the Hebrews was in the past regarded by many authorities as written by Paul, most modern scholars have come to reject the idea. However, if I may weigh in on this debate, judging only by the tortuous style of most of the a fortiori arguments in this book (compared to the straightforward style of the arguments found in Matthew and Luke), the hypothesis of Pauline authorship of Hebrews looks rather probable to me. Alternatively, Hebrews was written by someone else, but he cited a fortiori arguments by Paul or he wrote the arguments in the style used by Paul.

Let us now look at some of the arguments in more detail. First, let us look at the two arguments by Paul that I have classed as invalid. Romans 5:15 reads: "If through the offence of the one [i.e. Adam's original sin] many died, how much more did the grace of God, and the gift that came by the grace of the one man, Jesus Christ, overflow to many." That is to say, more formally: If offence by one (Q) caused many to die (S1), all the more grace by one (P) caused many to receive grace (S2). Note that the minor term (Q) and the major term (P) are opposites ('offence' *v.* 'grace'), even if they have a common factor ('by one'); also, the subsidiary term has opposite values ('to die' *v.* 'to receive grace') in the minor premise (S1) and conclusion (S2), even if there is a common factor ('many').

Similarly, Romans 5:17 reads: "If by one man's [Adam's] offence death reigned through that one; much more shall those who receive abundance of grace and the gift of righteousness reign in life through the one, Jesus Christ." The argument is much the same: Q is identical ('offence by one') and P is similar ('abundance of grace and the gift of righteousness', instead of just 'grace'); also, S1 is similar ('death to reign', instead of 'many to die') and S2 is similar ('reign in life', instead of 'receive grace'). So we can treat these as one and the same thought, slightly differently verbalized.

In either case, what is sure is the invalidity of the argument. The argument is presented as an a contrario one, with contrary minor and major subjects ("the offence of one man" and "the gift of grace of one man") and contrary subsidiary predicates ("the death of many" to "the grace of many"). Such reasoning by inversion seems reasonable enough, at first sight. The minor and major terms, though contrary, might well be placed in a continuum running from negative to positive values; and likewise, the subsidiary terms, though contrary, might well be placed in a continuum running from negative to positive values. Even though this double a crescendo idea is not in itself objectionable, the conclusion cannot be claimed to follow from the available premises, because the parallelism between the two negative values (in the minor premise) and the two positive values (in the conclusion) cannot be proposed as a premise without begging the question. That is, the argument inevitably involves a circularity, needing the conclusion to make possible deduction of the conclusion.

Such complicated argument, trying to express many thoughts at once, in tortuous ways, seems to me rather typical of Paul[10]. However, it should be pointed out that there are instances of similar invalid reasoning in the Talmud. For

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[10]   See for instance: Romans 5:10: "If, when we were enemies, we were reconciled to God by the death of his Son, how much more, having been reconciled, shall we shall be saved by his life." Here, the argument is valid, although the subjects 'who were enemies' and 'who have

instance, in Mishna Makkoth 3:15, which reads: "R. Hananiah the son of Gamaliel said: If in one transgression a transgressor forfeits his soul, how much more should one who performs one precept have his soul granted him!" So, while the form of Paul's reasoning here does suggest some mental confusion, it does not prove (as Maccoby has insisted) that he had no Pharisaic influence.

Let us now look at Hebrews 10:28-29, which fails to apply the rabbinical *dayo* principle. It reads: "He who despised Moses' law died without mercy on the testimony of two or three witnesses: How much more severe punishment do you suppose he shall deserve, who has trodden under foot the Son of God, and has counted the blood of the covenant, with which he was sanctified, an unholy thing, and has affronted the Spirit of grace?" More formally put, the argument runs: If one who breached Mosaic law (Q) was punished with death (S1), all the more he who has done all these un-Christian things (P) will be more severely punished still (S2).

This argument clearly intends an a crescendo movement from 'punished with death' (S1) to 'more severely punished' (S2). Since it is an inference from alleged Torah law (although where in it does 'despising Mosaic law' entail 'death without mercy' is not specified), it should not (according to the rabbinical rule of *dayo*, to repeat) conclude with a more severe punishment (for whatever greater sin). So this argument is invalid under Judaic logic, even if it could be regarded as sound (that is, if we grant the implicit major premise) under general logic. Thus, whoever (it was Paul, I suspect) formulated it cannot claim to be reasoning in accord with Pharisaic standards. Moreover, of course, there is no mention in Mosaic law or lore of any "Son of God," or "blood of the covenant, with which he was sanctified," or "Spirit of grace." These being all Christian concepts and values, quite foreign if not contrary to the Torah context, it is absurd to appeal to the Torah in relation to them.

Let me also here comment on Luke 16:11, in which Jesus says: "If you have not been faithful with worldly wealth, who will entrust you with true riches?" Although this sentence uses no special key words or phrase, it is obviously a fortiori. More formally put, it says: If you (S) have not been [trustworthy (R) enough to be] faithful with worldly wealth (Q), then you (S) will not be [trustworthy (R) enough to be] faithful with spiritual wealth (P). This is a valid purely a fortiori argument, of negative predicatal form.

I found this case through mechanical search for the key word *much*, which led me to Luke 16:10, viz. "He who is faithful in that which is least is faithful also in much: and he who is dishonest in the least is also dishonest in much." These two sentences are clearly not a fortiori arguments. In my opinion, they are even non-sequiturs – for to my mind one may be scrupulously honest in little things, but be tempted into dishonesty by the prospect of a big gain; or inversely, be merrily dishonest in little things, but steadfastly refuse to engage in a big crime. Nevertheless, it is while looking at Luke 16:10 that I noticed the a fortiori argument in 16:11. However, here too I would like to point out that the word "therefore" which links these two verses is logically quite unjustified. At best, it indicates some further logical confusion; at worst, it reveals a manipulative intent.

Paul makes a positive predicatal argument in Romans 11:24 (which we shall analyze further on) and another one in 1 Corinthians 6:3. The latter reads: "Do you not know that we shall judge angels? how much more, things that pertain to this life?" This is a valid purely a fortiori argument: If we (S) can [i.e. have the authority (R) to] judge (R) angels (P), all the more we (S) can [i.e. have the authority (R) to] judge things of this world (Q). I will not here bother to unpack all a fortiori arguments found in the NT, as I think most of the others can be sorted out without too much difficulty by the interested reader.

I have to say that, although I have many years ago read the whole Christian Bible in the way of an intellectual duty, I did not greatly enjoy doing so and remember little of it. I can see that it contains some wisdom and good, but there are also in it many things that I find hard to swallow. As a philosopher, I find the idea that God might have a son of flesh and bones untenable. I also find references by Jesus to demons possessing people quite silly. I understand that people at that time did believe in such things; there are echoes of this even in the Talmud, although such belief does not play any role in Judaism today. Additionally, I am rather put off by Jesus' occasional fits of bad temper and verbal abuse of people[11]. Such behavior does not look very 'high-minded' to me.

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been reconciled' are contrary, the predicates 'reconciliation with God' and 'being saved' are two degrees of the same thing, even if they are given an a contrario flavor by their further qualifications, viz. "by the death of his Son" and "by his life." We could here too argue that the conclusion cannot be drawn from the premises without taking the conclusion as a premise; but because of the vagueness of the contrast between the predicates here, we can generously say that the conclusion is not really needed as a premise to draw the conclusion, especially if we ignore the said further qualifications. On this basis, I have counted the argument as valid; but it is admittedly a borderline case. Romans 5:8-9 is not very different, though a bit less unsure. Note that although I have counted it as purely a fortiori, it could be considered as a crescendo.

[11]   Haï Bar-Zeev, in *Une lecture juive du Coran* (Paris: Berg, 2005), on p. 62, cites some anti-Semitic statements by Jesus: "You brood of vipers. How can you speak good, when you are evil?" (Matt. 12:34), "O faithless and perverse generation, how long shall I be with you? how long shall I suffer you?" (Matt. 17:17), "O ye of little faith" (Luke 12:28), "Ye are of your father the devil" (John, 8:44), "All [i.e. the rabbis] that ever came before me are thieves and robbers" (John, 10:8). Obviously, these are not pondered, empirical and rational judgments, but emotional outbursts. Note also the episode when Jesus initiated physical violence against certain merchants and caused them material losses (see Mark 11:15-16; Matt. 21:12; Luke 19:45; and John 2:13-16). One cannot avoid reflecting on the many Christians who, over the past two thousand years, have felt justified by such statements and stories to kill and otherwise persecute many, many innocent Jews.

Moreover as a Jew, I find Paul's frequent denigration of Jews and Judaism quite offensive and painful, since clearly lacking in objectivity and truth. He obviously had a personal axe to grind in this matter. I also suffer at the thought of all the innocent Jews that were persecuted through the centuries due to the unflattering narrative concerning "the Jews" given in the Christian Bible as a whole. For these reasons, it is with great reluctance that I wrote the present chapter, quoting some passages from this document. I did not want to give the impression I was endorsing it.

Anyway, I think I have managed here to give a new impetus to logic research in Christian sources. I hope other people, more at ease in this particular field than me, will take up the challenge and look further into the matter. It is not a religious issue, but has to do with the history of logic.

2. Jesus of Nazareth

In the present section, we will focus on one of the ten (or six, if we exclude repetitions) a fortiori arguments attributed to Jesus in the Christian Bible, and discuss its substance as well as its form. I will show some of the complications that may surround the reading of such arguments. Needless to say, although I am a Jew, I have no desire to engage here in religious polemics against Christians or any other denominations. Jews do not normally try to convert non-Jews to their views[12]. My interest here is entirely logical. The proof is that I am not always critical. When I am critical, it is with an impartial, scientific spirit – the same spirit I apply to assessment of a fortiori and other forms of reasoning found in Jewish texts.

In an essay entitled "Comparing Gospels and Rabbinic Writings: a Halakhic Instance"[13] Jacob Neusner, a Jew, draws attention to the following argument by the founder of Christianity, Jesus son of Joseph (ca. 7-2 BCE to 30-36 CE), in Matthew 12:10-12, which may be construed as a fortiori:

> "And they [the Pharisees present] questioned Jesus, asking, 'Is it lawful to heal on the Sabbath?' – so that they might accuse him. And he said to them, 'What man is there among you who has a sheep, and if it falls into a pit on the Sabbath, will he not take hold of it and lift it out? How much more valuable then is a man than a sheep! So then, it is lawful to do good on the Sabbath.'"

This exchange occurred as Jesus was about to "heal" a man's withered hand in a synagogue on a Sabbath. As Neusner points out, this passage displays ignorance of Jewish law, which in fact definitely allows and indeed recommends saving a person's life on the Sabbath, intervening however necessary even if the life is not directly and imminently endangered (citing Tosefta Shabbat 15:11-12); and as for a sheep fallen in a pit, the law allows and recommends that it be cared for where it is and later pulled out (Tosefta Shabbat 14:3); moreover, in the Halakhah "healing" does not constitute "work" forbidden on the Sabbath; so it is unlikely that "they" (i.e. some "Pharisees," i.e. some rabbis[14]) would "accuse him" on account of "healing on the Sabbath!"

There are other passages in the Christian Bible pointing to the same story, by the way, although they do not repeat the a fortiori argument. In Luke 6:9, Jesus says: "Then said Jesus unto them, I will ask you one thing: Is it lawful on the Sabbath to do good, or to do evil? to save life, or to destroy it?" And in Mark 3:4, "Then he turned to his critics and asked, "Does the law permit good deeds on the Sabbath, or is it a day for doing evil? Is this a day to save life or to destroy it?" The latter adds: "But they wouldn't answer him," which shows that the "Pharisees" concerned could not have been learned rabbis, but at best common people ignorant of Jewish law; for rabbis would have easily answered this tendentious query. Note the logically misleading phrasing of Jesus' questions: "to do good, or to do evil?" "save life, or destroy it?" etc. – as if there is no middle ground, as if the refusal of certain types of doing good implies a permission or imperative to do evil, as if such choices are the fundamental issue throughout every Sabbath.

One wonders if Jesus ever personally kept the Sabbath. Had he done so, he would surely have experienced the peace of it and known that it is not a time when one feels like getting involved in emotional disputes. Certainly, at the time of the episode here discussed Jesus was not, or at least his disciples were not, Sabbath observant. This is made evident in the text, just before the above mentioned exchange in the synagogue: "Jesus went on the sabbath day through the grainfields; and his disciples were hungry, and began to pluck the heads of grain, and to eat." Some Pharisees reprove them, but he signifies that he is above the law, saying of himself: "the Son of man is Lord of the Sabbath"[15].

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[12]   However, I would want to challenge the Christian inclinations of some contemporary Jews, those who call themselves "Jews for Jesus" or "messianic Jews." Christianity is not a sect of Judaism, but a quite different religion, even if the two have some common beliefs and one is historically an offshoot of the other to a large extent. Similarly, the differences between Judaism and Islam, and between Christianity and Islam, are sufficiently marked to be significant, even if Islam was originally largely plagiarized from its two predecessors.

[13]   In *Rabbinic Literature: An Essential Guide* (Nashville, Tenn.: Abingdon, 2005), pp. 144-149.

[14]   *Mepharshim* – interpreters of Scripture.

[15]   Matt. 12:1-8. Similarly, Luke 6:1-5. If Jesus and his disciples traveled further away from human habitation than the law permits, they were breaking the Sabbath. In any case, plucking grain standing in a field is definitely breach of Sabbath law. (Moreover, although Deut. 23:26 permits "plucking ears with thy hand" in "thy neighbour's standing corn," the rabbis interpret this permission as applicable only to laborers

But more to the point to the present logical inquiry, as Neusner also remarks, with reference to the argument in Mathew: "Saving life is not at issue in the story, only doing good;" and, further on: "The Halakhic definition of doing good on the Sabbath is feeding the beast in the pit, not raising it up." That is to say, aside from the issue of the truth or falsehood of its premises, the a fortiori argument (presumably intended here, since the signal expression "how much more" is used) is logically questionable, because its apparent conclusion does not quite follow from its premises. This can be seen if we try rewriting the argument more formally:

> A man (P) is much more valuable (R) than a sheep (Q).
> A sheep (Q) is valuable (R) enough to be lawfully lifted out a pit on the Sabbath (S1).
> Therefore, a man (P) is valuable (R) enough to be lawfully healed on the Sabbath (S2).

Note that it is I who has added the words "valuable (R) enough to be lawfully" to the minor premise and conclusion, so as to make the argument true to a fortiori form and thus logically credible; but I think no one would contest this addition. The implicit major premise must have an appropriate middle term R, such that a man has more R than a sheep; and the vague term "value" seems appropriate in this context. As for the addition of "lawfully," this is inserted to reflect the question "Is it lawful to heal on the Sabbath?" asked by the Pharisees and Jesus' conclusion that "It is lawful to do good on the Sabbath;" more will be said on this specification further on.

What we have here, evidently, is an attempt at positive subjectal a fortiori argument. Notice that the major and minor terms (P and Q) are subjects, the minor premise and conclusion are positive, and the inference goes from minor to major. Since the subsidiary term (S) is not identical in minor premise and conclusion, the argument intended must be a crescendo. If the argument intended were purely a fortiori, the correct conclusion would simply be that "a man is valuable enough to be lifted out of the pit on the Sabbath (S1)." Here, Jesus (according to Matthew) concludes that "a man is valuable enough to be healed on the Sabbath (S2)" – not exactly the same thing.

So we must assume a hidden additional premise to the effect that: "S varies in proportion to R." What is "S," here? That is, what is the common ground between "being lifted out of a pit" (S1) and "being healed" (S2) on the Sabbath? It is, as the proof-text has it, "being saved" from some danger on the Sabbath, or more broadly to be the recipient of some "good" deed. So the hidden premise is that the amount of "saving" (S) permissible is proportional to the "value" (R) of the creature being saved. If a sheep, which is worth less than a man, is worth saving, then all the more is a man worth saving and in more ways. This is the a crescendo reading of the argument.

I above say that Jesus draws a conclusion more specifically about healing, with reference to the question posed to him, viz. "Is it lawful to heal on the Sabbath?" to which he is presumably implicitly answering when he compares sheep and men. In fact, the conclusion explicitly drawn by Jesus, viz. that "it is lawful to do good on the Sabbath," is more general. We could alternatively, then, read the argument as purely a fortiori, by referring to the common ground of the two apparent predicates as the real predicate. That is, we could formulate the argument rather as follows:

> A man (P) is much more valuable (R) than a sheep (Q).
> A sheep (Q) is valuable (R) enough to be legitimately saved in some way on the Sabbath (S).
> Therefore, a man (P) is valuable (R) enough to be legitimately saved in some way on the Sabbath (S).

This argument is not strictly correct, because though the predicate "saved in some way" is made to seem the same in the minor premise and conclusion, it is in fact different below the surface. This difficulty could be overcome by suggesting, instead, that what the speaker had in mind was to generalize from the more specific formal conclusion, "lifting a man out of a pit on the Sabbath," to *any* act of "doing good" or "saving" on the Sabbath," and then to apply this general principle to the more specific case of "healing a man on the Sabbath." If we view his reasoning thus, we might justify it as a logical chain (a sorites) comprising a purely a fortiori deduction, a generalization (induction) and an application (syllogistic deduction).

Still, let us go back and look at the argument as it is presented. Jesus is not actually saying that the sheep legally *may or ought to be* lifted out. Rather, he is suggesting that a sheep owner *would anyway*, out of self-interest, irrespective of the law, pull out the sheep. That is, even if Jewish law forbade such action (which it does, in fact, as we have

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working in that field – and not as an invitation to all passersby to serve themselves without the owner's permission.) The text does not say that Jesus plucked grain, but only specifies his disciples as doing it. However, would his disciples have done that if their leader disapproved of such acts? Obviously, Jesus' unconcern with Sabbath laws does not only relate to "healing" sick people; it looks like a general indifference to this most important area of Jewish practice. The Sabbath is one of the first commandments given to the Jews in Sinai (Ex. 16:23), and has been considered by Jews throughout history as one of the most precious features of Judaism. It is the very heart of this belief system, constituting a symbolic acknowledgement of God as creator of the world and as active liberator of the Jews from Egyptian bondage.

seen), the sheep owner can be expected to be so attached to his material possession that he will ignore the prohibition and save his sheep. There is an implicit insult there, a suggestion that most if not all religious Jews are hypocrites; and indeed that very word is used in a similar context (Luke 13:15). The image thus projected of Jews as 'interested' (i.e. as essentially materialists, not spiritually inclined) has no doubt been very satisfying to anti-Semites in the past two millennia.

Granting this reading, the argument is not really about law – i.e. that saving a sheep's life is legal and therefore, all the more, saving a human life must be legal. The implication is rather that saving a sheep (by pulling it out of a pit) is *not* legal, but is nevertheless done in practice. Whence the conclusion ought logically to be that even if saving a human (in the same way) is likewise not legal, *it will likely be done in practice*. Such a conclusion would not answer the question posed, viz. "Is it *lawful* to heal on the Sabbath?" – but it would serve to make the legal question seem irrelevant. So this is really revolutionary discourse, aimed at encouraging that the law of the land (or religious law) be ignored or discarded.[16]

That may explain the (alleged) reaction of "the Pharisees" (as e.g. reported in the parallel episode in Luke 6:11) – viz. "But the Pharisees and the teachers of the law were furious and began to discuss with one another what they might do to Jesus." We may suggest that they were not angry because he had done something illegal on the Sabbath by healing, but because he had defied the authority of the law as such by means of false premises and invalid reasoning. It was no doubt his *hostile attitude*[17] that worried them the most, for mere doctrinal disputes are commonplace and generally accepted among Jews. So the purpose of the story is to give a false impression regarding the reaction of the Pharisees: to make them seem intolerant; whereas they were in fact reacting normally to an unfair attack.

I personally do not believe stories like the one about Jesus healing people miraculously. But let us suppose for the sake of argument that it was true. We can first ask what his motive was in performing such healing. If it was simply kindness, why did he choose to do it in such a demonstrative manner and precisely on the Sabbath? Presumably, if he had done it in private and on a weekday, he would not have provoked such negative reactions. Certainly, the person he healed could have waited one more day, having presumably been sick for years. So, we must assume that Jesus' intent was confrontational. It was not merely to heal, but to publicly contend. He was out to show his (alleged) miraculous powers and thus claim for himself a religious authority above that of the Pharisees.

However, in Deuteronomy 13:2-4, it is expressly stated: "If there arise in the midst of thee a prophet, or a dreamer of dreams – and he give thee a sign or a wonder, and the sign come to pass, whereof he spoke unto thee – saying: 'Let us go after other gods, which thou hast not known, and let us serve them'; thou shalt not hearken unto the words of that prophet, or unto that dreamer of dreams." "If there arise in the midst of thee, etc." refers to anyone who challenges the religious status quo, for whatever reason – even a Jew. "Going after other gods" refers to any proposed deviation from the Torah, as traditionally understood by the Jews. So the Pharisees were under no Torah-given obligation to submit to Jesus following such fancy demonstrations. On the contrary, their religious duty was to be extremely cautious with such a man.

Many Christians who read such stories simply cannot understand why the Jews rejected Jesus – in Christian eyes the performance of a miracle like the healing above described should have sufficed to convince the Jews to accept him. For such Christians, the Jews' negative response could only be explained as obstinacy and bad faith. Indeed, Jesus' own negative statements on several occasions, concerning Jews who did not accept his claims, shows he felt unjustly rejected by them[18]. What he and Christians have failed to understand is that Jews are positively *obligated* by their Scriptures to be very suspicious of anyone who makes extravagant claims, even if he comes on with impressive miracles[19].

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[16]   A Christian apologist presents this argument as follows: "Jesus exposes the fallacy in his critics' logic using an a fortiori argument. He points out that they would be willing to work in order to rescue a distressed sheep on the Sabbath. If that is true, then how much more should they be willing to restore a man who is created in the image of God." Norman L. Geisler in *Christian Apologetics* (Grand Rapids, MI: Baker, 2009), p. 71. It should be clear from my above analysis that Jesus did not expose "the fallacy in his critics' logic," but on the contrary engaged in fallacious reasoning!

[17]   Jesus' petulant behavior towards the rabbis reminds me of present-day anti-religious secularists, some of who go out of their way to upset religious people; it seems that the anti-establishment attitude of some Jews is not a new phenomenon. In my view, there is nothing wrong in challenging the establishment – I myself often criticize orthodox views, and so would hardly object; what is wrong is the antagonistic tone some people use to do that.

[18]   See Jesus' harsh words in Matt. 12:34, 17:17and John 8:44, 10:8. Surely, if Jesus acknowledged the Torah, he would have known that Deut. 13:2-4 and 18:21-22 fully justified the rejection of his claims by most Jews. As Bar-Zeev points out (pp. 57-64), the Jews who rejected Jesus were typically those most knowledgeable of Torah, while his followers were mostly ignorant people or social outcasts, "the poor in spirit" (Matt. 5:3), "tax collectors and sinners" (Matt. 9:10), "the lost sheep of the house of Israel" (Matt. 10:6). I should add that, as far as I can tell (I could be wrong), Jesus did not enjoin or even give leave to his future followers (i.e. Christians) to persecute Jews. His words were bitter, but not intended to incite hatred and violence.

[19]   In Sanhedrin 107B (in the uncensored editions of the Talmud) Jesus is presented as a pupil of the Pharisee sage R. Joshua ben Perachya, who publicly "excommunicated him" following some inappropriate behavior (apparently, just judging a woman by her looks); the

Let us return now to Neusner's critique. As he points out, no one denies Jesus' explicit principle that "It is lawful to do good on the Sabbath." The issue is: what constitutes "good" in the present context? For Jesus, it is (in a generous reading) pulling the sheep out of the pit; whereas for the rabbis, it is feeding the sheep in the pit until the Sabbath ends. Precisely because a sheep is not as valuable as a human being, it is not worth breaking the Sabbath for; nevertheless, as one of God's creatures, it should be treated in a kindly manner. But as regards a human being, according to the halakhah (i.e. established Jewish law), if one falls in a pit, he not only may but must be pulled out as soon as possible and using any means, and as Tosefta puts it: "it is not necessary to get permission [ad hoc] from a court" (i.e. the general permission suffices).

Clearly, the legal context and related rabbinical attitudes are very different from what Jesus here assumes and implies them to be. Lifting a sheep out of a pit is a good deed that is not allowed, though that does not exclude a less drastic good deed from being done (viz. feeding the sheep where it is for a while). Lifting a man out of a pit is a good deed that is allowed and indeed prescribed – not by a fortiori inference from the sheep, as Jesus seems to think, but for an entirely other reason, viz. (if I am not mistaken) that Jewish law is intended to preserve and prolong our life and not needlessly endanger or sacrifice it (except in certain very special cases[20], of which the present case is not one).

So, not only is Jesus' apparent a fortiori argument built on very unflattering insinuations and on quite false legal premises, but its conclusion is a non-sequitur and irrelevant to the question asked! It is more a sophistical exercise is misrepresenting facts and sowing doubt than a serious attempt at legal proof. If Jesus was as some claim a rabbi, even a dissident one, he would not have indulged in such confused and misleading discourse. All the more so if he was God incarnate or the son of God, as some claim[21].

If we try to understand Jesus' discourse from his own point of view, we must declare his incoherence. On the one hand, he gets angry at and insults some Jews for taking care of their animals – thus suggesting that he believes Jewish law to be against all such actions on the Sabbath (which it is not, in fact). On the other hand, he is willing for his part to dismiss the law's interdiction (as he wrongly assumes) of "healing" and more broadly "doing good." This is inconsistent discourse – he surely cannot both defend and dismiss the law at his convenience.

Perhaps the thrust of his argument is that even if "healing" is in actuality illegal it is in principle permissible; i.e. he is effectively advocating that the law currently accepted be changed. But if this was his purpose, a local synagogue was hardly the right forum – he ought to have addressed himself to the accredited lawmakers (ultimately, the Sanhedrin). But the legislative process is obviously not the center of interest here. Note that nowhere are the "Pharisees" in question named, as is customary in legal debate among Jewish rabbis. They are just presented as stereotypes, typical representatives of a monolithic class slated for contempt. No counter-arguments by them are cited, either; no exposition or explanation of their legal posture.

In any case, as Neusner rightly stresses and explains in detail, "healing" (by 'miraculous' or 'magical' means, like 'pronouncing prayers or incantations' or 'laying on of hands') does not in itself fit into the definition of "work" that Jewish law prohibits on the Sabbath, and so it is not and never was forbidden. It is only concrete acts that fall under one or more of the 39 categories of work (Heb. *melakhah*) which constitute breach of the Sabbath. It is true that the rabbis decreed that in a situation that is not life-threatening medical intervention should be avoided; but here their main concern was that the doctor or patient would likely prepare herbs for medication purposes (for this would involve *melakhah*). The rabbis were certainly never opposed to curing the sick! As regards chronology, these 39 categories were certainly known before Jesus' time, being given in the Mishna[22]. When after the above speech Jesus (allegedly) dramatically "heals" a man who had a "withered hand," using no material equipment, he has in fact done no forbidden work!

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master was criticized by his peers for his rigor in "repulsing Yeshu with both hands," saying: "Let the left hand repulse but the right hand always invite back." It is also there said that Jesus "practised magic and led Israel astray." (See the Soncino ed. footnotes 13 and 17.) In Berachot 17B (uncensored ed.) Jesus is referred to as someone "who disgrace[d] himself in public." (See the Soncino ed. footnote 5.) However, the Sanhedrin 107B story does not seem factually credible, because: (a) it contains an anachronism, since it is said to have taken place at the time of king Alexander Jannai (c. 107-78 BCE); and (b) it is unclear what Jewish law would justify excommunicating a Jew for merely judging a woman by her looks! (Maybe this explanation of Jesus' excommunication is intended as a metaphor; but I cannot imagine what that metaphor might be.) There may be some truth to the story, but the details were apparently forgotten and then fancifully filled in. Nevertheless, the said passages of the Talmud are still noteworthy, as they illustrate how *later* rabbis, at least, looked upon the founder of Christianity. Note that, contrary to what some apologists claim nowadays, there is no suggestion here or elsewhere that Jesus was regarded as a rabbi (even if a dissident one), let alone that he was ordained as one.

20   One should rather die than engage in idolatry, murder or illicit sexual relations (*Pesachim* 25a-b).
21   See e.g. John 3:17, 10:30. The concepts of God incarnating or having a son are, needless to say, totally foreign to Judaism; so we can well cite Deut. 13:2-4 in this context. These concepts are clearly imported from other cultures. In Greek and Roman mythology, for instance, gods (including the chief among them, Zeus or Jupiter) often visit humans under human guise and often beget children with human partners.
22   See Mishna Shabbat, and the corresponding Talmudic tractate (for the figure of 39, see on p. 69a: "The primary forms of labour are forty less one"), available in English at: www.halakhah.com/shabbath/index.html. A brief exposé on this topic can be read on Wikipedia at: en.wikipedia.org/wiki/Activities_prohibited_on_Shabbat. Though the Mishna was completed in about 200 CE, the discussions in it date from at least about 100 BCE.

In view of this, it is absurd to suppose that the rabbis would ever *have even asked* Jesus the specific question: "Is it lawful to heal on the Sabbath?" or even the vaguer question: "Is it lawful to do good on the Sabbath?" Why would they have done so, if a positive answer from him would have been correct? And why would they have been angered by his right answer and sought to obstruct him? It doesn't make sense. Whoever reported this incredible episode obviously misunderstood what was going on. More likely, the whole narrative is made up ex post facto so as to throw opprobrium on "the Pharisees." It can reasonably be considered an imaginary tale.[23]

3. Paul of Tarsus

Hyam Maccoby, a Jew, wrote several interesting books expressing his ideas about the narratives in the Christian Bible that many Christians, though presumably not all, have found objectionable. One of these books was *The Mythmaker: Paul and the Invention of Christianity*[24]. Basically, as I see it, Maccoby's intent was to harmonize as much as possible Christian lore with Jewish doctrine, and his means for this end was to place the responsibility for the divergences between them mainly on Paul of Tarsus (5-67 CE). I have no interest in the present context in Maccoby's larger theories or in the objections of some Christians to them, although needless to say I would likely be more receptive to the former's viewpoints.

What interests me here is Maccoby's objections to Paul's claim to having a "Pharisee," i.e. rabbinical, background. This claim is made in Acts 22:3: "I am a Jew, born in Tarsus, a city in Cilicia, yet brought up in this city at the feet of Gamaliel, and taught according to the strict manner of the law of our fathers…"[25]. Presumably this refers to Gamaliel the Elder (d. ca. 50 CE), grandson of Hillel the Elder and grandfather of Judah the Prince (the redactor and editor of the Mishna); if so, Paul is claiming a very high-level master, who was probably presided over the Sanhedrin at some time. Maccoby's discussion is mainly to be found in his book's chapter 7: "Alleged Rabbinical Style in Paul's Epistles" (pp. 62-71). There, Maccoby considers some of Paul's arguments, principally those apparently in the style of rabbinic *qal vachomer*; and he attempts to show that most are technically faulty to a degree that excludes the possibility that Paul may indeed have been trained as a Pharisee. Essentially, Maccoby argues that Paul constructed his arguments rhetorically, more in the manner of the sophists in the surrounding Hellenistic culture than in accord with rabbinic norm and practice.

Obviously, the evaluation of Paul's claim to rabbinic antecedents, cannot be based solely on the style of his discourse, but must focus mainly on his knowledge of Jewish law. I do not propose to here look into this matter, which requires a lot more study than I am willing to invest[26]. But offhand, I would certainly doubt that Paul was very knowledgeable. The ease with which he dropped out of normative Judaism and adopted religious ideas and attitudes, some of them pagan, from other traditions is indicative of a certain lack of grounding in Jewish belief and law. His defection sounds all the more incredible, considering his claim to have been a student of no less a personage than Gamaliel I[27]. Of course, he may have studied in the latter's academy (*yeshiva*) for a short while, without this implying that he reached a high level in his studies. That he sat "at the feet of Gamaliel" does not necessarily mean that he was a star student, or at all remarkable.

As I said earlier, based on inspection of all of Paul's a fortiori discourse, I do not entirely agree with Maccoby's sweeping assessment. I do agree with him that Paul's a fortiori arguments are often distinctive, often confused, and often incredible. Paul had apparently a tendency to pack many thoughts into a single statement, and his was not exactly an orderly mind. But comparable errors are found, even if very infrequently, in rabbinical discourse. So we cannot draw a hard and fast conclusion concerning Paul's Pharisee background or qualifications from his psycho-epistemology. He certainly resorts more than any other speaker in the NT to argument that looks a fortiori – at least

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[23]   I must say some passages in the Christian Bible give me the impression of recounting a bad dream or a hallucination, the kind where people speak verry slowly in loww-pitched voices and say and do things quitte out of touch with ordinary reality and logic.

[24]   London: Weidenfeld, 1986. I have found three excerpts from this book posted on the Internet. One, called "The Problem of Paul," which presents Maccoby's general view of Paul, is at: www.positiveatheism.org/hist/maccoby2.htm; and the second, called "Paul's Bungling Attempt at Sounding Pharisaic" is at: www.positiveatheism.org/hist/maccoby3.htm. The third is at: books.google.com/books?id=co_CxizRbTAC&printsec=frontcover&dq=Hyam+Maccoby+Paul+and+the+Invention+of+Christianity&hl=en&ei=BRmHTeJriYI6t4_w7wg&sa=X&oi=book_result&ct=result&resnum=1&ved=0CDAQ6AEwAA#v=onepage&q&f=false.

[25]   And again in Philippians 3:5, where Paul describes himself as "Circumcised the eighth day, of the people of Israel, of the tribe of Benjamin, a Hebrew of Hebrews; regarding the law, a Pharisee." However, Paul (to my knowledge) gives no exact dates regarding his alleged Judaic studies. How old was he when he started? How long did he study? His conversion to Christianity is dated as ca. 31-36 CE, i.e. when he was about 26-31 years old. That is not old enough to have made very advanced rabbinical studies (to be sure, there have occasionally been some brilliant young rabbis – but Paul does not claim or appear to have been one).

[26]   There is no doubt already plenty of literature on the subject. The writers would need to be very knowledgeable in both Jewish law and Christian literature.

[27]   A Christian tradition claims that this Gamaliel was eventually baptized, and remained a secret Christian even while he sat in the Sanhedrin so as to protect the new sect. This claim strikes me as an utterly incredible ex post facto fabrication.

thirteen times (and four more if the Epistles to the Hebrews are attributed to him). In only two cases, viz. Romans 5:15 and 5:17, would I declare his argumentation hopelessly invalid. Nevertheless, Maccoby's criticism has some justification.

Maccoby cites four examples of apparent *qal vachomer* from Paul, all from the Epistles to the Romans. Their analysis here is my own, independently of Maccoby, whose thinking will be examined further on. The first example runs as follows:

> Romans 5:10. "For if, when we were God's enemies, we were reconciled to him through the death of his Son, how much more, having been reconciled, shall we be saved through his life!"

This looks like some sort of *a crescendo* discourse: (a) God's enemies are, through the death of his son, reconciled to him; (b) those who have been reconciled [to God] are, through the life of his son, saved by him. This would be logically okay if presented as two independent statements of fact. But the difficulty arises due to use of the expression "how much more," which ordinarily implies an a fortiori type of argument, in which (a) would be a premise and (b) a conclusion. Six terms are mentioned here: enemies; death of son; reconciled; reconciled; life of son; saved; the premise and conclusion have one term in common, viz. reconciled, though this term is predicate in the premise and subject in the conclusion. This is not a known format of a fortiori argument.

The movement of thought involved here seems somewhat akin to first-figure syllogism: A is B and B is C, where B is the middle term ("reconciled") and A and C (God's enemies and friends, respectively) are the major and minor terms. But there is no intent at drawing the conclusion A is C. Rather, the intent seems to be: A is B is the premise, and B is C is the conclusion. So this is not syllogism, either. What is involved is step by step increase in proximity to God: from enmity to reconciliation, and from reconciliation to salvation. One difficulty is that, though the two propositions are apparently intended to be in chronological sequence, life comes after death. Presumably, then, 'death' here refers to the crucifixion and 'life' to the resurrection; or alternatively, maybe, 'death' refers to the Christian's reflection on the crucifixion and 'life' refers to his or her remembrance of Jesus' life.

Albeit these difficulties, let us try to formulate an a fortiori argument, by making a number of changes in the given data. I would say that Paul intended to argue as follows:

> Friends of God (P) are more redeemable (R) than enemies of God (Q);
> and, enemies (Q) are redeemable (R) enough to be reconciled through the crucifixion (S1);
> also, proximity to God (reconciliation/salvation) (S) is proportional to redeemability (R);
> therefore, friends (P) are redeemable (R) enough to be saved through the resurrection (S2).

This is a formally acceptable, positive subjectal a crescendo argument. The term "friends" here introduced is taken to be applicable to people who "have been reconciled" (in the minor premise), and is suggested by opposition to "enemies." The crescendo movement is suggested by the use of different predicates, and by the fact that the first (in the minor premise) is relatively negative (referring to death) while the second (in the conclusion) is relatively positive (referring to life). The premise about proportionality is needed to give the argument more formal validity. Presumably, Paul had it in mind as he formulated his argument.

This is a generous rewriting by me on Paul's behalf, to align his argument with a standard form. Whether any of the premises is true or not need not concern us here, we are only verifying formal validity. The use of parallel contrasts between terms is rather typical of Paul's a fortiori discourse. It is conceivable that he intended to say this, but at the same time wanted to add other ideas, and so spoke with his logical mouth a bit too full. Is this argument by Paul rabbinic in style? Truthfully, I do not remember coming across a rabbinic argument of similar form, although further research might discover one or more. Note that a very similar argument by Paul, not mentioned by Maccoby, is Romans 5:8-9. This reads:

> "But God shows his love toward us, in that while we were yet sinners, Christ died for us. Having been justified by his blood, how much more shall we be saved from the wrath of God through him."

Here again, Paul seems to intend an a crescendo argument, viz.: if people who are still sinners (Q) are redeemable (R) enough to be justified by the death of Jesus (S1), then converts (i.e. those already "justified") are redeemable (R) enough to be saved from punishment through him (S2). It is all a bit confusing, but can with a bit of effort be standardized.

I should add that both these arguments, i.e. Romans 5:8-9 ad 5:10, are not strictly speaking formally valid, but they are at least potentially so. They lack a clear middle term and major premise; and they lack a premise on proportionality, and furthermore a clear formula for calculation of the proportional change. For all that, I have accepted these two concrete arguments as (more or less) 'valid', for reasons that I have already laid out in my earlier theoretical treatment of such arguments. My position is, simply put, that we are not here concerned with the scientific truth of the premises and conclusion, or with the logical precision of the argument put forward, but merely with a

rough estimate of its general credibility as a unit of ordinary discourse. If these arguments were Talmudic, I would accept them as reasonable in this loose sense; therefore, even if I do not agree with their content, I must to be fair grant their form the same 'pass' status.

The second example Maccoby gives is the following:

> Romans 5:17. "For if, by the trespass of the one man, death reigned through that one man, how much more will those who receive God's abundant provision of grace and of the gift of righteousness reign in life through the one man, Jesus."

This takes a while to understand, but when one looks at the context it becomes clearer. The minor premise refers to the 'original sin' by Adam – when that one man (Adam) trespassed (ate the forbidden fruit), death reigned (humanity became mortal) through that one (thanks to Adam). The conclusion refers to the benefits, according to Paul, of following Jesus – when the recipients of grace and righteousness (the Christian converts) follow the other one man (Jesus), life will reign (for them, thanks to him). Whether or not we agree with what he is saying, this seems to be Paul's thought, although it is expressed by him in unnecessarily tortuous fashion. The way Paul puts it is rather confusing at first sight, but if we reshuffle the terms a little his intent becomes clearer. Thus, his argument might be construed as a positive subjectal a crescendo, as follows:

> Following Jesus (P) is more powerful (R) than the trespass by Adam (Q);
> and, the trespass by Adam (Q) was powerful (R) enough to cause many to die (S1);
> also, existential consequences (life/death) (S) are proportional to the power (R) of causal acts;
> therefore, following Jesus (P) is powerful (R) enough to cause many to live (S2).

Not all details of the original argument are carried over, but the gist of it is clearly there. I have inserted a middle term (R), viz. the "power" of causal acts, and used it to reconstruct the tacit major premise (which requires Christian faith in Jesus) and the tacit additional premise about proportionality (which seems reasonable enough as a generality). Now, the question is: is this valid reasoning? I would say—no.

Although this argument somewhat resembles the preceding, the intention here is clearly a contrario, i.e. if Q causes S1, then P, the opposite of Q, must cause S2, the opposite of S1. These two causations may happen to be both true, but it cannot be said that one necessarily proceeds from the other. The given premises by themselves do not allow us to infer the putative conclusion. Even if the crescendo from S1 to S2 is conceivable, it is not easy to provide an additional premise which guarantees that the switch from Q to its opposite P is *precisely correlated* with the switch from S1 to its opposite S2 – this degree of precision is very difficult to demonstrate. The argument is effectively a circular one, because we only imagine the correlation by virtue of the conclusion. Thus, the argument must be considered invalid.

It is true that the preceding argument, Romans 5:10 also seems a contrario, since it goes from enemies being 'reconciled through a death' to friends (the opposite of enemies) being 'saved through a life'. But there, even though the contrast between death and life does give the argument an a contrario flavor, the two predicates are not strictly-speaking antithetical, because reconciliation and salvation are within the same polarity even if the latter is more positive than the former. Although such argument could also be rejected as tenuous on formal grounds, I have as earlier indicated let it pass as reasonable ordinary discourse. Similarly for the comparable argument of Romans 5:8-9. On the other hand, in Romans 5:17, the two predicates (death and life) are diametrically opposed, and the argument is a clear case of a contrario, and therefore invalid. The same goes for another argument by Paul, not mentioned by Maccoby, namely Romans 5:15. This reads:

> "If through the offence of the one [i.e. Adam's original sin] many died, how much more did the grace of God, and the gift that came by the grace of the one man, Jesus Christ, overflow to many."

More formally put: If the offence by Adam (Q) was powerful (R) enough to cause many to die (S1), then the gift by Jesus (P) will be powerful (R) enough to cause many to receive grace (S2). Here again, notice the radical opposition between Q and P and between S1 and S2, and the consequent difficulty of formulating some additional premise that would justify the simultaneous switchover from negative to positive of both these variables. For this reason, the argument must be viewed as invalid.

Thus, as regards the second example of a fortiori argument by Paul that Maccoby draws attention to, we have to agree with him that it is invalid, even if the reasons we have given are (as we shall see) different from the reasons he gives. As regards the question as to whether similar reasoning is found in rabbinic discourse, the answer this time is—yes. I have found at least one a contrario argument in a Jewish source. It is in the Mishna Makkoth 3:15, which reads:

> "R. Hananiah the son of Gamaliel said: If in one transgression a transgressor forfeits his soul, how much more should one who performs one precept have his soul granted him!"

It is interesting to note that Paul's thought in Romans 5:17 (and indeed in 5:15) is essentially much the same as this Mishnaic saying, except that Paul has injected Adam and Jesus into the equation. It is conceivable that Paul had this very maxim in mind when he formulated his own[28]. In any event, in my opinion (I could yet be wrong on this one), the said argument in the Mishna is formally as invalid as the said two in Romans[29]. Thus, although Paul's formulation is a bit more complicated than R. Hananiah's, he may have made it under Pharisaic influence!

Regarding the third example put forward by Maccoby:

> Romans 11:15. "For if their [the Jews'] rejection is the reconciliation of the world, what will their acceptance be but life from the dead?"

Here, Paul seems to suggest that God's rejection of the Jews brought about His reconciliation with the world, and therefore when He accepts them again something even better will occur, namely a general resurrection of the dead. Or maybe, that the Jews' rejection of Jesus made possible his acceptance among the Gentiles, and therefore if the Jews decided one day to accept Jesus something even better will occur, namely a general resurrection of the dead. Or something like that – the subjects and objects intended are far from clear. Either they were not clear in Paul's mind, or he had difficulty expressing his thought. Be that as it may, what we need to note is that Paul is saying that *since* "rejection" leads to "world reconciliation," *it follows that* "acceptance," which is obviously a more friendly attitude, must lead to proportionately more than mere "world reconciliation," i.e. to "life from the dead." More formally presented, Paul's argument can be construed roughly as follows:

> Acceptance (P) is a more positive attitude (R) than rejection (Q);
> and, rejection (Q) is positive (R) enough to result in world reconciliation (S1);
> therefore, acceptance (P) is positive (R) enough to result in life from the dead (S2).

Formally, what we have here is a positive antecedental a crescendo argument, since it progresses from "world reconciliation" to "life from the dead." So, an additional premise about proportionality is required and presumably implicitly involved; something like "the desirability of the consequences (S1, S2) is proportional to the positivity (R value) of the antecedent events (Q, P)," which is not unreasonable[30]. So, the argument as such can be said to be valid, even if its terms are far from lacking in ambiguity and its minor premise, however it is interpreted, is open to doubt.

This argument, like that in Romans 5:10, has an a contrario flavor, insofar as its subjects (vaguely put, "rejection" and "acceptance") are antithetical and its predicates look a bit like opposites (because "life from the dead" includes mention of death, whereas "world reconciliation" does not). But of course, it is in truth not a contrario, since the term "life from the dead" is in fact positive and indeed more positive than the term "world reconciliation." So it can, like Romans 5:10, be considered valid. This is, of course, after generous rewriting by me on Paul's behalf, for the purpose of standardization. Here again, notice Paul's signature style, depicting a rising progression from negatives to positives and from lesser things to greater things. I do not know if similar rhetoric occurs in in rabbinical discourse; but if it does, it must be quite rare. So the form of the argument could perhaps eventually pass as rabbinical, even if its content would definitely not.

Note that a very similar argument by Paul, not mentioned by Maccoby, is Romans 11:12. This reads:

> "If their [the Jews'] fall means riches for the world, and their failure riches for the Gentiles; how much more will their fullness bring?"

This sentence in fact contains two arguments with the same thrust: If the Jews' fall means riches for the world, then their fullness will bring about something better. And: If the Jews' failure means riches for the Gentiles, then their fullness will bring about something better. Here again, some negative remarks by Paul about the Jews, claiming them to have fallen and failed, presumably because they did not acknowledge Jesus as divine. The statement is no doubt viewed by Paul as positive, since he considers that their recalcitrance did not prevent (or maybe enabled) spiritual enrichment for the world, and he predicts that if they change their mind, even better things will result (this being tacit, but obviously intended in the rhetorical question). The use of a rhetorical question in lieu of a definite conclusion is stylistically very rabbinic.

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[28]   I do not think that this R. Hananiah was a son of the Gamaliel referred to earlier as Paul's Pharisee teacher; that Gamaliel (the Elder) does not seem to have had a son of that name. It looks like the reference may be to the Tanna of the 1st-2nd cent. Hananiah/Hanina b. Gamaliel II, i.e. to a great-grandson of the aforesaid Gamaliel, since Gamaliel II was a son of Shimon, the son of Gamaliel I. In that case, the said Mishnaic statement would be later than Paul's. But Paul may have heard a similar statement from earlier lips, maybe even from another rabbi with the same name. In any case, the fact that Paul's statement is more complicated the Mishnaic one suggests that it came later, since the reverse direction of influence is extremely unlikely.

[29]   As you can see, I am not playing favorites. Although I personally accept the content of R. Hananiah's statement and do not accept the content of Paul's statements, I recognize that their forms are the same, i.e. equally a contrario, and thus judge them equally invalid.

[30]   Note in passing that there is no occasion to apply the *dayo* principle to this a crescendo argument for the simple reason that the conclusion is more positive than the minor premise.

The fourth example of Pauline a fortiori argument considered by Maccoby is:

> Romans 11:24. "After all, if you were cut out of an olive tree that is wild by nature, and contrary to nature were grafted into a cultivated olive tree, how much more readily will these, the natural branches, be grafted into their own olive tree!"

We see at once that this statement makes sense, and would be acceptable in rabbinical disputation. It can easily be recast in the following, positive predicatal form (note how it proceeds from major to minor, from the more difficult act to the easier one):

> More compatibility (R) is required to graft cuttings into another olive tree (P) than to do so into the same olive tree (Q);
> and, a wild olive tree cutting (S) is compatible (R) enough to be grafted into another, cultivated tree (P);
> therefore, a wild olive tree cutting (S) is compatible (R) enough to be grafted into its own, wild tree (Q).

Note that this argument is purely a fortiori (since the subsidiary term, S, remains constant), whereas all the preceding instances were a crescendo. Although it can be considered valid, Paul's habitual mental gymnastics are evident in it: a branch is "grafted," as against "cut out," in a manner "contrary to nature," as against "natural," into a "cultivated," as against "wild by nature," olive tree. It is clear that Paul enjoys such entangling thoughts.

We have thus far looked at seven of Paul's a fortiori arguments, four mentioned by Maccoby and another three not mentioned by him. These are all the a fortiori arguments in Romans. There are another five a fortiori arguments by Paul in: 1 Corinthians 6:3; 2 Corinthians 3:7-8, 3:9, 3:10-11; and Philemon 1:15-17. There are additionally four a fortiori arguments which *might* be (could well be, in my view) by Paul in Hebrews 9:13-14, 10:28-29, 12:9, 12:25. None of these arguments are mentioned, or taken into consideration, by Maccoby. I will not list them all here; but if the interested reader looks them up, he or she will see that most of them (including those in Hebrews) have features very similar to those highlighted above. Paul's distinctive style is easily recognized in them.

The following is just one example, found in Hebrews 9:13-14: "For if the blood of bulls and of goats, and the ashes of a heifer sprinkling those who are unclean, sanctifies them so their flesh is clean: How much more shall the blood of Christ, who through the eternal Spirit offered himself without spot to God, cleanse your consciences from dead works to serve the living God?" Notice the contrasts: between animal sacrifices and Jesus' self-sacrifice; between the uncleanness of those who are sprinkled with ashes and the spotlessness of Jesus; between the cleansing of the flesh in the first instance and the cleansing of consciences in the second; between dead works and the living God. The underlying argument form is simply a crescendo; but the suggestive play of light and shade looks very Pauline.

As regards the four arguments in Romans brought to our attention by Maccoby, following our own analysis above we can say the following about them in the way of a summary. In all four cases, the wording is far from straightforward, and not entirely clear; but with a bit of effort and imagination the intended arguments can be recast in standard forms. The first two can be construed as positive subjectal, the third as positive antecedental, and the fourth as positive predicatal. The first three are a crescendo arguments, and the fourth is purely a fortiori argument. Three of the arguments may be considered valid; but one is invalid. The validity of the first and third arguments is here accepted, even though in a stricter perspective it is open to debate. The invalid argument is the second, and its invalidity is due to its peculiar a contrario form. As for whether Paul's discourse has or lacks "rabbinical style" – the question is difficult to answer conclusively. He has his own peculiar style – that is all that can be said with certainty.

Maccoby's assessment of the four arguments differs considerably from mine, due to his different theoretical understanding of the nature and conditions of validity of a fortiori argument. He judges the first three of the said arguments by Paul as invalid and the fourth as valid. He regards the three invalid arguments as invalid because the predicate in each putative conclusion is never identical to the predicate in the minor premise. This, according to Maccoby, does not conform to the rabbinic *dayo* (sufficiency) principle, which he sees as interdicting all 'proportional' a fortiori argument (i.e. all a crescendo argument).

Maccoby is right in his observation that, as regards the first three arguments, the predicates in the minor premise and conclusion do not match. But in my opinion he is wrong in his assumption that this is necessarily a breach of the *dayo* principle. This principle, as I have shown in earlier chapters (7-8), only forbids the inference, from the penalty prescribed in the Torah for a certain crime, to a greater penalty (not prescribed in the Torah) for a greater crime. It is not a general proscription of proportionality. If we look at the three arguments in question, viz. Romans 5:10, 5:17 and 11:5, we see that none of them are to do with inference of a penalty – they all conclude with what Paul perceives as an increased good.

In fact, looking at all 28 a fortiori arguments found in the Christian Bible, only one might be construed as involving a breach of the *dayo* principle. This is the argument in Hebrews 10:28-29 – of which Paul could well be the author, in view of its tortuous style. I have analyzed this argument in more detail in a previous section (10.1). Suffices here to point out that it argues from an (alleged) death sentence for breach of Mosaic law to a "more severe punishment" for various unchristian attitudes or acts. Insofar as Mosaic law is mentioned in the premise, and an increase in punishment is mentioned in the conclusion, this may be said to be a breach of *dayo*.

Of course, no rabbi would accept this inference anyway, since the conclusion concerns matters that have nothing to do with Judaism. That is, the rabbis would strongly object to its (tacit) major premise, which places Christian values above Jewish ones. But as regards application of the rabbinical *dayo* principle, this is the only place where it might conceivably be formally applied; and its effect would be to declare the conclusion excessive, i.e. lacking conformity with Judaic standards of inference. But this constitutes a religious norm – not as Maccoby imagines a logical one. Logically, the argument is passable. This does not mean that its content is necessarily true, but simply means that if the required major premise and premise about proportionality were given, the conclusion would follow from the minor premise. The missing premises are, however, open to doubt.

Furthermore, it should be said that it would be inaccurate to say that the *dayo* principle is regarded by the rabbis as an absolutely unbreakable rule. We have seen in our study of the Baba Qama 25a that some rabbis allowed its occasional breach. Maccoby fails to mention that—not, I think, out of dishonesty, but because he regards the *dayo* principle as identical with the principle of deduction (i.e. the logical rule that the conclusion of a deductive inference cannot contain information not given explicitly or implicitly in the premises). This belief of his is based only on the Sages' objection to the first argument of R. Tarfon in the Mishna, without regard to their objection to R. Tarfon's second argument (which Maccoby fails to notice and take into consideration).

Due to his limited understanding of the *dayo* principle, Maccoby does not give credence to the Gemara which throws some doubt on it. He does not admit the possibility that it might have been Torah-decreed, as claimed by the Gemara. Maccoby rejects the Gemara as a late interpolation by some comparatively ignorant Amora. He does not notice the fact that the Gemara presents this thesis as being of Tannaic origin (i.e. as a *baraita*), in which case it was historically much earlier than he supposes. Anyway, since the Gemara was settled, its account is accepted as kosher; that is to say, rightly or wrongly most rabbis do accept that a fortiori argument may occasionally be performed without regard to *dayo*. Indeed, they generally believe, following the Gemara, that the natural conclusion of an a fortiori argument is 'proportional', and that an artificial decree is necessary to prevent such conclusion. Maccoby does not convincingly take these givens of Judaism into consideration.[31]

Anyhow, returning to Maccoby's criticism of Paul, his analysis proceeds as follows. Since only Paul's fourth argument can be considered as valid, a success rate of one out of four can hardly be regarded as skillful performance. As well, every student in rabbinic academies knows the *dayo* principle; so Paul, who did not apply this rule, cannot have been a Pharisee. But as we have seen, of the three arguments by Paul considered invalid by Maccoby, two are in fact valid; and the remaining invalid argument is not invalid for the reason given by Maccoby. Moreover, the *dayo* principle is not an issue in any of the arguments that Maccoby focused on. So Maccoby's criticism here was unjustified.

Of course, Paul can still be criticized on other grounds, mainly the evident confusion in his way of thinking and verbal expression, not to mention his unorthodox religious ideas and values. A Christian commentator, one James Patrick Holding[32], unhappy with negative judgments of Paul by Maccoby, engages in ad hominem and other fallacious attacks on him, and then complains as follows:

> "Maccoby gives Paul a failing grade on 3 out of 4, accusing him of 'woolly, imprecise reasoning' and going 'far beyond the conclusion warranted' – the bottom line being, Paul cannot be a Pharisee or a rabbinic exegete, because he 'was arguing for a doctrine of which the Pharisees would have disapproved strongly.' (pp. 65-6). Now, did the reader catch that? Paul can't be a Pharisee or a rabbinic exegete, because he comes to conclusions that are false by Pharisee thinking... i.e., because he asserts that Christianity is true. All 4 of these arguments, in fact, are quite sensible if what Paul argues is based on what is true; but that is the very point at issue, and Maccoby has merely started by assuming from the get-go that Christianity as we know it is a Pauline fraud. Once again, all he does here is argue in circles."

This is an unfair counterargument disingenuously posing as logical criticism. Holding is saying that Maccoby denies Paul's Pharisee credentials simply because he dislikes his Christian conclusions. But this is evidently not Maccoby's

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[31]   I am greatly simplifying the issues here; the reader is referred to the earlier chapters on Talmudic a fortiori argument (7-8) for more precise treatment. Funnily enough, Maccoby and the Gemara have in common the failure to have noticed the significance of the second argument of R. Tarfon and the Sages' objection to it.

[32]   "Hyam Maccoby: A Critique" posted online at: www.tektonics.org/lp/maccobyh01.html. I do not know when this essay was published – obviously sometime between 1986, when Maccoby's book was published, and 2011, when I found the essay.

approach. Maccoby clearly bases his rejection of Paul's Pharisee pretentions on his (alleged) demonstration of Paul's inability to reason correctly and in accord with rabbinic standards and practices (as he sees them). There is no prejudice on Maccoby's part, no circularity in his argument. His denial of Pharisee status to Paul is Maccoby's conclusion, not his premise. He does not primarily question Paul's concrete conclusions, but the process through which Paul drew them, which he judges (albeit incorrectly) to be pseudo-logical. It is not the content of Paul's discourse Maccoby attacks, but its form.

Contrary to Holding's claim, the invalidity of Paul's arguments (according to Maccoby) does not depend on the truth of the premises – it is an issue of process. Whether the premises are true or false, conclusions obtained by such means are invalid. Invalid does not mean false – the conclusions may still be true for other reasons – but they cannot be true for the reasons advanced, since the process is faulty. This is elementary logical doctrine, which Holding has evidently not yet grasped. Let us recall that Maccoby has demonstrated his intellectual integrity with regard to Judaism, too – he does not fear to criticize apparent wrong reasoning by the Amoraic writer of the Gemara Baba Qama 25a. He evidently tries to be an unbiased observer. So he is not some Jewish fanatic blindly attacking Christian doctrine, as Holding tries to depict him.

Maccoby's critical attitude of Paul is, in my opinion, most fitting with regard to Romans 7:1-6, which reads as follows:

> "You cannot be unaware, my friends – I am speaking to those who have some knowledge of law – that a person is subject to the law so long as he is alive, and no longer. For example, a married woman is by law bound to her husband while he lives; but if her husband dies, she is discharged from the obligations of the marriage-law. If, therefore, in her husband's lifetime she consorts with another man, she will incur the charge of adultery; but if her husband dies she is free of the law, and she does not commit adultery by consorting with another man. So you, my friends, have died to the law by becoming identified with the body of Christ, and accordingly you have found another husband in him who rose from the dead, so that we may bear fruit for God. While we lived on the level of our lower nature, the sinful passions evoked by the law worked in our bodies, to bear fruit for death. But now, having died to that which held us bound, we are discharged from the law, to serve God in a new way, the way of the spirit, in contrast to the old way, the way of a written code."

I agree with Maccoby's analysis of this passage and his conclusion that Paul is "muddle-headed." Paul here claims an analogy between a widow who consorts with a man (since her husband is dead, her act is not adultery), and converts (i.e. ex-Jews, presumably) who have taken up a new religion (Christianity). Whereas it is the widow's husband's death that frees her from the law against adultery, in the case of the converts it is claimed that it is *they* who have died "to the law," and that "having died" they are "discharged from the law." Moreover, the man ("another husband") they consort with is someone who "rose from the dead," so that the predicate of death originally applied to the widow's husband is now implicitly applied to the widow's *consort*. Such discourse can rightly be characterized as "muddle-headed."

Of course, Paul is saying something comprehensible. He is saying that the converts being no longer bound to their "lower nature," having adopted the "way of the spirit," have no need of the "old way, the way of a written code" (i.e. the Torah), which was designed to control their "sinful passions" and indeed perhaps "evoked" them. But the issue here is not *what* he is saying, but *how* he is saying it. His form of discourse is faulty, whatever its content might be – and that reveals something negative about his intellectual abilities, i.e. his psycho-epistemology. In short, Paul does not have a very logical mind[33]. An amusing comment in this regard is passed on by Solomon Schechter[34]: "Harnack makes somewhere the remark that, in the first two centuries of Christianity, no man understood Paul except that heathen-Christian Marcion, and he misunderstood him."

This study of Paul's attempts at logic is of course not exhaustive, and an exhaustive study would of course be welcome. But enough evidence has been adduced for us to draw a rather negative overall conclusion as suggested by Maccoby. Personally, I have always found Christian discourse a bit befuddling; I now better understand why. I do however understand that Paul's language may sound pleasant to Christians. To their ears, its illogical structure is not

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[33]   Boaz Cohen, in "Letter and Spirit in Jewish and Roman Law," in: *Essential Papers on the Talmud*, ed. Michael Chernick (New York: NYU Press, 1994), pp. 399-428, seems to agree with this proposition, when he says: "Paul contradicted himself, when he claimed that the promise made to the seed of Abraham could not be annulled. Using the argument *a fortiori*, he argued as follows: 'Brethren, I speak after the manner of men. Though it be but a man's testament, yet if it be confirmed none disannulleth, or addeth thereto' (Galatians 3:15). How much more is it true, he argues [further on], of the promise given to Abraham, which was confirmed before God" (p. 35). However, while Cohen may be justified in accusing Paul of inconsistency, I cannot confirm his claim that an a fortiori argument is involved here: he is using the King James NT, and the words "how much more" do not appear there (or in any other version I looked at). Note that Cohen's paper originally appeared in *Jewish and Roman Law: A Comparative Study* (New York: JTSA, 1966).

[34]   *Selected Writings*. Oxford: East and West Library, 1946.

a problem, but music and poetry. The message of love and salvation is, finally, all that matters to them. They are not concerned with technical issues.

To conclude our brief study: it is not possible to judge whether Paul did or did not receive a Pharisee education merely by looking at his a fortiori argumentation. The only way to really answer this question is to examine the degree of Paul's knowledge (or lack of knowledge) of Jewish law, which we have not done here. What is evident from Paul's discourse is that he had, after his conversion to Christianity, a very negative opinion of Judaism. This can be seen for instances in the two a fortiori arguments in 2 Corinthians 3:6-9:

> "Who has also made us able ministers of a new covenant; not of the letter, but of the spirit: for the letter kills, but the spirit gives life. For if the ministry of death, written and engraved in stone, was glorious, so that the children of Israel could not steadfastly behold the face of Moses because of the glory of his countenance; and that glory fading: Shall not the ministry of the spirit be even more glorious? For if the ministry of condemnation had glory, far more does the ministry of righteousness exceed it in glory."[35]

Notice the very negative characterizations of Judaism as "the letter [that] kills," "the ministry of death," and "the ministry of condemnation." Whether this strong antipathy was the result of a close contact with Judaism in the past which ended in deep disillusionment or was the result of very superficial past acquaintance with it needs investigating. My personal impression offhand – I claim no expertise in the matter – is that Paul's criticism of Judaism was very overall, very vague: it was not the criticism of an ex-scholar, i.e. someone able to formulate detailed and conclusive arguments. As regards Paul's substantive accusation, here, viz. that Judaism is a religion concerned with 'the letter' (i.e. soulless rituals) instead of 'the spirit' (i.e. soulful consciousness of God), this can easily be refuted.

While it is true that Jewish worship is always in danger of being an empty shell, it is also true that we are always expected to overcome this natural tendency and put some life into our worship. This problem of form without content is present in all religions, because it is part and parcel of the human condition. It is present even in Christian ritual – someone can regularly go to church, yet spend his or her time there idly chatting with a neighbor; or someone can be always nattily dressed as a bishop, yet behind the scenes engage in despicable pederastic orgies. Even in Zen meditation, which is in principle devoid of ritual, it is easy to lose one's concentration and be carried away by random thoughts. The human mind is fickle and readily wanders off; repeated effort is required to produce and sustain full presence of mind. Sometimes, the mind perversely does the opposite of what one wants it to do.

In Judaism, as in other religions, external observance without inner commitment is not regarded very highly. In Isaiah 29:13, God is reported as complaining that: "this people draw near, and with their mouth and with their lips do honour Me, but have removed their heart far from Me, and their fear of Me is a commandment of men learned by rote" [36]. Jewish teachers today are well aware of the danger, and they enjoin people to add *kavanah* (intention) to their observance, and not just practice in a mindless manner. That is, when praying to be aware of the words and to mean them; when donning phylacteries to do it attentively, aware of the significance of the act; and so forth. No doubt it was the same in Paul's day, i.e. during the Mishnaic period. However, Judaism does not condemn inattention outright, as Paul seems to have done. Performing the *mitzvot* (commandments) in a mechanical manner is not recommended, but it is certainly accepted as better than not performing them at all. Judaism is realistic and aware that it is not easy for most people to attain the ideal of full attention and intention.

Another common interpretation of Paul's letter-spirit dichotomy is that, whereas Judaism was a religion which put the emphasis on "works," Christianity was to stress "faith." To be "saved," a Jew had to "perform," whereas a Christian needed only to "believe." This, I would say, was a lure, a sales pitch – for I cannot conceive of anyone being accepted as a good Christian who does not eventually behave in a certain way, a way Christians consider acceptable. Of course, entrance into the fold has to be free of charge, or almost so, to attract converts; but once in, the convert must to a large extent conform to the norms and mores of the group he or she has joined, or face rejection. This is true in all religions. Faith and conduct cannot logically be dissociated: good conduct expresses sincere faith; whereas bad conduct expresses insincere faith, which means: lack of faith. Conduct is the seal of truth and measure of faith. Actions speak louder than words. One can, of course, argue about which actions are best; but some sort of action is necessary.

The importance of faith (in God and in the Torah) is not denied in Judaism, but on the contrary it is emphasized. For instance, the Gemara (Makkoth 24a) teaches, in the name of R. Simlai: "But it is Habakuk who came and based them

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[35]   For the record, these arguments are both positive subjectal, since they go from minor to major. Also, the words "even more glorious" and "exceed it in glory" suggest they are intended as a crescendo. Note in passing Paul's usual rhetorical resort to opposites: the letter kills *vs.* the spirit gives life, the ministry of death *vs.* that of the spirit, the ministry of condemnation *vs.* that of righteousness. However, the arguments are formally valid, because they are not a contrario, i.e. their subsidiary term (glorious, having glory) remains the same in minor premise and conclusion, varying only in degree.

[36]   Mendell Lewittes mentions this passage, and others drawn from the Tanakh and the Talmud to the same effect, in his *Principles and Development of Jewish Law* (New York: Bloch, 1987), p. 7.

[the 613 commandments] all on one [principle], as it is said: 'But the righteous shall live by his faith'." This passage (Habakuk 2:4) just means that faith is necessary *for* religious practice, not that it replaces it. The 613 commandments are "based on" faith; it is they, and not merely the faith underlying them, which make a man "righteous" and make possible for him to "live" spiritually. Faith is indeed part of the spiritual path, but it cannot by itself take a man very far along it. Additional acts are required, on the material, mental and spiritual planes, to progress further.[37]

So, Paul's criticism of Judaism is unfair. I am not saying that he had no right to criticize it, note well. I would and do say that Judaism deserves criticism in many respects. But I just do not think Paul zeroed in on precisely what, in it, deserves criticism. Note also that his criticism is not logically applicable to all Jewish law. Although many Jewish laws have to do with ritual (laws relating to animal sacrifices, permitted/forbidden foods, ritual purity, and so forth), many Jewish laws have nothing to do with ritual. They are aimed at ensuring justice, peace and social cohesion; they are about murder, theft, damages, commerce, inheritance, marriage, divorce, charity, and so on. Surely Paul was not against such laws, which are necessary (in some form or other) for any functioning society! Certainly, Christian societies also had to and do have such laws. So his comments lack precision in this respect too.

We might further speculate that Paul was a man ahead of his time, who found the rigid regulation by Judaism of all aspects of people's lives all the time to be oppressive and antithetical to genuine spirituality. This was the rule (i.e. the dominion) of the "letter"– always having to follow some regulation or other – and he advocated in its stead the rule of the "spirit" – a freer, more spontaneous approach to worship of God. Many of today's Jews would, in truth, agree with this more secular vision of religion, although those with some experience of Judaism know this to be something of a caricature. Some religious people are indeed spiritually inert; but others manage to lead inspired, lively lives. But was Paul really as modern as this reading suggests? Remember that before his conversion to Christianity Paul was by his own admission an extremist, persecuting Christians[38] against the advice of his Pharisee teacher Gamaliel. Judging by the negative tone of his subsequent statements against Judaism and Jews, it does not seem that his character radically changed through conversion. He was still antagonistic; only the target of his animus changed.[39]

Judging by his harsh words, I think Paul had a personal ax to grind. He verbally degrades Jews, speaking of their "fall," "failure," and "rejection." The term "ministry of condemnation" is perhaps indicative of his feeling rejected by fellow Jews. The term "ministry of death" is perhaps indicative of the deep pain such rejection caused in him. These terms may also allude to the condemnation and killing of Jesus, and thus perhaps intend a blood libel, but their main intent is clearly criticism of the Torah, the doctrine "engraved in stone." Considering Paul's Jewish roots, one can't help comparing his behavior to that of 'self-hating' Jews of the present day, like Noam Chomsky, George Soros or Yariv Oppenheimer, to name but three, who due to some obscure personal resentment against other Jews, perhaps merely wounded pride, go abroad sowing seeds of ill-will against the Jewish people. The irony of their position is that it is precisely because they are born Jews that their words are given weight. Such Jews forget the Torah's admonition (Leviticus 19:18):

> "Thou shalt not take vengeance, nor bear any grudge against the children of thy people, but thou shalt love thy neighbor as thyself."

Note that this is a purely spiritual instruction, without an iota of ritual in it. One cannot fulfill the letter of it without fulfilling the spirit of it. One cannot fake it, or go through the motions of it half-heartedly. It enjoins us to be conscious of our passions and learn to actually dominate them. This is not easy to do, when one's feelings are hurt and one yearns to get back at those who hurt them. Did Paul practice this commandment? I suspect not. It is doubtful, anyway, that he took it and others like it into consideration when he accused Judaism of consisting of "letter" without "spirit."

But the bottom line in any Jewish criticism of Paul is his Christian belief that Jesus was "the Son of God." This was a radical break from Judaism. The concept is not, has never been and will never be part of Judaism. The idea of a "messiah" is Jewish – but this refers to a human being, not to an incarnation or offspring of God; a spiritually exceptional man, to be sure, but still a man. The moment any Jew accepts the idea of a "son of God," he places himself outside the bounds of Judaism; he belongs to a very different religion, called Christianity. This is not "a sect" of Judaism, even if historically rooted in it and sharing many beliefs and values with it; it is something apart, with its

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[37]   Of course, faith in whom or in what is another question to ask here. In Judaism, the faith needed is faith in the existence of God and in His having revealed the Torah. In Christianity, the faith needed is faith in the divinity of Jesus and in his saving power. In Islam, Buddhism and other religions, the faith needed in each case is something else again.

[38]   Bar-Zeev makes the interesting suggestion that Paul may have, in Acts 22:4, exaggerated his persecution of Christians ("to the death, binding and delivering to prison both men and women") so as to make his conversion appear all the more radical. We have, after all, only his word for it.

[39]   As far as I know (I might be mistaken), however, Paul did not instruct or even merely permit Christians to persecute Jews. Anti-Semitic acts were probably a later development.

own course and destination. It is not a Jewish path at all, even though there are some people today – the "Jews for Jesus" and other Christian missionaries – who pretend that it is.

4. In later Christian discourse

Knowing that a fortiori argument is frequently used in the Christian Bible, and indeed in the Jewish Bible which Christians also study often, and moreover that a fortiori argument was often enough used in the Greek and Roman worlds where Christianity took root and evolved, it is natural to expect that such argument would be commonly used in later Christian literature. I searched mechanically for 19 key phrases[40], which are usually indicative of a fortiori discourse, in a couple of important Christian writings to confirm this prediction.

In the *Confessions*[41] of Augustine of Hippo (354-430 CE), I found 10 possible cases: how much more (2), how much less (2), so much less (1), how much the more (1), so much the more (3), and even more (1). In the *Summa Theologica*[42] by Thomas Aquinas (1225-1274 CE), I found 176 cases: a fortiori (3), all the more (54), all the same (2), how much more (15), how much less (2), so much more (4), so much the more (32), so much the less (4), still more (40), still less (12), even more (8). Note that I found no case of the expression 'a fortiori' in Augustine's book, and only three cases in that of Thomas Aquinas.[43]

The exactitude of the statistics given here is of course not very important. For this reason, I did not verify that each case counted was indeed an a fortiori argument. Also note, it may well be that some a fortiori arguments were not counted, because they do not involve the key phrases used as search strings. I did notice that a few of the a fortiori arguments in Thomas Aquinas' book were actually quotations from the Jewish and Christian Bibles; but that is an interesting finding in itself, so I did not exclude those cases.

Also noteworthy is that neither Augustine nor Thomas Aquinas *discuss* the a fortiori argument in these two books – they only *use* it. Well, that was to be expected, since these books are not inquiries into logic theory. I do not know where in Christian literature (as against literature by Christians) I might find discussion of the a fortiori argument as such. This is of course a more interesting topic of research than merely counting how often the argument is used. Wiseman[44] mentions a book that might be helpful in this regard:

> "A similar example to the lenient dayo is from the Digest of Justinian, no 49, in Albert Gautier, *Introduction to Roman Law for Studies in Canon Law*, (Rome: Faculty of Canon Law, St. Thomas University, 1994), page 154: *In poenis bensignior est interpretatio facienda.* 'In penalties, the more benign interpretation is to be applied.'"

I have not looked at this book, but judging from this brief citation it seems to point to a principle in Christian (or is it earlier Roman?) law that resembles the rabbinic *dayo* principle. Where it differs, perhaps, is in its apparent general applicability to penal law (note the unqualified reference to "interpretation"), whereas the rabbinic sufficiency principle is usually considered as specific in Jewish law to 'proportional' a fortiori argument (although I would view it as more broadly applicable to any 'proportional' argument).

"In penalties, the more benign interpretation is to be applied." This is the kind of text worth looking into. There are, of course, many potential resources in the Internet, including full listings of Catholic and other canon law codes. However, I will not attempt to research this issue further. It is something that should be dealt with by someone with more than my minimal interest in and knowledge of Christian hermeneutics. This job surely has yet to be done – because it can only be properly done in the light of my novel work in a fortiori logic, so whatever was done before it is inadequate.

5. Additional findings

Surfing again through the Internet more recently, after writing all the above, I discovered (to my dismay) that there are many passages of the NT that are considered as a fortiori and that I have not included in my list. I found these additional instances in various websites, where they are used for preaching purposes; but I did not note down the names of the websites. There are, I have little doubt, more instances to be found; but I did not pursue the matter

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[40]   Namely, briefly put, using the first letter of each word: *af, atm, atl, ats, hmm, hml, smm, sml, hmtm, hmtl, smtm, smtl, sm, sl, stm, stl, sts, em, em.*
[41]   *The Confessions of Saint Augustine*, a Project Gutenberg Etext (2002) prepared by Robert S. Munday.
[42]   A Project Gutenberg Etext (2006) produced by Sandra K. Perry, with corrections and supplementation by David McClamrock, based on the Complete American Edition, translated by Fathers of the English Dominican Province. (This eBook is divided into four parts, with a total of 3735 pages.)
[43]   Of course, these documents are translations from Latin, and the precise choice of words in them depends on the whim of the translator(s).
[44]   In *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions*, p. 165.

further. The instances I found were: Matthew 10:29-31, Luke 16:10 (2 instances, of which 1 invalid) and Luke 16:12, Luke 18:6-8, Romans 8:32, John 7:23 and John 10:35-6. Of these 8 instances, as we shall now see, 7 are technically valid while one is invalid.[45]

In **Matthew 10:29-31**, Jesus says: "Are not two sparrows sold for a penny? Yet not one of them will fall to the ground outside your Father's care. And even the very hairs of your head are all numbered. So don't be afraid; you are worth more than many sparrows." The a fortiori argument here is: if a sparrow (Q), whose is worth (R) little, is under God's protection (Sq), then you (P), who are worth (R) much more, is under God's protection even more (Sp) – so don't be afraid. This is clearly an a crescendo argument of positive subjectal form, going from minor to major, and therefore valid.

Concerning **Luke 16:10 and 12**. In Luke 16:1-9, Jesus tells the parable of a manager who is asked by the property owner, who suspects him of "wasting his possessions," to give an account of his work; so the manager uses his position to cancel a portion of the owner's debtors' debts, so as to gain their favors in case he loses his job; whereupon (surprisingly[46]) the owner commends the manager for shrewdness! Presumably, then, the owner was initially dissatisfied with the manager, not because he wasted his possessions in worldly economic terms, but because he wasted them on material pursuits instead of using them in spiritual pursuits; for Jesus uses this story to illustrate the dictum: "use worldly wealth to gain friends for yourselves, so that when it is gone, you will be welcomed into eternal dwellings." Then in Luke 16:10-12, Jesus makes four statements, apparently connected to this parable, which seem intended as a fortiori arguments. The problem with them is that while the last two are valid, only one of the first two can be valid.

Verse 10 states: "Whoever can be trusted with very little can also be trusted with much, and whoever is dishonest with very little will also be dishonest with much." These would appear to be two purely a fortiori arguments: (a) If someone (S) is trustworthy (R) with very little (Q), then he (S) is trustworthy (R) with much (P); and (b) if someone (S) is not trustworthy (R) with very little (Q), then he (S) is not trustworthy (R) with much (P). Granting that "very little" and "much" refer to *responsibilities*, then the argument of v. 10(b) would be valid since it is minor-to-major and negative predicatal; but the argument of v. 10(a) would be invalid since it is minor-to-major and positive predicatal. If we tried to fix this problem by interpreting "very little" and "much" as referring to *demands for rewards*, say, then argument (a) would be major-to-minor and valid, but (b) would be minor-to-major and invalid. Therefore, Jesus is here contradicting himself, however we interpret his words in v. 10.[47]

Verses 11-12 state: "So if you have not been trustworthy in handling worldly wealth, who will trust you with true riches? And if you have not been trustworthy with someone else's property, who will give you property of your own?" These two arguments are purely a fortiori, negative predicatal in form, and apparently intended to go from minor to major, which means they are valid. Moreover, they resemble argument (b) of v. 10 – which suggests that the missing word in v. 10 is 'responsibilities'; in which case we can say with some certainty that it is argument (a) of v. 10 which is invalid. In any case, it is sure that Jesus commits an error of logic somewhere in this passage. Obviously, someone who makes such an error cannot claim to be infallible (or at least, the document where the error is made cannot be claimed to be an accurate report). Note that I have already mentioned Luke 16:11 in my earlier list, so I do not count it as an additional instance here.

Concerning **Luke 18:6-8**. In Luke 18:1-5 Jesus tells the parable of "a judge who neither feared God nor cared what people thought" who was repeatedly approached by a widow demanding justice in a case against someone; at first the judge refuses to get involved, but finally decides to do her justice so as to get her off his back. Jesus uses this story to encourage his disciples to "always pray and not give up." Then in Luke 18:6-8 he proposes the following argument: "Listen to what the unjust judge says. And will not God bring about justice for his chosen ones, who cry out to him day and night? Will he keep putting them off? I tell you, he will see that they get justice, and quickly." This can be read as a purely a fortiori argument, as follows: if even an unjust human judge (Q) is (say) wise (R) enough to eventually relent and do justice if insistently appealed to (S), then God who is just (P) is wise (R) enough to eventually relent and do justice if insistently appealed to (S). We could also read the argument as a crescendo, if we mention the "widow" in the subsidiary term in the minor premise and the "chosen ones" in the subsidiary term in the

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[45]   All quotations are from the New International Version, given at: www.biblegateway.com.

[46]   This is a weird parable. A man gives away, without permission, his boss' money to third parties, in order to gain favor with them – and his boss would praise him? Surely, there is a big moral difference between being generous with one's own possessions, and stealing and distributing someone else's wealth to make friends.

[47]   His discourse is partly nonsensical, either way. If we say that the intent was 'responsibilities', then argument (a) is questionable, since obviously just because an employee can handle easy tasks, it does not follow that he can handle difficult ones. If we say that the intent was 'demands for rewards', then argument (b) is questionable, since obviously just because an employee is unreliable when dissatisfied with the rewards, it does not follow that he will be unreliable when rewarded as he wishes. Note well I say 'it does not follow', which does not exclude that the antecedents and consequents may on occasion occur together.

conclusion; and it seems clear that the latter interpretation is the more accurate one. In any case, the argument is positive subjectal, and goes from minor to major, and is therefore valid.

In **Romans 8:32**, the author (Paul) argues: "He who did not spare his own Son, but gave him up for us all – how will he not also, along with him, graciously give us all things?" The a fortiori argument intended here is: if God (S) cared for humans (R) enough to give up his son for them (P), then He cares for them (R) enough to graciously give them all things (Q). This is purely a fortiori, positive predicatal argument, going from major to minor, and therefore valid. We could throw some doubt on Paul's reasoning here, if we consider that the predicate of the minor premise is "give up his son" and that of the conclusion is originally "give up his son and graciously give people all things" – for then the latter is clearly "more" caring than the former, and the argument becomes minor-to-major and therefore invalid. However, we can just retort that the conjunction between "give up his son" and "graciously give people all things" occurs after the valid a fortiori argument has concluded.

In **John 7:23**, Jesus argues: "if a boy can be circumcised on the Sabbath so that the law of Moses may not be broken, why are you angry with me for healing a man's whole body on the Sabbath?" That is to say: if circumcision on the Sabbath (Q) is important (R) enough not to be contrary to Mosaic law (S), then healing a man's whole body on the Sabbath (P) is important (R) enough not to be contrary to Mosaic law (S). This is purely a fortiori argument, of positive subjectal form, going from minor to major, and therefore technically valid. Regarding the material issue at hand, as I explain in an earlier section of the present chapter (10.2), the dispute seems to have been unnecessary, since Mosaic law does not in fact forbid, but rather encourages (when the danger is serious enough), curing a sick person on the Sabbath.

In Judaism, the protection of life is considered a paramount value, which makes all other values possible; there are some exceptions to this rule, but Sabbath observance is not one of them. Moreover, it is doubtful that "healing" in the sense here used constitutes Sabbath "work," in which case it is not forbidden as such (though it might be forbidden as sorcery). So what is Jesus trying to prove, by means of this a fortiori argument? Not that Mosaic law allows healing people on the Sabbath, but that he is above Mosaic law. According to him (or to the book's author, John), Mosaic law does – absurdly, inhumanely – forbid healing on the Sabbath, and Jesus – who is "sent" by God (v. 18) – decrees otherwise. Thus, the a fortiori argument is not intended as a proof within the Mosaic system of law, but as a rejection of that system (at least in the said instance). Note Jesus' adversarial attitude, indeed his paranoia, implied by his statement (in v. 19) "Why are you trying to kill me?" – to which the crowd incredulously replies (in v. 20) "Who is trying to kill you?"

In **John 10:34-6**, Jesus argues: "Is it not written in your Law, 'I have said you are gods'? If he called them 'gods', to whom the word of God came—and Scripture cannot be set aside—what about the one whom the Father set apart as his very own and sent into the world? Why then do you accuse me of blasphemy because I said, 'I am God's Son'?" The a fortiori argument intended here is: if people to "whom the word of God came" (Q) are sufficiently exalted (R) to be called 'gods' (Sq), then a person "whom the Father set apart as his very own and sent into the world" (P) is sufficiently exalted (R) to be called "God's Son" (Sp). This is clearly an a crescendo argument, of positive subjectal form, going from minor to major, and therefore technically valid.

However, it should also be pointed out that the argument is circular, or at least 'custom made' to yield the desire conclusion. The context for it is given in verses 30-33 – there we see Jesus claiming: "I and the Father are one." Some people then pick up stones to stone him. He apparently does not understand why they would want to do that, saying: "I have shown you many good works from the Father. For which of these do you stone me?" To which they reply: "We are not stoning you for any good work, but for blasphemy, because you, a mere man, claim to be God." Whereupon Jesus offers the said a fortiori argument. But it is clear that this argument does not prove him to be "God's son," since it is premised on the already controversial idea that he is a person "whom the Father set apart as his very own and sent into the world;" and moreover, though the purely a fortiori conclusion that he is to be called a 'god' might follow from that, we have only his say-so that the argument ought to be a crescendo and conclude with a proportionately higher title.

Summing up. Thus, to conclude, there are at least eight more a fortiori arguments in the NT to add to our earlier list of such arguments. Of these, 1 is in Matthew, 4 are in Luke, 1 is in Romans and 2 are in John. One of those in Luke is, as we have shown, invalid, while all the others are formally valid (even if we may disagree with their contents). All these arguments are spoken by Jesus, except the one in Romans which is authored by Paul.

As regards the forms of argument used[48]: 4 are positive subjectal, 2 are positive predicatal and 2 are negative predicatal; also, 5 are purely a fortiori, while 3 are a crescendo. As regards the language used in them to suggest a fortiori reasoning: in Matthew 10:29-31, the words "even" and "more" seem to play that role; in Luke 16:10, the

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[48]   Matthew 10:29-31 {+s &}, Luke 16:10 {+p, –p}, Luke 16:12 {–p}, Luke 18:6-8 {+s &}, Romans 8:32{+p}, John 7:23 {+s}, John 10:35-6{+s &}.

words "very little," "much" and "also" are involved; in Luke 16:12, the rhetorical "if not, then who?" is used; in Luke 18:6-8, it is "will not?"; Romans 8:32 similarly has: "how will he not also?"; John 7:23 resorts to "why?"; and John 10:35-6 to "what about?". Evidently, the language used is not distinctively a fortiori, even if we clearly intuitively see in each case that the intent is so.

I have not tried to merge the results of my earlier and more recent research more seamlessly, partly so as to avoid rewriting the present chapter altogether, and partly so as to show readers how to proceed with new findings – since it is possible if not probable that there will be still more instances found in the future. But I can briefly sum up as follows: we have thus far discovered a grand total of 28 + 8 = 36 instances of a fortiori argument in the NT.

11. Islamic 'logic'

In this chapter, I am called upon for the sake of comprehensiveness to comment on some of the a fortiori discourse found in Moslem literature. I must stress that I do not intend the following treatment to be exhaustive. I am merely breaking ground for a more extensive treatment by others. Being personally not very interested in the Moslem religion, I am not sufficiently motivated to do a thorough job on the subject. I do hope someone else will take up the challenge and do the necessary research.

Needless to say, although I am a Jew, I have no desire to engage here in religious polemics against Islam. Jews do not normally try to convert non-Jews to their views. My interest here is entirely logical. The proof is that I am not always critical. When I am critical, it is with an impartial, scientific spirit – the same spirit I apply to assessment of a fortiori and other forms of reasoning found in Jewish texts or texts of other traditions.

It should of course be unnecessary for me to make such a disclaimer, but we sadly live again in an age where tempers easily flare in relation to religion (which is understandable) and even sometimes lives are threatened (which is inexcusable). Some decades ago, this was not the case (at least not in the Western world), although a few centuries ago the threat against free speech was indeed high (in Europe as well as in Islamic regions). But nowadays, there are unfortunately some dangerous fanatics around, so speaking one's mind freely takes a bit of guts. I am determined to do so, and not let myself be intimidated.

1. Logic in the Koran

A bit of systematic research. I have carefully searched through an English translation (from Arabic) of the Koran, looking for instances of a fortiori argument, or indeed any other logical argument I might find, by means of the same key words and phrases used in other contexts – and to my great surprise I found exactly *none!* The Koran seems to be a long harangue, with no resort to logical argument, let alone a fortiori argument.

The translation of the Koran I relied on is that by John Medows Rodwell, which dates from 1861, with a 2nd ed. in 1876. I acquired a Kindle edition (2011) of this document, which can be searched through using the Kindle reader. The following is a summary of the results of this research by means of key words or phrases in the Koran (to the exclusion of hits in the introductory and explanatory notes by the translator):

I first looked for idiomatic words or phrases that are often indicative of a fortiori argument. I found no instances of the specific key phrases *much the* (more/less) or *much more/less*; the vaguer key word *much* occurs 14 times, but none of these constitutes an a fortiori. The key phrases *all the more/less/same* were likewise not found; the vaguer phrase *all the* occurs 8 times, but again none of these involves a fortiori. The key phrases *still more/less* were likewise not found; the vaguer phrase *still* occurs 17 times (including 2 instances of *still the*, though none of *still the more/less*), but again none of these involves a fortiori. I found one instance of each of the key phrases *even more, less*, but neither involves a fortiori. There were no hits for the key phrases *more so, less so*, although each of the key words *more, less* registered over 100 hits (I did not look through the latter).

Next I looked for descriptive expressions, which might signal a fortiori argument or some other form of argument. The expression *a fortiori* occurs zero times. *Argue, argued, arguing* never occur; *argument(s)* occurs twice. *Prove* occurs 31 times (including *proved* twice and *proven* twice); *proof(s)* occurs 48 times[1]. *Deduce* and its derivatives never occur, except for *deduction* which occurs once (in "make no deduction"). *Infer* and its derivatives never occur. *It follows* never occurs, and *not follow* occurs 3 times. *Logic* or its derivatives never crops up. *Therefore* occurs over 100 times. However, although words or phrases relating to logical processes do (rarely, as these statistics show) occur – none of the cases found turned out to concern logical processes (I examined all the instances here mentioned, though only a sample of those with *therefore*).

There are no logical arguments ("since this and that, therefore so and so") – there are only *rhetorical* claims. For example: "Say: I only follow my Lord's utterances to me. This is a clear proof on the part of your Lord, and a

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[1]   Note that this includes cases where the meaning of 'to prove' is to test the faith or loyalty of *a person*, rather than to show the logic of a proposition.

guidance and a mercy for those who believe." The mere use of a term (such as "proof") normally associated with logic is not proof that logic is being used. Thus, it would appear from this research effort that there is no logic use in the Koran. The sweet voice of reason is never actually used. This is quite a shocking finding, which goes some way to explain the dogmatic style of Islam.

Note that this conclusion does not exclude the possibility that closer reading might reveal some use of logic, because it is based on mechanical search of key words and phrases. However, in similar research efforts elsewhere – in the Jewish Bible, or the works of Plato or Aristotle, or the Christian Bible – such mechanical search has always yielded some results, even if admittedly incomplete results. So, it looks as if there is no use of logic in the Koran.

In truth, after writing the above I discovered that there is in the Koran at least one passage that can reasonably be admitted as a fortiori, namely **36:78-79**:

> "He [man] says, 'Who will give life to bones while they are disintegrated?' Say [to him], 'He [God] will give them life who produced them the first time; and He is, of all creation, knowing.'"

Although here there is no key phrase indicative of a fortiori argument, there is a connection between the sentences in the fact that the first is a question and the second is an answer to it. Moreover, since the reply "He will give them life" would have sufficed, it is obvious that the clauses "who produced them the first time" and "He is, of all creation, knowing" are intended as additional explanations for that reply. The argument here is clearly that if God (S) was powerful (R) enough to create man in the first place (P), He (S) is surely just as able (R) to resurrect him long after he dies (Q). This is a positive predicatal argument[2], since the subsidiary term S (God) is the subject of the minor premise and conclusion. It would be counted as *a pari*, since the premised act P (initial creation) is not presented as more or less difficult than the concluding act Q (resurrection)[3]. Indeed, the additional comment that God fully knows creation implies that both these acts are equally easy for Him[4]. Lastly, the argument is purely a fortiori, not a crescendo, since the subject (God) is the same in the minor premise and conclusion.

So there is, after all, at least one a fortiori argument in the Koran. Maybe there are others, but so far this is all I have found – a pretty poor harvest, anyway.

Logic in the Koran spotted by al-Ghazali. Additionally, there are some syllogisms and other arguments in the Koran. The Moslem commentator Abu Hamid al-Ghazali (11th-12th cent. CE) draws attention to a number of them in his book *al-Qistas al-mustaqim* (The Correct Balance)[5]. Incidentally, he wrote this book, not as one might expect in defense of rationalism, but in order to show that logic was used in the Koran long before Greek logic made its way into Islamic discourse. Nevertheless, it is evident from his systematic treatment, starting with the first three figures of Aristotelian logic and continuing with the hypothetical and disjunctive arguments of Stoic logic, that he was himself strongly influenced by that logic. Despite this and other logical works, he is regarded, rightly, in view of his overall philosophical and religious orientation, as an anti-rationalist.

A syllogism is suggested in **Koran 2:258**, where Abraham says to Nimrod: "God brings up the sun from the east, so bring it up from the west." Ghazali explains that Abraham argued thus because "Nimrod claimed divinity" and he wanted to prove him wrong, so his argument in full would have been: "Whoever can make the sun rise is God; but my God can make the sun rise; [therefore] my God is God – and not you, Nimrod" (1/AAA). Ghazali justifies the major premise by saying that "'God' is a designation for the omnipotent, and making the sun rise belongs to the totality of those things [which he can do]; this principle is known by convention and agreement." He justifies the minor premise by saying that "'The one who can make the sun rise is not you [Nimrod]' is known by seeing."

The first figure argument proposed by Ghazali is formally valid. Its major premise can be justified as he suggests on purely rational grounds: given the definition of God as omnipotent (not to mention omniscient and all-good), it would follow that He can well cause the sun to rise (from the east or west or anywhere). But Ghazali's justification of the minor premise is clearly fallacious. For granted that Nimrod can evidently not make the sun rise (in the west or anywhere else), it does not follow that Abraham's God can do so. To assume the minor premise on this basis would be to argue in a circular manner. The argument implied by the Koran rather proceeds as follows. First, we reason:

2   Note that the identification of the argument as predicatal in form is mine; I have found no evidence so far that Islamic commentators are at all aware of the differences between predicatal and subjectal arguments. As we shall see further on, they seem to have only noticed the subjectal form.

3   The major premise of the argument is clearly: "As much power is required to produce new life as to recover past life." But it could be "More power is required, etc."

4   This Koran argument from one power of God to another is reminiscent of some in the Jewish Bible: Psalms 78:20, which states that if God is powerful enough to draw water from a rock, then He is powerful enough to feed His people with bread and meat; and Psalms 94:9-10, which states that if God is powerful enough to implant the ear and form the eye, then He is powerful enough to hear and see, and if God is powerful enough to chastise nations, then He is powerful enough to reprove individuals.

5   Trans. Richard J. McCarthy. Boston: Twayne, 1980. The title is drawn from a statement in the Koran. A free .pdf version of this work is available online at: www.ghazali.org/books/jb-4.pdf. Chapters 2, 3 and 4 concern respectively 1st , 2nd and 3rd figure syllogism; chapters 5 and 6 concern respectively hypothetical and disjunctive apodoses. More on Ghazali further on.

"Whoever cannot make the sun rise is not God; Nimrod cannot make the sun rise (differently than it does, i.e. in the west instead of the east); therefore, he is not God (1/EAE)[6]. This syllogism does not in itself prove that the God Abraham believes in is one and the same as the God defined as omnipotent. For that conclusion we must take for granted the Koran's implicit disjunctive proposition: Either Abraham's God or Nimrod is God. On this basis we can then argue: Since Nimrod is not God (as just proved syllogistically), then Abraham's God must be God.

Thus, the argument implied by the said Koran passage is not as Ghazali describes it. A syllogism is implied, but not the one he proposes. Additionally, there is a disjunctive apodosis he has not discerned. Moreover, the premise that either Abraham's God or Nimrod is God remains unproved, although obviously presented approvingly by the Koran. It can only be truly proved by eliminating all other possibilities, including the thesis of atheism. In other words, merely observing the sun rise in the east does not suffice to prove that it is God as Abraham conceives or experiences Him, or even the omnipotent Being men generally define as God, Who in fact *made* the sun rise. We might well assume so on faith, but to scientifically prove it is more difficult. Therefore, the Koran cannot truthfully be said to have produced a proof of the existence of God, or even a proof that Abraham's God is God, through this argument[7].

Another example of syllogistic reasoning is found in **Koran 6:75-78**:

> "And thus did We show Abraham the realm of the heavens and the earth that he would be among the certain [in faith]. So when the night covered him [with darkness], he saw a star. He said, 'This is my lord'. But when it set, he said, 'I like not those that disappear'. And when he saw the moon rising, he said, 'This is my lord'. But when it set, he said, 'Unless my Lord guides me, I will surely be among the people gone astray'. And when he saw the sun rising, he said, 'This is my lord; this is greater'. But when it set, he said, 'O my people, indeed I am free from what you associate with Allah. Indeed, I have turned my face toward He who created the heavens and the earth, inclining toward truth, and I am not of those who associate others with Allah'."

Although Abraham's thinking here is not made fully explicit, it is reasonable to suppose it was syllogistic. The three syllogisms involved here would have the form 2/EAE: God does not disappear; the stars, moon and sun do disappear; therefore, these cannot be God. So well and good; except that the content of these arguments is not very convincing. We can criticize them by pointing out that disappearance does not necessarily imply cessation of being – something may disappear by merely ceasing to be visible. To be hidden from view is not to be non-existent. For example, since God is formless, He is not perceivable through the senses[8], yet this 'invisibility' does not imply His non-existence. Thus, the proposed middle term is inaccurate, and the major premise ought rather to be: Since God is eternal, He cannot be *impermanent*. In that case the minor premise would need to be: the stars, moon and sun are *impermanent*. Only then would the putative conclusion that 'these cannot be God' be logically justified.

Abraham could well on the basis of observation say that the stars, moon and sun disappear daily; but to state that they are impermanent, he would have to rely on extrapolation – i.e. on the assumption that when these heavenly bodies disappear they actually cease to exist. Such extrapolation would, of course, constitute an inductive act – a generalization from experience. In his day, and maybe even at the time the Koran was written, such a supposition might have seemed credible. But of course, nowadays we know it is nonsense. The stars, moon and sun do not cease to exist when they disappear – we just no longer see them from where we happen to stand, although they remain or become visible elsewhere. They are indeed impermanent, but not for the reason given – i.e. not because they disappear daily. They are impermanent because they undergo changes (visible with telescopes) all the time, and because they will cease to have their present forms (as stars, moon or sun) in a few billion years.

Of course, Abraham could hardly know that. Maybe also the much later author of this story in the Koran could hardly be expected to have this knowledge, which we have thanks to modern astronomy and all the technology it is based on. Still, the author of this story ought to have realized the logical weaknesses in it, if he knew logic. This story seems designed to establish that knowledge of the existence of God can be known through intellectual means, i.e. that it is rationally obvious. But as we have just shown, it is not very successful in demonstrating that possibility. This shows that, even if Abraham, or at least the author of the Koran, could think syllogistically, he was not sufficiently skilled in logic to see that the proposed argument was not watertight and needed improvement. We can still say there is some (although not very much) syllogistic reasoning in the Koran, but it cannot be said that these three occurrences are demonstrative of the Divine source of the document, since God, being omniscient, would not make errors of fact or of reasoning.

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[6]   The argument is valid even though the middle term ("cannot make the sun rise") is negative in content, because it is the same in both premises.

[7]   Note well, I am not personally denying the proposition, but merely showing it to be unproved. See § 17-19.

[8]   Although, to be sure, it is said that God makes his presence known to prophets indirectly through sounds or sights, and perhaps also to ordinary people who are open to it through intuition. So, in one sense He can be said not to appear and therefore not disappear, and in another sense He can be said to both appear and disappear. Indeed, God does eventually appear to Abraham in various ways: the Lord "said unto Abraham" (Gen. 12:1, 13:14); "appeared unto Abraham" (12:7); "came unto Abraham in a vision" (15:1); and so on.

Note that Ghazali only mentions 6:76, regarding the moon; but it is clear that the Koran additionally contains two similar arguments, regarding the stars and the sun. Moreover, while he correctly quotes the Koran as there saying "I love not the things which set," for some reason he seems to assume this refers to the moon, instead of the stars; for he then describes the argument as: "The moon is a thing which sets; but God is not a thing which sets; therefore, the moon is not a God" (2/EAE). Be that as it may, he admits that this argument is not fully laid out in the Koran, since he adds that the latter is "its foundation by way of concinnity and ellipsis." He claims to draw its two premises, viz. that the moon sets whereas God does not, from the narrative. But in my view, it would be more accurate to say that the narrative implies the minor premise (the moon sets, from "when it set") and the conclusion (the moon is not God, from Abraham's stated rejection of "things which set"), while the reader must provide the major premise (God does not set), which is obvious enough.

However, Ghazali does not see the latter as obvious, saying: "that God is not a thing which sets I know neither necessarily nor by sensation." He then argues that Abraham must have known the latter indirectly through knowledge that "God is not a thing which changes [a changer]. And… setting is changing." Here, he is introducing a new syllogism: God is not subject to change; and setting is changing; therefore, God does not set (2/AEE). This comment of Ghazali's is interesting, in that it brings into play the more radical concept of *change*. As I point out above, the proposed syllogism, even if formally okay, is contentually weak without this narrower middle term. Nevertheless, Ghazali introduces this term only in order to establish that God does not set, and not in the way of a criticism of the proposed argument as I did. This shows he has not fully understood the issues involved.

In fact, it cannot rightly be said that "setting is changing." Although setting is disappearing and changing is disappearing, it does not follow that setting is changing (this would constitute a 2nd figure syllogism with two positive premises, which is invalid). Things may disappear (or more specifically, set) without changing: stars disappear in daylight because the strong light from the sun eclipses their light, not because they cease to be or go away; the sun disappears at night because it goes over the horizon, due to the rotation on its own axis of the earth and not to revolution of the sun round our planet; the moon's disappearances are due partly to its own movements and partly to ours. Thus, the minor premise of Ghazali's second syllogism is false, and the Koran's argument in the name of Abraham is wobbly. Even so, we can grant Ghazali's main claim that a syllogism[9] is (or rather, a set of them are) embedded in the said passage of the Koran.

After that, Ghazali lays claim to two more examples of syllogistic reasoning in the Koran, and hints that there are more of them besides these. More precisely, he states that Allah taught Mohammed "to weigh by this balance in many places in the Koran, to follow the example of his father, the Friend [i.e. Abraham];" and he adds: "Be content with my calling attention to two places and seek the rest in the verses of the Koran."[10]

The first additional example he gives is **Koran 5:18** – "But the Jews and the Christians say, 'We are the children of God and His beloved'. Say, 'Then why does He punish you for your sins?' Rather, you are human beings from among those He has created." Ghazali's comment on this passage is that the Jews and Christians "claimed to be the sons of God," so Allah taught Mohammed how to "expose their error by means of the correct balance [i.e. the appropriate argument]," namely: "Sons are not chastised; but you are chastised; therefore you are not sons" (2/EAE). He adds that the major premise of this syllogism is "known by experience," its minor premise is "known by seeing," and "from these two necessarily follows the denial of son-ship."

Here again, while the argument is formally okay, its content is open to much criticism. When and where do Jews and Christians claim to be "the sons of God" instead of human beings subject to chastisement? If this is an argument against the Jews' belief they are 'the chosen people'[11], it is nonsensical since this concept is not believed to exclude punishment for sins (but on the contrary makes it more likely)[12]. The Koran argument can only be credibly directed at the Christian belief in the divinity of Jesus and that he was crucified; yet elsewhere (4:157) it denies Jesus died on a cross, so that would be inconsistent. Clearly, the fact that the Koran includes some syllogistic reasoning should not be taken to imply that its reasoning is materially sound.

Note moreover that while Ghazali makes a show of specifying the empirical sources of the premises of the syllogism ("by experience," "by seeing"), he takes for granted without questioning it the Koran's unsubstantiated claim that Jews and Christians say: "We are the children of God and His beloved." This issue of historicity is not incidental.

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[9]   Which he clearly identifies as being in the second figure, since he defines its principle as: "that of which is denied what is affirmed of another is different [distinct] from that other." This discussion is found in § 36-40.

[10]   § 42-43.

[11]   As is written or implied in the Tanakh countless times; e.g. in Deut. 14:2 – "For thou art a holy people unto the Lord thy God, and the Lord hath chosen thee to be His own treasure out of all peoples that are upon the face of the earth."

[12]   See, for instance: Amos 3:2 – "You only have I known of all the families of the earth; therefore I will visit upon you all your iniquities." The covenant between God and the Jews is conceived as a demanding régime of *noblesse oblige* rather than as one (such as the Moslems claim for themselves) of supremacist privilege and domination immune from judgment. According to it, the Jews are given more responsibilities rather than more pleasure and power.

The Koran's syllogism is formulated with the express purpose of contradicting the statement attributed to Jews and Christians. If, as a matter of historical fact, they never made such a statement – then the whole argument is spurious rant. A logician is duty-bound to be scientific all the way, not just as convenient to his convictions. Fiction cannot be treated as fact.

The second additional example Ghazali gives is **Koran 62:6-7** – "Say, 'O you who are Jews, if you claim that you are allies of God, excluding the [other] people, then wish for death, if you should be truthful.' But they will not wish for it." He explains this as follows: the Jews "claimed friendship [with God]," and as everyone knows "the friend desires to meet his friend;" yet "it was also known that they did not desire death, which is the cause of the meeting:" whence "it follows of necessity that they are not the friends of God." He then proposes the following more formal presentation: "Every friend desires to meet his friend; but the Jew does not desire to meet God; therefore he is not the friend of God" (2/EAE). Excuse me for laughing out loud at this anti-Semitic drivel!

For a start, the latter presentation is formally invalid, since the middle term is differently formulated in the major premise (friend) and minor premise (God); a syllogism cannot have four terms. Ghazali's preceding explanation is closer to formally correct; this is in fact a succession of two arguments (a sorites). The first argument is: Whoever wants to meet God must desire death; but the Jews do not desire death; therefore, they do not want to meet Him. The second argument is: Whoever one is a friend of is someone one wants to meet; but the Jews do not want to meet God (as just concluded); therefore, they are not friends of God (contrary to what they claim). Both these arguments are substitutive syllogisms of form 1/AEE (or negative apodoses, *modus tollens*). Therefore, Ghazali did not manage to correctly pinpoint the logical structure of the Koran's argument.

Moreover, the argument's content is absurd. The Koran suggests that one has to die to meet with God. Yet, it also evidently claims that Mohammed met with God (if only through an angel) without having to die. Therefore, the Koran is using double standards. It is inventing an excuse for pouring on the Jews scorn that they do not deserve. The Koran's 'reasoning' here is ridiculous: why would truthful friends of God need to wish for death? The Jewish perspective is that God wants us to love life; to love life is to show God appreciation for His kindness in giving it to us. We do not yearn for death, for we believe that we can well through virtuous behavior "meet with" God in the midst of this very life. Indeed, that is the purpose of life and the reason for our creation. Therefore, the major premise of the first argument is, in Jewish eyes, wrong – not only factually erroneous, but also morally reprehensible. It is obviously just an expression of the Koran's hatred for Jews, a wish for their death.

A bit further on[13], Ghazali does in fact propose one more example of syllogistic reasoning, in **Koran 6:91**. This passage is yet another moronic diatribe against "the Jews." The relevant part reads: "And they [the Jews] did not appraise God with true appraisal when they said, 'God did not reveal to a human being anything'. Say, 'Who revealed the Scripture that Moses brought as light and guidance to the people?'" According to Ghazali, the argument's premises are: "Moses is a man" and "Moses is one upon whom the Book was sent down;" and its conclusion is: "some man has had sent down upon him the Book" (3/AAI). The first premise is "known by sensation;" and the second is "known by their own admission," since the Koran says of them: "You [Jews] make it into pages, disclosing [some of] it and concealing much." The particular conclusion they yield suffices to refute the Jews' "general claim that Scripture is not sent down upon any man at all."

Ghazali's third figure argument is sound, but quite incidental. The Koran's focus here is on the statement "God did not reveal to a human being anything." This proposition is of course one more fabrication by the Koran, expressing the anti-Semitic feelings of its author(s). When and where did the Jews say that God did not reveal anything to anyone, as the Koran here claims? They may well have said that God did not reveal anything to Mohammed specifically, but they surely never ever collectively made that general statement (at least in those days, even if some Jews nowadays are skeptics). The rebuttal of the said proposition is not as Ghazali claims a syllogism. It is simply a rhetorical question by the Koran, viz. "Who revealed the Scripture that Moses brought as light and guidance to the people?" The assertoric implication of this rhetorical question is: God revealed the Scripture to Moses, and it is this implication that contradicts the allegation that God has made no revelation to anyone. This is the essence of the argument here, and not Ghazali's syllogistic proof that revelation to Moses is revelation to some man.

The Koran's logic here is therefore oppositional rather than syllogistic. This is also in itself interesting, of course. Ghazali's 3rd figure syllogism is, admittedly, present in the background of the Koran's argument – but only in the sense that syllogistic logic is always present when we apply or deny a generality. So we can say that Ghazali is, in this instance, a bit artificially reading a syllogism into the Koran. Note moreover that here again he shows no aptitude for historical criticism. He takes for granted without questioning it the Koran's unsubstantiated claim that the Jews denied the occurrence of any revelation by God to humans. Nevertheless, albeit the deficiencies in Ghazali's understanding pointed out in the present analysis, his contribution to the search of logic in the Koran is very valuable.

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[13]   § 47-48.

Ghazali additionally points out three examples of counterfactual hypothetical argument (negative apodoses, *modus tollens*). They are: **Koran 17: 42** – "If there had been with Him [other] gods, as they say, then they [each] would have sought to the Owner of the Throne a way;" **Koran 21:22** – "Had there been within the heavens and earth gods besides God, they both [the heaven and earth] would have been ruined;" and **Koran 21:99** – "Had these been gods, they would not have come to it [Hell], but all are eternal therein." He rightly explains the reasoning involved as follows, for instance: "If the world has two gods, heaven and earth would have gone to ruin. But… they have not gone to ruin. So [the] necessary conclusion [is] the denial of the two gods."[14] Finally, Ghazali also points out an example of disjunctive argument, namely **Koran 34:24** – "We or you are either upon guidance or in clear error." He rightly interprets this as: "We or you are in manifest error. But… We are not in error. So… you are in error."[15]

I think this exhausts the examples of reasoning in the Koran pointed out by Ghazali in his *Qistas*[16]. If these examples are all the logic there is to be found in the Koran, it is not very much. But of course, there may be other instances, which may have been pointed out by Ghazali elsewhere or by other commentators, but which I have not come across to date. Note that Ghazali does not mention a fortiori argument in his *Qistas*, even though I have found one instance of it in the Koran. This shows two things: (a) that his treatment in this work is not exhaustive; and (b) that he was not very aware of a fortiori argument. In conclusion, having found in the Koran a total of about a dozen arguments, including one a fortiori argument and a few arguments of other forms, we cannot say that there is no logic in Islam's founding document, but we can still say that there is rather little. This is surprising, considering that this document dates from the 7th century CE or later, over a thousand years after Aristotle.

Sticks and carrots. There is rather little logic in the Koran – unless, that is, we count *threats of punishment* and *promises of reward* (*wa'id* and *wa'd*, in Arabic) as logical arguments. For of such 'stick and carrot' arguments, there are many in the Islamic Koran, as indeed in the Jewish Tanakh and the Christian New Testament. For example: Koran 9:5 teaches that idolaters who consent to convert to Islam should be granted freedom, whereas those that do not should be killed. These arguments take the following forms:

- If you do this vicious deed (or don't do that virtuous deed), then such and such negative consequences (punishments) will befall you; therefore, don't do this (or do do that)—and such and such harm won't happen to you.
- If you do this virtuous deed (or don't do that vicious deed), then such and such positive consequences (rewards) will befall you; therefore, do do this (or don't do that)—and such and such benefit will indeed come upon you.

Quite often, the promised reward or threatened punishment referred to in such statements is, respectively, heaven or hell. This is rather convenient, since there is no way to empirically verify such otherworldly claims, at least not till one dies! Sometimes, however, the reward or punishment referred to is an earthly one. But even then, such statements have to be taken on faith, since the reward or punishment follows, not mechanically, but only on condition that God wills it to. Since God may occasionally choose, for His own reasons, not to make the consequences follow, there is no way for people to empirically verify such earthly claims. Sometimes, the reward or punishment is not specified, but left tantalizingly or terrifyingly vague. Thus, when such statements concern Divine retribution, for good or bad, they necessarily rely on faith. Sometimes, of course, as in the example given above (9:5), such statements are intended to be realized by human agency – that is, the specified reward or punishment for the stated action or inaction is to be effected by some specified or even unspecified person(s), such as a court of law, or maybe the monarch, or even some self-appointed executor(s). But even the latter statements, unless they fit in with our natural sense of justice, are proposed as "revelations" to be taken on faith – there being no way to prove them true, let alone to prove their source to be Divine.[17]

From a formal perspective, statements of this sort, which threaten or cajole, are indeed logical arguments insofar as they involve apodosis: either the *modus ponens* 'If X, then Y, and X, therefore Y' or the *modus tollens* 'If X, then Y, and not Y, therefore not X'. We are enjoined to behave in certain ways *in order to* obtain certain desirable things and/or avoid certain undesirable things; and action in conscious accord with purposes is a prerogative of all conscious beings (animals), and most obviously of human beings (rational animals). However, the logical aspect of such discourse is only the surface of it. The purpose of such statements is not to invite rational deliberation and decision, but essentially to preempt or banish all thoughtful reflection and bring about blind compliance. This may be

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[14]   § 53-54.
[15]   § 60.
[16]   It is worth mentioning that Ghazali takes a passage of the Koran as a justification for the use of reasoning, namely 16:125– "Invite to the way of your Lord with wisdom and good instruction, and argue with them in a way that is best." (See chapter 9.)
[17]   The point being made here is not denial that God exists or denial that God instructs mankind, but only to emphasize our inability as mere human beings to determine God's existence and will with the utter certainty claimed by sundry "prophets." Such claims can only be taken on faith, and therefore must always be taken with a healthy dose of skepticism if we are to avoid fanatic excesses. It is doubtful that God, whose intelligence is surely the highest conceivable, wishes people to behave like idiots and believe whatever they are told without asking questions and demanding credible answers. Faith is valuable and necessary, but blind faith is dangerous.

characterized as irrational argument; it is appeal to emotions – namely, fear (*hawf*) or hope (*raga*). The message is not really: 'think about it carefully and do what you think is right', but more radically: 'just do and obey'. The talk of good or bad consequences of action or inaction is only intended to exploit the worries and appetites of people, and make them do what they are told to do and not-do what they are told not to do. There are no ifs or buts about it—it is commandment and interdiction.

This is, to be sure, the very nature of law. Even in a democratic state, when representatives of the people freely convene to enact just laws, after they have debated an issue, they make a decision in accord with the procedural norms, and once a ruling is handed down the citizens are required to abide by it. If the process has been truly democratic, individual citizens or groups of citizens are not expected to short-circuit the legislative process and point-blank refuse to abide by the decisions of the majority, even though in a democratic society the dissenters may well try by all legal means to have the laws they regard as unjust reviewed by the legislative body and possibly repealed. All the more so, in a non-democratic state, laws are intended as orders to be executed by the populace, like it or not; although here, of course, the laws, being tyrannous due to the way they have been enacted, are inherently unjust, and citizens indeed ought to rebel against them on principle, whatever their content, until the people's natural rights as human beings are clearly upheld.

As regards the Koran, it is not hard to see that its purpose in formulating threats and promises is simply to ensure compliance to the decrees of some human legislator(s) claiming to speak, directly or indirectly, on behalf of God. No discussion is allowed regarding ends, though the means may occasionally require some reflection and debate. The Koran, typically, claims that its commands and prohibitions, and indeed its permissions and exemptions, to be Divinely-ordained. But this is just the Koran's say-so; there is no "proof" for such claims to revelation. It may be argued that such a document instills fear and love of God in people, and makes them do good and eschew evil, and thus improves society. But this is just a circular argument, in that what the speaker (Mohammed or whoever) regards as good or evil is merely his personal assumption and certainly not something he has scientifically proved, or even could conceivably prove. There may be some truth in it; but there may also be a lot of falsehood. The only way to test and judge the matter is with reference to reason. No one may claim something so important arbitrarily, without being subject to rational examination and evaluation. To uncritically accept claims that are so consequential is to invite disaster somewhere down the line for sure.

It is easy to see in this context, regarding the issue of legitimacy of laws, the importance of freedom of conscience (to choose this or that faith, or even non-faith), free thought and free speech, as against "blasphemy" laws. The latter laws, which play a major role in Islam, are clearly intended to block at the outset all attempts to question and challenge Islamic belief in general and the temporal hegemony of its ruling classes in particular[18]. To physically enforce laws having to do with spiritual belief is in direct contradiction to the claim that such laws are 'ethical' – for what is ethical is by definition a matter of free choice under the guidance of reason. The Koran's legal philosophy is to *coerce* everyone, Moslems and non-Moslems alike, to submit to its will (which it calls the will of Allah or of his messenger Mohammed); this is what the word "Islam" literally means: utter submission, with no right of dissent or review. But mindless conviction and compliance, like an automaton, out of oppressive fear of punishment or abject hope for rewards, is surely the antithesis of human dignity, the very negation of human spirituality. It is the depths of darkness, the death of the light of life.

2. About the Koran

The paucity of logic in the Koran is seen to be all the more predictable considering the declamatory, peremptory, and mostly rancorous, tone the author adopts (or authors adopt) throughout the document. The 'voice' heard in the Koran is very different from that heard in the Jewish Bible. The Allah of the Koran does not sound like the God depicted in the Tanakh, who (besides) is differently named and described. The word 'Allah' (etym. *al-ilah*, the god) was, before the advent of monotheistic Islam, the name of a deity worshiped by idolatrous Arabs. Although the word is etymologically close to the Hebrew words 'El' and 'Elohim' – it is not necessarily equivalent to them. The Koran

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[18]   Even negative comments directed at their alleged prophet, Mohammed, or at the Koran, are considered "blasphemy." Apostasy, adultery, and many other violations of Islamic law are, it seems, also sometimes characterized as "blasphemy." Clearly, this word has for Moslems a wider applicability than speaking ill of God. It should be pointed out that such expansion of meaning is not innocuous. To denote criticism of Mohammed or of the Koran as "blasphemy" is in effect to *deify* the said person or book. To deify a mere person or book is nothing less than *idolatry*, since the implication is that God is not the One and Only. This is surely the very essence of blasphemy – an insult to God. Thus, to expand the meaning of blasphemy as Moslems do is itself an act of blasphemy.

verbally claims its god to be great and merciful; but the effective message of this document is one of pettiness and antagonism – perpetual enslavement for Moslems and implacable hostility towards all non-Moslems[19].

Admittedly, the word 'Allah' has since the advent of Islam been generally taken – even by non-Moslems – as referring to what everyone means by the English word 'God', and we shall here use the two terms as equivalent in various contexts. But I want to first briefly draw attention to the discursive difficulties this terminological equation presents. It is not innocuous, for if we say that 'Allah' is equivalent to 'God', we seem to accept as true the Moslem claim that their deity is indeed God (as understood in other traditions, notably the Jewish and Christian, and in Western philosophy). If, instead, on the basis of evident differences in the name, character, behavior and sayings of the Islamic deity, we say that 'Allah' is never equivalent to 'God', we would be in error, for there is much philosophical discourse in Islam, whether right or wrong, which is effectively about God although (naturally, since it is in Arabic) it is said to be about Allah. Thus, we must say that the term 'Allah' is, objectively, *sometimes but not always* equivalent to the term 'God'. The term 'Allah' of course always refers to God for Moslems; but for non-Moslems (yours truly included) it need not do so – it depends on the precise context.

The Koran (or *Qur'an*), the holy book of the Moslems, is traditionally regarded as having been composed by their alleged prophet, Mohammed (Arabia, ca. 568-632 CE), mostly under dictation from the angel Gabriel[20]. However, it was put together much later. Some twenty years later, during the reign of the Rashidun caliph Uthman (644-656 CE), according to Moslem tradition. More like as of some sixty years later, during the reign of the Umayyad caliph Abd al-Malik (685-705 CE), according to some modern critics, who also raise doubts as to the authorship of the document[21]. Some of the latter suggest the document was largely fabricated[22], for essentially political purposes, as convenient ideology for an already established (not yet Moslem) Arab empire. They point out the lack of solid historical evidence for an earlier date of composition. On the contrary, the little historical evidence found suggests the non-existence of a religion called Islam and its founding document the Koran till the late 7th or early 8th century CE. The personage of Mohammed described in it might, therefore, be partly based on a vaguely remembered past teacher or even be entirely mythical.[23]

The Koran contains many internal and external inconsistencies. That is, contradictions between propositions in it, and possibly some illogical propositions in it; as well as discrepancies between it and documents it refers to (mainly the Jewish and Christian Bibles), and between it and various scientific and historical facts[24]. This is not very surprising, being true to varying extents of all religious texts (including the Jewish and Christian Bibles). Of course, the frequency and nature of these inconsistencies are significant, and need to be closely examined. In any case, internal contradictions and contradictions with scientific and historical facts must surely be considered as unerring signs that the document is not, or at least not wholly, of Divine origin – since it is inconceivable that God makes such errors. Just as the external contradictions in a document can only be due to human ignorance, so the internal contradictions in it are indicative of human fallibility. Many of the contradictions within the Koran are between earlier and later laws; Moslems presumably view such developments as implying that God changed his mind, but this is an essentially absurd notion. An Omniscient Being would surely forewarn that a temporary or otherwise circumscribed law is so intended when promulgating it.

Islamic jurisprudence has developed complex hermeneutics for dealing with internal contradictions in the Koran (and other recognized sources). When two texts are found to be in conflict, various means may be used to reconcile them: they might upon further scrutiny be found to be more harmonious than they seemed at first sight, or one might be

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[19] One can only feel pity for Moslems as human beings, for the mental, social and political prison Islam condemns them to from day one and for their whole life. Even their rabid Jew-hatred is pitiful, indicative of their great inner confusion and turmoil. There seems to be no way out for them. Very, very few have the wit and courage to break free.

[20] Koran 2:97 – "Jibrael [Gabriel], for indeed he has brought it [this Quran] down to your heart by Allah's permission." Also, 53:5 – "He has been taught [this Quran] by one mighty in power [Jibrael]." And 53:10 "So did [Allah] convey the inspiration to His slave [Muhammad through Jibrael]." These are Muhsin Khan translations; note that the material in square brackets is not present in the original but constitutes interpretation by the translators and presumably by Moslem commentators before them.

[21] The editor, and perhaps largely the author, of the Koran seems to have been Hajjaj ibn Yusuf, governor of Iraq under Abd al-Malik. He then distributed this Koran throughout the Moslem world.

[22] If you think such outright fabrication is unthinkable, consider *The Urantia Book*, which was produced anonymously probably in the second quarter of the 20th cent. (first published in 1955). This strange book (which I read once, out of curiosity) is designed to look like a new revelation. Though this has not happened so far, one can well imagine a group of people adopting it as their scripture and founding a new religion with it; thereafter, some centuries later, when people have forgotten how it initially emerged, they will look upon it as a holy book. More details on this book at: en.wikipedia.org/wiki/Urantia.

[23] In this context I highly recommend Robert Spencer's very interesting books on Islam: *The Truth about Muhammad.* (Washington, D.C.: Regnery, 2006). *The Complete Infidel's Guide to the Koran.* (Washington D.C.: Regnery, 2009). *Did Muhammad Exist? An Inquiry into Islam's Obscure Origins.* (Wilmington, Del.: ISI, 2012). Spencer refers in the latter to many other, similarly skeptical, past and present authors.

[24] See the very interesting list and discussion of contradictions and confusions in the Koran at: answering-islam.org/Quran/Contra/index.html. See also in this regard the very interesting work of Haï Bar-Zeev, *Une lecture juive du Coran* (Paris: Berg, 2005).

considered an exception to the other, or their scopes might be particularized to exclude each other, or the two might be somehow merged into one; alternatively, as a last resort, one might be considered as abrogating the other[25]. In the latter event, the decision as to which supersedes the other is mainly made with reference to chronology, the later text being considered as intended to replace the earlier[26]. Of course, it is not always easy to establish chronology, but various criteria are agreed on as reasonable. Note that the Koran is *not* arranged in chronological order[27]. We need not go into more detail here, but only remark that all this goes to show that Moslem commentators admit the existence of internal contradictions within their source documents (and indeed between them). This shows commendable respect for logic on their part; but it also shows the logical imperfection of their proof-texts.

As regards discrepancies with earlier religious documents, although the Koran is manifestly (as anyone can see by making comparisons) largely based on the Jewish Bible (Torah and Nakh), and to a lesser extent on the Talmud and some Midrashim, as well as on the Christian Bible and some related books, it is obvious that whoever wrote the Koran he or they had a very superficial knowledge of these various books. The snippets of these books referred to in the Koran are obviously only known second-hand, by hearsay, or from casual perusal, not from intensive personal study and mastery. Either those who taught the Koran's author(s) parts of these books were themselves not very knowledgeable, or the Koran's author(s) had acute problems of attention and memory! This is evident from the ridiculous inaccuracies and bloopers in it, such as the anachronistic confusion between Miriam, the sister of Aaron, and Mary, the mother of Jesus, who existed some thirteen centuries apart[28]; or the unsubstantiated claim that Jews were wont to kill their prophets[29]! There are many details in Judaism and Christianity that the Koran displays ignorance of; and a lot of the information it apparently relays from them is erroneous[30]. How then can this document be regarded as credible?

To protect itself from this accusation of ignorance and confusion, the Koran claims that Jews and Christians falsified their own Scriptures[31], and that Islam is older than Judaism and Christianity, whose prophets it claims were really Moslems![32] This is like hijacking a couple of vehicles, and then accusing their real owners of theft. The main purpose

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[25]   I have described the formal logic of these different responses in a 1998-9 paper entitled "Islamic Hermeneutics," which was posted in my website in 2001 as an annex to my *Judaic Logic*, and then published in my *Ruminations* in 2005. This essay is still online at: www.thelogician.net/3_judaic_logic/3_islam_1.htm.

[26]   This harmonization is reminiscent of the 13th hermeneutic principle of Rabbi Ishmael (Israel, 90-135 CE), though they are not identical. At: wikiislam.net/wiki/List_of_Abrogations_in_the_Qur%27an there is a list of verses abrogated and verses they were abrogated by.

[27]   In my past essay on Islamic hermeneutics, I wrongly stated that the Koran "is supposed chronologically ordered." Perhaps I meant to say: "is supposed*ly* chronologically ordered"? In any case, it is generally agreed that it is *not* chronologically ordered, and besides that it is not always possible to determine the temporal order with certainty. For assumed order, see: wikiislam.net/wiki/Chronological_Order_of_the_Qur%27an.

[28]   Koran 19:16-34 and 66:12. Another confusion I find hilarious is the Koran's presentation of Haman as a contemporary of Pharaoh (28:4 and 8), and as being ordered by the latter to "make for me a tower that I may look at the God of Moses" (28:38). The idea of a tower reaching up to God comes from the Tower of Babel episode (Gen. 11:4), which occurs in the Babylon region some 500 years before Moses' time. The character of Haman comes from the Book of Esther (as of 3:1), which tells of events in Persia over 900 years after the Exodus. Yet another instance I find revealing is the conflation in the Koran (2:67-73) of two unrelated passages of the Torah (Num. 19, on the red heifer, and Deut. 21:1-9, on another heifer altogether). And there are many more such mix-ups.

[29]   Koran 2:61, 91; 3:21, 112, 181, 183; 4:155. None of these passages mention which prophets were supposed to have been killed; indeed, they seem to be saying that *all* the prophets were killed. Two more passages suggest that *only some* prophets were killed; namely, 2:87 and 5:70. Perhaps the author of the Koran here again demonstrates his confusion, and has in mind the killing by the prophet Elijah of the 'prophets' *of Baal* in 1 Kings 18:40? Or maybe he is referring to 1 Kings 19:10, where Elijah says: "the children of Israel have… slain Thy prophets with the sword"? But this passage in fact relates to Ahab, the renegade king of the northern kingdom, or more precisely to his non-Jewish (Zidonian) Baal-worshipping consort Jezebel, as it is written: "Jezebel cut off the prophets of the LORD" and "Jezebel slew the prophets of the LORD" (1 Kings 18:4, 13). Or maybe the Koran author has Jesus in mind, and thinks the Jews killed him, unaware of the role the Romans played in that episode; but then why in 4:157 would it say that Jesus was *not* killed (by anyone)? Clearly, this repeated accusation, that Jews are either habitually or occasionally prophet-killers, is a deliberate lie aimed at making the Jews look as bad as possible. It is sheer calumny, mere hate speech.

[30]   Note that the Koran does not say: "the story told in the Jewish Bible or the Christian Bible is thus and thus, but I inform you that it was really so and so" – the Koran just tells a story, obviously unaware of its contradicting the older sources, i.e. thinking it is in accord with them!

[31]   Jews and Christians (at least those who disagree with Islam, which means almost all of them) make "statements other than that which had been said to them" (2:59); they "conceal testimonies" (2:140); they "alter the Scripture" and "speak untruth" (3:78); they "distort words" (5:13); they "forget portions" (5:14); and so forth. This thesis of falsification was, according to Bar-Zeev (p. 134), later much stressed by Ibn Hazm (a convert from Christianity in Andalusia, 994-1064), Samuel al-Mograbi (a convert from Judaism, 12th cent.) and Ibn Taymya (d. 1328, who is often referred to by modern Salafists and Wahabists). Bar-Zeev also suggests (p. 135) that the original intent of such passages of the Koran may have been to refer to the false prophets mentioned in 2 Kings 17:9, Ezek. 13, Jer. 14:13-15 and Jer. 23.

[32]   Consider the 'logic' of such claims. In 3:65: "O People of the Scripture, why do you argue about Abraham while the Torah and the Gospel were not revealed until after him? Then will you not reason?" This is a claim that, since Abraham preceded the Torah and the Gospel, his religion cannot have been theirs. Fair enough; but then the Koran infers that, since Abraham was neither Jewish nor Christian, he must have been Moslem! This is the intent of 3:67: "Abraham was neither a Jew nor a Christian, but he was one inclining toward truth, a Muslim" (Tr. Sahih International). Notice the double standard: the fact that Abraham preceded the Koran, too, is blithely ignored. Another fallacy in this argument is that of equivocation: it could well be said that Abraham was a *musliman* with a small m, meaning someone submissive towards God; but it does not follow from that that he was a Muslim, meaning a member of the religion of Islam, which did not yet exist. No one denies that the patriarch Abraham preceded the Torah – the Torah itself affirms it, and indeed is the source of our knowledge of his existence. Moreover, the Torah does

of the Islamic claim to an earlier date is of course to deny that it was largely derived from Judaism and to a lesser extent Christianity; i.e. to occult the plagiarism on which it was founded[33]. The claim that Judaism and Christianity falsified their Scriptures is thus a subsidiary one, designed to explain differences in detail found in the Koran. Since Judaism and Christianity in fact, judging by concrete historical evidence, including third party records, antedate Islam by some 2,000 and 600 years, respectively – this is a claim that whole peoples for two millennia or several centuries had nothing better to do than combat and conceal a religion that they had never even heard of! Can such a wacky retroactive argument (to justify the Koran's misinformation) be taken seriously by anyone with intelligence and good faith? Surely this accusation of falsification is a cynical attempt at falsification by Moslems![34]

Imagine what would happen if the methodology thus proposed by Islam were to be applied in courts of law. In the abstract, it is conceivable that an innocent man be wrongly accused of some crime after some people maliciously hiding evidence in his favor and planting evidence against him. But no judge in his right mind would consider such abstract possibility as relevant in a trial where zero evidence is brought to bear that substantiates it in the particular case under consideration (and moreover where much evidence is available with opposite effect). Indeed, in a sane society, a judge who based his judgments on such fantasy would surely soon lose his job. If this fanciful argument was allowed by historiology, historicity would disappear from historiography. There would be no reliable history, only fictional accounts. While it is true that history books cannot be fully objective, and entirely based on facts demonstrable through documents and other physical traces, to say this is a far cry from regarding all allegedly historical accounts as equally valid (or equally invalid).

Some interpretation is inevitable and indeed necessary in history, but this must always be done within the framework of unbiased methodological criteria. History is an inductive discipline, subject to empirical evidence, critical verification and other rational considerations; it cannot be allowed to become the product of arbitrary assertions in the service of some ideology. Some accounts are, therefore, more credible than others. The notion that a book could have traversed centuries or millennia in a subterranean manner, leaving no mark on history, no mention anywhere, no archeological vestige, is one found in many religions. In Buddhism, for instance, Mahayana sutras are routinely pre-dated to Buddha's time. Just as I do not accept such Buddhist claims, or the unsubstantiated claim by some Jews that the Zohar, which appeared in 13th century CE Spain, originated in 2nd century CE Israel, I would not grant any credence whatsoever to the Islamic claim as to the antiquity, let alone perennity, of the Koran. The same criteria of evidence apply to all.

Moreover, the Koran's central thesis is preposterous. It claims that God, i.e. the God of the Jews, the children of Israel, sent a non-Jewish messenger to them (as well as to the Arabians, and everyone else eventually); and that when "the Jews" – i.e. only the few hundred or thousand Jews living in a certain corner of Arabia at that time, note well – refused to believe this alleged messenger, God became super angry with Jews in general, insulting them and cursing them all forever![35] Such a narrative is logically incredible for anyone truly acquainted with Jewish Scriptures, which

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not go against the teachings of Abraham, as the Koran suggests, but on the contrary lovingly transmitted them and was inspired by them. The Koran seems to be claiming that if someone antedates a religious document, having inspired it (as in the case of Abraham, Isaac, Jacob and his sons) or written it (as in the case of Moses), he may not be counted as a member of that religion, since it did not yet exist. But if we accept this idea, not only were the Israelites mentioned in the Jewish Bible not Jews, but Jesus and his Apostles were not Christians and Mohammed and his Companions were not Moslems! Surely, anyone able to reason would see the absurdity of such claims.

[33]   I can only here refer the reader to the important work of Bar-Zeev, *Une lecture juive du Coran*. This rich and illuminating study by a learned rabbi (the author adopted a pseudonym, no doubt to avoid becoming a victim of Moslem insults and threats) shows in detail the Judaic sources of much of the Koran's alleged prophesies. Moreover, it proposes a credible detailed theory regarding how the Koran was probably composed and put together from these sources. Ironically, Moslems proudly challenge others to "produce a *sura* like it," unaware of the foreign literary sources of their holy book. This may rightly be called plagiarism, in that material is drawn from other sources without acknowledging those sources.

[34]   And they keep up the tradition of lying and pretending to this day, claiming that Arabs are the original inhabitants of the holy land, which all historical records show the Jews inhabited long before any Arab arrived, and claiming that Israelis persecute them, while the exact opposite is the truth. They destroy archeological evidence of the Jewish Temples under the Temple Mount, and assert no such edifice ever existed, blithely claiming Jerusalem as their eternal capital. And the witless and wicked mass media, BBC, CNN, and all their ilk, and even the British Museum and sundry Western universities, shamelessly pass on such nonsense as undeniable fact. Another shocking example of this disregard for facts by Moslems is their lately concocted claim that Moslems discovered America before Christopher Columbus! This new invention is being propagated ostensibly in order to suggest that Moslems have from its beginnings been part of the USA and therefore they have an equal share in what was until very recently thought to be only a "Judeo-Christian civilization." But its ultimate purpose is obviously to stake a Moslem claim of ownership on that country, and indeed the rest of the continent, in order to eventually turn it into the 'Amerabia' province of the world caliphate which it is their stated ambition to create. Distortions of history are never innocent.

[35]   See 2:89, 3:181, 5:13, 5:41, 5:60, 5:64, 98:6. The Koran of course claims the Jews to be accursed so as to reassign the role of 'chosen people' to the Moslems. However, such replacement is impossible according to many passages of the Tanakh. See for instance Jer. 31:34-36: "Thus saith the LORD, Who giveth the sun for a light by day, and the ordinances of the moon and of the stars for a light by night, who stirreth up the sea, that the waves thereof roar, the LORD of hosts is His name: If these ordinances depart from before Me, saith the LORD, then the seed of Israel also shall cease from being a nation before Me for ever. Thus saith the LORD: If heaven above can be measured, and the foundations of the earth searched out beneath, then will I also cast off all the seed of Israel for all that they have done, saith the LORD;" indeed, read the whole chapter, a beautiful prophetic promise of eternal love by God for the Jewish people.

teach God's justice and mercy, his patience and benevolence. Moreover, nowhere in them does God indulge in rude language, as when the Koran refers to Jews as "apes and pigs"[36]. The Koran's Jew-hatred is certainly not God's. It is the emotional reaction of some quite ordinary person(s) filled with resentment of some sort[37].

Throughout the Tanakh, God professes *eternal love* for the people of Israel[38]. It is not conceivable that He would then, ever, change His mind. When the Jewish people fail to sufficiently obey His Torah, He may for a while seem angry with them. But then (at least in ancient times, according to the Tanakh), He sends them a prophet to call them to order. Always a Jewish prophet, one of their own brethren; never a foreigner. The Torah explicitly commands it[39], and the whole Tanakh repeatedly confirms it (i.e. all prophets and leaders of Jews therein were Jewish). All Jewish prophets were well versed in the Torah, and considered particularly wise and virtuous; and they had to be to have credibility in the Jewish people's eyes. It is unthinkable that God would suddenly choose to send the Jews someone ignorant of Jewish law and lore, and demand that they obey him. Someone who, to boot, engaged in highway robbery, murder, wife-stealing and pedophilia, to mention only some of the remarkable 'achievements' attributed to Mohammed by the Koran itself as well as by later reports[40].

God would surely have anticipated that the Jews were not likely to follow foreign religious leadership, all the more someone of doubtful morality. Indeed, they are specifically forbidden to do that, according to the Torah[41]. Only one of their own can lead them spiritually, and it must be someone of proven spiritual elevation. So it can hardly be claimed that He sent them an Arab messenger, and then got terribly upset when the Jews did not accept him as a prophet! Moreover, God had no reason to be angry with the Jews at that period of history, the early 7th century CE. They were doing rather well spiritually – learning, praying and following the Torah assiduously (this was the period of the Geonim in Babylonia, remember) – so why would God resent them? So, the whole scenario concerning them that the Koran blithely projects is absurd.

Funnily enough, unfortunately—Mohammed may have been Jewish![42] If Mohammed was indeed Jewish through his mother, it does not follow that he was qualified to preach to the Jews. His evidently sketchy knowledge of Torah and Talmud, and his immoral personal behavior, naturally disqualified him from such a mission. If he was a Jew, it can be said, in view of his many anti-Jewish statements, that he was a 'self-hating' Jew, i.e. a Jew who for whatever reason (in this case probably due to feeling rejected by his mother's tribe) hated the Jews in general (and therefore, by implication, himself too). The irony of all this is that when Moslems express their hatred for Jews in general they may be abusing their own leader as well as his relatives! This is something Moslems ought to think about.

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[36]   See 5:60; also 2:65 and 7:166. Lest this seem like a misunderstanding of the Koran's intention, note the characterization in 2010 by the Moslem Brotherhood president of Egypt, Mohammed Morsi, of Jews as "the descendants of apes and pigs." Needless to say, he was not referring to Darwin's theory of evolution!

[37]   If the Koran is attributed to Mohammed, then the insults and curses can be explained with reference to his having been slighted by the Jews of Medina. If the Koran is a later product, the anti-Semitism evident in it was probably due to more diffuse Christian and other cultural influences. In any case, we see how rudely and hotly many Arabs and Moslems still today react when their pride is hurt.

[38]   Needless to say, it is not my purpose here to defend the idea of Jews as the 'chosen people', but only to show the absurdity of the Islamic (and before that Christian) attempt at 'replacement theology'. The point made is that since such attempts refer to Jewish Scriptures they cannot consistently ignore what is in them.

[39]   See Deut. 13, 17:2-20, and 18:9-22. Online at: www.mechon-mamre.org/p/pt/pt0513.htm. Note especially: 17:15 – "One from among thy brethren shalt thou set king over thee; thou mayest not put a foreigner over thee, who is not thy brother;" and 18:15 – "A prophet will the LORD thy God raise up unto thee, from the midst of thee, of thy brethren," and 18:18 – "I will raise them up a prophet from among their brethren."

[40]   As regards murder, the Moslem historian Ibn Ishaq (8th cent. CE) reported that, after the Jewish Banu Qurayzah tribe surrendered to Mohammad, their men "were brought out to him in batches," and he "struck off their heads," thus massacring with his own hands at least 600, maybe as many as 900, unarmed innocent people. (Quoted in Spencer's *The Truth about Muhammad*, pp. 129-131. See the same book for details Mohammed's other 'achievements'.)

[41]   See references in preceding footnote. E.g. "Neither shall he multiply wives to himself, that his heart turn not away; neither shall he greatly multiply to himself silver and gold." (Dt. 17:17.)

[42]   This speculation is based on a report by the Islamic historian Ibn Hisham (d. ca. 833), following of a report by his predecessor Ibn Ishaq (ca. 704-761/7 CE), whose works are lost. These sources are both generally respected by Moslems. It seems that Mohammed's mother may not have been Amina (an Arab), as orthodox tradition has it, but was an unnamed Jewish woman, sister of Waraqa Ibn Naufal. The latter, according to these historians, "belonged to the religion of Moses, before embracing that of Jesus." Mohammed's Arab father, Abdallah, died before his son was born. Still young, Mohammed went to live with Waraqa, who referred to him as his nephew. Waraqa seems to have been Mohammed's main teacher in religious matters. When Mohammed was six, his mother took him to Medina, to visit her family in the Jewish clan of the Beni al Najjar. Later, when Mohammed left Mecca for Medina, he first went to live with this clan. All this information is drawn from the book by Bar-Zeev, p. 17. Needless to say, the thought that Mohammed might have been a Jew is not a source of pride for us, but – in view of the havoc and bloodshed he has caused in the past 1400 years and continues to cause today – a source of acute shame. Hopefully he was not Jewish; but if he was, we have much reason to be sorry – as with Karl Marx and other lasting trouble makers.

3. Logic in the hadiths

As we have seen, not only does the Koran involve almost no use of logic, i.e. of rational argument, but also the Koran involves a great deal of illogic. In view of this, we have to wonder how a bit of logic did come to appear in Islam at a later stage. A full study of this question would require us to first look for all a fortiori arguments and other logical processes in the *hadiths*[43]. These are the next layer of Islamic material, in principle closest in time and in perceived holiness to the Koran, being allegedly statements of the companions of Mohammed, purporting to recall things he (and to a lesser extent his companions) said and did. In view of the contradictions between some of these accounts, not to mention other absurdities, many hadiths are considered even by Moslems to be unreliable; but Moslems do believe many of them. This is rather optimistic on their part, seeing as these sayings and stories only began to appear on the stage of verifiable history in the late 7th cent. and early 8th cent. CE; that is, many decades after the purported date of Mohammed's death. Many hadiths are of much later date than that; some perhaps are from as late as the 9th cent. CE.

The following are two commonly given examples of logic in the hadiths[44]:

> "Ibn Abbas narrated: A woman said, 'O Messenger of Allah, my mother died owing a vow to fast; should I fast for her?' He said, 'What if your mother owed a debt and you paid it back for her, would that settle it?' She said, 'Yes.' He replied, 'Then, fast for your mother.'" "Abdullah ibn Zubair narrated: A man from Khath'an [a tribe] came to the Messenger of Allah and said, 'My father embraced Islam at an old age, and he cannot ride the camel and at the same time he is obligated to perform Hajj [the pilgrimage to Mecca]. Should I perform Hajj for him?' The Prophet said, 'Are you the eldest son?' He said, 'Yes.' The Messenger replied, 'What if your father owed a debt and you paid it back, would that settle it?' He said, 'Yes.' The Prophet said, 'Then perform Hajj for him.'"

These are both, of course, simple arguments by analogy, and their resemblance (both refer to paying off debt) is noteworthy. They constitute inductive, rather than deductive, logic – since the conclusion, though reasonable enough, is not necessary; i.e. we could well conceive Mohammed giving the contrary answers to the questions put to him without being guilty of illogic. An inductive argument is one whose conclusion can be assumed true on the basis of the given premises, *unless or until* some contrary information is found that puts it in doubt. Such simple argument by analogy may be all the logic that Moslems have found in the hadiths, judging by the fact that they are often given as the justification and illustration of Islamic hermeneutic techniques. They are considered as justifying the use of reasoning to develop the law, because they show Mohammed in the act of using such reasoning and therefore apparently inviting imitation by later authorities. Although some commentators did not accept this implication of the examples, arguing that while Mohammed could well do it, it does not follow that his successors were qualified to do it, the mainstream posture has been to accept some development of the law through reasoning.

However, as we shall see further on, Islamic jurisprudence in fact usually resorts to a more complex form of analogical argument. In the above examples, a religious obligation, whether voluntary or fixed, is simplistically *likened* to a financial debt; so the conclusion is based on a mere impression of similarity[45]. In the more complex form of the argument, however, the two things compared are considered to have an alleged or demonstrated *common ground*; so the conclusion is based on a more intricate rational process. In simple analogy, the comparison between the two terms involved is unmediated, direct; whereas in complex analogy, it is mediated, indirect. I do not know whether examples have been found in the hadiths themselves of such more elaborate form of argument by analogy. There may be other examples. There may also be examples of other logical processes – this question can only be answered through close study of all the hadith collections by competent logicians.

But judging from the data I have some far come across offhand, logic does not seem to be much more present in the hadiths than in the Koran. The following story tells us something about the level of logic to be expected in them.

Muhammad Ibn Ishaq, author of *Sirat Rasul Allah* (*Biography of the Prophet of Allah*), wrote about an alleged rabbi of Medina, called al-Husayn, also known (possibly after his conversion to Islam) as Abdullah bin Salam, who asked Muhammad "about three things which nobody knows unless he be a Prophet." Follow three silly questions which I will not bother repeating, to which Muhammad readily gives three silly answers, which again are not worth the trouble of retyping. Whereupon, highly impressed for his part, the questioner immediately converts to Islam, testifying that Muhammad is the messenger of Allah[46]. Obviously, the purpose of this story is to 'prove' Muhammad's status as an envoy of God, by having a Jewish rabbi test him and testify to his having successfully

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[43]   The plural of hadith, in Arabic, is *ahadith*; but here we shall give the word an English plural, hadiths.
[44]   The references given for these examples in the source I used are, respectively, "Moslem" and "Ahmed," without further specification.
[45]   Such simple analogy seems to be called *qiyas al-shibh*, judging by a comment by Arnaldez, p. 43 (see reference further down).
[46]   This story is reported and discussed in some detail by Spencer in his *The Truth about Muhammad*, pp. 92-5. He analyzes it further in his more recent book *Did Muhammad Exist?* pp. 107-9.

passed the test. Why a Jewish rabbi? Because that would connect Muhammad's mission to earlier Scriptures, and thus enhance his authority.

But what is the 'logic' of this attempted proof? First, there is no evidence whatsoever that this story is historically true; without our having any means to verify the fact, we have to keep in mind that it could have been invented by Ibn Ishaq or someone before him. Who is this rabbi? He must have been an important fellow, to have been entrusted with such an important secret. Yet no Rabbi al-Husayn of Medina is known to us Jews, or to historians at large. Second, even supposing that this Jew existed and was a rabbi and did indeed ask Muhammad those questions and did indeed find his answers correct – how can we be sure that he did not simply invent the questions with the intent to admit Muhammad's answers whatever they were (in order to please him and gain his favor)? In other words, what tests did al-Husayn first pass to prove *his own* reliability as an examiner? None that we know of. The story told, of course, implies that al-Husayn knew the three questions to ask of a prophetic candidate and the three answers to them, from his own, Jewish tradition; but we know of no such tradition in Judaism[47] (the Moslems would of course reply that we lie when we say that, claiming that we have falsified our tradition in order to hide its anticipation of Muhammad's mission). Third, there is an internal inconsistency to this story. If (as al-Husayn claims) the answers to the three questions are knowable *only* to a prophet, how does al-Husayn (who is *not* claimed to be a prophet) know them? To claim that someone *not* privy to information *is* nevertheless privy to it is a self-contradiction.

From this parody of logic we can conclude without doubt that the story is made up. Whoever made it up, either he did not himself have the intelligence to notice its inherent paradox, or he was confident that his target audience was composed of simpletons who would not spot its absurdity. I do not suppose any Moslem commentator through the ages ever belied this story on logical grounds.

The dating of hadiths is of course very relevant to the issue of the sources and development of Islamic logic. A considerable effort of collection and translation of non-Islamic texts into Arabic began already in the Omayyad period (661-750 CE). This effort increased greatly during the Abbasid period (750-1258 CE). Thus, the influx of foreign philosophy and logic into Islamic culture accelerated over time. Knowing this, we may expect the hadiths appearing in the later period to involve more logic than those in the earlier period. This is just a speculative prediction on my part, which may or may not be empirically confirmed. In any case, even before any translations of texts occurred, there was bound to be some measure of cultural osmosis from the population and institutions of the conquered peoples to their conquerors. The conquerors took over the existing institutions, without at first modifying them greatly. It is only over time that they tailored them to their own philosophy.

In any event, Islamic law (called the sharia) is often based to a large extent on material found in hadith collections, rather than in the Koran. These collections constitute the 'oral law' of Islam, as against the 'written law' given in the Koran. It is reasonable to suppose that some logic might be found in the hadiths, though this question can only, to repeat, be answered empirically by actual detailed research in these compilations, some of which are massive. However, it is safe to predict that most of the logic that eventually makes its appearance in Islam, in legal discussions leading to the formulation of laws (constituting the sharia), was learned from Jewish, Christian, and eventually Persian and Greek, and later Indian, logical traditions[48]. This could have occurred by observation during discussions with non-Moslems of their logical practices, as well as through learning from oral and written theoretical teachings. But I suspect that logic came into Islam mainly though diverse converts to it, who brought it with them as cultural baggage. In any case, while Islam was apparently little touched by logical thought in its presumed Arabian cradle, it was very soon in close contact with the rich traditions of the countries the Arabs and their successors conquered with the sword, and from then on could absorb and assimilate much of the knowledge in these other cultures.

It should be kept in mind that the Arabs produced no philosophical reflection, at least not in writing (even though they had an alphabet very early on), till the late 8th or early 9th century CE, when the bulk of writings in Greek, Persian and Syriac, and possibly Hebrew, among others, were translated into Arabic (notably under commission of the Abbasid caliph al-Mansur, who reigned in Baghdad 754-775 CE). Indeed, the Arabs hardly had any literature till the Koran appeared, as they themselves admit when they refer to the earlier era as the "period of ignorance" (*Jahiliyyah*). In short, to put it bluntly, the Arabs were not exactly an intellectual people, at least not till very late in human history compared to other peoples in their region. It is therefore not very surprising to find almost no logic,

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[47]   If we have any tradition concerning prophets it is that they must be extremely 'disinterested' – devoid of lust for political power or material possessions or sexual gratification. See for examples, regarding worldly gains, Numbers 16:15 (where Moses declares: "I have not taken one ass from them, neither have I hurt one of them.") and I Samuel 12:3-4 (where Samuel asks: "Whose ox have I taken, or whose ass have I taken? Whom have I defrauded or whom have I robbed? From whom have I taken a bribe to look the other way?"). Very different was the behavior of Mohammed, according to Moslem sources.

[48]   Hallaq considers that logic "made an entry to legal theory" after the 11th cent. CE (p. 257). But I would say that though this may well be true of conscious efforts of application of formal logic to Islamic law, logic must have be intuitively used and/or seeped in from the outside two or three centuries before that, as of the start of legal discourse, for the simple reason that no such discourse is possible without "reasoning" of some sort.

and much illogic, in the Koran and in many hadiths. And it is accordingly not very surprising that the Arabs, and likewise later conquerors[49], were greatly impressed by the discourse of the Koran and hadiths: they simply knew no better!

That the Arabs and later conquerors were willing to learn from their subject-peoples is certainly to their credit, and it made possible their eventual entry into the field of philosophy. However, while Islamic philosophy flourished for a while[50], between the 9th and 12th centuries CE, with the likes of al-Kindi (Arab, ca. 801-873), al-Farabi (Turk, ca. 872-950) and ibn-Sina (aka Avicenna, Persian, ca. 980-1037), under external influences, it soon came to an abrupt halt due to strong fundamentalist reaction. This reaction began early on, with the anti-rationalism of al-Ash'ari (Arab, ca. 874-936), and came into full force later on, through al-Ghazali (Persian, 1058/9-1111). Even if ibn-Rushd (aka Averroës, Andalusian, ca. 1126-98) made a last-ditch effort to rebut Ghazali, his writings had little effect on Moslem thought in the East. Free thought and free speech in Islamic philosophy effectively never recovered. There were also, of course, political causes for this reversal, notably the Mongol conquests in the 13th century. Islamic philosophy was thenceforth largely limited to the task of theological defense of faith against rational doubt (and Islam against other faiths, as well as disputes between Sunni and Shia Islam), and this has remained its essential role to this day.

This course of events may be described as follows in more sociological terms. A backward people (the Arabs) were suddenly confronted with bits and pieces of the thoughts of more advanced societies (mainly Jewish and Christian to start with, then many others), spurring them into a period of considerable spiritual and cultural progress. However, when they reached the limits of the developmental potential of their core doctrine (the Koran and hadiths), rather than question it and go beyond it they clung to it and erected it into an unassailable dogma. At that critical juncture, Moslems effectively gave up evolving intellectually and chose permanent stagnation instead. For this reason, their societies stagnated politically and economically thereafter, prospering intermittently only by looting other societies. For a while, in the past couple of centuries, it looked a bit as if Western (i.e. European and American) modernity might stimulate them into reviving their own slumbering spirits[51]. But in the last few decades we have seen a violent reaction to such liberating influences, in the form of 'Islamism'.

Not having looked into the hadith collections, or studied subsequent developments in Islamic law, I cannot for the time being propose a more precise analysis of how logic filtered into Islam. However, I propose to next briefly look into a modern work on Islamic law and legal reasoning, and see what we can learn from it about use of a fortiori argument, and eventually other forms of argument, in Islam. In this regard, I will refer mainly to a work by Hallaq[52], a contemporary scholar whose books seem at first blush particularly clear and instructive.

Sources of Islamic law. A fortiori and other arguments are studied in Islam, as in Judaism, in the context of jurisprudence (*fiqh*, in Arabic). The principles of Islamic jurisprudence (*usul al-fiqh*) today acknowledge four main sources of Islamic law (*Shariah*): the Moslems' holy book (*Qur'an*), their oral traditions (*Sunnah*), the consensus (*ijma*) of their religious authorities, and various arguments (*qiyas*) these used to infer information from the preceding sources. There is actually relatively little law in the Koran[53]; most Islamic law is drawn from the other sources. The Sunna[54] refers principally to the sum total of the hadiths accepted within a given Islamic community, or school of Islamic law. Hadiths, as we have seen, are alleged reports of the sayings and doings of Mohammed. Different communities may acknowledge (or reject) different hadiths. Moslem jurists of a given community first attempt to determine the reliability of hadiths (in their view, according to certain standards set by them); then they examine them closely, looking for answers to new legal questions they face – looking for clues in Mohammed's precise words, or even in his silences, on some matter, or in his practical responses in different situations, or in his habitual behavior patterns. He is regarded as "the best exemplar," someone who can do no wrong, and therefore whose words and acts can and should be used as bases for imitation and legislation[55].

The term Sharia – which means, the path or way (which Moslems are required to follow) – refers to the net result of all legal decision-making. It is thus equivalent in concept and etymology to the Hebrew word Halakha. The process of legal decision in Islam is called *ijtihad*. Naturally, in an organized religion like Islam, such decision making cannot be personal (*ra'y*), but must obtain the stamp of approval of the community as a whole (so to speak, meaning

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[49]   The Turks who later conquered Arab territories adopted Islam because it seemed great in their eyes compared to what *they* had before. The Mongols were less inclined to become Moslems, remaining largely aloof rulers – Moslem in name only, if at all.

[50]   Of course, this refers to the Moslems collectively. The philosophers were no doubt a small élite, the masses of the people remaining very ignorant. Still, some knowledge must have trickled down.

[51]   The Al-Nahda (Renaissance) movement is a notable example of such attempted awakening.

[52]   Arab, b. 1955, in Nazareth, Israel. More on this author at: en.wikipedia.org/wiki/Wael_Hallaq.

[53]   Hallaq tells us that though the Koran text "comprises some 500 legal verses… these cover a relatively limited number of legal issues and, furthermore, treat of them selectively" (p. 10).

[54]   "The term *sunna* means an exemplary mode of conduct." (Hallaq, p. 10.)

[55]   This of course goes a long way to explain the lack of wisdom in a great many Islamic laws and customs.

in practice the most authoritative religious scholars); this is the meaning of *ijma*, 'consensus'[56]. Lastly comes *qiyas*[57] which is usually translated as 'argument by analogy', although, while this may be accurate etymologically, the term in practice seems (as we shall see) to refer more broadly to any sort of deductive or inductive inference used to derive laws from Koran, Sunna or consensus. *Qiyas* thus corresponds somewhat to the hermeneutic principles (*middot*, in Hebrew) used in Jewish jurisprudence, or potentially even more largely to logic in general (*mantiq*, in Arabic).[58]

There are two main branches of Islam, the Sunni (to which the majority of Moslems adhere) and the Shia (or Shi'ite, the adherents to which are mostly Iranians). These differ considerably in their specific legal sources, methodologies and decisions, although they both adhere to the general pattern just described. Shia scholars refer to *'aql* (the rational faculty), rather than *qiyas* (arguments), to explain their inferences. There are also different communities within each of these larger groups that have their specific authorities and traditions. There are in Sunni Islam today four main schools of interpretation (*madh'hab*): the Hanafi, the Maliki, the Shafi'i, and the Hanbali; in Shia Islam, there are mainly two: the Ja'fari and the Zaidiyyah. These various schools all began roughly in the period from the mid-8th to mid-9th centuries CE. For historical reasons, their influence is essentially regional, although the Hanafi school is the most widely influential.

4. A fortiori in fiqh, based on Hallaq

We have already, in the previous section, briefly presented various concepts and facts we need to know before we can study Moslem use and discussion of a fortiori and other forms of argument. We shall now turn our attention to this interesting investigation, which is our main goal, referring mainly to **Wael B. Hallaq**'s *A History of Islamic Legal Theories: An Introduction to* Sunni usul al-fiqh[59]. This author devotes a goodly number of pages (pp. 83-107) to the topic of *qiyas*, including a few (pp. 96-99) to a fortiori argument in particular.[60]

Two moods of a fortiori. Hallaq informs us that Islamic jurists were aware of a fortiori argument in both its *a minore ad maius* (from minor to major) and *a maiore ad minus* (from major to minor) forms. First thing to note here is that he does not mention Islamic jurists having a specific name for a fortiori argument, nor tell us as of when they became aware of this argument at all. Secondly, we must note that the names of the two forms of a fortiori argument that Hallaq gives are, of course, Latin; he does not tell us what names Islamic jurists give them, and as of when they became aware of the distinction between them. I found the probable Arabic names of these two arguments in Islamic websites: they are *qiyas al-awla* (analogy of the superior) and *qiyas al-adna* (analogy of the inferior)[61]; however, it is not from there immediately clear which is which, i.e. whether 'of' is intended to mean 'from' or 'to' (see my analysis of this issue further on).

According to Hallaq (on p. 29), this nomenclature was first proposed by **Muhammed ibn-Idris al-Shafi'i** (b. Gaza, 767 – d. Fustat, 820 CE), who is generally considered to be the founder of Islamic "legal theorization" in its more systematic form. Shafi'i "lived in the major centers of learning, where he became exposed to all the influential trends of legal learning." He probably finalized his treatise known as *al-Risala* in Egypt, in about 813 CE. In this work, he offered the following illustration of the said two moods of a fortiori argument:

> "When God forbids a small quantity of a substance, we will know that a larger quantity is equally forbidden… and if He permits a large quantity of something then a lesser quantity of the same thing is *a fortiori* permitted."[62]

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[56]   Ordinary Moslems do not make their own decisions but follow the instructions of their religious authorities (this is called *taqlid*). *Ijma* is perhaps comparable to *rov* (majority decision in rabbinic jurisprudence).

[57]   *Qiyas* is a singular word, even though it ends in *s*. We shall nevertheless use the same word for the plural. I have not found the plural form of the word in Arabic – apparently no one uses it in English.

[58]   The passages to be used as premises in *qiyas* must first be evaluated, by checking that they are not in conflict with any others. If there seems to be some conflict, then the texts concerned must be reconciled somehow, whether by particularization, merger or abrogation (as already explained). *Qiyas* come into play only after such preparatory processing. These preliminaries are of course also hermeneutic acts.

[59]   Cambridge: UP, 1997. See especially pp. 96-99. This book is partly on display in Google books, at: books.google.ch/books?id=pB5mxMebvDUC&pg=PA99&dq=fortiori&hl=en&sa=X&ei=YptlT5LmJYfrOYylhf0H&redir_esc=y#v=onepage&q=fortiori&f=false.

[60]   A difficulty we face here is making a distinction between the author's personal analysis, and the analysis of Moslem traditions. He himself admits this distinction (p. 101). We shall do our best to remain aware of this difficulty and overcome it. Though he is a Moslem, Hallaq is also a modern Western university professor influenced by Western thought; so, his understanding is not purely Islamic. Our primary interest here is, of course, not in his synthetic or syncretic viewpoint, but in purely Moslem thought.

[61]   Moreover, this source mentions a third mood, *qiyas al-musawi* (analogy of equals); this obviously refers to *a pari* argument, which Hallaq does not discuss. An example given for this is Koran 4:2. I have not found out *when and by whom* these three expressions were introduced into Islamic logic.

[62]   This is the formula given by Hallaq on p. 29, between quotation marks. However, since he nowhere says what the expression "a fortiori" looks like in Arabic, I suspect this phrase is a translator's interpolation. On p. 96 he paraphrases this same illustration (without quotation marks) in the following words: "When God or His messenger forbids a small quantity of a certain matter, we conclude that a larger quantity of the

Thus, based on Hallaq's account, we can suppose that Shafi'i was, in the early 9ᵗʰ century CE, the first Moslem author to have consciously focused on a fortiori argument, by naming and exemplifying two of its moods and presumably discussing their operation somewhat[63]. This tells us something about the level of knowledge on this subject in Islam at that time. Since, as we shall presently show, these examples are valid, we can say that Shafi'i was capable of reasoning correctly in this mode. However, assuming he was able to express his understanding only in such relatively material terms, rather than in formal terms, we have to say that though he had mentally isolated the argument, he had not fully grasped how it works. Be that as it may, we may put these arguments in standard form for him as follows:

> A large quantity of a substance (P) is more legally serious (R) than a small quantity of it (Q), whence:
> given that a small quantity (Q) is legally serious (R) enough to be forbidden (S),
> it follows that a large quantity (P) is legally serious (R) enough to be forbidden (S).

> A large quantity of something (P) is more legally serious (R) than a small quantity of it (Q), whence:
> given that a large quantity (P) is legally serious (R) *not* enough to be forbidden (S),
> it follows that a small quantity (Q) is legally serious (R) *not* enough to be forbidden (S).

Note that both of these arguments are purely a fortiori – they are not a crescendo; that is to say, the predicate in the conclusion ("forbidden") is identical ("equal") to that in the minor premise – no proportionality is claimed. The first argument is a straightforward positive subjectal one, going from minor (Q) to major (P). The second argument, being also subjectal, has to be in negative form to go from major (P) to minor (Q) – otherwise it would not be valid. Therefore, the way it is originally formulated, as inference from permissibility of the greater quantity to permissibility of the lesser quantity, is a bit misleading. It is not a very serious error – but it is indicative of the unawareness by the person formulating the argument (Shafi'i) that his argument has to be negative in form to succeed. Positive permissibility may be educed from non-prohibition after the negative a fortiori conclusion has been drawn.

Another thing to note is the absence of an explicit major premise (the comparison between the larger and lesser quantity on some scale). Also notable here is the absence of an explicit middle term – it is I who has introduced the term "legally serious" needed here (some scale of comparison R between the major and minor terms is required); and moreover, the absence of the crucial factor of 'sufficiency' of the middle term in the minor premise and conclusion. The latter is a *sine qua non* to enable the inference: it is *because* the quantity in the minor premise is R enough (or not enough) for prohibition that the quantity in the conclusion is R enough (or not enough) for prohibition. Furthermore, note, the two moods of argument illustrated are both subjectal – there is no display of awareness of predicatal a fortiori argument[64]. Moreover, the examples are both copulative; no example is forthcoming of implicational argument. Thus, at least the way Islamic tradition has presented a fortiori argument thus far reveals a significant deficiency of knowledge. However, as we shall see further on, Hallaq himself does show awareness of the tacit major premise and middle term, although not of the other missing information.

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same matter is also forbidden. Similarly, if the consumption, say, of a large quantity of foodstuff is declared permissible, then a smaller quantity would also be permissible." Note here the addition of "or His messenger" and the specification of consumption of foodstuff, as well as the absence of the words "a fortiori."

[63] Note that there is not mention here of egalitarian (*a pari*) a fortiori argument. The reason may simply be that it is difficult to cast such argument in similar terms. If we try to state it as: "When the Koran forbids or permits a certain quantity of something, it follows that an equal quantity of the same (kind of) thing is equally forbidden or permitted, as the case may be."—it looks like mere tautology.

[64] Even though the one example of a fortiori argument seemingly to be found in the Koran is predicatal in form, Islamic scholars do not seem to have noticed the fact.

Notice as well that the subject-matter of these arguments is not general, but specifically legal prohibition or non-prohibition. This specificity is of course understandable – the arguments are formulated in a legal context. But the question may be asked whether Islamic jurists were or are aware that a fortiori argument can be formulated in non-legal contexts. Actually, it seems that they may well have been, judging by an example given further on (although that example involves value-judgments). Another reflection worth making here is how similar these examples are to rabbinic examples. Consider for instance the following formula for the rabbinic hermeneutic principle of *qal vachomer* proposed by R. Feigenbaum:

> "Any stringent ruling with regard to the lenient issue must be true of the stringent issue as well; [and] any lenient ruling regarding the stringent issue must be true with regard to the lenient matter as well."[65]

This definition of a fortiori argument is proposed by a contemporary rabbi, of course. I have not managed yet to discover exactly when some such abstract formulation first appeared in Jewish history. But I have managed to identify a passage in the Mishna which may be its source (namely, *Beitzah* 5:2)[66]; and there are probably others like it. I have not found quite similar arguments in Greek or Roman discourse. I have found in a work by Cicero, the famous Roman jurist (2nd cent. CE), a clear statement of two moods, indeed three, of a fortiori argument, viz.:

> "What is valid in the greater should be valid in the less … Likewise the reverse: what is valid in the less should be valid in the greater... Likewise, what is valid in one of two equal cases should be valid in the other."[67]

However, while this statement has the ideas of argument from major to minor and from minor to major – and additionally, from equal to equal – it is formulated more vaguely than the Jewish and Islamic statements. "What is valid, etc." is pretty vague – for legal purposes one has to be more specific and mention that what is under discussion is whether the court is *allowed or forbidden* to react in certain ways to defined situations. The reference to such predicates is what makes the a fortiori discourse specifically legal discourse. Moreover, Greek and Roman laws were not essentially deductive, but rather inductive; they were based on commonsense judgments, in the context of relatively hazy cultural traditions. On the other hand, Jewish and Islamic laws are predominantly deductive, even if they do have inductive elements – they are usually derived from some fixed proof-text.

For these reasons, and because Judaism preceded Islam in recorded history, I suggest that Islamic jurists probably learned these forms of a fortiori argument from Jews, either in discussions with learned Jews or through reading some Judaic texts or through Jewish converts to Islam bringing with them their knowledge. Remember that the Mishna had been in existence over 600 years and the Gemara had been around for over 300 years when Shafi'i wrote his treatise. Alternatively, the transmission may have occurred indirectly via Christian intermediaries. It is also possible, of course, that Islamic jurists came upon these forms of reasoning independently; but it seems unlikely. I say this without chauvinism, as an honest opinion based on the overall picture of development of a fortiori argument presented in the present book.

Hallaq next gives two examples of the above described two forms of a fortiori argument, drawn from Islamic tradition.

A crescendo argument passes unnoticed. To illustrate argument from minor to major, Shafi'i offered the following passage of the Koran (99:7-8): "Whoso has done an atom's weight of good shall see it, and whoso has done an atom's weight of evil shall see it." He then remarked (roughly, in Hallaq's words): "From this verse, it is understood that the reward for doing more than an atom's weight of good and the punishment for doing more than an atom's weight of evil are greater than that promised for an atom's weight." But clearly, this is an illicit process on the part of Shafi'i. The two arguments he formulates in his comment (one about good and one about evil) are indeed a fortiori – but these arguments are not given, or even hinted at, in the Koran passage it is based on. See for yourself. The said

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[65]   *Understanding the Talmud*, p. 88. Here, 'stringent ruling' refers to prohibition and 'lenient ruling' refers to permission. The 'stringent issue' is the major term and the 'lenient issue' is the minor term.

[66]   "All these things they [the rabbis] prescribed [as culpable] on a Festival, how much more [are they culpable] on Sabbath." This refers to inference of specific prohibitions from a less important religious ritual (Festivals) to a more important one (the Sabbath). This is a negative predicatal mood; the corresponding positive mood follows by reductio ad absurdum; and two other moods follow from these two in turn by negation of the subsidiary term. See the discussion in the section 'Qal vachomer without dayo' in the chapter 'In the Talmud, continued' (8.5) for more on this topic.

[67]   *Topics*, §23; in the original Latin: "*Quod in re maiore valet, valeat in minori… Quod in minori valet, valeat in maiore... Quod in re pari valet valeat in hac quae par est.*"

passage does not contain any a fortiori argument, nor suggest any sort of inference is intended; it is merely being used by the commentator to construct two arguments external to it.

Notice now that the proposed arguments are a crescendo, and not purely a fortiori. That is to say, they clearly yield *proportional* conclusions. Shafi'i shows no awareness that these two arguments are thus significantly different in structure from the preceding two examples he gave; or at least Hallaq does not mention any such awareness. Put in standard form, these two arguments would look as follows:

> More than an atom's weight of good (or evil) (P) is more consequential (R) than atom's weight of good (or evil) (Q), whence:
> given that an atom's weight of good (or evil) (Q) is consequential (Rq) enough to bring about a certain reward (or punishment) (Sq),
> *and given that S varies in proportion to R*,
> it follows that more than an atom's weight of good (or evil) (P) is consequential (Rp) enough to bring about a *greater* reward (or punishment) (Sp).

The mood is, here again, positive subjectal. The major premise seems obvious enough not to need further proof. Note well here the additional premise about proportionality of S to R (shown in italics), and the fact that the predicate Sp in the conclusion is quantitatively different from (greater than) the predicate Sq in the minor premise on which it is based. Without the tacit additional premise, the proportional conclusion cannot be validly drawn. However, this pro rata premise is a reasonable inference from the given information; obviously, if each atom's weight of good (or evil) engenders a certain quantity of reward (or punishment), then two or more such atom's weights will cumulate two or more such quantities. Hallaq, and therefore presumably Shafi'i, does not mention these important details. They have apparently passed unnoticed in Islamic tradition.

Furthermore, Shafi'i does not show awareness that these two arguments differ from the preceding ones in another significant respect. Namely, that although they involve value-judgments (about good or evil), they are not per se legal arguments[68]. They do *not* conclude that something is permitted, imperative, forbidden or exempt – they only tell us of the expected (natural or Divinely-willed) consequences of good or evil, viz. some reward or punishment, respectively. Of course, it is tacitly intended that, since good brings about a reward and evil brings about a punishment, we *should* do good and abstain from doing evil. The latter are indeed imperatives, or recommendations of wisdom, but they stand outside the a fortiori arguments. They are subsequent conclusions, which are obtained by hypothetical syllogisms from the major premises "if something is beneficial to you, then pursue it" and "if something is detrimental to you, then avoid it." The good being advantageous is thus recommended, and the evil being disadvantageous is thus discommended. But, to repeat, all this comes after the a fortiori arguments as such, and is not part of them.

There are a couple more criticisms we can level in this context. First, the major premise of the first proposed a fortiori argument could be put in doubt by pointing out that an excess of good can sometimes turn to bad. However, this objection can be rebutted by saying that when something that is good in lesser quantities becomes bad due to excess, it ceases to be classed as 'good' and becomes subsumed under 'bad', so that the major premise remains unaffected. Second, the Koran passage can be used to construct syllogisms instead of a fortiori arguments, since any weight of good or evil is composed of many atom weights thereof. The arguments would thus run as follows: any amount of good (or evil) entails some reward (or punishment); so and so is good (or evil); therefore, so and so entails reward (or punishment). The issue of proportionality is not mentioned here, but it could be by stating the major term as 'entails a proportionate amount of reward (or punishment)'. It could be that Shafi'i had these syllogisms in mind, rather than a fortiori arguments.

Concluding more than the conclusion. Again, as example of argument from major to minor, Hallaq offers "the Quranic permission to kill non-Muslims who engage in war against Muslims" (he unfortunately does not specify where in the Koran this is stated), and then adds: "From this permission it is inferred that acts short of killing, such as the confiscation of the unbelievers' property, are also lawful." Here again, it is clear that the a fortiori argument put forward is not explicitly found in the Koran, but is a construct proposed ex post facto by commentators. It seems, judging from these two examples, that Hallaq and no doubt other Islamic jurists do not have a clear idea of the

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[68]   To his credit, Hallaq does notice this discrepancy, saying (on p. 29): "the use of this inference derives from ethical rather than strictly legal subject matter."

difference between literal reading and interpretation. I have so far only found one a fortiori argument in the Koran, and the two examples here offered certainly do not mitigate this poor showing.

Now, assuming that Hallaq has correctly represented Islamic law in this context, the practical purpose of this argument is patent: to provide a legal framework for the lawful expropriation of enemies of Islam, be they still alive or already dead. The argument goes: since the Koran explicitly says they may be killed, then obviously it tacitly intends that they may also be expropriated. This argument can be put in standard negative subjectal form, as follows:

> With regard to non-Muslims who engage in war against Muslims, killing
> them (P) is legally more grave (R) than confiscation of their property (Q),
> whence:
> given that killing them (P) is legally grave (R) *not* enough to be forbidden
> (S),
> it follows that confiscation of their property (Q) is legally grave (R) *not*
> enough to be forbidden (S).

Note that this argument is purely a fortiori like the earlier examples and not a crescendo like those immediately preceding it. The argument has to be put in negative subjectal form because it goes from major (killing) to minor (confiscating property)[69]. Put like that, the argument looks technically flawless; but as we shall see, this a fortiori discourse is only the tip of the iceberg: there are unspoken aspects to the proposed argument.

Needless to say, the content of this argument is open to much criticism. For a start, the characterization (by Hallaq, at least) of non-Muslims as "unbelievers" is ridiculous – disbelief in Islam does not imply disbelief in God, as here implicitly suggested. The intent of this characterization is, of course, to devaluate non-Moslems, and thus make way for their mistreatment in Islamic law. Secondly, the implication that non-Muslims who engage in war against Muslims are criminal, i.e. deserving of legal sanctions in Islamic law, is also ridiculous, since it makes no distinction between wars of aggression and wars in self-defense (at any rate, Hallaq mentions no such distinction here). Judging by this broad term of reference (viz. "non-Muslims who engage in war against Muslims"), Hallaq and Moslems in general think that non-Moslems have no right to defend themselves against Moslem aggression; this is, of course, resorting to a double standard.

Thirdly, according to commonsense and the legal standards of civilized nations, even if one may lawfully kill an aggressor in self-defense, it does not follow that one may lawfully "confiscate," i.e. loot, his property. All the more, of course, one may not steal from a non-aggressor, i.e. from a person who was merely trying to defend himself from one's aggression. This means that the proposed argument is flawed somehow, even if it looks technically okay on the surface. If we were to follow its 'logic' thoroughly, then non-Muslims who engage in war against Muslims may not only be killed and have their property confiscated, but may also be tortured, sodomized, and have their wives, daughters and sons raped – they lose all rights as human beings – for each of these brutal acts is, like confiscation of property, considered to be less severe punishment than killing! This is no doubt the general conclusion consciously or subconsciously sought by this argument – a blanket permission to Moslems to do as they please with non-Moslems with the excuse that they dared to fight Moslems.

What, then, is the logical flaw, here? Obviously, a crime cannot have *unlimited* legal repercussions. It cannot be said that if a crime X1 deserves punishment Y1, then it *also* deserves *all* punishments (Y2, Y3, etc.) that are less severe than Y1. The underlying idea of the said a fortiori argument is rather that the less severe penalty (e.g. loss of property) may perhaps be *substituted for* the more severe penalty (death); it is not an argument that justifies an accumulation of penalties. Just because death is the most severe penalty, it does not follow that it *subsumes* all lesser penalties. It is a maximum penalty, not a genus for all penalties. A crime may well deserve death without deserving a mass of other penalties. Ideally, the penalty reflects the crime, measure for measure: murder is punished by execution, theft is punished by restitution of equivalent property and possibly more (i.e. a fine), and so on – although imprisonment is often used as a substitute penalty, and in some situations financial compensation is considered a fair compromise. There is no concept in rational jurisprudence of all-out vengeance for a single crime, even one deserving capital punishment. Thus, the Islamic interpretation of the said a fortiori argument is abusive: it concludes more than its conclusion. The argument might conceivably be used to justify the imposition of a lesser penalty *instead of* a greater one, but it cannot be used to stack any number of lesser penalties *on top of* a greater one.

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[69]   The same argument could also of course be put in positive predicatal form, i.e. worded: "If non-Muslims who engage in war against Muslims (S) are criminal (R) enough to be lawfully killed (P), then they (S) are criminal (R) enough to be lawfully divested of their property (Q)." However, it is clear from Hallaq's presentation that he does not conceive it in this form; and since we have no evidence so far that predicatal argument is at all noticed by him, or by Islamic jurists in general, we cannot analyze the discourse in this form.

Furthermore, note well, even if we can argue from the *permissibility* (i.e. the non-prohibition, as above) of the major penalty to that of the minor penalty, it does not follow that a similar inference is possible from the *imperative* of the major penalty to that of the minor penalty. In fact, such reasoning would be invalid: we cannot argue from the premise that a certain crime must be subject to a severe penalty to the conclusion that that same crime *must* be punished by a less severe penalty (whether in addition to or instead of the more severe one). Yet only such a (fallacious) inference would justify the accumulation of penalties that the said Islamic argument seems to advocate. We can further show the absurdity of the proposed argument by substituting the term "no penalty" for the term "confiscation of property," and say: since killing a criminal is more grave a penalty than letting the criminal off scot-free, then if killing him is permissible, setting him free at will is permissible. Arguing this way would, of course, be the end of the human justice system, which is based on the concept of retaliation (the *lex talionis*). But moreover, if we assume that the said a fortiori argument allows the minor penalty to be added to the major penalty, as above suggested, then in this case we would obtain a self-contradiction: both the penalty of death and no penalty at all, at once.

In short, the proposed a fortiori argument per se is not formally flawed; what is flawed is the Moslem attempt, consciously or subconsciously, to *subvert* that argument for conclusions that it does not formally afford. In other words, the argument actually proposed, though partly a fortiori, is not entirely a fortiori. Islamic jurists read the conclusion of the a fortiori part in ways that a fortiori reasoning per se does not justify; i.e. they implicitly add layers of meaning to it that it does not logically have. They use it not for strict inference, but to spin their desired final conclusion. Thus, the full argument though disguised as a fortiori, is not really a fortiori. It is not genuine logical discourse, but is just a bit of pseudo-logic designed to provide a legalistic pretext for theft by Moslems of non-Moslem property. This is in addition to the killing by Moslems of non-Moslems, which is allowed (if not called for) by the Koran in the event of any war.

This is, of course, essentially religious discourse, insofar as the protagonists and antagonists are defined by their religious membership. The argument is of course put forward so as to justify Moslem spoliation of non-Moslem property, even after they have killed the owner (and all the more so if they have not killed him); it is an expression of materialist greed. Whether such illogic is cynical manipulation or the result of intellectual confusion finally matters little: the fact remains that Moslems proceed to act on its basis. Ironically, while Moslems evidently consider non-Moslem property theirs to "confiscate" under whatever pretext, they quickly and vehemently object when what they consider to be their property is forcibly taken from them, even if this property was previously "confiscated" by them and is merely being restored to its original owners or their families and descendants or their people! This is of course the situation in Israel – where Arabs claim Jewish lands as their own, conveniently forgetting that they originally stole those lands from the Jews, or from other people who had stolen them from the Jews. This is the meaning of the fashionable characterization of the ancestral land of the Jews as "occupied Arab land."

It is worth observing, in passing, that Hallaq dishes out the above example of a fortiori argument quite uncritically. He does not raise any of the objections regarding the form or content of the discourse that I have just raised. He lets it all pass without comment as if it is obvious and innocuous. Non-Moslems are "unbelievers;" non-Moslems who engage in war against Moslems may have their property "confiscated." We would expect a Humanities professor at a prestigious American university at least to question such outrageous assumptions, if not the 'reasoning' that is put at their service; but he does not. Surely, one cannot let such things pass without comment by claiming they are outside the scope of one's study. Presumably, that means that he approves of them. Pity for him; because he piously suspended his critical faculty in this matter, he missed out on discovering an interesting twist in fallacious discourse – viz. the misinterpretation of a legitimate conclusion in order to fit it in with some preconceived agenda.

Attempts to underpin a fortiori argument. Hallaq gives some interesting information concerning how different Islamic jurists have tried to underpin a fortiori argument by describing its modus operandi. Some considered that "no inference is involved [in it] and the matter is purely linguistic," while others regarded it as "the most compelling form of *qiyas*." This is, really, an issue of formalization and validation. If we do not know how and why an argument 'works', we can hardly rely on it to prove anything.

The issue is discussed by Hallaq and others in relation to the following example. The Koran enjoins Moslems: "Say not 'Fie' to them [i.e. your parents] neither chide them, but speak to them graciously" (17:23). 'Fie' is an interjection (nowadays obsolete) expressing disapproval or even disgust[70]. From this injunction, Islamic commentators infer a fortiori that "actions of the same kind as well as actions exceeding in strength the uttering of 'fie', such as striking one's parents, are prohibited." There of course is no denying the wisdom of respect for parents, and the necessity to

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[70]   The word used in the Arabic original is "uff" – clearly just a sound expressing annoyance, an onomatopoeia. The nearest modern English equivalent would be "ugh" – and indeed one translator has opted for this word (Shakir). Other translators simply resort to description: "a word of disrespect" or "of contempt."

avoid verbally abusing or physically them; this is not the issue here. To clarify what the issue is, precisely, let me restate the proposed a fortiori argument more fully and precisely, in standard form. It is obviously a positive subjectal argument, going from minor (Q) to major (P):

> Physically striking one's parents (P) is morally more serious (R) than verbally abusing them (Q), whence:
> given that verbally abusing them (Q) is morally serious (R) enough to be forbidden (S),
> it follows that physically striking them (P) is morally serious (R) enough to be forbidden (S).

This is, note, a purely a fortiori argument (the subsidiary predicate S, "forbidden," being the same in minor premise and conclusion). The middle term (R), viz. "moral seriousness," has been introduced by me, but is clearly the minimum tacitly intended (if Islam enforces a law relating to parental abuse, the middle term would rather read "legally serious"). Note also that this a fortiori argument is not literally given, or even hinted at, in the Koran. Rather, the Koran passage provides the minor premise, while commentators provide the major premise (which is reasonable enough, because almost everyone would agree to its truth on the basis of normal human sensibility) and draw the conclusion (which is morally undeniable).

The argument is intuitively valid, and indeed since we can put it in standard form we know it can be formally validated (using the methods of validation developed earlier in the present work). This is my own formalization and validation for the reader's benefit. A fortiori argument undoubtedly appears intuitively valid to Moslems, as it does to all other people. But Moslem commentators are not entirely clear as to how and why it seems valid, or at least they disagree on this issue. It is the debate between Islamic jurists that concerns us, here; it allows us to measure their levels of understanding of this type of argument. Hallaq presents a couple of theories on the subject, plus one of his own.

The purely 'linguistic' explanation, regarding how the prohibition of saying 'fie' to parents in the above passage of the Koran gives rise to a wider prohibition including violent acts against them, was adopted for instance by the Hanafi jurist Ibn Sahl Sarakhsi (Persian, d. 1096 or 1101 CE). His view was, according to Hallaq, that "the full extent of the meaning of 'fie' is in fact encompassed by the meaning of harm." Thus, the word naturally ranges from expressing disapproval to striking parents, and no inference is involved in applying the Koran's rule not to say 'fie' to other forms of harm. The 'linguistic' explanation was opposed by other commentators, such as the Shafi'i jurists Abu Ishaq as-Shirazi (d. 1083[71]) and Abu al-Hasan al-Mawardi (Arab, 972-1058 CE). These emphasized that the language in such case is not conclusive, and inference must be involved. As Hallaq puts it, the conclusion is "deduced from the intention behind the prohibition of uttering the expression 'fie', and not intuitively conceived from the very word itself."

This was the position of **Abu Hamid al-Ghazali** (Persian, 1058-1111), also of the Shafi'i school. In his view, Hallaq tells us, "we deduce this prohibition through the *ratio* behind the necessity to respect parents, and the knowledge that uttering 'fie' runs counter to such respect." This is, for Ghazali, "the very course of reasoning known as *qiyas*," here using the term *qiyas* in its core sense of argument by analogy[72]. Hallaq reduces Ghazali's argument to the following syllogism: "All harmful acts (directed against one's parents) are prohibited" and "striking is a harmful act," therefore, "striking (one's parents) is prohibited." In this schema, which I will call 'syllogistic', "harm" is assumed by the jurist as the "*ratio legis*," i.e. the reason behind or underlying the law, which makes possible the inference from the law against saying 'fie' to parents to one against using violence against them. According to Hallaq, this argument is an enthymeme (i.e. incompletely stated), because "the jurists… supplied the implied premise" that saying 'fie' is a form of harm.

Hallaq's own perception of the argument is more in line with what we would regard as a fortiori argument. In his view, the inference is not subsumptive, as Ghazali's schema suggests, but transitive. It is "asyllogistic," as it involves "a comparison in terms of 'more', 'greater', 'smaller', etc." Thus, in the example under consideration, "striking parents is *more objectionable than* saying 'fie' to them" (his italics). This is, of course, a more accurate rendering of a fortiori argument; and Hallaq deserves credit for the clarity of his awareness of this premise, which most

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[71]   Presumably he was Persian, judging from his name. Hallaq does not say.
[72]   More precisely, in the sense of the rabbinic hermeneutic principle of *binyan av* (see further on).

commentators leave tacit. Thus, Hallaq presents three theories regarding the functioning of a fortiori argument. Two of them are drawn from Islamic tradition; and one is apparently his own, more modern explanation.[73]

Now let me react to these proposals. In my view, the 'linguistic' interpretation (in this particular example, at least[74]) is manifestly a cop-out – a mendacious attempt to evade the issue of a fortiori argument by denying the occurrence of an inference. In English common usage, at least, the word 'fie' *only* denotes verbal abuse, and certainly does *not* connote physical violence. The latter meaning is *read into* the text, rather than *read out of* it. To pretend that the word 'fie' equally intends verbal insults and acts of physical violence is therefore to lie. To label something in a convenient manner does not change its real nature. The mental movement from the former to the latter can only be honestly interpreted as inference, and specifically inference from minor to major. This inference is best represented as a fortiori, as above shown; but there is another way to look upon it.

A case could be made for its representation as movement from the thought behind or underlying the saying of 'fie' to parents, namely disrespect for them, to a non-verbal and more forceful expression of that same thought, viz. striking parents. We can describe this process more precisely as follows. We start with the Koranic law that "Saying 'fie' (to one's parents) is forbidden" and the commonplace that "Saying 'fie' is disrespectful." From these two premises, we form *a third figure syllogism* (3/AAI), whose conclusion is: "Some disrespectful acts (to one's parents) are forbidden"[75]. This conclusion is next *generalized* to give: "*All* disrespectful acts (to one's parents) are forbidden"[76]. This general law is next used as the major premise of *a first figure syllogism* (1/AAA), whose minor premise is "Striking (one's parents) is a disrespectful act" and whose conclusion is "Striking (one's parents) is forbidden." The latter is the desired final conclusion. The logical process leading to it thus consists of three stages: a deductive act, followed by an inductive act, followed by another deductive act.

This is a more precise rendering of the analogical process Ghazali must have had in mind than the single "deductive inference" proposed by Hallaq. Hallaq does not show he realizes that what he calls the *ratio legis* (viz. the prohibition of acts harmful[77] towards parents) is the product of a generalization, an inductive act capable of formal articulation. Hallaq does acknowledge that the ratio depends on a prior insight, when he says that "the jurists… supplied the implied premise," but he does not clearly describe this prior act. However, although Hallaq does not say so explicitly, he does realize that Ghazali's analogical schema is not a faithful rendition of what we call a fortiori argument.

Note this well: although Ghazali's reading is a credible alternative to a fortiori argument in this context, it is not a fortiori argument. Even though it can equally well explain arguments like the 'fie' example above given, it does not follow that it describes a fortiori reasoning. For whereas a fortiori depends on quantitative comparisons, the Ghazali formula works equally well with non-quantitative comparisons. From this insight, we can understand why Islamic tradition classifies a fortiori argument under the heading of *qiyas*, i.e. analogical argument: it is not because they consider a fortiori argument as analogical (which it is, somewhat), but because the authoritative commentator Ghazali misread its form. This is implied in the continued use of the generic name '*qiyas*' when referring to what is in fact more specifically a fortiori argument, something quite distinct. There does not seem to be a special name in Arabic for a fortiori argument.

In today's atmosphere of Islamist intolerance, Hallaq's approach is brave, in that it is somewhat critical – of both traditional explanations of a fortiori argument, i.e. the 'linguistic' of Sarakhsi and the 'syllogistic' of Ghazali. The criticism is evident in the fact that Hallaq proposes an alternative, and indeed more accurate, schema – involving the comparative major premise "striking parents is *more objectionable than* saying 'fie' to them". Unfortunately, he does not make his argument fully explicit, as he could have by saying: if saying 'fie' to one's parents is sufficiently objectionable that the Koran explicitly prohibits it, then striking one's parents must be sufficiently objectionable that the Koran implicitly intended to prohibit it. That is, although Hallaq's major premise has an explicit middle term ("objectionable"), he does not show awareness of the role played by this middle term in the minor premise and conclusion, i.e. that latter can be drawn from the former due to the sufficiency of the middle term. This is, of course, a serious deficiency in his treatment.

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[73]   Hallaq does not say whether this insight is his own, or derived from some other person's work. This book was published in 1997; but I do not think he was influenced by me, because his treatment lacks many important details found in my *Judaic Logic* (which was first published in 1995). I am assuming here that Hallaq does not intend to pass off his theory, here, ex post facto, as an originally Islamic idea – it is obviously one influenced, directly or indirectly by, Western perceptions.

[74]   There are no doubt cases where the 'linguistic' interpretation is the most appropriate, e.g. because the language is so vague or metaphorical that a general intent can be assumed immediately because appeal to any more precise form of argument (such as analogy or a fortiori) would be forced. But this is clearly not the case with the example of "saying 'fie'."

[75]   Note that 'some' here does not necessarily mean 'many', but merely means 'at least one'.

[76]   Such generalization is valid if and only so long as no counter-example is found; i.e. provided there is no textual evidence that "Some disrespectful acts (to one's parents) are *not* forbidden."

[77]   Note Hallaq's use of "harm" as the middle term, whereas I prefer to use "disrespect." This difference is not very significant; either term will do the trick. I think mine is more accurate and his is too broad.

Moreover, Hallaq shows some confusion in his concluding remarks, when he says that a fortiori inference is neither deductive (since its "premises have not been originally stipulated but are themselves conclusions of yet another inference") nor inductive (because, whereas "in analogy the inference proceeds from a particular to another particular," in a fortiori argument "there is no such parity"). Both these claims (and the reasons for them) are inaccurate (non-sequiturs). In truth, a fortiori argument per se is deductive, insofar as the premises together definitely imply the conclusion; yet it is also usually inductive, insofar as some elements in it are based on extrapolation or generalization. Additionally, in his view, whereas analogy is based on "a similarity" between the cases compared, in a fortiori argument the conclusion is drawn without appeal to such similarity. This too is inaccurate. *All* argument, including a fortiori, and not only argument by analogy, is based on appeal to similarities. The similarity always lies in the middle term – whether this is a particular or a general term.[78]

Nevertheless, on the whole, Hallaq's presentation is a valuable contribution to the historical study of a fortiori argument. Assuming that his reading of Islamic tradition is reliable and complete, it reveals to us that past Islamic scholars were in fact *not* able to theoretically understand a fortiori argument correctly, even though they apparently were able to intuitively use it in practice. They either effectively denied that it is inference, dishonestly claiming it is merely a 'linguistic' clarification; or they misconceived it as a kind of 'syllogistic' inference, from a more general principle to specific cases. Although Hallaq does come up with a third alternative, using a comparative major premise, he does not attribute it to some past Islamic scholars, but presents it as his own.

We could of course say that Hallaq is himself an "Islamic jurist" and thus conclude that Islam has at least lately (through him) produced a credible, albeit partial, explanation of a fortiori argument. However, this is a rather forced conclusion, considering that Hallaq even if well-versed in Islamic laws and methods is no doubt also someone much influenced by his modern Western education, which seems to have begun in Israel and continued in North America. Moreover, the question must be asked whether other Moslems approve of his innovation, or reject it as unorthodox. What is evident anyway, judging from his account, classical Islamic jurists did not manage to truly grasp the distinctive features of a fortiori argument. *They could apparently intuitively use the argument (some forms of it, at least) correctly, but they could not pinpoint how and why it works.* At least, that is my conclusion based on the information obtained thus far.

On second thought. After writing the above, I had the following reflections. Intuitively, the Koran passage seems to call for the said reading or inference. Why does the Koran mention the lesser sin, and say nothing of the greater sin? Obviously, because if it had only mentioned the greater sin, the lesser one would have remained unknown; but by mentioning the lesser sin, the greater sin was logically included albeit tacitly. Can one imagine saying that "saying 'fie' to one's parents is forbidden" without as well intending that "striking one's parents is forbidden"? Clearly, one cannot reasonably do so. But how can this be proved more formally? To correctly assess Koran 17:23, it is wise to look at the whole of it. Hallaq, as we have seen, uses the following translation: "Say not 'fie' to them neither chide them, but speak to them graciously." This seems to be largely based on the Pickthall translation, which presents the whole passage in the following terms:

> "Thy Lord hath decreed, that ye worship none save Him, and [that ye show] kindness to parents. If one of them or both of them attain old age with thee, say not 'Fie' unto them nor repulse them, but speak unto them a gracious word."

There is a difference, as can be seen, in that Pickthall's "nor repulse them" is rather vague. Hallaq's "neither chide them" suggests a more specifically verbal act of reproach, and some other translations concur in this respect, saying "nor chide them" (Shakir), "nor shout at them" (Muhsin Khan), "nor scold them" (Dr. Ghali)[79]. Thus, the expressly forbidden acts (saying 'fie', chiding) are *verbal* ones; there is no hint of a larger intention there. Indeed, the rest of that sentence, "but speak to them graciously," confirms that this Divine decree is concerned, at least primarily, with forms of *speech*. It just says: do not speak ungraciously (by saying 'fie' or chiding), but on the contrary do speak graciously (to your parents). If the sentence had been intended more broadly, it would have included some word or phrase indicative of such intent, e.g. "do not *so much as* say 'fie' to them." Indeed, some translations[80] include these additional words in round or square brackets. The fact that they do so may mean the words are felt to be intended – but they may also mean that they are felt to be needed and yet lacking.

However, if we look beyond that sentence, and notice the preceding sentence, which introduces it, we can assume a larger intent of "kindness to parents." This is the wording in the Pickthall translation; other translations refer

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[78]   Or the middle thesis. Hallaq also says of *reductio ad absurdum*, which he classifies as a third type of *qiyas* (besides analogy and a fortiori argument), that it "like the *a fortiori*, lacks all analogical features" (p. 101). But in my view, since such reduction relies on apodosis (specifically, as Hallaq himself describes it, on the *modus tollens*), it also calls for comparison (in this case, the contrast between the minor premise and the consequent of the major premise that it denies).

[79]   See various translations at: quran.com/17/23.

[80]   Sahih International, Shakir.

variously to: "good treatment," "dutiful[ness]," "goodness," and "fairest companionship," to parents. Thus, we do in fact have an *explicit* textual basis for broadening the application of the decree from mere speech to other forms of action. That is, it can reasonably be claimed that not only ungracious speech, but all forms of unkindness are forbidden. Ungracious speech is only given as an example of the more general prohibition of unkind deeds; thus, it does not exclude, but rather includes, more marked forms of unkindness such as "striking one's parents." In view of these insights, the easiest course in this specific example is to syllogistically deduce the interdiction to strike one's parents from the prohibition of unkindness to them, which is simply the obverse of the command to be kind to them. No generalization is needed, note well, since the general law is already explicitly given[81].

However, if this is true, we can equally easily infer syllogistically the interdiction to say 'fie' to one's parents and to chide them from the prohibition of any unkindness to them! In that case, what need had the Koran to specify the obvious conclusion about speech in particular? The answer to that could simply be: in order to exemplify the kind of deduction that should be carried out. But then, one might ask, why exemplify by means of the least form of unkindness, rather than by means of a more harsh form, such as striking one's parents? It is only at this stage that appeal to a fortiori argument becomes relevant again: because prohibition of unkind speech a fortiori implies prohibition of more unkind deeds (such as physical violence), whereas the reverse is not true. Thus, if a fortiori argument is at all involved in this passage of the Koran, it is not in order to deduce the prohibition of harsher forms of unkindness from that of unkind speech (for both these prohibitions are already implicit in the general prohibition of unkindness), but only in order to explain why unkind speech is given as example rather than more unkind deeds.

From here on, we must again judge the pertinence of the various explanations of a fortiori argument. The 'linguistic' explanation by Sarakhsi is seen to be inadequate, because even though it is correct in considering that the text explicitly includes the information (viz. the prohibition of all unkindness) needed to obtain the proposed law (viz. prohibition of unkind deeds), this explanation wrongly considers this information to be merely a connotation of the prohibition of unkind speech (whereas it is in fact given in the preceding sentence on the necessity of kindness), and moreover it cannot also explain why the Koran specifically mentions the lesser example (viz. prohibition of unkind speech). As regards the 'syllogistic' explanation by Ghazali, it is correct in referring to a *ratio legis*, since one is explicitly given in the text (viz. the necessity of kindness), and from its obverse we can – without even resorting to any generalization – syllogistically deduce the proposed law (viz. prohibition of unkind deeds); however, this explanation does not correctly identify the source of the middle term (viz. the preceding sentence), and moreover, here again, fails to explain why the Koran specifically mentions the lesser example (viz. prohibition of unkind speech).

Thus, only a fortiori argument, correctly conceived – as Hallaq in part succeeds in conceiving it – is capable of explaining the Koran's specific mention of prohibition of unkind speech and how it implies prohibition of unkind deeds. Whence, we return to the same conclusion; namely, that past Islamic scholars did not succeed in clarifying the nature and validity of a fortiori argument. They could use the argument, but apparently – assuming the accuracy of Hallaq's historical account – they were not able to grasp its exact conditions and functioning. This is an important finding for historians of logic to note. As regards Hallaq's own – much better, though incomplete – interpretation of a fortiori argument, we have already suggested that it was probably influenced (even if unconsciously or subconsciously) by his acquaintance with more modern Western reflections on this subject.

Modern usage. Let alone Hallaq's contemporary interpretation of a fortiori argument, even mere usage of a fortiori argument in today's context is of questionable origin. Considering the worldwide use of radio, television, telephonic and internet technologies, including their use by very traditional Moslems, we cannot immediately assume when we observe a Moslem speaker or writer using a fortiori argument that he learned this form of argument exclusively from Islamic sources, even if the content of his discourse is entirely Islamic. He may well have absorbed the form from Westerners or from people who absorbed it from Westerners. To illustrate this issue, I give you the following two examples I came across by chance. These two statements were originally in the Arabic language by Islamic commentators. Leaving aside for a moment the obscenity of what they are actually saying, we can see that these arguments are formally valid.

The first argument is an attempt to defend the sexual appetite of the prophet of Islam, Mohammed (who married at least nine women, including a child of six, a captive Jewish married woman and the wife of his adopted son). This sexual appetite, Muhammad Husayn Haykal assures us[82], "constitute[s] no flaw in the prophethood of Muhammad, in his own greatness or that of his message," because "the rules which are law to the people at large do not apply to the

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[81]   Thus, my reference to generalization in my earlier analysis of Ghazali's interpretation was unnecessary. It was unnecessary in this specific example because the required major premise is already textually given; but in other examples, where the required major premise is *not* textually given, the stage of generalization is essential.

[82]   In his *The Life of Muhammad*, translated by Isma'il Razi A. al-Faruqi (1968). Quoted by Robert Spencer in his *The Truth about Muhammad*, p.70.

great. *A fortiori*, they have no application on prophets, the messengers of God" (my emphasis). Of course, reason would have it that the standards of morality for spiritual leaders should be more stringent than those for common folk of whatever level; but, supposing the apologetic premise of Haykal that the opposite is true, the a fortiori argument would pass: If great men are great enough to be exempt from ordinary morality, then all the more so are prophets (who are greater than great men) exempt.

The second argument, spoken by the Islamic cleric Muhammad Al-'Arifi[83], is an explanation as to why Moslem men should beat their wives (as a last resort) only lightly: "Beating in the face is forbidden, even when it comes to animals… If this is true for animals, it is *all the more* true when it comes to humans. So beatings should be light and not in the face" (my emphasis). Here again, without accepting the speaker's premises that wives may be beaten, or that women may be compared to animals even if favorably, we can admit that the argument is formally passable: If animals have enough dignity that it is forbidden to strike their faces, then women (who have still more dignity) should not be so struck.

From these examples we see, by the way, that a fortiori reasoning can be used for the most nefarious purposes! Their content is definitely Islamic (Judaism and Christianity do not admit such outrageous conduct) – but is their form Islamic? Where did these two Moslems learn to argue a fortiori? Was it solely from Islamic tradition or as a result of Western influence, however indirect? This question is difficult to answer offhand.

5. Other presentations and issues

I found in the Internet what could be considered a different Moslem attempt to understand a fortiori argument[84]. Hallaq does not mention this alternative theory in his study; that is surprising and suggests his exposé is incomplete somewhat. However, judging from its frequent repetition in the Web, we may assume that this theory is commonly taught in at least some circles. However, I cannot tell when and by whom it was first expounded, as those who use it do not mention its original author.

Definitions and analyses. The theory is apparently founded on the following more detailed definition of the word *qiyas*. This means, firstly, "measuring or ascertaining the length, weight or quality of something. Scales are called *miqyas*." Also, it means "comparison – equality or similarity between two things." This twofold definition is interesting in that its first fork seems to clearly refer to a fortiori argument, while the second fork seems to refer to argument by analogy in a larger sense (which includes comparisons of quantity) or more narrowly in a residual sense (referring to non-quantitative comparisons). Moreover, note that the Arabic term *qiyas* as here explicated seems to correspond exactly to the Hebrew term *midah*, which likewise refers both specifically to a measurement and more broadly to a hermeneutic principle; this suggests a historical connection between Islamic interpretative techniques and the earlier rabbinic hermeneutic principles. The Arabic word for analogy, as a logical process, is actually *tamthil*. However, as we shall presently see, this definition promises more than is later delivered (in the said webpages, anyway). The theory proceeds by naming three "varieties" of *qiyas* (analogy); they are: *qiyas al-awla* (analogy of the superior), *qiyas al-adna* (analogy of the inferior), and *qiyas al-musawi* (analogy of equals). We might at first sight interpret these terms as referring to a fortiori argument, since that likewise includes three essential forms: from minor to major, from major to minor and from equal to equal (i.e. *a pari* or egalitarian a fortiori). However, while it is clear that analogy of equals may refer to *a pari* argument, it is not clear with regard to the other two varieties which name refers to which argument. Does "of" here mean 'from' or 'to'? Do the terms "superior" and "inferior" correspond to the terms 'major' and 'minor', respectively – or is the reverse true?

The answer to this question may be forthcoming if we consider how the three varieties are defined. *Qiyas* is described as an argument with four components: an "original case (*asl*)," a "new case (*far'*)," an "effective cause (*'illah*)" and a "rule (*hukm*)." The rule is known to apply to the original case on the basis of a source text, principally the Koran or the Sunna; the new case is a subject for which a ruling is sought, seeing it is not directly dealt with in any source text; a common ground between original and new cases provides a bridge from the former to the latter over which the rule may be carried. Thus, the major premise of the argument is the common ground (which Hallaq termed the *ratio legis*, the reason for the law) between the two cases; the minor premise is the application of the rule to the original case; and the conclusion is the application of the rule to the new case.

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[83]   In 2007, on Saudi and Kuwaiti TV. Quoted by Spencer in his *Infidel's Guide*, p.173.

[84]   See for instances: ufaoil.blogspot.ch/2008/02/rule-of-qiyas-its-meaning-justification_09.html, which lists the basics; www.islamicboard.com/worship-islam/4985-usul-al-fiqh-qiyas-analogical-deductions-detailed-description.html, which is more detailed; and www.theislamicseminary.org/articles/article002.html#article and related pages, which give additional details in some areas. The author(s) of these and similar webpages are not clearly identified, so I do not attempt to name them here. In any case, their names are not very important since they seem to all be following roughly the same line – which implies that they have some source(s) in common. The reader should study these and similar webpages, which contain many interesting details and conditions not here specified.

From this description, it would seem that what is here meant by *qiyas* is inference based quite simply on analogy, or perhaps rather the more complex form of analogical inference we earlier saw used by Ghazali. There is no description of the distinctive features of a fortiori argument. Thus, although the initial definition of some *qiyas* with reference to measurement, and the subsequent distinction between superior, inferior and equal *qiyas*, both seem to allude to a fortiori argument, the analysis of *qiyas* into four components (*asl, far', 'illah* and *hukm*) does not seem to have been made with a fortiori argument in mind. We could admittedly regard the *ratio legis* in its case to be the middle term used to quantitatively compare the major and minor terms; but this would be forcing things, for the said analysis is clearly syllogistic rather than quantitative. That is, it formally proceeds as follows:

> (a) 'Original case X is (textually given as) subject to rule S'. But new case Y is not textually given as subject to rule S. (This is based on literal reading of proof-text.)
> (b) Items X and Y both have property R (general knowledge, or textually given).
> (c) From (a) and (b) it follows by substituting 'some R' for 'X' that 'some things that are R are subject to S'. This is a deductive argument (substitution, 3/AAI).
> (d) From (c) it follows by generalization that 'all things that are R are subject to S', *provided* there is no counter-evidence, i.e. no case of R *known* not to be subject to rule S. This is an inductive argument (note well its negative proviso).
> (e) From (b) and (d) it follows syllogistically that 'New case Y is subject to rule S'. This is a deductive argument (syllogism, 1/AAA).

Notice that in this argument it makes no difference whether the new case Y is quantitatively larger, smaller or equal in some respect to the original case X – *the argument works just as well anyway*. And it works equally well without reference to any quantitative relation between X and Y. In Talmudic logic, this form of argument is called *binyan av* (lit. father construct, argument by analogy based on a common cause), and we shall here refer to it by this name. In the list of thirteen hermeneutic principles attributed to R. Ishmael ben Elisha (Eretz Israel, 90-135 CE), this is the third principle, after a fortiori argument and *gezerah shavah* (lit. equal rulings, argument by analogy based on synonymy or homonymy, i.e. on similar wording or meaning). Thus, the argument above described by Moslem commentators (initially by Ghazali, it seems) was one that had been in use for many centuries, very frequently and quite consciously, in rabbinic discourse. And to repeat, this argument is not to be confused with a fortiori argument, in which the middle term R is a scale of quantitative comparison and the conclusion (regarding being subject to rule S) *depends* on the positions of X and Y on that scale (if X is less than Y, the conclusion follows – but if X is more than Y, it does not).

Most people who try to analyze *binyan av*, whether Jewish or Moslem, think of it in the following terms: although it is given in the text that the original case X is subject to rule S, *the reason why* X is S is (it is claimed) that X is R and all R are S; this being so, since new case Y is also R, it follows that Y is also S. In this supposed modus operandi, it is assumed that the major premise 'all R are S' is knowable through some sort of direct insight; although the text only explicitly gives the more specific 'X is S', it is assumed to thereby implicitly intend the more general 'all R are S' from which 'Y is S' may be deduced. But this is of course a naïve view: as we have just shown the logical process through which we arrive at the desired conclusion is more complex and quite capable of formal representation; it is not an issue of direct insight but a set of deductive and inductive steps.

In view of the above, the three terms 'analogy of the superior', 'analogy of the inferior' and 'analogy of equals' become quite mysterious. Since as we have just clearly shown it makes no difference in *binyan av* reasoning whether the new case Y is greater or smaller than, or equal to, or not quantitatively related to, the original case X, since the conclusion is based on generalization and syllogism and not on quantitative comparison, these three terms seem to be quite out of place in *binyan av*. It appears that Moslem jurists tried to introduce them artificially, mimicking their use in Jewish, Greek, Roman, and Christian discourse in relation to a fortiori argument, but they did not know quite what to make of them, since they had no clear idea of the distinctive form of a fortiori argument, compared with their essentially non-quantitative analogical arguments (*qiyas*).

We cannot even assign to these terms epistemic instead of ontical values, for though it could well be said that the conclusion (about Y) is *less sure* than the premise (about X), due to an inductive act (generalization) intervening between them, it could never be said that the conclusion is *more* sure than or even (usually) *as* sure as the premise. Thus, while the term 'analogy from the superior (to the inferior)' might thus be given meaning (viz. from the more sure to the less sure), the terms 'analogy from the inferior (to the superior)' and (to a lesser extent) 'analogy from equal (to equal)' would be meaningless.

What, then, are these terms supposed to designate in Islamic jurisprudence? They are presented as follows. In 'analogy of the superior', the *ratio legis* (*'illah*) is "more evident" in the new case than the original case, and in

'analogy of the inferior', it is "less clearly effective" or "present less clearly" in the new case than the original case," whereas in 'analogy of equals', it is "equally effective" or "equally present" in both. Notice the description of the *ratio legis* as more or less "evident" or "present" (the Arabic word intended seems to be *zahir*). These terms, "evident," "present," or the like, could be intended as ontical or as epistemic – they are not in themselves clear on this point. The main question to ask here, though, is whether or not these definitions are applicable to a fortiori argument. To answer these several questions, we must closely examine some of the examples usually given as illustrations of these definitions.

Illustrations. The example usually given for 'analogy of the superior' is Koran 17:23; that is, the argument based on the prohibition of saying 'fie' to one's parents and concluding with the prohibition of striking one's parents. As we saw in the previous section, if the whole of this verse of the Koran is taken into consideration, there is no need of a fortiori argument to draw the conclusion – it can be drawn syllogistically from the explicitly given general command to be kind to one's parents; a fortiori argument can still, however, be used incidentally to explain the Koran's choice of example. But if, for the sake of argument, we ignore the said general command, the desired conclusion can be drawn in two ways: either as by means of a *binyan av* type of argument (as Ghazali did, though he did call it thus) or by means of an a fortiori argument (as Hallaq much later suggested). In the latter event, if we use the comparative major premise proposed by Hallaq, and ourselves add the formally required minor premise and conclusion, the argument would look as follows:

> Striking one's parents (P) is more objectionable (R) than saying 'fie' to them (Q),
> and saying 'fie' to them (Q) is objectionable (R) enough to be forbidden (S);
> therefore, striking them (P) is objectionable (R) enough to be forbidden (S).

This is a valid positive subjectal a fortiori argument, going from minor term (Q) to major term (P), passing through the middle term (R) to the subsidiary term (S). Thus, from an *ontical* perspective, this argument can well be labeled 'analogy of the superior' in the sense of 'analogy *to* the superior (from the inferior)', i.e. *to the more* objectionable from the less objectionable. Since this is deductive argument, the conclusion is in principle as sure as the major and minor premise combined; i.e. if they are both 100% sure, then it is also 100% sure. However, since only the minor premise is textually given, whereas the major premise is proposed by the interpreter (even if everyone would agree that it is very reasonable indeed), the conclusion may be said to be possibly slightly less sure than the minor premise. Thus, in an *epistemic* perspective, this argument can only be labeled 'analogy of the superior' in the sense of 'analogy *from* the superior (to the inferior)', i.e. *from the more* sure to the less sure. Thus, here, the ontical and epistemic senses of the term 'analogy of the superior' are in disagreement. Nevertheless, the example constitutes a credible illustration of a fortiori argument.

Having thus sorted out all the possibilities of interpretation, let us reassess the proposed theory that in 'analogy of the superior' the *ratio legis* (*'illah*) is "more evident" in the new case than the original case. If the term "evident" is interpreted in an ontical sense, this sentence means that the new case (striking) contains more of the *ratio legis* (objectionable qualities) than the original case (saying 'fie') – this is true. If the term "evident" is interpreted in an epistemic sense, the same sentence would mean that the new case (striking) has more *certainty* than the original case – which is usually both in principle and in practice untrue. Therefore, judging by the given definition of 'analogy of the superior', we can only credibly interpret it as ontical rather than as epistemic in intent.

We can similarly quickly deal with 'analogy of equals'. An example often given of this is the following: "according to a Hadith, a container which is licked by a dog must be washed seven times. The Shafi'is extend the same ruling by analogy to a container which is licked by swine.[85]" This example suggests that although dogs and swine are quite distinct animals, they are equal with respect to 'impurity' (in Moslem eyes[86]). Thus, any rule concerning 'impurity' relative to the one must equally apply to the other. This argument can be equally well conceived as *binyan av* or as a fortiori. In the *binyan av* approach, the law regarding vessels licked by dogs is generalized into a law for all 'impure' animals; after which this generality, now our major premise, is applied syllogistically to swine. In the a fortiori approach, the major premise declares dogs and swine 'impure' to the same degree; thence it follows that if dogs are impure enough that vessels licked by them must be washed seven times, the same applies to swine. However, it is clear from the above quotation that the Moslem interpretation in fact proceeds "by analogy" (i.e. by *binyan av*), and not by a fortiori argument. Anyway, as regards the expression 'analogy of equals', it is clear that the equality

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[85]   "The Hanafis, however, do not allow this Hadith in the first place."
[86]   Oh, how I pity Moslems for disliking dogs and missing out on the joys of interacting with 'man's best friend'!

intended here is ontical, because from an epistemic perspective the conclusion is pretty well bound to be less sure than the minor premise in view of the uncertainties inherent in the major premise.[87]

Next, let us consider 'analogy of the inferior', which is the most dubious of the three concepts. Remember that this was defined by saying that in such context the *ratio legis* is "less clearly effective" or "present less clearly" in the new case than the original case. The following illustration is usually given for this argument:

> "The rules of *riba* prohibit the exchange of wheat and of other specified commodities unless the two amounts are equal and delivery is immediate. By analogy this rule is extended to apples, since both wheat and apples are edible (according to Shafi'i) and measurable (according to Hanafi)… But the *'illah* of this *qiyas* is weaker in regard to apples which, unlike wheat, are not a staple food."[88]

One author remarks, by way of explanation of the term 'inferior', that "it is not quite so obvious whether the new case falls under the same ruling which applies to the original case." The argument can be restated as follows. Under certain conditions (namely, "unless the two amounts are equal and delivery is immediate"), a certain commercial operation relating to one sort of commodity (namely, "exchange of wheat and of other specified commodities") is forbidden (by "the rules of *riba*"). Islamic jurists seek to extend the same interdiction to another sort of commodity (namely, apples). For this purpose, they resort to "analogy" (this is how the argument is characterized in the quoted illustration), through the intermediary of a common property the two commodities (wheat and apples) have in common. One authority (Shafi'i) suggests that the effective common property might be "edibility," while another authority (Hanafi) suggests that it might rather be "measurability." However, someone raises the objection that, in either event, the analogy is imperfect, because wheat is a staple food, whereas apples are not. Thus, though the conclusion about apples (the new case) can be drawn from the premise about wheat, etc. (the original case), the conclusion is "weaker" than, or "not quite so obvious" as, that premise.

