FUTURE LOGIC:

Categorical and Conditional Deduction and Induction of the Natural, Temporal, Extensional, and Logical Modalities.

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Abstract

Future Logic is an original, and wide-ranging treatise of formal logic. It deals with deduction and induction, of categorical and conditional propositions, involving the natural, temporal, extensional, and logical modalities.

(Simply put, deduction and induction are inferences of more or less certainty; propositions refer to relations between things; modalities are attributes of relations like necessity, actuality or possibility.)

Traditional and Modern logic have covered in detail only formal deduction from actual categoricals, or from logical conditionals (conjunctives, hypotheticals, and disjunctives). Deduction from modal categoricals has also been considered, though very vaguely and roughly; whereas deduction from natural, temporal and extensional forms of conditioning has been all but totally ignored. As for induction, apart from the elucidation of adductive processes (the scientific method), almost no formal work has been done.

This is *the first work ever* **to strictly formalize the inductive processes of generalization and particularization**, through the novel methods of factorial analysis, factor selection and formula revision.

This is *the first work ever* **to develop a formal logic of the natural, temporal and extensional types of conditioning** (as distinct from logical conditioning), including their production from modal categorical premises.

Future Logic contains a great many other new discoveries, organized into a unified, consistent and empirical system, with precise definitions of the various categories and types of modality (including logical modality), and full awareness of the epistemological and ontological issues involved. Though strictly formal, it uses ordinary language, wherever symbols can be avoided.

Among its other contributions: a full list of **the valid modal syllogisms** (which is more restrictive than previous lists); the main formalities of **the logic of change** (which introduces a dynamic instead of merely static approach to classification); the first formal definitions of **the modal types of causality**; a new **theory of class logic**, free of the Russell Paradox; as well as a critical review of modern metalogic.

But it is impossible to list briefly all the innovations in logical science -- and therefore, epistemology and ontology -- this book presents; it has to be read for its scope to be appreciated.

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1. INTRODUCTION.

1. What is Logic?

a. Definition.

Logic is first of all an instinctive *art*. We all, from an early age, try to 'sort out' our experiences and 'make sense' of the world around us — and this thought process is to varying degrees 'logical'. It is logical to the extent that we try to consider the evidence, avoid contradictions, and try to understand. We call this using 'common sense'.

On a higher level, logic is a *science*, which developed out of the self-awareness of thinkers. They began to wonder why some thoughts were more credible, forceful, and informative than others, and gradually discerned the patterns of logical intelligence, the apparatus of reasoning. A logic theorist is called a logician. Note that we also call 'a logic', any specific field of or approach to logical science.

Logic as a field of inquiry has two goals, then. On a *practical* level, we want it to provide us with a guide book and exercise manual, which tells us how to think straight and trains us to do so efficiently. On a *theoretical* level, we seek the assurance that human knowledge does, or can be made to, conform to reality. How these methodological and philosophical tasks are fulfilled, will become apparent as we proceed.

Logic is of value to all individuals, bettering their daily reasoning processes, and thus their efficacy in dealing with their lives, and their work. It teaches you organization, enabling you to arrive at the solution of problems more efficiently. It helps you to formulate more pondered opinions and values.

Be you an artist, a parent, a university professor, a doctor, a psychologist, a civil engineer, an auto-mechanic, a bank manager, an office worker, an investor, a planner, an organizer, a negotiator, a lawmaker, judge or lawyer, a politician or journalist, a systems analyst, a statistician, a computer or robot programmer, whatever your profession or walk of life — you are sure to find the study of logic useful.

It is of value to scientists of all disciplines, helping them to clarify issues and formulate solutions to problems. There is no area of human interest or endeavor where logic does not have a say, and where the study of logic would not be effective in improving our situation.

Logic is worth studying also, for the sheer esthetic joy of it. There is no describing the mind's response to this beautiful, colorful achievement of the human spirit. I hope the reader will have as much fun reading this book, as I had writing it. It can be hard work, but it is rewarding. My own favorite topic is *de-re* modality; I find it closer to earth than logical modality.

b. Method.

Logic teaches us to pursue and verify knowledge. It is based on an acknowledgment of the possibility of human error, but also implies our ability to correct errors. Where veracity or falsity is hard to establish, it tells us at least how 'reasonable' or 'forced' our judgments are.

It is essentially a holistic science, teaching us to take everything into consideration when forming judgments. Truth is not to be found in a limited viewpoint, but through a global perspective, an awareness of all aspects of an issue, all proposed answers to a question.

Logical science shows us what to look for in the course of knowledge acquisition, by listing and clarifying the main forms of relation among things and ideas (whence the name 'formal logic'). It is the 'systems analysis' of human thought.

Logic is concerned with the formalities of reasoning, without so much regard to its subject-matter. It allows for objective assessments of inferential processes, precisely because its principles make minimal references to specific contents of thought. It is emotionally detached, it has no double standards, it is open-minded and fair.

Logic is a tool of interpretation, understanding, and prediction. It is a method for drawing the maximum amount of useful information from new experiences, or enveloped in previous knowledge, so as to fully exploit the lessons of the world of matter and mind, appearing all around us all the time.

What logic does is to help us to take all impressions and intuitions in stride, and resolve any disagreements which may arise. What is sure, is that, in reality, things themselves can never be in contradiction. It is ideas which conflict with each other or with primary experiences. Sometimes it is the idea that there is a conflict which turns out to be wrong.

The job of logicians is, not to reword what is already known, but to uncover and enhance the logical capabilities of everyday language. This is achieved by first singling out any concept which seems to infiltrate all fields of human interest. Often, the colloquial expression relating to it has many meanings; in such case, we make an agreement to use those words in only the selected sense, which is usually their most common connotation. Once all risk of ambiguity or equivocation is set aside, we can develop a clear and rigorous understanding of the logical properties of the concept under consideration.

The so-called logical order of development is satisfying to trained logicians (from the general to the particular, as it were), and has also some didactic value. But it is often the opposite of the way an individual or a researcher normally arrives at knowledge (building up from specific discoveries, then formulating a comprehensive theory); sometimes, replicating the natural order is a more effective teaching method.

Sometimes these two kinds of orders coincide. In the last analysis, they are always to some extent both involved, working in tandem; logical practice is an integral part of logical theorizing.

As for the historical order, it follows the natural order pretty closely, though with some redundancies. Some other consciousness must precede self-consciousness. Logic has developed on both the deductive and inductive sides alternately, and not in a systematic fashion.

c. Goals.

The goal of logic is to make the facts and their relations *transparent*; it teaches us to focus the object until its most firm manifestation is captured. Logic cannot immediately solve all problems, but it always brings us closer to the solutions.

For the individual, this self-discipline is the source of realism and understanding. ‘Think for yourself’, do your own thinking, ‘use your head’, be creative, think things through. The goal is not a mind a-buzz with words, a slave to words; but the inner peace and self-respect of efficacy.

In communication with others, transparency means expressing one’s thoughts clearly, so that, as far as possible at the time, there is no doubt or ambiguity as to just what one is trying to say, and on the basis of what processes. ‘Say what you mean, and mean what you say’. Information is freely and helpfully shared; points or areas of ignorance or error are easily admitted.

This is the idea of ‘*glasnost*’, transparency, a mutual respect and openness policy, a cooperative attitude, without unnecessary frictions. Too often, politicians, media, and others, use words to hide or distort, and do not in turn pay attention to input. You may prove something to them incontrovertibly; they remain unfazed, *comme si de rien n’était*.

Clarity of expression, accuracy of observation and thought, passing knowledge on honestly, reasonableness on all sides, are essential to vibrant democracy and social peace. Logic is a civilized way to resolve disputes.

This means self-criticism, the ability to review one’s own proposals, and anticipate possible objections, and try to deal with them as well as one can. We often gloss over possible problems in our own ideas, hoping no one will spot them; but this wastes one’s time, and everybody else’s. Logic is taking the time to double check one’s projects, shifting them this way and that way, to see how well focused they are in the largest context.

On the other hand, when receiving ideas, one’s should not look at them with an overly-critical eye, at least until one has properly understood them. Like rigid bone, hasty and excessive skepticism can inhibit the growth of knowledge. 'Stop, look, listen', hear, consider, make the effort to assimilate it. Learn before you try to teach.

While I am not of the opinion that logic is relative and arbitrary, there is more often than not at least some helpful truth to be found in other people's concerns. One should not reject offhand, though still reserve one's judgement. One should neither fool nor be fooled. Be humble, but keep your standards high.

2. What Logic is Not.

I get some very funny reactions from people at the mention of the word 'logic'. One should not reject logic offhand, because of a mistaken notion of what it is about.

Logic is not a method of inferring all knowledge from a limited number of abstract premises; it is not a magical tool of omniscience. It depends for its action on moment by moment impressions or intuitions, which in some cases turn out to be unfounded. Nor is logic merely a mechanized manner of pursuing solutions to specific problems.

People often wrongly regard and use logic as a square-headed, narrow-minded activity. But in my opinion, logic is, straight and tough on a level of details, but overall very broad and open minded. Obstinacy and prejudice, are rather attributes of people unwilling to listen to reason, not even to at least consider alternative viewpoints. This is the very antithesis of a logician's attitude.

People often oppose 'logic' to feeling; they believe it discards the emotional side of life. But logic does not mean ignoring feelings, but rather recommends taking the feelings — including their inner meaning, their intuited significance — as one set of data among others in the total picture; rationalistic data must also, however, be given their due weight.

Some people complain that 'logic' sometimes leads to evil conclusions. But value-judgments involve inferences from standards. So either the norms are unsound, or they have not been given their due weights in comparison to other norms, or the proposed means are not the exclusive ways to achieve the norms. Thus, the failure involved may precisely be a weakness in logical abilities, rather than any inherent coldness of logic.

Logic is only a tool — it cannot be blamed for errors made in its name, nor can it control the moral choices of individuals who utilize it. Its only possible danger is that the efficacy it endows on thought and action may be used for nefarious ends. But even then, a person who sees things truly clearly, with the broad conception logic gives, is less likely to have twisted values.

Logic is an important component of both mental health and moral responsibility. It requests that we face facts and listen to the voice of reason: this does not exclude having a heart or paying attention to one's intuition. A person who does not keep in close touch with reality, can easily develop unhealthy emotions and make counter-productive choices. Rationality is a sign of maturity.

Another wrong impression people have of logic is that it is a meaningless manipulation of symbols, or at best a branch of mathematics. One man recently told me the following sad story. He thought of himself as a 'logical person', and being inclined to constantly improve his education, he enrolled for a University course on the subject in San Francisco. He was so put off by the lessons he attended, that he now hesitates to call himself 'logical'!

3. Modus Operandi.

a. Title.

This is a book on logic, a formal and detailed study.

I called it 'Future Logic' to dedicate it to the future, to suggest its potential for improvement of human thinking and doing. It is also a logic about the future, aimed at knowledge of the possible and necessary. Lastly, it is futuristic, in that it is new, not of the past, unbound by previous limits. Hence, it is a young and optimistic logic, for and of the future, full of strength and energy.

I also called it 'Future Logic', because writing it has seemed an endless process. And it is really without end; I have left many things unsaid, only hinting at directions future logicians may take.

I would subtitle the book 'modal logic', to stress that all logic is 'modal', but not to imply that it concerns a specific sector of logic. The book ranges over virtually the whole of logic, constructing a well-integrated and fruitful *system* of logic, by means of an investigation of modality. A 'system' in the grand, traditional sense, not in the narrow sense used by modern logicians with reference to certain manipulations of limited scope.

'Modality', simply put, refers to concepts like possibility and necessity, which pervade knowledge in many different senses. Thought without modality is very limited in scope; much of our thinking depends on conceiving what the alternative possibilities are.

Modality is an incredibly creative force, which, like a crystal instantly solidifying a liquid, rushes through every topic and restructures it in new and interesting ways. I want to show how logic is forcefully pushed in a multitude of directions, as soon as modality and its ramifications are taken into account.

b. Targets.

In writing this book my ambition was to invigorate logic — to contribute to the science, and to revive interest in it by all segments of society.

Thus, it is intended equally for laypersons and scientists, for students and educators, and for professional logicians. It is equally a popularizing book, a text-book, and a research report.

My goal is not only to explore new avenues for the science of logic, but especially to make its teachings accessible to a wide public. For this reason, even while attempting to write a scholarly treatise, I do my best to keep it readable by anyone.

The book is full of ground-breaking discoveries, which should impress any logic theorist, and perhaps put him or her back to work. I mean, not just a peppering of incidental insights, but entirely original areas of concern, directions, and techniques, as will be seen. Though well-nigh encyclopedic in scope, it is not a compilation, but presents a unified system.

Although addressed to a wide audience, this is not an elementary work; it is an attempt to transmit advanced logic to everyone. My faith is that we have all reached a level of education high enough to absorb it and use it.

The book moves from the more obvious to the less so, from the simple to the complex, and from the old to the new, so that a layperson or student lacking any previous acquaintance with the subject-matter can grasp it all, granting a little effort. The order of development is thus natural and didactic, rather than strictly 'logical' in the sense of geometry. It is easy, at the end, when we know what we are talking about, to review the whole, and suggest a 'logical' ordering which consolidates it.

c. Strategies.

My approach is strongly influenced by Aristotle; all I do is push his methods into a much broader field. The primary purpose of logic should be to teach people to think clearly. For this reason, I try to develop the subject in ordinary language, and avoid any excessive symbolization.

Modern logicians have managed to overturn the very spirit of the discipline of logic, and made it cryptic, obscure, and esoteric. This was a disservice to the public, depriving it of an important tool for living, since most people lack the patience to decipher symbols.

Logical science as such has also suffered from this development. Logic has no intrinsic need of symbols other than those provided by ordinary language. An artificial language in principle adds nothing to knowledge, just as renaming things never does. Symbolization as such is just a quaint footnote to logic, not a real advance.

Symbols are to some extent valuable, *to summarize information in a minimum of space, or to discover and highlight patterns in the data*. But taken to an extreme, symbolism can lock us into simplistic mind-sets, and arrest further insight, limiting us to making trivial embellishments.

Worse still, it can distance us from empirical inputs, turning logic into a game, a conventional, mechanical manipulation of arbitrary constructs, without referents, divorced from reality.

Also, I try as much as possible in this volume to avoid philosophical issues and metaphysical speculations, anything too controversial or digressive — and to concentrate on the matter at hand, which is formal logic. Some comments on such topics are inserted at the end, for the record.

A logician is of course bound to get involved in some wider issues. Every logical analysis intimates something about 'thought processes' and something about 'external reality'. Logic somehow concerns the interface of these parallel dimensions of epistemology (the study of knowing) and ontology (the study of being), and it is hard to draw the lines.

d. Tactics.

I try to be brief. But I also try and touch on all relevant topics. Every issue is of course many-faceted, and capable of interminable treatment, with every layer uncovered seemingly more crucial than the previous. I still have great quantities of unused manuscript, and therefore know how much more remains to be said. But the reader may find that his questions at any stage, are readily answered in a later stage, in a wider context. One cannot do everything at once.

Often I am obliged to stop the further development of ideas. If I feel that an idea is already drawn clearly enough, and there would be boring repetitions of previously encountered patterns, I merely indicate the expected changes in pattern, and call on the reader to explore further on his or her own. This may be likened to the use of perspective and shading in artwork. Knowledge is infinite anyway, and as the saying goes 'there is no end to words'.

The informed reader may find that there is too much elementary logic — but I am forced to include some at first, to make the discussion comprehensible to all, and to show the more advanced developments in their proper context. In any case, even in a discussion of traditional logic, an expert may find novel details or viewpoints, as the various aspects of a topic are unraveled.

I apologize to the novice for my failure to give many examples, but this disadvantage seems to me outweighed by the advantage of brevity. I assume the reader capable of searching for appropriate examples, and it is a good exercise. The neophyte reader is warned to beware of our use of many words in selective, specialized senses, which may be based on common connotations or even be neologisms; the context hopefully always makes the intention clear.

I also keep historical notes to a minimum in the course of the text, more intent on being a logician than a historian of logic. However, an effort to attribute authorship of the main lines of thought, is made towards the end, when I seek to place my own contributions in their historical context. My critical evaluations of modern trends in logic are also included at that stage.