All this seems reasonable enough at first glance. However, let us look deeper. First, let us note that the common attributes of "edibility" and "measurability" are objectively true of all foods and all physical bodies, respectively; whereas the differentia being or not-being a "staple food" is an issue of custom, i.e. somewhat a subjective attribute. Thus, whereas the former might be confidently presented as candidates for the role of *ratio legis*, the latter constitutes a somewhat artificial objection. Nevertheless, it was reasonable to look for an objection, because there was a need to explain why wheat was specifically mentioned, and not (say) apples, or foods in general. That is to say, the argument presented is subject to contrary forces: on the one hand, there is a tendency to extend it to other foods (from wheat to apples); and on the other hand, there is resistance to such extension, due to the need to explain why wheat specifically was initially mentioned and not all foods, or at least all foods of a certain kind. It is this tension which generates debate.

Now, what is the logical form of the argument? Is it an argument by analogy (whether simple or of the more complex *binyan av* type) or is it an a fortiori argument (in the true sense of the term)? Obviously, the commentators above quoted consider this an argument "by analogy," which depends on an "*'illah*" (a *ratio legis*), and whose conclusion is relatively "weak." This is precisely why the argument is referred to 'analogy of the inferior' – to signify that the conclusion is "not quite so obvious" as the main premise it is based on, there being some doubt regarding the "effectiveness" of the "effective cause." Clearly, this argument nowhere appeals to a quantitative comparison between the original case and the new case; apples are not presented as being more or less edible or measurable than wheat. However, the reference to staple foods could be regarded as introducing a quantifiable factor, in that wheat could be said to be more "staple" (i.e. more commonly cultivated and consumed) than apples.

If we take the argument to be analogical, the expression 'analogy of the inferior' has no quantitative *ratio legis* to refer to and therefore must be understood in its epistemic sense – as indicative of the relative *uncertainty* of the conclusion in comparison to the given premise. But this, as we have seen, is something true of almost all argument, whatever its form: the conclusion is always a bit less reliable in some way than the premises giving rise to it. Moreover, we have shown above that the epistemic interpretation of 'analogy of the superior' (and likewise of 'analogy of equals') is incredible, since the conclusion can never be more reliable than the premises it is derived from. The latter means we cannot use the term 'analogy of the inferior' in an epistemic sense anyway, since we cannot use the term 'analogy of the superior' (or 'of equals') in an epistemic sense – otherwise, our terminology would be incoherent. Therefore, logically, the argument proposed can only be interpreted as a fortiori, if we wish to label it as 'analogy of the inferior'!

As an a fortiori argument, the above example would read: since wheat (P) is more staple (R) than apples (Q) are, and wheat (P) is staple (R) not enough to avoid subsumption under the rules of *riba* (S), it follows a fortiori that apples

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[87]   A similar analysis can be made relative to another often-given illustration of 'analogy of equals', based on Koran 4:2. However, I chose not to discuss this example here because it contains many ambiguities which are not of direct concern to our purposes here.

[88]   References given here are: "Muslim, Sahih Muslim, p. 41, Hadith no. 119; Ibn Hazm, Ihkam, VII, 54-55; Abu Zahrah, Usul, p.195-196. Zuhayr, Usul, IV, p. 44."

(Q) are staple (R) not enough to avoid subsumption under the rules of *riba* (S). The 'rules of riba' are apparently that a food cannot be exchanged, except for an equal amount and with immediate delivery. Notice that the minor premise and conclusion are formulated in a double negative manner, saying: "*not* enough to *avoid* subsumption under," rather than positively as "enough to be subsumed under." This is intentional and essential. For if the argument were formulated positively, it would be formally invalid, being positive subjectal and yet going from major to minor. The argument therefore must be formulated negatively, in which case being negative subjectal and going from major to minor, it is formally valid. Clearly, this argument can be credibly interpreted as 'analogy of the inferior' in an ontical sense, since it goes for major to minor; and this interpretation is consistent with the ontical interpretation of 'analogy of the superior' we arrived at earlier. In other words, this interpretation makes the proposed terminology both meaningful and coherent.

Concluding remarks. It evident that the Islamic jurists who developed the above examples were unaware of the central role played by a fortiori argument in the terminology they appealed to. If the stated examples are interpreted as arguments by analogy, as they indeed interpreted them, then the terminology of 'analogy of the superior' and 'analogy of the inferior' is not coherent. The examples can indeed be interpreted as analogical arguments, but in that case the proposed terminology cannot be applied to them, because if it is regarded as epistemic in intent it cannot differentiate between the two arguments, since both go from more sure premises to a less sure conclusion, and if it is regarded as ontical, quantitative comparison is not inherent to analogical argument, so there is no basis for reference to 'more' or 'less'. It is *only* in the context of a fortiori argument, properly conceived, that the terms 'of the superior' and 'of the inferior' become significant and needed.

Evidently, while Islamic jurists were vaguely aware of the existence and efficacy of a fortiori argument, they did not – as far as I can tell from the material found so far – actually integrate this knowledge in their sharia deliberations. They could well have done so, as we have above shown – but in fact they did not. This means that their thinking in these contexts was rather vague and uncertain – lacking in logical skill and understanding. Note in particular, as an added indicator, the fact that both the examples they gave (for analogy 'of the superior' and 'of the inferior') were positively formulated. This shows that they did not realize that arguments from minor to major and those from major to minor are correlative, the former being positive and the latter accordingly negative (in subjectal moods[89]). While it is undeniable that Shafi'i did correctly formulate some a fortiori arguments of both the minor-to-major and the major-to-minor types (we saw this earlier, in Hallaq's presentation), it does not follow that the a fortiori form of argument thereafter really played a great role in Islamic jurisprudence. This conclusion is in accord with our earlier conclusion based on Hallaq's presentation.

It appears from the examples we have just seen, and others like them, that in practice a fortiori argument was never clearly differentiated from other forms of analogical arguments and never really understood. Moreover, the fact that no distinct name (other than the generic word *qiyas*) is given to a fortiori argument tends to confirm this overall negative conclusion. Even though the terms 'analogy of the superior', 'analogy of the inferior' and 'analogy of equals' seem to us to refer to a fortiori argument, it does not follow that Islamic jurists using them are aware of their real meaning. I suggest that in their minds these three terms refer to three types of analogical argument in a generic sense, and not to three types of specifically a fortiori argument. I would however say that these terms were 'borrowed' by them from a fortiori argument, because in fact (as we demonstrated) they cannot be meaningfully and consistently applied to mere analogical argument. Thus, these terms are used more for the purpose of intellectual posturing than out of real intellectual comprehension.

Islamic jurists might be said to have mastered, more or less, mere argument by analogy, in its more complex form as well as in simpler form, but it seems that they never truly came to grips with a fortiori argument.[90]

Further confirmation. I made additional efforts to test this conclusion, that a fortiori argument though known in Islam and sometimes used, was never theoretically differentiated from complex analogy (i.e. *binyan av* type reasoning), and found it again and again confirmed. Of course, such 'spot checks' are not as conclusive as an exhaustive study of Islamic literature would be – but they certainly buttress our thesis considerably.

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[89]   In predicatal moods, the opposite is true – the negative mood is from minor to major and the positive mood is from major to minor. But no need to mention that, since Islamic jurists seem to have only noticed subjectal moods, even though the apparently sole example of a fortiori argument in the Koran (36:78-79) is positive predicatal in form. Each negative argument is validated by reductio ad absurdum to the corresponding positive one.

[90]   It is interesting to note that I came to similar conclusions in my first essay concerning Islamic logic, back in 1999, even though my approach at the time was much less critical. There I refer to simple and complex analogy as "extension by direct analogy" and "by indirect analogy," respectively, and show them to be inductive processes. Furthermore, I compare the former to the rabbinical rule of *gezerah shavah* and the latter to *binyan av*. I also say: "Talmudic logic includes valid forms, like the a-fortiori argument, which are (to my knowledge so far) absent in Islamic methodology, at least at a self-conscious level."

One book I read with this purpose in mind was: *A Summary of Logic* by Sayid Sadiq Shirazi (1964)[91]. Although this book does not claim to be an exhaustive study of logic knowledge in Islam, it does claim to be a summary of its main features.

It refers to three types of reasoning: Induction (*istriqra*), Analogy (*tamtheel*) and Syllogism (*qiyas*), which respectively infer universals from particulars, particulars from particulars and particulars from universals[92]. As regards induction, it distinguishes complete and partial enumeration; and although it does not specify the negative condition for the latter, it does point out that the assumption of generality is sometimes incorrect. As regards syllogism, it includes under this heading: firstly, the positive and negative moods of apodosis, i.e. of hypothetical and disjunctive[93] arguments; secondly, the universal and particular, positive and negative moods of Aristotelian syllogism in four figures; and thirdly, a variety of 'equational' arguments, such as what is equal to something is also equal to whatever that thing is equal to, or what is part of something is also part of whatever that thing is part of.

As regards analogy, it analyzes it as usual in terms of four components, viz. the root (*asl*), the branch (*far*), the reason (*illa*) and the ruling (*hukm*). The ruling (e.g. a prohibition) is transferable from the root to the branch, due to their having in common the reason; if any of the necessary conditions for such inference is lacking, the inference is of course not feasible. The example given is the usual one: the prohibition against wine may be passed on to any other intoxicating substance. What is noteworthy here for our purpose is that *there is no mention*, or even example, of a fortiori argument – let alone any theoretical consideration of its nature. Seeing the degree of detail with which 'induction' and 'syllogism' are dealt with, one would expect the varieties of analogical argument to be listed too. But 'analogy' is presented in a monolithic manner, without any thought of subdivision. This confirms my thesis, that Islamic logic has not clearly assimilated a fortiori argument.

I have briefly looked at other Islamic sources and found them equally silent on the subject of a fortiori argument.

Another book I looked into carefully was a study by **Roger Arnaldez** (France, 1911-2006)[94], in which he discusses Ghazali's take on *qiyas*. The basis of analogical inference is the cause (*illa*). Ghazali points out errors that may occur in relation to the supposed cause. It may in fact not be a cause, or at least not the relevant cause (in God's eyes); or it may be wrongly thought to be the whole of the cause, while it is only a part; or it may be the whole, but be wrongly thought to be only a part; or it may indeed be a part, but tied in thought to the wrong other part; or it may relate to the root and branch in different ways; or it may indeed be thought to be the cause, but without certainty, or without clarity. In such events, the proposed analogy is of course not reliable. In any case, the issue according to him is not merely formal or to be resolved by personal insight – the source text and its traditional interpretations have to be referred to.

In this context, Ghazali examines the argument a fortiori, which "draws from what is said (*al-mantuq*) what is not said (*al-maskut*)." For example, if saying 'Fie' to one's parents is forbidden, then all the more so is insulting or hitting them. Again, if sacrificing a one-eyed sheep or a sheep missing a leg is forbidden, then sacrificing a blind sheep or a sheep with two legs missing is a fortiori forbidden. Some say no *qiyas* is needed to draw such conclusions: to them, these are immediately obvious. Others, however, note that the conclusion has some additional (*ziyada*) factor, which extends the law beyond its literal meaning. For example, if expiation is required of someone who has killed unintentionally, it is also required of someone who has killed intentionally, for in the latter case the sin of hostility is added to that of killing. Again, if the testimony of the impious (a Moslem) is disqualified, all the more so is that of the infidel (a non-Moslem), since the latter is not only impious but also ignorant of the (Islamic idea of) truth.

But such additional factors cannot be proposed with scientific certainty; they are mere opinions. Except where *a quantity* is involved, in which case the conclusion is immediately obvious. If the testimony of two witnesses is admitted, then that of three must be; similarly, a blind sheep is twice as inadequate for sacrifice as a one-eyed sheep. However, in some cases the difference is *qualitative*, rather than quantitative, and the conclusion is not so sure. For example, it is conceivable that expiation is possible for unintentional killing but not for intentional killing, because they differ qualitatively. Admittedly, in some cases the qualitative difference does not affect the conclusion. For

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[91]   This book was originally written in Arabic and published in 1964, under the title *al-Mujaz Fi al-Mantiq*; it was translated into English by Ali Abdur-Rasheed in 2006, and posted in pdf format in the Internet at: www.al-islam.org/hawza/mantiq/A_Summary_Of_Logic.pdf. The author seems to be an Iraqi Shi'ite.

[92]   I am here changing the wording somewhat, but the intent is clearly as I present it. I do not here want to engage in a critique of the author's or translator's terminology.

[93]   The three forms of disjunction are dealt with: either-or, and/or and or-else. Invalid arguments are mentioned as well as valid ones, but there is no serious effort of validation (other than by example).

[94]   *Fakhr al-Din al-Razi: Commentateur du Coran et philosophe* (Paris: Vrin, 2002), pp. 40-43. There is a Google books preview of this book. I here paraphrase him very roughly. Arnaldez was a well-known Islamicist (academic student of Islam); see more on him: fr.wikipedia.org/wiki/Roger_Arnaldez.

instance, a difference in sex or color or size might be irrelevant in a given case. But can such inferences be regarded as *qiyas*? Some doubt it, and say that a reason for ignoring the difference would have to be adduced.

I will stop my account of Arnaldez's commentary on Ghazali here, having reached the crux of the issue concerning us – which is whether or not Islamic logic has drawn a clear differentiation between argument by analogy and a fortiori argument. The previous paragraph seems to suggest it has, although it is not clear to me whether the distinction between quantitative and qualitative *ziyada* is made by Ghazali or by Arnaldez. But, whoever made that distinction, it is inadequate. There is some truth in it; but it is too vague to accurately reflect the formal differences between the two types of argument.

For a start, it cannot be said that quantitative argument is more obvious than qualitative argument. Neither argument is obvious. Whether the terms involved differ in quantity or quality, a premise (be it made explicit or left implicit) is always formally needed to express their difference. That premise may in some cases be obvious to all, but it is still relevant to the argument as a logical process. For instance, the premise that 'three is more than two' is obvious to all, but it is still needed to infer a fortiori that the acceptability of three witnesses follows from that of two. Likewise, the premise that 'intentional killing is more grave a crime than unintentional killing' may seem obvious to all, but it still needs to be acknowledged as playing a role in drawing a conclusion concerning these qualitative terms. Furthermore, the distinction between quantity and quality is rather tenuous in this context. In fact, as common usage demonstrates, a fortiori argument is frequently possible with terms that are qualitatively different as well as with those that are quantitatively different.

For example, it was claimed above that the reason why "it is conceivable that expiation is possible for unintentional killing but not for intentional killing" is "because they differ qualitatively." But this explanation is wrong, for we can easily find a respect in which these two terms do differ quantitatively, such as 'gravity of the crime'. We can then use this comparative proposition as the major premise of a valid a fortiori argument, as follows: given that intentional killing is more grave a crime than unintentional killing, it follows that if unintentional killing is (grave enough to be) not-expiable, then intentional killing is (grave enough to be) not-expiable; and it also follows that if intentional killing is not (grave enough to be) not-expiable, then unintentional killing is not (grave enough to be) not-expiable. But we cannot validly deduce from the fact that unintentional killing is expiable that intentional killing is also expiable, because we conceive a crime to be expiable in *inverse* proportion to its gravity[95]. Similarly, in some cases differences in sex or color or size might be relevant and in some cases they might not be. But none of this affects the forms of the arguments.

In all the examples above, proposed by Ghazali or Arnaldez, the differences are not formal but material. As I have shown in my formal treatment of the issue earlier in the present volume, the differences between a fortiori argument and argument by analogy are so marked that *neither can be reduced to the other*, although the latter may be characterized as a subaltern of the former. Moreover, analogical argument is not necessarily qualitative – it may also be quantitative. Thus, the mere presence of a quantitative distinction is not proof that a fortiori argument is involved. A fortiori argument has two factors one or both of which are absent in argument by analogy: (a) a 'comparative' major premise that places the major and minor terms in a common continuum, signified by the middle term, and (b) a 'suffective' minor premise that sets a certain quantity of the middle term as precondition for access by its subject to its predicate. Quantitative analogy has (a) but lacks (b), while qualitative analogy lacks both (a) and (b).

Obviously, neither of these conditions for a fortiori argument can be mechanically satisfied given an analogical argument (whether simple or complex). For, even if the qualitative terms P and Q are known to be alike with respect to R, and therefore can theoretically be placed along a quantitative continuum R, we cannot in practice formulate the major premise, without further input of information as to whether P $\geq$ or $\leq$ Q. Moreover, even if we manage to formulate the major premise, it does not follow that the minor premise is readily true – i.e. that the subsidiary term S is predicable of the minor term Q (in the case of positive subjectal argument), or the major term P is predicable of the subsidiary term S (in the case of positive predicatal argument), *through the intermediary of* a certain measure or degree (a threshold value) of the same middle term R. The crucial factor of sufficiency of R for Q to be S or for S to be P (as the case may be) is always absent in analogical argument; and very often absent too is the information regarding the hierarchy along some middle item R between P and Q.

It is these formal issues that differentiate the arguments, and not a mere quantitative-qualitative distinction. For this reason, the above proposal, whether due to Ghazali or to Arnaldez, is wrong. Therefore, I maintain that Islamic logicians have apparently not managed to formally distinguish between the said two types of argument. Evidently, they vaguely imagined that a fortiori arguments they came across in practice could somehow be cast in the form of complex analogical argument – but they did not realize that doing so would not only cause waste of information but

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[95]   That is, the minor premise "if unintentional killing is (grave enough to be) expiable" or "if intentional killing is not (grave enough to be) expiable" would not make sense and therefore could not be used. The inverse proportionality is implied by Arnaldez's (or Ghazali's) own objection.

also decrease in certainty. For, whereas well-formed a fortiori argument is deductive, argument by analogy is always inductive (since, as already explained, we must resort to generalization before a conclusion can be drawn). Thus, though in practice Islamic jurists might intuitively use a fortiori argument correctly, they lacked a correct theoretical understanding of that reasoning process.

6. The dayo principle and more

There is good reason to believe that the hermeneutic principles used in Islam were, at least in part, derived from those used by rabbis in the Mishna and the Talmud to develop Jewish law on the basis of the written and oral Torah. This is reasonable, first of all, because the problem of exegesis faced by Islamic jurists was similar to that faced by the rabbis. Secondly, because it is evident that their solution to that problem was, at least in part, the same – namely, use of simple and complex analogy, i.e. of *gezerah shavah* and *binyan av*, and (to a lesser extent) use of a fortiori argument, i.e. of *qal vachomer*. It can hardly be claimed that Islamic jurists fortuitously 'rediscovered' these very same methods, which had been in use in Jewish jurisprudence for about a millennium before and were still being regularly used in countries (notably 'Palestine', Syria, Iraq and Persia) that the Arabs conquered. No, the Moslems no doubt learned these methods from the Jews, and possibly others[96], and adapted them to their own needs. They emphasized what was relatively clear to them (notably, complex analogy – i.e. *binyan av*), but only partly assimilated other things they did not fully understand (notably, a fortiori argument).

The latter difference is of course not insignificant. Its consequence is that whereas the forms of human reasoning considered most representative of Greek and Roman thought were Aristotelian syllogism and Stoic apodosis (*modus pollens* and *tollens*), and most representative of Jewish thought were a fortiori argument and to a lesser extent analogical arguments (including *gezerah shavah* and *binyan av*) and syllogistic ones (notably the *midot* of inclusion and exclusion and those of harmonization) – in the case of Islamic thought, analogical arguments (whether simple or complex, qualitative or quantitative) stand out undoubtedly as most representative, so much so that it is debatable whether the word '*qiyas*' applies to any other form of argument. The most elaborate form of Islamic argument is complex analogy (i.e. *binyan av*), rather than a fortiori argument. This concentration and dependence on one form of argument above all constitutes, of course, a limitation on the scope and potential of Islamic logic.

It should be said that I have here omitted to mention and deal with many interesting and relevant details. It would be unfair to say that what we have looked into here is all there is to Islamic hermeneutics. It is clear from even the few documents and webpages that I have read on the subject that Moslem jurists have given much thought to these issues, and have done so in a generally intelligent manner and in an atmosphere of considerably free debate. I do not want to seem to be dismissing these centuries of hard work offhand. However, I do believe that the analysis proposed above, of a fortiori argument in an Islamic context, is a fair assessment of the main issues. As regards some of my conclusions, I realize that many Moslems (in their ideological anti-Semitism) are sure to be indignant at the very thought of a debt that Islam may have to Judaism as regards hermeneutics. But this is a prejudice they need to get over.

Already at the level of the Koran, the debt of Islam to Judaism is enormous; see for example the detailed study by Haï Bar-Zeev to that effect. Moslems do not realize it, because they generally refuse to study Judaism (unlike Christians, who have on the whole maintained a profound interest in it). If they but read the Torah, they would surely be shocked to discover just how much of the Koran was derived, sometimes to be sure *haphazardly and in a distorted manner*, from the Torah; and, by the way, this would indeed help them better understand the Koran. Naturally, as a consequence, Islam shares many beliefs and values with Judaism, though they also disagree on many points. There are also, very probably, many correspondences to be found between the Islamic oral tradition given in the hadiths and various sources in Judaism.

There are likewise a great many convergences, as well as divergences, between later Islamic law (sharia) and the much earlier Jewish body of law (halakha). An extensive and detailed study of this matter needs to be done by someone – perhaps there are already published studies that I am not aware of. But even at a glance, the similarities are striking and undeniable, even if there are some significant differences in some details. Clearly, seeing their number and degrees, such similarities cannot be honestly attributed to mere chance. The same applies to the specific domain of hermeneutics. The Islamic principles of jurisprudence are very often almost identical to those developed in Judaism. This is no surprise, for the reason I already stated: similar problems often give rise to similar solutions.

One example that can be given offhand is the rule in Islam that "one qiyas may not constitute the asl of another qiyas;" that is to say, the conclusion of one *qiyas* cannot be used as the premise of a subsequent *qiyas*[97]. There is a

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[96]   A fortiori argument was present in other cultures, but I do not think (offhand) that *binyan av* was.

[97]   This rule was upheld by the majority of Islamic jurists, we are told, although "Ibn Rushd (Malikis) and some Hanafis opined otherwise."

similar rule in Judaism. Again, we find a rule in Islamic hermeneutics very similar to the *dayo* principle in Judaism. Consider the following "restrictions on the operation of qiyas in regard to crimes and penalties:"

> "There is a Hadith which provides: 'drop the hudud [penalties] in cases of doubt as far as possible. If there is a way out, then clear the way, for in penalties, if the Imam makes an error on the side of leniency, it is better than making an error on the side of severity'."[98]

With regard to penalties, says this hadith, leniency is preferable to severity; for if there is an error of fact or of judgment, it is better to punish too little than too much. This could of course be compared to the rule found in Christian canon law (possibly based on Roman law, or possibly on Jewish law): "*In poenis bensignior est interpretatio facienda*;" that is, in English translation: "In penalties, the more benign interpretation is to be applied."[99] Nevertheless, it is interesting that the same rule is found in Judaism; and that at least as of Mishnaic times, i.e. long before Christian canon law was developed (and possibly even before the equivalent, if any, in Roman law – though this yet needs investigation). In Judaism, the rule is worded as: "*Dayo lavo min hadin lihiot kenidon*;" which in English means: "It is quite sufficient if the law in respect of the thing inferred is equivalent to that from which it is derived."[100] This means that it is forbidden to infer a *greater* penalty for a greater crime from the penalty for a lesser crime textually given in the Torah; only *the same* penalty may be inferred. The focus here, note well, is on inference – and more specifically, apparently (in view of the original context), on a fortiori inference. The Islamic rule is not exactly identical, but clearly very similar. Clearly, there is every reason to expect some causal connection (if only by way of inspiration) between the rule in Islam and the comparable rules in earlier systems of law, and in particular the Judaic system which is generally the most similar.

To be sure, not everyone agrees with this hermeneutic restriction in Islam, any more than in Judaism. In Judaism, it appears that R. Tarfon disagreed with his colleagues, the Sages, in this regard; at least, this is the impression given by the relevant Mishna, even if the later Gemara relative to it attempts to claim they all agreed in principle. In Islam, too, there seems to be some disagreement. There is in Islam some resistance to the inference of even an *equal* penalty for a more serious crime, let alone more severe penalty. Thus, some scholars (Abu Hanifah is mentioned) argued that: "The Lawgiver has prescribed the amputation of hand for committing theft, but he has not prescribed it for making correspondence with the infidels in the enemy territory, although the latter is more serious." Such rejection of logical inference is, of course, a fundamentalist and anti-rationalist posture. But a majority of scholars (including, we are told, Al-Shafi'i, Ahmad, Ibn al Qassar) supported use of *qiyas* (i.e. inference by analogy) in such cases, arguing it did not apply in the said particular instance because "as regards theft and correspondence with the infidels, there is a difference between the two."

But of course, the *dayo* principle relates principally to *proportional* reasoning. We find such reasoning being both used and challenged in Islam, as the following anecdote illustrates: "In this authentic Hadith which is from Aban Ibn Taghlib from Abi 'Abdillah Al-sadiq who said I asked the Imam, 'What do you say sir, about a man who has cut one of the fingers of a woman and how much must he pay in compensation?' The Imam replied, 'The compensation is ten camels.' I asked, 'What if he cut two fingers?' The Imam replied, 'Twenty camels'. I asked, 'What if he cut three fingers?' The Imam replied, 'Thirty camels'. I asked, 'What if he cut four fingers?' The Imam replied, 'Twenty camels.'" The questioner is understandably befuddled by the last reply, but the Imam explains that, by "decree of the holy Prophet," there is an upper limit after which the compensation decreases.

The latter explanation makes no sense (to me at least). All it really means is that Mohammed arbitrarily decided to deny constant proportionality between the crime and the punishment in this case. There is a pro rata relationship between them only part of the way. The compensation varies in direct proportion to the number of fingers cut (10 camels for 1 finger, 20 for 2, 30 for 3), but then (as of 4 fingers) the proportionality breaks down, and is even reversed. One could argue convincingly that the compensation of thirty camels is a ceiling – so that it is the same for more than three fingers as for three fingers. But I do not see how one might argue that the compensation decreases for four fingers! It does not seem just – it seems to reward increased cruelty. Irregular proportionality (involving ups and downs) is not inconceivable – but it does not make sense in this particular case.[101]

The dangers of dogmatism. We can no doubt adduce many more examples of parallelism between Islamic and Judaic jurisprudence, but we shall not pursue the matter further here, as it is not the main object of our research and a very large study would be needed to fully document this thesis. I am not saying that Islamic religion and jurisprudence are entirely second-hand; but I am saying that they are far from original – and that this is something

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[98]   References given: "83. Tabrizi, Mishkat, II, 1061, Hadith No 3570; Abu Yusuf, Kitab al Kharaj, p. 152; Ibn Qayyim, I'lam, I, 209."

[99]   From the Digest of Justinian, no 49, in Albert Gautier, *Introduction to Roman Law for Studies in Canon Law* (Rome: Faculty of Canon Law, St. Thomas University, 1994), page 154. Quoted by Wiseman, in *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions*, p. 165. Note the reference to "interpretation," implying that the rule relates to deriving a penalty from some given text or tradition.

[100]   Mishna Baba Qama 2:5.

[101]   Note anyway that no a crescendo argument is possible here, since the additional premise about proportionality is denied.

that Moslems should be made acutely aware of. The reason they urgently need to know this is that such knowledge would considerably dampen their excessive pride and feelings of supremacy, which cause them to think, speak and behave arrogantly and insultingly towards others.

There is an enormous amount of material about Islam on the Internet. If for instance you search for the word '*qiyas*' in Google, it returns about 675,000 results. Some search strings yield results numbering in the millions. This is I venture to say out of all proportion to the importance of Islam in world culture; but, I suspect, much of it is probably financed directly or indirectly with oil money for propaganda purposes. Be that as it may, much of the Islamic educational material is repetitive, and repetitiveness is indicative of a deficiency of critical and novel thought. Moreover, the material is presented by its authors with great solemnity, using modern methods of orderly presentation – textbook-style lay out, with numbered and lettered paragraphs, and conflicting opinions by Islamic commentators carefully and respectfully reported. The message conveyed by this sort of stately discourse is that the Islamic religion as a whole, and especially Islamic jurisprudence, constitutes a profound and final 'science'.

At first, this sort of hubris is comical, being manifestly naïve; but upon reflection it is a very worrying phenomenon, being so totally devoid of humor. I suspect such dogmatic certitude leads many an ignorant Moslem, eager to do the will of Allah and thus earn a choice place on earth and in heaven, to believe that Islam is the last word on truth and therefore can and must be unconditionally and thoroughly obeyed, even to the point of killing innocent people. I do not say all Moslems react in this idiotic and selfish manner. Some are presumably more balanced mentally. But evidently hundreds, if not thousands, or even maybe hundreds of thousands, of Moslems today do so react and thence engage in a terrorist career, doing great harm to world peace and prosperity, not to mention our planet's ecology[102]. There is no denying the fact: we are pounded with it every day in the news. So, the matter is serious.

Thus, it is very important for thinking Moslems to realize and publicly admit that Islam in general, and Islamic jurisprudence in particular, is not a body of knowledge productive of utter certainties. It is a faith, which must be received if at all with the caution befitting a faith. Ideally, faith implies intellectual humility and modesty, i.e. inner and public admission of ignorance. Moslems must learn to approach their religion with a more scientific spirit; that is, with willingness to ask difficult questions and demand convincing answers, and a healthy-minded refusal to be bamboozled into submission through spine-chilling threats and fanciful promises. This is a mature and intelligent attitude, which ensures that truth is upheld and not smothered. As we have shown in this paper, even if Islam has resorted to some logic, its logical scope has been rather limited. There are many logical processes – notably, a fortiori argumentation – which it has not fully understood and assimilated. It is important for Moslems to grasp this fact.

The all-importance of history. Religion has both good and bad impacts on individuals and society. This is true of all religions, though no doubt in different ways and to varying degrees. To personally and collectively overcome the negative influences of any religion, it is important to know and understand that it has a history. Its holy book has a history; its oral traditions have a history; the laws derived from these sources have a history; the means used to derive these laws have a history; everything to do with religion, as with any other human endeavor, has a history. That history may not always be known, and even may not be entirely knowable. But every effort must be made to increase our collective and individual knowledge of it.

The reason for that is that history introduces into human affairs the crucial dimension of time. It tells us of the evolution and development of the things we face today. It adds *depth* to our perceptions of things. Without historical perspective, everything looks fixed and final; things have always been so and therefore will always continue to be so; no idea is therefore open to review or revision. Whereas, when time and change are taken into account, when we realize that the situation was not always what it is today and need not remain as it is today, we become free to rethink and remake the world. Awareness of historical development is liberating. It enables our wise judgment. It frees us from compulsively repeating the mistakes of the past.

With regard to the four sources of Islamic jurisprudence, the first question to ask is: what are *their* sources? And indeed, what is the source of this doctrine of four sources? It is not enough to piously acquire flat knowledge of the final product. We need to know its history, how it emerged out of the deeps of time, so as to be able to evaluate it correctly. The doctrine evolved gradually over more than a century, starting about one century after Mohammed's death; it was not handed down ready-made by him. In this perspective, a historical study like Hallaq's is very welcome and illuminating; his work is a lucid presentation of the development of Islamic (or at least, Sunni) legal thought. Nevertheless, even though his work is not dogmatic or apologetic, but constitutes a sincere research report, Hallaq does not question Islam radically enough.

With regard to the Koran, for a start, though he acknowledges that doubts have in recent times been sown by academics regarding its age and authorship, he dismisses these attempts at critical history rather shortly. As far as he is concerned, the Koran is a reliable document, which "originated during the lifetime of the Prophet" and "reflected

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[102]   They also, in fact, harm themselves enormously – spiritually, psychologically and materially.

events and ideas that occurred then." Whatever it says, he takes to be an "authentic representation" of things as they in fact were, and he justifies this "assumption" by "the absence of noteworthy evidence to the contrary." This is clearly a very conservative stance, which has been shown to be quite unjustified by recent studies (summarized by Spencer). There is much evidence within and outside the Koran to suggest that it is a much later production. By taking the Koran for granted at the outset, Hallaq throws considerable doubt on his scientific credentials. His dedication to impartial truth is clearly not total and unconditional. This means he must be considered as, at least to some extent, a Moslem apologist.

With regard to the Sunna, his position is, at least in theory, a lot more open; but in practice he does not return to this issue very much. In other words, he on the whole accepts the judgments of Islamic jurists as to the authenticity of this or that hadith, without greatly questioning the credibility of their standards of judgment. He does conceive as rather "historiographically problematical… the authenticity of the reports about the deeds and utterances of the Prophet," and mentions studies by "Goldziher, Schacht and Juynboll, among others" which suggest that "Prophetic reports were fabricated at a later stage in Islamic history" and "gradually projected back to the Prophet;" but he immediately adds that more recent research and critical studies showing that "while a great bulk of Prophetic reports may have originated many decades after the Hijra, there exists a body of material that can be dated to the Prophet's time."[103]

Thus, he concludes, though he does not "*a priori* preclude" all the reports as unauthentic, he does not either "accept their majority, even though many may have been admitted as authentic (*sahih*) by the Muslim 'science' of *hadith* criticism." This suggests Hallaq is willing to think for himself to some extent, and not necessarily tow the orthodox line. Here he is, of course, on less dangerous ground, since there have always anyway been controversies within Islam regarding the hadiths. With regard to 'consensus' and to '*qiyas*', he is again not sufficiently critical. Overall, we may say that, though Hallaq is occasionally willing to criticize the form of Islamic law, he rarely displays the *chutzpah* needed to judge or even merely question its content.

7. The essence of Islamic discourse

According to the testimony of a police informant, the terrorist Abu Khalid Abdul-Latif (aka Joseph Anthony Davis), referring to the murder of 13 people at Fort Hood, Texas, by another terrorist a couple of years earlier, argued a fortiori as follows (presumably in English): "If one person could kill so many, three attackers could kill many more!" He intended "to storm a Seattle military recruitment center with machine guns and grenades," but was arrested before he could carry out his plot[104].

Evidently, although Islamic terrorists, and the 'theoreticians' of Islam who encourage them, are mentally deranged, suffering from severe dysfunctions of their emotional and rational faculties, they are quite capable of making inferences within the framework of their deadly belief-system. Just as the Nazis or Communists were capable of reasoning to some extent, enough to produce their tanks and concentration camps, so today's Islamists are able to manipulate mobile phones and hi-tech guns. This is reason in the service of madness.

Everyone, or almost everyone, has the capacity to reason to some extent – but the question is always: to what extent? Obviously, people whose commitment to reason is very tenuous and circumscribed, consisting of occasional ad hoc reasoning acts, cannot rightly be called rational. Only people whose commitment to reason is firm and consistent can rightly be characterized as rational. This brings us to the issue of the essence of Islamic discourse: is it rational or irrational? Although, as we have seen, Islam has a tradition of genuine logic, it must be said that the overarching 'arguments' of Islam, the principal means of its propagation, are and have always been violence and threats of violence, and lying and pretending. We shall now expose this fact.

In any study of the logic of a particular culture, it is important to draw a distinction between its essential means of discourse, through which the culture is principally formed and sustained, and the relatively incidental aspects of its discourse. The logician cannot merely list forms of argument, without critical consideration of their effective hierarchy. His analysis must aim for the essence of the matter. Let us, then, be frank. The essence of Islamic discourse is, evidently, violence and the threat of more violence, and lying and pretending. Comparatively, all other forms of argument used in Islam pale into insignificance. It is not an accident that Islamic discourse today is so charged with anger, hatred and aggressiveness – these qualities are already manifest in Islam's founding document, the Unholy Koran. Today's Islamic discourse is just reviving and mimicking the tone and tactics exemplified in the Koran.

The primary 'argument' of Islamic logic is—**physical intimidation**, i.e. the explicit or implicit threat of violent reprisal if you disagree with Islam in any significant way. This 'might is right' argument is evident in many

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[103]   Op. cit. pp. 2-3.
[104]   He was "arrested in June 2011 and indicted the following month." This information is drawn from a report by Reuters on Dec. 6, 2012.

statements and stories in the Koran and the hadiths (the oral traditions of Islam)[105], and in consequent Islamic law (*Sharia*), and the consequent behavior throughout history and today of Islamic régimes and groups[106]. The physical intimidation argument is used on Moslems (e.g. apostates risk execution, so-called honor killings are tacitly encouraged), and against non-Moslems when these are under their power (Jews and Christians are forced to convert or at least pay 'protection money' (*jizya*) or be executed, while other non-Moslems[107] must either convert or face death). When non-Moslems are not under their power, they threaten them with deadly riots or terrorist acts. Of course, verbal threats have no persuasive power if not backed up by actual violence now and then.

And it works! Witness the way today most Western governments, academics and journalists are cowed into praising Islam and disapproving of anyone who criticizes it. Of course, these people do not admit themselves as intimidated; their mentality is that of people with the 'Stockholm syndrome'. They are hostages who are so thoroughly cowed that, instead of turning their wrath on the thugs victimizing them, curse and oppose the heroes who try to resist and oppose the thugs. Such people think that denial, passivity, appeasement and accommodation will save them – whereas it is precisely such timid responses that will cause their perdition and enslavement to the proponents of this evil doctrine, which aims at world domination and totalitarian control of everyone's life, and not incidentally intends to attempt another Holocaust of the Jews[108].

Another 'argument' of Islamic logic is—**deception**, i.e. Moslems lying and pretending so as to gradually subdue their potential victims without their true intentions and identity being evident[109]. They may lie by commission (*takkiya*) or by omission (*kitman*). This non-violent tool of persuasion is used in conjunction with that of violence and threats of violence, so as to psychologically allow the submissive victims (who may already be characterized as *dhimmis*, the name given to second-class citizens in a Moslem state) to maintain an illusion self-esteem. In this context, the predators pose as innocent victims of those who resist them and appeal to the scared portion of their prey for protection against the resistors[110].

One of the most successful acts of deceit is recent times has been the cunning redefinition of 'terrorism' by Moslems (and their 'leftist' sympathizers), as referring to acts of violence aimed at civilians, or at least with civilian casualties (even if they are incidental to a military operation). But this is not the essence of the matter. What distinguishes terrorism is that it involves *the initiation of violence*, or even the initiation of a threat of violence. It is the use of violence as an 'argument' against people who were not using force against the perpetrators. Thus, a soldier may be a victim of terrorism. When the intended victims are civilian, this is just a greater degree of terrorism. But violence aimed at combating terrorism is not terrorism. There is no 'moral equivalency' between thugs and police forces. The issue is always: who started it, really? Islamic terror, although it pretends to be defensive, is always aggressive – simply because it is triggered and driven by a perpetual religious imperative of global conquest. Terrorists usually give pretexts for their heinous acts, but their real motive is always ideological, viz. "holy war" (*jihad*).

The purpose of such redefinition is to blur the distinction between terrorism and anti-terrorism in the public's mind. Those who murder in the name of Islam are presented as freedom fighters struggling against imperialism and colonialism, or racism, while their victims are depicted as callous oppressors who 'had it coming'. Using their positions of influence and considerable propaganda skills, Western leftists[111] help to create and confirm this upside-down narrative. Moreover, anyone who dares to challenge it, demanding truth and justice, is slandered and diabolized as a 'right-wing extremist' or 'Islamophobe'. This is social intimidation, aimed at silencing opponents through fear of disrepute and ostracism. This is also a form of terror, of course—terror being action intended to cause

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[105] See examples at: wikiislam.net/wiki/List_of_Killings_Ordered_or_Supported_by_Muhammad.

[106] Such violent tendencies are admittedly apparent in all the major religions at different periods of history; but what distinguishes Islam is that that it has them still *today*, in the 21st century, and *in a big way*. So much so, that it is a direct threat to world peace and security, affecting everyone's lives and liberty daily. See the arguments at: wikiislam.net/wiki/Are_Judaism_and_Christianity_as_Violent_as_Islam%3F.

[107] This refers to non-monotheists – that is: pagans, polytheists, idolaters, Buddhists, Hindus, atheists, etc. When the Moslems conquered India, they exceptionally did not universally enforce the 'convert or die' rule, simply because there were too many Hindus around. One blogger (Bill Warner) has estimated that Islam has over the fourteen centuries of its existence caused the death of some 270 million non-Moslems, including 120m Africans, 60m Christians, 80m Hindus and 10m Buddhists: www.politicalislam.com/tears/pages/tears-of-jihad/.

[108] Islam is as rabidly anti-Semitic as Nazism was, one of its oft-declared ultimate goals being the mass murder of all Jews on this planet. A hadith has Muhammad saying that "the last hour" will not come unless the Muslims fight the Jews and kill them all (*Sahih Muslim*, book 41, no. 6985). There are many Muslims today just aching to carry out this genocidal programme, judging by declarations published (in writings, recordings or videos) on the Internet. Ultimately, all non-Moslems risk persecution, to be sure; but Jews are the most passionately hated.

[109] See some recent examples at: wikiislam.net/wiki/Muslims_Caught_Using_Deception.

[110] Read for instance the work of Robert Spencer on this topic, *Stealth Jihad: How Radical Islam is Subverting America without Guns or Bombs.* (Washington D.C.: Regnery, 2008)*,* and look for daily updates on the subject in his website, www.jihadwatch.org, and that of his close colleague Pamela Geller, www.atlasshrugs.com, for examples (see other links there). For a deeper understanding of the essentially political nature of Islam from its inception, do also read Spencer's works: *The Truth about Muhammad*, *The Complete Infidel's Guide to the Koran*, and *Did Muhammad Exist?*

[111] Let us call the non-Moslem de facto defenders of Islam 'leftists', for we need to give them some name, and most (though not all) of them come from the left side of the political spectrum, i.e. they incline towards socialism or communism.

governments and populations to *fearfully* submit to overt or stealthy aggression. Nowadays, even our judges and police are not immune to such insidious pressures, allowing illegal acts from Moslems they would never allow from others[112].

Note well the inversion: Islam, an utterly oppressive and intolerant, fascistic and bloodthirsty movement is presented as 'politically correct', while those who bravely denounce it and oppose it are reviled. Those who do the reviling categorically refuse to admit, or even consider the possibility, that the religious and political teaching of Islam is *the cause* of the crimes of Moslems who engage in jihadi terrorism or other inhumane acts[113]. Such people shunt aside all evidence brought to their attention of the criminality of those they passionately defend, preferring instead to 'shoot the messenger', i.e. to attack those who present them with the facts. They conceal the facts from people and even make efforts to excuse crimes, so as to grant them no reality. Not psychologically disposed to admit their cowardice, they simply deny its object.

Leftists who perform this sleight of hand are scared stiff that if they acknowledged that Islam is a menace, they would be obliged to do something about it, which would stir up still more trouble and make yet more demands on them. They apparently think that if they are sweet to the fundamentalist Moslems, they will be spared from their wrath. As a result, 'Islamization' is proceeding apace, unhindered, indeed aided by them. Europe and North America are hurtling towards what promises to be one of the darkest, most horrific periods in human history. When the Moslems come to power, as could happen within a few short years at the rate things are going, the leftists will be among the first victims of their cruel tyranny. The leftists' collaborationism will certainly not save them, since their ideology is objectively incompatible with that of Islam. They will be remembered by those they helped to come to power as 'useful idiots'.

Physical intimidation and deception are 'arguments' in the sense that they are used to make others submit to certain demands or adopt certain beliefs and practices. But of course, they are not *rational* arguments[114]. People resort to such irrational means when they believe reason to be incapable of moving their respondents, because these lack the requisite intelligence or honesty – or when they have no rational arguments to offer them. Moslems, believing that Islam is Divinely-ordained, think that all people who refuse to submit to its dictates must be stupid or evil. But of course, their blind faith in Islam is not rational justification for the violent and dishonest means they use to propagate it. Therefore, it is they, rather than the victims of their intolerance, who are deficient in intelligence and/or goodness.

The logical flaw, the contradiction, in the Islamic rationalization is that you cannot force or trick a volitional being into being moral, since morality depends on the *free* exercise of freewill. An act of will performed under coercion or ruse cannot constitute a moral act; people must *choose* to be moral. It is therefore unthinkable that God enjoins forcing or tricking people into (allegedly) moral behavior. Moreover, if Islam does not believe in human freewill, as seems to be the case, it cannot claim to be an instrument of morality. If human behavior is, as Islam apparently advocates, predestined (Moslems often say *maktoob* – meaning, 'it is written'), then there is logically no such thing as good or bad. Then anything goes, i.e. force and trickery are not reprehensible, although not morally justifiable. Thus, underlying Islamic use of coercion and ruse is a profound contempt for humankind, a denial of human dignity. Either way, Moslems who indulge in such practices, and those who effectively support them, cannot claim to be defending morality.

Islamic logic does also make use of rational or rational-seeming arguments in circumscribed contexts, but its overarching arguments – the arguments through which it has taken power and continues to spread – are the said irrational ones. These are very powerful arguments – even I, as I write these few lines, wonder whether some mad-as-hell Moslem fanatic will try to murder me when I publish them, or whether some people in authority subject to the Stockholm syndrome will try to suppress their publication. People who submit to such fears, and allow others to intimidate or fool them without reacting, effectively rush headlong like lemmings to their own destruction. Where there's no will, there's no way. By refusing to confront the ongoing world jihad head on, by first acknowledging its religious source and motive and then reacting as necessary, they hasten the realization of the nightmare global dictatorship it aims for.

Some reflections on current events. If things go on like this, future historians will say that the super-powerful West was defeated by a mere handful of Islamist terrorists in a single day, on September 11, 2001. True, the Americans

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[112]   And as everyone knows, when the law is not equitably enforced, the end of the rule of law is in sight. When Moslems commit violence or threaten it, the authorities blame their victims and turn a blind eye to the aggressors. Seeing such criminals treated with kid gloves by the authorities, ordinary citizens feel more and more unprotected, and naturally some of them are eventually tempted to take the law into their own hands.

[113]   For example, the increasingly common kidnap and gang-rapping of non-Moslem children or youths to "groom" them for sex-slavery or prostitution.

[114]   They are fallacies. Forcing and threatening force are traditionally referred to as the argumentum *ad baculum*. Lying and pretending might be classified as *ignoratio elenchi*, in that they aim at disinformation. Those who use such means think that they have 'won the argument' if they have succeeded in killing or cowing or fooling their opponents.

and some of their European allies at first responded forcefully to the aggression, by attacking Afghanistan and later Iraq. But under pressure of their mainstream media, their defiance and determination soon flagged, and they chose instead a course of resolute self-deception and self-paralysis. They opened the way to their own total defeat, by refusing to even name their enemy (Islam) let alone take all necessary action to decisively neutralize him. They now behave like prey frozen with fear, ready meat for the ruthless predator.

Such fearfulness is contrary to logic, which teaches that before one can solve a problem, one has to acknowledge its existence and identify its true nature; then one must respond accordingly, or face defeat. The lessons of history are clear on this point: denying reality and refusing to act in accordance with it is a sure way to self-destruction. A danger won't go away by being carefully ignored; it will only grow and grow. The enemy advances step by step; each step is too little to make much of, but the advances soon add up. This is clear with the now proverbial Munich agreement of 1938, between Chamberlain and Hitler[115].

The policy of appeasement and accommodation towards Islam started with U.S. President George W. Bush, even though his opponents paint him as a warmonger. Bush was of course only reflecting American 'public opinion', which was well orchestrated by the major (mostly leftist) media. These same forces soon replaced Bush with the more photogenic and likeminded Barack Hussein Obama. Even if Obama claims to be a Christian, the fact remains that he is a Moslem according to Islamic law, because his father was one. Moreover, he had some Moslem education in his youth (in Indonesia), and has never publicly renounced Islam[116]. Indeed, instead of denouncing its brutality and hegemonic ambitions, he loudly praises it. He does not seem very concerned with the persecution of Christian and other minorities in Moslem lands. But most importantly, his behavior since he became U.S. President (in 2009) shows he has considerable sympathies with Moslem causes both nationally and internationally. Thanks to his policies, the Islamic grand project is moving forward on all fronts.[117]

The election of an Islam-friendly President after 9/11 was surely no 'accident', but a statement by the people who brought him to power that they were duly impressed by that mega terrorist act and similar expressions of hatred, and wished to express their 'friendliness' to those responsible. I am not suggesting that Obama was a 'Manchurian candidate' injected by pro-Moslem conspirators; his is more likely a lone-wolf plot. But I am saying that his election was uncannily timely, effectively allowing the cowed portion of the American public to send a message of capitulation and abject submission to their aggressors[118]. It is significant that those who had doubts about Obama's constitutional eligibility to run for the office of President (due to questions regarding his birthplace and his father's nationality) were never allowed their day in court. Amazingly, no one in the U.S. judiciary was the least curious about what might very well turn out to be *the biggest scam and usurpation of power in world history*! We may well wonder, in view of this, what future there is for the rule of law and democracy in America.

The above remarks are perhaps as much a commentary on the sick 'logic' of Western cowards as on Islamic 'logic'. As regards the latter, physical intimidation and deception have, to repeat, proved very successful. As for rational or pseudo-rational arguments, they are used within Islamic societies for purposes of control, and not as in Western culture for the purpose of discovering truth or developing useful gadgets. Their main use is of course to develop Islamic law, based on the Koran and the hadiths; but these are composed of dogmas immune to rational criticism, forever closed to revision or repeal. Within this limited framework, there may well be some logical thinking[119] – although we must never forget that even if the form is logical, it does not follow that the content is logically sustainable let alone objectively true. The use of some logic does not vindicate the whole enterprise.

Looking at the history of Islamic societies, what is evident is how little they have contributed to the welfare and advancement of mankind. They have, to be sure, made some contributions to philosophy and mathematics way back in the Middle Ages, following their encounters with Greek and Indian thought. But their rationalism, creativity and

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[115]   But looking back further, consider the feeble response of Montezuma, the Aztec emperor of Mexico, who could muster an army of some 300,000 men, to the invasion by a Spaniard force of *only some 600 men* led by Cortés in 1519-20. Reflecting on this enigma, in his account of the episode, Hammond Innes wrote: "Moctezuma's policy was one of appeasement, but now he must have begun to realize that the road of appeasement is a long one that leads finally to degradation." Moctezuma sought "to buy time and wait upon events;" "but the door of appeasement, once opened, is not easily shut." And: "The wretched king was now so deeply involved with the Spaniards that he was apparently willing to buy peace at any price." *The Conquistadors* (London: the Book Club Associates, 1972): Pp. 148, 149, 151.

[116]   Which is perhaps why we almost never hear Moslems accusing him of apostasy.

[117]   For a detailed exposé of Obama's Islamic connections and tendencies, I recommend Pamela Geller's *The Post-American Presidency* (New York: Simon & Shuster, 2010). And of course her website, at: www.atlasshrugs.com. Concerning Obama's failure to produce a birth certificate and other documents, see: atlasshrugs2000.typepad.com/atlas_shrugs/obamas_birth_certifcate_forgery/; reading these blog posts leaves one with zero doubt that this president is illegitimate.

[118]   Needless to say, though alleviating the terrorist threat was an important (albeit largely subconscious) motive for the election of Obama in 2008, it was not the only motive; another was, of course, the desire by many voters to turn America into a socialist state. The latter motive was even more evident with his reelection in 2012, although by then Americans should have been intensely alarmed by his Islamic inclinations.

[119]   Although in a past essay and in the previous sections of the present chapter I have detailed various logical practices and doctrines found in Islam, and this material is of interest to logic history, it should not be used to dignify Islam as a rational doctrine. Islam remains an essentially emotional doctrine, driven by hatred.

productivity soon petered out, for the simple reason that their founding doctrine is essentially reactionary and repressive. Without freedom of thought, freedom of speech, and freedom of conscience (choice of this or that faith or even of non-faith), human welfare and progress are impossible. Logic can only properly function within a free mind, in a free society. To mention one statistical index – consider how very few Nobel prizes (just two) have been awarded to Moslems for contributions to science or technology[120]. It is shocking that a cultural group that is currently at the very back of the class nevertheless wishes to take over the world and rule it. And of course, loot it.

Prospects. As we have seen, Islamic discourse, whether intended for internal or external consumption, consists mainly of violent acts and threats of more violence, and lying and pretending. These are its main 'arguments', through which it has progressed in the past and through which it lately again hopes to take over the world and rule it. Whether out of intellectual mediocrity or out of moral cowardice, Moslems very early in their history lost their intellectual and other freedoms. They had only themselves to blame. Now, driven by the intense inner suffering their religion causes them, by confining them in one of the dreariest forms of existence ever devised by men, more and more of them want to impose the same suffering on the rest of humanity. People who feel miserable are always looking for others to blame for their pain, and so are easy targets for hate-mongers. A happy person simply cannot feel the sort of malevolence they are taught.

Admittedly, Moslems are not all the same. As in any religion, the adherents range in degree of commitment from zero to extreme. A few, a very few, born Moslems have the wisdom and courage to criticize and even actively combat Islam[121]. Many Moslems are Moslem only in name – their religious practice is perfunctory, if not nil. Many others, no doubt, practice out of conformism, i.e. to signal their inclusion in the society around them. Still others, no doubt, feel an emotional attachment to God, and know of no other way to express it than through practicing the religion they were born into. Some of the preceding may become more devout out of personal need for self-discipline and a purpose in life. Some of those may go much further and become fanatics, driven by the desire to surpass and dominate their neighbors. Evident in the latter are grave psychological problems: lacking a sense of identity, they seek to be acknowledged members of a group; and lacking self-respect they yearn to be feared by everyone.

Clearly, not all Moslems are actual terrorists, or even incipient ones. Some Moslems may well be considered as "moderate," *not* because normative Islam is moderate, but because they wisely don't really buy it. However, ultimately the true test of moderation is opposition to extremism. If ordinary Moslems remain silent and passive in the face of the jihadists' excesses (to put it mildly), they become effectively accomplices. It is primarily their responsibility to loudly condemn and resolutely eject from their midst the bad apples, who are enemies of all humanity. In any event, some Moslems, though not themselves actual terrorists, do ideologically or financially positively support terrorism, or at least secretly take pride in the actions of the terrorists; these certainly cannot be called "moderate," even if there is no visible blood on their hands – they are de facto accomplices to terrorism. And a big problem for the world right now is that the latter group seems to be growing. Moreover, even if most Moslems do not yet engage in terrorism, the fact remains that most terrorists nowadays are Moslems[122]. Since they are people who engage in terrorism in the name of Islam, it is true to say, and it would be a lie to deny, that there is a causal connection between Islam and terrorism.

And indeed if one examines Islamic doctrine honestly, one cannot but be struck by its extreme virulence. Moslems who become terrorists are radicals – people who take the imperative of violent jihad literally and wish to put it in practice fully. Therefore, the doctrine they refer to is *the main cause* of their savagery; it is not merely an accidental conjunct, as wishful-thinking apologists claim. The problem with Islam is not the divinity and religious rituals it advocates – who cares, nowadays? Everyone has a right to believe and worship as they see fit, *so long as they hurt no one else*. The problem with Islam is that its most radical adherents, the so-called Islamists, try to force less convinced Moslems as well as all non-Moslems to believe or do as they want them to. This is the problem – *force*. These people have not yet understood that other people have natural rights to life and liberty, and to their own conscience. That is why Islamists and those who cheer them on deserve to be described as uncivilized. To be intolerant of Islam does not

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[120]   Here, a distinction must obviously be made between Moslems educated in the West and those trained in Moslem countries. Only two Moslems have to date won a Nobel Prize in physics or chemistry, and both were based in the West. Do not be fooled by the image propagated by certain media of rich Arabians with engineering degrees from prestigious American and European universities driving fancy cars and using mobile phones. Arabian wealth derives principally from oil revenues, and thereafter to some extent from commerce; it is not based on scientific research and technological creativity. There are only some 600 universities in the 57 countries with Moslem majorities, for a combined population of 1.4 billion. About six out of ten Moslems are still today illiterate (i.e. cannot even read). Compare these figures to Western ones, or even to those for India and China, and you realize the immensity of the problem these people pose in today's world.

[121]   See at: wikiislam.net/wiki/Notable_Former_Muslims.

[122]   Moslem terrorists have murdered over 20'000 people across the world in the less than twelve years since Sept. 11, 2001. This statistic is given at www.thereligionofpeace.com, which has been keeping track of terrorist acts. This is 95% of all terrorism in the world; there is no comparable figure for any other violent group. See also: wikiislam.net/wiki/Muslim_Statistics_%28Terrorism%29.

constitute religious or racial intolerance – it is wise intolerance of brutal intolerance; the action and the reaction are morally very different.

Jihadists evidently imagine that God is pleased with their orgies of massacre and mayhem. They could hardly be more stupid. God is surely utterly disgusted by their wanton destruction of human life and property, and all the more so because it is alleged to be done in His name. God is not vain; He does not need the flattery of criminals. These people must be quite deluded to think that God, Who created this world and humanity out of love, and Who delights in the peace and welfare of His creatures, would condone such insane behavior[123]. In the Torah of Moses, God commands: do not kill, do not steal, do not covet[124]; do not hate, do not bear a grudge, but love your neighbor as yourself[125]. A psalm[126] of David urges: seek peace and pursue it. The Mishnaic sage Hillel taught: do not do to your fellow that which is hateful to you[127]. Likewise, the Talmud teaches that indulgence in anger is equivalent to idolatry[128]. It also teaches that wisdom departs from an angry sage, and his prophetic powers depart from an angry prophet[129]. Truly, the angry are mad – anger is madness. God certainly prefers a peaceful "unbeliever" to a murderous "believer."

Islam has, through the misdeeds of its fanatics, given itself a lasting bad name. These people claim to be fighting for God and religion, but in fact the effect of their cruel deeds is to turn decent people away from religion and even, by association, from God[130]. Unfortunately, for various reasons already mentioned, Islam seems to be currently waxing rather than waning. I solemnly pray to God that Moslems who take part in this calamitous revival wake up soon, and shake off the ignorance and blind hatred that currently afflicts them, and keeps them very far from spiritual enlightenment and liberation. They need to muster a healthy dose of critical thought, and have the courage to question and reject the many elements of mainstream Islamic doctrine that enjoin propagation by brute force. Do not allow your minds to be poisoned by this ideology. Be honest and think rationally. The antagonistic spirit of radical Islam may have seemed normal a millennium ago, but it is certainly outdated and utterly wrong today. God would surely never enjoin such animosity and ferocity; it is the invention of men craving to kill, plunder, rape, and enslave.

I often reflect that Moslems could learn much from Buddhism, regarding the futility of worldly desires and the value of self-control of one's passions[131]. To give an example, briefly: instead of hiding women from the sight of men, so as to avoid illicit sexual relations, they should teach men to control their own lust on the grounds that women are human beings with independent rights. They (the men in power) force the women (all of them) to hide their faces and stay indoors, when it is the men (mostly) who act irrationally. This means that they preemptively imprison the potential victims, and let the potential aggressors wander around with all their sick inner urges intact! This is patently unfair, and a very bad solution to the problem at hand. Another example is, of course, that of wrath. Buddhism rightly teaches the ugliness, evil and self-destructiveness of hatred, anger and violence. Yet, Islam goes to great lengths to cultivate these very vices in Moslems, teaching them: to consider themselves as superior to non-Moslems, and to hate them all and especially Jews; to engage pitilessly in "suicide bombings" and other forms of terror, with a view to ultimately conquer and enslave the world for Islam; to kill apostates, heretics and "blasphemers," i.e. the critics in their midst; to stone, burn, amputate or whip people they disapprove of; to mutilate the sex organs of Moslem women, and to beat and imprison them, so as to tame them and use them at will; to indulge in "honor" killings of relatives, or even of strangers[132]; so forth. Such inhumane acts are not only permitted, but enjoined by Islam; and they currently frequently occur[133].

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[123]   The Torah decries human sacrifice, e.g. the interdiction of sacrificing children to the pagan god Moloch (Lev. 18:21). But Islam still today entices its sons and daughters to prove their faith by killing innocent people; in view such terrorist tendencies, Islam can surely be characterized as a religion that practices human sacrifice, i.e. ritual murder. The people who thus kill fellow human beings out of selfish desires (for promised material or even spiritual rewards) have not "misunderstood" Islam, as the PC and *takkiya* crowd would have us believe; they have *all too well* understood it and taken it at its word.

[124]   Ex. 20:12-13.

[125]   Lev. 19:17-18. All the commandments and words of wisdom reproduced here are lessons in morality intended for all mankind, not just for Jews. And the list here proposed is, of course, far from exhaustive.

[126]   Ps. 34:14.

[127]   Shabbat 31a. Or more accurately said: Do not do unto others that which you would not have them do unto you. Notice that the golden rule is *not*: Do to your fellow what you would have them do unto you. It is not an instruction to lay your trip on others, but one to respect the space of others.

[128]   Shabbat 105b. Anger is comparable to idolatry because it is effectively a denial of God's control of the world.

[129]   Pesachim 66b. See also Rashi's commentary to Num. 31:21.

[130]   There is Judaism a concept known as *Chillul Hashem* – desecration of the name of God. This is the grave sin of people who commit atrocities allegedly in God's name, and so turn people off from spiritual pursuits, thinking that such disgusting behavior is where such pursuits inevitably lead. This is based on Lev. 22:32.

[131]   They should read, for a start, the *Dhammapada*, a treasury of wise insights attributed to the Buddha.

[132]   See: wikiislam.net/wiki/Honor_Killing_Index.

[133]   It could be said that Islamic preachers who encourage Moslems to hate, kill, plunder, destroy, enslave, rape, etc. their non-Moslem neighbors, and even engage in various crimes and depravities against other Moslems, are cunningly exploiting the lower natures of their fanatic

Clearly, Moslems need to cultivate the fine art of self-improvement; and for that, they must practice humble self-scrutiny. This is most easily achieved through regular meditation. If one wishes to become wise, one must be already at least intelligent enough to be able to discern which teachings are wise, and to avoid foolish ones. Buddhist psychology and ethics point the way to self-understanding and wise conduct. I am not, needless to say, advocating that Moslems bow down to Buddha statues. Buddhism is spiritually deficient with respect to such idolatry and, in my view, to its rejection of monotheism[134]. But these are minor sins, to be sure, compared to the major sins of unchecked rage and cruelty common in Islam. When in the Jewish Bible prophets condemn idolatry, their main focus is always on the *misconduct* of the idolaters who in those days practiced human sacrifices, sexual depravity and other immoralities. Buddhism, though idolatrous in some respects, repudiates such lowly practices. Islam, on the other hand, though not idolatrous (though that is debatable, in view of the Moslems' de facto excessive veneration of Mohammed), countenances some very immoral behavior patterns, as just explained (and of course, this is largely due to their imitation of Mohammed). God has surely made clear throughout the Scriptures of His love of harmonious living, and His profound contempt for troublemakers. This is by far more important to Him than worship, although worship of Him is certainly recommended as highly beneficial spiritually.

I also fervently pray that Europeans, North Americans and other Westerners soon become aware of the great danger to their civilization and personal freedom that today's Islam constitutes, and do whatever is necessary to prevent its implantation in the West. In particular, the Leftists who effectively support the current Islamic onslaught need to look in the mirror and ask themselves what motivates them to do this. Up till now, they posed as opponents of religion, "the opium of the people." Suddenly, they have become its most passionate defenders, so long as it is called Islam. They cannot but fail to see that every concession they make to Moslem demands relinquishes oh so precious freedoms of ours, which it took Western civilization centuries of courageous struggle to win and defend. It is true that the Left yearn for omnipresent world government, but do they really want to end up living in a totalitarian Islamic society?

The political left in the West has, in the past century, and especially since the 1968 cultural upheaval, notwithstanding the collapse of Marxism in 1991, managed to sell itself to the public as 'progressive'. This fantasy image has attracted a great many people with very conventional minds, who saw in adherence to its values a quick and easy way to feel and to seem avant-garde, intelligent and firmly on the moral high ground. Leftists are essentially dull and narcissistic people, which is why they have great difficulty facing reality and stopping to cling to the dogmas they have built their identity around. One can only hope and pray they will wake up and get wise before it is too late. Oh, have pity on your children! Consider that they will lose all human rights, and be at the mercy of merciless barbarians. If you do not take stock of what is happening and do something about it in time, our civilization is very likely to collapse and be replaced by a truly horrible régime – namely, the global Caliphate that many Moslems so fervently yearn for and some of them kill for.

If (God forbid) this historical disaster occurs, it will have done so, not because of cultural or moral inferiority by the West, as the Islamic supremacists claim, but because of the West's absurd self-delusion, self-abasement and lack of political will in the face of open and covert aggression.

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disciples. The fanatics respond enthusiastically, because they perceive in the call of Islam opportunities to engage in all sorts of obviously vicious deeds *with a good conscience*. That is surely one aspect of their motivation, one of the twists and turns in their sick psyche.

[134]   Needless to say, that is their own business: it does not give Moslems a moral right to engage in violent acts against them.

12. A fortiori in China and India

1. Zen logic in general

Zen logic, as is well known, is no logic, but a sort of **anti-logic**, an antithesis of logic[1]. It thrives on paradox and even contradiction, at least apparent if not real. A major feature of Zen logic, though this may not be distinctive to Buddhist or even to Indian or Chinese logic, is its belief in the 'tetralemma' (or *catuskoti*). According to this viewpoint, not only a thesis alone (A and not not-A) or alternatively its antithesis alone (not-A and not A) may in fact be true, but there is a real possibility that *both* the thesis and its contradictory (A and not-A) are true, or *neither* the thesis nor its contradictory (not A and not not-A) are true – or even, eventually, that two or more of these four compounds are true together or all false together.

For example, the "two truths" doctrine, formulated by the Buddhist philosopher Nagarjuna (India, ca. 150-250 CE), which distinguishes the "relative truth" of conventional minds and the "absolute truth" of enlightened minds may be classified under the tetralemma category of "neither A nor not-A," since relative truth is neither absolutely true nor untrue, but something in between. Again, the doctrine that "Nirvana and Samsara are one" may be classified under the tetralemma category of "both A and not-A," since it proposes a mixture of opposites. These two doctrines are paradoxically considered as mutually supportive, but of course that is quite illogical: if truth is twofold, its two aspects cannot be one; you can't have it both ways. In scientific Western thought[2], truth is one; if it is merely 'relative', it is simply untruth. Again, if two things are opposites, they cannot overlap.

Moreover, Buddhists argue that existents have no identity of their own, being merely aggregates, constantly in flux, and thoroughly dependent on causes and conditions. They apply this idea to mind as well as matter, and deny existence of the self or soul. Such claims are logically patently absurd. To deny the self or soul is to deny the existence of someone doing the denying[3]. If literally everything is aggregated, then the elements that are aggregated must also be aggregated, ad infinitum[4], in which case there is ultimately nothing to aggregate. There cannot be such a thing as aggregation without something *non*-aggregated to aggregate; the buck has to stop somewhere. Similarly, to the Buddhist doctrine of impermanence, which claims that literally everything is constantly in flux, we must ask: flux of what? Change must be change *of* something *to* something, with an least momentary stationary existence before and after the change. There cannot be such a thing as change without something static that changes or emerges from

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[1]   I should reiterate here that though I repeatedly criticize Buddhism for its illogic, my purpose is not to totally discredit it. I greatly respect this philosophy of life, and am myself positively influenced by it daily. However, there is much in its philosophical discourse that needs to be revised. Its cavalier attitude to logic is simply untenable.

[2]   The 12th century CE Islamic philosopher Averroës (or Ibn Rushd) tried to introduce a similar notion of "double truth" (one for common people and one for the élite). Some Christian philosophers, possibly including Boethius, tried to follow suit. But such tendencies were ultimately rejected in both cultures, as it was realized that if religion was cut off from reason, it ultimately implied that religion is irrational and therefore untrue. More recently, most Christians have gradually adapted their beliefs to empirical science and history (though many still resist, e.g. with regard to Darwinism). Islam, on the other hand, is still firmly marooned in the Middle Ages.

[3]   This goes against Descartes' phenomenological dictum: "I think, therefore I am," which means that as of the moment one acknowledges the phenomenon of thought by venturing some proposition, one logically must acknowledge the existence of someone having that thought. 'Consciousness' presupposes some sort of subject and some sort of object, being a special *relation* between two things, the conscious one being called 'subject' and the one the subject is conscious of being called 'object'. The difficulty of fathoming this relation, due to its ontological distinctiveness and therefore primacy, does not make it any the less real; there are a great many things we cannot fathom, but must take for granted. Knowledge must start with some irreducible primaries; it cannot be grounded in an infinity of definitions and proofs. To make a demand for endless grounding is to claim that demand as an irreducible primary; it is self-contradictory. Buddhists consider that what we call the self is simply the totality of our sensory and mental experiences at any given moment of time: for them, there is no one *having* those experiences – they just are, forming a changing bubble of manifest being (which they call 'consciousness'); this bubble being particular gives the illusion of selfhood. But the question remains: *who* has this illusory idea of being a self? How can a non-self imagine that it is a self? They have no answer to such questions, and avoid to ask them, being dogmatically attached to the idea of no-self.

[4]   Moreover, how can a human mind go *all the way to infinity* and observe that aggregation continues there, before making such a bold claim?

the change. The same applies to the Buddhist idea of interdependence, or co-dependence of everything[5]: one thing cannot depend on another if that other thing is as devoid of independent existence as itself. Dependence presupposes something more firmly rooted in being, which can be depended on. Simultaneous mutual dependence is unconscionable. Thus, Buddhist discourse is built on stolen concepts, ignoring their conceptual basis.[6]

Such Buddhist beliefs are contrary to the laws of thought discovered by Aristotle, namely the laws of identity, of non-contradiction and of the excluded middle. For Buddhists, all existents are ultimately "empty" of any nature. But the law of identity is that every existing thing has an identity, a specific nature (whatever that happens to in fact be[7]): it is not 'just anything' and it is not 'nothing'. Every existent is something in particular, with features and behavior peculiar to it. Moreover, a fact is a fact: while it occurs, its constituents, its history and its causes and conditions, if any, are irrelevant to the fact of its existence: it just is. Moreover, identities, facts, are mostly objective givens, not products of mind; to claim otherwise is to affirm one's claim itself to be imaginary and thus untrue. The law of contradiction is that an existent cannot at once have and not-have a particular identity; presence and absence are incompatible. The law of the excluded middle is that an existent cannot at once neither have nor not-have a particular identity; there is nothing besides presence or absence. These laws, properly understood, are absolute; they are not subject to any exceptions, under any circumstances whatsoever.

These laws – which have been foundational for Western logical thought and the source of its successes (although in today's atmosphere of willful unreason many people do take a perverse pleasure in disowning them) – were never, it seems, very influential further East. The tetralemma was evidently freely used very early on in East Asia and the Far East, since it is so pervasive in later literature. The "reasoning" behind this irrational belief is that all ordinary human cognition is necessarily "dualistic." According to its proponents, as soon as anything comes to mind, through perception or conception, its negation must also be considered, even if we tend to either ignore it or arbitrarily reject it. Thus, a positive is unthinkable without a corresponding negative. In one author's words:

> "In Buddhist logic, it is said that all concepts are based upon exclusion. As soon as we affirm something by saying 'It is this,' we automatically exclude so many other things it might have been. By imposing a conceptual limitation we fabricate an idea. The suggestion here is that it is just an idea – it is not an open experience."[8]

But of course this 'reasoning' is quite fallacious. Knowledge starts with pure perception of positive phenomena; negatives are never pure percepts but are *necessarily* products of conception. Note well: positives come before negatives; and negatives are inconceivable without positives. I can cognize a positive through perception and therefore without any reference to its contradictory; but I cannot do the same with a negative. In the latter case, I must first have some idea however vague or hypothetical of the positive, before I can even think of, let alone check out, the negative. Thus, though exclusion is indeed eventually part of the knowing process, it is certainly not a primary act: it is only possible after the pure perception of some things and the subsequent imagination of their possible negation.

Some Buddhist philosophers go still further and, appealing to the notion of "emptiness" (*shunyata*), claim the ontological primacy of negation over affirmation. But here again the question they do not ask is: "negation of what?" If as they suggest there is nothing there at all, then even negatives have no foot to stand on. The negation of a nothing does not produce a something. What needs to be understood by such people is that the word 'not' is more akin to a verb than to a noun. It expresses the Subject's mental *act* of rejection of a proposed object. It is therefore *necessarily conceptual, and never perceptual*. Moreover, such claims invariably ignore the positive existence of the claim, and of

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[5]   Here again, how can a human mind know the dependence of literally *all* things on each other? To have such knowledge, of all things past, present and future throughout the universe and their exact relations to each other, is conceivable for God – but how can a mere mortal obtain it?

[6]   Although Buddhists claim that enlightenment brings about omniscience, such a claim is not empirically justified. For a start, Buddhism has made and still makes many claims about the physical world and the history of life and men that are rejected by modern science; e.g. that the world and life have existed forever. More specifically, consider the following blooper: Zen master Dogen, after attaining enlightenment in 1227-8, wrote in an essay dated 1231 that the Buddha was active about 2000 years before, whereas we know that he lived in circa the 6th-5th centuries BCE, i.e. some two to four hundred years later than Dogen thought. See: *Beyond Thinking: A Guide to Zen Meditation*, Ed. Kazuaki Tanahashi (Boston, MA: Shambhala, 2004), p. 31. Dogen claims having attained enlightenment in another essay (p. 13).

[7]   How the identity of things is to be known is the question the science of logic seeks to answer. The short answer is, of course, by means of our senses and our reason. That is, empirically and logically, inductively and deductively. Not all identities are necessarily knowable; but we must admit that some are, for otherwise we would be involved in self-contradiction (claiming knowledge and denying it at once).

[8]   See Traleg Kyabgon Rinpoche, "The Path of Mahamudra," in *The Best Buddhist Writing 2005*, Ed. Melvin McLeod and the editors of the Shambhala Sun (Boston, MA: Shambhala, 2005), p. 98. Although he refers specifically to conception, the implication of such statements is usually taken to be that all affirmation implies negation, i.e. even affirmation based exclusively on perception. Note however, the contrary statement by Eleanor Rosch, in the same collection of essays, p. 114: "According to Buddhist teachings, while all of the interdependent past can be causally gathered into the microcosm of the moment of present experience, that does not mean that the basic mode of apprehending the present moment is somehow filtered or distorted or abstractly representational." In other words, Buddhists do ultimately admit of unadulterated percepts (if only in the context of the enlightenment experience).

someone doing the claiming, and of someone receiving the claim. Such people imagine they can speak in a vacuum, without acknowledging the existential context of their speech. This is illogical.

If anything, it is the Buddhist proponents of "paraconsistent" logic who are dualistic and divorced from reality. They fail to take note of the actual order of knowledge development from positive percepts to negative concepts. Indeed, even at the level of perception, one precedes two. Contrary to what many philosophers imagine, we perceive a whole before we mentally divide it into parts. Here, the confusion involved is to conflate a given moment of perception and perception over time. In any given moment, what we happen to perceive is a whole and this is quickly and mostly automatically divided into parts by the conceptual faculty (note that the perception precedes the subdivision, and only the latter involves negation, i.e. saying 'this part is not the same as that part'); but of course, over time, many such moments of perception, or more precisely their memories, are added together (again by the conceptual faculty) to form a larger whole. These two operations of the conceptual faculty – viz. conceptual dissection of a present perceptual whole and integration of many past percepts into a conceptual whole – should not be confused.

See **Appendix 7.3** for more on the topic of the tetralemma.

I did not, unfortunately, note down every use of such deviant logic that I came across over the years in Buddhist literature. But I do still remember one relatively early instance in the *Dhammapada*, traditionally attributed to the Buddha (India, ca. 563-483 BCE)[9], "He for whom there is neither this nor the further shore, nor both…." The tetralemma plays a very important role in the Madhyamika philosophy of Nagarjuna[10], regarded as a forerunner of Zen. As for later, specifically Zen writings, in China and then Japan, they are full of it. Consider, for instance, the words of the third patriarch of Zen, Seng Tsan (China, d. 606 CE):

> "*What is, is not; what is not, is.* If this is not yet clear to you, you're still far from the inner truth. One thing is all, all things are one; know this and all's whole and complete." (Italics mine.)[11]

Eihei Dogen (1200-53 CE), who founded the Japanese Soto Zen sect, often seems (to me, at least) maddeningly obscure, if not insane, due to his frequent breaches of the laws of thought. He indulges without hesitation in self-contradictory statements, such as "There is sitting letting go of body-mind, which is not the same as sitting letting go of body-mind." Likewise, the law of the excluded middle is no obstacle to his way of thinking. Consider, for instance, this statement: "Active buddhas are neither originally enlightened nor enlightened at some particular time, neither naturally enlightened, nor without enlightenment" – what are they, then, I ask? Or again: "practice-realization is neither existence nor beyond existence" – what's left, I ask? Surely, if all logical possibilities are exhausted (as seems to be the intention, here), then there are no other possibilities! Dogen pays no attention to such obvious restrictions, making his discourse incomprehensible nonsense.[12]

Although Zen discourse is often antinomic, its favorite form seems to be "neither this nor that." That is to say, although contradictions and exclusions of the middle are both viewed as possible and do occur in practice, the main emphasis is on denying any thesis whatever, and 'logically' enough also the contradictory of any thesis whatever. For ultimate reality is considered by Zen philosophers as essentially out of this world (even while in it) – therefore, whether phenomena point to the existence or to the non-existence of something, anything, is irrelevant. No proposition is true, because none is capable of describing reality as it really is. The phenomenal world is inherently paradoxical; only beyond it can all opposites be harmonized.[13]

This is the gist of the argument, however self-inconsistent and unconscionable it seems to us who are not enlightened. Of course, some sense can be made of it by thinking of ultimate reality as the 'common ground' of conflicting phenomena – and this sort of explanation is often used (for example, see the above quoted statement by Seng Tsan). So the 'noumenon' (i.e. that which is beyond the phenomenon) may be thought of as both transcendent

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[9]   *The Dhammapada* was probably compiled in the third century BCE.

[10]   See my book *Buddhist Illogic* on this subject. It should not be thought that Nagarjuna's perverse thought has had no equivalent in the West. For example, the Megaric school (founded by Euclides of Megara in 4th century BCE Greece) argued much like him that predication is either wrong (if the predicate "differs" from the subject) or useless (if the predicate is "the same as" the subject), ignoring the fact that such a statement is itself an act of predication. I have over the years spotted many such similarities between Eastern and Western philosophies. This is a topic that still needs extensive study, though there may already be good books on it that I am unaware of.

[11]   In his "Affirming Faith in Mind," given in full in Roshi Philip Kapleau's *Zen Merging of East and West* (New York: Doubleday, 1980), pp. 184-189. It is hard for me to believe that illogic, the suppression of reason, is compatible with enlightenment, let alone conducive or essential to it—just as it is hard for me to believe that idolatry, the worship of inanimate objects, is compatible with enlightenment, let alone conducive or essential to it. Yet these are recurring theoretical teachings within Zen Buddhism. Even so, paradoxically, I do believe that Zen has much good to offer mankind on a practical level!

[12]   *Beyond Thinking*, pp. 51, 79, 80. I should additionally draw attention to the frequent use of *tautology* in some Buddhist texts, as if this was informative. For example, Dogen also enjoys tautologies like "sitting is sitting;" he also, I notice, takes pleasure in reversing statements, as in "sitting is buddha-dharma and buddha-dharma is sitting" (p. 51); and reshuffling terms, as in "zazen is invariably the intention to become buddha, and… zazen is invariably becoming buddha with intention" (p. 39). Such discourse may of course be informative, but I suspect the intention is more poetic.

[13]   The *Vimalakirti Nirdesa Sutra*, a Mahayana text some consider as dating from about 100 CE (although there is no mention of it till after Nagarjuna's time, i.e. about a century later), is a veritable litany of antinomies.

and immanent. But a true Zen master would disdainfully reject all such philosophizing as misleading babble. Any resort to words as a means of rational description or explanation is regarded as useless when it comes to the "matter" of enlightenment. Consider for instance the following remarks in *The Blue Cliff Record*[14]:

> "It's wrong to say either that he had words or didn't have words; nor will it do to say that his answer neither had nor didn't have words. Chao Chou left behind all the permutations of logic. Why? If one discusses this matter, it is like sparks struck from a stone, like flashing lightning. Only if you set your eyes on it quickly can you see it. If you hesitate and vacillate you won't avoid losing your body and life."

All the above tends to the conclusion that Zen 'logic' is illogical. However, that judgment can be considerably mitigated, if we understand Zen 'anti-dualistic' discourse not as theoretical but as pragmatic. Its purpose is not to formulate a true philosophy, in the Western sense, but to push people to a transforming mystical experience. Thus, when a Zen advocate states: "This is neither true nor false" or "This is neither good nor bad" or "This is neither desirable nor repugnant" – his intent is really, respectively: "*Do not think or say* that this is true and do not think or say that it is false," "Do not think or say that this is good and do not think or say that it is bad," and "Do not think or say that this is desirable and do not think or say that it is repugnant." For example, Seng Tsan says:

> "When you *assert* that things are real, you miss their true reality. But to *assert* that things are void, also misses reality. The more you *talk and think* on this, the further from the truth you'll be." (Italics mine.)[15]

In other words, the Zen advocate is not really making logical, prescriptive or descriptive judgments, but advocating *the suspension of all judgments, all discourse*, in order to arrive at the ultimate "truth." There is no great inconsistency in doing that. We may, of course, point out that in claiming to be free of concepts he is using concepts and that that is an inconsistency. However, he would reply that he is doing that only in order to communicate with us in our language, in an attempt to allude to things beyond its scope. He is able to function in both the phenomenal and noumenal worlds, whereas we are not – so he has to find some way to reach out to us. In that case, we can only criticize him for being rather gauche in his discourse. He should make it more precise, as just demonstrated. It would then be possible to speak of Zen logic, without inverted commas.

Nevertheless, although a statement like "neither claim it is nor claim it is not" is intended as a non-claim, it objectively definitely does contain a factual claim – viz. the claim that following this advice will facilitate or result in enlightenment ("the truth"); and such a claim is, of course, subject to assessment as true or false, whether the Zen advocate admits it or not.

I would say a very representative example of Zen logic is the *koan* of Te-shan (China, 9th century CE), the Zen master famous for presenting his students with the following predicament: whether they 'uttered a word' (i.e. showed some evidence through word or deed of their understanding of Zen) or not, they would get thirty blows. Another master, Lin-chi (the founder of the Rinzai sect), sent one of his own followers to him with specific instructions. He told him to ask Te-shan why someone who said a word would nevertheless get thirty blows; then, when Te-shan struck him, the student was to grab the stick and push Te-shan back with it. When the student did as instructed, Te-shan responded by simply walking away.[16]

What we have here is a logic of action, rather than words[17]. There is, to start with, a seemingly inescapable dilemma – whether you speak (rightly or wrongly) or abstain from speech, the result will be the same: you will be in error and punished by blows. There is, however, a logical possibility of escape – grab the stick as it comes down and push it back. This could be described as a martial arts response to the attempted physical blow. Logically, the dilemma has by this means been effectively dissolved. There seemed to be no way out, judging by Te-shan's statement; but there was in fact a way out, perceived by Lin-chi. The opponent is neutralized, prevented from producing the threatened consequences (blows) to either antecedent (speech or silence).

Earlier in the present volume, in an attempt to more accurately depict the logic of a fortiori reasoning, I developed the notion of relative terms, say R1 and R2, such that more R1 and less R2 (and vice versa) are logically equivalent. This idea, I showed, can be extended to the special case of complements, say R and not-R. Although complements, taken as absolute terms, are mutually exclusive – if we take them as relative terms, they are compatible, indeed imply each other. That is, we can define R and not-R so largely that each includes the other, in the same continuum but in opposite directions, i.e. in such a way that more R is less not-R and less R is more not-R. This logical artifice of course changes the meaning of R and not-R, but it is useful for the development of a fortiori logic.

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[14]   *Pi Yen Lu*, a Chinese Ch'an Buddhist classic. These remarks were made by Yuan Wu K'e Ch'in (1063-1152), relative to Case 59 (p. 339). Boston, MA: Shambala, 2005. Tr. Thomas & J.C. Cleary.
[15]   *The Blue Cliff Record.* (I forgot to note the page number.)
[16]   See D. T. Suzuki, *The Zen doctrine of no-mind* (Boston, MA: Weiser, 1972), p. 87.
[17]   Notice that the student did not try to dissuade Te-shan, saying "if you try to hit me, I will grab the stick and push it." Rather, he waited for Te-shan to actually strike and then grabbed the stick and pushed it.

After I worked this idea out, it occurred to me that it could help explain Zen logic. It could be that Oriental philosophers who conceive of A and not-A as being compatible are really thinking in relative terms. Perhaps we in the West think of A and not-A in absolute terms, while they in the East think in relative terms. This may explain, at least in part, why the conjunction of A and not-A does not repel the Oriental mind to the same degree as it does the Western mind. Although, to be sure, this theory is somewhat belied by the fact that Orientals also accept the possibility of neither A nor not-A, which this theory cannot explain.

Needless to say, the said insight does not change the fact that A and not-A, taken in their absolute senses, are incompatible; the Aristotelian law of non-contradiction remains true and unassailable. Relative terms are logical artifacts that function consistently within that universal framework – they do not erase it. The law simply changes form, becoming a distinction between 'more' and 'less': What is more R is less (and *not* more) not-R, and what is less R is more (and *not* less) not-R. Moreover, it is interesting to note that when A and not-A are intended as relative terms, everything falls under both of them; there is no further possibility beyond them. That is to say, the law of the excluded middle also remains operative for relative terms, although it too is stated slightly differently.

2. A fortiori use in Zen

But we also find in Zen literature some quite ordinary logic. For example, Dogen mentions an earlier master, Guixing, as saying: "... mind arrives, but words do not; … words arrive, but mind does not; … both mind and words arrive; … neither mind nor words arrive"[18]; this saying of course shows awareness of the four logically possible combinations of two propositions and their negations. Zen writings do on occasion, when it suits their purposes, appeal to the law of non-contradiction. Hakuin Ekaku (Japan, 1686-1768) argues against 'do-nothing' Zen as follows: "If you are right about 'being as you are' in this 'plain bowl' suchness of yours, then the stages of Bodhisattva practice that were set forth long ago are mistaken. If the stages passed down from the past are correct, then being as you are like a 'plain bowl' is wrong"[19].

There are also some instances (few and far between) of a fortiori argument. The following is a list of instances found by me by chance. This is of course not a comprehensive list, but some insights can be gained from it. Note that I give a hint as to the form of each argument by using the abbreviations +s, –s, +p, –p, and &, and I summarize the findings at the end.

The argument can already be found in *The Dhammapada*, an Indian Buddhist document, in the Pali language, traditionally attributed to the Buddha (c. 563-483 BCE): "'These are my sons. This is my wealth.' In this way the fool troubles himself. He is not even the owner of himself: how much less of his sons and his wealth!" (–p)[20]. The argument also appears quite early in China, judging by its use in Taoist literature, notably in Lao Tzu's *Tao Te Ching* (traditionally dated 6th century BCE, though more probably late 4th century or early third century BCE): "If even heaven and earth cannot go on for ever, much less can man" (–s)[21]. We can also mention its occurrence in Sun Tzu's *The Art of War* (also traditionally dated to the 6th century BCE, but probably of later origin): "Thus do many calculations lead to victory, and few calculations to defeat: how much more no calculation at all!" (+s)[22].

Later, the argument appears in *The Diamond Sutra* (an important Mahayana Buddhist text, first translated from Sanskrit into Chinese in 401 CE), section 12: "Wherever this sutra or even four lines of it are preached, that place will be respected by all beings… How much more [worthy of respect] the person who can memorize and recite this sutra [for the benefit of others]!" (+s); or again, in section 15: "If someone… on hearing this discourse on Dharma,

[18]   *Moon in a dewdrop: Writings of Zen Master Dogen*, p. 81. Ed. Kazuaki Tanahashi (New York: North Point Press, 1985).

[19]   *Wild Ivy: The Spiritual Autobiography of Zen Master Hakuin*. Tr. Norman Waddell. Boston: Shambhala, 2010. (P. 56.) It is interesting to compare Hakuin's writing (18th cent.) with that of Dogen (13th cent.). While the latter's discourse often seems muddled and incomprehensible, Hakuin's (at least in this book) seems quite rational and straightforward. Maybe the five centuries separating them explains this – but more likely, I think it just goes to show how much the particular personality of individuals affects their discourse. In other words, each person has a mental framework that he brings to his philosophical discourse; and this must be taken into consideration when judging a school of philosophy as a whole. I do not think that the explanation is to be found in the fact that these two masters were from different Zen sects, because examples can be adduced from each sect that point in the opposite direction (i.e. there have been less irrational Soto masters than Dogen, and less rational Rinzai masters than Hakuin).

[20]   Anonymous. *The Dhammapada*: *The Path to Perfection.* Tr. Juan Mascaró. (London: Penguin, 1973), v. 62.

[21]   See Lao Tzu, *Tao Te Ching*, trans. D. C. Lau (London, UK: Penguin, 1963), p. 80 (23:51a). Regarding the dating of the text, see the translator's comments on p. 147. I assume this translation is literal and reflects actual use of a fortiori argument in Chinese language. I posed the question to a correspondent, Prof. Minghui Xiong of Sun Yat-Sen University, who replied that "a fortiori language or argument [is] really present in Chinese," and is indicated by the words "何況 (*he kuang*)."

[22]   (1:15.) Also: "But if neither time nor place be known, then the left wing will be impotent to succour the right, the right equally impotent to succour the left, the van unable to relieve the rear, or the rear to support the van. How much more so if the furthest portions of the army are anything under a hundred Li apart, and even the nearest are separated by several Li!" (+a) (6:20). Feedbooks (from Paxlibrorum.com, 2008-9).

were to accept it with a believing heart, the merit acquired by the latter would far exceed… How much more the merit of one who would copy, memorize, learn, recite and expound it for others!" (+s &)[23]

Much later, the Chinese Zen masters use a fortiori to reply to queries; thus, Hui Hai (8th cent. CE): "Even idealism is not Mahayanist, how much less so realism!" (–s)[24] and Huang-Po (d. ca. 850 CE): "If solid things do not exist, how much the less can we make use of reflections" (–s)[25]. Another Chinese Zen master, Kuei-shan (771-853 CE), when asked for a Buddhist verse responded by means of a fortiori discourse: "To try to express this [i.e. the key to enlightenment] from person to person is foolish, how much more if you try to put it on paper" (+s)[26]. A similar argument is used by the Japanese Zen master Bankei (1622-93), in response to a question asked of him through an intermediary: "This matter of Zen is difficult to convey even by direct question and direct answer; it is all the more difficult to convey by messenger" (–p)[27].

I have also found a fortiori arguments in *The Blue Cliff Record* (a 12th century compilation of *koans*, some of which date from the 10th century or before). "Gold dust is a cataract on the eye; the jewel in one's robe is the defilement of the Dharma. Even one's own spirit is not important; who are the Buddhas and Patriarchs?" (+s)[28] The point here made is that true enlightenment involves relinquishing all trace of 'self-consciousness' (in the pejorative sense) – even thoughts of Buddha Dharma (the teaching of the way to enlightenment; here "gold dust") and Buddha-nature (the substratum of all existence; here "the jewel"). If even one's spirituality is forgotten, how much more do the teachers ("Buddhas and Patriarchs") pale into insignificance. Again, "If they [saints, *arhats*] have themselves cut off the eighty-one kinds (of passion), how much more so the three poisons [greed, hatred and delusion]!" (+p)[29]

Dogen (whom we already mentioned in the previous section) makes the following a fortiori remark in one of his essays: "Such [deluded] people existed in the past; are there not even more today?" (+s &)[30] – inferring, from the common knowledge that some people were deluded in the past, that many contemporaries are deluded. In another essay he says: "Buddha said, 'if you are to practice giving to yourself, how much more so to your parents, wife, and children'" (+s)[31]. In yet another: "What the tenzo told me corresponded with Xuedou's poem. So I knew all the more that the tenzo was truly a person of the way" (+s)[32]. And again: "When you long for enlightenment… even taking sand and offering it to Buddha is beneficial…. How much more so to be in the position of tenzo" (+s)[33].

I can also quote you an a fortiori argument by the current Dalai Lama of Tibet; reflecting on the inevitability of death for all of us, he writes: "Even Buddhas have left their bodies behind, so what can be said of ordinary beings?" (+s)[34]

All these a fortiori arguments are well formed (albeit not fully explicit), showing that the people formulating them were able to be logical as well as illogical. Of the 13 instances here listed, 10 are positive subjectal (marked +s), 3 are negative subjectal (–s), 1 is positive predicatal (+p) and 2 are negative predicatal (–p). Moreover, 13 of these arguments are purely a fortiori, while 2 of them (marked &) seem to have been intended as a crescendo. These statistics show that all forms of a fortiori argument (at least, of the copulative variety) are represented in Zen Buddhist discourse. Of course, these are only isolated *uses of* the argument form. I know of no philosophical or logical *reflection on* a fortiori argumentation in Buddhist or Zen literature. They just used it instinctively and took it for granted.

3. The Indian kaimutika

A fortiori argument is called *kaimutika* or *kaimutya nyaya* in Indian logic. *Kaimutika* (कैमुतिक) or *kaimutya* is derived from the words *kim uta* which literally mean 'what to say of' (i.e. 'what is to be said of'), and are used like

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[23]   See Mu Soeng, *The Diamond Sutra* (Somerville, MA: Wisdom, 2000), pp. 111 and122.
[24]   See John Blofeld, Trans., *Zen Teaching of Instantaneous Awakening* (Totnes, Devon, UK: Buddhist PG, 1987), pp. 158-9. First publ. 1962.
[25]   See John Blofeld, Trans., *The Zen Teaching of Huang Po* (New York: Grove, 1958), p. 60.
[26]   See Audrey Yoshiko Seo, *Ensō: Zen Circles of Enlightenment* (Boston, MA: Weatherhill, 2007), p. 9.
[27]   See Thomas Cleary, Trans., *Zen Antics: 100 Stories of Enlightenment* (Boston, MA: Shambhala, 1993), pp. 17-18.
[28]   *The Blue Cliff Record*, p. 348. This is from a commentary by Yuan Wu K'e Ch'in (already mentioned), but he is quoting some unstated earlier master. The explanation I here offer is based on those given in the Translator's Notes. Needless to say, I assume the a fortiori form of the English sentence to be true to the Chinese original.
[29]   *The Blue Cliff Record*, p. 520.
[30]   *Beyond Thinking*, p. 18.
[31]   *Moon in a Dewdrop*, p. 45. I read this a fortiori argument as positive subjectal, because its intention seems to be (knowing Mahayana Buddhism's preference for altruism) from minor to major: "if giving to yourself is important enough for you to practice, how much more so giving to your parents, wife, and children."
[32]   *Moon in a Dewdrop*, p. 60. A *tenzo* is a monastery cook.
[33]   *Moon in a Dewdrop*, p. 64.
[34]   *The Essential Dalai Lama*, p. 119. Ed. Rajiv Mehrotra. (London: Hodder & Staughton, 2006.)

the expression 'how much more' (or how much less')[35]; *nyaya* means inference. In the *Vyavahāramayūkha* of Nīlankaṇṭha[36], it is written: "*kaimutika nyaya* is a maxim used where a conclusion will *a fortiori* follow as regards certain matters when it is conceded that it does follow in certain other *less* important or less obvious matters." This obviously refers to inference from minor to major; but as we shall soon see, there is also inference from major to minor. The following are examples of a fortiori argument in religious Indian discourse that I found in the Internet:

- "It has been said also in SANdilya-smriti: 'There may be doubts concerning the redemption of those who serve AchArya, but there is absolutely no doubt about the redemption of those who delight in the service of His devotees' (1-95). So, in the case of those who depend solely on the AchArya, there is no doubt at all concerning the fruition of prapatti, by the principle of 'kaimutika nyAya'. (Will not the Lord, who saves those who take refuge in His devotees, save those who take refuge in their AchAryas? Will not a benefit, which is got by one who is not qualified, be obtained by one who is qualified?)[37]. The intended a fortiori reasoning in this example seems to be: if those who serve mere devotees are meritorious enough to be redeemed (or to attain "the fruition of prapatti," i.e. of devotional surrender), all the more so those who serve the teachers of the devotees (the "AchArya"); this is *positive subjectal* (from minor to major) argument.

- "That Prabhu who bestows even upon His most antagonistic enemies a result that is supremely difficult for great yogés to attain must necessarily, according to the logic of 'inferential partial illustration' (kaimutika-nyäya), bestow the highest fruit of all upon His devotees who incessantly cultivate favorable activities and moods in His service."[38] Here again, the argument seems to be *negative subjectal* (from major to minor) a fortiori: if his enemies are not disqualified enough to be deprived of great benefits from Prabhu, all the more so his devotees. The intent here may even be a crescendo, in which case the benefits for the latter would be greater than those for the former.

- "This Seshatva, being the servant of the Supreme Lord, will automatically be reflected in his character as well as in his appearance. This results by the principle of 'kaimutika nyAya'. When a greater thing is known, will not the lesser great thing be known? That will be obvious. Here, the knowledge of SEshatvam is the greatest one. When it is known firmly to a person, will it not reflect in his behaviour and appearance? There is no need to mention it separately as it is obvious."[39] The explanation given within this example points to *positive predicatal* (from major to minor) a fortiori argument: if someone is high enough to know the greater thing, all the more so a lesser thing[40].

- "Here an argument *a fortiori* (*kaimutika-nyaya*) is advanced. Even in the world, without a knowledge of something to be done (*karya*), there is no eligibility. It goes without saying, therefore, that this must be so with regard to sacred matters."[41] This looks like a *negative predicatal* (from minor to major) a fortiori argument: If one is not sufficiently qualified to be eligible in relation to mundane matters, all the more so in relation to sacred matters.

From the above four examples, we can see that Indian a fortiori reasoning may go from minor to major or from major to minor. Indeed, all four moods of the argument appear to be used – the positive and negative subjectal and the positive and negative predicatal. Moreover, we can say that Indian discourse does not merely use a fortiori argument, but uses it consciously (as evidenced by appeal to "the principle" or "the logic" of *kaimutika nyaya*) and it seems that some discussion of it has occurred at some time (as the reference to "inferential partial illustration" suggests).

However, it is noteworthy that my Google searches did not yield many more examples than the above four (which are repeated frequently, in different Hindu websites)[42]; and as regards reasoned reflections on *kaimutika nyaya*, or even explanations of the expression "inferential partial illustration," I have not found any. We might from this surprising paucity of information infer that nowadays a fortiori argument is not very often used in Hindu discourse and the argument is very rarely if at all discussed.

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[35]   At: www.sanskritdictionary.com. In modern Hindi, the word for a fortiori is *nishchayapurvak* (निश्चयपूर्वक), according to: dict.hinkhoj.com/words/meaning-of-A%20FORTIORI-in-hindi.html.

[36]   Pandurang Vaman Kane, 1933. P. 262. The author, Nīlakaṇṭhabhaṭṭa, was active the second half of the 17th century. In Varanasi.

[37]   At: www.ibiblio.org/sripedia/oppiliappan/archives/jan08/msg00298.html.

[38]   At: www.iskcondesiretree.net/forum/topics/question-about-swarup-siddhi?xg_source=activity.

[39]   At: www.srihayagrivan.org/ebooks/086_ss_Part4.pdf.

[40]   However, I do not see the surrounding text as meaning this; it rather seems to be saying that if someone has "knowledge of SEshatvam," then this knowledge "will automatically be reflected in his character as well as in his appearance." This is not a fortiori argument, but indicative of a causation.

[41]   The *Saṁbandha-vārtika* of Sureśvarācārya (University of Madras, 1972), p. 12.

[42]   Actually, I found only more three examples, in Google Books: in *The Journal of Oriental Research, Madras*, Volume 36, p. 11; in John Muir's *Original Sanskrit texts on the Origin and History of the People of India, their Religion and Institutions*, Volume 4, p. 44 (London: Trübner, 1863); and in *Gleanings from Vedic to Puranic Age*, collected papers of Sadashiv A. Dange, p. 193 (New Delhi, Aryan, 2002). These three examples seem to relate to Hindu law.

As regards historical conclusions, it is impossible for us to draw any from the above four examples for now, since their original authors are not readily identifiable and dated (though further research might allow us to do that, of course). All we can say at this stage is that Hindu discourse today, whether based of ancient Indian sources or on capabilities more recently acquired from whatever (e.g. perhaps British) sources, includes valid a fortiori reasoning. Nevertheless, a fortiori argument is manifestly present in quite ancient Indian literature, as the following two examples demonstrate[43].

There is a case in the Pali-language *Dhammapada*, a collection of aphorisms traditionally attributed to the Buddha[44]: "'These are my sons. This is my wealth.' In this way the fool troubles himself. He is not even the owner of himself: how much less of his sons and his wealth!" – meaning: if the fool (S) has insufficient power (R) to claim ownership of himself (Q), all the less has he (S) power (R) enough to own his sons and wealth (P) (negative predicatal, from minor to major).

There are also four cases in the Sanskrit-language *Bhagavad-Gita* traditionally attributed to a sage called Vyasa[45]: "These I would not consent to kill, though killed myself, even for the kingdom of the three worlds; how much less for the sake of the earth?" (1:35) – meaning: if the three worlds (P) are of value to me (R) not enough to consent to kill these people to gain rule of them (S); all the less, an earthly kingdom (Q) (negative subjectal, from major to minor); "This world is not for him who offers no sacrifice; how, then, any other world?" (4:31) – meaning: a man who offers no sacrifice (S) has merit (R) not enough to have a share in this world (Q); all the more so, in the higher world (P) (negative predicatal, from minor to major); "Those who take refuge in Me, though they are lowly born, women, merchants, as well as workers—they also attain the highest goal—how much more, then, holy Brahmins and devoted royal saints?" (9:32-33) – meaning: the lower castes who take refuge in Me (Q) are wise (R) enough to attain the highest goal (S); all the more, the higher castes who do so (P) (positive subjectal, from minor to major); "None is equal to Thee; how then could there be one greater than Thee in the three worlds? (11:43) – meaning: no one (S) is great (R) enough to be equal to Thee (Q); all the less so, to surpass Thee (P) (negative predicatal, from minor to major).

These are of course only two texts in the long list of classics of Indian religious literature, including the four Vedas, the Vedangas, the Upanishads, the Ramayana and Mahabharata epics, the Sutras, the Puranas, and many more. A full study of the use of a fortiori discourse in ancient India would have to systematically examine all extant texts, looking for and counting all occurrences of the key words and phrases of such discourse in them. I will not do the job here, albeit its importance, but leave it to others[46].

I have looked through extracts from *The Nyaya Sutras of Gotama* (namely[47], 1:1-2, 2:1-2, 5:1-2). There is, there, a definition of comparison or analogy (*upamana*) as "the knowledge of a thing through its similarity to another thing well known" (1:1:6, p. 359); and it is later stated that knowledge by this means is "right" provided the "comparison is established through similarity in a high degree" (1:2:44-45, p. 368). There is also an apparent reference to syllogism, an example given being "Sound is non-eternal… because sound has the character of being a product; as a matter of fact, everything that is a product is non-eternal" (1:1:32-35, p. 362)[48]; and possibly also a reference to generalization (1:1:36-38, pp. 362-3). But I have not in these extracts found use or discussion of a fortiori argument. Of course, this being a very scant research effort it proves nothing.

As regards Indian logic in general, not having studied it in any depth I will not here attempt to discuss it further. But, based on the little I know of it, I can say offhand that while there are of course some ideas and doctrines in it that are open to criticism, it has a great many interesting and instructive features which we in the West would no doubt gain much from studying. An example of interest, found in the above-mentioned *Sourcebook*, is the understanding in *The Padarthadharmasamgraha of Prasastapada* that "Negation… is mere inference; just as the appearance of the effect becomes 'indicative' of the existence of the cause, so also does the non-appearance of the effect become 'indicative'

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[43] These books may have been compiled as late as the 4th or 3rd cent. BCE, but using material possibly centuries older. I have added the standard symbols P, Q, R, S, so as to clearly indicate the major, minor, middle and subsidiary terms in these two examples.

[44] Anonymous. *The Dhammapada*: *The Path to Perfection.* Tr. Juan Mascarò. (London: Penguin, 1973), v. 62.

[45] The passages here quoted are drawn from: *A Sourcebook of Indian Philosophy*, edited by Sarvepalli Radhakrishnan and Charles A. Moore (Princeton, N.J.: Princeton UP, 1957), which contains the whole *Gita*, translated by S. Radhakrishnan. I have left out words not directly relevant to our purpose, for brevity's sake. Note the use of 'how then' (ht) in two cases to indicate a fortiori argument; this expression is, of course, not reserved for such argument; I have not looked for it elsewhere.

[46] See Appendix 3 for my guidelines for such research. Many Hindu texts in English can be found at the Internet Sacred Text Archive: www.sacred-texts.com/hin/index.htm.

[47] These are the passages included in the excellent collection, *A Sourcebook of Indian Philosophy*, already referenced. According to the editors, Gotama's Nyaya Sutras date from the 3rd century BCE. The translation used here is that by S. C. Vidyabhusana, in "Sacred Books of the Hindus, VIII (Allahabad: The Panini Office, 1930)."

[48] Note that this example is actually a commentary drawn from *Gautama's Nyayasutras* with Vatsyayana's *Bhasya*, translated by Ganganatha Jha (Poona: Oriental Book Agency, 1939).

of the non-existence of the cause. (ix.ii.5)" (6:110, p. 415)[49]. This very important insight into negation as an inductive product rather than a direct experience is something that many Western logicians have not yet, to this day, realized and assimilated.[50]

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[49]   This text of the Vaisesika school dates from the 4th century CE. Trans. by Ganganatha Jha (Allahabad: E. J. Lazarus & Co., 1916). See also the small print commentary on this passage drawn from the *Nyayakandali* of Sridhara (991 CE).

[50]   The reader can find more information on Indian logic, to begin with, in the Web. See for a start: en.wikipedia.org/wiki/Nyaya and en.wikipedia.org/wiki/Navya-Ny%C4%81ya. "Nyāya … is the name given to one of the six orthodox or *astika* schools of Hindu philosophy— specifically the school of logic. The Nyaya school of philosophical speculation is based on texts known as the Nyaya Sutras, which were written by Aksapada Gautama from around the 2nd century BCE." "The Navya-Nyāya [school] … was founded in the 13th century CE by the philosopher Gangeśa Upādhyāya of Mithila. It was a development of the classical Nyāya darśana. Other influences on Navya-Nyāya were the work of earlier philosophers Vācaspati Miśra (900–980 CE) and Udayana (late 10th century). It remained active in India through to the 18th century." Look also in Google Books: https://www.google.com/search?tbm=bks&q=indian+logic&btnG=.
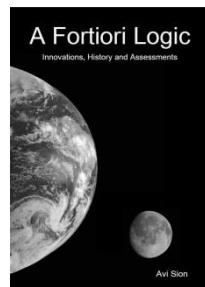
PART III – MODERN AND CONTEMPORARY AUTHORS

13. Moses Mielziner

Moses Mielziner (Poland-USA, 1828-1903) was an American Reform[1] rabbi and author. His work *Introduction to the Talmud* is definitely a classic of the genre, one of the best I have seen[2]. It starts with a "historical and literary introduction;" then it deals in considerable formal and material detail with the "legal hermeneutics of the Talmud;" this is followed by an extensive study of "Talmudical terminology and methodology;" and lastly various additional essays and notes. I wish I could take the time to review it all, but here is obviously not the place for this job; such a major digression from our central concern here would be unjustified. I will therefore confine my analysis here mainly to Mielziner's treatment of "the inference of kal ve-chomer" (part II, chapter I).

1. Description of the argument

Although he states at the outset that this hermeneutic rule "has quite a logical foundation, being *a kind of* syllogism" (my italics) – this cannot be taken to mean (as some[3] have done) that he equates a fortiori argument with syllogism. That is evident from what comes after. All he means here, I believe, is that a fortiori argument is, like syllogism[4], *mediate* inference, i.e. inference through a middle term (as distinct from immediate inference). Moreover, he makes clear that though the terms "kal" and "chomer," referring respectively to things of lesser and greater legal importance or significance, are called the minor and major terms, these words should not be confused with the same ones commonly used in regard to syllogism[5].

Mielziner describes the *qal vachomer* argument as follows:

> "The principle underlying the inference of [kal ve-chomer] is, that the law is assumed to have the tendency to proportionate its effects to the importance of the cases referred to, so as to be more rigorous and restrictive in important [matters], and more lenient and permissive in unimportant matters. Hence, if a certain rigorous restriction of the law is found regarding a matter of minor importance, we may infer that the same restriction is the more applicable to that which is of major importance, though that restriction be not expressly made in the law for this case. And on the other hand, if a certain allowance is made by the law regarding a thing of major importance, we may properly conclude that the same allowance is the more applicable to that which is of comparatively minor importance" (pp. 130-131).

Reading the first sentence, we might well assume that Mielziner identifies a fortiori argument with a crescendo, which yields a 'proportional' conclusion. But his next two sentences clearly refer to purely a fortiori argument, which yields an identical conclusion. He gives as his prime example the argument in the Mishna *Beitza* 5:2 (which we have dealt with in detail in an earlier chapter, viz. 8.5) – namely (briefly put) work permitted on the Sabbath is "more permissible" on a Festival, and vice versa, work forbidden on Festivals is "all the more imperatively to be forbidden" on the Sabbath. Judging by the language used in this example, it appears that Mielziner has not realized the difference between purely a fortiori and a crescendo argument. This is an important lacuna in his analysis.

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[1]   Note that "Reform" Judaism in his day was less radical than it is today; it was perhaps more akin to today's "Conservative" denomination.

[2]   The edition I have in hand is the third, published in 1925. But the original edition dates from 1894. He is thus just a few years ahead of Schwarz, at least with regard to analysis of the *qal vachomer* argument. This work is certainly worth perpetuating. I wish I had read it while writing my *Judaic Logic*; the latter work would have been greatly enriched.

[3]   Notably Schumann, p.7.

[4]   The etymology of syllogism is *syn+logism*, i.e. confluence of discourses.

[5]   In syllogism, the minor term is found in the minor premise and conclusion, and the major term in the major premise and conclusion; as for the middle term, it appears in the two premises but not in the conclusion. In a fortiori argument, the minor and major terms are both found in the major premise, and either one may appear in the minor premise while the other appears in the conclusion; as for the middle term, it appears in both premises and in the conclusion; moreover, here there is a fourth term, the subsidiary term (which is in fact two terms with a common ground, in the special case of a crescendo argument).

As we have seen in earlier chapters (1-2) of the present work, despite the misleading suggestions of commonly used phraseology of a fortiori argument like "all the more," there is in fact no 'proportionality' intrinsic to such argument. The conclusion of a purely a fortiori argument logically must needs refer to exactly the same subsidiary term as the minor premise of it does; otherwise, the argument effectively contains five terms instead of four, and the principle of deduction (according to which no more information may be found in a deduced proposition than was explicit or logically implicit in the propositions it was deduced from) is breached. To draw a 'proportional' conclusion, an additional premise about 'proportionality' must be admitted. This is sometimes readily forthcoming, but not always.

Mielziner rightly perceives a fortiori argument as a sort of argument by analogy. He parenthetically mentions the use of analogous reasoning in modern jurisprudence, quoting the maxim[6]: "*Quod in minor valet, valebit in majori; et quod in majori non valet, nec valet in minori*" – meaning: "what avails in the less, will avail in the greater; and what will not avail in the greater, will not avail in the less." This maxim seems to be based on Cicero's statement in *Topica* 23, though the wording is more polished here. But though this external analogy is apropos and interesting, Mielziner does not remark on its evident lack of 'proportionality', i.e. on there being no difference of degree in "availing" mentioned in it.[7]

Mielziner goes on to give the example of *qal vachomer* in Numbers 12:14 concerning the isolation of Miriam[8] as the "Biblical prototype" of such argument, like the Gemara does (Baba Qama, 25a). But he does not immediately question the Gemara's interpretation of the argument, as prescribing a "fourteen days" penalty that is thereafter mitigated to "seven days" by means of the *dayo* principle (the principle of sufficiency formulated by the Sages in the corresponding Mishna). Only further on does he introduce the latter principle as a "restriction in the application of inferences," and further analyze the Miriam example. But here again, Mielziner does not show that he realizes that a fortiori argument is not intrinsically proportional. All he says is:

> "Thus, in the inference made in Scripture in regard to Miriam, we might have expected that the time of her exclusion from the camp should be more than seven days, since the Lord's disfavor is of more consequence than a human father's; nevertheless, Scripture says, 'Let her be shut out from the camp seven days,' which is just as long as she would have felt humiliated if her father had treated her with contumely. On this passage the restrictive rule just mentioned is founded" (p. 135).

Notice the claim: "we might have expected… more than seven days." Whence, it is clear that Mielziner buys the traditional superficial reading of the Gemara, lock stock and barrel, without much reflection on the subject. For him, then, *qal vachomer* is naturally proportional (as his initial description partly suggests) and it is only due to the restriction imposed by the rabbinical (and ultimately Scriptural, in the rabbis' view) *dayo* principle that the conclusion is kept the same in magnitude as the minor premise. Nevertheless, despite this naivety, Mielziner displays considerably more awareness of the structure of a fortiori argument than many other commentators, even later ones.

2. Structural analyses

He clearly analyzes the structure of a fortiori argument as consisting of three propositions. The "first premise" (what I have called the major premise) informs us that "two certain things, A and B, stand to each other in the relation of major and minor importance." It should be noted that many commentators tend to ignore this crucial information, and Mielziner is rather exceptional is mentioning it explicitly as a premise. However, he fails to mention *in what respect* the two terms A and B are differently "important" – that is to say, he does not identify the presence of a specific middle term.

The "second premise" (what I have called the minor premise) tells us that "with one of these two things (A) a certain restrictive or permissive law is connected." The conclusion is that "the same law is more applicable to the other thing (B)." Here, we can congratulate Mielziner on stating that the conclusion is "*the same* law" as given in the second premise, although his qualifying it as "*more* applicable" is, as already stated, inaccurate (though a common error). However, here again, there is no mention of the middle term, which is left tacit in both the minor premise and conclusion – and no mention that it is by virtue of having *enough of* this middle term that the law (i.e. the subsidiary term) is connected.

Note that although A and B are in the first premise described as the items of major and minor importance (apparently respectively), A is always placed in the second premise and B is always placed in the conclusion – so A and B should

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6   On p. 131, footnote 1. Mielziner gives as reference: "quoted by Coke on Littleton, 260." Concerning Cicero, see the section devoted to him in an earlier chapter of the present volume (6.5). Cicero calls this "argument by comparison;" but he obviously refers to a fortiori argument, since he distinguishes between moods that go from major to minor; from minor to major; and from equal to equal.

7   Moreover, notice, the Latin statement does not mention the threshold or sufficiency of the middle term that logically enables the inference.

8   Which we have already analyzed in detail in an earlier chapter, viz. 7.4).

not be viewed as always symbolizing the same (major or minor) term. This becomes clear when Mielziner goes on to describe the different possibilities (i.e. moods) of *qal vachomer* argument. He points out that it is "is usually expressed by two compound propositions, one of which is the *antecedent* and the other the *consequent*" (his italics), i.e. if the minor premise, then the conclusion, effectively; and he describes two directions of inference.

"Inference from minor to major" goes from a minor thing A being "subject to a certain severity of the law" (which may be, he says it explicitly in Hebrew, a prohibition or an imperative, i.e. a negative or positive severity) to a major thing B being "more subject to the same severity" (again that extraneous and misleading "more" wording!). "Inference from major to minor" goes from a major thing A being subject to "a certain allowance" (which may be, he again says it explicitly in Hebrew, a permission or an exemption, i.e. a negative or positive allowance) to a minor thing B being "more" (grrr!) subject to "the same allowance." What is good here is Mielziner's awareness that severity and allowance may each be positive or negative.

What is missing, however, is an explicit statement of the square of opposition between *chayav* (imperative), *assur* (forbidden), *mutar* (permitted) and *patur* (exempted), which would more fully clarify that the 'minor to major' and 'major to minor' moods Mielziner lists here are related to each other. "If A which is minor is forbidden (or imperative), then B which is major is forbidden (or imperative)" implies, by contraposition, that "If B which is major is *not* forbidden (or imperative), i.e. is permitted (or exempted), then A which is minor is *not* forbidden (or imperative), i.e. is permitted (or exempted)." The two moods are related, they are not two separate phenomena. No doubt Mielziner is tacitly aware of this to some extent, but he does not say it out loud.

More serious than this lack of a distinction between positive and negative moods, is the absence of a distinction between subjectal and predicatal moods, and another between copulative and implicational moods. The forms of a fortiori argument that Mielziner explicitly acknowledges are essentially all copulative, subjectal and positive. He is apparently not aware that the terms A and B may sometimes be predicates instead of subjects; that the propositions they are in may sometimes be negative instead of positive; and that the argument may sometimes be organized around theses instead of terms. Moreover, he does not make any attempt at validation of *qal vachomer* argument, but takes it as intuitively obvious.

To sum up, although Mielziner gives us a competent basic guide to *qal vachomer* reasoning in the Talmud, he does not proceed far beyond the traditional approach and systematically develop the field. What I have done here is only a bare sketch of Mielziner's presentation, for the purpose of determining where he stood with regard to *qal vachomer* proportionality and the *dayo* principle, and how far he went in formal analysis of the argument.

3. Concerning the jus talionis

Mielziner gives some interesting illustrations of *qal vachomer* use in the Talmud and related documents. One of these concerns the so-called *jus talionis* (the law of 'talion', or retaliation), the law given in Exodus 21:23-25: "life for life, eye for an eye, tooth for tooth, etc." As is well known, the rabbis interpreted this Torah law in an indulgent manner, prescribing monetary compensation rather than a literal application, at least for eyes and teeth (contrary to what detractors of Rabbinic Judaism have claimed, starting with the Sadducees[9]). To justify this ruling, the rabbis made use, among other arguments, of a *qal vachomer*; this was, as Mielziner puts it (p. 134):

> "Referring to the law (Exodus xxi. 29-30), by which, under certain circumstances, the proprietor of a beast which is notably dangerous and which has killed a person, is judged liable to the death penalty; but the capital punishment could be redeemed by money. Now, if the law expressly admits a pecuniary compensation in a case where the guilty person deserved capital punishment, how much the more is a pecuniary compensation admissible in our case where it does not concern capital punishment. (Mechilta to Exodus xxi. 24.)[10]

Mielziner rightly classified this reasoning as "inference from major to minor," as can be seen by recasting it in standard (positive predicatal) form. Notice how such reformatting clarifies an argument and makes manifest its validity:

> More severe penalties (middle term, R) are required for heavier offences (major term, P) than for

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[9]   In this respect, I read an interesting comment somewhere that prior to the appearance of the law of "an eye for an eye," people probably practiced "two eyes for an eye" or other such arbitrarily disproportionate revenge or punishment. The law of talion was thus originally not, as some contend, a law of harsh retaliation, but rather one of kindly mitigation of retaliation. It introduced concepts of 'rule of law' and 'just proportionality' into the penal system. And this is taking it literally, as it was presumably originally intended. The rabbis, perhaps much later, went further in this humane direction, by taking the principle metaphorically rather than literally, and introducing monetary compensation instead of (as the case may be) capital punishment, blinding eyes or breaking teeth.

[10]   See also *Baba Qama*, 83b-84a.

lighter ones (minor term, Q).

Redemption by pecuniary compensation (subsidiary term, S) is (explicitly according to the Torah) a penalty severe (R) enough in the case detailed in Ex. 21:29-30, which is in principle a capital offence (P).

All the more so, redemption by pecuniary compensation (S) is (implicitly, by way of this inference) a penalty severe (R) enough in the case detailed in Ex. 21:24-25, which is in principle an offence liable to loss of limb (Q).

Several things are worth remarking here. The first is that, though Mielziner did not realize this, the argument used here being positive has to be predicatal[11]; argument from major to minor can be subjectal only if negative in form. Secondly, the major premise of this argument is not mentioned in the Torah, but is intuitively obvious, or can be deemed an application of the general principle of justice known as 'measure for measure' (*midah keneged midah*). Indeed, the law of retaliation (eye for eye, etc.) might be viewed as a special case of this very principle. Its literal application would perhaps be strict justice; but an effort is being made here to mitigate it: it is too extreme, lacking in mercy. Still, paradoxically, the general principle is used as a premise in the argument designed to mitigate it.

Thirdly, the argument is obviously major to minor only in cases of "limb for limb" (Ex. 21:24-25) – but less obviously so in the case of "life for life" (Ex. 21:23)[12]. Is the latter statement intended as very general? It seems doubtful that it includes cases of intentional homicide, which are dealt with elsewhere. Rather it seems to concern *un*intentional homicide, which presumably does not deserve capital punishment. Perhaps it is even narrower in intent, and concerns only, as the context (viz. Ex. 21:22) suggests, the unintentional killing of an unborn baby; in such case one might well ask: is the court supposed to execute the culprit's baby? And what if he doesn't have a baby?[13] It is very doubtful that such drastic retaliation would be even considered. Thus, upon reflection, inferring monetary compensation in cases of "life for life" is also argument *a majori ad minus* (and not *a pari* as might superficially seem).

Fourthly, there is no mention of the *size* of the monetary compensation. There is no attempt at explicit proportionality in that respect; nor at a contrary application of the *dayo* (sufficiency) principle. At least, Mielziner does not mention the issue here, though it would seem very appropriate to do so. The amount of monetary compensation is obviously to be determined by the courts. Presumably, they would make it proportional, i.e. charge the culprit less for a limb than for a life; in other words, I presume, they would not apply the *dayo* principle in this case. Why so? Again, I answer the question presumptuously (i.e. I do not know what the rabbis actually say on this subject) – because the inference is from major to minor. If *dayo* were here literally applied, the monetary compensation for loss of limb would have to be equal to that for loss of life – which is intuitively absurd.

This example seems to confirm what I suggested in my treatment (in an earlier chapter, viz. 8.2-3) of the *dayo* principle, namely that it can only be reasonably applied in arguments from minor to major. That is to say, it is obviously intended as a principle of mitigation of penalty – not of aggravation of penalty. This example also seems to confirm my contention that any proportionality that is admitted is not due to an operation of the *qal vachomer* argument as such but to an external application of the principle of *midah keneged midah*. A fortiori argument cannot be manipulated, being a logical 'law of nature'. But the measure for measure principle certainly can be adapted to different circumstances; and indeed it ought to adapt, if justice is to be precisely served. The measure for measure principle should be regarded as a general guideline, a rough rule of thumb, a heuristic principle, rather than as a mathematical law to be applied indiscriminately.

Although, as already said, Mielziner briefly discusses the *dayo* principle, his definition of it as "the law transferred to B (the major), must never surpass in severity the original law in A (the minor), from which the inference was made" (p. 134) is focused on *qal vachomer* inference from minor to major; he does not ask the question how it might be applied to inference from major to minor. This may well be due to there being no such discussion in the Talmud and other rabbinic literature, either (I do not know if there is any).

Moreover, and most significantly, Mielziner's definition of the *dayo* principle corresponds to the first type of *dayo*, that used by the Mishna Sages in reaction to R. Tarfon's first argument. He makes no mention of the second type of *dayo*, used by the Mishna Sages in reaction to R. Tarfon's second argument, which is significantly different. Here too, he may be excused as having been misled by the Gemara, which also failed to notice the difference. However, since commentators later than the Gemara and before Mielziner did notice the difference, he has less excuse.

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[11]   That is, the major and minor terms in it, P and Q, are predicates.

[12]   One might propose an *a pari* a fortiori argument from pecuniary compensation in lieu of death in relation to Ex. 21:29-30 to same in relation to Ex. 21:23.

[13]   Similarly, of course: if he is blind, how can "eye for eye" be applied? If he is toothless, how can "tooth for tooth" be applied? And so on. The law would be difficult to apply literally in some cases, and therefore may well be taken as having been intended metaphorically.

4. Restrictions and refutations

Mielziner discusses three Talmudic "restrictions in the application of inferences" of *qal vachomer*. The first is the *dayo* principle: "It is sufficient that the result derived from an inference be equivalent to the law from which it is drawn." This, he informs us, is founded on the Scriptural inference in regard to Miriam (Numbers 12:14) and there is an ample application of it in Mishna Baba Kamma II. 5. He does not remark that the Scriptural source is identified in the Gemara commentary to that Mishna, and there given as a *baraita*. Also, as we just pointed out, he does not remark that the *dayo* principle has two distinct senses, corresponding to the two uses of it by the Sages in the Mishna in response to the two arguments by R. Tarfon.

The second restrictive rule is: "The inference from minor to major is not to be applied in the *penal* law." Mielziner explains it as follows: "The reason for this rule lies in the possibility that the conclusions drawn by the inference might have been erroneous, so that the infliction of a penalty derived from such a conclusion would not be justified." He points out that similar provisions can be found in modern legal statutes. He informs us that "an application of [this] rule is made in Talmud Maccoth 5b." He does not clarify the difference between this second rule and the first one, which also forbids application of proportional penalties based on inference. Presumably, though they overlap somewhat, they differ in scope to some extent[14].

The third restriction he mentions is: "No inferences must be made from traditional laws to establish a new law." This is "laid down in Mishna Yadaim III. 2." However, he tells us, R. Akiva did not accept this rule. By "traditional laws" is here meant laws not given in the written Torah but considered to have been orally transmitted since the Sinai revelation, presumably, as well as later legal developments. There are other restrictions relating to inference, but Mielziner does not mention them – at least not here.

Mielziner goes on to describe "refutation of inferences." As he puts it: "Not every kal ve-chomer offered in Talmudic discussions of the law is correct and valid. We sometimes find there very problematic and even sophistical inferences set forth merely as suppositions or hypotheses; these are, however, finally refuted." Such refutation is called *pirka* (meaning: objection). It can be done in one of two ways: either a premise or the conclusion may be challenged. More specifically, the antecedent item "A which was supposed to be of minor importance is in some other respects really of major importance;" or again "the peculiar law connected with A can not be transferred to B[,] as it is not transferred to [some third item] C, which is in certain respects like B."

He gives Talmudic examples of such refutations, namely: "Chullin 115b; Mechilta to Exodus xxiii. 19." for the former, and "Mishna Pesachim vi. 1, 2." for the latter. We shall examine these examples presently. Moreover, Mielziner teaches us: "When an inference has been refuted in one of the two ways just mentioned, the attempt is sometimes made to defend and retain it by removing the objection raised in the refutation. If the arguments proffered for this purpose are found to be correct, the original inference is reinstated; if not, the refutation is sustained and the inference finally rejected." He gives an example of such discourse, too; but I will not here give further details on that.

The first question that pops into my mind in this context is: how is it possible for any of the rabbis involved in such debates to even momentarily propose an idea or argument that turns out to be wrong? I (and indeed anyone with a very logical mind) would regard such discursive conflicts as concrete evidence from the Talmud itself that the rabbis were neither omniscient nor infallible. But neither traditional commentators, nor for that matter Mielziner (here), reflect on this larger implication. We could, of course, say that the refuted rabbi was playfully putting forward an argument he knew full well would be refuted by a colleague, so as to teach us (future onlookers) that this argument would not stand scrutiny. And indeed this may be true in some cases. But, surely, in some cases such argumentation and counter-argumentation is evidence that the thinking of the rabbis, collectively speaking, was inductive – i.e. ordinary human trial and error. In which event, the rabbis can hardly claim absolute authority for their dictates.

The second question to ask is: how does such refutation proceed *in more formal terms*?

Let us consider the first example given by Mielziner, illustrating "**refutation of a premise**." Regarding *orlah* fruits ("the fruits of a tree during its first three years"), one rabbi argues: "If those fruits, regarding which no law has been violated, are forbidden to be used in any way, ought not meat and milk, which, in violation of a law have been boiled together, the more forbidden to be used in any way?" Mielziner explains the counterargument as follows: "The premise in this inference is that *orlah* is of *minor* importance compared with meat and milk; but this premise is disputed [by another rabbi] by demonstrating that in certain respects it was, in fact, of major importance, since those fruits had at no time before been permitted to be used, while in regard to meat and milk there had been a time (namely, before being boiled together), when the use of each of these components was allowed."

What is the formal basis of this particular refutation? It is not exactly as Mielziner here suggests (and no doubt many before and after him) that "*orlah* fruit" is in one "respect" less important, and in another "respect" more important, than "meat and milk." Rather, I would say, the first rabbi claims that meat-and-milk is *always* more unlawful than

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orlah-fruit, and constructs his *qal vachomer* on that basis, whereas the second rabbi denies his colleague's major premise, i.e. demonstrates that it is *sometimes* not true, and so neutralizes the a fortiori reasoning. Their arguments look like this:

> First rabbi: (positive subjectal a fortiori argument)
> Meat-and-milk (P) is (always) more unlawful (R) than *orlah*-fruit (Q) is.
> So, since *orlah*-fruit (Q) is unlawful (R) enough to be forbidden to use (S),
> it follows that meat-and-milk (P) is unlawful (R) enough to be forbidden to use (S).

> Second rabbi: (denies major premise of preceding)
> Meat-and-milk is sometimes (before mixing) *not* more unlawful than *orlah*-fruit.
> So, even though *orlah*-fruit is unlawful enough to be forbidden to use,
> it does not follow that meat-and-milk is unlawful enough to be forbidden to use.

Note that whereas the former tries to prove that meat-and-milk should be forbidden to use, the latter does not try (at least not here) to prove the contradictory conclusion but is content to block his colleague's attempted proof. The effective middle term, say "degree of unlawfulness," does not change from one argument to the next; so there is not really a change of "importance" in different "respects." What changes, rather, in this example at least, is *the modality* of the proposed major premise, which goes from (tacitly) "always" to (explicitly) "only sometimes." This modality may be viewed as temporal, i.e. as relating to the life cycle of the conjunctive term "meat and milk." Or it may be viewed as extensional, i.e. as distinguishing two physical kinds of "meat and milk" conjunctions – namely "meat and milk *separately*" and "meat and milk *together*."

The upshot of this analysis is that the language usually used to describe or explain such refutation is misleading. The impression given is that the initial a fortiori argument is rebutted by an equally cogent or more cogent a fortiori argument (much as one dilemma may be checkmated by another). But this is not the actual scenario in cases of this sort. When the critic shows that the relative importance of the two terms in question can in some circumstances be reversed, he is not proposing a counteracting a fortiori argument – he is simply discrediting the major premise of the initial argument. Note, finally, that the minor premise is not attacked in the above example; but, clearly, this would be an equally viable alternative approach to refutation.

Let us now examine the second example given by Mielziner, illustrating "**refutation of a conclusion**" (p. 138). It concerns a Passover festival that happens to fall on the eve of a Sabbath, so that the laws of both are somewhat in conflict. R. Eliezer argues: "If slaughtering [the paschal lamb], though a real labor, abrogates the Sabbath, ought not things not regarded as real labor the more abrogate the Sabbath?" To which R. Joshua replies: "A common holiday proves that this conclusion is not admissible, for on such a day some real labors (as cooking, baking, etc.) are permitted, while at the same time certain actions, which fall under the category of [things not regarded as real labor], are positively forbidden."

The phrase 'real labor' used here refers to *melachah*, while 'not regarded as real labor' refers to *shevut* (these are technical terms in Hebrew for these two categories of action). The expression "abrogates the Sabbath" means that some law normally applicable on the Sabbath is not applicable on the Sabbath under consideration; which means that something normally forbidden is here permitted. The Sabbath discussed here is that coinciding with the eve of Passover.

The arguments can be presented more formally as follows:

> First rabbi: (positive subjectal a fortiori argument)
> *Shevut* (P) is generally more lawful (R) than *melachah* (Q).
> So, since some *melachah* (such as slaughtering the paschal lamb) (Q) are lawful (R) enough be permitted on the Sabbath [coinciding with Pessach eve] (S),
> it follows that [all] *shevut* (such as carrying the lamb to the temple) (P) are lawful (R) enough to be permitted on the Sabbath [coinciding with Pessach eve] (S).

> Second rabbi: (denies conclusion of preceding)
> Even though *shevut* is in principle more lawful than *melachah*,
> and indeed, some *melachah* (such as cooking, baking, etc.) are [lawful enough to be] permitted on a Festival,
> nevertheless, some *shevut* (such as certain private affairs) are *not* [lawful enough to be] permitted on a Festival.

Let us now analyze these arguments more closely. The first rabbi's argument is clearly intended to be a fortiori, moving from minor to major. The major and minor premises must be as above shown, since the desired conclusion is permission of *shevut* on the specified Sabbaths. The middle term "lawful" is chosen by me offhand to fit the bill; it is possible that the rabbi who formulated the argument had another, better one in mind. The second rabbi's argument is not intended as a rival a fortiori, but as a critique of the first rabbi's a fortiori. While he apparently admits the former's major and minor premise, he denies *the generality* of his conclusion. That is, he points out that, while some *melachah* (activities forbidden on a Sabbath) are permitted on a Festival, not all *shevut* are permissible on a Festival (and therefore, a fortiori, not all *shevut* are permissible on a Sabbath coinciding with a Festival).

From which it follows that the major premise and/or the minor premise must be wrong in some respect(s). The second rabbi's statement that some *melachah* are permitted on a Festival seems to be intended as an acknowledgment of the first's minor premise, even though logically that some *melachah* are permitted on a Festival does not imply that they would be permitted on a Festival coinciding with a Sabbath. We can suppose that "Festival" here refers to "Festival coinciding with a Sabbath," since he does not explicitly claim that no *melachah* is permitted on a Festival coinciding with a Sabbath. The second rabbi's objection seems rather to be directed at the first's major premise. That is, his purpose must be to deny that *shevut* is *generally* more lawful than *melachah*, even if he grants that it is *in principle* (i.e. usually) more so. It is rather by this means that he inhibits the first rabbi's conclusion.

Thus, while the first rabbi's conclusion seems to concern all *shevut*, the second rabbi argues that it can only concern some *shevut*, and therefore by implication the former's major premise must also be less general than it seems. For the first rabbi "*Shevut* is more lawful than *melachah*" is literally a general proposition, whereas for the second rabbi it is only true 'in principle' since exceptions to it exist. In other words, it is not entirely accurate to describe this case as one of "refutation of the conclusion." It is true that the first rabbi's conclusion is contradicted by the second rabbi, since the latter puts forward a proposition that conflicts with it. But by doing that the latter is effectively also mitigating the major premise that gave rise to it, and thereby blocking the whole argument.

Thus, in the last analysis, what is touted as 'refutation of a conclusion' is really no different from 'refutation of a premise'. Logically, it is impossible to refute a conclusion if we admit all the premises and, of course, the validity of the deductive process used[15]. What is here (in the above example, least) called 'refutation of a conclusion' is *denial* of a conclusion coupled with denial of the generality of the major premise. We could say that the denial of the conclusion is the means through which the generality of the major premise is denied; and in that sense the denial of the conclusion comes first. In other words, what is involved here is indirect attack on a premise (or both of them), *through* an attack on the conclusion. The latter clarifies why this process can reasonably be labeled "refutation of a conclusion."[16]

Lastly, Mielziner draws attention to "some sophistical inferences of kal ve-chomer mentioned in the Talmudic literature, which are simply refuted by an argument *ad absurdum*," giving a number of illustrations. The example I found intriguing and amusing was that from *Derech Eretz Rabba*, chapter I. A layman "tried, in the presence of R. Gamaliel, to ridicule the application of inferences in ritual laws by the following paralogism:

> 'If the marriage with one's own daughter is prohibited, although the marriage with her mother is permitted, how much more unlawful must it be to marry another married woman's daughter, since the marriage with her mother, a married woman, is positively prohibited?'"

To which Mielziner comments: "the fallacy in this inference is that the conclusion contradicts the premise… But R. Gamaliel answered caustically…" (p. 140). Mielziner is of course right to point out that premise and conclusion were at odds, but I would like to see more precisely how the conclusion was arrived at. The answer has to be that an excessive generalization is involved. We are given that: (1) marriage with one's wife [one's daughter's mother] is permitted, (2) marriage with one's wife's [one's own] daughter is prohibited, and (3) marriage with another man's wife [a married woman] is prohibited; and the putative conclusion is: (4) marriage with another man's wife's [a married woman's] daughter is prohibited. This argument can be put in *qal vachomer* form, as follows:

> Marriage with a married woman's daughter (P) is more unlawful (R) than marriage with the married woman herself (Q).
> Marriage with another man's wife (Q) is unlawful (R) enough to be prohibited (S).

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[15]   Sometimes, it is the validity of the deductive process, rather than either premise, which is open to criticism. A case in point is in the Mishna Zebahim 7:4, where R. Yehoshua attacks R. Eliezer's a fortiori argument by pointing out that the subsidiary term is not really as uniform as it seems in minor premise and conclusion.

[16]   It should be noted that the dispute in question does not end where I ended it, but is pursued further with the first rabbi defending his position against the second, and then the second rabbi being in turn defended by a third (R. Akiba), and the rabbis finally opting for rejection of the first rabbi's conclusion. But these additional details do not impinge on what is being said here.

> Therefore, all the more, marriage with another man's wife's daughter (P) is unlawful (R) enough to be prohibited (S).

Clearly, the major premise of this argument is *generalized* from givens (1) and (2), i.e. from *one's own* wife and daughter to *all* wives and daughters. The a fortiori argument itself is formally valid; i.e. its conclusion does logically follow from its premises. But its conclusion is, as Mielziner points out, an absurdity, since it effectively implies that all marriage is unlawful, and therefore that even one's own wife, who was naturally another man's wife's daughter, is prohibited, even though we are initially given that it is permitted[17]. It follows that one of the premises must be false. It cannot be the minor premise, which is a Scriptural given. Therefore, it must be the major premise. What is wrong with it? Obviously, it was an overgeneralization from the given information. The prohibition to marry one's daughter is a special case, which cannot be generalized to all people's daughters without leading to absurdity.

To sum up, then. What is "sophistical" about this example (at least) is simply the false impression it gives of veracity due to its outward form being perfectly logical. But the validity of the explicitly claimed deduction conceals an invalid *previous* act of induction, which is not explicitly admitted. In other words, the proposed argument can readily be refuted by attacking its unstated major premise. So in fact, this refutation is formally not much different from the one used in an earlier example (concerning meat and milk and *orlah* fruit). This example again shows that if we know logic well, and we analyze particular cases carefully, we can never be fooled by any sophistry.

In my view, the epithet of sophistry is best reserved for *non-sequiturs*, i.e. arguments whose conclusions do not follow from their premises. In the case of a fortiori, this would refer a positive subjectal or negative predicatal argument from major to minor, or to a positive predicatal or negative subjectal argument from minor to major; and similarly, of course, with regard to implicational moods. Whether such cases actually occur in the Talmud, I cannot say; offhand, I would suspect they occasionally do. Thus, to refute an a fortiori argument, we may deny either or both of its premises, either directly or by attacking the conclusion, or we may deny that the conclusion logically follows from the premises.

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[17]   Maccoby suggests in this context that all marriage would be forbidden *except* "to the daughters of unmarried [i.e. single] mothers, widows or divorced women." I am not sure whether this exception is true; I always assumed that, in Jewish law, having sex with a currently unmarried (Jewish) woman is tantamount to marrying her. But in any case, the argument must be revised to precisely adapt it to this interesting issue; I leave the job to others.

14. Adolf Schwarz

The equation or assimilation of a fortiori argument with syllogism may be much older, but it is often – or at least, usually in literature on Judaic logic – attributed to Adolf Schwarz[1] with reference to his work *Der Hermeneutische Syllogismus in der Talmudischen Litteratur* (published in Vienna in 1901, by Israelitisch-Theologischen Lehranstalt). This was of course written in German. However, the work has been translated into Hebrew by Mikhal Berkovitsh under the title: *Midat ḳal ṿa-ḥomer ba-sifrut ha-talmudit: pereḳ be-toldot ḥokhmat ha-higayon be-artsot ha-ḳedem* (published in Cracow in 1905, by Y. Fisher)[2]. Unfortunately, the work has apparently to date not been translated into English, so I have not read it (and I suspect many people who mention him nowadays have not, either)[3].

Based on other readings (see chapter 9.9), it seems safe to say that the identification of a fortiori argument with syllogism occurred much earlier than the early 20th cent., maybe in the middle ages or perhaps even in ancient times. This being the case, it may be that Schwarz's originality lies in the degree of attention and conviction he gave to this thesis. However, not even having read Schwarz's work, I cannot say anything on these historical matters for sure[4].

But I did happily find an article written (in French, in 1897) by Lajos Blau[5], a contemporary of Schwarz's, about another work of his, *Die hermeneutische Analogie in der talmudischen Litteratur*, focused on the hermeneutic rule of *gezerah shavah*. Judging by this article, this earlier work unfortunately says little or nothing about the rule of *qal vachomer*; nevertheless, Schwarz appears in it as critical and innovative, a powerful researcher and sagacious commentator, so I would not hastily discount his opinions.[6]

1. Equation to syllogism

Many logicians and commentators, still today, identify a fortiori argument with syllogism. For instance, the Oxford Dictionary of Philosophy gives the following alleged example of a fortiori argument: *if all donkeys bray then a fortiori all young donkeys bray*. That not everyone has this opinion may be seen in the Internet site of the Philosophical Dictionary, which gives the following example: *Frank can't run to the store in less than five minutes, and the restaurant is several blocks further away than the store. Thus, a fortiori*, *Frank can't run to the restaurant in less than five minutes.* Needless to say, I consider the first example inappropriate, and the second correct.

As regards Schwarz, not having read his work or seen a detailed description of it, I cannot actually quote him or say for sure what motivated the opinion attributed to him, viz. the equation of a fortiori argument to syllogism. If that was truly his opinion, I don't suppose he thoroughly understood a fortiori. Rather, supposedly, he reasoned by an analogy of sorts: vaguely aware of the quantitative undercurrents in both a fortiori argument and syllogism, he equated the two offhand. It may be that Schwarz was influenced by Jewish writers of the medieval era, who opted for this viewpoint due the dominant position of Aristotelian syllogism in the logic of their time. But without seeing Schwarz's actual text it is of course impossible to say who (if anyone) influenced him. Maybe he frankly tells us who.

However, we can safely say that Schwarz did *not* arrive at this opinion following Aristotle. We have already in an earlier section (6.4) looked into the question of the latter's opinion on this issue, and found that we could not be sure

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[1]   Hungary-Austria, 1846-1931. He wrote many works on various aspects of Talmudic logic and methodology. His Hebrew name was Arieh (some of his books appear under this name).

[2]   This Hebrew version is available online at: openlibrary.org/works/OL15863565W/Midat_%E1%B8%B3al_%E1%B9%BFa-%E1%B8%A5omer_ba-sifrut_ha-talmudit. Wiseman mentions what might be yet another work on this subject in his bibliography: *Midat Kal Vachomer*, published in Cracow in 1914/5, unless this is just a later edition of same.

[3]   I take this opportunity to loudly recommend the translation into English and publication of Schwarz's relevant works. They are evidently of permanent interest and value to the field of Talmudic studies.

[4]   However, see Jacobs' comments on this issue, described further down.

[5]   Hungary, 1861–1936.

[6]   In addition to the said books on "Syllogism" (i.e. presumably, a fortiori argument) and "Analogy" (i.e. gezerah shavah, etc.), he produced four similarly named books: on "Induction" (i.e. presumably, *binyan av*) in 1909, on "Antinomy" in 1913, on "Quantitative Relations" in 1916, and on "Context" in 1921, as well as a number of other works. See list here: openlibrary.org/search?q=adolf+schwarz, where some works are freely available in e-book form (in German and Hebrew).

whether or not Aristotle formally equated these two kinds of argument to any extent. Some might contend that he (Aristotle) regarded syllogism as the essence of all rational argument; but I think an overall consideration of his work would suggest he did not have such a sweeping prejudice. The fact of the matter is that, although he studied syllogism in great detail, he only (so far as we know) glossed over a fortiori argument (even if he did so with considerable accuracy), and so could not have developed an informed opinion either way.

2. Jacobs' critique

Louis Jacobs, in his interesting work *Studies in Talmudic Logic and Methodology* (1961), reports the following concerning Schwarz (p. 3):

"Adolf Schwarz, in his well-known work, *Der Hermeneutische Syllogismus in der Talmudischen Litteratur* [(1901)], suggests in the title, and develops in the work itself, the idea that the Talmudic hermeneutic mode of *qal wa-homer*[7] is identical with the Aristotelean Syllogism. It will be shown here that not only is there no connection between the two forms of reasoning but that an analogy to the Syllogism is found in Talmudic literature [*ha-kol*] as something quite different from the *qal wa-homer*. The refutation of Schwarz's view is important because all too many scholars uncritically follow Schwarz in his identification."

Two items stand out, here. According to Jacobs, Schwarz considered a fortiori argument as "identical with" syllogism. If this is true, we have to repudiate Schwarz's theory, for we have definitely established, on formal grounds, that these two types of reasoning cannot be equated. However, we cannot endorse Jacobs' own contention that there is "no connection between the two," since we have found that syllogism and a fortiori argument can to some extent be correlated in both directions. Jacobs' rejection of Schwarz's alleged thesis is correct; but the justification for such rejection and the alternative theory that he offers are incorrect. We shall further on show more precisely why when we examine Jacobs' own theory of a fortiori argument; but the following can already be noted.

Although Jacobs begins his exposé by categorically stating that there is "no connection between the two forms of reasoning," he further on says: "of the two types of *qal wa-homer* it is the simple one which has affinities with the Syllogism" – so his rejection of Schwarz's equation is not as thorough as it first appears. It is only, according to him (by implication), the complex version of a fortiori argument that is thoroughly distinct from syllogism.

Jacobs goes on to argue that the fact that "the Pentateuch and other parts of the Bible in which this argument [i.e. the simple form of a fortiori argument] appears can hardly have been influenced by Aristotelian logic," in contrast to the complex form, which being a later development (according to him) might well have been so influenced, "is in itself a valid refutation of Schwarz's view" [that a fortiori argument is "identical with" syllogism]. However, I would not agree with Jacobs that the Pentateuch contains no instance of what he calls "complex" a fortiori argument; a case in point would be Deuteronomy, 31:27. So that part of his critique is useless.

Furthermore, I doubt that Schwarz ever said or had in mind that Aristotle affected reasoning within the Bible! But this attempted refutation of Schwarz by Jacobs is anyway absurd in my opinion. If the question we are asking is purely logical (i.e. can a fortiori argument be formally identified with syllogism?), then the answer is not affected by temporal issues. Of course, if we only think in historical terms, it is obvious that since Aristotle came after the Bible in time he cannot have affected it; but since he came before the Talmud he might conceivably have affected that. It must have been Talmudic and rabbinic reasoning that Schwarz had in mind. It is conceivable that Greek logic in general (and not just Aristotle's syllogism) may have had *some* influence on the early rabbis' thinking, since they lived in the Greco-Roman world; but the issue is *how much* influence?[8]

Further on[9], Jacobs suggests that Schwarz was in this regard influenced by **Adolf Jellinek**[10]. The latter speculated that Aristotelian logic may have inspired the Talmudic hermeneutic principles of Hillel (via his teachers Shemaiah and Abtalion). Jacobs affirms that Jellinek's hypothesis "has since won wide acceptance;" but I beg to differ. (a) Although such influence may well have occurred indirectly by a process of cultural osmosis and in a very watered-down echo, I very much doubt that Hillel or his teachers actually ever studied Aristotelian logic. Had they done so, the impact of his teaching on their way of thinking would have been much more apparent. (b) We must also take into consideration the fact that Aristotle did not *invent* the syllogism – he only *discovered* it, i.e. he merely noticed that human beings used it in their thinking (and of course went on to analyze it). If syllogistic thinking is found in other cultures after Aristotle's discovery, it does not mean that such thinking is due to Aristotelian influence.

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[7] This is the Hebrew technical term for a fortiori argument. It means 'light and weighty' or 'easy and difficult' or 'minor and major'.
[8] Jacobs (p. 19, footnote 3) refers us to two books on this question: "S. Lieberman: 'Hellenism in Jewish Palestine' N.Y., pp. 47ff. and D. Daube: 'Rabbinic Methods of Interpretation and Hellenistic Rhetoric' in *HUCA*, Vol. XXII, pp. 239-264." These are dealt with in later chapters of the present volume (15 and 30.5).
[9] In chapter 2 of the same work.
[10] Moravia-Austria, 1821-1893.

Another argument that Jacobs attempts in refutation of Schwarz is that "an analogous mode to the Syllogism is used in the Talmudic literature [*ha-kol*], but it is not a *qal wa-homer*. This mode is found frequently when the Talmud attempts to show that a given case falls under the heading of a more general principle" (p. 7). But here again we can contend that showing that the Talmud has another tool (viz. the *ha-kol* formula) for subsuming particular cases under more general principles does not in itself demonstrate that a fortiori argument is not also essentially syllogistic. Even if we largely agree with Jacobs' conclusion that a fortiori argument is not syllogistic, we must say that his second attempt to refute Schwarz is not very convincing.

Certainly, cases of *ha-kol* cannot conceivably cover all Talmudic use of syllogistic reasoning. The *ha-kol* label is explicitly applied to certain significant cases; but most syllogistic reasoning in the Talmud remains implicit and unacknowledged. Syllogism is a movement of thought that is ubiquitous and unavoidable in human cognitive functioning on a conceptual level; no discourse is possible without it, like it or not. There is no need to have studied Aristotle in order to think syllogistically (though studying Aristotle may well improve one's syllogistic thinking). Syllogism was in use among human beings in all cultures long before Aristotle appeared on the scene and analyzed and formalized the argument in the 4th Century BCE.

3. Kunst's critique

Jacobs mentions (p. 6) another commentator, **Arnold Kunst[11]**, who allegedly "finally disposes of Schwarz' viewpoint" when he remarks that:

> "The mistake (in Schwarz's reasoning) is that Aristotelean Syllogism… is the relation of the species to the genus, both being nothing but names, whilst… *qal wa-homer* deal[s] with *sentences*."

Kunst is here apparently basing his alleged refutation of Schwarz on comparison between categorical syllogism (e.g. A is B and B is C, therefore A is C) and some here unspecified samples of a fortiori argument (probably e.g. If A is B, then all the more A is C), and saying that whereas the former deals with terms (A, B, C), the latter deals with propositions (A is B, A is C).

But from our point of view Kunst's objection is in error – he fails to take into account non-categorical forms of syllogism (e.g. A is B implies C is D and C is D implies E is F, therefore A is B implies E is F), which concern "sentences" (i.e. propositions). Furthermore, he has obviously not analyzed a fortiori arguments very carefully, for we could equally say of some of them (viz. the copulative) that they deal in "names" (i.e. terms, in contrast to others – viz. the implicational – which deal in propositions). So Kunst's critique of Schwarz is also based on the wrong reasons.

Jacobs, by the way, interprets Kunst's statement differently, as meaning that:

> "…in the Syllogism the inference concerns the relationship between genus and species; we are saying that since Socrates belongs to the class *man* then he must share the characteristics of that class. Whereas in the *qal wa-homer* inference we do not say that a weighty precept belongs to the class *light precepts*; it obviously does not. We say that what is true of light precepts is true of weighty precepts."

Jacobs here shows a better understanding of both forms of argument than Kunst. Earlier, too, he stresses that "the element of 'how much more so' is lacking in the Syllogism." However, to repeat, I will show further on that he does not fully understand a fortiori argument, because his approach to it is – though somewhat formal – not formal enough.

4. Wiseman on Schwarz

Allen Wiseman, it seems, has not (to date) actually read Schwarz, since he speaks of "how Schwarz might have seen the matter"[12]. He supposes that Schwarz argued that a fortiori argument was "a form of" the categorical syllogism, because the former "resembles" the latter. However, Wiseman is careful to avoid a definite judgment, saying (QC and CS are his abbreviations for *qal vachomer* and categorical syllogism): "Whether Schwarz's actual claim was that the QC can only be a CS is a matter for further study,"[13] adding: "I shall… only focus on it as a possible CS."

Wiseman places Schwarz's apparent position in a historical context, with reference to early 20th century issues of *Mind*, where there was an ongoing debate as to "whether one needs to assume a universal 'all' to

make the *a fortiori* valid" or alternatively "one can dispense with it (as in transitivity) and just accept the particular terms as sufficient." The matter was not finally resolved and just drops out of sight (by about 1920), perhaps because the new, quantificational predicate forms expanded what constituted logic. In particular, it changed the nature of the classical paradigm of logic, the categorical syllogism, to become just a part of the newer, more generalized understanding."[14]

As regards this discussion, I have shown in an earlier chapter (3.2) that a fortiori argument may either concern individuals or classes, and I have there detailed in formal terms the various ways the singular argument can be quantified. The issue is not very difficult to resolve, once the form of a fortiori argument is identified. The reason why the issue was hotly debated in *Mind* is simply that the argument had not yet been formalized. If as Wiseman suggests the debate petered out due to a change of paradigm in logic, it was only (I would say) because attentions were drawn to other topics. For classical logic, which refers to ordinary language, was in fact as capable as the new, symbolic logic, of resolving the issue. Indeed more so, since as it turns out modern logic was not the first to resolve it but classical logic was.

In any event, Wiseman suggests that "Schwarz realized that with the latent, universal premise, one could construct some valid QC's as CS's." But this of course only means that Schwarz may have realized that some correlations are possible between the two argument forms, without going so far as to claim that they can be equated. In this regard, I again refer the reader to my own work, earlier in the present volume, where all the ways the two argument forms can be formally correlated are analyzed. Thus, I do not think that Wiseman is right in relating Schwarz's presumed syllogistic posture with the said debate in *Mind* (all the more so since that debate occurred later[15]).

Note that Wiseman's own position is that the two arguments "relate only in part." Wiseman speculates that Schwarz may have taken his cue from mathematics[16], and reasoned by analogy from the hierarchy of natural numbers that "if a lower case on a continuum within a special category has an inherent feature, every like, higher case has it too." Apparently he sees that reasoning as syllogistic, since he adds "Since a CS is not valid with particular premises, one needs a universal." But in fact syllogism *is* possible with singular premises; for example: Joan's husband is the man who built this house, and Tom is Joan's husband, therefore Tom is the man who built this house[17]. So, if Schwarz's reasoning was as Wiseman has it, it was not accurate.

More interesting is Wiseman's suggestion, "in Schwarz's defense" that "if his standard was mathematical proof, perhaps he saw in the CS the strongest form of deductive logic possible for the a fortiori *at the time of the Rabbis*. Even if he was familiar with quantificational predicate relations, the Biblical writers and later Rabbis did not likely argue in those terms. To propose such an advanced grasp of logic might be anachronistic." Now, that is a good point. We should not forget that Schwarz's books on the subject were primarily concerned with rabbinical logic, and not logic in general. Many contemporary commentators fail to make this distinction, and think that by making *qal vachomer* comprehensible to themselves (by means of intricate symbolic logic) they explain how it was understood by the rabbis.

5. Why a fortiori is not syllogism

Finally, let me make the following clear.

We have in an earlier chapter called Comparisons and Correlations (5), investigated in great detail the exact relationship between a fortiori argument and syllogism. We saw there that there have historically been many viewpoints on this issue; we made theoretical enquiries into the possible correlations between any two forms of argument; we engaged in structural comparisons between the two forms here of interest to us; we looked into the formal possibilities of turning syllogisms into a fortiori arguments and, conversely, a fortiori arguments into syllogism, and even followed through with reiteration of such translations. Our conclusions, after this thorough research work, were the following:

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[14]   Wiseman, on p. 28, tells us of "two opposing views:" that of C. A. Mercier, "that one need not assume or start with a universal, but that a universal (or better, a general) expression for the *a fortiori* arises only *a posteriori* from examples," and the view of F. C. S. Schiller and others, "that universals are implicitly necessary or involved." He lists several articles in *Mind*, which are all dated between Oct 1915 and Apr 1919.
[15]   The articles in *Mind* mentioned by Wiseman are dated between 1915 and 1918, whereas Schwarz's works on a fortiori argument are dated 1901 and 1914.
[16]   He was, Wiseman tells us on p. 13, a mathematician.
[17]   The distinctive feature of such singular syllogism is that the middle term, as well as the minor term, is singular; the major term may be singular (as in this example) or plural (e.g. 'a great architect'). A singular term may be viewed as a class with only one member. Needless to say, a proposition involving two such terms is not a tautology, but an informative statement equating two such classes. We may not know initially that they correspond. For the same reason, syllogism involving such propositions is useful, and it is commonly used.

"Syllogism and a fortiori argument are very different movements of thought. They are structurally different, and each serves a different rational purpose; so they are not equivalent or interchangeable. Although they can be formally interrelated in various ways, they remain logically distinct and irreplaceable in important ways. Syllogism orders terms or theses by reference to the inclusions (or exclusions) between them, while a fortiori argument orders them by reference to the measures or degrees of some property they have (or do not have) in common. Neither function can be substituted for the other. We can (however awkwardly) reword one form of argument into the other; but such translations are not exactly transformations, because significant information cannot be passed on from one form to the other; therefore, neither form of argument can be fully reduced to the other. Thus, both forms are needed by reason to pursue its business; they are complementary instruments of reasoning."

With regard to Schwarz, we can consequently say the following, abstractly. If his thesis was indeed, as many seem to affirm, that a fortiori argument is identical to, or essentially the same as, or equivalent to, or entirely reducible to syllogism – his thesis was demonstrably wrong. If however, his thesis was softer, less extreme, claiming only a web of partial relationships between the two forms, there may have been some truth in it. I would need to see the actual text of his theory in English to judge exactly what his position was and assess it concretely.

15. Saul Lieberman

1. Hermogenes

Saul Lieberman (Belarus, 1898 – Israel, 1983), in a celebrated 1950 essay called "Rabbinic Interpretation of Scripture"[1], discusses, among other topics, the seven "norms of interpretation" expounded by Hillel and the thirteen enumerated by R. Ishmael, which were used for *halakhic* (i.e. legal) purposes. "Many modern scholars have investigated these rules in detail," he tells us, giving Adolf Schwarz as example, but none "has been able to discover definite Greek influence in them."

Lieberman then quotes the 12th century Karaite author **Judah Hadassi** (ben Elijah), who, in *Eshkol haKofer*[2], compared the thirteen rabbinic rules to "twelve norms" in the "rules and laws" of "the sages of Greece" called "*ergasias kai epicheiremata* [executions and arguments]." Lieberman identifies this as an indirect reference to Hermogenes[3], who counted six "arguments," namely "place, time, way (manner), person, cause, [and] fact;'" and six "executions," by which every argument is elaborated, viz. "from a parable [a possible illustration], from an example [an actual case], from something smaller, from something bigger, from something equal, [and] from something opposite."

What mainly interests us here is Lieberman's following comment (where the terms 'arguments' and executions' must, of course, be understood in their specific sense just mentioned):

> "The arguments have certainly nothing to do with the rabbinic rules; we therefore shall consider the executions only. A comparison between the executions and the thirteen hermeneutic rules of R. Ishmael will demonstrate that they have only the *kal va-homer* and the analogy in common" (p. 433).

In an endnote, Lieberman clarifies: "Of R. Ishmael's rules the Karaite cited here only the first two, the *kal va-homer* and the *gezerah shavah* (analogy), and added 'etc.' […] Perhaps Hadassi was struck by the *verbal* similarity of the executions with some of the norms contained in the so-called thirty-two hermeneutic rules of the *Aggadah*, which he reproduced in his book (58b). … But the similarity is only verbal, as can be seen from the instances given in the Hebrew source, and quoted by Hadassi himself, to illustrate the rules."

I would here like to make a few comments regarding the twelve categories listed by Hermogenes. The first six, called "arguments," apparently refer to the different means through which something can be specified or explained, which we today commonly list as: 'Who? What? Where? When? How? Why?' Clearly: who = person; what = fact; where = place; when = time; how = way; and why = cause. The next six, called "executions," seem to refer to various ways a thesis might be supported or justified: by adducing a hypothetical or actual instance, by comparisons or by contrast.[4]

As regards Hermogenes' list of six "arguments," one can easily see it overlaps somewhat with Aristotle's list of ten "categories" (substance, quantity, quality, relation, place, time, action, passion, position, and state). The latter is also known as "the predicables;" so the concept generating the two lists is more or less the same. Comparing the two more specifically, however, we see that though they explicitly have in common place and time, the other items are more difficult to equate. Surprisingly absent in Hermogenes' list is any mention of quantity. As for the other items in Aristotle's list, we might correlate them collectively with Hermogenes' "way (manner), person, cause, [and] fact," but this would be forcing things a bit, I think. This said in passing.

Returning to the six questions, 'Who? What? Where? When? How? Why?' – it should be evident that this is not an exhaustive listing. As already stated, notably lacking in it are questions of quantity, mainly 'How much?' and subsidiaries of it like 'How many? How frequently? What size? How far? How soon? etc.' The question of place 'Where?' may be supposed to include more specific ones like 'At what place? In what direction? From where? Through where? To where?' Similarly, the question of time 'When?' supposedly includes: 'At what time? Since

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[1]   Originally published in: *Hellenism in Jewish Palestine* (New York: Jewish Theological Seminary, 1950.)
[2]   P. 124b. Book written in Constantinople, starting in 1148. He proposed, in this work, sixty grammatical rules and eighty exegetical rules.
[3]   In his "*peri evreseos* (III.7)." Presumably this refers to Hermogenes of Tarsus (fl. 2nd century CE). It is interesting to note he is classified as a sophist.
[4]   Said in passing, in today's speech we would rather call the "arguments" questions and the "executions" arguments.

when? Till when?' The questions of causality 'Why? How?' also have many forms: 'By whose action? From what motive? For what purpose? By what means? Through what process? Under what conditions? etc.' To these we should perhaps add questions of consequence like 'With what effect?' The questions 'What? Who?' can be taken to include: 'What sort of thing/person is it? What constitutes it/him? What are its/his attributes? and so forth'. There are no doubt many more questions that can be asked.

As regards Lieberman's statement that the six "arguments" of Hermogenes "have certainly nothing to do with the rabbinic rules" – I wonder if this is accurate. For a start, no one can deny that the rabbis ask the questions "Who? What? Where? When? How? Why?" (and so on) on many occasions, even if they do not identify such questions with particular rules. So even if these "arguments" have nothing to do with rabbinic rules, they do naturally have much to do with rabbinic thinking. But moreover, there are surely aspects of these six questions (or seven, if we add that of quantity) scattered in different hermeneutic principles.

The principle of *binyan av* (third rule in R. Ishmael's list) is concerned with finding a factor underlying a law, so as to propose a similar law in another context having the same underlying factor; this is causal reasoning (why, how). The principles relating to inference from context, namely the exegetic methods known as *heqesh* and *semukhim* (regarded as part of rule No. 2) and those known as *meinyano* and *misofo* (twelfth rule), are based on observations of the relative locations of sentences in the proof-text (where, when). The principles relating to the scope and classification of terms, such as *miklal uphrat* and *miprat ukhlal* (fourth and fifth rules) among others, are concerned with identifying things under discussion (what, who). So a case can be made that contradicts Lieberman's sweeping statement[5].

It is also obvious to anyone who has studied logic and rhetoric that Hermogenes' list of six "executions" is far from exhaustive; there are many more ways a thesis might be supported or justified.

As regards Lieberman's claim that the said "executions" and the hermeneutic principles "have only the *kal va-homer* and the analogy in common" – I again wonder how accurate a statement that is. It is also here evident from their practice that the rabbis used them all. In their processes of research or proof, they used possible and actual examples, comparisons to smaller, greater or equal things, and contrasts to opposite things. Such use is inevitable, since these are commonplace human thought processes. Moreover, the rabbis used these processes consciously and had names for them, even if they did not include them all in their lists of hermeneutic principles. So here again, Lieberman's remark is a bit hasty and misleading.

Anyway, we gather from the above that both Hadassi and Lieberman equated the three items "from something smaller, from something bigger, from something equal," to a fortiori argument (*qal vachomer*) and argument by analogy (*gezerah shavah*)[6]. Let us take it for granted. We could well read "from something smaller" to refer to *a minori ad majus* (*miqal lechomer*) and "from something bigger" to refer to a *majori ad minus* (*michomer leqal*). As regards the expression "from something equal," it could be interpreted in two ways. It may well refer to analogy in a broad sense, if the word "equal" is intended to mean same or similar, in a more qualitative sense (i.e. as against different or dissimilar). But if "equal" is understood quantitatively, then "from something equal" would specifically refer to the *a pari* (egalitarian) form of a fortiori argument.

So, how we interpret the category "from something equal" depends on whether we mentally relate this term to the preceding two, quantitative terms (something smaller or bigger), or to the next, qualitative term (something opposite). Thus, it may be true that this item corresponds to the rabbinical rule of analogy or *gezerah shavah*; but then again it may not: it may conceivably just refer to the third (*a pari*) type of a fortiori argument or *qal vachomer*. Or maybe we should read it more vaguely as "equal quantitatively and/or qualitatively." Perhaps the original Greek is less equivocal. In any case, it is interesting to note in passing the apparent historical fact that a 2nd century Hellenistic rhetorician, Hermogenes, listed two (or three) variants of a fortiori argument.

As regards the lists of Hillel and R. Ishmael, we must remark that both just mention *qal vachomer*, without making distinctions as to direction (from minor to major, or major to minor, or equal to equal). Some commentators have taken the expression *qal vachomer* as literally referring only to inference from minor to major. This is obviously wrong, since the rabbis do reason every which way in practice (though most often in the positive subjectal mood); but on the other hand, there may be some truth in it, for it is not evident that the rabbis were from the first *consciously* aware of the distinction. Lieberman, for his part, implicitly equates this rabbinic rule to at least two items (viz. "from something smaller" and "from something bigger"), if not all three.

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[5]   Note that the questions 'why, how, where, when, what, who' that I have in this paragraph apposed in pairs next to the rules of causal, contextual and classificatory inference are only intended as rough indications., For instance, causal inference is most obviously conceptually related to why and how; but it is also true that some *binyan av* are constructed in relation to where, when, what or who. Similarly, contextual inference is not limited to where or when, and classificatory inference is not limited to what or who. Much depends on the angle of vision.
[6]   Lieberman claims the Greek translation of these two terms to be: *apo meizonos kai elattou* and *apo syncriseos pros ison* (p. 435).

Regarding the unidirectional term *qal vachomer*, I rather like the explanation put forward by **Lajos Blau**[7]. The latter suggested that the expression *qal vachomer* refers, not to the inference *process* 'from minor to major' (as commonly assumed), but to the two kinds of *conclusion* that may be reached, viz. either "*qal*" (with the minor term in the conclusion) or "*chomer*" (with the major term in the conclusion). If so, there was from the start no need for a 'from major to minor' alternative; it was always implicitly understood. That would mean the *vav* conjunction in *qal vachomer* means 'or' rather than 'and', and it is useless to switch the terms (i.e. *chomer veqal* would mean exactly the same thing). Blau's idea may well be effectively true, but I would want to see textual evidence that the rabbis were always fully aware of the two possible directions of meaning.

2. Influences on rabbis

Returning to Lieberman's article, we must now ask what the apparent points of similarity between Hermogenes' list and the rabbinical lists signify historically. Granting that they have a fortiori argument and argument by analogy in common, does that mean the former influenced the latter, or vice versa, or perhaps that both stem from a third source text? Of course not! Granting that these are essentially natural ways of thinking for humans in general, it is not surprising to find them in two different cultural contexts, and such coincidence is not necessarily indicative of causal connection. Or, perhaps more precisely, there is already a satisfactory causal explanation in the fact of the common possession by all humans of a rational faculty; so no additional connection can be assumed offhand, i.e. without adducing further evidence.

This, of course, does not exclude the possibility that the rabbis may indeed have learned a fortiori argument and argument by analogy, and other logical techniques, from Greek logic. But the reason I doubt this thesis in general is that the rabbis were not that well-informed as regards formal logic. They do not seem to have actually studied Aristotelian or later formal logic. At best, we can assume some degree of "cultural osmosis," since the rabbis living in Israel were, after all, part of the Hellenistic world (the Greek and Roman empires, over several centuries)[8]. On the other hand, let us not forget that the rabbis of Babylon were to a large extent (after the destruction of the Second Temple) outside that sphere of influence, in a more oriental cultural context[9]. The latter, to be sure, had itself been subject to Greek influence since Alexander's visit in the 4th century BCE; but it must have been considerably different anyway.

Lieberman asserts: "It goes without saying that any thinking person who was acquainted with Greek logic and who heard something of the nature of rabbinical exegesis of the Bible would be inclined to associate it in some way with the former." He does not, however, draw any definite conclusion as to causality. He quotes Eusebius, who mentioned "the logical style of the Hebrew philosophers" (*Praep. Ev.* 513a), but he regards him as noncommittal, admitting that "his words only suggest that the Jews had their [own] system of logic." Lieberman's conclusion is simply: "We can safely assert that the Jews possessed their rules of logic for the interpretation of the Bible in the second half of the first century B.C.E." (p. 434). I thought we knew that already – well, this independently confirms what we knew.

In his 1949 paper "Rabbinic Methods of Interpretation and Hellenistic Rhetoric," David Daube[10] boldly claimed, with reference to Hillel's hermeneutic rules, that "rabbinic methods of interpretation derive from Hellenistic rhetoric." According to Gary Porton, "Daube was able to show that… the rabbinic program of biblical exegesis could be viewed as part of a larger cultural phenomenon in which Jewish and non-Jewish scholars of ancient texts participated. Daube bolstered this perception by arguing that the seven exegetical principles attributed to Hillel found parallels, sometimes verbal parallels, in the Hellenistic rhetorical tradition."

It seems clear that Lieberman's posture is more cautious than Daube's. Porton also wrote, concerning Lieberman's essay here reviewed, that his "main point was that while the rabbis did not borrow their exegetical rules from the Greeks, they most likely did borrow the 'formulation, terms, categories and systematization' of these rules which the Greeks developed. Throughout the essay Lieberman finds the Hebrew equivalents of a number of Greek terms and phrases. Thus, by 1950 it became widely accepted that rabbinic exegetical techniques placed the rabbis squarely

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[7]   See: "*Die hermeneutische Analogie in der talmudischen Litteratur* par Adolf Schwarz" (Vienna, 1897), in: *Revue des Études Juives*, Tome 36e. Paris: Durlacher, 1898. Pp. 150-159.

[8]   R. Ishmael, who authored the list of 13 hermeneutic principles, sojourned as a captive in Rome (or thereabouts) for a while (how long is not said) in his youth, until he was ransomed by Yehoshua ben Chananiah and brought back to Israel. Some rabbis sojourned in Alexandria, which was of course more subject to Hellenism than the Holy Land. For instance, the Tanna R. Yochanan Hasandlar (3rd century CE), a disciple of R. Akiva. The mere fact that some Greek words appear in the Mishna and Talmud of the Land of Israel is indicative of some influence. Nevertheless, the fact that the rabbis strongly despised Hellenism could well have totally prevented significant influence.

[9]   Including not only strictly Babylonian influences, but later on (when Babylon came under Persian hegemony) also Persian influences. Maybe there were even, through the latter, trickles of influence from further east, i.e. from India. Note that the Babylonian Talmud contains some words of Babylonian and Persian origin. We can also point to interest in angels, demons, sorcery, etc. as obviously influenced by regional beliefs.

[10]   Regarding comments by Daube specifically on *qal vachomer*, see the section devoted to him in a later chapter (30.5).

within the framework of Hellenistic culture."[11] In view of our above reflections, I think that Porton's conclusion is a bit exaggerated.

Lieberman does confirm that many of the rhetorical devices used by the rabbis for Biblical exegesis were also in common use in the surrounding non-Jewish cultures. He cites, for example, the "rearrangement of a verse" (or change of position of a word), so as to resolve a difficulty of interpretation; this is called *anastrophe* in Greek hermeneutics and *seres* (inversion) in Hebrew (pp. 438-9). Of course, the rabbis' interpretations were not limited to such devices. Lieberman shows, for instance, the distinctive and particularly sound philological practices of the rabbis in interpreting "rare and difficult terms," by referring to their more explicit occurrences in other Biblical contexts or to similar terms in other languages (pp. 430-1).

The more artificial techniques were especially used in *aggadic* (non-legal) contexts[12] (pp. 440-5). Among these Lieberman lists: "*mashal*, i.e. parable or allegory or symbol; paronomasia, amphiboly, i.e. playing with homonymous roots; *gematria*, *isopsepha*, i.e. computation of the numeric value of letters; substitution of letters, the so called *Athbash* alphabet; and *notaricon*," the interpretation of a word's letters or parts as standing for other words.[13] What is interesting is that, according to Lieberman, such devices were commonly used in the interpretation of dreams and oracles! They certainly added drama and mystery. He explains: "The cleverer the trick, the deeper the impression on the inquirer;" and cites examples from the *Onirocriticon* of Artemidorus, as well as rabbinic literature.

Such methods, he insists, "were invented neither by the Jews nor by the Greeks. They go back to hoary antiquity." And he very convincingly concludes: "Had the Rabbis themselves invented these rules in their interpretations, the 'supports' from the Bible would be ineffective and strange to the public." Whereas, as "instruments accepted all over the civilized world of that time," they would be "understood and appreciated by their contemporaries." There is much truth in that – though I wager that there were some logicians and philosophers, and possibly rabbis, who were skeptical about them anyway.

Furthermore, it must be said that most people tend to accept arguments that merely seem logical, without making any great effort to verify their validity. Sophists make a living exploiting such intellectual laziness, after all. So the mere fact of acceptance does not prove there was a sense of familiarity. Such methods may well have come down to the Jews and Greeks from their primitive common forebears, but it is not their antiquity which made them palatable. It was rather their power to mystify listeners and trick them into belief. The prestige of authority or social prominence must also not be discounted. Just as today when a university professor tells people something they do not fully understand they believe him anyway, so in those days the words of religious mavens were taken for granted uncritically.

3. Reassessment

After writing the above initial assessment, I found I still had some nagging doubt that the expression "from something smaller, from something bigger, from something equal" truly points to *qal vachomer* (and, according to Hadassi and Lieberman, to *gezerah shavah*). Why would Hermogenes have concentrated so much of his attention on a fortiori discourse, which was relatively little used in non-Jewish literature? Lieberman gives no examples or explanations from Hermogenes' work that might shed light on this question, or even definitely confirm that he had a fortiori reasoning in mind. As a rule, experience shows, it is not sufficient to look at abstract lists – or even definitions (which are usually imperfect) – to determine people's thoughts: we must look at concrete illustrations they give, to see whether they correctly conceptualized them.

Looking further into the matter, I did find a clue to Hermogenes' use of "from the greater;" he gives as an example of it: "Not even if someone gave ten thousand talents, nor even cities nor whole peoples, will I take in exchange for Demosthenes" (*On Invention*, III, 10). I would *not* look upon this example as a fortiori discourse! The speaker is just saying in a poetic manner that "*nothing* is sufficient to ransom Demosthenes." There is also, in the same place, an example of "from the smaller," which reads "She is in love with an image. If she loved some man [… or] you yourself, would I not put the blame on you?" Also an example of "from similarities": "For if she was not in love but had contracted some other disease in the body, would I not then too bring you to trial?" I cannot claim to understand

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[11]   These two quotations are from Gary G. Porton "Rabbinic Midrash" (p. 225) found online at: books.google.com/books?id=azrc9jU4YaUC&pg=PA270&dq=Saul+Lieberman+%E2%80%9CRabbinic+Interpretation+of+Scripture%E2%80%9D+in+Hellenism+in+Jewish+Palestine&hl=en&ei=5vxgTd6SM9H44Aba7rCkCg&sa=X&oi=book_result&ct=result&resnum=5&ved=0CDsQ6AEwBDgU#v=onepage&q&f=false.

[12]   Though they also sometimes appear in *halakhic* contexts, in the way of allusion, *zekher la-davar*, and support, *asmakhta* (p. 437).

[13]   Note that the Hebrew terms *gematria* and *notaricon* are obviously of Greek origin.

what these two examples are about, but they certainly do not look like a fortiori arguments to me! It follows that the said expressions need not always (if ever) refer to a fortiori discourse.**14**

Furthermore, I asked myself why Hermogenes had limited his list of "executions" to just the said six processes. Greek logic knew of a great many more ways of supporting or proving a thesis. To ask the most obvious question: where should syllogism, i.e. the basic mental process of classification, be placed in his list? In truth, Hermogenes dealt with much more than the "twelve arguments and executions" in his works. In my opinion, Lieberman should certainly have looked more deeply into them, and not limited himself to Hadassi's findings. In any case, Hermogenes' approach seems to be essentially that of a rhetorician, rather than a jurist. Although rhetoricians do show some concern with legal matters, their interest seems to be primarily in how to win litigations rather than in truth and justice. They write guidelines for orators and lawyers, rather than for lawmakers and judges. Their thinking is therefore more often sophistical than logical.

4. Cicero

Then it occurred to me: why fixate on Hermogenes? Were there not more important Hellenistic writers in the Talmudic period than him? Furthermore, why refer to such a late writer (2nd century CE)? Why not, for instance, rather focus on the earlier and more important **Cicero**? We can surely through the latter`s writings find many interesting parallels between Roman and rabbinic legal thinking. Marcus Tullius Cicero (106-46 BCE) was, after all, a contemporary of Hillel the Elder (110 BCE – 10 CE) to whom tradition attributes the first rabbinic list of hermeneutic principles. It is not unthinkable that Hillel heard of Cicero's work, even if he did not personally study it. There was at the time very probably some intellectual as well as other commerce between their countries. Although famous as an orator, Cicero was a thoughtful jurist, as his work *Topics***15** testifies. There he writes:

> "8. … Of the topics under which arguments are included, some are inherent in the very nature of the subject which is under discussion, and others are brought in from without. Inherent in the nature of the subject are arguments derived from the whole, from its parts, from its meaning, and from the things which are in some way closely connected with the subject which is being investigated. Arguments from external circumstances are those that are removed and widely separated from the subject….
> 11. Arguments are also drawn from circumstances closely connected with the subject which is under inquiry. But this class has many subdivisions. For we call some arguments 'conjugate,' others we derive from genus, species, similarity, difference, contraries, adjuncts, antecedents, consequents, contradictions, cause, effect, and comparison with events of greater, less or equal importance."

He then discusses in some detail all the arguments he lists, giving examples from Roman law practices. Just looking at this list of "topics," one can easily anticipate numerous parallels to the rabbinic *midot*, some of which may be close, while others are doubtless very loose. For instance, Cicero defines the term "conjugate" as referring to "arguments based on words of the same family" (§12); this seems to me very similar to the rabbinic *gezerah shavah* argument which proceeds by verbal analogy. Likewise, the topic "comparison with events of greater, less or equal importance" obviously (see §23 and 68-71) corresponds to a fortiori argument, i.e. the rabbinic *qal vachomer* rule**16**. However, we should note in the present context that there is no mention in Cicero's account, in whatever terminology, of the distinction between purely a fortiori argument and a crescendo argument. The issue of proportionality is simply not raised.

There is also no mention by him here of a restriction that could be equated to the rabbinical *dayo* (sufficiency) principle, which appears in Jewish legal discourse at the latest at the time R. Tarfon was active, i.e. roughly in 70-135 CE. However, Roman law seems to have had a very similar principle, although I do not know as of when it made its appearance. Therefore, I cannot say whether Cicero was aware of it, even if he does not mention it. If it existed in Roman law during his lifetime, it is safe to assume that he knew about it. The Roman law principle was: "*In poenis bensignior est interpretatio facienda,*" which means: in penalties, the more benign interpretation is to be applied.**17**

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14   Hermogenes is here discussing "hypodiaresis" – whatever that means. See p. 111 of the edition used. A copy of this page may be found online   at: books.google.com/books?id=1QPpgorkLKIC&printsec=frontcover&dq=hermogenes+inventions&hl=en&ei=eZR8TeioJon14Qa06OikBg&sa=X &oi=book_result&ct=result&resnum=1&ved=0CDcQ6AEwAA#v=onepage&q=%22from%20the%20smaller%22&f=false.

15   *Topica*. Trans. H. M. Hubble. Cambridge, Mass. Harvard UP, 1949. The full text of this book in Latin, with an English translation, may be read online at: www.scribd.com/doc/45159491/Cicero-Topica.

16   Note here that the "equal" definitely refers to the *a pari* variant of a fortiori argument, whereas there was a doubt with regard to that term's intent in Hermogenes.

17   This was found by Wiseman (p. 165). The reference he gives is: Digest of Justinian, no 49, in Albert Gautier, *Introduction to Roman Law for Studies in Canon Law*, (Rome: Faculty of Canon Law, St. Thomas University, 1994), page 154.

As regards the a fortiori argument, we should not look upon the contemporaneity of Hillel and Cicero as indicative of any significant influence of the one upon the other or vice versa. Hillel's awareness of the argument was without doubt due primarily to its use within the Tanakh; and Cicero's awareness was without doubt due to its use and discussion in Greek and Roman literature, notably in Aristotle's *Topics* and *Rhetoric*. Note moreover that the written Torah, however it is dated, certainly existed many centuries before the earliest Greek literature. It is conceivable that Hillel vaguely heard of Cicero or vice versa; but it is very doubtful that either of them greatly affected the thinking of the other; each existed in his own cultural bubble, subject to distinct intellectual roots. The fact that a fortiori argument arose in both cultures does not prove a causal connection between them, but may be explained as due to the common basic rationality of all human beings.

Looking beyond the a fortiori argument, we should not expect a great deal of parallelism between rabbinic and Greek and Roman legal hermeneutics, in view of a fundamental difference between the three legal systems. Rabbinic thinking is strongly tied to religious *proof-texts*, that is to say, to texts to which most if not all proofs must ultimately come down to, and to a lesser extent to traditions, common perceptions and conventions. The main proof-text is of course the Torah, i.e. the Five Books of Moses; next in importance comes the Nakh, the collection of Prophets and Writings; thirdly, come the Mishna and other authoritative sources of the same period; fourthly, rabbis must refer to the Gemara and other authoritative sources of the same period; fifth, to the early post-Talmudic rabbis, the Rishonim; sixth, to the later rabbis, the Acharonim[18].

So rabbinic thinking is essentially deductive in its orientation, even if it contains many inductive movements of thought. Or to put things another way: rabbinic thinking is indeed largely inductive rather than purely deductive in form (i.e. it involves generalization, adduction, etc.); but the term 'inductive' here should not be taken to connote empiricism in the strict sense, for the data base it refers to is not as in the case of scientific thought the full range of sensory and introspective material but mainly the information encapsulated in Jewish Scriptures and later Jewish commentaries thereon, much of which is far from empirical. Note that the reference to past religious literature and oral tradition is compulsory, and not merely voluntary.

In Greek and Roman law, on the other hand, while there is also some adherence to religious proof-texts and various traditions, this is not the defining feature. There is a far greater appeal to 'nature', as perceived by people of the time, i.e. to common-sense (which may, to be sure, include many unconscious prejudices, such as the belief in the superiority of the ruling classes over the plebeian class), and of course to arbitrary decisions by the rulers (in defense of their own interests). Such a context calls for more inductive, i.e. trial and error, forms of thought. Thus, rabbinic legal hermeneutics are mostly concerned with drawing information from the applicable proof-texts, i.e. interpreting them or using them for further expansion of the law. Whereas, Greek and Roman legal hermeneutics are basically concerned with the world at large, society and human nature, as they appeared to be in the eyes of the people concerned. This basic difference would explain many differences in the hermeneutics used.

Another preliminary observation we must make is that comparisons to Judaic hermeneutics should not be limited to explicit lists such as Hillel's or R. Ishmael's, but should range far and wide and consider all thought processes actually used by the rabbis. In this context, we might for instance refer to Louis Jacobs' brilliant essay "The Talmudic Argument" (1984), where he identifies nineteen "formal types or patterns" of Talmudic arguments (see also the corresponding footnotes, where he gives the Hebrew or Aramaic phrases that distinguish these arguments), namely:

> "Argument from authority; argument by comparison; argument by differentiation; either/or argument; on the contrary argument; acceptance of an argument in part; argument based on an opponent's position; argument exposing the flaws in an opponent's argument; argument based on historical or geographical conditions; argument based on the analysis of states of mind; readmission of an argument that has been previously rejected; argument against a statement of the obvious; argument to resolve a contradiction between sources; argument by textual emendation; argument from the principle of literary economy; different versions of an argument; argument presented by different teachers; consequences of different arguments; limited application of an argument."

I will not myself here even attempt to compare Cicero's presentation of the principles of legal reasoning in Roman jurisprudence to Hillel's and other lists concerning Jewish law – for two reasons. Firstly, such research is beyond the scope of the present work, which is essentially concerned specifically with a fortiori reasoning (and this we have already dealt with above). Secondly, because I think this job would best be done by a jurist, i.e. someone who would understand the legal issues involved better than a mere logician like me might. It is possible that someone has already done this job. There may be many parallels or there may not be. Either finding would be interesting. If there are many parallels, we can say that the two legal systems have some significant common epistemological roots even if

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[18]   The distinction between earlier and later authorities is made relative to the publication in the mid-16th cent. of the authoritative code of law called the *Shulchan Arukh.*

they eventually diverge in their concrete laws. If there are few parallels, we can conclude that they differ both in their approaches and in their results.

In any case, I think I have said enough here to show that Lieberman's approach here[19], based on Hadassi's earlier research, which focused specifically on Hermogenes, is not the most interesting route for us to follow. Certainly, Cicero would be a more fruitful subject of particular study. We have, for instances, found much clearer parallels in the latter's work to rabbinical *qal vachomer* and *gezerah shavah*. But ultimately, of course, all such relevant literary sources ought to be investigated, so that our conclusions are based on the full sweep of history, and perhaps a time line can be established for each item. Moreover, we should avoid drawing radical or sweeping conclusions, however entertaining they may be, and aim for scientific exactitude, however pedestrian it may seem.

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[19]   I am of course aware that Saul Lieberman was a high-level scholar. My concern here is with this one paper, not with all his works. It is possible that he wrote a lot more on this subject in other papers or books.

16. Louis Jacobs

Louis Jacobs[1] was an ordained orthodox Rabbi who was repudiated by his more orthodox colleagues for his view that the Torah is partly of human origin[2], and later founded the British Masorti (equivalent to the American Conservative) branch of Judaism. He wrote many thoughtful and interesting books on Judaism. Always knowledgeable, intelligent, honest and courageous, he tries in them to bring Jewish doctrine in phase with modern science, history and good sense. His theory of a fortiori reasoning appears in the first chapter of his *Studies in Talmudic Logic and Methodology*[3]. I will here detail and critically analyze it.

1. The simple and complex types

Let us go straight to the main issue in this work – Jacobs's attempt at formalization of a fortiori argument[4]. He distinguishes two types of *qal vachomer*:

> "*Simple*: If A has x then B certainly has x.
> *Complex*:   If A, which lacks y, has x, then B which has y certainly has x."

Jacobs' "simple" *qal vachomer* has the form: "**If A has x, then B certainly has x.**" He apparently considers all ten of the Biblical examples given in Genesis Rabbah 92:7 to fit into this form, rather than in the complex form. His simple form only refers to subjectal argument, since the subjects (A, B) differ, while the predicate (x) is the same, in the premise (A has x) and conclusion (B has x). The premise mentioned is of course the minor premise – there is no mention of the major premise. The subjects A, B can be identified as the minor and major terms (Q, P), respectively; and the predicate x as the subsidiary term (S). There is no mention of the middle term (R), which would be required in the major premise, and in the minor premise and conclusion. Put in standard form this argument would look like this:

> B (P) is more ? (R) than A (Q) is,
> and A (Q) is ? (R) enough to be x (S);
> therefore, B (P) is ? (R) enough to be x (S).

Jacobs' "complex" *qal vachomer* has the form: "**If A, which lacks y, has x, then B, which has y, certainly has x.**" His prime example of this type is a *baraita* found in Hullin 24a, viz.: "If priests (A) who are not disqualified for service in the Temple by age (lack y) are yet disqualified by bodily blemishes (have x), [then] the Levites (B) who are disqualified by age (have y) should certainly be disqualified by bodily blemishes (have x)"[5]. Here, an additional term is brought into play, viz. 'disqualification for Temple service due to age' (labeled "y"), which could serve as the middle term (R), if we consider y as referring to a range of values including zero. In that event, "lacks y" means 'y = 0' and "has y" means 'y > 0', so that B can be said to have (or be) 'more y' than A does (or is). We can then put this argument in standard form as follows:

> B (P) is more y (R) than A (Q) is,
> and A (Q) is y (R) enough (albeit no more than zero y) to be x (S);
> therefore, B (P) is y (R) enough (being more than zero y) to be x (S).

We see then that, logically, *the only difference* between the simple and complex arguments is that the former has an unstated middle term we have marked as '?' (R), whereas the latter has an explicit one labeled 'y' (R). In all other

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respects, the two arguments are formally identical. It follows that Jacobs' distinction is not one of logical form, but merely one of degree of explicitness. His forms are two discursive forms, i.e. two ways (among many more) that a fortiori argument may be formulated in practice. They are not logical forms, i.e. not formulas that reveal all the elements underlying and relevant to the discourse. They describe only the surface appearance of the discourse; they do not plunge into the deeper aspects needed for true understanding of it. Logically, the two forms are one and the same.

But Jacobs conceives these two arguments as more radically distinct, as his following statements (in pp. 3-4) make clear:

> "The simple *qal wa-homer* has a long history – its use is traced by the *Midrash* (*Gen R.* 92.7) to the Bible itself…. The complex *qal wa-homer* is of later Halakhic origin and is found in a *Baraitha* for example as follows (*Hull.*, 24a): 'If priests who are not disqualified for service in the Temple by age are yet disqualified by bodily blemishes the Levites who are disqualified by age should certainly be disqualified by bodily blemishes."

We see here that Jacobs regards all Biblical samples as falling under the category of simple a fortiori argument[6], and complex a fortiori argument as a later development in Judaic logic – as of Talmudic times. He repeats this conviction further on (p. 5): "The complex *qal wa-homer* is, as we have noted of late origin and appears to be an halakhic-methodological development from the simple one found in the Bible." There is some truth in this, insofar as it is difficult to find Biblical examples that are explicit enough to be readily fitted into Jacobs' complex formula; but it is not strictly correct.

Most Biblical examples leave out the middle term. For instance, in Exodus 6:12, Moses says: "Behold, the children of Israel have not hearkened unto me; then how shall Pharaoh hear me?" This means: if the Israelites, who have much faith, have not had enough of it to listen to Moses, then the chief of the Egyptians, who has far less faith (if any), will not have enough of it to do so. The middle term here is 'having faith', or something of the same sort by which we can explain the difference between the behavior of the Israelites and that of Pharaoh[7]. But this term is left tacit, presumably because it is obvious enough.

However, some Biblical examples, even among those listed in Genesis Rabbah (and thus known to Jacobs), do hint at the middle term. For instance, in Ezekiel 15:5, God says: "Behold, when it [the vine-tree] was whole, it was not meet for any work; how much less when the fire hath devoured it and it is burned, shall it then yet be meet for any work?" This means: if when whole the vine-tree was not meet (i.e. in good enough condition) for any work (i.e. to be useful); then now, when severely damaged, it is certainly not meet for any work. Here, note well, the intended middle term ("meet") is explicitly given.

Clearly, this argument can be cast in Jacobs' "complex" form, since the major, minor and subsidiary terms are also clearly given, being respectively "the whole vine tree," "the damaged vine tree," and "for work." However, we must change the polarities involved, because this argument happens to be negative rather than positive, and thus goes from the major (B) to the minor (A). The result is as follows: "If the vine tree when whole (B), which has good condition (has y), lacks utility (lacks x), then same when damaged (A), which lacks good condition (lacks y), certainly lacks utility (lacks x)."

It is evident from this example and should be stressed that Jacobs' formulae for *qal vachomer* did not take into consideration negative forms of the argument. These would obviously be as follows. The negative simple form would be: "If B lacks x, then A certainly lacks x;" and the negative complex form would be: If B, which has y, lacks x, then A, which lacks y, certainly lacks x." These arguments can, of course, be derived from the preceding by *reductio ad absurdum*. The example we have adduced for the complex type clearly fits into the latter form, without any artifice.

We have thus demonstrated, empirically, that Jacobs' claim that "the complex *qal wa-homer* is of later Halakhic origin" is not correct. This is not to deny that a fortiori arguments in later Jewish literature, notably in the Mishna and the Gemara, are not very often more complex in form than the Biblical ones. However, the fact remains that Jacobs' complex type is found in the Bible, as well as his simple type. He should have examined Biblical a fortiori argument more closely before making generalizations. Moreover, no doubt, the simple type is also found in Talmudic contexts.

Clearly, Jacobs is misleading himself and others when he says that the use of simple a fortiori argument "is traced by the *Midrash* to the Bible." All the Midrash does is point out ten cases of *qal vachomer* in the Bible, without referring

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[6]   This is further confirmed in a 1972 paper of Jacobs', viz. "The *Qal Va-Ḥomer* Argument in the Old Testament" (*Bulletin of the School of Oriental and African Studies*, 35:221-227. Cambridge University Press), where he, presumably speaking of all Biblical a fortiori, says: "The argument runs: if A is so then B must surely be so; if the 'minor' has this or that property then the 'major' must undoubtedly have it."

[7]   This is not, note well, something arbitrary. We must ask what tacitly differentiates the children of Israel and Pharaoh. The difference between them is obviously that the Israelites are spiritually close to Moses, whereas Pharaoh is not – so we would expect Pharaoh to ignore Moses' advice more readily than the Israelites do. Precisely how we word this difference is not important; but we must verbalize it.

to any distinction resembling Jacobs' between simple and complex forms. He makes the same error again when he says that the complex variety is "of later Halakhic origin and is found in a *Baraitha*," since that later source[8] too makes no mention of such distinction but refers to *qal vachomer* alone. Jacobs is unconsciously projecting his interpretation into the facts.

The underlying methodological error here is to confuse use of a form of argument with awareness and discussion of its use. A fortiori argument is *used* in the Bible (not only ten times, but at least thirty times according to my enumeration in *Judaic Logic*[9]) and in the much later Talmudic and post-Talmudic literature (umpteen times, yet to be enumerated). The Midrash shows *awareness* of this kind of argument by naming it and giving examples of its use, and the *baraita* shows *additional awareness* of it by listing it as a halakhic hermeneutic principle, although neither of these sources (or any other) theoretically describes or validates it. Such more logical work occurs much later in history.

None of these facts in themselves justify the proposed simple-complex distinction: it is not a traditional doctrine but Jacobs' interpolation. There is nothing wrong, of course, with making a new distinction – unless of course that distinction happens not to be accurate.

Missing forms. We have also shown that the two argument types Jacobs formulated, being positive, do not suffice to represent Biblical *qal vachomer*; he should also have formulated two corresponding negative forms (as we just did for him). Another criticism we must level against Jacobs' alleged formalization of a fortiori argument is that he fails to take into consideration *predicatal* argument. That is, he ignores arguments in which the minor premise and conclusion have the same subject but different predicates. His forms, "If A has x, then B certainly has x," and "If A, which lacks y, has x, then B, which has y certainly, has x," are inherently subjectal in orientation.

To be thorough, he should also have proposed the forms: "If x has B, then x certainly has A," and "If x has B, which requires y, then x certainly has A, which does not require y," in which x is the subject, and A, B are the predicates. Note that, here, we must refer to y being *required or not-required* (rather than to having or lacking y), due to the distinctive structure of predicatal argument. The negative equivalents of these two predicatal forms would therefore be: "If x lacks A, then x certainly lacks B," and "If x lacks A, which does not require y, then x certainly lacks B, which requires y."

Note that here, in the positive form of predicatal argument, the movement is from B to A (from major to minor), whereas in positive subjectal argument it is from A to B (from minor to major). Accordingly, in the negative form of predicatal argument, the movement would be from A to B, just as in negative subjectal argument it is from B to A. This is not a mere theoretical prediction. There are arguments of all four kinds (positive and negative subjectal and predicatal) in the Bible[10], and they must all be taken into account in any theory. As we saw earlier, the Ramchal was fully aware of these four forms back in 1741.

Also lacking in Jacobs' treatment is a consideration of *a crescendo* argument, i.e. 'proportional' a fortiori argument. His two (subjectal) forms are clearly purely a fortiori, since the predicate in them (x) remains constant. In subjectal a crescendo argument, the predicates in the minor premise and conclusion differ in magnitude (in accord with a ratio determined in an additional premise). In predicatal a crescendo argument, of course, it is the subjects in the minor premise and conclusion that differ in magnitude (again, as per a ratio specified in an additional premise).

There are several examples of a crescendo argument in the Tanakh, and several in the Mishna, and no doubt many more in the Gemara, Midrash and later rabbinic writings. No doubt it was due to misunderstanding of the *dayo* principle that Jacobs ignored a crescendo argument; the Mishna Baba Qama 2:5 seems to reject such argument. But the truth is that the Gemara Baba Qama 25a clearly accepts it, and Jacobs for some reason did not pay attention to that.

Furthermore, Jacobs' formulae only refer to copulative a fortiori argument and do not take into consideration *implicational* a fortiori argument. Some Biblical arguments, and many more arguments in subsequent Talmudic and rabbinic literature, are more precisely implicational in form. We might suppose that he viewed his two types as prototypes for all the missing forms we have enumerated; but there is no hint to that effect in his text: he seems to consider his treatment as exhaustive.

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[8]   Presumably he is referring to the Baraita of R. Ishmael given at the beginning of *Sifra*, which lists the thirteen rules of hermeneutics of R. Ishmael mostly derived from the seven rules of Hillel.
[9]   I was pleased to learn from Jacobs in his footnote 4 (p. 3) that "The commentaries state that far more than ten are to be found." He there refers us to H. Hirschensohn's *Berure HaMiddoth* and H.L. Strack's 'Introduction to the Talmud and Midrash'. More will be said on this issue in a later section of the present chapter (16.4).
[10]   See **Appendix 1** for a full list.

Jacobs' verbal formula for "complex" a fortiori argument bears comparison to the symbolic formula H. S. Hirschfeld proposed in 1840, viz. "A - a = A + x :: B + a = B + x"[11]. Jacobs may have been inspired by it, or may have produced his own independently; but in any event Jacobs' formula is much clearer and less ambiguous.

Do not confuse. As we have seen, Jacobs' simple form of a fortiori argument is essentially identical with his complex one, except that some information found explicitly in the latter is left tacit in the former. To understand Jacobs' view of a fortiori argument, therefore, we must especially focus on the complex form, "If A, which lacks y, has x, then B, which has y, certainly has x." It should, for one thing, be seen that this form implies that "A is (always) not-y" and "B is (always) y;" We might well specify 'always' (or 'in all cases'), so as to make clear that these implications are categorical and not conditional.

We have seen that there are Biblical examples of this form (e.g. Ezek. 15:5), and moreover that it is found frequently used in later Judaic literature (e.g. Hull. 24a); and indeed, I might add, in non-Jewish literature. However, this form should not be confused with the following form, which is very superficially similar and also exemplified in the Bible and Talmud: "If z, *when it* lacks y, has x, then z, *when it* has y, certainly has x." The latter form is really a special case of Jacobs' simple form, viz. "If A has x, then B certainly has x." For here, A = 'z when not y' and B = 'z when y'; i.e. A and B are compounds, which though different have two elements in common, 'z' (the same in both, note well) and 'y' (denied in the one and affirmed in the other).

An example of the special simple form would be Deuteronomy 31:27, where Moses says: "Behold, while I am yet alive with you this day, ye have been rebellious against the Lord; how much more after my death [ye will be rebellious]!" This means, in more standard form: if the people (z) during Moses' lifetime (not-y) are unfaithful (R) enough to rebel (x), then they (z) after his death (y) will be unfaithful (R) enough to rebel (x). Here, the minor (z + not-y), major (z + y) and subsidiary (x) terms are all explicit; but the middle term (R) is not specified and must eventually be added in.[12]

Note well how this form significantly differs from Jacobs' complex form. In the latter, as we saw, A and B are distinct terms, one of which always lacks y while the other always has y; therefore, not-y and y essentially stand outside A and B, since the latter items can be thought of without referring to the former. In the special simple form we are now considering, on the other hand, the relation of z to not-y or y is *conditional or occasional*; therefore, A and B cannot be thought of without reference to not-y and y, because the latter are differentia of the former (which, for the rest, share element z).

That is to say, one subject, A, is about an individual z in circumstances when it is not-y, or about instances of z that are not-y; while the other subject, B, is about an individual z in circumstances when it is y, or about instances of z that are y. There is here, obviously, no implication that "A is (always) not-y" and "B is (always) y," as this would result in a contradiction, since A and B have z in common. So the form here considered is clearly very different from Jacobs' complex form, even though their wording might seem similar at first sight. Another way to state the difference is to point out that in Jacobs' complex form, the elements y and not-y point to the middle term, whereas in the special simple form they point (in conjunction with a recurring element z) to the major and minor terms.

2. Deficiencies in Jacobs' forms

It should also be emphasized that Jacobs does not, in his attempt at formalization, explicitly acknowledge the major premise, which is necessary if we are to characterize one term as the major and the other as the minor. Moreover, the major premise has to include a middle term, which tells us *in respect of what* the major term is 'more' and the minor term is 'less'. Furthermore, the middle term must appear in the minor premise and conclusion, because it is only due to a subject having *enough of it* that a predicate can be predicated of it. Jacobs does not at all mention the middle term and the crucial roles it plays in the deduction.

Jacobs is certainly to be congratulated for having attempted to formalize a fortiori argument, by using abstract symbols like A, B, x, y, in lieu of concrete terms. But his formalization was certainly deficient. He identified a minor premise ("A has x" or "A, which lacks y, has x") and a conclusion ("B has x" or "B, which has y, has x"), but he failed to identify the major premise involved. Without a major premise, how would we know which term to call the 'major' and which to call the 'minor'? Obviously, there has to be a proposition at the back of our minds which makes possible that distinction! It is not "self-evident."

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[11]   In *Halachische Exegese* (Berlin, 1840), p. 227 (cited by Samely, p. 177). See the section devoted to Hirschfeld in a later chapter (30.1) for discussion of this formula.
[12]   Another example would be Prov. 21:27, where Solomon says: "If [even brought with a 'sincere' intent] the sacrifice of the wicked is an abomination; how much more brought with a wicked intent [is it abomination]?" That is: if the sacrifice of the wicked (z) brought with a 'sincere' intent (not-y) is abominable (R) enough to be rejected (x), then the sacrifice of the wicked (z) brought with a wicked intent (y) is abominable (R) enough to be rejected (x). Here, the middle term (R) is explicit, while the subsidiary term (x) must be added on.

At the outset, Jacobs explains his simple-complex division as follows (p. 3):

> "The simple *qal wa-homer* is a plain argument *de minore ad majus*, in which the severity of the major over the minor is self evident. The complex *qal wa-homer* is one in which this severity has to be proved by reference to external factors."

Jacobs likewise tries, in the Encyclopaedia Judaica article on Hermeneutics that he authored, to explain his distinction by saying that in the simple form of a fortiori argument "the 'minor' and 'major' are readily apparent," whereas in the complex form "an extraneous element has to be adduced to indicate which is the 'minor' and which the 'major'." These explanations are very vague indeed, and he does not manage to make them more precise.

I am not denying the value and validity of these remarks by Jacobs, i.e. that in some cases we can readily see which of the two terms (A and B) refers to the lesser quantity and which to the greater quantity, whereas in other cases we have to make a marked intellectual effort to pinpoint some specific difference (such as lacking y *versus* having y) before we can clearly discern which term is the lesser and which is the greater quantity. This insight of his can be taken as an often useful 'rule of thumb' (a heuristic guideline). But it cannot be understood and justified without reference to an underlying quantity comparison (which is to be brought out in the major premise). Jacobs nowhere explicitly does that. Though he uses the traditional terms 'major' and 'minor', he does not emphasize their quantitative significance.

In truth, as we have already shown, the difference between Jacobs' simple and complex forms is merely one of explicitness. That is to say, they are essentially one and the same logical form, but in one case less is said out loud than in the other. Therefore, the complex form – "If A, which lacks y, has x, then B, which has y, certainly has x." – may be taken as Jacobs principal contribution to the field of a fortiori logic.

Now, this form does give a lot of the information needed to reason a fortiori, but the problem with it is that it does not give *all* the information needed. If we examine it carefully, we see that it contains the four terms needed to draw the conclusion. But it does not interrelate these four terms with the necessary subtlety, in the major and minor premises and thence in the conclusion.

Jacobs no doubt induced this form by looking at actual cases. There is no doubt that "If A, which lacks y, has x, then B, which has y, certainly has x." is a recurring form of discourse with a fortiori intent, not only in the Bible and in the Talmud, and in other Jewish literature, but in non-Jewish literature and indeed in current oral expression. However, this is in fact a special case of a wider form, which is also (note well) widely used, namely: "If A, which *has less y*, has x, then B, which *has more y*, certainly has x." The clauses "lacks y" and "has y" are just special cases of the clauses "has less y" and "has more y."

That A lacks y is indicative of A being the minor term, and that B has y is indicative of B being the major term, is due to the possibility of placing 'zero y' and 'more than zero y' along the same quantitative continuum. This scale of comparison is what the so far tacit major premise, which says "A is more y than B," contributes to the argument. Clearly, the term 'y' here plays the formal role of middle term. For this reason, we can say that Jacobs' form does indeed contain the middle term. However, we can also say that Jacobs did not realize that this was the role played by his term 'y'.

This, incidentally, explains why Jacobs (quite rightly) specified that the minor term A lacks y whereas the major term B has y, rather than specifying that A has y whereas B lacks y. Of course, the latter combination can be contrived by substituting a negative term for a positive one and vice versa. But the underlying meaning has to be that lacking y (or some 'positive' equivalent) is in some significant way *less than* having y (or some 'negative' equivalent). For example, lack of intelligence and stupidity are equally 'less capable of understanding than' intelligence and lack of stupidity.

Furthermore, Jacobs' form is deficient in that it does not make clear the other function of the middle term (y), which is to set *a threshold as of which* a subject (here, the minor and major terms, A and then B) may gain access to a predicate (here, the subsidiary term x). It is because A (which still, though lacking any y) is *y enough to be* x, that B (which having some y, has more y) is likewise y enough to be x. This is another crucial feature of a fortiori argument, without which the conclusion just cannot logically be drawn from the minor premise.

Thus, a fortiori argument is essentially an argument about magnitudes or degrees. Jacobs' form does not bring out this *quantitative* nature of the argument (less y, more y), but makes it seem to concern a contrast of opposite *qualities* (lacks y, has y). Had he reflected as bit longer, and examined more examples, he might well have realized this clearly. But he stopped too soon in his reflection. Without sufficient training in formal logic, it is difficult for someone to know just what is being sought in the process of formalization.

It is also difficult for someone insufficiently trained in logic to develop a broad, exhaustive theory. Notice, in the above quoted statement by Jacobs, his reference to "argument *de minore ad majus*." Having mentioned inference from minor to major, he should have immediately asked himself: "what of inference from major to minor – is there such a thing and what shape would it have?" He might then have at least discussed negative subjectal argument, and

maybe even with further effort come upon the also important predicatal forms of the argument. But he nowhere does that.

This is surprising, considering that Talmudic logic (as of precisely when in history, I still do not know) does acknowledge *michomer leqal* argument as well as *miqal lechomer* argument, and that there are examples of both types throughout Jewish and non-Jewish literature. Jacobs must have known that, but somehow did not make the connection (at least in this book). Perhaps he was too focused on the general Hebrew term for a fortiori argument, viz. *qal vachomer*, and forgot the variations on that theme.

It should also be said that Jacobs, through his narrow vision of a fortiori, effectively leaves many arguments encountered in practice without a model to refer to. For instance, some a fortiori arguments are manifestly about degrees of a common property, and cannot readily be made to fit into Jacob's yes-or-no model. Take for example 1 Kings 8:27, where Solomon prays: "*heaven and the heaven of heavens cannot contain thee; how much less the house that I have builded?*" Here, the volumes of the heavens and of a mere building on earth are compared. It is not said or implied that one item *has* volume and the other *lacks* it – the intent is clearly that both have volume though *in different degrees*. The argument is that just as the larger volume (of the heavens) cannot contain God, so the smaller volume (of the First Temple) cannot do so.

To conclude: there is no viable substitute for the analysis of a fortiori argument that I proposed in my *Judaic Logic* (and reproduced and developed in the present work), which by introducing the middle term into the equation demystifies all such inference in a uniform manner. Whether an a fortiori argument looks simple or complex to us, we must in fact always look for the underlying middle term. If we refer as Jacobs does, and indeed almost all traditional commentators do, only to the 'major' and 'minor' terms, without consciously specifying a common ground relative to which they are compared as greater or lesser (in a range of values that may stretch from minus infinity through zero to plus infinity), we have not fully grasped this form of reasoning and are therefore unable to justify it.

Comparison to syllogism. Jacobs' comparisons and contrasts between a fortiori argument and syllogism are largely correct, but not entirely. He rightly insists that "there is no connection between the two forms of reasoning" (p. 3) – but at the same time he seems to allow for some equation between them. Consider his following remark (p. 4-5):

> "Of the two types of *qal wa-homer* it is the simple one which has affinities with the Syllogism in that both the simple *qal wa-homer* and the Syllogism draw their conclusion from a major premise without having recourse to any external factors."

According to this statement, the simple and complex forms of *qal vachomer* differ so radically that the former has "affinities" with syllogistic reasoning, whereas the latter does not. It is not clear what affinities he has in mind when he says this. The statement that they "draw their conclusion from a major premise without having recourse to any external factors" is far from clear, not to say confused.

In syllogism, typically, the major premise 'B is C' would allow us to draw the conclusion 'A is C', given the minor premise 'A is B'. But Jacobs' simple a fortiori argument is claimed to be immediate inference from 'A is x' to 'B is x' – i.e. as a direct insight, somehow, of the conclusion in the single premise. Presumably he regards 'A is x' (which I would call the minor premise) as the 'major premise' here, since there is no other premise on hand in his model. It is doubtful that Jacobs thought of syllogism as likewise an immediate inference (without the help of the minor premise 'A is B') from 'B is C' to 'A is C', since it is clear from an example he adduces further on that he is well aware that syllogism (as its name implies) involves two premises.

Perhaps the clue is his reference to "external factors." By this expression he presumably intends the clauses "lacks y" and "has y" which occur in his complex form of a fortiori. It may be that he realizes subconsciously that these clauses provide the middle term needed to construct major premise of complex a fortiori argument. Evidently, he does not realize that his simple a fortiori argument is just as much in need of a major premise, even if the middle term in their case is not suggested. No, the only "affinities" that I can imagine he had in mind is the fact that syllogism and simple a fortiori involve only three terms each (ABC and ABx, respectively) whereas "complex" a fortiori has a fourth term (y/not-y).

His above statement is therefore based on very superficial resemblance, and can be ignored. As we have already shown, Jacobs' simple and complex forms cannot in fact be logically differentiated, even if they are differently worded. There is certainly no conceivable way to draw the conclusion 'B has x' if all the information we are given is that 'A has x'. Logic admits of no such magical shortcuts; it demands a plausible explanation for any inference, a formal array that can prove the inference.

We can thus move on, and consider his more interesting discussion of the differences between complex *qal vachomer* and syllogism. He rightly stresses that the former has an element of "how much more so" that is lacking in the latter. He rightly explains the difference as follows (p. 6):

> "…in the Syllogism the inference concerns the relationship between genus and species; we are saying that since Socrates belongs to the class *man* then he must share the characteristics of that class. Whereas in the *qal wa-homer* inference we do not say that a weighty precept belongs to the class *light precepts*; it obviously does not. We say that what is true of light precepts is true of weighty precepts."

However, he does not manage to formally explain why, for instance, "what is true of light precepts" should be "true of weighty precepts" – he does not realize he is appealing to a tacit major premise that quantitatively orders precepts into lighter and weightier, and that this tacit premise (which should also tell us what lighter and weightier mean more precisely, i.e. should reveal the operative middle term) is the missing link between the minor premise and the conclusion.

The probable reason why he failed to see this is hinted at in his attempted formalization of complex a fortiori argument as "If A, which lacks y, has x, then B which has y certainly has x." Jacobs apparently conceived the argument as a sort of *a contrario* (or more precisely, obverted inversion), rather than a fortiori, movement of thought, with one term (B) *having* something (y) that the other (A) *lacks*.

Somehow, but he does not explain *how*, the fact that the one (A) who lacks y has x is supposed to convince us that the one (B) who has y also has x. But if we reflect, it is obvious that this cannot always be true. Generally put, any two subjects A and B may have in common any number of things and yet also differ in any number of respects. There is no formal reason why some particular difference (with regard to y, for instance) should imply some particular sameness (e.g. with regard to x). We must be given some precise *additional* information that allows such inference; the information may well in practice be implicit, but we must be able to make it explicit to justify the argument. In other words, the three propositions "B is y and A is not y and A is x" do not together formally imply the proposition "B is x": there is no way to validate such a general claim.

In short, Jacobs' formalization was incomplete. Although he deserves credit for his effort to formalize, and for noticing the feature of difference in properties between the subjects (no mean feat), the fact remains that he did not succeed in disclosing the whole underlying discourse. Had he persevered in his effort of formalization he might have discovered that the difference between *having* and *lacking* something is not the essence of the arguments he labeled complex. The essence is, rather, an implicit or explicit *quantitative* comparison of some sort between those two distinct qualifications – or any others for that matter. Once this is understood, we see that there is no essential difference between so-called simple and complex a fortiori argument – and that a fortiori argument cannot be clarified by such a distinction.

The importance of validation. Many contemporary writers continue to refer to Jacobs' theory of a fortiori argument as their standard of judgment. Some of these may be excused, since they have not heard of or read my work on the subject. But those who are apparently acquainted with my work (e.g. since they mention it in their bibliography) and yet persist in referring to Jacobs' theory are clearly not being scientific. I greatly respect Jacobs as a theologian and Talmudist; but in the matter of a fortiori logic he is less than expert. The authority of a writer is not based on how widely known he is, but on rational considerations. The nature of a fortiori argument is not my word against Jacobs' – it is an objective matter to be decided by reason. There is no ongoing 'debate' – so that any commentator can opt for one or the other contestant at will. The matter was settled already over fifteen years ago, as of when I published my *Judaic Logic*.

Why so? Because I there formally *validated* my proposed a fortiori forms, whereas Jacobs has only given a couple of examples of his. What is validation? It is to show by formal processes that the given premises do indeed (together, if they are more than one) logically *imply* the putative conclusion. The given premises and putative conclusion may well have been correctly formalized by looking at a number of examples; but this is only the first step in the logician's work of validation. The second step is to show that the assumed premises are *sufficient* to imply the putative conclusion. It may well be that, though the assumed premise(s) is/are *among* the propositions needed to obtain the conclusion, some *additional* unstated premise(s) is/are also required to obtain it. Of course, in some cases the proposed premises, or at least parts or aspects of them, are completely useless to the putative conclusion; but what some people forget is that they may be useful but insufficient.

The third step in the formalization and validation procedure is to verify that the form(s) of argument one has postulated do indeed cover the whole field being studied. A theory has to be exhaustive in its applications. It is not enough to explain *some* practical examples; one has to make sure one has succeeded in explaining *every* sort of practical example. For instance, if one has only taken note of positive forms, but ignored negative ones, one's theory is inadequate, its scope being too limited. I can tell you through large experience in logic theorizing that one can never be sure of one's theory of some argument till one has actually investigated all aspects of the question. A theory of argument is always in flux until it grows to full maturity; an initial insight or two are never enough to guarantee its correctness or completeness.

As I have argued above, although Jacob's proposed forms, the simple and the complex, may seem reasonable at first sight, they are upon closer scrutiny found wanting in many respects. Jacob's two forms cannot be validated because, as already shown, their stated premises on their own just *do not imply* their putative conclusions. Furthermore, although his forms superficially cover some examples, they leave many examples without explanation. That being the case, Jacob's two forms cannot be upheld by later writers as if they were valid and exhaustive. Such writers are satisfied with a 'quick fix' and make little effort to verify the matter. In this way, untruths and partial truths are perpetuated.

I do not say any of this out of disrespect for Jacobs (I have read a number of his books and found myself happily in agreement with most of the things he says) or out of egotistic competitiveness – my sole interest is in the truth of the matter at hand. If Jacobs' theory (to which my attention was drawn only recently, years after my own theory was completed) was better than mine, I would frankly admit it. I am rich enough in logic discoveries not to be avaricious. I have often made independent discoveries and then found them already made before me – and in such cases have modestly named my predecessors as the true discoverers. Occasionally, a reader finds an error in my work – in such cases I thank him profusely, grateful to have been saved from seeming foolish to other readers. I try to be scientific, in the best sense of the term.

At the time (1961) Jacobs presented his theory of a fortiori argument, it was without doubt a notable contribution and a valuable reference. But the fact is, since publication of my theory in *Judaic Logic* (1995), for the reasons above given, its significance is only historical and people should not continue to refer to it as if it were the final word on the subject.

Some time after writing the above lines, I discovered the work of Moshe Chaim Luzzatto on a fortiori argument. In his *Sepher haHigayon* (1741), he succeeds in defining the four moods of a fortiori that I much later labeled as positive and negative subjectal, and positive and negative predicatal. Therefore, if anyone should be mentioned as the first to have clearly formulated a fortiori argument in formal terms it is in truth R. Luzzatto (also known as the Ramchal), rather than myself or Louis Jacobs or any other later writers. Note however that R. Luzzatto did not fully formalize nor attempt to validate a fortiori argument; for a detailed analysis of his contribution, see the earlier section (9.10) devoted to him of the present work. I hope that in the future due regard will be given to this historical finding.

3. More comments on Jacobs' work

Allow me a few more comments, while on the subject of Louis Jacobs' *Studies in Talmudic Logic and Methodology*. This is a very interesting work, but it is not intended as a systematic, let alone exhaustive, analysis of Talmudic logic. Jacobs devotes only four chapters (about a quarter of the book) to the latter subject. In the first chapter, he deals with a fortiori argument (the first of the thirteen hermeneutic rules of R. Ishmael), as we saw above. In the second chapter, he examines the *binyan av* (the third rule); more on that topic below. In the third chapter, he deals with *svara* (or *sebhara*, in his transliteration), which refers to arguments[13] based on the rabbi's "common sense" or "reason," in contrast to arguments based more intentionally on Scriptural givens or traditional transmissions. The fourth chapter deals with rabbinical use of "*reductio ad absurdum*," which (by the way) here refers not only to arguments leading to a contradiction, but also to arguments leading to a result obviously contrary to experience, or to common knowledge or opinion. That's it for logic[14].

The rest of the work is devoted to methodology, by which Jacobs means modern literary and historic analysis of Talmudic *sugyot* (segments dealing with specific topics) and their arrangements in tractates. Jacobs' detailed study, in this book and others (notably in his 1991 work, *Structure and form in the Babylonian Talmud*[15]), of the way the Talmud must have been redacted is fascinating and worth reading. He manages to show, by consideration of the dates when the various rabbis involved were active and by consideration of the progressive presentation of arguments, and occasionally by comparisons to parallel material in the Jerusalem Talmud, how the redactors produced somewhat artificial constructs designed to retransmit information in the most dramatic and memorable way. This is as against

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[13]   Actually, in my opinion, it would be more accurate to characterize many *svara* as propositions or premises, rather than as arguments. For instance, "one who is in pain visits the house of the doctor" is (to R. Ashi) a reasonable proposition. Such a *svara* is used within an argument, but it is not itself an argument. But as Jacobs makes clear, the term *svara* is also often taken in the larger sense of human reasoning without direct Scriptural authority such as *qal vachomer* or *binyan av* inferences.

[14]   Thus, only two of the 13 hermeneutic principles of R. Ishmael are dealt with. As for *svara* and *reductio ad absurdum*, although not among the 13 explicit principles, they are rabbinical practices and so implicit principles, so it is legitimate and desirable to analyze them. But here again, these are only two of many logical practices unlisted by R. Ishmael or others. Clearly, Jacobs made no attempt to be exhaustive here.

[15]   Some of this book may be read online at: books.google.ch/books?id=Q7oXP0dPelgC&printsec=frontcover&dq=louis+jacobs&hl=en&sa=X&ei=F8KMUbGbOceI4AS1jYHIDw&redir_esc=y.

the traditional notion that the Talmud is a verbatim transcript of rabbinical discussions in the order they historically occurred.

What is manifest, incidentally, when reading Jacobs' descriptions of various *sugyot*, is how frequently and how competently the rabbis use hypothetical logic in very complex knots. There are complicated comparisons of the implications of different theses (e.g. the theories of different rabbis on some issue of law): If thesis A, then such and such are the implications and the implications of the implications; if thesis B, then so and so; if thesis C, then thus and thus. There is ample use of nesting, whose intricacy is sometimes mind-blowing[16]. Apodoses are used to eliminate theses with materially false or rationally absurd implications (*reductio ad absurdum*), or to confirm theses with true and consistent implications. Disjunctive arguments and dilemmas are also found in ample use, to decide between theses.

We should also mention here Jacobs' essay "The Talmudic Argument" (1984)[17], where he masterfully identifies nineteen "formal types or patterns" of Talmudic arguments (see also the corresponding footnotes, where he gives the Hebrew or Aramaic phrases that distinguish these arguments), namely:

> "Argument from authority; argument by comparison; argument by differentiation; either/or argument; on the contrary argument; acceptance of an argument in part; argument based on an opponent's position; argument exposing the flaws in an opponent's argument; argument based on historical or geographical conditions; argument based on the analysis of states of mind; readmission of an argument that has been previously rejected; argument against a statement of the obvious; argument to resolve a contradiction between sources; argument by textual emendation; argument from the principle of literary economy; different versions of an argument; argument presented by different teachers; consequences of different arguments; limited application of an argument."

All this is significant – because if we want to talk about Judaic logic, we must look not only at the *explicit principles* of rabbinical hermeneutics, but at the *implicit practices* of the rabbis. The latter greatly expands the field, and shows rabbinical logic in action to be much broader a field than rabbinic logic in theory. The distinction (stressed in my *Judaic Logic*) between the art of logic and the awareness and discussion of logic is of course very pertinent in this context.

A digression on *binyan av*. Returning to logic, a word now on Jacob's treatment of *binyan av* inference. I was very pleased to discover that our views on this topic are very close. In his *Studies* (chapter 2, pp. 9-10), Jacobs writes:

> "… a more fruitful way of explaining the principle [of *binyan 'abh*] is to compare it with John Stuart Mill's method of agreement, to which it bears a striking resemblance. It goes without saying that we are not suggesting any kind of anachronistic 'anticipation' of Mill by the Rabbis. All we suggest is that the Rabbis, in their attempt to discover general principles behind the laws of the Torah, used, apparently, a method similar in form to that classified by Mill as a means of discovering the laws of nature."

After which Jacobs goes on to give some Talmudic examples of the inference and tries to formalize it. I independently made a similar comparison to Mill's method of agreement, many years after him, in my *Judaic Logic* (chapter 10.2), saying:

> "Inferences of the *binyan av* type (Rule No. 3) seem to be a Rabbinical attempt at causal inference.… Causal inference has been much clarified in more recent times by John Stuart Mill, who identified the 'methods of agreement and difference'…. However, the Rabbinical attempts at formulation of this natural principle stressed more the side of 'agreement' than that of 'difference'."

After which, I analyzed *binyan av* reasoning in some detail, again in ways comparable to Jacobs'. However, certain differences in our approaches must be stressed. What is evident here, as with his treatment of a fortiori argument, is that though Jacobs makes some effort to formalize rabbinic thinking, he does not make sufficient effort to validate his forms. Not really being a formal logician (though he no doubt could have been had he tried to be), he does not realize the importance of validation – not only for the purpose of justification, but even for the purpose of increased accuracy in formalization.

We could say that Jacobs' comparison of rabbinical *binyan av* reasoning to Mill's method of agreement, is effectively putting forward the latter as a justification (albeit *ex post facto*) of the former; i.e. he uses this analogy as a validation argument of sorts. In his formal descriptions of the rabbinic thought processes, Jacobs' emphasis is on the *positive* aspect of causal relations, just like that of the rabbis. The negative aspect of the reasoning is mostly left unsaid, effectively ignored. This explains why Jacobs refers specifically to Mill's method of *agreement* and no other.

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[16] It is easy enough to follow a Talmudic discussion under the guidance of a modern master like Louis Jacobs. The experience is very different if one tries to do so alone or even under the guidance of purely traditional guides, for the convictions and ways of thinking of the ancient rabbis are very different from ours today, and their speech is telegraphic and dense and often obscure. They unfortunately apparently never discovered or used flowcharts!

[17] In: *Essential Papers on the Talmud*. Ed. Michael Chernick. New York: NYU Press, 1994. Pp. 429-460.

In my above mentioned study of *binyan av*, on the other hand, I refer rather to Mill's methods of agreement *and difference*. I was well aware, even then[18], that reasoning about causation requires consideration of both positive and negative aspects. Especially, to fully understand a causal relation, we must clarify not only what the effects are when the putative cause is present, but also what they are (or are not) when it is absent. Rabbinical emphasis in their practice of *binyan av* reasoning is, to repeat, rather on the positive side (though, to be sure, not always: sometimes they do take pains to ensure the negative side) – and Jacobs unconsciously follows suit in that respect in his more formal treatment.

Consider his proposed formalization of the *binyan av* argument[19]:

> AB results in 'a', and AC results in 'a';
> therefore, A is the cause of 'a'.
> Whence, if A is found in some new context, 'a' may logically be applied there.

This argument proceeds by comparing the components of different contexts, such as AB and AC (say), where a certain law 'a' is known to apply. Suppose it is found that these contexts have property A in common (and no other), and they differ in that one has B (and lacks C) while the other has C (and lacks B). We can infer from this information that the common property A is the source of the law 'a' relative to them. If now some new context (say, AN) is found to also have the property A, we may reasonably apply law 'a' to that context too.[20]

This formula is initially convincing, but it is incompletely formulated and therefore difficult to validate as it stands. We may assume from the wording that AB lacks C and AC lacks B, and moreover that there is nothing besides A common to contexts AB and AC (the latter condition is a tall order, to be sure, but let us suppose it)[21]. Even so, we cannot prove that A implies 'a' (and not-A does not imply 'a') from these premises, for A may not be a complete cause of 'a' but only a partial cause in conjunction with either B or C (or some other conjunct like N). Therefore, the inference that "A is the cause of a" is inductive rather than deductive. It involves a tacit *generalization* from the given contexts AB and AC to 'all contexts involving A'. Thereafter, on this basis, we may infer by simple syllogism that a new context with A (such as AN) is subject to law 'a'. Thus, although the proposed argument looks superficially deductive, it is in fact essentially inductive.[22]

The same reflections apply to the more complex form of *binyan av*, which the rabbis routinely use and Jacobs here also formalizes. He describes this process as follows: we start with knowing only that 'ABD results in a' and 'ACD results in a'. Since these two implications have not only one but two factors in common, viz. A and D, we are in a quandary and cannot decide which of these two items is 'the cause' of a. We must therefore look for and find a third implication, say that 'AE results in a', which *has* A but *lacks* D. In that event, we can conclude that 'A implies a'. In truth, this argument does not formally differ from the preceding, except in the number of contexts in which law 'a' is known to apply. We still need to generalize from the given contexts (ABD, ACD and AE) to all contexts involving A, to be able to apply law 'a' to any new context with A (such as AN).

However, we must here first realize that item D, contrary to initial appearances (in contexts ABD and ACD), plays no role in the causation, and this is achieved through the discovery of context AE, which lacks D, where law 'a' nonetheless applies. Jacobs rightly refers to the process as "the method of elimination," meaning that having eliminated hypothesis D we are only left with alternative hypothesis A, which may therefore be considered 'the cause' of 'a'. But note that the elimination is a preliminary to the *binyan av* inference, and not really part of it. If we

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[18]   I should mention that a few years later, after developing my own theory of causation in my book *The Logic of Causation*, I analyzed Mill's methods more carefully and became much more critical of his work. See the Appendix to Phase I of that work; actually, though first written in 2003, it was largely rewritten in 2005, and so really belongs to Phase II.

[19]   I have substituted plain English for his relational symbols. He uses a long arrow for 'results in' and a long dash for 'is the cause of' – but he does clearly at the outset define his symbols in these words. I would not use a causal phrase like 'results in' in the premises (unless it is specified in the source text), because the neutral phrase '*occurs in conjunction with*' suffices (if the argument is valid) to establish the causal conclusion ('is the cause of'). The whole point of causal argument is to construct a causal proposition from *non*-causal ones!

[20]   In my *Judaic Logic*, I formulate the essentially same argument in different terms. Jacob's AB, AC, A and 'a' are the same as my X, Y, A and B, respectively. Thus, our A symbols correspond, but his 'a' symbol is symbol B in my book. The new context, which Jacobs does not name (but which I here call AN), I there call Z.

[21]   Jacobs does show his awareness of this negative condition in his presentation of a material example and when he cites Mill's method of agreement and stresses: "Mill is careful to write 'have only one circumstance in common'" – but he (Jacobs) unfortunately does not explicitly integrate this knowledge into his forms.

[22]   It should be noted that there is a paradoxical aspect to *binyan av*. The source text contains at least three contexts with A (viz. AB, AC and AN) of which two occur in conjunction with law 'a' (viz. AB and AC) and one (viz. AN) *does not* do so. Taking *all three* of these contexts into consideration, we *cannot* logically conclude that A is the cause of 'a', since we have evidence that A is sometimes *not* textually associated with 'a'! The only way we can arrive at the desired conclusion is to momentarily *turn a blind eye* to the context AN (which lacks 'a') and base our conclusion about causation of 'a' by A on generalization from the remaining contexts (AB and AC); thereafter we return to AN and apply the conclusion to it. Thus, our reasoning is based on the (debatable) premise that 'not textually associated' is not the same as 'textually dissociated'.

did not find a context AE devoid of D, we would simply conclude that the cause of 'a' is the joint feature AD! This is not mentioned by Jacobs, because he develops the argument in relation to a specific Talmudic example, where a single differentia is sought by the rabbis. Clearly then, there is no significant difference between Jacobs' simpler and more complex versions.

I should add that whereas Jacobs refers to A as "the cause" of law 'a', I would prefer to label A as "a complete cause" of law 'a'. I would do that due to awareness that there may be more than one complete cause: there may be two or more *parallel* complete causes, provided they all imply each other and thus behave in the same ways. Also, by specifying complete (i.e. sufficient) causation, I mean to remind us that there are also the formal possibilities of *partial* causation, and of *necessary* and *contingent* causation[23]. Jacobs does not mention these in his formalization, possibly because he has not noticed them occurring in the Talmud, but more probably simply out of unawareness of these alternative forms of causation. I do not affirm it as established fact, but I would be very surprised if we did not indeed find in the Talmud use of these other determinations of causation. Of course, I refer here to *de facto* use of such forms; I do not expect them to be discussed by the rabbis (though they may well do so somewhere), much less formalized and validated.

Certainly, in view of its frequency in human thought, the *sine qua non* (without which not) form must occur often. This may not seem like *binyan av* reasoning at first sight, because instead of looking for a common property (a positive) we would be looking for a common privation (a negative). But apart from the changed polarity of the terms, the form of necessary causation is essentially similar to that of complete causation. Very likely, too, partial and likewise contingent causations occur at least occasionally. Such occurrences may be difficult to find, as the form of reasoning is more complex, since it refers to the composite action of two or more factors which cannot produce the effect separately. Jacobs mentions (pp. 13-15) examples from the Gemara where *binyan av* inference is attempted (without unanimous rabbinical approval, however) when the cases compared have *no* apparent common factor! I suggest as a possible solution to the problem they pose that such cases be closely examined with a view to find partial (or alternatively, contingent) causation in them.[24]

To my mind, some of the common factors that the rabbis come up with are very contrived. The alleged common factor may be a somewhat subjective take on the subject, or something quite incidental to the subject. One would think, looking at the formal presentation, that the common factor (A) is an essential element of the contexts compared – but I do not think that is always the case. Rather, I think, the rabbis want to apply law 'a' to the new context: if they cannot readily find a reason that naturally fits the bill, they try to make one up, i.e. they look for a *pretext* however flimsy for the preconceived desired result. It is only when even such last ditch effort fails that they admit there seems to be no common factor; and even then, they might still *assume* one exists though admittedly they cannot pinpoint it explicitly. My point here is that the common factor, in Talmudic reasoning, need not be a particularly significant property; it may be something otherwise irrelevant to the matter at hand.[25]

Before closing this parenthesis, I would like to briefly discuss the issue as to whether *binyan av* is induction or deduction. Jacobs begins his chapter on this hermeneutic principle by remarking:

> "In the nature of things there can have been little correspondence in the Talmudic literature with modern *inductive* logic. Schwarz, in his work on the principle of *binyan 'abh*, remarks on this and seeks to explain this principle as a form of analogy…"

Jacobs goes on "to use his [Schwarz's] own terminology, as a type of 'Species Induction Reference' or 'Genus Induction Reference'" – but since I do not know what Schwarz means by these terms, I cannot comment on them. I

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[23]   I refer you at this point to my book *The Logic of Causation* (chapter 2), where these various concepts are fully defined and discussed. Note that even Mill was not entirely clear about these different concepts, so there is no reason to expect Jacobs to have adequately dealt with them.

[24]   The *binyan av* argument formalized by Jacobs (from two or more contexts) is the main one used by the rabbis. But, as I point out in my *Judaic Logic* (chapter 10.2), there is another version of *binyan av* (from only one context), which some rabbis advocate and use. This may be expressed as "if a context with feature A prescribes law 'a', then another context with feature A may also be assumed to prescribe law 'a' (or, more precisely, may be used to justify such a ruling), and is called *chada mechada* (Aramaic, meaning 'one from one'). It should be clear that this reasoning is far more tenuous, since (a) it does not establish that there is no common factor other than A between the two contexts concerned, and (b) even if A were indeed their only common factor, A is not thereby proven to be the reason why law 'a' is given in the first context, and so we cannot be sure that law 'a' applies in the second context. There may be some other factor, say B, which is found in the first context but not in the second, and which is the true reason for law 'a'. Indeed, the *chada mechada* reasoning can be shown fallacious by pointing out that if it is used with regard to B in relation to some third context, which has B but not A, we would be saying that the law 'a' has two distinct reasons. That is, we would be applying the same law to contexts A+B, A without B, and B without A. It is not unthinkable that a law might be *given* with such various applications – but the claim of *inference* from case AB to the other two cases is very dubious. I suggest that rabbis use such weak reasoning as a last resort, when they have difficulty constructing a proper *binyan av*, but still want to justify a certain foregone conclusion. It occurs to me that such inference might best be classified as *gezerah shavah* (hermeneutic rule #2 of R. Ishmael), rather than as *binyan av*.

[25]   The rabbis sometimes use frequency of occurrence as a predicate. For instance in *B.Q.* 2b: "'Falling and Kicking'; are not these derivatives of Foot? — No; the damage of foot occurs frequently while the damage of these does not occur frequently." Sometimes, the differentiation is made with reference to subjective perceptions (i.e. how people commonly view the matter or imagine it). These are clearly incidental properties, which would not be regarded as significant in scientific discourse.

should add here that, as already pointed out in the chapter devoted to him (14), Schwartz seems to confuse historical and logical questions. He thinks that Talmudic arguments can be referred to Aristotelian logic because the latter preceded them in time, and cannot be referred to modern inductive logic because the latter came later. But this viewpoint absurdly assumes that forms of human thinking are not and cannot be used before they are discovered by logicians! The truth is, it does not matter when in history (and indeed where in geography) syllogism or induction were intellectually discovered – being natural human means of knowledge they were doubtless (though to varying degrees) used long before (and everywhere else), at least as of when (and where) modern man biologically evolved from a less rational species. Logical comparisons are feasible independently of historical or geographical issues.

As I have shown at length in my *Judaic Logic*, Talmudic thinking is very frequently inductive; i.e. it involves 'trial and error'. To say so is not to claim that the rabbis were in advance of our time (though they may well have been in some or even many respects). For the practice of logic, whether deductive or inductive, is natural to all human beings since our species appeared on Earth, and considerably independent of theoretical reflection on logic. Theory may and usually does improve practice, but practice can proceed apace without theory. Indeed, the art of logic is ultimately more important than the theorizing on logic, for we cannot arrive at true theories without competent practices. Thus, it is quite consistent to say that the rabbis were practicing induction as well as deduction very ably, without implying that they were in theoretical terms on a level comparable to modern thinking or even to ancient Greek thought.

I would agree with Schwarz that the *binyan av* is (as Jacobs puts it) "a form of analogy," inasmuch as the rabbis proceed essentially by arguing as follows (using Jacobs' above symbols): since all contexts where law 'a' is *known* to be applicable have factor A and only factor A in common, it follows that if a new comparable context where law 'a' is *not known* to be applicable also has factor A, we may reasonably assume that law 'a' is likewise applicable to it. This is analogy, insofar as we have copied the law in question from areas where it is given and pasted it in areas where it is not given. Although this process may to some people look like deduction, it is clearly induction since it involves a tacit generalization. If generalization was forbidden, we could not pass information from known cases to unknown cases however much the contexts resembled each other.

Additionally, the premise of the analogical argument is somewhat inductive. It involves an assumption that all relevant cases in the text have indeed been considered and thoroughly analyzed. This depends on human perceptiveness and to some extent on human judgment. Note that in Judaism it is assumed that the rabbis know all the Scriptures by heart, but it is not supposed that they individually know and have fully assimilated all the traditions and judgments handed down by earlier teachers – whence arise discussions and disagreements among them, till a consensus is established. As Jacobs point out, the argument "is not infallible. There is always the possibility that a common factor [besides A] has been overlooked." Of course, once a rabbinical decision is handed down it is thereafter treated as well-nigh infallible.

So, all things considered, we should rather regard *binyan av* as inductive, or more precisely as a logical process with both inductive and deductive stages. It should be added that this conclusion of mine is not peculiar to *binyan av*. As I explain in my *Phenomenology*[26], even syllogism, the paradigm of deductive inference, is in fact only partly deductive; i.e. it is partly inductive. When we infer from "All men are mortal, and Caius is a man" that "Caius is mortal," we can claim the conclusion to be new knowledge because the major premise was based on generalization rather than on an enumeration of all cases including that of Caius. The conclusion is a prediction from the hypothesis of general mortality; we cannot in fact be absolutely sure Caius is mortal till he actually dies; moreover, when he is found empirically dead, his case becomes a further confirmation of the truth of the major premise.

4. A more recent contribution

Several months after I wrote most of the above, I discovered that a more recent work by Louis Jacobs, *Rabbinic Thought in the Talmud* (2005), contains an essay devoted to "The Qal Va-Homer Argument in the Old Testament"[27]. We shall here analyze the contributions made in this late essay of his (Jacobs *z"l* passed away in 2006). Having by then read many of his works, and developed a true admiration for this scholar, I was very pleased to see more input from him. One thing that saddened me about it, though, was that Jacobs makes no mention in it of my contributions to the same subject in my *Judaic Logic* (1995), even though it was published about ten years before his essay. I am sure he would have been stimulated by it, had he read that work.

Enumeration. We shall start by comparing the list of Biblical *qal vachomer* drawn up by Jacobs in his latest essay with the list in my *Judaic Logic* and later findings[28]. I feel obliged to engage in this accounting, so as to give

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[26]   See chapter 7.4 there.
[27]   London: Vallentine Mitchell, 2005. See chapter 12, pp. 109-116.
[28]   Namely, 1 Samuel 17:37, the case given in Addendum 4, which was pointed out to me orally by Mark Leroux (from South Africa, a colleague at an office where I worked) in 2001. Before that (in 1998), I found a further case, namely: Jonah 4:10-11, by happenstance. More

everyone his due. If we merge the two lists together, we obtain a grand total of at least 46 instances (all presented in **Appendix 1**). Jacobs' list is apparently based, largely if not entirely, on an early 19th century work by **Wolf Einhorn** of Grodno[29], which I have not seen, but which reportedly contains 40 instances[30]. Jacobs presumably rejected some of the latter, since he only lists a total of 35 instances; but he does not say which instances he rejected or even just why he did so[31]. Nor does Jacobs tell us whether any of the instances he lists are his own findings, or all are included in Einhorn's list.[32]

We both have	24
I have, he lacks	14
He has, I lack	8

Jacobs and I have 24 instances in common. These of course include the famous ten instances given in *Genesis Rabbah* 92:7; namely, Genesis 44:8, Exodus 6:12, Numbers 12:14-15, Deuteronomy 31:27, 1 Samuel 23:3, Jeremiah 12:5 (2 cases), Ezekiel 15:5, Proverbs 11:31, and Esther, 9:12. Interestingly, probably because the Genesis 44:8 instance is spelled out first and then R. Ishmael says: "This is one of the ten instances of *qal va-homer* in the Torah," Jacobs suggests that only the first of these ten instances was originally in the Midrash, saying: "In what is in all probability an editorial, or even later, gloss, the Midrash gives the other nine" after R. Ishmael's remark[33].

Jacobs uncovers another two instances mentioned in the same Midrash but not listed among the ten, namely: Genesis 4:24 ("If Cain shall be avenged sevenfold, truly Lamekh seventy and sevenfold."), which I already knew of thanks to Rashi; and Genesis 17:20-21, which I did not know about, and so have included under the category of "He has, I lack" further down[34]. It is interesting that the Midrash lists only ten instances of a fortiori argument in a later page, even though the very same volume mentions another two earlier on! Such inconsistency certainly suggests that there was successive editing of the work.[35]

The failure of the Midrash to list its own 12 instances at once is otherwise inexplicable – unless its author used some unspecified selection criteria. Probably, the Gen. 4:24 case was left out as an "evil" case and Gen. 17:20-21 was left out as an "implicit" case[36]; another possible explanation is that there was later addition of these two cases. The question posed here is of course part of a larger one, which I already asked in my *Judaic Logic*: how is it that the author of the Midrash, who presumably knew the Tanakh by heart and was not half asleep, missed out on the numerous other a fortiori arguments that we have lately found there? This is a mystery. Jacobs acknowledges this mystery, saying: "the commentators to the Midrash and other scholars are puzzled by R. Ishmael's reference to only ten Scriptural cases."[37]

Note also in this context that I do not consider Esther 9:12, which the Midrash list includes, as a credible, sufficiently explicit instance of a fortiori argument. This example, which reads: "The Jews have slain and destroyed five hundred men in Shushan the castle, and the ten sons of Haman; what (*meh*) then have they done in the rest of the king's provinces!"[38] uses language that is to my knowledge nowhere else connected with a fortiori argument. The

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recently (in Aug. 2012), I found yet another case, Ezekiel 14:13-21, by means of a search for key phrases at www.mechon-mamre.org/p/pt/pt0.htm. Incidentally, the latter search only yielded a total of 19 cases: 13 cases with 'how much more and 6 cases with 'how much less'; there were no cases with the key phrases 'all the more/less' and '(how/so) much the more/less'.

[29]   Zeev Wolf Einhorn (Maharzav), Grodno, Lithuania, 1813- 1862. *Sefer Midrash Tannaim*, 1838.

[30]   Jacobs adds that "other commentators [have] come up with similar results" – but he does not say which commentators, nor what these results were nor compare them.

[31]   He does tell us that some of the instances proposed by various researchers "must be rejected as far-fetched and dubious," but he unfortunately does not perform his triage in public, and all too confidently declares that his list "contains all the definite references."

[32]   Goltzberg, in his 2010 essay "The A Fortiori Argument In The Talmud" (which I review further on), mentions "the forgotten a fortiori arguments," without however saying how many he thinks there are or listing them. Apparently, he draws this information from Moshe Koppel's *Meta-Halakha. Logic, Intuition And The Unfolding Of Jewish Law* (Northvale, NJ, Jason Aronson, 1987). Not having seen the latter work, I cannot say if it is any more informative than that. Note that Jacobs does not mention Koppel's book, which is earlier than mine, either.

[33]   It is interesting that R. Ishmael does not here rather mention Num. 12:14-15, which plays such important role in the Gemara explication of Mishna *Baba Qama* 2:5.

[34]   But only, as it turns out as an implicit case; not as an explicit case.

[35]   Jacob's comment is based on a note by Theodor-Albeck, the editor of the *Genesis Rabbah* edition that he refers to. In that edition, 92:7 is on pp. 1145-6, and 4:24 and 17:20-21 are on p. 225. Jacobs also informs us (in endnotes, p. 116) that *Yalkut*, 1 Sam. 132 "refers to ten but lists only nine" (he does not say which one is left out); and that Gen. 4:24 is also mentioned as a *qal vachomer* in *Avot de-Rabbi Nathan* (version B) 44 and in JT *Sanh*. 10:1 (27d).

[36]   Wiseman claims "evil" *qal vachomer* to be a rabbinical category, pp. 174-6. As regards the "implicit" case, see further on.

[37]   In this context Jacobs mentions (in an endnote) A. Schwarz, Ch. Hirschensohn, Jofe Ashkenazi (in *Yephe toar*) and H. Strack.

[38]   This quotation, as indeed all those from the Bible in the present section, is taken from the Mechon Mamre website at www.mechon-mamre.org/p/pt/pt0.htm, which is based on the 1917 edition of the Jewish Publication Society. Note that the word "then" is an interpolation by the translator; it is not found in the original Hebrew. Likewise, the exclamation mark is an addition; a question mark may have been more appropriate. Obviously, the translator was influenced by the tradition that this statement is a fortiori.

interpretation of the word *meh* as meaning "how much more" therefore seems a bit forced to me. I would at best consider this as an "implicit" a fortiori argument, in the sense of one read into the text (more on such arguments later), since no number greater than 500 is actually specified in the conclusion (which has the form of a question). Nevertheless, because this argument is so universally accepted as a fortiori, just because it is one of the main ten listed in the Midrash, I do exceptionally count it as an explicit a fortiori.

The remaining 13 instances *we have in common* are: 1 Samuel 14:29-30, 2 Samuel 12:18, 2 Samuel 16:11, 1 Kings 8:27, 2 Kings 10:4, Jonah 4:10-11, Proverbs 15:11, Proverbs 19:7, Proverbs 19:10, Proverbs 21:27, Job 4:18-19, Job 15:15-16, Job 25:5-6. That these instances were found independently by two or more parties is of course no surprise. Anyone looking out for arguments of a certain kind, who has some idea as to how they go about, will notice them as he reads through the Bible. In my case, the research was more systematic. I looked at the wording of known instances of a fortiori discourse, and then sought other Biblical passages with the same wording using a concordance.

As regards Jonah 4:10-11, where God says: "Thou hast had pity on the gourd, for which thou hast not laboured, neither madest it grow, which came up in a night, and perished in a night; and should not I (*vaani lo)* have pity on Nineveh, that great city, wherein are more than sixscore thousand persons that cannot discern between their right hand and their left hand, and also much cattle?" Although this case lacks distinctive a fortiori language, it clearly has a fortiori intent. I did not have this case in the early editions of my *Judaic Logic*, but after finding it by chance added it on (as a final footnote to chapter 6) as of June 1998.

Because of my use of a concordance, no doubt, I found numerous cases apparently previously unknown. The 14 instances *I have but Jacobs lacks* are: 1 Samuel 17:37, 1 Samuel 21:6, 2 Samuel 4:10-11, 2 Kings 5:13, 2 Kings 18:23-24 and its repetition in Isaiah 36:8-9, Ezekiel 14:13-21, Psalms 78:20, Psalms 94:9-10 (3 instances), Daniel 2:9[39], 2 Chronicles 6:18[40], 2 Chronicles 32:15. Note that since Jacobs only mentions 35 cases and Einhorn enumerates 40, it may well be for all I know that some of these 14 cases were known to the latter and rejected by the former. But it seems unlikely – why would Jacobs reject any of these cases, which are all pretty clear and explicit?

Note that 1 Samuel 17:37 was publicized and analyzed in Addendum 4 of my *Judaic Logic* (as of 2001); I did not myself discover it, but had my attention drawn to it by a reader named Mark Leroux (from South Africa). Ezekiel 14:13-21 was not mentioned in my *Judaic Logic*: I only recently discovered it (in 2012). It may be paraphrased as saying: "More spiritual credit is required to stop more numerous negative Divine decrees than fewer ones; therefore, if holy men, like Noah, Daniel or Job, lack sufficient spiritual credit to prevent the execution of the four separate decrees of the sword, famine, evil beasts, and pestilence, then they lack enough credit to stop all four of these decrees together"[41]. The latter case was not easy to spot, because it is spread out over several verses; what helped me find it was the key phrase "How much more" (*af ki*, in Heb.) used in it.

As regards 2 Kings 18:23-24 and its word-for-word repetition in Isaiah 36:8-9[42], they are mentioned in my *Judaic Logic*, but I had considerable skepticism concerning them and so did not at the time count them as sure cases[43]. However, reviewing the argument involved at a later date, its a fortiori intent became clearer to me. Rab-shakeh (emissary of the king of Assyria) says: "Now therefore, I pray thee, make a wager with my master the king of Assyria, and I will give thee two thousand horses, if thou be able on thy part to set riders upon them. How then (*ve-ekh*) canst thou turn away the face of one captain, even of the least of my masters servants? and yet thou puttest thy trust on Egypt for chariots and for horsemen!" The Assyrian spokesman thinks that king Hezekiah is hoping for Egyptian chariots and horsemen; so he says to him: 'even if your force was increased by 2000 horses (which I am willing to give to you), you could not find warriors to ride them and therefore could not hope to defeat the invaders; all the more so, without such additional force, you cannot hope to defeat the invading force, even the least fraction of it'. I do, therefore, henceforth class these two identical cases as surely a fortiori.

Let us now look at the cases *Jacobs has but I lack*. Since I have never before analyzed these, I will do so now. I will first list the 8 instances I accept as explicit a fortiori argument, and thereafter deal with the instances he mentions that I consider as only at best implicit.

Judges 14:16. "And he (Samson) said unto her (his wife): Behold (*hine*), I have not told it my father nor my mother, and (*ve*) shall I tell thee?" This is a clear case of *qal vachomer*, using keywords (*hine/ve*) found elsewhere. So much so that I am surprised I missed it! The reason I did so was probably that the word *hine* is very often used in contexts where there is no a fortiori intent, so I did not closely examine every occurrence of it.

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[39]   As I explain in my *Judaic Logic* (6:3), although the given text of Daniel 2:9 is not directly a fortiori argument, since it consists of an order by the king coupled with a reflection as to its utility, the reasoning used by the king is indubitably a fortiori argument, so much so that it can be counted as effectively explicitly so.
[40]   This is a repetition of 1 Kings 8:27, which Jacobs does have.
[41]   The original wording of this argument is given in Appendix 1.
[42]   Such copy-and-paste repetition is surely useful for purposes of "higher criticism."
[43]   In chapter 6, where I write: "Note also: 2 Kings 18:23-24, and its repetition in Isaiah 36:8-9, might at first glance be construed as a-fortiori in style. But try as I might, I have not been able to make a clear a-fortiori argument out of it, however artificial and logically improbable."

Isaiah 66:1. "The heaven is My throne, and the earth is My footstool; where (*eizeh*) is the house that ye may build unto Me? And where (*eizeh*) is the place that may be My resting-place?" This passage obviously echoes the message of 1 Kings 8:27 and 2 Chronicles 6:18, though the wording differs somewhat; viz. that God is too great to be housed in an earthly abode. I perhaps missed it because the Hebrew operator used in it, *eizeh* (אֵי-זֶה), meaning *what?* (or *which?* rather than *where?* as this JPS translation has it) does not to my knowledge occur in other a fortiori contexts. Nevertheless, it is quite credible as a case of a fortiori discourse.

Jeremiah 25:29. "For, lo (*hine*), I begin to bring evil on the city whereupon My name is called, and (*ve*) should ye be utterly unpunished? Ye shall not be unpunished; for I will call for a sword upon all the inhabitants of the earth." Here again, we have the keywords *hine/ve*, sometimes indicative of *qal vachomer* intent. The a fortiori argument is that if God is willing to bring evil on the city whereupon His name is called, he is certainly willing to utterly punish less important kingdoms.

Jeremiah 45:4-5. "Behold (*hine*), that which I have built will I break down, and that which I have planted I will pluck up; and this in the whole land. And (*ve*) seekest thou great things for thyself? seek them not; for, behold, I will bring evil upon all flesh." Here again, we find the keywords *hine/ve* used. The a fortiori argument is that if God is willing to break down what He has built, etc., he is certainly willing to prevent the success of endeavors by Baruch ben Neriah.

Jeremiah 49:12. "Behold (*hine*), they to whom it pertained not to drink of the cup shall assuredly drink; and (*ve*) art thou he that shall altogether go unpunished? thou shalt not go unpunished, but thou shalt surely drink." Here again, note use of the keywords *hine/ve*. The a fortiori argument is that if God is willing to punish those who do not deserve it, he is certainly willing to punish those who do. Note the similar form of the three a fortiori arguments of Jeremiah mentioned here – it is indicative of their common authorship.

Ezekiel 33:24. "They that inhabit those waste places in the land of Israel speak, saying: Abraham was one, and he inherited the land; but (*ve*) we are many; the land is given us for inheritance." In this case, there is no keyword indicative of a fortiori intent; but that happens. There clearly is an a fortiori intent, even if the argument is logically rather weak. Why should 'many' be more assured of inheritance than just 'one'? Indeed, this is precisely what the next two verses (25-26), which are spoken by God, contend – that it is not quantity but moral quality that determines ownership of that land:

> "Ye eat with the blood, and lift up your eyes unto your idols, and shed blood; and (*ve*) shall ye possess the land? Ye stand upon your sword, ye work abomination, and ye defile every one his neighbour's wife; and (*ve*) shall ye possess the land?"

This case is very interesting, because it provides a Biblical example of *rebuttal* of a weak a fortiori argument by attacking the major premise. Note well that God's reply is not itself an a fortiori argument, but an objection to such argument. This form of counter-argument is later practiced routinely by the rabbis of the Talmud, under the heading of *pirka* (in Aramaic) or *teshuvah* (in Hebrew). No doubt there are many such counter-arguments in the Bible, which we should henceforth lookout for and register. I have not looked for or noticed such rebuttals in the past.

Job 9:13-14. "God will not withdraw His anger; the helpers of Rahab did stoop under Him. How much less (*af ki*) shall I answer Him, and choose out my arguments with Him?" Job considers himself as less worthy than "the helpers of Rahab," therefore he is more than them bound to incline before God's judgment. This is a clear case of *qal vachomer*, using keywords (*af ki*) found elsewhere. I am very surprised I did not spot it!

Nehemiah 13:26-27. "Did not Solomon king of Israel sin by these things? yet among many nations was there no king like him, and he was beloved of his God, and God made him king over all Israel; nevertheless even (*gam*) him did the foreign women cause to sin. Shall we then (*ve*) hearken unto you to do all this great evil, to break faith with our God in marrying foreign women?" The gist of the argument is: If even Solomon could be caused to sin by foreign women, will not the lesser men of today be likewise caused to sin? This is clearly a fortiori argument, even if the operators (*gam, ve*) are rarely used.

We have thus drawn eight new, credible and pretty explicit, a fortiori arguments from Jacobs' (or Einhorn's) list. Six use known operators: four use *hine/ve*, one uses *af ki*, one uses *gam/ve*; one involves only the ubiquitous conjunction *ve*; and one involves the previously unheard of operator *eizeh*.

Jacobs lists in his paper another three Biblical passages that in his opinion involve a fortiori arguments. The first of these, which he has found mentioned in *Genesis Rabbah*, is:

Genesis 17:20-21. "And as for Ishmael, I have heard thee; behold, I have blessed him, and will make him fruitful, and will multiply him exceedingly; twelve princes shall he beget, and I will make him a great nation. But (*ve*) My covenant will I establish with Isaac, whom Sarah shall bear unto thee at this set time in the next year." Jacobs casts this in a fortiori form as follows: "If Ishmael, the son of the handmaiden will be blessed in this way then all the more will Isaac, the son of Sarah, be blessed."

Although I can see that such an a fortiori argument can certainly be read into the text, I do not agree that it is the only way the passage can be read. God may simply be saying to Abraham: I have blessed Ishmael thus and thus, but My covenant I will not establish with Ishmael but only with Isaac. The emphasis in this alternative reading is clearly different, and not a fortiori. Note moreover, that whereas Jacobs' a fortiori interpretation makes no mention of the covenant, it is central to my reading. For this reason, I would say that the proposed a fortiori argument qualifies as implicit rather than explicit. This is using the word "implicit" in the sense Jacobs uses it when presenting the next two cases. These instances are mentioned in the so-called *Baraita* of R. Eliezer b. R. Jose the Galilean:

Psalms 15:4. "He that sweareth to his own hurt, and changeth not." This verse could be read, as Jacobs has it, as saying that if he (i.e. the good man that the psalmist is describing) doesn't go back on his word when it is hurtful not to, then he certainly won't do so when it is for his good. But we could simply read this as saying that when the good man utters an oath, he sticks to it no matter how strong the pressure to break it increases. There is no necessity for the a fortiori interpretation; it is read into the text, rather than drawn from it. There is no call for it, because if the man utters an oath which causes him pleasure rather than pain, he obviously will be under no pressure to break it. This is not a conclusion obtained by a fortiori inference, but something everyone can confirm by introspection. So, really, this a fortiori reading is rather artificial. No doubt, it was concocted simply because its author needed some examples for teaching purposes.

Psalms 15:5. "Nor taketh a bribe concerning (*al*) the innocent." Two translations of this verse are possible, the Hebrew word *al* (meaning: on) being a bit equivocal (even in English).

Let us first consider the translation used by Jacobs: "Nor taketh a bribe *to side with* (*al*) the innocent." This verse can be read, as Jacobs has it, as saying that if he (i.e. the good man) won't take bribe to rule in favor of an innocent person, then he certainly won't do so regarding a guilty party. But we could simply read this as saying that the good man would not take a bribe even if he was being bribed to judge a matter as he would without being bribed, i.e. in favor of the innocent. It is true that, in this case (in contrast to the previous one), the a fortiori argument just indicated can additionally be constructed, and it makes a valuable prediction. So here we are justified in referring to an "implicit" a fortiori argument. It is not explicit, because the text can be read in a first phase without an a fortiori thought. But it is implicit, in that, if we dig deeper into it, we can indeed use it to make a useful a fortiori inference.

Let us now consider the alternative translation of the same verse given in the JPS 1917 edition: "Nor taketh a bribe *against* (*al*) the innocent." We can interpret this translation like the previous one, albeit with an interpolation: if he (i.e. the good man) won't take a bribe *not* to rule against an innocent person, then he certainly won't do so regarding a guilty party; and we can say more about it as before. However, a quite different, more literal approach to this translation is also possible: we could simply read it as saying that the good man would not take a bribe against an innocent person, i.e. in favor of a guilty one. We might now attempt the following a fortiori argument: if he (i.e. the good man) won't take a bribe to rule against an innocent person, then he certainly won't do so regarding a guilty party. But this argument is a *non sequitur*, since someone might well refuse to rule against the innocent for a bribe, but accept to rule against the guilty for a bribe, thinking that since he intended to rule against the guilty anyway, no harm is done by taking a bribe for it. Therefore, in this translation and reading there is no a fortiori adjunct, whether explicit or implicit.

Thus, if we qualify as "explicitly" and "implicitly" a fortiori argument, respectively, "a text that can only be read as a fortiori" and "a text that can be read as a fortiori but can also readily be read otherwise (i.e. more simply)" – we would have to say that Gen. 17:20-21 is implicit, that Ps. 15:4 is not a fortiori at all, and that Ps. 15:5 is implicit if read one way and not a fortiori at all if read another way. This is contrary to Jacobs, or rather to the rabbinic sources he refers to, who read these arguments as respectively explicit, implicit and implicit. As for the two examples of explicit a fortiori, which Jacobs mentions as given in the said *Baraita*, namely Jeremiah 12:5 and Esther 9:12 – we would for our part agree that the Jeremiah instances are explicit, but insist (for reasons already put forward) that the Esther example is (if at all a fortiori) at best implicit[44].

Clearly, we have here a serious divergence of views. I have to say that I have in the past, until I read Jacobs' present article, assumed that the distinction made by Eliezer ben Jose, between a fortiori arguments that are *meforash* (explicit) and those that are *satum* (implicit), was referring to how much of the argument's elements are laid out in the text at hand. If the a fortiori premises and conclusions, with all their terms or theses, are all fully laid out in the given text – then that text is a fully explicit a fortiori argument. If one premise or the conclusion are missing, or some of the terms or theses involved are missing – then that text is partly implicit to varying degrees. By that standard, of course, most if not all arguments in Scripture are partly implicit.

But Jacobs' article suggests that the rabbis' "explicit" means *sufficiently* explicit that there can be no interpretation other than an a fortiori one, while their "implicit" means *not so* explicit that there can be no interpretation other than

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[44]   Although, to repeat, I am still counting the Esther example as explicit, so as not to go against this too well-established tradition.

an a fortiori one. At least, this is how I now understand these expressions. It could be that the rabbis do not draw the lines so clearly, and understand them sometimes this way, sometimes that way. In any case, to conclude this discussion, the three arguments above listed are – as far as I am concerned, in the light of the above analyses – not to be listed among the explicit a fortiori arguments. They are possible interpretations of the texts, or artificially read into the texts, but the texts in themselves allow of readings that are not a fortiori.

Why is this issue important? Because it relates to attribution and dating. When a Biblical text clearly has an a fortiori intent, we may regard it as an explicit instance of Biblical a fortiori argument. If, however, the a fortiori intent of the Biblical text is not so obvious, and has only been brought out later in time by a rabbinical or other commentator, we must count it as only implicitly a fortiori, and attribute the a fortiori argument as such to the historically later commentator. It is not an issue of who discovers the a fortiori argument, note well, but of whether or not the author of that passage of the Bible worded it with a manifest a fortiori intent. If the a fortiori argument has later been read into the text, rather than found in it, then its author is really the person who proposed the interpretation. This is commonsense hermeneutics.

Of course, the rabbis consider that whatever they read into a Biblical text was indeed intended by that text, since God – its ultimate author – is all-knowing. But, even granting their premises, their conclusion does not follow. That is to say, God may well have foreseen the rabbinical interpretation, but that does not make it any the less an interpretation. Such foreknowledge is not indicative of an actual or direct intent, but only at best of a potential or indirect one. The explicit text constitutes the primary message; other information that can eventually be derived from that message is not strictly part of it, but at best an implicit adjunct to it. Sometimes, of course, it is highly debatable that the original text allows for a certain interpretation; and in such case, the interpretation must be characterized as forced rather than implicit[45]. It would be irrational to accept unquestioningly whatever the rabbis claim; they are, after all, just human beings.

Jacobs additionally mentions (in an endnote) three Biblical passages presented as a fortiori arguments by **Chaim Hirschensohn**[46]. Jacobs rejects these examples as "extremely doubtful," and I incline to agree with him. To my mind, they are at best implicit a fortiori arguments, but certainly not explicit ones. The first two texts in question are the following:

Genesis 3:22. "Behold (*hen*), the man is become as one of us, to know good and evil; and now (*ve-atah*), lest he put forth his hand, and take also of the tree of life, and eat, and live for ever. Therefore (*ve*), [He] sent him forth from the garden of Eden, to till the ground from whence he was taken."

Genesis 11:6-7. "Behold (*hen*), they are one people, and they have all one language; and this is what they begin to do; and now (*ve-atah*) nothing will be withholden from them, which they purpose to do. Come (*habah*), let us go down, and there confound their language, that they may not understand one another's speech."

It is interesting that the two verses, though chapters apart, use the same language (*hen/ve-atah*) and have similar form. Also note that in both cases, one of the operators used (*hen*) is sometimes indicative of a fortiori discourse, and the argument is concerned with increasing magnitudes of something. The argument involved can be paraphrased as follows: *This event is bad enough, therefore to avoid an even worse event we had better take certain precautions.* This is an interesting form of reasoning in itself, but it is clearly causal and ethical, rather than a fortiori as Hirschensohn reportedly claims.

We could admittedly formulate an a fortiori argument from it as follows: P is worse (R) than Q, and Q is bad (R) enough to be combated (S); therefore, P is bad (R) enough to be combated (S). But where in the text does it say that Q was fought against? It only says that P is to be fought against. So this a fortiori argument, if at all implied, must be characterized as implicit. It is anyway not the essence of the explicit discourse facing us, which has it that a minor problem (the "bad enough" clause) *could well eventually develop into* a major problem (the "even worse" clause), and for that reason *some preemptive action* against the latter is called for before it happens. The said a fortiori argument is perhaps implied by the text, but the text evidently tries to communicate considerably more than just that.

The third text is: **Genesis 17:17**. "Then Abraham fell upon his face, and laughed, and said in his heart: Shall a child be born unto him that is a hundred years old? and shall Sarah, that is ninety years old, bear?" It is difficult to perceive the a fortiori argument Hirschensohn had in mind here. Perhaps his thought was that it is unlikely enough for a hundred year old man to have a child, and therefore *even more* unlikely for a ninety year old woman to do so. But frankly, was that Abraham's thought? No, he was simply saying that it is unlikely for both a hundred year old man and a ninety year old woman to have a child. It is a statement, not a process of inference.

Certainly, in this case, the literal reading is not a fortiori, so that if an a fortiori argument be read into the text, it is at best implicit. But moreover, the a fortiori reading seems rather forced. Since we can fully understand the text without it, it serves no purpose other than to inflate the list of Biblical a fortiori arguments. Therefore, I would not even

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[45]   An important case in point that we have seen is the Gemara (*Baba Qama* 25a) interpretation of Numbers 12:14-15.
[46]   Israel, 1857 – 1935. The work cited is *Berure Ha-Middot* (Jerusalem, 1929), pp. 40-45.

include this case as an example of implicit a fortiori. Thus, while the first two examples could conceivably qualify as involving an a fortiori discourse implicitly, the third is much less credible. In any event, none of these three cases is explicit.

General observations. Let us now take a closer look at various general observations in Jacobs' essay. To begin with, it is evident that even in 2005 he had not yet grasped the actual form of a fortiori argument, since he here still describes it very superficially as "If A is so then B must surely be so; if the 'minor' has this or that property, then the 'major' must undoubtedly have it." This is what he has in the past called 'simple' a fortiori argument. He still, note also, fails to detect the use of what he has called 'complex' a fortiori argument in the Bible. He rightly remarks that the Rabbis learnt this form of inference from its occurrences in the Bible itself, and then used it "as one of their hermeneutical principles by means of which they expand and elaborate on the Biblical teachings." And this fact stimulates his present research into the actual examples of the argument in the Bible.

However, it is very surprising to see Jacobs assert (in an endnote) that "There does not appear to be, in fact, any real parallel to the *qal va-homer* in Greek thought." This is, as demonstrated in the present volume, quite off the mark – a fortiori argument is quite present, and consciously so, in Greek (and then Roman) literature, even if not as frequently as in rabbinical literature. He is here going further than he has in the past, where he only (and rightly) contended, against the apparent claims of Adolf Schwarz, that the identification of this form of argument with Aristotelian syllogism is "untenable." That a fortiori argument is not syllogistic does not imply that it was not used by the Greeks! The latter did not only think syllogistically, any more than the rabbis only thought by means of a fortiori argument. Moreover, to admit that the Greeks used a fortiori argument is of course not the same as to claim that the rabbis learnt it from them. So Jacobs' position in this matter, even if expressed offhandedly, is very surprising.

It is a pity that Jacobs did not push his analysis of the a fortiori arguments he lists more deeply. While he acknowledges the rabbis' debt to the Biblical occurrences of *qal vachomer*, he does not sufficiently examine how they actually interpreted those arguments. Notably, while he reads the *qal vachomer* in Numbers 12:14 correctly, saying: "if when [Miriam's] human father showed his disapproval of her actions she would hide herself in shame for seven days then when the Lord shows His disapproval all the more should she be shut away for seven days," he does not look further into the matter and discover the significantly different interpretation given by the Gemara in Baba Qama 25a, and the Pandora's Box of interesting problems (and opportunities) that the latter creates.

Nevertheless, Jacobs makes some valuable general observations:

> "From all that has been said it is surely well established that the argument from the minor to the major is used frequently throughout the Old Testament. Its use is not limited to any single phase in Israel's history but, it would appear, was employed in all periods. Neither is the usage confined to any single book of the Old Testament nor to any particular document, stratum, and trend… Moreover, as in many of the examples quoted, its use is generally of a formal nature, beginning with *hien* or *hinneh* and concluding with *'eykh* or *'aph*."

I came to similar conclusion in my *Judaic Logic*. But Jacobs takes the reflection further, raising "important questions, hitherto barely considered by Old Testament scholarship, regarding the use of rhetoric in ancient Israel." He cites O. Eissfeldt[47], who suggested that there were men and women "specially skilled in speech," using argumentative techniques acquired through "tradition and 'training'," resorting to "certain fixed forms for speech" and rhetorical devices such as "first obtaining from the person addressed an admission which does not appear to be relevant to the matter in hand," which "then compels him to grant the request which is really involved." Jacobs concurs, in view of the evidence provided by his listing of Biblical *qal vachomer*.

Such tradition and training is of course evident in the rabbinic period, Jacobs adds, when "formal argument was consciously and extensively cultivated" and "there are certain stereotyped rules" of argumentation. He then asks: "Was there anything like this in the Old Testament period?" and replies: "there seems to be no doubt that the answer should be in the affirmative." He admits, however, that "it is hard to find anything like an explicit reference anywhere in the Old Testament to schools in which rhetoric was taught." He could have buttressed his case by adducing that the rabbis did and do believe that such schools (*yeshivot*) existed throughout the past. The patriarch Jacob is said to have studied in the tents of Shem and Eber (*Genesis Rabbah* 63:10); king David is said to have studied with his counselor Ahithophel (*Pirqe Avot* 6:2); and so on.

In my view, to be honest, such claims are largely anachronistic, projecting later mores onto earlier times. It is not inconceivable that there were, very early on, educational institutions of sorts that organized common study of and reflection on knowledge inherited from the past. The issue is, as of when such institutions can be credibly claimed, and what it is that was studied in them. While transmission of knowledge and skills by village adults and elders to children and youth can be classified under the heading of education and is as old as mankind, it is less certain as of when in history the formal study of Torah and related argumentative skills began in Israel. I would say it developed

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[47]   In *The Old Testament: An Introduction* (Oxford, 1965), pp. 12-15.

apace in the period after the Return from Babylon after the First Exile, i.e. the formative period of the rabbinical doctrine and class. This is suggested, for instance, in the Mishnaic *Pirqe Avot*, which refers (1:1) to the *Knesset Hagedolah* (the Great Assembly).

This hypothesis seems most likely, in view of what was happening at the same time in other nations near and far. What is evident when we study world history is that cultural developments tend to be (increasingly over time) worldwide rather than local. Many major developments occurred as of the middle of the first millennium BCE, as if a new phase in human intellectual evolution was taking place. Suddenly, it seems, existing civilizations burst with newfound energy, producing religious and philosophical thoughts more sophisticated (at least on the surface) than ever before. Though scattered, they awoke simultaneously, in various directions, but also somewhat comparably.

In India, the ancient Vedic religion began its transformation into Hinduism, and Buddhism was founded. In China, Confucianism and Taoism emerged. In Greece, philosophy flowered in earnest, with the advent of Socrates, Plato, Aristotle, and many others. In Israel, Judaism came increasingly under the authority of Torah scholars[48]. This 'rabbinic' Judaism was apparently planted at the beginning of the Second Temple period and gradually grew and took shape in the following generations, till it fully flowered in the Mishna (which then stimulated the Gemara and subsequent rabbinic works). The intellectual growth in Israel, involving increasingly legalistic thinking, and therefore to some extent logical reflection, was thus rather typical of that epoch, and can only with difficulty be projected backwards into earlier ones.

Be that as it may, Jacobs ends his reflections with an interesting suggestion, also drawn from Eissfeldt, that the Hebrew root *dbr*, used to refer to 'speech', may in fact often be used with the intention to mean 'argument'. An example he gives is Gen. 44:18, where Judah begins his plea before Joseph by saying: "O my lord, let thy servant, I pray thee, speak a word in my lord's ears." Jacobs comments: "Since the expression *yedabber dabhar* is used, should it be translated as 'present an argument'?" Similarly, in other passages, 'speak rightly' might be taken to mean 'argue convincingly', 'these are the words' might be taken to mean 'these are the arguments', and so forth. This insight seems credible to me. All this goes to show in what directions and how far Jacobs' research into Biblical use of *qal vachomer* drove his reflections.

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[48] Perhaps starting with "Also we made ordinances for us," in Nehemiah 10:33. The Hebrew word used is *mitsvot*, which is usually translated as commandments. The laws gradually enacted by Jewish lawmakers were, it is worth noting, distinctively based on Torah law. They were not arbitrary, but guided and circumscribed by the strong moral standards already instituted by that document. The 'legalism' involved here is of a very different sort than that found in the same period of history in, say, China.

17. Heinrich Guggenheimer

Heinrich Guggenheimer wrote a paper called "Logical Problems in Jewish Tradition" (1966)[1] which is often referred to by others. Hyam Maccoby in *The Philosophy of the Talmud* (2002)[2] made the following remarks concerning it:

> "Heinrich Guggenheimer (pp. 181-85) gives a cogent account of the dayyo rule in terms of pure logic, saying that, in virtue of this rule, the qal va-homer argument is 'an admirable solution (the only one known to me) of the problem of making analogy an exact reasoning'. Guggenheimer also gives a rendering of the qal va-homer in the terminology of modern mathematical logic. He does not mention, however, the Amoraic discussion which takes the rule of dayyo out of the realm of logic, or the considerable medieval discussion based on b. Bava Qamma 25a."

1. Tout un programme

I have not managed to find a copy of Guggenheimer's essay to fully comment on it here. Actually, I did read it many years ago, though I do not recall seeing in it the clarifications and proofs Maccoby claims to have seen in it. Of course, what you see or don't see depends on your own level of logical comprehension[3]. I do mention Guggenheimer's essay in my *Judaic Logic* (in a long footnote to chapter 8.2) – very critically – in relation to his statements "The inner logic of the Law... is definitely hostile to modalities.... The Talmud avoids all attempts at modal logic" (pp. 179, 193). So inaccurate a statement makes me very skeptical at the outset as to his overall comprehension.

However, I notice that Allen Wiseman[4] quotes Guggenheimer at some length and analyzes his viewpoint in some detail, so we can here look further into the latter's theory of a fortiori argument. What appears from this third person account is that Guggenheimer has no precise formula and proof for such argument. He makes some general observations and vague speculations, couched in the language of modern logic with liberal use of fancy Greek symbols, but the bottom line is missing.

Guggenheimer begins by conceiving "the sentences of Scripture, as far as they have legal relevance" as constituting "a system of propositions $\sum_0$." This is, in a way, stating the obvious: taking any document as a whole, all the sentences in it form an object of study. But do they form a logically coherent "system"? Those who believe the Talmud infallible may think so. But if we look at the Talmud in detail, there are many controversies in it at many levels; so, it can hardly be viewed as a unitary whole.

Be that as it may, Guggenheimer then proposes "for the sake of clarity" that we "assume that all propositions are of the form '$\varphi_0(a)$ *is true*' where *a* is an element of a given set (of relevant subjects) and $\varphi(x)$ a predicate." Now, this is less clear than it seems. If by this form we presume that all the propositions have the specific form of predication of φ to a – we might ask on what basis the author makes such a simplistic assumption at the outset. We know that not all sentences in our minds, or even in the Talmud, fall in this category. Therefore, we must take his form to mean more vaguely that predicate φ stands 'in some relation or other' to subject a.

He then presents the problem of logical analysis of the Talmud in the following words: "Our problem is to find a set of predicates $\varphi(x)$ for which the *provable* propositions '$\varphi_0(a)$ *is true*' under the rules of propositional logics and certain other operations generate a logical system $\sum$ which consists of the true statements of Talmudic Law." He does not say how exactly these propositions are to be "proved," but simply assumes that means will be found. Presumably from outside the system, since he later says that such systems are "incomplete." In any event, these propositions will be used to build up a "logical system" that will churn out "the true statements of Talmudic Law" (p. 181).

This ambitious programme is, note well, partly descriptive and partly prescriptive. He apparently aims to order the data found in the Talmud, distinguishing source material (the "provable" propositions) and derived material ("the

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[1]   In: *The Great Society: Confrontations with Judaism. A Symposium*. Ed. Philip Longworth. London: Blond, 1966.

[2]   London: Routledge Curzon, 2002. See online excerpts at: www.mucjs.org/qalvahomer.htm.

[3]   As regards the deficiencies in the Gemara mentioned by Maccoby, the latter as we have seen was not really in a position to judge the matter since he had not himself fully correctly understood it.

[4]   *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions* (Waterloo, Ont.: University of Waterloo, 2010), pp. 89-91.

true statements of Talmudic Law"); but there is no guarantee offhand that the statements he concludes as "true" (or not so) are those that the authors of the Talmud would consider "true" (or not so).

Indeed, Guggenheimer implies a divergence, when he points out that "We know that the system of statement of $\sum_0$ contains contradictory statements and therefore the usual rules of logics cannot be used in it," and insists that "We shall ask that the system $\sum$ be free of contradictions if the usual rules of logics are applied to its statements." He predicts that the outcome will be "not a single system $\sum$ but a whole tree of systems." Maybe because there is likely to be many alternative resolutions of the contradictions?

So far, to my mind, Guggenheimer has said nothing of any great moment. All he has done is to propose the Talmud as a field of study, in a perspective comprehensible to adherents to 'modern logic'. He has certainly never gone ahead and executed his programme, traversing the whole Talmud with the systematic ordering he has in mind. It is just something he vaguely imagines as possible on purely intellectual grounds. It remains to the end an untested hypothesis, a speculation.

He apparently assumes he knows all there is to know about logic in advance, and does not foresee any snags on the way – other than occasional contradictions, which he does not tell us how he will resolve. But in truth the 'modern logic' conception of logic is very naïve. It oversimplifies complex operations, so as to force them into preconceived forms. This programme is doomed to failure from the start.

2. Theory of a fortiori

Turning now to the more specific problem of a fortiori argument, Guggenheimer says: "This is a fundamental procedure, so much so that it is called *din*, that is, logic….an admirable solution (the only one known to me) of the problem of making analogy an exact reasoning. As such it is valid not only as a rule of transfer, but also as a rule of derivation within the new system. It is the essential extension of Talmudic logics over propositional (Aristotelian) logics. It works because all [such] systems are incomplete." He is right in the observation that a fortiori argument is a more exact form of reasoning (derivation) than mere analogy (transfer).

I do not know what he means when he says that a fortiori argument is the essential "extension" of Talmudic logics "over" propositional (Aristotelian) logics. Does he mean that Aristotelian logic historically lacked a fortiori argument, or that it is logically incapable of explaining it? Either statement would be inaccurate. Aristotle and other non-Talmudic players were aware of a fortiori argument and did discuss it, even if not as thoroughly as they discussed other forms of argument. And furthermore, as I have shown, a fortiori argument is reducible to previously known forms of argument, namely hypothetical syllogism and inference of quantitative comparisons. I also do not know what he intends when he says: that a fortiori argument "works because" Talmudic systems are incomplete. Why should the incompleteness of Talmudic systems favor the working of a fortiori argument?

These statements of Guggenheimer's do not clearly ask or answer the essential questions: what is the exact form (or what are the exact forms) of a fortiori argument? And: is a fortiori argument as such, whether used in the Talmud or elsewhere, valid or not (or which of its possible forms are valid and which not)?

But fortunately, Guggenheimer gets more a bit specific. He observes that predicates may be compared and thus ordered; one predicate may be "stronger" than another. For example, "the severity of the penalty for an offence"[5]. He points out that "Mathematically, the main property of an order relationship is its transitivity;" that is, given comparable predicates, if one is stronger than another, and the latter is stronger than a third, then the first is stronger than the last. Such "ordering," he warns us, concerns predicates, not whole "statements."

From this[6] we learn that Guggenheimer was aware that a fortiori argument somehow involves quantitative comparisons. But this is not yet, by far, a thorough understanding of a fortiori argument. Finally, he attempts the following definite thesis:

> "If there exists a predicate $\chi(x)$ and elements a and b such that '$\chi(b)$ is true' is a *provable* proposition in the original system, but neither '$\chi(a)$ is true' nor '$\chi(a)$ is false' can be proven on the basis of the available data without recurrence to a new rule, and if a is stronger to[7] b, then the rule *kal vahomer* states that some statement '$\mu(a)$ is true' must hold for some predicate $\mu(x)$ which is not weaker than $\chi(x)$. By the axiom of definiteness, the only possible solution is $\mu(x) = \chi(x)$."

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[5]   As in Numbers 12:14-15, where the penalty prescribed for offending God would rationally seem to have to be more severe than that for offending one's father – although, in truth, the Bible prescribes the same penalty in both cases.

[6]   I have left out some of Guggenheimer's discourse, considering some ideas as too irrelevant to even mention. For instance, his "predicate of second type," which tells us that one predicate is stronger than another, is a useless complication. Again, his claim that transitive relations may be circular is absurd, the relative severities he gives as examples being mere figments of his imagination that are nowhere upheld in the Talmud and do not make sense anyway.

[7]   I do not know if the word "to" is found in Guggenheimer, or is an error by Wiseman.

What does that signify? In an attempt to decipher what he had in mind, let us try to recast the argument as he sees it in standard form. What is evident is that Guggenheimer is far from clear as to the roles of the terms he mentions. The "elements" *a* and *b* are presumably, since *a* is stronger than *b*, respectively the major and minor terms (P and Q). And they are subjects, since a predicate χ*(x)* is applicable to them. There is no mention of a middle term (R), i.e. no statement telling us in respect of what *a* is stronger than *b*. Presumably, predicate χ*(x)* plays that crucial role, somehow. The predicate μ*(x)* is apparently intended as the subsidiary term (S), since it figures in the conclusion. Granting these equations, we might propose the following positive subjectal a fortiori argument as representative of what Guggenheimer perhaps had in mind:

> *a* is more χ than *b* (is χ),
> and *b* is χ enough to be μ;
> therefore, *a* is χ enough to be μ.

From this we see that what he subconsciously meant when he says that "*a* is stronger than *b*" is that χ*(a)* is greater than χ*(b)*. This is effectively his major premise. When he says that "'χ*(b)* is true' is a *provable* proposition in the original system," he means that '*b* is χ to some degree' is a given. This is part of his minor premise. When he tells us that "neither 'χ*(a)* is true' nor 'χ*(a)* is false' can be proven on the basis of the available data without recurrence to a new rule," he means that '*a* is χ to some degree' is *not* a given. In fact, it *is* implied by the sentence "χ*(a)* is greater than χ*(b)*," which is the intent of his "*a* is stronger than *b*" – but he does not see that. What he is trying to say, anyway, is that the status of χ*(a)* will be settled as part of the conclusion, which has not yet been determined.

Now, when he refers to "the rule *kal vahomer*," he is *postulating* that there is an inference "rule" – whether in logic generally or in Talmudic logic – which allows us to draw *some* (yet unspecified) conclusion from the given premises. On what basis he assumes such inference logically permissible he does not say[8]. So his words "the rule *kal vahomer* states" should not be taken as suggesting that he has already brought to bear some known and authoritative "rule." For the moment, at least, he is merely describing a hypothetical process; he has not demonstrated its universal or local validity, note well.

The "rule" he proposes is "that some statement 'μ*(a)* is true' must hold for some predicate μ*(x)* which is not weaker than χ*(x)*." This is useful to us, not only to flesh out his putative conclusion, but also his minor premise. For as we have seen, his minor premise is still at this stage only partly formulated. It is thanks to the mention of μ here that we can formulate his minor premise, namely that "*b* is χ*(b)*, and so χ enough to be μ, specifically μ*(b)*." Although he does not specifically mention μ*(b)*, it is implied in his general term μ*(x)*. This tells us that '*b* is μ to some degree', and so amplifies the minor premise.

Where he refers to "some predicate μ*(x)* which is not weaker than χ*(x)*," he supposedly has it in mind that 'there is a threshold value of χ that must be attained before *a* or *b* (or any *x*) can be μ'. That is, to be μ to any degree, something *x* must be χ enough. This is quite a perspicacious insight on his part. His rule especially predicts that 'μ*(a)* is true' will appear in the conclusion – i.e. that the conclusion will include the information that '*a* is μ to some degree'.

Note that, although he forcefully says that this "must hold," he is still in the realm of speculative insight. Furthermore, his comparison of μ*(x)* to χ*(x)* betrays some confusion as to the respective roles played by these two predicates. Strictly speaking, his condition that μ*(x)* be "not weaker than" χ*(x)* is an error. These two terms may not be comparable. This gap in his understanding, as we shall next see, makes him formulate the conclusion erroneously.

Thus far, he has approximately identified the components of the putative conclusion, viz. '*a* is χ enough to be μ'; but he has not yet pinpointed a specific value for μ. Is the value to be μ*(a)*, as his statement "'μ*(a)* is true' must hold" seems to imply, or is the value to be μ*(b)*? This is where his "axiom of definiteness" comes into play, and prescribes the lesser predicate μ*(b)* to subject *a* in the conclusion. This is clearly the intent of his formula "the only possible solution is μ*(x)* = χ*(x)*." He instinctively opts for the known minimum value, and rightly so, even if he has not clearly verbalized his choice.

The correct explanation is as follows. Even though subject *a* is χ*(a)*, we have no given that χ*(a)* is enough to give rise to μ*(a)*. All we are given is that χ*(b)* is enough to give rise to μ*(b)*, and that χ*(a)* implies χ*(b)*. Whence, we can only deduce that *a* is μ*(b)*. This is what Guggenheimer means by "the only possible solution" – that we do not have enough information to deduce more than this minimum, and cannot conclude that '*a* is μ*(a)*'.

His reference to "μ*(x)* = χ*(x)*" is not correct. It again only reflects his personal confusion as to the precise role played by these two (and indeed all) terms. The subsidiary term obviously does not equal the corresponding middle term, which may be something quite different and incommensurable. What he was trying to say is that the value of μ in the

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[8]   To my mind, anyone who speaks of "rules" of inference in that way does not really know logic. Logic is not about abstract "rules" imposed on mankind by modern logicians; it rather has to do with *understanding*.

conclusion cannot exceed the value of μ in the minor premise (which is true, so long as no additional premise is brought into play to justify proportionality).

From this reading, we can comprehend Guggenheimer's "axiom of definiteness" as corresponding (in the present context, at least) to my "principle of deduction." There is an "axiom of definiteness" in some versions of set theory (though others eschew it), having to do with the equation of certain items. But I am not sure that Guggenheimer's "axiom of definiteness" strictly corresponds to the latter in the present specific context; I think he just put forward this impressively named idea to buttress his conclusion, i.e. to make it seem "scientifically" justified.

Guggenheimer had to appeal to a vague "axiom" – a seemingly unassailable first principle, however obscure – in order to rationalize his proposed conclusion, because he was not clear as to its precise justification. And he was not clear about its precise justification because he was not clear about its precise content. Knowing the latter, we can more clearly see that the minimalist conclusion μ(*b*) is all we can draw, because we do not have sufficient information to draw a more weighty conclusion, viz. μ(*a*).

Needless to say, all the above effort refers only at best to positive subjectal a fortiori argument. As with many other researchers, he does not mention negative subjectal, or predicatal, or implicational, moods of the argument. Also note that, though he rightly upholds the minimal non-proportional conclusion, he does not foresee the possibility of a proportional conclusion given additional information.

3. A faulty approach

Let us now review our examination of Guggenheimer's thesis. Although we are able to guess at what his intentions probably were, we can see after detailed analysis that he did not succeed in putting them down on paper convincingly. He perceived some relevant aspects of a fortiori argument, but not all. He understood some relevant aspects of a fortiori argument, but not all. He spotted the terms involved, but not their exact roles in the discourse. He roughly intuited the right conclusion, but due to his confusion was not able to formulate it correctly. On the whole, then, his analysis was incoherent and of little value.

We should also take note of the following faults in his approach. The exact forms of the premises and conclusions, and positions of the terms in them, are not exposed. Vague statements are considered sufficient. No logical explanation is given as to why the putative conclusion proceeds from the given premise. The inference is merely rationalized with reference to an arbitrarily declared "rule," instead of being justified by rational insight. Additionally, an obscure "axiom" is brought into play as convenient.

And lo and behold, the "axiom" proposed here just happens to correspond, at least in some circumstances, to the rabbinical *dayo* principle that Guggenheimer is trying to explain or justify: "This particular application of the principle of definiteness is known as *dayo*, *Baba Kama*, 24b. Its Biblical root is Numbers XII, 14 concerning the punishment of Miriam for her slander of Moses. The comparison is between punishment by the Deity and punishment by the father. Even though the first is infinitely stronger than the second, the punishment is quantitatively the same."

(I should add that Guggenheimer's consideration of the *dayo* principle here only covers one version of it, the objection of the Sages in reaction to R. Tarfon's first argument in the Mishna Baba Qama 2:5. The second objection of the Sages, in reaction to R. Tarfon's second argument, is significantly different, and remains unexplained by Guggenheimer's interpretation.)

What we have here is "custom-built" logic. From the point of view of formal logic, Guggenheimer's thesis is non-existent, a fantasy. He has taught us nothing. All he has said is that he believes there is, *somehow*, such a thing as a fortiori argument, and it does, *somehow*, as the rabbis seem to have advocated[9], yield a non-proportional conclusion. His 'logic' is one of vague and unsubstantiated claims.

Actually, Guggenheimer does not make these claims in his own name, but regards them as implied in the Talmudic system of logic. According to Wiseman (note that by "QC" he means *qal vachomer*):

> "Guggenheimer says that the QC as a rule 'transcends elementary logics' and derives from the Bible. 'Accordingly, the statement of the QC rule itself is an element in the system ∑ … limited to cases in which the statement χ(*b*) is true can be proven', both without the use of the QC and by it alone. In all, although he admits that the QC can fail, it can work in certain cases."[10]

Thus, Guggenheimer's claims are not intended by him as absolute, but as relative to Talmudic logic specifically. They are not universal logical principles, but descriptive of a particular world of discourse. This new claim is,

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[9]   As a matter of fact, the rabbis did not consistently advocate or apply the *dayo* principle. Guggenheimer apparently did not study the Gemara he cites very closely. As for the claimed axiom or principle of definiteness, Wiseman is quick to reject it, rightly pointing out that proportional a fortiori argument is a common occurrence in both general and rabbinic discourse (p. 94).

[10]   Guggenhiemer, p. 183; Wiseman, p. 90.

needless to say, quite untrue! A fortiori is a definable and demonstrable form of argument, of universal validity and widely used. Guggenheimer thought otherwise simply because he was unable to actually formalize and validate the argument.

Guggenheimer's essay has some prestige in certain quarters due to his appeal to 'modern logic'. But from our closer scrutiny of his work, it is evident that his recourse to symbolic logic did not aid him, and even on the contrary confounded him. As I keep repeating, there is no magic in symbols. The thought that they can somehow make an analysis clearer and more scientific is a lure. Symbols are just shorter words; they only serve to abbreviate longer discourses. They say nothing more than ordinary words do – indeed, they say less, and thus lose much of their semantic potential.

A symbolic treatment can never be more accurate than the underlying insight driving it. If a researcher has not understood the subject matter, his recourse to symbols won't help him – it will make things worse for him. There is no harm in symbolizing when one has attained understanding; whereas to symbolize before that is sure to lead to error. People who symbolize early generally do so because it allows them to conceal the triviality, if not the confusion and error, of their thinking. People who are impressed and attracted by symbolic treatment condemn themselves to being misled. There is no substitute for understanding and insight. It is interesting to note that (so far as I know) none of the advocates of modern logic who succeeded Guggenheimer have noticed the errors in his treatment.

18. Adin Steinsaltz

R. Adin Steinsaltz, aka Even Yisrael (Israel, b. 1937), translated the Talmud into Hebrew (and other languages) and published it with a new commentary, over many decades, starting in 1965. There is a website called The Aleph Society, where his biography[1] can be read, as well as his commentary on the Babylonian Talmud[2]. I searched there for his comments regarding (using the spelling there preferred) *kal va-homer* and the *dayyo* principle, and found[3] several posts which I will presently analyze.

I must say I am sorry my analyses turn out to be so critical, because I actually greatly admire R. Steinsaltz's *œuvre*. Nevertheless, emotions cannot be allowed to deter us from honest logical assessments. I take Biblical statements like the following as guidelines in such contexts: "Ye shall not… deal falsely, nor lie to one another" (Lev. 19:11), "Thou shalt not respect the person of the poor, nor favour the person of the mighty" (Lev. 19:15), "Ye shall do no unrighteousness in judgment, in meteyard, in weight, or in measure" (Lev. 19:36).

1. Qal vachomer and dayo

In this section, we shall look into R. Steinsaltz's descriptions of *qal vachomer* reasoning and the *dayo* principle in relation to it.

On Baba Kamma 25a-b. Having analyzed Baba Kamma 25a-b in great detail in an earlier chapter (7), I will not here say much about its content. Rather, I will concentrate on R. Steinsaltz's remarks on the subject and see where he personally stands. What is amazing throughout R. Steinsaltz's treatment here is the way he blithely ignores all the difficulties involved. He presents the matter very briefly, and on a very superficial level where everything seems obvious and harmonious, and he either does not realize or conceals the inherent difficulties.

As regards the Mishna, R. Steinsaltz only mentions and comments on R. Tarfon's first argument, without mention of the second. The differences between the two arguments, and between the Sages' *dayo* objections to them, are thus completely lost to him or at least skipped over. He interprets R. Tarfon's first argument as a 'proportional' a fortiori (unaware that it could also be read as a mere pro rata argument), and the Sages' *dayo* objection to it as a "ruling." He does not notice that R. Tarfon's second argument has the distinction that, whether interpreted as pro rata, a crescendo (proportional a fortiori) or pure a fortiori, it has the same conclusion, so that it is immune to the Sage's previous *dayo* objection, so that the Sages' renewed *dayo* objection must be understood differently. His definition of the *dayo* principle is therefore very simple:

> "Limiting the conclusions that can be reached by means of a *kal va-homer* in this manner is called *dayyo* – 'enough.' It is enough to learn a parallel *halakhah* from a *kal va-homer*, but not more than the original law itself."

With regard to the Gemara, he has only this to say:

> The Gemara explains that the concept of *kal va-homer* – and *dayyo* – stem from the story of Miriam who spoke inappropriately about her brother Moshe (see Bamidbar 12). As punishment, she was struck with *tzara'at* (biblical leprosy), and was forced to leave the encampment for seven days. The Torah explains that had her father banished her, surely she would have been embarrassed for seven days – now that she was banished by God, she will have to be removed for that length of time. Although logically banishment because of God's anger should have lasted twice as long, *dayyo* limits the punishment to the same amount of time that she would have been embarrassed by her father."

He does not notice that the Gemara takes for granted, on the basis of the *baraita*[4] it is quoting, which refers to the story of Miriam as its model, that *qal vachomer* is "logically" a crescendo in form, i.e. goes 'proportionately' from seven days banishment to fourteen days in the case under consideration, and that the *dayo* principle "limits" this

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[1]   www.steinsaltz.org/Biography.php.

[2]   The essays there posted are described as "based upon the insights and *chidushim* [i.e. novelties] of Rabbi Steinsaltz, as published in the Hebrew version of the Steinsaltz Edition of the Talmud."

[3]   At: www.steinsaltz.org/Search_Page.php?searchterm=kal+va-homer&submit=GO.

[4]   A *baraita* is a statement of Tannaic origin, i.e. antedating the Gemara.

penalty to seven days. This is of course, as we have shown, not true – *qal vachomer* may equally well be purely a fortiori argument, in which case there is no call for a *dayo* objection to it. This means that, in the Miriam example, the conclusion may well be immediately as the Torah has it seven days, rather than fourteen days reduced to seven as the Gemara naïvely claims.

Interestingly, although the Gemara does not explicitly say so, R. Steinsaltz claims that it says that "the concept of *kal va-homer*" – and not only that of *dayo* – "stems from the story of Miriam." This is inaccurate, since *qal vachomer* is found earlier in the Torah than in Num. 12:14-15; and as just explained, even the reading of the *dayo* principle into this passage of the Torah by the Gemara (or the *baraita* it quotes) is open to debate on logical grounds (though not impossible). These inaccuracies show that R. Steinsaltz has not studied the mechanics of a fortiori argument, and instead simply taken erroneous traditional views for granted.

To these criticisms we should add that R. Steinsaltz fails to mention and analyze all the subsequent issues arising in the Gemara. First, the troubling fact that the Gemara does not notice or take into consideration R. Tarfon's second argument in its explanation of *qal vachomer* and *dayo*; had it done so, it would have had to admit that a fortiori argument may be non-'proportional' and therefore that the *dayo* principle of the Mishna Sages is of two types. Second, in its headlong pursuit of proof that the *dayo* principle is "of Biblical origin," so that R. Tarfon must know it and essentially agree with the Sages, the Gemara makes up an intricate scenario about their different viewpoints, which upon detailed logical scrutiny turns out to be specious. Unfortunately, none of this is hinted at in R. Steinsaltz's treatment.

On Baba Kamma 63a-b. R. Steinsaltz's commentary on Baba Kamma 63a-b is essentially the same, repeating verbatim the above quoted paragraph about Miriam. What is added here is his definition of *qal vachomer*, and accessorily (though he does not here name it) the *dayo* principle, as follows:

> "The method of *kal va-homer* – usually translated as an A fortiori argument – allows us to learn one law from another by arguing that if the less stringent law included a stringency, we can conclude that the stricter law includes that stringency, as well. Although the method of *kal va-homer* is considered to be a powerful one, it is limited in cases where there is an attempt to derive more than the original law included."

This is of course a traditional rabbinic definition; we have previously seen other very similar statements. R. Steinsaltz is of course not claiming it as original, though he does not mention its historical author (perhaps because he is unknown[5]). We can better analyze his statement by presenting it more formally, as follows:

> Law P is more stringent (R) than law Q,
> and, law Q is stringent (R) enough to imply stringency S;
> therefore, law P is stringent (R) enough to imply that stringency S.

Notice firstly that this argument is purely a fortiori: the conclusion has *the same* ("that") stringency as the minor premise. R. Steinsaltz does not remark on the difference between such argument and the a crescendo form assumed by the Gemara in Baba Qama 25a, where the conclusion would be 'proportional', i.e. contain *a greater* stringency. Yet, R. Steinsaltz goes on in the same breath telling us that "the method of *kal va-homer*… is limited in cases where there is an attempt to derive more than the original law included." This is, as already pointed out, an allusion to the *dayo* principle. But then we have a contradiction, or at least a mix-up of genres! If the argument is as he depicts it here purely a fortiori (i.e. non-proportional), the *dayo* principle is irrelevant to it and should not be mentioned. If on the other hand the *dayo* principle is to be mentioned, then the argument must be presented as a crescendo (i.e. as proportional). He can't have it both ways.

Secondly, said in passing, the above definition of *qal vachomer* lacks the usual reverse statement:

> Law P is more lenient (R) than law Q,
> and, law Q is lenient (R) enough to imply leniency S;
> therefore, law P is lenient (R) enough to imply that leniency S.

This statement is implied, for examples, in R. Chavel's definition: "A form of reasoning by which a certain stricture applying to a minor matter is established as applying all the more to a major matter. Conversely, if a certain leniency applies to a major matter, it must apply all the more to the minor matter;" and again in R. Feigenbaum's: "Any stringent ruling with regard to the lenient issue must be true of the stringent issue as well; [and] any lenient ruling regarding the stringent issue must be true with regard to the lenient matter as well."[6]

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5   I suspect that such statements were derived from the Mishna *Beitzah* 5:2; but I do not know who did that first.
6   Chavel: *Encyclopedia of Torah Thoughts*, p. 27, n. 106. Feigenbaum: *Understanding the Talmud*, p. 88.

There is of course no doubt that R. Steinsaltz knows this; but he does not say it here. We do find a broader definition of *qal vachomer* in R. Steinsaltz's *Reference Guide* to the Talmud. There he says[7] that this hermeneutic rule sets up a parallel between two laws, one of which has some stricter aspects than the other. If the stricter law has a certain leniency, then the more indulgent law must have it too; and "vice versa," if the more indulgent law has a certain severity, then the stricter law must have it too.

Thirdly, note the change in the relative positions of P and Q, in the above two arguments. In the first, P is more stringent than Q; in the second, P is more lenient than Q. But as regards their form, both arguments are positive subjectal (or more precisely antecedental, since the subsidiary item S is implied). Therefore, both proceed (despite appearances) "from minor to major." It follows that R. Steinsaltz's definition of *qal vachomer* here, even if expanded as we have proposed, is too narrow, because it ignores the corresponding negative moods as well as all predicatal (or consequential) a fortiori reasoning. His definition is also too narrow because it is focused on legal matters, whereas in fact (even in the Bible and the Talmud) a fortiori argument can be used with regard to non-legal matters. But we can assume that R. Steinsaltz is well aware of the possibility of such wider use, since he quotes in his *Reference Guide* the example of Jer. 12:5: "If thou hast run with the footmen and they have wearied thee, then how canst thou contend with horses."

Moreover, R. Steinsaltz makes no effort at validation of a fortiori reasoning. He does not explain why it is indeed logical to reason in this manner. He takes it for granted without further ado, which attitude is quite curious for a man who was trained in the ways of modern science and mathematics. In his *Reference Guide*, he informs us that this is the exegetical rule most often encountered; but he does not go any deeper into the subject than that.

As regards the above definition the *dayo* principle, it looks commendably broad because it is sadly vague. Disappointingly, R. Steinsaltz does not delve into the nature, source and justification of this principle, nor analyze when it is applicable in any detail. However, in his *Reference Guide*, he lists various traditional specifications concerning *qal vachomer*, such as the possibility of applying it to new situations without the sanction of tradition, and (of significance to analysis of *dayo*) the impossibility to infer by a fortiori (as unanimously admitted) a prohibition or (according to some opinions) a punishment.

On Zevahim 69a-b. We will skip the legal minutiae dealt with in Zevahim 69a-b, which R. Steinsaltz does not develop in detail anyway, and rather focus on his general comments. He repeats here, as does the Gemara, previous comments regarding the *dayo* principle, and then adds:

> "One question raised by the *rishonim* is why logic would lead us to conclude that Miriam should have been banished for 14 days. Why not 8 days? Or forever?
> Rabbenu Tam is quoted as connecting this with the idea that there are three partners in the creation of a person – his mother, his father and God. Thus God is the equivalent of both mother and father and offense against Him deserves double banishment.
> Rabbenu Hayyim ha-Cohen suggests that Miriam deserved just a little extra banishment, but the minimum time that someone suffering from *tzara'at* is banished is a week, so any additional banishment must be for a full extra week.
> The Ramban argues that no explanation is necessary, since this is merely the way the *midrash halakhah* speaks; that since she deserves more the expression is that she needs twice as much."

This text is further confirmation that R. Steinsaltz – like many great rabbis before him – firmly believes that "logic would lead us to conclude" that an a fortiori argument yields a 'proportional' conclusion. He takes this for granted and merely like his predecessors questions why the Gemara specifies specifically 14 days as the logical inference from 7 days, and not more or less. Neither he nor they realize that (as I have explained in detail in an earlier chapter (8.2)) the issue of the quantity of punishment has nothing to do with *qal vachomer* as such, but relates to the separate operation of a principle of justice or of our sense of justice. Thus, though the question asked: "Why not 8 days? Or forever?" is pertinent, it is far less important than the unasked question: why not 7 days?

On Pesachim 81a-b. We need not here either be concerned with the legal details treated in Pessachim 81a-b; suffices for us to look at R. Steinsaltz's following remarks relating to the range of applicability of *qal vachomer* reasoning:

> "Although the Gemara on our *daf* (=page) tries to find a source in the Torah for this *halakha*, its conclusion is that there is no clear reference in the Torah for it, rather it is a *halakha le-Moshe mi-Sinai*, a law that was transmitted orally to Moses on Mount Sinai that was not recorded in the Torah.
> … Although Rabbah tries to apply the rule of *kal va-homer* (a fortiori) to this case..., the Gemara rejects this, arguing that we cannot learn a *kal va-homer* from a *halakha le-Moshe mi-Sinai*.

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[7]   I do not quote him verbatim, because I have before me the French version of his text. I presume the English edition says pretty much the same thing.

Although we usually perceive the rule of *kal va-homer* as being a straightforward logical one, it cannot be used in the case of *halakha le-Moshe mi-Sinai* because of the unique quality of such *halakhot*. In general, a law that appears in the Torah can be used not only for itself, but also as a source for other laws that can be compared to it. A *halakhah le-Moshe mi-Sinai*, even as its strength and severity are equal to those of a law written in the Torah, is not seen as being grounded in the same set of rules as the written *halakhot*, so we cannot extrapolate other laws from it."

This commentary contains three items of information: (a) a definition of the term *halakha le-Moshe mi-Sinai*; (b) the ruling of Rabbah that new laws cannot be deduced through *qal vachomer* argument from a premise characterized as *halakha le-Moshe mi-Sinai*; and (c) a highlighting of this logical phenomenon as exceptional. As commentaries go this strikes me as a bit thin, so I will now try to add my own reflections.

Regarding (a), what can be said (perhaps rather cynically) is that a law designated as *halakha le-Moshe mi-Sinai* is so labeled precisely because there is no written evidence that it was given to Moses at Sinai ! If anything, what we have here is an early example of the power of advertising, where the jingle counts for more than the product. What many modern commentators say (more moderately) is that such laws were so called simply because they were considered very ancient and already well-established in Jewish jurisprudence.

Regarding (b), the question logicians must ask here is: If X formally implies Y, does it logically follow that 'X is imperative' formally implies 'Y is imperative'? That is, if we can deduce Y from X, can we deduce the legal necessity of Y from the legal necessity of X? Answer: suppose Z is our ultimate standard of judgment (in the present context, say Obedience to Divine Will). Then our question is: if Z is impossible without X, does it follow that Z is impossible without Y? The answer is, clearly, yes: given X implies Y, and not-X implies not-Z, it follows that Z implies X, then Z implies Y, then not-Y implies not-Z[8]. Thus, as regards formal logic, we ought in principle to accept any *strictly deductive* inferences, including those made through properly formulated *qal vachomer* arguments. This cannot be disputed, as just demonstrated syllogistically.

Nevertheless, I do not deny that the conclusions of certain *qal vachomer* may be regarded as having less legal weight than their premises, in acknowledgment that the premises used usually have some inductive origins. In the context of Jewish law, laws that are evidently and incontestably Scriptural are treated as axioms (i.e. as purely deductive in origin), and therefore formal inferences drawn from them are likewise considered reliable; whereas laws transmitted orally are more inductive in nature and thus retain some measure of uncertainty[9], so that even if they are *per se* conventionally granted credence, laws derived from them *per accidens* may still credibly be refused equal weight. In other words, Rabbah's ruling is reasonable, even if it could have been otherwise.

Regarding (c), which is R. Steinsaltz's own commentary to the preceding, what I would like to remark on is its relative passivity and superficiality. He notes descriptively that although *halakha le-Moshe mi-Sinai* is considered as binding as written Torah law, what is logically derived from the former is not as binding as what is logically derived from the latter. But he does not make any effort to reconcile this surprising phenomenon with the universal implications of formal logic. Instead, he claims that each type of law is subject to a different "set of rules" – suggesting, without any formal demonstration, that such relativism is logically conceivable. My contention here is that today's more religious commentators must learn to overcome such intellectual restraint, and dare to ask difficult questions. They will find that the possible answers are usually not as frightening as they imagined. Credibility nowadays depends on readiness to question and if need be to honestly criticize.

2. A recurrent fallacy

In this section, we shall look into a couple of concrete applications, where the reasoning seems to be fallacious.

On Pesachim 23a-b. R. Steinsaltz presents the *qal vachomer* argument in Pessahim 23a-b as follows:

"The Gemara considers a number of cases of forbidden foods in an attempt to clarify whether an *issur hana'ah* – a prohibition against deriving benefit – is an inherent part of the *issur akhila* – the prohibition against eating something. One of the cases where we find a disagreement on this matter is *gid ha-nashe* (the sciatic nerve – see Bereshit 32:33), where Rabbi Shimon rules that we cannot derive benefit from it and Rabbi Yossi ha-Galili rules that we can.

The Gemara suggests that Rabbi Yossi ha-Galili learns this from a *kal va-homer* (an a fortiori argument) as follows: We know that the punishment for eating *helev* (forbidden fats) is very severe (*karet*), and that the

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[8]   I tacitly assume that the implications mentioned here are all normal; i.e. that not-X does not imply Y, X does not imply not-Z, and Y does not imply not-Z. The first two of these are tacit premises, and the third is a conclusion (demonstrable *ad absurdum*: if Y did imply not-Z, then since X implies Y it would follow that X implies not-Z – which is given as untrue).

[9]   Or, if you prefer, such oral laws are more subject to faith, because their emergence in the present cannot be exactly traced all the way back to Sinai. Of course, this is objectively true even of the written law, but a difference of degree can still be claimed.

punishment for eating *gid ha-nashe* is less severe (*malkot*). Since one is allowed to derive benefit from *helev* (this is clearly indicated in the Torah – see Vayikra 7:24), then certainly in the less severe case of *gid ha-nashe* one would be permitted to do the same."

This presentation would seem to be an accurate rendition of the Talmudic argument. The problem is that R. Steinsaltz accepts its claims uncritically. Notably, the claim by R. Yossi ha-Gelili[10] that he has put forward a valid *qal vachomer*. Notice the former's qualification of the conclusion as "certainly" following the premises.

However, on closer inspection, it is not obviously valid, because the terms used in the minor premise and conclusion (viz. deriving benefit from *helev* or *gid ha-nashe*) are not the same as those used in the major premise (viz. eating *helev* or *gid ha-nashe*). If there is indeed a valid *qal vachomer*, it must be less direct than it is made out to be; i.e. it must involve some tacit intermediate moves.

But further scrutiny shows that the putative conclusion cannot readily be derived from the given premises! Let us symbolize our terms as follows: P= *helev*, P1 = eating *helev,* P2 = deriving benefit from *helev*; Q= *gid ha-nashe*, Q1 = eating *gid ha-nashe,* Q2 = deriving benefit from *gid ha-nashe*; R = degree of punishment, so that R= 0 means 'allowed' and R > 0 means 'forbidden'. R. Yossi's argument can then be written as follows:

> P1 is more R than Q1;
> P2 is R not enough to be forbidden;
> therefore, Q2 is R not enough to be forbidden.

This is a negative subjectal a fortiori argument; it has to be so, since the terms P, Q are subjects throughout it and the movement of thought is from major (P) to minor (Q)[11]. But this is not a valid argument, as already stated, because the major premise concerns P1 and Q1, whereas the minor premise and conclusion concern respectively P2 and Q2. We can, still, try to make it valid by proving somehow that "P2 is more R than Q2."

(a)   Knowing that "P1 is more R than P2," we could through a generalization assume that "Q1 is more R than Q2;" but this does not permit us to infer that "P2 is more R than Q2." More specifically, we are given that "P1 is more R than P2," since eating *helev* is punishable (R), i.e. forbidden, whereas deriving benefit from *helev* (P2) is allowed, i.e. not punishable. From this, we could *by generalization* say: "for anything, deriving benefit is less punishable an act than eating." It follows by application of this generality that "Q1 is more R than Q2," i.e. that "eating *gid ha-nashe* (Q1) is more punishable than deriving benefit from it (Q2)." We are also given that "P1 is more R than Q1," i.e. that the punishment of *malkot* (lashes) is less severe than that of *karet* (excision). From this we can deduce that: "P1 is more R than Q2." But we still *cannot* deduce that "P2 is more R than Q2" – and without this proposition the a fortiori argument remains invalid. So this approach is not successful!

(b)   Alternatively, we could try *generalizing immediately* from the given major premise "P1 is more R than Q1" to "P is more R than Q," i.e. to "anything to do with P is more R than the same thing to do with Q," and thence by application infer the needed major premise that "P2 is more R than Q2." Although such more direct extrapolation is more far-fetched than the one tried previously, since it involves two distinct subjects in tandem, it at least yields the desired result!

Another way to approach this extrapolation would be to write the major premise as a hypothetical: "When (1) eaten, *Helev* (P) is more severely punished (R) than *gid ha-nashe* (Q);" then from this generalize to: "Under all conditions, *Helev* (P) is more severely punished (R) than *gid ha-nashe* (Q);" then apply the latter to: "When (2) deriving benefit, *Helev* (P) is more severely punished (R) than *gid ha-nashe* (Q)." We can now argue, regarding "deriving benefit": "if *gid ha-nashe* (Q) is not punished severely enough (R) to be forbidden, then *helev* (P) is not punished severely enough (R) to be forbidden." The problem with this approach is of course its credibility: it looks too much like deliberate manipulation to obtain the desired conclusion.

In sum: if deriving benefit from *helev* (P2) is not punishable (i.e. is allowed), *it does not necessarily follow that* deriving benefit from *gid ha-nashe* (Q2) is not punishable (i.e. is allowed). The latter conclusion is not logically impossible, and may even (as just shown) be produced by inductive means, but as far as deductive logic is concerned it is a *non sequitur*. There may be another proposition stated elsewhere or tacitly assumed in the Gemara, which makes possible the deductive generation of the required major premise "P2 is more R than Q2," but I have not found any such intermediary; therefore, as far as I am concerned, the argument has to be judged as formally invalid.

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[10]   Who is presumably no other than R. Jose, the father of R. Eliezer to whom the list of 32 hermeneutic rules is attributed.

[11]   R. Yossi's actual formulation, according to the Soncino Talmud, is: "If heleb, for which there is a penalty of kareth [if eaten], is permitted for use, how much the more the sinew [is permitted for use], for which there is no penalty of kareth [if eaten]" (brackets mine). Note in passing that the Socino Talmud wrongly refers to this as argument "*a minori*" (*ad majus*) – whereas it is clearly in fact *a majori ad minus* (from kareth to no kareth). Note also in passing that there is no issue of *dayo* in this context, and none is raised; yet no one marvels at the fact and its implication that the *dayo* principle is not always relevant.

Which means that R. Yossi was arguing in a fallacious manner. R. Steinsaltz, however, like the Talmud before him, takes R. Yossi's a fortiori argument as essentially valid, though open to rebuttal ("*ikka lemifrakh?* literally, 'you can break the argument'"). But note that this rebuttal is not an attack like mine above on the a fortiori process as such, but merely on one of its premises. He writes:

> "The Gemara records the response of Rabbi Shimon, who forbids deriving benefit from *gid ha-nashe*, as arguing that we cannot see *helev* as being more severe, since there are certain rules where *gid ha-nashe* is more stringent. For example, *gid ha-nashe* applies to all animals, whereas *helev* is limited to domesticated animals (*behemot*) and does not apply to wild animals (*hayyot*)."

The thrust of this counterargument by R. Shimon seems to be the rejection of "Eating *helev* (P1) is forbidden, whereas deriving benefit from *helev* (P2) is allowed." We are told that the interdiction concerning eating *helev* applies to domesticated animals, but not to wild ones. For the latter kind of animals, then, eating and deriving benefit are both allowed. Whereas the similar proposition on *gid ha-nashe* (Q) would have to apply to all animals. In short, the generality of "P1 is more R than P2" is not accepted by R. Shimon.

But anyway, as we have just shown, even if this generality were accepted, R. Yossi's argument would still not be valid, since we cannot deduce through it that "P2 is more R than Q2." Both R. Shimon and R. Steinsaltz do not seem to realize this more formal issue. This is a rather disappointing performance on the part of all three of these rabbis, and many others, which goes to show the importance of having formal models to go by.

On Baba Batra 111a-b. R. Steinsaltz describes the *qal vachomer* argument in Baba Batra 111a-b, after explaining how the premises were arrived at, as follows:

> "The Gemara suggests a *kal va-homer*… If a daughter, who has less rights of inheritance from her father's estate, nevertheless inherits her mother, certainly a son, who has stronger rights in inheriting his father's estate, will inherit from his mother."

If we try to present this reasoning in more formal terms, we get the following:

A son *of a man* (P1) has more (or stronger) rights of inheritance (R) than a daughter *of a man* (Q1),
and, a daughter *of a woman* (Q2) has rights of inheritance (R) enough to inherit from her (the mother) (S);
therefore, a son *of a woman* (P2) has rights of inheritance (R) enough to inherit from her (the mother) (S).

This argument looks at a glance like an a fortiori, but is not really one, since the major and minor terms are different in the major premise (P1, Q1) and in the minor premise (Q2) and conclusion (P2), although the middle term (R) and the subsidiary term (S) are uniform throughout. We can turn this argument into a genuine a fortiori, if we manage to deductively or inductively infer the required major premise: "A son of a woman (P2) has more rights of inheritance (R) than a daughter of a woman (Q2)." For a deductive solution, we need appropriate intermediate premises. For an inductive solution, we must accept the generalization of the given major premise, so that the needed major premise can be derived from it.

Alternatively, we could formulate the Gemara's a fortiori argument with uniform major, minor and subsidiary terms, as follows:

A son (P) has more (or stronger) rights of inheritance (*from father*) (R) than a daughter (Q),
and, a daughter (Q) has rights of inheritance (R) enough to inherit from her mother (S);
therefore, a son (P) has rights of inheritance (R) enough to inherit from his mother (S).

This would be a valid a fortiori argument if we could ignore the specification "from father" (which I have put in brackets) in the major premise. Otherwise, the middle term R ("rights of inheritance") would not be the same in the major premise (where "from father" is specified, as given) and in the minor premise and conclusion (where it is irrelevant, and therefore cannot be specified). In order to remove the specification "from father" in the major premise, we need to generalize the given proposition from "A son (P) has more rights of inheritance *from father* (R) than a daughter (Q)" to "A son (P) has more rights of inheritance *from anyone* (R) than a daughter (Q)" – i.e. to move from a relative proposition to an absolute one. In concrete terms, we must presume a son to be generally more privileged than a daughter in matters of inheritance.

Such generalizations are legitimate provided they are performed overtly and explicitly acknowledged to be inductive acts. From a deductive point of view they are of course akin to circular argument or tailoring a premise to obtain the

desired conclusion (see similar comments of mine relative to Pessachim 23a-b above, though the present case is a bit simpler). Thus, the author(s) of the Gemara containing this argument may be reproved, either for failing to realize and admit the inductive underpinning of the argument or for unconsciously engaging in fallacious deduction. In other words, there would have been *no logical inconsistency* if the Torah had prescribed that sons inherit from fathers more readily than daughters do, and daughters inherit from mothers more readily than sons do.

R. Steinsaltz next presents us with an attempted application of the *dayo* principle:

> "Following this argument, the Gemara continues and concludes that since both sons and daughters inherit their mothers, the sons have priority in this case just as they do in cases when their father passes away. This position is rejected by Rabbi Zekharia ben ha-Katzav who believes that sons and daughters should share equally in the mother's estate, because of the concept of *dayo*….
>
> The Gemara relates that several amora'im wanted to accept Rabbi Zekharia ben ha-Katzav's ruling, and the Talmud Yerushalmi reports that the Babylonian sages had a tradition that followed his teaching. Nevertheless, the *halakhah* follows the other opinion, and boys receive preference in inheritance laws also in the case of a mother's estate."

Apparently, the rabbis read the previously mentioned purely a fortiori argument as a crescendo, i.e. an argument involving a quantitative comparison, in this case a comparison of 'priority'. For instance, the second version of it may be supposed to contain an additional premise about 'proportionality' as follows:

> A son (P) has more (or stronger) rights of inheritance (*from father*) (R) than a daughter (Q),
> and, a daughter (Q) has rights of inheritance (R) enough to inherit from her mother with some 'priority' (S1);
> the 'priority' of inheritance (S) is proportional to the 'rights' of inheritance (R);
> therefore, a son (P) has rights of inheritance (R) enough to inherit from his mother with *greater* 'priority' (S2).

Thus, the subsidiary term (S) is different in the minor premise (S1) and conclusion (S2), with S2 > S1. According to R. Zekharia, this inference is to be interdicted by means of the *dayo* principle; whereas others accept it as is. Is this indeed, as the former claims, an argument subject to *dayo* application? We could say so, with reference to the daughter's position, since the conclusion *diminishes* what might be supposed to be her rights (to equal or even prior inheritance). On the other hand, from the son's viewpoint, since the conclusion *improves* his position, giving him first priority, *dayo* is not called for. So, there is room for debate[12].

Perhaps a few more words on this *sugya* would clarify matters a bit more. The Torah laws of inheritance (specifically, Num. 27:8) give the daughters of a man second priority compared to his sons: "If a man die, and have no son, then ye shall cause his inheritance to pass unto his daughter." The term "priority" used in this halakhic context refers to a "winner takes all" order of precedence.

The sons inherit (almost) *all* of their father's wealth, his wife and daughters being effectively excluded (except for certain provisions that need not concern us here). If a son predeceases his father, his own sons are next in line for his share, then his daughters[13] if sons are not available. If no sons, or male or female offspring of theirs, are alive, then and only then do the daughters (of the father), and their offspring, get the (whole) inheritance. Thus, the "right of inheritance" of daughters is potential, not actual. It is contingent: it is *conditioned* on there being no male child, or grandchild through a male child, available to receive the (whole) inheritance.

This concerns inheritance from a father. What of inheritance from a mother? The Torah does not explicitly answer this question; so, the rabbis try to answer it, in *Baba Bathra*, 111a, as follows[14]:

> "[It is written.] And every daughter that possesseth an inheritance in the tribes of the children of Israel; how can a daughter inherit [from] two tribes? — [Obviously] only when her father is from one tribe and her mother from another tribe, and both died, and she inherited [from] them. [From this] one may only [derive the law in respect of] a daughter."

Thus, as already mentioned, the rabbis first establish the rights of daughters to inheritance from their mothers. This serves as the minor premise of the a fortiori argument they use to derive the rights of sons to inheritance from their mothers:

> "Whence [may the law respecting] a son [he derived]? — One may derive it by an inference from minor to major: If a daughter, whose claims upon her father's property are impaired, has strong legal claims upon the

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[12]   I would say, however, that *dayo* is not relevant here, since what is at issue is not strictly speaking a *penalty* to be applied by the court.

[13]   This logically follows from Num. 27:8 by reiteration, i.e. applying the law first to the father and then to his sons.

[14]   I quote from the Soncino edition, because I do not have at hand R. Steinsaltz own English translation (if he has one). The "it is written" reference here is to Num. 36:8.

property of her mother, should a son, whose claims upon the property of his father are strong, not justly have strong legal claims upon the property of his mother?"

This purely a fortiori argument is thereafter, as already shown, turned into an a crescendo (i.e. a 'proportional' a fortiori) argument, and the question of *dayo* arises at this stage:

"And by the same argument: As there, a son takes precedence over a daughter, so here, a son takes precedence over a daughter. R. Jose son of R. Judah and R. Eleazar son of R. Jose said in the name of R. Zechariah h. Hakkazzab: Both a son and a daughter [have] equal [rights] in [the inheritance of] a mother's estate. What is the reason? — It is sufficient [etc.]" … "And does not the first Tanna expound. 'It is sufficient [etc.]'? Surely, [the exposition of] Dayyo is Pentateuchal!" … "Elsewhere he does expound Dayyo, but here it is different, because Scripture says, in the tribes, thus comparing the mother's tribe to the father's tribe: as [in the case of] the father's tribe a son takes precedence over a daughter, so [in the case of] the mother's tribe a son takes precedence over a daughter."

In this way, the law for inheritance from a mother is made to mirror that for inheritance from a father. Note that the sons are given precedence over daughters with regard to inheritance from the mother, even though the sons' rights are inferred from the daughters' rights; but this is not logically problematic, since the daughters' rights referred to are only potential and therefore not altogether displaced by the inferred sons' rights.

Here again we may express disappointment at the rabbis in general, and R. Steinsaltz in particular, for not analyzing the various logical issues dealt with in this section with appropriate rigor; and here again, their lack of formal understanding of *qal vachomer* and the *dayo* principle is to blame. I am of course not contesting the law[15], but only wish to point out that it is arrived at by means of logic which is not purely deductive (though it is made to seem so, somewhat) but which depends on some inductive leaps that we have above duly exposed. There is nothing wrong with induction, provided it is frankly recognized as such and not presented as deduction.

Various forms. To recapitulate: we examined the above two Talmudic examples of a fortiori argument to test R. Steinsaltz's understanding of such reasoning. Both arguments, though unrelated, displayed the same sort of fallacious thinking (judged by strict deductive logic standards) – so it looks like we stumbled upon a recurring fallacy in Talmudic logic, and no doubt in people's thinking in general. It is interesting to note that R. Steinsaltz did not notice these errors. To be fair, this was an easy mistake to make, even if a little careful reflection would have quickly gotten alarm bells ringing.

This fallacy has many possible forms. The forms we encountered above are the following. The most typical is an argument that resembles positive subjectal a fortiori, except that the major and minor terms (P and Q) are not uniform throughout – being greater in the major premise than in the minor premise and conclusion.

P1 is more R than Q1,
and P1 is greater than P2 and Q1 is greater than Q2.
"Therefore," if Q2 is R enough to be S,
then, all the more, P2 is R enough to be S.

This argument is fallacious because the givens before the "therefore" do not always imply the formally required major premise "P2 is more R than Q2," even if the latter is sometimes true. This formal requirement cannot be ignored or vaguely assumed; it has to be proved by some means.

We can show this argument is fallacious by simple mathematics. If we symbolize 'the value of R for some variable x' by $R\{x\}$, we can put it as follows: given that $R\{P1\} > R\{Q1\}$, and that $R\{P1\} > R\{P2\}$ and $R\{Q1\} > R\{Q2\}$, it does not follow that $R\{P2\} > R\{Q2\}$. This can be seen in the diagram below. All we can deduce is that $R\{P1\} > R\{Q2\}$; regarding $R\{P2\}$ and $R\{Q2\}$, the first may be greater (or equal) or lesser than the second.

Clearly, if we can somehow show that $R\{P2\} > R\{Q1\}$, then it would prove that $R\{P2\} > R\{Q2\}$. If on the contrary $R\{P2\} < R\{Q1\}$, the relation between $R\{P2\}$ and $R\{Q2\}$ remains undetermined; in such case, to prove the desired relation, we would need to refer to some other intermediary, say $R\{y\}$, such that $R\{P2\}$ implies it and it in turn implies $R\{Q2\}$.[16]

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[15]   Although, in all honesty, I personally find the Jewish law of inheritance unfair to women, denying them independence as human beings. It was, no doubt, at the time it was promulgated a reflection of the existing patriarchal society, and probably an improvement in the status of women. But times have changed, and in today's society women must be acknowledged as equals under the law.

[16]   Note in passing that, given $R\{P1\} > R\{Q1\}$, we would likewise be unable to establish that $R\{P2\} > R\{Q2\}$, if $R\{P1\} < R\{P2\}$ and $R\{Q1\} < R\{Q2\}$. On the other hand, if $R\{P1\} < R\{P2\}$ and $R\{Q1\} > R\{Q2\}$, it would follow that $R\{P2\} > R\{Q2\}$.
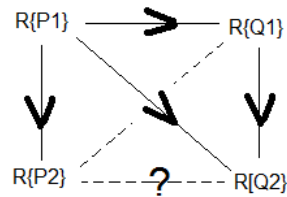
Diagram 18.1

Variants of this are: where the major and minor terms are uniform, but the middle term (R) is explicitly or implicitly relative (to some item X, say) in the major premise and the same relativity is not mentioned or intended in the minor premise and conclusion, or where the major premise is explicitly or implicitly conditional (again on some item X, say) while the same condition is not mentioned or intended in the minor premise and conclusion. That is to say:

> P is more R (relative to X) than Q,
> "Therefore," if Q is R (relative to something else) enough to be S,
> then, all the more, P is R enough to be S.

> (On condition X,) P is more R than Q,
> "Therefore," (on some other condition,) if Q is R enough to be S,
> then, all the more, P is R enough to be S.

In these two forms, the fallacy lies in ignoring that the tacitly intended 'relativity to X' or 'conditioning upon X' in the major premise may well not be also applicable in the minor premise and conclusion, whereas the minor premise would still be true if it applied to non-X but did not apply to X. The three arguments are, then, from a strictly deductive point of view, fallacious – i.e. the putative conclusion does not necessarily follow from the premises. This does not mean that there are not special cases where the required premises can be produced inductively or even deductively – it just means that they are not universally present.

We can from these positive subjectal forms predict three negative subjectal ones, where the minor premise is negative with the major term (P) as subject and the conclusion is negative with the minor term as subject (Q). We can also conceive of implicational and hybrid equivalents of these various forms. A similar set of fallacies can be expected to occur with predicatal a fortiori arguments. For instance, the following one would be typical:

> More R is required to be P1 than to be Q1,
> and P1 is greater than P2 and Q1 is greater than Q2
> "Therefore," if S is R enough to be P2;
> then, all the more, S is R enough to be Q2.

3. Lack of formalism

Although it is pretty obvious that the rabbis involved in the above described fallacious arguments were intending valid deductive arguments, we can 'save face' for them by suggesting that (though they did not say so out loud) they did not really mean their arguments as strictly a fortiori, but were consciously engaged in arguments that are '*roughly* a fortiori'. They are mere analogies that resemble a fortiori but are not truly so, being more inductive than deductive. They were 'pseudo-fortiori', or 'quasi-fortiori', but not really fortiori.

In any event, as I have shown in the present and other chapters (7-9), the main *cause* of problems in rabbinic use of a fortiori argument (and this is also true of other forms of argument, of course) is their eschewing formal logical studies. This lack of formalism is confirmed by R. Steinsaltz in his work *The Essential Talmud* (chapter 30)[17], where referring to the specificity of the "talmudic way of thinking" he writes:

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[17]   Tr. Chayah Galai. New York: Basic, 2006. Most of this work can be read, in Google books, at:
books.google.com/books?id=keXGJjd4ThcC&printsec=frontcover&dq=%22Adin+Steinsaltz%22++inauthor:%22Adin+Steinsaltz%22&hl=en&ei
=3qooTZepK4qD5AaOptTuCQ&sa=X&oi=book_result&ct=result&resnum=1&ved=0CCMQ6AEwAA#v=onepage&q=qal&f=false.

"A basic factor is the attitude towards abstraction. In the Talmud, as in most areas of original Jewish thought, there is deliberate evasion of abstract thinking based on abstract concepts. (…) The Talmud employs models in place of abstract concepts. (…) *Kal va-homer*, for example, is a method applied to a certain model in order to adapt it to another model. Thus there is a high degree of mechanical thought, and no attempt is made to clarify practical or logical problems per se; (…) it is not always possible to understand the convoluted methods of the operation itself" (p. 263).

R. Steinsaltz, of course, is not intending any radical criticism thereby. But I would say that this is the core problem, and if we ever hope to modernize and improve upon past Jewish legal thought, and credibly further develop it, we have to learn and adopt more formal methods of inquiry. There is in my view just no excuse for hanging on to ways of thinking that (occasionally, if not often) lead to error. Logic is not something arbitrary, which can be ignored or shunted aside when we dislike its results. Logic is a way to test if a theory is true, together with empirical data. If a theory goes against logic, and/or against empirical data, it must be rejected or at least reformulated. There is no credible escape from this methodological requirement. It is an absolute, applicable to religion as well as to everything else.

19. Jonathan Cohen

1. Formula for a fortiori

Alexander Samely tells us, in his *Rabbinic Interpretation of Scripture in the Mishnah*[1]:

> "In an informal conversation with me in July 1984 the Oxford logician Jonathan Cohen once worked out the following representation of the *a fortiori*, according to my notes: $(n)[(x)(y)(F(x,y) = n \rightarrow G(x,y)\ ^{=m}) \rightarrow (a)(b)(F(a,b) > n \rightarrow G(a,b)\ ^{>m})]$."

Samely does not further clarify or justify this symbolic formula, as if its meaning and truth are obvious. We shall assume here that he reported it fully and correctly.

Allen Wiseman proposes[2] the following interpretation of Cohen's formula in plain English: "For all n, and for any x and y, if they have a feature of equality in n, then another factor is equal to it as m, so that for any actual cases a and b, with the feature between them greater than n, then the other factor is greater as m." However, although he is a commentator well versed in issues of *dayo*, Wiseman does not discuss the validity of Cohen's formula.

My own reading in ordinary language of Cohen's formula would be as follows:

> *Given that:* a certain thesis F, involving terms x and y, containing some quantitative factor n, implies that thesis G, involving the same terms x and y, contains some quantitative factor m,
>
> *it follows that:* a like thesis F, involving terms a and b, containing some quantitative factor *greater than* n, implies that thesis G, involving the same terms a and b, contains some quantitative factor *greater than* m.

Clearly, what we have here is a premise and a conclusion, each of which has hypothetical form (i.e. consists of an if–then statement). Cohen is saying: Given the truth of the first if–then statement, it follows that the second if–then statement is true too. No proof of this is provided, note well; it is just an assertion. We are not told *why* the first statement should be taken to formally imply the second. More on this issue in a moment.

According to Wiseman, Cohen intends a and b as particular instances of abstractions x and y (respectively). This may well be the case. But if so, note well that the premise (involving x and y) is not identical in form with the conclusion (involving a and b) – so we do not here have a simple case of application of a general principle to a particular instance. It follows that nothing is gained, no progress is made toward validation, by having a and b as instances as x and y – if anything, we would be artificially limiting the applicability of the argument presented by Cohen if we in fact managed to validate it. In any case, as my analysis of a fortiori argument well shows, there is no rule concerning it, such as the one Cohen apparently assumes, that the items involved must be generalities and particulars.

In my opinion, therefore, mention of the terms x, y and a, b is superfluous, and only obscures the essence of the argument proposed. It is obvious that Cohen inserts these extra details in order to be in servile compliance with the pretentious conventions of modern symbolic logic. But they are encumbrances, useless complications, which do not in fact affect the process of inference of concern to us. We could considerably simplify the statement by referring to F(x,y) and G(x,y) as F1 and G1, and to F(a,b) and G(a,b) as F2 and G2. The suffixes 1 and 2 suffice to indicate that F and G with the same suffix have certain terms in common, and that the F and G in the premise are different in some way from the F and G in the conclusion. That is all that is needed to express the situation. Granting this clarification, Cohen's statement is effectively that:

> *Given that:* a certain thesis F1 involving a quantitative factor n, implies that thesis G1 involves a quantitative factor m,
>
> *it follows that:* a like thesis F2 involving a quantitative factor *greater than* n, implies that thesis G2 involves a quantitative factor *greater than* m.

Looking at this simplified statement, we could well suppose that what Cohen had in mind was not a sophisticated a crescendo argument, but the simpler pro rata form of argument. But if we grant him the benefit of the doubt and

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[1]   Oxford: Oxford UP, 2002. See footnote on p. 177. Samely would have been about 24 at the time of his "informal conversation." Presumably this refers to Laurence Jonathan Cohen (England, 1923-2006).

[2]   In *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions*, in a footnote on p. 129.

suppose that he indeed did have a fortiori argument in mind, the following would be the standard form (using our usual P, Q, R, S terminology) that would most adequately reflect such underlying thought:

> F2 (P) involves more n (R) than F1 (Q) does;
> and F1 (Q) involves n (R) enough to imply G1 to involve some m (S);
> and m (S) is proportional to n (R);
> therefore, F2 (P) involves n (R) enough to imply G2 to involve more m (>S).

Cohen's formula evidently refers to a positive antecedental a crescendo argument, going from minor to major. The sub-theses F1 and F2 (the antecedents in his premise and conclusion) are respectively the minor and major items (P and Q). The quantitative factor 'n' in common to them is the effective middle item (R), and the quantitative factor 'm' in common to the consequents in Cohen's premise and conclusion is the effective subsidiary item (S).

Cohen's formula is in fact not as sophisticated as I here present it on his behalf. He does not explicitly mention the major premise, though we can readily see that it was part of his thought process. Nor does he anywhere hint at the additional premise about proportionality between m and n; more on this serious lacuna in a moment. Nor does his formula contain the crucial notion of sufficiency, which gives the sense that there is a threshold of the middle item (n) to cross before the subsidiary item (m) can be applied. I have kindly filled in the gaps in Cohen's argument, to show what he was probably aiming at; but his formula as it stands did not hit the target, to say the least.

As regards the subsidiary item, more precisely put, it is not just 'm' but the whole implied proposition that 'G involves m'; this means that mention of sub-theses G1 and G2 was really an unnecessary complication. That is to say, substituting symbol M for 'G implies m', it would have sufficed for Cohen to say: *Given that:* 'F1 implies n' implies M1, *it follows that:* 'F2 implies more n' implies M2 (where M2 > M1). Cohen's mention of G and m is a fault, because it is a redundancy that conceals the essence of the argument; it introduces inessential elements, which make his formula more specific than it needs to be. The job of logicians is to reduce arguments to their bare essentials, avoiding all irrelevant embellishments.

The argument must be classed as 'antecedental' because it involves implications: 'F *implies* n' and this proposition in turn *implies* M. Cohen's formula is too narrow in that it includes only positive antecedental argument, and makes no mention of the negative antecedental form, or for that matter of the positive and negative consequental forms, of the arguments. Furthermore, Cohen's formula is too narrow in that it fails to mention the much simpler forms of copulative argument, involving the relation 'is' between terms instead of the relation 'implies' between theses. That is to say, he misses out on the positive and negative subjectal and positive and negative predicatal moods of a fortiori argument. To be general, his symbolic formula should have clearly included all possibilities.

2. Fallacy of diverse weights

But the most important issue to raise in relation to Cohen's formula is that it is in fact fallacious – it posits a *non sequitur*. That is, his conclusion does not deductively follow from his actual premise. He typically (as many other commentators unthinkingly do) infers a 'proportional' conclusion without justification. The a fortiori argument proposed by Cohen is actually not the one I have kindly proposed for him above but the following:

> F2 (P) involves more n (R) than F1 (Q) does;
> and F1 (Q) involves n (R) enough to imply M1 (S1);
> therefore, F2 (P) involves n (R) enough to imply M2 (S2),
> where M2 > M1 (presumably in proportion to n2 > n1).

We might call this 'the fallacy of diverse weights'. No reason is given here why M2 should be greater than M1 (presumably in proportion to the greater magnitude or degree of n2 over n1). The proportionality is taken for granted by Cohen, without provision of an explicit premise that would make it logically possible. In formal logic, the proportionality has to first be legitimized by means of an additional premise, before a 'proportional' conclusion can be drawn; without such additional premise, only a 'non-proportional' conclusion is logically legitimate. Cohen's formula shows him to be unaware of this indisputable logical rule. Had he been aware of it, his premise would have included a clause to the effect that 'm is proportional to n'. Without this addition, the only possible conclusion from Cohen's premise is that "F2 (P) involves n (R) enough to imply M1 (S1)" (note the suffix 1 in the subsidiary term).

Therefore, we must judge Cohen's formula to be invalid. If we take Cohen at his word, since his formula does not explicitly justify proportionality, we can say that he is advocating invalid 'proportional' a fortiori argument; that is to say, he erroneously imagines that the two premises of purely a fortiori argument suffice to yield a proportional

conclusion, which is wrong. If we put words into Cohen's mouth, and say that he tacitly intends a crescendo argument (i.e. a fortiori cum pro rata), we can ask why as a professional logician he failed to mention the crucial tacit assumption that 'm varies concomitantly with variation of n'. Thus, his formula is either fallacious or formally deficient. In any case, this means that Cohen has not understood that the essence of a fortiori argument is *non*-proportional; he clearly thinks the opposite.[3]

Moreover, although we have here done our best to form an a fortiori argument of some sort from his formula, his actual formulation is not in accord with the way people ordinarily think a fortiori thoughts; his statement is an artificial straitjacket, which misleads him as well as others. The job of a logician is not merely to propose a theoretical way to obtain some conclusion, but also to relate his proposal to the way people think in practice. Before logicians can prescribe, they must first be able to accurately describe.

3. No effort of validation

Finally, not only is Cohen's formula incomplete and inaccurate in the stated ways, but he apparently made no attempt to validate it. He just takes it for granted, as obvious on sight, without any attempt at explanation or justification. Perhaps he thought of it as an inductive leap, rather than as a deductive step. But if so, how come he uses the same symbol (an arrow) for inductive (merely probable) implication as he does for deductive (necessary) implication? It is of course possible that Samely recorded Cohen's formula incompletely or inaccurately, and without taking note of its validation. But I would confidently say that had Cohen in fact made an effort of validation, his formula would have turned out very differently. Its defects and errors, which we have highlighted above, are evidence of a lack of effort of validation. Modern logicians often function that way, thinking that it suffices to hand down a formula that seems 'intuitively' logical to them. No, my friends, everything has to be proved; and in the process of seeking proof one improves one's statement.

I should stress that the above comments are based on just one brief remark by Samely. It is conceivable that somewhere in Cohen's works there is a more complete and accurate theory of a fortiori argument. I have not read his works, which judging by their titles could be quite interesting. Needless to say, I would not presume to judge them on the basis of that one bit of hearsay evidence.

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[3] It is not surprising, in view of this lacuna in Cohen's treatment, that his student Samely was left with uncertainty as to whether the conclusion of a fortiori argument is proportional or non-proportional. We could conversely infer from Samely's evident uncertainty that Cohen's teaching on this issue was misleading, or at least unclear.

20. Michael Avraham

We shall again encounter Michael Avraham (or Abraham[1]), as co-author together with Dov Gabbay and Uri Schild of the 2009 paper "Analysis of the Talmudic Argumentum A Fortiori Inference Rule (Kal Vachomer) using Matrix Abduction" and a companion paper in Hebrew focusing in more detail on Talmudic a fortiori argument. But Avraham was evidently an old hand in the field by then, since he long before authored a paper called "The 'Kal Vachomer' as a Syllogism – Arithmetic Model" that was published in the 1992 issue of *Higayon* (vol. 2. Pp. 29-46)[2].

1. Model of a fortiori

I have a copy of this older article, which is in Hebrew. The article is summarized in English as follows:

> "A mathematical and conceptual analysis of the Talmudic Kal Vachomer is presented using the concept of 'rotation' of the Kal Vachomer. Several possible models which describe differing interpretations by the commentators are considered. It is argued that the Kal Vachomer – like the rest of the 13 'Middot' – is not a syllogism."

My Hebrew is simply not good enough to try to review it and assess all its contents independently, although I gather from a brief perusal of the Hebrew text that Avraham's wording for a fortiori argument (*qal vachomer*) is as follows[3]:

> "**If A is light in 'a' and heavy in 'b', then B which is heavy in 'a' will obviously be heavy in 'b'.**"

Note that, as far as I can tell, he fails to stress the relativity of the weights: he does not say lighter/heavier, but light/heavy (or also –/+). Judging from the word 'in' (*be*) he uses, I would say that 'a' and 'b' refer to subjects, and A and B refer to predicates. If so, then the argument would be positive *predicatal*, and the inference would be invalid since it is 'minor to major' (whereas it should then be 'major to minor'). Moreover, the number and functions of the terms involved is not correct. Although we could point to 'weight' (heavy or light) as the middle term (R), his construct has no subsidiary term (S) in common to the premise and conclusion.

Avraham's wording posits *mere analogy* between A and B, in 'a' and 'b': *Just as A is heavier in 'b' than it is in 'a', so B is heavier in 'b' than it is in 'a'*. So really, what we have here is just the argument: Just as A is X, so B is X, where A and B are two *subjects* and predicate X happens to refer to the comparative term "is heavier in 'b' than it is in 'a'"[4]. For this reason, the argument looks at first sight like a positive *subjectal* one, and thus like a valid 'minor to major' process, since a further given is that A is light in 'a' and B is heavy in 'a'. But on closer scrutiny it clearly is not even an a fortiori argument, let alone a valid one! It cannot be rendered in standard (PQRS) form. Although Avraham uses language suggestive of such argument, he does not correctly formulate it.[5]

The above was my first reaction, but returning to the enigma later, I thought of the following standardization. If we take B and A to be respectively the major and minor terms (P and Q), and "weight in 'a'" to be the middle term (R) and "heavy in 'b'" to be the subsidiary term (S), then we can construct the following valid, positive subjectal a fortiori argument:

> B (P) is heavier in 'a' (R) than A (Q) is; and
> A (Q) is heavy in 'a' (R) enough to be heavy in 'b' (S);
> therefore, B (P) is heavy in 'a' (R) enough to be heavy in 'b' (S).

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[1] The same Hebrew letter 'ב' may be read as 'v' or 'b'.
[2] Ed. Moshe Koppel and Ely Merzbach. Jerusalem: Aluma, 1992.
[3] See p. 30, below the first diagram. My translation.
[4] Note too that no justification is given for the claimed inference of X from A to B.
[5] Moreover, he seems to be complicating matters considerably by next introducing an additional term C, and engaging in a second, successive act of a fortiori argument. Why he does so, I cannot tell. This is presumably what he means by "'rotation' of the Kal Vachomer." I would guess that what Avraham had in mind here was what I later (in my *Judaic Logic*) called a fortiori arguments "in tandem." But if so, sorry to say, he does not manage to correctly formulate this logical phenomenon either. In any case, it is worth reminding that *rabbinical hermeneutics does not allow inference of a new ruling from a previously inferred ruling, i.e. use the conclusion of one a fortiori argument as a premise in the next*. So if Avraham predicts 'rotation' in Judaic logic, he is apparently ignoring this restriction. It is admittedly possible that he is not the first to do so, i.e. that rabbis have often in practice also ignored it.

Maybe Avraham had this thought at the back of his mind. Just maybe. But in that case, what need had he for the subsidiary term "heavy in 'b'"? Why this, rather than "light in 'b'" or "balanced in 'b'"? Indeed, any predicate would have fitted in here (e.g. "in 'b'" or just "b") – why did he specifically use a predicate to do with "weight"? He does not propose a 'proportional' conclusion, so that cannot be the reason why. Obviously, the specification "heavy in" relative to 'b' is a formal redundancy[6]. This peculiarity is indicative of a lack of clarity on Avraham's part as to the roles of the terms – which is why I suspect my preceding analysis of his thinking to be more likely. Thus, I maintain that he misunderstood how a fortiori argument is constituted.

2. Outlook on a fortiori

I would have left the matter at that, were it not for the fact that Allen Wiseman devotes considerable attention to Avraham's paper in his own 2010 treatise[7]. So we can briefly examine it further on this basis. As can be expected, Avraham presents in the 1992 paper the same basic theory of a fortiori argument as he (together with Gabbay and Schild) uses in the 2009 paper. The older paper may be said to lay the theoretical foundation for the later one. We might suppose Avraham's thought on the subject to have evolved over the years, but obviously we cannot suppose that his thinking was *more* accurate in the earlier paper than it was in the later one[8]. Therefore, we should not expect to find any insights in the former that radically overturn our critical assessment of the latter. That said, let us anyway take a look at Avraham's 1992 ideas as presented (presumably fairly enough) by Wiseman[9].

To begin with, he tells us, Avraham "writes the generic, Ostrovsky/Schwarz argument in predicate logic." I take this symbolic formulation to be Avraham's own interpretation of Ostrovsky and before him Schwarz, since Wiseman does not mention this formula in his earlier expositions of the latter two authors[10]. Avraham proposed formula is the following (I have changed his letter P to F[11]):

1st premise:	$(\forall x)(F(x) \rightarrow G(x))$	For all x, if x is F, then x is G
2nd premise:	$F(a)$	Specific instance (of x) a, is F
Conclusion:	$G(a)$	Specific instance (of x) a, is G

This is stated in the language of symbolic logic, which is supposed to be freer from ambiguities than ordinary-language logic. But to my mind, this is an overly rigid format, which complicates and obscures matters unduly[12], just to give an illusion of exactitude. A simpler and more natural statement of same would be:

If anything is F, it is G (or categorically put: All F are G),
and a certain thing (say, a) is F;
therefore, that thing (a) is G.

We thus see way that what we have before us is nothing more than a *modus ponens* apodosis; that is, an extensional conditional proposition, combined with a categorical one affirming its antecedent, concluding with a categorical proposition affirming the consequent. We can also look upon it as a categorical syllogism, as Wiseman has it, and as

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[6] If we look at his diagrams for 'rotation' (or what I presume this term to refer to, i.e. a chain of a fortiori arguments), we see that the item '+b' remains the same from one argument to the next. This means that we cannot explain his reference to "heavy in 'b'" as having something to do with 'rotation'.

[7] *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions* (Waterloo, Ont.: University of Waterloo, 2010), pp. 75-81.

[8] Unless, of course, he was not really involved in the writing of the later paper, but was counted as a co-author because some of his ideas were used by the other two co-authors. In that case, it could be that the latter misunderstood or misused his ideas.

[9] Pp. 75-81. Wiseman informs us, in a footnote: "His article I translate and reproduce here, mostly from his p. 35 onwards, sometimes paraphrased… All the main material is his." Thus, though he admits to largely skipping the first 6 pages of the paper, he maintains his presentation to be essentially accurate.

[10] Wiseman says that Avraham "states that Ostrovsky follows Schwarz in an attempt to construct a deductive argument based on the categorical syllogism" (p. 75). I present a brief study of Ostrovsky in a later chapter of the present volume (30.3).

[11] I do this primarily so as to preempt confusion with my standard lettering (where P refers to any major term). But also, because I think that when Avraham wrote P he meant F, as is evident from his use of G as the next letter. In Hebrew, the same letter can be read as P or F.

[12] The symbol 'x' is to my mind a useless interjection. Notice that the symbolic formula takes for granted, but does not explicitly say, that 'a is an instance of x' – yet it is only on that assumption that we *infer* that 'a is F'. So, before we can infer the conclusion 'a is G', we have to go through a preliminary apodosis. This is an unnecessary complication, which can be avoided simply by using 'anything' in lieu of 'x'. And indeed, the latter is the way we naturally think. The symbolic formula is a fabrication of people who confuse trivial conventions with intellectual breakthroughs.

no doubt Avraham thought of it, by formulating the major premise in categorical form (as shown in brackets). It is not very important how exactly we interpret Avraham's symbolic formula.

What *is* important is the fact that, as a somewhat perplexed Wiseman immediately remarks, "Avraham does not mention that the above form does not tell us what is more and less severe… Perhaps we are to know that [F] is less and G is more." In other words, what has the said formula to do with a fortiori argument if it does not even rank the terms involved as minor and major? It would seem that in Avraham's mind, the antecedent predicate F is the lesser quantity, and the consequent predicate G is the greater – so that the inference is from minor to major. But if he has not expressed that, we could equally well assume that the inference is from major to minor. So his formula is deficient.

Perhaps Avraham considers that the said formula merely underlays a fortiori argument, but does not wholly cover it. This is suggested by his recourse to what Wiseman refers to as "a chart" – actually a tabular representation. This is intended, he tells us, "to rectify matters." But the truth is that such a table also does not intrinsically bring out the relative ranking of the terms, unless verbal explanations are additionally given. What all this means is that Avraham in fact *does not in fact propose any formula*, whether in symbolic or ordinary language, *capable of entirely capturing and distinguishing* the a fortiori movement of thought, or even one mood thereof. This is the main thing to note.

Nevertheless, Avraham does have things to say about a fortiori argument. Wiseman informs us that: "He understands the overall argument as deductive, although it often comes as an induction." This comes as a surprise to me, knowing that in a later essay, co-authored with Gabbay and Schild, Avraham seems to have a more definitely inductive, as against deductive, view of a fortiori argument. At any rate, Avraham here distinguishes himself from Schwarz, who we are told insisted on the deductive aspect[13]. The way Avraham (or perhaps Wiseman for him) formulates the more inductive a fortiori argument is to qualify the premises as "possible givens," i.e. as somewhat uncertain; in which case the conclusion is at least equally unsure, "rather than certain or necessary."

I agree with this viewpoint: a fortiori argument is essentially deductive, although it is occasionally intended more as an inductive indicator. However, I would stress, to avoid all misapprehension, that the uncertainty in the conclusion is due to subsisting doubts in the contents of one or both of the premises, *rather than* doubts in the process of inference itself. The process of inference as such remains deductive – even if the conclusion has an inductive character in view of the (in most cases) inductive character of the premises it is based on. Moreover, this possibility of doubt is not peculiar to a fortiori argument, but is true of all deductive argument, including categorical syllogism. I agree with Avraham if this is what he meant by "possible givens."

So both Schwarz and Avraham can be considered right, assuming they indeed hold the opinions here attributed to them. The former is right in insisting that the *form* of a fortiori argument is deductive, and the latter is right in keeping in mind that the *contents* of the premises and therefore of the conclusion may still contain some doubt and therefore be merely inductive. All logicians know this distinction; but it is still worth reminding for the sake of novices. In Talmudic argumentation, putting the premises in doubt (either directly or through doubt in the conclusion) is called *teshuva* (in Hebrew) or *pirka* (in Aramaic).

Of course, a good question to ask here is: how would either Schwarz or Avraham know that a fortiori argument is essentially deductive, if they did not in fact correctly formalize the argument and validate it? We must, objectively, say that they posit this as an intuitive claim, without really knowing it by having proven it.

Of course, I hasten to add, even if the a fortiori process is in principle deductive and indubitable (once validated), it does not follow that all a fortiori arguments thought, spoken or written by human beings in practice are properly conceived and formulated. People do occasionally, if not often, make mistakes (or even fake things) while reasoning. They maybe do that more often when reasoning a fortiori, since this is more complicated. In this sense, even the process of a fortiori argument may be considered inductive, at least until we establish clearly that it was correctly done in the case under scrutiny. We do find instances in the Talmud where someone's process of a fortiori reasoning is questioned.

3. On Baba Qama 2:5

Let us now focus on Avraham's treatment of the famous debate in the Mishna (Baba Qama 2:5) between R. Tarfon and the Sages, which I have dealt with extensively in an earlier chapter (7.3). What is interesting at the outset is to see that Avraham is aware that there are two arguments and objections, and somewhat aware of the difficulties these present – which is more than can be said for some other commentators. Actually, we have elsewhere seen and

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[13]   As Wiseman puts it us in a footnote: "He says that Schwarz rejected the possibility claim (of "Korban Aharon") for that of certainty." *Korban Aharon* is an older commentary.

criticized Avraham's approach to this debate, re-enacted in the essay he co-authored with Gabbay and Schild 17 years onwards. So I will here only cursorily touch upon a few additional points.

Avraham approach here is very tedious, due to his excessive use of symbols, not to mention his repeated changes of symbols for the same items. Moreover, one gets the impression throughout that he is feeling his way around, trying to sort things out somehow – rather than presenting a fully matured product. He beats about the bush, and does not get to the essence of things. Many of the items he refers to are extraneous, even if to his mind they seem necessary to ensure the systematic character and generality of his conception of things. All this is complication, which obscures more than it enlightens.[14]

Avraham's essay is of course a serious effort. But try as I might, I cannot make out just what it is getting at. I fully understand every statement made, whether in symbols or words. But he does not here seem to be advocating some definite result that can be judged true or false. It could of course be that Wiseman's account of Avraham's paper is incomplete. That is, although what his account actually contains looks like an accurate rendition, he may have missed out some crucial information or insight that is clear enough in the original. The explanation may also be that Avraham wishes to take nothing for granted and leave everything open; even so, his position is far from clear.

Regarding the 2009 paper that we have already mentioned, we can say that, although the authors have not realized the standard form of a fortiori argument which I proposed in 1995, they have at least effectively identified such reasoning with argument by analogy, or more precisely pro rata argument, since the conclusions they draw from the givens are visibly proportional (even if we found errors in specific cases). In the 1992 paper, however, Avraham does not seem to yet propose any single final conclusion, but merely maps out different possible hypotheses that apparently peter out without a clear result. What stands out, anyway, is that Avraham has not here grasped a fortiori argument in the true sense.

His main inferences are, as the title of his paper implies, "arithmetical" – they concern quantitative comparisons: if one thing is greater than another, and that other is greater than a third, then the first is greater than the third[15]. This is what he apparently regards as the "deductive part" of the argument[16]. Beyond that, there is an "inductive part" of the argument, in which different possibilities have to be considered, before a final "estimate" is made as to the conclusion to draw. Since this process depends on human judgment-calls, it is inherently open to doubt. Its conclusion is therefore open to "refutation" or "disproof," meaning at least disagreement.[17]

Avraham views the Mishna debate through the prism of the following table, which is perhaps an innovation to his credit:

Penalties	In public domain	In private domain
For damage by bite or hoof	–	+
For damage by goring	+	(+)

The givens are that there is no penalty (–) for damage by 'bite or hoof' in the public domain, and some penalty (full damages) for such damage in the private domain, as well as some penalty (half damages) for damage by 'goring' in the public domain, and the question asked is: what is the penalty (+) for the latter sort of damage in the private domain? The plus sign in the corresponding cell suggests, of course, that the answer to that question is positive, or maybe full damages.

As I understand it, two directions of argument by analogy are possible. The first argument is "horizontal:" since 'bite or hoof' damage is greater in the private domain than in the public domain, it follows that 'goring' damage is greater in the private domain than in the public domain; conclusion: full damages (since more than full is excluded). The second argument is "vertical:" since in the public domain 'goring' damage is greater than 'bite or hoof' damage, it follows that in the private domain 'goring' damage is greater than 'bite or hoof' damage; conclusion: full damages, again.

Note that both these arguments of R. Tarfon's yield the same, proportional conclusion (full damages). Actually, R. Tarfon initially puts forward only the first argument; but the Sages reject his conclusion, saying *dayo* – it is enough to conclude with half damages for damage by 'goring' in the private domain, *as originally given* for damage by 'goring' in the public domain. Then R. Tarfon tries another tack, his second argument; but the Sages again reject his

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[14]   One is tempted to ask him the question: do you think that when people reason a fortiori they have to go through all the rigmarole you have laid out to arrive at a credible conclusion?

[15]   Wiseman refers to this as "quantificational logic."

[16]   The words in inverted commas are used by Wiseman, at least.

[17]   Note that if this reading is correct, it means that Avraham's idea of the "inductiveness" of a fortiori argument is not merely to do with uncertainty in the premises, as I earlier suggest, but also with doubts inherent in the process. But "the process" here is, of course, not the standard a fortiori argument, but the vaguer form of argument that Avraham has in mind and labels 'a fortiori argument'.

conclusion, repeating *dayo* (in identical words) – it is enough to conclude with half damages for damage by 'goring' in the private domain, *as originally given* for damage by 'goring' in the public domain.

R. Tarfon, interpreted the Sages' first objection as a demand that the quantity in the conclusion be equal to the quantity *in the last premise* (viz. half damages in this case), and so was surprised to find them objecting to his second conclusion, even though the quantity in it was equal to the quantity *in the last premise* (viz. full damages in that case). The Sages, on the other hand, were focused not on the technical issue of the last premise but on the content of *the original given* (about damage by 'goring' in the public domain), and for that reason were unfazed by his logical bravado. This means that the two *dayo* objections, though verbally the same, are technically different somehow. In the first argument, the original given happens to correspond to the minor premise; but in the second argument the original given is not an overt premise but is used to construct the major premise.

The job of any commentator on this topic is to notice this problem and to propose a credible solution for it (as I just did, for example). It seems as if Avraham has noticed the issue somewhat, and tried to grapple with it in some way. This is suggested in Wiseman's following statements (even if they do not directly concern the same topic)[18]. In the main text: "These two claims differ and are not two versions of the same claim as might seem in a cursory view. Thus, a disproof of either the vertical or the horizontal II statements will not apply to the other as before." And in a footnote:

> "The claim that one can escape disproof by changing directions looks very strange, as if the **h** and **v** seem to be differing formulations of the same claim, while the logical disproof does not care about the formulation. Both claim types derive from the same 3 givens and conclude with the same logic. So how can the disproof that attacks one not attack the other in principle? As the disproof of the QC [*qal vachomer*] comes inductively against the given claim and not the deductive part, one can simply disagree with the claimed givens of **h** and **v** as if wholly different; so the impression that they are the same claim presented in two ways is just an illusion."

However, it is difficult to say whether this is Wiseman trying to personally make sense of this issue, or he is reflecting Avraham's actual thinking. In any case, what seems clear is that the problem is not stated and dealt with by Avraham with the requisite clarity and thoroughness. His presentation is so padded and tortuous that one cannot readily discern just what he is trying to say[19]. I would like to make a fair assessment, and not dismiss Avraham's thesis offhand. But his presentation seems too unclear and messy to me. When one has nothing concrete to grab hold of, one can only say that one has nothing concrete to grab hold of.

What is certain, anyway, is that Avraham has not mastered a fortiori argument. We saw that from the beginning of his article he misapprehended what might constitute a fortiori discourse. Not surprisingly, he thereafter fails to produce a fitting and distinctive deductive formula for it. He is therefore reduced to looking for some inductive solution to the problem. But even that attempt leads him to an impasse.

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[18]   Wiseman, p. 78; with references to Avraham, p. 41-2.
[19]   Even regarding such a simple thing as what Avraham means by "horizontal" and "vertical," I have doubts. His "horizontal" (*meusan*) argument includes a statement (in symbols, of course) that the fine for damage by 'goring' in the public domain is greater than that for 'bite or hoof' in the public domain – yet if we look at his table, these two items are in the same column, not the same row. Accordingly, his "vertical" (*meunakh*) argument has a statement that the fine for damage by 'bite or hoof' in the private domain is greater than that for same in the public domain – yet these are in the same row, not the same column.

21. Gabriel Abitbol

Gabriel Abitbol's 1993 book, *Logique du droit talmudique*[1] is a detailed study of Talmudic logic (in French, obviously), including a long chapter devoted to *qal vachomer* argument (pp. 126-169). I found a copy of this book (which is, by the way, beautifully bound) by chance in a Paris bookshop in 1995, when my book *Judaic Logic* was almost finished. I did not read it all then, but only scanned through it enough to make sure Abitbol had not anticipated my findings. I will here look at his work more closely, especially as regards a fortiori argument.

1. Name and functioning

Abitbol begins his study in a promising manner, remarking that due to *qal vachomer* reasoning not being defined in Talmudic literature there are ambiguities regarding its application, and boldly stating his intention to discover the exact nature and logical structure of *qal vachomer* reasoning and, after examination of some examples of its use in Talmudic jurisprudence, to define it and determine the modalities of its application.

Terminology. He proceeds by first clarifying the expression *qal vachomer*, which may be identified with *a fortiori* argument. The word *qal*, which in Hebrew means "light" (i.e. of little weight), designates "a prescription that is less grave or easier to observe than another, or anything permitted, pure, without fault;" while the word *chomer*, which means "matter, heavy thing," has the opposite legal connotations. Since there are two inferences characterized by the conjunction *qal vachomer*, viz. *a maiori ad minus* and *a minori ad maius*, which serve respectively to attenuate or aggravate, it is clear that the expression does not indicate "the process through which we arrive at a conclusion," but simply recalls "the elements constituting the two premises of the reasoning."[2]

Here, I should say a few words. Abitbol is essentially right in this analysis of the *qal vachomer* expression, but wrong in characterizing the two parts of it as "premises." *Qal* and *chomer* in fact refer to *terms* found in the premises; they are not themselves premises. Both terms are found in the major premise, and only one of them appears in the minor premise while the other appears in the conclusion. In argument *a maiori ad minus* (i.e. from major to minor), the *chomer* is in the minor premise; while in argument *a minori ad maius* (i.e. from minor to major), the *qal* is in the minor premise.

Evidently, Abitbol thinks of the word *qal* as referring to the *proposition* containing the *qal* term, and the word *chomer* as referring to the *proposition* containing the *chomer* term; but these propositions cannot both be premises – one is a premise and the other a conclusion – so this is a bit thoughtless[3]. As regards the characterization of these two directions of argument as "attenuation" and "aggravation," he cites R. Samson de Chinon in *Sefer Keritut* as the source[4]. Personally, I think these characterizations are inappropriate in the context of a fortiori argument, since they signify *a change* of stringency and no change occurs in it – we just draw out implicit information.

Next, Abitbol informs us that the two "premises" (as he calls the propositions, again) are called in the Talmud *melamed* and *lamed*, i.e. the one that teaches and the one which is clarified by a teaching. He insists that these "must imperatively be constituted by prescriptions of the Written Law," and that any *qal vachomer* not based on Torah legislation (but, e.g., based on some oral tradition, even if that is attributed to Moses at Sinai) is not a *qal vachomer* in the sense intended in R. Ishmael's list of hermeneutic principles (but merely an "argumentative" *a fortiori*[5]). Here, of course, I agree with him, and say he is right to point it out, as some commentators do ignore or forget this truth. However, to be precise, it is only one of these two propositions, viz. the minor premise, which is inscribed in the

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[1]   Paris: Edition des Sciences Hébraïques, 1993.
[2]   Translations from French into English are my own, throughout the present chapter.
[3]   The word "premise" etymologically refers that which comes before, while the word "conclusion" refers to that which comes after. The same terminological error is found in other accounts, notably that of Louis Jacobs in his article on Hermeneutics in the Encyclopaedia Judaica (1971-2). Note also that Abitbol in this context makes an error of inattention: while rightly describing the major-to-minor process as "inference from a grave prescription to one that is less so," he wrongly describes the minor-to-major process as "passage from an important prescription to one that is less so." I mention this in passing, but it is unimportant since as is clear from his other statements he knows the correct reading.
[4]   New York, 1971 (publisher not specified), p. 3.
[5]   He gives a couple of examples of those, drawn from the set of ten or so found in the Torah – namely, Gen. 44:8, Ex. 6:12 and Dt. 31:27.

Torah; the other proposition, being the conclusion of the argument, is merely inferred. They are both Torah based, in a broad sense; but one is directly so, while the other is indirectly so.

Mechanism. After these preliminaries, Abitbol begins his analysis of *qal vachomer* in earnest, with reference to God's reaction to Miriam's gossip concerning Moses (Numbers 12:14), i.e. to the statement: "*If her father had but spit in her face, should she not hide in shame seven days? Let her be shut up without the camp seven days, and after that she shall be brought in again.*" According to Abitbol, the *qal vachomer* involved here is, as a *baraita* (given in *Sifra*, a legal commentary on Leviticus dating from Mishnaic times) asserts, that Miriam who offended God (and not merely her father) should have been shut up for fourteen days. Here, note well, the given predicate is "seven days" incarceration, whereas the inferred predicate is "fourteen days" of it.

In other words, he agrees with those commentators (including the Gemara, in Baba Qama 25a which refers to the same *baraita*) that a fortiori argument is inherently 'proportional'; i.e. that it is a crescendo argument. This does not mean that it is always 'proportional', since Abitbol also gives as example the *qal vachomer* proposed by Hillel to the Bene Bathyra in Pesachim 66a, which goes (in the Soncino Talmud): "*if the tamid, [the omission of] which is not punished by kareth, overrides the Sabbath, then the Passover [neglect of] which is punished by kareth, is it not logical that it overrides the Sabbath!*"[6] Here, the given and inferred predicates are clearly one and the same (viz. "overrides the Sabbath"). However, note well, Abitbol does not remark upon the difference between a 'proportional' conclusion (which the Miriam argument has, in his eyes) and a 'non-proportional' one (which the Hillel has, though he does not mention the fact).

Abitbol explains the "mechanism of the reasoning" concerning Miriam as follows: "comparing two situations," one where someone offends a neighbor (i.e. a daughter offends a father, in this case), for which the penalty is known, and the second where someone offends God, for which the penalty is not known, it is "deduced" from the fact that God is hierarchically above Man that the sanction is to be greater in the case of offense to God[7]. As regards Hillel's argument, the reason why it does not similarly conclude with a greater penalty is due to the fact that the hierarchy between God and Man in the first example is obvious, whereas that between the *tamid* (i.e. perpetual) sacrifice and the Passover (i.e. *pesach*) sacrifice has to be sought out.

Both these explanations are unconvincing; they are non sequiturs. As can be shown formally, the fact that two subjects are in a hierarchy does not in itself prove that they are subject to different predicates (or for that matter the same predicate); there has to be a separate justification for any claim of proportionality (otherwise, the minimum implied by non-proportionality must be assumed). Moreover, it does not matter how complex the process of justification is, i.e. whether the hierarchy is obvious or difficult to establish; what matters is only the end product of the process, i.e. the fact of the claim concerned being credible. Incidentally, the distinction made here by Abitbol between obvious and sought-out hierarchies calls to mind the distinction made by Louis Jacobs between simple and complex a fortiori.

Clearly, Abitbol (like many commentators before and after him) has not grasped the fact that a fortiori argument is essentially pure (i.e. non-proportional), and only occasionally (when an additional premise of proportionality is duly supplied) a crescendo. This means that he has not succeeded in his initially announced quest to discover the exact nature and logical structure of *qal vachomer* reasoning. As we shall see, Abitbol conflates a fortiori argument with analogical argument throughout his analysis.

2. Tabular representation

Abitbol presents a fortiori argument as a means of inference, from a textually given legal disposition, of an unknown (i.e. not textually given) legal disposition, through analogy to another pair of comparable legal dispositions. He proposes to graphically "represent" it in tabular form[8], essentially as follows (Table 21.1):

A fortiori	Situation 1	Situation 2
Subject 1	legal norm A	legal norm C
Subject 2	legal norm B	legal norm D

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[6]   "*Kareth*" refers to a Divine punishment, known in English as "cutting off."

[7]   Here Abitbol cites R. Aaron Ibn Chaim's *Midot Aharon* (Amsterdam, 1742) as asserting that comparison and hierarchization are the foundations of all *qal vachomer*.

[8]   Note that Abitbol is not the first (or last) to have opted for a tabular representation. As we have seen in the previous chapter, he was preceded in this by Michael Avraham (chapter 20), at least. We find similar tables used later, in the joint work of Abraham, Gabbay and Schild (chapter 25) and in that of Andrew Schumann (chapter 27).

Actually, Abitbol labels subjects 1 and 2 as P1 and P2, and situations 1 and 2 as S1 and S2, respectively. I avoid doing so, so as not to cause readers confusion with my standard symbols P and S. Moreover, he does not have the symbols A, B, C, D, but instead has a + or – sign in each of the four "legal norm" cells. Positive signifies "being permitted, easy, pure or without fault," and so on; and negative means "forbidden, grave, liable to, impure," and so on.[9] Thus, the "legal norms" are value judgments. However, since these signs vary from one example to the next, I have opted for the said variables. How is this table to be read – vertically or horizontally? It is evident that Abitbol is unsure on this issue, or at least unaware of the question and therefore inconsistent.

Judging by his stated intention ("the reading of this table is then as follows," p. 132), the reading should be vertical, i.e. as follows:

> Since in situation #1, subject #1 is A and subject #2 is B (*melamed*), it follows that in situation #2, where subject #1 is C, subject #2 will a fortiori be D (*lamed*). When A > B, then C > (or =) D (this is from major to minor); and when A < B, then C < (or =) D (this is from minor to major).

However, judging by most (though not all) of the examples he gives, he actually usually reads the table horizontally, i.e. as follows:

> Since subject #1 is A in situation #1 and is C in situation #2 (*melamed*), it follows that subject #2, which is B in situation #1, will a fortiori be D in situation #2 (*lamed*). When A > C, then B > (or =) D (this is from major to minor); and when A < C, then B < (or =) D (this is from minor to major).

Apparently, Abitbol does not fully realize the logical difference between these two processes. This is perhaps due to the fact that, in both the vertical and horizontal readings, if A, B and C are respectively +, – and –, then D will be –; and if A, B and C are respectively –, + and +, then D will be +. But these formulae are superficial, because they ignore the generalizations involved behind the scenes (more will be said on this further on). Note that the bracketed "or =" is my own addition, based on some of his examples, in which there is a legal maximum, so that "more" effectively means "as much"[10].

An example Abitbol uses for illustration is the a fortiori of Hillel to the Bene Bathyra (Pes. 66a), which we have already transcribed above. The way this example is originally worded (p. 130), with *tamid* sacrifice, *kareth* penalty and Shabbat mentioned first, followed by *pesach* sacrifice, *kareth* and Shabbat, suggests the following reading: since as regards the *tamid* sacrifice, the penalty of *kareth* for omission is not applicable, whereas overriding the Sabbath is applicable, it follows as regards the *pesach* sacrifice, for which the penalty of *kareth* for omission is applicable, that overriding the Sabbath must be applicable. This is reasoning from the minor to the major: the given *qal* and *chomer* being no-*kareth* and overriding of the Sabbath, and the inferred *qal* and *chomer* being *kareth* and overriding of the Sabbath. Abitbol represents this (p. 133) as a table with *tamid* and *pesach* as the "subjects" and *kareth* and Shabbat as the "situations;" therefore, his reading of the table here must be horizontal.

Another example he uses for illustration is from *Beitzah* 20b[11]. Its original wording is (roughly, p. 133): if on the Sabbath food preparation is forbidden while the *tamid* sacrifice is permitted, all the more on a Festival day, when such tasks are permitted, the *tamid* should be permitted. Here, the reasoning is from major to minor, with the given *chomer* and *qal* being Sabbath food preparation and *tamid* sacrifice, and the inferred *chomer* and *qal* being Festival food preparation and *tamid* sacrifice. Abitbol represents this (p. 134) as a table with food preparations and sacrifices as the "subjects" and Shabbat and Festivals as the "situations;" therefore, his reading of the table here must be vertical. This change of direction of reading goes to show that Abitbol tabulates the information in rather haphazard fashion, without attention to consistency. As we shall see, he sows additional confusion in this matter further on, when dealing with the two arguments of R. Tarfon. Although this issue is not crucial, it is indicative of a certain lack of professionalism.

Deficiencies. Abitbol's tabular representation is obviously incomplete, since it leaves much important information unspecified. When the inferred norm (D) is said to be is greater or lesser than the norm from which it is inferred (C or B, depending on the reading direction), we are not told by *how much* it is so; presumably, the change of magnitude would be proportional, but this is too vague a characterization for clear judgment. Very often, too, the inferred norm is in fact equal to the given norm (as the two examples just given illustrate). Moreover, the *major premise* is not explicitly given in Abitbol's table, even if it is implicit in the + and – signs placed in cells A and B, or respectively A

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[9]   Actually, Abitbol does not consistently stick to these definitions. Thus, in a later example (p. 138) he has + designating death sentence and – the opposite, i.e. + as the more severe sentence and – as the less so. Also, in the second argument of R. Tarfon, the symbols P1 and P2 change places with the symbols S1 and S2 (p. 145).

[10]   I could have added the special case of *a pari* (egalitarian) a fortiori argument, in which if A = B, then C = D – or if A = C, then B = D. Abitbol does not here mention this.

[11]   I have not found these exact words on that page, but I presume this is an interpretation of the sentence: "If when thy hearth is closed, the hearth of the Master is open, how much the more must the hearth of thy Master be open when thy hearth is open."

and C (obviously, if A is + and B is −, then A < B, and if A is − and B is +, then A > B; likewise, if A is + and C is −, then A < C, and if A is − and C is +, then A > C).

It cannot be said that Abitbol is unaware of the *major premise*, since he repeatedly refers to a hierarchy between the subjects. Indeed, further on (on p. 165), he explicitly refers to a judgment that subject #1 "is less (or more) grave than" subject #2, in which the two subjects are placed in a "scale of values" (*échelle évaluative*) in accord with their legal value (as permitted, forbidden, etc.). Moreover, he even there shows awareness of the *middle term* (which was thus far missing, or merely implicit) when he adds that the same subjects may be "ordered in another scale" (i.e. they may be differently "grave" in some other respect, in relation to some other middle term) and thereby turn out to have opposite relative values (i.e. the major term in one scale may be the minor term in another, relative to one and the same other term). But the problem is that his tabular scheme does not allow him to represent these subtleties.

But the main criticism I would level against Abitbol's treatment of a fortiori argument is his failure to realize the role of the middle term in the minor premise and conclusion. As just stated, he does acknowledge the middle term implicit in the major premise, in the context of the possibilities of "refutation" of a qal vachomer (as we shall see further on); but this does not imply that he is aware of its presence and crucial function in the minor premise. Nowhere does Abitbol explain the a fortiori inference of a conclusion as made possible by the passing of *a threshold value of the middle term* implied in the minor premise. Without this idea, he cannot be said to have grasped the full essence of a fortiori argument and what he refers to by that name is nothing more than mere analogy.

It is because of his failure to notice the need for a subject to have a *sufficient amount* of the middle term to gain access to a certain predicate (which condition is given in the minor premise, and makes possible the conclusion) that he remains stuck in the realm of mere *analogical argument*. At one point (p. 136) Abitbol seems about to hit upon this ignored element, but he surprisingly misses it. He says, much too vaguely, that the conclusion can be established only if two situations are shown directly or indirectly to be "tied by a certain relation" which allows the establishment in the second situation (i.e. between C and D) of the hierarchy observed in the first (i.e. between A and B). But he does not tell us what that "certain relation" might be, other than to refer to "reasoning by analogy." Thus, Abitbol cannot claim to have truly understood and correctly defined *qal vachomer* reasoning.

Standardization. These criticisms become clearer when we try to put Abitbol's effective formula for a fortiori argument in standard form for him. The following positive subjectal mood would correspond to his minor-to-major argument in the vertical reading:

> Since subject #2 (= P) is more "grave" (in some respect, R) than subject #1 (= Q)
> (as we know by generalization from situation #1, by comparing A and B),
> and, in situation #2, Q is (R enough to be) C (= S);
> (and S is constant, or varies in proportion to R;)
> it follows that P is (R enough to be) D (= S, or more than S).

As regards his major-to-minor argument, it would needs be put in negative subjectal form to be valid, even though he effectively presents it as equally positive reasoning. Note the reversal of order of the subjects in the major premise:

> Since subject #1 (= P) is more "grave" (in some respect, R) than subject #2 (= Q)
> (as we know by generalization from situation #1, by comparing A and B),
> and, in situation #2, P is (not R enough to be) C (= S);
> (and S is constant, or varies in proportion to R;)
> it follows that Q is (not R enough to be) D (= S, or less than S).

Evidently, Abitbol's two "subjects" are indeed logical subjects (the major and minor terms, P and Q), while his "legal norms" A, B, C and D are their predicates (C and D being the subsidiary term S). However, it is clear that the two "situations" stand outside the a fortiori arguments as such: they are actually linked by a generalization from the first situation to all situations and thereafter a subsumption from all situations to the second situation. Abitbol evidently does not realize this; i.e. he does not realize that the first situation, involving the predicates A and B, is relevant to the a fortiori argument only insofar as it helps us to establish, *by generalization*, its comparative major premise; but it is not in fact part of the a fortiori argument. As for the second situation, it is *the framework* within which the a fortiori argument per se occurs, but it plays no active role within the argument.

We could, of course, similarly standardize the horizontal reading. The minor-to-major mood would here read:

> Since situation #2 (= P) is more "grave" (in some respect, R) than situation #1 (= Q)
> (as we know by generalization from subject #1, by comparing A and C),

and, in subject #2, Q is (R enough to be) B (= S);
(and S is constant, or varies in proportion to R;)
it follows that P is (R enough to be) D (= S, or more than S).

And the major-to-minor mood would read:

Since situation #1 (= P) is more "grave" (in some respect, R) than situation #2 (= Q)
(as we know by generalization from subject #1, by comparing A and C),
and, in subject #2, P is (not R enough to be) B (= S);
(and S is constant, or varies in proportion to R;)
it follows that Q is (not R enough to be) D (= S, or less than S).

Here, note well, Abitbol's two "situations" are the logical subjects, while his two "subjects" are the outside conditions. Also, the generalization occurs with reference to subject #1, and is then applied to subject #2. Therefore, it seems to me, if we are going to use the terminology of "subjects" and "situations," we would be better off sticking to the vertical reading, where these terms are literally most appropriate. In other words, Abitbol's initial theoretical intent to go by the vertical reading was correct, whereas his subsequent frequent use in practice of the horizontal reading was not correct. To repeat, this is not a crucial matter – but it implies thoughtlessness and sows confusion.

More deficiencies. Anyway, from these more accurate formalizations, we can also see more clearly the missing or merely implied elements of Abitbol's representation. He lacks an explicit major premise and middle term; he lacks an additional premise relating to proportionality, where needed; and most important of all, he totally lacks the crucial clause "R enough to be" (or "not R enough to be," as appropriate). For these reasons, he cannot in fact logically validate the argument, even though he thinks he has done so by merely (roughly) representing it in a table and giving a few explanations. Moreover, Abitbol evidently does not realize that the above listed major-to-minor argument (i.e., in the vertical reading, when $C < D$, since $A < B$) is essentially negative in form. His failure to realize this is no doubt in part due to his opting for a graphic representation. It is difficult if not impossible to represent the distinction between positive and negative argument in a table; one might resort to 'crossing off' cell contents to signify their negation, but this is rather messy.

Furthermore, Abitbol does not show awareness of the distinction between predicatal argument and subjectal argument. I guess he would just tabulate predicatal arguments he came across in the same way, without highlighting the differences since he has no means to do so. That is to say, he would reword a predicatal argument he came across in superficially subjectal form[12], and thus manage to fit it into his tabular representation[13]. However, since none of the examples of a fortiori argument Abitbol gives happen to be predicatal, it is not possible to say for sure how he would react to such eventuality. Maybe it would make him change his whole outlook.

Non-universality. Moreover, as we shall now demonstrate, contrary to the impression that the above selected examples give, it is not always the case that we have two "situations" to deal with. We may well be given the general major premise at the outset, without having to derive it from a specific instance in this way. In other words, we do not always have to engage in preparatory work to obtain the needed major premise: it may be given in a text and taken for granted, or it may be intuitively obvious or generally believed to be so. Indeed, Abitbol himself earlier (pp. 130-1) effectively admits this when he makes a distinction between an obvious hierarchy (as that between God and Man implied in the Miriam example) and a hierarchy that has to be sought out (as that between the perpetual and Passover sacrifices involved in the a fortiori spoken by Hillel to the Bene Bathyra).

Therefore, Abitbol's table *is not universally applicable*, note well. Consider, for example, the following a fortiori argument presented by Abitbol (based on Makot 5b)[14]. It originally reads, roughly, as follows: "if false witnesses are liable to capital punishment if their lying is discovered by the court while the person they falsely accused is still alive, then *qal vachomer* they will be liable to capital punishment if their lying is brought to light after their victim has been executed."

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[12]   I have called this process traduction. See chapter 3.5 above. As I show there, this is merely verbal change – the argument's inherent form as predicatal is in fact unaffected.

[13]   There is no point my trying to do the job for him, since I do not think his tabular representation method is of much use, anyway.

[14]   See pp. 137-9, and further details on pp. 148-51. Note that the Sages eventually reject this argument, judging that if the accused is already dead the false witnesses need not be executed, based on the Torah (Deut. 19:18-19) saying "ye do unto him [any false witness], as he had *purposed to do* unto his brother" instead of "as he had *done*." I have changed Abitbol's wording a bit in the course of translation, only so as to briefly clarify the context. This argument is only alluded to in the Gemara ("…is there not an argument a fortiori?"), but the Soncino Talmud has a footnote making it explicit as follows: "If zomemim are put to death when their plot failed, it is surely all the more necessary that they should be where their plot had succeeded!" The word *zomemim* refers to false witnesses.

I would put this argument in standard form (minor premise and conclusion) as follows: 'if false witnesses discovered while the victim of conspiracy is yet alive (Q) are criminal (R) enough to be liable to capital punishment (S), then false witnesses discovered while the victim of conspiracy is already dead (P) are criminal (R) enough to be liable to capital punishment (S)'. This is clearly positive subjectal, minor-to-major reasoning, made possible by the tacit but intuitively obvious major premise that 'false witnesses discovered while the victim of conspiracy is already dead (P) are more criminal (R) than false witnesses discovered while the victim of conspiracy is yet alive (Q)' (since a successful criminal conspiracy is obviously worse than an unsuccessful one). Abitbol, on the other hand, tries to represent the argument in tabular form as follows[15]:

(Makot 5b)	Status of the accused (situation 1)	Status of the witnesses (situation 2)
Refutation before the execution (subject 1)	Not death sentence (A)	Death sentence (C)
Refutation after the execution (subject 2)	Not death sentence (B)	Death sentence (D)

This is manifestly a fabrication. He is *forcing* the given material a fortiori argument into a preconceived form, which is clearly not appropriate to it. For, where did he get the two "situations" from, here? Certainly the first situation is nowhere mentioned or needed in the original argument. What does "status of the accused" (the heading of the first column) mean exactly? We might say that if the witnesses are shown up to be liars before the accused is executed, then indeed he would not be sentenced to death; but we cannot logically say that if the witnesses are shown up to be liars after the accused is executed, then he would not be sentenced to death, since that is said and done (the poor man might have his name cleared, but his life is gone). The tacit major premise is intuitively obvious; it is not a generalization from one situation to another.

Note incidentally that Abitbol here again must be reading the table horizontally, for only in this way can the argument be said to be from minor to major (since A < C and B < D). If he read the argument vertically, he would be comparing A and B (which are equal) and applying the same relation (equality) to C and D; this would not be minor-to-major reasoning like the original argument, but *a pari*. This is one more inconsistency between his practice and his theory.

Abitbol tries to stuff the given argument into his tabular format by changing its terms somewhat. But his table doesn't make sense. If read horizontally, as he seems to read it, it says, in plain English: since in the event of the witnesses being discredited before execution of the accused, the latter is not liable to capital punishment whereas the former are so, it follows that in the event of discredit of the witnesses after execution of the accused, the latter is not liable to capital punishment whereas the former are so[16]. Clearly, this new argument is not the same as the given one! In fact, it is rather meaningless and unconvincing. The proposed re-presentation is thus very artificial and inappropriate. This is not due to some problem in the original argument, but simply means that Abitbol's tabular scheme is not fit to represent all conceivable a fortiori arguments. There are cases, like this one, that the scheme did not foresee, because they do not involve two "situations." Instead of realizing this and reviewing his scheme accordingly, Abitbol tried to fake compliance with it. Evidently, not knowing quite how to handle this argument, he forced things a bit, hoping to get away with it.

3. Treatment of dayo

Abitbol merits commendation for his taking into consideration, in his discussion of the *dayo* principle (pp. 140-8), both of R. Tarfon's a fortiori arguments in the Mishna Baba Qama 2:5 – some commentators (including the Gemara commentary in Baba Qama 25a-b) merely focus on the first. Both these arguments are necessary for a full understanding of the *dayo* ("it is sufficient") objections to R. Tarfon's conclusions by his colleagues, known collectively as the Sages, on which the *dayo* principle is based. However, as we shall see, he does not represent these

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[15]   Note that Abitbol has a + sign to denote "death sentence" and a – sign for "the contrary" (i.e. not death sentence). This is, incidentally, inconsistent with his earlier convention, according to which the *qal* (i.e. not death) should be a +, while the *chomer* (i.e. death) should be a –. This is not, of course, a big deal – except that it again shows some disorderly thinking on his part.
[16]   If read vertically, it runs as follows: since as regards the status of the accused, if the witnesses are discredited before his execution he is not liable to capital punishment, whereas if they are discredited after his execution he is not liable to capital punishment, it follows as regards the status of the witnesses, where witnesses discredited before their victim's execution are liable to capital punishment, that witnesses discredited after his execution are likewise liable to capital punishment.

two arguments in his tabular scheme quite correctly, and moreover does not grasp exactly where in each of them the *dayo* principle actually operates.

R. Tarfon's two arguments. R. Tarfon's two arguments are should by now be familiar to readers of the present volume, so we can here reproduce them very briefly, as follows. They concern damages caused by a bull, either by 'tooth & foot' (munching or trampling) or by 'horn' (goring), either on public grounds or on private property; and their purpose is to determine, from information given in the Torah regarding the appropriate compensation in some cases, what the appropriate compensation is to be in cases not mentioned in the Torah – more specifically, in the case of damage by horn in the private domain. R. Tarfon argues in two ways that the owner of the bull should pay full compensation for damages in that context. He argues as follows (I paraphrase):

- Argument #1 – since in the case of tooth & foot the owner needs pay nothing for damages in the public domain and must pay in full for damages in the private domain, it follows in the case of horn, where the owner must pay half for damages in the public domain, that he has to pay in full for damages in in the private domain. The Sages contend that the owner need only pay half damages by horn on private property, just as he does on public grounds[17].

- Argument #2 – since in the case of the public domain the owner needs pay nothing for damages by tooth and foot and must pay half for damages by horn, it follows in the case of the private domain, where the owner must pay full for damages by tooth & foot, that he has to pay in full for damages by horn[18]. The Sages contend that the owner need only pay half damages by horn on private property, just as he does on public grounds.

Now, before we look into how Abitbol represents these two arguments in his tabular scheme, let us see how we would represent them, given that ideally they are supposed to have "subjects" as row headings and "situations" as column headings and should be read vertically (i.e. first down column 1 and then down column two).

R. Tarfon's 1st argument	Tooth & foot damage (situation 1)	Horn damage (situation 2)
In the public domain (subject 1)	No compensation (A)	Half compensation (C)
In the private domain (subject 2)	Full compensation (B)	Full compensation (D)

R. Tarfon's 2nd argument	In the public domain (situation 1)	In the private domain (situation 2)
Tooth & foot damage (subject 1)	No compensation (A)	Full compensation (C)
Horn damage (subject 2)	Half compensation (B)	Full compensation (D)

Clearly, in the first argument, the "situations" are tooth & foot and horn and the "subjects" are the public and private domains; whereas in the second argument, the "situations" are the public and private domains and the "subjects" are tooth & foot and horn. In each argument, A, B and C are given (in that order) and D is derived (as the conclusion). We generalize the relation between A and B, and apply that generality to the relation between C and D.

Abitbol's tables, on the other hand, are turned the other way; i.e. he uses the first of the above tables to represent the second argument, and the second table for the first argument. This is not a big deal, but just means that his tables are to be read horizontally instead of vertically, for it is clear that he does understand the two arguments[19]. As we have seen, he does usually (though not always) set up his tables for horizontal reading, even though he initially (rightly)

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[17]   Note in passing that Abitbol wrongly interprets the Sages' *dayo* here as "reducing the payment by half" (p. 144). This is not strictly correct. Although *in this particular case* the payment advocated by the Sages is half that advocated by R. Tarfon, this proportion is mere happenstance. What the Sages are really saying is that the payment *should remain unchanged*. Clearly, if the original payment was, say, a third of full payment, R. Tarfon would have still concluded with full payment and the Sages would have concluded with a third (not a half). Abitbol is obviously influenced here by the Gemara's approach to a fortiori argument.

[18]   As Abitbol points out (p. 146), the conclusion could have been "*more* than the full payment," and thence "has to be at least full." He surprisingly does not go into the question why R. Tarfon concludes with full payment and not proportionately "more than full." The reason is, of course, that no punitive indemnity is intended, only restitution of financial losses.

[19]   Since he explicitly says (p. 145) that the "analogy" can be "either between the hierarchies defined by the situations, or between those defined by the subjects." However, when he adds "which makes possible the formulation of the same *qal vachomer* in two ways," it is clear that he considers both arguments as one and the same *qal vachomer*, whose terms are merely reshuffled. The latter perspective is not correct – the second argument differs more significantly from the first than Abitbol implies, due in fact to the different generalization preceding the major premise in each case.

defines the reading direction as vertical. Even so, this turning of the tables should be pointed out here as further indication that Abitbol experiences a certain amount of confusion in this issue. All the more so since in his first table the subjects are symbolized as P1 and P2, and the situations as S1 and S2 (as per his initial definitions of those symbols), while in his second table the subjects are symbolized as S1 and S2, and the situations as P1 and P2 (he obviously does this intentionally, so as to stress that the two pairs of terms have changed position, but this is not formally correct procedure[20]).

The Sages' two *dayo* objections. These few criticisms are not of great import, to repeat. But the fact that Abitbol does not interpret the relation between R. Tarfon's two arguments fully correctly and that he fails to see precisely what parts of them the Sages' two *dayo* objections are aimed at – that is of considerable import.

While Abitbol evidently (as we have just seen) is aware of the difference of direction in R. Tarfon's two arguments, he does not ask why the second argument constitutes a valid retort to the Sages' first *dayo* objection[21]. The answer to that question is that whereas R. Tarfon's first argument infers full damages (for horn in the private domain) *from half* damages (for horn in the public domain), his second argument infers full damages (for horn in the private domain) *from full* damages (for tooth & foot in the private domain). From this reformulation of his argument, using the same information in a new way, it is clear that R. Tarfon interpreted the Sages' *dayo* objection as an objection to the inference of full compensation from half compensation, and thus sought to avoid this weakness by proposing a renewed argument that infers full compensation from full compensation. In the latter case, the Sages' objection would seem to be successfully taken into consideration, for the conclusion refers to the same amount of compensation as *the minor premise* does.

Yet the Sages respond to this ploy by repeating the same objection, word for word (in what looks like a redactor's or editor's copy-and-paste job)! Abitbol here again fails to ask the obvious question: in what way does this repeated *dayo* objection interdict R. Tarfon's second argument? Obviously, the second *dayo* objection cannot really be identical to the first, since R. Tarfon's second argument was precisely designed to logically avoid what he thought was the Sages' first objection. Therefore, the Sages' second objection must be aimed at some other aspect of R. Tarfon's reasoning. Indeed, it is aimed at the formation of *the major premise* of his second argument. This premise relies on generalization from the given information (derived from Ex. 21:35) that compensation for damages caused by horn on public grounds is half.

Thus, the Sages' second objection must mean that, no matter how the given information is used in an a fortiori argument, whether as a minor premise (as in the first argument) or as the basis of a major premise (as in the second argument), the concluding amount (of compensation, in this case) cannot surpass the given amount. That is to say, the *dayo* principle is not merely that such information cannot be used as a minor premise in an a crescendo argument, but more broadly that such information also cannot be used as a major premise in a purely a fortiori argument. No conclusion can exceed in magnitude the given information from the Torah (concerning a penalty), *no matter how* (i.e. by what process of reasoning) such conclusion is drawn. That is the full meaning of the *dayo* principle, and that is why *both* of R. Tarfon's arguments are rejected by the Sages.

Abitbol has evidently not realized all this, because he makes no mention of it even though it is essential to true understanding of the Mishnaic discussion. It should be said that Abitbol is not alone in this failure of realization. The Gemara commentary relative to this Mishna is also totally unaware of these issues, and instead of addressing them gets bogged down in tangential issues of very doubtful relevance. It is only much later – I suspect thanks to some genial Tosafist (though I have not yet found who exactly, and when and where) – that Jewish tradition acknowledged a difference between the said two possibilities of *dayo* application. It was then stated that the Sages' *dayo* to R. Tarfon's first argument was aimed "at the end of the law" (*al sof hadin*), whereas their *dayo* to his first argument was aimed "at the beginning of the law" (*al techelet hadin*).

This traditional distinction does not fully clarify the issues, but it does take us a long way in that direction. It is surprising that Abitbol does not mention the distinction[22], since he refers in his bibliography (on p. 350) to the Encyclopedia Talmudit article on the *dayo* principle (in vol. 7, pp. 282-290) where this distinction is clearly mentioned. The explanation, I suggest, is that Abitbol is not aware that generalization is involved in the formation of the major premises of both of R. Tarfon's arguments. Due to the constraints of his tabular representation of the reasoning processes, he does not constantly have in mind the major premise (even though, as we saw, he somewhat

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[20]   The symbols should stick to the formal functions, not to the contents.
[21]   All he says about it is that "the conclusion of this *qal vachomer* was not upheld by the Sages due to the restrictive principle of *dayo*."
[22]   He does, further on, allude to this issue in his account of the Talmudic doctrine of refutations of *qal vachomer*; but he does not allude to it in the present context, in relation to the two arguments of R. Tarfon.

realizes its presence in the background), nor see how precisely it is derived from the Torah-given data. His tables, which are end products of reasoning processes, appear like raw givens[23].

To be fair, Abitbol does clearly say (p. 146-7) that the *dayo* principle is to be applied "whatever the hierarchy of reference defined at the start," adding by way of explanation: "what permits the interpreter to reason by *qal vachomer* is precisely the fact that the Torah has furnished a value that allows the definition of a hierarchy… as well in the first *qal vachomer* as in the second." By the latter explanation he means that since the major premise ("the definition of a hierarchy") is based on a "value" given in the Torah (i.e. the half payment for horn damage on public grounds), it would be wrong to exceed it. Furthermore, he says that the reason why the Sages "limit in one case the conclusion of the *qal vachomer*" and in another case "annul it" – presumably referring to respectively the first and second arguments of R. Tarfon – is their belief that a penalty should not be imposed on the basis of an interpretation (more on this topic further on).

He then rightly says that "all this cannot be understood if one does not turn towards the structure of the argument in question." However, his proposed structural explanation is incorrect. In his view, *qal vachomer* argument, which consists in reasoning from an admitted hierarchy to a contested one, "cannot in all cases lead to a complete certainty due to the analogy it involves;" such reasoning can only therefore result in a "probable" conclusion, which is neither "absolutely true" nor "absolutely false." The conclusion, according to him, "does not exclude the possibility of another decision, even one contrary to it." He cites in support of this view various medieval commentators[24].

But they are only pointing out in a general way the possibility of human error of reasoning and the danger of thereby wrongly punishing an innocent person, whereas he is claiming that *qal vachomer* is structurally (i.e. inherently) incapable of guaranteeing the conclusion. By thus qualifying *qal vachomer* as "reasoning with an uncertain result," Abitbol shows that he has not grasped the essentially deductive, as against inductive, nature of a fortiori argument. The reason he has not done so is of course, as we have seen, that he has failed to formalize and validate a fortiori argument. He thinks of it as mere analogy[25], and indeed his tabular representation of it is applicable to mere analogy. Thus, I am justified in saying, as above, that Abitbol has not fully understood the two arguments of R. Tarfon and the two *dayo* objections to them by the Sages.

The *dayo* principle. Let us now look at Abitbol's description and interpretation of the *dayo* principle as such (pp. 137-157). The latter principle, as we have seen, is based on the Sages' reply to both of R. Tarfon's arguments in the above described Mishnaic dialogue; it reads (in the Soncino Talmud): "it is quite sufficient that the law in respect of the thing inferred should be equivalent to that from which it is derived." Abitbol interprets it as: "it suffices for the conclusion of the reasoning to conserve the quantity of the premise." Although the original statement does not explicitly mention quantity (it says *ka*, meaning: as, like), this interpretation shows his awareness of the quantitative aspect of a fortiori argument.

Interestingly, Abitbol cites a second rabbinic principle as relevant in this context, viz. *ain onshin min hadine*, meaning literally, no punishment may be based on inference, or as he puts it: "the abrogation of a penal sanction based on a reasoning." An example of this that he gives is the above mentioned argument concerning false witnesses (based on Makot 5b). According to him, the conclusion of this argument, viz. that the false witnesses should be put to death if the accused has already been executed, is not abided by, in accord with the said principle of *ain onshin*, which he later tells us applies to all conclusions enjoining capital punishment or flagellation (whipping).

Thus, judging by Abitbol's presentation, it appears that *two* principles are involved in preventing penalties based on reasoning. The *ain onshin* principle would be the main principle, preventing the penalties of death or flagellation (which are irreversible), but not lesser penalties, based on any sort of inference; while the *dayo* principle would be called upon to prevent lesser penalties (such as imprisonment or financial compensation for damage), possibly only those proceeding from *qal vachomer* inference. This is new to me, but could well be true. Note however that Abitbol identifies the *dayo* principle with the "axiom well known in logic [that] the conclusion must not surpass the

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[23]   This goes to show, incidentally, how exclusive or excessive reliance on gadgets (like tables, graphics or symbols) can blind and mislead the researcher instead of enlightening and guiding him.

[24]   Namely the authors of *Halikhot Olam*, *Beit Ha-otsar* and *Magid Taalumot*. The first says: "since *qal vachomer* is pure reasoning, it may happen sometimes that a man unintentionally makes an error by formulating a faulty *qal vachomer*." The second says: "one may fear an error of reasoning which would allow for its refutation." The third says: "the interpreter can make a mistake in the logical reasoning and as a result his judgment will be open to refutation without his being aware of it." None of these statements affirms that *qal vachomer* is inherently refutable, but only that people might well err in formulating such argument (as indeed any argument).

[25]   This is forcefully brought home on p. 156, where Abitbol refers to the two *qal vachomer* arguments of R. Tarfon, and then points out that the conclusion of "more than half" of the damages could mean "the totality or more than the totality." From an analogical perspective, his comment is reasonable for both arguments. But from an a fortiori perspective, while his comment is reasonable for the first argument, which infers *a crescendo* from half to full compensation, it cannot apply to the second argument, which infers *purely a fortiori* from full to full compensation.

premises;" whereas of the *ain onshin* principle, he says that it can go against logic[26]. If these viewpoints are truly rabbinic, they are open to criticism.

Identifying the *dayo* principle with the principle of deduction (viz. that the conclusion cannot contain more information than explicitly or implicitly given in the premises), as Abitbol does[27], and as many other commentators do too, is incorrect. That is because, though in some cases (e.g. in the Sages' first objection) the principle is indeed aimed at the a fortiori argument as such, in other cases (e.g. in the Sages' second objection) it is aimed instead at the generalization preceding the a fortiori argument (which means that is not part of the a fortiori argument as such). This criticism may seem like a bit of hair-splitting (*pilpul*), if we think broad-mindedly of all pertinent *information* used in the construction and execution of a *qal vachomer* as "premises." But if we reflect that preparatory steps prior to the *qal vachomer* are *not* technically part of the a fortiori argument since it could well occur without them, and they without it, then we must either say that the *dayo* principle is not exclusively about *qal vachomer* as such, or we must reserve it for the Sages' first objection and assign the Sages' second objection another label, if not altogether deny it.

Furthermore, we must ask how far the *dayo* and *ain onshin* principles overlap. If we assign a general form to the *dayo* principle, and view it as interdicting *all* inference of penalties, including therefore the death penalty and flagellation, then we apparently have no need of the *ain onshin* principle, at least with regard to *qal vachomer* inferences. But of course this is not correct – the *dayo* principle does not interdict all inferred punishment, but more precisely the inference of proportionately increased punishment, whether due to *qal vachomer* as such or due to the generalization preceding it; whereas the *ain onshin* principle, if I understand correctly, interdicts all inference by whatever means of a death penalty or flagellation, *even if* there is no proportional increase[28]. Thus, their scopes differ quite a bit, and at the same time there must be some overlap between them.

The *dayo* principle only forbids inference of penalties greater than those textually given, whereas the *ain onshin* principle only forbids inference of the more serious penalties of death or flagellation. Thus, the *ain onshin* is needed where the *dayo* principle is technically inoperative because the inferred penalty is *not* greater than the textually given one, and yet the penalty (being death or flagellation) is too serious to be applied by virtue of a mere inference. Thus, *ain onshin* is operative beyond the scope of the *dayo* principle; and conversely, the *dayo* principle is applicable to cases (involving lesser penalties) where the *ain onshin* principle is inoperative.

Abitbol, as we have seen, equates *qal vachomer* with analogy, and as a result does not see precisely when and where in the overall thought process the principle actually operates. He can only vaguely say that the *dayo* principle "limits" or "annuls" the proportional conclusion the argument seems to warrant. Nevertheless, he must be commended for being aware of the quantitative aspect a fortiori argument and of the *dayo* principle's focus on "penal sanctions." His understanding is superior to that of many commentators before and after him, in that he is (as evident throughout his presentation) aware that the principle concerns specifically the inference by rabbis (legislators or judges) of a *greater penalty* for a tort or crime not accounted for in the Torah from the lesser penalty for a similar tort or crime accounted for in the Torah. As he points out, though the conclusion of a *qal vachomer* has "probable character," it does not "determine the degree of sanction to be applied."

One reason, according to Abitbol, why a penalty must not be based on an inference is that all sanctions must be preceded by warnings. For example, the Torah sanction against someone who works on the Sabbath (Ex. 31:15) is preceded by the warning not to work on the Sabbath (Ex. 20:9). He gives many such examples[29]. In other words, justice demands that the laws individuals are subject to be explicit (regarding both prohibitions and penalties for their breach). Abitbol points out that this principle of Mosaic law is also affirmed in the Latin adage *nulla poena sine lege* (no punishment without law), and more clearly and forcefully as of the 18th century, being stated in the 1789 Declaration of Human Rights (art. 8) as: "no one may be punished except by virtue of a law established and promulgated before the offence was committed and legally applied."

This explanation looks good, but it is logically wobbly. For, if the rabbis inferred prohibitions and penalties, *and duly decreed and publicized them*, then the reasonable demand for explicit law would be satisfied. Clearly, then, there must be another reason for not inferring laws and sanctions from the Torah. The main purpose seems rather to be, as Abitbol also points out, to limit the power of human legislators and judges (i.e. of the rabbis), so as to maintain the primacy and sanctity of the Torah; for people, no matter how wise and devoted, can never be at once omniscient,

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[26]   "This conclusion however logical it be is not retained by the jurisprudence due to application to the *qal vachomer* of the principle *ain onshin…*" (p. 139).

[27]   Although, as we saw earlier, he distinguishes between "limit[ing]" and "annull[ing]" the conclusion of a *qal vachomer*, still he identifies the *dayo* principle with the axiom that "the conclusion must not surpass the premises," which concerns limiting rather than annulling.

[28]   In the case of capital punishment, one cannot kill a person twice, but one could additionally torture him first or disrespect his body afterwards or harm his family or seize his property. In the case of flagellation, one could conceivably increase the number of blows or the way they are delivered.

[29]   I wonder whether it is true throughout the Torah that every sanction is preceded by a warning. This seems to go against the rabbinic principle that the Torah is not necessarily chronologically ordered.

perfectly just and merciful, and infallible like God, the giver of the Torah. Of course, historically this rule has not always been strictly upheld: the rabbis have in fact at times made innovations in laws and sanctions. Moreover, nowadays many Jews are not so sure that the Torah is entirely, if at all, of Divine origin, or for that matter that the rabbis are especially wise – so, the moral authority of both the Torah and the rabbis is doubted and we resort more readily to secular jurisprudence (which is far from always wise or just, I hasten to add).

4. Refutations

Further evidence of Abitbol's incomplete understanding of a fortiori argument can be found in his treatment of the doctrine of "refutations" of *qal vachomer* (pp. 157-66). There are, he correctly informs us, two sorts of refutation, as follows. Note that I use Abitbol's symbols here, i.e. P for subjects and S for situations (because I do not need to refer to my standard symbols P, Q, R, S).

The first process of refutation consists in demonstrating that the relation between the two subjects (P1 and P2) does not always hold. Given that in some situation S1 this relation is P1 > P2, it is inferred by *qal vachomer* that in second situation S2 the relation is similar (i.e. again P1 > P2); however, the latter conclusion can be refuted by showing that there is a third situation S3 where the relation between the subjects is known to be the reverse, i.e. where P1 < P2; in that event, the previously proposed conclusion concerning S2 is put in doubt. This process is in the Talmud called *pirkha a-iqara dedina*, meaning refutation directed at the starting point of the reasoning[30]. Abitbol proposes the following tabular representation of this process:

1st refutation	Situation S1	Situation S2	Situation S3
Subject P1	Permitted (+)	Forbidden (–)	Forbidden (–)
Subject P2	Forbidden (–)	Forbidden (–)	Permitted (+)

Abitbol refers to an actual example, drawn from *Chullin* 114a[31]. As can be seen (reading vertically), we cannot be sure that in S2, given that P1 is forbidden it follows that P2 is forbidden, as the state of affairs in S1 implies (since there P1 is permitted and P2 is forbidden), because there is a conflicting tendency implied by S3 (wherein P1 is forbidden and P2 is permitted). In other words, if we had started with situation S3 instead of S1 as our model, our conclusion concerning situation S2 would have been the opposite (from *chomer* to *qal* instead of from *qal* to *chomer*). This understanding cannot be gainsaid; but it cannot be said to be accurately represented. Drawing a table with six cells does not explain why the initial argument ceases to function; it would have been more instructive to draw two tables of four cells for the same subjects P1 and P2, one with columns S1 and S2 and the other with columns S3 and S2 (in that order).

What has to be stressed is that the new column S3 serves to deny generality to the first column S1, so that the content of S2 (i.e. the value of P2 to be inferred from that of P1 in this situation) becomes uncertain. This cannot be expressed within this tabular format, but only through full disclosure of the form of a fortiori argument. We have to recognize that the relation of P2 to P1 in S1 serves, after generalization, as the argument's major premise, from which together with the minor premise concerning P1 in S2, the putative conclusion concerning P2 in S2 is drawn. The refutation with reference to the opposite relation of P2 to P1 in S3 serves to deny validity to the generalization, i.e. to particularize the major premise – in which case, the putative conclusion can no longer be drawn. Furthermore, notice that there is no mention anywhere here of the operative middle term.

The second process of refutation is called in the Talmud *yokhiah mimaqom acher*, meaning refutation from another place, and is represented by Abitbol by means of the following table (please note that he, no doubt accidentally, here uses the + and – signs with the opposite senses, but I have for the sake of consistency stuck to the previous senses):

2nd refutation	Situation S1	Situation S2

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[30]   Abitbol says of the starting point that it is "at P1," but surely he means 'at S1', i.e. at the relation between subjects P1 and P2 in situation S1 (a reference to the major premise). Unless he means 'at P1 for S2' (a reference to the minor premise), which would be inaccurate (in most if not all cases).

[31]   Which I will not repeat here, for brevity's sake. Note that I have placed the terms in the table as he has. But looking at the original argument, it seems to me that the table should have been rotated (i.e. the terms called P1, P2 should have been S1, S2, and conversely; as for S3, it should have been called P3). And indeed, Abitbol reads his table horizontally (see top of p. 159). But there is no need for us to get into this issue here, as it has already been dealt with earlier.

Subject P1	Forbidden (–)	Permitted (+)
Subject P2	Permitted (+)	Permitted (+)
Subject P3	Permitted (+)	Forbidden (–)

This table was constructed with reference to another Talmudic example, drawn from *Qiddushin* 7b[32]. Here (reading the table vertically), the initial argument is that since in situation S1 subject P1 is forbidden while subject P2 is permitted, it would follow *qal vachomer* that in situation S2, where subject P1 is permitted, that subject P2 should be permitted; however, this cannot be, since in situation S1 subject P3 is permitted and in situation S2 subject P3 is forbidden, which means that if we similarly argued that since, under S1, P1 is forbidden whereas P3 is permitted, it would follow that, under S2, where P1 is permitted, P3 should be permitted, contrary to the given fact that it is forbidden. Clearly, here again, it would have been more accurate to illustrate the problem through two tables, instead of a single larger table.

Now, what is the exact nature of this second refutation? This can be better understood if we express the arguments involved in standard format, using some middle term (say, R). Clearly, just as it is true that given that P1 is more R than P2 and P1 is not R enough to be forbidden, it logically follows that P2 is not R enough to be forbidden, it would be equally sound to say that given that P1 is more R than P3 and P1 is not R enough to be forbidden, it logically follows that P3 is not R enough to be forbidden. Yet P3 is known to be forbidden! Therefore, there must be something wrong in one or both preceding a fortiori arguments (which are, notice, negative subjectal in form, going from major to minor). Granting that the middle term (R) is constant throughout, either the major premise or the minor premise or both must be wrong in at least one of the arguments.

We cannot offhand tell precisely what is wrong, but the refutation serves its purpose just by putting the original line of thinking in doubt. That is why in other accounts of the doctrine of refutation[33] we find the first sort of refutation labeled "refutation of a premise" and the second sort labeled "refutation of a conclusion." The difference between them is that the first sort constitutes a direct attack on the major premise, whereas the second sort constitutes an indirect attack (since it puts one or both of the premises in doubt simply by denying their joint conclusion). It should be said that Abitbol, although he does refer to the first sort as "the most direct process" of refutation, does not explain precisely how the second sort functions. His tabular representation is somewhat illustrative, but it is not explicative.

Finally, Abitbol describes – again by means of examples and tables – how such refutations might occasionally be neutralized by counter-objections. Judging by one example he gives, this is sometimes achieved by pointing out that something claimed to be permitted in a certain context is in fact not permitted in it, or that something claimed to be forbidden in a certain context is in fact not forbidden in it. In other words, the material truth of part or all of one or both of the premises may be challenged. There may be a counter-objection, and the initial premise may be reinstated, perhaps with slightly modified content. But all this, note well, is essentially not of formal significance – it has to do with the content. The rabbis are trying, through debate, to establish what premises to collectively adopt before drawing a final conclusion; the argument per se comes after.

It is at this point that Abitbol shows his clear understanding[34] that the argument depends on a "hierarchy" between two subjects, and that the same subjects may have opposite positions within different hierarchies. He uses this understanding to explain the process of rabbinic give and take, rightly identifying the positions of disputants at successive stages as "hypothetical." Each rabbi's position is relative to some information that he has taken into consideration, but another rabbi is free to bring to bear additional information that changes the initial judgment. For this reason, he believes, *qal vachomer* arguments are never final, and always potentially capable of refutation. However, he is amiss in not making a clear distinction between the form and the content of argumentation, which is essential for truly logical analysis.

5. Closing remarks

Clearly, Abitbol is not a formal logician. His study of Talmudic logic in general, and the *qal vachomer* in particular, contains interesting information and reflections, but it is not based on deep study of universal logic.

As we have seen, his tabular approach to a fortiori argument is of limited value, because it does not and cannot distinguish a fortiori argument from analogical argument. He fails to highlight all the factors involved in *qal*

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[32]   Which I will not repeat here, for brevity's sake. Note that I have placed the terms in the table as he has. But looking at the original argument, it seems to me that the table should have been rotated (i.e. the terms called S1, S2 should have been P1, P2, and conversely; as for P3, it should have been called S3). Abitbol seems to read his table horizontally (see p. 159-60). But there is no need for us to get into this issue here, as it has already been dealt with earlier.

[33]   See the discussion of this topic in the chapter on Mielziner (13.4).

[34]   Which we have drawn attention to previously (21.2).

vachomer reasoning, notably the middle term and the idea of a threshold value of it as determining the possibility of drawing a conclusion, not to mention the prior formation of the major premise. Furthermore, his effectively lumping together of material and formal issues makes him wrongly view the argument as "only probable and subject to contestation," even if he goes on to declare that despite its "inconveniences" it has "a certain logical security that guarantees the validity of its conclusion."

In his opinion, what distinguishes the *qal vachomer* argument from other rabbinic hermeneutic techniques is that it calls on any interpreter resorting to it to be extra careful in its elaboration so as to preempt all possible challenges such as those above described. This requires broad knowledge of the Torah, subtlety, attention to detail and nuances, rigor, a critical spirit. It is these controlling factors in the background of rabbinic discourse that give the final conclusion of such argument its logical credibility and legal value. There is, of course, much truth in that reflection, but it is not the whole story.

I will not here examine the rest of Abitbol's book, even though it looks interesting. What I can at a glance see, however, is that his treatment of the other rabbinic hermeneutic principles is even less formal than his treatment of *qal vachomer*[35]. He works essentially through traditional examples and rabbinic descriptions and rules, even if he goes into considerable detail in an effort to clarify the principles. Of course, the principles cannot all be formalized, or at least not to the same degree. But some can in fact be expressed in formal terms, i.e. using symbols like X and Y in place of terms or theses, and thereby shown to be logically valid or invalid as the case may be. I have definitely shown that to be possible in my book *Judaic Logic*, which however appeared a couple of years after the book by Abitbol here discussed. Nevertheless, to repeat, I think this book merits further attention.

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[35]   In particular, note that Abitbol does not attempt to formalize the *binyan av* principle (3rd in R. Ishmael's list of 13), or the harmonization principles (numbered 8-11, 13), which are the most susceptible to formal treatment besides *qal vachomer* (the 1st principle).

22. Hyam Maccoby

Hyam Maccoby (Britain, 1924-2004) wrote some interesting comments regarding a fortiori argumentation in the Talmud in one of his last works, *The Philosophy of the Talmud* (2002)**[1]**. The relevant parts of this – namely, chapter 14, "Talmudic Logic," and Appendix A, "Qal va-chomer in the Aggadah," were collected (presumably by himself) in an essay called "Some Problems in the Rabbinic Use of the Qal Va-Homer Argument" and published online**[2]**. We will in the present chapter review this essay.**[3]**

My interest in Hyam Maccoby dates from the time I started writing my book *Judaic Logic* in the early 1990s, when I learned the importance of the rabbinic *dayo* principle through his critique of the use of the a fortiori argument by Paul of Tarsus (1986). I will not, however, here examine this critique, since I do so in an earlier chapter of the present work (viz. 10.3).

Maccoby's more recent essay, on rabbinic use of a fortiori argument, is a mix of true insights and unfortunate errors. I refer the reader to the earlier chapters of the present volume (viz. 7-8) where I present my own detailed analysis of the Talmud's treatment of *qal vachomer* and the *dayo* principle; I take it for granted here that the reader has indeed studied that analysis, so as to avoid unnecessary lengthy repetitions.

1. Purely a fortiori argument

Hyam Maccoby nowhere formally defines *qal vachomer* in general and nowhere identifies its varieties (viz. positive or negative, copulative: subjectal or predicatal, or implicational: antecedental or consequental, etc.). He makes no attempt at formalization, and consequently cannot engage in formal validation. His approach is therefore, overall, rather vague and, as the saying goes, 'intuitive'; in other words, he refers to "common sense." Nevertheless, Maccoby is outstanding in his understanding of a fortiori argument and steadfast adherence to the logical rule (which I have much later labeled 'the principle of deduction') that its conclusion cannot be more informative than its premises, which he identifies with the rabbinical *dayo* (sufficiency) principle. Thanks to this lucid and uncompromising posture, he is able to find fault with many a fallacious argument billed as a fortiori. In the abstract of his essay, he writes:

> "The qal va-homer (a fortiori) argument is a logic of analogy, not of classes or sets (the subject-matter of Aristotelian logic), and this makes it suitable for legal, rather than scientific, argument."

In this general statement concerning the nature of *qal vachomer*, Maccoby shows his independence from the many commentators who claim this form of argument to be syllogistic. He rightly refers it to the more general category of argument by analogy. However, he fails to explicitly mention its quantitative aspect, which distinguishes it from qualitative analogy. Moreover, it is inaccurate to oppose analogy to classification, since the latter is also based on perceived or conceived similarities and differences. Also, that a fortiori argument is particularly suitable for legal thinking is true; but this does not make it any the less suitable for scientific purposes, if properly used.

Concerning the *dayo* principle, which note well (to repeat) he effectively equates to the principle of deduction, he writes:

> "What makes [qal va-homer] an exact reasoning is a special rule (unknown to Greek rhetorical use of a fortiori), namely the rule of dayyo**[4]**, which lays down that the conclusion must not contain anything that was not present in the premises."

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[1]   London: Routledge Curzon, 2002. Some excerpts of this book may be read online at:
books.google.com/books?id=0uHwGt1kypEC&printsec=frontcover&dq=Hyam+Maccoby+The+Philosophy+of+the+Talmud+%282002%29&hl=en&src=bmrr&ei=GZl4Tee7LoKV4gaMsNj6BQ&sa=X&oi=book_result&ct=result&resnum=1&ved=0CCwQ6AEwAA#v=onepage&q&f=false.
[2]   By the Centre for Jewish Studies, U. of Manchester, at: www.mucjs.org/qalvahomer.htm.
[3]   I was gratified to see that Maccoby cited my book *Judaic Logic* in a footnote to this essay, though disappointed that he apparently did not realize the full extent of my contribution there – namely, the formalization and validation of a fortiori argument. I assume from that that he did not actually read the book.
[4]   Maccoby reads this rule as: "dayyo lav'o min ha-din lihyot ke-nidon" and adds: "note that here din means 'conclusion' while nidon means premise; the transliteration, often found, nadon is incorrect, as this is Biblical, not Mishnaic, Hebrew, see Jastrow."

Here, we should note Maccoby's observation – which may be original with him – that the *dayo* principle formulated by the Mishnaic rabbis was unknown to the Greeks. I confirm that I have not so far found any comparable rule in Greek, Hellenistic or Roman logic or rhetoric. These cultures may conceivably have occasionally or even usually reasoned in unconscious accord with the principle of deduction, but in any case they apparently never explicitly said that the conclusion must mirror the minor premise and should not be made 'proportional' to it. As regards the *dayo* principle in its truly rabbinical sense, this concerns Jewish law in particular; so, though it is not inconceivable to find a similar restriction in some other law system, it is not surprising not to find one.

In truth, as I have shown in an earlier chapter (7), the rabbinical *dayo* principle correctly understood is not commensurate with the principle of deduction. Looking at the discussions Talmudic tractate Baba Qama, pp. 25a, we might at first make such an equation. That is because a possible and very common interpretation of the Mishna has it that R. Tarfon's first argument was an attempt at 'proportional' *qal vachomer* and the Sages' the *dayo* principle was intended to remind or inform him that the logical conclusion cannot surpass the given minor premise. However, this simplistic interpretation, which seems to be that favored by Maccoby, does not in fact correspond to the Gemara's viewpoint and subsequent tradition.

Furthermore, irrespective of the Gemara's viewpoint, the said interpretative hypothesis is definitely belied by the second exchange in the Mishna, where R. Tarfon proposes a modified *qal vachomer*, which is in perfect accord with the principle of deduction, and yet is struck down by the Sages using the exact same *dayo* objection! On this basis, the *dayo* principle has to be seen as different from the principle of deduction. Even if the two principles happen to have the same result in a particular context (viz. the first argument), they do not coincide in all contexts (notably not in relation to the second argument).

In fact, as I explain in my detailed analysis, the rabbinical *dayo* principle is not a logical principle at all, but a moral one, related to the principle of measure for measure (*midah keneged midah*) and other extra-logical considerations. As such, it need not apply to all a fortiori argument and may even apply to argument forms other than a fortiori (as indeed may the principle of deduction, of course).

Thus, Maccoby's understanding of the rabbinical *dayo* principle is not entirely correct, although his understanding of a fortiori argument is essentially correct. What he calls the *dayo* principle is really the principle of deduction, a purely logical rule known by rational insight. The *dayo* principle, in its true sense, is actually a rabbinical majority ruling presented as a Divine decree (based on Numbers 12:14-15, as the Gemara teaches). In fact, these two principles ought not to be confused; we may nevertheless verbally equate them in the present analysis so as to continue using Maccoby's own terminology.

Maccoby describes *qal vachomer* reasoning in general terms by saying: "If a conclusion is true in a weak situation, it is true 'all the more so' in a strong situation." This description, note well, only points to the most common form of the argument, viz. the positive subjectal mood; and he does not point to other moods elsewhere. For him, as already said, the *dayo* principle is what makes *qal vachomer* an exact form of reasoning. The example he proposes for it is very clear and accurate[5]:

> "If a moderately good child deserves one sweet, [then,] all the more so:
> – a very good child deserves one sweet (correct);
> – a very good child deserves two sweets (incorrect)."

However, to repeat, Maccoby does not formally explain why the first putative conclusion is correct and why the second is incorrect, other than to point to the fact that "the conclusion must not contain anything that was not present in the premises," which is true of all deductive processes. The structure of a fortiori argument is lacking in his treatment – he fails to distinguish the major premise from the minor premise and conclusion, and the middle term (R) from the major (P), minor (Q) and subsidiary terms (S); moreover, he does not have the idea of a threshold value of R as being "enough for" S.

If he had analyzed structural features, he would have formulated his argument more explicitly as follows. When an a fortiori argument is so precisely structured (in this case as a positive subjectal), it becomes easier to formally validate it and understand why the conclusion could not have been 'proportional' (namely, because S must be *identical* in the minor premise and conclusion):

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[5]   Note in passing that Maccoby would have been more accurate, as regards Talmudic logic (though not generic logic) to have given a negative example, say: "a moderately bad child deserves one slap; therefore, a very bad child deserves one slap too (not two slaps)." Why? Because, generally, the rabbinic *dayo* principle relates to penalties, rather than rewards. The rabbis were against proportionality of punishment, not of recompense. Their purpose was to limit the former, not the latter. *Dayo* perhaps applies more broadly to all negatives; i.e. not only to penalties but to the 'burdens' of mitzvoth. An inferred mitzvah should likewise not be made more burdensome than necessary on the basis of some misplaced 'proportionality'.

> A very good child (P) is more deserving (R) than a moderately good child (Q),
> and, a moderately good child (Q) is deserving (R) enough to get one sweet (S);
> therefore, a very good child (P) is deserving (R) enough to get one sweet (S).

Because Maccoby failed to do this (though he could have done it if he had studied my earlier work on the subject, *Judaic Logic*), he does not correctly perceive why, as he puts it: "a qal va-homer argument is not as unchallengeable as a syllogism, and the rabbis recognised various grounds of challengeablity." He explains this feature as follows:

> "The qal va-homer reasoning is open-ended, in that it depends on a distinction between 'light' and 'heavy' that is always open to question. This aspect, however, does not invalidate this type of reasoning, but differentiates it from the mathematical or logical kind of reasoning, where intuition or grasp of human values play no part."

That is to say, he thinks this form of argument is intrinsically open to debate, because it is concerned with human values. But that is not always the case: a fortiori argument is possible without reference to valuations[6]. He is thus wrong to differentiate it so radically. The truth of the matter is that given the truth of the premises, if we reason in accord with validated forms, the argument is unassailable. In a fortiori argument, there may be debate regarding the premises (and in particular, often, the major premise) and/or debate regarding the validity of the attempted process – but, *as in all logical argumentation*, once these issues are settled, the argument is mechanical and not open to further doubt. In other words, a fortiori argument is deductive and not merely inductive.

2. A crescendo argument

All that has been said so far only concerns *purely* a fortiori argument; that is, argument with two appropriate premises (the major and the minor): in such case, the conclusion can only be as Maccoby has it 'non-proportional'. However, contrary to what Maccoby believed, a 'proportional' conclusion from those very same premises is indeed logically permissible, provided *a third premise* is introduced that informs us of the general 'proportionality' between the subsidiary term and the middle term (I put the word proportionality in inverted commas to remind us that the intended proportions are in practice often rough rather than exact).

If such an additional premise is involved, a 'proportional' conclusion is in full accord with the principle of deduction, and therefore with the *dayo* principle as Maccoby effectively conceives it. In other words, Maccoby's presentation of *qal vachomer* reasoning and the *dayo* principle is all very well, so long as what we are referring to is specifically non-proportional a fortiori premises. However, if in a given case we admit, as well as the two premises of pure a fortiori argument, an additional premise about 'proportionality', the 'proportional' conclusion becomes quite valid and Maccoby's rejection of it cannot apply. Maccoby's sample argument would in such case look like this:

> A very good child (P) is more deserving (R) than a moderately good child (Q),
> and, a moderately good child (Q) is deserving (R) enough to get one sweet (S);
> and, reward in sweets to children (S) are to be proportioned to their deserts (R);
> therefore, a very good child (P) is deserving (R) enough to get two sweets (S).

Actually, unless we can provide a precise mathematical formula for the concomitant variation between S and R, the conclusion should rather read, more indefinitely, "more than one sweet;" or we should admit that the exact quantity is somewhat subjectively assessed.

Maccoby's error was to base his model of a fortiori argument, and consequently of the *dayo* principle, on only the *first* exchange between R. Tarfon and the Sages in the Mishna Baba Qama 2:5. Like many commentators before him and after him, including the Gemara no less – he completely failed to take into consideration the very different *second* exchange between R. Tarfon and the Sages. He did notice the latter (which is more than can be said of the Gemara), but he did not realize its significance. If one tailors one's hypothesis to fit only part of the data at hand, one is pretty well bound to go astray. Thus, though Maccoby did rightly conceive purely a fortiori argument, his ideas are too restrictive because he altogether missed out on what I have lately named a crescendo argument.

A crescendo argument, as a combination of purely a fortiori argument and pro rata argument, is (as I have formally demonstrated in an earlier chapter (2)) a fully legitimate form of deductive reasoning. A crescendo argument is a species of a fortiori argument, since it consists of purely a fortiori argument combined with pro rata argument.

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[6]   Whereas, it is true, the rabbinical *dayo* principle is always concerned with human values.

Moreover, many examples of its use can be found in everyday discourse, as well as in the Torah and in rabbinical literature. So, in the last analysis, Maccoby evidently strayed considerably in this matter.

3. Baba Qama 25a

The principal part of Maccoby's commentary naturally revolves around the famous Talmudic discussion on page 25a of tractate Baba Qama. Maccoby's position, briefly put, is that the Mishna and Gemara are at odds. He begins by detailing the Mishna, then remarks:

> "Both Rabbi Tarfon and his opponents the Sages accept not only the validity of qal va-homer, but also the validity of its limiting principle of dayyo. Where they differ, in this instance, is how to draw up the list of terms involved in the reasoning."

Actually, Maccoby's position here is logically untenable, since R. Tarfon's first *qal vachomer* argument is definitely not purely a fortiori argument and definitely not in accord with the Sages' first *dayo* objection interpreted as the principle of deduction. Funnily enough, in imagining that the Mishna opponents are in essential agreement, Maccoby mimics the Gemara, the very Gemara that he goes on to reject on the basis that it advocates 'proportional' a fortiori argument.

Maccoby is right to reject the Gemara, insofar as it appears to claim that all a fortiori argument is 'proportional' (i.e. a crescendo). But he is wrong to reject it, if all that the Gemara claims is that this particular case should be read as 'proportional'. As we have just seen, Maccoby thinks that valid *qal vachomer* is necessarily purely a fortiori; but as we have shown elsewhere a crescendo argument is quite possible, if an additional premise is granted. The Gemara here goes to the opposite extreme, seeming to claim that valid *qal vachomer* is necessarily 'proportional'[7], thus totally ignoring pure a fortiori argument (even though the Mishna it comments on does contain such argument).