My style of writing is no doubt not uniformly good. Repeated editing is bound to sometimes result in obscure discontinuities in the text. Little errors may creep in. I hope the reader will nevertheless be tolerant, because the substance is well worth it.

4. Scope.

The book is divided into 7 parts, with a total of 68 chapters; each of the chapters is split into on average 4 sections.

Part I starts with the three 'laws of thought', then presents the logic of actual categoricals (propositions of the form 'X is Y'), including their features, their oppositions and immediate inferences, and syllogistic argument. Most of the credit for this seminal work can be attributed to Aristotle, although many later logicians were involved in the further development and systematization of his findings.

Part II defines the modalities called '*de re*', and develops the logic of modal categoricals, following the same pattern as was established in the previous part. Although Aristotle wrote a great deal about concepts like potentiality, and described some modal arguments, he did not investigate this area of logic with the same thoroughness as the previous; nor have logicians since done much more, in my opinion. I introduce some new techniques, and arrive at some original results.

This part also, for the sake of completeness, analyses other forms of categorical proposition (among which, those concerning change) and other logical processes (such as 'substitution'), some of which seem to have been previously ignored or underrated.

Part III defines logical modality, and analyses logical conditioning. This concerns 'if-then' (and 'either-or') propositions, which have been dealt with in great detail by modern logicians. While my own results concur with theirs on the whole, my approach differs in many respects; especially different are the definitions of logical modalities, but there are many significant technical innovations too (such as 'production').

Part IV introduces '*de re*' conditioning, whose properties are found to be very distinct from those of logical conditioning. This is (to my knowledge) an entirely new class of propositions for logical science to consider, although commonly used in our everyday thought. The research emerged from the insights into modality obtained in part II, and provides us with original and important formal tools for the study of causality (and, incidentally, a better understanding of subsumption).

Part V begins with a new logic of classes (including a definitive solution of the Russell Paradox). Then I present the now-traditional discussion of scientific method (confirmation and discrediting of hypotheses), but from the viewpoint of logical modality.

Part VI contains an altogether original theory of induction based on 'factorial analysis'. Consideration of modality in its various senses gives rise to a need for a completely new area of logic: how to induce modal propositions and how to resolve contradictions between them. The problems of generalization and particularization are solved systematically, using very formal techniques. Every aspect of this research — the tasks set, and the ways they are fulfilled — is a major breakthrough for logic.

The practical importance of factorial induction cannot be overstated. How far and in what direction can one generalize any finding? What happens when conflicting data is uncovered — how far and in what direction should one retreat from previous positions? What is the middle ground or compromise position or synthesis between competing views?

Part VII considers some of the ontological and epistemological implications of all my previous findings, with a sketch of my theory of cognition. The last few chapters provide a critical, historical and philosophical review of the whole field; this segment is more of interest to academic logicians than to the ordinary reader. Finally, the work is summarized, and I point out some of the opportunities for further research.

The reader is invited to peruse the table of contents for a more precise overview. I recommend that you return to it from time to time, so as to place the topic you are studying in its proper perspective.

2. FOUNDATIONS.

Logic is founded on certain 'laws of thought', which were first formulated by Aristotle, an ancient Greek philosopher. We shall describe them separately here, and later consider their collective significance.

1. The Law of Identity.

The Law of Identity is an imperative that we consider all evidence at its face value, to begin with. Aristotle expressed this first law of thought by saying 'A is A', meaning 'whatever is, is whatever it is'.

There are three ways we look upon phenomena, the things which appear before us, however they happen to do so: at their face value, and as real or illusory.

We can be sure of every appearance, that it is, and is what it is. (i) *Something* has presented itself to us, whether we thereafter judge it real or illusory, and (ii) this something displays *a certain configuration*, whether we thereafter describe and interpret it rightly or wrongly. The present is present, the absent is absent.

Every appearance as such is objectively given and has a certain content or specificity. We can and should and commonly do initially regard it with a simple attitude of receptiveness and attention to detail. Every appearance is in itself neutral; the qualification of an appearance (thus broadly defined) as a 'reality' or an 'illusion', is a subsequent issue.

That statement is only an admission that any phenomenon minimally exists and has given characteristics, *without making claims about the source and significance* of this existence or these characteristics. The moment we manage to *but think* of something, it is already at least 'apparent'. No assumption need be made at this stage about the nature of being and knowledge in general, nor any detailed categorizations, descriptions or explanations of them.

Regarded in this way, at their face value, *all* phenomena are evident data, to be at least taken into consideration. The world of appearances thus offers us *something to work with, some reliable data* with which we can build the edifice of knowledge, a starting point of sorts. We need make no distinctions such as those between the physical/material and the mental, or sense-data and hallucinations, or concrete percepts and abstract concepts; these are later developments.

The law of identity is thus merely an acknowledgement of the world of appearances, without prejudice as to its ultimate value. It defines 'the world' *so broadly*, that there is no way to counter it with any other 'world'. When we lay claim to another 'world', we *merely expand* this one. All we can ever do is subdivide the world of appearances into two domains, one of 'reality' and one of 'illusion'; but these domains *can never abolish* each other's existence and content.

What needs to be grasped here is that every judgment implies the acceptance, at some stage, of some sort of appearance as real. There is no escape from that; to claim that nothing is real, is to claim that the appearance that 'everything is illusory' is real. We are first of all observers, and only thereafter can we be judges.

Reality and illusion are simply terms more loaded with meaning than appearance or phenomenon — they imply an evaluation to have taken place. This value-judgement is a final characterization of the object, requiring a more complex process, a reflection. It implies we went beyond the immediately apparent. It implies a broader perspective, more empirical research, more rigorous reasoning. But what we finally have is still 'appearance', though in a less pejorative sense than initially.

Thus, 'real' or 'illusory' are themselves always, ultimately, just appearances. They are themselves, like the objects of consciousness which they evaluate, distinct objects of consciousness. We could say that there is a bit of the real in the illusory and a bit of the illusory in the real; what they have in common is appearance. However, these terms lose their meaning if we try to equate them too seriously.

On what basis an appearance may or should be classified as real or illusory is of course a big issue, which needs to be addressed. That is the overall task of logic, to set precise guidelines for such classification. But the first step is to admit the available evidence, the phenomenal world as such: this gives us a data-base.

2. The Law of Contradiction.

The Law of Contradiction is an imperative to reject as illusory and not real, any apparent presence together of contradictories. This second law of thought could be stated as 'Nothing is both A and not-A', or 'whatever is, is not whatever it is not'.

We cannot say of anything that it is *both* present and absent at once: *what is present, is not absent*. If the world of appearance displays some content with an identity, then it has effectively failed to display nothing. Contradictory appearances cannot coexist, concur, overlap: they are 'incompatible extremes'.

We can say of something that it 'is' something else, in the sense of having a certain relation to something distinct from itself, but we cannot say of it that it both has and lacks that relation, in one and the same respect, at one and the same place and time.

It is evident, and therefore incontrovertible (by the previous law), that appearances are variegated, changing, and diverse. Phenomena have a variety of aspects and are usually composed of different elements, they often change, and differ from each other in many ways. However, for any respect, place and time, we pinpoint, the appearance as such is, and is whatever it is — *and not at once otherwise.*

The law of contradiction is not a mere rephrasing of the law of identity, note well, but goes one step further: it sets a standard for relegating some appearances to the status of illusions; in a sense, it begins to define what we mean by 'illusion'. It does not, however, thereby claim that all what is leftover in the field of appearance is real with finality; nor does it deny that some of the leftovers are real (as is assured us by the law of identity).

By the law of identity, whatever appears is *given some credence*: therefore, one might suggest, the coexistence of opposites has some credence. The law of contradiction interposes itself at this point, and says: no, such events *carry no conviction* for us, once clearly discerned. The first law continues to function as a recognition that there is an apparent contradiction; but the second law imposes on us the need to resolve that contradiction somehow.

The law of contradiction is itself, like anything else, an appearance among others, but it strikes us as an *especially credible* one, capable of overriding the initial credibility of all other considerations. It does not conflict with the message of the law of identity, since the latter is open to any event, including the event that some appearances be more forceful than others. The law of contradiction is precisely one such forceful appearance, an extremely forceful one.