Looking at the two arguments of R. Tarfon, Maccoby cannot consistently claim them to be both a fortiori by his definition, since the first of them is definitely not purely a fortiori (though the second could be so interpreted, and ultimately must be). Whereas, the Gemara can consistently claim both to be a fortiori by its definition, since both can be read as a crescendo (even if the second one could also be read as purely a fortiori, and ultimately must be). In either case, it cannot be said that R. Tarfon was in essential agreement with the Sages; their two objections were indicative of significant difference of opinion.

In Maccoby's view, R. Tarfon and the Sages are merely in disagreement about "the terms" of the arguments proposed. This is a traditional posture found in Tosafot, which is based on R. Tarfon's quick reformulation of his argument in a bid to satisfy the Sages' first *dayo* objection. Maccoby then explains the Sages' reticence to R. Tarfon's second argument by saying, somewhat lamely: "The Sages, however, see something illegitimate about this move, since, in the process, 'horn' has been surreptitiously promoted from 'half-damages' to 'whole-damages', which appears to be a breach of the principle of dayyo." Clearly, Maccoby does not understand exactly why R. Tarfon's change of terms does not change the Sages' resistance to his thesis. This forces Maccoby to conclude, rather mystically if I may say so:

> "We see from this that in a qal va-homer argument there may sometimes be an uncertainty arising from the choice of appropriate terms. This choice of terms may be a matter of intuition, rather than strict logic, and thus one person's valid qal va-homer may be another's fallacy. This does not mean that this method of argument should be condemned as subjective, but only that it belongs to the area of rationality rather than strict logic… a region of logic that transcends the usual parameters of Western logic."

In truth, from the perspective of a fortiori logic, R. Tarfon's second argument is logically stronger than his first, since the first is *only* valid as a crescendo (or even simply as pro rata) argument whereas the second is *also* valid as purely a fortiori argument. The issue is not therefore one of "choice of terms," as Maccoby vaguely suggests; it goes much deeper. The fact is that R. Tarfon's second try is *logically immune* to the Sages' first *dayo* objection; from which it follows that the Sages' second *dayo* objection must be directed at the inductive formation of the new major premise (with reshuffled terms) rather than at the additional premise of 'proportionality'.

Maccoby's difficulty in unraveling the problem is due to his simplistic identification of the rabbinical *dayo* principle with the principle of deduction. Regarding the first argument, this identification works out without apparent difficulty because there the two principles just happen to converge. But regarding the second argument, the two principles clearly diverge. This serves to show us that the Sages' *dayo* principle is a rejection of *any* extrapolation of legal penalties from Scriptural data, *even if* such inference is in accord with the principle of deduction. In other words, their second objection has to extend *dayo* restriction to *the inductive preliminaries* of R. Tarfon's second argument, since the subsequent deduction does not contravene their first objection. The Sages' *dayo* principle therefore cannot

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[7]   E.g. Inferring a conclusion of 14 instead of 7 days penalty for Miriam in the story at Numbers 12:14-15.

be limited, as Maccoby takes it to be, to *qal vachomer* inference of the kind used by R. Tarfon in his first argument, i.e. a crescendo. The Sages' *dayo* principle has to be broadened in such a way as to also interdict R. Tarfon's second argument, which is purely a fortiori.

Nevertheless, there is a bit of truth in Maccoby's insight concerning "choice of terms" – in the sense that it is interesting to observe that by judiciously reshuffling the given data R. Tarfon was somehow able to pass from an a fortiori argument not in accord with the Sages' first *dayo* objection to one in accord with it. But on closer analysis, this reconstruction is not as freewheeling as it seems. What differentiates the two arguments is the different directions of the preliminary generalizations which their respective major premises are based on. In this new context, we can view the Sages' *dayo* principle as a decree that when a possible generalization leads to a more presumptive conclusion, the safest course inductively and morally is to avoid it. Their principle is therefore, in this larger context, only incidentally related to a fortiori argument.

Maccoby's overly narrow reading of the Mishna explains his firm belief that *qal vachomer* reasoning is identical with purely a fortiori argument, and its attendant *dayo* principle is identical with the principle of deduction. His approach to both processes is evidently that they are natural and rational, even though not perfectly logical (as we just saw him say). Given this rationalistic attitude, Maccoby's strongly negative reaction to the Gemara is quite understandable and laudable. For him, the Gemara's explication of the Mishna is a manifest error:

> "The Amoraic discussion of the Mishnah (b. Bava Qamma, 25a) must be discounted, since it shows no comprehension of the logical force of the dayyo principle. Instead, it imagines that the rule is an arbitrary fiat of the Torah….
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> It seems that in the Amoraic period the rationale of the dayyo rule, perfectly understood in earlier times, had been lost. In earlier times, too, the derivation of the rule from Scripture (if made, which is doubtful) was not intended to give it the status of an arbitrary fiat, but to give authoritative approval to a deliverance of reason."

However, things are not so simple. Maccoby apparently did not know, or at least fails to mention[8], that the statement in the Gemara on which this position is based is there presented as a *baraita* – i.e. as a Tannaic statement not recorded in the Mishna but having an authority close to Mishnaic. Maccoby is thus, consciously or not, either denying the reliability of this particular *baraita*, or claiming that it is a fabrication, or perhaps even claiming most or all *baraitot* to be unreliable or fabricated.[9] Maccoby is not, however, doing this arbitrarily, note well. For him, the Gemara's claim (with regard to Numbers 12:14) that Miriam deserved fourteen days instead of just seven days isolation is contrary to natural logic, for only pure a fortiori argument can be valid[10]. It follows that, in his eyes, the Gemara must be wrong.

When I set out to analyze this Talmudic passage, before I studied Maccoby's article, my own first reaction was to, much like Maccoby, discount the Gemara's explication as a later misunderstanding. But when I realized that the Gemara's position was based on a *baraita*, I realized that a more subtle approach would be necessary to resolve the difficulty. In truth, the Gemara is only wrong if it thinks that *all* a fortiori argument is necessarily 'proportional'; but it is logically quite possible for it to claim that *a particular* a fortiori argument is 'proportional'.

In truth, if the *dayo* principle is none other than the principle of deduction as Maccoby thought, it is a redundancy. That the conclusion cannot go beyond what is given in the premises is true of both purely a fortiori argument and a crescendo argument, as indeed of all deductive argument, without any need to state it as a special principle; it is the very definition of deduction, as against induction or fallacious thought, and so the subtext of any deductive act. So, I thought, it is possible that the *dayo* principle concerns *qal vachomer* not *per se*, but only *per accidens*. The rabbis may well have thought it concerns *qal vachomer* as such, but judging from their reading of Numbers 12:14 it would seem that they were unconsciously mentally referring to the principle of justice.

In the Miriam story, we *do* have an intuitive sense that Miriam deserves more punishment for her act of *lèse majesté* towards God (in saying nasty things about her brother Moses) than a daughter who angered her father (a mere human being, after all) would deserve. So the Gemara *does* have a leg to stand on. What needs to be understood, however, is

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[8]   Unless we consider the statement "In earlier times, too, the derivation of the rule from Scripture (if made, which is doubtful)…" to be intended as a reference to the *baraita*.

[9]   It is not unthinkable that a *baraita* might be fabricated. Louis Jacobs, in his *Rabbinic Thought in the Talmud*, devotes a chapter to this issue in general. He mentions the thesis advanced by I. H. Weiss (in *Dor Dor ve Dorshav*) that many of the *baraitot* in the Babylonian Talmud may be "fictitious, i.e…. not authentic transmissions of tannaitic opinion but… invented by the Babylonian Amoraim as alleged support for their views." Though this view is later opposed by Abraham Weiss (in *Le-heqer ha-Talmud*), Jacobs considers it to have some credibility, and explains why with various examples. It does not follow, of course, that the particular *baraita* of concern to us here is fabricated. But, to repeat, it is not unthinkable that it might be.

[10]   However, Maccoby is not right when he ascribes to the Gemara that "the conclusion of a qal va-homer reasoning is cut in half (why by precisely a half is not explained)" by the *dayo* principle. First, because nowhere is the proportion of "half" literally mentioned as a general rule, even if 7 *happens to be* half of 14 in the Miriam example. Second, because later commentators have indeed proposed explanations (however flimsy) for this particular proportion in this particular case.

the source of this intuitive sense. It *cannot*, as the Gemara suggests, be due to *qal vachomer* necessarily having a 'proportional' conclusion (i.e. 14 days instead of 7), for this explanation can be proved indubitably wrong by means of formal logic. Therefore, it *must* be due to something else – viz. (I suggested) to the 'sense of justice', i.e. the belief that the punishment should be commensurate with the crime, or more intellectually put, the principle of 'measure for measure' (*midah keneged midah*). In that case, the *dayo* principle becomes an important tool, used to block excessively strict justice and impose some mercy.

Thus, the *dayo* principle can be viewed as essentially different from the principle of deduction, and the Gemara's claim that it is a Divine fiat can be justified. Maccoby, in his above quoted remark, expresses doubt as to the *dayo* principle's derivability from Scripture (i.e. from Numbers 12:14-15), though he grants it might at best be taken as confirming what is rationally obvious. He also excoriates the Amoraim for effectively denying the logical force of the *dayo* principle, since they effectively view it as an arbitrary fiat. These postures are simply due to Maccoby's inaccurate identification of the *dayo* principle with the principle of deduction.

Maccoby is, of course, quite justified in forcefully rejecting the thesis that the *dayo* principle, if it is taken as identical with the principle of deduction, can occasionally be bypassed. He presents the Gemara's suggestion that R. Tarfon claimed such exceptions as follows:

> "The Gemara then explains that Rabbi Tarfon, while acknowledging the rule of dayyo, had a variant view of it which would [have] excluded the present case (he considered that a qal va-homer that, by the application of dayyo, yields a result already derivable from other sources is not subject to the rule of dayyo). This implausible account ignores totally the plain reason which Rabbi Tarfon himself gives in the Mishnah: that he was using different terms as the basis for his reasoning from those used by the Sages. It ignores also the fact that Rabbi Tarfon's expressions in the Mishnah show that he is not arguing that this instance is exempt from the rule of dayyo but, on the contrary, that he is bringing it into the rule."

I agree with Maccoby's last sentence here to some extent: R. Tarfon was not, in his second argument, seeking exemption from the Sages's objection to his first argument, but on the contrary was seeking to conform to it. But Maccoby evidently did not realize that since the Sages countered this with a renewed objection, the *dayo* principle was thereby extended. I agree, too, with Maccoby's characterization of the Gemara's claim that R. Tarfon had a variant view of the *dayo* principle as "implausible." The variant view the Gemara imputes to R. Tarfon does not stand up to detailed logical scrutiny.

Nevertheless, if we take the *dayo* principle as referring to something other than a merely logical rule, namely to refraining from excess in the application of just 'proportionality' in retaliation, then occasional exception to the rule is not inconceivable or logically reprehensible. If the principle is a Divine fiat, then exceptions to it might also have been decreed. So the Gemara's position is not as indefensible as Maccoby thinks.

Although I have considerably criticized some of Maccoby's views, please do not get me wrong: I consider his contribution to the discussion as valuable. Although he inaccurately equates the *dayo* principle to the principle of deduction, his unswerving commitment to the latter and his courage to denounce those who indulge in deviations from that logical norm, makes him an inspiring figure.

4. Faulty qal vachomer

As already said, Maccoby had a good intuitive understanding of a fortiori argument. But he lacked the formal tools needed to fully support and express this understanding. As a result, he misperceived some parodies of *qal vachomer* reported in the Talmud as indicative of "vulnerability" in this form of reasoning. He cites the following as an example of parody:

> "In b. Sanh. 17a… it is said that one of the qualifications of a member of the Sanhedrin was the ability to prove that the body of a reptile (sheretz) was clean, i.e. did not convey ritual impurity. This would seem an impossible undertaking, since the Torah says explicitly that it is unclean. However, in response to the assertion, Rav offered to prove this impossible proposition by means of a qal va-homer argument. It ran as follows: the dead body of a snake does not convey impurity. Yet a snake is the means of spreading impurity, for it causes many deaths. How much more so should a reptile (which is harmless) be regarded as not causing impurity! This same argument is attributed to Ravina in b. Eruvin 13b. The Gemara immediately refutes this argument by denying the 'heaviness' of the heavy term. Being a cause of impurity indirectly by causing death has nothing to do with causing impurity by direct contact. Otherwise, we would have to regard a thorn as a cause of impurity by contact, since it may cause death to someone who becomes impaled on it."

However, if we examine this argument closely, we see that the problem with it is nothing to do with the mechanics of a fortiori inference; it is rather an obvious case of equivocation, the term "impurity" having a different sense in the

premise and in the conclusion. Purely a fortiori argument can readily be validated with reference to four terms (the major, minor, middle and subsidiary) – but it cannot be validated with reference to five or more terms[11]. This is not a weakness or flaw in the argument's logic – it is a rule in its logic. Just as in syllogism there is a "fallacy of four terms" – so with regard to purely a fortiori argument there is a "fallacy of five terms."

Maccoby admits as much, saying: "Of course, it is possible to produce parodies even of syllogistic arguments," adding: "perhaps we should attribute such parodies merely to high spirits, not to any flaw in the type of argument itself." His deficiency of formal equipment can also be seen in the other example of parody he gives, drawn from *Derekh Eretz Rabba*, 1. As it happens, I have already dealt with this sophistical argument in the chapter on Mielziner (13). As we saw there, the problem with it is not the a fortiori inference as such, but the overgeneralization used to inductively formulate its major premise. In fact, the absurdity of the a fortiori conclusion is logic's way of telling us that there is a problem in the premises (in this instance, the major). The parody therefore reveals, not a weakness in the argument form, but its strength as an instrument for ensuring internal consistency in our knowledge.

Nevertheless, Maccoby rejection of the first a fortiori argument in Mishnah Makkot 3:15 makes clear that his logical instincts are basically sound. This argument reads: "R. Hananiah ben Gamaliel said: If he that commits one transgression thereby forfeits his life, how much more, if he performs one religious duty, shall his life be given to him!" According to Maccoby, this involves an "apparently glaring infringement of the rule of dayo;" i.e. it constitutes an illicit process, since the predicate in the premise, "forfeits his life," is not identical with, but indeed contrary to, that in the conclusion "his life shall be given to him."

However, although Maccoby is right in rejecting this argument, his reason for its rejection is not correct. The problem is not essentially in the fact that the two predicates (subsidiary terms) are different, or even that they are antithetical. The problem lies in the a contrario form of the argument, i.e. in the fact that not only are the predicates antithetical, but also the subjects – viz. someone who "transgresses the law" and someone who "obeys the law" – are contrary, and these two contrarieties occur in lockstep.

The argument seems to have the following basic form: "If X, then Y; therefore, if not X, then not Y" – i.e. it refers to inversion. This is, of course, formally wrong – we cannot inverse a hypothetical proposition at will; i.e. the given premise does not formally imply the putative conclusion. It is true that, judging by the speaker's use of the expression "how much more," his intent is not immediate inversion, but something more, a sort of a fortiori argument. Of course, although this expression is conventionally understood to signal a fortiori inference, its use does not certify that a fortiori reasoning is actually involved.

But let us suppose an a fortiori argument is indeed intended. In that case, to draw the conclusion from the given (minor) premise, we would need a (major) premise that places the terms X and notX as minor and major in some scale (i.e. identifies them as bigger or smaller in some respect, say Z). Moreover, we must suppose that these subjects have enough of Z to receive predicate Y or notY, respectively. Furthermore, since the subsidiary terms Y and notY are not identical, we need some third premise about proportionality to justify an a crescendo inference from the one to the other.

But even if all these conditions are met, we would still not be able to draw the putative conclusion! For this argument is not just one with antithetical subjects or just one with antithetical predicates – it is one with *both* these features together. This means that the change from X to notX and the change from Y to notY must happen simultaneously. Yet the premises we have adopted are not sufficient to guarantee this simultaneity. We would need more information to guarantee it.

This sounds feasible offhand; but if we reflect we see that it is not easy to provide such additional information capable of justifying the desired inference. An attempt to do that would seem to be bound to end in circular argument[12]. And this is the reason why a contrario argument must be considered as invalid. Thus, R. Hanahiah's above argument is indeed invalid, but not for the reasons Maccoby gives. The problem is not that it constitutes an a crescendo argument; such arguments may well be valid. The problem is that it is an argument so demanding of further information that it forces us to beg the question.

Maccoby does not conceive the issue in such formal terms, but he does analyze this example at length and propose the following resolution of the problem as he sees it:

> "I suggest a very simple solution… The expression notel nafsho has been wrongly translated by all commentators as 'forfeits his life', but is much better translated as 'receives his life'… With this correction, the difficulties of continuity and logic disappear. The… saying now becomes: 'If he that commits one transgression (i.e. the transgressor of a karet law) receives his life (i.e. has his sentence commuted from

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[11]   Valid 'proportional' a fortiori (i.e. a crescendo) argument is, of course, exceptionally not subject to that rule, since it contains two versions of the subsidiary term (one bigger than the other) and so effectively five terms.

[12]   See more detailed analysis of this issue in Appendix 2.

death by the hand of God to flogging), how much more so will one who performs a commandment be given his life!'"

In this way, Maccoby harmonizes the subsidiary terms in the minor premise and conclusion, and changes the apparent a crescendo argument into a purely a fortiori argument. This example is, Maccoby admits, an extreme case. He grants, as most commentators do, that "the rules of the qal va-homer, and especially the rule of dayyo" ought perhaps not to be "so strictly applied in aggadic as opposed to halakhic reasonings." I am personally, I must say, not very tolerant of breaches in logic, even for homiletic purposes. But Maccoby is willing to grant some poetic license. He gives another example drawn from the same Mishna that seems less flagrant:

"R. Shimon ben Rabbi reasons: abstaining from blood, which causes revulsion, brings reward; all the more, abstaining from robbery and incest, which the soul longs for, should bring reward for all generations to come up to the end of the world. The last part of this reasoning constitutes a breach of dayyo, for nothing was said in the premise about all generations to come. Yet in this reasoning, faulty as it is by halakhic standards, the actual terms are not changed from premise to conclusion. There is only an intensification of the conclusion, in an enthusiastic, homiletic style."

This could be a correct reading. But of course, we could alternatively say that "abstaining from blood… brings reward" tacitly intends "for all generations to come up to the end of the world," in which case there would be no breach of the *dayo* principle! Very often, we read things unstated in the premise by means of the conclusion, as well as vice versa. For example, take Proverbs 21:27: "If [even brought with a 'sincere' intent] the sacrifice of the wicked is an abomination, how much more: brought with a wicked intent [is it abomination]?" The phrases in square brackets were added by me; but they are both clearly tacitly intended.

But in any case, this second example from Makkot 3:15 is not formally identical to the first. The second argument, the one by R. Shimon, is that abstinence from blood is psychologically less taxing, because blood is revolting, whereas abstinence from robbery and incest is more onerous, because they arouse desire; consequently, while the former (which is demanding enough) deserves some reward, the latter (which is more difficult) deserves a greater reward. While this argument resembles the first, in that both involve parallel continua, one for the subjects and one for the predicates, the subsidiary terms in the second argument are not in opposition but two degrees of the same thing (viz. "reward" and "greater reward"). Thus, in the latter case, there is no a contrario intent, and the argument is much easier to validate.

Maccoby does notice this important formal difference between the two arguments, but he does not formally explain why that difference should make one more invalid than the other. To him, both remain essentially invalid; and they are so for the same reason, viz. that they are 'proportional'. Thus, we see here again that Maccoby's judgments are negatively affected by his narrow view of a fortiori argument as necessarily non-proportional, and his failure to grasp the logical possibility of a crescendo argument if appropriate additional information is provided.

Nevertheless, to repeat, in view of his emphasis on 'rule of law' in a fortiori logic, he deserves recognition as one of the good guys in this field.

23. Alexander Samely

Alexander Samely published in 2002 a book called *Rabbinic Interpretation of Scripture in the Mishnah*, in which one chapter is devoted to a fortiori argument[1]. According to the Google Books blurb, "This volume offers a systematic and detailed description of early rabbinic hermeneutics as it can be reconstructed from the Mishnah (third century CE)"[2]. This is a very ambitious work, of wide scope and great detail, with many interesting innovations. I do not however propose to thoroughly review it, but will concentrate all my efforts on its treatment of a fortiori argument. I will also frequently refer to the author's associated website[3].

1. General definition

The following sentence may be taken as Samely's general definition of a fortiori argument:

> "The mechanism of the inference involves an assignment of (mostly halakhic) categories to the two subjects; a ranking of these categories in a dimension of comparison; and the transfer of what is known about one of them to the other based on its higher rank in the comparison of categories" (p. 174).

The "two subjects" in question are those generally called the major and the minor term, although Samely does not so call them; the "dimension of comparison" refers to what I have called the middle term; and "what is known about" one of them that is "transferred" to the other refers to what I have called the subsidiary term. Note that Samely's understanding of a fortiori argument is entirely focused on the positive subjectal mood, which goes from minor to major. His definition is pretty good for that mood, since it refers to four terms and correctly places them in relation to each other. But it misses out on the many other moods.

However, Samely is wrong when he then claims that it is the "differential of ranks" between the major and minor terms "which leads to the claim that for the second subject [i.e. the major term] the validity, certainty, or reasonableness of the *inferred* proposition is even greater than for the subject from which it is inferred [i.e. the minor term]" (p. 174, square brackets mine). He is here misled by colloquial expressions, like "all the more," which are commonly used to signal a fortiori argument, into believing that (given the major premise, of course) the conclusion is more reliable than the minor premise from which it is derived. This is untrue: logic simply cannot perform such a feat; in any kind of argument, the conclusion is always and of necessity no more reliable than the premise(s) it is derived from. This shows that Samely does not fully understand the nature of a fortiori inference, being misled by the wording we commonly use in it.

He is of course correct in pointing out that the function of "*a fortiori* inference, or *qal wa-homer*… in the Mishnaic discourse is often explorative rather than apodictic" (p. 174). And again further on: "the Mishnah uses the *a fortiori* more for probing the consistency of a normative position, or for exploring its consequences, than for categorically determining it" (p. 176); also: "the halakhic a fortiori is used heuristically, not apodictically" (p. 177). But that is not what is at issue as regards the validity of the process. Samely has evidently not studied the question of validity to sufficient depth.

The closest he comes to such general considerations is when he writes: "Although not syllogistic in nature, the *a fortiori* argument has the potential for being presented in an apodictic and formal manner" (p. 190). However, his presentation of this "apodictic and formal manner" is neither appropriately abstract nor valid. He merely proposes an illustration, which may be briefly paraphrased as: 'If single-sector consumer groups can influence things, then multiple-sector consumer groups can influence things even more'. The problem with it is the 'proportionality' of its conclusion – the suggestion that consumer groups can, in proportion to their numbers, jointly have *more* influence. That may be true empirically, but it is not a purely a fortiori deduction. No contradiction would have ensued if it was found that the influence is not so cumulative.

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[1] Oxford: Oxford UP, 2002. See chapter 7.
[2] Some excerpts of this book may be read online at: books.google.com/books?id=rf0q-ZkU2EcC&printsec=frontcover&dq=Alexander+Samely,+Rabbinic+Interpretation+of+Scripture+in+the+Mishnah&hl=en&ei=U73kTf6SJoODO ofF_LcG&sa=X&oi=book_result&ct=result&resnum=1&ved=0CCwQ6AEwAA#v=onepage&q=sion&f=false.
[3] mishnah.llc.manchester.ac.uk/home.aspx.

We can readily put the argument in standard form: the major premise would be: multiple-sector consumer groups (P) have more clout (R) than single-sector consumer groups (Q); whence: if single-sector consumer groups (Q) have clout (R) enough to influence things (S), then multiple-sector consumer groups (P) have clout (R) enough to influence things (S). But according to formal logic, there is no "even more" about it: the subsidiary term (S) cannot be greater in the conclusion than in the minor premise. By purely a fortiori argument, we can only infer that numerous consumer groups together will have at least as much influence as any of them singly (assuming no mutual interference). To draw an a crescendo conclusion as he does, we would have to append a pro rata argument. Samely has not clearly grasped this, and it colors all his analysis.

Samely does display considerable understanding of formal logic in general, when he rightly points out the flaws in Louis Jacobs' analysis of *a fortiori* argument (p. 177, footnote). Regarding Jacobs' "simple type," viz. "If A has x, then B certainly has x," Samely remarks that though this reproduces "the grammar" of the argument, it cannot do the same for its logical form, "since for the logical form some comparison between A and B is always necessary." As regards Jacobs' "complex type," viz. "If A, which lacks y, has x, then B, which has y, certainly has x," Samely comments: "the comparison at the centre of the argument is in any case not accurately represented by the idea of 'lacking' or 'having' y, but rather by 'having y in various degrees.'" However, though perspicacious, this comment is not entirely correct, in that the ideas of lacking and having can be placed in the wider context of the idea of degrees. This means that Jacobs' "complex type" can also be looked upon as a grammatical form rather than a logical one.[4]

However, Samely seems to basically distrust formal logic, when he writes: "And one should certainly avoid 'translating' a rabbinic argument so that it fits into a predetermined, 'known' logical format, in particular if that leads to the 'discovery' of Greek logic in the rabbinic procedures. In most cases a translation of this type does indeed succeed in reducing the ambiguities of the original argument. But the price that has to be paid for this is high, because the new language, 'logic', is so much poorer. A century of intense investigation of the relationship between logic and ordinary language has resulted in an attitude of extreme caution towards such translations even within a contemporary setting, let alone for arguments from culturally or historically distant contexts" (p. 181). While there is considerable truth in what he says here, it is not entirely accurate.

It is evident that he here has in mind Adolf Schwarz in particular, who is generally regarded as having had a preconceived and misconceived idea that a fortiori argument can be equated to syllogism, and thus that rabbinic procedures can be forced to fit into Greek logic. Samely is probably, subconsciously, also thinking more broadly of modern logic, which (in my view) is indeed so superficial and simplistic that it cannot reflect all the nuances and complexities of ordinary language. He suggests, on the authority of Wittgenstein, that "language is tied to the situations of its use (i.e. its illogicality, in a manner of speaking);" but in fact, this view however fashionable is not based on a correct understanding of the relation of formal logic to everyday thought and language.

Briefly put, though our theories of formal logic are based initially on intuitive use of logic by people in everyday discourse, the science of logic eventually transcends the practice of logic. That is to say, it passes from description to prescription. How people commonly verbally express their logical thoughts (which are not always in accord with formal logic, not only linguistically but also logically) then becomes a special object of study, which is relevant to the science of logic in a broad sense but not to be confused with the core field of study known as formal logic. Similarly, though the history of logic is part of the science of logic in a broad sense, it can never be a substitute for formal logic.

Certainly, there is no justification for taboos: we must refer to formal logic to evaluate given material arguments, however venerable they are. In other words, Samely's relative avoidance of formal logic (I say 'relative', because he does in fact appeal to it to some extent) is not based on a wide knowledge and understanding of the subject. Moreover, as we shall see, this prejudice constitutes an obstacle to insight in his work.

2. Descriptive formula

Samely conceives of a fortiori argument more specifically as having a "five-point structure, which takes the argument to have four premises and one conclusion": (1) There are two "norms" n and m, which (2) belong respectively to "norm types" N and M; and (3) when these are compared in a "dimension" X, the former are graded higher than the latter; given also that (4) to the lower grade (n, N) "applies feature A," it follows that (5) to the higher grade (m, M) applies feature A (p. 179).

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[4]   Regarding Samely's proposed distinction between grammatical form and logical form, I would say that a more accurate representation would be a distinction between surface form (the way an argument often appears in everyday discourse, which is often abridged and casually worded) and deep form (the way an argument academically ought to be presented, with all the elements relevant to its logical validity made manifest). Jacobs presented some (though not all) common surface forms of a fortiori argument, without succeeding or even trying to formulate deeper levels of discourse. Samely, for his part, as the present essay makes clear, tries to dig deeper but fails.

If now I try to recast this set of propositions in standard a fortiori format (in terms of P, Q, R S), I obtain the following valid positive subjectal argument:

> Norm m of type M (P) is graded higher in dimension X (R) than norm n of type N (Q) is [major premise, from "points" 1, 2, 3],
> and norm n of type N (Q) is graded in dimension X high (R) enough for feature A to be applied to it (S) [minor premise, from point 4];
> therefore, norm m of type M (P) is graded in dimension X high (R) enough for feature A to be applied to it (S) [conclusion, as in point 5].

There are a number of problems with this descriptive formula, which I will now elucidate:

Asymmetry. Actually, Samely's concluding point is worded as: "therefore: to norm m feature A should apply even more." But I do not see why he fails to mention there the norm type M, while he mentions the norm type N in the preceding point. Similarly, I do not see why he there says "feature A should apply even more" instead of simply saying "applies feature A" as before. The expression "applies" is vague enough as it is – there is no profit in adding ambiguity by changing the wording from "applies feature A" in the minor premise to "feature A applies" in the conclusion, giving a confusing impression (though I doubt it to be intentional) that A is a predicate in the former and a subject in the latter.

A redundancy. Another question one can ask is: why is it necessary to mention both "norms" (n, m) and "norm types" (N, M), as Samely proposes? If we want to classify the norms in their types, the major term (P) may be expressed as 'norm m of type M' and the minor term (Q) as 'norm n of type N' – and these may be quantitatively related via the middle term (R), viz. "dimension X" in the major premise. There is no need for three propositions when one suffices. Splitting the major premise up into three components is not only redundant[5], but causes confusion because it conceals the operative totality.

Alternatively, use the "norms" as the operative subjects and put the "norm types" in parentheses next to them. Actually, Samely's exact wording of the terms of the third (i.e. the major) premise are: "type M (norm *m*)" and "type N (norm *n*)" – which suggests he is not too certain as to whether the types or norms should be included here, so he fudges the issue. Possibly, what Samely intended here is that the major premise should be exclusively in terms of the "norm types," i.e. very broadly stated, and then the minor premise and conclusion should be exclusively in terms of the more narrow "norms."

In that case, there would have to be, at least implicitly, a substitutive syllogism that applies the given major premise 'M is more X than N' to the corresponding narrower terms (m, n), saying: since 'm is M and n is N, it follows that m is more X than n'. However, we must be careful here to engage in valid deduction. The proposition 'M is more X than N' should mean that 'every case of M is more X than any case of N', or at least that 'every M is more X than the N corresponding to it' – otherwise, even if m and n are known to be subcategories of M and N respectively, we cannot directly deduce that 'm is more X than n' from it.

In other words, it is conceivable that, in some cases, the vague generality 'M is more X than N' is true and yet the more specific 'this m is more X than that n' is not true – so we must be careful. In any case, when such deduction is indeed possible, it is clear that the proposition 'm is more X than n' would be the operative major premise of the a fortiori argument. In such cases, the original broad premise 'M is more X than N' would strictly speaking *not* be part of the a fortiori argument as such, but only serve a *preparatory* act of reasoning – so it should not be presented as part of the a fortiori structure *per se*. Samely has apparently not realized all that.[6]

In practice (I refer here to the Talmud in general and related documents, and not solely to the Mishna), the major premise is expressed (if at all) sometimes in terms of broader categories (like M, N) than those in the minor premise and conclusion, and sometimes in terms of the same categories (like m, n) throughout the argument. If only for this reason, the formal formulation of a fortiori argument must be in terms of the operative categories, i.e. those appearing or intended in the minor premise and conclusion, and not in terms of broader categories that may sometimes appear or be intended in the major premise. This is obvious, since the form we adopt as standard must cover *all* possible cases.

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[5]   It is evident that Samely found this gradual approach useful, to order things in his own mind; but we are here concerned with logic not psychology.

[6]   I should also here draw the reader's attention to the recurring fallacy identified in the chapter on Adin Steinsaltz (18.2), where six terms are also involved. It could be that some of the cases Samely had in mind belong under that fallacy. A fortiori argument as such must be thought of as a four-term argument form, to avoid all possible error. When more than four terms are involved, the required major premise must first be separately proved.

A deficiency. Another issue worth raising regarding Samely's formula is the absence of the notion of *sufficiency* in minor premise and conclusion, i.e. of the minor and major terms being X (i.e. the middle term, R) 'enough' for the application to them of feature A (the subsidiary term, S). Although it is true that this notion is very often absent in everyday and in Talmudic a fortiori discourse, this does not mean that it is at all expendable. It is in truth an essential part of the argument, without which the argument cannot be validated. Though it is not always explicit, it is certainly always at least implicit. What makes it possible for the norms n or m to have the feature A is that they have reached the level of dimension X *required for* A. Reaching or surpassing the value of X corresponding to the minor term is *what makes possible and justifies* the predication of the subsidiary term to the major term.

This value constitutes a benchmark or turning point. The argument tacitly intends that if the condition is not satisfied, the putative inference is *not* possible. Therefore, if the benchmark value of X (i.e. of the middle term, R) is not clearly made part of the minor premise (and thence, the conclusion), the involvement of X in the major premise *remains unexplained*. Samely has apparently not understood this crucial point, since he does not explicitly specify "X enough" in his formula. Perhaps we should consider his use of the vague relational expression "applies" as unconsciously intending this subtext. But such vagueness is unacceptable in formalization efforts.

Limitations. Another fault in Samely's structure, if it is intended as a general statement, i.e. one applicable to all possible cases of a fortiori argument, is that it is too limited, being restricted formally to the positive subjectal mood, without mention of the negative subjectal mood, or the positive or negative predicatal moods. While negative moods may be regarded as implicit in positive ones by *reductio ad absurdum*, predicatal moods are significantly different from subjectal ones with regard to the form of both premises and conclusion and therefore cannot be glossed over. Many commentators through logic history have like Samely ignored predicatal a fortiori; but it is alluded to in some rabbinic accounts.

Also not mentioned by Samely are the four implicational moods corresponding to these four copulative moods. Did he perhaps intend the vague word "applies" to suggest either predication or implication? It is possible. As we shall see in the next section, on hermeneutic techniques, Samely seems to be aware of implicational a fortiori argument, though he uses the Greek terms *protasis* and *apodosis* instead of the more common Latin terms antecedent and consequent. This is presuming I interpret his intention correctly. But in any case, he does not explicitly make and discuss the distinction. Furthermore, just as his copulative form is limited to the positive subjectal mood, so his apparently implicational form is limited to the positive antecedental mood.

Also missing in Samely's account is a distinction between superior, inferior and egalitarian a fortiori argument. That is, he does not notice that the major premise may be stated either as 'this term is more X than that one', or as 'this term is less X than that one', or as 'this term is as X as that one'. The superior and lesser versions may be converted to each other, but the egalitarian (i.e. *a pari*) version has both features (i.e. works as well in both directions). Samely does not apparently realize that this third version is commonly used (even in the Mishna) and formally significant.

I would also object to the apparent limitation by Samely of the subjects to "norms" – though this might be largely explained by his concentration on Mishnaic *qal vachomer*. But let us ask: are these two terms truly normative (in an ethical or legal sense)? I would say: usually not (though in some cases they happen to be). The usual pattern, *when there is a normative intent* (which is not always the case) is roughly as follows:

> Action P is better/worse (more R) than, or as good/bad (as much R) as, action Q (is R):
> and action Q is good/bad (R) enough to be (as the case may be) permitted, forbidden, exempted
> or prescribed (S);
> therefore, all the more (or equally), action P is good/bad (R) enough to be (as the case may be)
> permitted, forbidden, exempted or prescribed (S).

As can be seen in this general formulation, the major and minor terms, P and Q, e.g. two voluntary actions, are non-normative. But they are *compared in a normative scale*, viz. the middle term R; i.e. they are classified as in some sense good or bad. The conclusion may also be normative in the sense that the subsidiary term (S) permits, forbids, exempts or prescribes such behavior. In some cases, the conclusion is non-normative, with S simply denoting some physical or mental result. Thus, for Samely to call P and Q "norms" is misleading; it misplaces the possible normative elements, which are rather concentrated in R and S. His thinking here was too vague.

No validation. Most important, I see no effort by Samely here to formally validate the argument – i.e. to explain, other than merely 'intuitively' why the conclusion follows – indeed, *must* follow – from the premises. This is a major issue, which has to be addressed. We cannot just assert something, without proof. But maybe it is not of great interest to Samely, because he comes on more as a non-judgmental reporter of Mishnaic discourse than as a logician intent on evaluating it. He is perhaps more interested in the 'how' (the rhetoric) than the 'why' (the logic), i.e. in what

convinces some (possibly even many) people than in what ought to convince everyone (as it does all people skilled in logic).

Samely's use of "should apply" in the conclusion instead of "applies" suggests, however, that he does not consider the a fortiori argument to formally yield a deductive conclusion, but merely at best an inductive one. He is certainly marking some hesitation, thinking that in at least some cases the conclusion is open to doubt. This is indicative of a failure of understanding on his part – it means he is unaware that validity can be strictly demonstrated. It could of course be that he is merely trying to reflect rabbinic uncertainties in this issue. But then he should have clearly stated that such uncertainties are due to an erroneous conflation in their minds between doubts in a premise (usually the major, but sometimes the minor) and doubts in the process of inference (i.e. in the implication of the conclusion by the premises jointly).

As regards Samely's use of "even more" in the conclusion, this may merely be intended to reflect the expression "all the more," which is traditionally used to signal a fortiori reasoning. Or he may have observed that a fortiori argument in some cases seems intuitively to merit an a crescendo conclusion, without realizing that such cases are made possible by an unstated additional pro rata argument. Thus, his use of "even more" may also (and more likely does) signify that he is unsure as to whether the '*dayo*' principle (or more precisely put, the principle of deduction with which it is generally confused) is universally applicable, or maybe sometimes 'proportionality' is legitimate[7]. Such uncertainty is inevitable if no efforts at validation or invalidation are invested. Or maybe, again, he is just trying to reflect rabbinic vacillation in this issue. But then, again, he should have explicitly made his dissent clear, explaining on logical grounds why the conclusion (*qua* conclusion) must exactly mirror the minor premise.

Precedence. Albeit these various faults, Samely's descriptive formula is very good. It is so, because it correctly contains four explicit terms (n, m, X and A – corresponding to my P, Q, R and S), laid out in ordered premises and conclusion, including an explicit major premise with an explicit middle term (X) and a minor premise and conclusion with a clearly identified subsidiary term (A). These formal insights are so crucial and novel (in comparison to the proposals of the past authors he mentions) that I wondered where he got them from! The resemblance to the positive subjectal form presented in my book *Judaic Logic* is considerable. Yet Samely nowhere mentions my work on the a fortiori argument, though that book was published seven years before his. He does mention many other authors, so he must have made considerable efforts of research. It is surprising that he apparently never stumbled upon my work in the course of such research[8].

I put the question to Samely by e-mail recently, and he assured me that he had never before come across my work. I do not doubt his word, albeit the evident similarities, in view of the many *differences*, mentioned above and below, between his treatment and mine. In any case, for the record, I claim precedence as well greater accuracy[9].

It is a great pity that he did not discover and study my work before he engaged in his own research. It would have saved him from a lot of mistakes and made possible a much richer and more useful treatise. This remark applies primarily to a fortiori argument, but (knowing the originality of my findings) I expect it to equally apply to other Talmudic hermeneutic principles (notably those aimed at harmonization). As things stand, I would recommend to him to seriously study my *Judaic Logic* and the present work, *A Fortiori Logic*, and then review his whole treatise and do what is necessary to bring it up to date. I do not wish to sound condescending, but give this as friendly good advice.

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[7]   In the context of Samely's narrow view of a fortiori argument as positive subjectal or antecedental forms, 'proportionality' signifies that the conclusion's predicate is quantitatively greater than the minor premise's predicate. This is of course contrary to the findings of formal logic for purely a fortiori argument, but Samely apparently does not know it. For a 'proportional' conclusion to be justified, we must have additional information (i.e. a further premise).

[8]   I self-published my book *Judaic Logic* in the summer of 1995, and distributed many copies to various persons and libraries round the world, including a copy to the Jewish and National University Library in Jerusalem in October 1995 and one to the Bodleian Library in Oxford in March 1996 (which are still listed in their online catalogs today). An edition intended for the general public appeared in the spring of 1997, published by Editions Slatkine, Geneva, and many copies were sold. Furthermore, an extract of my book appeared in *Higayon: Studies in rabbinic logic*, Vol. 4 (1997), a journal edited by Moshe Koppel and Ely Merzbach of Bar Ilan U., and published by the Center for Jewish Public Policy / Alumna, in Jerusalem under the title "Forms of A-Fortiori Argument," which listed the main eight forms in bold type with appropriate explanations. Finally, I put the whole book online in my new website, www.TheLogician.net, as of May 2001, and it appeared very soon after in many search engines including Yahoo (which at the time was the foremost one). The title page of *Judaic Logic* had 1350 visitors in the first year of its exposure on my website. In other words, my work was not exactly unknown in 2002.

[9]   In any case, the Ramchal (R. Moshe Chaim Luzatto) can be credited with the main aspects of a fortiori argument formalization, back in the first half of the 18th century. See my essay devoted to his findings in the present volume. As I there explain, I only recently discovered his work, and was certainly not influenced by it when developing my theory of a fortiori. (The English translation of the Ramchal's *The Book of Logic* was published in Jerusalem the same year as my *Judaic Logic* appeared in Geneva. Had I known about it, I would certainly have looked into it and mentioned it, as I did for the Ramchal's *Ways of Reason*. In any case, I formulated my theory well before I published it. The book took me four years or so to complete, and the chapters on a fortiori argument were among the first I dealt with. I can probably find the exact date my formulations of the eight main moods of a fortiori argument crystallized, through old backup diskettes.)

3. Three alleged techniques

Samely has developed his own list of "hermeneutic resources" used in the Mishna, given in Appendix 1 of his book and also posted in his website[10]. This list, if I understand correctly, is not (or not exclusively) based on the traditional Jewish lists (namely, Hillel's, R. Ishmael's, R. Eliezer's and the Malbim's), but is an attempt to independently, systematically and exhaustively identify, in a modern perspective (i.e. scientifically, rather than dogmatically), the actual hermeneutic techniques of the participants in and compilers of the Mishna. This is, in principle, a very commendable enterprise. I presume, offhand, it has contributed a lot of insight and information to modern Talmudic studies. In truth, I have not read the whole work and so cannot judge it for good or bad, but at a glance it looks impressive and valuable.

All that interests me for the present are the three techniques of a fortiori inference that Samely lists in chapter 7 of his book. The following defines the technique he labels "Analogy4.2" (or "A4.2"), which obviously corresponds to the formula above reproduced and analyzed by me:

> "Inference by analogy that norm *m* possesses predicate A, in the following manner: If norm *n* which belongs to the category N, which category is lower on scale X, has predicate A; then norm *m* which belongs to the category M, which category is higher on scale X, logically also has predicate A (or: logically must have more of the quality A)."

The technique labeled "Analogy5" (or "A5") is similarly worded, with the following exception: instead of "norm" *n* or *m*, it speaks of "subject" *n* or *m.* These differences in wording suggest that Samely found that some Mishna *qal vachomer* discourse is not normative. The technique labeled "Analogy4.1" (or "A4.1") is also similarly worded, with the following exceptions: instead of "norm" *n* or *m* and of "predicate" A, it speaks of "the protasis" *n* or *m* and of "the apodosis" A. These differences in wording suggest that Samely found that some Mishna *qal vachomer* discourse is not copulative, but implicational.

Thus, Samely claims to have identified three distinct techniques of a fortiori argument in the Mishna. Even so, it is evident from this list and must be stressed that Samely has not identified all moods and types of a fortiori argument. Samely's listed arguments are apparently, to repeat, all of the positive subjectal (A4.2 and A5) or positive antecedental (A4.1) kind. Perhaps the reason is that he has found only these three varieties in the Mishna; but this needs empirical verification, with reference to the broader list of a fortiori moods given in my work. (As I show further on, such a claim is empirically inaccurate.)

Another difference in wording is that (my italics) A5 says "predicate A *applies* to subject m" and A4.1 says "the protasis of norm m *has* the apodosis A," whereas A4.2 says "norm m *possesses* predicate A." But all these are clearly intended as the same, as the relation "*has* [predicate or apodosis] A" is thereafter used throughout the definitions. They are mere stylistic variations in wording.

A detail to note in the three techniques is in the conclusion the bracketed clause: "or: logically must have more of the quality A" (in A4.2 and A5) or "or: logically must have an intensification of the apodosis A" (in A4.1). Notice the qualification "logically." This seems to confirm that Samely believes 'proportional' a fortiori arguments to be valid, or at least that he has found such apparent arguments in the Mishna. Of course, as I have shown through formal logic, 'proportional' a fortiori argument is in fact invalid. To draw a 'proportional' (a crescendo) conclusion, one needs *additional* information, besides the a fortiori premises, enabling us to formulate an appropriate pro rata argument.

My own reaction to the said three techniques is that making a distinction between them was unnecessary hair-splitting. True, there are noticeable differences between them; but (granting momentarily for the sake of argument that only these three moods exist) do they justify distinct categorizations? It is a bit like dividing the class 'flowers' into 'red roses', 'pink roses' and 'red tulips'! The differentia used are inessential; and the proposed division is obviously not exhaustive, since it lacks symmetry[11].

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[10]   At: mishnah.llc.manchester.ac.uk/docs/definitions.pdf.
[11]   Why is A5 not called A4.3? Surely, A5 is closer in form to A4.2 than A4.2 is to A4.1? Also, is A4.1 normative or non-normative or both? If only the one, why not the other? Or why is there not a form that is to A4.1 what A5 is to A4.2? These are issues of symmetry, which put the proposed list and labeling of just three types in doubt.

And why are these arguments not candidly called "*qal wa-homer*," and instead referred to as numbered "analogies"? One gets the impression that there was a deliberate intent to appear different and innovative. Samely's explanation for his use of artificial names ("Analogy4 and Analogy5") for the "Mishnaic *a fortiori* argument" is that "the hermeneutic argument known in rabbinic literature and in modern scholarship as *qal wa-homer*… is not clearly defined." Moreover, though this descriptive label is often used in the Mishna, it also frequently employs the vaguer expression *din*, meaning judgment or inference, with reference to the same structure of argument. The word *din*, he adds, may also be used for other types of inference. "More importantly," he argues, "although the Mishnah does not seem to use the terms *qal* and *homer* in conjunction with any [other] hermeneutic resource… the possibility of that happening somewhere in rabbinic literature cannot be ruled out" (p. 176).

These explanations are unconvincing. Although the concept of a fortiori argument is indeed insufficiently defined by the rabbis, and the terminology for it varies and may even not be exclusive to it, I see no reason for changing its accepted name. It would suffice to just point out these problems and specify one's more precise position, conventionally choosing one name (or more) from among the existing names. The real and only reason Samely changes the name is that he wishes to fit the argument into his wider system of classification and labeling. That being a methodologically acceptable reason, his stated lame excuses for doing it were unnecessary.

4. Bava Kamma 25a-b

To assess Samely's understanding of a fortiori argument more precisely, it is necessary to look at his interpretations of actual examples of Talmudic reasoning. Let us first look at Samely's analysis, given in his website[12], of Mishna Bava Kamma 2:5[13] and the related Gemara on pp. 25a-b of the Talmud tractate. He there says a few things I agree with, and many I disagree with. Regarding my own detailed analysis of this passage, the reader is referred to the earlier chapter of the present work, 'In the Talmud' (7). The reader must study that chapter in order to understand where I am coming from in the present critique. This saves me from having to repeat relevant findings here.

Samely's reading of this important *sugya* (i.e. Talmudic discussion on a specific topic) is original in some respects; but it is also seriously inadequate in certain respects. It is interesting to note that he realizes that R. Tarfon's two arguments need not be taken as a fortiori in form – as (to my recollection) all past commentators have assumed – but may, one or both of them, be viewed as mere analogy. He writes:

> "I take the two presentations of Tarfon's argument to be open to two different constructions: while the first sounds more like an a fortiori argument, the second sounds more like an 'analogy of relationships' argument. However, I am assuming that the argument in both formulations is the same, so that one of the two constructions is the meaning intended by both formulations."

Actually, I would say that the most likely scenario is that R. Tarfon's first argument is merely analogical (pro rata) whereas his second is (purely) a fortiori. It is however also conceivable that both R. Tarfon's arguments are analogical, or alternatively that his first is a crescendo (i.e. a fortiori *cum* pro rata) and his second purely a fortiori. But in any case, we cannot logically suppose that both R. Tarfon's arguments are purely a fortiori, unless we also suppose that R. Tarfon's first argument was fallacious (since it does not obey the principle of deduction, which in this instance corresponds to the *dayo* principle). Still less can we assume R. Tarfon's two arguments to be respectively a fortiori (the first) and analogical (the second), as Samely initially imagines.

Thus, Samely's just quoted interpretations are mostly incorrect, showing he does not clearly understand the differences between a fortiori argument and mere argument by analogy. The way he distinguishes the two types of reasoning is by saying:

> "If it is an analogy of relationships argument (A0.5), its terms happen to provide for the inference of an increase in punishment, but are otherwise quite independent of any 'all the more so' inference typical for the a fortiori."

What does this distinctive "all the more so" phrase do, exactly? He does not know or say. He seems to think that the distinction lies in preliminary generalization. It is to his credit that he realizes that the proposed a fortiori arguments are preceded by generalizations, which establish the needed major premise. However, he does not realize that identical generalizations precede the proposed mere analogies, as I have shown. As he puts it:

> "If it is a fortiori argument then it incorporates within itself … a tacit generalisation from a specific cited case to a whole class (the case of tooth/foot to the case of all damage where the "public domain versus

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13   I mainly use Samely's transliterations in the present chapter, so as to facilitate references in his own work. (The transliteration for this tractate's name that I use in other chapters is *Baba Qama.*)

private domain of the person injured" distinction applies); in either case, there is no involvement of Scriptural text."

Note that Samely views R. Tarfon's two arguments as essentially one and the same. He says so explicitly earlier ("the argument in both formulations is the same") and further confirms it here by using of the singular ("if it is…" etc.). However, in truth, not only are the structures of R. Tarfon's arguments potentially distinct (as already mentioned), but though both are derived from the same Scriptural givens their major premises are built on different directions of generalization from these Scriptural givens. In the first case, the greater stringency of the private over the public domain is generalized from 'tooth/foot' to 'all cases' – whereas in the second case, the greater stringency of horn over tooth/foot is generalized from 'the public domain' to 'all cases'. These preliminary inductive acts make the two arguments very different, whether they are conceived as analogical or a fortiori.

However (to repeat), in Samely's view R. Tarfon's two arguments are one and the same, and the only issue is whether the process is interpreted as analogical or a fortiori. Looking at his attempted explanations of the difference between these two processes of inference, however, I do not discern any formal difference! The conclusion is either way the same (as R. Tarfon indeed claims) – viz. that liability for "horn damage in private" property "should be" / "ought to be" "more severe" than in the public domain, i.e. "full damages." Evidently, he accepts 'proportionality' without reflection in both cases. Note also that his use of the wording "should be" or "ought to be" is indicative of some uncertainty in his mind as to the logical necessity such conclusion, either way. All this is evidence that Samely's understanding of the forms of reasoning he seeks to inventory is limited.

Moreover, it is not at all clear what Samely means when he writes "in either case, there is no involvement of Scriptural text." The generalizations are evidently generalizations from Scriptural givens. The whole discussion is an attempt to derive new laws from Scriptural givens (namely, Exodus 21:35 and 22:4).

As regards the Sages' replies to R. Tarfon, viz.:

> "They answered: It is enough if the inferred is as strict as that from which it is inferred: if in the public domain half-damages, so also in the private domain of him that was injured, half-damages."
>
> (אמרו לו דיו לבא מן הדין כנדון מה ברה"ר חצי נזק אף ברשות הניזק חצי נזק)

Samely deserves credit for noticing that they are identical in both cases, for this is often glossed over. This reiteration is surprising, since – as Samely also notices – R. Tarfon's two arguments proceed along different routes, as already seen. However, Samely's explanation of this is simply to say:

> "The Sages … do not accept his explanation. In their final reply they still depict him as inferring from the half-damages of the horn in public (the half-damages come only from the horn; had they taken on board what he said, they should have spoken about the full-damages of the tooth/foot)."

Evidently, Samely has not understood the difference between the Sages' first and second *dayo* objections. Though the language of the two objections is identical, their meanings differ significantly in relation to R. Tarfon's two formulations. Given that R. Tarfon's first argument goes from *half* damages for horn in the public domain (minor premise) to *full* damages for horn in the private domain (conclusion), the Sages' *dayo* objection can here be construed as a statement that the conclusion must mirror the minor premise (i.e. remain 'half'). But given that R. Tarfon's second argument goes from *full* damages for tooth & foot in the private domain to *full* damages for horn in the private domain, the Sages' *dayo* objection here obviously cannot be construed as a statement that the conclusion must mirror the minor premise, since it already does!

It follows that in the latter case, the Sages must – albeit using the very same language – be interdicting another part of the reasoning process, viz. the inductive formation of the major premise, i.e. more specifically, the generalization from "in the public domain, horn damage implies more legal liability than tooth & foot damage" to "in all domains (including the private), horn damage implies more legal liability than tooth & foot damage." If this generalization were not rejected or at least devalued by the Sages, R. Tarfon's conclusion of "full damages" would be logically inevitable, whether by analogy or a fortiori. By interdicting this generalization, the Sages thus ensured that the only possible conclusion was "half damages" – as before.

Samely, to repeat, did not grasp this reasoning, because he lacked precise analysis of analogical and a fortiori argument forms. His attempted explanation in fact does not explain anything. He should at least have read the explanation given by a Tosafot commentator, which clarifies things considerably. With regard to the Gemara relative to this Mishna, Samely remarks only: "The Gemara at bBQ 25a treats the whole as an a fortiori, naming it a qal wahomer but the Gemara focusses on the acceptance or rejection of the idea of dayyo." This is true – but of course says very little. Samely does not subject to any critical analysis and evaluation the Gemara's idea that *qal vachomer* is naturally 'proportional' and that the *dayo* principle is a Torah-based limitation on this logic. He seems to just take it all for granted.

Following this, Samely draws attention to the first of the three related a fortiori arguments proposed further on in the Gemara, saying: "A reversal of this argument is found in bBQ 25b bottom" (actually, this continues on p. 26a).

However, though he reads the argument correctly[14], and he derives from it the first argument of R. Tarfon[15], Samely makes no attempt to clarify the logical intricacies it involves. Moreover, he inexplicably makes no mention of the other two additional arguments in the Gemara.

All this leads me to conclude that Samely is not entirely clear with regard to a fortiori logic, whether theoretically or in practice. This conclusion naturally casts a shadow on his understanding of the Talmud's hermeneutic principles in general (and not only the first, "*qal vachomer*"). Sorry to say. On this basis, I would re-examine very carefully all the results of his research before granting any of it credence. It looks interesting enough offhand, but one cannot expect accurate results if one does not really understand the forms of discourse and reasoning involved. I would again recommend to that author to study my past and present work and then conscientiously review all his own past work. I cannot take the time to do all this verification and correction work for him; it is his responsibility.

5. Samely's online database

One very interesting aspect of Samely's research is that he has tried to list and analyze every hermeneutic act in the Mishna, categorizing them all in accord with his system and making the results publicly available through a search engine in his website[16]. I cannot, to repeat, say whether the work done was all appropriately and accurately done, but do wish to express approval of the task he set himself and the effort made anyway. Such empirical and thorough research is truly what is needed and deserves applause. I called for such meticulous and wide ranging research already in my book *Judaic Logic*. What I can say in the way of general criticism, off the cuff, is that he does not seem to give sufficient attention to the hermeneutic principles conceived by the rabbis themselves. If so, this is a serious lacuna, since we cannot possibly hope to understand their way of thinking if we do not pay close attention to how they themselves conceived it.

As regards *qal vachomer* specifically, Samely has (to date[17]) allegedly identified and analyzed 44 occurrences in the Mishna (including 25 of A4.1, 13 of A4.2 and 6 of A5). The following is an alphabetically ordered listing of the references.[18]

Alexander Samely's list of 44 *qal vachomer* cases in the Mishna

Arakin 8:4	Hullin 10:1 II	Negaim 10:2 II	Terumoth 5:4 I
Bekoroth 1:1 II	Hullin 10:1 III	Negaim 12:5 V	Terumoth 5:4 II
Bekoroth 9:1 I	Hullin 12:5	Pesahim 6:2 I	Yadayim 4:7
Berakoth 9:5 V	Makkoth 1:7 VI	Pesahim 6:2 II	Yadayim 4:8 I
Demai 2:2	Makkoth 3:15 II	Pesahim 6:2 III	Yebamoth 8:3 II
Eduyyoth 6:2 I	Makkoth 3:15 IV	Pesahim 6:2 IV	Yoma 8:9 III
Eduyyoth 6:3 I	Menahoth 8:5	Sanhedrin 6:5 II	Yom Tov 5:2
Eduyyoth 6:3 II	Nazir 7:4	Shebuoth 3:6	Zebahim 7:4
Eduyyoth 6:3 III-IV	Nedarim 3:11 VII	Sotah 6:3 I	Zebahim 7:6
Eduyyoth 6:3 V	Nedarim 10:7 I	Sotah 6:3 III	Zebahim 8:12
Hullin 2:7	Negaim 10:2 I	Temurah 1:1 III	Zebahim 12:3

My initial purpose here is merely to pass on the information, so as to make it more widely known. Looking at such a list, we need to answer the following questions. Are the 44 instances Samely found all indeed fortiori, or are some are not a fortiori at all? Are they all positive subjectal (or antecedental)

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[14]   Except that his wording of the conclusion, viz. "should even more be," seems to suggest a crescendo argument whereas the Gemara's finale is clearly purely a fortiori.

[15]   Effectively by *reductio ad absurdum*, since he speaks of "reversal" and after presenting the former he says "So:" and restates the latter. If we deny the putative conclusion that there is some liability for tooth and foot damage on public grounds, and combine this denial with the other premises, we obtain as conclusion that there is full liability for horn damage on private property. Actually, this is not quite accurate, because the minor premise of the "reversal" argument is that there is *some* liability for horn damage on public grounds, whereas that of the "derived" argument is that there is more precisely *half* liability. However, R. Tarfon's first argument would work as well with the vaguer minor premise; so that's ok.

[16]   At: mishnah.llc.manchester.ac.uk/search.aspx.

[17]   I say 'to date' because the number of cases identified by Samely has increased over time, as is evident from the fact that in his book (p. 414) he lists only 22 occurrences (including 14 of A4.1, 5 of A4.2 and 3 of A5). This is of course to his credit, signifying that his research is ongoing; but it also means there may be still more cases that he has not yet discovered. I examined the database in the Summer of 2011.

[18]   The roman numerals seem to be subdivisions added by Samely, though I have found no explanation by him. He also adds Arabic numerals in brackets, which having no idea what they might mean I have omitted.

as he claims – or do some of them have other forms? Are some of them negative and/or predicatal (or consequental)? Are they all superior (or inferior) in form, or are some egalitarian (*a pari*)? Were some a fortiori arguments missed by him, and if so what forms do they have?

Moreover, underlying these questions is another, more basic one: are all these attempted a fortiori arguments truly a fortiori, or are some of them merely *pseudo* a fortiori? Are there arguments that superficially may seem like a fortiori or be alleged to be so, and yet in reality are not so? Whoever tries to enumerate the occurrences of an argument form must be able to indubitably recognize such form, and tell real and valid cases of it from illusory and invalid cases of it. It does not seem that Samely has attained this level of awareness, and so his enumerations must be verified independently by more competent logicians before being accepted.

I have finally actually looked at all of his findings and his reflections on them, and can at the outset testify that not all of these are indeed instances of a fortiori argument (whether valid or invalid). Also, not all arguments have the forms he thinks they have. Moreover, his list is not exhaustive. Funnily enough, this inventory does not include Bava Qamma 2:5, the crucial examples and discussion of a fortiori argument in the Mishna (which we examined in the previous section); this may have been a mere error of inattention, because Samely does discuss these arguments[19]. But also excluded are four cases that I found independently: two in tractate Avot (1:5 and 6:3), one in Kelaim (8:1) and one in Pesahim (6:5)[20]; so, there may well be other oversights.

6. My critical researches

Having ascertained that Samely's theoretical and practical understanding of a fortiori argument is imperfect (to put it nicely), I decided reluctantly to closely examine the relevant material in the database posted in his website. I say 'reluctantly', because I could see that would be a daunting task, and I am as lazy as the next man. As it turned out, I did well to do so, because I discovered many more imperfections in his treatment. The job took me a few months to complete.

Samely has not helpfully provided visitors to his website with a downloadable one-document listing of all the data in his database. The material is buried in scattered places, to be recalled one bit at a time. This may be intended to make data theft more difficult, but it also constitutes an obstacle to ready verification of his results by his peers. Nevertheless, I went through the forty-five web pages where the data is posted, and collected it all into one document about twenty pages long (which I am holding on to as documentary evidence). This included the given "text" and his "analysis" of it.

As regards the original Mishna text, I take it for granted that he accurately transcribed it. I do have some doubts as to that, in view of the many errors I found in his typing, as well as many repetitive spelling mistakes (e.g.: aught i/o ought, purety i/o purity), all of which are indicative that he did not double-check his own work. Moreover, there are many explanatory interpolations in square brackets, and other kinds of brackets, and it is not clear whether they are *his own* additions, or commentaries by past authorities that were already attached to the text. Sometimes he seems to be quoting verbatim; sometimes to be paraphrasing.

Strictly, I ought to have gone back to his sources and verified all the Mishna passages in question to see whether he reported them correctly. However, I decided not to take on this big additional job, considering that my purpose here was primarily to verify Samely's interpretations, and not so much to definitively study the source document in question. In truth, I did some spot-checking (again with reference to the Soncino edition*)*, and found the half-dozen texts examined on the whole passable. I thus, to repeat, took the texts proposed by Samely for granted and proceeded from there. I then wrote down, more or less independently, my own analysis of each text; and then, after that, my analysis of Samely's analysis. I also tabulated for myself various features of these findings to obtain an overview.

These analyses of mine added over twenty-five more pages to the document I had constituted. I thought to attach this detailed document to the present book, knowing from experience that when one criticizes material displayed in a

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[19]   That is to say, when one searches the database for all instances of "A4.1," the results returned do not include mBQ 2:5. This could merely be a programming problem, because the database does have an entry with the arguments of R. Tarfon (labeled as "A4.1"). Even so, the absence of this reference is indicative of carelessness in Samely's work.

[20]   The two cases in Avot I found by chance, while reading in that book. The two other cases I found by means of a search for key phrases in a document downloaded from sacred-texts.com, called *Eighteen Treatises from the Mishna*, by D. A. Sola and M. J. Raphall (1843). That search yielded only 6 cases in all, namely 5 cases of 'a fortiori' (Chullin 10:1, Kelaim 8:1, Pesachim 6:2 and 6:5, and Beitzah 5:2) and 1 case of 'how much more' (Chullin 12:4-5). Still, it uncovered two new cases.

website (as against in a published book), the erroneous information is usually wiped out very soon after, without any admission of error or trace of correction. But I decided against such inclusion in view of its considerable bulk. Instead, what I have done is offer the reader of the present book various summaries of my main findings. I hope Samely will have the good grace to acknowledge his errors and thank me for finding them.

The main results of my research on Mishnaic a fortiori argument are exposed in detail and summarized in tables (A2.1 and A2.2) in **Appendix 2**. This exposition does not include my above mentioned critical notes on Samely's prior work, which are too rough for publication; but it is based on them. The comments I will now make are intended as a brief summary of my critique of his work, including some general remarks (which all interested readers should look at), backed by further details (which some readers may prefer to skip or skim over).

What I can say at the outset about this portion of Samely's work is that it is far from the scientific research project it is trumpeted to be. There is considerable lack of rigor in its methodology and slackness in its execution. All too often, the interpretations proposed seem quite subjective – just one man's reading, without convincing proof. Although he has manifestly invested much effort in the project, much more effort is needed on his part to make it fully credible. We shall presently go over the main faults in Samely's treatment, under various headings. But first, let me overview my findings statistically.

Summary of findings. Of the 45 Samely references that I examined, 2 had no relation to a fortiori argument (here henceforth abbreviated to 'afa') that I could discern; 6 did not contain any afa, but only one or more rebuttals of such; the remaining 37 did contain a total of 42 afa, of which 38 were valid and 4 were invalid. The 4 invalid afa included 1 positive subjectal and 3 negative subjectals. The 38 valid afa included 23 positive subjectals, of which 4 were *a pari*, plus 9 negative subjectals, of which 1 was *a pari*; 1 positive predicatal and 1 negative predicatal; 3 positive antecedentals and 1 negative antecedental. Most references (31, to be exact) are to a single valid afa, while two references are to 2 of them and one is to 3. Note additionally, as already mentioned, Samely misses out on 4 more valid positive subjectal a fortiori arguments[21].

Let us now examine more closely the main faults in Samely's treatment of Mishnaic a fortiori argument. We have already mentioned his inclusion of some non-afa in his listing of cases and his missing some genuine cases of afa. The following areas also deserve critical attention: the absence of accurate statistics; no effort at validation; peculiar categorizations; difficulties with the middle and subsidiary terms; difficulties with the major and minor terms; ignoring the sufficiency of the middle term; choosing the reverse of the effective middle term; failure to see and acknowledge unusual features, notably, negative arguments, 'major to minor' arguments, predicatal arguments and implicational arguments; failure to stick to the original wording and structure; confusion regarding 'proportionality'; and other problems.

Absence of accurate statistics. Our first impression, looking at the list of references relating to a fortiori argument in Samely's database – i.e. 25 cases of A4.1 (26 cases, if we add the missing reference to mBQ 2:5), 13 cases of A4.2 and 6 cases of A5, is that this is an enumeration of individual occurrences of the argument. However, when we look more closely at his data, we find that some of his references in fact contain no a fortiori argument, but either a mere objection to such an argument or another sort of discourse; others contain an invalid a fortiori argument; and others contain not just one, but two or even three valid arguments. Evidently, Samely needs to improve his mode of presentation, and give clear statistics on each type and subtype of argument.

As already mentioned, Samely's listing of afa (generated by his database search engine) does not include the two arguments in Bava Kamma 2:5 (though these arguments are present and treated in his database), as well as four arguments in the tractates Avot, Kilaim and Pesahim. Concerning the Samely cases I have above excluded from the modified total of 42 afa, the following explanations are in order:

- The two Samely references **unrelated to afa** were: Nedarim 3:11 and Yoma 8:9. The first case reads: "Great is the circumcision which for the sake of Moses the righteous was not suspended as much as an hour." Samely tries to make something of it, saying: "there may be an implied dimension of a fortiori here (if even for Moses it was not suspended even for one hour, all the less so for ordinary Israelites...)." Although we could construct a valid positive predicatal argument[22] from the given proposition, I do not see that this is the intent of the Mishna. The second case reads: "As the Miqweh cleanses the unclean so does the Holy One, Blessed be He, cleanse Israel." Presumably, Samely reads into this the a fortiori: "if the ritual bath can clean the unclean, then all the more so can God do so." But here again, this does not seem to be the intent of the Mishna, which is just making a comparative statement: "just as this, so that."

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[21]   He does mention the Avot 6:3 case in a footnote to p. 183 of his book, but does not say why he excluded it from his list. Note that this tractate is certainly regarded as part of the Mishna, even though there is no Gemara for it.

[22]   That is: "Stricter standards (R) are required of righteous individuals like Moses (P) than of ordinary Israelites (Q). If the command of circumcision (S) was strictly enough applied (that it was not suspended even for one hour) for Moses (P), then the command of circumcision (S) is to be strictly enough applied (that it be not suspended even for one hour) for ordinary Israelites (Q)."

- The 6 Samely references **not containing any afa**, but only one or more rebuttals of such, were: Eduyyoth 6:3 II & V, Hullin 10:1 III, Negaim 10:2 II, Pesahim 6:2 II & III. The problem here is simply that Samely sometimes treats the objections to an afa as equivalent to an afa, perhaps because they have 'to do with' afa. On the other hand, he does not follow this policy consistently. There are many cases where he does not treat the objections apart from the afa, such as: Eduyyoth 6:3 III-IV, Menahoth 8:5, Shebuoth 3:6, Sotah 6:3 I, Yadayim 4:7[23], Zebahim 12:3, to name some. It would be best for Samely to always group rebuttals together with actual afa.

Although Samely does somewhat distinguish arguments and objections, by usually labeling them oppositely (for example, if the argument is A4.1, then the objection to it is "non-A4.1"), it is evidently not always clear in his mind what the difference is between an argument of a fortiori form and the rejection of such an argument. He sometimes seems to regard the rejection as itself a sort of a fortiori argument. This is due to the fact that he has not formally analyzed the different ways a fortiori arguments may be rejected. Sometimes, we have to choose between rival a fortiori arguments; sometimes, the proposed a fortiori argument is rejected as an illicit process; but mostly, the opponent merely puts in doubt one or both of its premises, without denying validity to the process.

- In some cases, one senses a sort of perplexity in Samely, in that he is unable to mentally separate the a fortiori argument from its subsequent rejection. A case in point is Sotah 6:3 III, where he describes as: "non-A4.2 an a fortiori whose rejection is anticipated." That is to say, he is not entirely clear as to whether, when an a fortiori argument is proposed hypothetically, with the intention to immediately reject it, it is to be considered as an effective a fortiori argument or a nonexistent argument. The status of a counterfactual argument is uncertain for him. In fact, if one understands the formalities of a fortiori argument, one clearly sees the difference between rejecting an argument as badly-formed and rejecting a well-formed argument by denying one or both of its premises.

No effort of validation. Samely is basically forced to take all the rabbis' arguments at face value, because he has no technical means by which to judge their validity or invalidity. He almost never pronounces clearly "this argument is right/wrong," for the simple reason that he has found no theoretical way to tell the difference, or even sought for a way. He is not a formal logician, but only at best a reporter of rabbinic discourse. To his credit, he has effectively identified one form of a fortiori reasoning – namely, the positive subjectal (or antecedental) mood – and this helps him to standardize rabbinic discourse somewhat. This is no mean achievement, to be sure. But he is unable to deal with issues of validity: he lacks the tools for it.

We can see this for instance in his treatment of Nazir 7:4, for which Samely's only comment is "refuted by tradition" – implying that there is no need to analyze the argument independently since it is allegedly dealt with by tradition[24]. But it is especially evident with regard to the 4 references *containing invalid afa*, namely: Makkoth 3:15 II[25], Temurah 1:1 III, Terumoth 5:4 I & II.

- The Makkot 3:15 passage, reads: "If the one who commits one transgression has his life taken away, [then] all the more will the one who performs one commandment be given his life!" Although roughly positive subjectal in form, it is distinctive in that its subjects ('one who commits one transgression' and 'one who performs one commandment') are contrary and its predicates ('his life shall be taken away' and 'his life shall be given to him') are contrary; this means that the argument is a contrario, and therefore invalid[26]. Samely sees it as somewhat reasonable, saying: "the punishment for transgression is, measure for measure, smaller than is the reward for fulfillment," although he finally taxes it as "merely 'rhetorical'" on the ground that "Life or death are on/off qualities, so the idea of 'increase' is impossible (no person can have be given several concurrent lives)." Thus, although Samely does judge the argument to be invalid, he does not do so for the right reason.

The Temurah 1:1 passage contains two rival afa, of which one (by R. Yohanan ben Nuri) is invalid, because it is negative subjectal, yet minor to major. It goes like this: 'If the sin/guilt offering, which becomes priestly property only after slaughter (Q), is fully owned (R) not enough to be substitutable (S); then, all the more, the firstling (P) is fully owned (R) not enough to be substitutable (S)'. Samely does perceive and try to explicate the invalidity of this argument. He denies that a putative conclusion follows from the given premises, saying: "from the prohibition of sin and guilt offerings for substitutions *follows thus nothing* for the substitutability of the firstling" (my emphasis). However, he is not here engaging in independent criticism, but merely reflecting what the original speaker, R. Yohanan ben Nuri, is saying.

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[23]   This argument is mentioned in his book, p. 183.
[24]   Another telling example of such naivety is Bekoroth 9:1 I, where Samely comments: "The way the subsequent hermeneutic operation rejects this a fortiori argument is interesting: none of the 'premises' seem to be undermined: it is merely shown that the conclusion, point 5, is contradicted by Scripture."
[25]   This argument is mentioned in his book, p. 179.
[26]   See my fuller analysis of this argument in Appendix 2.

In fact, Samely has not fully understood the discussion. He fails to notice that before this non-sequitur, R. Akiva proposes a valid *a pari* negative subjectal that can be paraphrased as: 'The sin/guilt offering (P) is as much a gift to the priest (R) as the firstling (Q) is; whence, if the sin/guilt offering (P) is a gift (R) not enough to be substitutable (S), then the firstling (Q) likewise'. R. Yohanan counters this with an intentionally invalid argument, above described. This fake argument is put forward only to formally ridicule the previous; but in fact it does not do the job intended, because the previous argument is egalitarian, whereas this one is minor to major. This is why R. Akiva replies, if I understand correctly, with a claim that both offerings are equally holy and that holiness takes effect as soon as it comes into the owner's home, so that substitution can occur at once. Samely makes no mention of R. Akiva in this context.

As regards the two invalid afa in the Terumoth 5:4 passage: one, by Beit Hillel, can be stated as: 'If the clean heave offerings, which are forbidden to non-priests [but not to priests], (Q) are restricted (R) not enough to prevent them being outweighed by clean common food (S), then, all the more, the unclean heave offerings, which are forbidden [even] to priests, (P) are restricted (R) not enough to prevent them being outweighed by clean common food (S)'; the other, by Beit Shammai, can be stated as: 'If the "light" common produce, which are permitted to non-priests, (Q) are restricted (R) not enough to prevent them from being neutralized by clean common food (S), then, all the more, the "weighty" heave offerings, which are forbidden to non-priests, (P) are restricted (R) not enough prevent from being neutralized by clean common food (S)'. These arguments are both invalid, because they are negative subjectal, yet minor to major. The difference is that the first is apparently put forward in earnest, whereas the second is apparently put forward with the intent to show up the invalidity of the first. I should add that there are two implicit afa following the above two, which argue from the possibility of outweighing or neutralizing unclean heave offering by clean common food to a like possibility by clean heave offering. I have not counted these afa, though they are valid, because they are not explicitly formulated, even though they are necessary to obtain the desired final conclusions.

As regards Samely's treatment of this Terumoth passage, he does not declare the two arguments invalid. He tries to represent the argument of Beit Hillel as follows: His middle term for this inference is "the scale of restriction," relative to which unclean heave offering would be the major term and common food the minor term, and presumably (though he does not explicitly say so) clean heave offering would be an in-between term. On this basis he argues: If "unclean heave offering can be neutralized (even) by clean common food," then "unclean heave offering should all the more be capable of being neutralized by clean heave offering." Note that his minor premise and conclusion both have the major term as subject, and respectively the minor term and the in-between term as predicates. This gives the impression that the argument is positive predicatal, with the major term as the subsidiary term and the minor and in-between terms as respectively the major and minor terms. From a formal viewpoint, this spells utter confusion! The confusion is, I would say, due to Samely's attempt to telescope two arguments into one. As regards the argument of Beit Shammai, it leaves him perplex: "Is this meant as a counterargument, and reductio ad absurdum of the preceding A4.1? If so, it does not itself have the structure of an a fortiori?"

Samely's peculiar categorizations. Samely's categorization of a fortiori arguments, as A4.1, A4.2 or A5, is rather subjective and arbitrary. Based on his definitions of these labels, I identified them as possibly referring respectively to implicational, and normative and non-normative copulative, a fortiori argument, respectively, leaving aside gaps and overlaps for now. However, looking at the cases listed in his database, I must say that I do not see such assumed differences between them! Comparing his practice to his theory, it is evident that even he finds these categories confusing. His success rate here is a paltry 4/24, regarding A4.1, and 12/42, regarding A4.2 and A5.

- Samely's A4.1 category seems by its definition intended to refer to implicational a fortiori argument, yet of 24 cases he so labels, only 4 cases are indeed implicational (viz. Bava Qamma 2:5, 2 cases[27], Hullin 10:1 II and Zebahim 8:12), the 20 others being copulative. Samely's A4.2 category seems by its definition intended to refer to normative a fortiori argument, and the 12 cases he so labels are indeed normative – but then so in fact are all 30 other cases he lists! Samely's A5 category seems by its definition intended to refer to non-normative a fortiori argument, but the 6 cases he so labels are all normative. I take as normative any concept involving a value judgment, be it intuitive or based on Scripture. To my mind, "trustworthiness" (Demai 2:2), "transgression, fulfillment of commandment, reward, punishment" (Makkoth 3:15 II), "the dimension of value" (Negaim 12:5 V, 3 cases), "just, unjust, punishment" (Sanhedrin 6:5 II[28]) are all value judgments, which are all classed as A5,

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[27]   Which two cases Samely says might alternatively be construed as A0.5: "I take the two presentations of Tarfon's argument to be open to two different constructions: while the first sounds more like an a fortiori argument, the second sounds more like an 'analogy of relationships' argument [A0.5]. However, I am assuming that the argument in both formulations is the same, so that one of the two constructions is the meaning intended by both formulations."
[28]   This argument is mentioned in his book, p. 186.

are just as normative as "exempt from obligation" (Bekoroth 1:1 II), "commandment, reward" (Hullin 12:5), "transgression, fulfillment of commandment, reward, punishment" (Makkoth 1:7 VI), "forbid, reward" (Makkoth 3:15 IV), "requires, [having] to be used" (Menahot 8:5), "prohibition, authority" (Nedarim 10:7 I), "forbidden, sets aside prohibition" (Pesahim 6:2 I & IV, 3 cases), "need for ritual, bars, guilt" (Sotah 6:3 I & III, 2 cases), "respect due, can [i.e. is allowed to]" (Yadayim 4:8 I), which are all classed as A4.2. Indeed, notice, one case of A5 (Makkoth 3:15 II) involves exactly the same concepts as a case of A4.2 (Makkoth 1:7 VI).

Thus, though his definitions seem boisterously confident, their proper applications in practice are far from evident. He nowhere justifies himself, explaining why this case falls under one heading and that case falls under another heading. He does not demonstrate, as he should, the relevance and consistency of his labeling; indeed, he cannot. This is an inexcusably careless way to work. All this leads me to regard, regrettably, his categorizations as pseudo-intellectual window dressing, rather than seriously scientific observations.

Difficulties with the middle and subsidiary terms. Although in his theory Samely has clearly marked the difference between the middle term R (which Samely calls "the scale or dimension of comparison, X" between two items), and the subsidiary term S (which he refers to the "predicate or apodosis, A" which is "transferred" from one item to the other), his behavior in practice shows that he nevertheless often (though not always) confuses or conflates them. Such ambiguity means that his subsidiary term is usually anticipated in his major premise, and his middle term is usually left out in his minor premise and conclusion, so that his arguments are often in fact not true to a fortiori form. Usually, the difficulty he has with the argument is one of interpretation, for the original argument is itself clear enough, but he does not make enough of an effort to discern and depict its real intent. This concerns at least 8 cases.

- Thus, in Demai 2:2, he uses the compound term "scrupulous=trustworthy" for both terms, although the middle term is in fact "scrupulous" and "trustworthy" is the subsidiary; his using them both, in this equated form, in the major premise (given in his "points 1-3"), minor premise ("point 4") and conclusion ("point 5") shows that he was unable to decide which is which. In Yadayim 4:8, Samely's middle term in the major premise is "respect due to the names," and in the minor premise and conclusion his subsidiary term is "without this meaning irreverence/disrespect" and no middle term is mentioned, so that the argument appears invalid. In Eduyyoth 6:2 I, he does not show awareness that his middle term "uncleanness" refers to *potential* conveying of uncleanness, whereas his subsidiary term "conveys uncleanness" refers to *actual* conveying of uncleanness. In Eduyyoth 6:3 II, similarly, he should have clarified the difference between his middle term "corpse-uncleanness in persons alive and dead," and his subsidiary term "corpse-unclean;" the former apparently referring to 'potential or actual', and the latter to 'only actual', uncleanness[29]. We must assume the same ambiguity for Eduyyoth 6:3 III-V, which he claims without going into details to be "structurally exactly analogous" to the preceding.

  In Hullin 2:7, the division of labor between his middle term "susceptibility to invalidation" and his subsidiary "invalidation" is clear enough, but he again does not remark upon the implied difference between potentiality and actuality. Similarly, in Negaim 10:2 I, his middle term is "power to render unclean" and his subsidiary is "renders unclean;" in Sanhedrin 6:5 II, his middle term is "grievous… in the dimension of God's compassion" and his subsidiary is "grieves God;" in Zebahim 7:4, his middle term is "susceptibility to sacrilege" and his subsidiary is "subject to sacrilege" – these three cases again suggesting a potential-actual relation between the two terms. In Menahoth 8:5, Samely wrongly declares the middle term to be the generality "quality of material (purity)" and the subsidiary to be the more specific "pure beaten quality" (whereas the middle is in fact quite clearly: having or not-having "to do with eating"). This, together with the preceding examples, suggests that Samely imagines there has to be a relation of inclusion of some sort between the middle term and the subsidiary – which is untrue: the two concepts may be conceptually quite unrelated.

Difficulties with the major and minor terms. Again, although Samely's theoretical treatment suggests a relatively clear understanding of the positions and functions of the major and minor terms, in practice he sometimes indulges in some astounding mental gymnastics with them, as in the following 4 cases.

- Thus, in Arakin 8:4, his major premise suggests that his major and minor terms are the subjects "divine demands" and "human demands," whereas in his minor premise and conclusion the subjects are respectively "God" and "man" while "divine demands" and "human demands" appear as modifications of the predicate "taking care to preserve man's property," so that we are faced with a mongrel argument, half subjectal and half predicatal, with five terms instead of four. In Makkoth 1:7 VI, Samely admits two versions of each term: the minor term is "[person who commits] transgression" in the major premise, but "person joined to a transgression"

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[29]   In truth, Samely misconstrues both these terms. R. Eliezer's argument is really: since a limb severed from a living being (P) is just as *dead* (R) as a whole corpse (Q), then the *uncleanness* (S) of the latter (Q) is also true of the former (P). Samely senses that his a fortiori construction does not make sense, and therefore suggests that the argument may alternatively be "a fairly straightforward analogy (of part-whole relationships)."

in the minor premise; the major term is "[person who commits] fulfillment of a commandment" in the major premise, but "person joined to the fulfillment of a commandment" in the conclusion; moreover, the middle term is "God metes out punishment or reward" in the major premise, but "punished" in the minor premise and "rewarded" in the conclusion; and likewise, the subsidiary term is "actual transgressor" in the minor premise and "actual performer of the commandment" in the conclusion. This makes a total of eight terms instead of the standard four! Of course, a valid reading is possible[30], but Samely did not manage to sort it out.

In Makkoth 3:15 II, he has a similar major premise with two middle terms, and again a change (though slight) of minor term in the minor premise (to "one-off transgressor") and ditto (implied though not stated) for the major term in the conclusion, plus the subsidiary term is contrary in the conclusion ("gain or get back his life") to what it is in the minor premise ("lose his life as punishment"). Again, all this goes against a fortiori logic, i.e. it is formally invalid; and it demonstrates that Samely does not realize the importance of proper form for purposes of validation[31]. Admittedly, the confusion here is largely due to the original formulation by R. Hananiah, which is exceptionally a contrario; but in the next example this excuse does not apply.

I am referring to Terumoth 5:4 I, where the subjectal major premise has "common food" as the minor term and "unclean heave offering" as the major term; while the latter term is the subject of both the minor premise and conclusion, whose predicates are respectively "can be neutralized by clean common food" and "can be neutralized by clean heave offering." This means that the minor premise relates the major term and the minor term, while the conclusion relates the major term (again) and an additional term (presumably lying in between the major and minor, but not mentioned in the major premise)! The middle term is given in the major premise as "the scale of restrictions," but does not reappear in the minor premise and conclusion. The subsidiary term, common to the minor premise and conclusion, appears as "can be neutralized by;" but in fact, without the further clause saying by what neutralization occurs, such a term is incomplete, and as already seen it is differently completed in the minor premise and conclusion, so that we have two subsidiary terms in fact.

A veritable mixed salad! Samely's major premise has in fact the wrong minor term (it should have been "clean heave offerings") and his conclusion has the wrong subsidiary term (it should have been "can be neutralized by clean common food" as in the minor premise). Moreover, as I show in Appendix 2, the argument by Beit Hillel that concerns us here is even when properly formulated actually invalid, since it is negative subjectal, yet minor to major. Invalid also is the rival argument, in Terumoth 5:4 II, put forward by Beit Shammai for the purpose of showing up the invalidity of the preceding one. Samely, having apparently not understood this[32], tries to present the first argument in positive subjectal form (and thus as valid)[33]. But the truth is that his minor premise and conclusion, formulated with the phrase "are neutralized by," seem positive because they lack the middle term. If we insert the middle term (R), we see immediately that they are really negative propositions, with the phrase "are restricted (R) *not enough to be prevented from being* neutralized by."

Ignoring the sufficiency of the middle term. One important failure of Samely's attempted formalization of a fortiori argument is his missing out on the idea of the subject being "R enough" or "R not enough" for the predicate to be predicated of it. This idea is essential to understanding the minor premise and conclusion, and the logical relation between them and with the major premise. It is only by satisfying the condition of a certain threshold value of the middle term (R enough) that a subject and predicate become joined; and if that condition is not satisfied (R not enough), then they are not joined. Because he has not grasped that idea theoretically, Samely in practice usually makes no mention of the middle term in the minor premise and conclusion (though perhaps this is sometimes in part due, as we have seen, to his confusing or conflating the middle term with the subsidiary). He can get away with this in many cases, but in some 4 cases this results in his misunderstanding the argument.

- We have just seen a case in point, in our discussion of Terumoth 5:4 I, where the phrase "are restricted *not enough to be prevented from being* neutralized by" had to be used in lieu of Samely's "are neutralized by" in

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[30]   See Appendix 2.

[31]   See my full analysis of this argument in Appendix 2. Samely does characterize this argument as "rhetorical," but he does so not for formal reasons but because "life or death are on/off qualities, so the idea of 'increase' is impossible." Still, it is interesting that he appeals to the "measure for measure" principle to form his major premise: "the punishment for transgression is, measure for measure, smaller than is the reward for fulfillment." Although, note, this is an erroneous formulation of the principle as such, because there is no *a priori* reason why "the punishment for transgression" should be smaller than "the reward for fulfillment." One might declare the two as equal offhand – but one would have to give good reasons why one should be smaller than the other. The idea is in fact based on Scriptural passages like Exodus 20:5-6, where God states that he visits "the iniquity of the fathers upon the children unto the third and fourth generation" of them that hate Him, whereas he shows "mercy unto the thousandth generation" of them that love Him and keep His commandments. The latter is an additional principle, not identical with the principle of measure for measure.

[32]   Samely remarks: "(non-A4.1/A4.1) Is this meant as a counterargument, and reductio ad absurdum of the preceding A4.1? If so, it does not itself have the structure of an a fortiori? It is denied that the greater leverage attaches to the common food."

[33]   Since both arguments go from minor to major, they have to be formulated as negative subjectal to be indeed invalid. So, Samely's attempt to build a valid positive argument for Beit Hillel misses the whole point.

order to see the negativity and thence invalidity of the argument. Other cases worth mentioning include Yebamoth 8:3 II[34], where he should have had "are restricted *not enough to be prevented from being* allowed forthwith" instead of merely "are allowed forthwith;" Zebahim 7:6, where he should have had "… is able as carrion to convey of uncleanness etc. *not enough to prevent* its slaughtering from rendering clean the uncleanness of its terefah" instead of merely "the terefah of … is rendered clean by slaughter;" and Zebahim 8:12, where he should have had "is halakhically significant not enough to invalidate," instead of merely "does not invalidate." In these three cases, the extra words are necessary in order to make the 'major to minor' argument valid; in the first two, a double negative is introduced ("not enough to prevent"), while in the third, the negation is shown as applying to the sufficiency of the middle term ("not R enough to S") instead of directly to the subsidiary term ("not S").

As regards mere failure to mention R, without significant effect, see for instances: Shebuoth 3:6, Yadayim 4:7, Yom Tov 5:2 and Zebahim 12:3. In Berakoth 9:5 V, Samely mentions neither the middle term nor the subsidiary term, saying only "from shortcut to spitting" (meaning from minor to major, though no major premise is given). He does sometimes mention the middle term, perhaps as a qualification of the subsidiary term. An example of this is Hullin 2:7, where Samely argues: since "sacrificial slaughter cannot be invalidated by wrong intention of the owner," then "non-sacrificial slaughter should also not be invalidated by wrong intention of the owner." Here, objectively, the subsidiary term is "invalidated" and the middle term is "by wrong intention of the owner;" but, noting that the middle term is defined in the major premise as "susceptibility to invalidation by intention," we can suspect that the two terms are here again somewhat conflated in Samely's mind.

Choosing the reverse of the effective middle term. Another difficulty Samely has in practice with the middle term is that he does not realize that the direction of the relative term chosen as middle in the major premise affects the validity of the conclusion, since a major term in one direction is a minor term in the other direction, and vice versa. Such subtleties may look like linguistic manipulations to the untrained eye, but they are crucial to formulating valid a fortiori arguments.

- Thus, in Bekoroth 1:1 II, Samely's major premise states that (my italics) "the Israelites' first-born are *more distant* than the Levites' own first-born" (implying that the former term is the major and the latter term is the minor), whereas his minor premise and conclusion concern the first-born of respectively the Israelites and the Levites (implying that the former term is the minor and the latter term is the major). To form a valid argument, the major premise should have been "the Levites' first-born are *closer* than the Israelites' own first-born."

In Pesahim 6:2 I, it makes a big difference whether the major premise is stated as "Sabbath work rules have a *more stringent* status than Sabbath rest rules" or as "Sabbath rest rules have a *more lenient* status than Sabbath work rules." In the first formulation (which Samely chose), the works rules are the major term and the rest rules are the minor term, and the proposed argument being positive subjectal and from major to minor is invalid, whereas in the second formulation (which he should have preferred) the works rules are the minor term and the rest rules are the major term, and the proposed argument being positive subjectal and from minor to major is valid. Samely could have more easily spotted and avoided this error if he had put the middle term in the minor premise and conclusion. He would then have felt the important difference between "is stringent enough to set aside" and "is lenient enough to set aside," and realized the need to correct his major premise.

Similar comments can be made in relation to the two rival arguments in Pessahim 6:2 IV. Using the middle term "stringent" again, Samely has the first argument (by R. Eliezer) as positive subjectal yet major to minor, and the second argument (by R. Akiva) as negative subjectal yet minor to major. But in this guise, both arguments are formally invalid! Yet if he had reversed the polarity of his middle term, using "lenient" instead, as just explained in the previous example, he could have made both arguments valid. *Alternatively*, he could have fixed the two arguments by changing the polarity of the sufficiency of the middle term in the minor premises and conclusions, as earlier explained. That is, for the first argument, he could have used the negative "is not stringent enough to prohibit the setting aside of" instead of the positive "sets aside;" and for the second argument, he could have used the positive "is stringent enough to prohibit the setting aside of" instead of the negative "does not set aside."

Another case of this sort is Sotah 6:3 III, where Samely has as his major premise: "Compared for the duration of her being barred from her husband, the Sotah-initiating testimony is less weighty than the testimony establishing her guilt." This means that "the Sotah-initiating testimony" is the minor and "the testimony establishing her guilt" is the major, in the scale of "duration" – i.e. lengthiness of separation. Now, this major premise happens to work well in the previous case he considers (Sotah 6:3 I), where the argument from "the Sotah-initiating testimony" to "the testimony establishing her guilt" is a valid positive subjectal, from minor to major. But

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[34]   This argument is mentioned in his book, p. 184.

Samely foolishly sticks to the same major premise when formulating the Sotah 6:3 III argument, thereby producing an invalid positive subjectal, yet major to minor! Here again, he should either have reversed the polarity of the middle term (from *lengthiness* of separation to *briefness* of separation) or alternatively changed the polarity of his minor premise and conclusion (from "is lengthily separating enough to require only one witness" to "is lengthily separating *not* enough to require more than one witness").

Failure to see and acknowledge unusual features. Samely's theoretical model for a fortiori argument is essentially limited to the positive subjectal form. Consequently, he fails to see and acknowledge less typical forms; namely: negative arguments, major to minor arguments, predicatal arguments and implicational arguments. These unusual features are, of course, often interconnected – but it is well for us here to look on each of them separately.

*   *Negative arguments.* Although Samely formulates 6 arguments negatively (namely, Bekoroth 9:1 I, Demai 2:2, Hullin 2:7, the 2nd afa of Pesahim 6:2 IV, the 2nd afa of Temurah 1:1 III, Zebahim 8:12), he does not stop and reflect on such negativity so as to integrate it into his theoretical modeling[35]; as a result, he fails to notice the negativity of 5 other arguments (namely, Arakin 8:4, the 1st afa of Temurah 1:1 III, Terumoth 5:4 I & II, Yebamoth 8:3 II, Zebahim 7:6[36]). And of course, not having reflected on the issue, he does not realize that the negativity of an argument affects its conditions of validity. He does exceptionally, with regard to the 2nd afa of Temurah 1:1 III, deny validity to the inference (saying "follows thus nothing"); but the explanation he gives, though it refers to the major and minor terms, does not raise the issue of polarity. Consequently, since he lacks formal understanding, he does not criticize the 2nd afa of Pesahim 6:2 IV in the same manner, which he should have done.

*   *'Major to minor' arguments*. Although Samely formulates 13 arguments, rightly or wrongly, in major to minor form (namely, Arakin 8:4, Bekoroth 9:1 I, Demai 2:2, Hullin 2:7, Nedarim 3:11 VII, Nedarim 10:7 I, Pesahim 6:2 I, the 1st afa of Pesahim 6:2 IV, Sotah 6:3 III, Yadayim 4:8 I, Yebamoth 8:3 II, Zebahim 7:6, Zebahim 8:12), he usually does not notice himself doing so. In the 2 cases when he does notice (namely, Pesahim 6:2 IV and Sotah 6:3 III), it is without reflection and moreover he commits errors of logic[37]. Given such failure to notice and reflect, it is not surprising that he does not integrate such occurrences in his theoretical modeling; and given such failure of theoretical treatment, it is not surprising that he makes errors. It should also be noted that some arguments that he formulates as minor to major should have been formulated as major to minor (namely, the 2nd afa of Pesahim 6:2 IV) or as a pari (namely, Bekoroth 1:1 II, Eduyyoth 6:3 I, Makkoth 1:7 VI[38]).

*   *Predicatal arguments*. Samely unconsciously formulates 2 predicatal arguments, as is evident from his use of the words "required" and "due" in them. In Nedarim 10:7 I, his major premise defines the middle term as "the authority *required* to cancel [vows]" and his minor premise and conclusion have "he can cancel vows which…" ("he" being the subsidiary term, and "can" here meaning "has the authority required"); this is a well-formed positive predicatal argument, though Samely does not realize he is not engaged in the usual positive subjectal form of argument. In Yadayim 4:8, his major premise defines the middle term as "respect *due* to the names," but his minor premise and conclusion get a bit confused, making the argument look positive subjectal (and therefore invalid, since major to minor), whereas the intent is obviously positive predicatal (whence valid)[39]. A third predicatal argument, namely Demai 2:2 (which is negative, minor to major), is not similarly verbally marked by him (he presents it as effectively negative subjectal, major to minor).

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[35]   His comment concerning the afa in Demai 2:2, viz. "it has a negative form, but it is a positive analogy: he is unreliable concerning himself, how much more so concerning others," is an attempt to to dismiss the phenomenon rather than acknowledge and try to explain it.

[36]   In the first two of these six cases, Samely should have noticed that the original arguments in the Mishna are clearly negative; in the last four cases, Samely may be excused in that the rabbis themselves formulate the arguments positively, although it should have been obvious to him and them that they had to be formulated negatively (using the double negative "not enough R to prevent") to be made valid. The latter arguments could conceivably be recast in positive form, but such traductions are usually rhetoric rather than logic.

[37]   He does comment that the two arguments of Pesahim 6:2 IV are "going in opposite directions," but he does not clarify just what this means and what it implies regarding validity. And in fact, he formulates both arguments in the wrong direction – the one that should be minor to major as major to minor, and the one that should be major to minor as minor to major! Similarly, his comment concerning Sotah 6:3 III, viz. "Mirror-image of the earlier a fortiori inference: compared with mSot 6:3 I (4), propositions four and five have switched places" shows awareness that the minor premise and conclusion (i.e. more precisely put, the minor and major terms) have switched places – but Samely offers no explanation or validation for this phenomenon. Moreover, his formulation of the Sotah 6:3 III argument is invalid!

[38]   Another a pari argument, namely the 1st afa in Temurah 1:1 III, by R. Akiva, is not even noticed or mentioned by Samely.

[39]   His argument goes: Given more respect is due to the name of God than to that of Moses: if "the name of God appears in Scripture underneath that of the ruler without this meaning irreverence," then "the name of Moses can appear in the bill of divorce together with that of the ruler also/all the more without this meaning disrespect" – whereas it should have been: Given more respect is due to God than to Moses: if occurrence of name in a document (Scripture, in this case) side by side with a ruler's name (Pharaoh's) is respect enough for God, then occurrence of name in a document (a bill of divorce, in this case) side by side with a ruler's name (current ruler) is respect enough for Moses. Samely has the negative subjectal format "if PRS, then QRS" – whereas the correct format for positive predicatal argument is "if SRP, then SRQ."

- **Implicational arguments**. I would characterize 4 arguments in Samely's list as having implicational form. Namely: the two afa of Bava Qamma 2:5, Hullin 10:1 II, and Zebahim 8:12 – the first three being positive antecedental (minor to major) and the last being negative antecedental (major to minor); there are no consequental arguments, note in passing. The reason these arguments cannot be taken as subjectal rather than antecedental is simply that the subsidiary item in each case is not a predicate but a consequent of the minor and major items[40]. While Samely formulates these arguments in a way that is objectively implicational, he does not verbally mark their difference as such. As already seen, we cannot reasonably accept his categorization of these four cases as A4.1 as signifying implicational argument, since there are twenty other cases that are so labeled and yet are clearly not implicational. Thus, here again, we must consider that he is unconscious of formats other than the positive subjectal.

The distinction between "from minor to major" (Latin: *a minori ad majus*, Hebrew: *miqal lechomer*), "from major to minor" (Latin: *a majori ad minus*, Hebrew: *michomer leqal*) a fortiori argument is nothing new, but known with some clarity since antiquity in both the Greco-Roman and Jewish worlds. The ancients were also aware of egalitarian (*a pari*) argument and, though somewhat less clearly, of the possibility of negative argument. It is very surprising, therefore, to find Samely setting off on a systematic analysis of Mishnaic thought while apparently totally unaware of these different possibilities. The concepts of predicatal and implicational a fortiori argument being less widely known, Samely can be excused concerning them; but for the rest he needs to study the matter much more and revise his past work.

What is evident is that ignorance of the impact that polarity and orientation have on validity have caused Samely serious errors of logic. I would say that one practical reason he failed to notice such deviations from the forms he assumed to be general was that he did not actually label the terms (or theses) in his analyses. That is to say, at least in his database (as distinct from his book), he never inserts *his own* abstract labels N, M, n, m, X, and A, in the examples, but is content to just treat cases vaguely, in wholly concrete terms. Had he always inserted his labels, so as to demonstrate that the examples do indeed fit his forms, he would have quickly discovered that some of the examples in fact do *not* fit his forms[41]. This would have led him to improve his analyses and definitions. Thus, at the root of his theoretical and practical errors is a too casual approach, an absence of formalism.

Failure to stick to the original wording and structure. Another fault in Samely's approach is failure to stick as closely as possible to the wording and structure of the original text. This is, to be sure, not always possible or wise, for the job of an interpreter is not merely to reflect the surface presentation, but to bring out the deeper intent. An argument may appear to be invalid if taken too literally, but be found quite valid if an effort is made to capture the underlying logical intuition that gave rise to it. In everyday speech, we allow ourselves considerable brevity and poetic license; when such speech is examined more strictly, with logical evaluations in mind, some corrections are often called for. For instance, very often we have to propose an appropriate middle term, because it is left unstated in the original text. Nevertheless, we should not too readily depart from the original text, thinking to improve on it, for it may have unexpected lessons to teach us.

- For example, in Arakin 8:4 R. Eleazar ben Azariah says: "And if Man does not even have authority to devote to the High One everything he owns, how much more is Man obliged to protect his possessions!" This is naturally read as a negative subjectal a fortiori argument: 'Man's use of his possessions for holy ends (P) is more religiously valuable (R) than man's use of his possessions for profane ends [including the waste of his possessions without purpose] (Q); whence, if P is *R not* enough to be authorized without limit (S), then, Q is *R not* enough to be S'. Samely's interpretation of it is very similar, except that it is made in positive terms: "Compared in the dimension of a claim on man's property, divine demands are higher on the scale than human demands;" and "God has taken care to preserve man's property by limiting dedications to the Temple;" therefore, "man should take even greater care to preserve his property with regard to human wishes or priorities." In the original text, we find both the negative form "does *not* have authority to devote" and the positive form "is obliged to protect," so either way might seem okay *prima facie*. But if we reflect on the issue of validity, it is clear that the argument *has to be* formulated negatively, and not positively, since it is subjectal and major to minor. It is not sufficient to use positive terms with negative connotation like "limit" and "preserve," because this misplaces the negation, which formally must be applied to the "enough" of the middle term R.

  Regarding Bekoroth 1:1 II, Samely's interpretation is in a way closer to the rabbis' than mine. The Mishna claims a minor to major inference, and Samely tries to comply with that (though in a gauche manner, as already

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[40]   Thus, in the two Bava Kamma arguments, the damages brought about by an ox imply various legal liabilities on the owner of the ox (not the ox itself); in the Hullin case, offerings imply the payment certain dues to priests (the offerings do not pay, it is the one offering them who does); in the Zebahim case, wrong sprinkling of blood does not render the remaining blood unfit (the antecedent and consequent do not have the same subject, i.e. the same portion of blood).
[41]   See for instances Makkoth 1:7 VI and 3:15 II in this regard.

seen), whereas I propose an egalitarian major premise, seeing no reason offhand to treat the two parties involved differently. That is, I assume that all first-born (including Israelites and descendants of Levi) were together released from their traditional duties; and thereafter a selection of Levites (namely, Aaron and his descendants) were inducted for future priestly duties; whence, it is easy to argue from Israelites to Levites as regards the exemption from the earlier régime. On the other hand, Samely's formulation of the major premise introduces a notion of "distance" that does not reflect the original argument and would require complex additional explanations and justifications.

Samely formulates the Demai 2:2 argument as a negative subjectal, disregarding the negative predicatal format used by the rabbis when they say: "He is not trustworthy concerning himself, how can he be trustworthy concerning others?" Here, notice, "he" is the subsidiary term (S) and "trustworthy concerning himself" and "trustworthy concerning others" are respectively the minor and major terms (Q, P); since the latter are predicates, the argument ought to be formulated in predicatal form. In Eduyyoth 6:2 I, the rabbis argue: "For the living being, which is clean, a limb severed from it is unclean – so is it not to be inferred that for a corpse, which is unclean, a limb severed from it is unclean?" I read this simply to mean: 'If the limb severed from a living body (Q) is unclean (R) enough to be declared unclean (S); then the limb severed from a corpse (P) is unclean enough to be declared unclean'. Samely, however, effectively takes the subsidiary term to be "capable of conveying uncleanness." Unless he bases this on some Gemara or other traditional commentary (he does not mention any), I do not see the reason for this interpolation.

Similarly, Samely unnecessarily complicates the matter with regard to Eduyyoth 6:3 I. In his interpretation of Menahoth 8:5, Samely takes the middle term to be "quality of material ('purity')," whereas in the Mishna it is clearly "to do with eating." With reference to Nedarim 3:11 VII, Samely proposes an afa (saying, "there may be an implied dimension of a fortiori here") though there is no hint of such intention in the given text[42]. We have earlier come across other cases where the polarity of the argument is misconceived, or the middle or subsidiary term is badly chosen, or the middle and subsidiary terms are tied together; I shall not repeat my analysis of them here: suffice it to note that many of those cases are not only in error but fail to mirror the original text.

Confusion regarding 'proportionality'. Samely is just about silent on the theoretical issues relating to 'proportionality', and his recognition in practice of the difference between purely a fortiori argument and a crescendo (i.e. a fortiori cum pro rata) argument is far from manifest and consistent. Sometimes he judges correctly (24 cases), sometimes incorrectly (12 cases), and sometimes he fudges (6 cases). This demonstrates he does not have a clear and certain understanding of the subject-matter, even on an intuitive level, let alone on a formal level.

- He rightly suggests proportionality in his conclusion in 7 cases, namely: the first argument of Bava Kamma 2:5 (language "ought to be more"), Makkoth 3:15 II ("at least"[43]), Makkoth 3:15 IV ("should be even greater"), Negaim 12:5 V (3 cases, "even more") and Sanhedrin 6:5 II ("even more"). But he wrongly suggests it in 6 cases, viz. Arakin 8:4 ("even greater"), Demai 2:2 ("can only be less"), Shebuout 3:6 ("should cause even more"), Terumoth 5:4 I ("should even more"), Zebahim 7:4 ("should be even more subject") and Zebahim 12:3 ("should even more have"). In 1 case, Hullin 12:5 ("or even some greater"), Samely's language is tentative though it should have been definitely proportional. In 6 cases, viz. the second argument of Bava Qamma 2:5 ("should be more"), Eduyyoth 6:2 I ("should also or even more so"), Pesahim 6:2 I & IV (3 cases, "should even more/less") and Yadayim 4:7 ("should apply also/even more"), his language wrongly suggests proportionality as a possibility. In 3 cases, viz. Bekoroth 9:1 I ("should be even less substitutable, with even greater certainty not substitutable"), Makkoth 1:7 VI ("even more certainly") and Yom Tov 5:2 ("must be even more certainly"), the intent is epistemic rather than ontical. In 17 other cases, Samely rightly makes no suggestion of proportionality. And lastly, in 2 cases, namely Berakoth 9:5 V and Terumoth 5:4 II, he offers no analysis, so we cannot determine what he thinks (neither of these cases are in fact a crescendo).

If I had any doubt, looking at Samely's theoretical treatment, as to his posture regarding the validity of 'proportional' a fortiori argument, all such doubt disappeared when I saw his practical treatment. Samely has obviously not given any thought to this crucial issue. What is clear is that Samely is unsure what to make of it. Samely's numerous wrong suggestions of 'proportional' a fortiori argument are inexcusable, since he has come across and devoted some

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[42]   There is, to be sure, a rabbinic idea (see the 5th and 6th rules of R. Eliezer ben Yose haGelili) that a fortiori argument may be "*meforash*" (explicit) or "*satum*" (implicit); so Samely's suggestion is not in principle impossible. But it is not his task here to propose innovations. And besides, countless comparisons could similarly be turned into a fortiori arguments!

[43]   We can safely say that Samely here intends proportionality, even if the language he uses is indefinite, in view of the special difficulty of the case. As already stated, this argument is invalid, because it is a contrario. Nevertheless, it does involve 'proportionality', i.e. its predicates 'having life taken away' and 'having life given' can be viewed as two points along a common continuum. However, Samely's addition of "at least" in his conclusion is gratuitous.

attention to the rabbis' *dayo* principle, given in mBQ 2:5. Not once[44] does he again raise the issue in all the other examples of a fortiori argument that he deals with. Apparently, this encounter did not even awaken in him (as it did, for instance, in Maccoby) an awareness of the principle of deduction, i.e. that the conclusion of a deduction cannot claim more information than its premises logically provide. But in any case, he should have at least asked the question on the basis of Talmudic hermeneutics, i.e. in the way of a religious rule. Evidently, he did not internalize the *dayo* principle.

Additional criticisms. Some more criticisms, some general, some specific, can be leveled against Samely's treatment of a fortiori arguments in the Mishna. One worth mentioning is that Samely does not seem to be aware of the difference between the *davka* reading of a text (where what is not explicitly included is taken to be implicitly excluded) and *lav davka* readings (where things not explicitly included may yet be argued to be included). That is an important aspect of rabbinic hermeneutics which, judging offhand, he seems not to have grasped (see his comments on Hullin 10:1 III and Menahoth 8:5). But I think that enough has been said. We have found various sorts of faults: methodological, conceptual, interpretative, and most important—logical.

Samely has done a lot of work, and a lot of it deserves praise, but he clearly needs to overhaul his database – and before that, he needs to study a fortiori logic more carefully. Glancing very briefly at his "techniques of interpretation" *other* than those relating to a fortiori argument, I tend to suspect the rest of his work *also* needs to be thoroughly reviewed and revised. And for that purpose, Samely should further study works on the rabbinical hermeneutic principles (including my *Judaic Logic*), before trying to impose his own vision. One reason I engaged in such a detailed examination of Samely's work was to show him, his Oxford UP publishers, and indeed all readers, that my criticism of it is fair, being based on sustained study and not merely on general impressions. Certainly his effort deserves such proportional attention, whatever its shortcomings.

Moreover, it occurred to me as I analyzed his work that his many errors have some utility – they are lessons learned. By showing them up for all to see, I was providing future researchers with a valuable gallery of possible errors. When future researchers try to interpret and evaluate a fortiori argument in the Gemara or any other document, they will be more careful to avoid repeating such errors. A researcher must especially be sure to master the logic theory, before venturing to examine documents.

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[44]   Except in Zebahim 7:6, but only because the issue is there apparently raised by R. Yose. However, though the latter uses the language of *dayo*, saying "it is enough," I doubt he is really invoking the principle as Samely suggests. I see no quantitative or even qualitative change in R. Meir's proposed conclusion. R. Yose seems to be denying that the conclusion of the a fortiori argument follows from its minor premise (which he accepts), arguing: "[Even though] slaughtering the carrion of a beast renders clean, [still] nipping off [head of bird] does not [render clean]." This implies that he is doubting the major premise, somehow, rather than literally appealing to *dayo*.

24. Lenartowicz and Koszteyn

1. The form of the argument

Piotr Lenartowicz SJ and Jolanta Koszteyn, in their paper "On Paley, Epagogé, Technical Mind and *A Fortiori* Argumentation*" (Cracow, 2002)[1] refer to and comment on my theory of a fortiori argument in *Judaic Logic*. I would like here to respond to their comments. They begin promisingly, as follows:

> "We have decided to follow Avi Sion's version of the *a fortiori* argumentation (Hebr. *qal vachomer*). The formal structure of his version is complex. According to Avi Sion it involves two elements, the scheme of *a fortiori* "syllogism" and the Dayo (Sufficiency) Principle. The *a fortiori "syllogism"* is this:
>
> > P is more R than Q is R, and
> > Q is R enough to be S; all the more *(a fortiori)*
> > P is R enough to be S.
>
> It is not a typical syllogism. A typical syllogism has just three terms: P, Q (or a middle term) and S. Here we are dealing with four different concepts. The concept of R refers to a trait that decides whether something is S, or not. In the first 'premise' P is asserted to possess the trait R in a higher degree than Q. In the second 'premise' Q is asserted to possess the R trait in the sufficient degree to be S. In the conclusion P is asserted to possess the trait R in the sufficient degree to be S."

The authors here show they have clearly grasped the difference between Aristotelian syllogism ("typical syllogism") and a fortiori argument ("*a fortiori* syllogism") – one having three terms and the other four terms, whose functions in the inference are very different. Moreover, they have well understood that the fourth term R, found in all three propositions of a fortiori argument, "refers to a trait that decides whether something is S or not."

However, the form of a fortiori argument they list here is the positive subjectal, which is only one of many forms, albeit the most typical. It is therefore misleading to say as they did that this is "the" form of it. Moreover, it would be best, in order to avoid confusion, not to use the word "syllogism" at all with reference to a fortiori argument, although this is strictly speaking of course not erroneous, since the etymology of the word is syn + logism, meaning 'merged discourse'.

2. The dayo principle

Next, the authors comment on the rabbinic *dayo* (i.e. sufficiency) principle:

> "*The Dayo (Sufficiency) Principle* requires that the conclusion of an *a fortiori argumentation* be kept within the limits of the minor premise. Avi Sion's interpretation seems to reduce the classical *a fortiori* argument to the *a pari* argument. It changes the [form presented in the previous quotation] into
>
> > P is as R as Q, and
> > Q is R enough to be S. Therefore
> > P is R enough to be S.
>
> So, in our opinion, the Dayo (sufficiency) Principle might be called an Overcaution Principle. It gives a firm assurance of the right conclusion, although cognitively it is a wasteful, costly instrument."

Here unfortunately the authors display some confusion. Of course, I know where their criticism is coming from: they are thinking of 'proportional' a fortiori argument, i.e. what I have lately called a crescendo argument. In that specific context, the conclusion's subsidiary term (S2) is indeed greater than the minor premise's subsidiary term (S1), in

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proportion to the values of the middle term (R) for the major and minor terms (P and Q), respectively. However, what they do not realize is that such argument is not purely a fortiori, but a compound of a fortiori and pro rata arguments.

The argument they present in the above quotation, drawn from my book *Judaic Logic*, is clearly intended as purely a fortiori. Given *only* the two premises shown, one cannot but deduce the conclusion shown. There are no ifs and buts about it, no way to arrive at a 'proportional' conclusion. There is no "overcaution" in the non-proportional conclusion – it is an undeniable fact of logic. If we want to obtain a 'proportional' conclusion, we must provide an additional premise, which affirms that the subsidiary term (S) is 'proportional' (in some way: strictly or roughly, directly or inversely, or however) to the middle term (R). Given this third premise, which makes possible a pro rata argument, we can with the full blessing of logic deduce the putative 'proportional' conclusion. But in the absence of such added information, we are logically forced to remain content with the said non-proportional conclusion.

The definition of the *dayo* principle by Lenartowicz and Koszteyn, viz. that "the conclusion [must be] kept within the limits of the minor premise" is accurate enough in the context. This is the way I understood the Talmudic principle at the time I wrote *Judaic Logic*, although I have more recently come to the realization that the original rabbinical understanding of it was different. But for the moment here let us read it as the two authors do. In the context of purely a fortiori argument, the *dayo* principle may be equated to the principle of deduction, as I now label the logical rule that we cannot end up with more information in the conclusion than we are given in the premises. Granting this equation, as these authors do in their definition, they have no leg to stand on when they deny the principle, calling it "overcautious" and "cognitively wasteful," implying that it causes loss of information. It does not hide or jettison information, but merely prevents the unwarranted addition of information.

The principle of deduction is a truth of logic applicable not only to a fortiori argument but to all deductive arguments. As regards purely a fortiori argument, if we claimed that the difference in the magnitudes of R relative to P and Q implies that the S in the conclusion should be greater than the S in the minor premise (typically, though not necessarily or exactly, *in the same proportion as* P is more R than Q) – we would effectively have *two* subsidiary terms instead of S, viz. S1 (predicated of Q in the minor premise) and S2 (predicated of P in the conclusion). That is, we would have a total of five terms instead of four! Such 'proportional' reasoning is clearly fallacious, as can be shown by reviewing how such purely a fortiori argument (positive subjectal) is actually validated[2].

Many people find it difficult to be reconciled with this strict limitation set by deductive logic. Lenartowicz and Koszteyn are here evidently also recalcitrant, since they characterize it as overly cautious and wasteful. What of course they and others like them fail to realize is that the principle of deduction only insists that the value of the subsidiary term S in the conclusion of such an a fortiori argument (from only two premises) should be *at least* equal to its value in the minor premise; it does not *forbid* an increase in value based on other reasoning processes or on the basis of further observations. That is, there is no logical objection to discovering a change in the value of S *by other means*. All logic insists on is that the a fortiori argument *by itself* cannot effect such a change.

But such a change may well be deduced from other considerations, or be additionally proposed inductively. In the latter case, a hypothesis is formulated that the value of S in relation to P is greater than that in relation to Q, and perhaps (as is often the case) that these two values of S are in the same proportion as the values of R in relation to P and Q respectively. The a fortiori argument may even be viewed as *one* confirmation (among others, hopefully) of the said hypothesis, provided such confirmation is not confused with definitive (i.e. deductive) proof, since it takes us only part of the way towards the desired 'proportional' conclusion. Under such conditions, evidence might well still be adduced that refutes the claimed change in value of S, *without invalidating* the a fortiori inference of S at its initially given value.

It should be added that the principle of deduction applies equally well to a crescendo argument, though in a different way. Since there are three appropriate premises in a crescendo argument, the 'proportional' conclusion can be drawn in full accord with the principle of deduction. What the latter principle would here interdict, on the other hand, would be some fanciful conclusion other than the 'proportional' one that strict logic upholds. For instance, a conclusion that is manifestly *out of* proportion or that concerns a completely different predicate. Any inference beyond the bounds of the given data would be judged illicit.

One reason these authors, and many others like them, are confused in these matters is simply that they are misled by the traditional expression "all the more so" used to signal a "therefore" in a fortiori argument. This expression gives the impression that the conclusion may be quantitatively superior to the minor premise. But it is just hyperbolic. All it really means is that the conclusion is *as sure as* the minor premise, as is true of any deductive argument. We could equally well rhetorically say that the conclusion of an Aristotelian syllogism is "all the more" true given the premises. But we never do, reserving this phrase and others like it to flag a fortiori argument.

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[2]   See in my *Judaic Logic*, chapter 3.2.

Now, as regards the *dayo* principle. No doubt Lenartowicz and Koszteyn express skepticism towards it because they perceive it as of Talmudic origin, and being Christians they do not feel obliged to respect this source. And they are right to do so. This principle was of course not first formulated by me, but was put forward long before by Talmudic rabbis. At first I, and others like me (such as Maccoby), interpreted this principle as equivalent to the purely logical and quite universal principle of deduction, i.e. the idea that there cannot be more information in a deductive conclusion than was tacitly implied or explicitly mentioned in the premises. However, I have in the course of the present study found this interpretation inadequate and proposed a more accurate one.

In truth, the Talmudic *dayo* principle is quite different from the principle of deduction, although the two happen to intersect in some examples. The Talmudic *dayo* principle has a moral and not a logical significance, and is not applicable universally but in the particular context of Jewish law and even then under special conditions. Moreover, though in some cases its application consists in limiting the concluding predicate to the minor premise's predicate, there are other applications of it, notably the prevention of certain generalizations prior to a fortiori argumentation. It would be quite legitimate to claim that this *dayo* principle does not need to be adhered to outside Judaism.

We can apply the *dayo* principle to purely a fortiori argument, by limiting the conclusion to a mirror image of the minor premise (as the principle of deduction teaches anyway); we could also conceivably do so by (for some external reason) interdicting the formation of the major premise by generalization. However, the proper field of action of the *dayo* principle is a crescendo argument. For here it means, on the basis of its presentation in the Mishna (Baba Qama 2:5), either that the pro rata underpinning of the a crescendo argument is interdicted, or that the generalization through which the major premise is constituted is prevented. In either of those cases, the putative 'proportional' conclusion is stopped.

Thus, the resistance to the *dayo* principle can be justified, but not as attempted by Lenartowicz and Koszteyn. Their rejection is wrongful, because it is actually directed at the principle of deduction, i.e. it is an attempt to justify a 'proportional' conclusion from purely a fortiori premises, which is contrary to formal logic. But had they argued that the *dayo* principle in its true sense (as distinct from the principle of deduction) is a specifically Judaic legal principle – that would have been okay.

Additionally, these authors are completely wrong in their reading the *dayo* principle (i.e. the principle of deduction, in the present context) as implying that the initial a fortiori argument has been turned into an *a pari* (i.e. egalitarian) one. Certainly, I have never suggested such a thing, and they are wrong to think it. The said principle certainly does not require that, in purely a fortiori argument, P and Q be *equally R* (though in some cases this may happen). It only states that the subsidiary term *S must be the same* in both minor premise and conclusion (i.e. S is not affected by the difference between P and Q).

3. Epistemic substitution

Finally, Lenartowicz and Koszteyn propose to substitute an epistemic version of a fortiori argument for my more 'ontical' version, stating:

> "The true *a fortiori* argument therefore might be represented by the following scheme:
>
>> P is more evidently R [than] Q is R, and
>> Q is R evidently enough to be S. All the more *(a fortiori)*
>> P is R evidently enough to be S.
>
> In other words, if it is irrational and non-empirical to doubt that Q is S, then it is even more irrational and non-empirical to doubt that P is S."

Now, though this proposed epistemic a fortiori argument may seem reasonable at first sight, it can be severely criticized as follows.

Here, note firstly, the major premise *no longer* informs us that the quantity of R predicated of P is greater than the quantity of R predicated of Q, but instead that the amount of evidence available for the hypothesis that 'P is R' is greater than the amount of evidence available for the hypothesis that 'Q is R'. It may well be, in this perspective, that the quantity of R predicated of P is equal to or even less than the quantity of R predicated of Q, and it may even turn out that P is not at all R and/or that Q is not at all R. In other words, though it looks similar to the original, the major premise is now an entirely different proposition. As a result, *not only has given information been irretrievably lost, but also new information is being claimed without justification*. This is a very serious objection, but this is not the main objection.

The main objection is that the minor premise and conclusion are in fact *meaningless statements*. As I have explained in more detail earlier in the present work[3], it never happens in human knowledge that we say "X is R evidently enough to be S" (where X here stands for Q or P). What would such a statement possibly mean? Presumably, that if the probability that 'X is R' is true is great enough, then 'X is S' becomes true. Note well, 'becomes *true*', and not merely 'becomes as good as true for us, i.e. can be reasonably assumed to be true'. Yet we never in formal logic infer a *definite* truth from a *probable* truth. Such a thing is unheard of and not to be confused with the following sort of apodosis, where given that one thesis implies another we can pass on the probability of the first onto the second:

> If 'X is R', then 'X is S', and
> 'X is R' is probably (to some degree) true,
> therefore: 'X is S' is probably (to the same degree) true.

The conclusion here is, note well, only probable to the same degree. If it so happens that the given implication in the major premise (of this apodosis) is itself only probable then the probability of the conclusion is proportionately less than that of the minor premise. But whatever the case, we never have a probability implying a settled truth. Therefore, the minor premise and conclusion of the epistemic a fortiori argument proposed by these authors are not meaningful propositions. The argument may look like something, but it is not. Really, what the authors were trying to say, which is meaningful, is the following:

> a) There is more evidence that P is R than that Q is R.
> b) Given that 'Q is R' implies 'Q is S'; if there is some evidence that Q is R, then
> there is some evidence that Q is S.
> c) Given that 'P is R' implies 'P is S', if there is some evidence that P is R, then there
> is some evidence that P is S.

But this triad is not a valid a fortiori argument, indeed not even a valid argument! The proposition labeled (a) corresponds to their major premise. The proposition labeled (b) is my interpretation of their minor premise, "Q is R evidently enough to be S;" notice that this is an argument (apodosis) and not merely a proposition. The proposition labeled (c) is again my interpretation of the conclusion postulated by the authors "P is R evidently enough to be S;" this just mirrors my interpretation of the minor premise, note well. To validate the epistemological argument proposed by Lenartowicz and Koszteyn, we must show that the putative conclusion can be inferred from the given premises. My point is that such validation is not possible. Why?

Let us first look at proposition (a): all it tells us is that the amount of evidence in support of 'P is R' is numerically greater than that the amount of evidence in support of 'Q is R'. This does not imply, note well, that these two packets of evidence *overlap* at all; so the evidence in support of 'Q is R' is *useless* as evidence in support of 'P is R' and vice versa. Nevertheless, we can infer from this comparative proposition the two clauses: "there is some evidence in support of 'P is R'" and "there is some evidence in support of 'Q is R'," which clauses we do need in the next two segments. Segment (b) is an argument wherein the proposition "'Q is R' implies 'Q is S'" is the major premise; if this is granted, then using the clause "there is some evidence in support of 'Q is R'" found in proposition (a), we can infer that "there is some evidence in support of 'Q is S'." Segment (c) is likewise an argument wherein the proposition "'P is R' implies 'P is S'" is the major premise; if this is granted, then using the clause "there is some evidence in support of 'P is R'" found in proposition (a), we can infer that "there is some evidence in support of 'P is S'."

We could at most add the final conclusion that: "the amount of evidence in support of 'P is S' is numerically greater than the amount of evidence in support of 'Q is S'." This would serve to justify their statement that "if it is irrational and non-empirical to doubt that Q is S, then it is even more irrational and non-empirical to doubt that P is S." But in any event, the premises originally given by the authors do not suffice to draw their putative conclusion that "P is indeed S;" with those premises alone we do not have a valid argument.

Their proposed argument can be constructed, provided we grant two additional premises (namely, the major premises of segments (b) and (c)). However, if these additional premises are granted, the argument emerging is not a fortiori in form. For, though we do rely on the two clauses implicit in proposition (a), we do not at all make use of the comparison it effects between the respective amounts of evidence for these clauses; the "more evidence" for this element than for that element is redundant, and it plays no role in the inference actually made. Furthermore, segment (b) is totally useless in the process of drawing the conclusion of segment (c), i.e. the desired conclusion. All we need

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[3]   See the discussion of 'Certainty from mere probability' in the section 'Probable inferences' in the chapter 'Apparently Variant Forms' (4.5).

to draw that conclusion is the clause "there is some evidence in support of 'P is R'" implied by (a), plus the additional premise "'P is R' implies 'P is S'" separately granted in (c); clearly, (b) is redundant and plays no role in this inference.

Therefore, the scheme vaunted by Lenartowicz and Koszteyn as "the true *a fortiori* argument" is not an a fortiori argument and not an argument at all. It looks superficially meaningful and valid, but it is neither. In short, this proposal is mistaken and cannot be rationally upheld. Of course, its authors meant well; but they did not carefully think their proposal through. The lesson to draw from this episode is the importance of linguistic precision. It is very easy, when dealing with symbols, to get carried away by appearances and fail to connect to reality. And this is particularly true when logical or epistemological concepts are involved. Even so, there is a valid way to express what the authors were trying to say. They came close to it when they tried to explain their "true scheme," saying:

> "If it is irrational and non-empirical to doubt that Q is S, then
> it is even more irrational and non-empirical to doubt that P is S."

This statement of theirs can indeed be construed as a valid a fortiori argument, although I would have put it more positively as follows:

> Since thesis P is more evident (R) than thesis Q:
> if thesis Q is evident (R) enough to be believed (S),
> then thesis P is evident (R) enough to be believed (S).

Let us now analyze these two proposals, mine first and then theirs. My version is a hybrid-seeming a fortiori argument, involving two theses P and Q, respectively the major and the minor, compared by a logical middle term (R), referring to amounts of evidence, and an epistemic subsidiary term (S), referring to belief; this fits in the standard model and is valid. Now look at their version. What is its major premise? Presumably, it is that given in their "true scheme," viz. "'P is R' is more evident than 'Q is R'." But their minor premise and conclusion (rightly) only mention 'Q is S' and 'P is S'; they say nothing about a term R. So their major premise should really be: "'P is S' is more evident than 'Q is S'." Or, to use the same language as they use: "'P is S' is more 'irrational and non-empirical to doubt' than 'Q is S'." With this modified major premise, their conclusion can indeed be drawn from their minor premise.

In case the reader has been confused by all the Ps and Qs – let me now compare these two versions:

> Since 'P is S' (my 'thesis P') is more 'irrational and non-empirical to doubt' (term R,
> my 'evident') than 'Q is S' (my 'thesis Q'):
> if it is 'irrational and non-empirical to doubt' enough that 'Q is S' *is true* (term S, my
> 'believable'),
> then it is even more 'irrational and non-empirical to doubt' that 'P is S' *is true* (term
> S, my 'believable').

As you can see, the items I called 'thesis P' and 'thesis Q' correspond to the propositions 'P is S' and 'Q is S' (wherein P and Q are terms related to a third term S) of Lenartowicz and Koszteyn. Their middle term is really the epistemic qualification 'irrational and non-empirical to doubt' (corresponding to my 'evident') – this is what they should label R (though they do not label it at all). Their subsidiary term is, I suggest, the logical qualification 'true' (corresponding to my 'believable') – and this is what they should label S (though they have not mentioned it at all). Thus, we see that the authors got confused by the symbols involved, and for this reason produced a fallacious "true scheme."

Finally, let me again address their claim that the resulting a fortiori argument (after applying the various clarifications and corrections just seen) is the "true scheme." What they are saying is that the original ontical a fortiori format is not as interesting or valuable as the more epistemic format they propose. This claim is wrong, because (as already mentioned) in passing over from the ontical to the epistemic format much information has been lost on the way, and other information introduced *ex nihilo* in its stead! And furthermore, far from being the essence of a fortiori reasoning, the epistemic version is not even formally implicit in the ontical one. Consider the following two arguments:

Ontical version | Epistemic version

P is more R than Q is R, and	'P is S' is more evident than 'Q is S', and
Q is R enough to be S;	'Q is S' is evident enough to be believed;
so, P is R enough to be S.	so, 'P is S' is evident enough to be believed.

We see here that, although both these a fortiori arguments result in the same effective conclusion, viz. that 'P is S' – the ontical version has this conclusion as an established fact, whereas the epistemic version is only capable of yielding a probable conclusion, i.e. a less certain one. Moreover, the original (ontical) major premise, with all its information about P being R and Q being R, and the R of P being greater than the R of Q – all that data is absent in the revised (epistemic) version. There is therefore no wisdom in preferring the epistemic version to the ontical one.

Furthermore, given the ontical version does the epistemic version logically follow as these authors seem to think and suggest? Obviously not: the information given in the original version does not justify the revised version's claim that 'P is S' is more evident than 'Q is S'; nor for that matter that 'P is R' is more evident than 'Q is R'. We could say that the premises of the ontical version constitute evidence for its conclusion – but that does not mean that the conclusion is more evident than its premises. It is not with reference to evidence that the conclusion proceeds from the premises in the ontical version, but with reference to the given middle term R. So the proposed substitution of an epistemic format for the ontical a fortiori argument is an illicit process – i.e. it is fallacious.

This concludes my appraisal/criticism of the ideas of Lenartowicz and Koszteyn concerning a fortiori logic. They began wisely by understanding and adopting my standard model (or more precisely, one form of it) – but then they unwisely swerved off into denial of the principle of deduction, as well as misconception of epistemic a fortiori argument and preference for it over the ontical form. My purpose here, as throughout this volume, was of course not to publicize errors but to make sure they are not perpetuated.

25. Abraham, Gabbay and Schild

Michael Abraham (or Avraham), Dov Gabbay and Uri Schild have, in 2009, together written two long papers relating to a fortiori argument, namely: "Analysis of the Talmudic Argumentum A Fortiori Inference Rule (Kal Vachomer) using Matrix Abduction"[1] and "מידות הדרש ההגיוניות כאבני הבסיס להיסקים לא דדוקטיביים"[2] – the latter presented as the Hebrew companion of the English version, covering Talmudic analysis in more detail. I have copies of both them as I write the present review[3], but I will mainly refer to the English one, because my Hebrew is too elementary to deal with the companion piece in a serious manner[4].

1. Their opinion of past work

Naturally, the first thing I looked for in these papers is any mention of my past work on a fortiori argument – not out of personal vanity, but because I know that my findings are crucial to any discussion on this subject. There is no mention of it in the English paper. In the Hebrew paper, they mention it in a footnote, saying: "See also Avi Sion, Judaic Logic, Editions Slatkine, Geneva 1995," after which they state within the main text:

> "These sources offer a logical formalism that translates the inference into formal logical language, but it is not much more than a translation. Therefore it is difficult to see in these attempts a logical or mathematical model of the real" (p. 8)[5].

This very summary and quite unfair treatment suggests to me that they made no effort to actually study my work. On top of that they have the gall to declare sweepingly, without providing any proof, in their English paper:

> "The rule [of Kal-Vachomer] has never been properly formulated, though there have been many attempts" (p. 46).

They do go on to very briefly describe ideas on the subject by Louis Jacobs and Adolf Schwarz. Of the latter they say: "The simple Kal-Vachomer was analysed as an Aristotelian syllogism by A. Schwarz," and they "compare" a famous syllogism (*All men are mortal; Socrates is a man; therefore, Socrates is mortal*) and the a fortiori argument in Exodus 6:12 (*Behold, the children of Israel have not hearkened unto me; how then shall Pharaoh hear me, who am of uncircumcised lips?*), by casting both arguments in the same symbolic straightjacket, thus presumably demonstrating how 'modern' and 'scientific' their approach is. Of course, if anything is "not much more than a translation" it is surely this inane rewriting of ordinary words into esoteric symbols.

In plain English, what they have done is rewrite Moses' above argument as: "For all cases, if ListenIsrael is denied, then ListenPharaoh is denied; in the present case, ListenIsrael is denied, therefore ListenPharaoh is denied." What does ListenIsrael or ListenPharaoh mean? They do not say. Presumably the former expression means "the children of Israel hearken to Moses," and the latter means "Pharaoh does so." But these writers prefer to conceal than reveal, it seems. In any case, what is this argument of theirs? Is it a credible translation of a fortiori argument? Not at all! It is not even a syllogism! It is merely a *modus ponens* apodosis[6]. So this alleged symbolization is mere smoke in our eyes.

The authors do not, however, intend thereby to subscribe to Schwarz's theory, for they add: "Louis Jacobs refutes the similarity" (notice the word "refutes," connoting utter disproof), and then, in an authoritative tone: "Certainly the more complex cases of Kal-Vachomer are not syllogisms at all" (without any demonstrations to that effect). The

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[1]   *Studia Logica* 92(3): 281-364 (2009).

[2]   (Hebrew paper on Kal-Vachomer.) In the journal: *BDD (Bekhol Derakhekha Daehu)*. Ramat Gan: Bar Ilan University, 2009.

[3]   Note: the page numbering I use here refers to the 84 page pdf version I have of the English paper (#338); so my page 1 is really page 281 in the *Studia Logica* publication; so add 280 pages to any page number I use. The Hebrew paper (#340) pdf in my possession is 116 pages long.

[4]   I asked Dov Gabbay for a translation of the Hebrew paper, but he did not provide one; in my opinion, if one wants one's work to be considered internationally, one must provide translations of it into English (today's *lingua franca*). These three authors have also more recently together written in Hebrew a paper called "Logical Model for Talmudic Hermeneutics," which was published in *BDD* – in two parts (the first in No. 23 in 2010, and the second in No. 24 in 2011). This work, among other things, presents "a mathematical model representing… Argumentum a Fortiori (Kal Vachomer)," and studies "the methods of refuting such inferences." I have not read it to date.

[5]   My translation, partly based on a machine translation by Google.

[6]   That is, a hypothetical major premise, a minor premise affirming its antecedent and a conclusion affirming its consequent.

latter remark seems to imply that they consider that some simple a fortiori arguments may well be syllogisms. Regarding Jacobs' approach, which they seem to have more faith in, they tell us only that he "distinguishes two types of Kal-Vachomer. The simple one and the complex one." Their structures are respectively: "If *A* has x then *B* certainly has x;" and "If *A*, which lacks y, has x, then *B* which has y certainly has x."

They give as examples in the Torah of the simple type, the one in Exodus 6:12 just quoted and Deuteronomy 31:27, but as regards the complex type they state categorically (simply passing on an erroneous belief of Jacobs') that: "The Bible does not contain instances of the complex case. This has emerged later, after the Bible." They make no effort to assess Jacobs' theory, not even bothering to "translate" his two forms in symbolic terms as they did for Schwarz. They curiously do not wonder *why* A having x should imply B to have x (in the 'simple' type), or A lacking y but having x should imply B which has y to have x (in the 'complex' type). They apparently do not realize that proposing a formal representation is only half the work of a logician, the other half being to validate that form. If they disagreed with Jacobs, they should have shown his conclusions do not logically follow from his premises.[7]

That's it: Abraham, Gabbay and Schild mention no other theories on the subject other than their own, and their criticism of the theories they do mention is minimal, because stated in very vague terms. I will not bother here to analyze and criticize the work of Schwarz and Jacobs, as I have done so in considerable detail in separate chapters (14 and 16), showing there that a fortiori argument cannot be equated to syllogism and that Jacobs' simple and complex forms (though a bit better than Schwarz's apparent syllogistic proposal) are inaccurate non-sequiturs. My aim here is only to show the paucity and conventionality of the approach of our three authors in relation to past work, on which their above mentioned general negative statements are apparently exclusively based[8].

One thing is evident: they did not get acquainted with my past work on the subject, and therefore missed the opportunity to learn the various forms of a fortiori argument and how they may be logically validated. When they say: "It is difficult to see in these attempts a logical or mathematical model of the real" and "The rule [of Kal-Vachomer] has never been properly formulated, though there have been many attempts" – they are only referring to the work of Schwarz and Jacobs, and clearly not to that of Avi Sion. The latter does in fact offer a "logical model of the real" because it systematically presents, validates and explains the varieties of the argument.

2. Their erroneous basic premise

We shall presently try and analyze the three authors' own contribution, but first I wish to highlight the error of its basic premise. This is immediately apparent in the opening sentence of the Abstract to their English paper:

> "We motivate and introduce a new method of abduction, Matrix Abduction, and apply it to modelling the use of non-deductive inferences in the Talmud such as Analogy and the rule of Argumentum A Fortiori" (p. 1).

The error here, at the outset, is to regard "the rule of Argumentum A Fortiori," i.e. Kal Vachomer, as a "non-deductive inference." This starting point of theirs naturally colors and skews their whole approach. They understandably base this assumption on the failure of Schwarz and Jacobs to convincingly formalize, validate and explain a fortiori argument; but it is merely a hasty generalization from a limited sample.

In truth, as my own analyses show indubitably, *a fortiori argument is a strictly deductive inference*, although – as with all deductive inference, including syllogism – its premises may be open to discussion due to their inductive origins. Also, of course, we must take into consideration that many people reason incorrectly, in arguments which though labeled as "a fortiori" can in truth only be characterized as "*pseudo* a fortiori."[9]

Looking at Talmudic debates or other documents, involving a fortiori arguments, and in some cases fake a fortiori arguments, people are easily driven, into believing that it is not an essentially deductive form of inference. The shifting ground of intuitive insights and objections to them, of opinions and counter-opinions, of facts and conflicting

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[7]   In a perfunctory attempt at wider research, the three authors also give two examples of a fortiori argument in the "New Testament," as well as an example drawn from "Sanskrit logic (Nyaya)," where it is "known as Kaimatya Nyaya (or Kaimutika Nyaya, the even more so) rule." These three examples are okay, but they do not analyze them at all. They also give two examples from Islamic jurisprudence where such argument is "known as Qiyas (analogy)." However, while the second example they give is indeed akin to the Jewish technique of "Kal-Vachomer," the first is not so. Although the original author of the example (it is repeated in many websites, anonymously) correctly analyzes the thinking in it, our three authors do not explicitly identify it as akin to the Jewish technique of "Binyan Abh." This could be an oversight; but I suspect they lump these two kinds of reasoning by "analogy" together because they cannot tell the difference between them.

[8]   All this concerns the English paper – there may be more detailed analyses on these and other past authors in the Hebrew paper. But I doubt it, seeing the level of knowledge and understanding they display here.

[9]   As regards "Analogy," we discover later that what the authors mean by that is Binyan Abh (the third hermeneutic rule in R. Ishmael's list), which does not directly concern us in the present inquiry. Even so, it is worth noting in passing that even *binyan av* can be shown to be essentially deductive, with reference to the logic of causation.

facts, give the overall impression that this kind of reasoning is largely inductive. But this is a misleading impression, as closer analysis using formal tools shows.

Keep in mind that the *process* of inference is the relationship between given premises and putative conclusion. If *given* the premises the putative conclusion *necessarily follows*, then the process is indeed deductive, whether the premises given are true or false. If given the premises two or more alternative conclusions are logically conceivable, then each of these conclusions is by itself a deductive *non-sequitur*, and the selection of one of these conclusions in preference to the other(s) – for a host of other reasons, whatever they be – is indeed an inductive act.

To claim as do our three authors that a fortiori argument is "non-deductive inference" is just to admit being unaware of the formal validation processes for such arguments. If one looks at the work of Schwarz or Jacobs, one will of course not find formal validation of a fortiori argument. But if one takes the time to study my *Judaic Logic*, not to mention the present work, one will indeed find such formal validation. So the basic premise of Abraham, Gabbay and Schild is mistaken.

For them, then, a fortiori argument is inductive, and this motivates them to look for a way to find appropriate conclusions to such arguments. Having no formal understanding of the argument, they must instead look approximate solutions to a fortiori type problems. The way they propose to do this is "matrix abduction." In their view, "the Talmud is trying to abduce" (p. 9) a conclusion from certain premises. Further on, they explain:

> "The Kal-Vachomer rule (and the algorithms supporting it) are non-monotonic rules of induction. This means they are not absolute deductive rules but defeasible common sense rules. So we may use these rules to obtain a conclusion A, but further information and further arguments may force us to doubt A or even come to accept ~A [i.e. non-A]." (P. 12.)

In other words, the conclusions drawn from a fortiori arguments are always open to doubt, though one might at a given stage seem more credible to "common sense" than another. "Matrix abduction," the method they propose to use, visualizes the problem at hand as a matrix (a table with multiple rows and columns) in which most but not all the cells are filled in with given yes or no information (marked "1" or "0" respectively), and the solution as some mechanical (as against purely 'intuitive') way to predict the content of the empty cell(s), i.e. those for which yes or no information is not given (marked with a "?"). The result will be, effectively, a 'best guess' even if it is not absolutely sure.

Now, I have nothing against abduction (or adduction[10]) or the use of matrices (matricial analysis). I have used them for many years, and written much about them, and consider them essential to logic research[11]. *Nor*, note well, do I doubt the general value of the particular "matrix abduction" method proposed by our three authors, or (offhand) the accuracy of the complex diagrams and calculations it involves. What I *do* doubt, right from the start, is that this really has anything to do with a fortiori argument as such!

But we should anyway question the wisdom of resorting to the method of tabulation to resolve the complex issues posed by a fortiori argument, or even mere argument by analogy. Such argument is always based on some generalizations, and it is very difficult to represent this fact in tables. A table is a simplified picture of events, not to say a simplistic representation of them. The names of the rows and columns seem clear and definite, but this is only a superficial impression, concealing underlying complexities. To say that a cell (the intersection of a row and a column) has either a value "1" or "0," or eventually "?" is accurate only when dealing with *individual events*, which are either present, absent or uncertain. When dealing with *general propositions*, however, a fourth value must be introduced.

If the row label signifies a subject (or antecedent) and the column label a predicate (or consequent), then the conjunction of these two terms (or theses) may be true 'in all cases' (symbol 1), 'in no cases' (symbol 0) or 'in some cases yes and in others no' (say symbol *), or uncertainly (symbol ?). There are here clearly four and not just three alternatives: always yes, always no, sometimes yes and sometimes no, and unsure which of the preceding three is applicable. To put it another way, you cannot represent the family of propositions 'all A are B' (or 'if A, then B'), 'no A is B' (or 'if A, then not B'), 'some A are B and some are not B' (or 'if A, not-then B and not-then not-B') and

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[10]   I would like to express in passing my antipathy to the word "abduction." This word was, I gather, rather recently (early 20th century) introduced, by C. S. Pierce, and it commonly used by many modern logicians. I personally prefer the older (15th century) word "adduction." For me, abduction connotes kidnapping, whereas adduction connotes adding. We *adduce* information (data, reasons) in support of a hypothesis, we don't *abduct* it! If the English language has a perfectly adequate word for something, why introduce a new one? The motive is, I suggest, a sort of snobbery – one seems more intellectual if one uses esoteric language.

[11]   See especially my book *The Logic of Causation*. A crucial difference between my approach there and the approach of Abraham, Gabbay and Schild in the paper here examined, is that early on in my research (in phase II, 2003) I eschew the unknown "?" value and focus entirely on known "1" and "0" values through grand matrices that analyze all possible outcomes of various situations. This allows for systematic development of deductive arguments, and makes all the difference in the clarity and certainty of the results. They have apparently not realized the superior efficacy of this approach.

'unsure' by means of only three symbols. And things get still more complicated when natural modalities are taken into consideration!

Our authors do not realize this and cheerfully plunge into their analysis armed with only three alternatives. This is bound to lead them into error (and indeed, as we shall see, errors do arise), though the errors may remain well hidden below the surface of their discourse. Moreover, much depends on how accurately one selects the concepts to be put into rows and columns. As we shall see, it is not always easy to design an appropriate table. If one does not correctly perceive the data at hand, one may misconstrue the rows and columns needed for them. The truth of the matter is that, for all its usefulness, the tabular method of representation of facts is simplistic and awkward. It should only be used with full awareness of the dangers of misrepresentation it involves, treading carefully.

Moreover, I would like to draw attention to the overall complexity and consequent limited practical utility of the "matrix abduction" method. Humans have been reasoning a fortiori for millennia without any reference to the intricate techniques that Abraham, Gabbay and Schild here prescribe or even merely propose. The authors' approach, even if theoretically applicable and potentially usable with computers (which is open to doubt, as we shall see), has nothing to do with actual human practice. They do not reflect ordinary human ways of thinking a fortiori and therefore cannot effect improvements in these ways. The science of logic should not be considered as a purely theoretical exploration for purposes of computer programming, let alone as a game or a display of virtuosity; it is primarily aimed specifically at improving the art of logic, i.e. at helping real people to think straight.

The authors claim that "Matrix abduction is a new form of induction, arising from the Talmud, which can solve problems currently in the scientific community" (p. 60). As just mentioned, I do not regard the method they propose as entirely new or essentially related to *qal vachomer* in particular or, I might add, Talmudic reasoning in general. They may have gotten the idea while studying the Talmud, but I very much doubt that it can be said to have arisen from it, in the sense that the Talmudic rabbis used or discussed such a method. Tabulation of empirical data as a means of predicting missing information has been in use since the dawn of modern science. This is often accompanied by graphical representations, where a line is drawn in between all the points marked on the graph. Such a line is used to predict values otherwise not known, the assumption being that the variables compared are subject to concomitant variation.

Regarding the authors' statement that "rabbinic thinking is very similar to that of western science," as Menachem Fisch claimed (1997) and contrary to what Jacob Neusner claimed (1987), I would to some extent but not entirely agree. However, I would not go so far as to say as they do that "the logic of the Talmud is far richer and complex than currently available western logic." They do not say just what these "epistemological comments" of theirs are based on; it is more a hopeful credo of theirs than a demonstrated thesis. The truth is, Talmudic logic and Western logic have some things in common, and each has things to teach the other and things to learn from the other (see my detailed remarks in the conclusions[12] of my *Judaic Logic* on this issue, which they do not mention).

It is not my wish to dismiss offhand or belittle the work of Abraham, Gabbay and Schild, but only to assess its relevance to the issue at hand, i.e. a fortiori argument. If they have not discovered and understood *what* a fortiori argument is and *how* it works, i.e. its form(s) and validity – how can they possibly claim to find appropriate conclusions to premises? Think about that. Is what they do in their paper really pertinent to the issue of a fortiori argument, or are they just demonstrating impressive but unrelated skills? We shall have to answer this question with reference to the concrete cases they actually deal with.

Abraham, Gabbay and Schild deal, in their English paper (I do not here take into account their Hebrew paper, to repeat), with only two Talmudic examples. One, Baba Qama 25a-b, very briefly (on pp. 8-10); and another, viz. *Kiddushin* 5a-b, in some detail (on pp. 48-59). Let us now take a look at their treatment of these texts.

3. Some errors of logic

With regard to Baba Qama 25a-b, I have written much already in previous chapters (7-8) and will here assume the reader has already read them. Our authors present this as a "small example" of Talmudic *qal vachomer*, though they forgot to state the Talmudic pages concerned. As a "preview of the model," i.e. of their "matrix abduction" approach, Abraham, Gabbay and Schild represent the problem as a matrix with four cells, of which three have known content and one has unknown content, as follows[13]:

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[12]   Chapter 15, "Epilogue."

[13]   Note (as the authors acknowledge in a footnote) that here the alternative to 0, i.e. 1 in the sense of ≥ 0, has two values, viz. 1 (full compensation) and ½ (half compensation). Note (though the authors do not do so) that no value in between or beyond these two is admitted under Jewish law – thus ¼, ¾ or 2, say, are inadmissible.

	Public place	Private place
Foot action	0	1
Horn action	½	x = ?

They conceive the problem as one which can be approached from two viewpoints (intuitively or by maths, as they put it), each in two different directions (along rows or columns). According to them, it proceeds as follows.

- From the "intuitive" point of view. If we read the matrix horizontally, given x is to ½ as 1 is to 0, then x is more than or equal to ½ ("pay at least half"). If we read the matrix vertically, given x is to 1 as ½ is to 0, then x is equal to 1 ("pay in full").
- From the "maths" point of view. Horizontally, the answer must either be x = 1 or x ≤ ½. If we assume the latter (i.e. ≤ ½), the two columns are "incomparable." Whereas, if we assume the former (i.e. 1), we obtain an "ordering" result, which is "nicer." Vertically, the answer must either be x = 0 or x ≥ ½. If we assume the former (i.e. 0), the two columns "are not comparable." Whereas, if we assume the latter (i.e. ≥ ½), we obtain an "ordering" result, which is again "nicer." (The concept of niceness here used is intuitive – but to be defined more rigorously later, the authors promise us.)

Thus, the solution of the problem, by our three authors, whether by intuition or maths[14], is that x = ≥ ½ or 1 (or vice versa) – which means, more briefly put, that x = ≥ ½. However, they note without further comment: "The actual case is decided in Jewish law as x = ½." It is surely amazing that the discrepancy between their own expectation and the preferred result in the Talmud does not arouse any reconsideration on their part! Their theory is put in doubt, but they just blithely move on. This is anyway a very superficial treatment of the case…

Note, for a start, with reference to the Mishna, that the proposed horizontal reading corresponds to R. Tarfon's first argument and the proposed vertical reading corresponds to R. Tarfon's second argument. However, R. Tarfon concludes with a "1" in both cases, and the Sages conclude with a "½" in both cases. Neither party concludes anywhere with x = ≥ ½ as do our authors overall, and neither party concludes with x = ≥ ½ or 1 as in the intuitive point of view, or with x = 1 or ≥ ½ as in the maths point of view. Thus, the authors' conception of this debate does not correspond to, or precisely explain, the participants' conceptions, let alone arrive at the same results.

What their overall conclusion means, knowing that only three alternative penalties are allowed in the Talmudic conception (no fractions other than half, and no amount greater than full compensation), is that x ≠ 0 and x = ½ or 1. In one respect they agree with the rabbinical conclusions – in denying that x = 0. But they are unable to decide whether x = 1 as R. Tarfon claims, or x = ½ as the Sages claim. This suggests (assuming all three methods viable) that their method is less precise than theirs – it yields a vaguer result. Their solution x = ≥ ½ simply means "either R. Tarfon or the Sages may be right." In other words, they have found no way to decide the issue treated in the Mishna! But let us look at their reasoning again, more critically.

As regards the 'intuitive' approach, their vertical conclusion that "x equals 1" is credible, since just as ½ is more than 0, so x must be more than 1, and as 1 is the maximum allowed then x must be equal to 1. But their horizontal conclusion that "x is greater than or equal to ½" is unjustified, for just as 1 is more than 0, so x must be more than ½, i.e. x must equal 1, as R. Tarfon claimed. As regards the 'maths' approach, their horizontal conclusion that "x equals 1" is credible, because both "x equals 0" and "x equals ½" would be lopsided. But their vertical conclusion that "x is greater than or equal to ½" is again unwarranted, because here again both "x equals 0" and "x equals ½" would be asymmetrical; so the conclusion can only be "x equals 1" as R. Tarfon claimed.

From this review, it becomes clear that our authors *reasoned incorrectly twice* out of four. Judging *by their own standard* of "comparable ordering," their answer every which way should have been uniformly x = 1, like the answer of R. Tarfon. This result implies that their thinking was essentially argument by analogy (i.e. merely pro rata), rather than a fortiori (of whatever sort). Had they truly reasoned a fortiori, they would have at least been able to offer a formal explanation for the Sages' *dayo* objection in relation to R. Tarfon's first argument (their horizontal reading). They would also had been intrigued by the Sages' *dayo* objection in relation to R. Tarfon's second argument (their vertical reading), albeit the latter's conformity with their first objection.

Moreover, if we look again at their 'intuitive' and 'maths' methods, we see that when they are both correctly applied (as distinct from how they applied them), they yield exactly the same conclusion that x = 1, both horizontally and vertically. The reason for that is simply that they are one and the same technique! In their minds, they looked different; but in reality there is no difference between them. Both are simply pro rata argument, i.e. thinking based on

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[14]   They use the label "maths" to suggest a more scientific approach – but in truth, this has nothing to do with mathematics or mathematical logic. It is just another sort of "intuitive" approach, a bit more reasoned since it consists in trying the different possible solutions one by one and seeing if the "ordering" is "comparable."

ratios. The 'maths' method looks more sophisticated because it appeals explicitly to disjunctive reasoning; but in fact the 'intuitive' method is implicitly the same. So, the reference to two methods is just evidence of confusion; and it sows confusion. The authors got different solutions from them because they subconsciously wanted to. They had foregone conclusions in mind that they sought to justify. They were not entirely objective.

I would say Abraham, Gabbay and Schild made the two mistakes they made out of a desire to keep the option of x = ½ in the game, so as to somewhat resemble and not exclude the Sages' position. Had they not done so, their posture would have been evidently for all to see only representative of R. Tarfon's and therefore insufficient to explain the debate. In the 'intuitive' horizontal case, they erred by suggesting that the x = ½ option has a comparable relation to ½ as 1 has to 0. In the 'maths' vertical case, they erred by lumping the x = ½ option with the 1 option in the expression ≥ ½, whereas if they had treated it as a separate alternative they would have seen that it too can be eliminated. There was evidently, therefore, *a subconscious attempt at manipulation of premises to obtain a desired final result* (viz. "x is greater than or equal to ½"). But, fellows, you must be more careful, because there's always someone out there who will spot such maneuvers!

Moreover, as I show elsewhere in detail, there are various ways to view the Mishna debate between R. Tarfon and the unnamed Sages confronting him. As regards the first exchange, we can say that R. Tarfon's argument is either analogical (i.e. pro rata), where A is to B as C is to D, or a crescendo (i.e. a fortiori with a pro rata additional premise, and therefore a 'proportional' conclusion). Here, the Sages' *dayo* objection acts as an arbitrarily imposed (i.e. extra-logically, for moral reasons, albeit by rabbinical inference from the Torah), limitation on 'proportional' reasoning. This means that, if R. Tarfon's argument was a crescendo, the Sages confronted it with a purely a fortiori argument, and it explains why the former concludes with a "1" whereas the latter conclude with a "½."

With regard to R. Tarfon's second argument, whether we construe it as analogical (pro rata), a crescendo (a fortiori cum pro rata) *or purely a fortiori*, the conclusion is the same. So the Sages' new *dayo* objection, though worded like their preceding objection, must be interpreted differently (since the previous explanation is inapplicable). The only available explanation is that it is a limitation on the generalization preceding this second argument (whether taken as analogical or a fortiori). Here, then, the Sages are not confronting R. Tarfon with an alternative argument, which generates a conclusion of its own, but merely inhibiting him from inductively forming a major premise and thus drawing the conclusion he sought. Again, this explains their different "conclusions."

What is noteworthy, to repeat, is that the conception and results of Abraham, Gabbay and Schild do not square with any of these scenarios. As we have just demonstrated, their method of reasoning is essentially analogical (pro rata) throughout, and the reason why they do not get the same results as the traditional analogies is simply that they made errors of calculation. They have not found any other way to bring the Sages' opinion into the discussion than through such error. Their approach has in fact nothing to do with a fortiori argument, whereas only through understanding a fortiori argument (including the difference between the a crescendo and purely a fortiori forms) can the difference of opinion between the Mishna participants be convincingly explained (as just done above).

The authors make no mention of the distinction between mere analogy (pro rata) and a fortiori argument, or (more importantly) between a crescendo and purely a fortiori argument, which implies they are not even aware of these distinctions. They assume from the start, without reflection, the universal validity of 'proportionality'. Their method can only produce 'proportional' inferences. This means that it can only at best assimilate merely pro rata and a crescendo (a fortiori cum pro rata) arguments. There is no room in their logical universe for purely a fortiori argument; they have not intellectually assimilated this concept.

Also, they nowhere mention or discuss the Sages' *dayo* principle, and why and how it ought to apply in the present arguments. The authors do confront this issue in their Hebrew paper, but as far as I can tell their explanation is that the rabbis prefer the lowest value out of caution. This somewhat answers the why but not the how: they do not perceive where exactly the *dayo* principle operates. They cannot do so, really, because they lack the distinction between mere analogy and a fortiori argument, and between a crescendo and purely a fortiori argument. As well, they show no awareness of the generalizations through which the major premises of both of R. Tarfon's arguments are made possible, but take the results of such induction for granted. Yet such awareness is necessary to fully understand the Sages' *dayo* principle. This failure of awareness is due in part to their resort to tabular representation. This method is pretty well bound to lead to oversimplification and rigidity.

To top it all, the authors remain unfazed by the divergence between their vague final conclusion (viz. x = ≥ ½) and the two precise ones proposed in the Mishna (viz. x =1 for R. Tarfon and x = ½ for the Sages). They apparently imagine that, since their conclusion (which is anyhow, as we saw, in fact fabricated) logically includes, by way of disjunction, both of the Mishna conclusions, they have succeeded in proposing an interpretation of the Mishna. But in fact, all their indefinite conclusion shows (if anything) is that one of these finite conclusions must be correct; but it is incapable of sorting out which of the two is right. So it is as if nothing was said at all!

Thus, it appears from their treatment here (in the essay reviewed) that the claim by Abraham, Gabbay and Schild that their "matrix abduction" approach has something to do with the a fortiori thinking apparently used in the Mishna under scrutiny is not correct. A fortiori argument is a sophisticated *deductive* process with specific formal rules, yielding exact and reliable results. If the argument is well formed, its conclusion follows necessarily and entirely from the given premises. It is not as the authors assume an *inductive*, merely analogical process producing a tentative, educated guess, like their proposed "matrix abduction" method. Their method, however valuable in itself, is not relevant to a fortiori reasoning, since the latter is can be formally validated.

Their calling their essay "Analysis of the Talmudic Argumentum A Fortiori Inference Rule (Kal Vachomer) using Matrix Abduction" is therefore misleading advertising.

4. Mixing apples and oranges

Abraham, Gabbay and Schild claim that the a fortiori argument used in *Kiddushin* 5a-b is "one of the most complicated uses of this rule." Certainly, they here deal with only a part of the whole discussion, which they have subdivided into 16 or 17 "steps." The opening move in their selection is the following a fortiori argument:[15]

> "Rav Huna said: *Huppa* acquires a fortiori, since money, which does not allow one to eat *teruma*[16] does acquire, *Huppa* which allows one to eat *teruma*[17], how much more should it acquire."

Huppa refers to the wedding canopy, signifying the Jewish marriage ceremony. "Acquisition" here refers to *kiddushin*, the betrothal of a woman, i.e. the legal status of being engaged to be married[18]. "Money" here refers to a gift of value, such as a ring. *Teruma* refers to a food offering to priests, and a woman being allowed to eat it signifies that she is indeed married (to a Kohen). This passage would be rewritten by me in standard a fortiori format as follows:

> Huppa (P) allows teruma eating (R) more than money (Q),
> and, money (Q) allows teruma eating (R = 0) enough to acquire (S);
> therefore, huppa (P) allows teruma eating (R > 0) enough to acquire (S).

In the case of money (the minor term, Q), the allowance of eating teruma is nil (R = 0), whereas for huppa (the major term, P), the allowance exists (R > 0). Even so, note well, the middle term R, "allows teruma eating" (i.e. signifying, more broadly, being married, to repeat) can be expressed as ranging from zero up (i.e. '0' is effectively a positive value of R in this range). On this basis, we can construct our major premise, and justify the conclusion from the minor premise, as shown. The argument is thus positive subjectal, since it goes from minor to major. Our three authors, on the other hand, conceive the argument in tabular form, as follows[19]:

Original table	Marriage *(nissuin)*	Engagement *(kiddushin)*
Money	0	1
Huppa	1	x = ?

We can, of course, guess that the column regarding "marriage" is tacitly inferred from the original middle term "allowance of teruma eating" (money does not suffice to allow teruma eating, whereas huppa does) and the column for "engagement" refers to the original subsidiary term "acquisition" (money does acquire, and the question is whether huppa also does so), though the authors do not explicitly tell us all that. It seems, then, that the row labels (money and huppa) refer to *causes* and the column labels (marriage, engagement) refer to *effects*. Thus, the table tells us that money produces engagement but does not produce marriage, whereas huppa produces marriage, and it leaves open the question as to whether huppa results in engagement or not.

They explain that this argument (based on R. Huna's) is designed "to prove that the ceremony itself can do the job of the ring, i.e. it can do the engagement," and after drawing two "graphs" for x= 0 and x = 1, they consider that the

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[15]   This translation is stated by the authors to be "from the El-Am edition."
[16]   The Soncino edition explains in a footnote: "If a priest betroths an Israelite's daughter with money, she may not eat terumah until the huppah." This is found further on within the text, in R. Ulla's rejoinder.
[17]   See preceding footnote, where it says: "until the huppah."
[18]   I take acquisition to mean *kiddushin* (betrothal), following Tosafot to Yevamot 57b. Other authoritative commentaries, notably Tosafot HaRosh, however consider that R. Huna may have additionally intended *nisuin* (the final stage of Jewish marriage). See Note 35 in the Art Scrolls Talmud edition, p. 5a.
[19]   This is their Figure 46.

latter is "better" and "more connected," so their conclusion is that huppa causes engagement. As can be seen, their thinking is based entirely on considerations of symmetry. It is an argument by analogy – but the distinctive a fortiori underpinning of it has been completely hidden away or lost.

In passing, I would propose to them, instead, the following *modified* table. This would in my opinion be a more accurate rendition of the operative a fortiori argument. If we symbolize the four items involved, viz. huppa, money, permission to eat teruma (i.e. married status), and engagement status, by means of the four standard terms, P, Q, R and S, respectively, we get the following representative table:

Modified table	Enough of middle term, R	Presence of subsidiary term, S
Minor term, Q	1	1
Major term, P	1	1

Table 24.1

The logic of it is that since Q has enough R (even though 0) to imply S present, then P has more than enough R (since more than 0) to imply S present. Although this tabular schema is an improvement over the one proposed by our authors, it too is imperfect in that the major premise, which tells us that P has more R than Q, is still left tacit. It has to be artificially signaled by calling Q the minor term and P the major term, but it cannot be expressed within the matrix itself. Consequently, using this representation, we cannot see exactly how the conclusion is derived from the premises.[20]

Note that my first column has two entries '1' (meaning: 'yes, enough of R' for S), whereas the authors have a '0' and a '1' (meaning: 'not married' and 'married'). My second column has two entries '1' (meaning: 'yes, S present' due to R enough), whereas the authors have a '0' and a '?' (meaning: 'not engaged' and 'engagement status unknown'). What all this signifies is merely a difference in perspective: 'not married' is indeed a 'no' with regard to marriage (signified by allowance of teruma eating), but I call it a 'yes' since it is still 'sufficient to effect' engagement (acquisition). As for my having a '1' where they have a '?' – this is again not a disagreement between us, but only due to my saying the conclusion follows necessarily, whereas they consider it uncertain and needing to be "abduced."

Before reviewing and criticizing how our authors assimilate the Talmudic discussion that follows R. Huna's opening *qal vachomer*, I shall now briefly sketch the way I perceive that discussion, based on the very clear translation and commentary of the Art Scroll edition of this tractate. Whereas our authors present the discussion as a debate between a "proponent" (R. Huna) and an "opponent" (unnamed), the Art Scroll edition presents it as a pro and con reflection of "the Gemara," and I will do the same (finding this approach more realistic). However, I will number each "step" of the discussion as our authors have done, so as to maintain a reference to their work. But I will go through only about half the steps, just enough to prove the main point I will be making. If you do not want to read all the details of this exposé, you may skip it (the indented segment), and just proceed to the summary and conclusions further down.

> The opening a fortiori argument by R. Huna, already described above, is labeled step 1a. In step 1b, "the Gemara calls into question the premise of this *kal vachomer*" by objecting: "And is it really true that the giving of money for *kiddushin* does not enable a woman to eat *teruma*?"[21] This objection, note well, is clearly not itself a *qal vachomer*, but merely intends to put in doubt a premise of R. Huna's argument. He claimed that although money does not allow the woman to eat teruma, it still effects acquisition. This objection suggests that maybe the woman is allowed to eat, and goes on to argue that this is so according to Biblical law, although Rabbinic law forbids her to do so. In step 2, the Gemara "proposes a new explanation of this *kal vachomer*," which I would write as follows:
>
> > Huppa (P) completes marriage (R) more than money (Q),
> > and, money (Q) completes marriage (R = 0) enough to acquire (S);
> > therefore, huppa (P) completes marriage (R > 0) enough to acquire (S).
>
> This is actually a new *qal vachomer*, resembling the first in all respects except that the middle term has changed from "allowance of teruma eating" (a special indicative of marriage) to "completes marriage" (by

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[20]   That would be my proposed table for positive subjectal argument. For positive predicatal, the rows would be transposed, with that for P above that for Q. For the corresponding negative arguments, the first column would contain zeros (meaning: 'not enough of R' for S) and the second column would contain question marks (meaning: 'possibly S, possibly not S'). But in all these cases, the explanation for the a fortiori conclusion remains hidden, note well. Tabulation is thus not a fully revealing way to present such argument.

[21]   Note that when I here quote passages from the Art Scroll edition, I omit the bold type that is used to distinguish literal wording from the translator's explanatory interpolations.

effecting nissuin). From here on, this is rather the *qal vachomer* that seems to be referred to. In step 3, the Gemara then turns around and "suggests a refutation of this" second argument, by pointing out that "money is unique in that it can be used to redeem consecrated items [*hekdeshot*] and second tithe [*maaser sheni*]," whereas huppa cannot so redeem. This objection does not propose a third *qal vachomer*, note well, but only suggests something which puts the major premise of the preceding argument in doubt, so that the conclusion can no longer be confidently drawn from the minor premise.

In step 4, the Gemara responds to the latter objection with the remark that "cohabitation effects *kiddushin*" so as to "demonstrate that the ability to effect *kiddushin* is not limited to those methods of acquisition which can also redeem" – i.e. though cohabitation cannot redeem, it can still effect *kiddushin*; from which it follows that huppa's "inability to redeem should therefore not impede its capacity to effect *kiddushin*." In other words, the previous attempted refutation of the *qal vachomer* is denied, which allows the second argument to be maintained, at least temporarily.

In step 5, the Gemara then counters the preceding resolution with the following objection: cohabitation is "unique in that it acquires the *yevamah*" (this refers to a wife by levirate marriage) – whence, "it does not follow from the fact that cohabitation effects *kiddushin* that *chuppah* should do so as well." What this counter objection does is annul or weaken the previous objection, so that the second argument is again put in doubt. In step 6, the Gemara responds by arguing: "The fact that money effects *kiddushin* will demonstrate that the ability to effect *kiddushin* is not limited to those methods of acquisition which can also acquire a *yevamah*." What this does is remove the previously indicated doubt, i.e. it reaffirms the major premise by allowing that kiddushin might be effected otherwise than by money or cohabitation.

In step 7, the Gemara sums up that "neither money nor cohabitation carries the unique 'strength' of the other," so that (as stated in the Notes) "neither of the two methods can serve alone as a source that *chuppah* effects *kiddushin*, since each displays a 'strength' that *chuppah* does not share." The purpose of this comment is suggest that "the ability to effect *kiddushin* is not tied to either of their unique strengths" – i.e. that there is some unidentified underlying factor, in common to them, that makes them capable of effecting *kiddushin*. This common ground between them and with huppa is identified in the next step.

In step 8, the Gemara adds: "However, the characteristic common to both of them is that each acquires elsewhere, i.e. each has a capacity to effect a transaction other than *kiddushin* – and each acquires here, in the case of *kiddushin*. So too, by means of analogy, I will include *chuppah* in the class of ways a woman can be acquired, for it acquires elsewhere, and therefore should acquire here, i.e. should effect marriage without the need for prior *kiddushin*." In the Notes, this is further explained as: since "*chuppah* effects *nissuin*," which gives the husband certain rights, "it possesses the capacity to effect a transaction" and thus "should be able to effect *kiddushin*."

And so it goes, on and on; but I will stop here having accumulated enough data to make my point. My point is simply this: as you can see, most of the listed "steps" of the argument are not a fortiori arguments. The first step (1a) is a fortiori reasoning. The next (1b) simply intends to weaken a premise of that argument. Step 2 proposes a rival *qal vachomer*, which goes in the same direction but with a more reliable middle term. Step 3 raises a doubt concerning the major premise of this new argument. Step 4 attacks the preceding objection by pointing out an exception to it. Step 5 puts the preceding objection in doubt, which again weakens the proposed argument. Step 6 dissolves the preceding doubt, which again strengthens the (new) a fortiori argument. Steps 7 and 8 effectively abandon the attempt at a fortiori argument, and resort instead to an argument by analogy that I would rather classify as *binyan av* (a kind of causative argument). Subsequent steps (9-16) present objections and counter-objections to the latter argument, until the analogy is appropriately refined.

What is clear from this sequence of events is that it is totally misleading to refer to the totality of this discourse as "*qal vachomer*" or "a fortiori argument" as Abraham, Gabbay and Schild do. It involves one or two genuinely *qal vachomer* statements; but most of the other statements are attempts to either put in doubt one of the premises of a *qal vachomer* (and thus put the argument in doubt), or to object to the objection (and thus recover some credibility for the *qal vachomer*). Furthermore, some steps are neither *qal vachomer* themselves nor have nothing to do with any *qal vachomer*! Obviously, just because the opening argument in the series is an a fortiori argument, it does not follow that the whole series can be treated under the heading of a fortiori argument.

Yet our authors proceed precisely in this wrong manner, cheerfully lumping together apples and oranges in the same *qal vachomer* basket. Thus, for instance, the way they integrate step 3 into their tabular schema is simply by adding a third column (signifying redemption, *pidion*) to it, as follows[22]:

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[22]   This is their Figure 48.

	Marriage	Engagement	Redemption
Money	0	0	1
Huppa	1	x = ?	0

Since money redeems but huppa does not, whereas huppa effects marriage but money does not, it is now impossible on the basis of symmetry to decide whether x should be 1, as previously supposed, or 0, instead. In fact, as we have seen, the purpose of the Gemara's statement about the distinctive redemptive power of money is to weaken the major premise of the proposed a fortiori argument; but since our authors' tabular schema does not make possible a representation of the major premise, they are forced to express this objection by means of an additional column[23]. This expedient gives the impression that the a fortiori argument has been amplified, although in fact it has been dissolved.

After that, the authors continue representing the ongoing Talmudic discussion by means of additional tables. The third table is like the first (with 4 cells) except that its first row label is changed; the fourth table is like the second (with 6 cells) except that its first row and third column labels have changed; the fifth table has 12 cells, its 3 rows and 4 columns being all those used in the four preceding tables. These first five tables cover steps 1-8[24]. Two more tables, with respectively 15 and 24 cells, cover the remaining steps[25]. As the tables get bigger, their corresponding graphs of course get more complicated.

I will not here repeat their whole presentation in detail, nor make any effort to verify their specific interpretations or results, since (as will presently be made clear) there is no need to do so. Finally, the authors get the answer they were hoping for, which apparently matches that given in the Talmud (viz. x = 1). Whereupon, they conclude:

> "We built a matrix abduction model whose components and concepts use only topologically meaningful notions (and hence the model is culturally independent) and we used it to analyse an involved Talmudic argument and we got a perfect and meaningful fit."

However, if we observe the actual flux of these authors' matrix analysis of the *Kidushin* 5a-b discussion, building up representative tables by changing, adding or subtracting variables, i.e. rows and columns – it is clear that what is involved here cannot by any stretch of the imagination be conceived as a mechanical process. It is a process quite dependent on human agency, a process of ad hoc tailoring of overlays to match given data. The accuracy of this process depends primarily on the intelligence of the observer and interpreter of the data. It is not, to repeat, a mechanical process of inference, one that could be fed into a computer and proceed by itself. Therefore, one can well suspect that the authors obtained a final conclusion that perfectly fits (or so they claim) the Talmudic one, because they tailored their discourse to that precise end.

But my main question is, here again, what has all this to do with a fortiori argument? Is it essential, incidental or accidental to the issue at hand? The answer should be clear: purely a fortiori argument formally involves, and can only involve, four terms (the major, the minor, the middle and the subsidiary); a crescendo argument may be said to involve five terms, insofar as the subsidiary term has a different value in the minor premise and in the conclusion. Thus, *a fortiori argument can never involve more terms* than that, contrary to what the authors suggest when they draw tables with multiple columns and rows.

Therefore, they have missed the essence of a fortiori argument. This does not mean their tables are wrong; it only means that such tables are not representative of a fortiori argument *per se*. A fortiori argument can apparently (only very roughly, as already explained) be tabulated in the way they have done it; but—this changes the argument from a deductive to an inductive operation. The underlying logical power of a fortiori is lost, and a more diluted form of reasoning (which is mere analogy) is used in its stead. Note this well.

What is manifestly lacking in their account here, notice, is a credible general analysis of the Talmudic processes of objection (*pirka* or *teshuvah*) to *qal vachomer* argument. I have proposed a detailed analysis of this discursive phenomenon in the chapter on Moses Mielziner (13). The latter described such objections as "refutation of inferences," which may be achieved by attacking a premise or the conclusion. But as I showed there, even Mielziner's understanding of the process was inaccurate because too vague.

If premises A and B are considered as logically implying conclusion C, 'refutation of the inference' would consist in saying that A and B do not together imply C; but this is very rarely what is intended in Talmudic objections[26]. Usually, either premise A or premise B is denied somewhat (or even both A and B are denied, though that is rarer),

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[23]   The same expedient is used some years earlier by Gabriel Abitbol. I show in my critique of the latter's work why it is not very smart. As I explain there, rather than expand the initial table by adding a column (or a row), it would be wiser to illustrate the objection raised to the initial table by constructing a rival table. But in any case, tables cannot fully illustrate the evolving rabbinic discourse.

[24]   The latter three being referenced as Figures 50, 52 and 54.

[25]   These are referenced as Figures 56 and 58.

[26]   But it happens. A case in point is R. Yehoshua's objection to R. Eliezer's a fortiori argument in the Mishna Zebahim 7:4.

by some means (e.g. some other Biblical passage is brought to bear on the case at hand), whence it follows that conclusion C can no longer be inferred (which does not mean that other ways might not be found to maintain it, though it could also be denied); this is more prosaically, from a formal point of view, 'refutation of a premise'.

When a rabbi proposes a *qal vachomer* and another opposes it, the latter is usually not positing a *counter* a fortiori argument of his own, but merely putting the proponent's argument in doubt in some way. Usually, this is achieved by denying the truth of the major premise, or at least its general applicability, i.e. its applicability in the particular case at hand; alternatively (or as well), the proponent's minor premise might conceivably be attacked. Very often, too, the conclusion is apparently challenged; but closer examination reveals that this is not, as might be thought, a rejection of the proponent's reasoning process, but simply an explicit denial of his conclusion that implies a tacit denial of one of his premises. In very few cases is the reasoning process as such rejected as fallacious.

If we look at the series of arguments in *Kidushin* 5a-b that our authors have tabulated, we see that they do not constitute a chain (sorites) of a fortiori arguments and counter-arguments. Rather, what we have before us is an initial a fortiori argument or two; followed by a series of objections and counter-objections which apparently put a premise of an a fortiori argument in doubt to some extent, or recover its credibility to some extent; and then an argument which is not a fortiori at all but really *binyan av*; and so forth. This is, to repeat, not a chain of competing a fortiori arguments, but a trial and error process of individual reflection (or a give and take process of debate) through which a convincing conclusion is gradually sought and presumably found, which takes into consideration all data at hand in a consistent manner. This process does involve at least one a fortiori argument, but it is certainly not limited to that one form of reasoning.

Therefore, the presentation by Abraham, Gabbay and Schild is mostly beside the point. It is best characterized as tinkering. It may well (as they claim, but I have not checked it) succeed at generating the same final conclusion as the Talmud (at least in the example given), but that does not demonstrate that they understand the nature of a fortiori argument as such. This is evident, given the vagueness of their observations as to what is going on in these Talmudic discourses – i.e. the fact that they do not in their tabular schema reflect the significant differences between actual a fortiori arguments and objections thereto which are not themselves a fortiori arguments, or even arguments which are not at all related to a fortiori. They treat all elements of the dispute indiscriminately as "a fortiori," simply because the chain of reasoning that they selected started with an a fortiori argument. This is not a "meaningful" model.

5. Quid pro quo

Abraham, Gabbay and Schild also go through some non-Talmudic examples – but what is evident there again is that they have nothing or little to do with a fortiori argument *specifically*. Their applications are just attempts to fill in cells with missing data by extrapolation from information available in other cells of the same table. Quite possibly, identical results would be obtained if the sample arguments involved were mere analogies instead of a fortiori. To label something as "a fortiori" is not proof that it is in fact so; the qualification refers to a distinctive movement of thought that has to first of all be correctly perceived and understood.

We can, therefore, conclude by saying that, albeit their valiant effort, the three authors have not succeeded in (to use their own phrase) "properly formulating" this form of argument. As we have seen with reference to their analysis of the Baba Qama example, their approach is essentially analogical and therefore inductive. We also found that they wrongly processed even these simple arguments by analogy, so that their conclusions were anyway incorrect and for that reason in disagreement with the traditional ones for such form of reasoning. Possibly the same sort of unconscious manipulation and error might be found in their treatment of the *Kiddushin* example, though I did not take the trouble to look for it. I would certainly recommend to them to give their account a long and hard second look.

Moreover, their "matrix abduction" method (though no doubt valuable in itself as an aid to induction) does not truly represent a fortiori argument, because it is based on a false premise, a false stimulus; namely, the supposition that such argument is constitutionally not deductively certain and therefore must needs be dealt with by inductive means. And the reason they came to adopt that false premise became apparent in their treatment of the *Kiddushin* example, when they made no formal distinction between a fortiori argument and mere objections thereto or other forms of argument, and expanded the proposed matrix beyond the maximum allowable of four cells, making it a sort of *pot pourri* of disparate reasoning processes.

Moreover, it should be noted that the two Talmudic examples we have above considered both refer, at least in appearance (though not in reality), to positive subjectal a fortiori argument. The authors show no awareness of the positive predicatal form, and none of the negative forms of the argument. Presumably, these could be fitted in to their schema; but it was up to the authors to demonstrate it. Logicians must always aim for universality, and formally

consider all variations on a given theme. I can say from hands-on experience that such further research is often very helpful in deepening one's understanding of the initially identified form and avoiding mistakes.

Note, too, that the authors nowhere formally locate the various components of a fortiori argument (the terms and relations) in their tables – I had to do it for them (see above, my 'modified table' (24.1) with the row and column titles identified as P, Q, R and S). This again shows a lack of formalism on their part, which is unacceptable in scholarly discourse by logicians. Such informality is no doubt the root cause of their mistakenly opting for an inductive solution when deduction was in fact possible.

I should also here repeat that the science of logic itself is not a purely theoretical construct divorced from human practice. The work of logicians is not merely speculative or inventive, but requires *empirical observation of actual human thinking*. We first observe how human reason actually functions in different discursive situations; then we try to identify the precise conditions of validity of such thought processes. We do not approach the matter academically and impose artificial thought forms on the mind, especially when the mind already has natural ways and means. The "matrix abduction" method proposed by our authors is of course not in itself artificial – but it is artificial in relation to a fortiori reasoning.

I am tempted here to get heavy and quote Isaac Newton on the scientific method:

> "For the best and safest method of philosophizing seems to be, first to enquire diligently into the properties of things, and to establish these properties by experiment, and then to proceed more slowly to hypotheses for the explanation of them. For hypotheses should be employed only in explaining the properties of things, but not assumed in determining them, unless so far as they may furnish experiments."[27]

Newton was of course referring to physical science, not to the science of logic. But the methodological principles are approximately the same for all sciences, i.e. all consciously rigorous pursuits of knowledge. Many logicians nowadays tend to rush to judgment, and seek to develop tabular schemes, graphical models, symbolic formulas, and what have you, before having taken the trouble to carefully observe and examine the concrete data at hand. They aim their intellectual arrows in the approximate direction of the target, hoping to hit the bull's eye by luck – but that won't do. The form must exactly fit the content, no less and no more.

Abraham, Gabbay and Schild will of course retort that they used the scientific method, insofar as they tested their "matrix abduction" hypothesis on two actual Talmudic examples. Having in the more complex of the two examples obtained "a perfect and meaningful fit," they felt justified in claiming an "achievement." But the truth is, as they themselves admit, they did not get the same answer as the Talmud in their first, simpler example[28]. So that is a success rate of only one out of two, at best!

Moreover, I presume that they got the same answers as the Talmud in the second example because they say so – but not having myself analyzed their every step in detail, I cannot independently confirm that this is indeed the case. I stopped considering it necessary for me to follow our authors' interpretations of the Talmudic arguments and counter-arguments, remember, the moment I realized that they were lumping together one or two a fortiori arguments with objections and counter-objections thereto, as well as other types of argument, so that effectively they regarded as "a fortiori argument" a portion of discourse capable of containing more than four (or five) terms.

But anyway, even granting that the authors followed the second Talmudic debate correctly, the fact remains that they did not manage to get a likewise positive result for the first debate. As I show above, they actually made errors of calculation. Okay – anyone can make an error. Let us declare their errors are now corrected, so that their tabular approach now produces credible results matching R. Tarfon's. This would still be an inexact representation of the Mishna debate, since the Sages' part in it would remain unexplained. Moreover, even if both these examples had been correctly represented by the "matrix abduction" model, it would not follow that other applications of the proposed model would automatically be correct, because this model is far from mechanical, depending greatly on human interference.

And indeed, even generously assuming that both examples, and a few others besides, may be considered as successfully assimilated by their hypothesis, does it follow that their hypothesis is established? I think not. We cannot generalize from two instances, *especially not when the 'empirical data' at hand is itself tenuous*. For the fact is, as I have shown in my *Judaic Logic* and elsewhere in the present work, the rabbis do occasionally (if not often) engage in fallacious reasoning, including fallacious a fortiori argument[29]. I am sorry to say it, but it is demonstrably true. Therefore, Talmudic discourse is not strictly speaking 'empirical data' that can be safely relied on for confirmation of such a hypothesis.

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[27]   Quoted by Freely, p. 217.
[28]   Maybe they explain the discrepancy in their Hebrew paper, I don't know – but certainly they nowhere do so in their English paper.
[29]   See for examples my analysis of the *qal vachomer* arguments in *Pesachim* 23a-b and *Baba Batra* 111a-b, in the chapter on Adin Steinsaltz (18.2).

This being the case, what will our authors do with their "matrix abduction" hypothesis when its conclusions perchance do not match those of the Talmud? They cannot declare the Talmud is wrong, for their method is inductive and therefore constitutionally not capable of judging deductive validity or invalidity. Would they then declare their hypothesis wrong? But if the Talmud can be wrong, their hypothesis might still be right! So nothing is settled. Clearly, the authors cannot appeal to conformity with one or two Talmudic results as a gauge of the accuracy of their hypothesis. Such conformity proves nothing much, except by circular argument. If the "matrix abduction" method perchance obtained results that matched some Talmudic arguments that happen to be erroneous, it would wrongly be judged thereby to be scientifically confirmed on the false assumption that the Talmud's arguments are always reliable.

Finally, let us reiterate that the proposed method rests on mere analogy, and not as its proponents claim on a fortiori argument. It follows from such general methodological considerations that the "matrix abduction" method, whatever its merits as a general inductive technique, is in any case not particularly relevant to the solving the theoretical problem of a fortiori argument. What is needed is a deductive method of formalization and validation and invalidation, and I have already successfully developed that method in my *Judaic Logic* and further still in the present volume. Unfortunately for them, Abraham, Gabbay and Schild did not study my first work on the subject, though they knew of its existence.

26. Stefan Goltzberg

1. Source of his definition

Stefan Goltzberg apparently started writing quite recently, showing from the beginning considerable interest in a fortiori argument. We shall here review his essay in English "The A Fortiori Argument In The Talmud" (14p.), published in 2010[1]. He has written at least two more papers, in French, which also treat of a fortiori argument, though not exclusively[2]. However, they do not bring much more to the subject than the English one, so most of our attention here will be on the English one. All quotations come from the latter unless otherwise specified.

Goltzberg proposes the following, if only "provisional," definition of the argument:

> "The *a fortiori* argument… structure is that if p applies in case A, and since B is more x than A, then p applies *at least as much* in B. p is any category; A and B are situations; and x is the scalar feature of a situation by which a category applies."

Notice that he does not limit the argument to legal contexts. Notice, too, that he places in his discourse the minor premise before the major premise and conclusion; this is intentional on his part, because[3] he regards these as three "stages" of argumentation. This definition seems essentially correct, though as we shall see it is not all-embracing. Let us to begin with put it in standard format, to better understand it:

> B (P) is more x (R) than A (Q),
> and, A (Q) is x (R) enough to be p (S);
> therefore, B (P) is x (R) enough to be p (S).

We see that what Goltzberg has in mind is positive subjectal argument. His vague description of the subsidiary term p as "any category" that "applies" in "situations" A and B (the minor and major terms, respectively), and of the middle term x as "the scalar feature of a situation [i.e. A or B] by which a category [i.e. p] applies," may be viewed (generously) as perhaps an attempt on his part to expand his definition to include positive antecedental argument. But in any case, formally speaking, Goltzberg has here put his finger only on *a minori ad majus* argument, with no reference to *a majori ad minus* argument, although the latter is commonly known. More specifically, he mentions neither negative subjectal argument, nor positive or negative predicatal arguments, nor their implicational equivalents.

Additionally, though he does mention a middle term (x), his exposition lacks mention of the indicator of sufficiency (i.e. A or B is x *enough* to be p); yet, this indicator is necessary in order to signify that there is a threshold value of x as of which p is applicable. Like many other commentators, Goltzberg has not understood that without this crucial factor in the minor premise, the desired conclusion cannot be derived from it. Given that "B is more x than A," and merely that "p applies in case A," it does *not* logically follow that "p applies in B." In fact, Goltzberg's given premises are equally compatible with the contradictory of the desired conclusion, i.e. with "p does not apply in B." It is only with the minor premise "A is *x enough to be* p" that we can logically guarantee the conclusion "B is (x enough to be) p."

So, as regards the structure of a fortiori argument, Goltzberg's viewpoint is inappropriately narrow and sorely deficient. He could have remedied some of these imperfections by referring to the definition proposed (perhaps with other authors) by P. A. Lalande, in his *Vocabulaire technique et critique de la philosophie* (1926)[4], which I cited in my *Judaic Logic*:

> "Inference from one quantity to another quantity of similar nature, larger or smaller, and such that the first cannot be reached or passed without the second being [reached or passed] also" (my translation).

Here, we see that both directions of inference are mentioned, and moreover the idea of a threshold is emphasized. However, this definition too is imperfect because, as I remarked there, it "fails to specify that the positive movement from large to small is predicatal, while that from small to large is subjectal; and it ignores negative moods altogether, as well as differences between copulative and implicational forms."

Nevertheless, Goltzberg's definition is technically passable as a first attempt. But the question arises, *where did he get it*? Let us engage in a bit of 'higher criticism' and find out. Is he claiming it as his own discovery? One might think so, seeing how breezily he put it forward it, almost in passing, as if it is obvious. Or perhaps he regarded it as so commonly known that it does not require a reference.

At first glance, this definition reminded me of the one given by the *Encyclopedie Philosophique Universelle*[5], which I also cited in my *Judaic Logic*:

> "*A fortiori* argument rests on the following schema: x is y, whereas relatively to the issue at hand z is more than x, therefore a fortiori z is y" (my translation).

But of course, this definition was incomplete in that it did not specify a symbol for the "issue at hand" (i.e. the middle term). Perhaps Goltzberg read that and substituted the symbols A, B and p for x, z and y, respectively, and thought to add a fourth symbol (his x). Alternatively, he might have based his definition on the work of Moshe Chaim Luzzatto called *Sepher haHigayon* (1741), which was published in English translation under the title *The Book of Logic* in 1995. Four forms of a fortiori argument are there defined, including the positive subjectal: "Quantified commensurates [are terms that] share a common quality, but not in the same degree. One exhibits a Greater degree and the other a Lesser degree of the same quality…. Whatever is affirmed about the lesser will surely be affirmed about the greater"[6]. But there is no evidence he has read that book, either.

In any case, he does not refer to these possible sources of his definition, or any other. He does however say in a footnote to his definition: "Avi Sion puts forward that 'Aristotelian syllogism deals with attributes of various kinds, without effective reference to their measures or degrees' (Sion, 48)." So it is reasonable to suppose he got his definition from the presentation in my *Judaic Logic*, which (certainly some 15 years before him) clearly formulates a fortiori argument as comprising four terms (or theses) in three propositions (two premises and a conclusion). The questions arise: Why does he not say it? Why does he so carefully change the symbols used? Why did he only mention the positive subjectal form and no other? Why did he not mention the threshold involved? I suspect there was a conscious or subconscious desire to conceal my contribution, so as not to be overshadowed by it.

I resent such treatment. Such behavior is contrary to the ethics of academic work (and, for Jews, contrary to rabbinic teaching, see *Pirqe Avot*, 6:3,6). One should always try to acknowledge the sources of one's ideas. Even if one thinks one arrived at an idea independently: one may say so, but one must still admit that some other(s) arrived at it before – for the simple reason that one can never be sure not to have been indirectly and unconsciously influenced. One is duty bound, too, to pass on information correctly, and as fully as appropriate to the context. All the more so, one must not misquote someone, or quote him out of context in such a way as to make him appear to say something he did not say, or (worse still) the opposite of what he said. How can logical and other studies progress if people studiously avoid relaying new findings?[7]

2. Soundness of the argument

Goltzberg also needs to be reproved for his failure to mention, not only my formalization of the various moods of a fortiori argument, but also my original formal *validation* of those moods. How can something so important to the

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[4]   Paris: PUF, 1972.
[5]   Paris: PUF, 1990.
[6]   Pp. 89-90. Note that the two premises and conclusion, and the four terms in them, are all present and accounted for, although not symbolized.
[7]   In the same spirit, I believe, Goltzberg asserts: "Ten a fortiori arguments are to be found in the Tanakh" – without mentioning that I uncovered and listed an additional twenty or so cases! Though he does mention "the forgotten a fortiori arguments," with a reference to Moshe Koppel's *Meta-Halakha. Logic, Intuition And The Unfolding Of Jewish Law* (Northvale, NJ, Jason Aronson, 1987). Perhaps, then, someone preceded me in this matter. I have not seen what is actually said there, and so cannot judge. People should make the effort to more precisely relay information they refer to.

subject of study have been simply ignored? Obviously, he did not grasp its significance. He does, nevertheless, to his credit, on various occasions, acknowledge the logical force of the argument, saying, for instance:

> "I think that *... a fortiori ...* is … unrefutable (*sic*) … iff one accepts the premises and the hierarchy between the terms."**8**

That is to say, as I understand him, the argument is formally valid in itself, although of course (as in all argumentation) the truth of the conclusion depends on that of the two premises. It is not mere rhetoric, but logic. So well and good; we seem to be in agreement on this point. But this is not, as he seems to "think," a mere matter of opinion or authority – it can be formally proved! And Goltzberg fails to mention the important fact that I have indeed *demonstrated* its validity. For this reason, I get the impression that he merely glanced through my *Judaic Logic*, gleaning from it snippets of information without bothering to actually study the text as a whole. This impression is reinforced by the following passage:

> "Avi Sion has devoted many pages to *a fortiori* arguments within the Bible and the Talmud. At the end of a chapter on formalities of *a fortiori* arguments, Sion writes: "I did not *prove* the various irregular *a fortiori* to be invalid, but rather *did not find any proof* that they are valid" (Sion, 1997: 46). Sion claims that an *a fortiori* argument's validity, if not rebutted, is not yet demonstrated either. We do not claim to provide the reader with such a logical proof – Sion is right. The *a fortiori* argument is not only a logical but also linguistic device. This is why a logical approach to the *a fortiori* argument is insufficient to grasp its linguistic specificity."

Notice how, in his response to the statement of mine that he cites, Goltzberg drops out the word "irregular." I find this remark of his very irritating, both because of its unsubstantiated, pretentious claim that logic is insufficient in this matter, and because of its condescending, narcissistic tone. When I chanced upon it (in Aug. 2010), I wrote to its author the following complaint:

> "You have here taken what is effectively an endnote (to chapter 3.3 of my *Judaic Logic*), concerning 'additional details' within a fortiori logic – namely the status of certain arguments that resemble the a fortiori, but are doubtful (labeled 'irregular') – and you have made it seem as if I subscribe to your idea that the (regular) forms of a fortiori argument cannot be formally validated! You are careful not to mention the four (or eight) valid moods of a fortiori that I have originally identified (in chapter 3.1) and the validation procedures that I describe in detail (in chapter 3.2), though these are the crux of the matter. …"

It is bad enough to conceal relevant information. But in that statement Goltzberg depicts me as saying *the exact opposite* to what I intended in my book, apparently to make it look as if I effectively endorse his allegedly more discerning view. Forgive my harsh words, but this is clearly intellectual dishonesty on his part**9**. Let me emphasize: I firmly believe that the principal moods of a fortiori argument are demonstrable; there are no ifs and buts about that. This is not a mere opinion, but is the product of long and hard research work. Of course, as with all argumentation, one has to verify the truth of the premises and ensure the process is correctly carried out; but granting these precautions, if the mood concerned is formally validated, the conclusion is not subject to discussion. As for the irregular moods I mention in the said book, I have more recently developed the technical means to deal with them (i.e. formally validate or invalidate them) – namely, matricial analysis**10** – so the uncertainty Goltzberg relied on in his remark is in principle no longer present.

Let us move on. Goltzberg also mentions the work of Aristotle, saying:

> "As far as we can see, Aristotle does not explicitly look into the so-called *a fortiori* argument. He mentions the *topos*, '*He who can do more can do less*' in the books of the *Topics* (II, 10). …He presents the *a fortiori* argument as a *topos* among others, in other words, as a defeatable argument."

Actually, I have not found the maxim "He who can do more can do less" anywhere in Aristotle's *Organon*, but I do recall seeing it mentioned by Ventura in the name of Lalande**11**. Maybe it comes from some other Greek or Roman

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8   This said in a private e-mail to me dated 20/9/07, when he first made contact with me after discovering my book, *Judaic Logic*. He says about the same in French in his 2008 paper: "Les arguments indéfaisables [lit. which cannot be undone] incluent également l'argument *a fortiori*;" adding in a footnote: "Il va de soi que les arguments *a fortiori* ne sont contraignants que sous la clause *ceteris paribus* [i.e. all other things being equal]." And further on: "Pour être exact, un argument *a fortiori* est certes toujours défaisable, mais contrairement aux arguments *a pari* et *a contrario* qui peuvent être évincés sans autre forme de procès, la réfutation de l'argument *a fortiori* suppose que l'on rejette non seulement la conclusion mais toute l'échelle argumentative qui le sous-tend." I have not found a similar statement in his 2010 English essay, however.

9   When I said as much to him, Goltzberg replied: "I am not dishonest. I surely am not. It is always possible to misunderstand a text, but it is not necessarily the case when one quotes you. I did not want to distort your claim, which I've quoted. I did not even criticize your position and I do not behave 'as if' you said something else. I am not responsible for the hypothetical interpretations beyond what is said in my text." Readers may judge for themselves.

10   See my work *The Logic of Causation*, Phase III.

11   It was on p. 76 of *Terminologie Logique*, to be exact. (Paris: Librairie Philosophique, 1935.)

writer[12]. This said in passing – it is not very important. A more significant question is: is it accurate to say that Aristotle "does not explicitly look into *a fortiori* argument"? No – he describes the argument, or at least some aspects of it, pretty well albeit briefly in his *Rhetoric* (II, 23) and *Topics* (II, 10 and III, 6). It would, however, be true to say that, unfortunately, Aristotle does not analyze this form of argument in a systematic manner comparable to his study of syllogism: he does not formalize it, nor identify all its moods and figures, nor make any attempt at its validation.

A second important question is: does Aristotle consider a fortiori argument as "defeatable"? I would answer, emphatically: no. I quote him: "Another line of proof is the *a fortiori*. Thus it may be argued that if even the gods are not omniscient, certainly human beings are not." It is clear from this excerpt and the rest of his discussion that he (intuitively) regards the argument as deductively strong and not open to rebuttal. Note that he calls it a "line of proof" and says "certainly" in the conclusion. If he had considered the argument as inherently weak, he would surely have said so, and given at least one example of counterargument. Goltzberg's error here is to generalize too readily, imagining that just because many of the arguments described in Aristotle's *Topics* and *Rhetoric* are rhetorical in character it follows that none of them are logically forceful.

It should be said here that Goltzberg is a fan or disciple of Chaim Perelman[13]. I have not personally studied Perelman's work, but I gather from third party summaries that it is very interesting and influential. It is possible that Goltzberg's reading of Aristotle was influenced by Perelman. According to Goltzberg, the latter's position on a fortiori is as follows:

> "Among modern types of topical theories of argumentation, the new rhetoric of Chaïm Perelman deals with the *a fortiori* argument and considers it as a sort of analogy argument (Perelman 1977: 155) and stresses the fact that the *a fortiori* argument is not part of formal logic since there are laws that limit the use of *a fortiori* arguments (Perelman 1976: par. 33). Perelman, in this manner, renews the topical theory of argumentation. According to him, there is no undefeatable argument, not even the a fortiori argument. The *a fortiori* is then not set apart from the other types of arguments."[14]

Needless to say, I firmly disagree with Perelman's apparent exclusion of a fortiori argument from formal logic and view of such argument as "defeatable." That it is a sort of analogy is obvious – but it is a very sophisticated sort, which is particularly reliable because it is based on precise quantitative and implicational relations. That the fact that "there are laws that limit the use of"[15] such argument should make it "not part of formal logic" is a very mysterious notion; all of formal logic can be viewed as a study of the laws limiting the use of all types of arguments! It is only through such study that we can determine with certainty which arguments are valid logic and which are mere rhetoric. It would seem that Perelman did not understand the nature of logic. But anyhow, my sense is that, though Goltzberg admires Perelman, he does not entirely agree with him as regards a fortiori argument.

Incidentally, Gabriel Abitbol also mentions Perelman – with disapproval. Whereas Perelman's thesis is that legal dialectics "constitutes an argumentation or a set of proofs tending to persuade, if not convince, as to the legitimacy of the premises," Abitbol's view is that "Talmudic dialectic" is never about "the basis of premises," since in its case these "are axioms written in the Torah" (pp. 336-7). In other words, according to Abitbol, Perelman's analysis of legal debate, though it may reflect somewhat the discourse involved in practice in secular contexts, does not apply to Talmudic discourse. I would add that logic (inductive as well as deductive) is certainly preferred to mere rhetoric in all legal systems, including the secular and the Talmudic; the use of mere rhetoric is always only a last resort, when logical means, which more persuasive, are not found.[16]

3. The dayo principle

One feature of Goltzberg's above-cited definition that I have not mentioned so far is his inclusion of the qualification "at least as much" in the conclusion. As we shall see, this interpolation is crucial to his "theory of a fortiori," so it is no accident.

However, even though this qualification is in many cases tacitly intended, so that it is legitimate to make it explicit, it is an error to include it in the definition, for the simple reason that it is very often *not* applicable. Very often, the

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[12]   It has a Latin equivalent: *Qui magis potest minus potest*. This may be of Scholastic origin. But the maxim also seems proverbial in many other languages.
[13]   Poland, 1912 – Belgium, 1984.
[14]   The references are to: *Logique juridique. Nouvelle rhétorique* (Paris: Dalloz, 1976) and *L'empire rhétorique: Rhétorique et Argumentation* (Paris: Vrin, 1977).
[15]   Presumably this refers to the *dayo* principle (see next section).
[16]   Moreover, I would criticize Abitbol's position and say that even if Talmudists do not dispute the basic premises given by the Torah, treating them as axiomatic, it remains true that they are objectively open to dispute – and indeed have become hotly disputed in modern times. Moreover, even within the framework of the Torah as an uncontestable given, Talmudists do not always argue logically, but sometimes resort to mere rhetoric.

subsidiary term allows of no measures or degrees (if only because it is already the highest possible measure or degree, so that to say "at least" is a redundancy), so that what is said of it in the minor premise is bound to remain the same in the conclusion. For examples, something 'imperative' or 'black' cannot be more or less so – either it is so or it is not[17]. In such cases, to repeat, it is inappropriate to use the qualification "at least."

Goltzberg's emphasis on the expression "at least" is, of course, motivated by the desire to integrate the rabbinical 'dayo' (sufficiency) principle into the definition of a fortiori argument once and for all. This is commendable, but as already explained inaccurate and misleading. *Prima facie*, the *dayo* principle corresponds to the logical rule that we cannot bring more information into the conclusion than was given in the premises – which I have lately labeled 'the principle of deduction'. This rule is universal: it is not peculiar to a fortiori argument, but equally applicable to syllogism and all other forms of deduction. Of course, inductive reasoning is not subject to the same restriction. Induction is precisely the effort to extrapolate from given information and predict things not deductively implied in it.

Goltzberg apparently interprets the *dayo* principle in this manner when he says (all italics his, square brackets mine):

> "…this instruction aims at insisting on the fact that the second situation [i.e. the conclusion] deserves the judgment applied to the first situation [i.e. the minor premise], in a degree that is at least as great *but not greater*. …*Dayo*, as a claim that the second situation be treated *precisely as* the former, is not *added* to the a *fortiori* argument. It is simply inherent in it. The merit of the *Talmud* is not to have added this device but to have made it clear that one should respect the principle of the *dayo*."

Note well: according to him, *dayo* is not an artificially "added" limitation, but one naturally "inherent" in a fortiori argument. He denies the alternative interpretation, according to which:

> "The function of the *dayo* clause is [i.e. would, according to such alternative interpretation, be] the following: it prevents someone from applying a higher rate/price/praise/blame to a situation that obviously deserves it at least as much as the former *and probably more*, as one would want to continue the proposition. The Talmud would have added the *dayo* device and transformed thereby the very structure and use of a fortiori arguments."

Thus, Goltzberg is well aware that some people view the natural conclusion from an a fortiori argument to be 'proportional' and view the *dayo* principle as an artificial prohibition. But he disagrees in principle with them, though he adds in a footnote:

> "Several persons to whom I said this brought examples from the Talmud in which, according to them, some opinion stressed the fact that there was an a fortiori argument but the *dayo* was refused. In fact, this issue deserves a closer scrutiny. It is possible to make it clearer by the distinction between *de re* ('in fact') and *de dicto* ('supposedly'): someone may be said to claim (*de dicto*) that there is an *a fortiori* without *dayo*, but no one could possibly think that there is *de re* an *a fortiori* without *dayo*. This point merits wider examination."

Here we see that Goltzberg is not personally acquainted with the discussion in Baba Qama, 25a, but has relied on hearsay concerning it[18]. It is correct to say that there are instances in the Talmud where a fortiori argument is not subjected to *dayo*. But Goltzberg's attempt to explain this away by saying that perhaps those who think so confuse supposition with fact is not correct. I am afraid that I may be partly responsible for Goltzberg's misapprehensions in this matter, because of my emphasis in my *Judaic Logic* on the *dayo* principle as a formal limitation on a fortiori inference. In truth, I did there admit that the strict conclusion (in accord with *dayo*) could be surpassed by means of *additional* deductive or inductive argument. But I had not at the time yet developed precise formal conditions for this; I had not yet developed the concept of a crescendo argument.

I know now, after "closer scrutiny" of Talmudic a fortiori argument earlier in the present volume, that the *dayo* principle functions differently. Looking at the Mishna of Baba Qama 25a, whereas the *prima facie* idea that the *dayo* principle refers to the principle of deduction is consistent with the first argument of R. Tarfon, it is clearly not consistent with his second. This implies that the *dayo* principle is something else entirely, which, though in some cases it intersects with the principle of deduction, in other cases (as Tosafot remarked) it operates at a different level.

We could interpret both of R. Tarfon's arguments as mere analogical ones (pro rata, to be exact), and the two *dayo* objections by the Sages as ad hoc interdictions of proportionality. But this has nothing to do with a fortiori logic, with which these arguments are traditionally associated; so let us ignore it here. If we interpret the first argument by R. Tarfon as a crescendo, then the Sages' first *dayo* objection may be conceived as aimed at interdicting the additional premise in it that justifies 'proportionality', and therefore as advocating a purely a fortiori argument.

This scenario could be interpreted as an application of what I have lately called the principle of deduction, i.e. the rule of deductive logic that the conclusion cannot make claims that the premises cannot sustain. However, upon reflection it is perhaps not wise to here effectively identify the *dayo* principle with the principle of deduction, for the

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[17]   Goltzberg ignores this when he contrasts, for example, "forbidden" and "even more forbidden"!
[18]   As I did when I wrote my *Judaic Logic*, I must admit.

latter principle is applicable not only to purely a fortiori argument but also to a crescendo argument! If in the latter form of argument we advocate a conclusion that contains information not found (explicitly or implicitly) in the premises, then here too we would reject the putative conclusion as illegal (i.e. contrary to the principle of deduction). In any case, the *dayo* principle cannot be limited to its role in the first Mishna debate, of ensuring that the conclusion be quantitatively as well as qualitatively identical to the minor premise. It cannot, because in the second argument of R. Tarfon the conclusion is the same whether that argument is construed as a crescendo or purely a fortiori. In other words, here the first *dayo* objection by the Sages is already obeyed. So their second *dayo* objection must mean something else: it must (since there is no other explanation for it) be aimed at the generalization preceding the formation of the major premise of R. Tarfon's a fortiori argument. So, there are at least two types of *dayo* objection.

What is clear from this analysis of the *dayo* principle is that it is not a logical principle. It is not to be confused with the principle of deduction. It is a rabbinical decree, apparently based by the Gemara on a particular reading of a Torah passage (namely Num. 12:14-15). It is a legal posture, apparently aimed at preventing punishments based on inference to be in excess of those found in the source-text. It is a morally motivated principle, to temper justice with mercy. Maybe also an epistemological principle, to avoid errors of judgment based on errors of inference. This being so, the idea found in the Gemara that the *dayo* principle may in some circumstances be bypassed is not all that surprising.

All this is said to answer Goltzberg's uncertainties and doubts. We could relate all this to his definition of a fortiori argument as follows. We could say that the qualification "at least" found in it serves to include both purely a fortiori argument and a crescendo argument in a broad definition. When the minimum quantity is not surpassed, we have pure a fortiori argument; when it is surpassed, we have a crescendo argument. This would be a neat way to merge the two forms of a fortiori argument into a single, generic form. However, we would still need in such a broad definition to mention the additional premise that makes possible a pro rata conclusion in certain cases.

Goltzberg's definition as it stands does not specify under what conditions the minimum quantity may logically be surpassed; it would need to be modified appropriately to do so. This would result in a statement like: 'Given B is more x than A: if p applies in case A, then p applies at least as much in B; if it is additionally known that p is proportional to x, then p applies pro rata more to B than to A; otherwise, if such proportionality is not known to apply – or if it is known not to apply (e.g. through a *dayo* objection) – then p applies exactly as much to B as to A'.

4. His "two-dimensional" theory

All that we have presented and discussed so far seems to be, in Goltzberg's essays on the subject, merely introductory or incidental to his central thesis, which he calls grandiloquently the "two-dimensional theory of a fortiori." According to this insight:

> "Arguments are structured by two main parameters: orientation and strength. The four types of arguments may be analyzed through the following transitional keywords examples. Keywords may be co-oriented or counter-oriented and stronger or weaker."

	Weaker	Stronger
Co-orientation	At least	Or even
Counter-oriented	Even if	Unless

Goltzberg observes that in statements like "p or at least q" or "p or even q," thesis q is "co-oriented" with p, because it points in the same direction, differing perhaps only in measure or degree. Whereas in statements like "p even if q" or "p unless q," thesis q is "counter-oriented" to p, because it points in the opposite direction. That is to say, in the former cases, q is "an agreeing argument" with p; whereas in the latter q is a "counterargument" to p. Moreover, as regards "the strength parameter": in "p or at least q" or "p even if q," q being weaker strengthens p; in "p or even q" or "p unless q," q being stronger weakens p. He gives simple illustrations, like "He can run 10 miles or at least 5 miles."

The fact that Goltzberg repeats this theory in all his essays on the subject makes me call it his pet theory, and assume that it is his own for that very reason. He seems fascinated with its symmetry and versatility, and with the facts that theses (or clauses of theses or terms) may point in opposite directions as regards their meaning and that one may reinforce or undermine another. He calls this "two-dimensionality" to imply that both these sets of factors must be taken into consideration if we are to fully comprehend the relationship between the theses concerned. However, he does not precisely define or explain these various concepts, but only uses them intuitively.

Before further ado, let me say some things about the concept of "at least," which for his part Goltzberg hardly says anything about, though he uses and relies on the expression quite a bit. Underlying a fortiori argument is the known

mathematical field of comparisons of quantity. As I have shown in the context of validation procedures, the concepts of 'more than', 'less than', and 'as much as' are involved not only in the (commensurative) major premise but also in the (suffective) minor premise and conclusion. The expression 'at least' means 'not less than'; similarly, 'at most' means 'not more than' and 'exactly' means 'neither more nor less than'. So these expressions fall squarely in the said larger context, as *negations of* positive comparisons. Propositions involving them are members of one big family.

Furthermore, it is important to realize that these expressions are tacitly *modal*. When we say of something that it is 'at least' X, we intend X as a minimum quantity; we impose *the impossibility of* a lesser quantity and leave open *the possibility of* a greater quantity (than X). Likewise, 'at most Y' intends Y as a maximum, with lesser amounts possible and greater amounts impossible; and again, 'exactly Z' implies that Z is both minimum and maximum. These modalities, possibility and impossibility, may be intended in any of the various *modes* (or types) of modality[19]. When 'possibility' is intended in the logical mode, we mean that the event concerned is conceivable in the given context of knowledge; in the extensional mode, we mean that there are known cases of it; in the natural mode, that it occurs in some circumstances; in the temporal mode, at some times; in the spatial mode, in some places; and in the ethical mode, we mean that it is permissible, i.e. a means compatible with our ends.

As regards the "keywords" Goltzberg uses in the above table, let me add the following. The expression "p or at least q" means "the greater quantity (p) is possible, the lesser one (q) is sure;" whereas "p or even q" (i.e. "p or as much as q") means "the lesser quantity (p) is sure, the greater one (q) is possible." These two statements are very similar in form, though with different emphases; however, in view of the switching of symbols for greater and lesser, they are contraries[20]. Likewise, the expression "p unless q" means "if q, then not p;" whereas "p even if q" means "if q, not-then not p;" so these two statements are contradictories. These are four representative examples; but evidently they are not a symmetrical and exhaustive list of possibilities. Goltzberg does give a few other keywords in other essays, but he does not treat the matter systematically.

Now, as already said, Goltzberg does not seem to realize the exact meanings and sources of his concepts of orientation and strength. The idea of orientation is an old deductive concept – it is the idea that 'every thesis has a counter-thesis'. The idea of strength is an old inductive concept – it is the idea that 'a thesis may be more or less confirmed', ranging in probability from 0% (definitely denied, or impossible) to 100% (definitely affirmed, or necessary). And probability is a modal concept, which may be intended in any of the various modes of modality already listed above. As regards the concept of confirmation and its many relatives, I refer you for instance to my essay *Principles of Adduction*, where the seesaw between theses and counter-theses is well explained[21]. So these concepts are nothing new, and Goltzberg's use of them is neither innovative nor very profound.

But the bottom line is pragmatic. What is the practical value of Goltzberg's insight or observation regarding the "at least" factor in a fortiori discourse specifically? Its main function seems to be to institutionalize the *dayo* principle, which he understands as equivalent to what I have called the principle of deduction, but also apparently to clarify the role of the conflicting marker "all the more." The latter expression creates an expectation for a heavier claim, while the former principle signifies that we will nevertheless rest content in the conclusion with the lighter claim made in the minor premise. The conclusion neither surpasses nor annuls the premise it is based on, but modestly adapts to it, as it were. This approach relates to rhetoric rather than to logic, in accord with Goltzberg's underlying interests. From the formal point of view, it is neither useless nor harmful, but not of great moment.

Goltzberg, in a more recent paper[22], apparently generalizes his above-described idea that "arguments are structured by two main parameters: orientation and strength" to all forms of argument, and calls it "the two-dimensional theory of argument." Judging by this title, one would expect the idea to be as revolutionary as, say, "the special theory of relativity"! But as we have seen, there is not much to it. It is just old stuff repackaged with a bit of grandstanding. This remark is not, of course, intended to put down or discourage the author, but only to recommend more modesty.

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[19]   See my work *Future Logic*, chapter 11, on this topic.

[20]   p refers to the greater in one and the smaller in the other, while q refers to the smaller in former and the greater in the latter. To be more exhaustive, we would have to also consider: 'q or at least p' and 'q or even p'. Also, as above mentioned, 'at most' and 'exactly'. And negations of all these.

[21]   See also my 1990 work *Future logic*, chapters 46-49. This essay was written soon after that book (the same year) but only published in 2003 in my *Phenomenology* (as chapter 7.1). I do not of course claim being the originator of these concepts.

[22]   *Théorie bidimensionnelle de l'argumentation : Définition, Présomption, Argument A Fortiori*. I have not read this document (dated 2010-11), though it devotes a chapter to a fortiori argument, because it is not yet published. But its table of contents is posted online at: crfj.academia.edu/StefanGoltzberg/Papers/1441244/Theorie_bidimensionnelle_de_largumentation_definition_presomption_et_argument_a_fortiori.

27. Andrew Schumann

Andrew Schumann has edited two collections of essays by various authors, *Logic in Religious Discourse* and *Judaic Logic* (both published in 2010)[1], to which I was invited to contribute articles. Though I am grateful to him for this friendly gesture, it does not exempt him from honest criticism on my part for the serious faults in his approach to logic in general and his alleged theory of a fortiori argument in particular. I regret having to do it, but I must. If an engineer discovers dangerous deficiencies in a bridge designed by a fellow engineer, or alleged engineer, he is duty bound to blow the whistle. The same is true regarding logicians. It is not a personal matter, but a professional responsibility.

1. Interpretation of Baba Qama 25a

Let us examine and evaluate how Andrew Schumann understands the a fortiori argument. In the Introduction to the volume called *Judaic Logic* that he has edited (not to be confused with my much earlier work of the same name), he presents various ideas regarding Judaic logic and its relation to general logic to which we shall return further on. With regard to the a fortiori argument specifically, he begins (p. 7) by analyzing an example found in the Talmudic tractate Baba Qama[2].