Thus, though the world of appearances presents itself to us with some seeming contradictions, they appear *as* incredible puzzles — their unacceptability is inherent to them, obvious to us. We may verbally speculate about a world with real contradictions, and say that this position is consistent with itself even if inconsistent with itself. But the fact remains that whenever we are face to face with a specific contradiction (including that one) we are unavoidably skeptical — something seems 'wrong'.

The way we understand the apparent existence of contradictions is by viewing the world of appearances as *layered*, or stratified. Our first impressions of things are often superficial; as our experience grows, our consciousness penetrates *more deeply* into them. Thus, though each level is what it is (law of identity), parallel levels may be in contradiction; when a contradiction occurs, it is because we are superimposing different layers (law of contradiction). In this way, we resolve the 'general contradiction' of contradiction as such — we separate the conflicting elements from each other.

(Note in passing, as an alternative to the metaphor of 'depth', which likens consciousness to a beam of light, we also sometimes refer to 'height'. Here, the suggestion is that the essence of things is more elevated, and we have to raise ourselves up to make contact with it.)

That resolution of contradiction refers to the diversity and change in the world of appearance as due to the perspectives of consciousness. Thus, the appearance of the phenomena we classified as 'illusory' is due to the limitations of ordinary consciousness, its failure to know everything. This restriction in the power of consciousness may be viewed as a 'fault' of our minds, and in that sense 'illusion' is a 'product' of our minds. For that reason, we regard the illusory as in some sense 'imaginary' — this is our explanation of it.

On a more objective plane, we may of course accept diversity and change as real enough, and explain them with reference to the space and time dimensions, or to uniform and unchanging essences. In such cases, we are able to meet the demands of the law of contradiction without using the concept of 'illusion'; only when space, time, and respect, are clearly specified, does a contradiction signify illusion.

3. The Law of the Excluded Middle.

The Law of the Excluded Middle is an imperative to reject as illusory and not real, any apparent absence together of contradictories. This third law of thought could be stated as 'nothing is neither A nor not-A', or 'whatever is, either is some thing or is-not that thing'.

We cannot say of anything that it is at once neither present nor absent: *what is not present, is absent*. If the world of appearance fails to display some content with an identity, then it has effectively displayed nothing. There is no third alternative to these two events (whence the expression 'excluded middle'): they are exhaustive.

We may well say that some parts or aspects of the world are inaccessible to our limited faculties, but (as pointed out in the discussion of identity) we cannot claim a world beyond that of appearances: the moment we mention it, we include it.

It may be that we neither know that something is so and so, nor know that it is not so and so, but this concerns knowledge only, and in reality that thing either is or is-not so and so. Whatever we consider must either be there or not-there, in the specified respect, place and time, even if we cannot discern things enough to tell at this time or ever. There is an answer to every meaningful question; uncertainty is a 'state of mind', without 'objective' equivalent.

Moreover, *strictly speaking*, 'questions' are artificial attempts to anticipate undisplayed layers of appearance. As things appear now, if nothing is being displayed, *that* is the (current) 'answer' of the world of appearances; in the world of appearances there are no 'questions'. 'Questions' merely express our resolve to pursue the matter further, and try to uncover other layers of appearance; they are not statements about reality.

If we choose to, *loosely speaking*, regard doubts as kinds of assertions, the law of the excluded middle enjoins us to class them at the outset as illusory, and admit that in reality things are definite. Problematic statements like 'it might or might not be thus' are not intended to affirm that 'neither thus nor not-thus' *appeared*, but that what did appear (whether it was 'thus' or 'not-thus' — one of them did, for sure) was not sufficiently forceful to satisfy our curiosity.

Even if no phenomenon is encountered which confirms or discredits an idea, there must be a phenomenon capable of doing so, in the world somewhere, sometime. We have to focus on the evidence, and try and distinguish the appearance or nonappearance of that imagined phenomenon.

Thus, the law of the excluded middle serves to create a breach of sorts between the 'objective world' and the 'world of ideas', and establishes the pre-eminence of the former over the latter. The breach is not an unbridgeable gap, but allows us to expand our language, in such a way that we can discuss eventual layers of appearance besides those so far encountered, even while we admit of the evidence at hand.

Such an artifice is made possible by our general awareness from past experience that appearances do change in *some* cases, but should not be taken to mean that any given appearance *will* change. It is only the expression of a (commendable) 'open-mindedness' *in principle*, with no specific justification in any given case.

What we have done, effectively, is to expand what we mean by 'appearance', so as to include future appearance, in addition to appearances until now in evidence. Thus far, our implicit understanding was that appearance was *actual*, including present realities and present illusions. Now, we reflect further, and decide to embrace our anticipations of *'possible'* appearances as a kind of actuality, too.

Such hypothetical projections are also, in a sense, 'apparent'. But they are clearly imaginary, inventions of the mind. Their status as appearances is therefore immediately that of 'illusions'; that is their present status, whatever their future outcome. However, they are illusory with less finality than the phenomena so labeled by the law of contradiction; they retain some degree of credibility.

3. LOGICAL RELATIONS.

1. True or False.

Reality and illusion are attributes of phenomena. When we turn our attention to the implicit ‘consciousness’ of these phenomena, we correspondingly regard the consciousness as realistic or unrealistic. The consciousness, as a sort of peculiar relation between a Subject (us) and an Object (a phenomenon), is essentially the same; only, in one case the appearance falls in the reality class, in the other it falls in the illusion class.

Why some thoughts turn out to be illusory, when considered in a broader context, varies. For example, I may see a shape in the distance, and assume it that of a man, but as I approach it, it turns out to be a tree stump; this latter conclusion is preferred because the appearance withstands inspection, it is firmer, more often confirmed. A phenomenon always exists as such, but it may ‘exist’ in the realms of illusion, rather than in that of reality. The fact that I saw some shape is undeniable: the only question is whether the associations I made in relation to it are valid or not.

‘Propositions’ are statements depicting how things appear to us. Understood as mere *considerations* (or ‘hypothetically’), they contain no judgment as to the reality or illusion of the appearance. Understood as *assertions* (or ‘assertorically’), they contain a judgment of the appearance as real or illusory.

Assertoric propositions must either be ‘true’ or ‘false’. *If we affirm a proposition, we mean that it is true; if we deny a proposition, we mean that it is false.* Our definitions of truth and falsehood must be such that they are mutually exclusive and together exhaustive: what is true, is not false; what is false, is not true; what is not true, is false; what is not false, is true.

Strictly speaking, we call an assertion *true*, if it verbally depicts something which appears to us as real; and *false*, if it verbally depicts something which appears to us as illusory. In this ideal, absolute sense, true and false signify total or zero credibility, respectively, and allow of no degrees.

However, the expressions true and false are also used in *less stringent senses*, with reference to less than extreme degrees of credibility. Here, we call a proposition (relatively, practically) true if the appearance is more credible than any conflicting appearance; and (effectively) false, if the appearance is not the most credible of a set of conflicting appearances. Here, we can speak of more or less true or false.

The ultimate goal of logic is knowledge of reality, and avoidance of illusion. Logic is only incidentally interested in the less than extreme degrees of credibility. The reference to intermediate credibility merely allows us to gauge tendencies: how close we approach toward realism, or how far from it we stray. Note that the second versions of truth and falsehood are simply wider; they include the first versions as special, limiting cases.

Propositions which cannot be classed as true or false right now are said to be *‘problematic’*. Both sets of definitions of truth and falsehood leave us with gaps. The first system fails to address all propositions of intermediate credibility; the second system disregards situations where all the conflicting appearances are equally credible.

If we indeed cannot tip the scales one way or the other, we are in a quandary: if the alternatives are all labeled true, we violate the law of contradiction; if they are all labeled false, we violate the law of the excluded middle. Thus, we must remain with a suspended judgment, and though we have a proposition to consider, we lack an assertion.[1]

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1 I would like to mention here, in passing, the topic of the Logic of Questions, which some logicians have analyzed in considerable detail. Some of the features of interrogations are: they are signaled by a written question mark, or a certain intonation of speech. One question may conceal

2. Branches of Logic.

The concepts of truth and falsehood will be clarified more and more as we proceed. In a sense, the whole of the science of logic constitutes a definition of what we mean by them — what they are and how they are arrived at. We shall also learn how to treat problematic propositions, and gradually turn them into assertions.