The rabbis there (on p. 2b) distinguish three types of damages/*nezeqin* that an ox might cause, namely: by horn/*qeren* (i.e. goring), by tooth/*shen* (i.e. eating) or by foot/*regel* (i.e. trampling). They derive from the Torah (Ex. 22:4) that damages caused by foot or by tooth on public property result in no financial liability for the ox's owner[3], while for such damages on private property there is full liability. With regard to damages caused by horn, the Torah (Ex. 21:35) suggests that there is half liability on public property; while nothing is said about such damages on private property. The question the rabbis then ask (on pp. 24b-25a) is: what of the latter? A debate takes place in the Mishna between R. Tarfon and some unnamed Sages, with the former advocating full liability while the latter advocate half liability. The latters' answer, which appears to be the authoritative one, is considered as obtained by them through a *qal vachomer* (i.e. a fortiori) argument regulated by a principle referred to as *dayo* (sufficiency).

Now, Schumann represents this argument as follows:

> "In order to draw up a conclusion by *qal wa-homer*, we should define a two-dimensional ordering relation on the set of data: (i) on the one hand, according to the *dayo* principle, we know that payment for horn action in a private area [which amount is to be determined] *cannot be greater than* the same in a public area [which amount is given as 50%], (ii) on the other hand, payment for horn action at a private place [which amount is to be determined] *cannot be greater than* foot/tooth action at the same place [which amount is given as 100%]. Hence, we infer that payment of compensation for horn action at a private place is equal to 50% cost of damage." (My italics and my bracketed comments.)

Schumann is here saying that, based on the *dayo* principle (as he takes it): (i) horn in private places (unknown) is equal to or less than horn in public places (50%), and (ii) horn in private places (unknown) is equal to or less than foot/tooth in private places (100%); therefore, horn in private places = 50%. Though we shall later challenge his premises, let us accept them for a moment so as to show first that his conclusion does not follow from them. According to his premise (i), the amount for horn in private places must be between 0% and 50%; and according to his premise (ii), the amount for horn in private places must be between 0% and 100%. From these two premises, all we can conclude is the common ground that the amount for horn in private places must be *between 0% and 50%* – we *cannot* conclude, as Schumann does, that the conclusion is precisely 50% as the rabbis have taught.

Another flaw in Schumann's treatment is that neither of his premises (i) and (ii), which he announces grandiloquently to be "a two-dimensional ordering relation on the set of data," mention the given that foot/tooth damages in public

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places cost nothing (i.e. 0%); yet this fact is not generally considered irrelevant to the deduction. Perhaps Schumann has made an error of inattention while typing his two premises? If we look at the description of his argument in what he calls "a formal notation," we do find mention of the given 0% for foot/tooth damages in public places. Here, he writes[4]:

"	[Foot/]Tooth action		Horn action
public place:	pay 0%	$\leq$	pay 50%
private place:	pay 100%	$\geq$	pay x%
	$x = 50\%$	"	

This schema implies four premises, not two: the two referred to earlier in Schumann's text[5] and two more which we shall label as (iii) and (iv). Let us look at each in turn:

(i) Horn in private places (unknown, x%) has some *here unstated* relation to horn in public places (50%), although that relation was earlier specified as "cannot be greater than." It is not stated how this proposition is established.

(ii) Foot/tooth in private places (100%) is here symbolically specified as *more than or equal to* ($\geq$) horn in private places (unknown, x%); this is equivalent to the earlier specification that the latter "cannot be greater than" the former. It is not stated how this proposition is established.

(iii) Foot/tooth in private places (100%) and foot/tooth in public places (0%) have some here unstated relation; but since the numbers are given we can readily say that the former is *more than* the latter; note that the clause 'or equal to' is not an option here, since the exact quantities are known.

(iv) Foot/tooth in public places (0%) is here symbolically specified as *less than or equal to* ($\leq$) horn in public places (50%); however, this is an error, since the clause 'or equal to' is not an option here, since the exact quantities are known.

However, since the two latter premises, labeled (iii) and (iv), do not mention the unknown quantity x%, they cannot directly affect the conclusion we can draw, which (granting the previously specified relation for (i), even though it is here not mentioned) therefore remains as before, viz. that the amount for horn in private places must be *between 0% and 50%* – and *not* as Schumann claims (so as to appear in agreement with the rabbis' conclusion) precisely 50%. Note well that Schumann does not tell us precisely how his operative premises, viz. (i) and (ii), were established, other than by vague appeal to "the *dayo* principle."

Now, at first sight, this schema might be indicative of an argument by analogy[6]. However, upon scrutiny, it is obviously not so. Looking at Schumann's tabulation, one might think that (a) just as (iii) foot/tooth in private places is greater than same in public places (100% > 0%), so (i) horn in private places ought to be *greater than* same in public places (x% > 50%), from which we would conclude that x = 100% or more. Similarly, (b) just as (iv) horn in public places is greater than foot/tooth in public places (50% > 0%), so (ii) horn in private places ought to be *greater than* foot/tooth in private places (x% > 100%), from which we would again conclude that x = 100% or more. Yet, Schumann tells us that, in both cases (a) and (b), x cannot be greater than 50%.

In fact, looking at the Mishna debate, the analogies here proposed coincide with R. Tarfon's position, while the limitation given by Schumann attempts to reflect the Sages' position, which is based on a statement which has come to be called the *dayo* principle. According to the rabbis, only three options are conceivable, namely zero, half or full compensation for damages. For this reason, though the two analogies, (a) and (b), conclude with "x = 100% or more" – the 'or more' clause can be dropped without further ado. R. Tarfon and the Sages both agree that x = 0% is not the solution. So the issue is whether to conclude with "100%" like R. Tarfon, or with "50%" like the Sages. Schumann's conclusion of "50% or less" does not match either traditional position.

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[4]   It should be said that Schumann is not the first to represent a fortiori argument by means of a table. He is preceded in this by Michael Avraham in 1992 (chapter 20), by Gabriel Abitbol in 1993 (chapter 21), and by Abraham, Gabbay and Schild in 2005 (chapter 25). Also note that, as we have explained in our analysis of Abitbol's work, while such representation may be (roughly) applicable to some a fortiori arguments (notably those by R. Tarfon here discussed, where the major premises need to be constructed by generalizations), it is not universally applicable (it does not apply to an argument whose major premise is already given or immediately obvious).

[5]   Notice that these two premises are oddly placed in this tabular schema: one is represented by the *column* on the right, while the other is represented by the bottom *row*. This asymmetry is significant in that the reasoning involved is made more awkward by this table. Yet such devices ought to facilitate our thinking, not impede it. Furthermore, notice that while we can express comparative relations (like $\geq$) between elements horizontally (as in premise (ii)), we have no way of doing so vertically (as in premise (i)). Note also the asymmetry in comparisons, with the lower row having "$\geq$" while the upper one has "$\leq$" – note the reversal of direction. All this shows how artificial a construct Schumann's schema is.

[6]   Note that this is effectively the posture adopted by Abraham, Gabbay and Schild. However, they too, as we saw earlier, did not arrive at the logical conclusion (100%), but instead drew an erroneous foregone conclusion ($\geq$ 50%) so as to appear to reflect the Mishna!

Schumann's approach does not tell us what the *dayo* principle is all about. He does not mention R. Tarfon's apparent analogies at all, or explain why the Sages preferred the lesser conclusions on the basis of that principle. What he does instead is to take what he imagines to be the Sages' two intermediate *conclusions* (in reply to R. Tarfon's arguments (a) and (b)) as his own *premises*. Moreover, he does that erroneously, thinking that they concluded "$x = 50\%$ *or more*," whereas in fact they concluded "$x = 50\%$ exactly." Maybe he did this, consciously or unconsciously, in order to artificially imbed R. Tarfon's opinion into his representation. In any case, he does not at all perceive or analyze the two a fortiori arguments underlying the Sages' discourse, nor see where exactly in that discourse their *dayo* principle applies. All he does, to repeat, is calculate *the net result* of their two conclusions (as he sees them); and even that he does erroneously. This is not a fortiori argument, as we have seen.

This explains why Schumann has *only one* argument, namely (a), which corresponds to R. Tarfon's first argument. This also explains why he does not mention propositions (iii) and (iv) in his initial representation of the argument. Although they are pictorially implicit in his "formal notation" schema, he does not explicitly reflect on them and they in fact *play no role* in his pursuit of the final conclusion. I suggest that their presence here is just eyewash, to give the impression he is taking all known factors into consideration. In any case, note well, he in fact has not found a way to logically integrate them into his reading; i.e. he does *not* use all the given information traditionally considered relevant. Schumann has, in other words, like many other commentators before him, not taken due note of argument (b), i.e. R. Tarfon's second argument, let alone realized its crucial significance in the debate.

Actually, the derivation occurs in two different ways relative to the two *qal vachomer* arguments dealt with here. In relation to the first argument, the derivation has 'horn in public places' in the minor premise and 'horn in private places' in the conclusion. But in the second argument, though the conclusion is the same, 'horn in public places' is not present in the minor premise – but only in an inductive preliminary to the major premise. Thus, in fact, the *dayo* principle is not merely related to a fortiori argument *functioning*, but also to a fortiori argument *preparation*. It is therefore not to be confused with or equated to, as often done, the principle of deduction, according to which "the conclusion of a deductive argument cannot contain more information than what is already given in the premises," although they do happen to intersect occasionally.[7]

It is evident from the formulation of his premises that Schumann misconstrues the *dayo* principle. The Sages put it like this: "*It is quite sufficient if the law in respect of the thing inferred is equivalent to that from which it is derived.*" It is not as he claims that the amount in the conclusion merely "cannot be greater than" the amount in the premise it is derived from. The *dayo* principle is, rather, that *as far as the logical inference* is concerned the two amounts must be *identical*, no more and no less. If any premise predicates some amount, then so must the conclusion have it. In this case, since 'horn in public places' entails 50% penalty, then 'horn in private places', being derived from that information, must also entail 50%. There is no 'or more' clause attached to the Sages' conclusions.[8]

Schumann also evidently misconceives a fortiori argument, as inference based on *the intersection of two ranges of value*. This is why he defines it as "parallel concurrent deduction." We have seen that he does not by this method succeed in obtaining the expected conclusion (though he *fakes* the result to make it look as if he does). He claims the said intersection to be a single point (as his conclusion $x = 50\%$ implies), whereas in fact the two ranges overlap with one (0-50%) being entirely within the other (0-100%), so that his conclusion should have been the lesser *range*, i.e. $x = 0$-50%. The lesser, more precise range is preferred to the greater, vaguer one, because the former requirement *excludes* part of the latter (viz. the 100% part). If we correctly formalize his schema for him we obtain the following argument:

> x is equal to or less than A (50%).
> x is equal to or less than B (100%).
> A is less than B.
> Therefore, the range '≤ A' is wholly included in the range '≤ B'.
> Therefore, x is equal to or less than A (the common ground).

Schumann, to repeat, wrongly concludes that $x = A$. But in any case, we must ask, *is this a fortiori argument*? It may appear to be so, because he is verbally referring (albeit erroneously) to the *dayo* principle and because he is using comparative propositions (i.e. propositions involving quantitative relations like 'is equal to or less than'). But in fact it is not a fortiori argument – it does not have any acknowledged form of such argument. Schumann just labeled as 'a fortiori argument' something that vaguely seemed to him to resemble one.

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[7]   I did not know all this (and more) at the time I wrote my *Judaic Logic*, so Schumann can be excused for not having found it there.

[8]   However, this should not be taken to mean that the *dayo* principle excludes a priori the possibility that greater amounts might be discovered in some other Scriptural text. It just means that no *inference* is capable of guaranteeing greater amounts.

It is amazing to me that this writer, who I have been assuming has read my work on the subject, completely ignores it and presents a manifestly useless and false theory of a fortiori argument. In my book *Judaic Logic*, in a footnote to chapter 4.3, I explain clearly enough how the (first[9]) a fortiori argument in Baba Qama, p. 25a is to be construed:

> "…we have 3 *givens*: (1) relatively unintentional damages by animals (*shen* and *regel*) in the public domain imply owner's liability to pay none of the damages; whereas, (2) the same on private property imply his liability to pay all of the damages; and (3) relatively intentional damages by animals (*keren*) in the public domain imply liability to pay half of the damages. The first two givens serve to *induce* the major premise of our actual a-fortiori: make the comparison 'for unintentional acts, acts committed on private property imply more liability than those committed in the public domain', then generalize to 'for all acts, the same', then educe the new particular 'for intentional acts, the same'. This result, combined with the third given, which serves as minor premise, form *the a-fortiori argument proper*, whose conclusion is 'intentional acts on private property imply liability to pay half the damages'."

Notice that all available data is used. First, we induce the required major premise from the given data on foot and tooth damages; then, we combine it with the given minor premise on goring and draw the desired conclusion from a perfectly standard a fortiori argument, namely the following positive subjectal (from minor to major), where P is horn damage on private property, Q is horn damage on public property, the damages here being *intentional* damages because the goring ox intends (insofar as an animal can intend) to damage, R is the scale of liability for damages, S is the given or inferred amount to pay for the damages concerned:

> Major premise: [intentional damages committed on private property] (P) imply more [liability] (R) than [intentional damages committed in the public domain] (Q).
> Minor premise: [intentional damages in the public domain] (Q) imply enough [liability] (R) to [pay half of the damages] (S).
> Conclusion: therefore, all the more, [intentional damages on private property] (P) imply enough [liability] (R) to [pay half the damages] (S).

In that earlier account of the first argument in BQ 25a, I identify the fact that we can only infer in the conclusion the same amount (S) as given in the minor premise with the *dayo* principle. Upon reflection, this explanation is best attributed to what I have lately called the principle of deduction. This can be stated as: in a fortiori argument, as in syllogism or any other form of deductive inference, the conclusion cannot contain more information than what is already verbally or tacitly given in the premises. Given only the above shown two premises, and no additional information, if the term S in the conclusion was more or less than the term S in the minor premise, there would be illicit process – i.e. fallacious reasoning. Clearly, the principle of deduction is not an artificial, arbitrary or conventional limitation, but a natural, rational one.

Today, having reexamined the whole issue much more deeply, I would rather explain this debate as follows. R. Tarfon may be construed as having initially formulated an a crescendo argument – that is an a fortiori argument with an additional premise justifying proportionality – which concludes with full damages; while the Sages may be construed as having formulated the above described purely a fortiori argument in opposition – thus, effectively, by virtue of their *dayo* objection, rejecting the additional premise about proportionality. It just so happens in this first exchange that the *dayo* principle corresponds to the principle of deduction. But in the second exchange, this equation falls apart, because R. Tarfon's argument has the same conclusion whether construed as a crescendo *or as purely a fortiori*, which means that it is effectively the latter. In that case, the Sages' new *dayo* objection, though formulated in exactly the same words, must mean something different; namely, as already mentioned, it must concern the generalization of the major premise preceding the (second) a fortiori argument. The latter explanation is suggested by a Tosafot commentary, by the way.

Whence it follows that the *dayo* principle has to be understood as a principle specific to Judaic law, either revealed by the Torah or established by rabbinic authority, and not as a purely logical and therefore universal principle. I did not realize this at the time I wrote *Judaic Logic*. I only came to this realization in the present work, when I wrote the chapters called 'In the Talmud' (7-8). There, after careful examination of the Mishna and Gemara in question, I got to realize that the rabbinic viewpoint is not as monolithic and the *dayo* principle is not as uniform as depicted by me before. The views of R. Tarfon and the Sages in the Mishna, and of the later Gemara, cannot credibly be merged into

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[9]   To repeat, at the time I wrote my *Judaic Logic* I had not taken due note of R. Tarfon's second argument, let alone realized its crucial significance in the debate. But I had already correctly perceived the form of a fortiori argument in general, and thence the form of R. Tarfon's first argument.

one "rabbinic" opinion, as I earlier naïvely assumed. The divergences between them are rich in lessons for us about a fortiori argumentation and other interesting topics.

This is, briefly put, my revised viewpoint today. Because Schumann does not inquire into all these issues, his treatment of this important Talmudic passage is simplistic and prone to error.

2. Syllogism as a fortiori

Although the above examination already proves that Schumann has not grasped the nature of a fortiori argument, to be thorough in my critique of his views I would like us to look at the rest of his discussion of this subject. Without mention or rebuttal of objections to the idea by Louis Jacobs and others, he writes: "As A. Schwarz and M. Mielziner showed, an Aristotelian syllogism may be presented as the simplest case of *qal wa-homer*" (notice the word "showed" suggesting the matter is settled)[10]. He then proceeds to justify this claim with reference to the syllogism "All men are mortal. Socrates is a man. Therefore Socrates is mortal." – whose "*qal wa-homer* analogue" he presents as follows:

"general notion (predicate)	*(mortal)*	≥	*(x)*
particular notion (subject)	*(man)*	≥	*Socrates*

$$x = Socrates'\ mortality\qquad "$$

He explains this schema as follows:

> "Continuing our reasoning in the same way, we should define a two-dimensional ordering relation: (i) by the *dayo* principle, we know that the notion 'Socrates' is more general than the notion *x*, (ii) the notion 'mortal' is more general than this *x* as well. Hence, *x* is 'Socrates' mortality'."

Now, I (Avi Sion) ask you: Where did *x* come from, since it was not mentioned in the given syllogism? And how does "the *dayo* principle" inform us that Socrates and mortal are more general notions than the mysterious *x*? And where does concluding the equation of *x* to Socrates' mortality spring from? I would have thought by consideration of symmetry that *x* is like 'mortal' a predicate; indeed, according to the syllogism, *x* should be the predicate 'mortal'. Clearly, what we have here is the sight of Schumann working hard to load the dice in support of his utterly artificial schema. In what way does this schema of his render the reasoning involved more comprehensible or valid to us? It sounds impressive, but just what is a "two-dimensional ordering relation"? What has this schema of his to do with a fortiori reasoning, other than the fatuous appeal to a sort of omniscient "*dayo* principle" and the use of mathematical comparatives like '≥'[11]? How do we know what notion is "more general" than which, other than by reference to Aristotelian syllogisms? Of what additional use is Schumann's schema in this regard? Exactly how does the conclusion follow from the premises? The whole thing is contrived and useless – it proves neither the putative conclusion nor the claim that syllogism is "the simplest case" or "analogue" of a fortiori argument. I am sorry to say it, but it is just an empty show.

He goes on to propose to us, as "the [!] Biblical example of Aristotelian syllogism" (exclamation mark mine), and by implication as an example of a fortiori, the argument in Ex. 6:12: *Behold, the children of Israel have not hearkened unto me; how then shall Pharaoh hear me, who am of uncircumcised lips?* Schumann takes Moses' speech impediment as the syllogistic middle term and recasts the argument into his unnatural schema, as follows:

"(Jews do not hear Moses)	≥ *(because)*	*(Moses is of uncircumcised lips)*
(does Pharaoh hear Moses?)	≥ *(because)*	*(Moses is of uncircumcised lips)*

$$No,\ he\ does\ not\qquad "$$

Observe, now. There is no mention this time of a "*dayo* principle" setting relationships in order for us; and no explanation is given for this omission. Here, the symbol of comparative size '≥' is suddenly, without explanation, joined by or interpreted as the causative conjunction 'because'. There is no unknown quantity '*x*' this time. Instead,

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[10]   Schumann does not specify where such statements were supposedly made. As regards Mielziner, all I found him saying is that a fortiori is "a kind of syllogism" (p. 130); but it is clear from the rest of his treatment that he did not say or mean that "an Aristotelian syllogism may be presented as the simplest case of *qal wa-homer*" as Schumann suggests here, but was simply using the term etymologically (as 'syn' + 'logism', implying a combination of propositions bound by a common term).

[11]   Note by the way how the symbol of quantitative comparison has changed direction without forewarning or explanation. In the first schema, the top row had ≤ and the bottom row had ≥; whereas here and the Biblical example below both rows have ≥. Furthermore, in the first schema, the second premise referred to the bottom row; whereas here, it refers to the top row; and in the Biblical example below it refers to the bottom row again, though in the opposite direction to previously! Schumann manifestly manipulates his information as convenient to his ideas, hoping to fool everyone. Or alternatively, his thinking is very confused.

the two items on the right are one and the same proposition "Moses is of uncircumcised lips." One of the items on the left is an assertion "Jews do not hear Moses" and the other is a question "does Pharaoh hear Moses?" The putative conclusion confidently appears out of nowhere: "No, he does not." We are not told how it was derived from the apparent premises, or what role each element played in obtaining it. We are supposed to think that this schema is somehow a magical formula that "proves" whatever is forced into it. All this I submit is not logic, let alone syllogistic or a fortiori logic.

Also, Schumann has evidently read this passage of the Torah very superficially, as saying: "Because Moses has a speech impediment, the Jews do not understand him. For the same reason, Pharaoh won't understand him either." Even if this may at first sight seem what Moses was arguing, we should ask (here as always) how does the conclusion proceed from the premises? The simplest answer would be to say: by analogy; "*just as* the Jews are befuddled by Moses' speech impediment, *so* will Pharaoh be." But this is neither syllogism nor a fortiori argument. It is not even a deduction, since it is conceivable that Pharaoh could have been more tolerant or skillful in relation to people with speech impediments than the Jews allegedly were.

If we want to formally generate a syllogism from this material, we would have to begin with an induction. We would have to *generalize* the given proposition "Jews cannot understand Moses' speech" to "No one can understand Moses' speech." Using the latter as our major premise, we could then engage in the following syllogistic inference:

> No one can understand Moses' speech (by generalization).
> Pharaoh is someone (obviously).
> Therefore, Pharaoh cannot understand Moses' speech.

This would be a valid syllogism, subsuming Pharaoh under the previously induced general statement. But have we thereby engaged in a fortiori reasoning? Clearly, not. What is missing in this syllogism is the sense that Pharaoh would for some reason be *less prone* to listen to Moses than the children of Israel, who admittedly were recalcitrant enough (though, obviously, not entirely so, since they do listen to him quite a bit). Schumann could of course reasonably deny that such relative proclivity is intended in Moses' statement, since it is not explicitly mentioned in it. In that case, the argument would involve a syllogism (as just explained), but it would not be a fortiori (as he admits it to be). What is sure in any case, is that the rabbis interpreted Moses' statement as an a fortiori argument, and Schumann has tried but in no way succeeded in representing it as such.

I analyzed Ex. 6:12 in my *Judaic Logic*[12], explaining that the clause "who am of uncircumcised lips" could not be viewed as part of the a fortiori argument and that the argument required a fourth term more variable than that clause to serve as a fortiori middle term (R). I there proposed to use "fear of God" as middle term, but any other appropriate term would do as well (e.g. "spiritual closeness to Moses"). The resulting a fortiori argument was a negative subjectal, as follows:

> The children of Israel (P) fear God (R) more than Pharaoh (Q) does.
> Yet, they (P) did not fear Him (R) enough to hearken unto Moses (S).
> Therefore, all the more,
> Pharaoh (Q) will not fear Him (R) enough to hearken unto Moses (S).

We have here a perfectly natural and comprehensible a fortiori argument, capable of formal validation. It *explains* why Pharaoh will be less inclined to listen to Moses than the children of Israel have been. It interprets, as the overall context suggests, the phrase "hearken unto Moses" in a *moral* sense, as "taking Moses' instructions to heart and following them," and not merely as a physical issue of difficulty to understand Moses' speech due to his difficulties with speech. Moses' mention of his speech impediment is easily pushed aside as a lame excuse on his part, an expression of his humble hesitation to take on the gigantic political and spiritual task he is being given by God[13].

If we take the subsidiary term "hearken unto Moses" as an either-or predicate, then just as the children of Israel do not listen to Moses as much as they ought to (if at all), so Pharaoh can be expected not to listen to him as much as he ought to (if at all). This would be an acceptable purely a fortiori reading, suggesting that Pharaoh can be expected to disobey Moses at least *to the same degree* as the children of Israel do. We could however go further and read the argument as a crescendo. If we understand the subsidiary term "hearken unto Moses" as allowing of degrees, we can say that the suggestion that the children of Israel "do not" listen to Moses is intended hyperbolically rather than

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[12] Chapter 4.2. See also its mention in the present volume in the chapter on Louis Jacobs (16.1).

[13] See Ex. 4:10, where Moses says that he is "not a man of words" and he is "slow of speech and of a slow tongue," and then v. 13, where he begs God to send someone else on this mission. See also Ex. 6:9, where the word "to hearken" (*lishmoa'*) is clearly used in the mental sense of internalizing (which the children of Israel fail to do due to "impatience of spirit" etc.) – and not in a physical sense.

literally, and grant that since Pharaoh fears God less than the Jews, he is *more* prone to dismiss Moses' injunctions. In that case, we are tacitly admitting an additional premise that the degree of disobedience to Moses is inversely proportional to the person's fear of God.[14]

This is the correct explanation, which Schumann has totally ignored. How can anyone, having read the above and other examples, and copious explanations and proofs, manage to come up instead with such a confused concoction as his? Baffles me. What is clear that he has thoroughly misunderstood the nature of a fortiori argument. He misconceives it as some sort of table-filling mechanism, whereby given three cells, the fourth cell automatically follows; and he entirely ignores the key variable (viz. the middle term, R), the quantitative aspect underlying the other terms which makes the inference in fact possible[15]. Moreover, he imagines the *dayo* principle to be some sort of *premise*-formation tool, whereas its function is to limit the *conclusion* to information given in the core premises.[16]

And the main reason Schumann has misunderstood a fortiori argument is that he is trying to artificially impose on Judaic logic modes of thought or techniques (of anyway doubtful value) that he has supposedly gleaned in modern symbolic logic. Instead of going bottom-up from empirical study to fitting theory, he is functioning top-down from some rationalistic prejudice, forcing things into imaginary forms without regard to their actual content.[17]

3. Grandiosity without substance

We have thus far demonstrated that Schumann has grossly distorted the nature of a fortiori argument, not to mention his falsifying its applications in Judaic contexts. Allow me to continue with the present analysis, so as to better understand the methodological failures that have led that author into such serious error. For this purpose, I shall have to indulge in some literary and psychological analysis, in conjunction with purely logical analysis. Consider to start with the following paragraph:

> "All examples of *qal wa-homer* regarded above show that this inference rule cannot be presented in a linear form and assumes a multi-dimensional ordering relation (the simplest case of two-dimensional order was considered in instances above). As opposed to this, usual inference rules in modern logic suppose linearity, therefore by combining these rules we obtain conventional proof trees. *Qal wa-homer* provides us with an algorithm for massively-parallel proofs. Hence, a deductive system of Judaic logic may be presented as a hybrid cellular automaton[18]."

Here, Schumann communicates to us that he is not modestly content with his above *ad hoc* treatment, but has greater ambitions. He aims first to *generalize* his nonsensical findings and find ways to *mass produce* equally pseudo-logical "proofs." Observe the flashy verbiage he uses here and throughout. Its intent is of course firstly to excite the reader's awe and admiration. But moreover, its purpose is to suggest that he is fully plugged in to the latest trends in logic research and that his work fits right in there on the cutting edge; this serves to buttress his earlier, more specific claims, and to conceal their weaknesses. Psychologically, what is happening in Schumann's mind at this stage is that he is working himself up into a sort of intellectual frenzy. He is delighting his ego with the thought of the mind-boggling possibilities implied by his discoveries. Not only has he, in his mind's eye, solved the till now intractable problem of a fortiori argument, but he sees that he can take it all much farther and expand the horizon of logic into hitherto unexplored fields.

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[14]   As for the *dayo* principle, its application in this example would be as follows. If the argument is read as purely a fortiori, invoking the *dayo* principle in relation to it would mean no more than reminding us of the universal principle of deduction. If, on the other hand, the argument is read as a crescendo, invoking the *dayo* principle in relation to it would signify that the assumed proportionality is to be rejected. However, in truth, the *dayo* principle would not be invoked in such a case, because the a fortiori argument is not being used to justify application by a human court of a penalty by inference from a penalty found in Scripture.

[15]   Note too, he makes no mention of the distinction between subjectal and predicatal a fortiori or that between positive and negative a fortiori. All a fortiori look formally alike to him, it seems.

[16]   Another idea expressed by Schumann that shows his misunderstanding of a fortiori argument is found in his explanation of Hillel's first hermeneutic rule. Here, he writes (p. 14): "…one deduction of the set of concurrent deductions is much more certain. As a result, a certainty of that deduction is expanded to cases of other concurrent deductions." By "deductions" he here apparently means the propositions concerned (i.e. the minor premise and conclusion); this may be explained by remembering that in his mind they are inferred from the *dayo* principle. Anyway, what this citation shows is that Schumann thinks that a fortiori argument transmits the *greater certainty* of the (minor) premise to the heretofore uncertain concluding proposition. This is a common error people make – confusing an 'ontical' quantity with an epistemic one (see my treatment of this issue earlier on).

[17]   It is interesting to note that Schumann did not defend his thesis when I posted (in mid-December 2010) the first two sections of the present chapter in my website (I of course wrote to him telling him I did so). If he believed his theory of a fortiori argument was correct, one would have expected him to indignantly protest; but he did not. Alternatively, if my exposition of its faults was convincing to him, one would have expected him to publicly admit them, and indeed thank me for exposing them; he did not do that, either. From his silence we can infer a non-scientific attitude, and maybe that he consciously or subconsciously tried to fool people in the first place.

[18]   Here, we are referred to Schumann's article: Non-Archimedean Valued and *p*-Adic Valued Fuzzy Cellular Automata, in the *Journal of Cellular Automata*, 3(4) (2008), pp. 337-354.

With this goal in mind, Schumann now proposes a complex-looking symbolic formula for "the inference rule of *qal wa-homer*," again using impressively abstract and modernistic terminology, which I will spare you here[19]. I do not bother to reproduce it, for four reasons. First, if it is at all meaningful and comprehensible (which, I assure you, is open to debate), it is not at all clear that it has any relationship whatsoever (other than his say-so) with a fortiori argument. Second, assuming that it is as Schumann claims a generalization of a fortiori argument as he sees it, we can predict without further ado that it is bunkum. Why? Simply because, *if what is generalized is known to be devoid of substance and logically erroneous, it follows that its generalization is equally (if not even more) empty and mistaken*.

He, of course, is convinced that *the mere putting of a thought into symbolic language*, however superficial in content and logically fragile that thought may be, gives it profundity and legitimacy. This is very important to realize – this belief that symbolization is somehow, by some mysterious magic, able to confer on abstract ideas a truth and power that they lack in ordinary language. But this faith, though widespread in today's academia, is without basis. Third, Schumann makes no attempt to prove his proposed formula, i.e. to validate it from already established principles. He just asserts it, as if its truth is self-evident; and that just won't do, if logic is to be a scientific enterprise.

Fourth, he makes no effort to demonstrate on paper the practical unfolding of the "multi-dimensional ordering relation" that he claims performs "massively-parallel proofs." He just vaguely imagines this fantastic animal in fancy words and symbols, but does not actually show it in action or give us an example of its application and utility. Just what does a "multi-dimensional" a fortiori argument look like?[20] Exactly what are its premises and what is its conclusion? Those "massively-parallel proofs" that we are promised – they are proofs of what, exactly? All we are offered is an alluring chimera. Nevertheless, impressed with his linguistic *tour de force*, Schumann concludes:

> "As we see, Judaic reasoning may be formalized only by using the non-well-founded mathematics and process semantics, [which] both for the first time began to be used in computer science. In particular, we can assume that the (*sic*) Judaic semantics and Judaic formal logic can be developed within the framework of *interactive-computing / concurrency paradigm*. One of its means is the coinduction principle. For more details see [various references]. However, the concepts of coalgebra and coinduction have not yet had much impact in the pure logical investigations as well as in the logical-historical studies." (Italics his.)

Note here again the high density of glitzy words and the implication that all this is part of some larger *avant-garde* effort. Schumann is here revealing one aspect of his programme for Judaic logic, namely to extract from it hidden jewels of logic (like the a fortiori argument), and then inject them into general logic studies – of course in a more developed form capable of mechanically processing any number of elements, indeed even an "infinite" set of them. The a fortiori argument is important to him, but only as a stepping stone to bigger and better things. He is doubtless sincerely interested in Judaic texts and practices, but he also perceives them as potentially rich sources of new ideas for modern logicians to investigate. This goal is of course acceptable, and has long been pursued by many logicians. Nevertheless, the fact remains that, as we have above shown with regard to Schumann's work on a fortiori argument, he is unequal to the task.

What I find galling is Schumann's pretentiousness. After putting forward various "themes and objectives" for Judaic logic studies, he declares (my italics): "*These aims were not fulfilled in other books devoted to Judaic reasoning and methodology*." Earlier on in the same essay he dismisses offhand as "*informal traditional logic*" the work of "Ronen Reichman, Louis Jacobs, Avi Sion et all (*sic*)." I am not acquainted so far with the work of Ronen Reichman (who writes in German). Regarding Louis Jacobs, though it is true that the large majority of his work is informal (though very perspicacious), he has nevertheless made an effort at formalization of *qal vachomer* and *binyan av* Talmudic argumentation – an effort, I should add, much more interesting than Schumann's paltry attempt. As for my own work on Judaic logic, it is almost entirely formal[21], not to mention its wide scope and originality. I can only suppose that Schumann misrepresents it ("informal") and disparages it ("traditional," "aims… not fulfilled") in order to avoid having to compare his work to it and in order to claim originality.

As regards originality, I would like to point out that the phrase "Judaic logic" was, I think, coined by me; certainly, it was the title of my book on the subject published some 15 years before Schumann used it for his collection. Long before Schumann conceived it, my book explicitly set as its goal the study of the interaction of Judaic and general logic, as e.g. in its abstract[22]:

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[19]   Though I might mention some of its terms: "a hybrid cellular automaton for Judaic deduction," "finite or infinite set of elements, called the states of an automaton… collected from statements of the Pentateuch," etc. All this is, to my mind, bluff intended to conceal the underlying sophistry.

[20]   Perhaps his idea is that a fortiori arguments might be strung together, so that long chains of quantitative comparisons occur, like A > B > C > D > E…. Wow, revolutionary!

[21]   It is formal, though not symbolic – more on this distinction later.

[22]   For more detail, see chapter 15.1, Epilogue: Motives of the Present Research.

"*Judaic Logic* attempts to honestly estimate the extent to which the logic employed within Judaism fits into the general norms, and whether it has any contributions to make to them."

Even so, I did not and do not claim that such research was original with me. I was well aware then, and have become even more aware since, that many competent people have been making similar efforts at comparison and correlation between Talmudic reasoning and Aristotelian logic, since the Middle Ages if not earlier[23]. My point here is only that I resent the impression given by Schumann that this is some kind of new vision that he has personally come up with. A science cannot advance if those who wish to contribute to it do not duly acknowledge the significant work of others – or, if they disagree with it (as of course they may well), do not explain why they do so. It is silly for someone to pretend to be an innovator or a leader when he is not – the truth will come to light sooner or later.

Attempting to dig deeper into why Schumann failed in his attempt to formalize a fortiori argument, I would suggest that his methodological tool is fundamentally flawed. His reliance on the ways and means of "modern symbolic logic" is bound to lead him astray, for reasons I explain elsewhere (in **Appendix 7.1**). The fault is not his personally; it is to be blamed on the educational system that taught him; he is a victim of intellectual fashion, or more precisely perhaps of a new and infectious cognitive disease. My kindly advice to him (and others like him) is to quit the make-believe world of modern symbolic logic and learn some real logic – classical formal logic – e.g. by studying my books, starting with *Future Logic*. Ask yourself whether you want to *be* a logician, or just *appear* to be one; whether you want to merely talk the talk, or really walk the walk. Steer clear of posturing and imposture, and seek to master the fine and demanding art of true logical analysis.

Of course, Andrew Schumann deserves respect for his work as an editor, bringing together essays by many writers (of varying competence[24]) on logic relative to religion in general and Judaism in particular. Such initiatives stimulate often valuable research and writing, and acquaint writers with each other. I am sorry to have found it necessary to expose the shortcomings in his logic, here; it gives me no pleasure. I do believe he is well-meaning, and sincerely devoted to both logic and Judaism, values which I share. So, this issue was a hot debate in my conscience for weeks. But, I decided, intellectual integrity should be our paramount value, above all personal or ideological considerations.

4. Logic custom-made

My view is that logic, like philosophy, cannot be made a handmaiden to religion – any religion, my own included. It is not servile to any domain, be it secular or religious; it is our common precious tool for the objective and impartial rational judgment of all claims of human knowledge. Logicians cannot adapt logic to fit preconceived notions, however desirable they seem; that would be dishonest, unscientific and unethical. Logic is something universal; it is not the particular possession of any people, culture, class, place or time. Different people may well and do display different degrees of use, mastery and understanding of logic; it is possible to say this person is not as logical as that one. But it is not permissible to say that all 'logics' are on equal footing.

I am not sure that Schumann is of the same opinion on these issues, judging by his statements and performance in his contributions to the *Judaic Logic* book he edited. Already, in his Preface to it, he strikes me as excessively apologetic and relativistic, writing of Judaic logic as "a methodology for deducing religious laws [from the Pentateuch]" and as an "original logic that is not less deductive than Aristotle's logic." Admittedly, he is not here speaking in the first person, but his immediate emphasis on the deductive claim is clearly indicative of his own perspective. Indeed, this is confirmed further on when he proposes to "develop general approaches to formalizing Judaic logic" and adds: "This consideration of Judaic logic has relevance for modern logic and analytic philosophy and may be compared with the contribution made by the formalization of Ancient Greek logical systems to 20th-century logic and language philosophy."

My complaint here is firstly that he takes for granted that the explicit and implicit Talmudic *midot* (the rabbinic hermeneutic principles and practices) are all deductive and all valid, whereas I have shown systematically in my book *Judaic Logic* (see its chapter 12 for a summary of its conclusions) that, while some *midot* can truly be said to be deductive and valid (notably the a fortiori argument), other *midot* can only at best be conceived as inductive (and therefore compatible with alternative conclusions to those imposed by rabbinical fiat) and still others must be admitted to be invalid (i.e. phony logic or even contrary to logic). Many researchers agree with these overall conclusions from their own perspectives. It is therefore unacceptable that Schumann here abstains from critical

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[23]   Worth mentioning here as recent examples are the articles by numerous authors in the journal *Higayon: Studies in rabbinic logic*, edited by Moshe Koppel and Ely Merzbach of Bar Ilan University, which was published in 1989-2001 by Alumna, Jerusalem. It is a pity that this publication has been discontinued. Unfortunately, too, its articles (most of which are in Hebrew, and have not been translated into English) have not been posted on the Internet. See: u.cs.biu.ac.il/~koppel/higayon.html.

[24]   Most of those in *Judaic Logic* were interesting, but I particularly disliked the essay: "In Search of the Logic of Judaism: From Talmudic Chaos to Halakhic Linearity" by Tzvee Zahavy.

judgment and seems to give a blanket sanction to all of Judaic logic. This perhaps reflects an apologetic stance, or alternatively an ideological relativism.

Secondly, Schumann tries to put over that Judaic logic is "original," i.e. has made contributions unknown to Aristotelian logic. There is some truth to that claim, to be sure. The a fortiori argument, for instance, though not unknown to Aristotle and his predecessors and successors, was not extensively used or systematically studied by them, whereas it plays an important role in rabbinic discourse and the rabbis have over time made considerable efforts to understand its workings. Nevertheless, as I showed in my *Judaic Logic* (chapter 4.2), to formalize and validate a fortiori argument, we must refer to the methods developed by Greek logic. Some other *midot* can similarly be favorably compared, and found to contain some valuable novelty. One of the interesting findings in my study of the harmonization *midot* (see chapter 11 of my work) was that these arguments are built distinctively around four terms instead of three. Nevertheless, here again, it was necessary to refer to syllogistic logic to formalize and evaluate them.

Come to think of it, if I were to propose one *midah* as the most original in comparison to Aristotelian logic it would perhaps be the thirteenth rule of R. Ishmael, which can be defined in general terms as dialectical reasoning, i.e. thesis-antithesis-synthesis. The reason why this *midah* is worthy of attention is that it is distinctively inductive, whereas the a fortiori argument and the other harmonization rules are, or at least are claimed to be, deductive. This 13th rule teaches us (by implication) that if we come across a contradiction in our thinking, we should find a way to *reconcile* the theses concerned, either by modifying them both, or by preferring one over the other, or by rejecting them both in favor of some third thesis. Of course, Aristotle practiced such adaptation; but somehow he does not seem (as far as I can recall) to have enshrined it as a crucial principle in his theory of knowledge. Reconciliation of conflicting theses has remained a little noticed aspect of induction till modern times, and yet in rabbinic debate it played a very prominent and conscious role (not only through the thirteenth *midah*, but more frequently in the drama of *kushya* and *terutz* – difficulty and resolution – in rabbinical discussions).

If we leave aside this important inductive process, and here consider only deductive processes as Schumann does, we can surmise from what was said before that his claim to the originality of Judaic logic is exaggerated. I do not of course claim that the deductive aspects of Judaic logic are historically derived from Greek logic. Though there may have been some trickles of influence, there is no doubt that they emerged independently. What I am contending is that the distinctive deductive elements were not necessarily true logic – and that those that were true logic can only be so assessed by referring to Aristotelian logic. The rabbis originally never engaged in formalization and validation of their logical discourse; it is only much later in time under the influence of Aristotelian logic that some efforts were made to do so. Their justification of logical arguments was either revelation (see e.g. Baba Qama p. 25a: "Is not *dayo* of Biblical origin?"[25]), or rabbinic decision through rational insight (*svara*) or convention (*rov*, majority vote). Therefore their discourse, though it may well be characterized by us as logical, cannot strictly speaking be classified as an exercise in formal logic.

Schumann seems aware of that defect, since he proposes to formalize Judaic logic. However, what he means by "formalize" is "symbolize and axiomatize." As far he is concerned, even "Ancient Greek logical systems" were not formal and had to be formalized by "20th-century logic and language philosophy" – and he proposes to do the same job of updating for Judaic logic. He can thus ignore my attempts at formalization and validation of Judaic logic, and the attempts by others, since that is not what he means by formalization. What he has in mind, in fact, is symbolization and axiomatization[26]. We shall presently see what this means in practice. We have already in fact seen it with regard to the a fortiori argument. We saw how Schumann misconstrues the form of such argument and its justification. We saw how, when things do not fit into his conception of them, he forces them in and fakes the conclusion.[27]

Let us now look at another case in point – his treatment of the *klalim uphratim* (the hermeneutic rules referring to combinations of general and particular terms)[28]. In his Introduction, he writes:

> "In this volume we are going to present a modern logical analysis of basic *middot* used by Talmudists…. These rules have nothing in common with Ancient Greek logics. For instance, truth and falsity cannot be regarded as meaning of (*sic*) Scriptural passages. Evidently, the latter are viewed as absolutely true. By Judaic inference rules [here listed], other meanings of Biblical statements are introduced, namely the phrases may be either 'general' or 'particular.' As a result, logical connectives are defined in a unique way."

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[25]   That example is from the Talmud. Of course, long after the Talmud, Saadia Gaon (882-942 CE), in an effort to resist Karaite skepticism, claimed that all the *midot* were Divinely revealed to Moses at Mount Sinai.

[26]   Although Schumann does not mention Guggenheimer, his ambition to impose on Talmudic logic the terms and principles of modern symbolic logic is reminiscent of the latter's.

[27]   He even goes so far in manipulation as to *define* a fortiori argument as "parallel concurrent deduction" (in inverted commas, yet) in his listings (in an Appendix) of the hermeneutic rules of Hillel and R. Ishmael, as if this idea of his is obvious and already generally accepted.

[28]   See my detailed treatment of this topic in my *Judaic Logic*, chapter 10.3.

Needless to say, I do not agree that "These rules have nothing in common with Ancient Greek logics" – Schumann is only saying that to enhance the value of his forthcoming "formalization" of some *midot*[29]. As for his suggestion that since the rabbis viewed the Scriptures as "absolutely true," truth and falsity must have to them referred to "other meanings," viz. the distinction between "general" and "particular" – he is here obviously trying to give a modernistic spin to the rabbinic interest in the *klal-prat* distinction, again for the purpose of enhancement. In fact, the *klal-prat* distinction simply refers to the scope of terms. The interest of the rabbis is in interpreting *how far* a Biblical statement is applicable in practice. This is not, as Schumann hints, a theory of truth or of meaning.

Anyway, he then presents tables for the logical connectives "…and…," "…or…" and "if… , then…" (which he labels respectively as "Judaic conjunction," "Judaic disjunction" and "Judaic implication," because they seemingly have different properties to the corresponding ordinary, Western connectives) with reference to different combinations of "general" and "particular" contents. The first table, based on Hillel's fifth rule and R. Ishmael's rules 4, 5 and 6, looks like this:

A	*B*	*A* and *B*
Particular	Particular	Particular
Particular	General	General
General	Particular	Particular
General	General	General

Now what I notice here is that this table is both more extensive and less extensive than what is given in actual Judaic logic. It is correct that *klal uphrat* yields *prat* (rule 4 of R. Ishmael), *prat ukhlal* yields *klal* (rule 5 of R. Ishmael). But the table is more extensive, in that it mentions a conjunction of two particulars with a concluding particular (the first row) and a conjunction of two generals with a concluding general (the last row); these alleged conjunctions are, to my knowledge, nowhere treated in the listings of rules or anywhere else. Also, the table is less extensive in that it does not mention double conjunctions, i.e. those with three conjuncts. Rule 6 of R. Ishmael refers to *klal uphrat ukhlal*, and later additions to this rule (not listed) include *prat ukhlal uphrat, klal ukhlal uphrat* and *prat uphrat ukhlal.* The conclusions from these various conjunctions more or less follow the last conjunct – more or less, I say, because in practice this is somewhat subjective, depending on the rabbis' preferred interpretation in a given context. Schumann possibly intended his two interpolations (the first and last rows) to cover these various cases of double conjunction; but he does not say so. Moreover, he makes no mention of R. Akiva's *ribui umiut* and *miut uribui* alternatives to the R. Ishmael rules, although they play a prominent role in Talmudic discourse (initially as a rival system, but later as a complementary one).

Therefore, what appears to have happened here is that Schumann derived his first table from the single conjunctions of rules 4 and 5 of R. Ishmael, ignoring the double conjunction of rule 6 and its later variants. He added in the first and last rows of his table to fill in the blanks symmetrically, perhaps thinking to himself that rule 6 could be assumed to roughly fit in there somewhere. He skipped R. Akiva altogether, no doubt because distinguishing his approach from R. Ishmael's requires some rather subtle discussion. What Schumann wanted for his "formalization" demonstration was something manageable, like his first table, which would somehow reflect Judaic logic and also differ sufficiently from the conjunction rules of Western logic. Having more or less forced the given data into his simplistic scheme, he can now claim to have discovered an alternative "logic" in Jewish sources.

After that, Schumann proposes two more tables of similar construction, one for disjunction ("A or B") and another for implication ("If A, then B"), which I will spare you. Well, if you must know, the disjunction table is identical to the conjunction one, except for the first row, which tells us that "particular or particular" results in "general" (which makes sense only if we know that the two particulars are an exhaustive listing of alternatives). The implication table is identical to the conjunction one, except for the third row, which tells us that "if general then particular" results in "general" (which makes sense only if the terms of antecedent and consequent are the same).

Now, whereas the first table is in part based on explicit *midot*, Schumann gives no hint as to what Judaic texts or even just rabbinic practices these two additional tables are based on. He does give various examples as to how these tables might be interpreted, but these examples seem to be his own inventions, without basis (to repeat) in Jewish tradition. Furthermore, the examples he gives seem to me very arbitrary, with the sentence clauses he claims to be "particular" or "general" very doubtfully so. For my part, I have no recollection in my studies of any doctrine of disjunction or implication such as that which Schumann here proposes – correct me if I am wrong (I certainly do not

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[29]   Note, by the way, that since he makes no attempt to "formalize" *all* the *midot*, he can hardly claim to know for a fact that they "have nothing in common with Ancient Greek logics."

claim great erudition)[30]. So what we have here, again, are figments of Schumann's imagination presented as Judaic logic.

In short, in his effort to subject Judaic logic to "modern logical analysis" and to "formalize" it, Schumann *reinvents* "Judaic logic." He confuses fiction with fact. He lacks scholarship. He does not have a scientific, empirical approach, of patient sifting through a mass of given data and then tentatively summarizing it and then drawing conclusions from it. No: he decides the way it is, the way it ought to be, and that's that. He evidently does not have a clear idea in his mind as to the difference between description and prescription. Is he saying that these tables truly reflect the way the Talmudic rabbis actually thought – or are these tables intended to teach them how they should have thought? Is he accepting Talmudic logic at face value, or criticizing it? Perhaps he is not interested in history or religion, but is only intent in partly extracting from Talmudic sources a new logic for modern secular consumption. In that case, is he telling us that this new logic is better or truer than the one we have already – or is he just a relativist who considers all logics equally valuable or valid?

From what he tells us, with an impressive tone of authority, his view is that: "Judaic logic for inferring legislative statements from the Pentateuch is closed under connectives defined above." So it looks that Schumann takes it all very seriously and considers that he has correctly represented the logical connectives of Judaic logic. He then goes on to deal with the a fortiori argument as we saw earlier, first stating with characteristic bombast: "This inference rule differs from all conventional inference rules involved in a deduction procedure of classical and non-classical logics. It is connected with parallelism and non-recursiveness assumed in its using." So it also looks like he considers that he is teaching modern logic new tricks.

What is evident, in any case, is that Schumann is greedy for territory; he wants to stake his claim on Judaic logic, or the modern symbolic logic derivative of it, and take for himself the tacit title of its conqueror. If no single logician can do it, how can a pseudo-logician do it? His efforts remind me of a recent funny news item about some people laying claim to ownership of the moon or sun. With this, I am tempted to say, having already deconstructed his various claims: my case rests. But I notice he has another article, called "Judaic Syllogistics" (15p.), in the same volume, in which he proposes with his usual gusto "a formal syllogistics that verifies reasoning of the *Baba Qama*, one of the books belonging to the Talmud. The formal logical system of such kind is built for the first time. This system is a version of non-Aristotelian syllogistics."

Now, what can "non-Aristotelian syllogistics" possibly refer to? Presumably, it refers to syllogism of some sort – but to syllogism *not valid* by Aristotelian standards, i.e. under all three of his laws of thought (identity, non-contradiction and the excluded middle). What else could it mean, if appropriately named? Categorical syllogism (for example) is composed of two premises and a conclusion comprising three terms. There are four conceivable 'figures' of it, i.e. ways in which the terms might be disposed. Considering (for example) the four plural non-modal forms A, E, I, O, there are 64 (4x4x4) mathematically possible combinations of these propositions in each figure. As I show in my *Future Logic* (chapter 8-10), there are in all only 28 valid moods (13 primary and 15 secondary) in this sort of syllogism (out of 256 = 11%). All other moods are invalid, i.e. yield a non-sequitur or an antinomy. To claim any of the latter arguments valid, as Schumann apparently does when he boasts of "non-Aristotelian syllogistics," is not logic but sheer sophistry. Anyone who thinks by such means is not a logician but a sophist. Need anything more be said?

Or maybe by this term Schumann intends forms of *mediate inference* that were unknown to Aristotle (or at least not formally treated by him), perhaps having in mind hypothetical syllogism (as against Aristotle's categorical syllogism), hypothetical and disjunctive apodoses (i.e. *modus ponens* and *modus tollens*), and other sorts of inference through a middle term or thesis, including a fortiori argument. If so, why does he not say so; why does he opt for such a misleading term? Maybe he thinks that this makes him seem superior to Aristotle.

5. Not logic, but lunacy

Some months after writing the above analysis of Schumann's approach to a fortiori argument and Judaic logic, and logic in general, I happened to come across another essay of his posted on the Internet, which I think worth examining briefly because it is very revealing of the author's mental processes and motivations. The essay serves as

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[30]   I can here, offhand, point to one instance where the outcome of a disjunction is an issue. At one point in his *Studies* (chapter 8, p. 150), Jacobs discusses an expression found in a Gemara (*B.Q.* 2a-3b) that pertains to interpretation of a disjunction: "they are equally balanced and both of them can be included, for which will you exclude?" Just with reference to this case, one can see that the issue is more complex than Schumann pretends (i.e. it is not a mere issue of particularity or generality of the disjuncts). Moreover, looking at this example, it occurs to me that Schumann makes no distinctions between inclusive disjunction and exclusive disjunction, or between logical (*de dicto*) disjunction and factual (*de re*) disjunction.

Preface to a recent collective work called *History and Philosophy of Logic*[31]. Many of the things Schumann says in this essay are historically, philosophically and logically delusive nonsense, as we shall see. He thinks he is being clever, but is only making a fool of himself in public. The assertion of his that most caught my attention and incited my indignation[32] was the following:

> "In Aristotle's syllogistics we are moving from general knowledge to particular conclusions. For example, in the following syllogism
>
>> All men are mortal,
>> Socrates is a man;
>> therefore Socrates is mortal.
>
> we are concluding the facts, which can be verified by experience, from the less obvious general knowledge. Indeed, 'Socrates is a man' and 'Socrates is mortal' are singular facts which we can know from the direct observation or from testimonies borne by reliable witnesses. However, the statement that all people are mortal cannot be verified on experience. What is its truth then grounded on? Is that all people we knew did not live longer than 120 years? But we know not all people and on a broader scale it is a popular induction of type 'all swans are white' and 'all ravens are black'."

What we have here is a profound (though common enough nowadays) failure to understand the logical and epistemological role of the syllogism in human thinking. In Schumann's view, the singular minor premise and conclusion of the above syllogism are empirical facts, whereas the general major premise "cannot be verified on experience." According to him, then, all general propositions are groundless; claims of induction such as 'all swans are white' are unreliable (as is suggested by his "but we know not all people") and ultimately fallacious (as is suggested by his mention of 'all ravens are black', which is the statement used in Hempel's paradox of confirmation). There are many crass errors here. For a start, he presents the syllogism as a purely deductive tool ("moving from general knowledge to particular conclusions"), whereas in fact it plays a major role in induction. I have already explained this in some detail in the past[33] and will not repeat myself here. But briefly put, in the above example of syllogism, if the major premise were based on complete enumeration (as some claim), the argument would be circular, since the conclusion would be a necessary support for the major premise. The conclusion is indeed an inference from the major premise, because the latter is based on generalization. We believe that all men are mortal from past observation of all men dying; until Socrates dies, he cannot logically be used as a basis of such generalization, so his death is a prediction from it. Once, however, Socrates dies, he passes over from an inferred case to one of the many cases that buttress the generality. Notice the involvement of temporal factors in this understanding.

Schumann, on the other hand, presents the minor premise and conclusion as both known "from the direct observation or from testimonies borne by reliable witnesses." This means, for a start, that the syllogism he presents *involves no deduction at all*, since its "conclusion" that Socrates is mortal, being singular, is necessarily known by direct or indirect observation (for which, of course, we must wait till Socrates actually dies). The general major premise then becomes, in his way of looking at syllogism, totally inexplicable, playing no role in the emergence of our belief in the "conclusion." Moreover, he tries to put in doubt the validity of all generalization by pointing out (rightly or wrongly) that some people live for more than 120 years (as if this was the question, as if such people do not eventually die too), and giving as another example the claim that all swans are white (when of course we all know that some swans are black). As regards the justification of generalization, I have already written a lot about it[34], and will not repeat myself here.

Moreover, Schumann seemingly alludes to Hempel's "raven's paradox," trying to appear slick and knowledgeable. But as I have shown in the past[35] this so-called paradox is based on foolish misconceptions and easily resolved. Thus, Schumann's attack on "Aristotle's syllogistics" turns out to be just so much posturing, and a demonstration of his ignorance and stupidity. If we examine his own discourse, we observe that even he unconsciously uses syllogism. For example, when presenting the 'Socrates is mortal' syllogism as an example of syllogism in general, he obviously

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[31]   London: Taylor & Francis, 2011. The preface here examined (written: May 2010) is posted at: www.tandfonline.com/doi/pdf/10.1080/01445340.2010.506079. The full list of authors and articles in the book can be seen at: www.tandfonline.com/toc/thpl20/32/1.

[32]   I chided him about it by e-mail on 20-3-2012, but received no reply from him.

[33]   See the essay called 'Syllogism Adds to Knowledge' in my *Phenomenology*, chapter 7.4, posted online here: www.thelogician.net/2b_phenome_nology/2b_chapter_07.htm.

[34]   See for instance the essay called 'Generalization is Justifiable' in my *Phenomenology*, chapter 7.2, posted online here: www.thelogician.net/2b_phenome_nology/2b_chapter_07.htm.

[35]   See for this my *Logical and Spiritual Reflections*, book 1, chapter 8, posted online here: www.thelogician.net/6_reflect/6_Book_1/6a_chapter_08.htm.

expects readers to apply his critique concerning it to all other syllogisms they may encounter; and this is nothing other than an implicit act of syllogistic application!

The truth is that Aristotle's syllogism is essential to many aspects of human knowledge. It is obviously used (among other things) to include (first figure positive moods) or to exclude (first figure negative moods) observed percepts in/from concepts, or more narrow concepts in/from larger concepts. This function, as just briefly explained, may be deductive or inductive, according as we are using the syllogism to define the scope of a concept through sample percepts, or to make new predictions on the basis of past generalizations. Without this intelligent tool[36], we could not organize our knowledge or ensure its consistency, and would be faced with a mass of disorderly and potentially contradictory data. Syllogism is not something arbitrary, as Schumann attempts to imply. Aristotle did not *invent* this instrument of thought; he merely *discovered* it by observing his own thinking processes and the thoughts of others.

Schumann does further on say a few pious words about induction, but he does not identify this as the source of our major premises:

> "For the first time Karl Popper (1934) paid attention that there is an asymmetry between verification (modus ponens) and falsification (modus tollens): no general (empirical) statement can be proved or even definitively confirmed, but only it can be falsified [sic]. Thus, according to Popper giving priority to confirmation before falsification is not grounded logically. On the contrary, in scientific theories we should prefer modus tollens, instead of modus ponens."

Schumann, who poses (confusing name-dropping with scholarship) as a historian of logic and philosophy, should have known that it was, long before Popper, Francis Bacon[37] who "first" explicitly realized that a negative instance is sufficient to eliminate a scientific hypothesis, whereas an immense number of positive instances can only confirm it but never definitely prove it. We could also in this context mention the earlier Robert Grosseteste[38] and the later John Stuart Mill[39], and indeed many more. Even Immanuel Kant, whose main focus was (like David Hume's) the positive side of induction, shows some awareness of the negative side[40]. In a Jewish context we might mention Saadia Gaon[41], who wrote in *Emunot veDeot*: "Whatever leads to the rejection of the perception of the senses or rational faculty is false."

But in truth, we can probably find earlier roots than those[42]. The scientific method is a broad field of study, with a long and rich history; it certainly did not start from scratch with Popper as Schumann claims. Moreover, it is practitioners of science who have developed it the most even as they pursued their more specific scientific goals. Logicians and philosophers have developed their theories ex post facto, mostly by observation of scientists in action. Though sometimes, of course, the same person had both roles. Science developed for centuries, and indeed millennia, without need for Popper's guidance. And long before science, common man very often thought in a scientific manner, as he had to in order to at all survive on this planet. The scientific method is only a distillation of ordinary human cognitive means, a selection of the most reliable aspects of them.

In any case, from Schumann's presentation here it is clear that, though he mentions falsification[43], he has not grasped the significance of ***the absence of falsification***, i.e. of the power of confirmation inherent not only in positive instances, but in the fact of having *sought and not found* a negative instance. It is this absence of any negative instance that justifies generalization from the presence of positive instances. That is, *it is not the positive instances alone, but their conjunction with an absence of negative instance*, that logically allows us to generalize. This is

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[36]   Roughly speaking, the second figure serves to take note of differences between things and ideas or between ideas, while the third figure serves to notice common grounds between them.

[37]   England, 1561-1626; author of the *Novum Organon* (1620).

[38]   England, ca. 1168-1253, who (according to Freely, pp. 139-141) was "the first medieval scholar to deal with the methodology of science, which for him involved two distinct steps. The first of these was a combination of deduction and induction… The second step was what Grosseteste called verification and falsification… it was one of the basic tenets of his scientific method that if a theory was contradicted by observation it must be abandoned."

[39]   England, 1806-1873; author of *A System of Logic, Ratiocinative and Inductive* (1843). Note in his "method of difference" reference to "an instance in which a phenomenon does not occur" (see my *The Logic of Causation* for an analysis of his methods).

[40]   Prussia, 1724-1804. In *Lectures on Logic* (p. 176), he says: "By means of hypotheses one… assumes something, and investigates whether from it one can explain the known consequences or not; if the first occurs, then one accepts the hypothesis; if the latter occurs, one rejects it." (Tr. Michael Young. Cambridge: UP, 1992.)

[41]   Egypt-Irak, ca. 882-942. See my essay on this author earlier in the present volume.

[42]   We could even refer to the Torah, since the laws given Deut. 13:2-4 and 18:21-22 clearly exemplify respectively the positive and negative aspects of adduction, as I have explained in my *Judaic Logic*, chapter 2.2. See also Appendix 6 of the present volume.

[43]   We should not, by the way, get overly fixated on the fashionable word "falsification," or for that matter on the word "verification." These words, signifying the negative and positive aspects of research, are relatively recent (dating from about the 14th-16th centuries). Before their emergence, people no doubt used other words or phrases to signify that they were, by reference to experience and reason, checking out whether a certain idea was true or false, with the implicit intent to opt for it if found true and to drop it if found false. Also, the word "verification" is often intended in the sense of "falsification," or in a double (positive and negative) sense. Popper increased the separation between the two words so as to emphasize the difference between the positive and negative sides of research; but the underlying concepts they refer to were already there.

something known from way back, as evidenced by the statement by Jean Buridan[44] that the principles of natural science "are accepted because they have been observed to be true in many instances *and to be false in none*" (my italics). There are many people, still today, who do not understand this.

Schumann has obviously not understood this crucial point, since he presents the issue quite superficially, as a contest between the force of conviction of a negative instance and that of any number of positive instances. In truth, the modus ponens and modus tollens only occur simultaneously in the event of actual occurrence of a negative instance; as long as there is no such occurrence in a given case, only the modus ponens is relevant to it and the modus tollens plays no role. Thus, Schumann's claim that the priority of confirmation "is not grounded logically" merely reveals his own misunderstanding of inductive logic.

Positive instances are essential to induction, and the more of them the better; and most of the work of science relates to finding them, even if the scientist must always be on the lookout for any negative instance that might belie, or even just put in doubt[45], the generalities they suggest. The elimination of hypotheses that are inconsistent with empirical findings is just one aspect of induction, and certainly not all of it. Schumann's minimal interest in positive instances is indicative of his lack of acquaintance with the history of science. Positive data has always played a crucial role in the formulation and testing of theories. For instance, Johannes Kepler could not have traced the orbit of Mars in 1600 without the data previously collected by Tycho Brahe, and his more accurate prediction of the transit of Mercury in 1631 did much to gain support for his astronomical model as against Ptolemy's.

As regards Schumann's motivation in mentioning induction here, he apparently imagines that the Aristotelian method is antithetical to Popper's. He thinks that (as above exposed) Aristotle's syllogism is entirely deductive, and therefore irreconcilable with Popper's induction. But the truth is, they are complementary, Popper's induction providing the major premises needed for Aristotle's deduction; and indeed, in the opposite direction, syllogism is often used to make predictions from general hypotheses, in order to test them empirically. The reason why Schumann is unable to integrate the two is, ironically, because his own method is (as we have shown elsewhere) entirely "deductive" – being a figment of his imagination with no authentic relation to actual human reasoning processes. That is, while posing for the cameras as virtuously opposed Aristotle's deductive logic and favorable to Popper's inductive logic, his own method is even more blindly "deductive."

But aside from his questionable grasp of theory, let us take a look at Schumann's *practice* of induction. This is very telling. What we see him do right here, in the present essay (see further down), is hastily and erroneously generalize from a single example of syllogism with a totalitarian theme to "all totalitarian discourse is syllogistic," and moreover conversely to "all syllogistic discourse is totalitarian." There is no visible effort of falsification on his part. He does not ask whether any totalitarian discourse is non-syllogistic (whereas most of it is), nor whether any syllogistic discourse is non-totalitarian (whereas most of it is). On the basis of this one *content* of syllogism, and maybe a few more examples like it in his mind, he draws a general negative conclusion about the validity or value (the distinction is not mentioned by him) of *the form* of syllogism. Now, tell me – is this the practice of someone who has understood and assimilated inductive logic, or of someone who hasn't a clue? This is manifestly a tyro pretending to be an expert.

What becomes evident, watching the theatrical rhetoric of Schumann (and many others like him, to be sure), is that he is basically *against logic*. It is not just Aristotelian syllogism that he seeks to obliterate from our culture, but human reason as a whole. He poses as a hip and with-it proponent of non-Aristotelian logic – but Aristotle is not his real target; his real target is logic itself. Even though he labels what he does as logic, it is not only non-logic but anti-logic. He has obviously made no effort to study and understand true logic, but has all his life fed himself (or allowed himself to be fed) a diet of delusional nonsense dishonestly labeled as logic. Genuine logical theory, starting with Aristotelian logic, is an account of the means available to mankind to get in contact with reality. Schumann's pseudo-logical discourse is a formula for cognitive disaster; it does not get him or anyone in closer contact with reality, but sentences all who use it to the nuthouse. It is discourse emanating from and leading to acute mental derangement. It is not logic, but lunacy.

Evidently, he imagines it is all a power game that one can cynically play with impunity. He thinks that a theory of logic is something arbitrary established by authoritarian means. His failure to see the objective justification for the major premises of syllogisms is a cause of this attitude. He thinks that Aristotle's logic, comprising mainly the three laws of thought and the syllogism, was widely accepted merely because he spoke with authority. Schumann imagines that if he advocates another sort of logic, he may one day likewise be regarded as an authority. If all logic is arbitrary,

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[44]   France, ca. 1295-1358. He is quoted by Freely, p. 153.

[45]   I refer the reader to my essay "The Principles of Adduction" for a more detailed analysis of the nuances involved in the process of theory selection. It is not a mere matter of confirmation or rejection, but there are many situations in between, which strengthen or weaken a hypothesis in comparison with other hypotheses. See my *Phenomenology*, chapter 7.1, at: www.thelogician.net/2b_phenome_nology/2b_chapter_07.htm.

why not? Maybe he can get away with it. Look around: how many people who should know better are publicly objecting to what he says?[46] But what Schumann does not realize is that he is cognitively incapacitating and psychologically hurting himself as well as others. His writings are irresponsible.

We can see some of this in his ridiculous ideas about syllogistic argument being typical of Christianity and Communism, while excluded from Judaism:

> "While the Christian thinking is totalitarian and holistic, the Judaic thinking is massive-parallel and concurrent, it prefers a singular differentiation. For example, in Judaism there is differentiation between those animals which may be eaten and those which are unclean, between things devoted to the temple and not devoted, etc. This feature of the Judaic thinking has entailed the impossibility of using Aristotelian deductive syllogistics and in fact the latter has been never used in Judaism."

It should be mentioned that Schumann is from Belarus and of Jewish origin, so no doubt he and his forebears have suffered considerably at the hands of the totalitarian regimes that have plagued that country and region in the recent and more distant past. But that does not justify making empirically inaccurate statements. Firstly, Christianity is not intrinsically totalitarian[47]. Even if it has in the past been pretty nasty (to put it mildly) in various times and places, today in the various countries of Western Europe and North America that I have lived in, and in particular in Switzerland where I happily currently live, no such characterization is remotely appropriate. Most Christians (whether Protestant or Catholic), or people of Christian origin, in the West can safely be said to be essentially liberal and even libertarian; and let us not forget it is their recent forebears who conceived of and established our modern political freedoms[48], including the freedom to be Jewish without persecution or even discrimination.

I do not know about Eastern Europe nowadays, maybe things are different there; but in any case, Orthodox Christianity is not the whole of Christianity. I do not of course mean to imply that anti-Semitism has disappeared in the West. It is present enough, inexcusably so, mostly in the guise of rabid anti-Israeli propaganda and activism, in many mainstream media, among many (mostly leftwing) politicians and academics, and in many grassroots organizations, including some with Christian orientation. But this is not totalitarianism. If any religion in the world today is totalitarian, it is surely Islam[49]. Most of its preachers, it seems[50], promote the blind hatred, oppression and murder of Jews, Christians, and any other non-Moslems, and even recalcitrant Moslems, that their followers (who are relabeled as 'Islamists' once they concretely commit themselves to this course) can get their bloody hands on. Only God knows how long more this terrorist plague on our poor planet will last, for it will not stop till the Moslem peoples themselves get wise and energetically suppress all those in their midst who advocate or engage in such extreme intolerance.

Both Christianity and Islam – and indeed Judaism too – have given the world many famous scientists and philosophers of science. I have already mentioned Grosseteste, an important contributor to the scientific method, who was a Christian bishop[51]. We could also here mention Albertus Magnus[52], a monk and later a bishop, who (it is interesting to note in the present context) wrote: "Syllogisms cannot be made about particular natures, of which experience alone gives certainty," by which he surely meant that knowledge of nature cannot be obtained through reasoning alone but mainly relies on experience. And there are countless more, such as Gregor Mendel[53], the founder of the modern science of genetics, who was an Augustinian monk. To depict Christianity as inherently syllogistic, deductive, unscientific and totalitarian is surely a distortion. Islam, too, in the first few centuries of its existence produced important contributors to logic, philosophy and science, even if today it is more intellectually paralyzed than it has ever been, with some of its preachers even still claiming the earth is flat and the sun revolves around it![54]

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[46] I marvel at the incompetence of publishers who publish such drivel. It goes to show the low level knowledge and understanding of their editorial staff.

[47] I will not bother here to discuss Schumann's silly claim that Christianity is distinctively "holistic." For a start, he misuses that word, when by it he means monistic, or pantheistic if not monotheistic (since he quotes Colossians 3:11, "the Messiah is all, and in all," and the messiah in Christian belief is the same as God). Secondly, these ideas are not peculiar to Christianity; many similar ideas can be found in earlier religions (Judaism, Hinduism) and philosophies (Xenophanes, Parmenides). He obviously does not know what he is talking about. He should seriously study the history of ideas before having the pretention to pontificate on the subject. He should also improve his English.

[48] For instance, the founding fathers of the U.S.A. were Christians.

[49] *Sharia* (Islamic law) is nothing other than an instrument of total control, penetrating every nook and cranny of the lives of people living under its power.

[50] Judging by videos and texts seen on the Web over the years. See the many examples in jihadwatch.org, palwatch.org, and memri.org, among others. Note that these remarks of mine are written only a few days after the wanton murder of defenseless Jewish children by an Arab jihadist in Toulouse, France.

[51] Roger Bacon (c. 1214–1294), who is also credited with advances in scientific method, and who seems to have been a student of Grosseteste, was a Franciscan friar.

[52] Germany, ca. 1200-1280. He was canonized in the 20th century and made "patron saint of all those who cultivate the natural sciences."

[53] Austria-Hungary, 1822-1884.

[54] The paralysis of Islamic thought since the middle ages can be attributed mainly to the limitations on freedom of thought and speech, under the guise of anti-blasphemy laws. At first, Islam showed considerable intellectual energy; but in time the most reactionary elements in

With regard to the question as to why, despite the insights of medieval Christian philosophers of science like Grosseteste and Roger Bacon, science did not progress much at the time, Kneale and Kneale (who were genuine scholars and historians of logic) proposed the following answer: "The chief obstacle to steady scientific progress was not the influence of Aristotelian logic or anything else derived from Greece, but a lack of sustained curiosity about things which were not mentioned by ancient authors and did not appear to contribute in any way to salvation" (p. 241).

Secondly, use of the syllogism is not limited to totalitarian régimes as Schumann alleges. He gives the following inane illustration of the syllogistic thinking he considers typical of such régimes[55]:

> "Enemies of Soviet people must be taken into camp,
> Non-content with Stalin's policy are enemies of Soviet people;
> Therefore, non-content with Stalin's policy must be taken into camp."

But this is nothing special; it is just subsumption of a particular case under a general rule. Similar rules and reasoning occur in democratic societies. For instance: 'people who ride on a bus without a ticket must pay a fine; you did so, so you must pay up!' Similar arguments can be formulated in any field of human endeavor, even in sports. Only the terms are different; the form of reasoning is the same, and there is nothing intrinsically dreadful about it.

I would have rather thought that the difference between totalitarian and democratic régimes lies in people being forced to obey arbitrary decrees emanating from a powerful minority. Such obedience involves the following reasoning: 'if I do not do what I am told to do, I will be severely hurt; therefore, although I don't want to, I will do it'. This is apodosis, not syllogism. But even if we call it syllogism in a broad sense, it is certainly not the formal essence of syllogism, but merely a particular case of it, one with specific terms. Moreover, such reasoning concerns the victims; as regards the dictators and their henchmen, they rarely think rationally. The Nazis were furiously anti-rational, in the name of pre-Christian obscurantism. The Communists may have occasionally paid lip service to science and progress, but in actual practice were just as moved by unbridled passion, imprisoning or killing anyone they imagined to be a menace to their power.

Thirdly, Schumann's claim that such thinking is an "impossibility" in Judaism and has "never been used" in it is quite untenable. Judaism is certainly filled with rules (*mitzvoth*) that have to be obeyed under threat of punishment (and with promises of reward). Surely more than in the Christian religion, though perhaps less than in the Moslem one. Moreover, even if the various lists of hermeneutic principles developed during and after Mishnaic and Talmudic times do not explicitly mention syllogism, syllogistic thought processes underlie many of the rabbinic rules. As I explained in my earlier work, *Judaic Logic*, any inclusion or exclusion of a particular in/from a generality is a mental act of syllogism[56]. Moreover, several of the rules of Rabbi Ishmael having to do with harmonization are clearly syllogistic[57]. Many people fail to see that, because each such rule involves two syllogisms in series, with a total of four terms; but the fact is undeniable. Additionally, in earlier chapters of the present work, in a section on analogical argument (5.1) and in one on the syllogistic middot (8.8), I show how the rabbinic rules of *gezerah shavah*, *binyan av*, and others, rely on syllogism. Thus, syllogism is very present, even if often only implicitly, in rabbinic thinking.

Fourthly, contrary to Schumann's claim, the thought process of differentiation is not specific to Judaism, but found equally in Christianity, Islam, Communism, Nazism, Democracy, astrology, science, sports and what have you. To take an extreme example, Nazi "selections" at the gates of death involved differentiation. Just as no human thought is possible without syllogism, so none is possible without differentiation[58]. It is not an ideological matter, but a cognitive one. To classify things, we must make distinctions between them – observe what features each thing has and what each lacks. To make decisions, and proceed to act on them, we must be aware of differences as well as commonalities. Syllogism can be used equally well for integration and for differentiation. Schumann's attempt to reserve syllogistic reasoning to Christianity and Communism and differentiation to Judaism is totally unfounded.[59]

Islamic societies imposed their will, stifling any velleity of independent thought. This has been the situation in the countries concerned for hundreds of years.

[55]   Note that, though he refers to it as "often used," this example is made up by Schumann: he is not quoting any historical document or speech.

[56]   See discussion there in chapter 9.1 (on the elucidation of terms) and chapter 10.3 (on the scope of terms). These rules are not exclusively syllogistic, but certainly include much syllogistic thinking.

[57]   See chapter 11 there. That such rules are not always valid, i.e. in accord with the formal canons of syllogism, is another issue. An attempted syllogism is an object of study under the heading of syllogism (in a broad sense) even if it is found invalid.

[58]   In Schumann's own earlier example, concerning Soviet practice, there is differentiation between people "non-content with Stalin's policy" and the content, between "enemies of Soviet people" and non-enemies, between people "taken into camp" and those not taken. Or again, Schuman's distinction here between, as he has it, Christian syllogistic thinking and Judaic thinking by differentiation, is itself an act of differentiation, though not one given in Judaism.

[59]   As for his statement that "the Judaic thinking is massive-parallel and concurrent" – we have already seen in a previous section the meaninglessness and invalidity of this "massive-parallel and concurrent" thingamajig. The words refer to an imaginary multiplication of a logic formula he alleges is used in Judaism, which we however found to be a quite inaccurate representation of actual practice. It is a hasty

Therefore, Schumann's claims concerning Judaism are also just so much balderdash. Evidently, he is settling personal scores not only with Christianity and Communism, but also with Aristotelian logic, when he tries to associate them. As regards Judaism, he maybe imagines he is doing it a favor by associating it with his brand of non-Aristotelian "logic," which in his estimate is a more advanced level of logic; but he is in fact just giving Judaism a bad name, making it seem irrational and even anti-rational. He is rather in fact 'using' Judaism's good reputation, as at least somewhat logical, to try to plug his pseudo-logic as a logic with valuable practical applications[60]. The truth is that nowhere does he in a scientific manner substantiate any of his offensive assertions; all his fancy claims are arbitrary and moreover nonsensical. Like the contemporary avant-garde "artists" whose works are exhibited in costly museums, he relies on bluff.

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generalization from a false observation. In Schumann's mind, it is something established, as real to him as a perpetual motion machine is no doubt to the dreamer who, having seen it move repeatedly for a while in his mind's eye, thinks this implies that it can move perpetually in the real world. If anything is massive here, it is his incompetence.

[60]   Just as he 'uses' contributors to the collections of essays he edits and prefaces. His introductory remarks are given some appearance of legitimacy, by being published side-by-side (and indeed in a cadre position) with other people's more thoughtful work. Those thus 'used' don't mind too much, so long as they are getting published. And the publishers don't care, as long as they look set to make money. 'To use' here means 'to exploit', as against 'to serve'.

28. Allen Wiseman

Allen Conan Wiseman wrote in 2010 an extensive study of a fortiori argument, entitled *A Contemporary Examination of the* A Fortiori *Argument Involving Jewish Traditions.* This was not yet published in print form, but was made freely available in pdf format on the Internet[1]. I came across it there, quite fortuitously, in mid-October 2010; and I found it interesting enough to immediately pass the information on to some two dozen academic correspondents.

My present work was directly inspired by Wiseman's felicitous idea to devote a whole book to this one topic[2], surveying and discussing contributions to the field, and proposing insights and analyses of his own. Had he not written his treatise, I would not have written mine. I was aware at the time of a few papers on a fortiori argument[3]. But, although I disagreed with their authors in various ways, I felt no call to respond in writing to their work. It was only thanks to Wiseman's study that I realized that there were many more contributors to the field, and that I was duty bound (as a contributor to it myself) to respond to them all, including Wiseman.

1. Definition and Moods

Let us examine and assess Wiseman's contribution. Having carefully read the whole work, I can safely say that its main goal is to contest the rabbinical "*dayo* principle" (as he sees it), as an unreasonable and unfair limitation on natural a fortiori argument even in many religious contexts. More than half the book is devoted this topic, and the other half prepares the ground for its discussion. To defend his main thesis, Wiseman introduces and develops two distinctive general theses: a belief in "inductive a fortiori argument" and a belief in "proportional a fortiori argument." These overriding theses are both based on his peculiar definition of a fortiori argument – or, I should say it outright, his wrong definition of it. We shall first examine his definition of a fortiori argument, and the moods he develops around it, then the two theses he bases on it, and lastly his treatment of the *dayo* principle by means of them.

Wrong definition of a fortiori. Wiseman's working definition of a fortiori argument is consistent throughout his work. In the abstract, already, he describes the argument as follows: "Typically, the argument claims this: if a lesser (or greater) case has a feature, a correspondingly greater (or lesser) case has that feature too." In his introduction, he presents the argument as composed of two premises (P1, P2) and a conclusion (C), as follows:

> "P1:   **A is greater than B.**
> P2:   **B has [feature] f.**
> C: **A has f* too.**"

Regarding the "superlative relation" of A and B, he tells us that "one is ranked higher while the other lower *along a continuum within some common category*" (my italics) – which indicates a recognition that there is a middle term, even though this is not explicitly mentioned in the formula. However, his middle term appears (even if mostly implicitly) only in the major premise (P1); there is no reference to its

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[2]   Of course, Schwarz had long before devoted a whole book to *qal vachomer* argument (and another to *gezerah shavah*, for that matter), but I was not aware of his work till I came across Wiseman's.
[3]   Notably those of Lenartowicz and Koszteyn (2002), of Abraham, Gabbay and Schild (2009), of Goltzberg (2010), and of Schumann (2010).

important role in the minor premise (P2) and conclusion (C). Regarding the conclusion he adds: "The [feature] f* here may be *the same as or proportional to* f" (again, my italics). This formula for a fortiori argument recurs throughout the treatise[4].

At the end of his first chapter, he proposes the following "general definition" of a fortiori argument: "For congruent items, when one of the ranked items in the comparison has a feature, due to the (likely) heritable property, the other probably or surely has it to some degree too." His definitive, "reworked definition", which is presented in his Conclusion, goes like this: "The *a fortiori* argument compares two ranked items in a continuous, common category, one of which has a key feature, to conclude that the other likely has a form of the feature, which only in heritable cases is deductively valid and sound in that it surely has the same feature or its reasonable ratio."[5]

From these many sources, we can formulate the following composite picture he has of a fortiori argument: 'If A is more x than B, and B is y, then – probably or surely – A is equally y or more than y'. The 'probably or surely' part refers to the distinction between inductive and deductive a fortiori argument; while the 'equally or more than' part refers to the distinction between non-proportional and proportional a fortiori argument. Thus, his definition is intended to cover all contingencies, as he sees them, relating to such argument.

Additionally, Wiseman describes a fortiori reasoning by means of diagrams, and many examples. His main diagram consists of a horizontal line representing the "scale" (x) along which the major and minor terms (A and B) are compared – i.e. it is the middle term. The feature y being found at one point in this scale (B) can be expected to be found in equal (y) or greater measure (y+) at a later point (A). Thus[6]:

        Scale x ----- B ----- A ----- →
                      ┊   ┊
                      y   y+
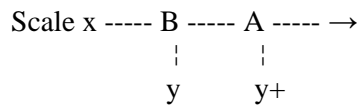
Further on[7], he draws two graphs to show the difference between a "same" conclusion (y) and a "proportional" one (y+). In the former, the x-y curve is a perpendicular line (meaning there is one value of y for all values of x), and in the latter the x-y curve is an inclined line (meaning there are different values of y for the various values of x).

It should be obvious by now why Wiseman's definition of a fortiori argument is sorely mistaken. Although, to his credit, he implicitly or explicitly does refer to the middle term (x) in the major premise (i.e. as the scale of comparison between the major and minor terms, A and B), he nowhere mentions the crucial role that this middle term is called upon to play in the minor premise and conclusion! His "a fortiori argument" is therefore no more than quantitative analogy.

He nowhere mentions that there is *a threshold value* of the middle term, which must be crossed before the subsidiary term (the feature y) can be predicated of either subject (B, and then A). Yet this is the basis of the whole a fortiori inference! The fact that 'A is more x than B' does *not* by itself allow us to infer that, if 'B is y', then (probably or surely) 'A is y' (or more than y). The inference is in truth based on the understanding that if 'B is *x enough to be* y', then 'A must be x enough to be y' (from which we can educe that 'A is y'). Given the major premise, if the "x enough to be" clause is missing from the minor premise, it is logically equally possible that 'A is y' or that 'A is not y'. Since 'A is y' is not formally implied by 'B is y', it follows that 'A is not y' is formally compatible with it.

Wiseman simply has not assimilated this crucial subtext. In one place, he does appear to realize it, when he gives as example the marks for an exam, taking a mark of 50 or less as a failure and any mark over 50

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[4]   See pp. 2, 47, 52, 66, 69, 140.
[5]   On p. 44 and p. 213, respectively. I marvel at the reference to a "heritable" property. Wiseman appears, on p. 92-3, to explain the concept with reference to mathematical recursion theory, naming Frege and Russell; but in the present context this is just a smokescreen. Do properties come with a label saying "I am heritable"?! How is this feature to be determined, exactly? He does not say. I view the use of this adjective as an attempt to give the impression that his definition formally accounts for deductive cases. It is a hidden circular argument, a sleight of hand. Surely, it is precisely the function of a fortiori argument to determine under what logical conditions properties are "heritable."
[6]   See pp. 37-40. Also, p.68, p. 130.
[7]   On p. 64.

as a pass, and then remarks: "Nothing matters except the mark above or below the critical turnover point"[8]. But in the rest of his work, and in his verbal and symbolic definitions of a fortiori argument, he does not take this lesson to heart. And this is, I must say, very surprising, considering that Wiseman has read and apparently understood my analysis of the argument, which clearly includes the issue of sufficiency (or insufficiency) of the middle term as an essential factor. It would seem therefore that, to his mind, the words "R enough to be" (or "not R enough to be") were redundant! He never internalized their crucial significance.

This is evident even in his diagrams. Although he states that his diagrams "adapt some of Samely and Sion's ideas," it looks like he relied on the former more than on the latter – because the diagram I had in my *Judaic Logic*, and which I reproduce in the present volume unchanged[9], included for each mood of a fortiori argument a comparison of *three* points (Rp, Rq, Rs) on the continuum (R). Even though Wiseman reproduces my three-point diagram for positive subjectal argument when he presents my theory[10], when he presents his own theory he only compares *two* points, completely ignoring the crucial third point which serves to correlate these two points. Yet, without an intermediary, there is no way to infer anything.

Even the examples he gives are deficient. For instance, he argues: "If I can lift a kilo with one finger, surely I can lift the same (or more) with my entire hand," whereas the a fortiori argument involved is really: 'A whole hand is (obviously) stronger than the fingers constituting it. If one of my fingers is strong enough to lift a kilo, then my whole hand is strong enough to lift a kilo (or more)'[11]. Wiseman often (too often!) refers to examples. He does so not only to illustrate but, I suspect, also to understand. Yet, as far as I can see, it did not occur to him to construct a deviant example, which would show up the absurdity of his definition. For instance: 'Jack (A) is stronger (more x) than Jill (B). If Jill (B) is beautiful (y), then Jack is surely as beautiful or more so (y)'. Obviously, such argument is nonsense. Why? Because no connection is indicated between the scale of comparison (x) and the predicated feature (y)[12].

The middle term must without fail be specified (or at least clearly tacitly intended) not only in the major premise (as the link between the major and minor terms), but also in the minor premise and conclusion (as the link between the subjects and predicates in them) – otherwise, we have no basis for inference, and the conclusion is arbitrary. We cannot be sure that the middle term mentioned (or intended) in the major premise plays any role whatever in the minor premise and conclusion; the latter may contain another 'middle term' or none at all, for all we know.

Judging by his definition of a fortiori argument, Wiseman has not fully understood this crucial point – and, as we shall see, this causes him countless problems throughout his treatise.

The moods of a fortiori. As regards the moods of a fortiori argument, the following should be added. Looking at Wiseman's above definitions, it is clear that what he has in mind is the paradigm of a fortiori argument, i.e. the positive subjectal mood. A and B are subjects and x and y are predicates. He does not notice, however, that many of his examples are in fact positive predicatal. For instance, the above given example, "If I can lift a kilo with one finger, surely I can lift the same (or more) with my entire hand," is positive predicatal in form, since the subject ('I' in this case) is the same in antecedent and consequent, while the predicates differ (lift a weight with one finger or with whole hand). In positive subjectal argument, remember, the subjects differ and the predicates are essentially one and the same (even if we admit of 'proportionality').

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[8]   On p. 68. On p. 40, he states that "the stronger can have or do what the weaker cannot or does not, because the stronger passes the weaker's (current) maximum value or threshold." But he does not take this momentary insight any further. He does not tell us precisely *what* the "maximum value or threshold" refers *to*. Instead, he offers as vague explanation an illustration: "human babies can have baby teeth, but they lack adult teeth." He apparently means that babies have lesser means of survival than adults – until they get strong teeth, at least. But he does not clarify or develop the idea of a threshold.

[9]   As Diagram 1.1, in chapter 1, in the section on Validation.

[10]   On p. 83.

[11]   This is putting the argument in positive subjectal form. In positive predicatal form, it would read: 'If I am strong enough to lift a kilo with one finger, then I am strong enough to do so (or more) with my entire hand."

[12]   That is, only given that 'Jill is (strong enough to be) beautiful' could we infer from the major premise that 'Jack is (strong enough to be) beautiful'.

The narrowness of Wiseman's formal definition comes as a surprise, considering that, in his analysis of my theory of a fortiori argument[13], he reviews my positive predicatal mood in as much detail as he does my positive subjectal mood. But I notice that he there misunderstands the expression 'from major to minor', which means from the major term (P, in the minor premise) to the minor term (Q, in the conclusion), as meaning: "from major (greater) to minor premise." The latter interpretation is of course absurd, and may be merely an error of inattention – but it is still indicative of some confusion on his part. In any case, his definition makes no mention of positive predicatal argument. Nor does it account for the negative moods of either subjectal or predicatal argument. Here again, although Wiseman does duly note in his description of my work that I include negative moods of a fortiori argument[14], he does not himself explicitly include them in his definition. It is also noteworthy that nowhere in his review of my work does he mention the derivation/validation of the negative moods by reduction ad absurdum from the positive moods. That he failed to notice out loud something so important is significant.

As regards his definition, I guess that Wiseman considers (though he nowhere tells us so) that when he says of some subject that it "has feature f," this includes cases where it "has feature *not*-f" – i.e. by the term feature, he tacitly intends negative as well as positive features. This may seem to exonerate him, but in fact it does not, because the positive and negative subjectal moods of a fortiori argument differ radically, in that the former proceeds from minor to major, whereas the latter proceeds from major to minor. This difference of direction is not a trivial detail, but makes all the difference between a valid argument and an invalid one. We could give Wiseman the benefit of the doubt here again, by referring to the definition in his abstract, viz. "Typically, the argument claims this: if a lesser (or greater) case has a feature, a correspondingly greater (or lesser) case has that feature too." Perhaps what he has in mind here is that in positive cases the inference will go from the lesser to the greater, and in negative cases it will go the other way.

However, this would be a too lenient judgment. First, because his more elaborate definitions (the "general definition" and the "reworked definition" above cited) *do not at all mention* the issue of direction of inference. They refer two "ranked items" and tell us that if one has a feature then the other one will (probably or surely) have it too, or "a form of" it, "to some degree." Secondly, because the graphs he uses to represent a fortiori argument, clearly allow motion in either direction (i.e. up or down the curve, even if it has outer limits). Indeed, he says it out loud: "Of course, we could reverse course, to begin higher and end lower down"[15]. And this is the way it *must* be, logically, if one's understanding of a fortiori argument does not include the concept of a threshold. Additionally, there are many more places throughout his work where it is obvious that Wiseman considers either direction of inference as okay.

It is true that his above-mentioned symbolic formula, viz. 'If A is greater than B, and B has f, then (probably or surely) A has f (or more),' suggests a unidirectional inference, from minor (B) to major (A). But this is clearly only intended as the typical or most common case, as is evident from Wiseman's subsequent listing of "alternate arrangements of QC terms" (QC refers to *qal vachomer*, which is the Hebrew term for a fortiori argument)[16]. There he presents what are, in his view, effectively, the various valid moods of a fortiori argument.

There, he mentions, as a valid mood: "If the lesser has the feature, surely (or very likely) the greater has the same, equal feature (or more of the feature);" and he also mentions, as an equally valid mood: "If the greater has the feature, surely (or very likely) the lesser has the same feature (or less of the feature)." The first of these is minor to major; but the second is, note well, major to minor. Wiseman also there proposes negative moods, which he refers to as "non-attaining cases." These are: "If the greater fails to attain a feature, surely (very likely) the lesser fails with the same feature (or fails it even more);" and "If the lesser fails to attain a feature, surely (or very likely) it fails equally (or more) with a greater feature." The first of these is major to minor; but the second is, note well, minor to major.

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[13]   Pp. 81-7.
[14]   P. 82, text and footnote.
[15]   P. 64.
[16]   Pp. 36-40. Most of the moods I mention here merge together two of his moods, for brevity's sake.

Notice that all these moods are subjectal in form. Thus, there is no doubt that, for Wiseman, a fortiori argument, whether positive or negative, may go in either direction. Basically, he accepts these moods, or "variants," because they seem intuitively reasonable to him. However, he does make a small effort at validation, in that he presents a diagram and an example for each mood. The problem is that these diagrams are, as already explained, logically deficient, in that they only compare two points on the scale, instead of three – i.e. there is no reference to a threshold. That explains why Wiseman is able to regard as valid reasoning the positive-subjectal, major-to-minor mood, and the negative-subjectal, minor-to-major mood. Such reasoning is characteristic of quantitative analogy or pro rata argument, but it is not possible with a fortiori argument.

To make matters worse, Wiseman proposes in the same context another set of positive and negative moods, in which the relations are more complex. One is: "If the greater has the feature, surely (or very likely) the lesser has more of the feature;" in this case, he is presumably thinking of inverse proportionality. Another is: "If the greater has the feature, surely (or very likely) the lesser fails to have that feature;" here, he apparently has in mind a breach in the continuity of the scale. Yet another is: "If the lesser has the feature *f* (as a defect), surely (or very likely) the greater has less (or fails to have) *f*;" this is really a more material case, because the concept of "defect" is not formal. Then there are the negative complements of these. These various additional moods are, however, superfluous. The first one, concerning inverse proportionality, is just a special case of direct proportionality. The second, concerning limited continua, can be dealt with by means of conditional a fortiori argument; i.e. within certain limits, this continuum applies, but beyond them, it fails. The third, as already pointed out, is a more material case. So this segment of Wiseman's attempt at a listing of moods is rather forced. And anyway, Wiseman's treatment of such special cases is far from exhaustive.

The upshot of all this is that Wiseman effectively considers all moods as valid. He does not identify certain moods as *invalid*. His view of a fortiori argument is so vague, so open, that anything goes. He would probably not admit it if confronted, but it summarizes what is implied by his doctrine of "alternate arrangements of QC terms." According to this doctrine, so long as one follows the given x-y curve, one can infer, positively or negatively, from A to B or from B to A. This result is, to repeat, due to Wiseman's signal failure to grasp the significance of the threshold value of the middle term. In other words, Wiseman has confused a fortiori argument with pro rata argument.

We shall presently go deeper into Wiseman's view of a fortiori argument, by analyzing his ideas of "inductive a fortiori argument" and "proportional a fortiori argument." We may view these ideas as derived from his broad, i.e. vague, definition – or, alternatively, we may view his definition as having been tailor-made to fit these ideas, which he needed to support his thesis concerning the rabbinical *dayo* principle. Probably, the genesis occurred both ways.

Stringent and lenient. But first, let us look more closely at another set of moods of a fortiori argument, which Wiseman develops further on[17]. This doctrine focuses on the words 'stringent' and 'lenient' commonly found in rabbinic formulations of a fortiori argument, such as that by R. Feigenbaum: "Any stringent ruling with regard to the lenient issue must be true of the stringent issue as well; [and] any lenient ruling regarding the stringent issue must be true with regard to the lenient matter as well"[18]. Wiseman tries to approach this formula more generally and systematically, by considering all conceivable permutations. He accordingly lists the eight "a fortiori possibilities" tabulated by me below:

| | Minor premise | | Conclusion | | My comment |
No.	From:	In matter:	To:	In matter:	Validity?
(1)	stringency	less serious	stringency	more serious	Minor to major
(8)	stringency	less serious	leniency	more serious	Contrary to (1)
(6)	stringency	more serious	stringency	less serious	Undetermined
(5)	stringency	more serious	leniency	less serious	Undetermined

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[17]   This doctrine is on pp. 113-4.
[18]   *Understanding the Talmud*, p. 88.

(4)	leniency	more serious	leniency	less serious	Major to minor
(7)	leniency	more serious	stringency	less serious	Contrary to (4)
(2)	leniency	less serious	leniency	more serious	Undetermined
(3)	leniency	less serious	stringency	more serious	Undetermined

Wiseman numbered the eight moods (1) to (8), but I have reordered them more logically, finding his list incomprehensibly disorderly (judge for yourself). The two shaded rows (numbered 1 and 4) represent the two rabbinical moods just cited. These two can be validated in the form of standard subjectal a fortiori argument, respectively positive (minor to major) and negative (major to minor):

> Mood (1): P is a more serious (R) matter than Q;
> and Q is serious (R) enough to be subject to stringency S;
> therefore, P is serious (R) enough to be subject to stringency S.
>
> Mood (4): P is a more serious (R) matter than Q;
> and P is serious (R) not enough to be subject to stringency S;
> therefore, Q is serious (R) not enough to be subject to stringency S.

Note that the negative predicate "is not subject to stringency" is subsequently read positively as "is subject to leniency." Also note that the subsidiary term S, whatever its level of severity, is identical in minor premise and conclusion, which suggests that no proportionality or *dayo* principle is invoked by the rabbis in these two cases.

Now, given the validity of mood (1), it follows that mood (8) is invalid, since it advocates the exact opposite conclusion. Likewise, given the validity of mood (4), it follows that mood (7) is invalid. The remaining moods cannot be similarly validated or invalidated; so their putative conclusions are all equally undetermined. This means that we may without contradiction come across the positive conclusion, or alternatively the negative one, in conjunction with the same premises. This is how I would present the eight moods; and it is clear from this presentation why the rabbis generally list only two of them – because they are the only two valid moods in this set of eight.

Wiseman, on the other hand, classifies the moods numbered (1) to (5) as "normally acceptable," and moods (6) to (8) as "unacceptable." He does this merely intuitively, without any formal process of justification – which is why he makes some errors, as we shall now show. Moods (7) and (8) are, I agree, "unacceptable," because they are formally invalid, since their conclusions contradict those of moods (4) and (1), respectively.

But what of mood (6) – what is wrong with that? Nothing at all, since it is quite conceivable and often happens that a stringency applies to all cases of a kind, the more serious and the less serious, indiscriminately. Indeed, mood (1) attests that stringency may be general. Moreover, if Wiseman considers mood (6) as "unacceptable," why not consider mood (2) as equally so? Symmetry would demand that both get the same treatment. There is, of course, also nothing wrong with mood (2), since it is quite conceivable and often happens that a leniency applies to all cases of a kind, the less serious and the more serious, indiscriminately. Indeed, mood (4) attests that leniency may be general.

The reason why Wiseman, "on grounds of fairness and justice," refuses mood (6) and accepts mood (2), is that he views, as it were, the former as passing stringency downhill and the latter as passing leniency uphill, and he believes, on the basis of an alleged general rabbinic injunction to "Be lenient in judgment[19]," that stringency should not be propagated, whereas leniency may be propagated. He argues that "punishment should fit the crime, rather than undervalue severer cases with minor penalties or commit injustice when minor misdemeanors receive overly severe judgements. To treat serious crimes lightly or

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[19]   P. 108. He refers to "*Pirkei Avot* (Sayings of the Fathers)," but does not cite chapter and verse. I have looked for but not found this precise sentence. Perhaps he had in mind 1:6, where Joshua ben Perahyah says "Judge all men charitably" (*dan et kol haadam lekhaf zekhut*). If so, the meaning is not the same; this is obviously intended as a general ethical recommendation, and not as an injunction intended specifically for rabbinical legislators and judges. Wiseman again mentions "be lenient" on pp. 168, with some other citations.

light crimes too harshly undermines the very notion of fairness and justice (unless deeper, ethically right and justifiable reasons override)."

But this interprets these two arguments as *generating* respectively increased stringency or leniency, whereas all they are doing is *discovering* respectively a stringency or leniency that is already there. It is of course very often true that different degrees of a crime deserve proportional punishments, measure for measure; but it is not always true – sometimes, the punishment is the same for all cases. For that reason, logic cannot impose proportionality indiscriminately, but must allow for equality. In any case, since in fact the given premises do not formally determine the said conclusions, there is no passing on of stringency or leniency in these two moods.

The next question is: why does Wiseman consider moods (3) and (5) as "acceptable"? The reason he does so is that he wants to legitimatize 'proportional' a fortiori argument, *contra* Maccoby who would "disqualify" such reasoning. And indeed, proportionality is not formally excluded. But this does not mean that a fortiori argument *per se* ever implies it; it just means it is compatible with it. So Wiseman's "acceptable" should not here be taken to mean valid. Nevertheless, Maccoby was wrong to regard these two arguments as formally invalid.

Notice that moods (6) and (5) have *the same* premises, yet the first yields an 'equal' conclusion (same stringency for the less serious matter) while the other yields a 'proportional' one (less stringency for the less serious matter). Similarly, moods (2) and (3) have *the same* premises, yet the first yields an 'equal' conclusion (same leniency for the more serious matter) while the other yields a 'proportional' one (less leniency for the more serious matter). This is why both moods in each pair must be classed as undetermined, because either outcome is logically possible.

From this analysis, we see the informality of Wiseman's approach to the moods of a fortiori argument and how such informality can lead to errors. As for Wiseman's concerns in this context, regarding proportionality and the *dayo* principle, more will be said further on.

2. Inductive a fortiori

One of Wiseman's main theses is that a fortiori argument ranges in reliability from the inductive to the deductive. As he puts it in his abstract: "While the argument aspires to be true and it can be deductively valid in those cases where heritable properties recur, it is more likely to be inductively probable." He expends quite some effort arguing in favor of induction; and he is right to do so, because many people still today do not realize the important role it plays in our lives. Induction produces various degrees of certainty, from low to high; and deduction is its summit:

> "Although fallible, induction is a broad, sufficiently reasonable form of argument. Indeed, because induction can cover deduction when 100% true or false, it is more inclusive in scope. On the other hand, one can view deduction as an ideal pattern that induction tries to approach"[20]. "Up to 50% of actual occurrence, we lack sufficient assurance… At 50%, we are undecided. For basic confidence, we need more than a 50% probability. Above 50%, the argument is more likely true than not" [21].

Moreover, the inductive probability of any proposition may change over time: "Although we start with some sense or estimate (of it being less likely, equally, or more likely than not), compared experience or compiled statistics can revise our earlier, partial intuitions or partial data accordingly"[22]. So well and good; all this is well known[23].

However, when we consider what Wiseman more specifically means by deductive and inductive a fortiori argument, we must express strong disappointment. It is evident from his definitions of a fortiori argument that his meaning is mainly the following:

Wiseman's *deductive* a fortiori argument: Wiseman's *inductive* a fortiori argument:
A is more (of something) than B;   A is more (of something) than B;
and B has f;   and B probably has f;
therefore, A surely has f (or more).   therefore, A should have f (or more).

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[20]   P. 47.
[21]   P. 49.
[22]   Footnote on p. 49.
[23]   I mention all that, for my part, in my first book, *Future Logic*, in chapters 21, 46, 64, and elsewhere.

As we have already pointed out, Wiseman's deductive form of a fortiori argument is invalid, since it lacks the crucial clause in the minor premise and conclusion about the sufficiency of the middle term (i.e. that B has enough of it, and therefore A has enough of it too). In other words, as it stands, its conclusion does not necessarily follow from its premises. As I have explained in an earlier chapter (3.3), an inductive argument must be based on a valid deductive argument to be declared valid from the point of view of formal logic. Since the proposed deductive form is invalid, the proposed inductive form must also be declared invalid. Strictly-speaking, then, both arguments are formally *non sequiturs*, and cannot be called deductive or inductive inference. We might call both arguments 'inductive' in a very loose sense of the term, but this is more misleading than useful.

In the said earlier chapter (3.3), based on my earlier researches, mainly in *Future Logic*, I list four types of formal induction: (a) generalization and particularization, (b) adduction, (c) induction based on deduction, and (d) induction *not* based on deduction. Wiseman's proposed "inductive a fortiori argument," and indeed his proposed "deductive a fortiori argument," clearly both belong under the last category (d): they are "induction" in a very weak and unreliable sense of the term. He gives no evidence of awareness of what truly constitutes deduction and induction, and how exactly they are related to and distinguished from each other. That is to say, he does not show understanding of the formal conditions of validity for alleged deductive and inductive arguments, since his deductive form and consequently his inductive form cannot be formally validated.

He is evidently unaware of formal induction, since he shows no awareness of the formalities of the first three types of induction in my list[24]. When he refers to inductive argument, he seems to intend argument which, though structurally sound, has uncertain premises so that the conclusion is likewise uncertain (we may refer to this as doubt concerning the *content* of the argument). But he does not realize that the deductive argument he has in mind as underpinning such induction is in fact structurally unsound (so that there is in fact doubt in the *form* of the argument). So, his inductive a fortiori argument cannot be said to be justified by his deductive a fortiori argument, because the latter is itself deficient and so unable to justify anything!

Wiseman often speaks of a fortiori argument as either deductive or inductive, as if these were two distinct forms of the argument. He repeatedly speaks of an "inductive form" of a fortiori argument[25]. But nowhere does he actually explicitly define such a form, distinguishable from the deductive form. What he actually proposes, as we have seen, is *a single, vague form* of argument, which may *in some unspecified circumstances* turn out to be deductive, and *in some unspecified circumstances* turn out to be inductive. This argument of his somewhat *resembles* strict a fortiori argument to some extent, but is manifestly not an accurate statement of it. Thus, his theory is in fact that one and the same form of a fortiori argument ranges from inductive to deductive applications. This is not formal logic, since there is no formal differentia by means of which we can tell a valid from an invalid argument, and a deductive from an inductive argument[26].

For him, "a fortiori" argument is typically this: given that A > B, then, if B has f, A *should* have f. The range of a fortiori argument is, then, the range of meaning of the qualifier "should." This may or may not indicate the inner certainty that its outward use suggests; i.e. sometimes it is used in a strong sense, and sometimes in a rather skeptical sense. It may be literally indicative of "a virtual 100% likelihood," in which case "should" becomes "surely;" or it may merely express our current expectation, even though we know that the probability is less than maximal, signifying merely inductive confidence.[27]

As an example of inductive a fortiori argument he gives us the following: "If this little apple is sweet, would a bigger one of the same family also be sweet?" The argument he is rhetorically considering is: If this little apple is sweet, then probably a bigger one of the same family would also be sweet. Now, the reason why such an argument would strike us as unconvincing is because the given information is far from sufficient to construct a valid a fortiori argument. Let us consider this example more precisely, by making explicit the *aimed at* standard format a fortiori argument (although, of course, Wiseman does not do that):

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[24]   In truth, he does once mention the induction by elimination of alternatives, on p. 150: "When two possible answers exist for an argument, in order to conclude a deduction validly and soundly, one must eliminate the other alternate. Similarly, for an induction, one must greatly diminish the alternate's likelihood. If neither is accomplished, the disjunct remains." He is saying this to criticize Maccoby for only considering the possibility of a non-proportional conclusion from a fortiori argument, without having first convincingly demonstrated the impossibility of a proportional conclusion. However, Wiseman does not make use of this principle anywhere else, and in particular not in his own theory of induction.

[25]   See pp. 9, 15, 39, 50, 71, 146, 213, 221. Sometimes he has it in the plural, as "inductive forms" – in such cases he has in mind the "sub-forms" of inductive a fortiori argument (which I discuss further down).

[26]   Indeed, he writes, on p. 146: "Inductive forms of the QC as analogical approximations are reasonable and, although tentative, can cover areas of nature, experience, and thought *beyond the current reach of strict, formal logic*" (my italics) – meaning that though he talks of "inductive forms" he does not see "strict, formal logic" as currently able to handle them.

[27]   For the "a fortiori" formula, see p. 69. For the grades of sureness, see p. 59. Needless to say, I agree that we often say that we are sure of something when we well know that we are in fact *not* so sure; what I am contesting is that such subjective notions can be used as logical differentiae.

The second apple (P) is bigger (R) than the first apple (Q);
so, if the first apple (Q) is big (R) enough to be sweet (S),
then second apple (P) is big (R) enough to be sweet (S).

Now, what information would we need to justify this argument? The major premise can be readily granted, assuming that we are able to measure the two apples compared (and so relate them as major and minor terms, P and Q). What is open to debate is the minor premise. We cannot construct such a proposition (which is of 'suffective' form) merely from the known facts that the apple has some size and tastes sweet: we must additionally show that sweetness (the subsidiary term, S) is causatively related to size (the middle term, R); or more precisely, that below a certain minimum size (say, Rq), sweetness is imperceptible. Only given such empirically determinable information (as to how much R is *enough*) could we then dare to infer the conclusion. Thus, the putative conclusion is not inductive because of some flaw in the process, but because of some deficiency in the minor premise. Moreover, from such purely a fortiori argument we could only deduce that the bigger apple is sweet; to claim it to be sweeter (than the smaller one), we would need additional empirical information to the effect that sweetness increases with size.

Thus, the reason why the proposed conclusion seems doubtful is that we are not given any empirical evidence that justifies the required minor premise. Wiseman says, "We cannot deduce it; but inductively, we can assume or test it;" and he refers to our varying "confidence levels." But he does not tell us what tests we should carry out. Anyway, the truth is, as regards the said minor premise, we would not normally assume it, being given merely that the apple has some size and tastes sweet; to do so would be very presumptuous. To assume the minor premise, we would need to have made some observations or even deliberate experiments, and found that apples smaller than a certain size are *not* sweet; in that case, our confidence in it would be proportional to how many apples were tested (since the initial conjunction of size with sweetness or sourness might be coincidental). At that stage, we could use the minor premise to deduce the conclusion, and our confidence in the latter would reflect our confidence in the former.

Note this well: we cannot generalize from "the first apple has some size and tastes sweet" to "the first apple has *enough* size *to* taste sweet," because the former proposition is entirely positive, whereas the latter proposition has an implicit negative component, viz. "if an apple does not have a certain minimal size, it will *not* taste sweet"[28]. Generalization can only proceed along the same polarity – it cannot produce information concerning the opposite polarity. Many people, still today, do not realize this important inductive principle. If the opposite polarity is assumed without specific justification, that is mere speculation – the very lowest type of induction. It is for this reason precisely that the above example seems particularly unconvincing – its minor premise is merely speculative rather than based on generalization. But all this has nothing to do with the form of the argument *sought for*, which is clearly a fortiori.[29]

From this example we see that what Wiseman means by "inductive a fortiori argument" is not something formally distinct from "deductive a fortiori argument," but merely an *incompletely formed* a fortiori argument, a merely 'potential' a fortiori argument. That is, some elements are known which *could eventually* crystalize as an actual a fortiori argument, but they *do not* in the present context of knowledge suffice to do so. We can perhaps foresee the valid, deductive a fortiori argument that we *wish* to formulate on the horizon – but we do not have all the information needed to formulate it – so, instead, we only claim an "inductive a fortiori argument," *even if in fact there is no genuine a fortiori argument at all, yet*. Thus, Wiseman applies the label "a fortiori" to some arguments without formal justification, on the presumption that they *might* eventually turn out to be a fortiori arguments.

As already suggested, the reason why Wiseman imagines that such *inchoate* argument may be characterized as "inductive a fortiori" is that he is *wrongly defining* a fortiori argument, omitting to mention the crucial factor of sufficiency (or insufficiency) of the middle term in the minor premise and conclusion, and thus making the concept more inclusive than it really is. This is evident in the definitions of a fortiori argument that we listed at the beginning. These definitions are far too vague for any formal development and validation.

A fortiori argument is essentially deductive; if it is not deductive, then it is not really a fortiori argument, or at least not yet so. To be characterized as a fortiori, an argument must be well-formed in accord with the standards of a fortiori logic. 'Well-formed' means that all the relevant elements are clearly laid out, and the process is fully

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[28]   In all fairness, I had not realized this clearly when I wrote my *Judaic Logic*; it is only in the course of the present, new study that I explicitly specified the negative aspect of suffective propositions.

[29]   A similar example is found on p. 58: "Mountain A is greater than mountain B; B has trees; so most likely, A has trees." Here, no reason is apparent why one should expect the size of a mountain to have anything to do with its tree cover. There is indeed some likelihood that any given mountain will have trees, since some mountains do; but there is no certainty it will, since some mountains don't. The degree of likelihood depends on the relative proportion of those two possibilities, worldwide or in the geographical region under consideration. But anyway, it is not through an a fortiori argument that this conclusion is arrived at. It does not logically proceed from the given premises. Without the "enough" factor, no claim can be made to a fortiori argument.

understood and validated. In practice, it is true, very often what we have before us is an argument of unclear content (with some elements left tacit), and therefore of uncertain form, so that the putative conclusion must be regarded as at best tentative and in need of review. Although strictly-speaking this does not qualify as valid a fortiori argument, or even as at all a fortiori argument, it is considered as 'in the same ball park', and *conventionally* labeled as 'a fortiori' so as to put a handle on it for discussion purposes.

In this sense, we might like Wiseman speak of an inductive a fortiori argument, when what we have before us is something that *more or less looks like* an a fortiori argument, but which may in fact not be one at all, or at least not be a well-formed one. Such argument being structurally deficient, it would be more accurate to speak in such cases of *an attempt at* a fortiori argument. The speaker aims for that ideal form, but does not yet fully succeed to attain it for lack of some needed information. Nevertheless, ultimately, only a fully explicit and obviously valid a fortiori argument, which can confidently be called 'deductive', can rightly be called a fortiori.

Needless to say, we can legitimately speak of an inductive a fortiori argument, when the attempted thought-process is clearly a fortiori deduction, but the premises have some material uncertainty. Since the premises, though correctly structured, have content that is less than 100% sure, the conclusion is accordingly unsure. There is no *formal* difference between such argument and deductive argument, note well – the difference is only in the factual reliability of the premises. In this sense, since human knowledge is rarely absolutely certain, we might say that almost all a fortiori arguments formulated in practice are effectively inductive, even if the relation between premises and conclusion is formally quite deductive.

Moreover, note well, the underlying principle, viz. that the probability of the conclusion reflects that of the premises, is not specific to a fortiori argument, but concerns all forms of argument, including categorical syllogism. A categorical syllogism yields a deductive conclusion if both its premises are 100% sure, whereas it yields an inductive conclusion if one or both of its premises are less than 100% sure. The same is true of apodosis, dilemma or what have you.

If, for instance, as is usually the case, the premises (of whatever deductive argument) are based on generalization from experience, then the conclusion, even though it *follows* necessarily from the premises, must be regarded as also inherently empirical in status[30]. We simply must not confuse the deductive character of the process with the usually inductive status of its conclusion. Thus, there is no need to justify such reasoning in the context of a fortiori argument specifically; it is a general problem of logic with a general solution.

But Wiseman's inductive a fortiori argument is not like that, since the deductive a fortiori argument it is based on is itself invalid. The problem with his inductive argument is not that it has material uncertainty; it is rather that it has formal uncertainty. Wiseman at one point seems to sense that the overly skeletal way he has defined a fortiori argument makes the relation between premises and conclusion very tenuous. In an attempt to forestall incorrect inductive inferences, he suggests that "we can strengthen the relevancy relation of the premises to allow only those that suit a sound, deductive *a fortiori*." He explains this concept in a footnote as follows:

> "The relevancy demand makes the argument a restricted deduction. Here, the premises (P's) are to be relevantly true and the conclusion (C) true for validity; if not, the claim is unacceptable. To be relevantly valid as an argument, it is not good enough to merely avoid the case of true P's and a false C, we want to disallow cases of false P's, and allow only true (P's) with true C. The relevancy condition, rather than general acceptability of the argument form, requires that the premises be inseparably related to the conclusion for the C to be valid."[31]

I am not sure just what he means, here. Is it possible that he does not realize that the validity or invalidity of an argument has nothing to do with the truth or falsehood of its premises? All we can say is that if the argument form is valid, then true premises yield a true conclusion; and if the premises are true and the conclusion is false, then the argument form must be invalid. An argument with false premises may still be valid; an argument with true premises may still be invalid. We cannot judge the argument form on the basis of the truth or falsehood of its conclusion, either. The argument form may be valid, yet have a false conclusion in a given case because of a false premise or two; the argument form may be invalid, yet have a true conclusion in a given case, whether its premises are true or false. We can say that of all argument forms, not just a fortiori.[32]

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[30]   And of course it is this fact that makes possible the rabbinical *dayo* principle – which is an interdiction to generalize particular givens in certain contexts.

[31]   P. 59.

[32]   Elsewhere (in a footnote on p. 66) Wiseman states: "Some Rabbis attacked the premises' truths, to reject the conclusion, which is an incorrect formal disproof procedure." But to my mind, it is quite okay to attack the premises of an argument to at least *put in doubt* its conclusion. The conclusion is not thereby proved false (whether the argument is valid or invalid), unless the negation of the premises formally implies (as may happen) the negation of the conclusion. But the conclusion is indeed put in doubt, assuming (as is often the case) that those premises were its only known source of credibility at that stage (i.e. that there is no alternative route for proving it).

I suspect that what Wiseman has in mind when he worries about "relevancy" is the practice that I have identified in the opening chapter (1.1) as the fallacy of two middle terms. The example I give there is, effectively: 'Since humans are more powerful than horses, it follows that if horses are powerful enough to run at a certain speed, humans must be powerful enough to do so too'. What is wrong with this argument is that the superiority intended in the major premise relates to mental capacities, whereas that intended in the minor premise and conclusion relates to physical capacities: so the argument involves two distinct middle terms. Here, obviously, the major premise is not relevant to the rest of the argument. But Wiseman's approach is here again too approximate to pinpoint the problem.

Whatever the case, Wiseman's above remark betrays his lack of clarity as to what exactly constitutes an "a fortiori argument." He does not see the exclusive connection between its premises and conclusion, and so is at a loss as to how to establish it. He apparently thinks it is all a matter of tinkering. This is also evident in his next words:

> "With the *a fortiori*, mere superior over inferior relation of general commonality is fine as far as it goes, but it is insufficient to grant certainty for the conclusion[33]. The less has to imply the more and vice-versa. This would narrow down the *a fortiori* to just those sorts of inclusions that work. By requiring a relevant relation and a true conclusion, we assure the argument's validity and increase its potential soundness… However, if the conclusion is well known, there is little point in making the argument. As such, relevancy may be too strong a requirement."

All this is very surprising, coming from someone acquainted with the treatment of a fortiori argument in my *Judaic Logic*. It is of course true that the major premise alone cannot guarantee the conclusion; we also need the minor premise for that. And the required minor premise is not simply about possession of some feature (the subsidiary term, S), but about satisfying a certain condition (a threshold magnitude of the middle term, R). In that case, the conclusion will be very specific and relevant, and follow necessarily; and seeing this is the point of the whole exercise.

3. Abduction and conduction

Wiseman considers that a fortiori induction has two "sub-forms," namely "abduction" and "conduction." These, together with analogy, are instruments of "practical," as against scientific, induction:

> "Induction expands the scope of reasonable answers to cover less universal and more particular issues. So too, when the greater rigour and assurance of scientific induction are unattainable, one can settle on largely successful, practical inductions or its sub-forms. These include the abductive (best choice), conductive (likelihood), and analogical (possibilities), sometimes combined."[34]

All three of these processes concern the "transfer" of information from one context to another, on the basis of indices insufficient for deduction.

The highest form of such induction, in Wiseman's view, is "abduction," because in its case the available alternatives are known and their relative ranks have been determined, so we choose "the overall best alternative." Thus, if some item B has some desirable feature and another item A has *more of* that feature, we would choose A as "that much better than" B. The reasoning he describes is, I agree (if we insert appropriate corrections into his formula), related to a fortiori argument in that we think: if the minor term B has enough of the decisive feature to be desirable, then the major term A has enough of the decisive feature to be desirable. This is a positive subjectal argument, with the "decisive feature" as the middle term (R) and "desirability" as the subsidiary term (S) (although Wiseman does not explicitly distinguish the latter two terms).

However, I do not see that we can conclude from the given premises, as Wiseman seems to, that A is proportionately more desirable than B, and thus automatically choose A over B. There may well be a ceiling for the decisive feature above which it is excessive[35] and not so desirable. As common wisdom has it, more is not always better; i.e. there is sometimes too much of a good thing. To draw a valid proportional conclusion, we would need an additional premise about proportionality. So, we could say that what makes the above "abduction" an inductive argument is only the speculative assumption of proportionality; for otherwise, the argument (if duly corrected as above suggested) would be deductive.

"Conduction," next in line in Wiseman's list, is comparatively a more epistemic form of reasoning, since it suggests the "more likely" choice for some purpose. Here, we are not sure whether A and B have the desirable feature, or to what extent they do; but we do know that A is more likely to have it than B, so we choose A over B. This thought

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[33]   It is amusing to note that in the very act of ignoring the suffective form of the minor premise and conclusion, here, Wiseman is expressing himself by means of a (negative) suffective, viz. "it is insufficient to grant certainty for the conclusion."

[34]   P. 71. These processes are detailed in pp. 51-2. Note that I often resort to paraphrase so as to keep the exposition short, because Wiseman often expresses his thoughts through examples.

[35]   Wiseman elsewhere (pp. 57-8) shows he is well aware that there may be limits, going so far as to propose a "fallacy of going beyond a limit." But here he forgets it, apparently; or maybe he considers that such forgetfulness is part of the "abductive" movement of thought.

process also, I agree (provided it is more correctly formulated), involves a fortiori thinking, except that the middle term (R) is now "*likely* to have the decisive feature," and the subsidiary term (S) is now only "*likely* desirability" (i.e. we somewhat arbitrarily pass on the likelihood of the middle term to the subsidiary term, for obviously something is actually desirable only on condition that it actually has the decisive feature). Obviously, with such unsure middle and subsidiary terms we cannot expect a very sure conclusion; but it is arguably better than no conclusion at all if we really must make a choice.

In any case, the conclusion here again can only be non-proportional, since Wiseman does not provide us with an additional premise about proportionality. So "conduction" is inductive argument in three ways. First, in view of the probabilistic character of its middle term; second, in view of the rather arbitrary passing over of the probability of the middle term to the subsidiary term in the minor premise (which makes the whole argument highly speculative); and thirdly, because of the assumption that the predicate in the conclusion will surpass the predicate in the minor premise in a proportionate manner, making a definite choice possible (which is also speculation, since not based on additional information).

The third and least reliable form of induction, in Wiseman's account, is mere "analogy," which "proposes some like feature from a given case that might apply to the new." More specifically, this means: "If A is more or less like B," and B has some feature, then A should have that feature. This is not, of course, a fortiori argument – and Wiseman does not claim it to be. Wiseman does, moreover, point out that we should not "just assume that superficial likeness to the given case will lead to the feature's reoccurrence" in the similar case, and he enjoins us to "try to defeat the argument." A claim is "much more likely to be true," if it has "weathered genuine attacks" than if it remains "untested." Well and good.

However, I do not see how this method (analogy) fits in with the preceding two: they are aids in decision-making, but in what way does analogy facilitate choice? Maybe Wiseman has in mind a sort of *a pari* argument: just as we chose B because of the decisive feature, then we should choose A because of it. But what of cases when we have to choose between A and B? Presumably, we would prefer B, which is a given, to A which is merely inferred by analogy (this is the thinking behind the rabbinical *dayo* principle, incidentally). But in that case, why bother making an analogy at all?

Anyhow, what should especially be noticed in this context is that Wiseman seems to opt for proportional conclusions in the two processes involving a fortiori argument that he describes. His "abduction" involves a proportional a fortiori argument whose additional premise concerning proportionality is not given, but merely speculatively assumed. His "conduction" involves a proportional a fortiori argument whose middle and subsidiary terms are both unsure, and which also involves an unsure proportionality. He thus "infers" choices to be made a bit hastily, without underscoring the additional assumptions they involve.

It is interesting that Wiseman conceives of induction as either abduction or conduction or analogy. This was inevitable, considering that Wiseman's vague definition of a fortiori argument, which made impossible the formal validation of his deductive form and thence all the more of his inductive form. Since his deductive form was itself too vague to be validated, his inductive form could not be claimed to be 'induction based on deduction', but only at best characterized as 'induction *not* based on deduction'. Indeed, even his so-called deductive form can only be so characterized. Thus, it is no accident that Wiseman's conception of induction refers to only the very weakest forms of induction – forms which are more guesswork that serious inference.

The strongest form of induction, according to Wiseman, is "abduction." This term is explained as follows in (for instance) a Wikipedia entry:

> "Abduction is a form of logical inference that goes from data description of something to a hypothesis that accounts for the data. The term was first introduced by the American philosopher Charles Sanders Peirce (1839–1914) as 'guessing'. Peirce said that to abduce a hypothetical explanation *a* from an observed surprising circumstance *b* is to surmise that *a* may be true because then *b* would be a matter of course. Thus, to abduce *a* from *b* involves determining that *a* is sufficient (or nearly sufficient), but not necessary, for *b*."

Judging by the definition here proposed, Pierce simply meant *guessing or surmising* what the explanation for something might be. This is technically mere speculation, of course. Effectively, the first possible explanation of a phenomenon that comes to mind is cheerfully relied on; at least, until some other explanation pops in one's head. However, judging by the way people commonly use the term "abduction" nowadays, including Wiseman, it is taken to refer to the *best* guess in a given context of information – i.e. to the most credible hypothesis given an amount of information too small to make a definite prediction.

And how is the *truly* best guess to be determined? Ideally, by 'the scientific method', of course – i.e. the formulation, testing, confirmation, comparison and elimination of competing hypotheses. There is no other reliable way. But Pierce only mentioned the positive aspect of this process, and missed out on (or at any rate did not sufficiently emphasize) the crucial negative aspect, namely that the hypothesis might upon further testing be eliminated either

because some contrary fact is discovered or because another hypothesis is found more appropriate in the current context of knowledge. What he apparently had in mind, then, was rather 'intuitive' guesswork, without checks and balances, with no trial and therefore no error (i.e. no awareness of error) and therefore no correction of error[36].

Thus, the correct usage for the term abduction is with reference to what we normally refer to as 'making an educated guess'. It is inaccurate to use the word abduction to refer to the intricacies of the scientific method. For the latter, and we should use the older appellation: adduction. In plain English, *we adduce, i.e. we add, we put forward for consideration, evidence or arguments in support of or in opposition to some thesis or counter-thesis.*[37] Such thinking is not mere guesswork, but thoroughly pondered judgment. We rigorously pursue the objectively most fitting hypothesis in our current context of knowledge, and indefatigably look out for improvements.[38]

In any case, Wiseman does correctly use the term abduction in the present context, since as we have seen his alleged inductive (and indeed deductive) a fortiori argument provides a very weak connection, if any at all, between the premises and conclusion. Therefore, he has to resort to guesswork to produce the semblance of a connection. The best method available for this purpose is abduction, because this bases a choice on some factual givens. In the absence of the latter, i.e. when the uncertainties involved are even greater, conduction is used instead. Both these methods inevitably involve guesswork, since they lack a firm formal foundation.

Even so, as already shown, Wiseman does not clearly discern the logical objects he refers to by means of the words abduction and conduction. His understanding of the thought processes he so labels is imprecise, due to insufficient analysis of what is logically going on in in them. Moreover, his claim that induction is either abduction, conduction or analogy is inaccurate. There are many more, and much better, means of induction than those he here lists. And those processes are more complex than mere conjectures, and most importantly they can be formally validated.

4. Proportional a fortiori

The second of Wiseman's main theses is that a fortiori argument may have proportional as well as non-proportional conclusions. Although I read a bit of Wiseman's work as soon as I got hold of it, enough to see where he was heading, I quickly decided to stop reading it, so as not to be overly influenced by it[39], until I had almost finished writing my own work. What I did note in my first cursory reading was that Wiseman is throughout very concerned with the issue of 'proportional' a fortiori argument.

My explanation for this common practice in my earlier work, *Judaic Logic*, was that, although a fortiori argument as such can only yield a conclusion with a subsidiary term equal in quantity to that in the minor premise, there might well in some cases be additional information that would allow, whether deductively or inductively, proportional inference; however, such inference would occur after the a fortiori argument, rather than as part of it. I there put it like this:

> "It is not the quantitative difference between the major and minor terms which is at issue; that is already given (or taken for granted) in the major premise. What is at issue is a quantitative evaluation of the remaining terms, the middle term and the subsidiary term, as they appear in the minor premise and conclusion. According to our theory, the outward uniformity of these terms in those propositions is a formal feature of a-fortiori argument. But this feature does not in itself exclude variety at a deeper level. Such specific differences are side-issues which the a-fortiori argument itself cannot prejudge. It takes supplementary propositions, in a separate argument, which is not a-fortiori but purely mathematical in form, to make inferences about the precise quantitative ramifications of the a-fortiori conclusion."[40]

Wiseman, on the other hand, considers that a proportional conclusion is, in some cases at least, as intrinsic to the a fortiori process as an equal conclusion. As he wrote back to me, in response to my congratulations for his work: "I believe there is one major disagreement, although that is at the core of the thesis – that proportionality must be accounted for, not just equality to the given." Although I thought my statement on the matter in *Judaic Logic*

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[36]   This would explain Pierce's takeover of a word which originally (1666) meant "unlawful carrying away of a woman for marriage or intercourse" (Merriam-Webster's Collegiate Dictionary); note the qualification "unlawful." That is, maybe he conceived of abduction as *kidnapping of information* in support of some arbitrary theory. (I am of course engaging in "abduction" as I emit this guess!)

[37]   According to Merriam-Webster's Collegiate Dictionary, "to adduce" means "to offer as example, reason or proof in discussion or analysis;" and adduction refers to "the act or action of adducing." These words are dated as being from the 14th or 15th century. My above definition differs in adding in the negative aspects, which are essential to the scientific approach. I suspect the reason some people nowadays prefer the newer word "abduction" is that it is more abstruse and therefore more impressive to pseudo-intellectuals.

[38]   That Wiseman thinks that the terms abduction and adduction mean the same is suggested on p. 134, where he writes: "…like Sion's majority vote or abductive decision (adductive as he calls it)." On the other hand, in a footnote on p. 150 he writes: "To cite the thing to which an argument or set of facts points is to adduce it. The process is an adduction. To take the stronger argument or set of facts is an abduction. Both are inductive in nature." Neither of these views is correct.

[39]   As it turned out, there was very little danger of that; but before having read Wiseman's whole book I could not know it.

[40]   Chapter 4.3.

sufficiently clear and accurate, I took Wiseman's criticism to heart, and resolved to respond more fully to his concern eventually.

I very early had the idea of coining the expression *a crescendo* argument to describe 'proportional' a fortiori argument. However, it is only when I was several months into the writing of the present book that it occurred to me to try and give a precise form to such argument. It was then that I had the good idea to explicitly formulate the 'additional premise' about proportionality, with which a fortiori argument would formally become a crescendo argument. It should be noted that, contrary to what I had assumed in my earlier work, this additional premise is not just an add-on at the end of the 'proportional' a fortiori argument, but is part and parcel of it, since the pro rata argument involving the said additional premise uses an implication of the minor premise and produces a quantitative change in the purely a fortiori conclusion. That is, a crescendo argument *intertwines* purely a fortiori argument and pro rata argument; it is not a mere succession of two such arguments.

This finding may now look simple, but it took me a while to realize it and properly develop it, and it had an enormous effect on my subsequent perceptions. I was forced because of it to spend several weeks laboriously rewriting much of what I had written until that point. For a start, my assessment of Maccoby's views had to be radically reviewed. Thus, although my notion of a crescendo argument as against purely a fortiori argument was developed independently, I can say with gratitude that Wiseman's loudly expressed concern had a direct role in the findings by stimulating this research.

Wiseman is of course right to refuse to brush off a crescendo argument. This is evident in a Judaic context: of the 46 a fortiori arguments found in the Tanakh (Jewish Bible), 6 are proportional (see **Appendix 1**); likewise, of the 46 a fortiori arguments found in the Mishna (the earlier compendium of Jewish law), 10 are proportional (see **Appendix 2**). It is also evident in a larger context: for instances, of the 15 a fortiori arguments found in Plato's works, 4 are proportional, and of the 80 a fortiori arguments found in Aristotle's works, 8 are proportional (see **Appendix 4**). Thus, 'proportional' a fortiori argument is quite present in human discourse. Moreover, the ratio being about the same in either context (respectively 17% and 13%), we can say that Judaism is not special in respect of use of a crescendo argument, compared say to Greek philosophy[41].

Let us now look more closely at what Wiseman says in theoretical terms about proportionality. His thesis seems to be that *a fortiori argument is essentially 'proportional', although it gives an 'equal' conclusion in specific cases where the ratio happens to be 1:1, or as a minimum in undocumented cases*. He says or implies this several times. For instance, in his abstract: "Logically, the *a fortiori's* conclusion can be either limited to the same feature given in one of its premises or else proportioned to it in a way that suits both premises. Mathematically, the same outcome is just one possible ratio." Again, later: "Overall, proportions [are] more natural, while fixing the conclusion to the level of the given is an exception in everyday situations"[42]; or again, further on:

> "Concerning the amount of the conclusion, logic on its own is neutral; it is satisfied with equality (as an interim minimum) and proportionality (as something more, which may or may not be exactly decidable). We have to decide which choice may be better in the actual context or accept both as reasonable, alternate solutions. Yet equality is just a special state of a more general proportionality."[43]

But I have in the present study definitively shown on formal grounds that the reverse is true; that is: purely a fortiori argument is the norm, whereas a crescendo argument occurs only in specific cases where *an additional premise* (whether explicitly put forward or tacitly intended) informs us that there is proportionality, so that a complementary pro rata argument can be constructed. Although (as already mentioned) I agree with Wiseman that a crescendo argument is often used in practice and may not be written off as illogical, the theoretical emphasis must in my view still be put on purely a fortiori argument.

The fact is, as we shall now show, *Wiseman nowhere succeeds in formalizing, let alone formally demonstrating, his idea that, as regards a fortiori argument, 'proportionality' is the rule and 'equality' is the exception.*

To formalize or buttress his thesis, Wiseman proposes "a treatment of *a fortiori* argument that utilizes quantificational predicate logic," abbreviated to "QPR"[44]. Using symbolic logic he tries to demonstrate that "in those cases that have the feature or property in question continuously, in that it is hereditary or ancestral or fixed… a feature of a category has the property throughout and so is necessarily true, universally or for a clearly defined range in which it holds." But, looking at his formulae, all I see is syllogistic reasoning, i.e. 'given a general proposition, it

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[41]   Even though a fortiori argument in general is, I would wager, more frequent (per block of discourse) in Judaism than elsewhere.

[42]   P. 63, footnote. To which he adds, "although in some Jewish contexts (the *Mishnah's*), equality is the norm."

[43]   P. 93.

[44]   See pp. 91-3. Wiseman additionally treats a fortiori argument with "two particular premises" in an appendix (E.4) on pp. 245-6. What he has in mind is presumably quantification of a fortiori argument, judging by his comments on p. 21, where he compares it to syllogism. He there seems to think that only a fully universal a fortiori argument would be valid. I deal with this topic fully in an earlier chapter of the present work (3.2).

applies to all particular cases'. Although he puts this in the redundant form of 'given a general proposition, if it applies to this particular case, then it applies to that particular case', this is still not distinctively a fortiori reasoning[45]. As an example, he takes "circles." First he tries to "prove" that if a little circle has a circumference, then a bigger circle has one too. Well, this is obvious by syllogistic reasoning: given that *all* circles have a circumference, then all circles *do* have a circumference, whether they are bigger or smaller![46] The mention of relative sizes does not turn a syllogism into an a fortiori argument, and is quite redundant. One could equally well argue that if a bigger circle has a circumference, then a smaller circle has one too; or even that if a blue circle has a circumference, then a red circle has one too. Moreover, the conclusion here that both bigger and smaller circles have a circumference does not imply that these circumferences are *at least equal* (if not proportional)[47] – whereas in truly a fortiori argument such minimal equality is indeed implied.

Next, he tries to "prove" that, given one circle is bigger than the other, the circumference of the bigger is proportionately greater than the circumference of the smaller. Here, we might ask him how he defines a "circle" – for surely this all-inclusive concept is not something separate from that of circumference. I would have thought that the problem posed is one to be solved by mathematics, not pure logic. Geometry refers to the radius (r), and teaches us that the circumference (c) of a circle is equal to $2\pi r$[48]. Therefore, yes: a bigger circle (radius) has a proportionately greater circumference than a smaller one. Also the converse is true; i.e. a smaller circle has a proportionately lesser circumference than a bigger one. What we have here is mere pro rata argument. We are not arguing in a fortiori style from minor to major, or even from major to minor, but just applying given ratios[49].

Wiseman does not apparently regard his proof of proportionality as determining, since he sums up somewhat lamely with: "both the basic fact of a circumference… and the greater amount… are equally valid conclusions." How so? If these are deductions, as they seem intended to be[50], how can their different conclusions be reconciled? Obviously, their premises must and do differ. In any case, it is evident that his attempted logical demonstrations have *foregone* conclusions – that is, they teach us nothing new but merely *apply* (in the tortuous fashion dear to modern logicians, who are all bureaucrats at heart) a principle already declared at the outset. This is, if not question begging, a very simple sort of syllogistic thinking which does not significantly clarify a fortiori argument.

I would say that Wiseman felt obliged to try out a "QPR" treatment – even though just a few pages before he has shown his acquaintance with my *Judaic Logic* exposé of a fortiori logic – simply because there is a belief out there, in today's academia, that putting things in symbolic terms somehow, magically, gives them greater credence. But as I keep repeating throughout this volume, the opposite is true. Symbols are bound to largely confuse those who resort to them, and prevent them from seeing reality clearly. They may be used to present results *ex post facto* in an abbreviated manner, but they should never be used before a thorough and convincing conceptual analysis has already obtained these very results. They are mere means, not ends in themselves.

In any case, Wiseman's formula for a fortiori argument predictably cannot be proved by "QPR," or any other symbolic hanky-panky, for the simple reason that the formula is wrong – i.e. its premises logically do not imply its conclusions. Wiseman's "QPR" treatment shows yet again that he does not realize that the concept of sufficiency (or insufficiency) is one of the *defining* features of a fortiori argument. Without the specification that the subject needs to "be (or not-be) R *enough* to" have the predicate, there is in fact no valid a fortiori argument. It is crucial. Even if Wiseman claims to "have relied upon [Sion] extensively throughout"[51], he has not fully internalized this *sine qua non*; and this causes him many problems.

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[45] Note moreover that although, given "All X are Y," it does formally follow that "if this X (e.g. a smaller instance of X) is Y, then that X (e.g. a larger instance of X) is Y," this merely means that if the universal "All X are Y" is true, then the particular "this X is Y" is true and the particular "that X is Y" cannot be false. The relation of implication from "this particular X is Y" to "that particular X is Y" must not be construed as independent of the universal given "All X are Y;" strictly speaking, that implication is no more than a conjunction. The illusion of independence makes the proposed inference essentially fallacious.

[46] As Wiseman earlier (on p. 67) explicitly admits, saying: "The relative sizes are superfluous facts once we know that all examples have the relevant characteristic (feature or judgement) anyway." Wiseman, by the way, there wrongly interprets the inference from "All apples are fruit" to "little apples are fruit" or "big apples are fruit" as one from a universal (A) to a particular (I) proposition (the latter being "formed directly" from the former); in truth, the inference involved is first figure AAA syllogism: 'Since all apples are fruit, and little (or big) apples are apples, then little (or big) apples are fruit'.

[47] As Wiseman later suggests in his earlier quoted summary, where he refers to an "interim minimum."

[48] Wiseman does mention the radius earlier on (p. 67), but not here. And more important, he does not mention the $c=2\pi r$ formula, i.e. that the proportionality in this case is established through geometrical theorems and not through his proposed a fortiori argument.

[49] This is also evident from his diagrams on p. 64, and his statements there that "we could reverse course" and "the situation can be reversed." Applying ratios signifies pro rata argument.

[50] Even if he claims to "generalize" each conclusion from an arbitrary case to all cases, this is just conventional language and technique – he is really thereby intending deduction not induction. That is, his result is meant to be not less than 100% sure, on abstract grounds.

[51] P. 65.

For example, he refers to the "transitivity argument: A > B, B > C, so A > C" as "a special *a fortiori*"[52]. But this argument is not at all a fortiori! I could construct one from it for him, as follows: A is greater than B; so, if B is big enough to be 'greater than C', then A is big enough to be 'greater than C'. The middle term here is the unspecified respect in which A is greater than B; and the subsidiary term is, note well, not the third subject 'C' but the predicate 'greater than C'. But the two arguments are formally very different logical entities, because the "transitivity argument" lacks the said crucial feature.

If we look at Wiseman's definition, on the same page, of "the more typical a fortiori argument," we see the same deficiency[53]. Such argument, he tells us, "has a new feature that it claims to transfer to the other case. This more frequent, *a fortiori* argument form says that if the lesser has a regularly associated factor, the greater should surely or probably have it too, minimally (A > B; B has f; A should have f)." Note well the failure to mention sufficiency of a middle term (or for that matter, the middle term)[54].

This omission is the main cause of his difficulties with regard to 'proportional' a fortiori argument. Nowhere in his treatment, note well, does Wiseman manage to give a credible formal expression to his (understandable) belief that a fortiori argument is essentially 'proportional', even if it is often more specifically 'equal' either because the applicable proportion happens to be even or because information is lacking to assume more than this even proportionality[55].

He does not manage it, because his working definition of a fortiori argument (i.e. the formula "A > B; B has f; A should have f") is too vague. In his mind, such vagueness is broadness; it is what makes possible greater variety of conclusion, ranging from the inductive to the specifically deductive, and from the proportional to the specifically equal. But in fact, such vagueness is what prevents him from identifying and developing the precise conditions of proportionality. He is thus condemned to deal with the matter by general appeals to reasonableness, concluding "We have to decide which choice may be better in the actual context or accept both as reasonable, alternate solutions"[56].

We have already examined Wiseman's theory of the moods of a fortiori argument. There is admittedly some formalism in it, since it refers to the terms and their relations in abstract ways (i.e. using words like "the lesser," "the greater," etc.). But remarkably, nowhere in it does he show under what precise circumstances the conclusion from a given premise might change. For instance, the two moods: "If the lesser has the feature, surely (or very likely) the greater has the same, equal feature," and "If the lesser has the feature, surely (or very likely) the greater has more of the feature," are placed side by side, with no formal explanation as to why the same premise has one conclusion one time and another conclusion another time, let alone any validation of these conclusions.

It seems that Wiseman thinks that the difference cannot be expressed in formal ways, but only through logical insight on a case by case basis. As he puts it, "Such inductive, *a fortiori* arguments involve empirical matters thought or known to be reliable." But that is not a teaching of formal logic – it is merely material logic. And the truth is, we refer to material (or informal) logic when we are unable to clearly discern the formal logic involved.

5. The dayo principle

Let us now look and see at Wiseman's treatment of the *dayo* principle, which I think is the central concern of his whole treatise.

Maccoby's *dayo*. Wiseman expends much effort contesting Hyam Maccoby, because the latter is on record as an uncompromising opponent of 'proportionality'[57]. Wiseman is of course right, and Maccoby wrong; but the question we must ask is whether the reasons Wiseman gives are sufficient to defeat Maccoby's viewpoint. Wiseman correctly

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[52]   P. 69.

[53]   See also Wiseman's attempt to differentiate a fortiori argument from ordinary analogy, on p. 63.

[54]   On the next page, Wiseman formulates the following example: "If A is a bigger cheat than B, and we hold B guilty, surely we also hold A guilty to some degree." Here, the middle term ("cheat") is indeed specified in the major premise, but it is still lacking in the minor premise and conclusion (which should specify that if B is enough of a cheat to be held guilty, then so is A).

[55]   Note that since, as we have seen, Wiseman distinctively believes a fortiori inference can in principle proceed from major to minor as well as from minor to major, the 'equal' conclusion has to be for him not a minimum assumption but rather a balanced one (i.e. the middle ground between greater and lesser).

[56]   P. 93. He proposes, on p. 65, a triad of "principles" that "operate in an *a fortiori* conclusion," namely "the Precedent [Principle] (represented by the *dayo*), the Practical Revision [Principle] (when an adjustment is better), and Proportionality [Principle] (to scale the variation appropriately)." But these notions are too loosely defined to serve as effective *guides*, whether logical or heuristic, through which one might judge which way to opt in practice; they could only at best serve to classify one's judgments *ex post facto*.

[57]   In *The Philosophy of the Talmud* (London: Routledge Curzon, 2002.)

reproves him, by pointing out that the latter nowhere proves his position, i.e. nowhere formally invalidates 'proportional' a fortiori argument. But the problem is that Wiseman's own approach is not sufficiently formal to definitely validate 'proportionality', even if only under certain specified conditions. For this reason, he has a hard time overcoming Maccoby's harder stance once and for all. He is forced to resort to reasonable examples, and even (to some extent) to appeal to authorities.

Maccoby's showcase for a fortiori argument was: given that "a moderately good child deserves one sweet," it is "correct" to infer that "a very good child deserves one sweet," and "incorrect" to infer that "a very good child deserves two sweets." As I have shown in the chapter devoted to Maccoby (22.1-2) this example is a mixture of true and false ideas. For a start, it lacks the crucial clause about sufficiency of the middle term, without which no formal validation of the argument is possible. Construed as a purely a fortiori argument, the example should have been formulated as follows:

> A very good child (P) is more deserving (R) than a moderately good child (Q),
> and, a moderately good child (Q) is deserving (R) enough to get one sweet (S);
> therefore, a very good child (P) is deserving (R) enough to get one sweet (S).

Secondly, while it is true that, given only these two premises, only the "one sweet" conclusion is logically permissible – it is also true that given an additional premise about proportionality, the "two sweets" conclusion becomes logically permissible. The argument would then have the following form:

> A very good child (P) is more deserving (R) than a moderately good child (Q),
> and, a moderately good child (Q) is deserving (R) enough to get one sweet (S);
> and, reward in sweets to children (S) are to be proportioned to their deserts (R);
> therefore, a very good child (P) is deserving (R) enough to get two sweets (S).

Actually, unless we can provide a precise mathematical formula for the concomitant variation between S and R, the conclusion should rather read, more indefinitely, "more than one sweet;" or we should admit that the exact quantity is somewhat subjectively assessed. In any case, the two above arguments, the purely a fortiori and the a crescendo, can be shown valid (as done in part I of the present volume). Therefore, as I said, Maccoby had some formal justification for his advocacy of non-proportional argument; but he was also formally unjustified in his categorical denial of proportional argument.

Now, let us compare Wiseman's stance. First, like Maccoby, he does not include the crucial clause about sufficiency of the middle term – a grave deficiency. Second, like Maccoby, he does not realize that the difference between proportional and non-proportional a fortiori argument lies in whether or not we have additional information regarding proportionality, i.e. in the above example the third premise: "reward in sweets to children (S) are to be proportioned to their deserts (R)." Thus, Wiseman is as unable to prove his position as Maccoby is unable to prove his. They are both too informal in their approach to be able to settle the matter. Third, both Maccoby and Wiseman stray by focusing on "rewards" for good deeds, whereas the Mishnaic *dayo* principle is more specifically concerned with penalties for offenses.

Wiseman's position could be considered closer to the truth, since he admits of both non-proportional and proportional a fortiori arguments, even if he does so on the basis of vague definition. On the other hand, Maccoby is more accurate, in that he claims his non-proportional argument as a deduction – whereas Wiseman views both types of a fortiori argument as essentially inductive (even if he admits that under some unspecified conditions they may become deductive). Maccoby fails to perceive under what conditions non-proportional deduction can become proportional deduction. Wiseman fails to perceive the formal conditions for both non-proportional and proportional deduction, so that his reasoning is always relative to material examples. This is confirmed by his concluding remark that "philosophically, Maccoby's denial of degrees was untrue; even if his argument was formally valid, it was still factually unsound"[58].

He proposes a "counterexample" to Maccoby's good children example[59]. He argues that "Although the *dayo* recognizes the value of persons as equal, still, it is often right to reward a better performance," as in the case of two students who get "differing grades or rewards for varied performances." While he does not deny that Maccoby's egalitarianism might be appropriate in some cases, e.g. to encourage behavior "pure and free of any ulterior motive" or to discourage "mere posturing or outward show for the sake of reward" or to avoid "quarrels," he insists that "by the principle of fairness or justice… much better work surely deserves more" reward. Sometimes, when the focus is

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[58]   P. 211.
[59]   See pp. 106-7, 137-140.

on an "essential quality" held in common by the subjects, we opt for a single lowest common denominator as predicate; other times, when the focus is on the "relative performance" of the two subjects, we opt for quantitatively different predicates.

But though this counterexample may seem reasonable enough as argument in favor of the 'measure for measure' principle, it is not really relevant to the issue of proportionality in a fortiori argument. The issue is not whether proportionality exists in nature and morality, which no one denies, but *whether it can be inferred by means of a fortiori argument*, and if so under what precise formal conditions. Even though Wiseman asks the crucial question "how could one credibly scale exact rewards to ill-definable levels of goodness?" – he does not in fact ever take it to heart and adequately answer it. Yet this is the crux of the matter, and ultimately the raison d'être of the rabbinical *dayo* principle. Wiseman does not realize that this principle was not formulated in ignorance of the measure for measure principle, but precisely in order to impede its overenthusiastic use.

The Mishnaic source. Moreover, Wiseman has much in common with Maccoby with respect to the interpretation of Mishna (Baba Qama 2:5), from which the rabbinic *dayo* principle apparently sprung. In this documentary source, the majority of rabbis (the Sages) seem to reprove a member of their assembly (R. Tarfon) for engaging in 'proportional' a fortiori argument, saying '*dayo*', i.e. it is enough to conclude with the given quantity, it is not legitimate to infer a proportionate quantity. This is interpreted by both Maccoby and Wiseman as a blanket interdiction by the Sages to engage in proportional a fortiori argument. Although Maccoby looks upon the Sages' *dayo* principle as supportive of his view of it as logical, whereas Wiseman considers it as unreasonable and unfair – they basically agree that this interpretation of the Sages' intention is correct. However, it is clear from this interpretation that neither Maccoby nor Wiseman has sufficiently studied the relevant sources.

Maccoby sustained the apparent position of the Sages, by declaring the non-proportional conclusion to be the only strictly logical conclusion. However, Maccoby based this judgment on a partial reading of the relevant Mishna, namely on only the first argument of R. Tarfon and the Sages' first *dayo* objection. He did not notice the second argument of R. Tarfon and the Sages' second *dayo* objection, which are significantly different and cannot be read the same way. Furthermore, he summarily dismissed the Gemara's treatment of the matter as the work of an ignorant Amora, having failed to notice that the Gemara attributes its view of a fortiori argument to a *baraita*, i.e. a Tannaic source, one as old as the Mishna.

Wiseman has likewise not grasped the significance of the second argument of R. Tarfon and the Sages' renewed objection to it. He somewhat agrees with Maccoby that, following the Mishnaic Sages, Judaic logic is principally, if not exclusively, in favor of non-proportional argument. However, he gives more credence to the deviations from that norm in the Tanakh, the Mishna, the Gemara and other rabbinic sources, since of course this confirms his own theory. However, it is evident that Wiseman has not actually studied the relevant Gemara (Baba Qama 25a). He too makes no mention of the *baraita* there presented, "How does the rule of *qal vachomer* work?" – which seems to claim the natural conclusion of all *qal vachomer* to be proportional, a position very close to Wiseman's.

The truth is, if we look dispassionately at the data, both 'proportional' and 'non-proportional' a fortiori arguments are used in Judaism. I have above provided some statistics relating to the Tanakh and the Mishna; I expect comparable figures obtain in the Gemara. The two arguments of R. Tarfon in this context are frankly 'proportional', although the second can also be read as 'non-proportional' (whereas the first cannot). The Gemara's position (at least *ad loc*) is consciously 'proportional', and its author goes to great lengths to demonstrate that the Sages have the same belief. However, the Sages' position is objectively subject to various interpretations. The most likely interpretation in my view is that their *dayo* objections were not intended to reprove R. Tarfon for any *logical* error, but merely meant to institute an *ethical* principle.

Now, Wiseman ultimately upholds at the same idea, that the *dayo* principle is more ethically than logically motivated. Nevertheless, not having thoroughly studied the Mishna and Gemara, not to mention later commentaries on them and other relevant sources, he is unable to fully substantiate his position. His position is right, but not solidly founded. He adheres to this idea simply because it makes sense to him, as a way to prevent the overly extreme judgment of Maccoby, who contrary to evident common practice rejects all 'proportional' a fortiori argument offhand as inherently illogical.

The rabbis' motive. Contrary to what Wiseman imagines, the *dayo* principle is not on the rabbis' part a dogmatic rejection of pro rata reasoning as such, but merely a humble admission that they dare not use such uncertain reasoning when human destinies are at stake. It is not an act of injustice on their part, but of justice. They do not deny that proportionality exists; they merely deny that they have the cognitive means to identify it correctly in the course of Torah interpretation, and so prefer to take no chances to make a wrong estimate. If they had the means to predict exactly what the right proportional penalty should be for each fault, they might well edict such penalties; but there is no exact science in these matters, so they prefer to rest content with the minimum penalty specifically mentioned in the Torah.

Thanks to the restraint enjoined by the *dayo* principle, the rabbis are sure not to offend God by handing down an unjust sentence. They prefer to sin on the side of caution and mercy, than to sin through excess of harshness and vengefulness. In both cases there is injustice, but more so in the latter than the former. True, some guilty persons may thus get away with less punishment than they deserve; but that is not as bad as causing even one innocent person to receive more punishment than he deserves. God can, if He later chooses to do so, more easily redress the wrong of insufficient punishment than that of excessive punishment. Earthly rabbinical justice is, in any case, not the end of the matter; God may also have His say, in this world or the next.

This is what the *dayo* principle is all about. It does not limit the extension of a law to analogous cases, but prohibits a fortiori inference based on 'measure for measure' or on generalization. Inference is permitted, but boldness is not.

Wiseman admits that: "in many situations it is safer to go with what you know than a more doubtful and perhaps, partly arbitrary, proportional conclusion, especially as relative differences between things are often too vague, hard to assess, or irrelevant to satisfy the need for a definite result"[60]. Moreover, he is well aware that "the Rabbis did not want to cause undue or unjust suffering"[61]. Yet, he is so intent on justifying his ideas concerning proportionality and the *dayo* principle that he does not realize the finality of these insights, and their decisive role in explaining the rabbis' perplexity and their preference in principle for *dayo* over proportionality.

Measure for measure. It is only recently, when looking through Wiseman's paper in order to review it, that I found out he mentioned the principle of 'measure for measure' before me. I did not realize this till then. Wiseman hinted it when he wrote to me, after his book was finished: "The issue of mercy that you noted was strengthened." But the truth is, although in my *Judaic Logic*, following traditional commentaries, I had pointed out that, in the example of Miriam (Num. 12:14-15), the penalty imposed on her was "out of sheer mercy" less severe than might be intuitively and logically expected, I made no reference there to the principle of 'measure for measure' (*midah keneged midah*). It is only in the course of writing the present book that I referred to this principle as having a role in understanding the *dayo* principle. That I came upon this insight independently of Wiseman, though he preceded me with it, is clear from the different emphasis we place on it.

The principle of measure for measure plays a much lesser role in Wiseman's treatment of the *dayo* principle, than it does in my present study. He appeals to the former principle in order to combat what he perceives as excessive reliance on the latter by the rabbis – whereas, in my account, the *dayo* principle is intended by the rabbis as a restraint on the measure for measure principle. Note well the difference. Wiseman rightly states that measure for measure "equitably relates a consequence to an action," implying that "identical acts have equal results, and differing acts have correspondingly scaled results." But he does not realize that this thought is precisely the tacit premise of proportionality in halakhic a fortiori arguments that the rabbis sought to harness when they formulated the *dayo* principle. They did not make that decision unaware of the morality of measure for measure, but instituted the *dayo* principle as a further refinement in morality in view of the uncertainties inherent in inference.[62]

In his analysis of the Miriam episode, Wiseman refers to the *dayo* principle to raise the question as to why she received a greater punishment than Aaron did for apparently the same crime. In this context, he is taking the principle to mean that 'for the same fault, the same punishment is due'. This seems to classify the *dayo* principle as a special case of the measure for measure principle, whereas the *dayo* principle is clearly not an application of that principle, but on the contrary a limitation on its applications. However, I think this is just equivocation on Wiseman's part – he tends to use the word *dayo* in variegated senses. He does in the main treat the *dayo* and measure for measure principles as antithetical.

For him, the measure for measure principle is an argument against the *dayo* principle, which he thinks was intended in too sweeping a manner[63]. Whereas for me, the *dayo* principle is an argument against the measure for measure principle, which the Sages considered potentially too sweeping.

Gezerah shavah. Wiseman tries to argue that if a *qal vachomer* argument is prevented by the *dayo* principle from drawing a proportional conclusion, it would be functioning as no more than a *gezerah shavah*. The latter term refers to the second of the rabbinical hermeneutic principles (*qal vachomer* being the first), and translated literally means "equal judgment." This consists in argument by analogy based on similar wording or meaning, making possible the inference of one law (not explicitly given in the Torah) from another (explicitly given in the Torah). Wiseman makes no mention of the third hermeneutic principle, *binyan av*, which is also an argument by analogy of sorts, passing

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[60]   P. 108.
[61]   P. 112.
[62]   See pp. 111-3.
[63]   See for instance p. 208, where he argues in favor of "a circumspect *dayo* principle applicable in the light of a normal, proportional justice, guided by a 'Measure-for-Measure' principle."

from common causal characteristics to common effects or vice versa. He also does not mention inferences from context, based on the textual proximity of topics (*heqesh*, *semukhim*, *meinyano* and *misofo*)[64].

Wiseman develops this argument[65] in an effort to show Maccoby's preference for non-proportionality to be a dead end. Briefly put, if this rule was true, "the QC would then reduce to a GS" ("QC" and "GS" are Wiseman's abbreviations for *qal vachomer* and *gezerah shavah* respectively). Again, "the same, given *dayo*, makes the *Mishnaic* QC function like a strict, Scriptural GS. This effectively erases the separate natures of the religious QC and the Scriptural GS as individual, hermeneutic rules, as promoted by the same *Mishnaic* authorities." Again, "Then the QC and GS are interchangeable when each result is the same. For all its acceptable, Jewish uses, therefore, such a religious QC reduces to a GS and does not really function as a unique rule of Biblical interpretation as so assumed. If they are distinct rules, equality is problematic."

But this viewpoint is, of course, wrong. A fortiori argument has to do specifically with comparisons of quantity and with thresholds – considerations which are absent in *gezerah shavah* and other *midot*. A crescendo (i.e. proportional a fortiori) argument and purely (i.e. non-proportional) a fortiori argument are both quite different from any other sort of argument by analogy (whether verbal/semantic, causal/effectual, or spatial/temporal). It is not the quantification of the subsidiary term that makes a fortiori argument different from other analogical arguments, it is the whole peculiar structure of it which makes it unique. An a crescendo argument to which the *dayo* principle is applied does not thereby cease to be an a fortiori argument; it just becomes a purely a fortiori argument, which still retains the special structure that differentiates it from other analogies.

Certainly, quantity is not by itself indicative of a fortiori argument: one could conceivably (but I do not know if this is ever actually done) effect a *gezerah shavah* or other analogy with reference to a quantitative term without intending a *qal vachomer*. Wiseman does not see all that, because his concept of the a fortiori argument is too vague. His attempt to involve other hermeneutic principles in the discussion reflects this lack of clarity. It is just an expedient for polemical purposes; it has no formal standing.[66]

Moreover, he is mistaken with regard to both *qal vachomer*, or more precisely the *dayo* principle relating to it, and *gezerah shavah*, in thinking of them as general principles of inference which could be applied *within* the Jewish Bible. This is evident, for instance, when he says that "Malachi 1:6 could be a QC, although on the face of it is a GS"[67]. No! A statement in the Tanakh is neither a *dayo* nor a *gezerah shavah*. These are rabbinic *hermeneutic* principles through which the rabbis, who come long after the Scriptures, try to infer Jewish laws *from* the Torah (maybe from the Nakh too, in the limit). They are not to be confused with use of a fortiori argument, whether proportional or non-proportional, or of argument by analogy, within the narrative.

Furthermore, strictly speaking, later rabbis are not allowed to apply the hermeneutic techniques, originally intended for inferences from Scripture (i.e. the Torah, or more broadly perhaps the whole Tanakh), to earlier rabbinic tradition, i.e. to the "oral law." I quoted Bergman (who refers to Rashi on *Shabbat* 132a) to this effect in my *Judaic Logic*: "The Oral Law cannot be interpreted with any of the thirteen hermeneutic rules." Even if this restriction is maybe not always obeyed in practice, it is an officially accepted one (at least for serious legal matters, i.e. *halakhah* – for homiletic purposes, i.e. *haggadah*, there is much more leeway). Yet Wiseman (following Samely) assumes "the Mishnaic QC" to be "a tool to uncover existing or hidden truths of Scripture *or tradition*" (my emphasis)[68].

6. The scope of dayo

Sometimes, Wiseman seems to consider that the Mishna Baba Qama 2:5 Sages intended the *dayo* principle as a general truth, applicable in the way of a rule of natural logic to all a fortiori argument whatever its content, whether secular or religious. In this, he reflects the view of his rival commentator, Maccoby, except that unlike the latter he does not agree with such a sweeping rule and seeks to combat it by means of rational reflection and examples.

Other times, Wiseman seems to consider that these Sages intended *dayo* principle as a general truth for Judaism specifically. In this, he perhaps reflects the view I (wrongly) took for granted in my earlier work, *Judaic Logic*. Here again, Wiseman responds in a combative manner, opposing the Sages on rational grounds, mainly by appeal to the

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[64]   He does mentions "context" as one of the bases of *gezerah shavah*; but that is an error.
[65]   Mainly in pp. 148,153-7.
[66]   He quotes various other commentators who have regarded a fortiori argument as an argument by analogy of sorts: Maccoby, Samely, and Moshe Weiss (whose paper "The Gezera Shava and the Qal-VaChomer in the Explicit Discussions of Bet Shammai and Bet Hillel" I have unfortunately not been able to find). He could have mentioned me; I too see it as analogical. But the issue is not one of authority – it is a formal one.
[67]   See footnote to p. 127. Note that the argument here is neither an a fortiori nor an argument by analogy. It is: Fathers are generally honored and I am their Father, therefore I would expect them to honor Me (syllogism) – yet they do not (contrary to rational expectation).
[68]   P. 127. He is perhaps also influenced by Daube in this matter; see footnote to p. 133. See also p. 218, where Wiseman describes the *dayo* principle as "pegging the conclusion to the given, Rabbinic tradition."

'measure for measure' principle, and by adducing examples from the Tanakh, the Mishna, and the Gemara, to show that even within Judaism such generality is not generally believed or adhered to. In his view, "both the same precedent and ratios" are "possible conclusions of an a fortiori," so that "either way, enduring moral truths" can be "applied to new cases"[69].

Wiseman's position, then, is that one way or the other the *dayo* principle is overextended, and he makes every effort to circumscribe its scope. This aim of his discourse is, to my mind, commendable. But, as should be evident, I do not agree with all the means he uses. Moreover, although I agree that the Sages' *dayo* principle has been interpreted too broadly, either as a logical generality or as a Judaic one, I very much doubt that the Sages themselves ever intended either of these interpretations. I believe that their intent was from the start very specifically halakhic, to interdict the inference, by rabbinical legislators and judges, of a greater penalty for a greater offense merely on the basis of analogical reasoning (or more specifically, a fortiori argument).

Wiseman, on the other hand, regards the Mishna Sages as having *themselves* intended a larger scope for the *dayo* principle. This is evident in the way he often directs his criticism in large part towards them[70], rather than towards subsequent interpreters of their thought. It is also implied in his frequent use of the word "*dayo*" as synonymous with 'equal' (i.e. non-'proportional') a fortiori argument. This is a rather funny usage, suggesting that any 'equal' conclusion is indicative of *dayo* application, even though by his own admission 'equal' conclusions are sometimes justified, whether because the proportionality involved happens to be 'equal' or because lack of information renders impossible the conclusion of a more accurate proportion. Moreover, *dayo* is supposed to be a principle that obstructs an attempt at proportional conclusion, not one that preempts any such attempt; it comes after, not before, the velleity of proportional reasoning[71].

That some of Wiseman's beliefs are in contradiction is evident, and I submit he does not clearly and convincingly resolve the contradictions and arrive at a consistent thesis. This may in part be due to the great quantity of material and issues that needed to be taken into consideration and dealt with. But it is also in large part due to his failure to develop the subject-matter in a sufficiently thorough and systematic manner. Though he tries to do so, he does not have all the needed methodological and logical tools in hand to get the job done. Moreover, he has not sufficiently studied the relevant texts; and of course, this affects his views. For example, because he is aware only of the first *dayo* objection, and not of the second, he is unable to immediately refute Maccoby's idea that the *dayo* principle is a general truth, or to adequately analyze the Gemara and the Miriam story mentioned in it.

Wiseman also deserves criticism for his tendentious interpretation of many Biblical and Talmudic examples. He freely reads meanings into given texts that are simply not there, which suggests that he has not fully understood the difference between rhetoric for polemical purposes and impartial scientific discourse, or between fiction and fact. When analyzing a narrative, one should seek to determine what the characters in it are actually described as thinking, saying or doing, and not merely project what one thinks they might have or ought to have thought, said or done. Rigorous textual analysis must precede and be consciously distinguished from speculative interpolations.

Wiseman's motive is of course primarily to show that there are many examples of proportional a fortiori argument in the Torah and the Nakh, and in the Mishna and Gemara. And indeed there are many examples – but he does wrong in trying to artificially produce many more. The reason he does that is that he desires to prove that proportional a fortiori arguments are *more frequent* than non-proportional ones. But this more specific thesis is not confirmed by the bare facts[72], so he tries to incline the data in the desired direction. Even within rabbinic hermeneutics, a distinction is made between explicit and implicit a fortiori arguments; he does not heed this warning, and indulges in freewheeling interpretation. He does admit this somewhat, e.g. when he writes: "Some of the claims around the *Mishnaic* era are *my reading into* these issues"[73]; but he shows little restraint in practice, especially when interpreting Biblical examples. We shall now examine some examples of this excess.

As regards **Biblical** instances of *qal vachomer*, Wiseman refers to 25 "main" cases[74]. His list is based on the one I developed in my *Judaic Logic*[75]. He examines these cases one by one, with a view to "evaluate" whether its intent is:

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[69]   P. 213. Notice his reference to "enduring moral truths" generally, and not specifically to Torah penalties for offenses.

[70]   Certainly, they deserve criticism for being insufficiently clear about their intentions. Or at least, the compiler(s) of the Mishna left unsaid many things that it would have been wise to say out loud. It is very doubtful that the Sages were actually as laconic as they are made to appear in the written record.

[71]   In the Mishna concerned, the Sages formulate their *dayo* objections *in reaction to* R. Tarfon's attempts at proportional reasoning.

[72]   He pays no heed to the interpretations proposed in chapter 6 of my *Judaic Logic*.

[73]   P. 11.

[74]   See pp. 173-194. Wiseman additionally mentions Esth. 7:4 and Isa. 49:15 as "implied QC", and as an "abductive comparison" Gen. 29:19 – but, frankly, I see no intended a fortiori argument in these passages. He also suggest that Gen. 3:1-5, in view of its use of the keywords *af-ki/lo*, may be intended as *qal vachomer*. But I see no such velleity here either: the serpent's argument seems to be (in paraphrase): since God did *not* say 'you shall not eat of *any* tree of the garden', then you *may* eat of this tree; to which Eve rightly replies 'He said we may eat of all trees *except* this one'; thus, the serpent attempted a fallacious inference from 'not all are forbidden' to 'this one is not forbidden'.

proportional (P), "*dayo*" (D), i.e. non-proportional, or a "tie" (T), the latter apparently referring to cases which are both or uncertain (i.e. D and/or P). Note his telling use of the word "*dayo*" to refer to any non-proportional a fortiori argument, rather than to a principle that turns certain proportional arguments into non-proportional ones.

The result of his detailed analysis seems to be that, of the 25 cases surveyed, 14 are P, 8 are D and 3 are T[76]. From these figures he infers that, in the Tanakh, proportional *qal vachomer* arguments are more frequent than non-proportional ones. And he uses this finding to demonstrate that the *dayo* principle cannot be as universal, or at least as widespread in Judaic contexts, as the rabbis seem to think. And more broadly, he generalizes it to suggest that proportional a fortiori argument is more common in human discourse than non-proportional a fortiori argument.[77]

My own assessment (in **Appendix 1**) is that, of 46 Biblical a fortiori arguments, only 6 are a crescendo. This means that non-proportional *qal vachomer* arguments are much more frequent than proportional ones; but it still supports the thesis that there are, in both the Torah and the Nakh, some samples of a crescendo argument. More specifically, of the 30 arguments considered by Wiseman, only 4 are a crescendo[78]. My accounting thus differs from Wiseman's in 13 cases (including the 3 "ties"). This great disparity shows that Wiseman's interpretation are, to put it mildly, much less conservative than mine. Let us now look more closely at some of Wiseman's readings.

As regards the "ties," Wiseman concedes the non-proportional interpretation, but regards the proportional one as equally if not more reasonable. For instance, for Ex. 6:12, he favors the reading that Moses expects a greater rejection from Pharaoh than he got from the Israelites, though he admits the possibility of equal rejection as the "default" expectation, speculating that the Israelites' rejection will incite greater rejection by Pharaoh. Similarly for 1 Sam. 23:3 and the three cases in Job. But Wiseman does not understand that such material speculations are beside the point: what matters is formal logic, i.e. what conclusions the given premises logically allow. That other possibilities exist no one denies – but the question to ask is what the specific data at hand implies.

In some cases, a proportional conclusion is manifestly justified by the language used in the given text. Thus, in Gen. 4:24, Lemekh explicitly argues from a premise of seven to a conclusion of seventy-seven; in 1 Sam. 14:29-30, Jonathan predicts "a much greater slaughter;" and in 2 Sam. 12:18, David's servants argue from his "not hearkening to our voice" to his "do[ing] himself some harm." These are three arguments clearly intended as proportional, proceeding from a lesser to a greater quantity. In Esth. 9:12, the issue is debatable, since the premise mentions 500 dead, whereas the conclusion is a mere question; but let us grant this ellipsis as signifying an expectation of *many more* dead (since this interpretation has become a venerable tradition). Yet, even if Wiseman rightly classifies these cases as proportional, his analysis of them is far from credible[79].

In other cases, Wiseman classifies arguments as definitely proportional without any basis in the text. Thus, for Deut. 31:27, whereas Moses only predicts the Israelites will continue to rebel after his death, Wiseman subjectively quantifies the prediction, claiming things are bound to go from bad to worse "because people tend to move away from God," and he cites subsequent events as proof. Again, in 1 Kings 8:27, Solomon argues that if the heavens are not big enough to contain God, then an earthly house is not big enough to do so; this being a negative argument, there is no call for a proportional reading; yet Wiseman gets into a discussion about the relative sizes of God, the heavens and an earthly building, and concludes that the proportionality (not just the a fortiori) is "impeccable." He treats such arguments as springboards for his own preachments or speculations, instead of focusing on what the speaker is actually saying. Even if what Wiseman says is true in the abstract, it does not follow that it is directly pertinent to the issue of whether the given argument is proportional or not.

Next, look at 2 Kings 5:13, where Naaman's servants advise him to obey Elisha's injunction to immerse in the Jordan river so as to cure his leprosy; the premise is that Naaman would have done whatever Elisha asks if the latter had asked for difficult things, and the conclusion is that Naaman should do whatever Elisha asks even if the latter asks for something easy. The consequence is essentially the same in both cases, viz. doing whatever Elisha asks to do. Yet Wiseman constructs *another* a fortiori argument, which refers not to the act of obedience, but to Naaman's pride – and since doing the easier thing requires more humility, Wiseman "opt[s] for proportionality."

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[75]   See e.g. the first table in chapter 5. Wiseman acknowledges the source of his list in a footnote, saying "much of this is Sion's." My list actually has 31 cases; but he counts some sets of cases as single cases (namely, the 2 cases in 1 Kings 8:27 and 2 Chronicles 6:18, the 3 cases in Job, the 3 cases in Psalms 94, and the 2 cases in Jeremiah); also, he leaves out one case (namely, 1 Samuel 21:6) without any explanation – presumably just an oversight due to inattention, because he does not explain the lapse; and he adds on the case in Esther 9:12, which I had there intentionally left out as doubtful, even though it is traditionally counted as a fortiori.

[76]   These figures are my own accounting, based on the contents *of the cells* in his table ("Diagram 9"). The totals given by Wiseman in the same table are 12-15 P, 7-10 D and 3 T, due to some cases being counted more than once. Moreover, in a comment below the table, he has the score as 12 P and 8 D. He gives no explanation for the inconsistencies.

[77]   Additionally, Wiseman draws a distinction between arguments with "natural" content, and those which are "revelatory" in content. Results: of the 25 arguments, 13 (8 P, 3 D, and 2 T) are natural; and 12 (6 P, 5 D, and 1 T) are revelatory. This goes to show that the distribution (of P, D and T cases) is about the same in both areas.

[78]   Namely, Genesis 4:24, 1 Samuel 14:29-30, 2 Samuel 12:18, and Esther 9:12.

[79]   His reading of Gen. 4:24 is particularly confused.

Next, consider 2 Kings 10:4, in which the rulers of Jezreel reflect: if the two kings were not strong enough to resist Jehu, then we are not strong enough to do so. Here again, the conclusion is obviously identical to the premise: there was and will be no resistance to Jehu. But Wiseman imagines how much more death and destruction would befall the speakers if they did resist, and so concludes with proportionality. Thus, whenever he can spin some proportional change as conceivably occurring in the background, he judges the argument as proportional. He does not pay attention to the terms of the given argument, but engages in fiction writing, adding elements that have no direct relevance to the logical form of the actual text under scrutiny.

Wiseman fancifully misreads even obvious arguments. He judges Ps. 78:20 as a proportional argument, because "meat, water and manna" is better than just "water and manna;" but that is not the given argument, which is that since it is more difficult to draw water from a rock than to provide food, the fact that God did the former is proof that He can do the latter. Again, Wiseman judges Ps. 94:9-10 as proportional arguments, because the Creator has "superior ability" compared to humans; but this is not the given arguments, which are that since God is powerful enough to create sense-organs, He is powerful enough to see and hear, and since He has the majesty to chastise nations, He can well reprove individuals. Note that these arguments are positive predicatal, major to minor in form.

Wiseman also tendentiously misinterprets Prov. 11:31, which says "Behold, the just man shall be recompensed on earth: and likewise the wicked and the sinner," as proportional – arguing that since a positive action is worth more than a negative one, it follows that if a positive action has positive consequences, a negative action must have negative consequences (at least, this is how I understand his symbolic statement "If A+, then C+; A+ > A–; so for A-, then C–"). But this is not the argument intended in the text; and anyway it is not a valid a fortiori argument, because it is positive subjectal in form and yet goes from major to minor, and because the predicate is not the same in the premise and conclusion. The argument has to be something like: If the just man is imperfect enough to be recompensed on earth, then the wicked and sinner are imperfect enough to be recompensed on earth; and this form is definitely non-proportional, since no quantity is mentioned.

Prov. 21:27, which reads "If the sacrifice of the wicked is an abomination; how much more brought with a wicked intent?" is interpreted by Wiseman as proportional, because "the bad intent of the evil perpetrator makes his offering an even more unacceptable sham, rather than just a reinforcement of the same degree of badness." Here again, his assumption is that the degrees in the subject must be reflected in the predicates. But this is not the given argument, which is rather: If the sacrifice of the wicked brought with a 'sincere' intent is abominable enough to be rejected, then the sacrifice of the wicked brought with a wicked intent is abominable enough to be rejected. The sacrifice will in either case be rejected (or be "unacceptable"). There are no degrees to rejection: it either happens or does not. The predicate is the same in both premise and conclusion. We could well imagine that the rejection in one case will involve some public ceremony of humiliation, and in the other case an even more humiliating ceremony – but nothing to that effect is remotely hinted at in the text.

Wiseman misinterprets the arguments in Jer. 12:5 as proportional, just because they *could* conceivably be so. Even if it is very reasonable to suppose that one will suffer proportionately more in more difficult circumstances, it does not follow that one can *infer* the greater suffering from the lesser one, unless one accepts the premise of proportionality as a given – and in this case nothing in the text suggests it (and on empirical grounds, one may well *not* suffer more, so there is no reason to expect such a premise); in other words, the intent may well not be quantitative. Anyhow, God (the speaker of those lines) is not forewarning Jeremiah about suffering, but telling him in a metaphorical way that, in view of his human limitations, he could not possibly grasp the answers to his questions as to why the wicked prosper. The arguments are thus essentially negative predicatal, minor to major, in form.

Finally, consider Dan. 2:9, where Nebuchadnezzar demands: "Tell me the dream, and I shall know that you can declare its interpretation to me." Wiseman reconstructs this to read: "Were I to tell you the dream… I could not be certain that your version was correct; so it is all the more necessary that you tell me both the dream and its meaning, so that I can trust your interpretation." He views this as proportional with reference to the degrees of certainty involved: "If I tell, then answer A1 is uncertain, but if I do not tell, answer A2 is certain." But once again, this is not the argument directly intended. Nebuchadnezzar is clearly arguing: if Daniel has powers sufficient to tell the dream, then he has powers sufficient to interpret it. It is the cognitive powers required of Daniel for the two feats which are compared. Note that this is a positive predicatal, major to minor argument.

We have thus looked at all of Wiseman's proposed reinterpretations, where he opts for proportionality, and found them all unconvincing. Some are incomprehensible and some are in manifest error. Some of his arguments are not even a fortiori in form. He invents alleged a fortiori arguments out of hand, with terms not directly given or implied in the text. He ignores the forms as well as the contents in the original text – reading negative arguments as positive ones, or predicatal arguments as subjectal ones, so that almost all his readings are predictably positive subjectal. He

hardly notices the difference between minor-to-major and major-to-minor argument, or realize its logical significance.[80]

Wiseman tried to obtain a maximum number of proportional examples, in order to buttress in larger thesis that proportional a fortiori argument is the rule and non-proportional a fortiori argument is the exception. He also sought thereby to buttress his narrower thesis against the rabbinical the *dayo* principle. But this effort was unnecessary, because the few demonstrable examples of proportional argument that there are suffice to prove the point – viz. that there are evident departures from the *dayo* principle, conceived as a general injunction applicable to all Jewish matters if not universally. These departures prove, not that the Sages' *dayo* principle is often ignored in practice, but that those who interpret it too broadly are wrong.

Properly understood, the *dayo* principle has no application in Biblical passages or in their interpretation. If (as I now suggest[81]) the *dayo* principle is an ethical-legal restriction, designed to obstruct the inference of penalties for offenses that are *not* explicitly found in the Torah from penalties for offenses that *are* explicitly found in the Torah, then we have no reason to expect the *dayo* principle to be used *in* the Torah. It might conceivably be used in the Nakh (the later books of the Bible), with reference to a Torah law and penalty – say, in a story where a king or some judges are trying to derive a law from the Torah (I have not tried to find out whether such use in fact occurs). But there is no reason to expect a ubiquitous *dayo* principle, i.e. to regard as Wiseman tends to all non-proportional a fortiori arguments as having been subjected to the principle, and all proportional a fortiori arguments as having erroneously escaped it.

Wiseman offers two examples of **Mishnaic** *qal vachomer*[82]. The first is not from the Mishna, but is attributed to a 2nd cent. CE Tanna (a Mishnaic Sage), R. Shimon bar Yohai, in the Talmud Yerushalmi[83]. It reads: "Not even a bird perishes without the will of heaven, how much less a man." I would interpret this argument as follows: A man is more precious to God than a bird; a bird has sufficient worth to be incapable of perishing without God's decree; therefore, a man has sufficient worth to be incapable of perishing without God's decree (positive subjectal, minor to major). There is no possibility here of proportionality: "cannot perish" means what it says and does not allow of degrees. If R. bar Yohai intended otherwise, he would have worded his minor premise more softly. Since no proportionality is attempted, the *dayo* principle cannot be called upon to oppose such attempt.

The second example is from the Mishna (Sanhedrin 6:5), and reads: "If God is troubled at the shedding of the blood of the ungodly, how much more at the blood of the righteous!" I interpret this argument as follows: God has for the righteous more concern than He has for the wicked; if for the wicked God has concern enough to be troubled at the shedding of their blood, then for the righteous God has concern enough to be troubled at the shedding of their blood (positive subjectal, minor to major). As it stands, the argument is purely a fortiori; but it could easily have been made a crescendo by adding the premise that the degree to which He troubled at the shedding of someone's blood is proportional to the degree of His concern for him. But even if proportionality was intended, it is hard to see how the *dayo* principle might be called upon to oppose it, since the conclusion does not infer a Torah penalty but merely informs that the blood of the righteous is (or is more) troubling to God.

Wiseman understands these two arguments somewhat similarly, even if less clearly. For the first, he remarks: "As a physical maximum, death is the severe end of life. We have little reason to seek some decision procedure here, or that the *dayo* is offered as a universal norm (although the end is clearly the same), or that one needs to know anything other than that the prior example elicits a similar result." And for the second, he remarks: "Despite our inability to account for the greater upset that God might feel, the most sensible view is that of an increase;" to which he adds: "why not a justly proportional conclusion in differing cases?" although he admits: "One does see a distinction between conceptual acceptance of this possibility versus practical punishment."

He says many more things, some of which I would criticize, but I will focus on just these insights. What they show is that Wiseman is well aware that the 'equal' conclusion (which he misleadingly characterizes as "*dayo*") is sometimes appropriate (as in the first example), while in other cases it is open to discussion (as in the second example). It is also

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[80]   Of his 25 cases, 14 are in fact (by my readings) major-to-minor (6 P, 5 D, 3 T). More precisely, 10 are positive subjectal, 5 are negative subjectal, 9 are positive predicatal, and 1 is negative predicatal. Wiseman does not notice these formal differences, or reflect on their implications.

[81]   I must confess that, in my earlier work *Judaic Logic*, I wrongly perceived the *dayo* restriction as intended as general to Judaism, although I considered such general restriction as unjustified and not always obeyed. For these reasons, in my analysis of Biblical a fortiori argument there, I remarked in the three cases I identified as proportional a fortiori argument: "Dayo ignored." Wiseman may have been influenced by these errors of mine, although he took them farther by reading proportionality into many more arguments.

[82]   Pp. 114-7. Wiseman also mentions (on pp. 119-125): the *qal vachomer* in Mishna Yadayim 4:7 (which I deal with in Appendix 2), as an example of how such arguments are "neutralized;" and the one in the name of Hillel in the Gemara Pessachim 66a (which I quote in the section on Gary Porton, further on), which is a *baraita* (i.e. claimed as Tannaic, but not given in a Mishna), as an example of how a *gezerah shavah* "comes to the aid of" a *qal vachomer*. And maybe others still. I will not however, for a lack of space, analyze and evaluate his every statement on this topic.

[83]   Zevachim 9:1, according to Wiseman (I have not checked).

interesting to see that he makes a distinction between "conceptual" expectation of proportionality and the "practical" difficulty of imposing punishment on that basis, though he does not pursue this thought as far as he ought to have. He sums up: "I simply propose that the *dayo* cannot be exclusive in every case, let alone in religious ones, especially when dealing with many practical issues that would make some outcomes unfair or even unbelievable. It is normal that a more severe crime deserves a more severe outcome, although an equal outcome might work."

This is true enough. The problem, as we have seen, is that his approach to the a fortiori argument is too vague to be able to formally predict when an equal or proportional outcome is the more appropriate. He is only able to deal with this issue on material grounds, on grounds of apparent "reasonableness." The other problem is his confusion of the term "*dayo*" with that of "equal" conclusion. Although former implies the latter, it does not follow that the latter implies the former. Moreover, while according to the measure for measure principle "a more severe crime deserves a more severe outcome," the *dayo* principle is precisely aimed at interdicting that way of thinking when it is an attempt to *infer* a harsher penalty (albeit for a worse offense) from a lighter Torah penalty (for a lighter offense). This is what the Mishna Sages *consciously* intended; it was not a thoughtless dictate. Logically, the Sages' position is unassailable; they break no law of logic in advocating their *dayo* principle. Yet Wiseman keeps trying to talk them out of it, as it were!

With regard to *qal vachomer* in the **Gemara**, it is unfortunate that Wiseman nowhere in his paper attempts a study of the crucial passage in Baba Qama 25a-b, which comments on the Mishna Baba Qama 2:5. He mentions this *sugya* of the Talmud a few times, but has obviously made no effort to actually look at it and study it. He relies entirely on hearsay concerning it, mainly Maccoby's; and Maccoby of course is an unreliable source, since he himself did not study it attentively. As a result, Wiseman constructs a somewhat fanciful view of what the Amoraim (i.e. the rabbis of the Gemara) said about *qal vachomer* and the *dayo* principle. Though he occasionally mentions "examples" of Amoraic proportional a fortiori argument[84], I have not found any cited in his paper. Nevertheless, let us consider his views.

Wiseman imagines that the Amoraim advocated proportional a fortiori argument[85]. This seems accurate, since a *baraita* is cited to that effect in the Gemara we just mentioned. He thinks the Amoraim did so in opposition to the Mishna Sages who instituted the *dayo* principle (the "majority") and in support of their opponent, R. Tarfon (the "minority")[86]. This is inaccurate; the Gemara actually seeks to justify the *dayo* principle ("Is not *dayo* of Biblical origin?") and to show that R. Tarfon believes in it too ("Does R. Tarfon really ignore [it]?"). Wiseman does not, however, claim that the Amoraim totally denied the *dayo* principle; he sees them as rationally circumscribing it[87]. There is truth in that; the Gemara makes it more conditional, in an effort to reconcile the opposing views in the Mishna.

Additionally, Wiseman suggests that the *dayo* principle was instituted and accepted in Mishnaic times because the surrounding dangers made unity, preservation and transmission imperative. On the other hand, the Amoraim, living in less disturbed times, could afford to be more open to the extension of laws through proportional a fortiori argument.[88] But this view is, in my opinion, incorrect. The Mishnaic Sages instituted the *dayo* principle not out of fear of external forces, but out of fear of God – the desire to avoid punishing people more harshly than specifically permissible under Torah law. And (to my knowledge) the Amoraim never theoretically reprove them for it, nor markedly deviate from it in practice.

Furthermore, Wiseman implies that the early rabbis who instituted the *dayo* principle did so out of rigidity of mind, rejecting the nuances made possible through proportional a fortiori argument, whereas the later rabbis were freer and more precise thinkers. But the Tannaim were just being morally responsible, very careful not to make mistakes

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[84]   On pp. 159, 170, 219.

[85]   Which he variously calls: "ratios," pp. 109, 150, 171; "proportional QC's," pp. 111, 210; "explicitly non-dayo QC's," p. 149; "scaled QC's" p. 154; "degrees," pp. 157, 168; "proportion," p. 161; "QC's with degrees," p. 170; "scaled examples," p. 219.

[86]   The argument is "reassessed" by them (p. 99); "the minority *Mishnaic* view (and the greater *Amoraic* latitude) differs from the majority one" (p. 99); the Amoraim "strengthened" the minority view (pp. 111, 149); "The Amoraic addition of proportional examples may well express their disagreement with the arbitrary dayo fiat" (p. 159); "the Amoraim sought a solution that brought harmony back" (p. 160); "One Rabbinic view can correct another" (p. 161).

[87]   "Like the later Amoraim, we should opt for a better, QC solution that allows for any rational result: whether a degree, sameness, or a compromise" (p. 156); "All told, the Amoraic position is preferable, for a simple weakening of the strong dayo to a principle alongside proportion removes its arbitrary character" (pp. 160-1); "Because the Amoraim concluded some QC's as ratios, they understood that the dayo was not a Divine truth about all QC's" (p. 161); they "saw the Mishnah's dayo claim as applicable under specific conditions only, while their own view worked under others" (p. 170).

[88]   P. 170. "Minority compliance may have been for reasons other than the *dayo's* partial sensibleness: they submitted because tradition was important, majority decisions were preferable, unity in difficult times was critical, their keen protests were recognized, and they feared excommunication." "What served the purposes of the Tannaim—preservation and transmission—was not the same as those of the Amoraim—classification and extension. So one expects that the freer Amoraim could systematize and generalize more than their predecessors, who needed to remember, record, and reiterate the tradition in times of persecution."

through fanciful extrapolations. Moreover, it cannot truly be said that the Amoraim, who considered themselves their humble disciples, were "freer" than the Tannaim in the interpretation of existing laws or formulation of new ones. Certainly Wiseman does not substantiate this claim in a systematic manner. In particular, he gives no examples of Amoraim freely using proportional a fortiori argument in halakhic contexts.

The problem here is, of course, one of perception. When Wiseman thinks of the *dayo* principle, he thinks of it as intended by the rabbis who instituted it very broadly, at least within the sphere of Judaism. Then, when he sees (or hears about) later rabbis apparently ignoring this broad *dayo* principle and indulging in proportional *qal vachomer*, it seems to him to constitute a reversal of previous policy[89]. But in fact, *dayo* was from the start *not* intended as so broad a principle, but one limited to specific inferences (as earlier described). So, before claiming that later rabbis sometimes ignore *dayo*, it is necessary to demonstrate that they ignore *this* limited principle, and not a conjectured broader one. It may be that they sometimes do; but Wiseman has not empirically demonstrated it.

7. Miriam and Aaron

Wiseman expends considerable effort criticizing the Torah and subsequent commentaries for not judging Miriam and her brother Aaron with the same leniency (or severity), regarding the events narrated in Numbers 12. Miriam and Aaron are there said to have spoken against their brother Moses, apparently for having married a foreign woman, and seemingly suggesting that their own level of prophesy was equal to his. God summons them and reproves them. When He withdraws, Miriam is struck with "leprosy;" but there is no mention of Aaron being so afflicted too. Thereafter, Miriam is condemned to a seven-day imprisonment outside the camp; but again Aaron is not so punished. As Wiseman puts it:

> "More severity falls on Miriam than on Aaron. If Aaron could get away with just a lesser rebuke, why did Miriam get more: a rebuke, leprosy, and seven days of separation? Not even Aaron could fathom why she got these heavier strokes. Nor did it appear right to Moses, who accepted Aaron's plea for their sister and asked God for a lessening (or healing). In other words, both men thought that she had received considerably more for no apparent reason, perhaps reasoning that a rebuke should be enough (*dayo*) for her too."[90]

To Wiseman, who interprets the episode as "a challenge to Moses' leadership" by both of siblings[91], this apparent double-standard is unreasonable and unfair. According to him, Miriam should have received the same light sentence as Aaron, by a *dayo* inference from the lighter sentence against him; alternatively, we might add (though Wiseman does not say it), Aaron should have received the same severe sentence as Miriam, by the same reasoning.

Wiseman acknowledges that the argument in the Torah concerning Miriam is from seven days for offense against a father to seven days for offense against God (or Moses, in his view). But he wonders why a similar inference is not made for Aaron too. He complains that past commentators seem to ignore the "disparities" between their sentences. All this makes God seem "inconsistent"[92]. Wiseman wonders whether perhaps God exempted Aaron because of his privileged position as High Priest, objecting that his priestly duties could have been temporarily carried out by his sons.

Wiseman has evidently not properly studied this episode. As Rashi reads the story, Miriam is mentioned before Aaron to indicate a leadership role on her part in this incident. Also, they mentioned Moses' wife to criticize him for causing her chagrin by no longer having marital relations with her, and they mentioned their own status to point out that although they had not given up on marital relations their prophetic powers were not diminished. God later replies to them that He ordered Moses to stay away from his wife, because he needed to be readily available at all times to receive His instructions – so there was no sin on his part, and they were wrong to criticize him.[93]

Wiseman should have realized that if Aaron was punished less severely than Miriam it was not because of unjust favoritism, but simply because their sins were different. Wiseman does momentarily admit the possibility that

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[89]   "Without stating why they [the Amoraim] shifted from the Tannaic *dayo* norm, they simply added scaled examples, to readjust the imbalance so that it correlated with the Tanach's earlier range of proper QC's" (p. 219).

[90]   P. 196.

[91]   In a footnote, he speculates that Miriam and Aaron had "personal and perhaps political motives." There was "a tinge of jealousy" in their criticism of their "younger brother," whom they had respectively rescued and helped in the past. They may have "wanted a more egalitarian, less hierarchal structure," which could be "construed as an attack upon God's theocratic rule through Moses, desiring a wider form of participation, although not necessarily a call for democracy." He is here confusing this story with that of Korach.

[92]   P. 213.

[93]   It is interesting to note in passing that this tradition, that Moses abstained from sexual relations with his wife to be ready for prophesy at all times, was apparently known to Philo of Alexandria, i.e. in the period of the Mishna. This is according to E. Starobinski-Safran, who cites "*Mos*. II, 68-69" (in a fn. on p. 169). I do not know whether earlier evidence of this tradition has been found; but it seems doubtful Philo was its author, as the rabbis do not ever, to my knowledge, refer to him for information or ideas.

Miriam "bore greater guilt" than Aaron, since she is "mentioned first as instigator, gossip"[94]; but he does not linger on that thought. Reading the story, the first explanation that comes to mind is that Miriam's sin was to talk, whereas Aaron's sin was to listen to her talk. And indeed, this is the explanation offered by a Midrash, according to a rabbi I questioned[95]. It is true that listening to "evil speech" (*lashon hara*) is considered as sinful as proffering it; nevertheless, a difference in degree may be assumed between the two acts (one being passive, the other active)[96].

I would add that Aaron seems to have confessed and repented more readily, since it is he alone who says to Moses: "we have done foolishly… we have sinned." Notice, too, that he altruistically speaks on her behalf, as well as his own. Moreover, he appealed to Moses for his sister's release from the disease, whereupon Moses uttered his famous prayer, "*El na refa na lah!*" Miriam was perhaps too shocked and afraid to say anything for herself, let alone for Aaron. In sum, there are many indices that could explain the difference in judgment besides the difference in initial sin.

So much for the material aspects of the case. What needs clarifying now are the formal aspects. The first thing to notice is that Wiseman introduces an argument of his own, namely, "If Aaron could get away with just a lesser rebuke, why did Miriam get more: a rebuke, leprosy, and seven days of separation?" This seems reasonable to him, because he assumes their sins to have been the same. His answer seems to be that their different treatment is a Divine fiat: "In essence, Divine revelation required more [for Miriam] than Aaron's judgement, not the same." Although human logic expects an equal sentence for the same offense, "God's-eye-view trumped human thinking that took Aaron's, lesser precedent as governing."

He then asks: "If this actual, prior given is not binding, how can the *dayo* arise from Miriam's case?" By this he means: if in the above argument the *dayo* principle is not applied (since Miriam's sentence differs from Aaron's, contrary to expectation), how can the *dayo* principle be derived (as the Gemara teaches) from this episode? Although he admits that the Torah refers to an inference from (a theoretical) offense against a father to an offense against God[97], he argues that this "still does not explain the unequal sentences between the co-participants." Why, he asks, does the same argument "not work with Aaron as well?"

Wiseman proposes an accounting of the penalties involved. Miriam is obviously more severely punished than Aaron, since he just gets a rebuke while she gets that plus leprosy and seven days quarantine[98]. In his opinion, "Aaron's rebuke for disrespect, as the greater person, should have been sufficient (*dayo*) for Miriam, the lesser;" and he is indignant that "The greater one gets less and the lesser more!" His answer to the question of equity is that "mercy in judgement" may be involved, and possibly "major, social concerns too, such as education." But as regards basing the *dayo* principle on this episode, he expresses strong doubt: "Logical necessity is not evident, however, in this *dayo* as a QC paradigm."

In reply, let us first note that Wiseman's first argument (that since Aaron's fault was equal to Miriam's, her punishment should by the *dayo* principle have been equal to his) is not in fact an application of the *dayo* principle. It is an egalitarian (*a pari*) a fortiori argument, as follows: 'Assuming Aaron's sin was the same in gravity as Miriam's, it follows that if Aaron's sin was not grave enough to merit leprosy and imprisonment, then Miriam's sin was not grave enough for the same penalties'[99]. No *dayo* application is called for, or even possible, in this reasoning. Moreover, it does not reflect the intent of the *dayo* principle.

Wiseman evidently wrongly imagines the *dayo* principle to function laterally as well as vertically. Laterally, meaning: by transferring a law from one case to another on the same plane. As against vertically, meaning: by applying a more ancient and authoritative source to a more recent quandary. In truth, as just explicated, the lateral function consists in *a pari* a fortiori argument; the law used is one and the same in both cases (viz. Aaron and Miriam). The vertical function, if we are to illustrate it by means of the episode in Numbers, consists in a minor to major (positive subjectal) a fortiori argument from the 'more ancient and authoritative' case of a penalty for offense

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[94]   "And," Wiseman adds, "even bully of her weak-willed brother Aaron." In fact, though Miriam is indeed mentioned first, and as a gossip (it is she who found out about Moses' abstinence from marital relations), she is nowhere implied to have bullied Aaron: the latter accusation is a bit of drama added on by Wiseman!

[95]   I did not ask him for exact references, because this explanation seemed to me the likely one even before I asked. The writer of the Midrash obviously had the same thought, long before.

[96]   Some rabbis (e.g. R. Shlomo Ganzfried in his *Kitzur Shulchan Aroukh*) say that listening to *lashon hara* is worse than speaking it; but in the present context we may take the more lenient view as applicable. In practice, it is not always possible to avoid hearing *lashon hara*, and one cannot always readily reprove the speaker.

[97]   He says "their father," but this is not in the text; and he says "Moses," but this is not the usual reading.

[98]   I was told that, according some Midrash, Aaron was indeed momentarily afflicted with "leprosy," while Miriam's "leprosy" disappeared soon after, as soon as Moses prayed for her. So the difference in their punishments was mainly in the seven days of imprisonment, which Aaron was spared but Miriam had to endure.

[99]   Significantly, as already pointed out, Wiseman does not realize that he could equally well have argued the other way: 'if Miriam's sin was grave enough to merit leprosy and imprisonment, then Aaron's sin was grave enough for the same penalties.' This means that the major premise, which claims the sins of both to be equal in gravity, does not by itself tell us whether to argue from Aaron to Miriam or vice versa.

against a father, to the 'more recent' quandary of the penalty for offense against God. The premise about a father is the given law (here given as human instinct, let us say) and the conclusion about God is the derived law (which human instinct could not predict).

The *dayo* principle as it emerges in Mishna Baba Qama 2:5 is aimed at the vertical function. Here, the penalty for some fault is clearly spelled out in the Torah, and the rabbis (as legislators and judges) need to know what penalty to apply to a comparable but not identical fault, which is not explicitly dealt with in the Torah. They argue by analogy, and are perhaps tempted to apply a proportional penalty, in accord with their intuition of 'measure for measure'. However, the *dayo* principle – i.e. the precedent set in the said Mishna example – teaches them not to indulge in such pro rata reasoning, but to stick to the given penalty.

This Mishnaic *dayo* principle may be illustrated and derived analogically from the Miriam case given in the Torah, but only to the extent that we accept the idea floated above that the penalty for offense against a father is *obvious* to all, whereas the penalty for offense against God is *not as obvious*. Clearly, we are not in that Torah example *literally* arguing from Torah law to 'rabbinically derived' law, since both penalties are in fact Torah given in that example. But we can say, as the Gemara effectively does, that this argument from the more obvious case (offense against father) to the less obvious case (offense against God) is *by analogy* an illustration of and a justification for the Mishnaic *dayo* principle, even though the latter strictly only applies to inference from a Torah-given penalty (obvious, since explicit in the Torah) to a 'rabbinically derived' penalty (less obvious, since not explicit in the Torah).

In truth, as Wiseman also has noticed, the argument used as the Torah illustration and justification of the *dayo* principle is not entirely non-proportional. Comparing the theoretical precedent of an offending daughter punished by her father with Miriam's punishment by God, Wiseman rightly points out that "being spit upon and getting leprosy" involve "a quantitative change of progressive increase." Indeed, if the punishments are not exactly identical, it cannot be said that this argument has a conclusion equal to the minor premise. The punishment is the same only as regards the seven days of isolation; but as regards the being spit upon and getting leprosy, "the difference in kind and degree stand out."

Wiseman suggests that maybe these different treatments were necessary to obtain "the same level of shame" (implying that it is easier to feel shame for offending one's father than for offending God – a debatable assumption). But "still, a psychological effect is one thing, the actual stroke quite another." Wiseman's conclusion is that the attempted grounding of the *dayo* principle is not so sure. As he puts it, "The Rabbis claim that this passage is a QC argument, although the actual, Biblical text is not that clear." The said objection is quite valid in my opinion. But Wiseman's conclusion is not inevitable.

As I have pointed out in my own analysis, we can only argue that the Miriam story is a fitting a fortiori argument, i.e. a purely a fortiori argument as I take it to be, or an a crescendo argument coupled with an application of *dayo* as the Gemara takes it to be, if we gloss over the said difficulty, and focus only on the seven days isolation penalty (ignoring also the difference between the voluntary isolation of the daughter and the enforced isolation of Miriam).

We can justify that, I believe, by pointing out that the leprosy part of Miriam's punishment was Divinely carried out, whereas the seven days isolation part is to be carried out by the earthly court of law. Since the *dayo* principle is intended for use by earthly judges, only this aspect of the Divine decree in the Miriam story needs be taken into consideration. Moreover, the leprosy part is already a done deed at the time that the a fortiori argument is pronounced; so the argument can only concern the not yet actual part, i.e. the sentence of seven days isolation of Miriam which Moses is ordered to execute.

Thus, contrary to Wiseman's view, the Miriam episode can be credibly used to illustrate the *dayo* principle, and even somewhat ground it Biblically, if it is read carefully. But it must be said that we cannot correctly perceive the *dayo* principle's place in the Miriam episode without referring to the Mishna formulation. So, strictly speaking, the Mishna formulation is conceptually prior to the Biblical interpretation. That is, the Miriam episode can be cited, but it is not by itself a solid foundation.

In his concluding remarks, Wiseman argues that the Miriam *qal vachomer* "could not qualify as a paradigm." It "turned out to have a mixed conclusion—at once the same, proportioned, and restrained. Her case was isolated and unique." Her judgment involved nuances that could not be adequately explained by the *dayo* principle. Instead, it was conceptually preferable to refer to God's mercy which "could moderate or modify the prior norm of proportional justice when it advanced the overall good."[100]

Wiseman's final conclusion that "the *dayo* is *non-universal*" remains true, even if the reasons he gives for that conclusion are not all entirely convincing and even though he does not clearly identify its intended scope. It is obviously wrong to consider the *dayo* principle as a general rule of reasoning, applicable universally or even only in

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[100]   P. 212.

Judaic contexts. That the principle is much more narrowly intended is clear from its initial, Mishnaic formulation. Its Biblical grounding is possible, but not simple.

8. Summing up

On other commentators. Wiseman devotes a good deal of his paper to detailing and analyzing the views of various, mostly contemporary, logicians or commentators, on the subject of a fortiori argument, myself included. This is a very important service that he has rendered to our field of study, bringing to wider attention many contributions to it and stimulating debate. Thanks to his mentions of Samely, for example, I discovered the work of that author and was moved to comment extensively on it. Again, Wiseman's translations of articles by Avraham and Brachfeld made possible my evaluation of these authors' ideas.

I have followed Wiseman's lead, and tried to say something about as many past contributors as I could, including many from past and present that he does not mention. My approach differs from his somewhat, however, in that I am much more critical than he has been. For instances, with regard to the Avraham and Brachfeld articles, Wiseman is content to report their views, whereas I am quick to point out that Avraham wrongly defines a fortiori argument and Brachfeld makes no attempt to define it.

Although Wiseman does on occasion offer value-judgments, his treatment of most commentators with the exception of Maccoby is overall not very critical. We could say that Wiseman is open-minded to almost all approaches, regarding them all as valuable contributions in various respects. But that open-mindedness is only due to the vagueness and uncertainty of his own perceptions: if he fully realized the logical conditions for a fortiori argument, he would be like me less tolerant of deviations from the norm.

I have already described and commented on Wiseman's treatment of Schwarz in the chapter devoted to the latter (14.4). His thoughts on Ostrovsky, Daube, Guggenheimer, Cohen, Avraham, and Brachfeld are incidentally apparent in the sections I devote to these authors in a later chapter (31)[101]. His reaction to Maccoby is described in the discussion of the *dayo* principle in the present chapter[102]; as there made evident, he strongly disapproves of Maccoby's position.

As regards **Alexander Samely**, Wiseman can be said to approve considerably of his views. The reason for that is, I think, that Samely's definitions of a fortiori argument are, like his own, sufficiently vague and uncertain as to allow for 'inductive' and 'proportional' a fortiori arguments. As I have shown in the chapter dedicated to Samely's work (23 – see my full analysis there), the latter's approach leaves much to be desired, involving as it does both important gaps and significant errors, in both form and content. Therefore, for Wiseman to lean on Samely to support certain ideas is not very secure.

Wiseman presents Samely's *qal vachomer* formula[103] ("somewhat simplified," i.e. in if–then form) as "If norm n, belonging to category N that is lower on scale X, has predicate A, then norm m, of category M, higher on scale X, logically has predicate A too." To which he adds the comment: "I take his term 'logically' to mean 'with good reason', rather than as a deductive truth." The reason why Wiseman does this is that, like Samely, he is unable to explain how the putative conclusion follows from the given premise(s). This being the case, he (they) must claim this conclusion to be, albeit not deductive, at least a reasonable expectation (i.e. inductive). By referring to Samely, Wiseman hopes to buttress his own idea that a fortiori argument as he conceives it may be claimed inductive.

As regards proportionality, although Samely states the conclusion in ambiguous terms, as "*therefore*: To *norm* m *feature* A *should apply even more*," Wiseman interprets this explicitly as possible proportionality, saying it could be A or A+ (i.e. more than A)[104]. I would say that Samely's ambiguity is deliberate, because he does not know exactly what to logically conclude. Wiseman's more explicit rendering is justified by the example under consideration, which does suggest an a crescendo conclusion[105]. But Wiseman is equally unable to formally differentiate the two possible conclusions. He does try to do so, with reference to a diagram; but since this makes no mention of the crucial clause "R enough to be" it is not a credible model. My point here is that although Wiseman's position here is more definite than Samely's, he is still relying on Samely's tergiversation to buttress his own allowance for proportionality.

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[101]   Wiseman on several occasions refers to Chaim Hirschensohn, author of *Beirurei HaMidot* (Jerusalem: Haivri Press, 1929). But his comments (at least those Wiseman mentions) seem to all relate to the conditions and limits traditionally set on a fortiori reasoning, so there is not much for me to say about him.

[102]   Additional comments on Maccoby can be found in the section on Daube in a later chapter (31.5).

[103]   Analogy 4.2 as Samely calls it (in his appendix, pp. 413-4). Wiseman, p. 129.

[104]   Actually, Wiseman allows for three possible conclusions: "A (the same), A+ (more), or A- (less)," in accord with his very 'open' approach. I do not mention his third, out of acute embarrassment at the very thought! In any case, Samely surely does not intend this additional alternative.

[105]   See my Appendix 2, Mishna Makkot 3:15 (b).

Wiseman, mirroring Samely, then offers the following justification: "In general, Samely urges caution in pressing a formal, logical schema on the Rabbinic QC, so that the rhetorical and informal nature of this language, culturally and historically based, can express itself. By allowing language its initial, creative freedom and vagueness, one does not force language artificially into overly restricted or truncated forms at the outset. Inasmuch as logical systems eventually can and do expand the range of things they cover, this more permissive approach has clear benefits for later advances in logic." This is of course just spinning an excuse for a cop-out. Obviously, the "later advances in logic" would allow us to judge the logical validity of historical examples – so why not effect these advances and judge?

Finally, let us look at Wiseman's references to **Avi Sion**, i.e. to my own past work on a fortiori argument (in my *Judaic Logic*). Although he devotes a number of pages of his book[106] to the study of my work, and he there seems to have understood it, in the rest of his work he in fact completely ignores crucial findings of mine, notably the factor of sufficiency (or insufficiency) of the middle term which makes possible the formal validation of such argument. He defines a fortiori argument without this crucial factor, and thus makes the argument seem much more vague and uncertain. He does this, as we have seen, in order to lend support to his beliefs in inductive and proportional a fortiori argument.

But I had previously clearly formalized and validated a fortiori argument – specifically with the crucial factor "R enough (or not enough)" – thus definitively showing its deductive status! How could he then claim such argument to be inherently inductive, and only in special (unspecified) cases deductive? Moreover, I had explained that a proportional conclusion was not justified from the given two premises, although it might be justified given additional information. How then could Wiseman maintain the possibility of a proportional conclusion from the two premises without specifying an additional premise?

Although Wiseman is not entirely wrong in speaking of "Sion and Abraham's contention that the QC was often inductive and not just deductive"[107], my view of the inductiveness of many arguments presented as a fortiori is not like Wiseman's, or for that matter Avraham's. Whereas they imagine that a fortiori argument *as such* can be inductive as well as deductive, my own view is that although many concrete arguments intended as a fortiori by their speakers have inductive status and value, as regards a fortiori argument *as such*, as an abstract logical entity, it is clearly deductive[108]. They effectively think that "inductive a fortiori argument" has a distinct form or is of uncertain *form*; whereas I insist that the inductiveness of some such arguments in practice is simply due to uncertainties in the *content* of one or more of their premises and therefore in their conclusion.

I think Wiseman presented my ideas on proportionality, as laid out in my *Judaic Logic*, with (on the whole) accuracy[109] and fairness[110], and he was right to demand that more stress be laid on the possibility of 'proportional' a fortiori argument. However, as already shown, his own attempt to give formal basis to such a fortiori argument is not successful. Here again, the main reason for his failure is his appeal to a very vague definition of a fortiori argument, devoid of a middle term or at least devoid of the issue of sufficiency (or insufficiency) of the middle term. The a crescendo argument that I have developed in the present study, comprising a purely a fortiori argument *combined with a pro rata argument*, is the correct and complete formal solution to the problem wisely posed by Wiseman.

As regards the *dayo* principle, Wiseman considers that his view "largely agrees with Sion['s]." That is somewhat true, but not entirely so. At the time I wrote *Judaic Logic*, I assumed the rabbinical *dayo* principle to have a broader intent than I do today. I think it seemed to me, as it did to Maccoby, to be essentially a logical principle. However, I differed significantly from Maccoby in admitting that given additional information a proportional conclusion might be drawn. Thus, it seems that I regarded the *dayo* principle as a rule intended more specifically for Judaic contexts, although I could see it is not always adhered to even there.

However, I did not (as I do today) further circumscribe the restriction, by making a distinction between punishments and rewards, or by specifying that the premise of such argument had to be a Torah given penalty, because I had not studied the relevant Mishna and Gemara texts. Wiseman too has evidently not studied the relevant texts. His

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[106]   Pp. 81-7.
[107]   P. 12. Similarly, on p. 35, "as Sion observes, the Rabbis approach was more inductive than deductive."
[108]   Obviously, I do not mean that all moods are valid. I refer here to the valid moods of it, like the minor-to-major positive subjectal, and do not deny there are invalid moods, like the major-to-minor positive subjectal.
[109]   Wiseman does make one error in his presentation, on p.86. He misquotes me as saying "Though P is R enough to be Q," whereas I say: "Though P is more R than Q." This is obviously just an error of momentary inattention. I immediately spotted it, but others may not – so Wiseman should fix it. All the more so, since he tries to construct an argument of his own on the basis of this wrong reading!
[110]   Except where, on p. 84, Wiseman wrote that my method "does not fully capture the significance of the two related items...; instead it focuses on the mutually same feature." I understand that this was a criticism designed to make room for a proportional conclusion. But in fact, the room was already there; as he well knew, since he refers to my openness to 'proportionality' on several occasions. For instance, on p. 94, "Sion allows either a proportion or the same amount as possible, although his method tends to favour the latter as truer." My conclusion was formally correct for purely a fortiori argument, and did not exclude the possibility of a crescendo argument.

treatment of the *dayo* principle seems partly influenced by mine, in its uncertainty as to the exact scope intended by the rabbis who instituted it. But his treatment differs from mine in that it is strongly focused on opposing and rebutting Maccoby's rigid view. And of course, he does this in ways that are too informal to be definitely convincing. Funnily enough, Wiseman at one point effectively accuses me of informality! Referring to my ordinary-language method of formalization and validation of a fortiori argument, Wiseman says: "Since he verbally explains the argument in an implicational form without symbols, I make a number of assumptions to symbolize them in propositional logic to show its possible deductive validity and certainty;" to which he adds in a footnote: "Sion also alludes to mathematical formalizations without showing them"[111]. Here, Wiseman is confusing formalization and symbolization.

Personally, I see no profit in replacing "If P then Rp" (or in other words, "P implies Rp") with "P → Rp" or with "p → r₁"! As far as I can see, all the good it does is save space on paper or on computer screens. At the cerebral level, on the other hand, there is a costly slowing down of thought. Such symbolization puts the reader unnecessarily one or two steps behind, since every time he sees an arrow he has to mentally translate it into a verbal "if-then" (or "implies") and likewise if the letters are changed, he must remember that the letters p and r₁ mean the same as the previous letters P and Rp. Tell me, how does this visual and verbal reshuffle "show" the "possible deductive validity and certainty" of the given text?

Symbolic logic is a myth, which many people fall for – but I refuse to. Ordinary language is already symbolic enough; nothing much is gained and much is lost by re-symbolizing it. Such activity is at best trivial, and more likely harmful. Instead of shedding light and facilitating, it obscures and misleads. That is why I avoid excessive symbolization. If I were to further symbolize my findings, it would not add any credence to them. All I would succeed in doing is make my work seem more inscrutable, and therefore more impressive to vain people who confuse inscrutability with depth. My goal in writing logic is not to entertain people, however, but to teach them to think more accurately and more efficiently.

Concluding remarks. Wiseman's goals in writing his dissertation were ambitious. His opening remark in his abstract is: "This study proposes to clarify the *a fortiori* argument's components, structure, definitions, formulations, and logical status, as well as the specific conditions under which it is to be employed, both generally and in a Jewish context." And further on, in his introduction, he states: "In sum, the overall purpose of the thesis is to advance the *a fortiori's* place as an acceptable reasoning method;" or again, in his concluding remarks, "The purpose of this dissertation has been to determine the reasonableness of the *a fortiori* argument and its parallel Jewish QC."

But, as we have seen in the present review, although he can be said to have advanced the cause of a fortiori argument, and this is a very commendable achievement, he did not demonstrate a thorough understanding of the nature of the argument, and therefore could not establish its rationality. Even though he was well aware of my formal definition and validation of a fortiori argument, he inexplicably opted for a vaguer definition that could not possibly be validated, since it made no mention of a 'threshold' for predication. He did so with two main motives in mind: to allow for "inductive a fortiori argument" and for "proportional a fortiori argument." But though he tried, he did not manage to work out credible formal supports for these ideas.

Wiseman's desire to expand and innovate in the field of a fortiori logic is praiseworthy; the problem is only that he was not equal to the task. We might here apply his own words: "Past and current good practices are preferable to poorer, new ones, especially where shortsighted tampering lacks sufficient prior knowledge or skill"[112].

Let me clearly state that my criticism of some of Wiseman's views is not criticism of his general attitudes. Unlike certain other contemporary commentators we have looked at, he does not come across as posturing and pretentious, but exhibits sincere interest in getting at the truth without prejudice. Moreover, "inductive a fortiori" and "proportional a fortiori" are worthy ideas. It is only the inadequacies of his explanations of them that I have here criticized. These ideas have an important place in a fortiori logic, but not exactly the place he assigned them.