The task of sorting out truth from falsehood, case by case, is precisely what logic is all about. What is sure, however, is that that is in principle feasible.

If thought was regarded as not intimately bound with the phenomena it is intended to refer to, it would be from the start disqualified. In that case, the skeptical statement in question itself would be meaningless and self-contradictory. The only way to resolve this conflict and paradox is to admit the opposite thesis, viz. that some thoughts are valid; that thesis, being the only internally consistent of the two, therefore stands as proven.

This is a very important first principle, supplied to us by logic, for all discussion of knowledge. *We cannot consistently deny the ultimate realism of (some) knowledge.* We cannot logically accept a theory of knowledge which in effect invalidates knowledge. *That* we know is unquestionable; *how* we know is another question.

Now, logical processes are called *deductive* (or analytic) to the extent that they yield indisputable results of zero or total credibility; and *inductive* (or synthetic) insofar as their results are more qualified, and of intermediate credibility. Deductive logic is conceived as concerned with truth and falsehood in their strict senses; inductive logic is content to deal with truth and falsehood in their not so strict senses.

This distinction is initially of some convenience, but it ultimately blurs. Logical theory begins by considering deductive processes, because they seem easier; but as it develops, its results are found extendible to lesser truths. Likewise, inductive logic begins with humble goals, but is eventually found to embrace deduction as a limiting case.

As we shall see, both these branches of logic require intuition of logical relations, and both presuppose some reliance on other phenomena. Both concern both concrete percepts and abstract concepts. Both involve the three faculties of experience, reason and imagination; only their emphasis differs somewhat. There is, at the end, no clear line of demarcation between them.

3. Tools of Logic.

The following are three logical relations which we will often refer to in this study: implication, incompatibility, and exhaustiveness. We symbolize propositions by letters like P or Q for the sake of brevity; their negations are referred to as notP (or nonP) and notQ, respectively.

a. ***Implication***. One proposition (P) is said to imply another (Q) if it cannot happen that the former is true and the latter false. Thus, if P is true, so must Q be; and if Q is false, so

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several subsidiary questions, whose answers together lead to the whole answer. Questions cannot as such be said to be true or false, though they often intend or logically imply some tacit assertion. ‘Every question delimits a range of possible answers’--a yes or no, a case in point or example, an instruction on how to do something (the New Encyclopaedia Britannica, 23:283). But some rhetorical questions are so constructed that only false answers to them are possible. A compound question which it is difficult to answer tersely correctly is a case in point (called the ‘fallacy of the many questions’). In such case, the question posed should of course be challenged. A teacher may well ask a leading question of a pupil, hinting at the true answer; but in some cases, this technique is abused, and we see for instance a journalist generating a false answer with propaganda value from an unaware respondent.

must P be — by definition. It does not follow that P is in turn implied by Q, nor is this possibility excluded. This relationship may be expressed as “If P, then Q”, or equally as “If nonQ, then nonP”. We can deny that Q is implicit in P by the formula “If P, not-then Q”, or “If nonQ, not-then nonP”.

When we use expressions like ‘it follows that’, ‘then’, ‘therefore’, ‘hence’, ‘thence’, ‘so that’, ‘consequently’, ‘it presupposes that’ — we are suggesting a relation of implication.

b. ***Incompatibility*** (or inconsistency or mutual exclusion). Two propositions (P, Q) are said to be incompatible if they cannot both be true. This relation is also called ‘exclusive disjunction’, and expressed by the formula ‘P or else Q’. Thus, if either is true, the other is false. The possibility that both be false is not excluded, nor is it affirmed. This relation can be formulated as “If P, then nonQ”, or equally as “If Q, then nonP”. The denial of such a relation would be stated as “If P, not-then nonQ”., or “If Q, not-then nonP”.

We can also say of more than two propositions that they are incompatible; meaning, if any one of them is true, all the others must be false (though they might well all be false).

c. ***Exhaustiveness***. Two propositions (P, Q) are said to be exhaustive if they cannot both be false. This relation is also called ‘inclusive disjunction’, and expressed by the formula ‘P and/or Q’. Thus, if either is false, the other is true. The possibility that both be true is not excluded, nor is it affirmed. This relation can be formulated as “If nonP, then Q”, or equally as “If nonQ, then P”. The denial of such a relation would be stated as “If nonP, not-then Q”., or “If nonQ, not-then P”.

We can also say of more than two propositions that they are exhaustive; meaning, if all but one of them is false, the remaining one must be true (though they might well be all true).

We note that whereas implication and its denial are directional relations, incompatibility and exhaustiveness and their denials are symmetrical relations.

Also, underlying them all is the concept of ‘conjunction’, whether or not one can say one thing with or without the other. Consequently, these expressions are interconnected; we could rephrase any one in terms of any other. For example, ‘P implies Q’ could be restated as ‘P is incompatible with notQ’ or as ‘notP and Q are exhaustive’.

The following table summarizes the above through analysis of the possibilities of combination of the affirmations and denials of two propositions, P and Q, which are given as having a certain logical relation, specified in the left column. ‘**No**’ indicates logically impossible combinations, ‘**yes**’ combinations specified as possible, and ‘**?**’ signifies that the status of the combination as it stands, without further specification, is undetermined by the logical relation concerned.

Table 3.1 Definitions of Logical Relations.

POSSIBILITY OF:	P+Q	P+nonQ	nonP+Q	nonP+nonQ
Implication	yes	no	?	yes
Incompatibility	no	yes	yes	?
Exhaustiveness	?	yes	yes	no
Unimplication	?	yes	?	?
Compatibility	yes	?	?	?
Inexhaustiveness	?	?	?	yes

We shall have occasion to review these relations in more detail later, and also define what we mean by logical possibility or impossibility. Their study is a big part of logic. For now, it is enough to just point them out, for practical purposes.

4. Axioms of Logic.

We can now re-state the laws of thought with regard to the truth or falsehood of (assertoric) propositions as follows. These principles (or the most primary among them) may be viewed as the axioms of logic, while however keeping in mind our later comments (ch. 20) on the issue of their development.

a. ***The law of identity***: Every assertion implies itself as 'true'. However, this self-implication is only a claim, and does not by itself prove the statement.

More broadly, whatever is implied by a true proposition is also true; and whatever implies a false proposition is also false. (However, a proposition may well be implied by a false one, and still be true; and a proposition may well imply a true one, and still be false.)

b. ***The law of contradiction***: If an affirmation is true, then its denial is false; if the denial is true, then the affirmation is false. They cannot be both true. (It follows that if two assertions are indeed both true, they are consistent.)

A special case is: any assertion which implies itself to be false, is false (this is called self-contradiction, and disproves the assertion; not all false assertions have this property, however).

More broadly, if two propositions are mutually exclusive, the truth of either implies the falsehood of the other, and furthermore implies that any proposition which implies that other is also false

c. ***The law of the excluded middle***: If an affirmation is false, then its denial is true; if the denial is false, then the affirmation is true. They cannot both be false. (It follows that if two assertions are indeed both false, they are not exhaustive).

A special case is: any assertion whose negation implies itself to be false, is true (this is called self-evidence, and proves the assertion; not all true assertions have this property, however).

More broadly, if two propositions are together exhaustive, the falsehood of either implies the truth of the other, and furthermore implies that any proposition which that other implies is also true (though propositions which imply that other may still be false).

Thus, in summary, every statement implies itself true and its negation false; it must be either true or false: it cannot be both and it cannot be neither. In special cases, as we shall see, a statement may additionally be self-contradictory or self-evident.

Some of these principles are obvious, others require more reflection and will be justified later. They are hopefully at least easy enough to understand; that suffices for our immediate needs.

Note in passing that each of the laws exemplifies one of the logical relations earlier introduced. Identity illustrates implication, contradiction illustrates incompatibility, excluded-middle illustrates exhaustiveness.

Although we introduced the logical relations before the laws of thought, here (for the sake of clarity and since we speak the same language), it should be obvious that, conceptually, the reverse order would be more accurate.