As regards the *dayo* principle, Wiseman was quite right in having sensed that this rule taken to extremes, as many commentators (and notably Maccoby) take it, is rationally untenable. And he was quite right in looking for ways and means to diminish its apparent scope. His arguments in this respect, that proportional a fortiori argument is often theoretically reasonable and often used in practice, are largely correct; the problem is only in accurately determining just how far to push back the *dayo* restriction. If Wiseman had studied the pertinent texts (Mishna, Gemara, etc.) more closely, he might well have attained a more precise (narrower) statement of the principle.

Not having done so, he misconstrues the intent of the Mishna rabbis who formulated the principle. He thinks their purpose was "to limit reason to prescribed bounds" and "curtail the range of possible opinions;" to "cajole, coerce,

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[111]   P. 17.
[112]   P. 215.

and control" out of "religious and political paternalism" and in defense of "orthodox Judaism"[113]. All this is groundless interpolation on his part[114]. The Mishna rabbis stuck to the revealed quantity (i.e. the penalty given in the Torah for a lesser offence) to avoid the risk of excessive extrapolation. It was not arrogance of power, but kindness and humble admission of their human limits that motivated them.

But anyway, Wiseman's projections of dubious motives onto the rabbis are not of great moment. What matters is the bottom line, viz. precisely delimiting the legitimate application of the *dayo* principle. He does succeed in delimiting it somewhat, at least by declaring it to be a Judaic (even specifically Mishnaic) principle rather than (as Maccoby presents it) a logical one. But he does not attain the needed pinpoint precision, i.e. the reference specifically to *penalties* given *in the Torah*.

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[113]   Pp. 216-8.

[114]   Fanciful too, in my opinion, is Wiseman's assumption that the Amoraim were basically at odds with the Tannaim with respect to the *dayo* principle. E.g. on p. 137, where he says: "a trend is evident through time, with an increasing contrast between the *Mishnah's* limit on reason and the *Gemara's* greater latitude."

29. Yisrael Ury

Yisrael Ury's *Charting the Sea of Talmud: A Visual Method for Understanding the Talmud*, published in 2012[1] is a work with a very interesting logical innovation. But, though it has a chapter on *qal vachomer* (a fortiori) argument, it is not very innovative in that particular domain. We shall begin our exposition of it with an analysis of his contribution to logic in general and then deal more specifically with his comments on a fortiori.

1. An ingenious idea

Ury describes his work as "a revolutionary visual method for understanding and summarizing Talmudic discussions, conclusions, and laws." His method is, as far as I know, indeed original; and moreover, it really does greatly clarify Talmudic discourse[2]. Of course, he applies it only to a few *sugyas* (Talmudic discussion units), by way of example. Ideally, he should have applied it to all the Talmud's discussions; or he or others should do that in the future in a separate work. If a sea is chartable, it should be charted.

But what is more amazing, to my mind, is that he has developed a truly practical way to represent all 'if–then' statements diagrammatically, a way hitherto unknown (so far as I know) to logical science. Ury deserves heaps of praise for this brilliant idea of general value. Everyone has heard of Euler diagrams. They are commonly used by logicians and students of logic. They allow us to express relationships of inclusion and exclusion visually, by means of circles within circles, partly intersecting circles and non-intersecting circles. Now, we shall have what we may henceforth call Ury diagrams. These are able to do things that Euler diagrams cannot do, though they cannot replace them.

A is not X   A is X
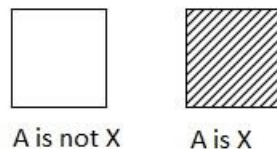
The simplest Ury diagram is a small square, or "box." This unit represents a subject, say A. If the subject is shaded a certain way, it means that A has a certain predicate, say X; i.e. it means that 'A is X'. If the subject is not so shaded, the meaning is accordingly that 'A is not X'. If the square is shaded some other way, e.g. with vertical stripes instead of diagonal ones, then another predicate, say Y, is intended. Thus, the square might be devoid of shading (in which case, A is neither X nor Y), or shaded one way but not the other (in which case, A is X but not Y, or Y but not X, as appropriate), or shaded both ways (in which case, A is X and Y).

A is neither X nor Y   A is X but not Y   A is Y but not X   A is both X and Y
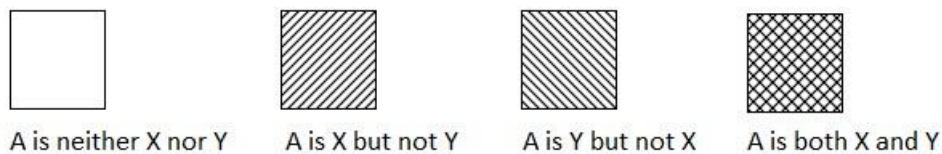
Note that, because he is dealing with Talmudic discourse, Ury himself thinks of the subject (e.g. A) primarily as a legal context, a situation subject to some law, and of the predicate (e.g. X) as the law applying (or not applying, as the case may be) to that context. More precisely, "A is X" would here mean "case A is subject to law x." However,

[1]   Jerusalem: Mosaica, 2012.
[2]   The reader can get an initial idea of Ury's method by visiting his website, www.talmuddiagrams.org, which contains informative videos.

he is clearly aware that the concept has broader application than legal discourse, since he gives an example dealing with fuel efficiency of cars[3].

Also note, even if Ury does not himself do so, that although the square (A) and its shading (X) are here used to refer to the two components of a copulative proposition, i.e. a subject and a predicate (terms), they could equally well be used for an implicational proposition, i.e. with reference to an antecedent and a consequent (theses). In such cases, instead of 'A is X' we would have 'A implies X'.
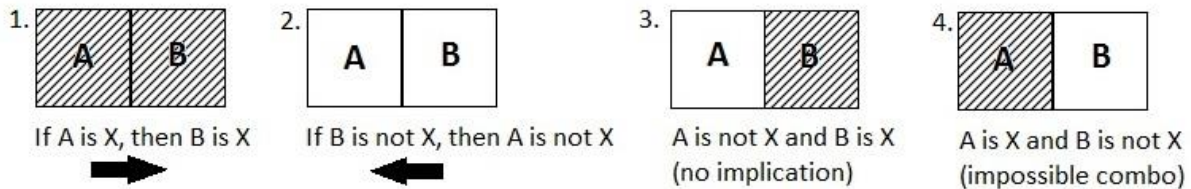
Now, the next development is the most significant one. An 'if–then' relationship can be visually represented by means of the following convention. The two subjects concerned are represented by two contiguous squares. The square for the antecedent subject is placed to the left of (or below, or more forward than) the square for the consequent subject (which is therefore on the right, or above, or behind). If we label the two squares as A and B, and a predicate concerning them as X, the implication intended is "If A is X, then B is X." This logically signifies the following three possibilities:

1. A and B are both shaded – this means we first discovered that 'A is X' and thence inferred that 'B is X'. We pass the shading of A on to B.
2. A and B are both blank, – this means we first discovered that 'B is not X' and thence inferred that 'A is not X.' We pass the non-shading of B on to A.
3. A is blank and B is shaded – this means that 'A is not X' and 'B is X' (neither of these implies the other). We cannot pass the non-shading of A on to B, or the shading of B on to A.

Note the dynamic intent these diagrams. As regards the fourth alternative, viz. A is shaded and B is blank, this is by definition impossible – i.e. the very meaning of "If A is X, then B is X" is that 'A is X' and 'B is not X' are incompatible.

Notice that an Ury representation of implication goes only one way – from 'A is X' to 'B is X', i.e. from left square to right one (or equally well, from lower to upper, or from closer to further), or vice versa (from 'B is not X' to 'A is not X'). A two-way (mutual) implication would mean that the third combination above (i.e. 'A is not X' and 'B is X') is also impossible. Of course, too, we can get fancier, and consider implications from X to not-X or vice versa, but there is no need here for us to get into such complications. The important thing is to realize the ingenuity and value of the diagram as a tool of representation and communication.

An Euler diagram cannot perform quite the same feat. We can put a smaller circle A within a larger circle B, and represent the above three alternatives by shading both circles or leaving both blank or leaving A blank and shading B – so that the impossible fourth case is that with A shaded and B blank. But here the meaning of the implication is significantly different – it is: if all (or even some) A are X, then *some* B are X; and by contraposition, if no B are X, then no A are X. Notice the reference to some of B, instead of to B as a whole. What is new in the Ury diagram is our ability to address B *as such*. This is very useful in certain forms of discourse.

Note that Ury does not explicitly identify the relationship between the squares as one of implication, as done here. Rather, he describes the square to the right as "more likely" than the one to the left, meaning that there is "more reason" for the law (i.e. predicate X) found in A to be found in B. But this is effectively implication, since we definitely *infer* (deductively, or eventually inductively) one proposition (B is X) from the other (A is X) on this basis. The defining feature of such implication, to repeat, is that the combination "A is X and B is not X" is being excluded. Note that the preceding diagrams refer to copulative implications, i.e. those between categorical propositions ('A is X' *implies* that 'B is X'). Although Ury does not mention this, they could also refer to implicational implications, i.e. those between hypothetical propositions ('A implies X' *implies* that 'B implies X'). This is said by me in passing, so as to be exhaustive.

Now, the above presentation of Ury's idea is only the beginning of it for him, though we shall essentially leave it at that. He goes on, and explains how such diagrams can be used to describe the theoretical positions of opponents in a dispute (e.g. rabbis debating some point of law) or the changing theoretical position of an individual (e.g. the Gemara, as it considers different options and possible objections). Usually, the theoretical positions are constituted

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[3]   On pp. 107-111.

by not one but two implications, so that the best way to represent them is by means of what Ury calls "two by two Diagrams." The following is an example of such a more complex diagram:
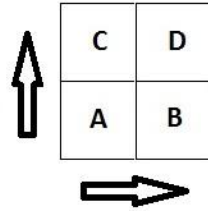
As Ury explains, there are sixteen possible ways to shade such a diagram, but only six of them are 'allowed' (i.e. logically consistent) and ten of them are not, given that the boxes to the left imply those to the right and the lower boxes imply those above them. He formulates the implicit restrictions in what he calls "The Shading Rule." This rule states: "In a Diagram, if a box is shaded, all boxes above it and to its right are also shaded. If a box is blank, all boxes below it and to its left are also blank." Later, he introduces three dimensional diagrams, though he also shows how these can be reduced to a set of two dimensional ones.
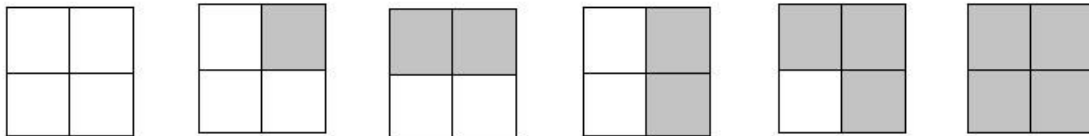
Six consistent combinations

By means of such two by two diagrams, Ury manages to greatly clarify theoretical debates or developments in the Talmud. Moreover, he shows how the diagrams constitute a new "language," insofar as, once we know the meaning of particular diagram (i.e. what its boxes and their shadings signify), we can abstract it from its labeling, and use the graphic form to quickly identify a theoretical position and compare it to others with comparable labeling. We can thus, by placing different diagrams side by side see in one glance how various positions differ; or we can show the inconsistency of a position or predict additional positions.

This is obviously a very valuable tool, not only for Talmud study, but for thought in general. However, as we shall see, its value is not unlimited.

2. Diagrams for a fortiori argument

The limits of Ury's diagrammatic tool become evident when we consider its application to a fortiori argument, even though to be sure this tool remains valuable in many contexts.

To begin with, it should be said that nowhere does Ury actually delve deeply into the nature of a fortiori argument. He defines the *kal vachomer* as "a logical argument that proves a proposition to be true under one set of circumstances based on its being true under a less compelling set of circumstances." As he sees it, "if a certain Law applies to a certain situation, we are allowed to apply that Law to a different situation where it is even more likely to apply." For him, "The *kal vachomer* is valid because it is one of the Thirteen Hermeneutic Principles by which Torah Law is derived, not simply because of 'logic'." Notice his putting the term logic in inverted commas. Let us examine these ideas.

First, Ury does not say how we know that one set of circumstances is more "compelling" or more "likely" than another. In some cases, this is obviously known 'intuitively'; e.g. as one knows that four apples weight more than two of them. In other cases, presumably, we have to assume that the rabbis were handed the information down by oral tradition since the Sinai revelation. But the use of words like "compelling" or "likely" can only in this context be taken as figures of speech. For, according to formal logic, the conclusion of an argument can never literally be more forceful than the premise(s) which give rise to it. By way of a given argument, the conclusion can only be *as or less probable* than the premise(s), even if it may turn out to be more probable through additional reasons external to the argument. Thus, Ury's language here is misleading, and shows that he has not reflected on this issue.

Second, he does not tell us by what means the law is passed on from one context to another. In his view, it suffices that the words "*kal vachomer*" are listed as one of the thirteen rabbinic "principles" of hermeneutics for the question to be answered. The reason why he puts 'logic' in inverted commas, is that though he sees intuitively that the argument is somehow logical, he cannot say exactly what it is that makes it seem so. For this reason, he falls back on the traditional argumentum. While Ury defines the *kal vachomer* as "a logical argument that *proves* etc.," the only proof he provides for it is the say-so of the rabbis – it is thanks to them (or ultimately the revelation at Sinai, which they claim to merely pass on) that "*we are allowed* to apply, etc." But as I have shown in my *Judaic Logic*[4], this "Sinai connection" argumentum does not stand up to closer scrutiny; it constitutes a circular argument.

To say that a certain rule of inference is, albeit not proven by formal logic, valid by virtue of a Divine decree, one would have to first show that there was a decree and that it was indeed of Divine origin. But the Tanakh (the Jewish Bible) contains no such explicit decree for a fortiori argument or any other rabbinic hermeneutic principles. There are many examples of a fortiori discourse in the Tanakh, which can be taken as teachings of logic; but such examples are not express decrees, they are subject to interpretation. Moreover, even if the Tanakh did explicitly decree such principles, since belief in the Tanakh's Divine origin is a matter of faith and not scientific proof, such content would not constitute proof. Faith is certainly necessary and valuable in religious matters, but it is not as reliable as proof. Likewise, there is no credible proof for the claim that the hermeneutic principles were Divinely-decreed orally, and then handed down by tradition, orally and then in writing, unscathed by temporal events. Our acceptance of this historical claim can only be based on faith, since there is no other way to justify it. Thus, the Sinai connection thesis is reasonably open to doubt.

Reason should never be made subservient to apologetic purposes; its independence, objectivity and integrity should always be defended if we really want to pursue truth. To be absolutely clear: if a certain form of argument, such as a fortiori argument, cannot be validated by logical means, then it is invalid; i.e. it is either a non sequitur (i.e. its conclusion does not follow from its premises) or antinomic (i.e. its conclusion contradicts its premises). There is no way around this principle – it is a law of nature. If an argument form *cannot* be validated by logic, then there is *no way* to validate it by other means; and conversely, if an argument form *can* be validated by logic, then there is *no need* to validate it by other means, such as written Divine decree or subsequent oral tradition. Reasoning is not an arbitrary matter, but a matter of logic.

Following rabbinic precedent, Ury identifies two forms of *kal vachomer* (lenient and stringent) argument: the first type argues that a lenient law (*kula*) that applies to a more stringent case (the *chomer*) is bound to apply to any more lenient case (the *kal*); and the second type argues that a stringent law (*chumra*) that applies to a more lenient case (the *kal*) is bound to apply to any more stringent case (the *chomer*). These two arguments may be graphically represented as follows:
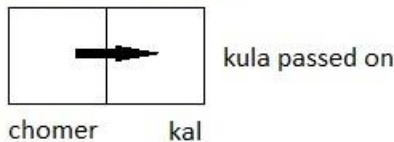
Notice the different order of the *kal* and *chomer* items, and the different shadings, in the two diagrams. In the first diagram, a certain leniency is passed from the *chomer* item to the *kal* item, so both boxes remain blank. In the second diagram, a certain stringency is passed from the *kal* item to the *chomer* item, so both boxes are shaded. If we take these diagrams as Ury does as simply statements *that* the application of the "law" concerned to one item implies its application to the other, they are quite okay. But if we ask *how come* the law is passed on from one item to the other, we are left completely in the dark. Or if we ask *why* a leniency (rather than a stringency) is passed from *chomer* to *kal*, and a stringency (rather than a leniency) is passed from *kal* to *chomer*, again no explanation is forthcoming.

Clearly, while this graphical approach is capable of recording the fact of implication in a useful manner, it is apparently not able to explain or justify the implication. Thus, Ury diagrams are heuristic instruments, rather than hermeneutic ones. They can be used to superficially 'describe' a fortiori arguments for us, but they do not tell us just

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[4]   See chapter 12.1 there.

how they function or why they work. They are, as he claims them to be, useful as learning or teaching tools, facilitating conceptualization and memorization; but they are not useful for theoretical exploration of a fortiori argument, at least not in the way they have thus far been presented to us.

It is evident that Ury conceives a fortiori argument as some sort of direct implication. The shading of the box to the left is passed on to the box to the right without an intermediary stage, i.e. without apparent reason. Even more, Ury seems to think that *all* implication is really a fortiori argument. As he puts it:

> "The principle of *kal vachomer* is built into the fabric of Diagrams. Every time we completed a Diagram, we did so using the principle of *kal vachomer*, and the Shading Rule itself can be viewed as nothing more than a restatement of the principle of *kal vachomer.*"

In this conception of all implication as essentially a fortiori argument he does not diverge from the traditional view – that is precisely why the rabbis of the Talmud regard a fortiori argument as the very essence of logic, and why such argument is so frequently used in Jewish discourse. But this is untenable from the perspective of formal logic.

If we examine actual a fortiori discourse, even arguments formulated in the Tanakh, in the Talmud and other rabbinic literature, it is evident that it does not consist merely of a minor premise and conclusion. There is also, of necessity, a major premise to take into consideration, which contains a middle term. Furthermore, the minor premise is necessarily more complex than it appears at first sight: it informs us of a threshold value of the middle term as of which the predication is (or is not) feasible. It is precisely this extra information, which is absent in Ury diagrams, that *makes possible* the putative conclusion. Even if the information is not always explicitly given, it is always implicitly assumed; otherwise, the argument would simply not be convincing.

The reason why Ury cannot validate a fortiori argument is that he does not take the trouble to look for all its implicit components, i.e. to first formalize it. We could say that his diagrammatic approach imprisons him in a simple conception that prevents him from seeing the logical nature of a fortiori argument, and moreover from seeing the other types of arguments that may underlie an implication. If he had pondered the obvious question, regarding the two types of *kal vachomer* argument, why a leniency is passed from the *chomer* to the *kal*, whereas a stringency is passed from the *kal* to the *chomer*, he might have begun to go deeper into the logic involved.

Clearly, these drawings are intended to represent a fortiori argument from minor to major and from major to minor, respectively. Stringencies and leniencies are not "laws" in the same sense. By "stringency" is meant some definite imperative or prohibition, whereas by "leniency" is meant a corresponding exemption (i.e. non-imperative) or permission (i.e. non-prohibition). Thus, strictly speaking, the two terms do not refer to different kinds of law, but respectively to the presence or absence of a law. That is, a *kula* is a non-*chumra*. If we put the rabbinic formulations in standard a fortiori form it becomes much clearer how such arguments proceed.

> "Type 2": The *chomer* (P) is more serious (R) than the *kal* (Q);
> and Q is serious enough to be subject to a certain stringency (S);
> therefore, P is serious enough to be subject to S.

> "Type 1": The *chomer* (P) is more serious (R) than the *kal* (Q);
> and P is serious *not* enough to be subject to a certain stringency (S);
> therefore, Q is serious *not* enough to be subject to S.

Note that I have placed Ury's type 2 before his type 1, because they are respectively positive subjectal and negative subjectal in form. Ury placed them in the reverse order because he did not realize that the type 2 argument is really negative in form, and therefore a derivative of the positive type 1 argument by means of *reductio ad absurdum* validation. So, to him, either order was okay, and he happened to choose the said order.

Note also my labeling of the four terms involved as P. Q, R and S, in accord with my standard practice. The *chomer* (P) is the major term and the *kal* (Q) is the minor term. The implicit more-less relationship (quantitative comparison) between these two terms has to be expressed in a separate proposition, called the major premise. Furthermore, that proposition must contain a middle term (R), which tells us in respect of what precisely P is more than Q. Although the middle term is not explicitly given here, it must still be provided – I use the vague term "serious" here, because no more specific term is suggested to us.

The subsidiary term (S) is explicitly given in type 2 as "a certain stringency;" this intends a certain legal imperative or prohibition. Although the subsidiary term (S) explicitly given in type 1 is "a certain leniency," meaning a certain exemption or permission, the real subsidiary is still "a certain stringency" because this argument cannot be positive, since it is subjectal and a positive subjectal cannot validly proceed from major to minor as this argument does. This issue of polarity presents no great problem, since we can obvert the conclusion of type 1 to "Q is not-S," and not-

imperative means exempted and not-prohibited means permitted. Nevertheless, to repeat, acknowledging the underlying negative form of the argument is essential to its validation.
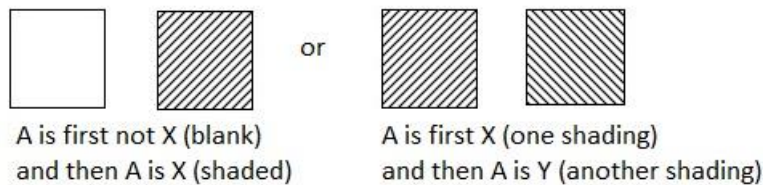
The middle term (R) must also appear in the given proposition, called the minor premise, that "Q is subject to stringency S" (in type 2) or "P is not subject to stringency S" (in type 1), as the case may be. For a certain value of the middle term (what is "R *enough* to be" S) serves as the *sine qua non condition* for a subject's access to the subsidiary term (S). It is only through our knowing this condition from the minor premise that we are able to conclude that the remaining term (P or Q, as the case may be) is also subject or not-subject (as the case may be) to the subsidiary term (S). Thus, even if the middle term is not mentioned or even hinted at anywhere in the argument, it has to be assumed present for any conclusion to be credibly drawn.

Needless to say, an Ury diagram does not comprise these various details and conditions concerning a fortiori argument. It is not a very subtle tool. All it tells us is: in type 2: "if the *kal* is subject to a certain stringency, then the *chomer* is so too;" and in type 1: "if the *kal* is not subject to a certain stringency, then the *chomer* is also not subject to it." There is no inherent explanation or justification of these processes. Furthermore, we are not told why we could not equally well say: "if the *kal* is subject to a certain leniency, then the *chomer* is so too;" and "if the *kal* is not subject to a certain leniency, then the *chomer* is also not."

The only way to understand why the former two moods are valid and the latter two are not is to refer to the middle term (R), which (as already stated) underlies both the major and minor premises and the conclusion. Rationally, the more "serious" (R) a context is, the more stringent the law concerning it is likely to be; and conversely, the less "serious" (R) a context is, the more lenient the law concerning it is likely to be. This is not a formal matter, but an expression of the principle of justice or of 'measure for measure'. In general, there might be inverse proportionality between the middle term and the subsidiary term; but in contexts of morality and law, such inversion would be contrary to reason. Since Ury's diagrams do not formally make room for the middle term, they cannot explain why two moods are valid and two are invalid.

In fact, we *could* integrate the middle term into his diagrams, as follows. Instead of saying, for instance (in the case of type 2): "if the *kal* is subject to a certain stringency, then the *chomer* is so too," we would say: "if the less *serious* context is *serious enough to be* subject to a certain stringency, then the more *serious* context is so too." That is, we would label *the boxes* as "the less serious context" or "the more serious context," i.e. in such a way that the intended middle term is clearly designated in them; and moreover we would label *their shading* as "serious enough to be subject to a certain stringency," i.e. in such a way that the necessity of a sufficient quantity of the middle term for access to the subsidiary term is clearly acknowledged. Thus, we can by such semantic artifice improve the accuracy of an Ury diagram, even though the diagram per se is technically simplistic.

Another weakness of Ury diagrams that we must consider is that they apparently refer only to *subjectal* argument, and make no mention of *predicatal* argument. Each box represents a subject and its shading (or lack of it) represents a predicate, and the inference consists of transfer of shading (i.e. predicate) from one box (subject) to another. Yet in real discourse – including Biblical, Talmudic and other rabbinic discourse – argument often proceeds in the opposite direction, by change of predicate for one and the same subject. In pictorial terms, the latter would mean that the same single box, say A, lacks and then has a certain shading (signifying X), or vice versa; or has a certain shading (X) at one time and another shading (Y) at another time:

A is first not X (blank)
and then A is X (shaded)

or

A is first X (one shading)
and then A is Y (another shading)

To try and resolve this problem, i.e. how to express predicatal arguments through Ury diagrams, let us look more closely at the arguments we wish to put in diagrammatic form. They are, mirroring those proposed above by Ury, the following:

'Type 4': More seriousness (R) is required to be subject to the *chumra* (P)
than to the *kula* (Q);
and a certain context (S) is serious enough to be subject to P;
therefore, S is serious enough to be subject to Q.

'Type 3': More seriousness (R) is required to be subject to the *chumra* (P)
than to the *kula* (Q);
and a certain context (S) is *not* serious enough to be subject to Q;
therefore, S is *not* serious enough to be subject to P.

These two arguments are respectively positive and negative predicatal. Notice that here the subsidiary term (S) is the subject of the minor premises and conclusions and refers to "a certain context," whereas the major and minor terms (P and Q) are now predicates and refer respectively to a *chumra* (stringency) and a *kula* (leniency); as we have seen, a *kula* is a non-*chumra*. The middle term (R) is the same as before ("seriousness"), but it now serves as the precondition for S to be (or not be) P or Q, as the case may be. Notice that the positive form goes from major to minor and the negative form goes from minor to major. I have numbered the arguments as 4 and 3 in accord with Ury's preference (even though the opposite order would be logically more accurate).

If we tried to illustrate these two arguments by means of Ury diagrams in the usual way, we would have to call both boxes S, and make the shading change from box one to the other. This would be from *chumra* shade to *kula* non-shade in type 4 argument, and from *kula* non-shade to *chumra* shade in type 3 argument. But clearly, this diagram would *not* be in accord with Ury's diagrammatic scheme and his Shading Rule. It follows that Ury diagrams are not able to represent predicatal a fortiori argument as such. Therefore, their power of representation is essentially limited to subjectal a fortiori argument.

We could however try and *rephrase* the minor premises and conclusions of the latter two a fortiori arguments as follows. For type 4: "If the *chumra* (P) is applicable to a certain context (S) – because S is serious (R) enough to be subject to it – then the *kula* (Q) is applicable to that context (S)." And for type 3: "If the *kula* (Q) is *not* applicable to a certain context (S) – because S is *not* serious (R) enough to be subject to it – then the *chumra* (P) is *not* applicable to that context (S)." This is leaving the major premise unchanged. In this way P and Q are made to appear as subjects and S as (part of) the predicate. The explanations – that S is or is not serious enough to be P or Q, as the case may be – are then effectively left out of the arguments, note well. In that event, we could propose the following Ury diagrams to illustrate them:

Kal vachomer type 3
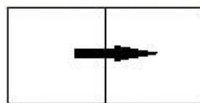
kula   chumra

Given that a kula is not applicable to a
certain context, then any relative
chumra must also be not applicable to it.

Kal vachomer type 4

chumra   kula

Given that a chumra is applicable to a
certain context, then any relative
kula must also be applicable to it.

In these diagrams, note well, the shading refers to the positive predicate "applicable to context S" and the lack of shading to the negative predicate "not applicable to context S." Although the arguments are really predicatal, they are *made to seem subjectal* in order to be fitted willy-nilly into Ury diagrams. Notice how these two arguments (types 3 and 4) differ from the previous two (types 1 and 2). Here, each argument is concerned with legal rulings of different severities relative to one and the same case, whereas previously each argument was concerned with legal rulings of one and the same severity relative to different cases.

As I have demonstrated formally and in detail in an earlier chapter (3.5) such 'traductions' (as I called processes of this sort) are artificial and somewhat misleading. In fact, a predicatal argument *cannot* be recast in subjectal form (nor, incidentally, can a subjectal argument be recast in predicatal form). The reason for that is that each argument form lacks some of the information needed to construct the other argument form. Although we can superficially, by verbal manipulation, make a predicatal argument look

subjectal (or vice versa), in fact its underlying rationale, which determines its structure, remains unaffected[5].

Nevertheless, Ury diagrams can, as just explained, be used in practice to visually represent predicatal arguments, because Ury diagrams are anyway rough tools, which cannot display the fine distinctions needed to distinguish differently structured arguments. In conclusion, although Ury has not addressed the question of diagrammatic representation of predicatal a fortiori argument in his book, we have here done it for him (albeit with appropriate warnings), and thus extended the utility of Ury diagrams.

All the above concerns copulative a fortiori arguments, but the same could obviously (possibly with appropriate adjustments) be repeated with reference to implicational a fortiori arguments.

3. No a crescendo or dayo

We saw in the previous section that there are definite limits to the power of representation of Ury diagrams when it comes a fortiori argument, because such argument is in fact not as simple and straightforward as it is commonly depicted to be. In the present section, we shall reinforce this observation with reference to a crescendo argument and the *dayo* principle. In his book, Ury gives no hint of being at all aware of the issue of 'proportionality', i.e. of the difference between purely a fortiori argument and a crescendo argument. Consequently, it is no surprise to see that he does not deal with the *dayo* principle, either.

Ury warns that "The *kal vachomer* also has certain special rules and limitations", but adds that he "will not deal with all the aspects" of the argument ("in this chapter" he says, but in fact he means in the whole book). He views it as "compelling… unless refuted," and gives as example of a refutable argument: "If it is healthy to consume a certain amount of a nutrient then is it certainly healthy to take a larger amount?" However, though this reflects the rabbinic view, it is not an accurate statement. When, as in this example, the argument does not work (in this particular case due to an erroneous major premise[6]), it is not due to a weakness in a fortiori argument as such, but due to the proposed particular a fortiori argument being not well formed. It is not the argument form that makes the example less compelling, but the fact that the example *fails to conform* to the argument form.

Ury mentions the "concept of *dayo*" only once, in passing, in a footnote, even though he there acknowledges it to be an "important concept."[7] The reason for Ury's silence on proportionality and the related *dayo* (sufficiency) principle is, I suggest, that they are not easy to fit into his diagrammatic scheme. The reason is not that these matters are unknown to rabbinic reasoning – indeed, they are very present in it. We find in the Mishna (Baba Qama 2:5) R. Tarfon reasoning a crescendo from half payment for certain damages on public grounds to full payment for similar damages on private property, and we find there his colleagues, the Sages, rejecting his conclusion by saying that "It is enough (*dayo*)" to conclude with the same penalty (i.e. half payment).

Then in the Gemara commentary on this issue (Baba Qama 25a), a *baraita* is cited, according to which the *dayo* principle is based on the example of Numbers 12:14-15, in which the prophetess Miriam is sentenced to only seven days punishment for a sin which, the *baraita* claims, deserves fourteen days. The Torah passage cited makes no mention of fourteen days, and could easily be read as purely a fortiori argument. Nevertheless, the Gemara insists on reading it in accord with the said *baraita*, and thus seemingly adopts the viewpoint that all a fortiori argument is essentially a crescendo argument, and therefore that the *dayo* principle is always needed to limit the conclusion to a non-proportional magnitude.

Never mind that many a fortiori arguments within the Tanakh, the Mishna, *and even the Gemara*, not to mention other rabbinical discourse, are clearly non-proportional – and indeed, some cannot be interpreted in any other way than as non-proportional – the Gemara's clear position in this *sugya* of the Babylonian Talmud, through which it explains the dispute between R. Tarfon and the Sages in the Mishna, is that a fortiori argument is essentially proportional. The Gemara's explanation is in fact deficient, since it only addresses one of the two arguments reported in the Mishna, as I show in an earlier chapter (7.5).

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[5]   Simply put, although the sentence "X is applicable to A" has X in the position of subject and A in that of predicate, if we consider the meaning of the relationship "applicable to" it is clear that X is still in fact the predicate and A the subject.

[6]   A certain quantity x of this nutrient may have beneficial effects on the organism, while larger quantity y (an overdose) of it may be detrimental, and for that matter a lesser quantity z (too small a dose) of it may be useless. We cannot infer from the fact that x is an amount sufficiently beneficial to cause health that y is also so, because we lack the needed major premise that y is more beneficial (as well as a larger quantity) than x. This example is mentioned by Ury on p. 95; but he does not attempt to understand why it does not work.

[7]   Ury also, in the same footnote, mentions in passing the concept of *tzad hashaveh*. This rabbinic term refers to the common feature between two items, which makes possible an analogical inference from one to the other. This concept is in fact more relevant to *binyan av* argument than to *kal vachomer*. See the section on analogical argument in an earlier chapter of the present work (5.1).

Be that as it may, it is evident from all this that it is impossible to claim to be able to visually represent Judaic (Biblical, Mishnaic, Talmudic or later rabbinic) a fortiori argument if one has not duly considered and assimilated the complications of 'proportionality' and *dayo*. Yet Ury does just that, blithely glossing over these crucial issues. He must have studied the Mishna and Gemara in Baba Qama 25a-b regarding these topics. Possibly he considered them, but found them intractable; but if so, he should have said that. Let us therefore look and see if we can apply his diagrammatic scheme to these issues.

The two a fortiori argument forms Ury deals with pictorially in his book are *purely* a fortiori positive and negative subjectal arguments. That is to say, the minor premise and conclusion in them have the *exact same* subsidiary term. The argument, as he sees it, consists in passing a given law from one context to another. If the 'law' is a leniency (i.e. it grants an exemption or permission) applicable to a *chomer* (more serious) case, it can be passed on to a *kal* (less serious) case; and if the 'law' is a stringency (i.e. it makes something imperative or prohibited) applicable to a *kal* case, it can be passed on to a *chomer* case.

In both these arguments, note well, the 'law' remains unchanged in the transfer from one context to the next. Pictorially, in the corresponding Ury diagrams, this means that the shading or non-shading of one box is carried over *unchanged* to the next box, as appropriate (i.e. as specified in the Shading Rule). This is purely a fortiori argument – it involves no 'proportionality,' and therefore no *dayo* restriction is applicable to it, since *dayo* can only be applied where a 'proportional' interpretation is attempted.

A crescendo argument differs from purely a fortiori argument in having a 'proportional' conclusion. That is to say, if the minor premise tells us that predicate S is applicable to the major or minor term, then the conclusion's predicate will have a magnitude *smaller or greater than S* (i.e. S– or S+), as the case may be, for the remaining term (the minor or major, as the case may be), *in proportion to* the different magnitudes of the major and minor terms (or more precisely, to the amounts of the middle term that they respectively have). This information, which differentiates a crescendo argument from purely a fortiori argument, is usually left tacit, but is logically essential to draw a 'proportional' conclusion, note well.

The *dayo* principle is applied in some though not all cases of a crescendo argument (specifically, in cases where a penalty for a greater crime is being derived from the penalty for a lesser crime given in the Torah). Its application consists in saying 'no' to the putative proportional conclusion (i.e. it demands that the concluding penalty remains quantitatively the same as the textually given penalty from which it is derived). This is not a rule of logic, but a rabbinic moral rule (or ultimately a Divinely-decreed one, according to the rabbis). Our sense of justice, or the moral principle of 'measure for measure' (*midah keneged midah*) we derive from it, would have us infer a proportional penalty; but the rabbis (rightly, I think) prefer to take no chances and avoid all possible errors of human judgment (which could occasionally result in unjust punishments) by interdicting such inference where applicable.

While, as we have seen, purely a fortiori argument can be expressed (briefly put, leaving aside important details) as: "If A is X, then B is X," the a crescendo form of such argument has the form: "If A is X, then B is Y, where Y is greater than X" (again, this is putting it briefly, leaving aside the middle term and the premise about proportionality). This means that the visual representation of a crescendo argument using Ury's diagrams would require that the shading within a box to the right of (or above or behind) another box be different. This would be contrary to his Shading Rule.

If A is X, then B is Y (where Y > X)

Note the arrow and the difference in shading in the above diagram. Of course, in cases where the *dayo* principle is applied, the shading would again be made the same, and the Shading Rule would be obeyed. Nevertheless, there are cases where a crescendo argument is used, and the *dayo* principle is not appropriate or merely not applied, which therefore continue to break the Shading Rule. Thus, Ury's scheme, as it appears in his book, is deficient in not having taken into account such a crescendo arguments, which are far from uncommon in Judaism or elsewhere.

This deficiency can possibly be remedied by changing the conventions involved in Ury diagrams. We could reformulate the Shading Rule, saying that in certain cases the shading might vary and be different on the left and right (or below and above, or in front and behind, depending on the orientation of the diagram), being perhaps made more or less intense in proportion to the magnitudes of the predicates in the minor premise and conclusion. Of

course, this greatly diminishes the visual impact of Ury diagrams, since whether its conventions have been fully obeyed or not cannot be known merely by glancing at the diagrams.

Indeed, this reminds me that there are examples of a fortiori argument where we argue from a negative to a positive or a positive to a negative. I have discussed such arguments in detail in an earlier chapter, under the heading of a crescendo argument with 'antithetical subsidiary terms' (3.4)[8]; and I there found them to be usually invalid but in certain cases valid. In valid such instances, the Ury diagram might thus have a shading on the left which is not passed on to the right, or a blank on the right which is not passed on to the left – in direct contravention of the Shading Rule. This is explicable with reference to 'proportionality' (i.e. the blank and the shading are in such cases effectively two degrees of shading).

Thus, to conclude our critique of Ury diagrams, it appears that, though they function well in simpler cases of a fortiori argument, there are not infrequently cases of a fortiori argument that are considerably more complex and which therefore cannot be adequately represented by means of such diagrams and in accord with the Shading Rule regulating their use. Furthermore, remember, while Ury diagrams do visually represent some of the information contained in ordinary a fortiori arguments, they do not visually represent *all* the information in them – so even with respect to simpler arguments (which can easily be fit into Ury diagrams and do obey the Shading rule) there are relevant aspects that remain invisible and therefore susceptible to misunderstanding or error.

Thus, in the final analysis these diagrams may not be as generally useful as initially thought. They are doubtless of use in many contexts, but are better avoided in certain contexts.

4. Kol zeh achnis

Actually, Yisrael Ury does mention and partly discuss Baba Qama 25a[9], but he does so only incidentally while dealing with another *sugya*. He does not deal with the issues it involves relating to 'proportionality' and *dayo*, but focuses principally on an argument proposed by Tosafot through the expression: "*kol zeh achnis bakal vachomer*," which Ury translates as: "I will fold all this into the *kal vachomer*."

I have formally analyzed the passage of Tosafot concerned, thanks in part to the explanation provided by Ury of the Ri's objection an a fortiori argument by R. Tarfon, and placed my analysis in an earlier chapter of the present volume (9.7), where it rightly belongs. I refer readers of the present chapter to that preliminary analysis, because its study greatly facilitates the present discussion. Here, what interests us primarily is to see how Ury diagrammatically expresses this Tosafot commentary, which he cites in full in both Hebrew and English.

The Tosafot commentary consists of three distinct stages. In the first stage (i), an argument by R. Tarfon is paraphrased (the argument is attributed to the Gemara, but actually it comes from the Mishna); this is his first argument: his second is not mentioned. In the second stage (ii), an objection by the Ri to the preceding is introduced, and then the Ri's own reply to such objection. In a third stage (iii), we are taught the *kol zeh achnis* argument proposed by the unnamed Tosafist, the author of the commentary, which argument is supposed to conflate the components of the previous two stages.

(i)   Ury's diagram for the argument by R. Tarfon (fig. 6.15a) is a two by two square similar to the following:
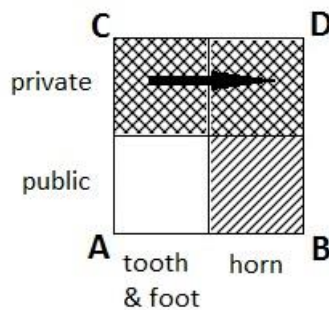
As can be seen, the horizontal distinction is that between tooth & foot damage and horn damage, and these are placed with the *kal* (more lenient) item on the left and the *chomer* (more stringent) item on the right. The vertical distinction

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[8]   A possible illustration given there is 2 Samuel 12:18. "Behold: while the child was yet alive, we spoke unto him, and he hearkened not unto our voice; then how shall we tell him that the child is dead, so that he do himself some harm?" The relevant antithesis here is between David 'being distracted' and 'harming himself'.

[9]   See pp. 113-118 in his book.

is that between damage occurring on public grounds and that on private property, and these are placed with the *kal* (more lenient) item below and the *chomer* (more stringent) item above. Two shadings are used here, single hash for half payment and double hash for full payment. No shading (i.e. blank) signifies that no payment is required. We are given three boxes (A, B and C), and asked to infer the fourth, namely the box in the top right hand corner (D). The conclusion proposed by R. Tarfon is that box D should have double hash shading, i.e. that horn damage on private property entails full payment. Thus, the shading in box C is passed on to box D, in accord with Ury's Shading Rule.

However, as far as I can see, the Shading Rule would equally well allow us to pass on the single hash shading in box B to box D, and thus conclude that horn damage on private property entails half payment. Ury does not make clear why we prefer to inferring D from C (as R. Tarfon indeed does) to inferring D from B (as the Sages do when they say "*dayo*—it is enough," although Ury does not mention them, and indeed the Tosafot passage at hand does not mention them). Is there some unstated additional clause to the Shading Rule that justifies such preference? He does not say. Clearly, he is content here to illustrate; he makes no effort to explain the a fortiori reasoning involved.

Actually, the argument by R. Tarfon that Ury is here illustrating is not his first, but his second. This is evident, since the arrow of inference in Ury's diagram goes horizontally across from box C to box D, i.e. from the *full* payment required in the case of damage by tooth & foot on private property (the minor premise) to *full* payment for damage by horn on private property (the conclusion). See for yourself:

> Horn damage (P) implies more legal liability (R) than tooth & foot damage (Q) [as we know by extrapolation from the case of public domain].
> Tooth & foot damage (Q) implies legal liability (R) enough to necessitate *full* payment for damage on private property (S).
> Therefore, horn damage (P) implies legal liability (R) enough to necessitate *full* payment for damage on private property (S).

The major premise of this argument is based on comparison of the legal liabilities for damage in the public domain by tooth & foot and horn, respectively. It is this premise, obtained by generalization, which makes the conclusion follow from the minor premise. The major premise is represented in Ury's diagram by the lower two boxes, A and B. The argument as a whole is purely a fortiori. Thus, Ury does not illustrate the argument actually cited by Tosafot, i.e. R. Tarfon's first, but R. Tarfon's second argument. R. Tarfon's first argument may be stated formally as follows:

> Private property damage (P) implies more legal liability (R) than public domain damage (Q) [as we know by extrapolation from the case of tooth & foot].
> Public domain damage (Q) implies legal liability (Rq) enough to necessitate *half* payment for damage by horn (Sq).
> The payment due (S) is 'proportional' to the degree of legal liability (R).
> Therefore, private property damage (P) implies legal liability (Rp) enough to necessitate *full* payment for damage by horn (Sp = more than Sq).

The Ury diagram for this argument could be the same as the one above, but here the arrow of inference would go vertically up from box B to box D, i.e. we would be inferring *full* payment for horn damage on private property (double hash shading) from *half* payment for horn damage on public grounds (single hash shading), by analogy to the transition from no payment for damage by tooth & foot on public grounds to full payment for damage by tooth & foot on private property. And this would, of course, constitute an a crescendo argument, i.e. a proportional a fortiori argument.

However, such representation would clearly be contrary to the Shading Rule formulated by Ury for his diagrams! Thus, the Shading Rule would have to be modified to assimilate a crescendo argument in Ury diagrams, as already pointed out. As things stand, Ury diagrams cannot handle a crescendo argument. So, it is no surprise that Ury opted unconsciously for R. Tarfon's second argument which is purely a fortiori, i.e. non-proportional. But it should be said that the more relevant argument in the present discussion, as we shall presently see when we deal with the next two stages of it, is in fact R. Tarfon's second. So Ury was thinking straight when he opted for the latter. It is Tosafot, not Ury, who is to blame for evoking R. Tarfon's first argument.

(ii)   The objection of the Ri (another Tosafist, called R. Isaac, whence Ri) is, briefly put, that the major premise of R. Tarfon's second argument could well be reversed, i.e. it may argued be that Horn damage implies *less, instead of more,* legal liability than tooth & foot damage. This alternative hierarchy of liability would be based on the idea that tooth & foot damage being rather common, the ox's owner should be especially vigilant to prevent it; in comparison, horn damage being rather uncommon, the ox's owner is not expected to be so careful. Given this new

major premise, the objection goes, it is not logically possible to infer (as R. Tarfon proposed) full payment for damage by horn on private property from the full payment required in the case of damage by tooth & foot on private property. This objection (*pircha*, in Aramaic) is illustrated by Ury (Fig. 6.15b) roughly as follows:
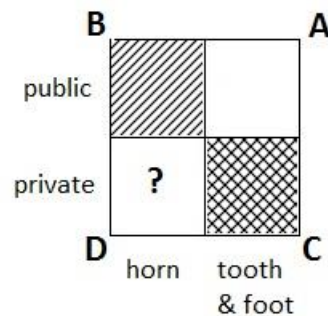

Here, horn is placed to the left of tooth & foot, so that (by the Shading Rule) we cannot transfer the double hash shading (i.e. full payment) in the latter to the former (i.e. from box C to box D).

Moreover, the private domain is placed by Ury below the public domain, so as to prevent transfer of the single hash shading for horn on public grounds to horn on private property (i.e. from B to D). The latter move is surely an overreaction by Ury, since it is not mentioned in the Ri's objection. Perhaps that is why Ury explains it in a footnote, rather than in his main text. His idea seems to be that if the columns were switched but the rows were left in their original order, we would be tempted (given the Shading Rule) to infer half payment for horn damage on private property from the half payment for horn damage on public grounds. But I do not see why he would want to preempt such inference, since it would be quite compatible with the Ri's objection and not legally problematic (since, in fact, the Sages end up with that conclusion through their *dayo* objection). Certainly, the payment for horn damage on private property cannot be zero (less than on public grounds), but must be at least half.

Furthermore, in the configuration thus proposed by Ury, we might be tempted (given the Shading Rule) to infer from the half payment for horn damage on public grounds, a like half payment for tooth & foot damage on public grounds (i.e. to pass single hash shading horizontally from B to A); or alternatively, to infer from the full payment for tooth & foot damage on private property, a like full payment for tooth & foot damage on public grounds (i.e. to pass double hash shading vertically from C to A) – whereas the Torah specifies exemption from payment for this case (i.e. no shading in box A is allowed)! Ury does not address this issue, but simply leaves box A blank.

In any event, Ury surprisingly does not at all mention the Ri's *own reply* to the objection. The Ri's retort is that if the reversed major premise that tooth & foot damage implies more legal liability than horn damage were adopted as the objection advocates, then we could infer from the half payment for horn damage on public grounds a like half payment for tooth & foot damage on public grounds (i.e. we could pass single hash shading from B to A) – which, as already said, would be contrary to the Torah specification of zero payment in such case. This retort by the Ri must, of course, be taken into consideration in our diagram. But since it refers to a horizontal inference, it does not matter to it whether we place the 'public' row above the 'private' row, as Ury has it, or vice versa.
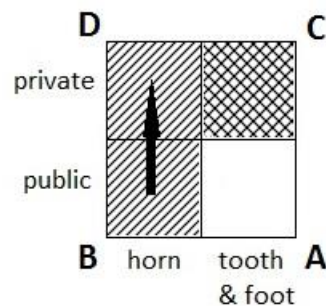
Thus, in conclusion, I would say that Ury erred in this matter, out of inattention (i.e. he did not think his diagram through far enough). He should have left the row B-A below the row D-C, and only switched the two columns around, as in our diagram above. By switching the two rows around also, unnecessarily, Ury created an additional problem (namely, the possibility of passing double hash shading vertically from C to A). Our drawing is preferable,

because it avoids this added problem. Moreover, it excludes the possibility of zero payment for damage by horn on private property, which Ury thoughtlessly allows. Ury also made a mistake in not mentioning the Ri's own reply to the objection, which is a crucial part of the whole commentary. It looks like he did not realize its exact significance.

The Ri's retort to the objection is in fact tacitly included in Ury's diagram for the objection, in that in the row labeled 'public' he has left the box on the right (for tooth & foot, here called A) blank, even though the box on the left (for horn, here called B) is shaded – whereas, according to the Shading Rule, given this configuration, the boxes should be either both shaded or both blank. But Ury does not point this out in so many words. He does, admittedly, write:

> "Even though *shein* and *regel* damages are commonplace, and therefore the owner should be liable, nevertheless the owner is exempt in the public domain, proving that it is very difficult to make the owner liable for *shein* and *regel* damages."

However, this statement is not made as a presentation of the Ri's explicit retort to the objection. Moreover, Ury here shows that he misunderstands the Ri's objection. It is not that it is "very difficult" to blame the ox's owner for tooth & foot damage – it is that it is *impossible* to do so, since doing so would lead through a fortiori argument to a conclusion of half payment for such damage in the public domain, which would contradict the Torah given of zero compensation. Thus, I would say, Ury does not duly take stock of and acknowledge the Ri's full intervention; he read the objection, but skimmed over the retort to it.

(iii)   Instead, he jumps fast forward to the *kol zeh achnis* argument proposed by the unnamed Tosafot commentator. He presents this pictorially (Fig. 6.15c) roughly as follows:

private

public

tooth & foot (real)   tooth & foot (original)   horn
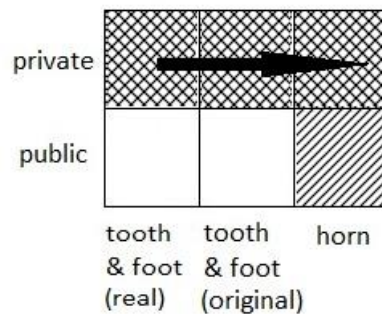
The column placed most to the left represents the *kol zeh achnis* argument, while the two columns to the right of it are the original two by two diagram. Ury refers to the new column as "reality," and to the middle column as the "original conception." Notice that the arrow in the top row goes across all three boxes in it. The message of this diagram is that the *kol zeh achnis* argument reinforces the initial argument, by rendering it immune to the objection raised by the Ri. However, this diagram far from clarifies just what Tosafot's new argument is! This is clear from the fact that the two columns in it labeled "tooth & foot" are visually *identical*, even though the argument in fact changes. We see from this the weakness of Ury's diagrammatic method, at least with regard to a fortiori argument. As already pointed out, we cannot by this method make fine comparisons and contrasts between arguments, and discern precisely where they agree and differ.

In fact, as I show in my own analysis, the *kol zeh achnis* argument proposed by Tosafot differs from the original argument by R. Tarfon in an important respect, namely in *the middle term* involved. The argument by the Tosafist can be stated formally as follows:

> Tooth & foot damage (P) is *more common* (R) than horn damage (Q) [since the former is common and the latter not so].
> Yet, tooth & foot damage (P) is common (R) *not* enough make the ox's owner exempt from full payment for damages on private property (S) [since tooth & foot damage on private property necessitates full payment].
> Therefore, horn damage (Q) is common (R) *not* enough to make the ox's owner exempt from full payment for damages on private property (S) [whence, horn damage on private property does necessitate full payment].

From this formulation, we can see clearly what purposes the argument serves and what novelties it contains. By using the middle term "common," Tosafot integrates both insights of the Ri, the possible objection regarding frequency of occurrence and the retort to it that this would lead to absurdity, since he avoids inferring the middle

term "legal liability" from such frequency. Moreover, so doing, Tosafot is able to arrive at the same conclusion as R. Tarfon, namely that damage by horn on private property calls for full compensation. In this sense, Tosafot manages to *allude* to the whole discussion in one argument.

However, though this *kol zeh achnis* argument is consistent with the earlier argument by R. Tarfon, it is no substitute for it. It cannot either be said that the Tosafot argument is needed, as Ury seems to think by placing it far to the left, to buttress R. Tarfon's case, since in fact the doubt sown by the Ri's objection is dissolved by the Ri's own reply to the objection. Once this dissolution occurs, any further mention of 'frequency of occurrence' (i.e. of what is common or uncommon) becomes redundant, because it in fact has no relevant consequence on the issue of legal liability. This means that the Tosafot argument is an embellishment, but not a very useful one. To fully understand its utility, we must still evoke the Ri's objection and his own retort to it. If anything is important in this context it is the Ri's objection and his own retort to it.

It is evident from Ury's characterization of the Tosafot argument as "reality" and as able to "prove forcefully," in comparison with R. Tarfon's argument, that Ury takes the words "all this I will fold into the *kal vachomer*" quite literally, i.e. as signifying that the Tosafot argument contains "all" the thoughts preceding it. But these words are an exaggeration – the *kol zeh achnis* argument is in fact qualitatively inferior to R. Tarfon's original argument because it does not have (not even implicitly) the latter's reference to "legal liability." Clearly, Ury allows the prestige of Tosafot to affect his judgment here. He allows himself no critical thoughts.

Moreover, according to Ury's explanation, Tosafot considered that "when R' Tarfon stated his *kal vachomer*, he was well aware of the presence of a potential *pircha*, but already had it included into his *kal vachomer* using the principle of *kol zeh achnis*!" But this seems to be just a projection of the thoughts of later commentators onto earlier protagonists. Such retroactive rationalizations are very common in traditional readings, giving the impression that there are ultimately no innovations in Torah discourse. But to my mind, they are dishonest (if their historicity is not established) and unnecessary. Certainly, R. Tarfon need not have thought of the said complications to formulate his own arguments; they stand quite well on their own two feet.

Note that Ury gives no evidence from the Mishna that R. Tarfon was ex ante facto "well aware" of the said problem and the said solution to it. He does not specify just what formal logical relations he is referring to when he says that R. Tarfon's Mishna argument "included" the *pircha* "using" the *kol zeh achnis*; these are overly vague expressions. He gives no evidence that the *kol zeh achnis* form of argument was ever used before the Tosafist's time (about 12th century CE, well over a millennium after R. Tarfon's time). Note also that he seems to think that R. Tarfon formulated only one argument ("his *kal vachomer*"), whereas in fact the latter formulated two quite distinct arguments (though they came to the same conclusion).

Ury's approach reflects a common fault in traditional hermeneutics – namely, anachronism. Reading his presentation, one does not see a clear distinction between the contributions of R. Tarfon, the Ri and the more recent Tosafist. They are treated synchronically instead of diachronically. Ury seems to think that R. Tarfon already anticipated and took into consideration both the Ri's and the later Tosafist's contributions. But, to repeat, there is no textual evidence to that effect, and no logical need to assume such foreknowledge on R. Tarfon's part. There is only the need to perpetuate the 'orthodox' fantasy that earlier rabbis were well-nigh omniscient, so that later rabbis are only involved in making explicit information that is timelessly already there implicitly.[10]

Furthermore, as I explain in my earlier detailed analysis of Tosafot's argument, this argument is much more radically in conflict with the Sages' *dayo* principle than R. Tarfon's two arguments were, because unlike them *it does not resort* to information concerning the ox owner's half legal liability following horn damage on public grounds to draw a conclusion on his full liability following such damage on private property. Tosafot's argument is thus unaffected by *dayo* objection, and could be used to challenge the *dayo* principle. Ury – like Tosafot and subsequent rabbis (as far as I know) – does not point this out, nor show how Tosafot's argument might be scrapped and the *dayo* principle might be saved.

In conclusion, although Ury's diagrams can no doubt be very useful in many contexts, they can only roughly depict the state of affairs in certain contexts, and not at all do so in still other contexts. It is noteworthy that he does not attempt to analyze in detail how he would pictorially handle other hermeneutic rules, besides *qal vachomer* (R. Ishmael's #1). What of *gezerah shavah* (#2) and the other contextual rules (#12), *binyan av* (#3), the inclusion and exclusion rules (#s 4-7), and the rules of harmonization (#s 8-11, 13)? How would he distinguish these various discursive techniques, and others still, from each other in his pictorial approach? He can only deal with 'implication'

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[10] Of course, religious debate can – just like philosophical debate – be considered as essentially transcending time. I am not denying that. Nevertheless, it cannot be taken for granted *without evidence* that earlier protagonists actually anticipated, took into consideration and then dismissed objections raised by later protagonists.

in a general way, without distinguishing its apparent varieties. Clearly, Ury's diagrams constitute too rough an instrument to reflect the complexities of more intricate forms of reasoning.

Logical precision is only possible through formal verbal discourse. Every component of every argument must be brought out into the open and credibly shown to be relevant, truthful and valid. Diagrams of various types may be helpful, but they can never suffice.

30. Hubert Marraud

The most recent attempt to study a fortiori argument may be that of Hubert Marraud, whose "From a Stronger Reason," a draft paper of 14p, was posted online in 2012[1]. From the outset, it is evident that this author's orientation is 'rhetorical' rather than logical. This rhetorical approach is, of course, not without value, and rather commonly used; but it does not go to the core of the issues, and for that reason – as we shall see – makes possible some *serious errors of logic*. I shall here only comment on Marraud's ideas directly concerned with a fortiori argument, and not deal with some of his views relating to argument in general.

1. Warrants and premises

Marraud begins his presentation by stating: "A fortiori argument (henceforth AF argument) is a traditional kind of argument that has been largely ignored by contemporary argumentation theorists." In fact, as we have seen in the present survey, quite a number of contemporary writers have ventured into the a fortiori field, although most of those who have done so have proved to be rather amateurish. What is most striking is the silence on this subject of big name modern logicians. In the course of his essay, Marraud cites some authors of papers concerned with a fortiori argument that I was unaware of. Not having found copies of their papers, I cannot assess them in the present volume. Notably absent from Marraud's references, however, is my own work on the subject, in my *Judaic Logic* (1995). It is a pity for him that he did not discover it, even though it has been freely available on the Internet since 2001; it would no doubt have improved his work and saved him much embarrassment.

He traces "the concept of" a fortiori "back to Aristotle's topic from more and less (Rhetoric II, 23)." I take it that Marraud here means that Aristotle was the first to draw our attention to the use of a fortiori as a distinct form of reasoning. But he (Marraud) does not explicitly mention that the argument can be found used in earlier literature. From his remark that "In fact most translators use *a fortiori* instead of *from more and less*," it is evident that Marraud looked for the literal translation of Aristotle's Greek words for this sort of argument. He further informs us that "Aristotle's examples include both theoretical arguments (If not even the gods know everything, human beings can hardly be expected to do so) and practical arguments (If Hector did well to slay Patroclus, Paris did well to slay Achilles)." But this theoretical-practical distinction is certainly not clear; and Marraud does not clarify it for us.

Be that as it may, let us examine Marraud's views concerning a fortiori argument and look at and evaluate his treatment of specific examples. Marraud tells us that he follows a "Toulmin model" for the description of different a fortiori arguments (p. 5). This model consists of premises and a conclusion—and a "warrant." This constitutes an "argument scheme." The framework used is: "*premises so conclusion since warrant*." The warrant is what explains how the premises "lead to" the conclusion. The "strength" of the argument depends, we are told, on the kind of warrant involved; but it is not made clear (as far as I could tell) just what "strength" is and just how the said dependence is to be determined[2]. Marraud presents his arguments in diagrams like the following (the diagram below is my own generalization from his various applications):

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[1]   See at: logicforum.org/PDF%20files/MARRAUD%20-%20From%20a%20Stronger%20Reason.pdf. I solicited a copy of this draft paper from the author (a third party had informed me of it), and posted it in The Logic Forum with his permission. Marraud subsequently informed me: "an improved version of my paper has been accepted for publication in the journal *Theoria*." But I cannot refer to this new version, because it is in Spanish. I take it the improvements in it are not significant enough to invalidate my present critique, since the author did not ask me to make any changes to the online English version.
[2]   Marraud cites Perelman and Olbrechts-Tyteca (1989), who describe argument "strength" as "a confuse but indispensable notion." And also Anscombre and Ducrot (1983), who define it by saying that if a reason B "is used for" a conclusion C, and reason A is "stronger" than B, then reason A "should be considered usable for the same conclusion." To exclude cases where A and B are of equal strength, they add the proviso that "there are circumstances where A, but not B, may be used for a particular conclusion C." But such "definition" is circular, and provides no information regarding the nature of "strength" and how it is to be logically established. Similarly, Marraud's own definition of "argument schemes" as "common patterns of transfer of acceptability from the premises to the conclusion" does not tell us what "acceptability" is and how it is to be proved.

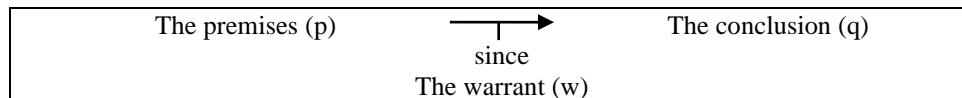| The premises (p)	$\longrightarrow$	The conclusion (q)
	since	
	The warrant (w)	

Diagram 30.1

The symbol he uses (an arrow on a stand[3]) apparently means that the premises (p) imply the conclusion (q) on the basis of the warrant (w) – the letters p, q, and w, for the three items involved, being added by me. Logically, of course, this diagram signifies: If the warrant is true, then the premises do together imply the conclusion; or, in other words: If w, then if p then q. But this nested hypothetical proposition in turn formally implies: If w + p, then q – which suggests that w is just another premise. However, Marraud is careful to advise us that the warrant, whether explicit or implicit, is not conceived as an additional premise. It is rather what Aristotle called the "principle" underlying the argument, as in the example:

> "The argument that a man who strikes his father also strikes his neighbours follows from the principle that, if the less likely thing is true, the more likely is true also." (*Rhetoric* II, 23[4].)

I must first comment on this proposed model and the ideas it enshrines. It is true, as Marraud and presumably before him Toulmin observes, that when people reason, they sometimes – though not always – explain or justify their reasoning. So, there is empirical basis for the concept of a "warrant." However, what is the difference between a warrant and a premise? Marraud does not look into this issue very deeply. As a result, as we shall see when we examine his specific examples, all of his alleged warrants are in fact premises! To avoid this confusion, we must clarify the difference.

A warrant differs from a premise in that the former is more abstract than the latter. A warrant concerns the form of an argument, whereas a premise tells us something about the content. In the symbolic terms, the content of a warrant w for an argument with specific premises p and specific conclusion q would be: "premises of form p do imply a conclusion of form q." In the example from Aristotle above given, "if the less likely thing is true, the more likely is true also" is a warrant, while "a certain man occasionally strikes his father" is a premise. The former principle is indeed a warrant, because it is a proven general statement about a certain form of argument; whereas the latter information is a mere premise, because it is a statement involving particular terms (namely: man, strikes his father), whose implication of the conclusion (that the man may strike his neighbors) needs to be established.

Actually, this definition of a warrant is not entirely correct. A more precise statement would be the following. It could be said that the whole science of logic is a search for argument forms and their warrants. Identifying an argument form is formalization, and validating it is finding out how to warrant it. The primary warrant of any argument form is *the process* through which it is validated. But once this initial validation is effected by some logician, the argument *form* becomes a warrant in its own right. For example, once Aristotle validated the moods of 2nd figure syllogism by reductio ad absurdum to moods of 1st figure syllogism (which were already validated by exposition), then anyone reasoning by way of 2nd figure syllogism could henceforth warrant his *particular* argument either by engaging in a *particular* reductio ad absurdum process, or by appealing to the already established *general* form of that syllogism without the need to repeatedly validate it.

Thus, particular arguments can in practice be warranted – i.e. explained and justified – in two ways. We may either use an appropriate logical validation process for that particular argument, which may be complicated and long-winded. Or we may simply evoke the form of the argument – in wordless intention, in words or using symbols – implying that this form of argument has already been validated in theoretical terms by someone for all time and there is no need to repeat the validation process in this concrete case. However, it should be kept in mind that the latter sort of warrant is an intellectual construct; it is something relatively artificial and static, although full of potential. The true warrant is always the underlying validation process, which calls for actual rational insights. If we lack these actual insights, and therefore cannot really understand and validate a certain principle, the principle is for us an empty shell and appeal to it does not in fact constitute a warrant.

Contrary to common misconception, the validation process does not consist in applying the 'laws of thought' (conceived as axioms, i.e. irreducible primaries) to particular arguments, but in the ad hoc use of rational insight. This ad hoc view of validation has to be upheld, since otherwise we could not explain or justify the supposedly *first* application of these laws. That is, we cannot appeal to syllogistic inference from axioms before we have validated

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[3]   I cannot resist once again railing the modern habit of resorting to symbols (like the arrow on a stand here used) as if this makes logical discourse more "scientific." At best, it is laziness; at worst, it is deliberate concealment and obfuscation. To be truly scientific, logical discourse must be in clear and unambiguous ordinary language. If symbols are used, as they might be to save space, they must first be named and precisely defined in ordinary language, and such definition must be adhered to throughout their use.

[4]   Marraud has chapters 15-18, but that must be an error of inattention.

such inference. The axioms are thus ex post facto summaries of what we effectively *do* – they are not a priori or arbitrary principles that we *apply*. Rational insight is an empirical act, whereas application of principles is conceptual reflection. Therefore, though it is true that we often think by appeal to established logical principles, we cannot be said to truly understand these principles if we are unable to validate them *without* resort to a prior warrant.

We cannot logically say that "p implies q" really means "p implies q, if and only if w;" i.e. that the former statement is always a truncated expression of the latter. This would mean that p is not sufficient by itself to imply q; i.e. that only in conjunction with w (or some other such warrant) can p imply q. And since this in turn means "(w + p) implies q," it calls for a further (anterior) warrant; and so on, ad infinitum. Therefore, we *must* admit the intervention, and *logical autonomy*, of primary rational insights at some stage. If we admit it immediately for "(w + p) implies q," we might as well admit it immediately for "p implies q." Thus, the Toulmin model is logically untenable if taken to extremes. The buck has to stop somewhere. We have to admit that not all reasoning calls for a warrant, let alone for further "backing" for the warrant. Otherwise, there would be no end to it – no possibility for rational knowledge at all.

2. The main form of a fortiori

Let us move on and analyze what Marraud refers to as "the positive form of arguments from more and less," which he presents as follows (p. 6):

$$\text{O is P; O is} \pm \text{R than O'} \quad \xrightarrow[\text{since}]{} \quad \text{So, O' is P}$$

$$\pm \text{ an object has property R,} \pm \text{it has property P}$$

Taking him at his word. The first thing to note is that Marraud's proposed warrant (the bottom line in his diagram) is not a warrant but a premise. It is not an abstract principle of logic which explains and justifies the proposed argument (the top line); it is a concrete proposition with specific terms, and is therefore part and parcel of the argument. Thus, the argument presented here does not involve an explicit warrant – it is only superficially in accord with the Toulmin model. This means that the argument should be rewritten as follows. Note that I have changed the symbols – putting, in place of Marraud's O, O', R, P, my by now standard symbols P, Q, R, S, for the major, minor, middle and subsidiary terms, respectively[5].

> (a) P is ± R than Q.
> (b) ± an object has property R, ± it has property S.
> (c) P is S.
> Therefore, (d) Q is S.

The first question to ask is: do the given premises (a), (b) and (c), indeed imply the conclusion (d) – that is, is the argument valid? The answer is: yes; and this can be proved as follows: Premise (a) implies that P is R and that Q is R. Premise (b) implies that whatever is R is S. Whence, we can infer that P is S and that Q is S. We have thus established that the putative conclusion (c) does indeed logically follow from the given premises, and they alone. This validation process constitutes the true warrant of this argument, and not the proposition (here labeled (b)) that Marraud claimed as the warrant.

Our next question is: was all the information given in the premises needed to obtain the conclusion, or was some of it redundant? Well, for a start premise (c) was redundant, since it can be inferred from premises (a) and (b), and anyway was not used to get to the conclusion. But moreover, the comparison inherent in premise (a), i.e. the fact that Rp > or < Rq, was redundant information, since we only used the incidental implications that P and Q are R. Similarly, the concomitant information inherent in premise (b), i.e. the fact that S varies in proportion to R, was redundant information, since we only used the incidental implication that All R are S.

Thus, the argument could have been formulated much more succinctly than it was, as: (a) Q is R, and (b) All R are S; therefore, (d) Q is S. This is just positive first figure syllogism. Thus, although Marraud's argument looks at first sight like a complex a fortiori argument, it turns out to be a very simple syllogism. This does not mean that a fortiori argument is syllogism, but means that what Marraud has presented as a fortiori argument is in fact not a fortiori argument. For one cannot claim to have formalized a fortiori argument if one has not, among other things, taken into

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[5]   One reason I do this is because Marraud's symbol O' has an apostrophe, which is easily confused with a quotation mark. But the main reason is to constantly evoke my standard forms of a fortiori argument in the reader's mind.

account the quantitative comparison, viz. Rp > Rq, which such argument is generally acknowledged to involve. Merely to have this information in a premise is not enough – it has to be actually used to infer the conclusion. That quantitative comparison is a defining aspect of a fortiori argument is acknowledged by Marraud through the very fact that he includes it in his schema. But his inclusion of it is very superficial – it in fact plays no active role in his argument.

Ignoring the so-called warrant. We could end our analysis of Marraud's thesis here, for what we have found so far is bad enough. But we shall explore his work further and show up its other faults. Let us for a while, for the sake of argument, ignore Marraud's "warrant" (i.e. premise (b)), and examine what he obviously regards as the core argument (namely, premises (c) and (a) and conclusion (d)). This is worth doing, since in his estimation the warrant stands apart from the argument, as evidenced by his diagram which has the former in the bottom line and the latter in the top one. Thus, we could say that for Marraud the essence of a fortiori argument is:

(a) P is ± R than Q.
(c) P is S.
Therefore, (d) Q is S.

Now, this looks more like what is generally considered as the form of a fortiori argument. However, it is not. Although it somewhat resembles the positive subjectal mood of such argument, there are two serious errors in this representation, one being an error of commission and the other being one of omission. The first serious error is that the major premise says "P is ± [i.e. more or less] R than Q," implying that positive subjectal a fortiori argument can proceed in either direction, which is wrong. I do not think this is an error of inattention, because he repeats it many times. He might have been unconsciously influenced into making this error by Aristotle's label for the argument as "from more and less." But unlike Aristotle, he displays no awareness of the difference between argument "from major to minor" and that "from minor to major" [6]. So I believe Marraud's use of "± R" is indicative of his uncertainty in this matter, and is designed to fudge the issue.

This is further confirmed by the following observations. I have applied the labels P and Q in the order of appearance of Marraud's objects O and O', and everywhere in my work these labels stand for the major and minor terms, respectively. It would appear, judging by Marraud's description, further on, of the corresponding "meta-argument with scalar warrant," where he has the premise "O is *more* R than O'," that he thinks of O as the major term and O' as the minor term; so, my choice of substitute labels is confirmed. However, this would imply, in the present context, where 'P is S' is the minor premise and 'Q is S' is the conclusion, that Marraud erroneously believes that positive subjectal a fortiori argument proceeds from major to minor! However, this supposition is belied by Marraud's description of "meta-argument from strength comparison," in which the movement is clearly (as it should be) from minor to major. So it looks like Marraud is not sure what to think or is confused, sometimes ignoring the issue, sometimes opting for major-to-minor, and sometimes opting for minor-to-major.

But even if we generously take the above core argument to read: "P is *less* R than Q, and P is S; therefore, Q is S," it is faulty. For, even with the correct major premise, the putative conclusion (Q is S) cannot logically be drawn from the given minor premise (P is S)[7]. The said conclusion can only be drawn from a minor premise which reads: P is *R enough to be* S. This is a very different proposition from the simpler 'P is S' (which it implies, however). Marraud is evidently unaware of this essential condition for validation, since he does not mention this missing factor in his formula. This lacuna again shows he does not fully understand a fortiori argument. We cannot assume that his "warrant" is indicative of awareness of this issue. For this proposition of his only affirms a concomitant variation between the middle term (R) and the subsidiary term (S). It clearly does not state that there is a *threshold value* of R for acceding to predicate S.

Reinserting the so-called warrant. Now, let us consider Marraud's "warrant," i.e. the proposition "± an object has property R, ± it has property S," which we chose to ignore momentarily. We have seen that, once that is treated as a premise (b), in conjunction with premise (a), i.e. 'P is ± R than Q', we can *without need of any further information* infer both that 'P is S' and that 'Q is S'. Thus, given the premise (b), it does not matter whether P is more or less than Q, i.e. premise (a) may well read 'P is ± R than Q', for the argument works as well both ways, yielding *both*

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[6]   As I show in the chapter on Aristotle (6.1), the latter had in mind an inference from negation of the more to negation of the less, and from affirmation of the less to affirmation of the more, i.e. the negative and positive moods of subjectal a fortiori argument, and not only the positive mood like Marraud here.

[7]   In other words, the propositions "P is less R than Q" and "P is S" are equally compatible with the contradictory propositions "Q is S" and "Q is *not* S." How do we know that? From the fact that we cannot prove that "P is less R than Q" and "P is S" together imply "Q is S" (or, for that matter, "Q is *not* S").

conclusions *either* way. Of course, that is not magic, but simply due to the fact – as already explained – that the comparative function of the major premise (viz. that Rp > or < than Rq) is in fact totally unused.

Another impact that consideration of the "warrant" would have is quantitative differentiation between the two conclusions. One would read 'P is Sp' and the other would read 'Q is Sq', since the value of R for P would be different from the value of R for Q, and S being proportional[8] to R the value of S for P would accordingly be different from the value of S for Q. But Marraud is unaware of this issue of quantitative variation in the predicate S, or at least he does not display awareness by using different symbols for the S of P and the S of Q. Even though he makes a big thing of the proposition "± an object has property R, ± it has property S," characterizing it as the warrant for the core argument, he in fact does not apply the proportionality inherent in it in this formula. This is surprising, considering his following statement:

> "The asymmetry underlying AF arguments is explained through the scalar principle or *topos* serving as warrant. Anscombre and Ducrot (1983) define a *topos* (plural *topoï*) as a general principle authorizing the step from premises to conclusion and consisting in a correspondence between two scales… This implies that the predicates in the *topos* admit degrees."

After saying that, it occurs to him that in one of Aristotle's examples, viz. "If not even the gods know everything, human beings can hardly be expected to do so," the predicate (i.e. the subsidiary term S, "omniscient") is not "gradual" (i.e. either one knows literally everything or one does not) – so, no proportionality is appealed to. His response to this "complication" is to ignore it "for the moment," although he does not return to it later as far as I can see. This suggests that he views a fortiori argument as essentially proportional, even though the conclusion of his above argument, as already pointed out, has the same predicate S, and not a proportional predicate, S+ or S–.

It could be supposed that Marraud's proposition "± an object has property R, ± it has property S" corresponds to what I have identified as the additional premise of a crescendo argument, which I formulated as "for whatever is R and S, S varies in proportion to R." But, whereas I make it clear that the variation of S is tied to that of R *as of* the threshold value of R which gives access to S (as the clause 'R enough to be' of my minor premise emphasizes), Marraud's proposition is quite general. Thus, while my premise about proportionality only applies to a limited range of R (namely, where an R is indeed S), and thus implies that *not all* R are S, Marraud's proposition is indiscriminate, i.e. it implies that *all* R are S. This distinction explains why my arguments are indeed a fortiori whereas Marraud's argument is merely syllogistic.

Moreover, whereas I use this premise about proportionality to distinguish a crescendo argument from purely a fortiori argument, Marraud apparently considers it as necessary (if only in the way of a "warrant") even for purely a fortiori argument, and he remains oddly silent concerning a crescendo argument. In truth, such a proposition about proportionality is *redundant* in purely a fortiori argument, and in applicable cases *necessitates* a crescendo argument. So while Marraud can be commended for having, like many before him, mentioned proportionality in the context of a fortiori argument, it cannot be fairly said that he grasped its full significance.

In any case, as already explained, the proposition about proportionality is not a true warrant but a mere premise, since it contains useful information with specific terms. And of course, once this proposition is taken as a mere premise, in conjunction with Marraud's other premises, the argument as a whole ceases to be a fortiori in type, and becomes syllogistic, so that the proportionality implied by it as well as the comparison implied by the major premise become completely irrelevant and unused. Thus, not only is it not a warrant, but much of the information it contains is not used as a premise. The true warrant of any material a fortiori argument is the standard form that corresponds to it, and the process through which that form (and thus any of its material instances) can be validated.

In any case, Marraud's treatment does not offer us a logical validation of a fortiori argument. Presumably, he regards the stated "warrant" as having a validating effect – but as it turns out, his "warrant" is inappropriate – it does not actually explain or justify the reasoning process in the way he means it to. This is true, whether we consider the implications involved in his schema – namely, the arrow symbol and the "since" – as deductive or inductive. I took them in my above analysis to refer to deductive implications, but the same critique holds if they are taken as more inductive. For, if the schema is not successful assuming the stronger deductive implications, it is all the more unsuccessful assuming weaker inductive ones. An inductive argument is only credible if it is formally based on a validated deductive argument.

The issue of origin. Even if some of the errors in Marraud's treatment above pointed out are inexcusable, he still deserves some praise for glimpsing some aspects of a fortiori argument as such, namely: that it involves a comparative major premise (his "P is ± R than Q"), a predication as minor premise ("P is S"), and a somewhat similar one as conclusion ("Q is S"). The details of his presentation are inaccurate, as already explained; but these

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[8]   Or even, in some cases, inversely proportional – although Marraud does not mention that.

rough ideas are still laudable (though far from novel in 2012, of course). The question is: are they his own ideas, or did he get them from elsewhere?

Marraud's above definition, leaving out the "warrant" part, looks very much like the deficient one proposed by Allen Wiseman in *A Contemporary Examination of the* A Fortiori *Argument Involving Jewish Traditions* (2010). But there is no evidence that he was aware of that dissertation, even though it was immediately published on the Internet. I suspect Marraud was probably influenced by Stefan Goltzberg's equally deficient definition: "The *a fortiori* argument… structure is that if p applies in case A, and since B is more x than A, then p applies *at least as much* in B," since he refers to the latter's essay, called "The A fortiori Argument in the Talmud," in his paper, even though he does not specifically mention Goltzberg's definition[9]. Needless to say, Goltzberg's definition was, as I show in the chapter devoted to him the present volume (26), itself influenced by earlier definitions.

Since Marraud in his reference to Goltzberg's essay mentions the collection called *Judaic Logic* edited by Andrew Schumann (2010), it is possible that he also read my essay in it, viz. "A Fortiori Reasoning in Judaic Logic" (pp. 145-175). This essay includes my clear and definitive definition of a fortiori argument in all its figures and moods, already presented in my earlier book *Judaic Logic* (1995). However, I doubt he did read that essay, or the earlier book, because I cannot imagine if he had how on earth he could have made the mistakes he made (detailed above)[10]. Moreover, it is noteworthy that Marraud's treatment of a fortiori argument, such as it is, focuses exclusively on the positive subjectal mood of it. He is apparently aware that this argument is also possible in negative form, since he says: "the positive form of arguments from more and less can be represented as follows" – but he nowhere goes on to specify what the negative subjectal mood might look like. In any case, he shows no awareness of the predicatal form of the argument, whether positive or negative. He is also unaware of the difference between copulative and implicational arguments.

3. So-called meta-arguments

Marraud next tries to add depth to his analysis, by distinguishing three "different though interrelated forms of a fortiori argument," three "senses" of the "argument scheme" so called. In addition to the above described basic or main sense of "an argument with a scalar warrant" (also called argument "from more and less"), he discerns two additional senses: "a meta-argument concluding that an argument is even stronger than a previous one" (also called meta-argument "with a scalar warrant" or "from more and less") and "a meta-argument establishing the sufficiency of some argument on the grounds that a weaker argument is sufficient" (also called meta-argument "from strength comparison"). He defines a "meta-argument" as "an argument about arguments." Let us now look into these two "more complex" forms of a fortiori argument.

Meta-argument from more and less. The argument he refers to as "sense 2" proceeds as follows (p. 6):

$$\text{O is more R than O'} \quad \xrightarrow[\text{since}]{} \quad \text{So, } A_2 < A_1$$

$$\pm \text{ an object has property R, } \pm \text{ it has property P}$$

The two "arguments" this meta-argument is about are A1: "O is R, so O is P" and A2: "O' is R, so O' is P". We may call these the "sub-arguments," as the author himself occasionally does. The conclusion is to be read as "argument A1 is stronger than argument A2."

The first thing to note, here again, is that Marraud's proposed warrant (the bottom line in his diagram) is not a warrant but a premise. This means that the argument should be rewritten as follows. Note that I have here again changed the symbols – putting, in place of Marraud's symbols O, O', R, P, my by now standard symbols P, Q, R, S, for the major, minor, middle and subsidiary terms, respectively.

> (a) P is more R than Q.
> (b) ± an object has property R, ± it has property S.
> Therefore, (c) argument A1 ("P is R, so P is S") is 'stronger' than argument A2 ("Q is R, so Q is S").

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[9]   Notice that Marraud, like Goltzberg, mentions the minor premise before the major premise, even if his symbols (O, O', R and P) are different from Goltzberg's (A, B, x and p). However, Marraud differs significantly from Goltzberg in having the vaguer "± R" instead of the more precise "more x" in the major premise; maybe he was trying to widen Goltzberg's statement! The letter they use in common, i.e. P or p, is obviously intended to signify a predicate.

[10]   Still, I wonder how come Marraud chose to use the symbol R for the middle term in his formula, i.e. precisely the letter I always use for that. One would have expected him to use the letter Q, which is next in line after O and P. Why did he skip Q and use the letter R instead?

Comparing this argument form to the preceding one, we notice a number of details. The major premise (a) here has it that P is "more R" than Q is (i.e. the major term P has more of the middle term R than the minor term Q does), whereas previously P was said more vaguely "± R" than Q. Marraud had to be more specific here, because his thesis is precisely that the argument involving the greater value of R (viz. P) is "stronger" than the one with the lesser value of R (viz. Q). The premise (b) remains the same, while the minor premise "P is S" (which was redundant, anyway) has been dropped. The conclusion is very different – before it was simply "Q is S," whereas now it is a complex comparison of two sub-arguments. Even so, the present meta-argument consists essentially of two simpler arguments.

Here again, from premise (a) we can educe that P is R and Q is R; and from premise (b) we can educe that All R are S. Thence we can deduce syllogistically that P is S and Q is S. Thus, a full statement of the two sub-arguments, A1 and A2, would be "All R are S, and P is R; therefore, P is S" and "All R are S, and Q is R; therefore, Q is S." Note that Marraud's definition of the two sub-arguments as "P is R, so P is S" and "Q is R, so Q is S" is deficient, since it does not specify the premise "All R are S" of the two syllogisms. The reason he does not make this premise explicit is probably, I submit, that he does not discern it; I suspect that he imagines that premise (a) suffices – without the active participation of premise (b), other than as "warrant" somehow buttressing the argument – to infer the two sub-arguments and their comparative strengths.

Thus, we can prove for Marraud that the two "sub-arguments" he refers to as A1 and A2 are indeed implied by his premise (a) and his "warrant" (i.e. his premise (b)). But this is not the purpose of his "meta-argument," which a claim that A1 is "stronger" than A2. The obvious next question to ask is: do the given premises (a) and (b) indeed imply the conclusion (c) – that is, is the argument valid? The answer is, resoundingly: no! For a start, Marraud does not prove his claim, but takes it to be obvious. In his mind's eye, evidently, the mere fact that argument A1 concerns an object (i.e. a logical subject) "more R" than argument A2, logically implies that A1 is "stronger" than A2. Yet, a moment's thought should have shown him the absurdity of this assumption, in view of the reversibility of comparative propositions.

For every term R1, we can posit a relative term R2, such that if P is more R1 than Q, then Q is more R2 than P, and vice versa. To give an example, if Jack is taller (*more* tall) than Jill, then Jill is shorter (*more* short) than Jack, and vice versa. Thus, the same "meta-argument" would yield conflicting conclusions, according as we read its major premise (a) in the form "P is more R1 than Q" or in the reverse form "Q is more R2 than P." In the former case Marraud would conclude "A1 > A2" and in the latter case he would conclude "A2 > A1." He would thus draw contrary conclusions from logically identical premises! This is manifest nonsense, and it formally proves that his conclusion about the relative "strength" of A1 and A2 is invalid. Anyway, why should a greater quantity of something ("more R") give rise to more certainty ("strength") than a lesser quantity of it? It is a silly notion[11].

Another proof of the absurdity of Marraud's claim is that the quantitative comparison inherent in the major premise "P is more R than Q," i.e. the implication that $Rp > Rq$ (i.e. that the quantity of R of P is greater than that of Q) is in fact *unused* in the formation of the two sub-arguments, A1 and A2. If it is unused, it cannot possibly affect the result; i.e. it makes no difference to these sub-arguments whether $Rp > Rq$ or $Rp < Rq$, or even $Rp = Rq$. The two sub-arguments only make use of the implications "P is R" and "Q is R" of the major premise; and these implications have no quantitative differentia. And indeed, Marraud betrays his uncertainty or confusion further on, when he refers to "greater or lesser presence of R" and "more/less R" even though in the diagram he only has "more R"[12].

Still further proof of the absurdity of Marraud's claim is that the two sub-arguments are *identical* in form (they are both syllogisms of form 1/AAA), which share the exact *same* major premise (All R are S, implied by "± an object has property R, ± it has property S") and whose minor premises (P is R and Q is R) were both obtained by the *same* process (immediate inference) from the very *same* source (namely, "P is more R than Q" – or even if we wished, "P is ± R than Q"). How can two processes, identical in all respects, be characterized as having different "strengths"? Logic is formal – it is ruled by law. Processes that are formally indistinguishable must be admitted to have the same "strength." Clearly, Marraud did not reflect on the untenable implications of his claim[13].

It should be added that all the above critique would be applicable equally well if Marraud had used the proportionality implied by premise (b) to infer different values of S for P and Q – i.e. to infer "P is S1" and "Q is S2." Had he done so, the contents of arguments A1 and A2 would be a bit more different – but their form and

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[11]   Already, some two and a half centuries ago, M. C. Luzzatto demonstrated clear awareness of the difference between the *de re* and *de dicto* comparisons when he remarked: "when a certain quality is exhibited to a greater degree, it is not, therefore, more likely to occur; in fact, it is often less likely" (p. 90).

[12]   Where he writes (I quote him verbatim): the strength of two such arguments can be compared in terms of the greater or lesser presence of R: if O is more/less R than O', the argument A1 "O is R, so O is P" will be stronger than the argument A2 "O' is R, so O' is P".

[13]   Or maybe he did think about it, but could think of no solution. This is perhaps why he writes: "Despite their close affinity, the cogency of an argument from more and less do[es] not entail the cogency of the corresponding meta-argument from more and less."

therefore their logical status would remain the same, i.e. it would still be formally impossible to conclude that A1 is "stronger" than A2. The only way Marraud could defend his thesis would be by introducing additional information that would somehow alter the logical status of one or the other of the arguments. But then the "meta-argument" would have a different form than the one he here proposes.

The term "stronger" used here supposedly refers to logical strength – i.e. to a probability rating. Maybe Marraud thought the sub-arguments are inductive rather than deductive, and therefore likely to differ in probability. But as we showed above, they are in fact both deductive, and even simply syllogistic, and based on the same premises. And it can be said for any deductive argument: if the premises are sure, so is the conclusion; the conclusion is exactly as sure as the two premises that give rise to it – neither more nor less. If the argument is well-formed, but one or both premises is/are less than sure, so that the argument is effectively inductive, the probability rating of the conclusion is the product of the probability ratings of the two premises. Probability ratings do not emerge haphazardly, but in accord with reasonable rules.

It could be that Marraud, like many commentators before him, was misled by the name "a fortiori" or by the traditional marker "all the more" into the belief that the argument he describes involves a change of strength. But it is hard to see how it reasonably could, since both these expressions, and others like them, suggest that the putative conclusion is 'stronger' than the minor premise it was based on. The Latin phrase *a fortiori ratione*, remember, means '*with* stronger reason'[14]. But this is just hyperbole, for as above explained, it cannot be literally correct. For sure, the phrase does not mean, as Marraud translates it in the title of his essay, '*from* a stronger reason'. That is, it does not mean that the minor premise is stronger than the conclusion; for in truth, the minor premise of *any* deductive or inductive argument, whatever its form and content, is always either stronger or equal in strength to the conclusion, and never of lesser strength, so to say 'from a stronger reason' would be a redundancy.

In any case, Marraud's thesis here is not that the conclusion of a fortiori argument is stronger than its minor premise, or vice versa, but that one sub-argument *as a whole* is "stronger" than the other sub-argument *as a whole*. There is clearly zero logical justification for that claim; it is simply a personal fantasy, which reveals the extent of its author's misunderstanding of logic in general and a fortiori argument in particular. In sum, there is *absolutely no logical basis* for Marraud's claim to have here identified a distinct form of a fortiori argument deserving to be characterized as a "meta-argument." His "meta-argument" cannot be claimed to be anything more than a conjunction of two quite ornery "arguments" – there is in fact no "argument about arguments" in it. To speak of a "meta-argument" in this context is, frankly, pretentious nonsense.

Meta-argument from strength comparison. Marraud next very briefly presents the argument he refers to as "sense 3," saying: "Once it has been established, $A_2 < A_1$ may serve as a principle for transferring sufficiency from A_2 to A_1," and depicting the argument as having the following form (p. 7):

$$A_2 \text{ is sufficient} \quad \xrightarrow{} \quad \text{So, } A_1 \text{ is sufficient}$$
$$\text{since}$$
$$A_2 < A_1$$

Marraud then explains: "we are bound to accept *a fortiori* an argument because of our prior acceptance of a weaker argument." And he calls this "a variant of arguments from strength comparison." It is not difficult to back-engineer the image at the back of Marraud's mind, which caused him to formulate this "meta-argument." I would say his subconscious thought was the following a fortiori argument, in which "sufficient" is taken to mean "sufficiently strong to be relied on":

> Argument A1 is stronger than argument A2 (Marraud's "$A_2 < A_1$"),
> and A2 is sufficiently strong to be relied on (his "A_2 is sufficient");
> therefore, A1 is sufficiently strong to be relied on (his "A_1 is sufficient").

This is a valid positive subjectal mood of a fortiori argument, assuming we manage to establish the premises somehow[15]. To my mind, it is an argument like any other argument – not a transcendent "meta-argument." It just so

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[14]   In French: *à plus forte raison*, meaning: *avec encore plus de bonne raison*. What is the added weight intended, here? Merely the fact that the major term is greater (in some respect) than the minor, or that the minor term is smaller (in some respect) than the major; nothing besides that. Thus, the phrase just signifies that *there is an appropriate major premise*, justifying the inference of the conclusion from the minor premise!

[15]   Clearly, Marraud views the proposition "A1 > A2" here as the conclusion of the previous "meta-argument." And he maybe invented that meta-argument *in order to* obtain this conclusion somehow. But we have just shown that meta-argument to be in fact incapable of providing such a conclusion. Nevertheless, such a conclusion could conceivably derive from other arguments.

happens that its subject-matter – i.e. its major and minor terms A1 and A2 – consists of 'arguments'. Its middle term (R) is 'logical strength' and its subsidiary term (S) is 'reliability' (or some such indicator of credibility or persuasiveness, which causes our "acceptance"). The expression 'sufficiently strong' or 'strong enough' indicates that the threshold at which logical strength has a magnitude or degree capable of generating 'reliability' has been *reached or surpassed*. To repeat, this is a normal, truly a fortiori argument – nothing special, nothing calling for a new name. Its content does not affect its form. What can be said to characterize it is that it is a logical-epistemic argument, rather than an ontical one.

If we now look at Marraud's rendering of this argument, we can discern various problems with it. First, as usual, what he claims to be the "warrant" of the argument (viz. "$A_2 < A_1$") is not an abstract, external warrant, but a concrete major premise without which the conclusion ("A_1 is sufficient") could not be drawn from the minor premise ("A_2 is sufficient"). It does not matter that this "warrant" is different from the preceding "warrants" (which affirmed proportionality); what matters is the role it plays in the inference. The true warrant of this inference is the standard form of positive subjectal a fortiori argument, or the validation process for that form.

Second, the minor premise as he has it is logically incapable of yielding the conclusion. In the proposition "A_2 is sufficient," Marraud does not say what he means by "sufficient." Sufficiency *of what* and *for what*? And how exactly is the degree of sufficiency to be determined? And what is the relation of this notion to that of strength? He does not say. In truth, this premise must be formulated as "A2 is sufficiently strong to be relied on," in accord with the standard formula "Q is R enough to be S" (here, the minor term P is A2, the middle term R is 'strength', and the subsidiary term S is 'reliable'). We can then draw the conclusion "A1 is sufficiently strong to be relied on" (which implies Marraud's "A1 is sufficient").

Notice that the reasoning here is, rightly, 'from minor to major' (i.e. from the minor term in the minor premise, to the major term in the conclusion). That is, from the weaker argument (A2) to the stronger one (A1). In the previous type of "meta-argument," however, Marraud conceived the conclusion as establishing one argument (A1) as "stronger" than the other (A2) – i.e. it seemed to go 'from major to minor', or at least to favor the stronger argument over the weaker. It is surprising that Marraud does not stop and wonder about this reversal of fortune. In any case, his adoption of the present argument, viz. "$A_2 < A_1$ and A2 is sufficient; therefore, A1 is sufficient," is quite intuitive – he does not attempt to validate it.

As we have seen, Marraud does not have the crucial factor of "R enough to be" in his primary scheme; i.e. he does not realize that his minor premise imperatively needed to be "P is *R enough to be* S" (rather than merely "P is S") to make his conclusion "Q is S" formally possible. Even so, to his credit, he shows an intuitive sense of the need for this factor somehow, when he says "the presence of (a certain amount of) R is a sign of S" and when he here mentions "transferring sufficiency." But this is not a scientific theory, with substance and consistency. It does not qualify as knowledge. Confusion is bound to ensue. This man needs to study logic *a lot*, before attempting any further ado in this demanding field.

4. Paulo minor argument

Apparently determined to innovate somehow, Marraud tries to introduce a fourth form of a fortiori argument (pp. 8-9), which he dubs as *paulo minor*, which is Latin for 'a little less'. Although that name hardly matches the three examples he gives[16], we shall see that it is appropriate.

He gives the following as its prime example: "If demigods are little more than humans, they are also slaves to their passions." Here, presumably, the words "little more" make him think this constitutes some sort of a fortiori argument. But in fact this argument is merely rhetorical, for the inference it proposes is fallacious. "Demigods are little more than humans" implies that "Demigods are more than humans," even if the difference is only a "little" bit. Therefore, one *cannot* logically infer from the fact that humans are "slaves to their passions" that demigods are also "slaves to their passions." This can be clearly seen if we cast the argument in standard form (using "godlike" as the middle term):

> Demigods (P) are more godlike (R) than humans (Q) are, and
> humans (Q) are not godlike (R) enough not to be slaves to their passions (S); so:
> demigods (P) are not godlike (R) enough not to be slaves to their passions (S).

As can be seen, the argument is actually a negative subjectal one, and yet it proceeds 'from minor to major' – therefore, it is invalid. In other words, even if the demigods are little more than human, they might still have more

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[16]   "A little less" is neither the same as "little more" nor the same as "almost as much as."

power over their passions than humans do; only creatures that are less than human may logically be expected to be as much or more enslaved to their passions. That Marraud finds the said discourse convincing shows his lack of logical acumen.

The second example looks nothing like the first, and moreover contains glaring inconsistencies due only in part to the author's bad English. He starts with the sentence: "Anne is almost taller as Betty" – I am not sure whether that means "Anne is almost as tall as Betty" (which implies she is not as tall) or "Anne is almost taller than Betty" (which implies she is perhaps as tall, though not taller). Then he states that "this sentence implies 'Anne is tall' and 'presupposes Betty is tall'" – showing he does not understand that 'presupposes' is effectively equivalent to 'implies'. Then he says that "the underlying reasoning can be reconstructed as an argument from more and less," and forces the argument into his usual diagram, with the premises "Betty is tall; Anne is almost taller as Betty;" and the conclusion "Anne is tall," with the alleged warrant: "x is taller than y, so x is tall"[17]. Notice how the "warrant" differs from the example: "y is tall" is missing from it and the wording "almost taller as" is replaced by "taller than." This is quite a mess.

In truth, "Anne is taller than Betty" formally implies both "Anne is tall" and "Betty is tall" by mere eduction (i.e. immediate inference), where "tall" may have any value. "Betty is tall" is not a premise for "Anne is tall;" and there is no need for a fortiori argument to draw the desired conclusion, and anyway the argument Marraud presents as a fortiori is not one. Additionally, Marraud tries to spin the argument as one "by strength comparison," saying: "The warrant correlates tallerness to tallness, so that an accrual of tallerness appears as *an accrual of reasons for* tallness … there are no degrees of tallness, and degrees of tallerness are to be correlated to *degrees of justified belief*." (The italics are mine.) Here again, as with his earlier "meta-argument from more and less," he is trying to tie epistemic credibility to ontical size – which is balderdash.

The third example Marraud gives us looks nothing like the preceding two. It reads: "If you're planning to become pregnant, taking certain steps can help reduce risks for both you and your baby. Proper health before deciding to become pregnant is almost as important as maintaining a healthy body during pregnancy." He depicts this pictorially as follows:

Proper health before deciding to become pregnant is almost ——⊤—▶ So, A_2 is almost as important as maintaining a healthy body during pregnancy   since   strong as A_1

± important, ± constraining

Where, "A_2 = You're planning to become pregnant, so eat a balanced diet." and "A_1 = You're pregnant, so eat a balanced diet." This is just more spin on Marraud's part. All the given discourse says is that proper health care before pregnancy is almost as important as proper health care during pregnancy. Presumably "importance" here refers to "importance for the future baby's health," i.e. to the power to cause health in the future baby[18]. In other words, if she wants to have a healthy baby, a woman should take care of her health before she actually gets pregnant as well as during her pregnancy. Health care during pregnancy plays a major role in ensuring the baby's health, but health care before pregnancy also plays a role even if a slightly lesser one.

There is no inference involved; it is just a statement of fact. The words "almost as important" used here are not indicative of a fortiori argument, let alone a "meta-argument from more or less" or an "argument by strength comparison" or a "*paula minor* argument." Indeed, if we try to formulate a standard a fortiori argument, all we get for our troubles is yet another non-sequitur, since the argument is positive subjectal and yet goes from major to minor: "Health during pregnancy (P) is a bit more important (R) than health before pregnancy (Q) is, and health during pregnancy (P) is important (R) enough to make it recommended (S); therefore, health before pregnancy (Q) is important (R) enough to make it recommended (S)." This is, to repeat, invalid reasoning.

To conclude: Marraud's idea of *paula minor* a fortiori argument is clear enough, even if he does not manage to express it clearly – but it is wrong if deductive argument is intended. Given, P is more R than Q, one cannot deduce, from "Q is 'almost but not quite' R enough to be S", that "P is likewise 'almost though not quite' R enough to be S." Similarly, one cannot deduce, from "P is 'almost but not quite' not R enough to be S", that "Q is likewise 'almost though not quite' not R enough to be S." A little less than enough is just not good enough – if the threshold for something is stated as Rx, than nothing less than the value Rx will do the trick. Deductive logic does not allow approximations or compromises.

One might, however, argue inductively that, given that P is R enough to be S, and Q is only a little less R than P is, there is a reasonable chance that Q is R enough to be S. This would be based on the thought that the threshold value

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[17]   Note that this "warrant" involves a material term (taller, tall). To be fully formal, it would have to read: "x is more z than y, so x is z."
[18]   "Importance" does not refer to "strength" of an argument, as Marraud tries to suggest.

of R for S (i.e. the minimum Rx) *might be* below both the values Rp and Rq, since that is often de facto the case. Similarly, given that Q is not R enough to be S, and Q is only a little less R than P is, there is a reasonable chance that P is not R enough to be S. This would be based on the thought that the threshold value of R for S (i.e. the minimum Rs) *might be* above both the values Rq and Rp, since that is often de facto the case. Of course, such thoughts are speculative; but they do suggest there is some probability in the proposed conclusion. The closer the values of Rp and Rq, the greater the probability; the wider the spread between them, the lesser the probability.

The above concerns subjectal a fortiori argument, but obviously similar reasoning can be applied to predicatal argument. As regards the positive mood: given that more R is required to be P than to be Q, one cannot deduce, from "S is 'almost but not quite' R enough to be P", that "S is likewise 'almost though not quite' R enough to be Q." Similarly, as regards the negative mood: one cannot deduce, from "S is 'almost but not quite' not R enough to be Q", that "S is likewise 'almost though not quite' not R enough to be P." These two arguments are of course also invalid, since 'almost though not quite' implies 'not'. But they could be taken as inductive. We could also formulate implicational equivalents of the four copulative forms mentioned above.

Thus, from an *inductive* perspective, positive subjectal (and similarly, negative predicatal) argument could proceed from major to minor, and negative subjectal (and similarly, positive predicatal) could proceed from minor to major. While this sounds reasonable, I wonder if such discourse can truly be referred to as inference, even if only as inductive inference. For ultimately, it seems to me, since we lack precise information, the probability is really always fifty-fifty. Strictly speaking, then, *paula minor* arguments are invalid. They might however retain some power of conviction as inductive inferences, although that is open to debate. Thus, if as Marraud suggests they have some rhetorical weight in ordinary discourse, it is at best merely inductively and at worst through sophistry.

5. Legal a fortiori argument

Marraud considers some examples of legal reasoning, drawn from actual law cases, and tries to show how they fit into his conception of a fortiori argument (or meta-argument) as comparison of the strengths of two arguments (or sub-arguments).

The first example of legal argumentation he presents (pp. 2-3) is put forward by an American judge in an actual case:

> "Possession by an accused of recently stolen property is sufficient to sustain a conviction of theft where a satisfactory explanation is not given, particularly where the nature of the items and their condition support an inference that they have been stolen."

This is a straightforward a fortiori argument, which can be put in standard (positive subjectal) form as follows. The tacit major premise is: Possession by an accused of recently stolen property where a satisfactory explanation is not given and where the nature of the items and their condition support an inference that they have been stolen (P) is more damning evidence (R) than Possession by an accused of recently stolen property where a satisfactory explanation is not given and where the nature of the items and their condition *do not* support an inference that they have been stolen (Q). The minor premise is explicitly: Q is sufficient (i.e. sufficiently damning evidence, R) to sustain a conviction of theft (S); and the conclusion is explicitly: P is sufficient (i.e. sufficiently damning evidence, R) to sustain a conviction of theft (S). Note that the word "particularly" used here serves merely to signal a fortiori argument, in the same way as expressions like "a fortiori" or "all the more" would do.

Marraud, however, conceives the judge's argument as consisting of two "arguments," the first being effectively that 'Q implies S' and the second that 'P implies S'. He considers the first argument as "cogent," but the second one as "a stronger argument, as the word 'specially' indicates." Then he adds: "So far as the first argument provides sufficient evidence, the second seems unnecessary. This leaves open the question of why an arguer would use such a redundant way or arguing." He speculates that the judge may by this means be "anticipating the rebuttal of someone rejecting the sufficiency of the first argument." That is why, he tells us, "it is often said that an AF argument reinforces a claim already established." In Marraud's view, the judge "thinks that the premise 'the nature and condition of the items support an inference that they have been stolen' can be suspended without ruining the argument."

In other words, for Marraud – who conceives of the overall argument as a comparison in "strength" of two implications, or if–then propositions – the first antecedent (Q) is strong enough to prove the consequent (S) and the second (P), though stronger (due to providing an additional reason), and therefore also able to prove S, is redundant. Marraud says this, about redundancy, because he considers the two antecedents, which I have labeled Q and P, to be two related theses: Q being "A" and P being "A + B" – both able to imply the conclusion "C" (my S). Clearly, his thought is that if A alone suffices to imply C, then the B in A + B is redundant. But these are not the true antecedents involved. The two these are in fact 'A + notB' and 'A + B'. 'A alone' here means 'A without B', and not as Marraud

thinks 'A, whether or not B'. Note well the difference[19]. Once this is realized, it becomes clear why there is no redundancy, but new information has been uncovered.

Thus, though Marraud's explication sounds reasonable on the surface, it is not at all a correct description of the overall argument. The two propositions 'Q implies S' and 'P implies S' are quite distinct and, from a logical point of view, equally cogent—equally "strong." Neither is more reliable than the other (at any rate, the second is not more reliable than the first, since it depends on both the minor and premises). The a fortiori argument, we might say, simply consists in logically deriving the second implication from the first. That is to say, the first helps us to discover and justify the second. But once that discovery and justification effected, the net credibility of the conclusion is neither greater nor smaller than the joint credibility of the given major and minor premises. There is no comparison of "strength" involved, and no part of the argument is "redundant."

Marraud presents his next example of legal argumentation (pp. 9-10) as follows, in his own words:

> "Justice Souter uses first an argument from precedent: the Court of King's Bench held that a private person needed no warrant to arrest a common cheater whom he discovered cozening with false dice; so in this case no arrest warrant was needed. Then he goes on to reinforce his argument with another: by a stronger reason a police officer can execute a warrantless arrest because a police officer has even more of a right to arrest than another person."

Before analyzing Marraud's take on this reasoning, let me say how I see it. In my view, the first argument is an argument from precedent, meaning that a past judgment of a court is held up as a case in point. The latter is implicitly generalized, meaning that all subsequent cases that are reasonably similar may be subjected to a like judgment. Then this generality is syllogistically applied, meaning that the present case is judged in accordance with the said rule. This is not a fortiori argument, but more akin to the rabbinical technique of *binyan av* – i.e. it is complex analogical argument. The second argument can be construed as a fortiori, positive subjectal in form, as follows:

A peace officer (P) has more 'right to arrest' (R) than a private person (Q).
If (as in the present case) a private person (Q) has right to arrest (R) enough to allow him to arrest without a warrant (S),
then, all the more, a peace officer (P) has right to arrest (R) enough to allow him to arrest without a warrant (S).

Marraud, for his part, analyzes the judge's reasoning in much more convoluted terms. He perceives the reasoning as a comparison between arguments of different strengths, and thus reconstructs the argumentation "as a meta-argument from more and less." This is presented diagrammatically as follows:

A police officer has more of a right to   ⟶   So, A$_1$ < A$_2$
arrest than a private person   since

Scale of arrestors
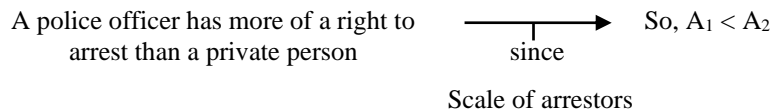
In this schema, argument A1 is that "that there are reasons X to allow a private person to execute a warrantless arrest," and argument A2 is that for the same reasons X, "a police officer can execute a warrantless arrest." And Marraud is here claiming to have proved, given the stated premise and warrant, that A2 is "stronger" than A1. But as we have seen before, this schema is logically incapable of proving any such thing. It should additionally be noted that Marraud is here, as usual, unclear about the difference between the "warrant" of an argument and a premise. The proposition "a police officer has more of a right to arrest than a private person" was previously described as a "warrant" and here placed in the role of premise. As regards the "warrant" in the diagram, simply stated as "scale of arrestors," it was previously presented with the words: "The implicit warrant will be some scale of arrestors (taking into account their status, the circumstances of the arrest, the nature of the offence, etc.) and placing police officers at the top." Clearly, these two say much the same thing, even if he labels them differently to make two propositions out of them, and project the first as a concrete premise and the second as a "principle."

After presenting this "meta-argument from more and less," Marraud suggests that "An associate argument from strength comparison leads to the main conclusion: police officers can arrest someone for a minor criminal offense without warrant." Though he does not detail this new argument, we can assume it is that since the (allegedly) weaker argument A1 is "sufficient," then the (allegedly) stronger argument A2 must be "sufficient." This argument is indeed

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[19]   If we followed Marraud's assumption that A implies C, then, since (A + B) implies A, it would follow that (A + B) implies C. But then the inference of the second implication from the first would be syllogistic, and not involve a fortiori argument at all!

a fortiori in intent, even though incompletely verbalized (the meaning of "sufficient" having not been clarified by him, as earlier explained). However, the argument is wrong anyway, since its major premise, viz. that "A2 is stronger than A1" has, as we just pointed out, not been proved by the preceding argument! In conclusion, even though Marraud tries to use the said example of legal reasoning as evidence for the utility of his two "meta-arguments" – it turns out that this is mere spin on his part. In fact, that example cannot be explained by these means, but only as I previously explained it.

Another example of legal reasoning proposed by Marraud (pp. 10-12) is the following:

> "Suppose a thief steals a wallet and the £20 note therein. His victim will undoubtedly have a claim against him in wrongs, more specifically, in the tort of conversion. […] What is a matter of debate is whether the victim can maintain a common law strict liability claim in unjust enrichment. The existence of such a claim is said to flow as a matter of deductive logic from the availability of strict liability common law claims in unjust enrichment for mistaken transfers. Mistaken transferors recover because their consent to the transfer was impaired. In the posited case, the victim of the theft gave no consent whatever to the 'transfer' of the wallet and note to the thief. He was 'ignorant' of it. His ability to claim in unjust enrichment is, it is said, a fortiori from mistake."[20]

As before, let me analyze it prior to considering how Marraud perceives it. In my view, the argument is clearly as follows: Since the transfer of the stolen property to the thief involves no consent whatever by the victim of the theft, it is *analogous to, or classifiable under*, mistaken transfer, i.e. transfer in which consent was impaired. Therefore, since mistaken transferors recover their property under a common law strict liability claim in unjust enrichment, it follows that victims of theft can recover their property under the same law. This reading implies two possible interpretations.

The first interpretation is syllogistic: it considers that "transfer of property with no consent" (i.e. through theft) is simply *a case of* (even if only a limiting case of) "transfer of property with incomplete consent" (i.e. mistaken transfer). That is, here, "incomplete consent" is understood to mean "less than complete consent," and thus include "zero consent"[21]. Thus, the argument takes the form of 1/AAA syllogism, with minor term P, middle term Q and major term S, as follows:

> Transfer of property with incomplete consent (Q) is subject to recovery of property under a common law strict liability claim in unjust enrichment (S).
> And transfer of property with no consent (P) *is* transfer of property with incomplete consent (Q).
> Therefore, transfer of property with no consent (P) is subject to recovery of property under a common law strict liability claim in unjust enrichment (S).

Alternatively, we can place "transfer of property with no consent" and "transfer of property with incomplete consent" on a common continuum, R (say, degrees of dishonesty). Here, "incomplete consent" implies "some consent" and thus excludes "no consent." We can then interpret the argument as a fortiori, of positive subjectal form, with major term P, minor term Q, middle term R, and subsidiary term S, as follows:

> Transfer of property with no consent (P) is more dishonest (R) than transfer of property with incomplete consent (Q)
> And transfer of property with incomplete consent (Q) is dishonest (R) enough to justify recovery of property under a common law strict liability claim in unjust enrichment (S).
> Therefore, transfer of property with no consent (P) is dishonest (R) enough to justify recovery of property under the same law (S).

Note the simplicity of both these interpretations. Marraud, for his part, makes something rather more complicated of it. He sees it as "a comparison of the strength of two arguments." The first argument is that since "a mistake is recognized by law as an unjust factor," then "suppose the transferor's consent to a transfer were vitiated by a mistake," it would follow that "the victim could claim in unjust enrichment." The second argument is stated as: "suppose a thief steals a wallet and the £20 note therein," then "the victim can claim in unjust enrichment." Both these sub-arguments are presented in the usual diagrammatic form. The "warrant" given for the first argument is, of

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[20]   This is drawn from "Swadling, 2008, pp.627-628." Marraud states that "Swadling purports to demonstrate the falsity of this claim." If so, Swadling is of course wrong.
[21]   If we take this to be true by generalization (from 'impaired consent' to all sorts of 'incomplete consent'), then the argument as a whole should be regarded as not merely syllogistic, but as complex analogy (i.e. as an argument of the type called *binyan av* in rabbinical hermeneutics).

course, as usual, not in fact a warrant but a premise; the argument being essentially syllogism (or apodosis), as Marraud admits when he characterizes it as "hypothetical." The second argument, on the other hand, contains a question mark in lieu of a "warrant." The first argument is "invoked to gain acceptance for" the second one, which is called "the main argument."

Marraud now searches for an appropriate warrant for the second argument. He needs to do this because the relative strengths of the arguments will depend on their respective warrants. The warrant of the first argument cannot, however, be carried over to the second argument, since the subject-matter is different. If "the missing warrant" were taken to be "theft is recognized by law as an unjust factor," this would amount to "begging the question." If the needed warrant were assumed to be "ignorance is recognized by law as an unjust factor," a debate would ensue as to the truth of this under English law, and anyway this would (for some unstated reason) "lend less force" to the conclusion of the 2nd argument than "mistake is a recognized by law as an unjust factor" does to the conclusion of the 1st argument. Therefore, Marraud considers that "some principle like 'mistake is less than ignorance'… is needed to transfer acceptability from one argument to the other."

After that, he thickens the plot further by introducing the notion of "backings" to "warrants," explaining that "backings may confer different degrees of force to the warrants they justify." To my mind, all this discussion – Marraud's lame search for a warrant, and then for a backing for it – is close to useless chatter, being based on a wrong perception of the reasoning involved in the said example. The reasoning has nothing to do with comparison of the "strengths" of different "arguments." The reasoning is, as above shown, simply syllogistic or a fortiori with reference to specific terms (given or implied). Marraud gets entangled in a complicated discussion, because he is unable to see the straightforward solution to the problem. And the reason for that is that he has not sufficiently studied formal logic, but evidently thus far only studied the 'rhetorical' approach to analysis of arguments.

The rhetorical description of human thought processes is interesting, but essentially superficial being incapable of judging validity. Only formal logic can provide a credible and deep understanding of reasoning in general, and legal argumentation in particular.

31. Various other commentaries

The following shorter assessments are ordered chronologically. They are comparatively brief, either because I lack the information needed to say more about the author's work, or because not much need be said concerning it. It should be said that this listing is no doubt far, far from exhaustive. Just as I seemed close to finishing the present book, I happened to type "a fortiori" in the search box of Google Books, and to my great surprise discovered that there are about 2.2 million books with this phrase in them! Of course, in many books the expression is used and not discussed; but surely also, in many something is said about it.

1. H. S. Hirschfeld

H. S. Hirschfeld is the author of two books, called *Halachische Exegese* (1840) and *Hagadische Exegese* (1847), dealing as their names imply with halakhic and haggadic exegesis. I have not read these books, but Samely mentions some of the contents of the first, and gives us some idea of Hirschfeld's views on the a fortiori argument. Thus, he informs us: "Hirschfeld … was perhaps the first explorer of the *qal wa-homer* to attempt a symbolic representation of its logic. The formula he devised, for one main version of the argument, is '$A - a = A + x :: B + a = B + x$.'"[1]

I can only guess what this symbolic formula is about, not having seen the context. Perhaps more information about it is given in Hirschfeld's book. But, quite offhand, it seems to be saying: "*If A without (or with less) a, has x, then surely B with (or with more) a, has x*"[2]. Hirschfeld's representation of a fortiori argument thus seems to be a forerunner of Louis Jacob's 'complex' a fortiori argument "If A, which lacks y, has x, then surely B, which has y, has x." Whether the latter based his formulation on the former's, or came to it independently, I do not know. In any case, this means that my criticisms of Jacob's idea can equally be applied to Hirschfeld's. In the last analysis, what we are offered in both cases is a description of the surface appearance of some a fortiori discourse, and not an explanation or validation of such reasoning. We are not told what the presumed relation is between A and B; nor how 'A without a' can tell us something about 'B with a'.

Consider an example: 'If a man without much money can find a wife, then surely a man with much money can do so.' The reason this argument sounds credible is because we consider that money is an advantage that most women would appreciate having. But an equally cogent argument of the same form could be formulated with opposite effect. 'If a man without children can find a wife, then surely a man with children can do so.' The reason this argument does *not* sound very credible is because we consider that children (from a previous relationship) are an added burden that most women would rather do without. Thus, the credibility of the argument depends on a tacit additional premise. It is not enough to state a minor premise and conclusion; we also have to specify the major premise. In these examples[3], the major premise would tell us which attribute of the man is more attractive to the potential wife. Having money is more attractive than lacking it – so the first argument works; it is minor to major. Having children is less attractive than lacking them – so the second argument is unconvincing; it is major to minor. Thus, though both arguments are positive subjectal in form, they impact differently on the rational faculty due to unspoken differences.

It follows that a formula like Hirschfeld's – or like Jacobs' – cannot suffice as a representation of a fortiori argument. Note moreover that the argument form they propose is just one (albeit the most common one) among many. They do not mention negative subjectal, or positive or negative predicatal, or the implicational moods. They mention purely a

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[1]   In a footnote on p. 177, referring to *Halachische Exegese* (Berlin: Athenaeum, 1840)*, p. 227. Samely adds: "But the reduction of complex conceptual relationships with the *qal wa-homer* to the meaning of the arithmetic minus and plus offers no adequate analysis of the argument's structure. Modern symbolic logic is certainly in a better position to capture the dynamics of the *a fortiori* argument, and Koch and Rüssmann, *Juristische Begründungslehre*, 259f, and Klug, *Juristische Logik*, 132-7, attempt to provide formulas." Further on, in a foortnote on p. 179, Samely tells us that Hirschfeld "identifies the *qal wa-homer* as an inference from *opposition* (Gegensatz', *Halachische Exegese*, 217f and 224)."

[2]   Most probably the plus and minus signs signify presence and absence; but they might also signify increase and decrease (as stated in brackets). However, in the latter case, the plus sign has both meanings, since it is used with one meaning relative to 'a' and with the other relative to 'x'.

[3]   Needless to say I am not here trying to recommend to women any specific attitudes; the arguments here proposed are just intended as illustrations, as possible opinions.

fortiori argument, but fail to mention a crescendo argument. It is a pity that neither of these researchers found out about the much broader and more accurate representations by Moshe Chaim Luzzatto (mid-18th century).

2. H.W.B. Joseph

I was inspired to start writing logic in my late teens after reading *Introduction to Logic*[4] by Horace William Brindley Joseph[5]. His long, free-wheeling discussions of general issues and particular points made the subject very interesting to me. I had already by then read some of Aristotle's work, but his dry, more technical approach did not have the same effect on me. Naturally, then, today, when I looked for past commentaries on the a fortiori argument, I looked into Joseph's book. Here is all that he says:

> "… there are certainly forms which have not been examined. For example, there is the a fortiori argument. 'He that loveth not his brother whom he hath seen', asks St. John, 'how can he love God whom he hath not seen?'" (Pp. 369-370.)

This is not much, but it tells us a couple of things. First, that H. W. B. Joseph was immediately aware that the a fortiori argument is not some sort of syllogism, but a distinct type of argument that had not yet been adequately examined (in 1916). Secondly, he gives us just one example of it, drawn from the Christian Bible, uncharacteristically without any attempt at analysis. Obviously, his statement was intended as a final note, outside the scope of his book.

The sample argument is indeed a valid a fortiori, as my formalization of it below shows. Specifically, it is a negative predicatal mood, whose implicit middle (R) would be a term like 'capacity for love (heart)' or 'spirituality', say:

> More spirituality (R) is required to love someone unseen (P) than to love someone seen (Q).
> Yet, he (some man) is not spiritual (R) enough to love his brother, whom he hath seen (Q).
> All the more (or equally), he is not spiritual (R) enough to love God, whom he hath not seen (P).

3. Moshe Ostrovsky

Moshe Ostrovsky wrote a study of the rabbinical hermeneutic principles, called in Hebrew *Hamidot She-haTora Nidreshet Bahem* (meaning in English: *The Rules that the Torah Requires*), which was published in Jerusalem in 1924/5[6]. I have not read the book, but some of its content relating to *qal vachomer* (a fortiori argument) are reported and discussed by Wiseman[7]. The latter translates Ostrovsky's description of the argument as follows: "The name Qal VaChomer applies only to teach what is a special judgement between two things that bear a graded difference, by means of which we judge what applies from one to the other"[8]. As Wiseman puts it, this is "a minimally adequate definition."

However, Ostrovsky gets more specific, and even waxes symbolic, when he analyzes the Mishnaic controversy between R. Tarfon and the Sages (i.e. Baba Qama 2:5). It is interesting to note that Ostrovsky views this controversy as one "between reason and the majority view," where R. Tarfon upheld "ordinary reasoning" which the Sages overruled with reference to "tradition"[9]; I would partly but not wholly agree with this understanding. According to Wiseman, Ostrovsky's approach is "based partly upon the prior work of Schwarz;" although, looking at the information Wiseman gives, I do not see exactly how Ostrovsky is rooted in Schwarz, if by the latter we take to mean a reference to syllogistic interpretation of a fortiori argument.

Ostrovsky's symbolic representation of the a fortiori argument is as follows[10]:

1st premise:	$A - a + b$	(item A has less of something in case 'a' and more of it in case 'b');
2nd premise:	$B + a$	(item B has a certain amount of it in case 'a');
conclusion:	$B + b$	(item B has a greater amount of it in case 'b').

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[4]   2nd ed. rev. Oxford: Clarendon, 1916. A full copy of this work may be downloaded at: ia700303.us.archive.org/8/items/introductiontolo00jose/introductiontolo00jose.pdf.

[5]   England, 1867-1943.

[6]   By K. I. Milman.

[7]   In *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions* (Waterloo, Ont.: University of Waterloo, 2010).

[8]   Ostrovsky, p. 41; Wiseman, p. 30.

[9]   Ostrovsky, p. 36; Wiseman, p. 73.

[10]   Ostrovsky, p. 68; Wiseman, p. 73-4.

This formula (the explanations in brackets are mine) is used to express the two examples given in the above mentioned Mishna (i.e. the two arguments of R. Tarfon), namely (the wording is mine, and the explanations in brackets, in terms of P, Q, R, S, are mine):

- If tooth & foot damage (A) engenders a lesser fine (less R; 0%) on public grounds ('a'=Q) and a greater fine (more R; 100%) on private grounds ('b'=P), and horn damage (B) on public grounds ('a'=Q) engenders a certain fine (less R; 50%), then horn damage (B) on private grounds ('b'=P) engenders a greater fine (more R; 100%) than that.
- Again, if on public grounds (A) tooth & foot damage Q) engenders a lesser fine (less R; 0%) and horn damage ('b'=P) engenders a greater fine (more R; 50%), and on private grounds (B) tooth & foot damage ('a'=Q) engenders a certain fine (less R; 100%), then on private grounds (B) horn damage ('b'=P) engenders a greater fine (more R; 100%) than that.[11]

I should first point out that, strictly speaking, in Ostrovsky's symbolic formula, the first premise seems to say that A has less 'a' and more 'b' (less/more, relatively to each other); the second premise seems to say that B has *more* 'a' rather than as I put it 'a certain amount of' (*more* here being apparently in comparison to the 'a' of A, since 50>0 or 100>0), and the conclusion seems to say that B has *more* 'b' (*more* here being in comparison to the 'a' of B, since 100>50 or 100">"100; and maybe also in comparison to 'b' of A, since 100">"100 or 100>50). From the vagueness and inaccuracy of these details alone, we can see that this symbolic formula contains uncertainties and confusions.

We may now, for purposes of clarification, compare Ostrovsky's symbolic formula for the Mishnaic illustrations to our standard form for them. For a start, we have to reason that since item A has less of a certain thing (R) in case 'a' and more of it in case 'b', then the same may (by generalization) be said for all items, including B; whence:

> For all items like A, case 'b' (P) implies more of something (R) than case 'a' (Q).
> So, for item B, if case 'a' (Q) involves R enough for a certain result (S),
> it follows that case 'b' (Q) involves R enough for the same result (S).

Comparing this standard form (positive subjectal, from minor to major) to Ostrovsky's symbolic formula, we see clearly that two terms are missing in it, viz. the middle (R) and subsidiary (S) terms. In his verbal description of the argument, he does have the middle term as "fine," but he does not give this term a symbol in his abstract formula – meaning that he does not realize its central role in the thought process. Furthermore, he lacks the idea of *a threshold value* of R, i.e. that *enough of* R gives access to a certain result (S) and without that amount of R no access is possible. Also, the result (S) is not mentioned in his symbolic representation – i.e. it is not stated that the resulting "fine" is the same in the minor premise and conclusion. On the contrary, the conclusion seems to be that the "fine" is bound to be greater in the conclusion than it was in the premise. This suggests that Ostrovsky, like R. Tarfon, conceived the a fortiori argument here as a crescendo. However, he offers no additional premise justifying such 'proportionality'. Rather, he seems to have mentally conflated the middle and subsidiary terms, and reasoned directly from the given that 'b' is greater than 'a' (as regards item A) to the conclusion 'b' from the given 'a' (as regards item B).

We can thus surmise that he conceives the argument as essentially analogical. His formula contains four terms (A, B, 'a' and 'b'), as appropriate for mere analogy (if for A, b>a, then similarly for B, b>a). These four terms are not those of a fortiori argument (only two of them, viz. 'a' and 'b', are used in the a fortiori proper – the other two, viz. A and B, being external conditions). He shows no awareness that a generalization (from item A to all items, including B) precedes the a fortiori argument. His first premise seems to play the role of major premise, even though in fact it is only a given instance from which the effective major premise is obtained by generalization. The second premise (B + a), which serves as minor premise, seems to say that B has or involves *more* 'a' – whereas it should say that B has or involves *a certain amount* of the middle term in case 'a'; and the conclusion (B + b) seems to say that B has or involves more 'b' – whereas it should say that B has or involves a certain amount of the middle term in case 'b', which is at least equal to (rather than necessarily greater than, as he seems to imply) the amount in case 'a'.

Moreover, his symbolic formula is structured in such a way as to give the wrong impression that A and B are subjects, while 'a' and 'b' are predicates (or possibly, A and B are antecedents and 'a' and 'b' are consequents). If this were so, i.e. if B was the subject of the minor premise and conclusion (rather than 'a' and 'b', respectively), then the argument would be positive predicatal (rather than subjectal) – but if this were true, then it could not validly go from minor to major (since positive predicatal a fortiori argument is only valid from major to minor). Clearly, the

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[11] Notice that this reading is essentially implicational, with antecedents 'engendering' consequents. We could also propose a copulative reading, by saying: 'if the fine for tooth and & foot damage is lesser on public grounds and greater on private grounds; etc." The difference lies in where we place the recurring word 'fine'. However, I would say the implicational version is more accurate, because the terms 'greater' and 'lesser' are relative to each other and to the term 'fine'.

argument must be structured as we have above proposed in our standard form rendering, with A and B as external conditions, and 'a' and 'b' as subjects. And for this, two additional terms must be introduced (viz. R for the middle term and S for the subsidiary). So, it cannot be said that Ostrovsky understood a fortiori argument.

Clearly, his symbolic formula is not even adequate as representation of the 'if–then–' givens, let alone as an explanation as to why the conclusion follows from the premises. The peculiar form of a fortiori argument is nowhere made explicit. There is no proposition in the formula that clarifies why what it tells us about item A is at all relevant to item B, and may be used as a source of inference; i.e. the major premise is missing. Moreover, if the inference is considered as deductive rather than merely inductive (as indeed seems to be the case in the examples given from the Mishna), there ought to be a credible validation procedure – which there is not. Ostrovsky's attempt at formalization is thus only superficially descriptive, lacking depth and justification.

Furthermore, although (as already mentioned) Ostrovsky rightly presents the difference between R. Tarfon and the Sages as one between rational argument and majority ruling, he does not explain how a convention (majority ruling) can conceivably overrule an apparent law of logic (rational argument). Moreover, his presentation does not bring out the significant difference between R. Tarfon's two arguments. Both his arguments yield a proportional conclusion, but whereas his first conclusion can be interdicted by the Sages by means of a rival purely a fortiori argument, his second argument yields the same conclusion whether it is read as a crescendo or purely a fortiori and therefore cannot be so easily interdicted. Because the greater cunning of R. Tarfon's second argument compared to his first is not brought out, the important difference between the Sages' two *dayo* ("it is sufficient") objections cannot be made manifest.

Consequently, Ostrovsky (judging from Wiseman's presentation) has to resort to non-formal explanations of the controversy. The Sages' *dayo* principle is then made reasonable as a reflection of the uncertainties that may arise in judgment. As Wiseman puts it: "Inasmuch as a judge may not have adequate evidence to determine the truth of the counter claims, he… operates more by a general rule of likelihood than a theoretical construct;" meaning that "the same level of fine as that given (the *dayo*) is the norm, barring any other testimony that can be trusted to alter that judgement. It is practical (legal) reasoning rather than a formal, theoretical logic"[12].

Note again that (as far as can be seen from Wiseman's account) Ostrovsky's seems to view a fortiori argument as exclusively a crescendo, since he does not anticipate purely a fortiori argument. Note also that, though he presumably intended his analysis as traditional in orientation, he did not address the more tortuous interpretation of the Mishna proposed by the corresponding Gemara (Baba Qama 25a-b). With regard to formalities, too, we should note the absence of distinction between implicational and copulative a fortiori argument, between subjectal and predicatal moods, and between positive and negative moods. Ostrovsky's presentation is (judging from what Wiseman reports) solely positive subjectal.

In conclusion, Ostrovsky did not formally solve the problem of a fortiori argument.

4. Pierre André Lalande

An outstanding definition of a fortiori argument that I have come across is that proposed in 1926 by Pierre André Lalande[13], perhaps with other authors, in his *Vocabulaire technique et critique de la philosophie*[14], viz.:

> "Inference from one quantity to another quantity of similar nature, larger or smaller, and such that the first cannot be reached or passed without the second being [reached or passed] also."[15]

This definition is very accurate, even if not perfect. The starting clause, "inference from one quantity to another quantity of similar nature, larger or smaller," refers to then major premise, which relates two terms – a "larger" (the major, P) and a "smaller" (the minor, Q) – which have a "similar nature," i.e. a comparable property in common (the middle term, R). The next clause, "such that the first cannot be reached or passed without the second being [reached or passed] also," refers to the minor premise and conclusion, and clearly explains that the inference depends on a *threshold* that once "reached or passed" in the one case (the larger or the smaller quantity) is necessarily reached or passed in the other case.

Note well the ontical reference to quantities (rather than epistemic reference to convictions), and the clear reference to the passing of a threshold as the explanation and justification of the inference. The realization that the passing of the threshold by one quantity necessitates that by the other is truly exceptional in its clarity of vision. Few commentators have come close to this degree of understanding of the argument; Lalande must have really thought about this a lot. Nevertheless, his definition is not perfect, for many reasons.

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[12]   Wiseman, p. 74.
[13]   France, 1867–1963.
[14]   Paris: Alcan, 1926. Paris: PUF, 1972.
[15]   My translation. I already called attention to this definition in my *Judaic Logic*.

Most important, Lalande's definition does not clearly specify that there is an implied threshold that is to be reached or passed, which is a certain value of the common property (which we have called the middle term, R) of the major and minor terms (P, Q) – which is specified in the major premise as their "similar nature." The way he has formulated it, the middle term is left tacit in the minor premise and conclusion, giving the impression that it is the quantities of the major and minor terms themselves that are reached or passed, rather than a certain quantity of their common factor. Thus, he has the notion of "sufficiency" in his definition (implied by the words "cannot be reached or passed without"), but he lacks a precise notion regarding sufficient *of what*. He has, effectively (in the subjectal mood): "if Q is enough, then P is enough" (or vice versa), instead of the more accurate: "if Q is R enough, then P is R enough" (or vice versa).

Furthermore, notably absent in Lalande's formulation is mention of the subsidiary term (S). He effectively says: "if Q is enough, then P is enough" (or vice versa), but he does not say *for what*. There is no predication. This may be due to an attempt to cover in the same statement both subjectal and predicatal argument; but he does not make that clear. It appears that his definition intends both directions of reasoning, since he apparently conceives the argument as proceeding from the larger to the smaller quantity (from major to minor) or vice versa (from minor to major), and at the same time only mentions a positive minor premise and positive conclusion (reaching or passing). If so, his definition is deficient in not mentioning negative subjectal and negative predicatal arguments (i.e. arguments involving *not* reaching or passing).

In truth, if what he had in mind in his above definition was positive subjectal and positive predicatal arguments, he would have seen fit to mention and emphasize the structural difference between them, and not left such an important matter tacit. More likely, what he had in mind was only positive and negative *subjectal* argument, and he did not realize that only the movement from minor to major is positive while that from major to minor is negative. It is also apparent from his definition that he only had in mind copulative forms of a fortiori argument, and did not become aware of the more complex implicational forms. Moreover, since he does not mention, let alone discuss, the subsidiary term, he could not have reflected on the difference between pure a fortiori argument and a crescendo argument (i.e. proportional a fortiori argument).

Note that he also describes[16] a fortiori argument as "an enthymeme that assumes a premise like the following: 'Who can do the more can do the less'." He is here referring to the Latin legal rule: "*Non debet, cui plus licet, quod minus est non licere,*" which is better translated as: "one who is [logically] permitted to do the greater, is all the more [logically] permitted to do the lesser." This is, of course, a more specific definition of a fortiori argument than the one discussed earlier, since it is limited to legal or eventually ethical contexts. Here, we might also (by contraposition, or more precisely by *reductio ad absurdum*) argue that "one who is *not* permitted to do the lesser, is all the more *not* permitted do the greater". Note that these formulae, the positive and negative, are predicatal in form; and that they go, respectively, from major to minor and from minor to major.

Clearly, Lalande should have realized that there are four (or even eight) moods of a fortiori argument, and not just two as his main definition seems to imply. Nevertheless, his definition is way ahead of those of many other commentators, and highly to be praised.

5. David Daube

Wiseman occasionally refers David Daube, author of the celebrated paper "Rabbinic Methods of Interpretation and Hellenistic Rhetoric" (1947), to buttress his own positions, although he does not agree with him entirely[17]. A Wikipedia post describes Daube[18] very enthusiastically, as "the twentieth century's preeminent scholar of ancient law," who "combined a familiarity with many legal systems, particularly Roman law and biblical law, with an expertise in Greek, Roman, Jewish, and Christian literature."

According to Wiseman, Daube "suggests that much (if not all) of Hillel's seven interpretative rules might have arisen from Hellenistic rhetoric, not just logic, as well as from its Roman incorporation in jurisprudence." He considers Daube as having "overrate[d] Greek and Roman rhetorical influences on Jewish rules and on the QC particularly," reminding that a fortiori argument (which he abbreviates as QC) appears in the Jewish Bible long before; however, he conceives it as possible that the orderly arrangement of the hermeneutic rules in lists may have been influenced by Hellenistic models. I agree with Wiseman, here.

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[16]   P. 32.
[17]   Wiseman, in *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions*, pp. 165-7. Daube's paper in *Collected Works of David Daube* (Cambridge: Cambridge UP, 1947), pp. 333-355. See also my brief comment on this paper in the chapter on Saul Lieberman (15.2).
[18]   Germany, 1909 – USA, 1999.

Regarding *qal vachomer*, Wiseman informs us that Daube stated that at least two clearly proportional cases display "the methodological elaboration of law and theology by means of the norm *a minori ad maius*." The two cases referred to are Matthew 12:10 and Romans 5:8[19] Daube, reports Wiseman, considered these two examples to be "legitimate, in form similar to Roman, juridical reasoning." Indeed, Daube considered that, whereas the Jewish Bible uses of *qal vachomer* are "popular," the New Testament (NT) cases – i.e. the two just cited, presumably – are "technical," meaning "academic, 'Halakhic' applications of Hillel's first rule of exegesis."

Wiseman tells us all that in order to pit the authority of Daube against that of Maccoby, since he concludes: "by Daube's reckoning, since these QC's express proper usage, Maccoby's critical view of the NT writers' misunderstanding of the QC is largely empty or rebutted." As for the case given by Paul that Maccoby dismisses as sheer "rhetoric," Daube calls it "[n]o less significant."

When I read this information about Daube I was, to put it mildly, nonplussed. Having earlier in the present study closely examined the a fortiori arguments in the NT, including the argument by Jesus in Matthew 12:10, and the four arguments by Paul in Romans 5:10, 17 and 11:15, 24 that Maccoby mentions, I can say with certainty that to claim these NT arguments as technically faultless, and indeed more skillful than the samples found in the Jewish Bible, is exaggeration if not deliberate disinformation. As for Romans 5:8, I do not see any a fortiori intent in it at all (but perhaps 5:10 is meant).

Note Wiseman's characterization of these cases as "clearly proportional;" after which he expresses disappointment that Daube "does not address the *dayo* limit directly." It is indeed surprising that Daube does not mention the distinction between proportional and non-proportional a fortiori argument, because it is very present in various ways in Mishna and Talmud, even though apparently unknown in ancient non-Jewish sources. If Daube was as thoroughly acquainted with both literary cultures as he is reputed to have been, he would surely have noticed that.

As regards the issue of proportionality, I agree with Wiseman that the a fortiori argument in Matthew 12:10 is intended as proportional. The one in Romans 5:10 (if that is the argument referred to as 5:8 by Wiseman) is perhaps intended as proportional, but being formally rather mixed up this cannot be said with certainty (the underlying a fortiori argument may be non-proportional). In any event – so what? How would adducing these NT arguments disprove Maccoby's claim that a fortiori argument cannot be proportional? Even if a thousand examples of proportional such argument were brought to bear it would prove nothing, since Maccoby is not contending that people do not so argue in practice but that logically people should not so argue.

It is therefore clear that Wiseman is attempting here to lean on Daube's prestige; this is argument by authority. Daube may have been an expert in many things, but he obviously did not understand logic, or at least a fortiori logic. It looks to me as if Daube was here intent on devaluating Judaism, and uplifting Christianity, for whatever motive. His attempt to thoroughly subsume Judaic hermeneutics under Hellenistic rhetoric is likewise suspect. He seems not to have been an impartial scholar, but someone with a partisan agenda[20]. Thus, Wiseman can hardly rely on him to defeat Maccoby's thesis. The latter admittedly had his own prejudices, but invoking Daube's views is not the way to neutralize him.

Additionally, Wiseman's attempt to oppose Maccoby by appealing to the concept of rhetoric defended by Daube is confused. 'Rhetoric', properly understood, is a wide concept that includes all human discourse, whether logical (formally validated) or illogical (whether non-sequitur or antinomic), aimed at convincing others of something for whatever purpose. In a narrower sense, when opposed to logic, 'rhetoric' refers to illogical discourse (i.e. sophistry). Some people cannot tell the difference. Some people, like Daube when he (according to Wiseman) "calls *all* the hermeneutical rules of the Rabbis rhetorical," seem to try to blur the difference. Maccoby, for his part, takes the hard line that non-proportional a fortiori argument (which he and Wiseman both equate with the *dayo* principle) is logical, whereas proportional argument is rhetorical in the sense of illogical.

Wiseman attempts to discredit Maccoby's stance by means of Daube's fuzzy concept. He reproaches Maccoby for not admitting that proportional a fortiori argument has at least rhetorical value. Here, he relies on the vague generic connotation, which is partly positive. However, Maccoby insistence that proportional a fortiori argument is illogical does not deny it to be rhetoric in the negative sense, but only in the positive sense. So Maccoby does not deny it to be rhetoric in the all-inclusive sense, and Wiseman is here indulging in fallacious reasoning. His additional reproach that Maccoby by dismissing proportional a fortiori argument offhand is himself engaging in rhetoric in the negative sense (i.e. illogic) is fair enough; but it does not cancel out Wiseman's own resort here to illicit rhetoric.

Anyhow, rhetoric is not the issue; the issue is logic. The questions to ask are obvious and essential. Can non-proportional and proportional a fortiori arguments respectively be formally validated? Or under what conditions can

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[19]   Matthew 12:10 – "And a man was there with a withered hand. And they asked him, "Is it lawful to heal on the Sabbath?"—so that they might accuse him." Romans 5:8 – "but God shows his love for us in that while we were still sinners, [Jesus] died for us." (English Standard Version, 2001.)

[20]   Daube was apparently of Jewish descent, but it is not stated whether he was in fact Jewish.

they be validated? And if so, exactly how? As Wiseman rightly points out, Maccoby does not formally validate his claims. As for Wiseman, he cannot formally validate his counter-claims because, as we have seen, his definitions of these arguments are too vague. For this reason, he is forced to resort to authorities and fuzzy concepts.

6. Meir Zvi Bergman

I often referred to R. Meir Zvi Bergman's 1985 work *Gateway to the Talmud*[21] when I wrote my *Judaic Logic*, having found him both scrupulously traditional and relatively clear in his expositions of the rabbinic hermeneutic principles. I did not at the time know of the work of Mielziner or other authors, so it would be fair to say that without Bergman I would not have progressed as far as I did at that time. In *Judaic Logic*, although I react somewhat critically to quite a few rabbinic doctrines relayed and defended by Bergman, my focus is not principally on critique. In the present brief exposé, I will examine his approach more critically, especially with regard to a fortiori argument.

Bergman devotes a few pages (pp. 121-129) to *kal vachomer*, the first of the thirteen "rules of Biblical exegesis." He defines it as follows: "In this interpretation, inference of a stringency is drawn from a lenient case (*kal*) to a strict one (*chomer*), and inference of a leniency is drawn from a strict case to a lenient one." This is of course the traditional rabbinic formula for such argument, which does not in fact cover all sorts of a fortiori arguments although it covers the main ones of interest to rabbis. Bergman has no theory of a fortiori argument to speak of other than that statement. By way of validation all he does is say: "the logic is compelling;" and he effectively repeats the same definition with the words "all the more… should" and "certainly… should" thrown in for emphasis. This is, needless to say, not formal proof, but mere declaration of intimate conviction.

Bergman then says: "Ten examples of this rule are explicitly mentioned in the Torah," and proceeds to list the ten examples cited by R. Ishmael in the Midrash *Bereshit Rabbah* (without mentioning the two more cited separately in that volume, namely Gen. 4:24 and 17:20-21[22]). Note that he says "in the Torah," whereas he should have said "in the Tanakh" (i.e. the whole Jewish Bible), since only four of the ten examples he lists are from the Torah proper (i.e. the Pentateuch). His mention of only ten examples suggests that he is unaware that there are many more instances scattered throughout Jewish Scripture. To be precise, according to my most recent listing there are 46 instances. In any event, listing examples is not a viable substitute for formal treatment. The examples may strike us as reasonable, but we still need formalization and validation to be sure.

Also, the mere fact that an argument is used in the Torah is not proof of its validity. Its validity must still be established by formal logic. It can be said, within the framework of Jewish faith, that the Torah is the authoritative source of information from which our premises are to be formulated; but we cannot refer to the Torah to evaluate a process of reasoning per se. The Torah may well provide *the content* of Jewish discourse; but *the form* of valid discourse is something universal and independent of any religious faith. The process of moving from premises to conclusion has nothing to do with religious faith, but depends on the science of logic which is based on the five laws of thought – the three laws of identity, non-contradiction, exclusion of the middle, and the principles of induction and deduction. The Torah could not credibly deny these fundamental laws[23].

Bergman does ask the question: what determines whether a case is stringent or lenient. But his answer that it may be either "logic" or "the laws of the Torah that pertain to the case" is much too vague. He gives an example for each of these possibilities, but these examples do not clarify his meaning. In truth, the distinction between the major and minor items (terms or theses) depends in each particular argument on a tacit or explicit major premise, which specifies which item is which *in relation to a specific middle term*. This major premise might be known through rational insight and/or empirical observation, or it might be given in the Torah and/or be decided by the rabbis – but it must in any case be acknowledged as underlying the argumentative process. Bergman, not having to begin with analyzed the components of a fortiori argument, makes no mention of this major premise. All that his said initial formula includes are the minor premise and conclusion, and those without explicit reference to a middle term.

Bergman of course details the traditional doctrine of "refuting a kal vachomer" (*pircha*), but here again he is handicapped by his lack of formal tools, so he does not distinguish between challenging the content of an argument and challenging its form[24]. He distinguishes, as traditionally done, between "challenging the origin" and "challenging

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[21] New York: Mesorah, 1985.

[22] No doubt he skips these two cases so as to avoid the embarrassing question as to why they were not included in the list of ten. Louis Jacobs has suggested that this discrepancy is evidence that the Midrash has been augmented over time (see 16.4).

[23] And it nowhere explicitly denies them, although it does at times seem to de facto contravene one or the other of them, as when there is an apparent contradiction between two passages of the Torah, or between the Torah and certain empirical findings by the scientific method. Even the rabbis understand that inconsistencies are a serious problem, and they always try to explain them away.

[24] Bergman at one point says that a "refutation based on logic… cannot rebut a *kal vachomer*" (p. 124-5). But he is in fact discussing the *kol ze assim* argument proposed by a Tosafist, which is not really a formal issue but still a material one. I have dealt with that in chapter 9.7 and

the conclusion;" but he does not realize that these "refutations" are concerned with the content of a given argument rather than with its form. The *kal vachomer* argument as such is never "undermined" – all that is put in doubt is the information it contains. This may occur directly, by pointing out that the proposed major premise ("the origin") is overly general, since there are cases where it evidently does not hold; or it may occur indirectly, by pointing out that the conclusion drawn is not in accord with certain known cases (which is a sort of *reductio ad impossibile* which calls for revision of a premise, usually the major though in some cases the minor); but in any event, this concerns the matter at hand not the logical process used.

As regards the *dayo* principle, Bergman says that "the full force of *kal vachomer* argument is qualified by" it. This is an inaccurate view, again due to absence of preliminary formal analysis. A fortiori argument, as a deductive process, is in fact unaffected by the rabbinical *dayo* principle; all the latter does is prevent the material formation of its major premise or of its additional premise about proportionality. Bergman also errs in following the Gemara's claim, developed in Baba Qama 25a, that the *dayo* principle is "derived from Scripture (*Num*. 12:14)." According to this view, a fortiori argument is necessarily 'proportional'; yet many arguments in the Tanakh, the Mishna and indeed in other parts of the Gemara and in later rabbinic literature are not 'proportional', so this is an absurd notion.

Bergman does acknowledge that the *dayo* principle may be "applied" in two ways – either, as he puts it, "on the origin" or "on the conclusion" of the *kal vachomer*. However, his explanations of these applications are not very clearly worded. He says that the first application refers to "the 'lenient' case," whereas the second refers to "the 'stricter' case." He does correctly say, regarding the Mishna Baba Qama 2:5, that the *dayo* objection by the Sages against R. Tarfon's first argument relates to "the conclusion," whereas the *dayo* objection by the Sages against R. Tarfon's second argument relates to "the origin" – but his explanations are not technically correct. He does not show awareness of the 'proportionality' involved in the first argument and the generalization involved in the second, which truly explain the difference between the two ways of *dayo* objection. Bergman does not mention that this distinction is not found in the Gemara, but apparently emerged much later, probably in the medieval period thanks to some Tosafist.

Note lastly that Bergman uncritically accepts the Gemara's invented debate between the Sages and R. Tarfon regarding the conditions of application of the *dayo* principle. This is of course to be expected from such a thoroughly traditional commentator. But in fact this Gemara can be subjected to scathing logical criticism[25].

Bergman's approach throughout his study is decidedly uncritical. He recites traditional positions as if they are indubitable. This means that whatever he says must be received with extreme caution. If a commentator would not, even if he found some serious difficulty in the texts he analyzes, ever dare to question the correctness of a rabbinical pronouncement or an implied idea of theirs, his overall reliability is surely in doubt. His attitude may prove his great piety and love of the Sages, but it is not reassuring as regards objectivity and scientific accuracy. Whatever the difficulty, he will consciously or unconsciously do his best to ignore it or gloss over it, without respect for facts. There is a big difference between apologetics and critical study.

7. Strack and Stemberger

This section concerns the book *Introduction to the Talmud and Midrash* by H. L. Strack and G. Stemberger. Hermann L. Strack (Germany, 1848-1922) was a Protestant theologian and an authority on Talmudic and rabbinic literature. He authored, among many other works, a book called *Einleitung in den Talmud & Midrasch*, which was (it seems) first published in Leipzig in 1887, and then went through many editions. The fifth edition was translated into English and published by the Jewish Publication Society in Philadelphia in 1931. I gather that the nominal 'co-author', Guenter Stemberger (Austrian, still living), arrived on the scene long after and considerably 'updated' Strack's work in successive editions. The first edition with his signature (in German) seems to have been in 1982. The first English translation of that seems to have been in 1991. According to Google Books, Gunter Stemberger later, together with Markus Bockmuehl, produced a revised edition of the latter in 1996. The edition I refer to here is a French translation and adaptation dated 1986[26].

Chapter 3, on rabbinic hermeneutics, presents the seven rules of Hillel, the thirteen of R. Ishmael, and the thirty-two of R. Eliezer ben Yose haGelili, in a standard manner, with some history and some examples. *Qal vachomer* is briefly described as "a reasoning that goes from the simple to the complex and vice versa" (my translation). There is no attempt at a more precise or formal definition, or to describe its varieties or workings, let alone at validation. It is

need not repeat my analysis here. Note that Bergman does not mention that this argument is from the Tosafot – because, I suspect, to him all rabbinic commentaries, from the Mishna to present day discourses, are timelessly Torah-given.

[25]   See chapter 7.5 and compare it to Bergman's presentation of this issue on pp. 128-9. I stress that there is no evidence of or call for any such debate in the Mishna; it is a gratuitous projection of the Gemara.

[26]   Tr. and adapt. by Maurice-Ruben Hayoun (Paris: Cerf, 1986.)

not clear whether by "simple" and "complex" the author refers to the major and minor terms, or to two values of the subsidiary term. In other words, the issue of 'proportionality', i.e. the distinction between purely a fortiori argument and a crescendo argument, is not treated here. However, since he adds "and vice versa," it is implied that he has at least vaguely become aware of the distinction between positive and negative subjectal argument, and/or between subjectal and predicatal argument.

With regard to history, it is suggested that Hillel, R. Ishmael and R. Eliezer probably did not author or compile their respective lists, but these were later attributed to them. Stemberger (obviously him, since this is very recent stuff) refers uncritically to the theories of Daube, Lieberman and Porton that we have examined (above and below) and found somewhat doubtful. One interesting (for me) item of information gleaned here is that the thirteen *midot* of R. Ishmael were believed by many doctors of Jewish law to be of Sinaitic origin as of the Middle Ages. They mention the *Midrash haGadol*, an anonymous work thought to date from the 14th century, as introducing the subject (in a comment on Exodus 21:1) as follows: "Rabbi Ishmael said: Here are the thirteen midot used for exegesis of the Torah, *which were transmitted to Moses on Mount Sinai*" (my translation and italics). However, there is a doubt as to whether this remark is as old as the work, because it was not included in fragments found in the Cairo 'geniza' (for this reason it has been excluded from some more recent editions).

8. Meir Brachfeld

Meir Brachfeld is the author of a paper entitled "A Formal Analysis of Kal VaChomer and Tsad Hashaveh," published in 1992 in the *Higayon* journal (vol. 2, pp. 47-55)[27]. I have a copy of this article, but it is in Hebrew, and my Hebrew is not fluent enough, so I shall here again refer to Wiseman, who offers in *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions* (Appendix E.3, pp. 242-5) a pretty thorough English translation of it, as well as his own discussion of it (on pp. 87-9). The article's English abstract in *Higayon* is as follows:

> "A section of 'Halichot Olam' [a rabbinical commentary] dealing with the mechanics of the principles of 'Kal VaChomer' and 'Tsad Hashaveh' is translated into the language of set theory. In this language, the principles can be concisely and precisely defined. Most interestingly, it emerges that a central concept underlying these principles is that of 'irrelevance' and that this concept is naturally defined using recursion."

What is immediately evident, looking at the article, is that it takes a fortiori argument for granted without analyzing its form or validating it. Thus, *there is no specific theory of a fortiori argument as such in it*.

Brachfeld's opening formula for the argument merely tells us that, under appropriate conditions, information (a *din*, a law or legal decision) is inferred from one context (the *melamed*, that which teaches) to another, similar context (the *lamad*, that which is taught), without specifying exactly how the process occurs[28]. His formula is vague enough to be applicable to any sort of argument. He does not try and find the precise mechanics of a fortiori inference. The purpose of his article is not to analyze the argument per se, but rather to describe the ways it is used and challenged in halakhic (Jewish law) discourse. That is, he only proposes a sort of flow-chart, in the language of set theory, of typical rabbinic arguments and counterarguments in relation to it. This is of course interesting stuff[29], but not our main topic of interest in the present work.

I have, anyway, already analyzed this topic in some detail in the chapter devoted to Mielziner (13.4). Briefly put, the rabbis conceive of two ways that a *qal vachomer* may be "refuted." It may be attacked by way of a premise or by way of the conclusion. However, their understanding of what is technically going on in such refutation is not entirely accurate. Both attacks in fact challenge a premise – the former rather directly, the latter rather indirectly. This means that the premise under attack is shown to be deficient in some way; usually, it is found not as general in scope as it was initially made out to be. When the conclusion is under attack, it is not *the process* of a fortiori inference that is being challenged, but a premise. Very rarely is the process of a fortiori inference itself challenged; and when it is, this simply shows that a *particular* argument was improperly formulated in some way, not that a fortiori argument in general is open to doubt[30].

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[27]   Ed. Moshe Koppel and Ely Merzbach. Jerusalem: Aluma, 1992.

[28]   We can put the process in standard form for him, of course. In positive subjectal (or negative predicatal) a fortiori arguments, the *melamed* and the *lamad* are respectively the minor and major terms (Q and P), the similarity between them is the middle term (R), and the *din* is the subsidiary term (S). In negative subjectal (or positive predicatal) arguments, the *melamed* and the *lamad* are respectively the major and minor terms (P and Q). But Brachfeld himself shows no awareness of the actual form(s) of a fortiori argument.

[29]   Nevertheless, we can question Brachfeld's implicit assumption that he can fully sort out the technical possibilities of counterargument without first understanding the nature of the argument itself. See the next paragraph.

[30]   Obviously, if a speaker uses an invalid mood, it does not follow that all moods of a fortiori argument are invalid. This is true of any form of argument, not just a fortiori.

The significance of these findings is that the belief by some rabbinical commentators that *qal vachomer* is not a deductive, always reliable, argument is based on incorrect analysis of the data they base their judgment on. Unfortunately, Brachfeld seems to take such erroneous assumptions for granted to a large extent. He is not, however, totally uncritical, since he points out that the traditional view creates certain logical problems. As Wiseman puts it, the author of 'Halichot Olam', is "also aware" of these problems, but "offers a mere excuse that just seems to brush aside the real issue, by delivering the comment of Rashi… that the rules [of interpretation, including *qal vachomer*] are handed down to us from Sinai."

9. Gary G. Porton

According to rabbinic tradition[31], Hillel I the Elder (Babylon, 110 BCE – Israel, 10 CE) drew up a list of seven *midot* (techniques of Biblical interpretation); later, R. Ishmael ben Elisha (Israel, ca. 50-135 CE) expanded this list to thirteen. It is generally considered that Hillel had learned these exegetic rules or principles, or at least the practices they represent, from his teachers Shemaya and Abtalion. Further developments in this field occurred through Nahum of Gimzo and R. Akiva, on the one hand, and by R. Ishmael, on the other hand. The latter's list was identical to Hillel's in some respects (namely, *qal vachomer* and *gezerah shavah*), but differed by merging certain rules, splitting others up, modifying still others, and adding at least one (namely the last).[32]

In an essay called "Rabbinic Midrash" (1995)[33], Gary Porton urges us not to (like Daube and Lieberman did) "take the existence, use, and attribution of the exegetical rules to specific sages as fact," and not to "accept the rabbinic texts' data as historically accurate and valid." He informs us that in an earlier paper of his, "Rabbi Ishmael and His Thirteen *Middot*"[34], he raised "serious questions about Ishmael's relationship to the thirteen exegetical rules attributed to him at the opening of Sifra and to the seven principles assigned to Hillel in several rabbinic documents." More specifically, he informs us that:

- There are three divergent versions of Hillel's so-called 'list of 7 rules': two having 8 rules and one having only 6 rules; and only 5 rules are found in common to them. (Porton does not here specify the core rules and the additional rules.)
- R. Ishmael's so-called 'list of 13 rules' contains 16 rules, written "in four distinct literary styles." (Porton seems to suggest here that the list had four different authors or compilers.)
- "We have no record of Hillel's ever using any of the exegetical techniques attributed to him." (Porton does not here say whether Hillel used other techniques or never publicly engaged in exegesis.)
- R. Ishmael uses only 8 of the 16 principles in the 'list of 13' attributed to him, and additionally 17 techniques "which do not appear in the list." (Porton does not here specify which 8 principles were used and what the 17 unlisted techniques consisted of.)
- R. "Ishmael's exegetical activity as recorded in the rabbinic corpus" is limited "with two exceptions… [to] exegetical principles which the two lists have in common." (Porton does not here tell us which principles he considers as in common to both lists, and which not.)

I have quoted and paraphrased him at length because I find this historical thesis very interesting; not only because of its novel negative conclusions, but especially because of the methodology it presumably was based them on – namely, systematic empirical research throughout the Talmud.

The conclusions as such are not too worrisome (to my mind, at least): they can probably be explained away. Perhaps Hillel did list seven rules, but their transmission was faulty. Some of R. Ishmael's thirteen rules might well have been lumped together from a longer list. Similarly, the lack of evidence of actual use by Hillel of any of the rules listed by him, or by R. Ishmael of half of the rules listed by him, can be explained by saying that their intent was not to describe their own personal practices, but the practices of their colleagues in general. Moreover, that R. Ishmael may have (with two exceptions) used only rules listed by Hillel is not too problematic, since after all his list is

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[31] The Jewish Encyclopedia gives as references: for Hillel's rules, Tosef., Sanh. vii; the introduction to the Sifra, ed. Weiss, p. 3a, end; Ab. R. N. xxxvii; and for R. Ishmael's rules, the *Baraita* of R. Ishmael (intro. to Sifra). Some say Hillel composed or at least compiled the list; others that he merely expounded it (implying that it existed before him).

[32] See Mielziner, pp. 123-128. Nahum of Gimzo was opposed by R. Nehunia ben Hakana, "who insisted upon retaining only the rules of Hillel." See also Neusner: *Rabbinic Literature: An Essential Guide*, pp. 56-63.

[33] In: *Judaism in Late Antiquity*. Ed. Jacob Neusner. Leiden, Netherlands: Brill, 1995. Pp. 217-238. The relevant part of this essay (p. 226) may be read online at:
books.google.com/books?id=azrc9jU4YaUC&pg=PA270&dq=Saul+Lieberman+%E2%80%9CRabbinic+Interpretation+of+Scripture%E2%80%9D+in+Hellenism+in+Jewish+Palestine&hl=en&ei=5vxgTd6SM9H44Aba7rCkCg&sa=X&oi=book_result&ct=result&resnum=5&ved=0CDsQ6AEwBDgU#v=onepage&q=Ishmael%27s%20and%20Hillel%27s%20Exegetical%20Techniques&f=false.

[34] In: *New Perspectives on Ancient Judaism Volume One: Religion, Literature, and Society in Ancient Israel Formative Christianity and Judaism*. Ed. J. Neusner, P. Borgen, E. S. Frerichs, and R. Horsley. Lanham: University Press of America, 1987. Pp. 3-18.

traditionally presented as a derivative of Hillel's. As for the use of techniques by R. Ishmael that are not included in these two lists – well, that comes as no surprise: we have always known that these lists were incomplete, since for instance R. Akiva often used techniques not listed in them, and it is readily evident when studying the Talmud that reasoning processes are used in it which are not explicitly acknowledged, or which though named are not listed.

It should be added that speculation concerning the composition of the two lists is nothing new[35]. I have also reported and made some suggestions of this sort in my *Judaic Logic*, with reference to R. Ishmael's list. For a start, the various moods of *qal vachomer* could be counted as separate rules (this would make four or eight or more moods, according to how we go about it). *Gezerah shavah* may be based on verbal or material analogy (homonymy or synonymy)[36]. *Hekesh* and *semukhim*[37] are early practices apparently not initially listed but later classified under that heading (i.e. as divisions of rule #2). *Meinyano* (Hillel's rule #5) and *misofo* (new in R. Ishmael's list) are two divisions of rule #12 (which in my opinion ought to be placed with or close to rule #2). *Binyan av* likewise has two varieties, *mi katuv echad* and *mi shnei ketuvim* (which were two rules #3 and #4 in Hillel's list)[38].

Rules #4-9, 11 may all be viewed[39] as developments of Hillel's #5 (which was itself two rules *klal uphrat* and *prat ukhlal* bunched together). To R. Ishmael's rule #6, *klal uphrat ukhlal*, were apparently later added three other variants[40], viz. *prat ukhlal uphrat*, *klal ukhlal uphrat*, *prat uphrat ukhlal*. Rule #7 has two divisions, *miklal hatsarikh liphrat* and *miprat hatsarikh likhlal*. Rule #8, *lelamed hadavar leshar haklal*, is recognized[41] as having two varieties (which I have named): the original *lelamed oto hadavar*, and an apparently later development, *lelamed hefekh hadavar*. Thus, R. Ishmael's rules, though conventionally counted as 13 in number, are well known by tradition to include more than 13 exegetic devices (we could, based on the above enumeration, put the number as, say, 26), and Porton's claim to have newly distinguished just 16 rules in them must be taxed as ingenuous and rather arbitrary!

Moreover, I am curious about the 17 techniques not on his list reportedly used by R. Ishmael. Are these techniques newly identified by Porton, or are they perhaps included in the list of 32 by R. Eliezer ben Yose haGelili or the list of 613 by the Malbim? It should be stressed that it is often very difficult to classify a given argument as falling under this or that heading. We have earlier seen, for example, how the arguments of R. Tarfon in Baba Qama 24b-25a could be construed as plain arguments by analogy rather than as a fortiori arguments. Porton's claim that R. Ishmael's practices can be classified in precisely 25 categories (8 listed by him and 17 not listed by him) sounds suspiciously confident, even if we assume he found just one case for each category.

Nevertheless, Porton's thesis is intriguing, because he seems to be claiming that he has systematically researched the whole Talmud. We would expect him to at least scan through the Mishna, since both Hillel and R. Ishmael are Tannaic; but in truth, he would also have to look through the Gemara, and indeed other literature of the Talmudic period[42], since they might be mentioned there even if not mentioned in the Mishna. He has apparently searched for exegetic activity by Hillel and found none, or at least none included in Hillel's list. And he has apparently examined all of R. Ishmael's exegetic practices, and classified each instance under this or that rule, or under no listed rule, and then drawn the various conclusions mentioned above. If that is indeed the case, i.e. if Porton or anyone actually did

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[35]   I have for instance a text before me, the introduction by Joël Muller (1897) to the *Oeuvres Complètes de R. Saadia ben Iosef al-Fayyoumi* (vol. 9), which recounts various ideas on this topic. For example, one Aharon ben Chayim suggested that the three last rules of R. Ishmael (based on verbal differences) may be later additions, and that the rules numbered 3, 4 and 6 should each be counted as two. Also mentioned are differences in the order of listing of the rules; for instance, Saadia Gaon places rule 9 in 5th place.

[36]   Mielziner moreover distinguishes exegetical, constructional and exorbitant varieties of *gezerah shavah* (pp. 143-150). Had I studied Mielziner's work before writing my *Judaic Logic*, I would surely have integrated his insights and classifications into it.

[37]   Mielziner lists *semukhim* (juxtaposition) as an "additional rule," which "has some similarity to *Heckesh*," and which "was probably introduced by R. Akiba;" he distinguishes two kinds of *semukhim* (pp. 177-179). In my *Judaic Logic*, I advocated lumping together all arguments by analogy under the loose heading of *gezerah shavah*. This was just a convenience measure, as R. Ishmael's list of thirteen *midot* is considered post-facto as symbolically representative of all rabbinic hermeneutics, even though R. Akiva's rules were originally competitive.

[38]   *Mah matsinu* is counted by some commentators as a third variety of *binyan av*. Mielziner considers that incorrect, explaining that this refers to analogy from one case to a *single* similar case, whereas *binyan av* refers to generalization from one or more special provisions to *all* similar cases (p. 159); and thus he seems to think of *mah matsinu* as closer to *gezerah shavah* (p. 142). In any case, *mah matsinu* has to be fitted in somewhere in the list of thirteen midot, if it is to be regarded as exhaustive.

[39]   See Mielziner, p. 127. R. Akiva's alternative set of rules *ribui umiut* might also be listed and counted in this context, even though they were originally a competing viewpoint, if we regard R, Ishmael's list as having become the symbolic representation of all rabbinic hermeneutics.See Mielziner, pp. 182-5.

[40]   Mielziner only mentions only two variants: *prat ukhlal ukhlal* and *klal ukhlal uphrat* (p. 168).

[41]   See Bergman (I do not have his book at hand right now, but I do quote him on this in my *Judaic Logic*, chapter 11). Bergman refers to these two variants as the particular teaching "about itself as well as the general law," or (not about itself but) "only about the general law". Examples are there given. Mielziner makes no mention of this variation.

[42]   Including the Tosefta, Mechilta, Sifra, Sifre, Baraitot, etc. I should add that, in my opinion, it is imperative to always specify the exact documentary source of each instance of hermeneutic practice found; I cannot imagine that the same credence be given to a statement attributed to a Tanna found in the Mishna, and one found in the Gemara (which is comparatively hearsay evidence), or in some other document of the Talmudic period or later. Obviously, there are degrees of credibility in proportion to the distances in time and place.

this detailed research, he is to be highly commended for his empiricism and scientific method! This is just the sort of thorough effort needed.

I wonder, however, if this systematic effort actually did take place. I have not personally read his paper "Rabbi Ishmael and His Thirteen *Middot*" – which is presumably his research report – but only read his summary of findings in his later essay "Rabbinic Midrash." However, I note that the earlier paper is only 15 pages long. That seems a bit slim, unless we assume that he did not publish all his findings (or maybe the data was scarce). For surely, to be scientific in his exposé, he would have to actually list all the arguments in the Talmud; or at least all those by R. Ishmael, since he claims there are none by Hillel, or at least none corresponding to any techniques on his list. It would not suffice for Porton to just give us a list of references[43]; he would have to actually quote the passages referred to, so as to show us that Ishmael is indeed mentioned in them and so as to convince us that each passage was correctly identified as falling under this or that rule or under none of the rules.

Just how many arguments of each type in the Talmudic sources did Porton find and analyze and how many did he attribute to R. Ishmael? E.g. as regards *qal vachomer* specifically, how many occur in the Talmudic sources and how many of them was R. Ishmael the author of? He does not here give us even such basic statistics. Is the exhaustive research data perhaps made available somewhere for peer review? I do not know; I have found nothing posted to this effect on the Internet. Be that as it may, it is interesting to see someone at least thinking in terms of systematic empirical research. This is the sort of research I advocate. Traditional lists are interesting, but not enough.

My doubts as to the actuality or reliability of Porton's alleged empirical research increased exponentially when, by coincidence, I came across a mention by Mielziner of Hillel applying, "among other arguments," the "constructional" variant of the rule of *gezerah shavah* in *Pessachim* 66a. I looked up this Talmudic page, and sure enough Hillel is presented in the Gemara as having applied his second hermeneutic rule (and also his first, *qal vachomer*) to the issue at hand:

> "They were told 'There is a certain man who has come up from Babylonia, Hillel the Babylonian by name, who served the two greatest men of the time [Shemaiah and Abtalyon], and he knows whether the Passover overrides the Sabbath or not. [Thereupon] they summoned him [and] said to him, 'Do you know whether the Passover overrides the Sabbath or not?' … He answered them, 'In its appointed time' is stated in connection with the Passover, and 'In its appointed time' is stated in connection with the *tamid* [sacrifice]; just as 'Its appointed time' which is said in connection with the *tamid* overrides the Sabbath, so 'Its appointed time' which is said in connection with the Passover overrides the Sabbath. Moreover, it follows a minori [ad majus], if the *tamid*, [the omission of] which is not punished by *kareth* [cutting-off], overrides the Sabbath, then the Passover, [neglect of] which is punished by *kareth*, is it not logical that it overrides the Sabbath!"[44]

We can infer from this error by Porton that his statistical claims must be received with the utmost caution. Maybe he looked throughout the Mishna only and did not scan the Gemara (let alone other literature of the period), forgetting that there are many statements in the latter too made in the name of the Tannaim. Many people, it seems, are tempted to make far out declarations so as to be noticed in the academic world; this motive may have played a role here.

10.   **Mordechai Torczyner**

R. Mordechai Torczyner is the creator since 1995 of www.webshas.org, a website "designed as a topical index to the Talmud." This interesting resource includes a webpage devoted to "Talmudic methods of analyzing the Torah's text," including traditional hermeneutics, including kal vachomer argument[45]. What he tries to do here is simply to list references for different aspects of kal vachomer use in Talmudic contexts.

Torczyner's definition of kal vachomer as "learning more obvious lessons from less obvious lessons" expresses his pragmatic orientation. But of course, it offers no answer to the theoretical question as to precisely how "less obvious" lessons can teach us something about "more obvious" ones. It is just a vague statement, without logical explanation.

As regards the *dayo* principle, Torczyner defines it as follows: "A Kal VaChomer can't teach a greater stringency from the lenient side to the stringent side, than the literal rule applicable to the stringent case ('Dayo'): Bava Metzia 41b; Zevachim 43b-44a." Surprisingly, he does not mention the Mishna Bava Kama 2:5 and/or the Gemara Bava Kama 25a-b! Yet these passages are crucial to rabbinical understanding of kal vachomer argument and the *dayo* principle.

Nevertheless, this is an interesting contribution, because of the references it gives. The following are some examples of the way information is presented:

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[43]   Most people would not go to the source text and check firsthand every case mentioned in a list of cases.
[44]   From the Soncino Talmud (with minor adaptations).
[45]   See www.webshas.org/torah/alpeh/midos.htm. While other topics dealt with there deserve further attention, we shall here only consider the treatment of "kal vachomer" (I use his spelling of the term). Torczyner is apparently American.

"Deducing a Kal VaChomer on one's own, without a received tradition: Niddah 19b.

A pasuk trumps a kal vachomer: Zevachim 3b….

The Torah writing out something which could have been learned from a Kal VeChomer: Pesachim 16b; Kiddushin 4a, 4b….

Punishment for a crime may not be deduced based on a Kal vaChomer (Ein Onshin min haDin): Makkot 5b, 14a, 17b; Temurah 9a; Keritot 2b-3a…."

11.   Ron Villanova

Ron Villanova, in his work *Legal Methods: A Guide For Paralegals And Law Students*[46], defines a fortiori argument as follows: "This 'with stronger reason' argument implies a comparison of values;" it is "grounded on the common sense (and logical) convention that within the same category the greater includes the lesser (or, if you will, the stronger includes the weaker)." However, he warns us not to be misled by the word "includes" – by this term he intends "comparison," as between the height of a taller man and that of a shorter one.

This definition is not very adequate, but it is improved somewhat by Villanova's admonitions, such as that "the comparison should be one of factually like things and be factually meaningful," and by the many legal examples he brings to bear. The comparative major premise of a fortiori argument is present in it; and it is understood that one cannot argue in any direction one pleases (e.g. negatively from a lesser predicate to a greater one). Also present are the idea of a middle term in it (implied by the words "within the same category") and the idea that this same term must remain operative throughout the argument (as evident through examples).

However, the idea of a threshold of the middle term, as of which predication occurs, and before which it cannot, seems to be lacking here. This is evident in the author's attempt to express a fortiori argument in the form of "conditional syllogism" (apodosis). He does this through examples, but we can do it for him more generally as follows:

If so and so is true of X, then (*a fortiori*) it is true of Y.
So and so is true of X.
(Therefore): it is true of Y.

Clearly, such presentation of a fortiori argument is simplistic. Just writing "a fortiori" in the consequent does not magically turn this argument into an a fortiori one! It does not tell us what a fortiori reasoning constitutes; it does not tell us just why and under what conditions the consequent follows logically from the antecedent.

No wonder then that Villanova sums up by claiming that "while *a fortiori* is a valid form argument, it is a form that is prone to weakness and, therefore, challenge." He gives examples of possible error with reference to conditional syllogism; but these concern conditional syllogism generally, and are not specific to a fortiori argument. Without first clearly identifying the form of a fortiori argument – indeed its various forms – it is impossible to clearly identify the errors specific to such argument.

12.   Giovanni Sartor

Giovanni Sartor, in *Legal Reasoning: A Cognitive Approach to the Law*[47], draws attention to some interesting forms of a fortiori reasoning. This is a large and interesting-looking treatise, but I will only here briefly look at its theory of a fortiori argument[48]. I have here changed some of the terminology and symbols used by Sartor so as to simplify my analysis of his ideas as much as possible. Sartor considers a fortiori reasoning in general as heuristic and analogical, for reasons that shall become apparent.

"Factor-based" a fortiori reasoning compares the pros and cons for a certain decision. Sartor is thinking in terms of a "decision" because his concern is with legal reasoning, i.e. with judges deciding what judgment to apply in a "new case" given some "legislation, doctrine or precedents." Suppose F1, F2 are reasons in favor of a certain decision D,

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[46]   Coral Springs: Llumina, 1999. Large parts of this volume can be consulted online at Google Books: books.google.ch/books?id=_mhnDRnoaOcC&printsec=frontcover#v=onepage&q&f=false.

[47]   This is volume 5 of *A Treatise of Legal Philosophy and General Jurisprudence*. Chief ed. Enrico Pattaro (Dordrecht, Netherlands: Springer, 2005). Large parts of this volume (but not all of it) can be consulted online at Google Books: books.google.ch/books?id=udmrNHt4wB8C&printsec=frontcover#v=onepage&q&f=false.

[48]   See chapter 8, pp. 221-240. I cannot fully analyze this work, because I do not have the whole text, and because I discovered it rather late, when my book was almost finished.

and G1, G2 are reasons against it; then: *given that* the pro reason F1 outweighs the con reasons G1 and G2 taken together, enough to decide D, it follows that:

a) the pro reasons F1 and F2 taken together would outweigh the con reasons G1 and G2 taken together, enough to decide D (additive a fortiori), since F1 and F2 taken together are "more inclusive" than F1 alone;

b) the pro reason F1 would outweigh the con reason G1 taken alone, enough to decide D (subtractive a fortiori), since G1 is "less inclusive" than G1 and G2 taken together;

c) the pro reasons F1 and F2 would outweigh the con reason G1 taken alone, enough to decide D (bidirectional a fortiori).

We can put these three arguments in standard form for him, as follows (notice that the values of P, Q and S vary from one argument to the next, as appropriate):

a) Since F1 + F2 together (P) are more weighty (R) than F1 alone (Q), it follows that if F1 alone (Q) is weighty (R) enough against G1 + G2 to produce decision D (S), then F1 + F2 together (P) are weighty (R) enough against G1 + G2 to produce decision D (S) (positive subjectal, minor to major).

b) Since G1 + G2 together (P) are more weighty (R) than G1 alone (Q), it follows that if G1 + G2 together (P) are *not* weighty (R) enough against F1 to prevent decision D (S), then G1 alone (Q) is *not* weighty enough (R) against F1 to prevent decision D (S) (negative subjectal, major to minor).

c) Since F1 + F2 together (P) are more weighty (R) than F1 alone (Q), it follows that if F1 alone (Q) is weighty (R) enough against G1 to produce decision D (S) (as implied by conclusion (b)), then F1 + F2 together (P) are weighty (R) enough against G1 to produce decision D (S) (positive subjectal, minor to major).

d) Since G1 + G2 together (P) are more weighty (R) than G1 alone (Q), it follows that if G1 + G2 together (P) are *not* weighty (R) enough against F1 + F2 to prevent decision D (S) (as implied by conclusion (a)), then G1 alone (Q) is *not* weighty enough (R) against F1 + F2 to prevent decision D (S) (negative subjectal, major to minor).

Note that, though Sartor thinks of the "bidirectional" argument as one (c), it in fact has two forms ((c) and (d)), whose conclusions are of course essentially the same, though expressed in different directions (and therefore with opposite polarity); furthermore these two arguments depend on the preceding two ((a) and (b)). In all cases, notice, the operative middle term (R) is the "weightiness" of the factors concerned in producing or preventing a decision D in a given set of circumstances. The latter complex term is of course the effective subsidiary term (S). It is interesting that Sartor explicitly refers to the factors as "sufficient" or not for the decision; so he has grasped that crucial aspect of a fortiori argument.

Thus, Sartor's "factor-based" a fortiori reasoning is valid a fortiori argument (of positive or negative subjectal forms). It is, however, just a special case of a fortiori argument, dealing with compounding or separation of terms ("factors"), assuming of course that such compounding or separation is in fact possible in a given case. By standardizing the reasoning we show its validity and also its ordinariness. Nevertheless, Sartor deserves credit for drawing attention to this particular sort of a fortiori reasoning, which is obviously valuable.

It should be pointed out, however, that Sartor considers the a fortiori conclusion to be "defeasible" because "there may be interference between the factors." This may well happen; but if it were true in a particular case, we could not affirm the major premise in the first place. From this we see that Sartor is not fully conscious of the major premise, and of the fact that *given* that premise and the appropriate minor premise (and thus having the building blocks needed for a fortiori argument) the conclusion is logically inevitable, i.e. not "defeasible." Of course, if we lack the needed premises, we do not have the means for engaging in a fortiori argument; but that would not constitute a weakness in a fortiori argument!

Moreover, Sartor conceives the above arguments as due to the relative inclusiveness or non-inclusiveness of the terms, i.e. (to use his exact words) as due to the fact that a compound is "at least as inclusive" as its components, and a component is "no more inclusive" than a compound with it. This is true specifically in "factor-based" reasoning; but of course it is not true in all a fortiori argument – so issues of inclusion should not be viewed as part of the essence of a fortiori argument.

But further on Sartor does enlarge his theory of a fortiori argument, stepping from "binary factors" to "dimensions (scalable factors)." He explains: "Categorising a situation as exemplifying or not certain binary factors is a superficial way of understanding how the features of that situation favour a certain outcome. In many cases, a binary categorisation results from transforming a deeper dimensional structure into a binary alternative." He adds that dimensions may be "continuous" or "discrete."

He then proposes a "reasoning schema" for "dimensional" a fortiori reasoning similar to that suggested above for (bidirectional) "factor-based" a fortiori reasoning. Briefly stated, he substitutes the notion of "dimensional strength" for what was previously regarded as a difference of "inclusiveness." That is to say, instead of saying that one term is more or less "inclusive" than the other, he says that it is more or less "dimensionally strong." This does the trick, but

I have to remark (without going into the details here) that he unduly complicates matters in trying to convey this change of perspective.

Even so, the net result of his effort is pretty good compared to many other people's attempts. His theory does acknowledge, albeit in a rather scattered way, the main formal components of a fortiori argument. There is no attempt at formal validation, but the argument is made comprehensible (albeit, to repeat, in a needlessly complicated manner). He does not seem to have come across my earlier work on the subject.

13.   And others still

I have no doubt that there are many more studies relating to a fortiori argument besides those dealt with above. I can at this time list three more, two of which I came across surfing the Internet and the third I found mentioned in a publication.

The first is a book in German by **Thomas K. Grabenhorst**, *Das argumentum a fortiori* (1990)[49]. It is described by the publisher as "a pilot study using the practice of decision justification". There is also a synopsis in German, which I translated very roughly using the Google facility. From it I gather that the author is claiming to have found a computer-assisted way to analyze a fortiori arguments used in legal contexts… or something of the sort. I cannot of course evaluate this claim without seeing an English translation of the book; but offhand I am skeptical. Why? Firstly, because I saw no mention in the abstract of preliminary formal work to determine the nature and varieties of a fortiori argument; and without such formal study, how could one program a computer to make correct assessments? Secondly, if even I, a human who has plenty of experience in formalizing material a fortiori arguments, must in each new case encountered make an intellectual effort to find the fitting form, how much more difficult would it be for a machine!

More recently, I came across a brief article by **Tomasz Zurek**, "Modelling of a fortiori reasoning" (2011)[50]. The abstract reads: "The paper presents the model of two variants of a fortiori reasoning applicable in the case of statutory law as well the example of the genuine law case, which has been modeled with use of established methodology. The model of reasoning assumes the existence of 'less-more' relation between the analyzed actions, which has been expressed by means of strict partial order and some additional assumptions. The paper also contains the implementation of the analyzed example." I have not read this paper either, but mention it anyway because it cites the chapter on the formalities of a fortiori logic in my *Judaic Logic*, which is a good sign in its favor!

Thirdly, there is in a recent issue of the journal *B. D. D. – Bekhol Derakhekha Daehu* (No. 24, published in 2011) – an article in Hebrew by **Avraham Lifshitz**, called "Kal VaChomer and its Disprovings" (pp. 73-88). The English abstract is as follows:

"This article seeks to study the nature of the Kal VaChomer by examining those issues for which the G'mara disproves the Kal VaChomer in different ways.

The argument raised in the first part of the article is not only that the Kal Vachomer is not a necessary condition, but that it also not a sufficient condition. This is the conclusion drawn from the distinction between two disprovings: an inner disproving which undermines the relations of the Kal VaChomer, and an external disproving which demonstrates from an external data that the Kal VaChomer does not work. The invalidation of the Kal VaChomer as a sufficient condition stems from the ability to disprove it in an external fashion. Thus we have solved a severe Kushiya [difficulty] of the MaHaRil Diskin about the Tashbitu commandment.

In the second part of the article we raise the new insight that every internal disproving is in fact also an external disproving to the contradictory manner of the Kal Vachomer. By doing so we propose an answer to the Kushiya why the G'mara does not save Kal Vachomer from being disproved by means of its reversal."

I have not read the article, but it seems clear from this description that its focus is specifically on Talmudic argumentation, and from its verbal expression that its approach is very traditional. When the author says that *qal vachomer* is neither a necessary nor a sufficient condition, he presumably means that such proof can be dispensed with and is in any case not decisive; I would be curious to see how he justifies those general claims. His distinction between inner and external "disproving" seems to refer to the commonly known distinction between invalidation of the process of inference and refutation (or at least putting-in-doubt) of a premise or two; I would have to see the article to judge his stated claims in that regard. The essay needs to be translated into English.

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[49]   Frankfurt am Main: Peter Lang, 1990.
[50]   *Proceedings of the 13th International Conference on Artificial Intelligence and Law* (New York: ACM, 2011), pp. 96-100.

32.   A fortiori in various lexicons

In the present chapter, we shall investigate the treatment (or non-treatment) of a fortiori argument in general – and of the more specifically Jewish concepts of *qal vachomer* and *dayo* – in various dictionaries and encyclopedias. This is not intended to be an exhaustive survey, but should give us a good idea of how this topic is regarded and how far it is understood by the academics, writers and editors, who produce the lexicons concerned.

1.   The Jewish Encyclopedia

The *Jewish Encyclopedia* (henceforth, JE) is an important English-language resource for information on Judaism and Jews, which was published in 1901-6 in New York. It can nowadays be consulted online[1]. The article in it which interests us here is called "Talmud Hermeneutics" (in vol. 12, pp. 30-33); it was apparently written by **Jacob Zallel Lauterbach** (1873-1942), with a bibliography by Wilhelm Bacher[2]. This article is wide-ranging, dealing with all sorts of interpretative techniques, as well as the more logical ones enshrined in the seven rules of Hillel and the later thirteen rules of R. Ishmael.[3]

We shall not here, needless to say, review the whole article but only focus on what it says regarding a fortiori argument, i.e. which it refers to as Ḳal wa-ḥomer, the first rule in the lists of Hillel and R. Ishmael. JE tells us that "The completed argument is illustrated in ten examples given in Gen. R. xcii." – however, this statement is not entirely accurate, because this set of ten examples is claimed in Genesis Rabbah 92:7 to be exhaustive; JE should have pointed out that there are a lot more than ten examples of such argument in the Tanakh (at least 46, according to more recent research).

JE identifies Ḳal wa-ḥomer as "the argument 'a minori ad majus' or 'a majori ad minus';" and states that "The full name of this rule should be 'ḳal wa-ḥomer, ḥomer we-ḳal' (simple and complex, complex and simple), since by it deductions are made from the simple to the complex or vice versa, according to the nature of the conclusion required." This is of course an important observation regarding the two directions of a fortiori thought; but JE does not clarify when it first appears in Jewish texts. Note also its translation/interpretation of the terms *ḳal* and *ḥomer* as respectively "simple" and "complex;" this is not the most accurate rendering of their meanings. For a start, we are not told *what* is qualified as "simple" or "complex," let alone what these expressions mean. This is not terminology used by the rabbis.

Note that JE gives no unified definition for a fortiori argument, i.e. it does not attempt to uncover what the two above forms of it have in common. Nor indeed what in fact distinguishes them (namely, that the first is positive whereas the second is negative). Moreover, JE makes no attempt to clarify how and why the a fortiori argument works, i.e. to formally describe and validate it. All it tells us regarding its structure is: "The major premise on which the argument is based is called 'nadon', or, at a later period, 'melammed' (that which teaches); the conclusion resulting from the argument is termed 'ba min hadin', or, later, 'lamed' (that which learns)." It is inaccurate to call the *nadon* the major premise; it is more accurate to refer to it as the minor premise.

Evidently, JE is not aware of the premise that compares the major and minor terms in relation to the middle term, which deserves the name of major premise. As we saw, it does not mention the major and minor terms as such (it mentions *ḳal* & *ḥomer* and simple & complex – but it does not say that these items are terms). Also, it does not even

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[1]   At: www.jewishencyclopedia.com. For more on this encyclopedia, see: en.wikipedia.org/wiki/Jewish_Encyclopedia.

[2]   See   at: www.jewishencyclopedia.com/articles/14215-talmud-hermeneutics.   The   JE   articles   on   Talmud www.jewishencyclopedia.com/articles/10739-methodology and on Bible Exegesis: www.jewishencyclopedia.com/articles/3263-bible-exegesis are also worth reading in this context.

[3]   Two remarks in this article are worth noting in passing. First, that "neither Hillel, Ishmael, nor Eliezer ben Jose ha-Gelili sought to give a complete enumeration of the rules of interpretation current in his day." Second, that "The antiquity of the rules can be determined only by the dates of the authorities who quote them; in general, they can not safely be declared older than the tanna to whom they are first ascribed. It is certain, however, that the seven middot of Hillel and the thirteen of Ishmael are earlier than the time of Hillel himself, who was the first to transmit them. At all events, he did not invent them, but merely collected them as current in his day, though he possibly amplified them. The Talmud itself gives no information concerning the origin of the middot, although the Geonim regarded them as Sinaitic…. This can be correct only if the expression means nothing more than 'very old', as is the case in many Talmudic passages. It is decidedly erroneous, however, to take this expression literally and to consider the middot as traditional from the time of Moses on Sinai."

hint at the middle term, nor realize that it has to be present (setting a threshold) in the minor premise for the conclusion to be possible. The predication in common to the minor premise and conclusion is not at all mentioned either. Furthermore, JE shows no awareness of the differences between subjectal and predicatal forms of a fortiori argument, let alone of the differences between copulative and implicational forms.

All in all, then, this article provides little information on the nature of a fortiori reasoning. It does mention the distinction, found as rules 5 and 6 in the list of Eliezer ben Jose ha-Gelili, "between a course of reasoning carried to its logical conclusion in the Holy Scriptures themselves ('ḳal wa-ḥomer meforash') and one merely suggested there ('ḳal wa-ḥomer satum')." But this is a relatively unimportant insight, as it refers to the degree to which the Biblical text is explicit or implicit rather than to the form of a fortiori argument as such.

As regards the *dayo* principle, JE describes it as follows:

> "The process of deduction in the ḳal wa-ḥomer is limited by the rule that the conclusion may contain nothing more than is found in the premise. This is the so-called 'dayyo' law, which many teachers, however, ignored. It is formulated thus: *dayo lavo min hadin lihiot kanidon* ('The conclusion of an argument is satisfied when it is like the major premise')."

This description, too, is inaccurate, in that it confuses the *dayo* principle with what I have called the principle of deduction, i.e. the logical rule applicable to all deductive argument that "the conclusion may contain nothing more than is found in the premise." As I have shown, the *dayo* principle is something else entirely: it is an ethical limitation on the inference of greater penalties for greater crimes, or relatively stringent laws, from information given in the Torah regarding lesser penalties for lesser crimes, or relatively lenient laws. It constitutes self-restraint on the part of rabbis, so as to avoid the risk of excessive punishment, or severity of duties, through erroneous human interpretation of Divine law.

JE should have realized that the reason why "the 'dayyo' law" could be "ignored" by "many teachers" is precisely that it is not a principle of deduction, but more like one of induction. JE also shows unawareness of the difference between inductive and deductive inference when it informs us that: "The discovery of a fallacy in the process of deduction is called 'teshubah' (objection), or, in the terminology of the Amoraim, 'pirka'. The possibility of such an objection is never wholly excluded, hence the deduction of the ḳal wa-ḥomer has no absolute certainty."

In truth, most such objections in practice refer to the content, rather than to the form, of the a fortiori argument. The deduction involved in the argument, if properly formulated, is quite certain – that is what the term "deduction" (when applicable) means. The questions raised in objections are occasionally whether the argument has indeed been correctly formulated, but more commonly whether the information or perspective it relied on is indeed reliable. Thus, we see, here again, that the author of the JE article was not too clear about many issues concerning general logic, as well as some issues relating to Talmud.

Nevertheless, the article as a whole remains very interesting historically and doctrinally. It certainly, albeit brief, contains a lot of valuable information on the topics it treats.

2. Encyclopaedia Judaica

The *Encyclopaedia Judaica* (henceforth, EJ) is a more recent English-language resource for information on Judaism and Jews, which was published in 1971-2 in Jerusalem and New York; over time, this was supplemented with several yearbooks and decennial volumes; a second edition, including many major revisions and updates, was published in 2006-7[4]. The article we shall take a look at here is called "Hermeneutics," and is found in the first edition (pp. 367-372). It is signed by **Louis Jacobs** (Britain, 1920-2006). This article was retained in the second edition, perhaps with some minor modifications (since it is there cosigned by David Derovan)[5].

Although the EJ article expounds the thirteen rules of R. Ishmael and other interpretative techniques in some detail, we shall only here be concerned with its exposition of *Kal va-ḥomer*, i.e. of the first of the thirteen rules. Since we have already, earlier in the present volume, devoted a whole chapter (16) to the views of Louis Jacobs regarding a fortiori argument, we need not go into great detail in the present context. It will suffice for us to briefly comment on a number of points.

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[4]   More on this encyclopedia at: en.wikipedia.org/wiki/Encyclopedia_Judaica. Note that a CD-rom version of it (presumably of the 1st ed.) exists.

[5]   Online at: www.jewishvirtuallibrary.org/jsource/judaica/ejud_0002_0009_0_08805.html. The only difference I have spotted in the section about *qal vachomer* in the 2nd ed. is the added statement: "Not all of the thirteen principles are based on logic as is the *kal va-ḥomer*. Some are purely literary tools, while the *gezerah shavah* is only valid if received through the transmission of a rabbinic tradition." This statement is interesting in that it explicitly characterizes *qal vachomer* as "based on logic."

To begin with, EJ defines "*Kal va-ḥomer*" as "an argument from the minor premise (*kal*) to the major (*ḥomer*)." This is, of course, wrong – *kal* and *ḥomer* refer not to premises but to terms within the premises and conclusion! It is true that in the *a minori ad majus* form of the argument the *kal* (minor) term is in the minor premise; but the *ḥomer* (major) term here intended is, not in the major premise as EJ implies, but in the conclusion. It is also to be found in the major premise, which compares the two terms, but this is not the proposition that EJ is referring to here. So this is an error, if only of inattention[6].

Furthermore, EJ does not mention the major premise, nor therefore the presence in it of a middle term that performs the comparison that determines which term is the major and which is the minor. Nor does EJ here mention the presence of the middle term in the minor premise and conclusion, and its crucial role in them in establishing the quantitative threshold as of which predication is possible. The subsidiary term (the predication) is likewise not highlighted. Note also that, whereas JE explicitly acknowledges both the *a minori ad majus* or *a majori ad minus* moods of a fortiori argument, EJ only considers the former (implicitly, without naming it) and ignores the latter.

So, what we find in the EJ article on the whole is a very limited understanding of the nature and scope of a fortiori reasoning. All it gives us is a very rough sketch of the paradigmatic form of such argument. Not surprisingly, it fails to distinguish between positive and negative arguments, between subjectal and predicatal ones, and between copulative and implicational ones.

However, further on, when EJ rightly strongly rejects the identification by Schwarz between a fortiori argument and Aristotelian syllogism, it offers a somewhat better definition inspired by Kunst[7]: "in the *kal va-ḥomer* it is not suggested that the 'major' belongs in the class of the 'minor' but that what is true of the 'minor' must be true of the 'major'". In this statement, the "major" and "minor" more clearly refer to subjects (or even antecedents), since the phrase "what is true of" them obviously refers to a predicate (or even a consequent). Thus, EJ here may be said to point to three of the four items of a fortiori argument. But the middle term (or eventually, thesis) is still missing, and this is of course a serious lacuna.

As regards use of a fortiori argument in Judaism, EJ lists the ten examples of it found in the Bible according to the Midrash (Gen. R. 92:7), and two more examples drawn from the Mishna (Sanh. 6:5) and Gemara (Ḥul. 24a). The article then proposes Jacobs' own theoretical distinction between "simple" and "complex" *qal vachomer* (which the preceding two Talmudic examples are taken to illustrate, respectively). In the "simple" type, "the 'major' and 'minor' are readily apparent," whereas in the "complex" type, "an extraneous element… has to be adduced to indicate which is the 'minor' and which is the 'major'."

The two types are symbolically represented as follows: "Simple: If A has x, then B certainly has x;" and "Complex: If A, which lacks y, has x, then B, which has y, certainly has x." As I have argued at length in the chapter devoted to Louis Jacobs (16), the symbolic formulae used to define the proposed distinction are very superficial, since they fail to clarify why or how the consequents should logically follow from the antecedents. In truth, a major premise is required in either case, whether it is readily apparent or takes an effort of reflection to formulate. This means that the simple type is tacitly complex, since its terms A and B are known to differ in some respect (say, in degrees of y). Thus, the distinction is formally inadequate.

Nevertheless, the "complex" form proposed by EJ does have considerable value, in that it does effectively allude to the middle term and the major premise of a fortiori argument. Here, "y" plays this mediating role, although this is just a special case of a more general form. A more general statement would have been: 'If A, which *has less y*, has x, then B, which *has more y*, certainly has x'. In that expanded formula, the major premise that 'B has more y than A does' is clearly implied. The case referred to by EJ is the special case where 'less y' is specifically 'no y', and 'more y' is specifically 'some y'. We do often in practice come across a fortiori statements both of the general kind (with 'less y' and 'more y') and of the special kind (with 'zero y' and 'more than zero y'). Thus, EJ cannot be said to have totally ignored the middle term and the major premise – it just did not clearly acknowledge them.

As regards "the principle of *dayyo* ('it is sufficient')," EJ presents it as "a qualification of the *kal va-ḥomer*," according to which "the conclusion [can] advance only as far as the premise and not beyond it." In symbolic terms, this means: "It must not be argued that if A has x, then B has x + y. The *kal va-ḥomer* suffices only to prove that B has x, and it is to go beyond the evidence to conclude that it also has y." This definition, though nice and clear, is partly inaccurate. It is a good statement of the principle of deduction for purely a fortiori argument – but that it not what the *dayo* principle is really about.

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[6]   In a 1972 paper, "The *Qal Va-Ḥomer* Argument in the Old Testament" (*Bulletin of the School of Oriental and African Studies*, 35:221-227. Cambridge University Press), Jacobs writes more accurately: "The argument runs: if A is so then B must surely be so; if the 'minor' has this or that property then the 'major' must undoubtedly have it."
[7]   A. Schwarz in *Hermeneutischer Syllogismus in der talmudischen Litteratur* (1901); A. Kunst in *Bulletin of the School of Oriental African Studies*, 10 (1942), 976-91.

EJ rightly refers the principle to the Mishna Baba Qama 2:5, where it is first formulated, but it wrongly interprets its discussion in the corresponding Gemara Baba Qama 25a. The latter does not exactly or merely teach that "R. Tarfon rejects the *dayyo* principle in certain instances." Rather the Gemara, relying on a *baraita* (known as The Baraita of R. Ishmael), advocates precisely the view that *qal vachomer* yields a 'proportional' conclusion. This is evident from the Biblical example regarded as the prototype of *qal vachomer*, viz. Numbers 12:14-15.

In this example, according to the Gemara, the correct inference by *qal vachomer* would be a punishment of fourteen days incarceration (for Miriam, for speaking out of turn concerning her brother Moses), even though only seven days incarceration is mentioned in the Torah. The *dayo* principle then serves to reduce the concluding penalty from fourteen days to back to seven. Thus, the Gemara's view is that *qal vachomer* is essentially a crescendo argument (rather than purely a fortiori argument as EJ implies it), and that the *dayo* principle diminishes its conclusion ex post facto (rather than denying that the 'proportional' conclusion can at all be drawn as EJ implies it). Thus, EJ does not represent the *dayo* principle as it is actually understood in Talmudic literature.

Evidently, EJ looks upon the *dayo* principle as a principle of formal logic, whereas it is more precisely put an ethical principle adopted by the rabbis to ensure their deliberations do not end up with excessively severe legal rulings. Moreover, the scope of the *dayo* principle is clearly not as wide as EJ has it, if we refer to its genesis in the said Mishna. It is not a general principle regarding any sort of conclusion (i.e. *any* A, B, x and y) – but more specifically a warning against inferring a harsher penalty for a greater crime from a Biblical text that imposes a certain (gentler) penalty for a specific (lesser) crime. Even though this warning might be extended somewhat to duties in general (instead of being applied only to penalties for crimes), it is by no means as general as EJ implies.

In conclusion, while the EJ article on hermeneutics is admittedly full of valuable information, it is seen on closer scrutiny to be open to considerable criticism. As the saying goes, "the devil is in the details." *Qal vachomer* and the *dayo* principle are certainly more intricate topics than EJ makes them out to be. And I suggest, based on my past study of the hermeneutic principles in my *Judaic Logic*, that EJ treatment of the other hermeneutic rules can similarly be criticized as oversimplified and not entirely accurate. Of course, one should not expect too much from a brief article in an encyclopedia.

Still, to my mind, there was a failure of adequate research by editors of this important encyclopedia. It is shocking that in the first edition (1971-72) there is no mention of the Ramchal's 1741 contribution to understanding of *qal vachomer* (he managed to describe the four main moods of the argument) and that in the second edition (2006-7) there is no mention of Avi Sion's 1995 clarification of *qal vachomer* (which included full formalization and validation of such reasoning).

There is in EJ, 2nd ed. (vol. 9) an article on "Interpretation" in which, under the heading of "Analogical Interpretation," can be found a brief explanation of *qal vachomer* (p. 819). This article was authored by **Menachem Elon**. The term analogical interpretation (*midrash ha-mekish*) refers to "the subject matter of the first three of the 13 middot enumerated by R. Ishmael." The first of these is "*kal va-ḥomer*," which refers to "a fortiori inference, a minori ad majus or a majori ad minus."

"The basis of this *middah*," we are told, "is found in Scripture itself (Gen. 44:8; Deut. 31:27) and the scholars enumerated ten pentateuchal *kallin va-ḥomarim* (Gen. R. 92:7)." This information is not quite correct. Though examples of *qal vachomer* occur in Scripture, it does not follow that they are its "basis;" a fortiori argument is rationally evident, and does not require revelation. Moreover, the ten examples given by the cited Midrash are not all "pentateuchal," i.e. in the Five Books of Moses (Torah), but range across the whole Jewish Bible (Tanakh); and besides, there are in fact many more instances of the argument in the latter than the ten mentioned in the Midrash.

This article defines "the rule of *kal va-ḥomer*" as "a process of reasoning by analogy whereby an inference is drawn in both directions from one matter to another, when the two have a common premise – i.e., it can be drawn either from the minor to the major in order to apply the stringent aspect of the minor premise also (BM 95a), or from the major to the minor in order to apply the lighter aspect of the major premise to the minor premise (Bezah 20b)." The two examples here mentioned are not quoted in the article, but I have looked them up (in the Soncino English ed.) so as to examine them.

The example in Baba Metzia 95a reads as follows: "You can reason a *minori*: if a paid bailee, who is not responsible for injury and death, is nevertheless liable for theft and loss; then a borrower, who is liable for the former, is surely liable for the latter too!" This argument is indeed from minor to major, being positive subjectal in form. Its major, minor, middle and subsidiary terms are, respectively, "a borrower" (P), "a paid bailee" (Q), "responsible for injury and death" (R, ranging from zero upwards), and "liable for theft and loss" (S). Note that what EJ refers to, here, as "the stringent aspect of the minor premise" is the subsidiary term, S; more precisely, this is a stringency applicable to (i.e. predicated of) the minor term, Q (which in this case happens to be located in the minor premise) which is passed on to the major term, P (located in the conclusion).

There are in Beẓah 20b two examples of *kal va-ḥomer* (they are there discussed and contested, but this need not concern us here). These are indeed arguments from major to minor, being negative subjectal in form. The first example reads: "If, when it is forbidden [to slaughter to provide food] for a layman, it is permitted [to slaughter] for the Most High, then where it is permitted on behalf of a layman, it is surely logical that it is permitted for the Most High." Its major premise is that "slaughter for a layman (P) is more restricted (R) than slaughter for the Most High (Q)," since the former is forbidden when the latter is permitted (this involves a generalization, note, from certain cases to all cases). The minor premise and conclusion predicate that P and Q, respectively, are "restricted (R) not enough to be forbidden (S)." The second example reads: "If when thy hearth is closed, the hearth of the Master is open, how much the more must the hearth of thy Master be open when thy hearth is open." It is very similar (and indeed is presented as "the same in another form"), except that here, P is "thy hearth," Q is "the hearth of the Master," R is "restricted," and S is "closed."

With regard to these arguments, EJ remarks that they go "from the major to the minor in order to apply the lighter aspect of the major premise to the minor premise." This tells us that the author of this article interprets the expression 'from major to minor' as referring not to terms but to premises; i.e. in his perspective, the "major premise" is the proposition containing the major term and the "minor premise" is the one containing the minor term. But obviously this perspective is incorrect – for the proposition containing the minor term here is not a premise but the conclusion! So this is a misinterpretation of the said expression. This reveals that the author is not very well versed in logic.

As regards the *dayo* principle, the article states: "Material to this rule is the principle *dayo la-ba min ha-din lihyot ka-niddon* (Sifra, loc. cit.; BK 25a, etc.), i.e., it suffices when the inference drawn from the argument (*ha-ba min ha-din*) is equal in stringency to the premise from which it is derived (the *niddon*), but not more so, not even when it might be argued that logically the inference should be even more stringent than the premise from which it is derived." This formulation is potentially interesting, in that the *dayo* restriction is placed in opposition to possible "logical inference." Unfortunately, the author does not discuss the implications of such opposition. If it disagrees with logical inference, then *dayo* is not a logical principle; in which case, we have to suppose that it is based on other considerations, perhaps moral ones. Moreover, we must ponder: is the logical inference necessarily, or only contingently, "more stringent"? If the logical inference is necessarily a crescendo, how can it be occasionally ignored by a *dayo* principle? And if the logical inference is only occasionally a crescendo, under what conditions does this occur, precisely? The author does not ask these questions.

Despite of such mistakes and deficiencies, the EJ article on Interpretation is on the whole, of course, very informative.

3. Encyclopedia Talmudit

The *Encyclopedia Talmudit* (henceforth, ET) is a Hebrew language encyclopedia whose purpose is to "summarize all the Talmudic halakhic issues and concepts, and all the opinions of halakhic scholars, from the completion of the Talmud to modern times, on every aspect of Jewish law"[8]. This ambitious task began over 60 years ago, and is still far from finished today (though 29 volumes have been published so far, 50 more are on the way). An English translation, called the *Encyclopedia Talmudica*, is also being published over time.

I looked for but nowhere found the Hebrew ET volume containing an article on *qal vachomer*[9]. I did however find the volume with an article on the *dayo* principle (vol. 7, 1990)[10]. It is this article that I will here comment on briefly. As would be expected from an encyclopedia devoted to halakhic thought in Judaism, it simply presents the main lines of the traditional view of the *dayo* principle relative to *qal vachomer* argument. There is no novel theoretical research in it, or criticism of existing doctrines and methods.

The *dayo* principle is stated by the Sages in the Mishna as: *dayo lavo min hadin lihiot kanidon*, which can be translated as: "it is quite sufficient that the law in respect of the thing inferred should be equivalent to that from which it is derived." ET explains this rule as teaching, with regard to inferences made by means of a fortiori, that "one cannot lay down (*lehatil*) more in the conclusion (*halamad*) than there is in the premises (*hamelamed*)." This is a reasonable explanation, except that it seems here intended too generally; that is to say, it does not specify what is

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[9]   Maybe the libraries I looked in did not buy that volume; or maybe it has not been published yet; or maybe it has not been written yet – I do not know.
[10]   I do not know who its author(s) is (or are). I hired someone to translate it for me. Actually, he only translated the main text, and ignored the footnotes. He was a Hebrew speaker, but (to put it mildly) not very good at English.

being "laid down," whereas (in my view) it should say that this rule specifically concerns the inference of *penalties or restrictive laws* (as is true of the Mishnaic example where the principle is first formulated).

ET goes on to say that "this principle of *dayo* is from the Torah;" and it presents its source, just as the Gemara (Baba Qama 25a) does, as the episode of Miriam's punishment (Numbers 12:14-15). According to the Gemara, even though the Torah only mentions seven days incarceration and makes no mention of fourteen days, the penalty of seven days for offending God was not directly inferred (by purely a fortiori argument) from the seven days for offending one's father, but was inferred indirectly (by a crescendo argument and then by application of the *dayo* principle) from fourteen days for offending God. After which, ET merely discusses the basis of the "fourteen days" interpolation, rather than any other (e.g. infinitely larger) quantity.

But the truth is that this narrative is a fanciful retroactive projection by the Gemara (based on a *baraita*). The *dayo* principle historically first appears in the Mishna (Baba Qama 2:5) which is being commented on by the Gemara. Here, R. Tarfon tries to prove in two different ways that the penalty for damage by an ox on private property is full payment, and his colleagues the Sages reject his two proofs, saying both times: "*dayo*—it is enough." The Gemara's explanation of this dispute is quite contrived and far from credible, as I have shown earlier in the present volume. Moreover, the Gemara only takes into consideration the first argument of R. Tarfon in its narrative. ET does not show any awareness of the logical issues involved, but accepts the traditional treatment quite unthinkingly.

ET does thereafter mention the two arguments of R. Tarfon, but only in order to illustrate the two types of *dayo* application. This is of course an important distinction[11]. The first type, called "at the beginning of the law" (*al techelet hadin*), is illustrated by the Sages' *dayo* to R. Tarfon's *second* argument; while the second type, called "at the end of the law" (*al sof hadin*), is illustrated by the Sages' *dayo* to R. Tarfon's *first* argument.

Both of his arguments attempt to prove that damage by horn on private property entails full compensation; and in both instances, the Sages reject his conclusion and limit the penalty to half. R. Tarfon first argues from the facts that damage by tooth & foot in the public domain entails zero compensation, while on private property it entails full compensation, and that damage by horn in the public domain entails half compensation. Then, seeing his argument rejected, he argues from the facts that damage in the public domain by tooth & foot entails zero compensation, while by horn it entails half compensation, and that on private property damage by tooth & foot entails full compensation. Both attempts, though significantly different, are blocked by the Sages using the exact same words: *dayo lavo min hadin lihiot kanidon*.

ET's explanation of the two terms seems to be as follows. The *dayo* objection to the second argument is characterized as "at the beginning of the law," because it is applied to the major premise (which comes first), while the *dayo* objection to the first argument is characterized as "at the end of the law," because it is applied to the minor premise (which comes last). The "beginning" type of *dayo* could not be applied to the first argument, because there the major premise does not mention horn damage; the "end" type of *dayo* could not be applied to the second argument, because there the minor premise (concerning tooth & foot) and the conclusion (concerning horn) have the same law (viz. full payment). This explanation is essentially correct in my view, though I would say more precisely that as regards the major premise, it is the generalization that precedes its formation which the *dayo* blocks; and as regards the minor premise, it is rather the formation of the additional premise of 'proportionality' that the *dayo* blocks.

After this, ET develops in some detail the notion, found in the same Gemara, of "nullification" of an a fortiori argument. It presents various views regarding when such nullification is possible or even necessary, and how this affects application of the *dayo* principle. ET also details various limitations imposed on the *dayo* principle in later rabbinic discourse. All this is presented with apposite examples. But the logical issues underlying such manipulations of human discourse are never raised. Thus, ET treatment of the subject-matter tends to remain on a rather superficial level, a mere presentation of traditional doctrines without any attempt to question them and dig deeper.

This, as already said, was to be expected from the sort of publication that ET is intended to be. Even so, the article can be faulted for lacking chronological information, i.e. for not tracing the historical development of ideas relating to the *qal vachomer* argument[12] and the *dayo* principle. Moreover, it fails to present certain ideas as clearly as it might and should have, preferring to faithfully reproduce the peculiar ways of expression found in rabbinic discourse.

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[11]   To my knowledge ET does not state precisely who first made the distinction; this information is historically important and must be sought. I suspect offhand the discovery was made by some Tosafist, though I have not found out just who and in what commentary precisely.

[12]   Perhaps some of that may appear in the ET article on *qal vachomer*, if there already is or ever is one. I would be curious to see, in particular, if ET there mentions the important contribution of the Ramchal to the categorization and thence understanding of *qal vachomer* argument. See my essay on this topic in an earlier chapter of the present book (9.10).

4. How to define a fortiori

It is not easy to briefly define a fortiori argument, in view of its complexity and variety. Ideally, one might put it as follows (I add in the usual symbols P, Q, R, S, to facilitate the reader's comprehension, but they would of course not be included in a lexicon): argument from the sufficiency of something lesser (Q) in some respect (R) for some attribute or consequence (S), to the sufficiency of another thing (P) greater in the same respect (R) for the same attribute or consequence (S); or from the insufficiency of something (P) greater in some respect (R) for some attribute or consequence (S), to the insufficiency of another thing (Q) lesser in the same respect (R) for the same attribute or consequence (S); or from the sufficiency of something (S) in some respect (R) for some greater attribute or consequence (P), to the sufficiency of the same thing (S) in the same respect (R) for a lesser attribute or consequence (Q); or from the insufficiency of something (S) in some respect (R) for some lesser attribute or consequence (Q), to the insufficiency of the same thing (S) in the same respect (R) for some greater attribute or consequence (P).

This definition would thus include all four (or eight) primary moods of a fortiori argument, namely the positive and negative subjectal (or antecedental) and the positive and negative predicatal (or consequental). Note that the first and fourth clauses concern argument 'from minor (Q) to major (P)', while the second and third clauses concern argument 'from major (P) to minor (Q)'. What is mentioned in each case is the minor premise and conclusion; there is no need to mention the major premise ('P is/requires more R than Q is/does') because it is implicit in the quantification of two items as 'greater' and 'lesser'. This definition applies to purely a fortiori argument; for a crescendo argument, one would have to mention the underlying proportionality (the variation of S with R in subjectal/antecedental moods, or R with S in predicatal/consequental moods). In any case, in my view it is very important to mention the middle item (R) and the fact that it is sufficiency or insufficiency of it that makes the inference possible; to fail to mention these details would miss the essence of the matter.

I am not a lexicographer, but it is obvious enough to me that the above 'ideal' definition would not be appropriate for a popular dictionary. It is much too technical and long-winded. For such a publication, the purpose of the 'definition' is to give readers a ball-park idea of the intent of the phrase. It would therefore suffice to mention the positive subjectal mood, together with a simple example. It is the paradigmatic form, and probably the most used in practice, even if (as demonstrated empirically in the present work) the other moods are also frequently used. For more academic purposes, such as a legal or philosophical dictionary, or a full-scale encyclopedia, the above full definition would seem to me worth stating, possibly together with examples. Thus, when judging the definitions proposed by various lexicons, which is what we shall attempt to do in the present chapter, we have to be understanding and flexible, and to consider the purpose and readership of the document concerned. Similar considerations apply, of course, to related topics that may be treated in certain lexicons.

5. Various dictionaries and encyclopedias

In the present section we shall focus on the description and (where applicable) explanation of a fortiori argument given in certain dictionaries and encyclopedias. This is not, by far, a study of all lexicons available in libraries (which work ought to be done eventually), but refers to some of the main ones, mostly among those that are present on the Internet[13].

Many dictionaries and encyclopedias do not even mention a fortiori argument; some display use of it, or mention it in passing within articles on other topics, but do not have a separate entry for it. Some lexicons, of course, do have an entry for 'a fortiori'; among these, some just give synonyms for the term, while others attempt a more descriptive, and even sometimes explicative, approach. Many give examples of the argument – some of which are correct, but some of which are not really a fortiori in form.

We shall critically examine the matter in three stages, dealing first with ordinary dictionaries and encyclopedias, intended for the public at large, then respectively with dictionaries and encyclopedias designated as legal or philosophical, which are intended more specifically for college students, academics and other professionals in those fields. Note that there are many dictionaries that I have looked into, but found nothing of interest in – these I do not mention at all. I do however mention some encyclopedias that have no entry on the subject concerning us, since that is inexcusable.

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[13]   As of late May 2013.

Ordinary dictionaries and encyclopedias

The Collins Dictionary[14] defines a fortiori as meaning "for similar but more convincing reasons." This statement mentions "more," the indicator of quantitative comparison; but it does not tell us what makes some "similar" reasons "more convincing"! The example given, "If Britain cannot afford a space programme, then, a fortiori, neither can India," suggests that it is because India is even poorer than Britain that it likewise cannot afford a space programme. Notice that the middle term here (poor) is not epistemic but ontical, whereas the proposed definition rather refers to an epistemic condition (conviction). Still the example is valid, its form being positive subjectal (from minor to major).

The Oxford Dictionary[15] states that a fortiori is "used to express a conclusion for which there is stronger evidence than for a previously accepted one." This definition differs from the preceding in that it is more empirical or objective, referring to "stronger evidence" instead of to "more conviction;" moreover, it speaks of "conclusion," suggesting a logical process. However, it is still essentially epistemic rather than ontical. Moreover, it suggests that a fortiori argument is inductive, rather than deductive. Indeed, it confuses a fortiori argument with induction, since it only refers to comparisons between conclusions, and does not refer to the less evident "conclusion" as the premise of the more evident one. Furthermore, the example given by this dictionary is: "They reject all absolute ideas of justice, and a fortiori the natural-law position." It sounds more high-brow, but it lacks a middle term (which would explain why the natural-law position is 'a fortiori' compared to the absolute ideas of justice). One might even wonder if this example is not in fact syllogistic, rather than genuinely a fortiori! Assuming the argument is a fortiori, it would be predicatal in form, since the subject is the same in premise and conclusion; whether it would be positive or negative depends on whether the author conceives rejection of all absolute ideas of justice as having more or less of the unstated middle term than rejection of the natural-law position; so the example is far from clear, anyway.

The Merriam-Webster's Collegiate Dictionary[16] entry for a fortiori is: "with greater reason or more convincing force – used in drawing a conclusion that is inferred to be even more certain than another;" and it proposes as example: "the man of prejudice is, a fortiori, a man of limited mental vision." Here again, it is not said why the inferred proposition is "even more certain" or "more convincing" than the other; this makes the comparison seem like a subjective decision. Moreover, the proposed example might in fact be syllogistic, instead of a truly fortiori; for the intended argument seems to be the apodosis: 'if a man has prejudice then, since prejudice is a sign of limited mental vision, he must have limited vision'. A more truly a fortiori interpretation of the argument would be, say: 'since prejudice is a more advanced ailment than limited mental vision, if a man is sick enough to be prejudiced then he must be sick enough to have limited mental vision'; this would be positive predicatal (from major to minor) a fortiori argument. Clearly, the proposed example is ambiguous.

Garner's Modern American Usage[17] proposes a terse definition of the phrase 'a fortiori': "by even greater force of logic; so much the more," without clarifying what would in this context constitute "even greater force of logic." However, it adds to that the following two interesting remarks: "the phrase is sometimes effective, but only if the intended readers are sure to get it;" and "the phrase is used illogically when the proposition following a fortiori is no stronger than the one preceding it." Each remark is accompanied by two illustrations drawn from actual legal discourse, which are further explained. The author of this has obviously understood a fortiori argument more than many, even if he has not attempted a proper definition.

The Macmillan British Dictionary[18] tells us that a fortiori is "used for saying that something that is true for one case is even more true in another case." The New World Dictionary[19] tells us that it is "said of a conclusion that follows with even greater logical necessity than another already accepted in the argument." In both these cases, no further explanation or example is given. We are not told why the result is "even more true," or why it has "even greater logical necessity." Some dictionaries are even more laconic[20]: the Encarta Dictionary has the pro-forma "for an even stronger reason;" the American Heritage Dictionary has "for a still stronger reason; all the more;" the Random House

[14]   At: www.collinsdictionary.com/dictionary/english/a-fortiori.
[15]   At: oxforddictionaries.com/definition/english/a-fortiori?q=a+fortiori.
[16]   On CD (version 2.5), 2000.
[17]   By Bryan A. Garner (New York: Oxford UP, 2009). See full entry at:
books.google.ch/books?id=FwmQpyibKkAC&pg=PA28&dq=fortiori&hl=en&sa=X&ei=h1KjUabRIMmGhQfQnYGoAg&redir_esc=y#v=onepage&q=fortiori&f=false.
[18]   At: www.macmillandictionary.com/dictionary/british/a-fortiori.   The   same   definition   is   found   at: www.merriam-webster.com/dictionary/a+fortiori.
[19]   Webster's New World College Dictionary (Cleveland, Ohio: Wiley, 2010).
[20]   See: www.memidex.com/fortiori; www.yourdictionary.com/a-fortiori; dictionary.infoplease.com/a-fortiori. Note that mimidex.com gives as example: "if you are wrong then, a fortiori, so am I." This example is truly no example; it gives no indication of the reasoning involved. Apparently, these people imagine that the mere use of the words 'a fortiori' is evidence that a fortiori argument is involved – this is far from true.

Dictionary: has the same, plus "even more certain." As for the Encyclopedia Britannica[21], it only briefly mentions (without giving an example) a fortiori argument (in contrast to *a pari* and *a contrario* arguments) in an article on rhetoric, defining it as "arguing from an accepted conclusion to an even more evident one." Such 'definitions' are obviously inadequate – they do not sufficiently circumscribe the term concerned for someone who does not know anything about it.

Note finally, in passing, the definition and illustration proposed by the Christian Apologetics & Research Ministry (CARM)[22]: "argumentum a fortiori is an argument based on stronger reason. It is when an argument is proposed where the more probable of possibilities are affirmed and a conclusion is arrived at. For example, "My wife likes diamonds so she will like any diamond I give her." This definition is ambiguously worded: it is not clear whether it intends that the "conclusion arrived at" is, or is derived from, the "affirmed more probable possibilities." In any case, whether the latter refers to a premise or to the conclusion, we are not told why some possibilities are more probable and worthy of being affirmed. Moreover, the example given is clearly not a fortiori argument, but mere syllogism.

Legal dictionaries and encyclopedias

Two online French-language websites have interesting definitions of a fortiori argument in legal contexts. The Faculty of Law of Université Paris Descartes[23] has: "a fortiori interpretation consists in extending a rule to a hypothesis not foreseen by it;" for example: "if a regulation forbids the presence of dogs in accommodations, it will be admitted that the presence of bears is equally proscribed." The legal dictionary of the Montreal law firm of Lecours-Hébert[24] has: "Latin expression: with stronger reason… aims at cases for which the field of application of a legal norm concerns one or more situations or conventions that were not originally aimed at." Of course, these definitions do not tell us just *how* the application of a rule may be enlarged, e.g. the example given would have been more enlightening if it was explained that bears cause more damage than dogs. Comparatively, the definitions given in English-language websites concerned with law are far less informative.

The dictionary of the Encyclopedia of Law Project[25] states that a fortiori is "applied to the argument that, because of the concession or establishment of a given proposition, another included in it is by the greater reason true." This definition is not very clear, and no example is proposed which might clarify it. It says that because some proposition is given, having already been conceded or established, another proposition "included in" it is "by greater reason" true. But this does not tell us what is meant here by these two phrases; a fortiori is not like syllogism about inclusion, nor even (despite its etymology) about comparison of reasons. The Law.com dictionary [26] states that the expression a fortiori "applies to a situation in which if one thing is true then it can be inferred that a second thing is even more certainly true," and gives as example: "if Abel is too young to serve as administrator, then his younger brother Cain certainly is too young." The example is clearly a fortiori argument (negative subjectal in form, from major to minor), and should have inspired a more precise definition than the vague one proposed.

Webster's New World Law Dictionary[27] proposes: "To draw an inference that when one proposition is true, then a second proposition must also be true, especially if the second is included in the first; and gives as example: "if a 19 year old is legally an adult, then a 20 year old is, too." Note that this example is similar to the preceding (though positive subjectal in form, from minor to major); and indeed the definition is similar, except for the clause: "especially if the second is included in the first." The latter clause is inappropriate here, since it suggests syllogistic rather than a fortiori reasoning. Cornell University Law School Legal Information Institute[28] puts it more succinctly as: "If a particular fact is true, then one can infer that a second fact is also true." Here, there is happily no added clause about inclusion; but there is no explanation or justification of the claim that "one can infer" – not even an example. How, then, is one expected to distinguish this form of reasoning from any other? A species definition, Aristotle taught, must contain both a genus and a differentia; all lexicographers should know that!

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[21]   At: www.britannica.com/EBchecked/topic/64/a-fortiori. Similarly in a CD edition (2004) that I have. The 1911 edition of this encyclopedia describes a fortiori argument as: "if A is proved less than B, and is known to be greater than C, it follows a fortiori that C is less than B without further proof." See full statement at en.wikisource.org/wiki/1911_Encyclop%C3%A6dia_Britannica/A_Fortiori. This is of course not a fortiori argument, but simply the mathematical argument that if B > A and A > C, then B > C.

[22]   At: carm.org/dictionary-argumentum-a-fortiori.

[23]   At: www.droit.univ-paris5.fr/HTMLpages/recherche/griad/regle.html. My translation. The example given is valid positive subjectal (from minor to major) a fortiori argument. Note also the definitions given here for *a pari* argument (extending the rule to a hypothesis not foreseen by it but judged to be analogous) and for *a contrario* argument (limiting the rule to the foreseen hypothesis so as to infer the contrary solution in all other hypothesis). "Hypothesis" here, of course, refers to a situation or type of case.

[24]   At: www.lecourshebert.com/dictionnaire-juridique/details/5686.

[25]   At: legaldictionary.lawin.org/a-fortiori/.

[26]   At: dictionary.law.com/Default.aspx?selected=2365.

[27]   At: law.yourdictionary.com/a-fortiori. (Hoboken, New Jersey: Wiley, 2010)

[28]   At: www.law.cornell.edu/wex/a_fortiori.

West's Encyclopedia of American Law[29] states, concerning a fortiori: "this phrase is used in logic to denote an argument to the effect that because one ascertained fact exists, therefore another which is included in it or analogous to it and is less improbable, unusual, or surprising must also exist;" no illustration is proposed. This definition of a fortiori argument looks a bit more sought out than usual, but upon further scrutiny it is obvious that its author did not grasp the reasoning involved. Although the clause "[is] analogous to it" could pass for a fortiori reasoning (since quantitative comparison is analogy of sorts), the clause "which is included in it" would only be appropriate for syllogistic reasoning. Also, in any case, the inference from something "ascertained" to another "less improbable" is not the essence of a fortiori argument.

Worth mentioning here is the definition of a fortiori given in a newspaper article on a particular law case[30]: "In case of absence of a text on the case at hand, the search for the presumed intent of the legislator is required by deduction, including reasoning by analogy, that is to give the case at hand the same treatment given to a past case when they are similar and united in the same causation; or the reasoning a fortiori, that is to apply to the case terms considered in another case to have a stronger causality than the case at hand, in other words when the conditions for the case at hand are more suitable for applying the law than those stipulated by the legislator, which is a logical device that emphasizes causation of stronger cases for application to weaker ones." The latter article is noteworthy for its attempt to differentiate a fortiori argument from mere analogical (or *a pari*) argument, with reference to "causation." This clearly refers to "the conditions for… applying the law," which may be more "suitable" in some cases than in others. This is, of course, *one possible* major premise for a fortiori argument. It does not constitute a general definition of such argument, but can be used as a general model for inference from a legal text.

Philosophical dictionaries and encyclopedias

I have already in the preceding chapter (31.4) analyzed the interesting definition of a fortiori argument proposed by Pierre André Lalande, perhaps in collaboration with other authors, in his 1926 opus, *Vocabulaire technique et critique de la philosophie*. So I will just repeat his definition in the present context, with a very brief comment. It went (my translation): "Inference from one quantity to another quantity of similar nature, larger or smaller, and such that the first cannot be reached or passed without the second being [reached or passed] also." What needs to be noted here is that: this definition refers explicitly to quantitative comparison, and suggests both inference from minor to major and that from major to minor; also, it hints at the middle term, by specifying that the quantities are "of similar nature;" and it alludes to a threshold which needs to be "reached or passed" for the inference to occur. However, though very good, this definition has imperfections, notably the non-mention of the subsidiary term (i.e. that which is being inferred); and the failure to distinguish subjectal and predicatal arguments and draw attention to negative moods.

Another interesting definition from a French source is that of the *Encyclopédie Philosophique Universelle*[31] (my translation): "*A fortiori* argument rests on the following schema: x is y, whereas relatively to the issue at hand z is more than x, therefore a fortiori z is y." This definition is also very good, in that it comprises the major premise (z is, relatively to the issue at hand, more than x), the minor premise (x is y) and the conclusion (z is y) of a fortiori argument. However, it is deficient in not having a symbol (say, w) for the middle term, being content to call it "the issue at hand;" and for failing to mention this term in the minor premise and conclusion, as the threshold condition underlying the inference. That is to say, it should have had: z is more w than x is, and x is w enough to be y; therefore, z is w enough to be y. This is, of course, positive subjectal argument (from minor to major), i.e. the simplest form of a fortiori. Missing here are negative subjectal as well as positive and negative predicatal forms of it; also, implicational forms.

It is interesting to compare these last two definitions. The latter has some improvement over the former, in that it includes a subsidiary term (y) and clarifies things by introducing symbols. But the latter fails to mention important elements of the former, namely the possibility of argument from major to minor and also the fact that there is a threshold condition for the predications in minor premise and conclusion. Nevertheless, taken together, these French-language definitions are clearly valuable contributions to the field, and far superior to those given in English-language publications that I have looked into.

The online edition of the Oxford Dictionary of Philosophy[32] simply defines a fortiori as a "phrase used for 'all the more' or 'even more so';" and it gives as example: "if all donkeys bray, then *a fortiori* all young donkeys bray." This

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[29]   At: legal-dictionary.thefreedictionary.com/A+Fortiori. 2nd ed. (Farmington Hills, MI: Gale, 2008.)

[30]   At: www.dailystar.com.lb/Law/Aug/13/Children-of-foreign-father-declared-Lebanese.ashx. Reference is there made to "Raymond Farhat, *Introduction à l'étude du droit*, Beryte ed., 62." I detect in this article's terms of reference an influence of Islamic hermeneutics, which is not surprising since it comes from Lebanon, even if not all Lebanese are Moslem.

[31]   Paris: PUF, 1990. I did not look for the name of the author of this definition; but it is clear that he deserves to be acknowledged by name.

[32]   At: www.oxfordreference.com/search?siteToSearch=aup&q=a+fortiori&searchBtn=Search&isQuickSearch=true.

is definition merely by synonym; and the example offered is not correct, being syllogism rather than a fortiori argument. It is shocking that such a prestigious publication should display such poverty of thought. A Dictionary of Philosophy[33], which I have a copy of, does no better. It similarly defines a fortiori as "a phrase used to signify 'all the more' or 'even more certain';" and it gives as example another mere syllogism: "If all men are mortal, then *a fortiori* all Englishmen - who constitute a small class of all men - must also be mortal." Another Dictionary of Philosophy[34], found online, just says, without giving an example: "A fortiori: a phrase signifying all the more; applied to something which must be admitted for a still stronger reason."

Some Internet sites do somewhat better. The philosophical dictionary provided by Philosophy Pages[35] explains the argument by saying: "we are bound to accept an *a fortiori* claim because of our prior acceptance of a weaker application of the same reasoning or truth;" and it gives as illustration: "Frank can't run to the store in less than five minutes, and the restaurant is several blocks further away than the store. Thus, *a fortiori*, Frank can't run to the restaurant in less than five minutes." The Fallacy Files Glossary[36] explains a fortiori as a "phrase is used when arguing that what is true of a given case because it possesses a certain attribute will certainly be true of another case which has more of the relevant attribute;" and proposes as an example: "Suppose that Tommy is Timmy's older brother. We can argue that if Tommy is too young to see a certain movie, then a fortiori Timmy is too young, as well, since he is younger than Tommy."

Both of the preceding examples are valid 'from minor to major' a fortiori arguments. But note that while the latter is positive subjectal, the former is negative predicatal. I doubt that the author of the Philosophy Pages entry realized that he was using a relatively unusual form of the argument; in any case, his explanation is not very accurate, being rather vague and epistemic. The Fallacy Files explanation is a lot better, because it is ontical and refers to a comparison between two cases, one having "a certain attribute" and the other "more of the relevant attribute." The two "cases" are the minor and major terms; the "attribute" they have in common to different degrees is the middle term, and the phrases "what is true" and "will certainly be true" refer to the subsidiary term – so all four required terms are present in it. However, the idea of a threshold value of the middle term is not sufficiently highlighted, although suggested somewhat by the conjunction "because." Also, of course, this definition refers to just one mood of a fortiori argument, and fails to mention the other three (or seven); moreover, it only refers to purely a fortiori argument, and fails to mention a crescendo argument.

It is sad to see that some reputed major encyclopedias of philosophy with an Internet presence do not have an entry for a fortiori argument, or do not at least mention it as a form of reasoning in articles on related subjects, even if the phrase is used in some articles; I here refer to: the Stanford Encyclopedia of Philosophy (SEP), the Internet Encyclopedia of Philosophy (IEP) and the Concise Routledge Encyclopedia of Philosophy. The Oxford Companion to Philosophy[37] is likewise silent regarding a fortiori argument.

6. Wikipedia

Wikipedia, the online "free encyclopedia that anyone can edit," is very often a useful source of information, which I for one happily look into when I need a quick answer to a question. However, as can be expected from the fact that it is open to all volunteers irrespective of their real knowledge of what they are talking about, it is often enough inaccurate, and therefore should always be viewed with caution (double checking information in other venues).

A fortiori argument. A case in point is the Wikipedia article on a fortiori argument[38]. Although this article tries to be more informative than those on the same topic in many dictionaries and encyclopedias, it is still inadequate. The

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[33]   London: Pan, 1975.

[34]   Edited by Dagobert D. Runes (1942). At: www.ditext.com/runes/a.html. The entry is signed "J.J.R."

[35]   At: www.philosophypages.com/dy/a2.htm#a-fo.

[36]   At: www.fallacyfiles.org/glossary.html. See also the discussion of "if X is good then more X is better," which is posted at: www.fallacyfiles.org/archive032009.html#03212009. This is really not an issue of a fortiori argument, but concerns pro rata argument. Given the major premise that 'good (say, for Y)' varies proportionally with 'X', then indeed it would follow that if for some value X1 of 'X' the corresponding value of 'good for Y' is Y1, then for a greater (or lesser) value X2 of X the corresponding value of 'good for Y' would be Y2 (the value of which to be calculated using the exact ratio between the terms). This is generally true; but a problem arises when the major premise is only true within a limited range of values. Then, of course, the reasoning becomes fallacious. Note also that if we did try to interpret "if X is good (or bad) then more X is better (or worse)" as an a fortiori argument, it would be positive predicatal in form, so that it would be fallacious to reason with it from minor to major. So the flaw inherent in "if X is good then more X is better" is formally explicable.

[37]   Oxford: Oxford UP, 1995.

[38]   At: en.wikipedia.org/wiki/Argumentum_a_fortiori. In my experience, when you try to correct errors in a Wikipedia, you may find your efforts erased; so, I never waste my time trying to teach anything in that forum. Note that the article here examined is that posted end May 2013; someone may of course make changes to it in the future. Hopefully, some changes will be made to it by someone, based on the comments in the present work.

article first defines the term as argument "from a stronger reason," which merely literally translates its Latin name: *a fortiori ratione*. An example is then given: "if it has been established that a person is deceased (the stronger reason), then one can with equal or greater certainty argue that the person is not breathing. This example is valid a fortiori argument (positive predicatal in form, going from major to minor). There follows three sections, devoted to usage, meaning and prevailing circumstances of use.

As regards "usage," we are told that "in the natural sciences and in social and other human sciences where statistics plays a large role, the phrase is used to mean 'even more likely' or 'with even more certainty'." This definition is incorrect: a fortiori argument is not inference of a more likely conclusion from a less likely premise; it is impossible through any sort of deduction, let alone of induction, to infer more certainty from less certainty. Moreover, this definition is inconsistent with the suggestion in the previous paragraph that the premise is to be called "the stronger reason."

Furthermore, the example given here, viz. that if two or more phenomena are observed to be conjoined for a certain amount of time, they will be "a fortiori" be present as much or more of the time, since they might occur separately as well as in conjunction (my paraphrase), is not really an a fortiori argument but rather a syllogistic one, being about the inclusion of a smaller set in a possible larger one. Moreover, this example, being 'from minor to major' (i.e. from a smaller portion of time to a possibly larger one), differs from the previous example, and the author of the article fails to take stock of the difference out loud and modify his initial definition accordingly.

Further on, another definition is proposed: "In classical logic, 'a fortiori' is a signal indicating an attempt to justify an inferential step by claiming that the point being proven follows 'from a[n even] stronger [claim]' or has been stated 'by means of [an even] stronger [assertion]'. That is, the phrase indicates that a) a proposition previously given or proven in the argument contains and implies a variety of 'weaker' or less contentful propositions and b) the proposition being proven is only one of the propositions contained and implied." This definition is also wrong. It is more akin to the first than to the second, in that the argument is thought here to go from a 'stronger' claim to a 'weaker' one; but it is also in part comparable to the second definition, in that the argument is here thought to be about inclusion (note the idea that the premise 'contains' the conclusion).

In the section on "meaning," the argument is presented as: "a proof in which one demonstrates a claim by invoking as proof an already proven, stronger claim." And the example offered here is: "if it is forbidden to ride a bicycle with an extra passenger, it is also forbidden to ride a bike with fourteen extra passengers." This example is a valid a fortiori argument, but not for the reason given. It is valid because riding with fourteen passengers is more dangerous than riding with only one passenger, and not because the prohibition of riding with one passenger is a "stronger claim." The reasoning here is from minor to major, and not from major to minor. The article says a few more things under the said three headings, but I do not repeat them here or comment on them as they are not directly relevant to our subject.

The article also informs us that "there are two types of the a fortiori argument: a maiore ad minus: 'from greater to smaller' [and] a minore ad maius: 'from smaller to greater'." If we follow the hyperlinks on these expressions, we find their definitions to be as follows. The former "describes a simple and obvious inference from a claim about a stronger entity, greater quantity, or general class to one about a weaker entity, smaller quantity, or specific member of that class;" and the latter "denotes an inference from smaller to bigger." The author believes that the former argument is "more universally known" and "also usually has a broader usage" and "is incomparably more general." Moreover, whereas he offers examples of the former, he apparently cannot think of one for the latter.

Here again, there is some confusion with syllogistic reasoning, in the application of the term 'from major to minor' to arguments "from general to particular" (e.g. "what holds for all X also holds for one particular X") or to those "from the whole to the part" (e.g. "if the law permits a testator to revoke the entirety of a bequest… then the law also permits a testator to revoke the portion of a bequest"). Traditionally, the term is only used for a fortiori argument, i.e. in relation to arguments "from stronger to weaker" (e.g. "if one may safely use a rope to tow a truck… one may also use it to tow a car").

Thus, to conclude, the author (or is it authors?) of the Wikipedia article on a fortiori argument has some idea of a fortiori argument, but not a very clear idea. He does not realize that the argument has four main moods: the positive and negative subjectal and the positive and negative predicatal, and that both the first and fourth of these go 'from minor to major' and both the second and third of them go 'from major to minor'. He does not mention the negative moods at all. He also does not mention the corresponding four implicational (antecedental or consequental) moods. He also does not raise the issue of proportionality (a crescendo argument).

Moreover, contrary to what the author imagines, the positive predicatal mood (which goes from major to minor) is not the most typical or the most used. If we look at actual usage statistics[39], we see that the positive subjectal mood

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[39]   The statistics are developed in greater detail elsewhere in the present work. Note that I here lump positive antecedental cases together with positive subjectal cases. There are no positive consequental cases for the texts mentioned, to my knowledge so far.

(which goes from minor to major) is the most typical and the most used. In Plato's works, there are apparently none of the former and at least 9 of the latter. In Aristotle's work, there are 5 of the former and 50 of the latter. In the Tanakh (Jewish Bible), there are 15 of the former and 14 of the latter. In the Mishna, there is only one of the former and 32 of the latter. In the Christian Bible, there are 3 of the former and 28 of the latter. In the Koran, there is only one of the former and none of the latter. Need one say more? The author obviously did not research the matter.

It is interesting to note that in another page of Wikipedia, listing Latin phrases[40], a fortiori is described as: "often used to lead from a less certain proposition to a more evident corollary." This of course refers to 'from minor to major' argument, demonstrating that another author has another opinion than the one above. Of course, this definition is also inadequate since it does not refer to argument 'from major to minor', and moreover since it wrongly considers the premise as "less certain" and the conclusion as "more evident."

It should also be noted that the main Wikipedia article on a fortiori argument seems to be based on very little research. It only mentions two obscure references, namely: "Grabenhorst, Thomas K.: *Das argumentum a fortiori* (Verlag Peter Lang, 1990)" and "Schneider: *Logik für Juristen*, S. 158ff." Note also that both these books are in German. I have not found further information on the second reference, but regarding the first, it is described as "a pilot study using the practice of decision justification." If the author of the Wikipedia article truly based his understanding of a fortiori argument on these two books, then they cannot have been too informative on the subject!

Note additionally: I came across a book in Amazon.com, entitled *A Fortiori Argument: Rhetoric, Truth-Value, Argument* (paperback, 96 pages), edited by Lambert M. Timpledon, Miriam T. Marseken, Susan F. Surhone, and published in late 2010 by Betascript[41]. This is billed as "high quality content in Wikipedia articles," and upon further research I found that Betascript was a label created in 2010 that "specialize[s] in publishing and selling Wikipedia articles in printed form via print on demand." I do not know how they managed to make a 96p book out of the brief article we have above reviewed; but in any case, based on this review, it is very doubtful that it has "high quality content"!

Kal va-chomer. Wikipedia has a brief definition of kal va-chomer[42], i.e. of a fortiori argument as used in Jewish literature, in its article on Halakha, as follows: "We find a similar stringency in a more lenient case; how more so should that stringency apply to our stricter case!" This is a correct rendition of rabbinic formulations of the principle; but missing here is the other half of such formulations, viz. if a leniency is applicable to a relatively stringent case, then it should also apply to a more lenient case.

However, the main discussion of kal va-chomer is to be found in the article on Talmudic Hermeneutics[43]. This article is a copy-paste of the article on the same subject in the Jewish Encyclopedia (JE), which we have already reviewed in the first section of the present chapter – to which some examples and explanations have been added. We need only here highlight the JE statement: "the process of deduction in the kal wa-chomer is limited by the rule that the conclusion may contain nothing more than is found in the premise. This is the so-called 'dayyo' law, which many teachers, however, ignored." "It is formulated thus… 'The conclusion of an argument is satisfied when it is like the major premise'." It should here be stressed that the author of the statement imposes his own interpretation, confusing the *dayo* (sufficiency) principle with the principle of deduction.

The Wikipedia article offers the following illustration for this principle: "If a parent will punish his or her child with a minor punishment should the latter return home with scuffed shoes, surely the parent will punish his or her child with a major punishment should the latter return home with scuffed shoes, ripped pants and a torn shirt." And it comments: "This is an illogical deduction; although it might be a fair speculation, it cannot be proven with logic. All that can be proven is at least the result of the lesser offense." All this is not quite correct. Although purely a fortiori argument logically demands an identical conclusion, a crescendo argument (which is a fortiori argument with an additional premise about proportionality) logically allows for a 'proportional' conclusion.

Moreover, a crescendo argument is frequent in the Tanakh (6 cases out of 46), in the Mishna (some 10 cases of 46) and in the Gemara (not yet counted, but note for a start the discussion in Baba Qama 25a-b). The point is that the *dayo* principle is not a logical rule, as both JE and Wikipedia wrongly assume it to be, but an ethical one, addressed to rabbis (legislators and judges) attempting to infer greater penalties for greater crimes, from lesser penalties for lesser crimes specified in the written Torah.

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[40]   At: en.wikipedia.org/wiki/Talk:List_of_Latin_phrases_%28A%29#a_fortiori.
[41]   At: https://www.morebooks.de/store/de/book/a-fortiori-argument/isbn/978-613-1-17240-3.
[42]   At: en.wikipedia.org/wiki/Halakha#Rules_by_which_early_Jewish_law_was_derived.
[43]   At: en.wikipedia.org/wiki/Talmudical_hermeneutics.

33. Conclusions and prospects

We shall here briefly recount the main theoretical innovations, historical findings, and critical assessments put forward in the present work (including the Appendices), and reflect on possible future directions of research.

1. My past errors and present improvements

In my book *Judaic Logic*, first published in 1995, I devoted four chapters (out of fifteen), or some 65 A4 pages (out some 300), to the a fortiori argument because of the important role such discourse plays in Jewish thinking. The first of these chapters is devoted to 'the formalities of a fortiori logic'; the second, to '*qal vachomer*'; the third, to a 'revised list of Biblical a-fortiori'; and the fourth, to 'the language of Biblical a-fortiori'. The present work devotes about ten times as much space to the same subject.

As regards the formalities, I distinguished in *Judaic Logic* eight primary moods of the argument. These were of two forms, the copulative and the implicational. The structure of the former could be subjectal or predicatal, and that of the latter could be antecedental or consequental. Furthermore, for each of these possibilities, the polarity of the argument might be positive or negative. There were thus four moods with the orientation 'from minor to major' and four with that 'from major to minor'. The distinctions and terminology there introduced have been retained in the present work and proven very useful. Moreover, the formal analysis that I made in the past – of each of the said eight moods, into a 'commensurative' major premise, together with a 'suffective' minor premise, yielding a 'suffective' conclusion – has endured and proven perfectly accurate in the present work. However, I have here gained a better understanding of implicational moods, after realizing that the 'middle thesis' in them, being a proposition, cannot strictly speaking have degrees (i.e. be implied more or less); so there was some inaccuracy in this specific area.

But the most important improvements have emerged in the area of validation. The idea of reducing a fortiori argument to a complex of implications and quantitative comparisons has been carried over. But, whereas my analysis of 'commensurative' propositions has stayed essentially the same, that of 'suffective' propositions has evolved considerably. Whereas previously I assumed the latter forms to be composed of three simpler propositions, I have here come to realize that a fourth constituent must be acknowledged to fully capture the crucial notion of 'sufficiency in relation to a threshold' involved. For instance, it is not enough to say that 'something X is R enough to be Y' implies, *inter alia*, the positive constituent: 'whatever is at least to a certain measure or degree R (say, Ry) is Y' – we must also acknowledge the corresponding negative constituent: 'whatever is *not* at least to that measure or degree R (i.e. is not Ry) is *not* Y'.

Another important finding in the present work is the idea that the middle item R may range in value from minus infinity through zero to plus infinity, i.e. that R need not necessarily be positive, but may well be zero or negative. This is something that I had some difficulty with before, but which I realized the need for when I tried to explain Louis Jacobs' distinctive construction of a fortiori argument. Once the idea of a full range of values emerged, it became very useful – in the development of secondary moods of the argument, in dealing with relative and antithetical middle items, and in efforts of 'traduction' (i.e. correlation of moods). Similarly, my theory of quantification of a fortiori argument was drawn up in response to the report by Wiseman of a hot dispute on this issue between Mercier, Schiller and others, in early 20th century (1915-1918) issues of *Mind*. As regards my correlations and comparisons between a fortiori argument and syllogism, they were of course proposed in opposition to Schwarz's reported claim that these forms of argument are identical or interchangeable. Similarly, my comparison and contrast between a fortiori argument and argument by analogy was made in reaction to the apparent equation of the two by several commentators, notably by al-Ghazali. My distinction between ontical, logical-epistemic and ethical-legal a fortiori arguments was likewise motivated by the wish to correct erroneous thinking found in various commentaries, such as that of Lenartowicz and Koszteyn.

Another important new breakthrough in the present work is the formal description and validation of a crescendo argument. I had in the past mentioned the possibility of 'proportional' a fortiori argument, but I had not tried to present and analyze such reasoning in formal terms. The formal study of this topic in the present work greatly clarifies the issues involved. Also new in the present work is a thorough examination of what constitutes valid

inductive a fortiori reasoning. Wiseman tried to give credence to proportional a fortiori argument, but in fact failed. He also tried to give voice to inductive a fortiori argument, but again in fact did not succeed. With all these new formal tools in hand, it becomes easier from now on to interpret a fortiori arguments one comes across in material discourse – i.e. to estimate the probable intent of the speaker or writer in each case – and of course to judge their validity. This emphasis on hermeneutics – in the widest sense – is a valuable aspect of the present work.

Turning now to my studies more specifically aimed at Judaic a fortiori argument, there are many significant improvements in the present work. For a start, thanks in part to Louis Jacobs' contribution, I now have an even longer list of Biblical *qal vachomer*. Moreover, thanks largely to Samely's hard work, I now have a long list of Mishnaic *qal vachomer*. But the most important novelty here is the greater understanding, and more precise pinpointing, of the *dayo* principle. It is not, as many have thought, including myself in my previous work, a general logical principle; it is a much more specific, essentially moral principle that rabbis should not infer, measure for measure, from the lower penalties given in the Torah for certain crimes, higher penalties for greater crimes. This understanding was made possible through detailed study of the Mishnaic origin of this principle, and is amply confirmed with reference to other uses of it in the Talmud.

Also to be found in the present work is a first attempt by me at listing the a fortiori arguments in later, Talmudic literature. For the time being, our study proceeds only in relation to Rodkinson's English edition of the Babylonian Talmud, which contains only part of that major cultural asset. But by this means we provide future researchers with valuable ways and means that they can apply in a more exhaustive study of both Talmuds and of other literature of the same period. Also new in the present work, as contributions to historical research: detailed listings of a fortiori discourse in the works of Plato and Aristotle, and in the Christian Bible and in the Koran, as well as evidence of such discourse in Chinese and Indian spiritual literature.

Additionally, in the final appendix, I present novel analyses of some topics in general logic, including symbolization and axiomatization, existential import, the tetralemma, the Liar paradox and the Russell paradox.

2. Historical research into logic

If we introspect, and watch our thought processes carefully, we can well see that rational insight can be non-verbal; indeed, logically, the insight must precede its verbalization, even if verbalization of an earlier insight greatly facilitates the next insight. The proof is that sometimes we have a thought in mind, but have difficulty putting into words (and in the opposite direction, we may hear or read words and yet not immediately understand them). This psychological sequence of events tells us something about the history of logic and of language. When did some pre-human species of animal evolve into the "rational animal" we call man? Two million years ago (when Australopithecus made and used simple tools)? Four hundred thousand years ago (when Homo Erectus used fire)? Or two hundred thousand years ago (when Homo Sapiens emerged)? It is hard to say. But we can say that even before linguistic abilities distinguished mankind from other animals, we must have had some simple logical thoughts. Since the understanding and use of words is itself a complex logical act, speech must have evolved later. It emerged as an additional species skill, greatly enhancing our reasoning powers. Much later still writing appeared, further distinguishing us from other species.

Logic is not only implicit in language, but in all human behavior that calls upon reason. All tool-making and much tool usage involves rational activity. To understand, invent, design and perfect, and even use, a hunting instrument (e.g. a trap, a spear, a bow and arrow) requires rational reflection, which need not be verbal. Similarly, fire-making, cooking, preparing pigments, agriculture and other such activities present in very early human societies, are all evidence of reasoning of sorts. Some animals can use natural tools, e.g. a stone to break a nut; but only man can make artificial tools. Perhaps if we analyzed precisely, through careful introspection today, the reasoning necessary for these creative activities, we could draw up an inventory of the earliest expressions of logic in mankind. By dating the earliest tools or their products, we might then be able to date the logic they imply. For instance, there are paintings in cave dwellings that are estimated to be some 40,000 years old. Obviously, man was already, at least that far back, a "rational animal."

In this way, by referring to our present mental and physical behavior, we can somewhat trace the historical development of logic. However, to say that logical abilities are inherited from our biological forebears does not mean that such potential is automatically actualized in us. For instance, the hypothetical (if–then) form of proposition is not some *pre-existing* genetically transmitted mental tool, but emerges initially, consciously or almost unconsciously, from reflection (which may be preverbal) on the laws of thought. The negations of conjunction found in the laws of non-contradiction and of the excluded middle move us to *invent and define* hypotheticals – in the ways I explained in my past work, *Future Logic*. If you look at the order of the chapters in that book, you will see that the chapter on

conjunction precedes the chapter on hypotheticals – it has to, because we cannot mentally truly understand hypothetical forms without first grasping the idea of conjunction.

I do not of course mean that each of us individually invents hypotheticals. Some individuals presumably did, probably in prehistory, in the hazy dawn of mankind. Eventually, these logical tools were passed on as cultural heritage, through language. But such discovery and transmission was not all done at once. There has been a development through historical times, which can indeed be traced to some extent. You can see it, for instance, by comparing the wording for if–then in the Tanakh and that in Talmudic literature; or again the latter to modern wording. Language changes over time in response to changing awareness of the nature of things. Verbal changes reflect changes in understanding. You can see also the growing awareness of the meaning and importance of hypotheticals in modern logical science, compared to earlier phases of logic. Aristotle hardly reflected on hypotheticals, although he used them liberally; whereas other logicians, starting with the Stoic Diodorus Cronus, made a big thing of them.

The same can of course be said concerning a fortiori discourse, which is more complex than if–then and syllogistic forms of discourse, since it appeals to them among others. It has a history, no doubt a very long and geographically scattered one. I have tried in the present work to trace some of that history. There is no doubt much more work to be done in this respect. The present treatise does not pretend to be a sweeping history of the a fortiori argument, of its use and discussion of it through all ages and in all cultures. Such a detailed, definitive history still needs to be researched and written – for a fortiori, and indeed for all forms of argument. Nevertheless, I believe the present work shows the way to carry out this mission, with some novel methods and applications. In particular, note well that it is not enough to just describe past ideas – they must also be judged[1].

We have seen in the present volume that Aristotle (4th century BCE) was probably the first to discuss a fortiori argument, though clearly not the first to use it. However, as far as we know he did not study such argument in as much detail as he did the syllogism. A fortiori argument appears in Jewish literature probably in the 13th century BCE; in Greek literature probably in the 7th or 8th century BCE; and in Indian and Chinese literature probably in the 5th century BCE. But these dates of course refer to *written* language; they do not exclude the existence of a fortiori *speech* centuries or even millennia before them. My own belief is that a fortiori discourse antedates the splitting of humanity into different linguistic groups. This is suggested by its wide geographical distribution. It should be sought in proto-languages, and in languages with little or no written component, though of course all languages change over time. Did aboriginal African, Asian, European, American and Australasian languages in the distant past include a fortiori phrases?

In any study of a fortiori logic, a large place must perforce be accorded to Jewish logicians. Because of the prime position of such argument throughout Talmudic and other rabbinic discourse, and in all lists of hermeneutic principles used in Judaism, it is natural for Jews to have had an especial interest in this thought form. These ancient practical and theoretical expressions of human thought, however imperfect some of them might have been found upon further scrutiny, were without doubt a strong stimulus to Jewish interest in logic in general and a fortiori logic in particular. A considerable expertise emerged over time, in the way of a 'family business', or more broadly put a 'tribal specialty'! (This is just a statistical observation, nothing more, I hasten to add.[2]) Doubtless, Christian and Islamic thought were strongly affected by Judaic logical practices in religious and legal contexts, even if other cultures also had their influences.

The present investigations show that whereas a fortiori expressions (like 'a fortiori', 'all the more', etc.) are found in all sorts of ancient Greek and other non-Jewish literature (philosophical and otherwise) – they are in such literature mostly used as 'turns of speech', i.e. as elegant rhetorical devices that could conceivably have been dispensed with, rather than as logical arguments crucial to proving some significant assertion. In this sense, it may be more appropriate to refer to such use more vaguely as a fortiori *discourse*, rather than argument. On the other hand, in Talmudic and rabbinic literature such discourse is usually indicative of an *argument*, in the true sense of the term. It is used to discover or prove something, something important to the speaker or writer (such as a legal ruling).

As regards religion, the following perhaps needs be said here. Much of human discourse over the centuries has occurred in religious contexts, so it is necessary for historians of logic to study religious documents in search of logic. This is, however, a delicate and touchy subject. My own approach to religion is, I think, objective. I do not hesitate to criticize where criticism is due. To my mind, spirituality is a natural dimension of man; man needs to earnestly reflect on the meaning and direction of his existence. Such reflection has historically produced a great

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[1]   The Kneales' *The Development of Logic* provides a good example of the critical approach to logic history.
[2]   A similar statistical observation I've made in the past (in a footnote to chapter 4 of my *Judaic Logic*): "I have an impression, for instance, that modern French discourse involves more use of a-fortiori than modern English discourse. To what extent that is true, and why it should be so, I cannot venture to say."

variety of man-made religions and sects. All religions (including the religion called atheism) claim omniscience of sorts, to establish their authority. All indeed contain some truth in some degree; but in fact none contains all truth or even only truth. This is evident, if only at the material level of logical and factual blunders; in many cases, there are also (in my personal opinion) serious spiritual blunders.

The logician or philosopher being a human being cannot be entirely objective on such crucial issues; but he or she has to try to. A good start is to notice and admit factual errors. There are many errors of that sort in all religious documents, though to varying degrees. The next thing to notice and admit is logical inconsistencies or gaps. Again, these occur in all religions, to various degrees. As regards spiritual blunders, their discovery is perhaps best ensured by comparative studies. If one studies only one religion (or no religion), one remains locked in a single conception of things; but when one broadens one's perspective, one is better able to see things differently, and eventually judge by oneself what is best point by point. That is 'natural religion' – one remains open to influence by the spirituality of our forefathers, but in a more lucid and rational manner, without fanaticism.

Nothing is further from true spirituality than blind faith, especially when this results in attempts to impose one's views on others by force. In this perspective, the religion called Islam has to be singled out, and regarded by all honest and moral men and women as a criminal organization, and treated as such. For Islam is exceptional in this day and age in its political ambition of world dominion. Whereas Judaism in the distant past (in its war against the Canaanites) and Christianity in the more recent past (in its persecution of Jews, for instance) have been guilty of religiously-motivated violence, Islam still today advocates and works towards mass conversion by coercion, and the enslavement or murder of those who refuse to submit to it. That Islam may contain some spiritually positive elements, which make it attractive to many people, does not change the fact that its political intentions and ways are evil. This fact must never be glossed over.

3. Assessing contemporaries

If philosophy – and its various branches, including epistemology and logic – is to be a scientific enterprise, it has to be subjected to strict standards of judgment, comparable to those of more materialistic disciplines. The way things stand, still today, is that anyone may say anything, however patently silly and contrary to fact, and hardly have to face any reasoned criticism from his or her peers. The reason is that many of those peers, even those with advanced university degrees and high academic positions, are themselves ignorant and/or confused, and so unable to tell the difference between wheat and chaff. As we have seen in the present volume, reputable journals, book publishers and encyclopedias publish papers and treatises that are full of glaring factual and logical errors. One wonders who sits on their editorial committees, till one realizes that the publishers themselves cannot tell the difference between those who have and those who haven't a clue, and so are content to rely on the qualifications 'on paper' of the staff they hire.

Too many of those who write articles or books simply do not know what they are talking about. Their motive is not pursuit of truth, but mere posturing. Rather than wanting to know, they want to appear as if they know. They seek glory, or at least position, not true knowledge. They are too proud to ask questions of those who know, because that would imply admission of ignorance. They would rather say nonsense than humbly face the fact of their nonsense and learn how to think straight. Sure, they do read – skim through – other people's work, but only enough to drop names. They especially like to sow famous names all over their discourses, so as to effortlessly bask in reflected glory. They think that they need only mention Kant, Frege or Wittgenstein to seem like cognoscenti. This is the way they were educated, how they gained their credits and diplomas. Their teachers do not know any better, having themselves been brought up that way.

I was constantly struggling with my conscience while writing the assessments in the present work, especially the assessments of the work of my mostly younger contemporaries, including Alexander Samely, Dov Gabbay, Stefan Goltzberg, Andrew Schumann, Allen Wiseman, Yisrael Ury, and Hubert Marraud, many of whom I have corresponded with and look upon with friendliness. Jewish ethics forbids the practice of *lashon hara* (the evil tongue). Telling people their faults to their face; reproving them in public and with harsh words – this is not moral behavior, even if no lies are told. Even just hurting a person's feelings is bad; but such speech is especially reprehensible if it may cause the person targeted loss of a job or of an economic opportunity. Yet, I finally gave myself free rein and allowed myself total frankness. My argument was that if these high moral standards were applied to scientific debate, it would spell the end of science, since science depends on open criticism. If I did not draw attention to the mistakes and foolishness I came across, I would be allowing them to persist indefinitely, causing many people harm. A distinction must be made between *ad hominem* argument and argument against a thesis. The former attacks the thesis *indirectly*, by defaming the person defending it; the latter attacks the thesis *itself*, even if doing so may cause vexation to its defender.

My whole writing career in logic and philosophy has been motivated by the desire to increase the use of logic in the world. I have spent, hours, days, weeks, months and years, unhealthily stuck at my desk, with this one goal in mind. Logic and philosophy today are still sorely lacking in intelligence and consistency. To become sciences in the best sense of the term, they need to be tamed by logic and empiricism. These fields should not remain playgrounds for dilettantes and poseurs. High standards of discourse must be imposed and maintained, as is done in mathematics or the physical sciences. I sincerely apologize to the people, alive or dead, that I have criticized, for hurting them personally in whatever ways. But in this conflict of values, the value of truth must take precedence over that of solidarity. The moment someone emits an opinion concerning matters of logic or philosophy, he must be ready to receive criticism. If the criticism is undeserved, he can and should defend his thesis; but if it is deserved, he should admit his intellectual transgressions without resentment, indeed with gratitude[3]. Criticism looks destructive; but it is necessary, because one cannot build the new without tearing down the old.

Here is the consolation: even as I criticize other writers for their errors, I must say that I am grateful to them for their errors, because were it not for their errors, I would not have thought of the corrections to their errors. It was by detecting and examining their mistakes that I found out how such mistakes could be avoided. These people were therefore to some extent my teachers or collaborators. Thus, people can contribute to a subject-matter even through errors. They may have made errors, but their work was not in vain. One thing they can be sure of, and that is that I have given their work a fair hearing. Very often, my analysis is longer that the passage or essay or treatise that I analyze. My criticism is never unjustified, always carefully reasoned. Remains to be seen how many of these people will have the grace to publicly admit their errors, and maybe even thank me for pointing them out! Those of them who were really curious about a fortiori logic, and wished to know the answers to questions at all costs, will study the present work and be grateful to me for it. As regards those whose foray into this field was entirely narcissistic, motivated by the desire to appear – to themselves if not to others – like great thinkers and discoverers: they will lose interest in the subject of a fortiori logic, and will not read the present work but instead be angry with me for having written it.

In any case, I take no ultimate credit for my work, considering that all intelligence, all insight and understanding, comes from God, since it says in the Torah: "In the heart of all that are wise-hearted, I have put wisdom" (Ex. 31:6). I naturally had to sacrifice time and generate effort, to discipline myself and persevere; but my abilities were given to me, as to everyone else, as a gift. Moreover, to repeat, the innovations made in the present work were not produced out of the blue. Many or even most of them emerged in reaction to the work of others, duly mentioned in it. Therefore, to repeat, it can be said of even the authors here greatly criticized that their work was far from useless and they participated indirectly in the formation of these innovations. Without their mistakes, I would have taken for granted many things that seemed obvious to me. Their errors or lacunae stimulated my responses, and turned them from potential to actual doctrines.

The theory of a fortiori presented in my *Judaic Logic* some 18 years ago was, I thought then, pretty thorough. However, in the past few years I came across some old and some new theories on the subject that I did not at that time know of – and these now needed to be commented on by me. When I first presented my own wholly original theory, I considered and claimed, implicitly if not explicitly, that other moods than those I validated were not valid and that other interpretations of the argument as a whole could no longer be upheld. Yet, still today, I find some writers trying to revive some old notions or to propose new ones inferior to or in conflict with mine. I had to respond to their implicit challenge, because after having read their theories I still viewed mine as the definitive and leading statement on this topic. I had no desire to 'defend my turf' egotistically – but I did feel responsible as a logician who has contributed considerably to this particular field.

It shocks me that some barely competent, if not quite incompetent, people try to make a name for themselves by formulating some new theory of a fortiori argument, thinking the field still virgin territory. They pursue innovation 'for its own sake,' even when there is no call for it. This is not a syndrome peculiar to a fortiori logic, but is unfortunately found in all fields of logic. I have often remarked in my works on the ridiculous tendency of some alleged logicians to look for some completely new approach to logic that would constitute the Copernican revolution for that field. Everybody seems to want to be the Einstein of logic theory, so as to impress the whole world with his incomparable genius. Such motivation is a sure formula for failure. A logician must be moved by awareness of his basic ignorance and stupidity, and a strong desire to improve his situation somewhat through hard work. He must regard himself as a humble servant, and seek to honestly clarify and understand the subject-matter at hand.

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[3]   I certainly practice what I preach here. Indeed, I beg all readers to draw my attention to any error they may happen to find in my work, be it typographical, grammatical, logical, or whatever. And when they do, I hasten to correct it (as soon as time allows), and to thank the readers for their solicitude.

4. Perspectives

I will finally briefly state what directions I think research in a fortiori logic should take – in general and in specific areas.

The first thing I would recommend to future researchers is to fully study and absorb the present work. Then they should continue collecting past ideas and commentaries on the subject of a fortiori argument, carefully assessing the contribution of each, looking to see if it offers any valuable new insights and cataloguing the errors and lacunae in it. In this way, future researchers can develop full understanding and skill in this field. Further empirical research into the history of use and discussion of a fortiori argument is also very important work. Finding evidence of this argument (and indeed others) in the phraseology of different languages – particularly exotic ones – would greatly help us determine how far back in human history it appeared. All of world literature should be scanned, with the aid of computers at least, for words or phrases indicative of a fortiori intent. Then each concrete argument found should be analyzed and duly classified.

Each cultural context should be studied separately, by knowledgeable researchers. Thus, for instance, Jewish literature, including the two Talmuds and all related works, requires specialized research. Similarly, Christian literature, Islamic literature, Buddhist literature, Hindu literature, etc. In order for results to be made readily available to all interested parties, they should ideally be made freely accessible in the Internet. The latter is a tall order, seeing as many people like the prestige and the income that ideally comes from publishing hardcopy articles or books. But if your motive is to accelerate the spread and pursuit of knowledge, nothing beats posting your findings for all to see on the Internet. In this context, translation into an international language – meaning English, today's *lingua franca* of science – is essential. Of course, the research work should preferably be done in the original language – but when it is done, make the results available to all through translation into English. Brief English abstracts are not enough. Without a common language progress is bound to be slow.

Many modern authors dealt with in the present volume attempt to define or describe a fortiori argument in symbolic terms – and they all, as we have seen, woefully fail to do so convincingly. This should not be viewed as a mere coincidence, but as a failure almost inevitable if a subject is approached through the medium of modern logic. People generally wrongly assume that what is stated in symbolic terms must have been "scientifically" pondered. The opposite is closer to the truth. Symbols more often than not are designed to hide the ignorance or triviality or confusion or errors of those who indulge in them. They are a smokescreen behind which those who have not understood a subject hide, hoping no one will find them out. One purpose of this book has been to demonstrate the superiority of ordinary-language logic over symbolic logic. It is best to use ordinary language to discover and deeply analyze logical forms; only after this main job has been completed is it permissible, although not necessary, to express the results by means of symbols.

Unfortunately, symbolism is all too often a refuge for pretense by not very bright people. But let the child in us blurt out that their emperor's new clothes are non-existent. The main sin of modern, symbolic logicians is to try to circumscribe and finalize something which is far too big for human control. The domain of logic is something that must always remain *open* for new insights and further evolution; it cannot be locked down with simplistic formulas akin to those of mathematics. Symbolic logic based on paltry human analysis can only produce paltry results. What counts is the logician's depth of understanding *before* he starts concocting symbolic formulas. Otherwise such formulas are merely the frozen evidence of the superficiality of his thought. There is room for symbolic logic in logic – but it is something to be done, if at all, at the tail end of the logician's work. Once a fortiori argument is fully understood, as in the present study, it is easy to reword the results in symbolic terms; but understanding must come first.

If universities continue to produce students who can restate and reshuffle worthless symbolic formulas ad nauseam, but are unable to intelligently analyze actual human discourse, there is little hope for the survival of logic in our culture. I would propose we replace the current fashion of 'modern, symbolic' logic with what might be called 'neo-classical' logic. The latter is a logic whose main goal is deep understanding, rather than superficial posturing.

As regards the development of intelligence, I would like to here say just how big a role sitting meditation has played in my writing of the present book. Most of the significant insights to be found in it were obtained in the course of my daily meditations. It is possible that many of the problems here solved would have eventually been solved anyway; but they were certainly solved more readily and efficiently thanks to meditation. When the mind is relatively quiet, it is able to concentrate with much greater force on any issue put before it. So, another methodological recommendation I would offer to anyone wishing to get into logic research is: learn and regularly practice meditation so as to sharpen your intellectual capabilities and develop patience.

My regular practice of tai chi and other sporting activities should also be mentioned as a source of health and equilibrium. Having mentioned meditation, I should also mention the Sabbath: this weekly day of obligatory rest has always been for me a source of renewed energy and inspiration. Finally, I wish to thank God for kindly allowing me

to complete this arduous task. I pray that this work, together with my other works, improves many people's minds and help them to live their lives more wisely.

APPENDICES

1. A fortiori discourse in the Jewish Bible

There are at least 46 instances of a fortiori discourse in the Tanakh (the Jewish Bible). Their dating is traditionally referred to the dates of the events in books concerned, viz. Genesis (Lamekh in 'antediluvian' times; Joseph's brothers in patriarchal period); Exodus, Numbers, and Deuteronomy (Moses in the Sinai), 1 & 2 Samuel (Davidic kingdom), and so forth (subsequent history till the 1st Exile period)[1].

Of the 46 cases listed below, only 11 are given in the Midrashic work *Genesis Rabbah*; these are here marked as "Listed by GR." In my *Judaic Logic* (1995), I listed another 24 cases[2] found through independent research, to which I later added 1 case in a footnote at the end of chapter 6 (which I found by chance in June 1998) and 1 case in Addendum 4 (which was suggested to me by a reader in 2001). However, I discovered while writing the present book that 13 of the additional cases listed in my book were in fact known to others (all apparently found by Wolf Einhorn in 1838, according to Louis Jacobs in 2005); these cases in common are here not marked at all. Still, the remaining 13 of the additional cases listed in my book seem to have been unknown to others; these are here marked as "New from AS." To these must be added 1 more case, that I only recently discovered, namely Ezekiel 14:13-21. On the other hand, Louis Jacobs listed 8 cases (presumably found in Wolf Einhorn) that I had not previously found; these are here marked as "New from WE."[3]

I have put the arguments in 'if–then' form, showing the minor premise and conclusion, but leaving the major premise tacit. If the subjects are different and the predicates are the same, the mood is subjectal. If the subjects are the same and the predicates different, the mood is predicatal. If there is 'enough of' the middle item, the mood is positive; if there is 'not enough of' the middle item, the mood is negative. I have formulated all the arguments in copulative form, even when an implicational form might have been closer to the original. If the mood is positive subjectal (marked +s), or negative predicatal (marked –p), the argument is from the minor term to the major term. If the mood is negative subjectal (marked –s), or positive predicatal (marked +p), the argument is from the major term to the minor term. Most arguments are purely a fortiori, but some are clearly intended as a crescendo (these are marked by a &). Note that some classifications here differ from those in my *Judaic Logic*; those here are more reliable[4].

The following table shows the number of instances of a fortiori argument in the Tanakh, classified by mood of copulative argument concerned. We see that the distribution is about even for the first three moods (14, 13, 15), while there are only 4 instances of the fourth (still, it is significant). It is interesting that 41% (19/46) of the arguments are predicatal, considering how little attention this form has received from most commentators. Only 13% (6/46) of the arguments are a crescendo, and yet many commentators have assumed this form to be the essence of a fortiori argument.

Mood	Instances	Of which &
Positive subjectal {+s}	14	3
Negative subjectal {–s}	13	3
Positive predicatal {+p}	15	0
Negative Predicatal {–p}	4	0

Table A1.1

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[1]   I do not here give traditional chronology, based on *Seder Olam* calculations, because modern secular historians do not advocate quite the same dates. The present work is obviously not the place here to debate this hot issue.

[2]   Although in chapter 6 of my *Judaic Logic*, I rejected three of the instances here counted as genuine a fortiori, I now see more clearly how they can be so construed. See 2 Kings 18:23-24 and its repetition in Isaiah 36:8-9, and 2 Chronicles 32:15.

[3]   The transliteration of Heb. words in "(*italics*)" are sometimes given here because they were helpful indices in the research process. The remarks in square brackets "[xxx]" are my own interpolations to facilitate the proposed a fortiori reading where necessary. I also on occasion reorder the clauses involved to accord with the perceived sequence of reasoning or show up the utility of an operator.

[4]   I there (in chapter 5.4) had Dt. 31:27 as +p (here it is +s), 1 Sam. 23:3 as +a (here it is +p), Jer. 12:5 as +a (here they are –p).

Genesis 4:24. Lemekh ben Methushael: "If (*ki*): Cain shall be avenged sevenfold, then (*ve*): Lemekh [shall be avenged] seventy and seven-fold." (Listed by GR.) {–s &} If an intentional killer is not abhorred enough to be punished immediately, then an unintentional killer will remain unpunished for a much longer time.

Genesis 44:8. Joseph's Brothers: "Behold (*hen*): the money, which we found in our sacks' mouths, we brought back unto thee out of the land of Canaan; then (*ve*): how (*ekh*) should we steal out of thy lord's house silver or gold?" (Listed by GR.) {+p} If the accused were honest enough to return found goods, then they must have been honest enough not to steal anything.

Exodus 6:12. Moses: "Behold (*hen*): the Children of Israel have not hearkened unto me; then (*ve*): how (*ekh*) shall Pharaoh hear me?" (Listed by GR.) {–s} If the Israelites, who have much faith, have not had enough of it to listen to Moses, then the chief of the Egyptians, who has far less faith (if any), will not have enough of it to do so.

Numbers 12:14. God: "[Granting that:] if (*ve*) her father had but spit in her face, should she not (*halo*) hide in shame seven days? [Similarly, since God is angry with her,] let her be shut up without the camp seven days." (Listed by GR.) {+s} If someone causing paternal anger is culpable enough to deserve seven days isolation, then someone causing Divine anger is culpable enough to deserve seven days isolation.

Deuteronomy 31:27. Moses: "Behold (*hen*): while (*be*) I am yet alive with you this day, ye have been rebellious against the Lord; how much more (*af*): in the time (*ki*) after my death, so (*ve*) [i.e. ye will be rebellious]?" (Listed by GR.) {+s} If the people during Moses' lifetime are unfaithful enough to rebel, then they after his death will be unfaithful enough to rebel.

Judges 14:16. Samson to his wife: "Behold (*hine*), I have not told it [the solution to my riddle to] my father nor my mother, and (*ve*) shall I tell [it to] thee?" (New from WE.) {–p} If Samson was not trusting enough to tell the secret to his parents, then he won't be trusting enough to tell it to his wife.

1 Samuel 14:29-30. Jonathan: "See (*reu*): because (*ki*) I tasted a little of this honey, how (*ki*) mine eyes are brightened. How much more (*af*): if (*ki*) haply the people had eaten freely today of the spoil of their enemies which they found, then (*ki*) would there not have been a much greater slaughter among the Philistines?" {+s &} If someone eating a little honey is energized enough to have his eyes brighten, then people eating lots of food are energized enough to do that and much more.

1 Samuel 17:37. David: "The Lord who saved me from the paw of the lion and the paw of the bear [= innocent animals], He will [surely] save me from the hand of the Philistine [= willful enemy]." (New from AS[5].) {+p} If David had spiritual credit enough to be saved from innocent creatures, then he has credit enough to be saved from evil ones.

1 Samuel 21:6. David: "Of a truth (*ki*): when (*be*) I came out, though (*ve*) it was but a common journey, yet (*im*) women have been kept from us about these three days, and (*ve*) the vessels of the young men were holy; how much more (*af*): when (*ki*) today there shall be holy bread in the vessels, so (*ve*) [i.e. have we avoided women and kept the young men's vessels holy]." (New from AS.) {+p} If we were virtuous enough to practice abstinence on a common journey, then we are virtuous enough to do so on a special day like today.

1 Samuel 23:3. David's men: "Behold (*hine*): here in (*be*) Judah [= our own territory], we are afraid; how much more (*af*): if (*ki*) we go to Keilah [= enemy territory], so (*ve*) [i.e. will we be afraid]?" (Listed by GR.) {+p} If we lack confidence enough that we feel fear while on our own territory, then we will lack confidence enough that we will feel fear when on enemy territory.

2 Samuel 4:10-11. David: "If (*ki*): [when] one told me saying, 'behold, Saul is dead' and (*ve*) he was in his own eyes as though he had brought good tidings, then (*ve*) I took hold of him and (*ve*) slew him in Ziklag in the way of reward. How much more (*af*): when (*ki*) wicked men have slain a righteous man in his own house upon his bed, then (*ve*) now shall I not (*halo*) require his blood of your hand and (*ve*) take you away from the earth?" (New from AS.) {+s}

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[5]   But actually found by Mark Leroux.

If someone who merely announced the death of Saul, David's respected adversary, was judged wicked enough to deserve execution, then the people who actually killed a respectable man, Ish-bosheth the son of Saul, who did David no harm, must be judged wicked enough to deserve execution.

2 Samuel 12:18. David's servants: "Behold (*hine*): while (*be*) the child was yet alive, [David's sorrow was so great that] we spoke unto him, and (*ve*) he hearkened not unto our voice; then (*ve*): how (*ekh*) shall we tell him that the child is dead, so that (*ve*) he do himself some harm?" {+s &} If David while his child still lived was sorrowful enough to be utterly distracted, then David now that the child has died will be sorrowful enough to cause himself some harm.

2 Samuel 16:11. David: "Behold (*hine*): my son, who came forth from my body, seeketh my life [still, I do not react]; how much more (*af*): in the case of (*ki*) this Benjamite now [who is less close], and curseth [me], then (*ve*) should I let him alone; for the Lord has bidden him." {+p} If David was self-controlled enough to avoid reacting under attack from his own son, then David will be self-controlled enough to avoid reacting under attack from a more remote enemy.

1 Kings 8:27. Solomon: "Behold (*hine*), heaven and the heaven of heavens cannot contain thee; how much less (*af*): in the case of (*ki*) this house that I have builded, will (*ki*) God in very truth dwell on the earth [i.e. be contained in this house]?" This is repeated in 2 Chronicles 6:18. {–s} If the heavens are not big enough to contain God, then an earthly house is not big enough to do so.

2 Kings 5:13. Naaman's servants: "Granting (*ki*): had the prophet bid thee do some great thing, wouldst thou not (*halo*) have done it? how much rather (*af*): he [merely] saith to thee: wash and be clean? then (*ve*) [you should do it]" (New from AS.) {+s} If the prophet making some difficult request would have seemed powerful enough in your eyes to succeed in healing you, causing you to obey him, then his making an easy request suggests he may be more powerful than you expected and indeed powerful enough to heal you, and should cause you to obey him.

2 Kings 10:4. The rulers of Jezreel in Samaria: "Behold (*hine*): the two kings [Joram and Ahaziah, who were powerful men], stood not before him [Jehu]; then (*ve*): [we, who are relatively weak,] how (*ekh*) shall we stand [before him]?" {–s} If the two kings were not strong enough to resist Jehu, then we are not strong enough to do so.

2 Kings 18:23-24. Rab-shakeh (emissary of the king of Assyria): "[Since] thou puttest thy trust on Egypt for chariots and for horsemen, I will give thee two thousand horses, if thou be able on thy part to set riders upon them. [But you are not able to do even that, and so cannot hope to defeat us.] How then (*ve-ekh*) canst thou [without gift of horses] turn away the face of one captain, even of the least of my masters servants?" (New from AS.) This is repeated in Isaiah 36:8-9. {–s &} If you had 2000 horses, you would not have enough power to defeat the Assyrian army, then without such a gift you do not have enough power to do so, not even to defeat a minor captain of it.

Isaiah 36:8-9. Rab-shakeh (emissary of the king of Assyria): "[Since] thou puttest thy trust on Egypt for chariots and for horsemen, I will give thee two thousand horses, if thou be able on thy part to set riders upon them. [But you are not able to do even that, and so cannot hope to defeat us.] How then (*ve-ekh*) canst thou [without gift of horses] turn away the face of one captain, even of the least of my masters servants?" (New from AS.) This is the same as 2 Kings 18:23-24. {–s &}

Isaiah 66:1. God: "The heaven is My throne, and the earth is My footstool; where (*eizeh*) is the house that ye may build unto Me? And where (*eizeh*) is the place that may be My resting-place?" (New from WE.) This is comparable to 1 Kings 8:27 and 2 Chronicles 6:18. {–s} If the heavens are not big enough to contain God, then an earthly house is not big enough to do so.

Jeremiah 12:5. God: "If (*ki*): thou hast run with the footmen and (*ve*) they have wearied thee, then (*ve*): how (*ekh*) canst thou contend with horses [and not be wearied]? and if (*u*): in the land of peace, thou dost [hardly] feel secure; then (*ve*): in the wild country of the Jordan, how (*ekh*) wilt thou do [feel secure]?" (2 instances, listed by GR.) (Both –p) If you are not strong enough to cope with the easier challenges, then you are not strong enough to cope with the more difficult ones.

Jeremiah 25:29. God: "For, lo (*hine*), I begin to bring evil on the city whereupon My name is called, and (*ve*) should ye [who is less virtuous] be utterly unpunished?" (New from WE.) {–s} If those calling on my name are not absolved enough to escape my wrath, then you less virtuous folk are not absolved enough to escape my wrath.

Jeremiah 45:4-5. God: "Behold (*hine*), that which I have built will I break down, and that which I have planted I will pluck up; and this in the whole land. And (*ve*) seekest thou [who is less valued] great things for thyself?" (New from WE.) {–s} If the things I worked for are valued by Me not enough to escape being undone, then the things you work for are valued by Me not enough to escape being undone.

Jeremiah 49:12. God: "Behold (*hine*), they to whom it pertained not to drink of the cup shall assuredly drink; and (*ve*) art thou [who is more guilty] he that shall altogether go unpunished?" (New from WE.) {+s} If people who are not reprehensible are implicated enough to be punished, then people who are reprehensible are implicated enough to be punished.

Ezekiel 14:13-21. God: "Son of man, when a land sinneth against Me by trespassing grievously, and I stretch out My hand upon it… and send famine upon it… [Or] if I cause evil beasts to pass through the land… Or if I bring a sword upon that land… Or if I send a pestilence into that land… though Noah, Daniel, and Job, were in it, as I live, saith the Lord God, they shall deliver neither son nor daughter; they shall but deliver their own souls by their righteousness… How much more (*af ki*) when I send My four sore judgments against Jerusalem, the sword, and the famine, and the evil beasts, and the pestilence, to cut off from it man and beast." (New from AS.) {–p} If such holy men lack sufficient spiritual credit to prevent the execution of each of the four negative decrees separately, then they lack enough to stop all four of these decrees together.

Ezekiel 15:5. God: "Behold (*hine*): when (*be*) it [the vine-tree] was whole, it was not meet for any work; how much less (*af*): when (*ki*) the fire hath devoured it and (*ve*) it is burned, shall it then (*ve*) yet be meet for any work?" (Listed by GR.) {–s} If when whole the vine-tree was not in good condition enough to be useful; then now when damaged it is not in good condition enough to be useful.

Ezekiel 33:24. God: "They that inhabit those waste places in the land of Israel speak, saying: Abraham was one, and he inherited the land; but (*ve*) we are many; the land is given us for inheritance." (New from WE.) {+s} If one man is important enough to inherit the land, then many men are important enough to inherit the land. (Obviously, though God is reporting this argument, He is not its author. It is not very credible, and rightly rebutted in the verses 25 and 26: it is not numbers but moral worth that makes possible inheritance of the land.)

Jonah 4:10-11. God: "Thou hast had pity on the gourd, for which thou hast not laboured, neither madest it grow, which came up in a night, and perished in a night; and should not I (*vaani lo*) have pity on Nineveh, that great city, wherein are more than sixscore thousand persons that cannot discern between their right hand and their left hand, and also much cattle?" {+s} If a mere gourd etc. can be appreciated enough to be cared for (as by Jonah), then a great city etc. can be appreciated enough to be cared for (by God).

2 Chronicles 6:18. Solomon: "Behold (*hine*), heaven and the heaven of heavens cannot contain Thee; how much less (*af*): in the case of (*ki*) this house that I have builded, will (*ki*) God in very truth dwell on the earth?" (New from AS.) This is the same as 1 Kings 8:27. {–s} If the heavens are not big enough to contain God, then an earthly house is not big enough to do so.

2 Chronicles 32:15. Sennacherib, king of Assyria (through his messengers) says: "For (*ki*): no god of any nation or kingdom was able to deliver his people out of my hand, and out of the hand of my fathers; likewise (*af*): therefore (*ki*), shall your God [presumed by the messengers as no different from other gods] not be able to deliver you out of my hand." (New from AS.) {–s} If other national gods were not powerful enough to deliver their respective nations, then the God of Judah is not powerful enough to deliver his nation. (This of course wrongly equates God with non-gods, but it is how the Assyrian king thinks.)

Psalms 78:20. Asaph: "Behold (*hen*): He struck a rock, then (*ve*) waters flowed and (*u*) streams burst forth. In that case (*gam*): bread He can give; is there any doubt that (*im*): He will prepare meat for His people?" (New from AS.)

{+p} If God is powerful enough to draw water from a rock[6], then He is powerful enough to feed His people with bread and meat.

Psalms 94:9-10. Moshe: "He who implanted the ear, does He not (*halo*) hear?" "If (*im*) He formed the eye, does He not (*halo*) see?" "He who chastises nations, does He not (*halo*) reprove [the individual]?" (3 instances, New from AS.) (All 3 +p) If God is powerful enough to implant the ear and form the eye, then He is powerful enough to hear and see. If God is powerful enough to chastise nations, then He is powerful enough to reprove individuals.

Job 4:18-19. Eliphaz the Temanite: "Behold (*hen*): He puts no trust in His servants, and (*u*) His angels he charges with folly; how much more (*af*): those who dwell in houses of clay, whose foundation is in the dust [does He distrust and charge with folly]?" {+p} If God is perspicacious enough to judge His servants/angels as untrustworthy and unwise, then He is perspicacious enough to judge mere human beings as untrustworthy and unwise.

Job 9:13-14. Job: "God will not withdraw His anger; the helpers of Rahab did stoop under Him. How much less (*af ki*) shall I answer Him, and choose out my arguments with Him?" (New from WE.) {−s} If Rahab's helpers were not worthy enough to argue with God, then Job is not worthy enough to do so.

Job 15:15-16. Eliphaz the Temanite: "Behold (*hen*): He puts no trust in His holy ones; and (*ve*) the heavens are not clean in His sight. How much less (*af*): one who (*ki*) is abominable and filthy, man, who drinks iniquity like water [does He trust or consider clean]!" {+p} If God is demanding enough to judge His holy ones as untrustworthy and the heavens as unclean, then He is demanding enough to judge mere human beings as untrustworthy and unclean.

Job 25:5-6. Bildad the Shuhite: "Behold (*hen*): even the moon has no brightness, and (*ve*) the stars are not pure in His sight; how much less (*af*): man, that (*ki*) is a worm [is bright and pure in His sight]?" {+p} If God is perfectionist enough to judge the moon as obscure and the stars as impure, then He is perfectionist enough to judge mere human beings as obscure and impure.

Proverbs 11:31. Solomon: "Behold (*hen*): the just man shall be recompensed on earth: how much more (*af*): the wicked and the sinner, so (*ki*) [i.e. shall be recompensed on earth]." (Listed by GR.) {+s} If the just man is imperfect enough to be recompensed on earth, then the wicked and sinner are imperfect enough to be recompensed on earth.

Proverbs 15:11. Solomon: "If (*ki*): hell and destruction are before the Lord; how much more (*af*): the hearts of the children of men [are before the Lord]?" {+p} If God is powerful enough to look into hell and destruction, then He is powerful enough to look into people's hearts.

Proverbs 19:7. Solomon: "If (*ki*): all the brethren of the poor do hate him, how much more (*af*): do his friends go far from him?" {+p} If the poor man is disliked enough that his brothers avoid him, then he is disliked enough that his friends avoid him.

Proverbs 19:10. Solomon: "If (*ki*): for a fool to have luxury is not seemly; how much less (*af*): for a servant to have rule over princes [would be seemly]." {+s} If for a fool to have luxury is inappropriate enough to be unseemly, then for a servant to have rule over princes is inappropriate enough to be unseemly.

Proverbs 21:27. Solomon: "If (*ki*): [even brought with a 'sincere' intent] the sacrifice of the wicked is an abomination; how much more (*af*): brought with a wicked intent [is it abomination]?" {+s} If the sacrifice of the wicked brought with a 'sincere' intent is abominable enough to be rejected, then the sacrifice of the wicked brought with a wicked intent is abominable enough to be rejected.

Esther 9:12. Ahasuerus says: "In (*be*) Shushan the capital, the Jews have slain and destroyed five hundred men and the ten sons of Haman; in (*be*) the rest of the king's provinces, what (*meh*) have they done? [i.e. surely many more!]" (Listed by GR.) {+s &} If the Jews in Shushan have found and destroyed as many as 500 anti-Semites, then the Jews in provinces have found and destroyed many more than 500 of their enemies. (Not sure of a fortiori intent, in my view; but kept because traditional.)

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[6]   The subject of "he struck a rock" could be Moses, but the cause of the water gushing from it must be God. Likewise, it is God that provides bread and meat. This is obvious from the Torah account (Ex. 17:6, 16:12).

Daniel 2:9. Nebuchadnezzar: "Thus (*lahen*): tell me the dream, and (*ve*): I shall know that you can declare its interpretation to me [since it is more difficult to tell it than to interpret it]." (New from AS.) {+p} If Daniel is powerful enough to tell the dream, then he is powerful enough to interpret it.

Nehemiah 13:26-27. Nehemiah: "Did not Solomon king of Israel sin by these things? yet among many nations was there no king like him, and he was beloved of his God, and God made him king over all Israel; nevertheless even (*gam*) him did the foreign women cause to sin. Shall we then (*ve*) hearken unto you to do all this great evil, to break faith with our God in marrying foreign women?" (New from WE.) {+s} If king Solomon, who was not very weak, was weak enough to be brought to sin by foreign women, then we, who are much weaker, are weak enough to be brought to sin by foreign women.

2. A fortiori discourse in the Mishna

If we wish to enumerate the use of a fortiori argument in the Talmud, we must first make a census of its use in the Mishna (closed ca. 200 CE) – and perhaps also the Tosefta (closed ca. 300 CE), if any – since the Mishna is a document found in both the Jerusalem and Babylonian Talmuds. Alternatively, when we list all the a fortiori arguments in each Talmud, we shall have to specify which of those arguments are Mishnaic and there will be overlap equal to that set. In any case, the two Talmuds do not cover the same Mishna divisions: the Jerusalem treats the first four and the Babylonian treats the second to fifth. Also, neither of them treats the last (sixth) division; so, the Mishna must in any case be researched separately (in this regard, see Neusner: *Rabbinic Literature: An Essential Guide*, p. 34).

In truth, even though the Mishna is habitually mostly looked at and seen through the prism of Gemara commentary, it is historically a separate document and should be treated as such. Three sorts of listing are possible. The simplest is just a list of locations where the argument can be found. A more thorough listing includes the relevant extract without comment. The best listing must include a full analysis of each case found (this of course requires acquaintance with traditional commentaries, notably the Gemara, but also Rashi, Tosafot and others). Ideally, this research should be done with reference to the original Hebrew and Aramaic texts; but English translations would be a good start. All this is of course a very big job, but it is feasible today with relative ease and speed thanks to the possibilities of computer assisted search for key words and phrases (although these should not be considered as yielding exhaustive results).

Alexander Samely, in about 2002, made a valiant and apparently original attempt to list and analyze all the a fortiori arguments in the Mishna, posting the results of his work on the Internet, in a database (which may be viewed at: mishnah.llc.manchester.ac.uk/search.aspx). As we saw in the main text, some of the cases he listed are not credible, and a few more cases need to be added to the list. On this basis, I propose the following tentative list of *qal vachomer* arguments in the Mishna, which includes 46 attempted cases of a fortiori argument, of which 42 are valid and 4 are invalid.

There is no guarantee that Samely's collection of cases is exhaustive, as he has found additional cases over time; and moreover he apparently missed, or for some reason disregarded, 4 cases, viz.: 2 in *Avot*, 1 in *Kelaim*, and 1 in *Pesahim* (6:5). Some of his analyses are essentially retained, though rewritten by me in standard form, but some cases are partly or entirely reanalyzed by me. I here take the "texts" proposed by him for granted, although I have some doubts that these quotations are all really accurate (in view of spelling mistakes, unexplained brackets, etc.). Hence, the exposition below (alphabetically ordered) is only tentative, a model and starting point for future research based on more reliable source texts.

The following table lists the results obtained:

Reference	Qty	valid or not	Q-P, P-Q or pari	Mood	Possible a crescendo
Arakin 8:4	1	valid	P to Q	-s	
Avot 1:5	1	valid	Q to P	+s	&
Avot 6:3	1	valid	Q to P	+a	&

Baba Qama 2:5 (a)	1	valid	Q to P	+a	&
Baba Qama 2:5 (b)	1	valid	Q to P	+a	
Bekoroth 1:1	1	valid	pari	+s	
Bekoroth 9:1	1	valid	P to Q	-s	
Berakoth 9:5	1	valid	Q to P	+s	
Demai 2:2	1	valid	Q to P	-p	
Eduyyoth 6:2	1	valid	Q to P	+s	
Eduyyoth 6:3 (a)	1	valid	pari	+s	
Eduyyoth 6:3 (b)	1	valid	pari	+s	
Hullin 2:7	1	valid	P to Q	-s	
Hullin 10:1	1	valid	Q to P	+a	
Hullin 12:4-5	1	valid	Q to P	+s	&
Kilaim 8:1	1	valid	Q to P	+s	
Makkoth 1:7	1	valid	pari	+s	
Makkoth 3:15 (a)	1	invalid	Q to P	+s	&
Makkoth 3:15 (b)	1	valid	Q to P	+s	&
Menahoth 8:5	1	valid	Q to P	+s	
Nazir 7:4	1	valid	Q to P	+s	
Nedarim 10:7	1	valid	P to Q	+p	
Negaim 10:2	1	valid	Q to P	+s	
Negaim 12:5 (a), (b), & (c)	3	valids	Q to P	+s, +s, +s	&&&
Pesahim 6:2 (a)	1	valid	Q to P	+s	
Pesahim 6:2 (b) & (c)	2	valid rivals	Q-P, P-Q	+s, -s	
Pesahim 6:5	1	valid	Q to P	+s	
Sanhedrin 6:5	1	valid	Q to P	+s	&
Shebuoth 3:6	1	valid	Q to P	+s	
Sotah 6:3 (a)	1	valid	Q to P	+s	
Sotah 6:3 (b)	1	valid	Q to P	+s	
Temurah 1:1 (a) & (b)	1 + 1	valid, invalid	pari, Q-P	-s, -s	
Terumoth 5:4 (a) & (b)	2	invalids	Q to P	-s, -s	
Yadayim 4:7	1	valid	Q to P	+s	
Yadayim 4:8	1	valid	P to Q	-s	
Yebamoth 8:3	1	valid	P to Q	-s	
Yom Tov 5:2	1	valid	Q to P	+s	
Zebahim 7:4	1	valid	Q to P	+s	
Zebahim 7:6	1	valid	P to Q	-s	
Zebahim 8:12	1	valid	P to Q	-a	
Zebahim 12:3	1	valid	Q to P	+s	

Table A2.1

The above list can be summarized as follows.

Mood of a fortiori argument	Number found	Of which, a pari	Possibly a crescendo

Positive subjectal {+s}	28	4	8
Negative subjectal (-s)	11	1	
Positive predicatal {+p}	1		
Negative predicatal (-p)	1		
Copulative	41	5	8
Positive antecedental (+a)	4		2
Negative antecedental (-a)	1		
Positive consequental (+c)	0		
Negative consequental (-c)	0		
Implicational	5	0	2
Total	46	5	10

Table A2.2

The detailed expositions that these tables are based on are given below.

Note that three arguments are invalid because they are negative subjectal, and yet go from minor to major; however, one of these invalids, viz. Terumoth 5:4 (b) is intentionally so, having been put forward only to illustrate the invalidity of its predecessor, viz. (a). A fourth argument, viz. Makkoth 3:15 (a), is invalid because it is a contrario. So there are, it seems (unless alternative explanations for them are put forward), two unintentional formal errors of a fortiori reasoning in the Mishna (2 out of 46 = 4.3%). But one of these errors is challenged by someone else within the text; so only one, viz. Temurah 1:1 (b), apparently passes unnoticed. Note also, in passing, that all the arguments listed involve normative judgment of some sort; none are purely factual in content.

Regarding the language used in these 46 arguments, a few observations are worth making. The main indicators or markers of a fortiori argument (in the English translation, which of course depends on the translator's choices of words) are: inference, inferred, logical[7] (19 cases), how much more, how much more so, how much the more, how can, all the more, still more (16 cases), light-heavy and the like[8], more stringent (10 cases), a fortiori (2 cases), as well as various other expressions: so, also, so also, just as … so, to these… and yet, *dayo*, etc. (16 cases); these add up to more than 46 cases because there are overlaps, i.e. some a fortiori argument are indicated by more than one marker.

It seems that, in the Mishna at least, a crescendo arguments are always indicated by expressions with 'more', viz.: 5 how much more, 2 how much more so, 1 how much the more, 1 all the more and 1 more stringently. However, note well, expressions with 'more' are not exclusively used for a crescendo discourse, but may also signal purely a fortiori discourse. As regards the *dayo* objection, it only appears in relation to the two arguments in Baba Qama 2:5, of which the first is a crescendo, while the second is purely a fortiori. There is no mention of *dayo* in relation to the remaining nine (possibly) a crescendo arguments[9], which anyhow do not involve inference of a penalty from the Torah. Assuming our list is exhaustive, there are no other applications of *dayo* in the Mishna (though certainly many more in the Gemara). This is quite surprising![10]

Arakin 8:4

TEXT: "A man may devote part of his flock or of his herd or of his Canaanite slaves and bondwomen, or 'of the field of his possession'; and if he devoted the whole of them they are not devoted - words of R. Eliezer. R. Eleazar ben Azariah said: And if Man does not even have authority to devote to the High One everything he owns, *how much more* is Man obliged to protect his possessions!"

MY READING: negative subjectal, major to minor**.**

> Man's use of his possessions for holy ends (P) is more religiously valuable (R) than man's use of
> his possessions for profane ends [including the waste of his possessions without purpose] (Q) is.

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[7]   These are formulated as negative questions: e.g. "Is it not an inference that?" I presume that such expressions refer to the phrase "*eino din*" in the original Hebrew. I do not at this time have the resources to verify translations.

[8]   These are all, I presume, literal translations of expressions like *qal vachomer* or *miqal lechomer*.

[9]   The expression "*dayo*" is used in Zebahim 7:6, but as I point out there this use is rhetorically and not literally intended.

[10]   This may explain why the lists of Hillel and R. Ishmael do not mention a *dayo* principle, though they mention a fortiori argument. Also, it suggests that the two *dayo* objections in the said Mishna may have been intended as merely ad hoc solutions, which were only later elaborated as guiding principles in the Gemara.

If man's use of his possessions for holy ends (P) is *not* religiously valuable (R) enough to be authorized without limit (S),
then man's use of his possessions for profane ends (Q) is *not* religiously valuable (R) enough to be authorized without limit (S).

Avot 1:5

TEXT: "Yose ben Yohanan of Jerusalem was wont to say:… Engage not in overmuch converse with a woman. If he said this of his wife, *how much more* does it apply to the wife of another."[11]

MY READING: positive subjectal, minor to major. Could be intended as a crescendo.

Speaking to another man's wife (P) is more to be avoided (R) than speaking to one's own wife (Q) is.
If speaking to one's own wife (Q) is to be avoided (R) enough to be discommended by Yose (S),
then speaking to another man's wife (P) is to be avoided (R) enough to be discommended by Yose (S).

An a crescendo reading would imply a stronger recommendation against speech in the latter case, in proportion to its moral unsuitability.

Avot 6:3

TEXT: "If David king of Israel, who having learned from Ahitophel but a couple of worldly matters yet called him his master… *how much the more* honor must be shown by the one who learns from his associate but a single chapter, law, or verse or saying, or even a single letter of Torah."

MY READING: positive antecedental, minor to major. Could be intended as a crescendo.

Learning holy matters from someone (P) entails honoring him (R) more than learning worldly matters from him (Q) does.
If learning worldly matters from someone (Q) entails honoring him (R) enough to justify calling him one's master (S),
then learning holy matters from someone (P) entails honoring him (R) enough to justify calling him one's master (S).

An a crescendo reading would imply still more demonstrations of gratitude in the latter case, in proportion to the greater honor due to the Torah.

Baba Qama 2:5 (a) and (b)

TEXT: "An ox which causes damage in the [private] domain to him that is injured - thus, if it gored, pushed, bit, lay down, or kicked in the public domain it pays only half-damages; but if in the private domain of him that was injured, R. Tarfon says: It pays full damages. But the Sages say: Half-damages. R. Tarfon said to them: What! If they have dealt leniently with damage caused by tooth or foot in the public domain, when no restitution is imposed, and stringently with like damage in the private domain of him that is injured, when full damages are imposed, then since they have dealt stringently with damage caused by the horn in the public domain, when half-damages are imposed, ought we not therefore to deal *the more stringently* with damage caused by the horn in the private domain of him that was injured, so that full damages shall be imposed? They answered: It is enough (*dayo*) if the inferred is as strict as that from which it is inferred: if [for damage] caused by the horn in the public domain half-damages [are imposed], so also [for like damage] in the private domain of him that was injured, half-damages [only are imposed]."

MY READING: The first argument by R. Tarfon, in view of its 'proportional' conclusion, may be taken minimally as an argument pro rata. It could alternatively be interpreted more elaborately as an a crescendo argument, i.e. as an a fortiori *cum* pro rata argument. But in any case, note well, it cannot be considered as a purely a fortiori argument, unless we are ready to suggest (unnecessarily) that R. Tarfon here argues in a formally invalid manner. The Sages counter this argument by saying that the penalty prescribed in the conclusion must not be greater than that given in the minor premise; this has come to be known as the *dayo* (sufficiency) principle. R. Tarfon's first argument, taken as pro rata, can be expressed as follows:

Just as, in one case (that of tooth & foot), damage in private domain (full payment) implies more legal liability than damage in public domain (no payment) –
so, in the other case (viz. horn), we can likewise say that damage in private domain implies more

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11   Louis Jacobs, in his *Studies* (in a footnote on p. 4), suggests that this a fortiori argument, being by Jose ben Johanan (c. 160 BCE), "may be the earliest reference in Rabbinic literature to the *qal wa-homer*." Then he adds that "the highly plausible suggestion has been made that the *qal wa-homer* does not belong to Jose's saying but is a later addition;" in support of which he mentions Schwarz and Daube. But I would not regard a suggestion by the latter two writers as highly plausible.

legal liability than damage in public domain (half payment).

Whence, R. Tarfon concludes with full payment for damage by horn in the private domain. The Sages may be construed to oppose this conclusion by means of a formally valid purely a fortiori argument, *without* pro rata extension, as follows:

> Private domain damage (P) implies more legal liability (R) than public domain damage (Q) [as we know by extrapolation from the case of tooth & foot].
>
> For horn, public domain damage (Q) implies legal liability (R) enough to make the payment half (S).
>
> Therefore, for horn, private domain damage (P) implies legal liability (R) enough to make the payment half (S).

Note that the Sages do not explicitly formulate this a fortiori argument, but may reasonably be assumed to intend it in view of their preferred conclusion. The *dayo* principle here simply corresponds to the principle of deduction, which is that the conclusion cannot contain more information than is given in the premises. For this reason, the *dayo* principle has been equated by many commentators (myself included, in the past) to the principle of deduction; but as we shall see this is only a point of intersection between them – they are not the same.

FURTHER TEXT: "He said to them: My inference is not from one case of damage caused by the horn to another case of damage caused by the horn, but from what applies in a case of damage caused by the foot to what should apply in the case of damage caused by the horn: If they have dealt leniently with damage caused by the tooth or foot in the public domain and stringently with damage caused by the horn [also in the public domain], then since they have dealt stringently [more stringently] with damage caused by the tooth or foot in the private domain of him that was injured [than in the public domain], ought we not, therefore, to deal *the more stringently* with damage caused by the horn [in the private domain]?! They answered: It is enough (*dayo*) if the inferred is as strict as that from which it was inferred: [as in the case of damage caused by the horn] in the public domain half-damages [are imposed], so also [for damage caused by the horn] in the private domain of him that was injured, half-damages [only are imposed]."

MY READING: The second argument by R. Tarfon can be read both as a mere argument by analogy (pro rata) like before, *and* as a valid purely a fortiori (as against a crescendo) argument that is in accord with the Sages' previous objection. The Sages nevertheless reject this restructured argument by saying that the penalty prescribed in the conclusion should not surpass that given in the raw data that was generalized into the major premise; this signifies an enlargement and complication of the *dayo* (sufficiency) principle, compared to its tenor in relation to the previous argument. R. Tarfon's second argument can be expressed as an argument pro rata as follows:

> Just as, in one case (that of public domain), damage by horn (full payment) implies more legal liability than damage by tooth & foot (no payment).
>
> So, in the other case (viz. private domain), we can likewise say that damage by horn implies more legal liability than damage by tooth & foot (full payment).

Whence, R. Tarfon concludes with full payment for damage by horn in the private domain. This 'proportional' conclusion is all the more credible, since the exact same conclusion can this time be obtained by regular (i.e. not a crescendo) a fortiori argument, as follows:

> Horn damage (P) implies more legal liability (R) than tooth & foot damage (Q) [as we know by extrapolation from the case of public domain].
>
> For private domain, tooth & foot damage (Q) implies legal liability (R) enough to make the payment full (S).
>
> Therefore, for private domain, horn damage (P) implies legal liability (R) enough to make the payment full (S).

The Sages remain unfazed by this new, double-barrel argument, repeating their previous "it is enough" objection, in exactly the same words. From this we must infer that their first objection did not consist merely in opposing a purely a fortiori argument to an argument pro rata or a crescendo, but referred to any source of information used in formulating the premises, limiting the conclusion drawn to such given information. In the present case, the limiting information is not apparent in the a fortiori argument as such, but is influential in the formation of its major premise. This interpretation is in accord with rabbinical commentary.

Note that both a fortiori arguments are positive antecedental in form.

Bekoroth 1:1

TEXT: "The priests and Levites are free (i.e. from the duty of the first-born, including unclean animals which are treated like human first-born in Num. 18:15), by reason of the light and heavy (*mi-qal wahomer*): If they exempted those (first-born) of the Israelites in the desert, it is an inference that they exempted their own (first-born)."

MY READING: positive subjectal, a pari (egalitarian)[12].

> The (first-born) of the priests and Levites (P) were as much released in the desert from their duties (R) as the (first-born) of the Israelites (Q) were.
>
> If the (first-born) of the Israelites (Q) were released in the desert from their duties as first-born (R) enough to be henceforth exempt from them (S),
>
> then the (first-born) of the priests and Levites (P) were released from their duties as first-born (R) enough to be henceforth exempt from them (S).

Bekoroth 9:1

TEXT: "The tithe of cattle applies in the land (of Israel) and outside the land... It applies to the herd and to the flock, and they do not tithe from one to the other; to sheep and goats, and they do tithe from one to the other; to new and old, and they do not tithe from one to the other. For, it could be an inference: Just as for the new and old, which are not diverse kinds (cf. Lev. 19:19), they do not tithe from one to the other, is it not an *inference* that for sheep and goats, which are diverse kinds, they (also) do not tithe from one to the other?"

MY READING: negative subjectal, major to minor.

> New and old (P) are more similar to each other (R) than sheep and goats (Q) are.
>
> If new and old (P) are similar to each other (R) *not* enough to be interchangeable (S),
>
> then sheep and goats (Q) are similar to each other (R) *not* enough to be interchangeable (S).

Berakoth 9:5

TEXT: "A man may not enter the temple mount with his staff, or his sandal, or his wallet; or with the dust upon his feet. Nor may he use it as a shortcut, let alone (*miqal wa-homer*) spit there. [Cambridge MS adds:] If it is forbidden to enter with shodden (*sic*) feet which implies lack of respect, *how much more* is spitting forbidden which implies contempt."

MY READING: positive subjectal, minor to major.

> Spitting there (P) is more disrespectful of Temple Mount (R) than entering it with staff, sandal, wallet or dust or using it as a shortcut (Q) is.
>
> If entering it with staff, sandal, wallet or dust or using it as a shortcut (Q) is disrespectful (R) enough to be forbidden (S),
>
> then spitting there (P) is disrespectful (R) enough to be forbidden (S).

Demai 2:2

TEXT: "He that takes it upon himself to be trustworthy must give tithe [for?] that which he eats and that which he sells and that which he buys [in order to sell again] and may not be the guest of an 'am haarets' (ignoramus). R. Yehudah says: The one who is a guest of an 'am haarets' is also trustworthy. They said to him: He is not trustworthy concerning himself, *how can he be* trustworthy concerning others?!"

MY READING: negative predicatal, minor to major.

> More scrupulousness (R) is required to be trustworthy concerning others (P) than concerning oneself (Q).
>
> If someone (S) is *not* scrupulous enough (R) (e.g. by abstaining to partake of the food of an ignoramus) to be trustworthy concerning himself (i.e. for the good of his own soul) (Q),
>
> then he (S) is *not* scrupulous (R) enough (e.g. by tithing food before selling it) to be trustworthy concerning others (i.e. for the good of their souls) (P).

Eduyyoth 6:2

TEXT: "[They said:] R. Yehoshua and R. Nechunya ben Elinathan of Kefar Ha-Bavli testified with regard to [the smallest] member of a corpse that it is unclean [in the sense of conveying uncleanness by overshadowing, mOhol 2:1], concerning which R. Eliezer says: They [i.e. the aforementioned two] have said this only of a member of living being [mOhol 1:7]. They [i.e. the first speakers] said to him: But is it not a [relationship between] *lesser and greater*: For the living being, which is clean, a limb severed from it is unclean – so is it not to be inferred that for a corpse, which is unclean, a limb severed from it is unclean?"

MY READING: positive subjectal, minor to major.

> A limb severed from a corpse (a part severed from an unclean whole) (P) is more unclean (R)

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than a limb severed from a living body (a part severed from a clean whole) (Q).

If the limb severed from a living body (Q) is unclean (R) enough to be declared unclean (S),

then the limb severed from a corpse (P) is (R) enough to be declared unclean (S).

FURTHER TEXT: "R. Eliezer … said to them [i.e. the first speakers]: They [the aforementioned two] have said this only of a member of living being. Another answer is: The uncleanness of living beings is greater than the uncleanness of corpses, for the living being makes what it lies and sits on convey uncleanness to men and to garments, and [makes] what is above it *maddaf*-unclean so that it conveys uncleanness to food and liquids, which is not how the corpse makes unclean."

MY READING: This is not an a fortiori argument, but an attempted refutation of the preceding a fortiori argument. The opponents correct the initial statement attributed to R. Yehoshua and R. Nechunya ben Elinathan of Kefar Ha-Bavli, saying they "testified with regard to [the smallest] member *of a living being* that it is unclean;" and they reject the major premise of the proposed a fortiori argument by reversing it, saying "The uncleanness of living beings is greater than the uncleanness of corpses."

Eduyyoth 6:3 (a)

TEXT: "R. Eliezer … said to them: We find that a member from a living being is like a whole corpse (mOhol 2:1); as an olive's bulk of flesh severed from a corpse is unclean, *so* an olive's bulk of flesh severed from the member of a living being will be unclean."

MY READING: positive subjectal, a pari (egalitarian).

An olive's bulk of flesh severed from a living body (P) is as dead (R) as an olive's bulk of flesh severed from a corpse (Q).

If an olive's bulk of flesh severed from a corpse (Q) is dead (R) enough to be declared unclean (S),

then an olive's bulk of flesh severed from the limb severed from a living body (P) is dead (R) enough to be declared unclean (S).

FURTHER TEXT: "R. Yehoshua and R. Nechunya … said to him: No. If you declare unclean an olive's bulk of flesh severed from a corpse, of which you have declared unclean a barleycorn's bulk of bone severed from it (mOhol 2:3), would you also declare unclean an olive's bulk of flesh severed from the member of a living being, of which you have declared clean a barleycorn's bulk of bone severed from it?"

MY READING: This is not an a fortiori argument, but an attempted refutation of the preceding a fortiori argument. The opponents argue:

Given that: if a barleycorn's bulk of bone severed from a corpse is *un*clean, then an olive's bulk of flesh severed from a corpse is *un*clean.

Does it follow that: if a barleycorn's bulk of bone severed from a member of a living body is *clean*, then an olive's bulk of flesh severed from the member of a living being is *un*clean? No!

What the opponents are saying is that the major premise ("a member from a living being is like a whole corpse"), is not as general as it is made to seem. A barleycorn's bulk of bone is unclean severed from a corpse, yet clean severed from a member of a living being. Therefore, they say, we cannot infer an olive's bulk of flesh severed from a member of a living being to be unclean from the fact that an olive's bulk of flesh severed from a corpse is declared unclean.

Eduyyoth 6:3 (b)

TEXT: "R. Nechunya … We find that a member from a living being is like a whole corpse (mOhol 2:1); as a barleycorn's bulk of bone that is severed from a corpse is unclean, *so* a barleycorn's bulk of bone that is severed from a member from a living body will be unclean."

MY READING: positive subjectal, a pari (egalitarian).

A barleycorn's bulk of bone severed from a living body (P) is as dead (R) as a barleycorn's bulk of bone severed from a corpse (Q) is.

If a barleycorn's bulk of bone severed from a corpse (Q) is dead (R) enough to be declared unclean (S),

then a barleycorn's bulk of bone severed from the limb severed from a living body (P) is dead (R) enough to be declared unclean (S).[13]

FURTHER TEXT: "They said to him: No. If you declare unclean a barleycorn's bulk of bone severed from a corpse, of which you have declared unclean an olive's bulk of flesh severed from it, would you also declare unclean a

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[13]   Note that this resembles the earlier a fortiori argument by R. Eliezer, except that the subsumed part is "a *barleycorn's* bulk of *bone*" instead of "an *olive's* bulk of *flesh*".

barleycorn's bulk of bone severed from the member of a living being, of which you have declared clean an olive's bulk of flesh that is severed from it?"

MY READING: This is not an a fortiori argument, but an attempted refutation of the preceding a fortiori argument. The opponents argue, as in the previous rebuttal:

> Given that: if an olive's bulk of flesh severed from a corpse is *un*clean, then a barleycorn's bulk of bone severed from a corpse is *un*clean.
>
> Does it follow that: if an olive's bulk of flesh severed from the member of a living body is *clean*, then a barleycorn's bulk of bone severed from the member of a living body is *un*clean? No![14]

Here again the major premise of the proposed a fortiori is put in doubt.

Hullin 2:7

TEXT: "R. Yose said: The things are *lighter and weightier*: Just as in the case (*maqom*) where the thought invalidates, regarding sacrifices (cf. mZeb 2:2), everything depends on the person officiating (i.e. not the owner), so in the case where thought does not invalidate, regarding profane slaughter, should not everything also depend on the slaughterer (i.e. not the owner, in this case a non-Jew)?"

MY READING: negative subjectal, major to minor.

> Sacrificial slaughter (P) is more dependent on the right thought of the owner (R) than profane slaughter (Q) is.
>
> If sacrificial slaughter (P) is dependent on the right thought of the owner (R) *not* enough to be invalidated (S),
>
> then profane slaughter (Q) is dependent on the right thought of the owner (R) *not* enough to be invalidated (S).

Hullin 10:1

TEXT: "{"the shoulder and the two cheeks and the stomach" (Deut. 18:3) ... applies to non-sacrificial slaughter but not to sacrificial slaughter.} For it might have been an inference [to say]: And if even non-sacrificial slaughter which is not liable to "breast and thigh" (Lev. 7:31) is liable to these [other] dues, is it not an *inference* to that consecrated animals which are liable to "breast and thigh" should be also liable to these [other] dues? {In this regard it is instructive that Scripture says: "[For I have taken the breast of elevation offering and the thigh of gift offering from the Israelites, from their sacrifices of well-being] and given them to Aaron the priest and his sons as a prescribed due for ever [from the Israelites]" (Lev. 7:34). You have there only what is said as stated.}"

MY READING: positive antecedental, minor to major.

> Sacrificial offerings (P) are more liable to priestly dues (R) than non-sacrificial slaughter (Q) is (this being generalized from 'breast and thigh' dues to all dues).
>
> If non-sacrificial slaughter (which has no priestly involvement) (Q) is liable to priestly dues (R) enough to necessitate payment of 'the shoulder and the two cheeks and the stomach' (S),
>
> then sacrificial offerings (which have priestly involvement) (P) are liable to priestly dues (R) enough to necessitate payment of 'the shoulder and the two cheeks and the stomach' (S).

The last remark "You have there only what is said as stated" seems intended to deny the conclusion of the *qal vachomer*, saying that Lev. 7:34 is to be read as exclusive of 'the shoulder and the two cheeks and the stomach' (i.e. as *davka*).

Hullin 12:4-5

TEXT: "A man may not take 'the dam together with the young' even in order to effect with them the purification of the leper [where one of whose birds is let go at the end of the ritual, cf. Lev. 14:7]. And if (concerning) a 'light' commandment, involving an Issar, the Torah says: 'So that it may be good with you and you have length of days', *how much more so* (*qal wahomer*) concerning the weightier commandments which are in the Torah!"

MY READING: positive subjectal, minor to major. Could be intended as a crescendo.

> Obeying weightier (more demanding) commandments (P) earns one more merit (R) than obeying lighter (less demanding) commandments (Q) does.
>
> If someone obeying a light commandment (demanding merely one Issar) (Q) earns merit (R) enough to get rewarded with good and long life (S),
>
> then someone weightier commandments (demanding more than one Issar) (P) earn merit (R) enough to get rewarded with good and long life (S).

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[14]   Notice that the "given" here is *the reverse* of the previous "given," and likewise the "does it follow that" antecedent and consequent are *reversed*! This seems to suggests that these rebuttals are quite hypothetical, and apparently not intended as factual.

An a crescendo reading would imply a greater reward (i.e. proportionately more good and longer life) in the latter case, in proportion to the greater merit of weightier commandments.

Kilaim 8:1
TEXT: "It is prohibited to sow kilaim [a certain mingling] of seeds, and to allow it to grow; but it is lawful to eat of it, and, *a fortiori*, to derive benefit therefrom."
MY READING: This is a Mishnaic a fortiori argument not mentioned by Samely. It is a straightforward positive subjectal case:

> Deriving benefit from something (P) is more innocuous (R) than eating of it (Q).
>
> If eating of something (Q) is innocuous (R) enough to be lawful (S),
>
> then, deriving benefit from it (P) is innocuous (R) enough to be lawful (S).

Makkoth 1:7
TEXT: "{'On the evidence of two witnesses or three witnesses, shall he that is to die be put to death' ...] R. Aqiva says: The third witness is here mentioned only that the same stringency shall apply to him also, and that his condemnation shall be made like to that of the other two.} If thus Scripture punishes the (person) who is joined to those who commit transgressions in the same way as those who (actually) commit transgressions, *how much more* will it reward the (person) who is joined to those who fulfill commandments in the same way as those who (actually) fulfill commandments."
MY READING: positive subjectal, a pari (egalitarian).

> Joining those who fulfill commandments (P) earns one as much merit (R) as actual fulfilling of commandments (Q) does.[15]
>
> If someone actually fulfilling commandments (Q) earns merit (R) enough to be rewarded in a certain way (S),
>
> then someone joining those who fulfill commandments (P) earns merit (R) enough to be rewarded in that way (S).

Makkoth 3:15 (a)
TEXT: "R. Hananiah ben Gamliel (also) said: If the one who commits one transgression has his life taken away, *all the more* will the one who performs one commandment be given [or restored] his life!"
MY READING: positive subjectal, minor to major. Strictly speaking, this attempted a fortiori argument is *invalid*, because the subsidiary term (S) is not the same in the minor premise and conclusion.

> The one who performs one commandment (P) deserves more credit (R) than the one who commits one transgression (who deserves not credit, but debit) (Q) does.
>
> If the one who commits one transgression (Q) deserves credit (R) enough to have his life taken away (S1),
>
> then the one who performs one commandment (P) deserves credit (R) enough to be given [or restored] his life! (S2).

We could regard this argument more generously as valid, if we look upon it as a crescendo, i.e. if we assume an unstated additional premise about proportionality to be tacitly intended. In that case, S1 (life taken away) and S2 (life given or restored) are viewed as two sides of a continuum S (life), the former being negative and the latter positive. This continuum being parallel to the continuum R ('credit' in a broad senses, ranging from actual debit to actual credit), we can reason proportionately. Just as Rq implies S1, so Rp implies S2.

However, even then the argument is of very doubtful validity, because it is essentially a contrario. Notice that not only the predicates (life forfeited, life restored) are contrary, but also the subjects (commits transgression, performs commandment) are contrary. This is not per se something inconceivable; however, the difficulty lies in the *coupling* of these two pairs of contraries. By what formal means does the speaker know that the switchover from the first predicate to the second is tied precisely to the switchover from first subject to the second? Obviously, he perceives a causative relation between the subjects and predicates, i.e. he believes that transgression causes life to be forfeited and performing commandments causes life to be restored. Fair enough; this may well be true as a pair of observations (or through further inductive and deductive arguments). But the problem is in the inference from the premises to the conclusion. If the conclusion is already known by observation (or however), and is appealed to in order to justify the said coupling of switchovers, then the proposed a fortiori argument constitutes circular reasoning. It begs the question, since its putative conclusion can only really be drawn if it is previously given.

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[15]   The major premise is obtained by generalization from potential and actual transgressions (bad) to all relations to commandments (bad and good), including fulfillment of commandments (good).

Makkoth 3:15 (b)
TEXT: "R. Shimon ben Rabbi says: Behold it says: 'Only be firm not to eat the blood, for the blood is the life...' And if the person who separates from the blood, from which man recoils, receives a reward; then the person who separates from robbery and forbidden sexual relations, which man covets and desires, *how much more so* will he acquire merit for himself and his generations and the generations of his generations until the end of all the generations!"
MY READING: positive subjectal, minor to major. Could be intended as a crescendo.

> A person who separates from robbery and forbidden sexual relations (P) has more inner resistance to overcome (R) than a person who separates from the blood (Q).
> If a person who separates from the blood (Q), from which (many a) man recoils, has enough inner resistance to overcome (R) that he merits to receive a reward (S),
> then a person who separates from robbery and forbidden sexual relations (P), which (many a) man covets and desires, has enough inner resistance to overcome (R) that he merits to receive a reward (S).

An a crescendo reading would imply a greater reward in the latter case, in proportion to the inner obstacles that had to be overcome. Although at first sight, R. Shimon's argument appears a crescendo, it can definitely also be interpreted as purely a fortiori. The latter is possible in two ways, in both of which the minor premise and conclusion have the same predicate (the subsidiary term, S): either (a) both propositions state that the subject vaguely "receives a reward" (for self and perhaps children), or (b) both propositions state that the subject "will he acquire merit for himself and his generations and the generations of his generations until the end of all the generations!" In the event of (a), R. Shimon's conclusion must be taken as mere hyperbole, and cannot be accepted literally as the logical conclusion of the a fortiori argument as such. In the event of (b), R. Shimon's conclusion must be taken as having been tacitly intended also in his more vaguely put minor premise. But, equally well, we may consider R. Shimon's conclusion as occurring *after* the a fortiori argument, the product of a subsequent pro rata argument, i.e. as an extrapolation (in time) based on other considerations (e.g. the principle of measure for measure), i.e. as an a crescendo conclusion.

To be sure, underlying this positive subjectal argument, with the middle term "having some resistance to overcome," is a positive predicatal argument, with the middle term "having some self-control." The latter may be formulated as follows:

> More self-control (R) is required to avoid robbery and incest (P), which arouse desire, than to avoid shedding blood (Q), which arouses aversion.
> If a person (S) has self-control (R) enough to avoid robbery and incest (P),
> Then that person (S) has self-control (R) enough to avoid shedding blood (Q).

However, this argument lacks the information about reward, and moreover proceeds from major to minor. For the inference of reward, the subjectal form used by R. Shimon seems more appropriate.

Menahoth 8:5
TEXT: "Also, for the meal offerings should be inferred that they require 'pure olive oil': Just as the Menorah which is not to do with eating, requires 'pure olive oil', so meal offerings, which are to do with eating, is it not an *inference* that they should require 'pure olive oil'? Scripture instructs by saying: '[olive oil] pure, beaten, for lighting...' - and not pure, beaten for the meal offerings."
MY READING: positive subjectal, minor to major.

> Oil for the meal offering (P) has more to do with eating (R) than oil for lighting the Menorah (Q).
> If oil for lighting the Menorah (Q), though not intended as food, has enough to do with eating (R) (R = 0) to need to be pure olive oil (S),
> then meal offerings (P), which are intended as food, have enough to do with eating (R) (R > 0) to need to be pure olive oil (S).

The final sentence ("Scripture instructs, etc."), read *davka*, i.e. as exclusive of anything not explicitly mentioned therein, is intended as a rebuttal of the putative conclusion. In this case, the problem lies not with the major premise (which seems credible enough), but with the minor premise (which seems forced anyway).

Nazir 7:4
TEXT: "R. Eleazar said in the name of R. Yehoshua: For whatsoever uncleanness from a corpse a Nazirite must cut off his hair [mishnah 2], for that too is a man culpable if he enters into the Temple; and for whatsoever uncleanness from a corpse a Nazirite need not cut off his hair [mishnah 3], for that too is a man not culpable if he enters into the

Temple. R. Meir said: Would there not thus be less stringency than [when uncleanness is contracted from] a creeping thing! [Lev. 5:2, Num. 19:20] R. Aqiva said: I argued before R. Eliezer: If because of the contact or carrying of a barleycorn's bulk of bone which does not render a man unclean by overshadowing a Nazirite must cut off his hair, *how much more*, then, ought he to cut off his hair because of the contact or carrying of a quarter-log of blood [cf. mishnah 3] which renders a man unclean by overshadowing! He said to me: What is this, Aqiva? We cannot here argue *from the lesser to the greater* [since it is the accepted ruling]. But when I came and declared these words before R. Yehoshua, he said to me: You have spoken well; but thus have they said as Halakhah."

MY READING: R. Akiva's argument is positive subjectal, minor to major.

> The contact or carrying of a quarter-log of blood (P) renders a man more unclean by overshadowing (R) than the contact or carrying of a barleycorn's bulk of bone (Q) does.
>
> If a Nazirite's contact or carrying of a barleycorn's bulk of bone (Q) renders him unclean by overshadowing (R) (R = 0) enough to make him have to cut off his hair (S),
>
> then a Nazirite's contact or carrying of a quarter-log of blood (P) renders him unclean by overshadowing (R) (R > 0) enough to make him have to cut off his hair (S).

R. Akiva seems to sustain R. Eleazar[16], by arguing a fortiori as described. Then R. Akiva explains that R. Eliezer objected to this a fortiori argument (as against Halakha), to this, whereas R. Yehoshua approved of it (i.e. as formally valid) but suggested the Halakha goes the other way anyway (like R. Eliezer).

Nedarim 10:7 I (2) = A4.2

TEXT: "If a man said to his wife 'Let every vow be established that you shall vow from this time forth until I return from such a place', he has said nothing; but if he said, 'Let them be void', R. Eliezer says: They are cancelled. But the Sages say: They are not cancelled. R. Eliezer said: If he can cancel vows which have already had [for a time, before he cancelled them] the force of a 'prohibition' (cf. Num. 30:3) [as any vow of his wife that he cancels], can he not *also* cancel vows which have not yet the force of a 'prohibition'? {They answered: Behold, it is written. 'Her husband may establish it and her husband may cancel it' - that which comes under the category of 'establishing' also comes under the category of 'cancelling', and that which does not come under the category of 'establishing' [also] does not come under the category of 'cancelling'.}"

MY READING: R. Eliezer' argument is positive predicatal, major to minor:

> More authority (R) is required to cancel vows which already had the force of prohibition (P) than vows which do not yet have such force (Q).
>
> If a husband (S) has authority (R) enough to cancel his wife's vows which already have the force of a prohibition (P),
>
> then a husband (S) has authority (R) enough to cancel his wife's vows which do not yet have the force of a prohibition (Q).

The Sages reject this conclusion, effectively by denying the major premise. By saying that the husband can only cancel vows that he can establish, they mean (if I understand correctly) that since he is away and not able to establish his wife's vows individually, he has no authority to cancel them collectively in advance.

Negaim 10:2

TEXT: "'Thin yellow hair' means uncleanness: clustered together or dispersed, surrounded or not surrounded, turned [yellow by the scall] or not turned - words of R. Yehudah. R. Shimon says: It only means uncleanness if turned. R. Shimon said: And it is a an inference: If the white hair, against which another hair does not afford protection, does not render unclean except when turned, then the 'yellow thin hair', against which another hair does afford protection (cf. Lev. 13:31), is it not an *inference* that it also does not shall render unclean except when turned?'

MY READING: R. Shimon's argument is positive subjectal, minor to major.

> 'Yellow thin hair' (P) is afforded more protection against uncleanness by another hair (R) than white hair (Q) is.
>
> If someone with 'white hair' (Q) is afforded enough protection against uncleanness by another hair (R) (R = 0) to not-render him unclean except when it is turned (S),
>
> then someone with 'yellow thin hair' (P) is afforded enough protection against uncleanness by another hair (R) (R > 0) to not-render him unclean except when it is turned (S).

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[16]   R. Eleazar's argument looks like mere analogy (actually, an argument by inversion, i.e. a contrario): if uncleanness of Nazirite from corpse is sufficient to impose haircut then it is sufficient to forbid Temple entry; likewise, if uncleanness of Nazirite from corpse is *in*sufficient to impose haircut then it is *in*sufficient to forbid Temple entry. R. Meir's reply to this argument is intended to put it in doubt.

TEXT: "R. Yehudah said: In every place where it was necessary to say 'turned', it [i.e. Scripture] said 'turned' (e.g. Lev. 13:3). But the scall, about which it is said: 'And there is no yellow hair in it', renders unclean [whether the hair] turned [yellow[17]] or whether it did not turn."

MY READING: R. Yehudah proposes a case which apparently belies or gives an exception to the conclusion of the previous a fortiori argument, i.e. he says that: whether it turned or did not turn, scall renders unclean. How is this a rebuttal? I am not sure. I would rather look at the above conclusion clause 'except when turned' and suggest that R. Yehudah is saying: the above conclusion says that *un*turned implies *not* unclean, whereas scall is a case where albeit *un*turned, nevertheless unclean *is* implied.

Negaim 12:5

TEXT: "About what then does the Torah take care? About his earthenware utensils, and about his flask and his [oil] vessels. If the Torah thus cares for his humble possession, *how much more* for his beloved possession! If [it thus cares] for his possession, *how much more* for the life of his sons and daughters! If [it thus cares] for those of the wicked, *how much more* for those of the righteous!"

MY READING: There are here three distinct a fortiori arguments, all of positive subjectal (minor to major) form. These could be intended as a crescendo. The first is:

> Beloved possessions (P) are more valuable (R) than humble possessions (Q) are.
> If humble possessions (Q) are valuable (R) enough to be taken care of by the Torah (S).
> then beloved possessions (P) are valuable (R) enough to be taken care of by the Torah (S).

Similarly the other two. Note that there is a progression in value, from humble to beloved possessions, from material possessions to life of children, from life of children of wicked to those of righteous. The conclusion of first is a springboard for the next, which is in turn a springboard for the third. A crescendo readings would imply more care taken by the Torah in each succeeding case, in proportion to the value of the possessions.

Pesahim 6:2 (a)

TEXT: "These acts pertaining to the Pesah offering override the Sabbath: slaughtering it, tossing its blood, scraping its entrails and burning its fat pieces. But the roasting of it and rising its entrails do not override the Sabbath. Carrying it [to the Temple] and bringing it from the outside to within the Sabbath limit and cutting off a wen [from the carcass] do not override the Sabbath. R. Eliezer says: They do override it. And is it not an *inference*: Just as the slaughtering which comes under [Sabbath] work overrides the Sabbath, those [activities] which come [only] under [Sabbath] rest - should they not [also] override the Sabbath?"

MY READING: This is positive subjectal, minor to major.

> Activities classed under Sabbath rest (P) are more leniently regulated (R) than activities classed
>   under Sabbath work (Q).
> If activities classed under Sabbath work (Q) are leniently regulated (R) enough to be permitted
>   on the Sabbath (S),
> then, activities classed under Sabbath rest (P) are leniently regulated (R) enough to be permitted
>   on the Sabbath (S).

FURTHER TEXT: "R. Joshua said to him: The festival day proves [it], for on it they have allowed [activities] under the category of work and [activities] under the category of rest are forbidden. R. Eliezer said to him: What is this, Joshua! What is a proof from that which is allowed to that which is commanded?"

MY READING: R. Joshua objects to R. Eliezer's argument by pointing out that on a festival some Sabbath work activities are permitted and some Sabbath rest activities are forbidden. This means that R. Eliezer's major premise about relative leniency of regulation is not universally true – and so the conclusion he draws cannot be drawn. R. Eliezer replies by claiming that R. Joshua is inferring something commanded from something allowed. I do not know to what he is referring specifically.

FURTHER TEXT: "R. Aqiva replied and said: The sprinkling [of the sin offering water on day 3 and 7 after attracting corpse-uncleanness] proves [it], for it is commanded and it comes under [Sabbath] rest, but it does not override the Sabbath. Thus also do not be astounded at those [other] ones, for they [too], despite being commanded and [only] under the category of rest, do not override the Sabbath."

MY READING: R. Akiva is saying: Sprinkling is commanded Sabbath rest, yet is forbidden on a Festival. Therefore, conceivably, other things may be commanded Sabbath rest, yet be forbidden on a Festival. This like the preceding objection is designed to neutralize R. Eliezer's a fortiori argument.

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[17]   The [yellow] interpolation may be Samely's.

Pesahim 6:2 (b) and (c)

TEXT: "R. Eliezer said to him: And on this [itself] do I base an inference: (And) if the slaughtering which is under the category of work overrides the Sabbath, the sprinkling which is under the category of rest - should it not be *inferred* that it overrides the Sabbath [also]? R. Aqiva said to him: Or the reverse! If the sprinkling which is [only under the category] of rest does not override the Sabbath, the slaughtering which is [under the category] of work - should it not be *inferred* that it [also] does not override the Sabbath?"

MY READING: Here we have for once two rival a fortiori arguments! This is worth mentioning as an example of such rivalry.

R. Eliezer's is almost the same argument as already seen, except that here 'slaughtering' and 'sprinkling' are specifically mentioned instead of the vaguer minor and major term. It is positive subjectal, going from minor to major.

> Activities classed under Sabbath rest (P) are more leniently regulated (R) than activities classed under Sabbath work (Q).
> If the slaughtering which comes under Sabbath work (Q) is leniently regulated (R) enough to be permitted on the Sabbath (S),
> then, sprinkling which comes [only] under Sabbath rest (P) is leniently regulated (R) enough to be permitted on the Sabbath (S).

R. Aqiva's retort is negative subjectal, going from major to minor, as follows:

> Activities classed under Sabbath rest (P) are more leniently regulated (R) than activities classed under Sabbath work (Q).
> If sprinkling which comes [only] under Sabbath rest (P) is leniently regulated (R) *not* enough to be permitted on the Sabbath (S),
> then, the slaughtering which comes under Sabbath work (Q) is leniently regulated (R) *not* enough to be permitted on the Sabbath (S).

What is the status of this controversy? The two arguments in fact formally imply each other, since they have the same major premise. What puts them in opposition to each other is that each speaker assumes himself to have a true minor premise, and therefore his opponent to have a false conclusion. Presented with the two arguments, and no other information, we have no way to choose between them. Though contrary, they are both equally cogent hypothetical scenarios, given their common major premise. It is a standoff. The answer is presumably given further on in the text.

Pesahim 6:5

TEXT: "R. Eliezer argues: If a person, when he has changed the name of the paschal sacrifice, which sacrifice he may slaughter on the Sabbath, is deemed to be guilty; does it not follow that when he had changed the names of other sacrifices which are already prohibited to be offered thereon as such, that he must *a fortiori*, be considered guilty?"

MY READING: This is a Mishnaic a fortiori argument not mentioned by Samely. It can be put in positive subjectal form:

> Changing the purpose of a sacrifice that must not be slaughtered on the Sabbath (P) is more culpable (R) than changing the purpose of a sacrifice that may be slaughtered on the Sabbath (Q).
> If a person who changes the purpose of a sacrifice (such as the paschal sacrifice) which may be slaughtered on the Sabbath (Q) is culpable (R) enough to be liable to a sin-offering (S),
> then, a person who changes the purpose of a sacrifice which (though eligible on Pesach) must not be slaughtered on the Sabbath (P) is culpable (R) enough to be liable to a sin-offering (S).

FURTHER TEXT: "To this R. Joshua answered: You cannot apply what is affirmed in respect to the sacrifice, when it was changed to that which it is unlawful to offer on the Sabbath, to other sacrifices where the name has been changed to what is lawful. R. Eliezer replied: The offerings brought for the whole congregation [of Israel] shall prove [my assertion,] for it is lawful to offer them on the Sabbath under their proper name; yet whoever brings other offerings under their denomination is declared to be guilty. Then R. Joshua answered: You cannot apply what is affirmed in respect to the offerings of the whole congregation which have a determinate number, to the paschal sacrifice which has no determinate number."

MY READING: R. Joshua apparently denies the major premise, saying that relabeling a sacrifice as equivalent to one unlawful on the Sabbath (e.g. changing the purpose of a paschal offering to some other) is not comparable to relabeling a sacrifice as equivalent to one lawful on the Sabbath (e.g. changing the purpose of some other offering to paschal). To defend his major premise, R. Eliezer retorts that whereas sacrifices for the whole congregation may be offered on the Sabbath under their name (i.e. as public offerings), other sacrifices cannot be offered on the Sabbath under that name (i.e. as public offerings); that is, the latter name change does not make them Sabbath compatible

(just as the name change to paschal sacrifice does not make an offering Sabbath compatible). But R. Joshua rejects that defense, saying that whereas the offerings of the whole congregation are limited in number, the paschal sacrifice is not (so no comparison between them is possible).

Sanhedrin 6:5
TEXT: 'R. Meir said: … says God (var. Scripture), I am pained at the blood of the wicked, *how much more* at the blood of the righteous!'
MY READING: positive subjectal, minor to major. Could be intended as a crescendo.

> God has for the righteous (P) more concern (R) than He has for the wicked (Q).
> If God has for the wicked (Q) concern (R) enough to be pained at their blood (S),
> then God has for the righteous (P) concern (R) enough to be pained at their blood (S).

An a crescendo reading would imply God's greater unhappiness in the latter case, in proportion to His greater love for the righteous.

Shebuoth 3:6
TEXT: "If he swore to cancel the commandment and did not cancel it, he is free (but see mShebu 3:8); [if he swore] to fulfill the commandment and did not fulfill it, he is free. Yet, it might be inferred that he was culpable, as according to the words of R. Yehudah ben Batyra. R. Yehudah ben Batyra said: If he is liable for [broken] oaths concerning that which is discretionary, for which no oath was imposed from Mount Sinai, is it not *logical* that he should be liable for [broken] oaths concerning commandments, for which an oath was imposed from Mount Sinai?"
MY READING: R. Yehuda's argument is a positive subjectal, minor to major.

> Broken oath about commandment (P) is more binding (R) than broken oath about discretionary
> item (Q) is.
> If broken oath about discretionary item (Q) is binding (R) enough to make one liable (S),
> then broken oath about commandment (P) is binding (R) enough to make one liable (S).

FURTHER TEXT: "They said to him: No; if you speak of an oath concerning what is discretionary, in which a No is as [valid as] a Yes, would you say the same for an oath concerning a [positive] commandment, where the No is not as [valid as] the Yes? (some mss add: So that a person taking an oath to cancel it, and did not cancel it, is not liable)."
MY READING: The rebuttal apparently denies the truth of the major premise – i.e. not all broken oaths about a commandment are taken that seriously. I gather from Samely that this refers to oaths against the commandment which are exempt from liability if not fulfilled.

Sotah 6:3 (a)
TEXT: "[mishnah 2] If [even] one witness said: I have seen her that she was defiled, she does not drink [the Sotah waters], and not only this, but even a slave, even a female slave, behold these are believed ... Her mother-in-law, the daughter of her mother-in-law ... behold these are believed ... [3] It could have been a [correct] *inference* [to say]: If the initial testimony, which renders her not forbidden forever [to her husband], cannot be established by less than two witnesses, should not that which does render her forbidden forever, [also] be established by a minimum of two witnesses? {In this regard it is instructive that Scripture says, 'and there is no witness against her' - any testimony regarding her.}"
MY READING: This is positive subjectal, minor to major.

> The later testimony (P) forbids wife to husband for longer time (R) than the initial testimony (Q)
> does.
> If the initial testimony (Q) forbids wife to husband (not forever) for long (R) enough to require at
> least two witnesses (S),
> then the later testimony (P) forbids wife to husband (forever) for long (R) enough to require at
> least two witnesses (S).

Sotah 6:3 (b)
TEXT: "There is an inference to be drawn *from the less to the more stringent* concerning the first testimony from this very fact [that only one witness is necessary]: Just as the last testimony which renders her forbidden forever, behold, is established by one witness [only], should not the first testimony which does not render her forbidden for ever *also* be capable of being established by one witness [only]?!"
MY READING: This is positive subjectal, minor to major.

> The initial testimony (P) forbids wife to husband for shorter time (R) than the later testimony (Q)
> does.

> If the later testimony (Q) forbids wife to husband (forever) briefly (R) enough to require only one
> witness (S),
> then the initial testimony (P) forbids wife to husband (not forever) briefly (R) enough to require
> only one witness (S).

This argument, take note, is intended to rebut – or at least to rival (being apparently an equally cogent alternative) – the preceding one. Notice that though the terms here are labeled by me similarly to those there (i.e. P, Q, R, S), the meanings are different. Here, the middle term is the relative of the previous middle term (referring to shortness of time instead of length of time), and consequently the roles of the initial and final testimony are reverse; moreover, the subsidiary terms has changed from "at least two witnesses" to "only one witness."

FURTHER TEXT: "{In this regard it is instructive that Scripture says: 'For he has found in her the indecency of a matter [and he writes for her a bill of divorce]', and above it says: 'According to two witnesses [or according to three witnesses] shall the matter be established'. Just as the 'matter' enunciated above is [established] according to two witnesses, so the 'matter' enunciated here is according to two witnesses also.}"

MY READING: The above counterargument (b) is rejected by reference to Scripture, which specifies two or more witnesses for the initial testimony. It does not follow, however, that the previous argument (a) is established. I suspect (though this needs verification) that the Mishna which advocates only one witness for the final testimony is maintained, somehow.

Temurah 1:1 (a) and (b)

TEXT: "{The priests may substitute what is theirs and Israelites substitute what is theirs. The priests do not substitute the sin offering, and not the guilt offering and not the firstling. R. Yohanan ben Nuri said: And why do they not substitute the firstling? R. Aqiva said to him: The sin offering and the guilt offering are a gift to the priest; and the firstling is a gift to the priest. *Just as* they may not substitute the sin offering and the guilt offering, *so* they may not substitute the firstling.}"

MY READING: R. Akiva's initial argument seems to be a negative subjectal, a pari (egalitarian).

> The sin/guilt offerings (P) are as much a gift to the priest (R) as the firstling (Q) is.
> If the sin/guilt offerings (P) are gifts (R) not enough to be substitutable (S),
> then the firstling (Q) is a gift (R) not enough to be substitutable (S).

FURTHER TEXT: "R. Yohanan ben Nuri said to him: What do I have [knowing that] there is no substitution of sin offering and guilt offering, for *to these* they have no right while they [the animals] are alive, *and yet* you are telling me regarding the firstling to which they do have a right while it is alive? {R. Aqiva said to him: And is it not already said: 'And it will be that both it and its substitute will be holy'? Where does its holiness take effect for it? In the house of the owner. So also substitution, in the house of the owner.}"

MY READING: R. Yohanan's argument is intended to rival the preceding one by R. Akiva. It appeals to an additional distinction between live and dead offerings, which makes the attempted a fortiori argument not egalitarian, and therefore *invalid*, because though it is negative subjectal, it is yet minor to major.

> What becomes priestly property while alive (P) is more fully owned (R) than what becomes
> priestly property only after slaughter (Q) is.
> If the sin/guilt offering, which becomes priestly property only after slaughter (Q) is fully owned
> (R) not enough to be substitutable (S),
> then the firstling, which becomes priestly property while alive (P) is fully owned (R) not enough
> to be substitutable (S).

R. Akiva apparently counters this invalid argument, if I understand correctly, with a claim that both offerings are equally holy and that holiness takes effect as soon as it comes into the owner's home, so that substitution can take effect at once. This is not a third a fortiori argument, but an attempt to neutralize R. Yohanan's rival a fortiori argument by denying his minor premise and conclusion. It is noteworthy that R. Akiva does not here (apparently) challenge R. Yohanan on more formal ground, i.e. by pointing out that his reasoning process is invalid.

Terumoth 5:4 (a) and (b)

TEXT: "If one seah of unclean heave offering fell into a hundred seahs of clean heave offering, the House of Shammai forbid the whole, but the School of Hillel permit it. The House of Hillel said to the House of Shammai: *Since* clean [heave offering] is forbidden to non-priests and unclean is forbidden to priests, if the clean can be outweighed cannot the unclean be outweighed too? The House of Shammai answered: No! If the *'light'* common produce, which is permitted to non-priests, neutralizes what is clean (cf. mTer 5:3), should the *'weighty'* heave offering, which is forbidden to non-priests, neutralize what is unclean?! After they agreed, R. Eliezer said: It should be taken up and burnt. But the Sages say: It is lost through its scantiness."

MY READING: There are in fact two a fortiori arguments here, both of them negative subjectal, and both invalid because minor to major (instead of major to minor). The first argument, by the House of Hillel, is intended as valid; the second argument, by the House of Shammai, is put forward as invalid: it is formulated in order to show up the invalidity of the first argument.

The Hillel House argument seems to be the following:

> What is forbidden [even] to priests (unclean heave offerings) (P) is more restricted (R) than what is forbidden to non-priests [but not to priests] (clean heave offerings) (Q).
>
> If the clean heave offerings, which are forbidden to non-priests [but not to priests], (Q) are restricted (R) not enough to be prevented from being outweighed by clean common food (= effectively turned into clean common food by mixture in 100 times more of it) (S),
>
> then the unclean heave offerings, which are forbidden [even] to priests (P) are restricted (R) not enough to be prevented from being outweighed by clean common food (= effectively turned into clean common food by mixture in 100 times more of it) (S).

This argument is fallacious: one can well imagine the clean being outweighed but the unclean not being outweighed. To say that the latter follows the former is a non-sequitur. This is apparently the intent of the objection by the Shammai House. They are not so much proposing a counter a fortiori argument as denying the process of the Hillel House proposal. Nevertheless, they modify the wording of the a fortiori argument as follows, presumably so as to show more clearly its absurdity: "If the 'light' common produce, which is permitted to non-priests, neutralizes what is clean (cf. mTer 5:3), should the 'weighty' heave offering, which is forbidden to non-priests, neutralize what is unclean?!"

> What is forbidden to non-priests ('weighty' heave offerings) (P) is more restricted (R) than what is permitted to non-priests ('light' common produce) (Q).
>
> If the 'light' common produce, which is permitted to non-priests, (Q) is restricted (R) not enough to be prevented from being neutralized by clean common food (= effectively turned into clean common food by mixture in 100 times more of it) (S),
>
> then the 'weighty' heave offerings, which are forbidden to non-priests, (P) are restricted (R) not enough to be prevented from being neutralized by clean common food (= effectively turned into clean common food by mixture in 100 times more of it) (S).

This argument differs from the preceding in that it concerns only non-priests, ranging from what is forbidden to them to what is permitted to them. This clarifies the logical issue a bit, removing complications in the terms. If what is permitted to them can be neutralized, then surely what is forbidden to them can be neutralized too? The logical answer is of course: no – one can conceive the former being true without the latter being true. So this is an illicit process – i.e. the argument is invalid, going from minor to major whereas it should have gone from major to minor (i.e. if the forbidden can be neutralized then yes, surely the permitted can be so too). So this second a fortiori argument is invalid too – but intentionally so, so as to emphasize the invalidity of the first a fortiori argument.

In my opinion, Shammai here beats Hillel; i.e. Hillel House has not proven its point and Shammai House has demonstrated that absence of proof (though that does not mean it proves the opposite point). The last sentence in this passage, "After they agreed, R. Eliezer said: It should be taken up and burnt. But the Sages say: It is lost through its scantiness." seems to say that the two sides agreed that Shammai House was right in its critique of Hillel House.

ADDITIONAL NOTE. Moreover that the conclusion of Hillel House is about outweighing by clean common food – but a further argument is tacitly implied, that if unclean heave offerings are outweighed by clean common food, then they are also a fortiori outweighed by clean heave offerings (which is the desired final conclusion), since the latter are more holy than the former. Similarly, the (ad absurdum) conclusion of Shammai House is about outweighing by clean common food – but a further argument is tacitly implied, that if 'weighty' heave offerings are outweighed by clean common food, then they are also a fortiori outweighed by clean heave offerings (which is the desired final conclusion), since the latter are more holy than the former. So we may say that we in fact have four a fortiori arguments here! The first two (explicit) are invalid, but the latter two (implicit) would be valid. I do not count the latter, since no one in the text has actually stated them.

Yadayim 4:7

TEXT: "The Sadducees say: We raise a complaint against you, o Pharisees, (for you say: If my ox and my donkey have caused damage they are culpable [making me liable]; but if my slave and female slave have caused damage, they are free [causing no liability for me]). Just as with regard to my ox and my donkey, concerning which I am not liable through commandments, behold I am culpable for damage, is it not *logical* that with regard to my slave and my female slave, concerning whom I am liable through commandments, I should be liable for damage?"

MY READING: The Sadducees propose the following positive subjectal, minor to major:

The owner of a male or female slave (P) is more liable through commandments (R) than the owner of an ox or donkey (Q) is.

If the owner of an ox or donkey (Q) is liable through commandments (R) (R = 0) enough to be culpable for damage (S),

Then the owner of a male or female slave (P) is liable through commandments (R) (R > 0) enough to be culpable for damage (S).

FURTHER TEXT: "They said to them: No. If you say this about my ox and my donkey that have no understanding, will you also say it about my slave and female slave who have understanding? So that if I provoke him he goes and sets fire to someone's stack of corn and I am liable to compensate?"

MY READING: The Pharisees object to the above a fortiori argument of the Sadducees by denying its major premise, saying: the owner is responsible for his animals because they cannot understand laws, but the owner is not responsible for his slaves because they can understand laws.

Yadayim 4:8

TEXT: 'A Galilean heretic (var.: Sadducee) said: I raise a complaint against you, O Pharisees, for you write the [name of the] ruler together with [the name of] Moses in a bill of divorce. The Pharisees say: We [*raise a complaint*] against you, O Galilean heretic, for you write the Name [of the] God together with the [name of] the ruler on [one] page, and not only that, but you write the ruler above and the Name beneath, (*var.* as it is said:) 'And Pharaoh said: Who is the Lord that I shall listen to his voice to let go Israel? [I do not know the Lord and also Israel I shall not let go]'. {And when he was smitten, what does he say? 'The Lord is righteous [and I and my people are the wicked ones].'}'

MY READING: The Pharisee argument is best expressed as negative subjectal, since the argument goes from major to minor.

God (P) is more worthy of being dissociated from earthly rulers (R) than Moses (Q) is.

If God (P) is worthy of being dissociated from earthly rulers (R) not enough to have his name excluded from a document with an earthly ruler's name in it (S) (specifically, in the Torah, with Pharaoh),

then Moses (Q) is worthy of being dissociated from earthly rulers (R) not enough to have his name excluded from a document with an earthly ruler's name in it (S) (specifically, in a bill of divorce, with any current ruler).

This a fortiori argument is put forward by the Pharisees, in order to arrive at a conclusion which contradicts the Sadducee's (or Galilean's) assertion (which is not an argument, notice) that we cannot write the name of Moses together with that of the ruler in a bill of divorce. They say, citing an instance from Scripture: not only can such names appear together, but the more honorable one can even appear beneath the less honorable one, and not only in bill of divorce but in any document.

Yebamoth 8:3

TEXT: "{'An Ammonite and a Moabite' is prohibited [to marry an Israelite] and their prohibition is an everlasting prohibition (cf. verse). But their females are allowed right away. An Egyptian and Edomite are only prohibited for three generations, males as well as and females.} R. Shimon allows the females right away. R. Shimon said: The things are *lighter and heavier*: If in a place for which it [Scripture] forbids the males with an everlasting prohibition, it allows the females right away, then in a place for which it forbids the males only for three generations, is it not *logical* that the females are allowed right away?! They answered: If this is Halakhah [which you have received] we receive it. But if it is but an inference [of your own] a counter-interference may rebut it. He answered: Not so, but I declare what is Halakhah."

MY READING: This argument is negative subjectal, since it goes from major to minor.

The females of peoples whose males are forbidden forever (namely, Ammonites and Moabites) (P) are more liable to exclusion (R) than the females of peoples whose males are forbidden for three generations (namely, Egyptians and Edomites) (Q) are.

If the females of peoples whose males are forbidden forever (P) are liable to exclusion (R) not enough to be prevented from inclusion forthwith (S),

then the females of peoples whose males are forbidden for three generations (Q) are liable to exclusion (R) not enough to be prevented from inclusion forthwith (S).

Yom Tov 5:2

TEXT: "Any act that is culpable on the Sabbath, whether by virtue of the rules concerning Sabbath rest (cf. Erub. 10:3, 15) or concerning acts of choice or concerning pious duties, is culpable also on a festival day. And these by virtue of the rules concerning Sabbath rest: no one may climb a tree or ride a beast or swim on water or clap the hands or slap the thighs or stamp the feet [or dance]. And these by virtue of the rules concerning acts of choice: no one may sit in judgment or conclude a betrothal or perform Halitsah or contract levirate marriage. And these by virtue of the rules concerning pious duties: no-one may dedicate anything or make a vow of valuation or devote anything or set apart heave offering or tithes (Deut. 14:22-29). All these things have they prescribed [as culpable] on a festival day: *still more so* [are they culpable] on the Sabbath. A Festival-day differs from the Sabbath in nothing but the preparing of necessary food (cf. mMeg 1:5)."

MY READING: This is a positive subjectal, minor to major.

> The Sabbath (P) is more restrictive (R) than any Festival day (Q) is.
> If a Festival day (Q) is restrictive (R) enough to prescribe certain listed actions (S),
> then the Sabbath (P) is restrictive (R) enough to prescribe the same listed actions (S).

We are given that everything prescribed on Sabbath is so on Festival day, *except* food preparation; and everything prescribed on Festival day is all the more so on Sabbath[, without exception]. The first proposition is almost general but exceptive. The second, which is the reverse if-then, is fully general. Both propositions are needed to fully express the relation between the two situations. As regards the a fortiori argument, it is an apparent redundancy, since we anyway know its conclusion before and independently of its premises. Nevertheless, it can be presented as a useful rule of thumb. That is, it may not be of hermeneutic/theoretical value, but it is heuristic/practical utility. In any case, the argument is formally valid and that is what concerns us here.

Zebahim 7:4

TEXT: "If the whole offering of a bird was offered below (the red line) after the manner of a sin offering and under the name sin offering, R. Eliezer says: The law of sacrilege still applies to it. R. Yehoshua says: The law of sacrilege no longer applies to it. R. Eliezer said: If the sin offering, which is not subject to the law of sacrilege when it is offered under that name, becomes subject to the law of sacrilege if it is offered under another name, *how much more* must the whole offering, which is subject to the law of sacrilege when it is offered under that name, be subject to the law of sacrilege when it is offered under another name."

MY READING: R. Eliezer's argument is positive subjectal, minor to major.

> The whole-offering (P), being subject to law of sacrilege under its own name, is more susceptible
> to sacrilege (R) than the sin-offering (Q), which is not subject to law of sacrilege under its own name.
> If the sin-offering (Q) is susceptible to sacrilege (R) (R = 0) enough to be subject to the law of
> sacrilege under another name (S),
> then the whole-offering (P) is susceptible to sacrilege (R) (R > 0) enough to be subject to the law
> of sacrilege under another name (S).

FURTHER TEXT: "R. Yehoshua said to him: No, as you argue of the sin offering, which when its name is changed to that of a whole offering thereby becomes changed to a thing subject to the law of sacrilege, would you also argue of a whole offering, which when its name is changed to that of a sin offering thereby becomes changed to a thing not subject to the law of sacrilege)?"

MY READING: R. Yehoshua objects to R. Eliezer's a fortiori argument, by considering changes of status from sin offering to whole offering and vice versa. He points out that in the former case, the change makes the offering become subject to the law of sacrilege; whereas in the latter case, the change makes the offering cease to be subject to the law of sacrilege. This denies the conclusion of the a fortiori argument, and thus puts in doubt the process of inference. What is formally wrong with that process? The answer to this question is that although superficially the subsidiary term is the same in the minor premise and conclusion, if we examine it more closely we realize that it is not really so. The words used are the same, but their underlying meaning is quite different. In the minor premise, the offering becomes *truly* subject to the law of sacrilege, whereas in the conclusion the offering becomes *not* subject to it. In the subsidiary term, we cannot use the same relative language as we use in the middle term. Whereas (R) = 0 and (R) > 0 can both count as (R), (S) = 0 and (S) > 0 cannot both count as (S). This is precisely the meaning of R. Yehoshua's objection.[18]

Zebahim 7:6

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[18]   After this, R. Eliezer counters R. Yehoshua's objection, by pointing out that in some cases name change of an offering subject to the law of sacrilege does not cause that offering to cease to be subject to said law – so this constancy may well apply to the whole offering. But R. Yehoshua rejects this analogy, pointing out certain differences between the proposed analogues.

TEXT: "If he had nipped off the head (of the bird) and it was found to be terefah, R. Meir says: It does not convey uncleanness of the gullet. R. Yehudah says: It conveys uncleanness of the gullet. R. Meir said: If in the case of a beast which as carrion would convey uncleanness by contact or carrying, slaughtering renders clean the uncleanness of an animal that is terefah, is it not an *inference* that, in the case of a bird which as carrion would not convey uncleanness by contact or carrying, slaughtering should render clean the uncleanness of an animal that is terefah? Just as we find with slaughtering that it renders fit for eating and renders clean the terefah from its uncleanness, so nipping off (the head) which renders fit for eating, renders clean the terefah from its uncleanness."

MY READING: R. Meir's argument negative subjectal, major to minor.

> A beast (which as carrion would so convey) (P) is more able as carrion to convey uncleanness by contact or carrying (R) than a bird (which as carrion would not so convey) (Q) is.
> If a beast (P) is able as carrion to convey uncleanness by contact or carrying (R) not enough to prevent its slaughtering from rendering clean the uncleanness of its terefah (S),
> then a bird (Q) is able as carrion to convey uncleanness by contact or carrying (R) not enough to prevent its slaughtering from rendering clean the uncleanness of its terefah (S).

Thereafter, R. Meir argues by analogy from slaughtering to nipping off head. It goes apparently: "Since slaughtering and nipping-off both render fit for eating, then just as the former renders clean the terefah from its uncleanness, so does the latter."

FURTHER TEXT: "R. Yose says: It is enough (*dayo*) to compare the carrion of a beast: slaughtering renders clean, nipping off does not."

MY READING: R. Yose objects to R. Meir's arguments. Though he uses the language of *dayo*, saying "it is enough," I do not think he is really invoking the principle of sufficiency, since there is no quantitative or other difference in the subsidiary term of R. Meir's proposed conclusion. R. Yose denies that the conclusion of the a fortiori argument follows from its minor premise (which he accepts), saying: "[Even though] slaughtering the carrion of a beast renders clean, [still] nipping off [head of bird] does not [render clean]." This implies that R. Yose doubts R. Meir's major premise, for some reason.

Zebahim 8:12

TEXT: "If the blood of a sin offering was received into two cups and one of them was brought to the outside [of the temple court], the one that remained inside is fit. If one of them was brought inside [the sanctuary], R. Yose Ha-Gelili declares the outside one fit [i.e. the one that is in the temple court], and the Sages declare it unfit. R. Yose Ha-Gelili said: If in the case where the thought renders unfit, [as when there is an intention to sprinkle] outside [the temple court, cf. mZeb 2:2], this [outside] does not render unfit the remainder [still inside the temple area] like the one that was brought out, is it not an *inference* that in a case where thought does not render unfit, [as when there is an intention to sprinkle] inside [the sanctuary], the one that remains outside is [also] not made like the one brought in [to the sanctuary, namely invalid, mZeb 8:11] ?"

MY READING: R. Yose's argument is negative antecedental, major to minor.

> Wrongly sprinkling blood outside the temple (which renders it unfit) (P) is more ritually problematic (R) than wrongly sprinkling blood inside the sanctuary (which does not render it unfit) (Q).
> If wrongly sprinkling blood outside the temple (P) is ritually problematic (R) not enough to render the remaining blood unfit (S),
> then wrongly sprinkling blood inside the sanctuary (Q) is ritually problematic (R) not enough to render the remaining blood unfit (S).

Zebahim 12:3

TEXT: "The hides of the lesser holy offerings belong to the owners; the hides of the most holy offerings belong to the priests. *Light and heavy*: Just as when in the case of a burnt offering for which they do not have the right to its flesh, they have the right to its hide, for the most holy offerings, for which they have a right to their flesh, is it not an *inference* that they also have the right to their hides?"

MY READING: This is a positive subjectal, minor to major.

> The most holy offerings (whose flesh does belong to the priests) (P) belong to the priests (R) more than the burnt offerings (whose flesh does not belong to the priests) (Q) do.
> If the priests have in relation to the burnt offerings (Q) ownership rights (R) enough to have the right to the hides (S),
> then the priests have in relation to the most holy offerings (P) ownership rights (R) enough to

have the right to the hides (S).

FURTHER TEXT: "The (case of the) altar cannot serve as standard (countering the inference), for it does not have the hide in any case."

MY READING: This is an objection: a denial of the conclusion, which puts in doubt a premise or the process.

DISCLAIMER: *I would like to emphasize that I am not a Talmudist. Being but an amateur, it is quite possible that I have partly or wholly misunderstood some of the texts. I do not pretend here to have fully and accurately explicated the Mishna passages listed – I am not knowledgeable in Jewish law enough to do that. All I have tried to do is to briefly interpret the a fortiori aspect of these discourses in standard form, as they appear without looking at the wider context. Of course, I should have devoted more study to this field, and even consulted an expert in it in order to confirm or correct my interpretations, but I chose not to do so, considering that I had my hands full already with more pressing matters. I would be very grateful to anyone who, finding errors in my treatment, tells me about them. I would certainly encourage anyone who can improve on my work to do so. If the latter wishes me to publish his or her commentary, please submit it to* www.logicforum.org.

3. A fortiori discourse in the two Talmuds

There is a great need for someone to go through all Talmudic literature, and in particular the two Talmuds, looking for all sorts of reasoning in it, and more specifically for applications of the rabbinic hermeneutic principles, and in particular for instances of a fortiori argument. This is a massive job, of course, which ideally ought to be carried out in relation to the original texts, in Hebrew and Aramaic. Many people need to get involved in this project, which is really worthwhile. We can never hope to fully and correctly understand and evaluate Talmudic logic without such thorough empirical research. I have no intention to do this important work, for the simple reason that I do not have the linguistic knowledge needed for it. But I here try and do a small part of it, specifically in relation to a fortiori argument and in English translation.

The Jerusalem Talmud (JT) was the first to have been closed, ca. 400 CE. I have almost no personal experience with this Talmud, but judging from what I have read about it, it is shorter and less disputative, and so we may expect it to contain relatively fewer a fortiori arguments. As Neusner wrote:

> "The Yerushalmi speaks about the Mishnah in essentially a single voice, about fundamentally few things…. [It] takes up a program of inquiry that is not very complex or diverse. The Yerushalmi also utilizes a single, rather limited repertoire of exegetical initiatives and rhetorical choices, etc." (*Rabbinic Literature: An Essential Guide*, p. 41.)

The Babylonian Talmud (BT), closed ca. 600 CE, is the document that will require the most research work. We can expect hundreds of a fortiori arguments in it, to be listed and eventually analyzed. Blau reports that Schwarz[19] estimates the statistics for the second hermeneutic rule, that of *gezerah shavah*, as follows:

> "…in the Babylonian Talmud alone, there has to be close to four hundred, in the Talmud Yerushalmi about one hundred and fifty and in the Tosefta thirty. If one adds to that the *gezerah shavah* in halakhic works and other sources, there would be, after deduction of the numerous parallel passages, a total of six hundred" (p. 156).

I do not know if Schwarz made similar estimates for the first hermeneutic rule, *qal vachomer*, in his book devoted to that subject. But it is a fair guess offhand that the statistics are in the same order of magnitude. Needless to say, we are not counting a fortiori arguments in order to discover who has argued a fortiori the most often; there is no competition to be won! The purpose of our counting them is to accurately determine the number of cases we have to eventually list and analyze.

In any study of a fortiori argument in the Talmud, we must of course distinguish the different *sources* of cases found in it. The Talmud may be quoting a passage from the Torah (the Five Books of Moses) or the Nakh (the rest of the Jewish Bible), or from the Mishna (redacted ca. 220 CE)[20], or from the Tosefta (compiled at about the same date or

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[19]   In his *Der Hermeneutische Analogie in der Talmudischen Litteratur*, pp. 84, 87, 89.
[20]   See the Jewish Encyclopedia article online: www.jewishencyclopedia.com/articles/10879-mishnah.

soon after)[21], or a *baraita* (a statement the Gemara claims as Tannaic, not included in the Mishna, though it may be in the Tosefta), or lastly the Gemara (which in turn has many layers). I will not do this here, but obviously it has to be done if we want to obtain an accurate picture of a fortiori use.

As regards English translations of the Talmuds, we have, I think only five sets to choose from. The three most recent are *The Schottenstein Edition of the Talmud Bavli* as well as *The Schottenstein Edition of the Talmud Yerushalmi* (New York: ArtScroll, various dates)[22], *The Steinsaltz Edition of the Talmud* (New York: Random House, 1989-99)[23], and *The Talmud of Babylonia. An American Translation* (Atlanta: Scholars Press for Brown Judaic Studies, 1984-95)[24]. To my knowledge, these editions are not available in a form that allows computer search, although the Steinsaltz edition is at least partly posted in the author's website[25].

An older translation is *The Soncino Edition of the Talmud* (London, 36 volumes, 1935-1952), edited by R. Isidore Epstein (1894–1962). This is freely available online, thanks to Halakhah.com (a Chabad project), in 63 pdf files[26]. This resource is potentially very useful, provided we take the trouble to merge all these files into one document so as to avoid repetitive work; the single file would of course need to be purged of all editorial content, such as introductory material and footnotes.

Still older is *The Rodkinson Edition of the Babylonian Talmud* (1903)[27]. This edition is freely available online thanks to the Internet Sacred Text Archive[28], in Kindle format[29]. I managed to convert this file into a Word file, from which I removed all extraneous material (i.e. all Rodkinson's introductions, synopses, footnotes, etc.). This edition contains all of the tractates in the Orders (Sedarim) of Moed (Appointed Seasons: 12 tractates) and Nezikin (Damages: 10 tractates)[30]. Thus, four entire Orders are missing in it, namely: Zeraim (Seeds: 11 tractates), Nashim (Women: 7 tractates), Kodashim (Holy Things: 11 tractates), Tohoroth (Cleannesses: 12 tractates). Clearly, the Rodkinson edition does not comprise the whole Talmud, so that any information gathered from it is likely to be incomplete.

But my purpose here is to launch *a pilot study*, to show the way we may obtain the desired information and statistics. I would have preferred to do this pilot study in relation to the Soncino edition, which is not only more complete but also more generally respected; but I decided to focus on the Rodkinson edition to save time and effort. This should suffice to show the way, even if the results obtained will not be as thorough and reliable. Anyway, even if Rodkinson's translations are not universally approved, this handicap hardly affects our study because it specifically focuses on a fortiori argument.

Pilot study. Ultimately, we need to actually list all passages of the Talmud that seem to have a fortiori intent, and see whether they can indeed be cast in standard form (whether valid or invalid). This can only be done exhaustively by going through the whole Talmud page by page, which I do not propose to do here. Instead, I propose to search for a number of key phrases which are usually, or even just often, *indicative of a fortiori discourse*. This is why I needed a single file, purged of all commentary. We cannot find key phrases and count instances in the Rodkinson edition by means of an Index, because it does not have one.

I did in the past, when I wrote *Judaic Logic* (1995), look into the Index Volume of the Soncino edition (1952), and there found 137 entries apparently indicative of a fortiori argument, which I tabulated as follows[31]:

Soncino BT index entries	#
A fortiori	52

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[21]   See the Jewish Encyclopedia article online: www.jewishencyclopedia.com/articles/14458-tosefta.

[22]   These seem to be complete. See: www.artscroll.com/Talmud1.htm.

[23]   This refers to BT and apparently only includes Bava Metzia, Ketubot, Ta'anit, and Sanhedrin. See: www.steinsaltz.org/learning.php?pg=Talmud_-_Books&articleId=1424.

[24]   This is translated (or edited?) by Jacob Neusner, Tzvee Zahavy and others. Complete.

[25]   At: www.steinsaltz.org/index.php. Since the search facility returns only 11 results for 'fortiori', I assume the data base is far from exhaustive.

[26]   This is found at: www.halakhah.com/indexrst.html. A Kindle edition is also available for a small price, at www.talmudicbooks.blogspot.ch/2012/05/amazon-kindle-oral-torah-in-36-volumes.html.

[27]   Michael Levi Rodkinson, previously Frumkin, was a Jew who emigrated to America (1845-1904).

[28]   At www.sacred-texts.com/jud/talmud.htm.

[29]   It is also available in eBook format which can be read using Adobe Digital Editions (ADE) reader. I should mention that, while the Kindle for PC reader has the advantage that its search facility lists 'all' the matching cases at once, it has a maximum limit of 100 hits; the ADE reader, on the other hand, has no maximum limit, but it only takes you to the matching cases one at a time.

[30]   "Plus some additional material related to these Orders;" namely: Ebel Rabbathi / Semahoth; Aboth of R. Nathan; Derech Eretz Rabba and Eretz Zuta.

[31]   As I pointed out at the time, this statistic cannot be taken at face value, "because the references are to page numbers, which may contain more than one argument of the same type; also, not having looked at them, I cannot guarantee that they are all legitimate cases. I would suspect offhand, on the basis of my minimal experience of Talmud study, that this list is incomplete (all the more so if we include the Commentaries)."

A minori ad majus	31
Kal wa-homer	34
Deduction, proofs by	2
Inference from minor to major	8
Major, inference from minor to	8
Minor, inference from major to	2
Total count of a fortiori references	137

Table A3.1

Research by means of search strings is bound to give us a more accurate picture of a fortiori use. The problem with it, of course, is that it allows for *overlaps*. For example, we might count twice the single argument "Aqiba then drew an *a fortiori* conclusion. He said: 'If the soft has so much power over the hard as to bore it (water over stone), *how much more* power will the Torah, the words of which are as hard as iron, have over my heart, which is flesh and blood?'" – once for the phrase "a fortiori" and once for "how much more." Such overlaps can only be eliminated at a later stage, when each argument is listed and examined closely. For the time being, we shall ignore this difficulty and aim for a rough estimate.

Incidentally, it is important to keep in mind when searching for such arguments that the relevant quantitative indicator has to be in the putative conclusion – not in a premise. In the above example, for instance, the relevant indicator is not the antecedent "so much" (which merely refers to an unspecified, impressive quantity), but the consequent "how much more" (which serves to signal a fortiori argument). Thus, an expression (such as the "so much" used here) might be counted as indicative of a fortiori argument and yet in fact not be so – because, though it would be indicative were it in the conclusion, it is not in the conclusion.

The first step in our research is to think of key phrases to search for. The expressions possibly indicative of a fortiori discourse are of two kinds. The first groups includes idiomatic markers like 'all the more', 'how much the less', 'so much more', and so on. The second groups descriptive markers such as 'a fortiori', 'from minor to major', 'inference', 'argue', 'logical' – to name just a few.

With regard to the first kind, we need to decide the order in which our search will proceed, so as to avoid unnecessary repetition. For that purpose, I have developed the hierarchical arrangement shown in the following diagram. The a fortiori phrases are there abbreviated, using the first letters of the words constituting them; for example, 'smms' means 'so much more so'. Note that for every expression with 'more', there is a similar expression with 'less'. The root of all these expressions is the top one, the comparative 'more' (or 'less', as the case may be); from this we derive 'much more', 'how much more', 'so much more', and also 'still more' and 'even more', and more specific verbal forms. A similar flowchart may be constructed starting with the subsidiary root 'the more', from which we derive 'much the more', 'how much the more', 'so much the more', and also 'still more' and 'all the more', and less generic verbal forms.

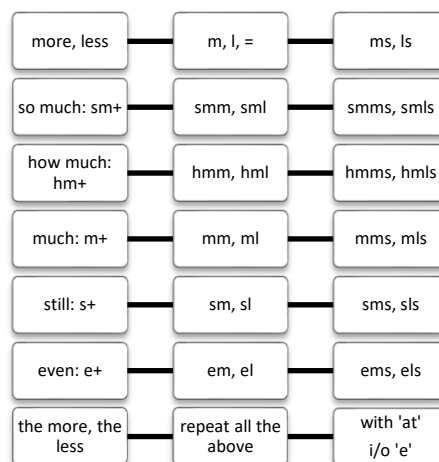| more, less | m, l, = | ms, ls |
so much: sm+	smm, sml	smms, smls
how much: hm+	hmm, hml	hmms, hmls
much: m+	mm, ml	mms, mls
still: s+	sm, sl	sms, sls
even: e+	em, el	ems, els
the more, the less	repeat all the above	with 'at' i/o 'e'

Diagram A3.1   Hierarchy of a fortiori expressions

In the above diagram, the most specific expressions (e.g. smms) are on the right; the more generic (e.g. smm) are in the middle, and the most vague (e.g. sm+) are on the left (examples of the latter are 'so much greater' or 'so much worse'). From this we see that the best way to search through a given document, to ensure a minimum of misses or overlaps, would be in the following order. First, we should look for derivatives of 'the more', starting with the most specific ones and ending with the most generic ones; second, we should look for derivatives of 'more', starting with the most specific ones and ending with the most generic ones. The full orderly list and the results obtained are given in the following table:

A fortiori wording	Count	A fortiori wording	Count
so much the more so	10	so much more so	0
so much the less so	1	so much less so	2
so much the more (residue)	152	so much more (residue)	11
so much the less (residue)	9	so much less (residue)	3
so much the (residue)	1	so much (residue)	6
how much the more so	0	how much more so	0
how much the less so	0	how much less so	0
how much the more (residue)	13	how much more (residue)	15
how much the less (residue)	0	how much less (residue)	4
how much the (residue)	0	how much (residue)	1
much the more so (residue)	0	much more so (residue)	1
much the less so (residue)	0	much less so (residue)	2
much the more (residue)	3	much more (residue)	2
much the less (residue)	0	much less (residue)	18
much the (residue)	0	as much as	2
still the more so	0	as little as	0
still the less so	0	much (residue)	4
still the more (residue)	0	still more so	3
still the less (residue)	0	still less so	1
still the same	1	still more (residue)	10
still the (residue)	0	still less (residue)	0
all the more so	2	still (residue)	?
all the less so	0	even more so	1
all the more (residue)	2	even less so	0
all the less (residue)	1	even more (residue)	15
all the same	3	even less (residue)	4
all the (residue)	0	even (residue)	?
none the less / nonetheless	18	more so (residue)	0
nevertheless	?	less so (residue)	0
the more so (residue)	8	no/not more	3
the less so (residue)	3	no/not less	0
the more (residue)	3	more (residue)	?
the less (residue)	1	less (residue)	?

Table A3.2

Please note well that this is almost raw data, yet to be fully processed by detailed analysis case by case. However, I have here made a small effort to narrow the field. As regards idioms that are almost sure to signal a fortiori discourse, I looked at most cases briefly, in an offhand manner, and eliminated obvious 'duds', by which I mean letter strings

that accidentally resemble a fortiori ones (for example: in 'a children-teacher who struck too much the children', the string 'much the' is obviously not intended as an a fortiori marker). As a general rule, in cases of doubt I counted possible cases as actual cases, without taking the trouble to closely examine the data further.

As regards the search strings labeled 'residue', my policy was to discount all cases but the most likely to be a fortiori discourse. The statistics for such more generic words or phrases exclude the counts for more specific phrases derived from them: whence the label 'residue'. For example, the count for 'so much the' excludes the counts for 'so much the more' and 'so much the less', which in turn exclude the counts for 'so much the more so' and 'so much the less so'. This allows us to see more precisely the a fortiori wording used, and also facilitates dealing with the vaguest residues. To give an example: the count for 'so much the more' (152) excludes the more specific cases of 'so much the more so' (10), and the more generic string 'so much the' (1 instance) excludes the cases of 'so much the more' and of 'so much the less'.

Similarly with other word strings. Obviously, the count of the residue of 'all the' must exclude cases falling under 'all the same', as well as those under 'all the more' and 'all the less'. The reason why phrases with 'the more' (first column) must be counted before those with just 'more' (second column) is that the residue of 'much' excludes all cases with 'much the'; and likewise, 'still' must exclude 'still the', and 'more/less' must exclude 'the more/less'.

The more vague the search string, the more instances it in fact includes; but the method of residues here used allows us to narrow the field somewhat. In the case of 'so much the', only one instance ("so much the firmer") was leftover, and this happened to indeed be a fortiori. In the case of the residue 'so much', only 6 instances out of 75 qualified at first glance as a fortiori (namely, those worded "in a so much larger degree" or "in so much greater a degree"). For 'how much', out 80 remaining instances only 1 qualified (worded "how much severer"). For 'much', out of 234 instances only 4 qualified (worded "much better" or "a much greater"). For the residue 'the more', out of 83 instances only 3 turned out to be apparently a fortiori ("I enjoyed myself the more because I fulfilled two religious duties," "the more should it be allowed…," "it applies the more to…").

Note that all admitted cases involve *a comparison* (e.g. firmer, larger, greater, severer, better, more enjoyable). In many cases, no potentially a fortiori instances were found (at least in my offhand reading of them). Thus, to illustrate: none of the 99 instances of 'no less' or 'not less' qualified; likewise, none of the residual 33 instances of 'still the' and none of the 1010 instances of 'all the' qualified (I looked). In certain residual cases, I did not take the trouble to look at the individual instances at all, expecting negligible results (i.e. close to zero). Thus, for 'nevertheless' (498 instances) 'still' (777), 'even' (3642), 'more' (1159) and 'less' (1949) – I put a question mark, and counted the results as zero. I should explain that the work of individual verification, even done as roughly as I did it, is extremely time consuming.

It is worth remarking that the 'so' of phrases ending in 'more/less *so*' obviously refers to a previously given predication. Phrases with '*the* more/less' are intended as more emphatic than those with just 'more/less'; likewise, '*so* much' is more emphatic than just 'much'; but these emphases are rhetorical: the logical weight is the same. Similarly, '*how* much' is a rhetorical question and therefore less emphatic than 'so much'; but their logical weight is the same. Looking at the above list of commonly used expressions, it occurs to me that, from a purely logical point of view, we could equally well use milder forms, like 'a bit more', 'quite a bit more', 'a little more', 'somewhat more', or even 'some more' – for it is clear that *the amount of* 'more' is irrelevant here. Our habit is to signal a fortiori intent by means of hyperbole (e.g. 'all the more'), but we could equally do so by understatement. However, looking for such milder expressions in Rodkinson's Talmud, I found no cases. Maybe some occur in the Soncino Talmud.

Notice that I add in the above table a number of search strings not included in the preceding diagram, namely: 'still the same' (1 out of 2 instances), 'all the same' (3/4), 'none the less' (18/18), 'nevertheless' (?/498), 'as much/little as' (2/114 and 0/0)[32], and 'no/not more/less' (3/117 and 0/99)[33]. These are all expressions which may be (though evidently often are not) indicative of *a pari* a fortiori argument (i.e. forms with an egalitarian major premise, from which we can equally well reason from minor to major or from major to minor). In any event, when we include these expressions in our listing, we realize that there is a continuity in the wording, ranging from 'all the more', through 'much the more' and 'much the less', to 'all the same' and 'none the less' (and other expressions possibly indicative of *a pari*).

To repeat, definitive statistics will only be possible when each and every case is actually listed and examined in detail – a massive job, which I will not here attempt to do. Having in the above table dealt with idiomatic a fortiori indicators, we should next deal with the more descriptive ones. The following table should, I think, cover most of the potential ground.

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[32]   The two apparent *a pari* in the sentence: "Yea, thou hast occupied thyself as much as R. Hyya, but thou hast not multiplied the Torah as much as he did" are perhaps more implicit than explicit. Paraphrasing: If you occupied yourself with Torah as much as he did, then if his credit is x, your credit would be x (as much as his); and if you spread the Torah as much as he, then your credit would be as much as his.

[33]   Wording: "not more rigorous" (2 instances of 3) or "no more than…" (1 instance of 17).

A fortiori wording	Count	A fortiori wording	Count
a fortiori	147	it is sufficient	73
a minori (ad majus)	0	sufficient (residue)	365
a majori (ad minus)	0	it suffices	13
from minor (to major)	0	suffice (residue)	144
from major (to minor)	0	it is enough	0
kal vochomer	0	enough (residue)	171
inference(s)	60	it follows	19
infer(s), inferred, inferring	984	not follow	23
deduction(s)	19	analogy, analogies	166
deduce(s), deduced, deducing	467	analogous	32
proof	70	analogical, analogue	0
prove, proved, disproved	142	likewise	121
argument(s), argumentation(s)	70	similarly	23
argue(s), argued, arguing	27	general	261
logical	10	particular	255
other wording	?	N.B. These counts are raw data.	

Table A3.3

Please note well that the statistics in this table are even more unprocessed than those in the previous table. I just give the raw numbers dished out by the search engine, without taking the trouble to look at individual cases. The total for this table is 3662, and this is not counting words indicative of inference like 'therefore' (1616), 'hence' (2270, including 861 'whence' and 42 'thence'), then (4729, including 'thence'), etc. Clearly, a lot of work is necessary to sort through all these.

I include 'kal vochomer' in this table, because Rodkinson used this phrase in a note[34]; but as it turned out he did not use it in the text proper. Nor does his translation, unlike the later Soncino translation, ever use the key phrases 'a minori/majori', or 'from minor/major'. His main descriptive term is, thus, 'a fortiori'; this may be used to signal a fortiori intent or to refer to an already proposed a fortiori argument. In any event, use of this key phrase cannot be indicative of anything other than assumed a fortiori discourse.

Nevertheless, many more a fortiori arguments *may* be found by means of the other key words listed in this table and others like them. The word 'infer' presumably usually corresponds to the Hebrew word *din*, which is in rabbinic discourse often used to refer to a fortiori; the same may apply to the words 'deduce', 'prove' and 'argue'. Note that these words often appear in a rhetorical negative question: 'is it not an inference that…', 'can we not deduce that…', 'is it not logical that…'.[35]

Obviously, some of these inferences, deductions, proofs or arguments must refer other hermeneutic principles, such as *gezerah shavah* and *binyan av*, since a fortiori is not the only form of reasoning used in the Talmud. I do not at this time have a clear idea as to how such other interpretative forms are actually worded in Rodkinson's edition, or anywhere else for that matter. Obviously, this question must eventually be answered. When we do that, our investigation will expand from specific concern with a fortiori argument to general concern with all the hermeneutic principles. However, I am not disposed at the present time to look further into this matter.

The main key phrases used by Rodkinson to refer to a fortiori argument are now seen to be the descriptive phrase 'a fortiori' (147 instances), and the various idiomatic phrases 'much the more/less' (189), 'much more/less' (71), 'even

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[34] Rodkinson's there (in Vol. 2, Part I) says: "This is a case of where the peculiar Talmudical expression of Kal Vochomer appears in the text. The literal translation is 'light and heavy', i.e., from the lighter to the heavier or from minor to major. In the Introduction to the Talmud by Prof. Dr. Mielziner an entire chapter is devoted to the explanation of this term (pp. 130-141). However, no general term can be found to express its meaning, and the expression must be varied according to the demand of the text." This remark is to my mind rather strange, given that the Hebrew expression *qal vachomer* has long been known to refer to a fortiori argument, and indeed Rodkinson freely uses the expression a fortiori elsewhere!

[35] Note that the 365 instances of 'sufficient' include 56 'not sufficient' and 5 'insufficient'; the 144 instances of 'suffice' include 20 'not suffice'; and the 171 instances of 'enough' include 15 'not enough'. Note also that besides the 19 instances of 'it follows' and the 7 of 'not follow', there are 281 other 'follows' and 82 other 'follow'.

more/less' (20), 'still (the) more/less' (15), 'all the more/less' (8), among others (36)[36]. The overall result is that the number of a fortiori arguments in the Rodkinson edition of the Talmud may be **about 500** (a round number). This is ignoring overlaps in the first and second tables (no doubt many), as well as all possibly a fortiori intents in the remainder of the second table (maybe numerous); perhaps these and those balance each other out somewhat. This is still a very rough and uncertain tally, of course; but it is better than nothing – an educated guess, let's say. Moreover, keep in mind that Rodkinson's edition includes only two of the six orders of the Talmud – so the final count may be three times this figure!

It should be emphasized that this statistic lumps together purely a fortiori arguments and a crescendo arguments. It is clear that a future fuller study has to distinguish them, i.e. identify how many cases of each of these two types there are. Moreover, each case must be classified as either positive or negative, and subjectal or predicatal (copulative) or antecedental or consequental (implicational), to be really understood. These various moods should then be counted separately. All this additional precision of course requires more detailed analysis of each individual case than here done.

Regarding the *dayo* principle. The key phrase 'it is sufficient' seems to be our main indicator of appeals to the *dayo* principle here; surprisingly, this occurs very rarely. From the data found through mechanical search for "it is sufficient" (73 instances) in the Rodkinson edition of the Talmud, there appears to be *only six passages* that explicitly appeal to the *principle* of *dayo* in some form, such as "it is sufficient that the result derived from an inference be equivalent to the law from which it is drawn," or more briefly as "the rule of 'It is sufficient,' etc." The passages concerned are the following: In tract Baba Kama: the Gemara concerning Mishna 2:1 (1 mention); the Mishna 2:5 (1 mention) and its Gemara (5 mentions); the Gemara concerning Mishna 4:3 (1 mention). In tract Baba Metzia, the Gemara concerning Mishna 3:6 (1 mention). In tract Baba Bathra, the Gemara concerning Mishna 8:1 (2 mentions). And in tract Shebuoth, the Gemara concerning Mishna 4:1 (1 mention).

One question to ask here is: do all these applications concern the inference of a penalty from Biblical law? The answer is clearly yes in cases 1-4, which all concern payment of damages. Case 6 concerns legal liability through making an oath, and so can also be viewed as proper for *dayo* application. Case 5 is open to debate: I have dealt with it in the chapter on Adin Steinsaltz, in the section called 'A recurring fallacy' (18.2), under the heading of 'On Baba Batra 111a-b', there pointing out that reference to the *dayo* principle may be misplaced because while for the daughter the proposed judgment is unfavorable, for the son it is favorable[37]. Thus, judging by the Rodkinson edition, in the 6 cases which explicitly appeal to the *dayo* principle, it is used to limit a penalty or responsibility or right. This accords with my theory of the intended scope of *dayo*.

This result, of course, does not exclude the possibility that there are cases other than those here enumerated, where the *dayo* principle is appealed to explicitly but using other wording than "it is sufficient," or in a more tacit manner, which might yield a different conclusion regarding the intended scope of the principle. To give an example, we saw in the chapter 'In the Talmud, continued', in the section 'Three additional Gemara arguments' (8.6), how the *dayo* principle (in both its versions) may be very present in the background of a discussion without being explicitly mentioned. Moreover, the Rodkinson edition is far from complete; so, some *dayo* applications may well be missed in it – for instance, the *dayo* principle is appealed to in Zebachim 43b-44a, but the Rodkinson edition lacks this tractate.

Note lastly that I have not here made an effort to determine the standard form(s) of the six arguments relative to which *dayo* was used. Cases 2 and 5, having been dealt with elsewhere, we know to be positive subjectal; but the other four cases have yet to be classified. Since the original *dayo* objections in the Mishna are of two sorts, applicable respectively to purely a fortiori argument or to a crescendo argument, we cannot predict how many of these two sorts occur in the Gemara. Furthermore, we should look and see whether the language used in proportional differs from that in non-proportional arguments. I leave these tasks to others.

We will end our pilot study here, without going into more detail or precision, having set an example of methodology and structure of research, and anticipated and dealt with some of the pitfalls that may be encountered.

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[36]   Note that here under 'more/less' I include cases of 'same' and other comparatives.
[37]   This may be why the halakhah in this case does not align with the *dayo* principle.

4. A fortiori discourse by Plato and Aristotle

I found a number of instances of a fortiori discourse by Plato (15) and Aristotle (80). So it is evident that such discourse was relatively common in the Greece of the 5th to 4th century BCE. Note that Plato (or Socrates before him, since Plato attributes most of his to Socrates) uses a fortiori language, but apparently does not reflect upon it anywhere; whereas Aristotle, in a couple of instances, namely *Topics* 2:10 and 3:6 and *Rhetoric* 2:23, engages in a bit of reflection on the subject.

This research was primarily based on fifteen key phrases commonly indicative of a fortiori argument, namely: *a fortiori*, *all the more/less*, *(how/so) much more/less*, *(how/so) much the more/less*. Additionally, to be on the safe side, I looked for cases involving the less specific phrases: *even more/less*[38], *more/less so* (and later also for: *still more/less, still the more/less*). Another important source of discovery was the Kneales's list of references in Aristotle's works, which points to many a fortiori arguments that do not involve any of the key phrases. It is of course conceivable that there are still more a fortiori arguments, differently phrased, in both Plato's and Aristotle's works.

It must be said, however, that not all occurrences of a string of words that includes a key phrase of a fortiori are necessarily indicative of a fortiori argument. Consider for examples, in Plato's LYSIS, the sentence "Suppose that I were to cover your auburn locks with white lead… They would only appear to be white… But that would not make them at *all the more* white, notwithstanding the presence of white in them;" and in his THE SYMPOSIUM, "Do you think that my head is so full of the theatre as not to know *how much more* formidable to a man of sense a few good judges are than many fools?" Although both these sentences contain word strings that usually signal a fortiori argument, neither of them intends such argument. In the first sentence, the string 'all the more' is accidental, part of the larger string 'not at all the more', which simply means 'not more'. In the second sentence, although there is a comparative proposition, it is not used as the major premise of an argument; an argument could be formed with it, but none is.

Each instance of apparent a fortiori argument found is listed below, with an appropriate reference (as available in my sources). I interpret the text, so as to more clearly bring out its a fortiori form, proposing the minor premise and conclusion (given these, the major premise is obvious and so I left it out). The original text is in inverted commas, while my interpretation is without. Needless to say, some interpretations are open to debate; but the exact number of cases is not of great importance, here.

The arguments are classified into moods by means of the following symbols in {} brackets: + for positive, – for negative, s for subjectal, p for predicatal, a for antecedental, c for consequental, and & for a crescendo. If the predicate (or consequent) is the same in premise and conclusion, the mood is subjectal (or antecedental); whereas, if the subject (or antecedent) is the same in premise and conclusion, the mood is predicatal (or consequental).

1. Plato

Regarding Plato (Greece, ca. 428-347 BCE), I first looked for the fifteen key phrases of a fortiori discourse in an eBook called *The Complete Plato*[39], and only found 13 instances: *a fortiori* (1), *all the more* (4), *much more* (1), *much less* (6), *so much the more* (1). I then tried out some vaguer word strings, and found 2 additional instances: *even more* (1), *more so* (1). So, assuming my interpretations are correct, there are in Plato's works at least 15 instances of a fortiori argument. There may, of course, be many more cases, which I have not discovered because I misinterpreted the said phrases in them or because they do not involve these phrases.

Of the 15 cases found, 9 are positive subjectal (including 4 a crescendo), 1 is negative subjectal, 4 are negative predicatal, and 1 is negative consequental; no other forms are exemplified. Note that all 15 arguments are ontical; 4 of them might be characterized as ethical (Crito, Theaetetus, Gorgias, Laws Bk 12); 3 others concern cognitive abilities, but they should not be classed as logical-epistemic (Meno, Cratylus, Laws Bk 7). Also note, 9 of the instances are attributed to Socrates (Greece, ca. 469-399 BCE), while 4 were spoken by an unnamed Athenian, 1 by Alcibiades, and 1 by Cebes. The following are the cases found:

CRITO (p. 39). Socrates speaks to Crito: "our country is more to be valued and higher and holier far than mother or father or any ancestor… and if he may do no violence to his father or mother, *much less* may he do violence to his

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[38] 'X is even more than Y' means 'Y is a lot (or sufficiently much), and X > Y'. Likewise, 'X is even less than Y' means 'Y is a little (or sufficiently little), and X < Y'.

[39] Feedbooks, from ebooks.adelaide.edu.au/p/plato/. Based on the 1891 translation by Benjamin Jowett (of *The Dialogues of Plato*?). I assume this edition is really complete.

country." {+s} If parents are to be valued etc. enough that no one may do them violence, then country is valued etc. enough that no one may do it violence. Notice in this example the explicit major premise that country is more to be valued than parents.**40**

GORGIAS (p. 195). Socrates speaks to Callicles: "If a man who is afflicted by great and incurable bodily diseases… is in no way benefited by him in having been saved from drowning, *much less* he who has great and incurable diseases, not of the body, but of the soul, which is the more valuable part of him." {+s} If a man whose body is very sick has a life bitter enough to be not worth saving, then a man who is spiritually very sick has a life bitter enough to be not worth saving. Notice the statement that the soul is more valuable than the body, from which derives the major premise.

MENO (p. 252). Socrates speaks to Meno: "For I literally do not know what virtue is, and *much less* whether it is acquired by teaching or not." {–p} If I am not wise enough to know what virtue is, then I am not wise enough to know how it is acquired.

EUTHYDEMUS (p. 298). Socrates speaks to Dionysorodus: "I am not a match for one of you; and *a fortiori* I must run away from two." {–p} If I am not skilled enough for one, then I am not skilled enough for two.

CRATYLUS (p. 341). Socrates: "Well, but do you suppose that you will be able to analyse them in this way? for I am certain that I should not." Hermogenes: "*Much less* am I likely to be able." {+s} If Socrates is not intelligent enough to do the analysis, then Hermogenes is not intelligent enough to do it.

PHAEDO (p. 381). Cebes speaks to Socrates: "I would therefore rather not rely on the argument from superior strength to prove the continued existence of the soul after death. For granting *even more* than you affirm to be possible, and acknowledging not only that the soul existed before birth, but also that the souls of some exist, and will continue to exist after death, and will be born and die again and again, and that there is a natural strength in the soul which will hold out and be born many times—nevertheless, we may be still inclined to think that she will weary in the labours of successive births, and may at last succumb in one of her deaths and utterly perish; and this death and dissolution of the body which brings destruction to the soul may be unknown to any of us, for no one of us can have had any experience of it: and if so, then I maintain that he who is confident about death has but a foolish confidence, unless he is able to prove that the soul is altogether immortal and imperishable. {–c} If the various possibilities I grant are not conclusive enough to imply impossible the death of the soul, then the various possibilities you grant are not conclusive enough for that.

PHAEDRUS (p. 430). Socrates speaks to Phaedrus: "the qualities of their god they attribute to the beloved, wherefore they love him *all the more*." {+s &} If someone ordinary has qualities enough to be loved, then someone godly has qualities enough to be loved – even more intensely.

THE SYMPOSIUM (p. 495). Alcibiades: "If Ajax could not be wounded by steel, *much less* he by money." {–p} If Ajax is not vulnerable enough to be wounded by steel, then he is not vulnerable enough to be wounded by money.

THEAETETUS (p. 528). Socrates speaks to Theodorus: "Let us tell them that they [rogues] are *all the more* truly what they do not think they are because they do not know it." {+s} If a rogue who is aware of his roguery is dishonest enough to be censured, then a rogue who is not aware of his roguery is dishonest enough to be censured.

THE REPUBLIC (Book 3, p. 858). Socrates to Adeimantus: "These [harmonies] then… must be banished; they are of no use, even to women who have a character to maintain, and *much less* to men." {–p} If they are not becoming enough to be of use to women, then they are not becoming enough to be of use to men.**41**

THE REPUBLIC (Book 6, p. 933). Socrates speaks to Adeimantus: "Why, I said, we know that all germs or seeds, whether vegetable or animal, when they fail to meet with proper nutriment or climate or soil, in proportion to their vigour, are *all the more* sensitive to the want of a suitable environment, for evil is a greater enemy to what is good than to what is not…. There is reason in supposing that the finest natures, when under alien conditions, receive more

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40   It is thanks to Wiseman's mention of this case that I was moved to look for cases with the vaguer key phrases 'much more/less', 'more/less so'.
41   I am not sure what the intended middle term is here; so I have opted for 'becoming'.

injuries than the inferior, because the contrast is greater." {+s &} If a coarse organism is sensitive enough to be negatively affected by an unsuitable environment, then a finer organism is sensitive enough to be negatively affected by an unsuitable environment – even more negatively.

THE LAWS (Book 1, p. 1058). "Athenian: In the first place, then, the revellers as well as the soldiers will require a ruler? Cleinias: To be sure; no men *more so*." {+s} If other men are dependent enough to require a ruler, then the revellers as well as the soldiers are dependent enough to require a ruler.

THE LAWS (Book 3, p. 1087). Athenian speaks to Cleinias: "The fewness of the survivors at that time would have made them *all the more* desirous of seeing one another." {+s &} If many survivors are lonely enough to desire seeing others, then few survivors are lonely enough to desire seeing others – even more intensely.

THE LAWS (Book 7, p. 1186). Athenian. "Any young man, and *much more* any old one, when he sees or hears anything strange or unaccustomed, does not at once run to embrace the paradox, but he stands considering…." {−s} If a young man encountering the unexpected is not sufficiently perplexed to refrain from reflection, then an old one encountering the unexpected is not sufficiently perplexed to refrain from reflection.

THE LAWS (Book 12, p. 1315). Athenian speaks to Cleinias: "And if he be seen to have come home neither better nor worse, let him be praised at any rate for his enthusiasm; and if he be much better, let him be praised *so much the more*." {+s &} If a man who is neither better nor worse is worthy enough to be praised, then a man who has improved is worthy enough to praised – indeed praised more.

I found no cases in the following books: Apology, Charmides, Laches, Lysis, Euthyphro, Menexenus, Ion, Protagoras, Parmenides, Sophist, Statesman, Philebus, Timaeus, Critias.

After writing the above, it occurred to me that I might find some more instances by searching for the four key phrases: 'still (the) more/less'. Looking into the same document, I found no occurrences of 'still the more' or 'still the less'. However, the string 'still more' occurred 38 times, and 'still less' occurred 8 times. Even so, I would not consider all these occurrences as pointing to a fortiori discourse. Very often, these expressions seem to intend no more than 'some amount more' and 'some amount less', respectively – i.e. their intent is only to signify some greater or lesser degree of something compared to something else, without any *inference* of the one from the other being claimed. Still, in some cases, inference does seem to be intended, and these may be considered as a fortiori arguments. The reader is encouraged to look for these and analyze them; I will give just one of them as an example, without further analysis:

THE APOLOGY (p. 28) Socrates: "Some one will say: Yes, Socrates, but cannot you hold your tongue, and then you may go into a foreign city, and no one will interfere with you? Now I have great difficulty in making you understand my answer to this. For if I tell you that to do as you say would be a disobedience to the God, and therefore that I cannot hold my tongue, you will not believe that I am serious; and if I say again that daily to discourse about virtue, and of those other things about which you hear me examining myself and others, is the greatest good of man, and that the unexamined life is not worth living, you are *still less* likely to believe me."

2. Aristotle

For research into Aristotle (Greece, 384-322 BCE), I used a pdf copy of *The Works of Aristotle*[42]. I searched through that for the fifteen key phrases of a fortiori discourse, and found 40 instances: *a fortiori* (12), *all the more* (22), *how much more* (2), *so much more* (1), *much more* (2), *much less* (1). I then tried out some vaguer word strings, and found 3 additional instances: *even more* (2), *more so* (1). Additionally, I found many instances (37) of a fortiori argument not involving these key phrases, mostly thanks to the already mentioned Kneales' list of references. So, assuming my interpretations are correct, there are in Aristotle's works at least 80 instances of a fortiori argument. There may, of course, be many more cases, which I have not discovered because I misinterpreted the said phrases in them or because they do not involve these phrases.

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[42] Ed. William David Ross. Chicago: Encyclopædia Britannica, 1952.

Of the 80 cases found, 48 are positive subjectal (including 7 a pari and 8 a crescendo), 22 are negative subjectal (including 4 a pari), 5 are positive predicatal, 3 are negative predicatal, and 2 are positive antecedental. Note that only 17 of these arguments are logical-epistemic, while the rest are ontical. All the relevant passages found are listed below[43].

POSTERIOR ANALYTICS (Tr. G. R. G. Mure). 3 instances, of which 1 is positive subjectal, 1 is positive predicatal and 1 is negative predicatal. All 3 arguments may be considered logical-epistemic.

1:1 (p. 222). "A man is asked, 'Do you, or do you not, know that every pair is even?' He says he does know it. The questioner then produces a particular pair, of the existence, and so *a fortiori* of the evenness, of which he was unaware." {+p} If this particular pair is manifest enough to convince you of the existence of which you were unaware, then it is manifest enough to convince you of the evenness of which you were unaware. This argument may be considered logical-epistemic.

1:3 (p. 228). "Two premisses constitute the first and smallest foundation for drawing a conclusion at all and therefore *a fortiori* for the demonstrative syllogism of science." {–p} If less than two premises are not informative enough to draw a conclusion at all, then less than two premises are not informative enough for the demonstrative syllogism of science. This argument may be considered logical-epistemic.

1:10 (pp. 242-3). "That which expresses necessary self-grounded fact, and which we must necessarily believe, is distinct both from the hypotheses of a science and from illegitimate postulate – I say 'must believe', because all syllogism, and therefore *a fortiori* [all] demonstration, is addressed not to the spoken word, but to the discourse within the soul, and though we can always raise objections to the spoken word, to the inward discourse we cannot always object." {+s} If syllogism is inward discourse enough to be necessarily believed, then other forms of demonstration are inward discourse enough to be necessarily believed. This argument may be considered logical-epistemic.

TOPICS (Tr. W. A. Pickard-Cambridge). 24 instances, of which 15 are positive subjectal (including 7 a pari), 9 are negative subjectal (including 4 a pari). At least 8 of these arguments may be considered logical-epistemic.

2:10 (pp. 372-3). "Moreover, argue from greater and less degrees…. If one predicate be attributed to two subjects; then supposing it does not belong to the subject to which it is the more likely to belong, neither does it belong where it is less likely to belong; while if it does belong where it is less likely to belong, then it belongs as well where it is more likely. Again: If two predicates be attributed to one subject, then if the one which is more generally thought to belong does not belong, neither does the one that is less generally thought to belong; or, if the one that is less generally thought to belong does belong, so also does the other. Moreover: If two predicates be attributed to two subjects, then if the one which is more usually thought to belong to the one subject does not belong, neither does the remaining predicate belong to the remaining subject; or, if the one which is less usually thought to belong to the one subject does belong, so too does the remaining predicate to the remaining subject. Moreover, you can argue from the fact that an attribute belongs, or is generally supposed to belong, in a like degree, in three ways, viz. those described in the last three rules given in regard to a greater degree. For supposing that one predicate belongs, or is supposed to belong, to two subjects in a like degree, then if it does not belong to the one, neither does it belong to the other; while if it belongs to the one, it belongs to the remaining one as well. Or, supposing two predicates to belong in a like degree to the same subject, then, if the one does not belong, neither does the remaining one; while if the one does belong, the remaining one belongs as well. The case is the same also if two predicates belong in a like degree to two subjects; for if the one predicate does not belong to the one subject, neither does the remaining predicate belong to the remaining subject, while if the one predicate does belong to the one subject, the remaining predicate belongs to the remaining subject as well."
Here, Aristotle has two sets of subjectal a fortiori argument, one in which one term is superior to the other, and the other in which the terms are equal. In the each set, three examples are given, each of them in positive and negative forms. Thus, there are a total of 12 distinct subjectal arguments, including 6 positives (of which 3 are a pari) and 6 negatives (of which 3 are a pari). That is counting a pari arguments in only one direction, though his wording allows

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[43]   It should be noted that often, when Aristotle makes observations of proportionality, using words indicative of relative quantity (like "more" or "less"), his intent is a pro rata statement rather than an a fortiori argument. For examples: "The greater [prosperity] is, the more exposed is it to risk" (Nicomachean Ethics, 8:1), "The more virtuous a city is, the more happy it is" (Politics, 7:2).

for two. For my more detailed interpretation of these arguments, see the main text. At least 6 of these arguments may be considered logical-epistemic.

3:6 (pp. 388-9). "Moreover you should judge by means of greater or smaller or like degrees: for if some member of another genus exhibit such and such a character in a more marked degree than your object, while no member of that genus exhibits that character at all, then you may take it that neither does the object in question exhibit it; e.g. if some form of knowledge be good in a greater degree than pleasure, while no form of knowledge is good, then you may take it that pleasure is not good either. Also, you should judge by a smaller or like degree in the same way: for so you will find it possible both to demolish and to establish a view, except that whereas both are possible by means of like degrees, by means of a smaller degree it is possible only to establish, not to overthrow. For if a certain form of capacity be good in a like degree to knowledge, and a certain form of capacity be good, then so also is knowledge; while if no form of capacity be good, then neither is knowledge. If, too, a certain form of capacity be good in a less degree than knowledge, and a certain form of capacity be good, then so also is knowledge; but if no form of capacity be good, there is no necessity that no form of knowledge either should be good. Clearly, then, it is only possible to establish a view by means of a less degree."
Here, Aristotle first presents an abstract example of negative subjectal a fortiori argument with unequal terms, then a more concrete illustration of it. Next, he illustrates a pari subjectal argument in positive and negative forms; but his illustrations go only in one direction. Finally, he offers an example of positive subjectal argument with unequal terms. Thus, there are a total of 5 distinct subjectal arguments, including 2 positives (of which 1 is a pari) and 3 negatives (of which 1 is a pari). For further comments on these arguments, see the main text.

4:6 (pp. 415). "On the other hand, the comparison of the genera and of the species one with another is of use: e.g. supposing A and B to have a like claim to be genus, then if one be a genus, so also is the other. Likewise, also, if what has less claim be a genus, so also is what has more claim: e.g. if 'capacity' have more claim than 'virtue' to be the genus of self-control, and virtue be the genus, so also is capacity. The same observations will apply also in the case of the species. For instance, supposing A and B to have a like claim to be a species of the genus in question, then if the one be a species, so also is the other: and if that which is less generally thought to be so be a species, so also is that which is more generally thought to be so."
Here, Aristotle first presents an example of positive subjectal a fortiori argument with equal terms. Next, with regard to positive subjectal argument with unequal terms, he offers us an abstract example followed by an illustration of it. Lastly, he presents two examples of positive subjectal argument, one a pari and one not so. Thus, there are a total of 5 distinct positive subjectal arguments (of which 2 are a pari). For further comments on these arguments, see the main text.

7:3 (pp. 496). "Moreover, look at it from the point of [*sic*] and like degrees, in all the ways in which it is possible to establish a result by comparing two and two together. Thus if A defines a better than B defines [b?] and B is a definition of [b?] so too is A of a. Further, if A's claim to define a is like B's to define b, and B defines b, then A too defines a. This examination from the point of view of greater degrees is of no use when a single definition is compared with two things, or two definitions with one thing; for there cannot possibly be one definition of two things or two of the same thing."
Here, Aristotle presents 2 examples of positive subjectal a fortiori argument, 1 with unequal terms and 1 a pari. For further comments on these arguments, see the main text. These 2 arguments may be considered logical-epistemic.

PHYSICS (Tr. R. P. Hardie and R. K. Gaye). 1 instance, which is negative subjectal.

3:5 (p. 663). "Further, how can the infinite be itself any thing, unless both number and magnitude, of which it is an essential attribute, exist in that way? If they are not substances, *a fortiori* the infinite is not." {–s} If number and magnitude are not concrete enough to be considered substances, then the infinite is not concrete enough to be considered a substance.

ON THE HEAVENS (Tr. J. L. Stocks). 3 instances, 2 being positive subjectal and 1 negative subjectal.

2:7 (p. 899). "Movement tends to create fire in wood, stone, and iron; and with *even more* reason should it have that effect on air, a substance which is closer to fire than these." {+s} If wood, stone, and iron are close enough to fire that their movement tends to create fire in them, then air is close enough to fire that its movement tends to create fire in it.

4:6 (p. 951). "He [Democritus] says that the warm bodies moving up out of the water hold up heavy bodies which are broad, while the narrow ones fall through, because the bodies which offer this resistance are not numerous. But this would be *even more* likely to happen in air – an objection which he himself raises. His reply to the objection is feeble. In the air, he says, the 'drive' (meaning by drive the movement of the upward moving bodies) is not uniform in direction." {–s} If air has not enough upward mobile warm bodies to prevent narrow bodies falling, then water has not enough upward mobile warm bodies to prevent narrow bodies sinking.[44]

4:6 (p. 951). "And these considerations apply with far greater force to air, since it is *so much more* easily divided than water." {+s} If water is easily divided enough that these considerations apply to it, then air is divisible enough that these considerations apply to it.

METEOROLOGY (Tr. E. W. Webster). 1 instance, which is positive subjectal.

2:2 (p. 1076). "All of which is plainly impossible on the theory, and the *more so* as it derives the sea from Tartarus." {+s} If the Phaedo theory, due to its inconsistency with various empirical facts (e.g. that rivers only flow downhill), even without mentioning that it derives the sea from Tartarus, is incredible enough to be rejected, then that theory, when its claim that the sea derives from Tartarus, is additionally taken into consideration, is incredible enough to be rejected. This argument may be considered logical-epistemic.

ON THE SOUL (Tr. J. A. Smith). 2 instances, 1 being negative subjectal and 1 negative predicatal.

1:3 (p. 1174). "Further, the cause of the revolution of the heavens is left obscure. It is not the essence of soul which is the cause of this circular movement – that movement is only incidental to soul – nor is, *a fortiori*, the body its cause." {–s} If the essence of soul is not powerful enough to be the cause of the revolution of the heavens, then the body is not powerful enough to be the cause of the revolution of the heavens.

1:5 (p. 1183). "The problem might also be raised: What is that which unifies the elements into a soul? The elements correspond, it would appear, to the matter; what unites them, whatever it is, is the supremely important factor. But it is impossible that there should be something superior to, and dominant over, the soul (and *a fortiori* over the mind); it is reasonable to hold that mind is by nature most primordial and dominant, while their statement [is] that it is the elements which are first of all that is." {–p} If something is not primordial enough to be dominant over the soul, then it is not primordial enough to be dominant over the mind.

ON SENSE AND THE SENSIBLE (Tr. J. I. Beare). 1 instance, which is negative subjectal.

5 (p. 1281-2). "But since even water by itself alone, that is, when unmixed, will not suffice for food – for anything which is to form a consistency must be corporeal – , it is still *much less* conceivable that air should be so corporealized [and thus fitted to be food]." {–s} If water is not corporeal enough to be food, then air is not corporeal enough to be food.

PARVA NATURALIA (Tr. J. I. Beare and G.R.T. Ross). 2 instances, 1 being positive subjectal and 1 negative predicatal.

7 (p. 1291). "If, then, the sensibles denominated co-ordinates though in different provinces of sense (e.g. I call Sweet and White coordinates though in different provinces) stand yet more aloof, and differ more, from one another than do any sensibles in the same province; while Sweet differs from White even more than Black does from White, it is still less conceivable that one should discern them [viz. sensibles in different sensory provinces whether co-ordinates or not] coinstantaneously than sensibles which are in the same province. Therefore, if coinstantaneous perception of the latter be impossible, that of the former is *a fortiori* impossible." {+s} If sensibles in the same province are separate enough that their simultaneous perception is impossible, then sensibles in the different provinces are separate enough that their simultaneous perception is impossible.

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[44]   Democritus is apparently saying that, although one might object that in air the upward mobile warm bodies, *being more numerous*, would be more likely to uphold things (i.e. narrow bodies as well as broad ones) than those in water do, this does not in fact occur because in air the upward motion is more scattered than in water. Whence the major premise of the a fortiori argument involved is: Air has more upward mobile warm bodies than water does, and the minor premise and conclusion are as stated (major to minor).

7 (p. 1294). "But if the Soul does not, in the way suggested [i.e. with different parts of itself acting simultaneously], perceive in one and the same individual time sensibles of the same sense, *a fortiori* it is not thus that it perceives sensibles of different senses. For it is, as already stated, more conceivable that it should perceive a plurality of the former together in this way than a plurality of heterogeneous objects." {–p} If the soul is not versatile enough to perceive simultaneously sensibles of the same sense, then the soul is not versatile enough to perceive simultaneously sensibles of different senses.

ON MEMORY AND REMINISCENCE (Tr. J. I. Beare). 1 instance, which is negative subjectal.

2 (p. 1307). "And since in the realm of nature occurrences take place which are even contrary to nature, or fortuitous, the same happens *a fortiori* in the sphere swayed by custom, since in this sphere natural law is not similarly established." {–s} If natural law in the realm of nature is not sufficiently established to prevent occurrences (seemingly) taking place which are even contrary to nature or fortuitous, then natural law in the sphere swayed by custom is not sufficiently established to prevent occurrences (seemingly) taking place which are even contrary to nature or fortuitous.

HISTORY OF ANIMALS (Tr. D'Arcy Wentworth Thompson). 10 instances, of which 7 are positive subjectal (including a crescendo), 1 is positive predicatal and 2 are positive antecedental.

3:2 (p. 1463). "Now, as the nature of blood and the nature of the veins have all the appearance of being primitive, we must discuss their properties first of all, and *all the more* as some previous writers have treated them very unsatisfactorily." {+a} If the primitiveness of the properties of blood and veins implies urgency enough for us to discuss them first, then their having been unsatisfactorily treated by past writers implies urgency enough for us to discuss them first.

5:14 (p. 1558). "If a sow be highly fed, it is *all the more* eager for sexual commerce, whether old or young." {+s &} If a moderately-fed sow of any age is sated enough to be eager for sex, then a highly-fed one is sated enough to be eager for sex – even more eager.

7:1 (p. 1652). "And among men, the breasts grow more conspicuous and more like to those of women, both in young men and old, when the individual temperament is moist and sleek and the reverse of sinewy, and *all the more* among the dark-complexioned than the fair." {+s} If the breasts of fair men are affected by the men's temperament enough to grow, then the breasts of dark men are affected by the men's temperament enough to grow.

7:4 (p. 1659). "Women who have connexion with their husbands shortly before childbirth are delivered *all the more* quickly." {+s &} If women who do not have intercourse etc. are ready enough to give birth quickly, then women who do have intercourse etc. are ready enough to give birth quickly – even more so.

8:7 (p. 1690). "Cattle grow *all the more* in size when they are kept from sexual commerce over a number of years." {+s &} If cattle which are allowed to have sex grow enough to reach a certain size, then cattle which are forbidden to have sex grow enough to reach a certain size – indeed, a greater size.

8:10 (p. 1692). "Sheep are fattened by twigs of the olive or of the oleaster, by vetch, and bran of every kind; and these articles of food fatten *all the more* if they be first sprinkled with brine." {+s &} If certain specified feeds without brine are fattening enough that they fatten sheep to some extent, then these feeds with brine are fattening enough that they fatten sheep to some extent – a greater extent.

8:19 (p. 1707). "Whilst rain is wholesome for most fishes, it is, on the contrary, unwholesome for the mullet, the cephalus, and the so-called marinus, for rain superinduces blindness in most of these fishes, and *all the more* rapidly if the rainfall be superabundant." {+p} If the eyesight of certain fishes (the mullet, etc.) is vulnerable enough to be affected slowly by a bit of rain, then their eyesight is vulnerable enough to be eventually affected rapidly by a lot of rain.

8:24 (p. 1714). "The bite is *all the more* dangerous if the mouse be pregnant when she bites, for the boils then burst, but do not burst otherwise." {+s &} If the bite of a non-pregnant mouse (which does not burst boils) is dangerous

enough to be avoided, then the bite of a pregnant mouse (which does burst boils) is dangerous enough to be avoided – even more so.

8:27 (p. 1717). "All insects, without exception, die if they be smeared over with oil; and they die *all the more* rapidly if you smear their head with the oil and lay them out in the sun." {+s &} If any insect smeared with oil in ordinary ways is affected enough to soon die, then any insect whose head is smeared with oil and which is laid in the sun is affected enough to soon die – even sooner.

9:1 (p. 1726). "And so between the aegithus and the ass, owing to the fact that the ass, in passing a furze-bush, rubs its sore and itching parts against the prickles; by so doing, and *all the more* if it brays, it topples the eggs and the brood out of the nest, the young ones tumble out in fright, and the mother-bird, to avenge this wrong, flies at the beast and pecks at his sore places." {+a} If the non-braying ass causes damage enough for the mother-bird to peck at it, then the braying ass causes damage enough for the mother-bird to peck at it.

METAPHYSICS (Tr. W. D. Ross). 4 instances, of which 3 are positive subjectal and 1 is positive predicatal. 1 of these arguments may be considered logical-epistemic.

1:8 (p. 2227-8). "But these thinkers are, after all, at home only in arguments about generation and destruction and movement; for it is practically only of this sort of substance that they seek the principles and the causes. But those who extend their vision to all things that exist, and of existing things suppose some to be perceptible and others not perceptible, evidently study both classes, which is *all the more* reason why one should devote some time to seeing what is good in their views and what bad from the standpoint of the inquiry we have now before us." {+p} If certain views are interesting enough to deserve study, then they are interesting enough to justify our devoting some time to develop value judgments concerning them.

3:4 (p. 2255). "Further, since the matter exists, because it is ungenerated, it is *a fortiori* reasonable that the substance or essence, that which the matter is at any time coming to be, should exist; for if neither essence nor matter is to be, nothing will be at all, and since this is impossible there must be something besides the concrete thing, viz. the shape or form." {+s} If the idea that matter (the concrete thing) exists without being generated is reasonable enough to be accepted, then the idea that the substance or essence (the shape or form) exists is reasonable enough to be accepted. This argument may be considered logical-epistemic.

4:4 (p. 2275). "The same account holds good with regard to 'not being a man', for 'being a man' and 'being a not-man' mean different things, since even 'being white' and 'being a man' are different; for the former terms are much more different so that they must *a fortiori* mean different things." {+s} If 'being white' and 'being a man' are different enough to mean different things, then 'being a not-man' and 'being a man' are different enough to mean different things.

4:4 (p. 2279). "And if this is not knowledge but opinion, they should be *all the more* anxious about the truth, as a sick man should be more anxious about his health than one who is healthy." {+s} If someone who has some apparent knowledge has enough doubt to incite him to verify it, then his having some apparent opinion should give rise to enough doubt to incite him to verify it.

NICOMACHEAN ETHICS (Tr. W. D. Ross). 4 instances, all positive subjectal (including 1 a crescendo).

1:13 (p. 2558). "But if this is so, clearly the student of politics must know somehow the facts about soul, as the man who is to heal the eyes or the body as a whole must know about the eyes or the body; and *all the more* since politics is more prized and better than medicine." {+s} If medicine, which cares for the body, is prized enough to be studied, then politics, which cares for the soul, is prized enough to be studied.

3:9 (p. 2601). "Death and wounds will be painful to the brave man and against his will, but he will face them because it is noble to do so or because it is base not to do so. And the more he is possessed of virtue in its entirety and the happier he is, the more he will be pained at the thought of death; for life is best worth living for such a man, and he is knowingly losing the greatest goods, and this is painful. But he is none the less brave, and perhaps *all the more* so, because he chooses noble deeds of war at that cost." {+s} If the man of low virtue, who has little to lose, is brave enough to choose battle, then the virtuous man, who has much to lose, is brave enough to choose battle.

7:12 (p. 2704) "Neither practical wisdom nor any state of being is impeded by the pleasure arising from it; it is foreign pleasures that impede, for the pleasures arising from thinking and learning will make us think and learn *all the more*." {+s} If without intellectual pleasure we are interested enough to think and learn, then with intellectual pleasure we are interested enough to think and learn.

10:5 (p. 2771). "This happens, similarly, in all other cases, when one is active about two things at once; the more pleasant activity drives out the other, and if it is much more pleasant does so *all the more*, so that one even ceases from the other." {+s &} If a moderately pleasant activity is absorbing enough to diminish conflicting activities, then an intensely pleasant activity is absorbing enough to diminish others, maybe even so much as to stop them.

POLITICS (Tr. Benjamin Jowett). 3 instances, all positive subjectal.

1:5 (p. 2796). "And if this is true of the body, *how much more* just that a similar distinction should exist in the soul?" {+s} If the distinctions between human bodies are significant enough to suggest different social roles, then the distinctions between human souls are significant enough to suggest different social roles.

2:5 (p. 2825). "The government, too, as constituted by Socrates, contains elements of danger; for he makes the same persons always rule. And if this is often a cause of disturbance among the meaner sort, *how much more* among high-spirited warriors?" {+s} If the meaner sort find government by the same persons unpleasant enough to cause disturbance, then high-spirited warriors will find government by the same persons unpleasant enough to cause disturbance.

7:15 (p. 3026). "Courage and endurance are required for business and philosophy for leisure, temperance and justice for both, and more especially in times of peace and leisure, for war compels men to be just and temperate, whereas the enjoyment of good fortune and the leisure which comes with peace tend to make them insolent. Those then who seem to be the best-off and to be in the possession of every good, have special need of justice and temperance – for example, those (if such there be, as the poets say) who dwell in the Islands of the Blest; they above all will need philosophy and temperance and justice, and *all the more* the more leisure they have, living in the midst of abundance." {+s} If the better-off people in ordinary times need philosophy and temperance and justice enough to make them necessary, then they will in times of security and plenty need philosophy and temperance and justice enough to make them necessary.

RHETORIC (Tr. W. Rhys Roberts). 21 instances, of which 11 are positive subjectal (including 1 a crescendo), 8 are negative subjectal, and 2 are positive predicatal. Of these arguments, 4 may be considered logical-epistemic.

1:9 (p. 3168-9). "If a man runs into danger needlessly, *much more* will he do so in a noble cause; and if a man is open-handed to any one and every one, he will be so to his friends also." {+s} If a man is brave enough to endanger himself needlessly, then he is brave enough to endanger himself in a noble cause. {+s} If a man is generous enough to be open-handed to strangers, then he will be generous enough to be open-handed to his friends also. (2 cases.)

2:3 (p. 3202). "The punishment of servants shows this: those who contradict us and deny their offence we punish *all the more*, but we cease to be incensed against those who agree that they deserved their punishment." {+s &} If someone who admits having offended is enervating enough to be punished, then someone who denies having offended is enervating enough to be punished, perhaps more severely.

2:6 (p. 3211). "Once we are on a level with others, it is a disgrace to be, say, less well educated than they are; and so with other advantages: *all the more* so, in each case, if it is seen to be our own fault: wherever we are ourselves to blame for our present, past, or future circumstances, it follows at once that this is to a greater extent due to our moral badness." {+s} If inferiority in any respect though not due to any fault is disgraceful enough to be despised, then such inferiority when due to a fault is disgraceful enough to be despised.

2:13 (p. 3226). "They love life; and *all the more* when their last day has come, because the object of all desire is something we have not got, and also because we desire most strongly that which we need most urgently." {+s} If people who are not close to death love life enough to cling to it, then people who are close to death love life enough to cling to it.

2:19 (p. 3232). "If a thing can be produced without art or preparation, it can be produced *still more certainly* by the careful application of art to it." {+p} If something is simple enough to be produced without art or preparation, then it is simple enough to produce with art and/or preparation.

2:23 (p. 3245). "4. Another line of proof is the '*a fortiori*'. Thus it may be argued that if even the gods are not omniscient, certainly human beings are not. The principle here is that, if a quality does not in fact exist where it is more likely to exist, it clearly does not exist where it is less likely. Again, the argument that a man who strikes his father also strikes his neighbours follows from the principle that, if the less likely thing is true, the more likely thing is true also; for a man is less likely to strike his father than to strike his neighbours. The argument, then, may run thus. Or it may be urged that, if a thing is not true where it is more likely, it is not true where it is less likely; or that, if it is true where it is less likely, it is true where it is more likely: according as we have to show that a thing is or is not true. This argument might also be used in a case of parity, as in the lines: Thou hast pity for thy sire, who has lost his sons: Hast none for Oeneus, whose brave son is dead? And, again, 'if Theseus did no wrong, neither did Paris'; or 'the sons of Tyndareus did no wrong, neither did Paris'; or 'if Hector did well to slay Patroclus, Paris did well to slay Achilles'. And 'if other followers of an art are not bad men, neither are philosophers'. And 'if generals are not bad men because it often happens that they are condemned to death, neither are sophists'. And the remark that 'if each individual among you ought to think of his own city's reputation, you ought all to think of the reputation of Greece as a whole'."

Here, Aristotle starts with a concrete example of negative subjectal argument, which he repeats in more abstract form. Next, he gives us a concrete example of positive subjectal argument, which he likewise repeats in more abstract form. Then he formulates in relatively abstract terms a negative subjectal and a positive subjectal. Finally, he points out that a fortiori argument can be used with equal terms, and offers us 7 subjectal examples (of which 5 are negative and 2 are positive) whose "parity" is, however, unclear. Thus, we may count a total of 13 subjectal arguments, including 5 positive ones and 8 negative ones (leaving open the number of a pari). Note that, although the whole passage is explicitly referred to as concerning a fortiori argument, none of these examples contain a key word or phrase signaling a fortiori argument. 4 of the arguments here may be considered logical-epistemic.

2:23 (p. 3252). "Thus Thrasybulus accused Leodamas of having had his name recorded as a criminal on the slab in the Acropolis, and of erasing the record in the time of the Thirty Tyrants: to which Leodamas replied, 'Impossible: for the Thirty would have trusted me *all the more* if my quarrel with the commons had been inscribed on the slab.'" {+p} If Leodamas was trustworthy to the Thirty enough with his name erased from the slab in the Acropolis, then he was trustworthy enough to them with his name recorded on it as an opponent of the commons.

3:11 (p. 3284). "Liveliness is specially conveyed by metaphor, and by the further power of surprising the hearer; because the hearer expected something different, his acquisition of the new idea impresses him *all the more*." {+s} If someone hearing an unsurprising new idea may be impressed enough to adopt it, then when hearing a surprising new idea is impressed enough to adopt it.

After writing the above, it occurred to me that I might find some more instances by searching for the four key phrases: 'still (the) more/less'. Looking into the same document, I found no occurrences of 'still the more' or 'still the less'. However, the string 'still more' occurred 35 times, and 'still less' occurred 7 times. Even so, I would not consider all these occurrences as pointing to a fortiori discourse. Very often, these expressions seem to intend no more than 'some amount more' and 'some amount less', respectively – i.e. their intent is only to signify some greater or lesser degree of something compared to something else, without any *inference* of the one from the other being claimed. Still, in some cases, inference does seem to be intended, and these may be considered as a fortiori arguments. The reader is encouraged to look for these and analyze them; I will give just one of them as an example, without further analysis:

NICOMACHEAN ETHICS 4:7 (p. 1676). "For the man who loves truth, and is truthful where nothing is at stake, will *still more* be truthful where something is at stake."

5. A fortiori discourse in other world literature

A thorough investigation of the use of a fortiori argument would imply looking at all literature in all cultures and through the ages. This would of course be interesting to do; but it is not really necessary. The most important question is when is the *earliest* use documented use of a fortiori argument in the world, and then in each culture. We may reasonably expect each culture to have its own earliest sample, and these may come at different dates. Even so, two pitfalls must obviously be avoided. (a) It would be erroneous to assume that the earliest sample we found in the world or in a specific culture is really indicative of the date when the argument first appeared in the world or in that culture. (b) It would be erroneous to assume that because the earliest sample in one culture precedes the earliest sample in another culture, it follows that the former produced the latter.

It is a reasonable hypothesis that the use of a fortiori argument, and indeed all the main forms of argument, is a general human phenomenon, which was a feature of oral culture long before it became one of written culture. For this reason, one can expect such use to appear at some time in all or most cultures, and the order of appearance is not necessarily indicative of real precedence or of antecedence. This does not mean that there may not be noticeable differences in the frequency of use in different cultures. Some cultures may exhibit little or even no use of it, while others may exhibit very frequent use. Obviously, the absence (or low score) of a fortiori discourse is as interesting as its presence (or high score). We should also note the variations in frequency of use within a given culture; a fortiori discourse may increase or decrease across time. Thus, as well as looking for the earliest use of such argument, we should try and assess how common it is in contemporary discourse.

Another problem researchers must be aware of is that of language. Looking at translations of foreign literature, we might assume the a fortiori argument to be in use when in fact it is the translator who has injected it into his translation. Often, translators use language that seems a fortiori in intent, but is only hyperbolic, the real intent being merely comparative (e.g. "all the more" sometimes means "a lot more"). Presumably this does not happen often, but it may conceivably do so. Thus, we should always qualify our findings by adding: "assuming the translation accurate."

Needless to say, much more significant than the *use* of a fortiori argument is the *discussion* of such argument. This is indicative of a 'self-consciousness' of the thinker in the act of thinking, an awareness that his thinking has a distinct form that is worthy of study in its own right irrespective of content. Such reflection, of course, signifies an intellectual breakthrough of historical moment. And of course, beyond that initial stage we must expect increasing degrees of awareness and understanding. The highest point of development is the ability to accurately distinguish valid from invalid argument. But in the present appendix, our concern is only with use of the argument.

Some of the research work may nowadays be done by means of computer-assisted searches. In English texts, we should first look for the fifteen key phrases commonly indicative of a fortiori argument (in brackets are my proposed abbreviations), namely: *a fortiori (af)*, *all the more/less (atm, atl)*, *how much more/less (hmm, hml)*, *so much more/less (smm, sml)*, other *much more/less (mm, ml)*, *how much the more/less (hmtm, hmtl)*, *so much the more/less (smtm, smtl)*, other *much the more/less (mtm, mtl)*. To spot *a pari* arguments, we might look for *as much as (ama), as little as (ala)*. Additionally, to be on the safe side, we should perhaps examine cases involving the following eight less specific phrases: *more/less so (ms, ls)*, *even more/less (em, el)*, *still more/less (sm, sl)*, *still the more/less (stm, stl)*. A further possibility is to try out the truncated phrases: *all the (at), how much (hm), much the (mt),* in the hope of finding cases with specific adjectives or adverbs, like: (all the / how much / much the) greater or lesser, better or worse, faster or slower, and so forth (i.e. mostly words ending in –er, but not always).

As we have seen with reference to the Tanakh and Mishna, and to Plato and Aristotle, such research cannot be done mechanically, because the various search strings are not necessarily indicative of a fortiori discourse. Moreover, although useful, such computer searches do not necessarily lead to discovery of all cases of a fortiori argument, because it is often enough conveyed without use of these indicative phrases. Eventually, if we want to be thorough, we have to carefully read through each book examined and record cases we come across. Nevertheless, computer search is very helpful, giving us some tantalizing results to start with.

With these thoughts in mind, I have looked for a fortiori discourse in a number of books, both through computer search and by actual reading. Most of this research is based on my readily finding a computer searchable eBook or pdf copy of a text, although a few of the cases listed I found through actual reading or because someone pointed them out to me. Some of the findings are given above, in the main text or in previous appendices of the present volume. Some more findings are given below, without any claim to systematic effort or thoroughness. To repeat, finding no instances or a certain number of instances of a fortiori argument does not mean that there are not in fact more instances. Moreover, we are here concerned with *intended* a fortiori arguments, whether these are validly or invalidly formulated.