First, come the intuitions of identity, contradiction, and excluded-middle, with the underlying notions (visual images, with velleities of movement), of equality ('to go together'), conflict ('to keep apart'), and limitation ('to circumscribe'). Thereafter, with these given instances in mind, we construct the more definite ideas of implication, incompatibility, and exhaustion.

4. WORDS AND THINGS.

1. Verbalizing.

A major function of the discipline of logic is to teach us to express our thoughts explicitly, clearly and unambiguously. We consider thought as serial, because words are strung together; but the underlying perception or conceptual insight is often more global.

A perception or conceptual insight may be wordless, even a logical process of thought and inference may occur in inner silence. We often feel in ourselves or see in other people a facial or bodily reaction, like a smile of assent or sardonic grin of doubt, and know some thought has taken place on a subconscious or unconscious level, though we cannot say what or why.

We can be aware of a phenomenon without labeling it; but we often label things, to mentally process or socially communicate a thought concerning them. Our thinking is usually expressed by the formation of sounds inside our heads, or we voice or write or even gesture our thoughts.

To label something, we need only *point to it*, physically or mentally, and utter a word; we then understand that henceforth this word is to direct our attention to that thing. When we point to something for our own purposes, we know immediately what we mean; but when we do so in an attempt to communicate our intention to others, we may of course be misunderstood.

If one does not understand the significance of 'pointing', one cannot grasp the intention of words. Physical pointing seems to be a sending out of 'energy' in the desired direction, enough to draw the respondent's attention along that line till it meets the object concerned. Animals seem not to comprehend it usually, though sometimes they seem to.

The *'meaning'* of a word, then, is primarily the phenomenon or group of phenomena we pointed to, one way or another, when we introduced it — and all its eventual manifestations. However, we may later narrow the meaning down, and gradually attach the word to a more distinctive and invariable aspect of it all.

Words are symbols. The mind usually assigns one word (if any) for each thing, though sometimes more than one word may be assigned to one thing (equivocation), or one word may be assigned to more than one thing (ambiguity).[2]

Any object of our consciousness may be distinctively named. But most literally single phenomena are ephemeral, and naming them all would be pointless and confusing. Mostly, we label things by reference to their *similarities and differences*. We look for *repetitive, yet distinct* experiences, and assign names to *groups of phenomena* which have some permanence and relevance to our lives. Even 'individual' things are groups of phenomena; 'kinds' of things are doubly so.

In the case of *proper names*, of persons or pets, all the manifestations of an individual entity are referred to; for example, 'Aristotle' refers to all the accumulated impressions of that person. In the case of *common names*, like 'man', a group of similar entities is intended, and all

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[2] Actually, the terminology is colloquially a bit confused. Here, I use the term equivocation as equivalent to **synonymy** (different words for the same thing), and the word ambiguity as equivalent to **homonymy** (the same word for different things). In common parlance, the words equivocal and ambiguous are used interchangeably, because both imply an uncertainty as to the meaning or interpretation. But clearly, there are two ways that uncertainty might arise. For this reason, the terms synonymy and homonymy are preferable, being clearer. I often use equivocation and ambiguity because they are more familiar to the general public. Looking at the etymology of the terms equi-vocation (equal speech) and ambi-aguity (roughly, both actions), it appears that these two terms could be used either way. I tried to freeze their senses, but I must admit I also sometimes revert to colloquial use.

their manifestations as individuals. We do not, of course, have to give proper names to every instance of a kind; we can distinguish them *indicatively*, as in 'this flower'.

In any case, the existence of *a continuity* is always presumed by our use of words; as is our ability to *recognize* such continuity, in spite of changes of the individual across time or differences from one individual to the next. Many differences are discounted. The labeling is open-ended, confident of our power to apply it as we proceed; if we managed previously, why not also subsequently?

We can limit our vocabulary further, by making statements involving strings of words, instead of inventing new words. Things appear to us not in isolation, but as having various relations. 'Relationships' are of course themselves phenomena, which we group and name if found interesting.

When we encounter a relational phenomenon, rather than viewing it as a unity, we distinguish the things related and the relation, and verbally express our perception or conceptual insight as a sentence. Still more complex phenomena may require finer analysis through the use of many sentences.

Thus, words serve first to capture our concrete or abstract experiences. When the phenomenon is relational, we may express it verbally through a sentence or series of sentences. A language is an agreed upon collection of words, a vocabulary, and a convention as to the ways the words may be put together into sentences, a grammar.

The mental or vocal sound, or written symbol, or gesture, acquires the status of a word, only if we once pointed to something (with the index finger, or saying 'look there!'), giving its 'coordinates', or address in space and time. Eventually, we could name something described in terms of other words previously based on such pointing. A 'word' without some ultimate points of reference is a meaningless entity.

A word establishes a conventional correspondence between word and thing. We may imagine a 'line of relation' joining word and thing, and call it 'meaning' or 'intent'. Once invented in this way, the word may be used as an instrument of thought. Henceforth the word becomes, as it were, our substitute for the thing, representing it like an ambassador. We can focus on it, manipulate it, store it away in or recall it from memory, or pass it on to other people (communicate it).

Simply put, 'memory' is any locale where words are laid to rest pending our resumption of attention to them. Words may be externally stored: written in a book or taped on a cassette; or they may be internally stored in our own 'minds'. However, memory must include not only the word, but also somehow what it refers to.

Recalling the word shape or sound would not constitute full remembering, unless we are also awakened to the meaning of the word as well. On the other hand, remembering may be wordless. Therefore, the essence of memory, however it works, is its ability to cause our awareness to return to the original object or some comparable re-enactment of it.

The words involved are incidental; what counts is the underlying act of consciousness. Still, words are useful instruments, not mere appendages. The words we read or hear act as 'switches', which re-trigger and direct our attention to specific experiences or reproductions of them.

2. Same and Different.

Note that we may decide together that this sound and that visual symbol will be 'the same word', and be used to refer to the same thing. For example, the sound 'dog' and the written 'd-o-g' are considered equivalent, though they are substantially different.

Furthermore, a 'word' is always *a class of symbols*: many individual sounds or visual symbols which resemble each other, or are accepted as one and the same 'word'. Any word that I utter or hear or write or read today is a different individual manifestation from its previous occurrence, yet their similarity of sound or look, allow me to recognize them as 'one' word.

For this reason, it is absurd to contend that 'the only thing which allegedly similar objects have in common is the name we assign them'. If nothing was similar to anything else, or we could not recognize things, then even words (as themselves objects) would have no resemblances, and be unrepeatable.

Thus, the existence of *some* similarity, and its knowability *in principle*, are inescapable. How we come to know that things are same or different is a big question, but it need not concern us at this stage, since logic assures us that we at least sometimes do manage to know it.

An entity is a unique complex knot of time, place, attributes, motions, relationships of various kinds. The 'boundaries' of an appearance are themselves usually given as a component of the total phenomenon, though occasionally we may delimit some arbitrary part of a continuum as a unit for consideration. Nothing seems to exist which appears unrelated in some way or other to other things. Something can always be said about anything.

Especially, the relations of sameness and difference seems to be pervasive; everywhere we look, we get these impressions of resemblance and differentiation. If the world contained absolutely only one uniform thing, there would be no call for concepts of similarity or difference. Such utter inimitable and undifferentiatable Unity perhaps concerns G-d, prior to Creation. But the world we know, the world of appearances, is given as a multiplicity of experiences, with more than one object and at least one subject of consciousness.

A world of many things, but which are entirely *without any similarities* between them, a world where nothing has anything in common with anything else and everything is 'an island unto itself', is unimaginable. If such a world contains more than one thing, they have in common at least 'existence', 'singularity', and 'dissimilarity'.

A world of many things, but which are entirely *without any differences* between them, a world where everything has everything in common with everything else and is an 'exact replica' of each other thing, is also unimaginable. If such a world contains more than one thing, they must differ at least in their space and time coordinates to be apart, to be 'many'.

In comparing two or more individual appearances, we may find that they seem to have certain distinguishable factors in common, and our response is to look upon these distinct similarities as significant enough to be named and treated as thought-units. In philosophy, the apparent common factors of things are called *'universals'*.

The simplest way to think of universals is to regard them as substances scattered throughout the world, mingling in different combinations, together constituting entities. Thus, greenness may color objects as distinct as a leaf or a computer screen; a leaf is a meeting point, a sum, of shape, size, color, temperature, and so on. This is the common-sense view, which we will accept as good enough for our purposes here.

When a word is assigned to a new appearance, we do so because the phenomenon seems distinct from any other previously encountered. If further experience shows this initial impression erroneous, because the phenomenon is not novel, then the word becomes an equivocation or falls into disuse.

Likewise, we may wrongly assign a previously created word, or combination of words, to a new phenomenon, which at first seemed to, but on closer inspection ceased to, resemble the old, so that ambiguity arises, or we must reclassify the experience under another word or formulate another sentence.

Thus, naming and verbalizing of our experiences suggests analogies which may later be found inadequate, or which may stand the test of time and further experience. In the former case, we judge the initial assumption illusory; in the latter case, real. But the experience in question remains what it was, however we judge it. Whether real or illusory, it is an 'appearance', something presenting itself to us as object of consciousness.

A big issue in philosophy is whether these intuited commonalities, these resemblances (re-appearances, seeming repetitions), are rooted in the mind somehow (subjective), or whether they exist out there in the object somehow (objective), or both somehow. How can something (a universal) be at once one and many? Theorists have suggested a variety of possible scenarios on either side, but never to everyone's full satisfaction. There may be truth in what they say, but further follow-up is needed.

From the point of view of Logic, no such theory can stand which concludes in the denial that these similarities have some status of reality. For such theory itself, being formulated in conceptual terms, would thereby imply itself untrue. Whatever our theory, the result must be to justify, rather than cause rejection of, the assumption of similarity; for only such result is logically tenable.

As far as concerns Logic, if there is an appearance of resemblance, it is to be considered at its face value. Logically, the appearance of resemblance cannot be declared wrong in principle, even though its exact nature is admittedly yet unclear to us. We may initially assume it to be realistic, without a priori excluding the possibility that *some* such appearances may (as any appearance may) turn out to be illusory.

On the basis of our apparent knowledge of similarity, we tend to group individual phenomena into classes, defined by some selected common factor. In what sense that common factor is itself essentially singular, while being scattered in the many class members, is a mystery. Logic leaves such issues to philosophers and moves on.

3. On Definition.

Some comments concerning definition are in order here. One way to define a word is to point to a material object with one's index finger and say the word; or we may mentally focus on something and think the word in our heads. Alternatively, we may notice that other people repeatedly use a word in the face of a certain experience, and thus we learn that this word refers to that experience.

Yet another way, is to describe something using other words, and assign the new word to this description. Effectively, such definition serves to draw the mind's attention to the object intended: it is not a mere equation of words. We may later realize that the description we gave was not accurate, and propose a new verbal definition. The word can stay unchanged, we 'know' what we were trying to mean by it, only now we have a clearer description of that phenomenon.

Were definition a mere conventional equation of words, a definition would be unchangeable, since the meaning of the word would change when the definition was altered, and we would be talking about a different object than originally intended. But because a definition is an attempt at description, merely designed to direct the mind towards an object of wordless consciousness, it is changeable.

Definition is an attempt to express what appears to be the 'essential' character of the object concerned. Nonetheless, it must be stressed that the assumed essence is itself only an appearance: it may at a later stage appear unessential, or even be found to not always be displayed by the object, and other definitions may replace it. Although definition is, like any other aspect of knowledge, flexible, that does not make it any less useful or valid.

In principle, note well, not everything is definable. To suggest that every word must be defined in other words, is to make an impossible demand for an infinite chain of derivations. There has to be some primary meanings, known directly, on which later descriptive meanings may be built. The phenomenon in question may be so fundamental, that we cannot discern any simpler components in it, but can only discern it as a component of more complex phenomena.

Thus, there is no rational basis for forbidding 'circular' definition as such. Some definitions are merely formulated to clarify, but make no claim to being much more than tautologies. Even as we make one, we may know that the words we are using are not themselves definable, and may just be other words meaning the same things. But the definition may still be useful in directing the mind more precisely where we want it, by linking together disparate pointings and namings.

5. PROPOSITIONS.

1. Terms and Copula.

Logic looks upon sentences as attempts to record or predict reality, which may or may not be correct. For this reason, it calls them *propositions*, to stress their fallibility. Logic develops by scrutiny of ordinary thought and language, but also sets especially rigid structural standards in order to be able to develop systematically.

Looking at many propositions, we see that irrespective of their particular contents, they appear to share certain ‘forms’. Our job is to analyze each form, how it is structured, what it means and implies, what are its interrelationships with other propositions, and how it can be known to be true.

Our study begins with one form shared by many propositions, ‘S is P’. Propositions of this sort are characterized as *categorical*, meaning that they are unconditional. We call ‘S’ the *subject*; ‘is’, the *copula*; and ‘P’ the *predicate*. The subject and predicate are both called *terms*. The copula relates the terms together in a certain way. We may view the subject as our center of interest, while the predication (copula and predicate) provides us with additional information concerning it.

Note well how the terms are treated as ‘variables’, while other features such as the copula (so far) are kept ‘constant’, like in algebra. In this way, we can theoretically concentrate on the properties of a kind of proposition, without regard to the specific ‘values’ which might take the place of the symbols S and P. Form is released from content.

We owe this artifice to Aristotle’s genius. In one stroke, it made possible the development of a science of logic, because the study of relations and processes was thereby greatly facilitated, as we shall see.

We will concentrate mainly on categoricals called *classificatory*. Here, the subject and predicate are classes, and their copula informs us that they contain members in common. Typically, in a general proposition, the subject is a species and the predicate a genus; for example, ‘trees are plants’. Other forms will be dealt with eventually.

2. Polarity and Quantity.

Propositions may be distinguished by the *polarity* of their copula. Thus, ‘S is P’ is said to have a *positive* copula; ‘S is not P’, a *negative* one. (Polarity is traditionally also known as ‘quality’, note, but since this word has other meanings it will be avoided here.)

We could view ‘is’ and ‘is not’ as two distinct relations (which happen to be contradictory), or as respectively the presence and absence of the same relation of ‘being’ (so that ‘is-not’ means ‘not-is’); logically, these viewpoints are equivalent.

The characterization of propositions as affirmations or denials has accordingly two senses, one absolute and the other relative. Normally, an assertion with a positive copula is called affirmative, and that with a negative copula is called denying; but also, we say of either polarity that it affirms itself and denies the other.

Another relevant distinction between propositions refers to their *quantity*. This primarily concerns the subject, clarifying how much of it we intend by our statement. The quantity is often left tacit in everyday discourse, but for the purposes of science, we have to be more explicit.

If S is a specific, recognizable individual, we use the designation ‘this S’, and the proposition is said to be *singular* (and indicative). Any proposition which is not singular may be called *plural*. If S refers to the whole class, we say ‘all S’, and the proposition is called *general* or universal. If S is a loose reference to some unspecified member(s) of the class, we say ‘some S’, and the proposition is called *particular*.

Other quantifiers define 'some' more precisely. Thus, 'a few' or 'many' mean, a small or large number; 'few' or 'most' mean, a minority or majority, a small or large proportion. These for most purposes have the same logical properties as particulars, though the latter two sometimes require special treatment.

By combining these different features, the various polarities and quantities, we obtain the following list of classificatory propositions. These are traditionally assigned symbols as shown to facilitate treatment (from the Latin words Aff**IR**mo and n**EGO**, which serve as mnemonics).

A	All S are P	E	No S is P
R	This S is P	G	This S is not P
I	Some S are P	O	Some S are not P

The other quantities are also applicable to the two polarities, of course, as in 'Few or Most S are or are not P', but have not been traditionally symbolized.

All such propositions are called *actual*, because they suggest the relation they describe as taking place in the present. In that case, they imply that the units which their terms referred to do exist, i.e. that there are S's and P's in the world at the time concerned. This claim is open to debate, but will be taken for granted for now — later, we will clarify the issues involved, and look into the implications of not making such an assumption.