1. Ancient literature

I found *no* instances of the fifteen key phrases of a fortiori discourse in various ancient documents or collections, namely: the stories of Gilgamesh (Sumerian king, 2500 BCE)[45]; an anthology of ancient Egyptian literature[46]; the Code of Hammurabi (Babylon, c. 1772 BCE) and several Roman legal documents[47] listed at the Avalon Project[48]; Homer's *The Iliad* (7th or 8th century BCE); two collections of ancient fragments[49]; *The Analects* of Confucius (China, 551–479 BCE); the sayings of Epictetus (Greece, 55-135 CE). However, I did find a number of ancient works with one or more instances of the fifteen key phrases of a fortiori discourse. Of course, many more ancient sources could and should be similarly scanned.

In *The Odyssey* by **Homer** (Greece, 7th or 8th century BCE, maybe earlier), I found one case (ml): "Fear not a losing game; Phæacian none Will reach thy measure, *much less* overcast."[50] This is an important finding, being the earliest case I found in Greek literature.

In *The Fables* by **Aesop** (Greece, c. 620–564 BC)[51], I found one case (hmm): "The fox and the Leopard disputed which was the more beautiful of the two. The Leopard exhibited one by one the various spots which decorated his skin. But the Fox, interrupting him, said, 'and *how much more* beautiful than you am I, who am decorated, not in body, but in mind.'"

In *The History of the Peloponnesian War* by **Thucydides** (Greece, c. 460-395 BCE)[52], I found seven cases (2 atm, 2 mm, 2 ml, 1 smtm), viz.:

- 1:5. "It would be difficult for any system of fortifications to establish a rival city, even in time of peace, *much more*, surely, in an enemy's country, with Athens just as much fortified against it as it against Athens."
- 2:6."And if our more remote ancestors deserve praise, *much more* do our own fathers, who added to their inheritance the empire which we now possess, and spared no pains to be able to leave their acquisitions to us of the present generation."
- 3:9. "Our revolt, however, has taken place prematurely and without preparation--a fact which makes it *all the more* incumbent on you to receive us into alliance and to send us speedy relief, in order to show that you support your friends, and at the same time do harm to your enemies."
- 5:17. "Melians: Is that your subjects' idea of equity, to put those who have nothing to do with you in the same category with peoples that are most of them your own colonists, and some conquered rebels? Athenians: As far as right goes they think one has as much of it as the other, and that if any maintain their independence it is because they are strong, and that if we do not molest them it is because we are afraid; so that besides extending our empire we should gain in security by your subjection; the fact that you are islanders and weaker than others rendering it *all the more* important that you should not succeed in baffling the masters of the sea."
- 6:19. "I do not well see how they could avoid annihilation if they brought with them another city as large as Syracuse, and settled down and carried on war from our frontier; *much less* can they hope to succeed with all Sicily hostile to them."
- 6:20. "I would have him bear in mind that he will fight in my country, not more for mine than for his own, and by *so much the more* safely in that he will enter on the struggle not alone, after the way has been cleared by my ruin, but with me as his ally."
- 8:24. "After its late misfortunes it could hardly be justified in voluntarily taking the offensive even with the strongest force, except in a case of absolute necessity: *much less* then without compulsion could it rush upon peril of its own seeking."

I have elsewhere mentioned a fortiori discourse in some Indian and Chinese classics. For instances, I mention one case (hml) in *The **Dhammapada***, which is traditionally attributed to the Buddha (c. 563-483 BCE), and four cases (hml, hmm, 2 ht) in the ***Bhagavad-Gita*** (India, c. 5th-2nd centuries BCE). Also, I mention one case (ml) in Lao Tzu's

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[45]   These are given online at etcsl.orinst.ox.ac.uk/cgi-bin/etcsl.cgi?text=c.1.8.1*#.

[46]   *Ancient Egyptian Literature: An Anthology*. Tr. John L. Foster. Austin, Tx.: U. of Texas, 2001.

[47]   Namely: The Twelve Tables (450 BCE), The Acilian Law on the Right to Recovery of Property Officially Extorted (122 BCE), The Agrarian Law (111 BCE), The Edicts of Augustus and Decree of the Senate on the Judicial Process in Cyrene (64 BCE), The Julian Law on Agrarian Matters (c. 58 BCE), The Law of Caesar on Municipalities (44 BCE), and The Charter of Urso (44 BCE).

[48]   The Yale Law School Avalon Project avalon.law.yale.edu/subject_menus/ancient.asp. I tested all Roman law documents shown there, but not *The Athenian Constitution*, because it is included in the works of Aristotle (dealt with in Appendix 4).

[49]   *Ancient Fragments, Containing What Remains Of The Writings Of Sanchoniatho, Berossus, Abydenus, Megasthenes, And Manetho* (Isaac P. Cory, undated – Forgotten Books, 2010); and *Ancilla To The Pre-Socratic Philosophers* (Kathleen Freeman, 1948 – Forgotten Books, 2010).

[50]   Wiseman, in a footnote on p. 131, claims an example of a fortiori argument is to be found in "Homer 386;" maybe it is the same case.

[51]   Tr. by George F. Townsend. Kindle, undated.

[52]   Tr. by Richard Crawley. Kindle, 2008-9. The references here given are the book and chapter numbers (although the book numbers are continuous).

Tao Te Ching (traditionally dated 6th century BCE, though more probably late 4th century or early third century BCE), and two cases (hmm) in Sun Tzu's *The Art of War* (also traditionally dated to the 6th century BCE, but probably of later origin). Note that the main four moods of a fortiori argument (+s, –s, +p, –p) can be found in such ancient literature collectively.**53**

Cicero (Rome, 106-43 BCE)**54** uses a fortiori form at least twice (hmm, mm): in a speech to the Senate, he says: "If, twenty years ago, I declared in this house that death could never be said to have come before its time to a man who had been consul of Rome, with *how much more* truth, at my age, may I say it now;" in a letter to Paetus, he writes: "You seem to insinuate that when grandees are so moderate, *much more* ought a poor ex-consul like myself so to be."

Marcus Aurelius (Rome, 121-180 CE) uses the form at least three times (hmm, mm, ml) in his *Meditations***55**:

- 3:10. "Short then is the time which every man lives, and small the nook of the earth where he lives; and short too the longest posthumous fame, and even this only continued by a succession of poor human beings, who will very soon die, and who know not even themselves, *much less* him who died long ago."
- 11:18. "Consider *how much more* pain is brought on us by the anger and vexation caused by such acts than by the acts themselves, at which we are angry and vexed."
- 11:29. "Neither in writing nor in reading wilt thou be able to lay down rules for others before thou shalt have first learned to obey rules thyself. *Much more* is this so in life."

From the above examples, and many other cases given earlier in the course of the present study, it is evident that a fortiori discourse has been in common use in both the West and the East for a long time. Judging by the data I have at hand, the earliest known record of such use appears to be the Torah, i.e. possibly as early as 1300 BCE, if the traditional date is accepted (and earlier still, if we consider the probable dates of the stories told within the Torah that contain an a fortiori argument). In Greek literature, the argument appears as early as the 7th or 8th century BCE, maybe earlier, in Homer's *The Odyssey*. In India and in China, the earliest date is probably around the 5th century CE. But these dates should not be considered definitive, since they are not based on thorough examination of all extant world literature.

2. More recent literature

Let us now take a look at a fortiori discourse in more recent literature. We shall here focus principally on philosophical literature. I have here again only looked for the fifteen key phrases, and not bothered to look for vaguer phrases or other indices. Moreover, I have only briefly, in an offhand manner, examined the instances found. Needless to say, to the authors mentioned below must be added the authors mentioned in the rest of the present study. In other words, the listing below is not intended by itself as an exhaustive study, but merely as a set of examples taken more or less at random.

Niccolo **Machiavelli** (Italy, 1469-1527), in *The Prince***56**, uses a fortiori discourse at least 3 times (atm); for example: "And for this very reason the Prince ought the less to fear, because after a few days, when the first ardour has abated, the injury is already done and suffered, and cannot be undone; and the people will now, *all the more* readily, make common cause with their Prince from his seeming to be under obligations to them, their houses having been burned and their lands wasted in his defence."

Francis **Bacon** (Britain, 1561-1626), in *The Advancement of Learning***57**, uses a fortiori discourse at least 4 times (1 hmm, 1 smm, 2 mm):

- "If the invention of the ship was thought so noble, which carrieth riches and commodities from place to place, and consociateth the most remote regions in participation of their fruits, *how much more* are letters to be magnified, which as ships pass through the vast seas of time, and make ages so distant to participate of the wisdom, illuminations, and inventions, one of the other?" (1:8:6.)
- "So of degenerate and revolted spirits, the conversing with them or the employment of them is prohibited, *much more* any veneration towards them." (2:6:2.)
- "And if the government of the countenance be of such effect, *much more* is that of the speech, and other carriage appertaining to conversation." (2:23:3.)
- "As in nature, the more you remove yourself from particulars, the greater peril of error you do incur; *so much more* in divinity, the more you recede from the Scriptures by inferences and consequences, the more weak and

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53   See chapter 12.2-3.
54   *Cicero. Ancient Classics for English Readers.* Ed. by W. Lucas Collins. Kindle, 2004.
55   *Thoughts of Marcus Aurelius*. Kindle, 2004.
56   Feedbooks, undated.
57   Kindle, 2004 (based on Cassel, 1893 ed.).

dilute are your positions." (2:25:12.) It is interesting to note that this idea is implicit in the rabbinical *dayo* principle.

Regarding William **Shakespeare** (Britain, 1564-1616), in all of his works[58], I found only 13 instances of the fifteen key phrases of a fortiori discourse (3 hmm, 1 smm, 6 mm, 2 ml, 1 smtm). A bit disappointed, I exceptionally to be on the safe side tried out a couple of other phrases, and found 2 more instances of the argument (1 ama, 1 at). The following are 7 selected examples[59]:

- *The Sonnets* (54). "O *how much more* doth beauty beauteous seem, by that sweet ornament which truth doth give!"
- *Macbeth* (1:7). "What beast wast then that made you break this enterprise to me? When you durst do it, then you were a man, and, to be more than what you were, you would be *so much more* the man."
- *Much Ado About Nothing* (3:3). "I would not hang a dog by my will, *much more* a man who hath any honesty in him."
- *The Winter's Tale* (5:1). "To greet a man not worth her pains, *much less* th' adventure of her person?"
- *King Henry the Eighth* (2:3). "Alas, poor lady! She's a stranger now again. *So much the more* must pity drop upon her."
- *The Merry Wives of Windsor* (1:1). "I' faith, I'll eat nothing; I thank you *as much as* though I did."
- *As You Like It* (1:2). "With his mouth full of news. Which he will put on us as pigeons feed their young. Then shall we be news-cramm'd. *All the* better; we shall be the more marketable."

David **Hume** (Scotland, 1711-1776), in *An Enquiry Concerning Human Understanding*[60] uses a fortiori discourse at least 6 times (1 hmm, 2 mm, 3 ml), and does so very clearly:

- "When we reason a priori, and consider merely any object or cause, as it appears to the mind, independent of all observation, it never could suggest to us the notion of any distinct object, such as its effect; *much less*, show us the inseparable and inviolable connexion between them." (Sect. 4, part 1, §27.)
- "Neither can they dwell with constancy on his mind, even though undisturbed by the emotions of pain or passion; *much less* can they maintain their ground when attacked by such powerful antagonists." (Sect. 8, part 2, §79.)
- "I need not mention the difficulty of detecting a falsehood in any private or even public history, at the place, where it is said to happen; *much more* when the scene is removed to ever so small a distance." (Sect. 10, part 2, §97.)
- "No testimony for any kind of miracle has ever amounted to a probability, *much less* to a proof." (Sect. 10, part 2, §98.)
- "So far as the traces of any attributes, at present, appear, so far may we conclude these attributes to exist. The supposition of farther attributes is mere hypothesis; *much more* the supposition, that, in distant regions of space or periods of time, there has been, or will be, a more magnificent display of these attributes." (Sect. 11, §106.)
- "Yet nothing appears more certain to reason, than that an infinite number of them composes and infinite extension. *How much more* an infinite number of those infinitely small parts of extension, which are still supposed infinitely divisible." (Sect. 12, part 2, §124, fn. 32.)

Regarding Immanuel **Kant** (Prussia, 1724-1804): in *The Critique of Pure Reason*[61], I found 8 instances (2 atm, 2 mm, 4 ml) of the fifteen key phrases. In *The Critique of Practical Reason*[62], I found 3 instances (2 ml, 1 smtm). In *Lectures on Logic*[63], I found 11 instances (10 atm, 1 ml). I list most cases found, out of interest in their subject-matter[64]. Note that in none of the cases found is there any discussion of the a fortiori argument as such – these are only examples of its use.

- "For it is a necessary condition of every cognition that is to be established upon a priori grounds that it shall be held to be absolutely necessary; *much more* is this the case with an attempt to determine all pure a priori cognition, and to furnish the standard—and consequently an example— of all apodeictic (philosophical) certitude." (*Pure*, preface to 1st ed.)

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[58]   *The Complete Works of William Shakespeare*. Kindle, 1994.
[59]   The 8 other cases found are in: *The Sonnets* (2), hmm. *King Henry the Eighth* (5:3), hmm. *King Henry the Fourth, Second Part* (1:3), mm. *The Life of King Henry the Fifth* (5, prologue), 2 mm. *Measure for Measure* (5:1), mm. *The History of Troilus and Cressida* (5:6), mm. *The Tempest* (3:1), ml.
[60]   Kindle, 2006 (based on a 1902 ed. based on 1777 ed.), fn 32.
[61]   Kindle, 2011.
[62]   Kindle, 2012.
[63]   Tr. Michael Young. Cambridge: UP, 1992.
[64]   The cases not listed are in *Logic*, p. 146 (3 atm), p. 318 (atm), p. 366 (atm), p. 474 (ml). Note that the systems of reference in the two *Critiques* are very confusing, at least in the editions I have used; I do the best I can but without full certainty that the references I give are accurate.

- "The early success of logic must be attributed exclusively to the narrowness of its field, in which abstraction may, or rather must, be made of all the objects of cognition with their characteristic distinctions, and in which the understanding has only to deal with itself and with its own forms. It is, obviously, a *much more* difficult task for reason to strike into the sure path of science, where it has to deal not simply with itself, but with objects external to itself." (*Pure*, preface to 2ⁿᵈ ed.)

- "But of what kind is this intuition? Is it a pure a priori, or is it an empirical intuition? If the latter, then neither an universally valid, *much less* an apodeictic proposition can arise from it, for experience never can give us any such proposition." (*Pure*, part 1, sect. 2, sub-sect. 9, 1)

- "So that thus we should not be men, but belong to a class of beings, the possibility of whose existence, *much less* their nature and constitution, we have no means of cognizing." (*Pure*, part 2, 1ˢᵗ division, book 2, Remark on the amphiboly of the conceptions of reflection.)

- "If – as often happens – empiricism, in relation to ideas, becomes itself dogmatic and boldly denies that which is above the sphere of its phenomenal cognition, it falls itself into the error of intemperance – and error which is here *all the more* reprehensible, as thereby the practical interest of reason receives an irreparable injury." (*Pure*, part 2, 2ⁿᵈ division, book 2, chapter 2, section 3, 2.)

- "And, how far soever we have to travel upon the path of experience to discover some fact or event, this idea requires us to believe that we have approached *all the more* nearly to the completion of its use in the sphere of nature, although such completion can never be attained." (*Pure*, part 2, 2ⁿᵈ division, book 2, chapter 3, section 7, appendix.)

- "Negative judgements—those which are so not merely as regards their logical form, but in respect of their content—are not commonly held in especial respect. They are, on the contrary, regarded as jealous enemies of our insatiable desire for knowledge; and it almost requires an apology to induce us to tolerate, *much less* to prize and to respect them." (*Pure*, part 2, 2ⁿᵈ division, book 2, chapter 3, section 8, 2.)

- "No power of the understanding could infer from the conceptions which we previously possessed of these substances; *much less* is there any a priori law that could conduct us to such a conclusion, which experience alone can certify." (*Pure*, part 2, 2ⁿᵈ division, book 2, chapter 1, section 2.)

- "These, on account of the uniformity of conduct, exhibit a natural connection, which however does not make the vicious quality of the will necessary, but on the contrary, is the consequence of the evil principles voluntarily adopted and unchangeable, which only make it *so much the more* culpable and deserving of punishment." (*Practical*, book 1, chapter 3.)

- "A scholar who has not come so far as to guide himself, *much less* to guide others." (*Practical*, book 2, chapter 1.)

- "In all cases where the want is founded on inclination, which cannot necessarily postulate the existence of its object even for the man that is affected by it, *much less* can it contain a demand valid for everyone." (*Practical*, book 2, chapter 2.)

- "Through an excess of learnedness people often become *all the more* absurd and completely unfit for judging *in concreto*. The healthy understanding, which is small but correct, involves *simplicity*. It remains on the ground of experience and does not love chimerical ideals. This very simplicity makes the understanding *all the more* correct, certain, and reliable than science." (*Logic*, p. 10).

- "Many prejudices are such and of the kind that if one dared to attack them, to extinguish them, to try to clear them away, they would nonetheless make [the] mind of man *all the more* embittered, indeed, would deafen man to listening and attending to true doctrines and important *dogmata*, which have great consequences for them." (*Logic*, p. 134-5).

- "There are cases where precision cannot appropriately be sought, and where it causes indistinctness, although it provides *all the more* distinctness for a capable mind." (*Logic*, p. 302).

- "In the case of unity, I infer that because many consequences fit with one ground, it is *all the more* probable that [this] ground is true and is the right one." (*Logic*, p. 335).

- "He who hates science but loves wisdom *all the more* is called a *misologist*. Misology arises commonly out of an emptiness of scientific cognitions and a certain vanity bound up with that." (*Logic*, p. 539).

Hendrik **Lorentz** (Holland, 1853-1928), in *The Einstein Theory of Relativity*[65], has one a fortiori argument (atm): "It is not necessary to give up entirely even the ether. Many natural philosophers find satisfaction in the idea of a material intermediate substance in which the vibrations of light take place, and they will very probably be *all the*

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[65]   Kindle, 2004.

more inclined to imagine such a medium when they learn that, according to the Einstein theory, gravitation itself does not spread instantly, but with a velocity that at the first estimate may be compared with that of light."

Kaiten **Nukariya** (Japan, c. 1913)[66], in *The Religion of the Samurai: A Study of Zen Philosophy and Discipline in China and Japan*, uses the argument at least 4 times (2 atm, 2 hmm): For example: "Similarly, it is the case with religion and morality. If we admit extreme idealism as true, there can be nothing objectively real. God is little more than a mental image. He must be a creature of mind instead of a Creator. He has no objective reality. He is when we think He is. He is not when we think He is not. He is at the mercy of our thought. *How much more* unreal the world must be, which is supposed to have been created by an unreal God! Providence, salvation, and divine grace--what are they? A bare dream dreamed in a dream!" Also: "The believer of Buddha is thankful even for death itself, the which is the sole means of conquering death. If he be thankful even for death, *how much more* for the rest of things!"

Moreover, I found instances of a fortiori discourse using the fifteen key phrases in the following books. (I give no examples in these cases, but the reader may readily find them.)

- J.-J. Rousseau (Geneva-France, 1712-1778), in the *Confessions*[67], three cases (1 hml, 2 ml).
- Adam Smith (Scotland, 1723-1790), in *The Wealth of Nations*[68], four cases (2 hml, 1 smtg, 1 smtl).
- Thomas Paine (England-America, 1737-1809), in *Common Sense*[69], two cases (1 mm, 1 ml).
- In *The Federalist Papers* (USA, 1787-8)[70], written by Alexander Hamilton, James Madison, and John Jay, under the pseudonym of Publius, one case (hmm).
- Arthur Schopenhauer (Germany, 1788-1860), in a collection of his essays[71], two cases (hmm).
- John Stuart Mill (Britain, 1806-1873), in *A System of Logic*[72], seventeen cases (4 af, 1 atm, 1 hmm, 3 mm, 8 ml); in *On Liberty*[73], four cases (1 af, 3 atm).
- W. Stanley Jevons (Britain, 1835-1882), in *The Principles of Science*[74], one case (atm).
- Friedrich Nietzsche (Germany, 1844-1900), in *Beyond Good And Evil*[75], four cases (2 atm, 2 ml); in *Thus Spake Zarathustra*[76], two cases (ml).
- Wilhelm Windelband (Germany, 1848-1915), in *History of Philosophy*[77], forty-six cases (45 atm, 1 ml); in *An Introduction To Philosophy*[78], five cases (all atm).

On the other hand, using the fifteen key phrases I looked for a fortiori discourse in the following books, and found *no* instances in them: *The Book of Five Rings* by Miyamoto Musashi (Japan, c. 1584-1645); *Discourse on Method* by Rene Descartes (France, 1596-1650); *Candide* by Voltaire (France, 1694-1778); *The Age of Reason* by Thomas Paine (England-America, 1737-1809); *The U.S. Constitution* by James Madison (USA, 1751-1836); *The Origin of Species* by Charles Darwin (Britain, 1809-1882); *Dream Psychology* by Sigmund Freud (Austria-England, 1856-1939); *The Book of Tea* by Kakuzo Okakura (Japan, 1863-1913); *The Problems of Philosophy* and *Mysticism and Logic and Other Essays* by Bertrand Russell (Britain, 1872-1970).

Summarizing the above information somewhat, we obtain the following results regarding the language used and its frequency: in the 16th - 17th cent., 4 atm, 4 hmm, 2 smm, 7 mm, 2 ml, 1 smt; in the 18th cent., 2 atm, 2 hmm, 5 mm, 3 hml, 12 ml, 2 smtm, 1 smtl; in the 19th cent., 5 af, 5 atm, 3 hmm, 3 mm, 12 ml; in the 20th cent., 48 atm, 2 hmm, 1 ml. We cannot, of course, draw any definite conclusions from such scattered data. But we can say that the verbal expression of a fortiori discourse has remained widely used and pretty uniform in modern times.

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[66]   Kindle, 2004.
[67]   Feedbooks, undated.
[68]   Feedbooks, undated.
[69]   Feedbooks, undated.
[70]   Feedbooks, undated.
[71]   *The Essays of Arthur Schopenhauer; Religion, a Dialogue, Etc.* Kindle, 2004.
[72]   *A System of Logic, Ratiocinative and Inductive*. Kindle, 2009.
[73]   Feedbooks, undated.
[74]   *The Principles of Science: A Treatise on Logic and Scientific Method*. London: MacMillan, 1913.
[75]   Feedbooks, undated.
[76]   Feedbooks, undated.
[77]   Tr. James H. Tufts. 2nd ed. London: MacMillan, 1914.
[78]   Tr. Joseph McCabe. London: Fisher-Unwin, 1914.

6. Logic in the Torah

There is evidently quite a bit of logic – inductive as well as deductive – to be found in the Tanakh (the Jewish Bible). Although this document aims, of course, primarily to convey narratives (they did this, they said that) and legislation (do this, don't do that), it also incidentally – whether intentionally or not – contains quite a few lessons in logic. Perhaps for this reason, Jews have traditionally been rather logical-minded people. However, to date no one (so far as I know) has made a systematic study of this topic, looking for all information of logical interest in the Tanakh. The term 'logic' should here be taken in its most comprehensive sense, including not only inductive and deductive processes capable of formal representation, but also logical intuitions, methodological guidelines and epistemological theories, and indeed anything that may conceivably improve cognitive efficacy, for that after all is the main purpose of logic.

As we have seen, researchers have over time found 46 instances of a fortiori argument, including 5 in the Torah proper (i.e. the Pentateuch). In my earlier work, *Judaic Logic*, I focused on passages in the Torah which contain a couple of very important principles of inductive logic. But there is bound to be much more material of logical significance than that. If a fortiori argument, a relatively subtle and complex argument, is so frequent, we can reasonably expect a great many simpler and more common arguments to be found. Noticing them and correctly identifying them requires some logical skills, of course; not everyone can do this job.

With this thought in mind, I have read once through the Torah looking for new material of logical significance and commenting on it. My findings are given below. The result is a grab bag of miscellaneous logical techniques or principles; it is clearly not an exhaustive toolbox, let alone a systematic teaching. Still, these findings are worth noting, as they may well have influenced and continue to influence Jews and others, consciously or otherwise, into more logical thinking. The findings listed below are, to repeat the findings from merely a first reading of the Torah; more readings will very likely yield additional findings, some perhaps more subtle and profound than those initially identified.

Moreover, I have not yet looked into the rest of the Jewish Bible (the Nakh) for similar material; no doubt quite a bit more should be found there. Nevertheless, I think the work below should pave the way for further research of the same sort in the future, by myself and others.

The first words of **Genesis 2:16**, "And the Lord God commanded the man, saying," suggest the possibility of communication between God and mankind, i.e. of prophecy. From a secular point of view, this possibility is not very significant, since most of us do not nowadays lay claim to prophetic powers. But from a religious point of view, it is of course of major significance, and it is implied not only here but throughout the Torah and Tanakh.

In **Genesis 2:16-17**, God gives Adam the first of all commandments, viz.: "Of every tree of the garden [of Eden] thou mayest freely eat, but of the tree of the knowledge of good and evil, thou shalt not eat of it; for in the day that thou eatest thereof thou shalt surely die." This passage is of interest to logic, because of the form of its discourse. The second part of it may be stated as: If you eat of this tree, the consequence will be death; therefore, do not eat of it; and the first part as: If you eat of any other tree, no such consequence will follow; therefore, you may eat of it. More symbolically put: if you do X, Y will then occur, and Y is undesirable, therefore avoid X; and if you do something else X', Y will not then occur, therefore, no need to avoid that other thing X'. Thus, this passage makes use of an if–then proposition, and implicitly of an inverse if–not-then proposition. Moreover, it teaches that if something undesirable would follow a certain action, one should not do it; whereas if nothing undesirable would follow a certain action, one may do it. The latter lesson concerns ethical logic.

Genesis 2:19 tells us that after God formed the beasts of the field and the fowl of the air, he paraded them before Adam "to see what he would call them; and whatsoever the man would call every living creature, that was to be the name thereof." Now this obviously a statement with logical significance; but what does it mean, exactly? Well, the simplest reading would be that words are arbitrary labels (initially composed of sounds, and later of written symbols) that man, for his own purposes, mentally attaches to the things he perceives or conceives. And this corresponds to the commonsense view of words; so we can say that the author of this statement was saying the obvious. The implication of this simple view, note well, is that words *in themselves* tell us nothing about the objects they refer to: they are conventional.

Very different is the mystical interpretation of this verse that has developed among Jewish commentators[79]. According to them, language is originally made up of Hebrew letters, which were used by God during Creation to fashion the things created. When Adam named things, he was actually seeing into their essence and intuiting the verbal elements constituting them. This is clearly a much more complex theory of words, with metaphysical as well

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[79]   Notably *Radak* (David Kimchi – Southern France, 1160–1235), according to a comment ad loc. in *The Stone Chumash* (ArtScroll, NY).

as epistemological implications. And, note well, it is in direct opposition to the simpler, commonsense view, since it claims that words (or at least, the Hebrew words used by God and Adam) are significant in themselves, as against conventional.

My point here is not to judge the matter, but merely to draw attention to the philosophical issues raised by this statement. Nevertheless, I would personally opt for the non-mystical theory, in view of the practical difficulties inherent in trying to prove the mystical one. The latter can only be classed as speculative, since (as far as I can see) we have no way to ever inductively prove it. Of course, we can conceive the things of this world as consisting of vibrations (of force fields, presumably) – but how would we scientifically establish that these vibrations correspond to those of Hebrew letters? Anyway, the commentators who make this claim have not demonstrated the correspondences. Therefore, by elimination, the commonsense view is more convincing.[80]

Genesis 2:21-24 tells us that God made the first woman (Eve) by taking one of the ribs of the first man (Adam), then declares: "Therefore (*al ken*) shall a man leave his father and his mother, and shall cleave unto his wife, and they shall be one flesh." This passage is logically interesting because it says "therefore," implying that an inference is taking place. But what inference is intended here is far from clear! Why should the fact that the first man and woman were "of one flesh" imply that men should, or even merely would, thenceforth leave their parents and cleave to their wives? For a start, Adam and Eve had no parents, so they provide no example to follow. Secondly, women after the first were not formed from a rib of their husband, but by ordinary reproduction; so the unity of later couples is not as literal.

Thirdly, leaving aside those first two objections, why would couples who are "of one flesh" *not* stay with the husband's parents? In fact, they often do stay with his parents (or at least, did so till modern times), and even sometimes with hers (though this is not excluded here). No explanation is given, in the text or in subsequent commentary. Sure, one can readily see the purpose of the statement – to teach that the new couple is, or is to be, a new psychosocial unit, distinct from the parental units. But this is a supplementary insight, rather than a deduction; so why say "therefore"? The translation of *al ken* into "therefore" seems sound; the literal meaning of the phrase is 'on yes' – i.e. on this basis, because of this, this implies.

So we may have here an early instance of 'dud' inference; that is to say, of something presented as inference, which is in fact not at all inference. The use of a marker like "therefore" makes it look like inference, but it is not really so. In short, this is rhetoric, rather than logic. It is interesting to find this mode of communication being used in the Torah. Such use is perhaps innocent – not intended to fool anyone, but merely to make the narrative seem more continuous.

Another tack. Needless to say, all the above is said within the framework of the Torah world view. But of course, modern science has a very different world view. The Torah view is one claimed to be based on revelation, and deduction and to a lesser extent induction therefrom; whereas the scientific view is one based on publicly accessible empirical data, and induction and to a lesser extent deduction therefrom. According to modern science, there was no first man and woman, and the world was not created less than 6000 years ago. The Big Bang, which started the expansion of the material universe, occurred some 13.8 billion years ago (based on complex astronomical measurements). Our planet, Earth, was born some 4.6 billion years ago, soon after the Sun (long after most stars visible in the sky, note) and before the Moon. Life on Earth began about 3.9 billion years ago (we do not know exactly how). Mankind is the product of a long evolution of life forms since that time. The age of our species is difficult to gauge, simply because it gradually emerged in an evolutionary continuum; but we can say, even if somewhat conventionally, that it is about 200,000 years old (which means, very, very recently).

As regards sexual differentiation, it has a long and rich natural history. Certainly, males did not precede females in our species, but both existed in earlier species well before our species arose. Therefore, the Torah here, as in many other contexts, is a world apart – a factually inaccurate report. This is, of course, indicative of logical errors; especially, not enough inductive logic was used. The Torah account may seem convincing to some people, due to their limited scientific knowledge; but the conclusion of inaccuracy and error is inevitable to anyone who studies the matter objectively. I take no pleasure is saying this, and I am certainly not the first to say it. Therefore, we must view many of the Torah claims – especially concerning prehistoric times – as myths, legends and opinions. We can still learn many valuable lessons from this venerable document, which is after all a message from our ancestors; but we should remain coolheaded. It may conceivably have been Divinely inspired, but some admixture of human ideas must have occurred in the process, since it contains factual errors.

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[80]   It should be added that the Biblical view that all languages are derived from Hebrew (since the incident described in Gen. 11:6-7) is open to doubt. In this regard, I would cite for instance John McWhorter's *The Power of Babel: A Natural History of Language* (London, Heinemann, 2002). This explains the technical difficulties inherent in such reductionism, no matter what the initial language is assumed to be.

In **Genesis 3:1-5**, the serpent tries to tempt Eve by means of the following argument (here paraphrased): "since God did not say 'you shall not eat of any tree of the garden'[81], then you may eat of this tree;" to which Eve rightly retorts, briefly put: "He said we may eat of all trees *except* this one." We can discern in this a teaching of logic, namely that the serpent's inference from 'not all X are Y' (i.e. 'some X are not Y') to 'this one X is not Y' is fallacious, and learn from Eve's reply that a proposition may be general and exceptive, i.e. have the form 'all X except this one are Y'. It is easy to see how readers of such discourse absorb, over time, if only incidentally, lessons in logic.

Genesis 4:15 provides the first Biblical example of syllogism and apodosis in the Torah. In response to Cain's complaint (in the preceding verse) that "whosoever findeth me will slay me," God declares: "whosoever slayeth Cain, vengeance shall be taken on him sevenfold." The form of this statement is: anyone who does X, will be subject to consequence Y. Understanding such a statement requires the ability to subsume a particular case under a generality Although this is not explicitly exemplified here, it is understood that if ever any individual does indeed do X, the consequence Y will befall him. That is to say:

> Any man who does X will receive Y;
> and this man did X,
> therefore, this man will receive Y.

This is syllogism (1/ARR) if the major premise is viewed as categorical, or apodosis (*modus ponens*) if it is viewed as conditional. Clearly, the argument may be described as application of a general rule to a particular case. The same arguments are implicit in all the Biblical statements that detail a negative consequence of disobeying a commandment. For example, in Exodus 12:15: "whosoever eateth leavened bread from the first day until the seventh day, that soul shall be cut off from Israel." There is no need to list all cases.

This is admittedly not a fully explicit demonstration of syllogism and apodosis in the Torah, since the minor premise and conclusion are not stated, but obviously the Torah takes for granted that we understand the word 'whosoever' and can grasp the practical significance of an if–then statement. We do have an example of concrete application of an abstract rule, in the case of breach of the Sabbath, described in Numbers 15:32-36, where a man disobeys the interdiction to work on the day of rest and is consequently put to death[82]. Further on, e.g. in Leviticus 21:21, we do find more explicit apodoses.

In **Genesis 4:23-24** we find the Torah's first example of a fortiori argument, which is (more precisely) an a crescendo argument. Lamekh, who was the father of Noah (5:28-29), argues: "If Cain shall be avenged sevenfold, truly Lamekh seventy and seven-fold." The statement about Cain being "avenged sevenfold" is a reference to 4:15, which reads: "whosoever slayeth Cain, vengeance shall be taken on him sevenfold." This interpretation and translation[83] is not very clear. The statement is by God, who intends thereby to prevent people from slaying Cain, who murdered his brother Abel. Fair enough; but how would the person who slays Cain be punished "sevenfold" – he can only be executed once, unless we admit of reincarnation! For this reason, I gather, Rashi (and other commentators follow suit) reads the statement as meaning that the punishment of Cain must be put off for seven generations.

Given this interpretation, we can better understand Lamekh's thinking. Rashi[84] reads his statement as meaning: "If Cain killed intentionally, [and yet] his punishment was delayed for seven generations, [then] I, who killed unintentionally[85], surely will have my punishment deferred for many periods of seven generations." This is, as already mentioned, an a fortiori argument; or more precisely, it is an a crescendo argument, since the conclusion has a larger quantity (77) than the premise (7). And the argument is logically credible: unintentional killing is far less culpable than intentional killing, and so deserves proportionately less punishment, and/or longer deferral of punishment. All this may be viewed as, incidentally at least, a teaching of logic.

There are four more examples of a fortiori argument in the Torah, namely: **Genesis 44:8**, **Exodus 6:12**, **Numbers 12:14**, **Deuteronomy 31:27**. Each of these contains some interesting lesson of logic, since their forms vary. The first is positive predical in form, the second negative subjectal, and the last two are positive subjectal. All four are (in my opinion) purely a fortiori, unlike the above argument by Lamekh. The argument in Numbers plays a big role in Talmudic discussion, and is there considered by the Gemara author(s) as a crescendo. I need not here say more about them, since I deal with them in plentiful detail elsewhere.

Genesis 8:6-12 may be viewed as a lesson in inductive logic, since it describes a deliberate experiment. This passage is part of the deluge story: Noah, while still in his ark, together with his family and samples of all animals, at some point "wanted to see if the waters were abated from off the face of the ground." To test the matter, he first "sent forth

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[81] He puts it as a rhetorical question: "Hath God said 'you shall not eat of any tree of the garden'?" – suggesting that God did *not* say that.
[82] The rule is given earlier, in Ex. 31:14: "everyone that profaneth it [the Sabbath] shall surely be put to death." Nevertheless, in the episode of Num. 15:32-36, the question is put to God before the rule is applied.
[83] It is the American-Jewish (A.J.) translation, given in *The Soncino Chumash*.
[84] Based on *Tanchuma Bereshit* 11.
[85] The story traditionally told in this context is that Lamekh, who was blind, shot an arrow at what he thought (because his son Tubal-Cain suggested it) was a deer – but his arrow shot and killed Cain (and then, when he discovered his mistake, he killed his own son).

a raven;" this bird "went forth to and fro, until the waters were dried up from off the face of the earth" – meaning that the raven kept flying around, either close to the ark or elsewhere, without returning into the ark[86]. Noah then "sent forth a dove," but she "found no rest for the sole of her foot, and she returned unto him to the ark, for the waters were on the face of the whole earth." A few days later, "he again sent forth the dove out of the ark;" and this time she "came to him at eventide, and lo in her mouth an olive-leaf freshly plucked." In this way Noah "knew that the waters were abated from off the earth." A few days later, he "sent forth the dove" again, "and she returned not again unto him any more."

This story gives its readers a clear example of experimental method. Noah sends birds out to scout the surrounding countryside, and judges the level of water there by observing their subsequent behavior. The experiment seems to be a flop with a raven: its behavior is not conclusive. He instead tries a dove, three times. When she first returns, he assumes this means that she found no rest elsewhere (this conclusion is a bit doubtful, if doves behave like homing pigeons or if they are attached to their mates; moreover, the world is a big place, which it would take a long time for a dove to cover and a dove's energy is not unlimited). When she next returns with an olive leaf in her beak, he assumes this means she found some dry land or at least an olive tree branch above the waters (though strictly it might have been floating on the water, even if it seemed fresh to him). The last time, she does not return, and he presumably concludes that it went off to get on with its life (though strictly it might have died – it is after all surprising that it did not return for its mate). The conclusions Noah draws are reasonable, even though only probable.[87]

Of course, in view of its divergences from scientific fact, we can hardly doubt that this story is legendary[88]. For a start, where would the quantity of water required to cover the earth's surface to the peaks of the highest mountains come from? Gen. 1:6 mentions "waters above the firmament" and "waters below the firmament," and 7:11 states that "the fountains of the great deep [were] broken up, and the windows of heaven were opened." As regards the "fountains of the great deep," people in antiquity[89] believed the waters of rivers and seas were overflows from subterranean reservoirs of water. As regards the "windows of heaven," our forefathers thought that rain came from a heavenly reservoir of water[90]. But these beliefs are now known to be empirically inaccurate. This is not said to deny God and his kindly providence – it only means that the Bible's particular theory as to how that providence proceeds is fanciful. The text cannot be taken literally; it is at best a poetic statement.

Secondly, there are no geological or other traces of a worldwide flood about 4000 years ago. Of course, God may have erased all traces of the event – but why would He do that? Note that four millennia are not so long ago; yet, there is no evidence that history was suddenly interrupted. Peoples across the globe did not disappear, to be replaced by descendants of Noah, as is obvious from genetic and cultural evidence[91]. Thirdly, I have not made the calculation, but I very much doubt that two specimens of each and every animal species existing in the world at the time concerned would have fitted into a space the size of Noah's ark. People at the time this story was told were evidently not aware of the enormous number of animal species in the world. What of animals in the Americas, in Australia and other far off places – how did they get to the ark? Of course, God might have performed miracles, teleporting the beasts, and shrinking them or expanding the ark (and of course, undoing all that after the flood) – but to affirm this would be sheer conjecture and interpolation, for it is nowhere hinted at in the proof-text.

Genesis 18:16-32 can be viewed as a lesson in logic, if we understand the term logic in a broad sense including methodology. What is taught here, by means of an example, is the need to *ask questions*, and not passively accept things that seem unreasonable, if we want to get to know reality. Inversely, we should know when to stop asking questions, for we cannot expect answers to an infinite number of questions. When an answer satisfies one's rational faculty, there is no need to press further. One may of course press a little bit further, just to make sure; but at some

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[86]   Rashi, referring to the haggadah of [chapter] Chelek [from Sanh. 108b], reads "to and fro" as meaning that "the raven kept returning to the ark, because it [the raven] suspected him [Noah] concerning its mate [i.e. as having sexual interest in the raven's mate]." This interpretation strikes me as so fanciful as to be ridiculous.

[87]   Notice that Noah uses inductive means; he does not try to deduce reality from previous Divine utterances, using *gematria* (numerology) or any other mystical means.

[88]   The only way this story might conceivably be sustained is by claiming the apparent world to be illusory, and the reality behind it to be as described in the Bible. But that is really a farfetched defense!

[89]   At least until Aristotle objected to the idea (see his Meteorology 2:2, given below in Appendix 4). Note that Ex. 20:4 and Deut. 4:18 mention "the water under the earth." There is no mention anywhere of ice at the earth's poles melting, and indeed they did not.

[90]   They apparently did not realize that rainclouds are formed by evaporation of water from the seas, lakes and rivers. See also Deut. 11:17 and 28:12.

[91]   For instance, there are Amerindian tribes in North, Central and South America which are genetically proven descendants of a human whose bones were unearthed on the west coast of the U.S.A. and found to be 13000 years old. At that time, the Bering Strait was frozen, and people could cross over it (on a coastal route) from Asia. Today's tribes could not have descended from Noah, since the Bering Strait was no longer frozen 4000 years ago. Think likewise of Australia's Aborigines, and many other remote peoples. As for cultures, they continued without a break; for instance, in nearby Egypt (where, by the way, most pyramids, including the three big ones at Giza, were already in existence). The flood story obviously doesn't hold water.

point one must stop. It may, however, be that some time later, one does feel the need to ask more questions, because one's context of knowledge has changed somewhat and so one's reason requires revised answers. Different people may, of course, due to having different contexts of knowledge, ask different questions and different numbers of questions.

All this can be seen in the story told in this passage. God tells Abraham that He may destroy Sodom and Gomorrah if their sin is as grievous as the cry of it suggests. There is a moment of silence, during which Abraham reflects on this and decides to ask a question: what if there are some righteous men among the wicked people there – would God refrain from destroying the cities for their sake? Abraham first asks the question with reference to 50 righteous men, and God answers He would forgive for their sake. Abraham then tries 45, then 40, then 30, then 20, then 10 righteous men – God answers every time that in such case He would not destroy[92]. Below 10, Abraham asks no further question – he is satisfied with that number as a minimum; there has to be some justice, after all.

It is noteworthy that God does not preempt Abraham's question-asking outburst by specifying at the outset that He would refrain from destruction even if there are only 10 righteous men. Nor does God answer Abraham's first to fifth questions that way. Nor does He state that the answer to Abraham's sixth question is the last. This suggests that God's relationship to Man is interactive. God leaves blank spaces in His discourse, inviting Man to ask questions about them and eventually to try filling them in. This preference of God for dialogue is also evident in the passages relating to the daughters of Zelophehad (later on, in Numbers). God did not create human beings as automatons, never questioning anything, born only to shut up and obey. He endowed each one of us with a rational faculty and some intelligence, giving us means to think for ourselves, to ask questions and look for answers.

Genesis 23:18-27 provides another lesson in inductive logic. The patriarch Isaac asks his son Jacob to draw near so that he may feel him and tell whether he is indeed Esau (Jacob's brother, whom he is impersonating). This is also experiment – using one's senses to answer some material question. This story could be viewed as a forewarning that while one can test ideas empirically, one must be careful to construct the trial in a sufficiently conclusive manner. For here, Isaac was indeed (luckily) fooled by Jacob, being led to believe he was Esau by means of fake hair. And this, even though Isaac suspected something was wrong, since he reflected that the voice of the person before him sounded more like Jacob's than Esau's. From a logical perspective, he should have investigated the matter further before drawing a conclusion, even if it was in this case a good thing that he did not.

Exodus 2:11-14 may be construed as illustrating legal reasoning by analogy, and more specifically a possible illustration of the hermeneutic principle of *gezerah shavah*. Here, we are told that when Moses "saw an Egyptian smiting (*makeh*) a Hebrew, one of his brethren" he "smote (*vayakh*) the Egyptian, and hid him in the sand." Later on, seeing two Hebrews "striving together," he says to the bad guy: "Wherefore smitest thou (*takeh*) thy fellow?" To which, the man retorts: "Who made thee a ruler and a judge over us? Thinkest thou to kill me, as thou didst kill the Egyptian?" This causes Moses to worry that his earlier summary execution of the Egyptian might be common knowledge; and eventually, when Pharaoh hears of the matter and orders Moses be killed, he flees the country.

Now, the analogy here is proposed by the bad Hebrew man, but he is suggesting that Moses might be thinking along those lines. The argument conceived is: just as Moses smote the Egyptian who smote a Hebrew slave, so he might smite the bad Hebrew who smote another Hebrew. The situations might superficially be construed as analogous, in that in both cases there is one man striking another and a third comes along and kills the striker. The analogy is perhaps not perfect, since the first case seems to have to do with political injustice (an Egyptian oppressor smiting an oppressed Hebrew), while the second case seems to concern a more private dispute (a Hebrew, supposedly unjustly, smiting another); but both events seem to involve a bully severely beating up and perhaps killing an innocent person. The repeated use of the same verb to smite (*lehakot*) suggests the hermeneutic process of *gezerah shavah*. To begin with, since the violence by the bad Hebrew (*harasha*) is described with the same verb (to smite) as the violence by the Egyptian, we may suppose that the two crimes were the same in their severity – this is the main *gezerah shavah*. But what was the crime involved – was it merely a severe beating, or was it blows resulting in death? We are not told whether the two victims died or not. Yet Moses evidently considered the Egyptian sufficiently criminal to kill him. Moreover, the bad Hebrew asks whether Moses intends to kill him too. What legal principle was Moses appealing to? Two explanations of Moses' response are possible. One is to suppose both victims to have been actually killed; the other is to suppose that Moses intervened to prevent their likely murder. In the former case, Moses was punishing or about to punish a murderer with the death penalty[93]. In the latter case, Moses was acting preemptively, on the principle that a 'pursuer' (*rodef*) can be killed if that is the only way to save the life of his victim[94]. The Torah's use of the verb to smite to characterize Moses' killing of the Egyptian may suggest that this word is intended to refer to killing; and this may be viewed as confirmation that the two aggressions (the Egyptian smiting his victim and the bad

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[92]   Note in passing that forgiveness is only promised for 50 righteous men, whereas for 45-10 such men only non-destruction is promised.
[93]   See Ex. 21:13: "He that smiteth a man, so that he dieth, shall surely be put to death."
[94]   This is based on Deut. 22:25-27, according to BT *Sanhedrin*, 73a.

Hebrew smiting his) were also killings. Alternatively, smiting may here mean, more loosely, striking – possibly, but not necessarily, to the extent of killing – in which case, Moses' smiting of the Egyptian may be known to have been a killing by inference from the fact that he thereafter buried the corpse.

That Moses' approach is legalistic seems implied by the bad Hebrew's question: "Who made thee a ruler and a judge over us?" The bad Hebrew, of course, is not aware of the Torah's use of the verb to smite. When he asks Moses: "Thinkest thou to kill me, as thou didst kill the Egyptian?" he characterizes Moses' killing of the Egyptian – which he presumably witnessed, or maybe just heard about – as killing, using the less ambiguous verb *leharog*, and he uses that same verb for what he anticipates to be Moses' reaction to his own deeds. That this argument by analogy offers a template for the hermeneutic technique of *gezerah shavah* is evident from the important role that *verbal* equation plays in it. This is quite an interesting finding, suggesting that *gezerah shavah* has indeed, like *qal vachomer*, a Biblical origin.

Exodus 3:2-3 contains an important teaching of inductive logic; namely that if you have some theoretical belief in mind, and it turns out to be belied by empirical observation, you should look for a way to harmonize the two (either by particularizing the theory or by revising the initial observation by means of additional observations).

Here, Moses comes across a bush that apparently burns with fire and yet is not consumed; and naturally he wonders "why the bush is not burnt." All of us would, on the basis of past experience, firmly expect the burning of a bush (cause) to be invariably followed by its consummation (effect). If this expectation seemed belied by the facts, we would wonder what optical illusion was involved: maybe the fire we glimpsed was not really fire, but a dance of colorful butterflies; or maybe the bush was not really a wooden bush, but a metal replica of one; or maybe both the fire and the bush were real, but their conjunction was illusory, made possible by some arrangement of mirrors. How would we find out? Normally, we would do so by more careful observation of the facts on the ground.

And this is what Moses was evidently about to do, moving towards the burning bush to observe it more closely. But in this particular case, a miracle was involved, and he was advised not to come closer. A miracle, by the way, would not annul the categorical statement that 'all burning bushes are consumed' or the hypothetical statement that 'if a bush is burning, it must be consumed' – for these statements are generalizations from experience in a natural context, and miracle is by definition non-natural or supernatural. In miracle, God willfully prevents nature from taking its normal course, so this is outside the range of ordinary human knowledge. But we can anyway, with reference to Moses' initial reaction to the surprising phenomenon, learn from this episode a valuable lesson about inductive reasoning – viz. to research the matter further, when something unexpected occurs.

Exodus 18:13-26 exemplifies the logical process of formulating causal propositions relating means to ends. Such propositions are composed of if–then clauses. Although there are, of course, many if–then propositions in the Torah, before and after this passage, here we witness the reasoning through which one was constructed. Jethro, Moses' father-in-law, observes that Moses spends much of his time, and the time of his people, sitting in judgment in particular cases. This is, in Jethro's eyes, "not good," because it will unnecessarily fatigue both Moses and the people. He recommends that Moses, instead of this, concentrate on obtaining from God the general laws, and appoint and train qualified men to deal with their application to particular cases. These men are placed in a hierarchy, with Moses at the top. In difficult cases, where all lower-level judges experience doubt, Moses might still be called upon to decide. Moses follows his advice.

From this story we learn that, seeing a faulty state-of-affairs (say, not-Y), one must look for ways (say, X) to correct it (i.e. produce Y). That is, one must find the causes (X) that give rise to the desired effect (Y). This is causal logic: 'if not-X, then not-Y, but if X, then Y'; that is, 'if the corrective measures (X) are not done, the negative situation (not-Y) will endure, whereas if the corrective measures are indeed done, the negative situation will be eliminated'. Of course, the appropriate measures cannot be chosen out of context. This is made clear by Jethro when he wisely adds: "If thou shalt do this thing, *and God command thee so,* then…" (my italics), which may be taken to mean: *but remain aware of the wider context, i.e. your ultimate values* (in this case, God's will). To avoid collateral damage from the pursuit of particular goals, one must not proceed 'with tunnel vision', but must act in harmony with the totality of one's value-system. In the event of conflict between values, obviously the higher values ought to be preferred to the lesser ones. Such reasoning is central to ethical logic.

Exodus 20:12 and **Exodus 23:1-3, 6-8**, and similar passages in Leviticus and Deuteronomy, proffer guidelines or rules addressed primarily to judges functioning within a formal justice system. But they can also be read more broadly as applicable to anyone at all times, since we are all throughout our lives called upon to judge between truth and falsehood. There are two aspects to this cognitive imperative: accuracy in collection and reporting of facts and wisdom in judgment. Regarding the first aspect, viz. ensuring the factual accuracy of our knowledge, we can be inspired by the following instructions of the Torah:

> Ex. 20:12 – "Thou shalt not bear false witness against thy neighbour." Ex. 23:1 – "Thou shalt not utter a false report; put not thy hand with the wicked to be an unrighteous witness." Ex. 23:2 – "Thou shalt not

> follow a multitude to do evil; neither shalt thou bear witness in a cause to turn aside after a multitude to pervert justice." Ex. 23:7 – "Keep thee far from a false matter; and the innocent and righteous slay thou not; for I will not justify the wicked."

We may be factually inaccurate either through negligence in our cognitive ways or through more or less deliberate prevarication. This may seem without consequence to some people, but that is because they do not realize how much they harm themselves as well as others through such policies. In the short term, it may seem innocuous; but over time, the harmful consequences come round. Logic is founded on the idea of utter respect for reality; on the idea that knowing reality is certainly a major component of the highest good, if not the *summum bonum* itself. To divorce one's mind (and eventually the minds of others) from reality is to sentence oneself (and others) to a sort of prison, where the soul is out of contact with the world at large – that is, to a delusional state[95]. The business of cognition is a delicate matter, requiring constant attention and conscientious observation and memorization. Moreover, one should have respect for and confidence in one's own faculties, and not let one's perceptions and conceptions be influenced unduly by the say-so of other people, be they a few authorities or a big crowd. One may of course learn from others, but one should do so in a careful manner and not irrationally. By training oneself, one can develop a mind that is both independent and balanced. Regarding the second aspect, viz. judging perspicaciously, we can be inspired by the following instructions of the Torah:

> Ex. 23:3 – "Neither shalt thou favour a poor man in his cause." Ex. 23:6 – "Thou shalt not wrest the judgment of thy poor in his cause." Ex. 23:8 – "And thou shalt take no gift; for a gift blindeth them that have sight, and perverteth the words of the righteous."[96]

This can be taken to mean: Don't let your judgment be swayed by extraneous considerations, but judge honestly in accord with the facts of the case at hand. Good judgment is an exercise in common sense. The poverty, or the richness, of a man has nothing to do with his innocence, or with his guilt – for a man, whether outwardly rich or poor, is subject to the same inner constraints and liberties, and may do good or bad equally. Similarly, ideological preference for the underprivileged or for the elite, or greedy desire for social or material rewards, should not be allowed to impinge on the impartiality of one's judgments. Emotion is all very nice, but when it comes to judgment one should try to be cool-headed. Objectivity is necessary to good judgment, and every effort should be made to ensure one has it.

Leviticus 10:9-11 teaches us a principle which has to do with epistemology, even though not with formal logic. This reads: "Drink no wine nor strong drink… that ye may put the difference between the holy and the common, and between the unclean and the clean, and that ye teach the children of Israel all the statutes, etc." This passage is followed by a great many laws relating to allowed and forbidden foods, to ritual purity and impurity, and so forth – so it may be viewed as an introduction to that field of religious study. Nevertheless, we can also regard it more broadly as a recommendation relating to the pursuit and transmission of knowledge in general.

If we want to distinguish things accurately, we must have a clear head. Alcohol can, at least when consumed in excess, negatively affect our cognitive abilities. Moreover, although the text does not explicitly say so, we can generalize from it and say the same for all psychotropic drugs – marijuana and the like. We can reasonably do that, since the Torah explicitly tells us *why* alcohol is undesirable – because (that, *ve*) it affects our ability to discriminate and to teach. Thus, this passage can be viewed as a general warning that, if we want to discover and transmit truth (in particular religious truth, but more generally any truth), it is wise to avoid alcohol (and more broadly, any substance abuse that affects the mind adversely).

The reasoning through a common cause proposed here suggests that the rabbinic hermeneutic principle of *binyan av* (lit. 'father construct') has some root in the Torah. When the reason for a Torah commandment is explicitly given, as here, or (presumably, by extension) implied with sufficient clarity, the same reason might be used to generate other commandments. This would constitute inductive reasoning, since we are in effect generalizing from the given Torah commandment (whether positive or negative) to a new ruling; i.e. we are assuming that the reason given (or implied[97]) is applicable not only to the specific Torah commandment, but to other, similar situations. Such assumption is inductively reasonable, provided no counterevidence makes it unreasonable; i.e. unless or until we find the apparently general rule belied elsewhere in the proof-text.

Leviticus 10:16-20 provides us with another epistemological guideline. This passage recounts how Moses "diligently inquired" regarding the correct performance of some ritual, and "he was angry" when he discovered that an error had been made. He asks Aaron and his sons why they did things as they did, and Aaron gives him a reply he

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[95] This is not to be confused with meditative states, where one temporarily distances oneself from sensory and mental phenomena in order to better get in touch with aspects of reality that are accessible more intuitively.

[96] A similar message is given in Deut. 1:17 – "Ye shall not respect persons in judgment; ye shall hear the small and the great alike; ye shall not be afraid of the face of any man; for the judgment is God's." This is commented on further on.

[97] Obviously, reliance on a relatively implicit reason is even more inductive than an explicit reason.

finds satisfactory. Finally, "when Moses heard that, it was well-pleasing in his sight." We learn two things, here, of value to all pursuit of knowledge: (a) it is virtuous to diligently inquire into every significant issue, and not just let apparent problems pass without trying to solve them; and (b) when one discovers one was wrong in some contention, one should be happy to have learnt the truth, and be grateful rather than resentful to one's opponent.

Leviticus 11:3-4 – and a similar passage in **Deuteronomy 14:6-8** – is of some logical interest. This passage allows Jews to eat beasts that "chew the cud" *and* "part the hoof," but forbids them those that "*only* chew the cud" or "*only* part the hoof." There is, however, no explicit mention here of beasts that neither chew the cud nor part the hoof: can they be eaten or not? Presumably, the implicit intent is that subsequent verses will clarify the matter – i.e. some may be eaten, and some not.

In any case, this passage can be used to exemplify the various ways two items X and Y and their negations can be combined: in "X and Y," in "X but not Y" (i.e. "only X"), and in "Y but not X" (i.e. "only Y"). This list is missing a fourth possibility, viz. "neither X nor Y" (that is, "both not X and not Y"). Thus, the list of three combinations really refers to the inclusive disjunction "X and/or Y." Although the Torah contains no explicitly formal logic, by providing this example it teaches people immersed in it how to think of the different possibilities of combination between any two items, X and Y.**[98]**

Leviticus 13 is of logical interest in that it describes how the plague of 'leprosy' in people or objects may be recognized through certain indices by a qualified priest. Needless to say, I am not here granting credibility to the description of this so-called leprosy and its symptoms. All that I wish to draw attention to is the example of looking at alleged symptoms of an alleged disease, and deciding whether they are indeed indicative of the disease or not. It is this *logical relation between a sign and what it signals* that are of interest from a logical perspective. Under what conditions do symptoms point to a disease?

The answer, according to causative logic, is as follows. The disease X may be said to be the cause of its symptoms Y1, Y2, Y3, etc. That means that the relation: "if X, then Y1 and Y2 and Y3, etc." is true. It does not follow from this that the symptoms Y1, Y2, Y3, etc., individually (i.e. separately) imply their underlying cause, or even that they collectively (i.e. in conjunction) do so. For it may be that these very phenomena, Y1, Y2, Y3, etc., individually or collectively, are also implied by a cause other than X. However, in some cases we might know by observation (or, in a Torah context, by revelation) that these symptoms, either individually or collectively, are *exclusively* implied by the cause X. In such event, the presence of any one of them, or certain combinations of them, or all of them, as the case may be, may be taken to imply the presence of the cause X.

Although the Torah does not say all that in the referenced chapter, it does sufficiently hint at it in its precise descriptions of what signs are or are not conclusive for the diagnosis of 'leprosy'. Note that in some cases the passage of a certain amount of time stands as one of the indicators. The reader who pays attention will take note of the intricacies involved, and absorb a lesson about causative logic in more abstract terms.

Leviticus 18:24, which reads: "Defile not ye yourselves in any of these things; for in all these the nations are defiled, which I cast out from before you," contains a teaching of logic – namely, that the quantity 'all' does not necessarily mean 'all together' (all collectively), but may in some situations mean 'every one of' (all distributively). Although in the Hebrew original the same word (*kol*) is used in both instances, the context makes clear that there is no need to do literally *all* the deviant sexual acts referred to in the preceding verses (viz. 6-23) to be defiled; doing *any one or more* of them is enough to be defiled. This is evident from the repetition of the word 'defiled' in a couple of the listed cases (namely, in v. 20 and 23). Note that this understanding of the passage is explicitly upheld by the rabbis in the Talmud (Makkoth 24a).

Leviticus 19:11 has obvious logical implications. "Ye shall not steal; neither shall ye deal falsely, nor lie one to another." When we lie to other people, whether outright or by intentionally misleading them in our discourse, we deal falsely with them and steal their credulity. Lying implies dishonesty in the speaker and disrespect for the one spoken to. A person with respect for reality and for other people avoids such behavior, knowing that in the long run it is sure to cause him confusion and loss.

Leviticus 19:15 and 19:35 are passages of the Torah primarily addressed to judges, but which can be interpreted more generally. The first verse reads: "Ye shall do no unrighteousness in judgment; thou shalt not respect the person of the poor, nor favour the person of the mighty; but in righteousness shalt thou judge thy neighbour." This is a teaching that we should not allow our judgment to be distorted by favoritism for this or that prejudice, but judge matters in a lucid and honest manner.

The second verse reads: "Ye shall do no unrighteousness in judgment, in meteyard, in weight, or in measure." This gives us two very important teachings in inductive logic. The lesson of the second verse is that empirical studies must be accurate: if we want to discover reality and not get lost in illusion, we must make thorough and precise

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[98]   The Torah does make use of disjunctions with more than two alternatives. To give an example at random, Leviticus 22:19 says: "Ye shall offer a male without blemish, of the beeves, of the sheep, or of the goats."

observations or experiments. The lesson of the first verse is that rational interpretation of the data thus collected must be objective, impartial and intelligent; we have no right to ignore or twist facts and fake conclusions. Truth is a paramount value, not to be cynically compromised for whatever motive. (Similar lessons can be found in Exodus and Deuteronomy.)

In **Leviticus 21:21**, we are given an explicit positive apodosis (*modus ponens*), i.e. an argument consisting of a hypothetical major premise, a categorical minor premise that affirms its antecedent, and a categorical conclusion that affirms its consequent. This passage first tells us that no priest who has a blemish "shall come nigh to offer the offerings… made by fire" – this is the major premise; then it adds: "he hath a blemish; he shall not come nigh to offer the bread…" – these are the minor premise and conclusion. Although in truth the major premise concerns offering "the offerings… made by fire" and the conclusion concerns offering "the bread," so there is a change of term that is not strictly deduced, nevertheless I would say that a partial deduction (as regards a priest with a blemish not coming nigh) is indeed intended, because of the categorical form of the statement "he hath a blemish." This statement clearly serves as an explanation for the inference from the previous sentence to the next.**[99]**

Numbers 13:17-20 may be looked at as example of planned research – i.e. as a lesson in methodology. Here, Moses tells the twelve men chosen to spy out the promised land what information to look for: they are "to see the land, what it is; and the people that dwelleth therein, whether they are strong or weak, whether they are few or many; and what the land is that they dwell in, whether it is good or bad; and what cities they are that they dwell in, whether in camps or in strongholds; and what the land is, whether it is fat or lean, whether there is wood therein, or not." Thus, when we have a research project, we should start by thinking about the kind of material information we shall need to collect in order to arrive at an answer to the question asked.

Numbers 14:11-16 has significance for inductive logic, in that it shows that an event might have different explanations. Here, God is angry at the children of Israel for doubting in His capacity to get them into the promised land; and He proposes to wipe them out and replace them with another people, descended from Moses. Moses argues with Him that if He carries out this threat, other peoples will think that He killed the Israelites because He was unable to do with them as He had sworn, i.e. unable to give them the promised land. Moses thus points out that the destruction of the Jewish people by God (the event referred to) would be wrongly interpreted by the nations: instead of understanding God's true motive (exasperation with the Israelites' weak faith), they would assume that God was motivated by lack of power (inability to achieve what He set out to do).**[100]**

Symbolically put, an event E may have alternative causes C1, C2, etc. Each cause implies the same event: i.e. 'if C1, then E'; 'if C2, then E'; etc. are all true. Thus, observing the occurrence of E, one cannot logically tell whether its cause is C1 or C2 or what. That is to say, positive apodosis from the presence of the consequent to that of the antecedent is logically invalid. We can also learn from this that the absence of any one antecedent (say, C1) to the absence of the consequent (E) is invalid, since the consequent might occur through the agency of another antecedent (say, C2). In other words, if C1 implies E, it does not follow that E implies C1, or that not-C1 implies not-E; and similarly for C2, etc. Thus, to correctly identify the causes of events, one must remain aware that there may alternative explanations and not rush to judgment. We should not assume the first hypothesis that comes to mind to be correct, but consider what other hypotheses might be equally (or more or less) credible.

Numbers 16:28-33 can be viewed as another lesson in inductive logic. Moses tells the children of Israel to observe what will befall the rebels Korah, Dathan and Abiram: "If these men die the common death of all men… then the Lord has not sent me. But if the Lord make a new thing, and the ground open her mouth, and swallow them up, with all that appertain unto them, and they go down alive into the pit, then ye shall understand that these men have despised the Lord," by doubting that He chose Moses. And indeed the latter prediction is realized. This argument can be clarified by putting it in more symbolic terms. There are two conflicting theories: that God chose Moses (T) and that He did not (not-T); and they make two conflicting predictions: that the rebels will perish in ordinary circumstances (not-P) and that they will perish in an unusual manner (P). The argument is, then, as follows: If not-P, then not-T; but if P, then T. Since P came true, rather than not-P, it follows that T is true, and not-T is false.

The argument is formally valid and its content is reasonable (assuming that natural means like high explosives were not available to Moses and that these events indeed took place). However, it should be noted that this is a special case, in that the relation between the theories and their predictions is here exclusive and exhaustive; this is what makes possible the inference from the predictions to the theories. In most cases, the problem is more complicated in that the theories may number more than two and they may have some predictions in common, and their other predictions may differ but not be known to be in conflict. In such cases, inference from a prediction to a theory is not logically possible – and we decide between the theories on the basis, rather, of comparatively how many successful

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[99]   There may be better examples of explicit positive apodosis, and maybe also of negative apodosis. But at least this one is a start.
[100]   Moses therefore asks God for mercy for the Jews; and God pardons them to some extent, agreeing not to wipe them out immediately, but at the same time resolving to wait for the next generation before taking them into the promised land (v. 17-24).

predictions they respectively make, and especially with reference to unsuccessful predictions if any. In other words, the truth of a prediction is less decisive that the falsehood of a prediction, for whereas the former cannot prove the theory implying it (unless only that one theory is compatible with it), the latter makes possible immediate elimination of the theory implying it (or at least calls for modification of the theory).

The latter, which are principles of adduction, are implied more explicitly in the Torah in Deuteronomy 13:2-4 and 18:21-22 (see further on).

Numbers 17:16-24 provides us with yet another example of inductive logic. In order to preempt further doubt and rebellion, Moses, under instructions from God, had the princes of the tribes of Israel prepare rods, each man's rod with his name on it, including Aaron the high priest, of the tribe of Levi; the twelve rods were then placed overnight in the tent of meeting, and the next morning Moses found that the rod of Aaron "was budded, and put forth buds, and bloomed blossoms, and bore ripe almonds," which miracle demonstrated that God's choice for the post of high priest was definitely Aaron.

Even though this story relates to a miracle (and of course it contains a weakness in that the miracle occurred behind the scenes, so that the onlookers could not be sure Aaron's rod was not switched by human hand), it can be viewed as a lesson in controlled experiment. The conflicting predictions made by different theories are tested empirically: the winning theory is the one with the most successful prediction. Here, there were twelve theories regarding which prince merited the high priest's position, with twelve predictions regarding which rod would stand out from the rest. The miraculous budding, blooming and bearing fruit of just one rod, namely Aaron's, indicated that the latter was selected for the job.

Numbers 27:1-11 and the follow-up in **Numbers 36:1-12** demonstrate again the methodological importance of asking questions. A group of women, the daughters of Zelophehad, of the tribe of Manasseh, approach Moses demanding that they be given the portion of the land of Israel that would have been given to their father, who died earlier on without sons. Moses brings their cause before God, who agrees with the women and decrees that women may inherit from their father if he has no son. Further on, the leaders of the family grouping to which the daughters of Zelophehad belong come to Moses and complain that if any of these women married outside her tribe, the land she inherited would go to her husband's tribe and so be lost to her father's tribe. Once again, Moses consults with God on the issue, and the latter agrees and decrees that the women may marry who they wish but only within their own tribe.[101]

In these passages we see, as we did in relation to Genesis 18:16-32, that God often waits for a question before giving us its answer. If the daughters of Zelophehad and the leaders of their family grouping had not had the curiosity and courage to ask their respective questions, they (and we) would not have known the answer. People, women as well as men, are graced with thinking minds; and they have to use them to get at the truth. The truth of a matter will not always be placed in their laps ready-made. Research is usually necessary. If this is true with regard to matters subject to revelation, it is all the more true with regard to natural issues. Notice also Moses' open-mindedness. He does not reject the appeals offhand, but gives them serious consideration, even going so far as to bring them before God.

Numbers 32:24 – "Do that which has proceeded out of your mouth!" – may be viewed as a lesson in methodology, insofar as harmony between word and deed, or more broadly between thought and action, or consciousness and will, is essential to proper functioning of the human mind and to human relations. Someone who often indulges in lies, to self and/or others, is ultimately bound to suffer psychologically, losing contact with reality and self-confidence, and socially, losing credibility and respect. Of course, that does not mean one should be rigid-minded at all costs – sometimes it is wise and good to change one's mind. But as a general rule of behavior, integrity is a key to mental health and social well-being. One's word, to oneself or to others, should be one's bond.

In **Numbers 35:30**, and further on in **Deuteronomy 17:6** and **19:15**, we are told that a matter cannot be established by the testimony of only one witness, there has to be more than one witness for credibility. This rule is of course stated in the Torah primarily with regard to issues that come before a court of law; but we can also assume it to have a broader intent. What does 'witnessing' mean? Many events in the world occur without having been observed by any man or woman. Some events are only observed by the person or persons participating in them. Some events are observed by non-participant third parties. A 'witness' is someone who has observed an event, and thus can and eventually does describe it to other people. The observation may occur through any combination of the five senses; and it may be direct or made through the intermediary of some instrument.

The alleged witness may or may not be reliable. Reliability depends, firstly, on the witness' skills in observation, memory and verbal expression. If someone lacks these abilities, then no matter how earnest their testimony, it is unreliable. Secondly, reliability depends on the witness' personal interest in the matter, i.e. on whether he or she has

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[101]   Women are evidently not treated entirely as equal to men in these contexts. They do inherit from their fathers, but only if they have no brothers. They can freely marry within their tribe, but not outside it. Or maybe (I do not know) the law is that they may marry into other tribes, but in that case they lose their share of their father's estate.

something to gain (e.g. money) from the testimony or is proffering it purely as a public service. It is of course theoretically possible for someone to be able to speak truthfully in spite of interests to the contrary; and indeed this is ethically ideal. But few people are able to do this; and it is not always possible for third parties (who must judge the matter) to tell whether they are trustworthy or not. For these reasons, the testimony of one witness is rarely enough evidence to establish a matter; and in serious cases, there has to be more than one.

In science, such reflections give rise to the principle of reproducibility of experiments. In most matters of internal experience, scientists have to rely on the say-so of people. If someone claims to believe or disbelieve something, or to love or hate something, scientists can hardly contradict him or her (although they might check the claim by observing the person's actual behavior over time). However, in matters of external experience, scientists generally do not believe the say-so of people, but endeavor to recreate the experience for themselves (and thus become first-hand witnesses too). For instance, if a scientist claims he observed so and so in a certain experiment, he is in principle not believed until at least one other scientist has made the same experiment with the same results. Thus, we can say that testimony just provides confirmation for a hypothesis – never proof. The more such confirmations, the more confidence we have in the hypothesis; but it is never definitely proved.

In **Deuteronomy 1:17** and **16:18-20**, Moses reminds the appointed judges some of the ways to attain just judgment, saying: "Ye shall not respect persons in judgment; ye shall hear the small and the great alike; ye shall not be afraid of the face of any man;" and again: "Judge the people with righteous judgment. Thou shalt not wrest judgment; thou shalt not respect persons; neither shalt thou take a gift; for a gift doth blind the eyes of the wise, and pervert the words of the righteous. Justice, justice shalt thou follow." As we remarked in relation to similar passages in Exodus and Leviticus, such injunctions may be taken as general cognitive principles, for everyone to follow in all circumstances. If we want to get in contact with reality, and remain there, we must all always make every endeavor necessary to that end, and not allow any consideration to lower our determination and deflect us from the truth. Knowledge of the truth is a great value, not to be abandoned out of fear of some loss or out of hope for some gain. Integrity is a virtue not only for judges, but for all people.

Deuteronomy 5:17, which forbids us to "bear vain witness" against our neighbors, is a repetition of the same commandment in Exodus 20:12 against bearing "false" witness, i.e. of the ninth of the 'ten commandments'. The intent is primarily to refrain from lying in a court of law. Commentators explain the difference in wording here by saying that even if the lie seems harmless, it should be avoided. But here again, we may generalize the imperative to all cognitive contexts. The importance of scrupulous accuracy, honesty and integrity in the pursuit and transmission of knowledge cannot be overemphasized.

In **Deuteronomy 5:21-23**, referring to Moses statement: "we have seen this day that God doth speak with man and he [i.e. the man spoken to] liveth [i.e. survives the experience]," the children of Israel retort: "if we hear the voice of our Lord our God any more, then we shall die. For who is there of all flesh, that hath heard the voice of the living God speaking out of the midst of the fire, as we have, and lived?"[102] Now, these statements are somewhat in conflict, in that Moses points out that in fact God spoke to the Israelites and they survived, yet the latter fear that if God addresses them directly any longer they risk dying nevertheless, since in their view this is ultimately impossible[103]. The resolution of that conflict is of course simply that the Israelites were momentarily graced with an exception to the rule, but they are not confident that they merit further direct revelation after that brief preview.

What is interesting in this passage, for the purposes of logic theory, is that it contains an effective *definition of implication*, albeit in material terms. The first statement of the Israelites: 'if we hear the voice (etc.), then we shall die' signifies an implication of the form 'if X, then Y'. Their next statement: 'For no one who hears the voice (etc.) survives' constitutes an explication of the first statement, since it says 'for' (*ki*, in Hebrew), and it has the form of a negation of conjunction 'not-(X and not-Y)'. Thus, we have here a clear hint that 'if X, then Y' means the same (to the ancient Israelites, or at least to the author of that Biblical passage) as 'not-(X and not-Y)', even though that equivalence is stated in material terms. Moses' earlier empirical statement may be symbolized as 'X and not-Y', and so apparently belies the Israelites' statements. That being the case, the implication advocated by the Israelites must be viewed as more limited in scope than it seems at first – i.e. as applicable *thenceforth* (i.e. after Moses' statement) – to avoid inconsistency.

All this may be taken as a teaching of logic in a book of the Torah, a document traditionally dated as ca. 1300 BCE (though some modern commentators consider it as having been composed some 500 years later). This is of course (in any event) quite a while before the time of Megarians Diodorus Cronus (d. ca. 284 BCE, some say later) and his

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[102]   Similarly, in Ex. 20:16, the people ask Moses to speak with them instead God – "lest we die."
[103]   The people believed this perhaps based on tradition; or maybe it seemed obvious to them for some reason. For although in Ex. 33:20 God says: "man shall not see Me and live," this is well after the ten commandments episode (which occurs in Ex. 20 and is retold in Dt. 5). Ex. 33-34 details God's revelation to Moses of the thirteen attributes of mercy, after the episode of the golden calf, when he went back up Mount Sinai with two new stone tables on which to write the ten commandments again.

disciple Philo (fl. ca. 300 BCE), as well as their successor Chrysippus of Soli (279-206 BCE). I am not, of course, saying that the Torah teaching was as thorough as that of these Greeks[104]; but I am saying that it was potentially as effective. I am not saying, either, that the said finding in the Torah excludes the possibility of similar findings in more ancient Greek or other literature; obviously, the latter remains possible (though of course, if we accept the traditional dating of the Torah, we would have to look into very early literature). It is just an interesting finding. So far as I know, the rabbis never consciously discussed the logical aspects of this or similar passages of the Torah. In any case, they do not include the definition of implication among their hermeneutic principles. But one can well suppose that they were at least subconsciously affected by this passage and others like it.

Deuteronomy 6:7, "thou shalt teach them diligently unto thy children," refers primarily to teaching the commandments of the Torah to one's children. But we can take this verse as exemplifying the importance of *education* more generally. Human knowledge depends largely on education. Human knowledge is of course primitively an individual enterprise, meaning that we can and must to some extent engage in self-education. But human knowledge would be very limited if it were limited to that one source or process. In fact, human knowledge is a social enterprise, stretching in time as well as space. Much of what we know we have learned from others, people from other countries as well as people in the past we descend from. The riches of our individual knowledge were made possible by all the information and skills we were fortunate to receive from others.

A well-rounded education would consist of what we might call 'The Six Rs' – Reading, wRiting, aRithmetic, Reasoning (logic), Righteousness (ethics), and Religion (spirituality). Through literacy, we can receive knowledge from others (reading) and transmit knowledge to others (writing). Though it is true that receipt and transmission of knowledge of various sorts can be done orally, oral communication is relatively limited in scope and also not entirely reliable (since information might more easily be forgotten or modified or added on). Writing and reading are manifestly more powerful means overall, opening the way to a vast pool of world literature that everyone can contribute to and draw from – more than ever nowadays due to technological developments. Arithmetic, and more broadly mathematics (including geometry), is needed to understand quantitative relations. Logic trains the mind to think correctly and avoid error, thus improving one's own production of knowledge and also learning to be critical and selective in one's absorption of knowledge from others.

Ethics is needed not only to develop worthwhile values, but even for the pursuit of purely factual knowledge. To obtain true knowledge of any kind, we have to observe accurately, report honestly, make an intellectual effort, be realistic, be open-minded, be courageous, and so forth. Methodology is impossible without appeal to many ethical imperatives of this sort. And of course, ethics in a broader sense also serves to build within us the mental health necessary to knowledge. In this context, the value of meditation should be stressed: it calms the mind and ensures its most efficient working. Also, the social and political offshoot of ethics, jurisprudence, serves to ensure optimum conditions around us for knowledge. Without freedom of thought and speech, for instances, we obviously cannot hope to attain and pass on true knowledge. Again, a society where no one cares for his fellows, or worse still where violence is widespread, is obviously not conducive to education.

Last but not least is spirituality, which consists in awareness that we humans are not merely bodies with minds, but moreover spirits or souls – i.e. that we have a dimension of being that transcends the material living organism with some mental capabilities that we seem to be or inhabit at first sight. Such awareness is needed to take into consideration facts relating to consciousness, volition and valuation. Many scientists doggedly refuse to take these facts into consideration, out of fear of spirituality. This is understandable in view of the fact that religions have often misled mankind, by dogmatically denying material facts. But there is no wisdom in throwing out the baby with the bath-water. Human spirituality is also a fact – to ignore it and opt for a narrow materialistic viewpoint is to condemn ourselves to another sort of stupidity. Open-mindedness on both sides is essential to true cognition.

Deuteronomy 13:2-4 and **18:21-22** contain two *extremely important principles* of inductive logic. I drew attention to these two passages of the Torah and explicated them in detail, years ago in my *Judaic Logic* (chapter 2.2). The first passage clarifies the positive aspect of adduction, viz. that a thesis which makes a correct prediction regarding certain

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[104]   For Diodorus Cronus a conditional proposition was true only if the consequent (Y) *always* followed the antecedent (X), whereas for Philo, it sufficed for such a proposition to be true if the consequent followed the antecedent *at a given time*. These two kinds of implication are today labeled respectively 'strict' and 'material', or necessary and actual. Given that actual implication means 'It is at this time false that X and not-Y', we might ask how it can be known – for knowledge of such 'negation of a conjunction' cannot be arrived at independently. Either (a) we know this because we know (either by generalization from past experiences or as the resolution of some paradox) that 'it is impossible for X to be true and Y to be false together', or (b) we know it through specific knowledge that 'X is true and Y is true' or that 'X is false and Y is true' or that 'X is false and Y is false' is true at this time, or (c) *someone else* knows one of the preceding facts and tells us its implication that 'X is true and Y is false' is false at this time. Thus, though so-called Philonian implication is conceivable, it is a rather artificial form. Chrysippus went further than his predecessors by clarifying the arguments (which I call 'apodoses') that could be formulated with conditional propositions, such as the *modus ponens* (affirming the antecedent, and concluding with the consequent) and the *modus tollens* (denying the consequent, and concluding with denial of the antecedent).

empirical events is thereby *confirmed* – although this does not mean that it is proven: it is only made more probable than it previously was. The second passage clarifies the negative aspect of adduction, viz. that a thesis which makes a wrong prediction regarding certain empirical events is thereby *disproved* – and not merely undermined, i.e. not just made less probable than it previously was. These are, to repeat, two crucial logical principles.

I'd like to add here a few remarks concerning the form of reasoning called, in Latin, *post hoc ergo propter hoc* (meaning, in English, 'after this, therefore because of this')[105]. Aristotle drew attention to such reasoning as fallacious, in his *Rhetoric* (2:24.7): it "consists in representing as causes things which are not causes[106], on the ground that they happened along with or before the event in question. They assume that, because B happens after A, it happens because of A." With regard to Deut. 13:2-4, this means that reasoning from "a sign or wonder" that "comes to pass" to the authenticity of an alleged prophet, would be fallacious[107]. However, it should be stressed that this severe judgment by Aristotle is made specifically with reference to deductive logic: we cannot *deduce* from B following A that B is caused by A.

However, from an inductive perspective, we might well and often do *induce* from B following A that B is caused by A. On this basis, popular confidence in a prophet is naturally encouraged by the accuracy of his predictions. Such reasoning can be justified by generalization, i.e. by assuming that since A is followed by B in this case, and maybe a few more similar individual instances, we may expect A to be followed by B in all cases, or all cases of a certain sort, which would indeed imply that A causes B, at least for cases of that sort. Such generalization may, of course, turn out to be wrong – and often enough does. But then, in those cases where it fails, we would simply particularize; i.e. we would retract the generalization. This is indeed the teaching of Deut. 18:21-22, according to which if what an alleged prophet's prediction "follow not, nor come to pass," then he may confidently be considered not to be a true prophet.

It should also be pointed out that there are two distinct areas of causality where reasoning *post hoc ergo propter hoc* might be used. One is causation and the other is volition. When by 'cause' we mean causation, then the argument from 'after this' to 'because of this' always relies on generalization, as above detailed. However, when by 'cause' we mean volition (i.e. acts of will), we rely on such generalization only in some situations; very often, we rather rely on introspection (and then, of course, generalize from there). The reason we have to do so is that volition, unlike causation, is inherently a form of causality that is not necessarily repetitive. To will something means to cause it, but we could equally have willed the opposite thing; this is the meaning of 'freedom of the will'. Every act of will is individual, rather than the product of a law of nature as in causation. Therefore, in volition, the notion that "like causes have like effects" does not always hold[108].

In volition, there is room for real variety emerging from one and the same cause (i.e. the person doing the willing). Will is usually motivated, but it may occasionally be quite devoid of purpose. Motives are variously influential, but never determining; which is why the person willing may logically (and morally and legally) be held responsible for the act. Therefore, we can often only guess at the motive of an action, by resorting to introspection (or more broadly, by asking a sample of people to introspect and answer our questions) and assuming that the action occurred in such and such a context. This is also inductive reasoning, note well, for we may well later be proven wrong by discovering that the person concerned could not have had that particular motive. In the latter case, we might then doubt that the person did the action, or we might suspect that he or she did it but with some other motive in mind. Clearly, causal reasoning becomes more speculative and complex when dealing with volition. Needless to say, the issues involved are very significant in legal contexts when, for instance, judges attempt to determine who committed a crime and why; but they are also relevant in everyday life.[109]

Deuteronomy 13:15, "then shalt thou inquire, and make search, and ask diligently; and, behold, if it be truth, and the thing certain…," is of course stated in a specific context, with intent to prevent idolatry in Israel. But we can also learn from it the attitude of *respect for truth*: we should not base our beliefs on mere imagination or hearsay, but

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[105]   When the intent is 'with this, therefore because of this', the Latin phrase would strictly speaking be *cum hoc ergo propter hoc*; but we may take *post hoc* as including this special case. Note that a distinction might also be drawn between events that *are* successive or simultaneous, and events that *are noticed* successively or simultaneously, and the discussion further deepened.

[106]   Note that Aristotle's wording here suggests that *post hoc ergo propter hoc* is a specific fallacy within the larger category of *non causa pro causa* (taking a non-cause for a cause). We might include under the latter heading not only unjustified assumptions of causality, but also mistaken causal identities – by which I mean, for instances, taking a partial cause for a complete (sufficient) cause or taking a contingent cause for a necessary (*sine qua non*, lit. 'without which not') cause.

[107]   Clearly, if a predictor is a genuine prophet, then his predictions should "come to pass;" but if his predictions "come to pass" it does not logically imply that the predictor is a genuine prophet. A scientist may repeatedly make correct predictions without thereby being considered prophetic, for he remains naturally capable of human error like anyone else. An alleged prophet, on the other hand, has to be invariably right to be accepted as genuine.

[108]   The statement "like causes have like effects" is sometimes referred to as the law of causation. I call this a mere notion, because it is too vague: it does not clarify how much alike "like" means. Furthermore, it refers to things as causes or effects without having first proven them to be causes or effects. The statement is nevertheless useful as a practical guide.

[109]   For a more detailed study of this topic see my book *Volition and Allied Causal Concepts*.

make inquiries, do all necessary research, and also pay attention to what others have said or say on the subject, and then only, *if* and only if the empirical evidence and rational considerations in favor of the belief are sufficiently convincing and comparatively strong may we opt for the belief in question. Our beliefs should be products of pondered judgment, not fanciful prejudices.

Deuteronomy 18:10-11 states "There shall not be found among you… one that useth divination, a soothsayer, or an enchanter, or a sorcerer, or a charmer, or one that consulteth a ghost or a familiar spirit, or a necromancer." This may be read as an epistemological guideline: truth is not to be found through such superstitious means. Here, the Torah is way ahead of other documents in its civilizing influence. Of course, it recognizes prophesy as a means of knowledge; but that is a much grander route, to which these various superstitious means cannot be compared.

Deuteronomy 25:13-15, which commands us to have perfect and just, and not diverse, weights and measures, is a passage that has been cited by some commentators in support of the application of logic to legal matters. This makes sense, in that logic is an essential tool to get at the truth and avoid falsehood in any and every context. The command regarding weights and measures is, of course, primarily aimed at accuracy in perception and in factual reporting. A dishonest merchant attempts, by means of adulterated weights and measures, to buy more quantity of a good for less money or to sell less quantity for more money. In the same way, by analogy or by generalization, we can say that a dishonest speaker attempts, by means of fallacious arguments, to defend a false thesis or to condemn or put in doubt a true one. The value of logic is its ability to spot and unmask such dishonest attempts to fool people. And it is important not only as a protective shield against being fooled by others, but also against being fooled by oneself. For we do often out of various motives, fool ourselves. For this reason, training in logic is essential. Even though it does not guarantee perfection, it safeguards us against much dishonesty and foolishness.

Deuteronomy 27:18 says: "Cursed be he that maketh the blind to go astray in the way." There are in this world people who, when asked by a blind person for directions, think it is very funny to misguide him or her; and this is of course sick behavior[110]. We can generalize this moral judgment to all people who knowingly misinform their neighbor – and of those there are a lot more. Think of all the politicians and journalists who daily, for whatever motives, grossly lie to the people: hiding relevant information, inventing patent falsehoods, and distorting what they know to be the truth; they are indeed despicable. The Torah here again, then, stresses the importance of commitment to truth – for this is one of the foundations of social cohesion and peace. It is one of the prime expressions of benevolence towards one's fellows.

Everyone, including philosophers and sundry academics, and indeed religious teachers, are required by the ethics of logic to pursue and propagate truth, because we are all responsible for the collective knowledge of mankind as well as for our individual state of knowledge. This means that we should always admit our ignorance or uncertainty when applicable, and not give other people the false impression that we have knowledge or certainty when we in fact lack it. Most if not all of us are unaware of very many things, and thus deserving to be qualified as "blind" to varying degrees. When stupid or malicious people propagate falsehood, it is like "the blind leading the blind." If they do not know for sure something to be true, they should not pretend they do; all the more, if they do know for sure something to be untrue, they should not dish it out as true.

The *Sayings of the Fathers* counts these virtues among the seven characteristics of the true scholar: "Where he has heard no information he says: 'I have not heard'" and "He admits that which is true;" to which it adds: "The contrary of these attributes marks the boor" (5:10). Another saying worth quoting here is: "Who is wise? He who learns from all men" (4:1).

Conclusion. We can summarize the above information, if only roughly, concerning the Torah, as in the following table. As can be seen, we found altogether some 50 passages of broadly logical interest (respectively 11, 6, 9, 9 and 15, in each of the books of Moses); and these we roughly classified as: deduction (15), induction (10), causative logic (2), ethical logic (2), methodology (16) and, more vaguely, epistemology (6). This may not seem like a very rich harvest, but it is considerable anyway in view of the great antiquity of the document concerned.

Reference	Content (roughly)	Type (roughly)
Gen. 2:16	Possibility of prophecy	epistemology
Gen. 2:16-17	If-then discourse	ethical logic
Gen. 2:19	Theory of words	epistemology

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[110]   Lev. 19:14 – "You shall not curse the deaf nor place a stumbling block before the blind." – is a broader warning not to take malevolent advantage of people's weaknesses. The reference to the blind in both passages (and to the deaf in the one from Lev.) is obviously metaphorical – and has indeed been so taken by Jewish commentators. "Plac[ing] a stumbling block before the blind," being more calculating and active, seems generally a bit more criminal than "Mak[ing] the blind to go astray in the way," although of course in some cases losing one's way might be more harmful than being tripped over.

Gen. 2:21-24	Dud 'therefore' marker	deductive logic
Gen. 3:1-5	'Not all are' does not imply 'this one is not'	deductive logic
Gen. 4:15	Syllogism (fig. 1) or apodosis (*ponens*)	deductive logic
Gen. 4:23-24	A crescendo argument (+s)	deductive logic
Gen. 8:6-12	Experiment	inductive logic
Gen. 18:16-32	Asking questions	methodology
Gen. 23:18-27	Experiment	inductive logic
Gen. 44:8	A fortiori argument (+s)	deductive logic
Ex. 2:11-14	Analogy (*gezerah shavah*)	deductive logic
Ex. 3:2-3	Harmonization of theory with observation	inductive logic
Ex. 6:12	A fortiori argument (-s)	deductive logic
Ex. 18:13-26	Means and ends	ethical logic
Ex. 20:12	Factual accuracy	methodology
Ex. 23:1-3, 6-8	Factual accuracy, perspicacious judgment	methodology
Lev. 10:9-11	Stay sober and analogy (*binyan av*)	method, deduction
Lev. 10:16-20	Diligent inquiry and admitting one's errors	methodology
Lev. 11:3-4	Inclusive disjunction	deductive logic
Lev. 13	Sign and signaled	causative logic
Lev. 18:24	'All' as collective or distributive	deductive logic
Lev. 19:11	Truthfulness	methodology
Lev. 19:15	Unprejudiced judgment	methodology
Lev. 19:35	Factual accuracy	methodology
Lev. 21:21	Apodosis (*modus ponens*)	deductive logic
Num. 12:14	A fortiori argument (+s)	deductive logic
Num. 13:17-20	Planning research	methodology
Num. 14:11-16	Alternative explanations	causative logic
Num. 16:28-33	Theory and predictions	inductive logic
Num. 17:16-24	Controlled experiment	inductive logic
Num. 27:1-11	Asking questions and open-mindedness	methodology
Num. 36:1-12	Asking questions and open-mindedness	methodology
Num. 32:24	Word and deed	epistemology
Num. 35:30	Testimony	inductive logic
Deut. 1:17	Unprejudiced judgment	methodology
Deut. 5:17	Factual accuracy	methodology
Deut. 5:21-23	Definition of implication	deductive logic
Deut. 6:7	Education	epistemology
Deut. 13:2-4	Positive adduction	inductive logic
Deut. 13:15	Respect for truth	epistemology
Deut. 14:6-8	Inclusive disjunction	deductive logic
Deut. 16:18-20	Unprejudiced judgment	methodology
Deut. 17:6	Testimony	inductive logic
Deut. 18:10-11	No superstition	epistemology
Deut. 18:21-22	Negative adduction	inductive logic
Deut. 19:15	Testimony	inductive logic
Deut. 25:13-15	Factual accuracy and use of logic	methodology
Deut. 27:18	Commitment to truth	methodology
Deut. 31:27	A fortiori argument (+s)	deductive logic

Table A6.1

So much, for now, as regards logic in the Torah[111]. I will not here propose a list of cases in the rest of the Tanakh, but I predict that many more cases are there to be found. As regards a fortiori argument alone, 41 instances have been identified.

The following is, offhand, one example which provides evidence of *syllogistic reasoning* in the Tanakh. In **Malachi 1:6**, God argues: "A son honors his father, and a slave his master. If I am a father, where is the honor due me? If I am a master, where is the respect due me?" This may be rendered as: "Fathers/masters are generally [to be] honored by their sons/slaves" (major premise, 'All Y should receive Z from their counterparts') and "I am their Father/Master" (minor premise, 'X is Y'), therefore "I ought to be honored by them" (tacit conclusion of the syllogism, 'X should receive Z from his counterparts') – "yet they do not honor Me" (i.e. 'X does not receive Z from his counterparts', contrary to the tacit conclusion of the syllogism, i.e. to rational expectation). There is no other way to understand this discourse than through syllogism.

I believe I have over the years noticed many other Biblical passages that may be likewise viewed as logic teachings of various kinds. The Tanakh did not wait for Aristotle or other Greeks to engage in logical thought. Such thought is an integral part of the human condition – something that existed long before logicians brought it into full light by distinguishing the form of logical argument from its content. Certainly, once logicians did highlight argument forms, ordinary people thereafter made use of them more easily, correctly and frequently. Similarly, though the examples of reasoning offered in the Tanakh were not new to mankind, nevertheless they helped Jews and other readers of that document to fortify their reasoning powers.

7. Some logic topics of general interest

Although the present work is not a general study of logic, but essentially restricted to a fortiori logic, I have in the course of writing it reflected on more general issues, and wish to share these reflections here rather than hold on to them till a future work, all the more so since they significantly either correct or amplify my comments on these issues in my past works.

1. About modern symbolic logic

Since the later decades the 19th century, and more and more so throughout the 20th century, "modern symbolic logic" has gradually discarded and displaced "classical formal logic." What is the essential difference between them? Classical formal logic, which was discovered or invented by Aristotle (4th century BCE) and further developed and improved on over time by many successors, is based on the idea of studying the logical properties of propositions by replacing material propositions with formal ones. A categorical proposition is formal, if its terms are variables instead of constants – e.g. "All X are Y" is formal, because the symbols X, Y represent in theory any terms that might arise in practice. A hypothetical proposition is formal, if its theses are variables instead of constants – e.g. "If X, then Y" is formal, because the symbols X, Y represent in theory any theses that might arise in practice.

Now, whereas classical logic symbolized terms and propositions, it did not similarly symbolize the other components of propositions, such as their quantities, their modalities or their relational operators. In "All X are Y," the words "all" and "are" remained in ordinary language (in our case, plain English). Similarly, in "If X, then Y" the words "if" and "then" were not symbolized. In modern symbolic logic, on the other hand, the trend developed to symbolize every aspect of every proposition[112]. This was, to be sure, *a new school* of logic, which considered that only in this way could utter precision of language be achieved, and all ambiguity or equivocation be removed from human discourse. Modern symbolic logic, then, advocated the adoption of an altogether artificial language comparable to the language of mathematics.

Some of the pros and cons of this approach are immediately obvious[113]. One advantage of symbolization, already mentioned, is the sense of precision sometimes lacking in natural languages. However, this impression is surely illusory – for if one's understanding of the matter at hand is vague and uncertain to start with, how can symbols

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[111]   Note that some passages cited are only intended as representative, without attempting to find and list all similar instances.
[112]   For examples: the word "all" became an upside down capital A, the word "some" became a laterally inversed capital E (for existence, as in 'there are'), the words "if–then" (implication) became an arrow pointing from antecedent to consequent, and so forth.
[113]   For more on this topic, see my *Future Logic*, chapter 64: Critique of Modern Logic.

improve on it? A patent disadvantage of symbolization is the esoteric nature of artificial language. Logic was originally intended as a teaching for the masses, or at least the intellectuals, to improve their daily thinking. Nowadays, logic has become the exclusive domain of a few specialists, and has little to do with human cognitive practice. Moreover, communication is not always easy even among symbolic logicians, because each of them quite naturally prefers a different set of symbols, so that there is in fact not one artificial language, but many of them. Another disadvantage is the slow adaptability of any artificial language to forms of discourse newly discovered in everyday usage. An example of this is the a fortiori argument, which is still without convincing symbolic expression.

The main activity of 'modern logicians' nowadays seems to be *to translate* ordinary language into their favorite symbolic language. Most of the time, they seem content to rewrite a perfectly comprehensible plain English sentence into a purely symbolic one, as if this is some great achievement that will earn them their place in history, or at least in the profession. Just that act of translation or rewriting in symbolic terms seems to satisfy and thrill them tremendously, *as if it confers scientific status onto the sentence*. Additionally, they resort to pompous terminology for window-dressing and intimidation purposes. One gets the impression that symbols play for them the role of magic incantations in ancient times – 'abracadabra!' they would chant in pursuit of mystical insights and powers.

But, think about it a moment. In truth, when modern logicians rewrite a sentence in symbolic terms they have achieved *exactly nothing* other than to use shorter 'words' (i.e. the symbols they invent) in place of ordinary words, and (ideally) drawn up a table telling us what symbols correspond to what ordinary words. All they have done is *abbreviate* the given sentence. Apparently, they are too lazy to write long sentences and prefer concise ones. They have produced *no new information or insight*. They cannot credibly argue that the ordinary language statement was essentially deficient, since it must have been understandable enough for them to have translated it into symbols. If it was understandable enough for them, why not for everyone else? What absolute need have we of the artificial language(s) they so insistently try to sell us?[114]

Moreover, note this well, whenever we (and they) read the symbolic statement they have concocted, *our minds have to translate it back into ordinary language in order to understand it*. We have to mentally repeatedly refer back to the ordinary language definitions of the symbols. We have to remember: "Oh! This funny symbol means so and so, and that weird doodle means this, and the zigzag means that," and so forth. This means that our mental process of understanding is made more difficult and slowed down considerably. We are further from the object of study than we were to start with – more removed from the reality we are trying to think about. This increased distance and waste of time is not accidental or incidental, however – it serves to cloud the issues and prevent critical judgment. Errors are hidden from sight, and if we spot them we hardly dare point to them for fear of admitting we may have missed something. In this way, foolishness is perpetuated and spreads on.

But all that is not the worst of it. The worst of it is twofold. First, modern logicians usually *symbolize much too soon*, when their level of insight and analysis is still in its early stages. They typically do not give the subject-matter studied time to develop and mature in their own minds, but impatiently rush into their more orderly looking and comfortable world of symbols. The result is that their symbols are usually representative of a very naïve, elementary, immature level of understanding of the object at hand. Secondly, once their symbolic representation is done, *it freezes all subsequent work* at that childish level. Since the symbolization is already settled, all they can do is play around with it in different ways. All they can do is manipulate and reorder and recombine their symbols this way and that way, and this is what they pass the rest of their time doing. They cannot feed on new experiences and insights from the world out there or actual human discourse, since they have already confidently and reassuringly separated themselves from all that. Their symbols thus blind them and paralyze them. The paucity of their results testifies to it.

Another aspect of modern symbolic logic important to note is its pretensions of 'axiomatization'. In classical formal logic, the Laws of Thought (Identity, Non-contradiction and the Excluded Middle) were sovereign; these were axioms in the original sense of irreducible primaries of rational knowledge, together constituting the very essence of logic. In syllogistic validation by Aristotle, all syllogisms could be reduced directly or *ad absurdum* to a minimum number of primary moods – mainly the first figure positive singular syllogism: "This S is M, and all M are P, therefore, this S is P." The latter argument was not perceived as an axiom in its own right or even as an arbitrary convention, but as a logical insight in accord with the laws of thought that what is claimed applicable to a concept must be acknowledged to apply to the things it subsumes. The relation of human reasoning, and more deeply of formal logic, to the laws of thought was progressively 'systematized', but it was allowed to remain essentially *open and flexible*. Though integrated, it was not rigidly fixed, so as to allow for its constant evolution and adaptation as knowledge developed.

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[114] Ideally, of course, symbols are useful to summarize large amounts of information. I would dearly love to develop terse symbolic formulas that summarize my findings in the logic of causation. So I am not entirely rejecting symbolism. What I am saying here is that it is not necessary (i.e. we can well do without it) and indeed can be a serious hindrance to logical thinking and logic theory.

Modern logicians, on the other hand, focusing on the *more geometrico*, the method of proof used in Euclidean geometry, sought a more predictable and definitive arrangement of knowledge. Their simplistic minds demanded rigid rules and perfect orderliness. A hierarchy was established between thoughts – with those at the top of the hierarchy (the laws of thought) being viewed as 'axioms' and those lower down (syllogisms, and eventually similarly other arguments) as 'theorems'. This may work well for mathematics, which is a relatively special science, but it caused havoc in general conceptual logic, which is the science of science. The question naturally arose as to where those apparent axioms came from and whether they could be replaced by contrary ones as was done in non-Euclidean geometry. It did not take long for these simpleminded people to decide that logic was a conventional mental game, with no apparent connection to the empirical world. This philosophy (known as Logical Positivism) was largely justified by Immanuel Kant's analytic-synthetic dichotomy[115], so it could hardly be doubted.

What is lacking in this model of knowledge is the understanding that formal logic is not deduced from the laws of thought. The laws of thought *are not premises* of formal logic; they are not contents from which other contents are deduced. The laws of thought refer us to the autonomous logical insights through which we naturally judge what constitutes appropriate inference. They are what justifies *the processes* of deduction (and more broadly, of induction) from premises to conclusions. The forms of syllogism and other arguments are not deduced from the laws of thought. The forms are induced from actual thought contents. The thought contents can be judged correct or not without reference to the forms, using ad hoc logical insights. What formal logic does is simply collect under a number of headings recurring types of thoughts, so that again using ad hoc logical insights we can once and for all predict for each type of thought (e.g. syllogism 1/AAA) whether it is correct or not. There is in fact no appeal to general 'laws of thought' in this validation (or invalidation) process; honest ad hoc logical insights are sufficient. The 'laws of thought' are merely ex post facto typologies of ad hoc particular acts of logical insight. For that reason, they are not top premises is a geometrical model of knowledge.

The question these modern pseudo-logicians did not ask themselves, of course, is why *their* allegedly logical insights in the course of 'axiomatization' (including their skepticism towards the objectivity of the laws of thought) should be preferred to the logical insights of the 'non-axiomatization' logicians. Is any discussion of logic possible without use of logic? Can logicians ever rightly claim to transcend logic? Can they logically deride and nullify logic? For instance, some have argued that appeal to the laws of thought is either circular argument or infinite regression. They stopped their reflection there, and never asked themselves why the rejection of circular argument or infinite regression should be considered primary logical acts not needing justification, while the laws of thought are to be rejected precisely on the ground that (according to them) there are no logical acts not needing justification. Is that not a double standard (another primary logical insight)?[116]

The radical blunder of the Kantian legacy is the belief that there is such a thing a 'purely analytic' or 'a priori' knowledge. Logicians influenced by this inane idea remain blind to the empirical aspects of all knowledge development. Even apparently purely symbolic systems of logic rely on perceptions. Some symbols refer to concrete objects (e.g. individuals a, b) and some to abstract ones (e.g. classes x, y); but every symbol is, as well as a sign for something else, *in itself* a concrete object (whether as a bit of ink on paper, or of light on a computer screen, or as a shape conjured in our mind's eye). If follows that symbolic formulas, whether inductively or deductively developed, always depend on some empirical observation. The observation of symbols is not a transcending of experience; it is an empirical process just like the observation of cows; it requires physical or mental perception. Thus, if I count symbolic entities or I imagine them collected together, that is not purely analytic work – it is quite synthetic work. Moreover, such logicians tend to ignore the countless memories, imaginations and rational insights that form the wordless background of all discourse concerning logic.

Clearly, axiomatization was a con-game on a grand scale, through which shallow but cunning pseudo-logicians wanted to take power in the domain of logic. And they have indeed managed to do that almost completely. But the fact remains that primary logical insights like the laws of thought, or the rejection of circularity, infinite regression and double standards, or again the acceptance of subsumption (syllogistic reasoning) and the many, many other foundations of human thought, are not open to doubt. No amount of 'axiomatization' can either prove or disprove them, because all proof and disproof depends on such insights *to at all convince us*. The conclusion to draw from that is certainly *not* relativism or conventionalism, for *that too* would be a claim to the validity of some logical insight – i.e. the insight that all insights are arbitrary must itself be arbitrary. Logic cannot be refuted by logic. Logic can however be justified, by honest acknowledgment that some thoughts are primary logical insights. And these insights,

[115]   See my *Logical and Spiritual Reflections*, book 2, chapter 2, posted online at:
www.thelogician.net/6_reflect/6_Book_2/6b_chapter_02.htm.
[116]   We should of course in this context mention Kurt Gödel, who showed the incompleteness of axiomatic systems like that of David Hilbert.

which together constitute what we call 'human reason', cannot all be listed in advance, but emerge over time as knowledge develops.

Aristotle said all that long ago, but many have preferred to ignore him or misrepresent him because they dearly want to belittle him and supplant him, being envious of his achievements. Consider for instance the following statement about the law of non-contradiction drawn from his *Metaphysics* (Book 4, part 3. Translated by W. D. Ross.):

> "For a principle which every one *must have who understands anything* that is, is not a hypothesis; and that which every one must know who knows anything, he must already have when he comes to a special study. Evidently then such a principle is the most certain of all; which principle this is, let us proceed to say. It is, that the same attribute cannot at the same time belong and not belong to the same subject and in the same respect." (Italics mine.)

As a result of symbolization and axiomatization, *modern logic is essentially a deductive logic enterprise*. However complicated or complex it may look, it is inevitably superficial and simplistic. *Even when modern logicians pretend to discuss induction, they are stuck in deductive activities.* Their 'logic' is thus more and more *divorced from reality*. For this reason their thinking on issues of metalogic is thoroughly relativistic. Things have to be the way they think they are, since symbols are somehow omniscient and omnipotent. They see no idiocy or harm in 'paraconsistent logic' (i.e. in breach of one or more of the laws of thought), since to them it is all a game with conventional symbols with no connection to any reality. When things do not fit into their preconceived schemes, they blithely *force* them in and use florid terminology to keep critical judgment at bay. They do not look upon practical deviations from their arbitrary theoretical constructs as problems, as signals that they have made mistakes somewhere on their way; they just add more symbols and make their theories more abstruse. Please don't think I am exaggerating – that's the way it is.

Why are so many people drawn to and impressed by modern symbolic logic? Part of the problem is of course that this is what the universities demand from their teaching staff and teach their students; papers have to be written in symbolic terms to be even considered. But why this preference? Perhaps because pages filled with esoteric symbols seem more 'scientific', reminding readers of mathematical formulae in the physical sciences. It matters little that in logical science the subjacent subject-matter becomes less transparent and comprehensible when translated into symbols. Indeed, part of the aim is to befuddle and intimidate the reader, so as to conceal weaknesses and faults in the treatment. The grandiloquent language is similarly useful as eyewash. Modern symbolic logic boasts of superiority to classical formal logic, to give itself authority; but the truth is that most good ideas the former has it has stolen from the latter, reworking them a little and renaming them to seem original and independent. The whole enterprise is a massive ongoing fraud; or, alternatively, a collective delusion of epidemic proportions.

I am not, of course, saying (as, no doubt, some will rush to accuse me of saying) that everything modern symbolic logic tells us is false and irrelevant, or stolen. What I am saying is that whatever is true and significant in it is certainly not *due to* symbolization and axiomatization, and can equally well be (could be and probably was) developed by classical formal logic. Moreover, to repeat, excessive symbolism tends to simplistically lump things together and gloss over important nuances, and condemns its users to rigid and abstract thinking processes out of touch with the empirical domain.

We have seen, in the course of the present treatise on a fortiori logic, how some budding or experienced logicians strayed or failed due to their attempt to solve problems by means of modern symbolic logic. In the following pages, I present some more examples of the relative ineffectiveness of modern symbolic logic compared to classical formal logic. I show how the issue of 'existential import' is far less significant that it is touted to be; how attempts to bypass the laws of thought are futile; how the liar paradox is not only due to self-reference; and how the Russell paradox is due to the acceptance of self-membership.

2. The triviality of the existential import doctrine

A term is, nowadays, said to have 'existential import' if it is considered to have existing referents; otherwise, it is said to be 'empty' or a 'null class'. For examples, 'men' has existential import, whereas 'dragons' does not. This concept is considered original and important, if not revolutionary, in modern symbolic logic; and it is often touted as proof of the superiority of that school over that of classical formal logic. We shall here examine and assess this claim. As we shall see, although the concept has some formal basis, it is in the last analysis logically trivial and cognitively not innocuous.

The founder of formal logic, Aristotle, apparently did not reflect on the issue of existential import and therefore built a logical system which did not address it. The issue began to be raised in the middle ages, but it was not till the latter half of the nineteenth century that it acquired the importance attached to it today by modern logicians.

a.   Based on Aristotle's teaching, classical formal logic recognizes six basic categorical forms of proposition: the general affirmative, "All S are P" (A), which means that each and every S is P; the general negative, "No S is P" (E), which means that each and every S is not-P; the particular affirmative, "Some S are P" (I), which means that each of an indefinite number (one or more) of S is P; the particular negative, "Some S are not P" (O), which means that each of an indefinite number (one or more) of S is not-P; and the singular affirmative, "This S is P" (R), and the singular negative, "This S is not P" (G), which refer to a specifically pointed-to or at least thought-of individual instance. Note that general (also called universal) propositions and particular propositions are called plural, in contradistinction to singular ones[117]. The labels A, E, I, O, R and G come from the Latin words *AffIRmo* and *nEGO*; the first four are traditional, the last two (R and G) were introduced by me years ago[118].

The symbols S and P stand for the subject and predicate. The verb relating them is called the copula, and may have positive (is or are) or negative (is not or are not) polarity[119]. In the present context, the copula should be understood very broadly, in a *timeless* sense[120]. When we say 'is' (or 'is not') we do not mean merely "is (or is not) *now*, at this precise time," but more broadly "is (or is not) *at some time*, in the past and/or present and/or future." The expressions 'all', 'some' and 'this' are called quantities. Obviously, the general 'all' covers every single instance, including necessarily 'this' specific instance; and 'all' and 'this' both imply the particular 'some', since it indefinitely includes 'at least one' instance. The 'oppositions' between the six forms, i.e. their logical interrelationships, are traditionally illustrated by means of the following 'rectangle of oppositions':
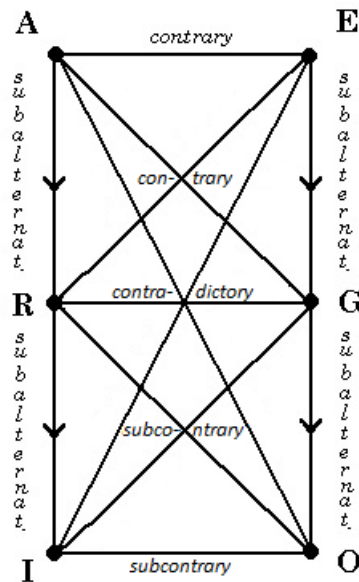
Diagram 7.1 – Aristotelian oppositions

Although Aristotle did not, to our knowledge, represent the oppositions by means of such a diagram, we can refer to it as a summary his views. It is taken for granted that, on the positive side A implies R and R implies I (so, A implies I), and on the negative side E implies G and G implies O (so, E implies O), although these implications cannot be reversed, i.e. I does not imply R or A, and R does not imply A, and so forth. This is called subalternation[121]. The core opposition in this diagram is the contradiction between R and G; from this assumption, and the said subalternations, all else logically follows[122]. A and O are contradictory, and so are E and I; A and E, A and G, E and R, are pairs of contraries; I and O, I and G, O and R, are pairs of subcontraries. Two propositions are contradictory if they cannot be

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[117]   Singular propositions are often called particular, but this usage is inaccurate, since they refer to an indicated individual.
[118]   One can remember these six labels by means of the phrase ARIEGO.
[119]   What I have called 'polarity' is traditionally called 'quality', but the latter term is inaccurate and confusing and should be avoided.
[120]   This approach allows us to momentarily ignore the issue of modality, and reflects common usage in many contexts. A fuller treatment of categorical propositions must of course deal with modality; I do that in my earlier work, *Future Logic*.
[121]   The implying proposition being called the subalternant and the implied one the subaltern, and the two being called subalternatives.
[122]   If A is true, then R is true, then G is false, then E is false; whence, the contrarieties shown on the diagram. If I is false, then R is false, then G is true, then O is true; whence, thus the subcontrarieties shown. Since R and G are incompatible (cannot both be true) and exhaustive (cannot both be false), it follows that A and O, and likewise E and I, whose instances overlap somewhat, must be contradictory, since, if they were both true or both false, R and G would in at least one case be accordingly both true or both false (this is proof by exposition).

both true and cannot be both false; they are contrary if they cannot be both true but may be both false; they are subcontrary if they may be both true but cannot be both false.

b. Shockingly, the above traditional interpretation of the basic categorical forms (Diagram 7.1) has in modern times been found to be problematic. The above listed propositions are not as simple as they appear. The form "Some S are P" (I) means "Something is both S and P," while the form "All S are P" (A) means "Something is both S and P, and nothing is both S and not-P;" similarly, the form "Some S are not P" (O) means "Something is both S and not-P," while the form "No S is P" (E) means "Something is both S and not-P, and nothing is both S and P." Seeing the forms I, A, O, E, in this more detailed manner, we can understand that A implies I since I is part of A (and likewise for E and O), but then we realize that A and O are not truly contradictories (and likewise for E and I).

The exact contradictory of "Something is both S and not-P" (O) is "Nothing is both S and not-P" (i.e. only part of A, with no mention of its I component) and the exact contradictory of "Something is both S and P, and nothing is both S and not-P" (A) is "Nothing is both S and P, and/or something is both S and not-P" (i.e. a disjunction including O, but also E). Note this well[123].

It should be pointed out that "All S are P" (A) can be defined more briefly as: "Something is S, and nothing is both S and not-P;" for given this information, it follows logically that the things that are S are P (for if this was denied, it would follow that some things are both S and not-P), Similarly, "No S is P" (E) can be defined more briefly as: "Something is S, and nothing is both S and P," without need to specify explicitly that "Some things are both S and not-P." Thus, all four forms A, E, I, O, imply, or presuppose (which is logically the same), that "some S exist(s)." Also, the positive forms, A and I, imply that "some P exist(s)." On the other hand, the negative forms, E and O, do not imply that "some P exist(s)," since the negation of a term is not informative regarding its affirmation[124].

Thus, in the above diagram, the diagonal links between the corners A and O, and between E and I, should not be contradiction but contrariety. For, while to affirm one proposition implies denial of its opposite, to deny one proposition does not imply affirmation of the other. To remedy this real problem of consistency, modern logicians have proposed to *redefine* the general propositions A and E as the exact contradictories of O and I, respectively. That is to say, the new meaning of A is only "Nothing is both S and not-P" and the new meaning of E is only "Nothing is both S and P." It follows from this measure that A (in its new, slimmer sense) no longer implies I, and likewise E (in its new, slimmer sense) no longer implies O. This redefinition of symbols A and E can, to my mind, lead to much confusion. In my view, it would be better to *re-label* the forms involved as follows:

- Keep the traditional (old) labels A and E without change of meaning; i.e. old A = A, old E = E.
- Label the modern (new) senses of A and E as respectively not-O and not-I.
- That is, new 'A' = not-O ≠ old A. Whereas, old A = new 'A' plus I = I and not-O.
- Likewise, new 'E' = not-I ≠ old E. Whereas, old E = new 'E' plus I = O and not-I.

Thus, when we say A or E in the present paper, we mean exclusively the traditional A and E; and when we wish to speak of the modern 'A' and 'E' we simply say not-O and not-I, respectively. Note this convention well[125]. Actually, such propositional symbols are effectively abandoned in modern logic and the propositions are expressed by means of a symbolic notation, including the existential and universal quantifiers, ∃ (there exists) and ∀ (for all), respectively; but we do not need to get into the intricate details of that approach here, because we can readily discuss the issues of interest to us in plain English. Now, let us consider the formal consequences of the above findings in pictorial terms.

One way for us to solve the stated problem is to merely modify the traditional rectangle of oppositions, by showing the diagonal relationships between A and O and between E and I to be contrariety instead of contradiction; this restores the traditional diagram's consistency, even if it somewhat dilutes its force (Diagram 7.2). Another possibility, which is the usual modern reaction, is to change the top two corners of the rectangle to not-O and not-I, instead of A and E respectively; this allows us to retain the contradiction between diagonally opposed corners, although now the lateral relation between the top corners is unconnectedness instead of contrariety, and the vertical relations in the upper square are unconnectedness instead of subalternation (Diagram 7.3).[126]

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[123] The Kneales propose a similar analysis of the problem in *The Development of Logic* (Oxford, London: Clarendon, 1962), chapter II, section 5. Further on (on p. 211), they say that Peter Abelard "should have the credit of being the first to worry about the traditional square of opposition, though he did not work out all the consequences of the change he advocated."
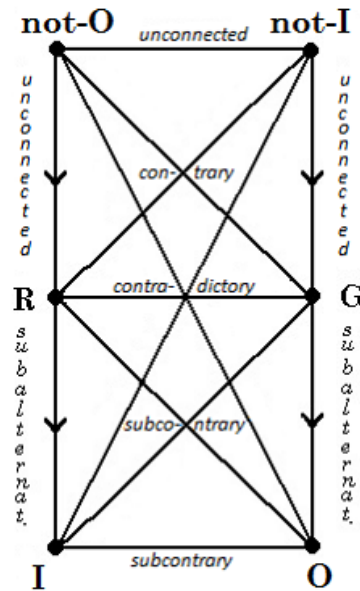
[124] We could say that nothing in the world is conceivably P, without affecting the truth of "Some S are not P" or "No S is P." Clearly, in the special case where "nothing is P," the latter propositions are true for any and every value of S.

[125] Of course, we could introduce modified symbols for the new A and E, such as A' and E', but I prefer to stress their underlying meanings, viz. not-O and not-I. In my view, it is dishonest and misleading to redefine the symbols A and E *themselves* as meaning only not-O and not-I. This is like a hostile takeover, permanently blocking further reflection and debate.

[126] A third possible approach is, of course, to draw a rectangle with A and E in the top two corners, and not-E and not-A (instead of I and O) in the bottom two corners. In that case, it is the lower square that would suffer changes, with not-E and not-A as unconnected to each other and to R and G respectively. This possibility is however not very interesting, as the forms not-E and not-A are disjunctive. That is, not-E = not-(O and
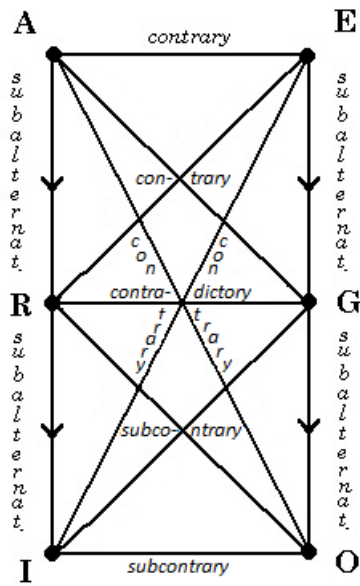
7.2 – modified traditional   7.3 – modern version

Notice that the lower square of the modern version is unchanged. This is due to the judgment that the forms I and O, i.e. "Something is both S and P" and "Something is both S and not-P," both imply that "some S exist" (or "some things are S" or "there are things which are S") meaning that if they are true, their subject 'some S' has existential import. Moreover, in the case of I, the predicate P is also implied to have existential import, since it is affirmed; but in the case of O, the predicate P is not implied to have existential import, since it is merely denied. Until now, note well, we have not mentioned the issue of existential import in our formal treatment. Now, it comes into play, with this interpretation of particular propositions.

The same applies to R and G – their subject 'this S' has existential import, whereas the predicate P has it if affirmed but lacks it if denied. On the other hand, since not-O (as distinct from A) is a negative statement, i.e. means "Nothing is both S and not-P," it has no implication of existential import. Similarly, since not-I (as distinct from E) is a negative statement, i.e. means "Nothing is both S and P," it has no implication of existential import. Clearly, if not-O was thought to be contrary to not-I, then if not-O were true, it would imply the negation of not-I, i.e. it would imply I; but this being erroneous, not-O and not-I cannot be contrary, i.e. they must be unconnected. Similarly, if not-O was assumed to imply R, it would then imply I, since R still implies I; therefore, not-O must also be unconnected to R; and similarly for not-I and G. On the other hand, not-O remains contrary to G, since if not-O is true, then O is false, in which case G must be false; similarly as regards not-I and R.

It is now easier to see why the traditional rectangle of oppositions (7.1) seemed right for centuries although it was strictly-speaking wrong. It was tacitly assumed when drawing it that the subjects of general propositions *always* have existential import, i.e. imply that "some S exist (s)." *When this condition is granted*, then in combination with it not-O becomes A and not-I becomes E, and A implies I and E implies O, and A exactly contradicts O and E exactly contradicts I – in other words we happily return to the original rectangle of oppositions (7.1). The problem is that this condition is *not* always satisfied in practice. That is, not-O or not-I can occur without their subject S having existential import.

Effectively, the forms "Nothing is both S and not-P" (not-O) and "nothing is both S and P" (not-I) signify conditional propositions ("Whatever is S, is P" and "Whatever is S, is not P") which, without the minor premise "this is S," cannot be made to conclude "this is P" or "this is not P" (respectively). In other words, they record a 'connection' between an antecedent and a consequent, but they have no 'basis', i.e. they contain no information affirming the antecedent, and thence the consequent. Obviously, if that information is provided, the condition is fulfilled and the result follows. Once we realize that *the traditional rectangle remains true in the framework of a certain simple condition (viz. that some S exist)*, we see that its hidden 'inconsistency' is not such a big problem for formal logic.

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not-I) = not-O and/or I; and not-A = not-(I and not-O) = not-I and/or O. Note that this position is historically found in Peter Abelard, who insisted on distinguishing between "Not all S are P" (not-A) and "Some S are not P" (O), and who apparently denied that "No S is P" (E) implies anything to be S let alone P (even while regarding "All S are P" (A) as implying that something is S); see Kneales, p. 210.
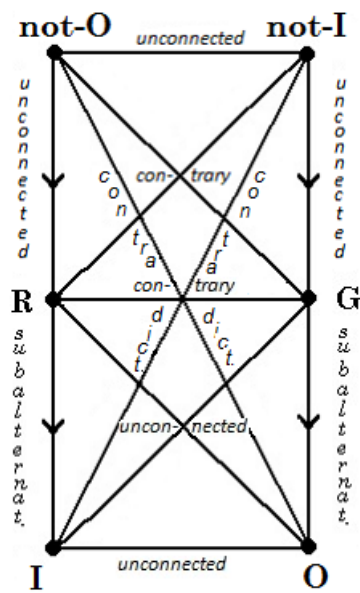
It is interesting to also consider the significance of the above revisions in the field of eduction (i.e. immediate inference). Whereas A, which implies I ("Some S are P"), is convertible to "Some P are S" – not-O, which does not imply I, is *not* so convertible. Also, whereas not-I is convertible to "No P is S," since "Nothing is S and P" and "Nothing is P and S" are equivalent and have no implication of existential import for S or P – E is *not* likewise unconditionally convertible, since in its case even if we are given that "some S exist" we cannot be sure that "some P exist" (but only that "some not-P exist"). Note well, just as O does not imply predicate P to have existential import, since it merely negates it, so is it true for E; therefore, the traditional conversion of E is really only valid conditionally. We can also look into the consequences of the above revisions in the field of syllogistic reasoning; the main ones are pointed out further on.

c.   Let us now go a step further in the possible critique of Aristotelian oppositions, and suggest that *all* terms may be denied to have existential import, whatever the forms they occur in, and whatever their positions therein. That is to say, not only the subjects of general propositions, but even the subjects of singular or particular propositions might conceivably lack existential import. Although R and G, and I and O, do formally imply that some S exist(s), it is still possible to deny them in pairs without self-contradiction. That is, R and G cannot be claimed strictly-speaking contradictory, because if "this S exists" is false then they are both false; this means that their traditional relation of contradiction is valid only conditionally (i.e. provided "this S exists" is true) and their absolute relation is in truth only contrariety. Similarly, I and O are only relatively subcontrary and their unconditional relation is really unconnectedness.

Indeed, it happens in practice that we reject a singular subject altogether, when we find that some predicate can be both affirmed and denied of it. This is dilemmatic argument: finding both that 'this S' is P and that it is not P, we must conclude that either one of these predications is wrong, or both are wrong because 'this S' does not exist. Particulars, of course, do not necessarily overlap; but if we can show by other means that "no S exists," we can be sure that neither the set of S referred to by I nor that referred to by O exist, and thus deny both propositions at once. Granting all this, the above diagrams (7.2 and 7.3) can be *further* modified as follows:
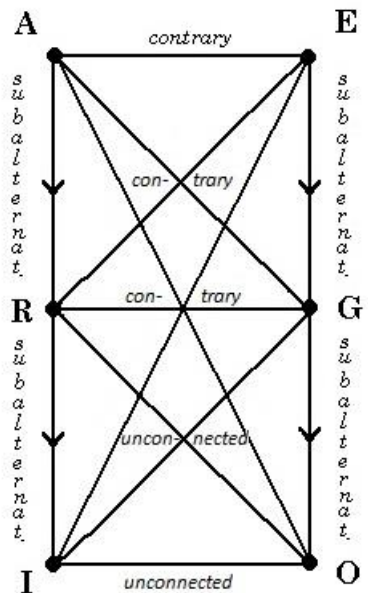
7.4 – re-modified traditional   7.5 – modified modern version

In both these diagrams (7.4 and 7.5), all relations are the same as before, except the one between R and G (contrariety), and those between R and O, G and I, and I and O (which are now unconnected pairs). Notice that in the second diagram (7.5), although R and G are no longer contradictory, the pairs not-O and O, and not-I and I, remain contradictory, since if we deny that "Something is both S and P" (I) on the basis that "No S exists," we can all the more be sure that "Nothing is both S and P" (not-I), and likewise regarding O and not-O.

d.   We have thus proposed two successive dilutions (weakening revisions) of the traditional rectangle of oppositions. In the first, we followed modern logic in no longer assuming with Aristotle that the subjects of universal propositions have existential import. In the second, we went further and additionally denied that singular and particular propositions may well lack existential import. Clearly, if our goal is to formulate an absolute logic, one

applicable equally to propositions with existential import and those without, the successive dilutions of the Aristotelian diagram are justified and important. But are such logics of anything more than academic interest – are they of practical interest? The answer must clearly be no, as I will now explain.

A difficulty with the ideas of existential import and emptiness is immediately apparent: these are characterizations that may be true or false. Different people at the same time, or the same person at different times, may have different opinions as to the existential import or emptiness of a certain term. Some people used to think that dragons exist, and maybe some people still do, yet most people today think dragons never existed. So, these characterizations are not obvious or fixed. Yet modern logicians present the question of existence or non-existence as one which has a ready answer, which can be formally enshrined. They fail to see that the issue is not formal but contentual, and thus in every given material case subject to ordinary processes of testing and eventual confirmation or disconfirmation.

It follows that the issue of existential import is not as binary as it is made out to be. The issue is not simply existence or non-existence, as modern logicians present it. The issue is whether at a given time we know or not that existence or non-existence is applicable to the case at hand. A term with existential import may be said to be 'realistic', in that it refers (or is believed to refer) to some existing thing(s). An empty term, i.e. one without existential import, may be said to be 'unrealistic', in that it refers (or is believed to refer) to a non-existent thing. In between these two possibilities lies a third, namely that of 'hypothetical' terms, for which we have *not yet* settled the issue as to whether they are (in our opinion) realistic or unrealistic. Moreover, this third possibility is not monolithic like the other two, but comprises a host of different degrees.

Our knowledge is mostly based on experience of physical and mental phenomena, though also on logical insights relating to such experience. Roughly put, we would regard a term as realistic, if we have plentiful empirical evidence as to the existence of what it refers to, and little reason to doubt it. We would regard a term as unrealistic, if we have little empirical evidence as to the existence of what it refers to, and much reason to doubt it. And we would regard a term as hypothetical if we are thus far unable to decide whether it should be characterized this way or that. In any case, the decision is usually and mostly inductive rather than purely deductive as modern logicians effectively imagine it.

How are terms formed? Very often, a term is formed by giving a name to a circumscribed phenomenon or set of phenomena that we wish to think about. Here, the definition is fixed. More often, a term is applied *tentatively* to a phenomenon or set of phenomena, which we are not yet able to precisely and definitively circumscribe. In such case, we may tentatively define it and affirm it, but such a term is still vague as well as uncertain. Over time we may succeed in clarifying it and making it more credible. Here, the definition is variable. Thus, the formation of terms is usually not a simple matter, but *an inductive process* that takes time and whose success depends on the logical skills of the thinker(s) concerned.

Of course, as individuals we mostly, since our childhood, learn words from the people around us. This is effectively fixed-definition terminology for the individual, even if the term may have been developed originally as a variable-definition one. In this context, if we come across an obscure ready-made term, we cannot understand it till we find some dictionary definition of it or someone somehow points out for us the referent(s) intended by it. But even then, inductive acts are needed to understand the definition or the intent of the pointing. When you point at something, I cannot immediately be sure exactly what it is you are pointing at; I may have to ask you: 'do you mean including this, excluding that?' and thus gradually zero in on your true intent.

Each of us, at all times, retains the responsibility to judge the status of the terms he or she uses. The judgment as to whether a term is realistic, or unrealistic is not always easy. *In practice, therefore, most terms are effectively hypothetical*, whether classed as more probably realistic or more probably unrealistic. Even so, some terms are certainly realistic or unrealistic. All terms that are truly based exclusively on empirical evidence or whose denial is self-contradictory are certainly realistic, and all manifestly counterfactual or self-contradictory terms are certainly unrealistic. So, all three of these characterizations are needed and effective.

Let us suppose the formation of realistic terms is obvious enough, and ask how imaginary ones are formed. Imaginary terms are not formed ex nihilo; they are formed by combining old terms together in new ways. A new term T is imagined by means of two or more existing terms T1, T2…. We would call term T realistic, if all the terms (T1, T2…) constituting it are realistic and their combination is credible. But if all the terms on which T is based are realistic, but their combination is not credible (e.g. we know that no T1 is T2, so the conjunction T1 + T2 is contrary to fact), we would call T unrealistic; and of course, if one or more of the terms constituting T is/are unrealistic, we would call T unrealistic. If T is made up of hypothetical elements or if its elements are realistic but their combination is of uncertain status, we would call T hypothetical.

Now, our thinking in practice is aimed at knowledge of reality. That is to say, when we come across a term without existential import, i.e. when we decide that a term is unrealistic, whether because it goes against our empirical observations or because it is in some way illogical—*we normally lose interest in it and drop it*. We certainly do not

waste our time wondering whether such a subject has or lacks some predicate, since obviously if the subject is non-existent it *has no* predicates anyway. If we regard a term as empty, the oppositions of its various quantities and polarities in relation to whatever predicate are henceforth totally irrelevant. An empty term, once established as such, or at least considered to be such, plays no further role in the pursuit of knowledge. This attitude is plain common sense, except perhaps for lunatics of various sorts. For this reason, the oppositions between propositions involving empty terms are *trivial*. That is, the above detailed non-Aristotelian models of opposition are insignificant.

The net effect of the successive 'dilutions' is to make the strong, Aristotelian rectangle of oppositions (concerning propositions with existential import) seem like a special case of little importance, and to give the weaker, non-Aristotelian rectangles (concerning variously empty propositions) a disproportional appearance of importance. The reason why this occurs is that the weaker oppositions represent the lowest common denominator between the Aristotelian and non-Aristotelian oppositions, which we need if we want to simultaneously discuss propositions with and without existential import. But the result is silly, for the Aristotelian diagram (7.1) is the important one, teaching us to think straight, whereas the non-Aristotelian ones are really of very minimal and tangential academic interest.

Practical logic is focused on terms that are believed to be realistic or at least hypothetical – it is not essentially concerned with empty terms. Contrary to the accusations made by modern logicians, Aristotelian logic is not only concerned with realistic terms. It is in fact mainly used with hypothetical terms, since (as already pointed out) most of the terms which furnish our thoughts are hypothetical – tentative constructs in an ongoing inductive enterprise. We do not think hypothetical thoughts by means of some special logic – we use the same old Aristotelian logic for them. That is to say, *in accord with* **the principle of induction**, *we treat a hypothetical term as a realistic term until and unless we have reason to believe otherwise*.

The reason we do so is that a hypothetical term, i.e. one not yet proved to be realistic or unrealistic, is a candidate for the status of realism. This being the case, we treat it as we would any realistic term, subjecting it to the strong, Aristotelian model of oppositions, rather than to any watered-down model with wider aspirations, in the way of an inductive test. If the hypothetical term is indeed deserving of realistic status, it will survive the trial; if, on the other hand, it does not deserve such status, it will hopefully eventually be found to lead to contradiction of some sort. In that event, we would decide that the hypothetical term should rather be classed as an unrealistic term, and we would naturally soon lose interest in it. Thus, there is only one significant and useful model for oppositions between propositions, namely the Aristotelian one.

Indeed, we sometimes use Aristotelian logic even for unrealistic terms. Very often, we remove the stigma of unrealism by rephrasing our statement more precisely[127]. Alternatively, we might just keep the imaginary intent in mind: say a novelist wishes to write about fictional people, or even science-fiction creatures, he would not logically treat his subjects as empty terms – but rather subject them to the logic applicable to realistic terms, so as to enhance the illusion of realism in his novel. Thus, the logic applicable to empty terms which we have above investigated is in practice never used.

Whatever the alleged existential import of the terms involved, our thoughts remain guided by the demanding model of Aristotelian oppositions. The rational pursuit of knowledge still indubitably *requires* the clear-cut logic of Aristotle enshrined in the traditional rectangle of oppositions (diagram 7.1). The reason why Aristotle took the existential import of the subjects of categorical propositions for granted is, I suggest, because *naturally, if there is nothing (i.e. no subject) to talk about (i.e. to predicate something of) we will not talk about it; and if we are talking, then that presumably means we do have something to talk about, i.e. a subject as well as a (positive or negative) predicate*. This is manifest common sense.

If Aristotle – as far as we know, or at least as far as readers of his extant works have so far managed to discern, or so we are told by historians of logic – did not ask the question regarding the existence of the subject, it is probably simply because he quite intelligently had no interest in empty subjects. He was rightly focused on the pursuit of knowledge of the world facing him, not some non-existent domain. Modern logicians are rather, I suggest, more intent on impressing the yokels with their intellectual brilliance. With that overriding purpose in mind, they fashion systems of no practical significance whatever. They make mountains out of molehills, presenting trivia as crucial discoveries, so as to draw attention to their own persons.

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[127]   For example, we might say (instead of "unicorns are horses with a horn") "the imaginary entities called unicorns look like horses with a horn on their forehead" or (instead of "some unicorns are white, some black") "some of the unicorn illustrations I have seen involve a white horse, but some involve a black one". Note that *both* the initial propositions (given in brackets) have empty terms, even though one is general and the other is particular. Clearly, after such corrective rephrasing the two propositions do have existential import, although they do so with reference to imaginary (mental) entities rather than to real (physical) ones. Consequently, while the initial propositions cannot be said to be true, the more precise ones replacing them can be said to be true, and we can apply Aristotelian logic to them without qualms. Note also in passing that even a seemingly eternally imaginary entity may one day become real – for example, we might by artificial selection or by some genetic manipulation one day produce real unicorns.

e.   Modern logic is a complex web of static relationships, most of them irrelevant. It ignores the dynamics of human thinking, the fact that our knowledge is constantly in flux. It is, we might say, a science of space irrespective of time. In an effort, on the surface praiseworthy, to formally acknowledge the issue of existential import, it gives undue attention to empty terms, elevating them from a very marginal problem to a central consideration. Instead of dealing with existential import parenthetically, as a side issue, it erects a logical system that effectively shunts aside some of the most important logical processes in the human cognitive arsenal.

The traditional universal propositions are cognitively of great importance. They cannot just be discarded, as modern logic has tried doing under the pretext that formal logic had to be expanded to include consideration of counterfactual terms. There are logical processes involving these propositional forms that are of great practical importance, and which logic must focus on and emphasize. It is absurd to henceforth effectively ignore these venerable and indispensable forms while making a big thing of a theoretical consideration of no practical significance whatever. The universals A and E cannot be retired under any pretext; they are not mere conventional conjunctions of more primitive forms.

For a start, universal propositions are essential to the crucial logical processes of *subsumption and non-subsumption*, which are enshrined in Aristotle's syllogistic. First figure syllogisms serve to include an instance in a class or a subclass in a wider class; they teach us the notion that 'all X' includes every individual 'this X' and any possible set of 'some X'. If, instead of an argument such as "All X are P and this S is X, therefore this S is P" (1/ARR) we propose the modern major premise "Nothing is X and not-P," with the same minor premise, we obviously (even though the minor premise implies the existential import of an X) *can no longer directly draw the desired conclusion*! We are forced to stop and think about it, and infer that "this S is not not-P" before concluding that "this S is P." Similarly, second figure syllogisms serve to exclude an instance from a class or a subclass from a wider class, and third figure syllogisms to identify overlaps between classes; and the moods of these figures become inhibited or greatly distorted if universal propositions are reinterpreted as modern logicians suggest.

Again, universal propositions are essential to the crucial logical processes of *generalization and particularization*. If 'this X' and 'some X' are not implied by 'all X', then we cannot generalize from the former to the latter. Of course, given 'this X' or 'some X', we do have existential import, and thus can anyway generalize to 'all X'. But the fact remains that if, in accord with modern logic, we conceive our generalization as a movement of thought from "This/Some X is/are Y" to "Nothing is X and not-Y," we miss the point entirely, even if admittedly the existential import of X is implied by the premise. For in such case, *the formal continuity between premise and conclusion is lost*, there being two inexplicable changes of polarity (from something to nothing and from Y to not-Y)! Similarly, particularization requires formal continuity. To move freely from I to A, and then possibly to IO, we need the traditional opposition (contradiction) between A and O.

Another issue that is ignored by modern logicians is *modality*. Although modern logic has developed modal logic to some extent, it has done so by means of symbolic notations based on very simplistic analyses of modality. Although it has conventionally identified the different categories of modality (necessity, impossibility, actuality, inactuality, possibility, unnecessity), it has not thoroughly understood them. It has not clearly identified and assimilated the different types of modality (the logical, extensional, natural, temporal, and spatial modes), even if human discourse has included them all since time immemorial. Notably lacking in its treatment is the awareness that modality is an expression of conditioning and that the different types of modality give rise to different types of conditioning.

Consideration of modality is manifestly absent in the doctrine of existential import. The latter (as we saw) is built around the timeless (or 'omnitemporal') forms of categorical proposition, which are non-modal. It does not apply to modal categorical propositions, *for these do not formally imply (or presuppose) the actuality of their subject but only its possibility*. Thus, a universal proposition with natural-modality, "All S can (or must) be P," does not formally imply that "Some things *are* S" but only that "Some things *can be* S;" likewise, one with temporal modality, "All S are sometimes (or always) P" does not imply that "Some things *are* S" but only that "Some things *are sometimes* S;" and so forth.

This may be called 'existential import' in a broadened sense, acknowledging that being has degrees; but it is certainly not the actual sense intended by modern logicians: they apparently imagine that use of such modal propositions implies belief that "Some things *are* S." And of course, the *modality of subsumption*, as I have called this phenomenon in my book *Future Logic* (chapter 41), is very relevant to the processes of opposition, eduction (immediate inferences), syllogistic deduction (mediate inference) and induction. Regarding the latter, see my detailed theory of factorial induction in the said work. Thus, we may well say that the proponents of the doctrine of existential import constructed an expanded system of logic based on a rather narrow vision of the scope of logic. Even if their expansion (for all it is worth—not much, I'd say) is applicable to non-modal propositions, it is not appropriate for modal ones.

f.   The critique of the Aristotelian rectangle of oppositions began apparently in the middle ages, with Peter Abelard (France, 1079-1142). According to the Kneales, further input on this issue was made over time by William of Shyreswood, by Peter of Spain and St. Vincent Ferrer, and by Leibniz. They also mention Boole's interest in it, and many people attribute the modern view of the issue to this 19th century logician. However, E. D. Buckner suggests that the modern idea stems rather from Franz Brentano (Austria, 1838-1917), in a paper published in 1874[128]. And of course, many big name logicians such as Frege and Russell have weighed in since then.

Even though the new logic that ensued, based on the concept of existential import, is today strongly entrenched in academia, the switchover to it was epistemologically clearly not only unnecessary but ill-advised. The doctrine of existential import has been woefully misnamed: it is in fact *not* about existential import, but rather about *non-existential* import. It gives to empty terms undue importance, and thus greatly diminishes the real importance of non-empty terms. To be sure, this innovation fitted the anti-rational 'spirit of the times', and it kept many people happily busy for over a century, and thus feeling they existed and were important – but it was in truth emptiness and vanity.

Apparently, none of these people reflected on the obvious fact that once a term is identified as empty, it is simply dumped – it does not continue affecting our reasoning in any significant manner. This being so, there is no need to abandon the universal forms A and E because they imply (presuppose) the existential import of their subject. Even if the Aristotelian framework, which is built around non-empty terms, occasionally 'fails' due to the appearance of an empty term in discourse, such event is taken in stride and dealt with by summarily eliminating the discredited term thenceforth, and certainly *not* by switching to a non-Aristotelian framework as modern logicians recommend to do. In any case, the issue of existential import does not apply to modal logic, and so lacks generality.

Moreover, these people failed to realize that Aristotelian logical processing relates not only to realistic terms, but more significantly to hypothetical terms, i.e. *terms in process*. They viewed logic as a deductive activity; they did not realize its *essentially inductive* character. If, due to an immoderate interest in empty terms, the science of logic abandons the universal forms A and E, it deprives people of a language with which to accurately express the movements of thought inherent in the processes of syllogistic inference and of generalization and particularization. The science of logic must acknowledge the forms of actual human thinking, and not seek to impose artificial contraptions of no practical value. Otherwise, natural processes essential to human cognition cannot be credibly expressed and logic will seem obscure and arbitrary.

Modern logic has sown confusion in many people's minds, turning the West from a culture of confident reason to one of neurotic unreason. The purpose of logic studies ought to be to cognitively empower people, not incapacitate them. If logicians err in the forms of thought they describe and prescribe, they betray their mission, which is to intelligently and benevolently guide and improve human thinking. If they err, whether out of stupidity or malice, they turn logic from a responsible science and a fine art to a vain and dangerous game. They do not merely cease benefitting mankind; they positively harm people's minds.

3. The vanity of the tetralemma

The most radical assault on reason consists in trying to put in doubt the laws of thought, for these are indeed the foundations of all rational discourse. First, the law of identity is denied by saying that things are never quite what they seem to be, or that what they are is closer to grey than black and white. This is, of course, an absurd remark, in that for itself it lays claim to utter certainty and clarity. Then, the laws of non-contradiction and of the excluded middle are denied by saying that things may both be and not-be, or neither be nor not-be. This is the 'tetralemma', the fourfold logic which is favored in Indian and Chinese philosophies, in religious mysticism, and which is increasingly referred to among some 'scientists'. To grasp the vanity of the tetralemma, it is necessary to understand the nature of negation and the role of negation as one of the foundations of human logic.

The first thing to understand is that everything we experience is *positive phenomenon*. Everything we perceive through our senses, or remember or imagine in our minds, or even cognize through 'intuition' – all that has to have some sort of content to be at all perceived. Each sense organ is a window to a distinct type of positive phenomenon. We see the blue sky above, we hear birds sing, we smell the fresh air, we taste a fruit, we feel the earth's texture and warmth, etc. Similarly, the images and sounds in our heads, whether they come from memory or are produced by imagination, are positive phenomena; and even the objects of intuition must have some content that we can cognize. Secondly, we must realize that many positive phenomena may appear together in space at a given moment. This is true for each phenomenal type. Thus, the blue sky may fill only part of our field of vision, being bounded by green

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[128]   For Buckner's account of the history, see: www.logicmuseum.com/cantor/Eximport.htm. Notice his pretentious characterization of "the traditional 'syllogistic'" as "a historical curiosity." Brentano's position is to be found in his *Psychologie vom empirischen Standpunkt*, II, ch. 7. The Kneales do mention the latter reference in passing, in a footnote on p. 411.

trees and grey buildings; we may at once hear the sounds of birds and cars; and so on. Thirdly, many positive phenomena may at any given time share the space perceived by us. Thus, superimposed on visual phenomena like the sky may be other types of phenomena: the sound of birds in the trees, the smell of traffic in the streets, the feelings in our own body, and so on. We may even hallucinate, seeming to project objects of mental perception onto physical space. For example, the image of one's eyeglasses may persist for a while after their removal. Fourthly, each positive phenomenon, whatever its type, varies in time, more or less quickly. Thus, the blue sky may turn red or dark, the sounds of birds or traffic may increase or decrease or even stop for a while, and so forth.

In order to express all these perceptual possibilities – differences in space and in time and in other respects, we need a concept of negation, or more precisely an act of negating. Without 'negation', we cannot make sense of the world in a rational manner – it is the very beginning of logical ordering of our experience. Thus, in a given visual field, where (say) blue sky and trees appear, to be able to say 'the sky ends here, where the trees begin' we need the idea of 'negation' – i.e. that on one side of some boundary sky is apparent and on the other side it is not, whereas on the first side of it trees are not apparent and on the other they are. Likewise, with regard to time, to be able to describe change, e.g. from blue sky to pink sky, we need the idea of 'negation' – i.e. that earlier on this part of the sky was blue and not pink, and later on it was pink and not blue. Again, we need the idea of 'negation' to express differences in other respects – e.g. to say that 'the sounds of birds singing seem to emanate from the trees, rather than from buildings'. Thus, negation is one of the very first tools of logic, coming into play already at the level of sorting of experiences.

Moreover, negation continues to have a central role when we begin to deal with abstractions. Conceptual knowledge, which consists of terms and propositions based directly or indirectly on perceptual phenomena, relies for a start on our ability to cognize similarities between objects of perception: 'this seems to resemble that somewhat' – so we mentally project the idea of this and that 'having something in common', an abstract (i.e. non-phenomenal, not perceived by any means) common property, which we might choose to assign a name to. However, to take this conceptual process further, we must be able to negate – i.e. to say that 'certain things other than this and that do not have the abstract common property which this and that seem to have', or to say that 'this and that do not have everything in common'. That is, we must be able to say not only that one thing resembles another in some way, but also that these or other things do not resemble each other in that way or in another way. Thus, negation is essential for making sense of information also at the conceptual level of consciousness.

Now, what is negation? To answer this question we first need to realize that *there are no negative phenomena in the realm of experience*. Everything we perceive is positive phenomenon – because if it was not we obviously would have nothing to perceive. We can only 'perceive' a negative state of affairs by first mentally defining some positive state of affairs that we should look for, and then look for it; if having looked for it assiduously we fail to find it, we then conclude inductively that it is 'absent', i.e. 'not present'. Thus, positive phenomena come before negative ones, and not after. Existence logically precedes non-existence. Negative phenomena are 'phenomena' only metaphorically, by analogy to positive phenomena – in truth, negative phenomena are not: they do not exist. 'Negation' is not a concept in the sense of an abstraction from many particular experiences having a certain property in common. Negation is a tool of the thinking observer, as above described. It is an act, an intention of his.

To illustrate how confused some people – even some scientists – are with regard to negation, I offer you the following example drawn from Richard Dawkins' *The Greatest Show on Earth: The Evidence for Evolution*[129]. He describes an experiment by Daniel J. Simons, in which some people are asked to watch a brief video and observe how many times a certain event takes place in it; but at the end they are asked another question entirely, viz. whether they noticed the presence of a man dressed up as a gorilla in the course of the movie, and most of them admit they did not[130]. According to Dawkins, we may infer from this experiment how "eye witness testimony, 'actual observation', 'a datum of experience' – all are, or at least can be, hopelessly unreliable."

But this is a wrong inference from the data at hand, because he confuses positive and negative experience. The people who watched the video were too busy looking for what they had been asked to observe to notice the gorilla. Later, when the video was shown them a second time, they did indeed spot the gorilla. There is no reason to expect us to actually experience everything which is presented to our senses. Our sensory experiences are always, necessarily, selective. The validity of sense-perception as such is not put in doubt by the limited scope of particular sense-perceptions. The proof is that it is *through further sense-perception* that we discover what we missed before. Non-perception of something does not constitute misperception, but merely incomplete perception. 'I did not see X' does not deductively imply 'I saw the absence of X', even though repetition of the former tends to inductively imply the latter.

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[129]   New York: Free Press, 2009. Pp. 13-14.
[130]   The video can be seen at: www.theinvisiblegorilla.com/videos.html.

A negative 'phenomenon' is not like a positive phenomenon, something that can directly be perceived or intuited. A negation is of necessity the product of indirect cognition, i.e. of an inductive (specifically, adductive) process. We mentally hypothesize that such and such a positive phenomenon is absent, and then test and confirm this hypothesis by repeatedly searching-for and not-finding the positive phenomenon[131]. If we were to at any time indeed find the positive phenomenon, the hypothesis of negation would immediately be rejected; for the reliability of a negation is far below that of a positive experience. We would not even formulate the negation, if we already had in the past or present perceived the positive phenomenon. And if we did formulate the negation, we would naturally retract our claim if we later came across the positive phenomenon. Therefore, the content of negative phenomena is necessarily always hypothetical, i.e. tentative to some degree; it is never firm and sure as with (experienced) positive phenomena.

Negative assertions, like positive assertions, can be right or wrong. If one looked diligently for a positive phenomenon and did not find it, then one can logically claim its negation. Such claim is necessarily inductive – it is valid only so long as the positive phenomenon is actively sought and not found. The moment the positive phenomenon is observed, the negation ceases to be justified. If one did *not* look for the positive phenomenon, or did not look with all due diligence, perhaps because of some distraction (as in the example cited above), then of course the claim of negation is open to doubt; certainly, it is inductively weak, and one is very likely to be proved wrong through some later observation.

How, then, is negation to be defined? We could well say that negation is defined by the laws of non-contradiction and of the excluded middle. That is, with regard to any term 'X' and its negation 'not-X', the relation between them is by definition the disjunction "Either X or not-X" – which is here taken to mean that these terms (X and not-X) cannot be both true and cannot be both false, i.e. they are exclusive and exhaustive. What do I mean here by 'definition'? – is that an arbitrary act? No – it is 'pointing to' something evident; it is 'intentional'. Here, it points to the instrument of rational discourse which we need, so as to order experience and produce consistent conceptual derivatives from it. The needed instrument has to be thus and thus constructed; another construct than this one would not do the job we need it to do for us. That is, the only conceivable way for us to logically order our knowledge is by means of negation defined by means of the laws of non-contradiction and of the excluded middle. Without this tool, analysis of experience is impossible.

Suppose now that someone comes along and nevertheless objects to the preceding assertion. Well, he says, how do you *know* that the dilemma "either X or not-X" is true? You just arbitrarily *defined* things that way, but it does not mean it is a fact! Could we not equally well claim the tetralemma "Either X or not-X *or both or neither*" to be true? The reply to that objection is very simple. Suppose I accept this criticism and agree to the tetralemma. Now, let me divide this fourfold disjunction, putting on the one side the single alternative 'X' and on the other side the triple alternative 'not-X or both or neither'. I now again have a dilemma, viz. "either 'X' or 'not-X or both or neither'." Let me next define a new concept of negation on this basis, such that we get a disjunction of two alternatives instead of four. Let us call the complex second alternative 'not-X or both or neither' of this disjunction 'NOT-X' and call it 'the super-negation of X'.

Thus, now, the objector and I agree that the disjunction "either X or NOT-X" is exclusive and exhaustive. We agree, presumably, that this new dilemma cannot in turn be opposed by a tetralemma of the form "Either X or NOT-X or both or neither" – for if such opposition was tried again it could surely be countered by another division and redefinition. We cannot reasonably repeat that process ad infinitum; to do so would be tantamount to blocking all rational thought forever. Having thus blocked all avenues to thought, the objector could not claim to have a better thought, or any thought at all. There is thus no profit in further objection. Thus, the tetralemma is merely a tease, for we were quite able to parry the blow. Having come to an agreement that the new disjunction "Either X or NOT-X" is logically unassailable, we must admit that the original disjunction "Either X or not-X" was logically sound from the first. For I can tell you that what I meant by not-X, or the 'negation of X', was from the beginning what is now intended by NOT-X, or the 'super-negation of X'!

I was never interested in a relative, weak negation, but from the start sought an absolute, strong negation. For such utter negation, and nothing less radical, is the tool we all need to order experience and develop conceptual knowledge in a consistent and effective manner. In other words, whatever weaker version of negation someone tries to invent[132],

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[131]   Not-finding is the non-occurrence of the positive act of finding. Objectively, note well, not-finding is itself a negative phenomenon, and not a positive one. But subjectively, something positive may occur within us – perhaps a sense of disappointment or continued relief. See more on this topic in my *Ruminations*, chapter 9.

[132]   There are people who say that the law of non-contradiction is logically necessary, but the law of the excluded middle is not. Clearly, this claim can be refuted in the same way. If they claim the three alternatives "Either X or not-X or 'neither X nor not-X'" – we can again split the disjunction into two, with on one side "X" and on the other side "not-X or 'neither X nor not-X'" – and then proceed as we did for the tetralemma. The same can be done if anyone accepts the law of the excluded middle but rejects the law of non-contradiction. All such attempts are fallacious nonsense.

we can still propose a strong version such that both the laws of non-contradiction and of the excluded middle are applicable without doubt to it. If such negation did not exist, it would have to be invented. No one can destroy it by denying it or diluting it. Those who try to are merely sophists who do not understand the source, nature and function of negation in human discourse. They think it is a matter of symbolic manipulation, and fail to realize that its role in human discourse is far more fundamental and complex than that. Negation is the indispensable instrument for any attempt at knowledge beyond pure perception.

4. The Liar paradox (redux)

I dealt with the Liar paradox previously, in my *Future Logic*[133], but now realize that more needs to be said about it. This paradox is especially difficult to deal with because it resorts to several different discursive 'tricks' simultaneously.

a.   The statement "This proposition is false" looks conceivable offhand, until we realize that if we assume it to be true, then we must admit it to be indeed false, while if we assume it to be indeed false, then we must admit it to be true – all of which seems unconscionable. Obviously, there is a contradiction in such discourse, since nothing can be both true and false. But the question is: just what is causing it and how can it be resolved? We are not 'deducing' the fact of contradiction from a 'law of thought' – we are 'observing' the fact through our rational faculty. We cannot, either, 'deduce' the resolution of the contradiction from a 'law of thought' – we have to analyze the problem at hand very closely and creatively propose a satisfying solution to it, i.e. one which indeed puts our intellectual anxiety to rest. As we shall see, this is by no means a simple and straightforward matter.

The proposition "This proposition is false" is a double paradox, because: *if it is true, then it is false; and if it is false, then it is true*. Notice the circularity from true to false and from false to true. The implications we draw from the given proposition seems unavoidable at first sight. But we must to begin with wonder *how we know these implications* (the two if–then statements) to be true. How do we know that "it is true" implies "it is false," and that "it is false" implies "it is true"? Apparently, we are not 'deducing' these implications from some unstated proposition. We are, rather, using ad hoc rational insight of some sort – i.e. in a sense directly 'perceiving' (intellectually cognizing) the implications of the given proposition. But such rational insight, though in principle reliable, is clearly *inductive*, rather than deductive, in epistemological status. That is to say, it is trustworthy until and unless it is found for some reason to be incorrect. This means, there may be one or more errors in our thinking, here; it is not cast in stone. And indeed there must be some error(s), since it has led to double paradox. Therefore, we must look for it.

Perhaps use of the pronoun "it" is a problem, for it is a rather vague term. Let us therefore ask the question: *more precisely what does the pronoun "it" refer us to*, here?

At first sight, the "it" in "if it is true, then it is false; and if it is false, then it is true" refers to *the whole given statement*, "This proposition is false." In that event, we must reword the double paradox as follows: *if 'this proposition is false' is true, then 'this proposition is false' is false; and if 'this proposition is false' is false, then 'this proposition is false' is true*. Here, the subject of the two if–then statements is more clearly marked out as "this proposition is false," and so remains constant throughout. But this clarification reveals abnormal changes of predicate, from "true" to "false" and from "false" to "true," which cannot be readily be explained. Normally, we would say: if 'this proposition is false' is true, then 'this proposition is false' is *true*; and if 'this proposition is false' is false, then 'this proposition is false' is *false*. The reason we here reverse the predicates is that we consider the original proposition, "this proposition is false," as instructing such reversal.

However, whereas a proposition of the form "'this proposition is false' is true" is readily interpretable in the simpler form "this proposition is false," a proposition of the form "'this proposition is false' is false" cannot likewise be simplified. How would we express the double negation involved? As "this proposition is true"? Clearly, the meaning of the latter is not identical to that of the former, since the subject "this proposition" refers to different propositions in each case. So the formulation of the liar paradox in full form, i.e. as "if 'this proposition is false' is true, then 'this proposition is false' is false; and if 'this proposition is false' is false, then 'this proposition is false' is true," does not make possible the reproduction of the initial formula expressed in terms of the pronoun "it."

b.   Let us therefore try something else. If the pronoun "it" refers to *the term* "this proposition", then the double paradox should be reformulated as follows: *if 'this proposition' is true, then 'this proposition' is false; and if 'this proposition' is false, then 'this proposition' is true*. But doing that, we see that in each of these two if–then statements, though the subject ("this proposition") remains constant throughout, the predicate ("true" or "false," as

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[133]   See there chapter 32.2. (See also *Ruminations* 5.1.)

the case may be) is not the same in the consequent as it was in the antecedent. There is no logical explanation for these inversions of the predicate. Normally, the truth of a proposition P does not imply its falsehood or vice versa.

We might be tempted to use the given "This proposition is false" as a premise to justify the inference from the said antecedents to the said consequents. We might try to formulate two apodoses, as follows:

If this proposition is true, then it is false (hypothesis),	If this proposition is false, then it is true (hypothesis),
and this proposition is false (given);	and this proposition is false (given);
therefore, this proposition is true (putative conclusion).	therefore, this proposition is true (putative conclusion).

Obviously, in the first case we have invalid inference, in that we try to deny the antecedent to deny the consequent, or to affirm the consequent to affirm the antecedent. In the second case, the putative conclusion does follow from the premises; but we can still wonder where the major premise (the hypothetical proposition) came from, so we are none the wiser. So, this approach too is useless – i.e. it proves nothing.

Alternatively, we might try formulating the following two syllogisms:

This proposition is false (given),	This proposition is false (given).
and this proposition is true (supposition);	and this proposition is false (supposition);
therefore, this proposition is false (putative conclusion).	therefore, this proposition is true (putative conclusion).

Clearly, these arguments are not quite syllogistic in form; but they can be reworded a bit to produce syllogisms. The first two premises would then yield the conclusion "there is a proposition that is true and false" (3/RRI), which is self-contradictory (whence, one of the premises must be false); the second two premises, however, being one and the same proposition, would yield no syllogistic conclusion other than "there is a proposition that is false and false" (3/RRI), which is self-evident (and trivial). But these are not the conclusions we seek, which must concern "this proposition" and not merely "some proposition."

A better approach is to look upon the latter two arguments as follows. In the first case, the premises "this proposition is false" (given) and "this proposition is true" (supposition) seem to together imply "this proposition is both true and false;" and the latter paradoxical conclusion in turn indeed suggests that "this proposition is false," since contradiction is impossible. And in the second case, the premises "this proposition is false" (given) and "this proposition is false" (supposition) agree with each other that "this proposition is false," and so this is their logical conclusion. Since both arguments conclude with "this proposition is false," the latter must be the overall conclusion.

However, the latter result is not as conclusive as it seems, because upon closer scrutiny it is obvious that "this proposition is false" and "this proposition is true" do not refer to the same subject, since the predicate changes. The first "this proposition" refers to the proposition "this proposition is false" and the second "this proposition" refers to the proposition "this proposition is true." So, these two propositions in fact have different subjects as well as different predicates (viz. false and true, respectively). The subjects superficially look the same, because they are verbally expressed in identical words; but their underlying intent is not the same, since they refer to significantly different propositions (propositions with manifestly different, indeed contradictory, predicates). This means that when the predicate changes, the subject effectively changes too. When the predicate is "true," the subject means one thing; and when the predicate is "false," the subject means something else. Although the words "this proposition" are constant, their underlying intent varies. That is to say, the term "this proposition" does not have a uniform meaning throughout, and therefore cannot be used as a basis for the inferences above proposed.

c.   Let us now try another angle. If we examine our initial reasoning in terms of the pronoun "it" more carefully, we can see what is really happening in it. Given that 'this proposition is false' is true, we can more briefly say: 'this proposition is false.' Also, given 'this proposition is false' is false, we can by negation educe that 'this proposition is *not* false' is true, which means that 'this proposition is true' is true, or more briefly put: 'this proposition is true'[134]. In this way, we *seem* to argue, regarding the subject "this proposition is false," from 'it is true' to 'it is false', and from 'it is false' to 'it is true'. But in fact the use of the pronoun "it" or the term "this proposition" as abbreviated subject is a sleight of hand, for the underlying subject changes in the course of the second transition (that ending in "this proposition is true"). When abbreviation is used throughout, we seem to be talking about one

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[134]   Some logicians have tried to deal with the liar paradox by denying that true and false are contradictory terms, i.e. that not-true = false and not-false = true. Such a claim is utter nonsense; the attempt to shunt aside the laws of non-contradiction of the excluded middle so as to resolve a paradox is self-contradiction in action.

and the same proposition throughout as being both true and false. But seeing that this is based on hidden equivocation, the paradoxes disappear.

It is interesting to note that when the reasoning is viewed more explicitly like that, the proposition "this proposition is true" also becomes paradoxical! We can argue: if 'this proposition is true' is true, then obviously 'this proposition is true'. And: if 'this proposition is true' is false, then its contradictory 'this proposition is *not* true' must be true, which means that 'this proposition is false' is true, i.e. more succinctly: 'this proposition is false'. Here, superficially, there seems to be no paradox, because we seem to argue, regarding the subject "this proposition is true," from 'it is true' to 'it is true', and from 'it is false' to 'it is false'. But if we look at the final conclusion, viz. "this proposition is false," we see that it *corresponds to* the liar paradox![135] And here again, the explanation of the double paradox is that the apparent subject "it" or "this proposition" changes significance in the course of drawing the implications.

Notice that, in both these lines of reasoning, the first leg is ordinary self-implication, mere tautology, while the second leg is the operative self-contradiction, the paradox. If the given proposition (whether "this proposition is false" or "this proposition is true") is true, we merely repeat the proposition as is (without need to add the predication "is true"). But if the given proposition is false, we cannot drop the additional predication (i.e. "is false") without changing the original proposition. Thus, we could say that the two propositions, "this proposition is false" or "this proposition is true," present no problem when taken as true; and it is only when they are hypothetically taken as false that the problem is created. So we could say that the way out of the liar paradox (and its positive analogue) is simply to accept the two claims as true, and not imagine them to be false!

We could furthermore, if we really want to, argue that "this proposition is false" and "this proposition is true" differ in that the former explicitly appears to put itself in doubt whereas the latter does not do so. On this basis, we could immediately reject the former and somewhat accept the latter, even while admitting that the latter is equally devoid of any useful information. That is to say, since the former appears 'more paradoxical' than the latter, the latter is to be preferred *in extremis*. But this, note well, ignores the equally insurmountable difficulties in it. It is better to resolutely reject both forms as vicious constructs.

d.   To grasp the illusoriness of the liar paradox, it is important to realize that the two forms, "this proposition is false" and "this proposition is true," are *not* each other's contradictory; and that, in fact, neither of them *has* a contradictory! This is *a logical anomaly*, a fatal flaw in the discourse of the liar paradox; for in principle, every well-formed and meaningful proposition is logically required to have a contradictory. If a propositional form lacks a contradictory form, it cannot be judged true or false, for such judgment depends on there being a choice. We do not even have to limit our propositions to the predicates "true" or "false" – any predicate X and its negation not-X would display the same property given the same said subject. That is, "this proposition is X" and "this proposition is not-X" are *not* each other's contradictory, and are therefore *both* equally deprived of contradictory.

We could, of course, remark that "this proposition is X" can be denied by "*that* proposition (i.e. the preceding one) is not X," or even introduce *a symbol* for the original proposition in the new proposition. In such case, although the subjects would be verbally different, their intents would surely be the same. But the form "that proposition is not X" is more akin to the form "'this proposition is X' is not X," in which the whole original proposition is given the role of subject and its predicate is given the role of predicate. However, though these two forms are somewhat equivalent in meaning to each other and to the original proposition, their logical behavior patterns are not identical with that of the original proposition, as we have already seen. The fact remains that "this proposition is not X" is not the contradictory of "this proposition is X."

Clearly, any proposition involving the special subject "this proposition" exhibits a very unusual property, and may be dismissed on that basis alone. The reason why such a proposition lacks a contradictory is that its subject refers to the proposition *it happens to be in*, and that proposition is evidently different when the predicate in it is the term "false" and when it is the term "true" (or more generally, any pair of predicates 'X' and 'not-X'). When the predicate changes, *so does the subject*; so the subject cannot be pinned-down, it is variable, it is not constant as it should be. The term "this proposition" has a different reference in each case, which depends on the predicate; consequently, each subject can only be associated with one predicate and never with the other (i.e. its negation).

From this we see that when at the beginning we thought, looking upon the statement "This proposition is false," that if we take it at its word, then it is must be regarded as false, and so we have to prefer to it "This proposition is not false," i.e. "This proposition is true," and so forth, we did not realize that we were in fact, due to the ambiguity inherent in the term "This proposition" or "it," changing its meaning at every turn. This change of meaning passes by unnoticed, because the term used is by its very nature not fixed. The pronouns "this" and "it" can be applied to anything and its opposite without such change of meaning being verbally signaled in them. They are not permanently

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[135]   That 'this proposition is true' is implicitly (if only potentially) as paradoxical as 'this proposition is false' is, so far as I know, a new discovery. Note well how *both* paradoxes occur through quite ordinary eductions: viz. if 'P is Q' is affirmed, then P is Q; and if 'P is Q' is denied, then 'P is not Q' is affirmed, then P is not Q (where P stands for 'this proposition', and Q for 'false' or 'true' as the case may be).

attached to any object, but are merely contextual designations. In the technical terminology of linguistics, they are characterized as 'deictic' or 'indexical'.

Thus, it appears that the liar paradox arises, however we understand its terms, as a result of some sort of equivocation in the subject. Although we seem superficially to refer to one and the same subject in the antecedent and consequent of our if–then reasoning, there is in fact a covert change of meaning which once we become aware of it belies the initial appearance of contradiction. The suggested impossible implications are thus put in doubt, made incredible. The contradictions apparently produced are thus defused or dissolved, by virtue of our inability to make them stick.

e.   Another, and complementary, way to deal with the liar paradox is to point out *the logical difficulty of self-reference*. This is a tack many logicians have adopted, including me in my first foray into this topic in *Future Logic*. The argument proposed here is that the term "this proposition" refers to an object (viz. "This proposition is false" or "This proposition is true") which includes the term itself. A finger cannot point at itself, and "this" is the conceptual equivalent of a finger. Effectively, the expression "this" has no content when it is directed at itself or at a sentence including it. It is empty, without substance. It is as if nothing is said when we indulge in such self-reference.

Thus, "This proposition is X" (where X stands for false, or true, or indeed anything) is in fact meaningless; and a meaningless sentence cannot be true or false. Such a sentence can reasonably be described as neither true nor false, without breach of the law of the excluded middle, because neither of these logical evaluations is applicable to meaningless sentences. "This proposition is false" looks meaningful because its four constituents (i.e. "this," "proposition," "is" and "false") are separately normally meaningful. But in this particular combination, where one of the elements (viz. "this") does not refer to anything already existent, the sentence is found to be meaningless.

The apparent contradictions that self-reference produces help us to realize its meaninglessness. And it is through the intellectual realization of the meaninglessness of self-reference that we explain away and annul the apparent contradictions. On this basis, we can say that even though the sentence "This proposition is true" does not at first sight give rise to any paradox (as people think: "if it is true, it is true; and if it is false, it is false"), nevertheless, since it involves self-reference as much as "This proposition is false," it is equally meaningless and cannot be characterized as true or false. In fact, as I have shown above, "This proposition is true" does also give rise to double paradox.

Someone might object: What about the propositions: "this statement is self-referential" and "this statement is not self-referential"? Surely, we can say that these are meaningful and that the former is true while the latter is false! The retort to that objection is that the two propositions "this proposition refers to itself" and "this proposition does not refer to itself" are *not* mutual contradictories, because (just like in the liar paradox) their subjects differ radically, each referring to the proposition it is in and not to the other. Thus, while the positive version may seem more self-consistent than the negative one, and therefore to be preferred *in extremis*, they are in fact both fundamentally flawed, because (just like in the liar paradox) neither of them *has* a contradictory, and without the logical possibility of negating a discourse it is impossible to judge whether it is right or wrong.[136]

f.   Not long after the preceding reflections, I happened to come across another interesting example of paradoxical self-reference, namely "Disobey me!"[137] This involves the 'double bind' – *if I obey it, I disobey it and if I disobey it, I obey it*. To resolve this paradox, we need to first put the statement in more precise form, say: "you must disobey this command!" We can then disentangle the knot by realizing that the order being given *has outwardly imperative form but inwardly lacks content*. It does not define a specific, concrete action that is to be done or not-done. If we wished to obey it, or to disobey it, we would not know just what we are supposed to do or not-do! It is therefore an order that can neither be obeyed nor be disobeyed. Ruminating on this case led me to what I now believe is the trump card, which convincingly finalizes the resolution of the liar paradox, even as the preceding reflections all continue to be relevant.

It occurred to me then that this is precisely the problem with the liar paradox. It says "this proposition is false" – but *it does not tell us anything about the world that can be judged as true or false*. A 'proposition' is a statement that makes some claim about the world. If the statement makes no such claim, if it 'proposes' nothing, it cannot be logically assessed as true or false. If it refers to nothing – whether physical, mental or spiritual, perceptual, intuitive

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[136]   Another objection (which was actually put to me by a reader) would be propositions like "this statement has five words" and "this statement has six words" – even though they contain the demonstrative "this," the former looks true and the latter false! Here, we might in reply point out that though the propositions "this statement has five words" and "this statement does not have five words," seem to mean opposite things, they cannot be contradictories, since both appear true. Also compare: "this statement has five words" and "this statement does have five words" – the former is true while the latter is false, though both *mean* essentially the same. Clearly, the behavior of these propositions is far from normal, due to their unusual dependence on the wording used in them. On one level, we get the message of the proposition and count the number of words in it, and then check whether this number corresponds to the given number: if yes, the proposition is judged 'true', and if no, it is judged 'false'. But at the same time, we have to be keep track of the changing reference of the demonstrative "this," which complicates matters as already explained, and additionally in this particular context we must beware of the impact of wording. The Kneales give "What I am now saying is a sentence in English" as an example of "harmless self-reference" (p. 228).

[137]   I found this example in Robert Maggiori's *La philosophie au jour le jour* (Paris: Flammarion, 1994); the author does not say whether it is his own invention or someone else's (p. 438).

or conceptual – it has no meaning. A meaningless statement does not qualify as a 'proposition'. The attributes of 'true' or 'false' are not ordinary predicates, like 'white' or 'black', which can be attached to any subject and then judged to be truly or falsely attached. The attributes of 'true' or 'false' require a precise claim to be made before they can at all be used.

The truth of this explication can be seen with reference to the 'propositional forms' used in logic theory. Take, for example, "All X are Y." Such a propositional form cannot be judged true or false because it manifestly has no content. Only when such an abstraction is given some specific content, such as "All men are mortal," can we begin to ask whether it is true or false. A propositional form is too vague to count as a proposition. It does not tell us anything about the world, other than implying that there are (or even just that there may be) concrete propositions which have this form. Just as we cannot disobey or even obey an imperative without content, so we cannot judge a purely formal expression true or false.

The same applies to the liar paradox: like a formal proposition, it has no concrete content, and therefore cannot be judged true or false. The liar paradox has no content partly due to its having a self-referential subject ("this proposition"). But the truth is, *even if its subject was not self-referential*, it would still have insufficient content. This is so, because its predicate "false" (and likewise its opposite, "true") is not an ordinary predicate; it is more like a formal predicate. It can only be used if another, more concrete predicate has already been proposed for the subject at hand. For example, "this proposition is interesting" could be judged true or false (if it was not self-referential) because it already has a predicate (viz. "interesting"). Thus, the problem with the liar paradox is not only the self-reference it involves but also its lack of a predicate more concrete than the logical predicate "false" (or "true").

All this illustrates how the 'laws of thought' are not axioms in the sense of top premises in the knowledge enterprise from which we mechanically derive other premises. Rather the expression 'laws of thought' refers to recurring insights which provide us with some intellectual guidance but cannot by themselves determine the outcome. The individual in pursuit of knowledge, and in particular the logician, is *driven* by the obviousness or by the absurdity of a situation to look for creative solutions to problems. He or she must still think of possible solutions and test them.

5. The Russell paradox (redux)

Logic is what helps us transmute scattered concrete perceptions into well-ordered abstract concepts. Human knowledge, or opinion, is based on experience, imagination and rational insight. The latter is a kind of 'experience' in the larger sense, a non-phenomenal sort of experience, call it logical 'intuition'. Reason was for this reason called by the ancients, in both West and East, the 'sixth sense' or 'common sense', i.e. the sense-organ which ties together the other five senses, those that bring us in empirical contact with phenomenal experience: colors, shapes, sounds, smells, tastes, touch-sensations, etc., whether they are physically perceived or mentally imagined. The five senses without the sixth yield chaotic nonsense (they are non-sense, one cannot 'make sense' of them); and conversely, the sixth sense is useless without the other five, because it has nothing about which to have rational insights. Imagination reshuffles past experiential data and reasoning, making possible the formation of new ideas and theories which are later tested with reference to further experience and reasoning.

Elements of class logic. Logic initially developed as a science primarily with reference to natural discourse, resulting in what we today refer to as predicate logic. In natural human discourse, we (you and me, and everyone else) routinely think of and discuss things we have perceived, or eventually conceived, by means of categorical propositions involving a subject (say, S) and a predicate (say, P) which are related to each other by means of the copula 'is'. Such propositions have the form "S is P," which may be singular or plural, and in the latter case general (or universal) or particular, and positive or negative, and moreover may involve various modes and categories of modality[138].

A proposition of the form 'S is P' is really a double predication – it tells us that a thing which is S is also P; thus, S and P are really both predicates, though one (the subject S) is given precedence in thought so as to 'predicate' the other (the predicate P) of it[139]. Primarily, S refers to some concrete phenomenon or phenomena (be it/they physical, mental or spiritual), i.e. an individual entity or a set of entities, and P to a property of it or of theirs. For examples, "John is a man" and "All men are human beings" are respectively a singular predication (about one man, John) and a plural one (about all men).

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[138]   We need not go into the details of these distinctions here, for they are well known. There are also many fine distinctions between different sorts of terms that may appear in propositions as subjects or predicates; but let us keep the matter simple.
[139]   'Predication' refers to the copula and the predicate together as if they were an action of the speaker (or the statement made) on the subject.

Additionally, still in natural discourse, the subject of our thoughts may be predicates *as such*, i.e. predicates in their capacity as predicates; an example is: "'men' may be the subject or predicate of a proposition." The latter occurs in specifically philosophical (or logical or linguistic) discourse; for example, in the present essay.

Now, logicians through the ages, and especially in modern times, have effectively found natural discourse somewhat inadequate for their needs and gradually developed a more artificial language, that of 'classes'[140]. This type of discourse exactly parallels natural discourse, but is a bit more abstract and descriptive so as to facilitate philosophical (or logical or linguistic) discourse and make it more precise. In this language, instead of saying "this S *is* P," we say "this S *is a member (or instance) of* P" (note well the lengthening of the copula from 'is' to 'is a member (or instance) of'. If 'this S' symbolizes a concrete individual, then 'P' here is called a '*class*'; but if 'this S' symbolizes an abstract class, then 'P' here is called a '*class of classes*'.

A class, then, is an abstraction, a mental construct in which we figuratively group some concrete things (be they physical, mental or spiritual). Although we can and do temporarily mentally classify things without naming the class for them, we normally name classes (i.e. assign them a distinctive word or phrase) because this facilitates memory and communication. Naming is not the essence of classification, but it is a great facilitator of large-scale classification. The name of a class of things does not 'stand for them' in the way of a token, but rather 'points the mind to them' or 'draws our attention to them'; that is to say, it is an instrument of intention.

A class in the primary sense is a class of things in general; a class in the secondary sense is more specifically a class *of classes*. Membership is thus of two kinds: membership of non-classes in a class, or membership of classes in a class of classes. Alternatively, we may speak of first-order classes and second-order classes to distinguish these two types. There are no other orders of classes. When we think about or discuss more concrete things, we are talking in first-order class-logic; when we think about or discuss first-order classes, we are talking in second-order class-logic, and the latter also applies to second-order classes since after all they are classes too. The two orders of classes should not be confused with the hierarchy of classes within each order.

The relation between classes of classes and classes is analogous to the relation between classes and concretes; it is a relation of subsumption. When a lower (i.e. first-order) class is a member of a higher (i.e. second-order) class, it does *not* follow that the members of the lower class are also members of the higher class; in fact, if they are members of the one they are certainly not members of the other. Thus, for example, you and me, although we are members of the class 'men' because we are men, we are not members of the class 'classes of men' because we are not 'men'. Also, the class 'men' is not a man, but is a member of the class 'classes of men'. The members of the class 'classes of men' (or more briefly put, 'men-classes'), which is a class of classes, are, in addition to the broad class 'men', the narrower classes 'gardeners', 'engineers', 'sages', 'neurotics', and so on.[141]

Hierarchization, on the other hand, refers to classes *within a given order* that share instances, not merely by partly overlapping, but in such a way that all the members of one class are members of the other (and in some but not all cases, vice versa). For example, since all men are animals, though not all animals are men, the class 'men' is *a subclass (or species) of* the class 'animals', and the class 'animals' is *an overclass (or genus) of* the class 'men'. If two classes have the same instances, no more and no less, they may be said to be *co-extensive* classes (a class that serves as both species and genus in some context is said to be *sui generis*). If two classes merely share some instances, they may be said to be *intersecting (or overlapping)* classes, but they are not hierarchically arranged (e.g. 'gardeners' and 'engineers'). If two classes of the same order have no instances in common, they may be said to be *mutually exclusive* classes.

It is important to grasp and keep in mind the distinction between hierarchy and order. Since you and I are men, each of us is a member of the class 'men'; this is subsumption by a first-order class of its concrete instances. Since all men are animals, the class 'men' is a subclass of the class 'animals'; this is hierarchy between two classes of the first order. Since 'men' is a class of animals, it is a member of the class 'classes of animals' (or 'animal-classes'); this is subsumption by a second-order class (i.e. a class of classes) of its first-order-class instances (i.e. mere classes). Since all 'classes of men' are 'classes of animals', the class 'men-classes' is a subclass of the class 'animals-classes'; this is hierarchy between two classes of the second order, i.e. between two classes of classes. The relation between classes of the first order and classes of the second order is never one of hierarchy, but always one of subsumption; i.e. the

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[140]   The following account of class logic is based on my presentation in *Future Logic*, chapters 43-45. The word 'class' comes from the Latin *classis*, which refers to a "group called to military service" (Merriam-Webster). I do not know whether the Ancients used that word in its logical sense, or some such word, in their discourse, but they certainly thought in class logic mode. Examples of class thinking are Aristotle's distinction between species and genera and Porphyry's tree.

[141]   Note that saying or writing the word men without inverted commas refers to a *predicate*. When we wish to refer to the corresponding *class*, we say the class of men, or the class men; if we are writing, we may write the same with or without inverted commas, or simply 'men' in inverted commas. When dealing with classes of classes, we say the class of classes of men, or the class of men-classes, or the class men-classes, and we may write the same with or without inverted commas, or simply 'classes of men' or 'men-classes' in inverted commas.

former are always members (instances) of the latter, never subclasses. Hierarchies only occur between classes of the same order.

Thus, in class logic, we have two planes of existence to consider. At the ground level is the relatively objective plane of empirical phenomena (whether these are physical, mental or spiritual in substance); above that, residing in our minds, is the relatively subjective plane of ideas (which are conceived as insubstantial, but do have phenomenal aspects – namely mental or physical images, spoken or written words, and the intentions of such signs), comprising ideas about empirical phenomena and ideas about such ideas. Classes are developed to facilitate our study of empirical phenomena and classes of classes are developed in turn to facilitate our study of classes – for classes (including classes of classes) are of course themselves empirical phenomena of sorts. Classification is a human invention helpful for cognitive ordering of the things observed through our senses or our imaginations or our introspective intuitions. Although classes are products of mind, this does not mean that they are arbitrary – they are formed, organized and controlled by means of our rational faculty, i.e. with the aid of logic.

Clearly, to qualify as a class, a class must have at least one member (in which case the sole member is "one of a kind"). Usually, a class has two or more members, indeed innumerable members. A class is finite if it includes a specified number of instances; if the number of instances it includes is difficult to enumerate, the class is said to be open-ended (meaning infinite or at least indefinite). What brings the instances of a class together in it is their possession of some distinctive property in common; the class is defined by this property (which may of course be a complicated conjunction of many properties). A class without instances is called a null (or empty) class; this signifies that its defining property is known to be fanciful, so that it is strictly speaking a non-class.

Thus, note well, the term 'class' is a bit ambiguous, as it may refer to a first-order class (a class of non-classes, i.e. of things other than classes) or a second-order class (a class of classes, i.e. a mental construct grouping two or more such mental constructs). A class (of the first order) is not, indeed cannot be, a class of classes (i.e. a class of the second order). There is, of course, a class called 'non-classes'; its instances are principally all concrete things, which are not themselves classes; for example, you and I are non-classes. 'Non-classes' is merely a class, not a class of classes, since it does not include any classes. Thus, 'non-classes' may be said to be a first-order class, but does not qualify as a second-order class.[142]

The realm of classes of classes is very limited as an object of study in comparison to the realm of mere classes. For what distinctions can we draw between classes? Not many. We can distinguish between classes and classes of classes, between finite and open-ended classes, between positive and negative classes[143], and maybe a few more things, but not much more.

An apparent double paradox. Bertrand Russell (Britain, 1872-1970) proposed a distinction between 'a class that is a member of itself' and 'a class that is not a member of itself'. Although every class is necessarily co-extensive with itself (and in this sense is included in itself), it does not follow that every class is a member of itself (evidently, some are and some are not). Such a distinction can be shown to be legitimate by citing convincing examples. Thus, the class 'positive classes' is a member of itself, since it is defined by a positive property; whereas the class 'negative classes' is not a member of itself, since it is also positively defined (albeit with general reference to negation). Again, the class 'finite classes' is not a member of itself, since it has innumerable members; while the class 'open-ended classes' is a member of itself, since it too has innumerable members.

What about the class 'classes' – is it a member of itself or not? Since 'classes' is a class, it must be a member of 'classes' – i.e. of itself. This is said without paying attention to the distinction between classes of the first and second orders. If we ask the question more specifically, the answer has to be nuanced. The class 'first-order classes' being a class of classes and not a mere class, cannot be a member of itself, but only a member of 'second-order classes'; the members of the 'first-order classes' are all mere classes. On the other hand, since the class 'second-order classes' is a class of classes, it is a member of itself, i.e. a member of 'second-order classes'. Thus, the class 'second-order classes' includes both itself and the class 'first-order classes', so that when we say that the wider class 'classes' is a member of itself, we mean that it is more specifically a member of the narrower class 'classes of classes'. As regards the class 'non-classes', since it is a class and not a non-class, it is not a member of itself. Note however that Russell's

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[142]   Note that, whereas positive terms are easy enough to translate into class logic language, negative terms present a real difficulty. For example, whereas the term men refers only to non-classes, its strict antithesis, the term non-men in its broadest sense, includes both non-classes (i.e. concrete things other than men) and classes (i.e. more abstract things). Again, whereas the term finite classes refers only to classes, its strict antithesis, the term non-finite-classes in its broadest sense, includes both open-ended classes (abstracts) and non-classes (concretes). Thus, we must, for purposes of consistency, admit that some terms do cover both non-classes and classes (including classes of classes). Practically, this means we have to make use of *disjunctives* which reveal the implicit alternatives. This of course complicates class logic considerably.

[143]   Positive classes are defined by some positive property and negative classes are defined by a negative one. For examples, 'men' is defined with reference to rational animals (positive), whereas 'bachelors' is defined with reference to not yet married men (negative).

paradox does not make a distinction between classes of the first and second orders, but focuses on 'classes' indiscriminately.

Russell asked **whether "the class of all classes which are not members of themselves" is or is not a member of itself**. It seemed logically impossible to answer the question, because either way a contradiction ensued. For if the class 'classes not members of themselves' *is not* a member of the class 'classes not members of themselves,' then it is indeed a member of 'classes not members of themselves' (i.e. of itself); and if the class 'classes not members of themselves' *is* a member of 'classes not members of themselves,' then it is also a member of 'classes which are members of themselves' (i.e. of its contradictory). This looked like a mind-blowing double paradox.

The solution of the problem. The pursuit of knowledge is a human enterprise, and therefore one which proceeds by trial and error. Knowledge is inductive much more than deductive; deduction is just one of the tools of induction. There are absolutes in human knowledge, but they are few and far between. When we formulate a theory, it is always essentially a hypothesis, which might later need to be revised or ruled out. So long as it looks useful and sound, and does so more than any competing theory, we adopt it; but if it ever turns out to be belied by some facts or productive of antinomy, we are obliged to either reformulate it or drop it. This is the principle of induction. When we come upon a contradiction, we have to 'check our premises' and modify them as necessary. In the case at hand, since our conception of class logic is shown by the Russell paradox to be faulty somehow, we must go back and find out just where we went wrong. So, let us carefully retrace our steps. We defined a class and membership in a class by turning predication into classification, saying effectively:

If something is X, then it is a member of the class 'X', and not a member of the class 'nonX'.

If something is not X, then it is not a member of the class 'X', but a member of the class 'nonX'.

Where did we get this definition? It is not an absolute that was somehow cognitively imposed on us. We invented it – it was a convention by means of which we devised the idea of classes and membership in them. Knowledge can very well proceed without recourse to this idea, and has done so for millennia and continues to do so in many people's mind. It is an idea with a history, which was added to the arsenal of reasoning techniques by logicians of relatively recent times. These logicians noticed themselves and others reasoning by means of classification, and they realized that this is a useful artifice, distinct from predication and yet based on it somehow. They therefore formally proposed the above definition, and proceeded to study the matter in more detail so as to maximize its utility. The 'logic of classes', or 'class logic', was born.

However, at some stage, one logician, Bertrand Russell, realized that there was an inherent inconsistency in our conception of classification, which put the whole edifice of class logic in serious doubt. That was the discovery of the paradox bearing his name. That was a great finding, for there is nothing more important to knowledge development, and especially to development of the branch of knowledge called formal logic, than the maintenance of consistency. Every discovery of inconsistency is a stimulation to refine and perfect our knowledge. Russell deserves much credit for this finding, even if he had a lot of difficulty resolving the paradox in a fully convincing manner. Let us here try to do better, by digging deeper into the thought processes involved in classification than he did. What is classification, more precisely?

If we look more closely at our above definition of a class 'X' and membership of things in it by virtue of being X, we must ask the question: what does this definition achieve, concretely? Are we merely substituting the phrase 'is a member of' for the copula 'is', and the class 'X' for the predicate X? If this is what we are doing, there is no point in it – for it is obvious that changing *the name* of a relation or a term in no way affects it. Words are incidental to knowledge; what matters is their underlying intent, their meaning. If the words change, but not the meaning, nothing of great significance has changed. No, we are not here merely changing the words used – we are proposing a mental image.

Our idea of classification is that of *mental entities* called classes in which things other than classes (or lesser classes, in the case of classes of classes) are *figuratively collected and contained*. When we say of things that they are members of class 'X', we mean that class 'X' is a sort of box *into* which these things are, by means of imagination, stored (at a given time, whether temporarily or permanently). That is to say, our 'definition' of classification is really a formal convention used to institute this image. What it really means is the following:

If something is X, then it is *in* the class 'X', and out of (i.e. *not in*) the class 'nonX'.

If something is not X, then it is out of (i.e. not in) the class 'X', but in the class 'nonX'.

Clearly, to 'be' something and to 'be in' (within, inside) something else *are not the same thing*. Our definition conventionally (i.e. by common agreement) decrees that if X is predicated of something, then we may think of that thing as being as if contained by the mental entity called class 'X'. But this decree is not an absolute; it is not a proposition that being subject to predication of X *naturally and necessarily implies* being a member of class 'X'. For the whole idea of classification, and therefore this definition of what constitutes a class and membership therein, is a human invention. This invention may well be, and indeed is, very useful – but it remains bound by the laws of nature.

If we find that the way we have conceived it, i.e. our definition of it, inevitably leads to contradiction, we must adjust our definition of it in such a way that such contradiction can no longer arise. This is our way of reasoning and acting in all similar situations.

As we shall presently show, since the contradiction is a consequence of the just mentioned defining implication, we must modify that implication. That is to say, we must decree it to have limits. Of course, we cannot just vaguely say that it has limits; we must precisely define these limits so that the practical value of our concept of classification is restored. We can do that by realizing that our definition of classification with reference to something 'being in' something else means that *class logic is conceived of as related to geometrical logic*. This is obvious, when we reflect on the fact that we often 'represent' classes as geometrical figures (notably, circles) and their members as points within those figures. This practice is not accidental, but of the very essence of our idea of classification. Classification is imagining that we put certain items, identified by their possession of some common and distinctive property, in a labeled container[144].

Let us now examine the concept of self-membership in the light of these reflections. What is the idea of self-membership? It is the presupposition that a class may be a member of itself. But is that notion truly conceivable? If we for a moment put aside the class logic issue, and reformulate the question in terms of geometrical logic, we see that it is absurd. Can a container contain itself? Of course not. There is no known example of a container containing itself in the physical world; and indeed we cannot even visually imagine a container containing itself. So the idea of self-containment has no empirical basis, not even in the mental sphere. It is only a fanciful conjunction of two words, without experiential basis. For this reason, the idea strikes us as illogical and we can safely posit as a universal and eternal 'axiom' that self-containment is impossible. A nonsensical term like 'the collection of all collections' is of necessity an empty term; we are not forced to accept it, indeed we are logically not allowed to do so; we can only consistently speak of 'the collection of all *other* collections'[145].

A container is of course always co-extensive with itself, i.e. it occupies exactly the space it occupies. But such 'co-extension' is not containment, let alone self-containment, for it does not really (other than verbally) concern two things but only one; there is no 'co-' about it, it is just extended, just once. We refer to containment when a smaller object fits inside a larger object (or in the limit when *another* object of equal size neatly fits inside a certain object). The concept of containment refers to two objects, not one. There has to be two distinct objects; it does not suffice to label the same object in two ways. To imagine 'self-containment' is to imagine that a whole object can somehow fit into itself as a smaller object (or that it can somehow become two, with one of the two inside the other). This is *unconscionable*. A whole thing cannot be a part (whether a full or partial part) of itself; nothing can be both whole and part at once. A single thing cannot be two things (whether of the same or different size) at once; nothing can simultaneously exist as two things.

You cannot decide by convention that something is both whole and part or that one thing is two. You cannot convene something naturally impossible. You can only convene something naturally possible, even though it is unnecessary. Thus, the concept of self-containment is meaningless; it is an inevitably empty concept, because it assumes something impossible to be possible. There is no such thing as self-containment; a container can never contain itself. If this is true, then it is of course equally true that no class includes itself, for (as we have seen) classification is essentially a geometrical idea. Given that a container cannot contain itself, it follows that the answer to the question as to whether a class can be a member of itself is indubitably and definitely: No. Because to say of any class that it is a member of itself is to imply that a container can be a content of itself. Just as no container which is a content of itself exists, so *no class which is a member of itself exists!*

Now, this is a revolutionary idea for class logic. It applies to any and every class, not just to the class 'classes not members of themselves' which gave rise to the Russell paradox. Moreover, note well that we are here denying the possibility of membership of a class in itself, but not the possibility of *non*-membership of a class in itself. When we say that no container contains itself, we imply that it is true of each and every container that it does not contain itself. Similarly, when we say that no class is a member of itself, we imply that it is true of each and every class that it is not a member of itself. What this means is that while we acknowledge the subject of the Russell paradox, namely the class 'classes that are not members of themselves', we reject the notion that such a class might *ever*, even hypothetically for a moment, be a member of itself (and therefore paradoxical) – for, we claim, no class whatever is ever a member of itself.

How can this be, you may well ask? Have we not already shown by example that some classes are members of themselves? Have we not agreed, for example, that the class 'classes' being a class has to be a member of the class 'classes', i.e. of itself? How can we deny something so obvious? Surely, you may well object further, if the class

'classes that are not members of themselves' is not a member of itself, then it is undeniably a member of itself; and if it is a member of itself, then it is undeniably not a member of itself? To answer these legitimate questions, let us go back to our definition of classification, and the things we said about that definition. As I pointed earlier, our definition of classes and membership in them has the form of a conventional implication. It says:

> If and only if something is X, then it is a member of the class 'X'.

Now, since this conventional implication leads us inexorably to paradox, we must revise it, i.e. make it more limited in scope, i.e. specify the exact conditions when it 'works' and when it ceases to 'work'. What is essentially wrong with it, as we have seen, is that it suggests that a class can be a member of itself. For example, since the class 'classes' is a class, then it is a member of 'classes'; in this example, the variable X has value class and the variable 'X' has value 'classes'. But, as we have shown, the claim that a class can be a member of itself logically implies something geometrically impossible; namely, that a container can be a content of itself. So, to prevent the Russell paradox from arising, we need to prevent the unwanted consequences of our definition from occurring. Given that our concept of classification is problematic as it stands, what are the conditions we have to specify to delimit it so that the problem is dissolved?

The answer to this question is that when the subject and predicate of the antecedent clause are one and the same, then the consequent clause should cease to be implied. That is to say, if the antecedent clause has the form "if the class 'X' is X" then the consequent clause "then the class 'X' is a member of 'X' (and thus of itself)" *does not follow*. This 'does not follow' is a convention, just as the general 'it follows' was a convention. What we have done here is merely to draw a line, saying that the consequent *generally* follows the antecedent, *except in the special case* where the subject and predicate in the antecedent are 'the same' (in the sense that predicate X is applicable to class 'X' which is itself based on predicate X). This is logically a quite acceptable measure, clearly. If an induced general proposition is found to have exceptions, then it is quite legitimate and indeed obligatory to make it less general, retreating only just enough to allow for these exceptions.

Since the initial definition of classification was a general convention, it is quite permissible, upon discovering that this convention leads us into contradiction, to agree on a slightly narrower convention. Thus, whereas, in the large majority of cases, it remains true that if something is X, then it is a member of the class 'X', and more specifically, if a class (say, 'Y') is X, then it (i.e. 'Y) is a member of the class of classes 'X' – nevertheless, ***exceptionally, in the special case where the class that is X is the class 'X' (i.e. where 'Y' = 'X'), we cannot go on to say of it that it is a member of 'X'***, for this would be to claim it to be a member of itself, which is impossible since this implies that a container can be a content of itself. Note well that we are not denying that, for example, the class 'classes' *is* a class; we are only denying *the implication* this is normally taken to have that the class 'classes' *is a member of* the class 'classes'. We can cheerfully continue saying 'is' (for that is mere predication), but we are not here allowed to turn that 'is' into 'is a member of' (for that would constitute illicit classification).

In this way, the Russell paradox is inhibited from arising. That is to say, with reference to the class 'classes not members of themselves': firstly, it is quite legitimate to suppose that the class 'classes not members of themselves' is not a member of itself, since we know for sure (from geometrical logic) that no class is a member of itself; but it is *not* legitimate to say that this fact (i.e. that it is not a member of itself) implies that it is a member of itself, since such implication has been conventionally excluded. Secondly, it is *not* legitimate to suppose, even for the sake of argument, that the class 'classes not members of themselves' is a member of itself, since we already know (from geometrical logic) that no class is a member of itself, and therefore we cannot establish through such supposition that it is not a member of itself, even though it is anyway true that it is not a member of itself.

As can be seen, our correction of the definition of classification, making it less general than it originally was, by specifying the specific situation in which the implication involved is not to be applied, succeeds in eliminating the Russell paradox. We can say that the class 'classes not members of themselves' is not a member of itself, but we cannot say that it is a member of itself; therefore, both legs of the double paradox are blocked. In the first leg, we have blocked the inference from not-being 'a member of itself' to being one; in the second leg, we have interdicted the supposition of being 'a member of itself' even though inference from it of not-being one would be harmless. Accordingly, the answer to the question posed by Russell – viz. "Is the class of all classes which are not members of themselves a member of itself or not?" – is that this class is not a member of itself, and that this class not-being a member of itself does not, contrary to appearances, make it a member of itself, because no class is a member of itself anyway.

Thus, to be sure, though it is true that the class 'classes' is a class, it does not follow that it is a member of itself; though it is true that the class 'classes of classes' is a class of classes, it does not follow that it is a member of itself; though it is true that the class 'positive classes' is a positive class, it does not follow that it is a member of itself; though it is true that the class 'open-ended classes' is an open-ended class, it does not follow that it is a member of itself; though it is true that the class 'classes that are not members of themselves' is a class that is not a member of

itself, it does not follow that it is a member of itself. As for the class 'classes members of themselves', it has no members at all. It should be emphasized that the restriction on classification that we have here introduced is of very limited scope; it hardly affects class logic at all, concerning as it does a few very borderline cases.

The above is, I believe, the *correct and definitive resolution* of the Russell paradox. We acknowledged the existence of a problem, the Russell paradox. We diagnosed the cause of the problem, the assumption that *self-membership* is possible. We showed that self-membership is unconscionable, since it implies that a container can contain itself; this was not arbitrary tinkering, note well, but appealed to reason. We proposed a solution to the problem, one that precisely targets it and surgically removes it. Our remedy consisted in *uncoupling* predication from classification in all cases where self-membership is assumed, and only in such cases. This solution of the problem is plain common sense and not a flight of speculation; it is simple and elegant; it is convincing and uncomplicated; it does not essentially modify the concept of class membership, but only limits its application a little; it introduces a restriction, but one that is clearly circumscribed and quite small; it does not result in collateral damage on areas of class logic, or logic in general, that are not problematic, and therefore does not call for further adaptations of logic doctrine. Note moreover that our solution does not resort to any obscure 'system' of modern symbolic logic, but is entirely developed using ordinary language and widely known and accepted concepts and processes.

A bit of the history. Let us now look briefly at some of the history of the Russell paradox, and see how he and some other modern symbolic logicians dealt with it[146].

Georg Cantor had already in 1895 found an antinomy in his own theory of sets. In 1902, when Gottlob Frege (Germany, 1848-1925) was about to publish the second volume of his *Grundgesetze*, he was advised by Russell of the said paradox. Frege was totally taken in and could not see how to get out of the self-contradictions inherent in "the class of classes that do not belong to themselves." He perceived this as very serious, saying: "What is in question is … whether arithmetic can possibly be given a logical foundation at all." Frege first tried to fix things by suggesting that there might be "concepts with no corresponding classes," or alternatively by adjusting one of his "axioms" in such a way that:

> "Two concepts should be said to have the same extension if, and only if, every object which fell under the first *but was not itself the extension of the first* fell likewise under the second and vice versa"[147].

Clearly, Frege's initial suggestion that there might be "concepts with no corresponding classes" can be viewed as an anticipation of my uncoupling of predication and classification in specific cases. However, Frege did not identify precisely in what cases such uncoupling has to occur. This is evident in his next suggestion, which, though it points tantalizingly to the difficulty in the notion of self-membership, does not reject this notion outright but instead attempts to mitigate it. He speaks of two concepts instead of one, and tries to conventionally exclude the extension as a whole of each from the other, while of course continuing to include the objects falling under the extension; this shows he has not realized that self-inclusion by an extension is not even thinkable.

It should be stressed that Russell's paradox pertains to a certain class (viz. that of all classes not members of themselves) being or not-being a member of *itself* – not of *some other* class. Frege tries to resolve this paradox with reference, not to a single class, but to a pair of equal classes, even though (to my knowledge) he has not demonstrated that co-extensive classes result in a paradox comparable to the Russell paradox. It follows that his attempted solution to the problem is not germane to it. Moreover, Frege seems to have thought that if all items that fall under one class (say, 'Y') fall under another class (say, 'X'), then *the class* 'Y' may be assumed to fall under the class 'X'; and vice versa in the event of co-extension. This is suggested by his attempt to prevent such assumption, so as to avoid (in his estimate) the resulting Russell paradox. But in truth, it does not follow from the given that all Y are X that the class 'Y' is a member of the class 'X' – it only follows that the class 'Y' is a subclass of the class 'X', or an equal class if the relation is reversible. Thus, it appears that Frege confused the relations of class-membership and hierarchization of classes, using a vague term like 'falling' to characterize them both.

We may well ask the question whether an equal class, or a subclass, or even an overclass, might be a member of its hierarchically related class. Offhand, it would seem to be possible. For example, all positive classes are classes and therefore members of the class 'classes', and the class 'positive classes' is a subclass of the class 'classes'; however, although not all classes are positive classes (some are negative classes), nevertheless the class 'classes' is a positive class (being defined by a positive statement), and so is a member of the class 'positive classes'. But although this example suggests that an overclass might be a member of its subclass (and therefore, all the more, an equal class or a subclass might be a member of its hierarchical relative), we might still express a doubt by means of *analogy*, as Frege perhaps intended to do. We could argue inductively, by generalization, that if a class cannot be a member of itself,

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[146]   I am here referring principally to the account by William and Martha Kneale in *The Development of Logic* (Oxford, London: Clarendon, 1962), ch. XI.1-2.
[147]   Kneale and Kneale, p. 654. Italics theirs.

then maybe it cannot be a member of any coextensive class (as Frege suggests), and perhaps even of a subclass or an overclass. For the issue here is whether the instances referred to by the first class can be thought to occur twice in the second class (as members of it in their own right, and as constituents of a member). So, Frege may have raised a valid issue, which could lead to further restrictions in class logic. However, this need not concern us further in the present context, since (as already explained) it is not directly relevant to resolution of the Russell paradox.

Russell described his paradox in his book *Principles of Mathematics*, published soon after. Although at first inclined to Frege's second approach, he later preferred Frege's first one, proposing that there might be "some propositional functions which did not determine genuine classes." Note here again the failure to pinpoint the precise source and remedy of the problem. Subsequently, Russell thought that "the problem could never be solved completely until all classes were eliminated from logical theory." This, in my view, would be throwing out the baby with the bath water – an overreaction. But then he found out (or rather, he thought he did, or he convinced himself that he did) that the same paradox could be generated without "talk of classes," i.e. with reference to mere predicates – that is, in terms of predicate logic instead of in terms of class logic. As Kneale and Kneale put it (p. 655):

> "Instead of the class which is supposed to contain all classes that are not members of themselves let us consider the property of being a property which does not exemplify itself. If this property exemplifies itself, then it cannot exemplify itself; and if it does not exemplify itself, then it must exemplify itself. Clearly, the nature of the trouble is the same here as in the original paradox, and yet there is no talk of classes."

But even if classes are not explicitly mentioned here, it is clear that they are tacitly intended. How would a property 'exemplify' itself? Presumably, property X would be 'a property which exemplifies itself' if property X happens to be one of the things that have property X. That is to say, X exemplifies X if X is a member of the class of things that are X. We cannot talk about properties without resorting to predication; and once we predicate we can (given the initial definition of classification) surely classify. So, this attempt is just verbal chicanery; the same thought is intended, but it is dressed up in other words. It is dishonest. Moreover, the way the paradox is allegedly evoked here does not in fact result in paradox.

We cannot say, even hypothetically, "if this property [i.e. the property of being a property which does not exemplify itself] exemplifies itself" for that is already self-contradictory. To reconstruct a Russell paradox in 'property' terms, we would have to speak of '*the property of all properties* which do not exemplify themselves'; for then we would have a new term to chew on, as we did in class logic. But clearly, this new term is quite contrived and meaningless. Here again, we must mean '*the class of all properties* which do not exemplify themselves' – and in that event, we are back in class logic. Thus, note well, while Russell was right in looking to see whether his paradox was a problem specific to class logic, or one also occurring in predicate logic, and he claimed to have established that it occurred in both fields, in truth (as we have just demonstrated) he did not succeed in doing that. In truth, the paradox *was* specific to class logic; and he would have been better off admitting the fact than trying to ignore it.

In response to certain criticisms by his peers, Russell eventually "agreed that the paradoxes were all due to vicious circles, and laid it down as a principle for the avoidance of such circles that 'whatever involves all of a collection must not be one of the collection'." Thus, Russell may be said to have conceded the principle I have also used, namely that a collection cannot include itself as one of the items collected, although in truth the way he put it suggests he conceived it as a convention designed to block incomprehensible vicious circles rather than a logical absolute (notice that he says 'must not' rather than 'cannot'). He viewed the paradoxes of set theory as "essentially of the same kind as the old paradox of Epimenides (or the Liar)." This suggests that, at this stage, he saw his own paradox as due to self-reference, somehow. It does look at first sight as if there is some sort of self-reference in the proposition 'the class of all classes that are not members of themselves is (or is not) a member of itself', because the clause 'member of itself' is repeated (positively or negatively, in the singular or plural) in subject and predicate[148]. But it cannot be said that self-reference is exactly the problem.

A few years later, in a paper published in 1908, Russell came up with a more elaborate explanation of the Russell paradox based on his 'theory of types'. Russell now argued that "no function can have among its values anything which presupposes the function, for if it had, we could not regard the objects ambiguously denoted by the function as definite until the function was definite, while conversely … the function cannot be definite until its values are definite"[149]. In other words, the question "the class of all classes that are not members of themselves, is it or is it not a member of itself" is inherently flawed, because the subject remains forever out of reach. We cannot take hold of it till we resolve whether or not it is a member of itself, and we cannot do the latter till we do the former; so, the conundrum is unresolvable, i.e. the question is unanswerable. Effectively, the subject is a term cognitively impossible to formulate, due to the double bind the issue of its definition involves for any thinker.

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[148]   Note that if self-reference were the crux of the problem, then the proposition 'the class of all classes that are members of themselves is (or is not) a member of itself' would be equally problematic, even though it apparently does not result in a similar paradox.

[149]   Quoted by the Kneales, p. 658.

Here, we should note that the purpose of Russell's said explanation was effectively to invalidate the negative class 'classes not members of themselves', since this is the class giving rise to the double paradox he was trying to cure. The positive class 'classes members of themselves' clearly does not result in a double paradox: if we suppose it is *not* a member of itself, self-contradiction does ensue, but we can still say without self-contradiction that it *is* a member of itself. In fact, if Russell's explanation were correct, the positive class ought to be as illicit as the negative one. For if we claim the impossibility of a class referring to something that is *not yet settled*, as Russell did with reference to the negative class, then we must admit this characteristic is also found in the positive class, and we must reject it too. Russell does not seem to have realized that, i.e. that his remedy did not technically differentiate the two classes and so could be applied to both. For this reason, his attempt to solve the Russell paradox with reference to circularity or infinity must be judged as a failure. In my own theory, on the other hand, it is the positive class (that of self-membership) which is invalid (and empty), since it is geometrically unthinkable, while the negative class (that of non-self-membership) remains quite legitimate (and instantiated), as indeed we would expect on the principle that all claims (including that of self-membership) ought to be deniable.

Anyway, Russell concluded, briefly put, that a function could not be a value of itself; and proposed that function and value be differentiated as two 'types' that could not be mixed together indiscriminately. But this theory is, I would say, too general, and it complicates matters considerably. As we have seen, we cannot refuse to admit that, for instance, 'classes' *is* a class; the most we can do is to deny that this implies that 'classes' *is a member of* itself. This is a denial of self-membership, not of self-predication or of self-reference. As regards the notion of 'types' and later that of 'orders within types', these should not be confused with the more traditional ideas of hierarchies and orders of classes, which we laid out earlier in the present essay. In truth, the resemblance between Russell's concepts and the latter concepts gives Russell's theory a semblance of credibility; but this appearance is quite illusory – these are very different sets of concepts. Russell's notion of 'types' is highly speculative and far from commonsense; while it might appear to solve the Russell paradox, it has ramifications that range far beyond it and incidentally invalidate traditional ideas that do not seem problematic[150]. In short, it is a rough-and-ready, makeshift measure, and not a very convincing one.

Every paradox we come across is, of course, a signal to us that we are going astray somehow. Accordingly, the Russell paradox may be said to have been a signal to Frege, Russell, and other modern logicians, that something was wrong in their outlook. They struggled hard to find the source of the problem, but apparently could not exactly pinpoint its location. All the intricacy and complexity of their symbolic and axiomatic approach to logic could not help them, but rather obscured the solution of the problem for them. This shows that before any attempt at symbolization and axiomatization it is essential for logicians to fully understand the subject at hand in ordinary language terms and by means of commonsense. To my knowledge, the solution of the problem proposed in the present essay is original, i.e. not to be found elsewhere. If that is true, then the theory of class logic developed by modern symbolic logicians, which is still the core of what is being taught in universities today, needs to be thoroughly reviewed and revised.

A bit of self-criticism. As regards the resolution of the Russell paradox that I proposed over two decades ago in my *Future Logic*, the following needs to be said here. While I stand, in the main, by my theory of the logic of classes there (in chapters 43-44), I must now distance myself somewhat from my attempted resolution of the Russell paradox there (in chapter 45).

I did, to my credit, in that past work express great skepticism with regard to the notion of self-membership; but I did not manage to totally rule it out. I did declare: "Intuitively, to me at least, the suggestion that something can be both container and contained is hard to swallow," and I even postulated, in the way of a generalization from a number of cases examined, that "no class of anything, or class of classes of anything, is ever a member of itself," with the possible exception of "things" or "things-classes" (although it might be said of these classes that they are not members of any classes, let alone themselves[151]); but still, I did not reject self-membership on principle, and use that rejection to explain and resolve the Russell paradox, as I do in the present essay.

This is evident, for instance, in my accepting the idea that "'self-member classes' is a member of itself." The reason I did so was the thought that "whether self-membership is possible or not, is not the issue." Superficially, this is of course true – the Russell paradox concerns the 'class of all classes that are not members of themselves', and not 'the class of all classes that are members of themselves'. But in fact, as I have shown today, this is not true; acceptance of self-membership is the true cause of the Russell paradox, and non-self-membership is not in itself problematic.

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[150]   See for a start the Kneales' critique of the 'theory of types' in ch. XI.2.
[151]   Note that in this context I come up with the idea that the definition of membership might occasionally fail. But I did not at the time pursue that idea further, because I did not then analyze what such failure would formally imply.

Anyway, not having duly ruled out self-membership, I resorted to the only solution of the problem that looked promising to me at the time – namely, rejection of 'permutation' from "is (or is not) a member of itself" to "is (or is not) {a member of itself}" (notice the addition of curly brackets). That is to say, I proposed the logical interdiction of changing the *relation* of self-membership or non-self-membership into a predicable *term*. Now I see that this was wrong – it was an action taken *too late* in the process of thought leading up to the Russell paradox. It was a superficial attempt, treating a symptom instead of the disease. I did that, of course, because I thought this was "of all the processes used in developing these arguments, [the] only one of uncertain (unestablished) validity." But in truth, it was not the only possible cause of the effect – there was a process *before* that, one of deeper significance, namely the transition from 'is' to 'is a member of'. I did not at the time notice this earlier process, let alone realize its vulnerability; and for that reason, I did not attack it.

Clearly, I was on the right track, in that I sought for a place along the thought process at which to block development of the Russell paradox. But my error was to pick a place too late along that process. In fact, the right place is earlier on, as advocated in the present essay. The Russell paradox does not arise due to an illicit permutation, but due to the illicit transformation of a predicate into a class in cases where a claim of self-membership would ensue. And while the remedy proposed is even now in a sense 'conventional', the flaw it is designed to fix is quite real – it is that self-membership is in fact impossible and therefore can never be assumed true. My previous proposed solution to the problem only prevented the Russell paradox; it did not prevent self-membership, which is the real cause of the paradox. Thus, the solution I propose in the present essay is more profound and more accurate.

Main References

The following are the main references made in the present work. There are, in fact, many more references scattered throughout the present volume, in footnotes. The selection of some under the heading of 'main references' is somewhat subjective. References made through Internet links, even if important are usually excluded here – as are references to works quoted rather incidentally. Note that this listing is not intended as a bibliography, at least not as an exhaustive one.

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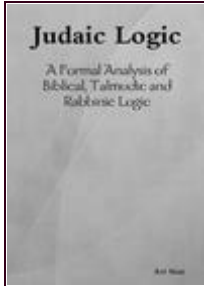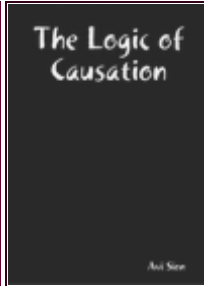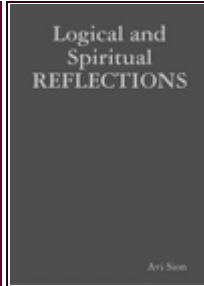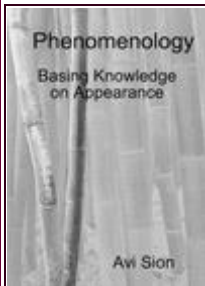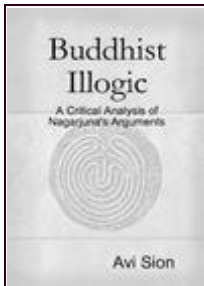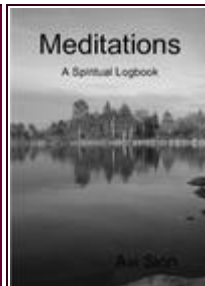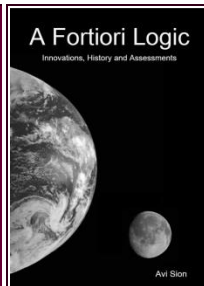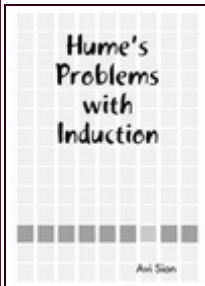
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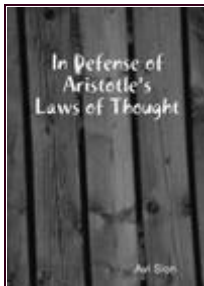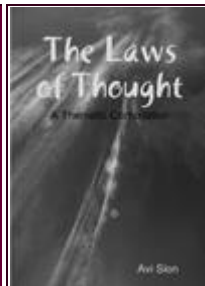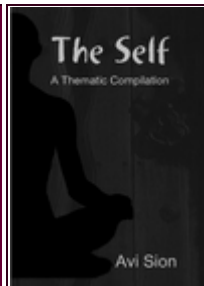
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