3. Distribution.

Plural propositions normally refer us to their class members *each one singly*; the plural is simply a shorthand statement of a number of independent singulars. Each individual, subsumed by the subject, and included in the all or some enumeration, is separately and equally related to the predicate. The predication is intended to be 'dispensively' applied; meaning severally, not jointly or collectively.

Thus, 'All S are P' or 'Some S are P', here means 'S1 is P', 'S2 is P', 'S3 is P', … and so on; 'No S is P' or 'Some S are not P' here means 'S1 is not P', 'S2 is not P', …etc. — until every S, this one, that one, and the others, which are included by the quantity have been listed.

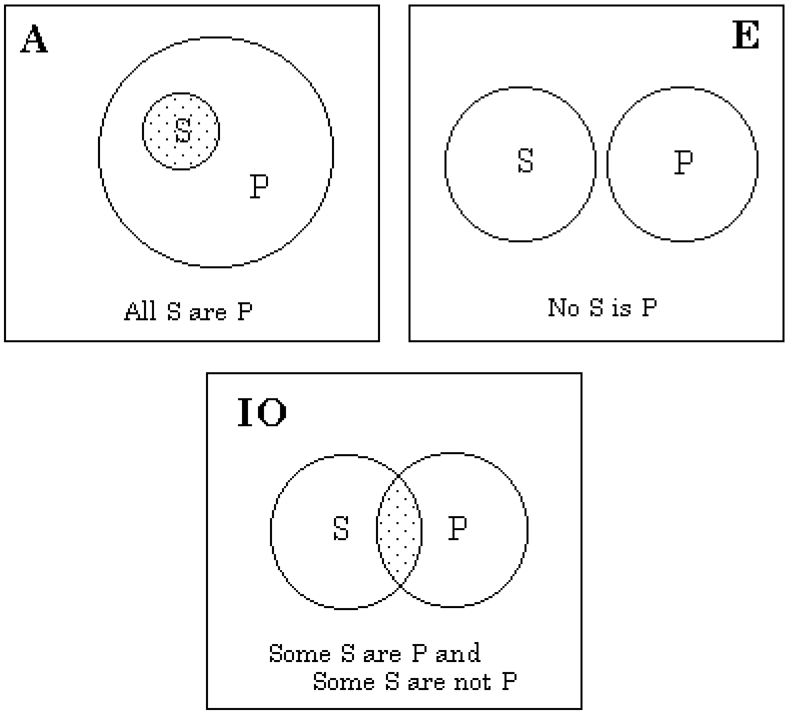
The doctrine of distribution is that if all the members of a class are covered, the term is called 'distributive'; otherwise it is not.

This means that the subjects of universals, **A** and **E**, are distributive; whereas those of particulars, **I** and **O**, are not, since the instances involved are not fully enumerated. With regard to singulars, **R** and **G**, they are effectively distributive, insofar as they point to unique subjects.

What of the distribution of predicates? The predicates of negatives, **E**, **G**, and **O**, are distributive, because P is altogether absent from the cases of S concerned ; while in affirmatives, **A**, **R**, and **I**, the predicates are undistributive, since things other than the cases of S concerned might be P.

These properties can be illustrated by means of Euler diagrams, named after the Swiss logician who invented them. In these, S and P are represented by the areas of circles, which overlap or fail to overlap to varying degrees. The reader is invited to explore these analogies. (Very similar are Venn diagrams, named after another logician; the latter differ in that they stress the areas outside the circles, the areas of nonS or nonP.)

Diagram 5.1 Euler Circles.



In **A** propositions, the S circle is wholly within the P circle, and smaller or equal in size to it. In **E**, the circles are apart, whatever their relative sizes. In **I** propositions, the two circles at least partly intersect, whether each covers only a part of the other's area, or S is wholly embraced by P, or P by S, or they both cover one and the same area. In **O**, the two circles at least partly do not overlap, whether each only covers only a part of the other's area, or neither covers any part of the other's area.

The forms in current use, listed above, are so designed that we can specify alternate quantities for the predicate, if necessary, simply by making an additional statement, in which the original predicate is subject and the original subject is predicate, with the appropriate distributions.

As a result of the distribution doctrine, there have been attempts to invent forms which quantify the predicate, but they have not aroused much interest, being artificial to our normal ways of thinking.

4. Permutation.

Classification is a special outlook, but one we can use to develop Logic with efficiently, because it allows us to standardize statements. Classification is more mathematical in nature, and so easier of treatment, than other relations. The process of rewording a proposition, so that its terms are overlapping classes, is called 'permutation'.

Note that, in formal logic, the word 'universal' is used in a quantitative sense, to apply to general propositions, which address the totality of a class. But in philosophy, a 'universal' is understood as the common factor, resemblance, similarity, which led us or allowed us to group certain units into a class; in this sense every term is a universal for its members, and even a particular proposition contains universals, except that they happen to be only partially addressed.

Likewise, the word 'particular' refers to less than general propositions, in formal logic; whereas, in philosophy, it is understood to mean concrete individuals, as distinct from abstract essences. Normally, the context makes clear what sense of each word we intend.

a. The equivocation of the word 'universal' is not entirely an historical accident. A proposition may have a 'quality as such' as its subject, and only incidentally imply a quantifiable subject-class. Thus, for example, 'greenness is a (kind of) color' and 'all green things are colored' do not mean quite the same, though their truths are related.

Propositions which have as their subject a quality as such, a universal in the philosophical sense, are virtually singular in format. To be quantified, their subject must be reworded somewhat. This is called permutation of the subject.

b. As Logic has developed, it has come to focus especially on the classificatory sense of 'is', because attribution, and other relations, can be reduced to it. Colloquially, the 'is' copula first suggests that the subject has a certain attribute, viz. the predicate, as in 'trees are green'. But attribution is a more complex and qualitative relational format than classification, requiring more philosophical analysis.

Many propositions which normally are thought without the classifying 'is' copula, can be restructured to fit into it, while more or less retaining the same meaning. Thus, in our example, we would shift from the sense 'trees have greenness' to the sense 'trees are greenness-having-things'. This is called permutation of the predicate.

Most logical processing of categoricals assumes that the statements involved have been permuted into classificatory form. Note well that permutation merely *conceals* the previously intended relationship in a new term, it does not annul or replace it. The difficult relation is once-removed, put out of the way; it is not defined.

6. OPPOSITIONS.

1. Definitions.

By the 'opposition' of two propositions, is meant: the exact logical relation existing between them — whether the truth or falsehood of either affects, or not, the truth or falsehood of the other.

In this context, note, the expression 'opposition' is a technical term not necessarily connoting conflict. We commonly say of two statements that they are 'opposite', in the sense of incompatible. But here, the meaning is wider; it refers to any mental confrontation, any logical face-off, between distinguishable propositions. In this sense, even forms which imply each other may be viewed as 'opposed' by virtue of their contradistinction, though to a much lesser degree than contradictories. Thus, the various relations of opposition make up a continuum.

Now, upon reflection, the logical relations of implication, incompatibility, and exhaustiveness, defined earlier, are found to be incomplete insofar as they leave certain issues open. There is therefore a need to combine them in various ways, to obtain a list of seven fully defining kinds of 'oppositions':

a. **Mutual Implication** (or **implicance**): is defined as the relation between two propositions which are either both true or both false. Each is called an implicant and is said to implicate the other. P implies Q, and Q implies P; and, nonQ implies nonP, and nonP implies nonQ.

b. **Subalternation**: is the relation between two propositions which are either both true or both false, or one — called the subalternant — false and the other — called the subaltern — true; the occurrence of 'subalternant true and subaltern false' being excluded by definition. The subalternant and subaltern may be referred to jointly as the subalternatives. This relation is, therefore, one-way implication. P implies Q, but Q does not imply P; and, nonQ implies nonP, but nonP does not imply nonQ.

Subalternation, may be counted as two distinct relations, subalternating, and being subalternated, each of whose direction must be specified. This is in contrast to the other five oppositions, which are symmetrical.

c. **Contradiction**: exists between two propositions which cannot be both true and cannot be both false. If either is true, the other is false; and if either is false, the other is true. They are said to be contradictories. Their affirmations are incompatible and their denials are incompatible. P implies nonQ, and nonP implies Q; and, Q implies nonP, and nonQ implies P.

d. **Contrariety**: two propositions are contrary if they cannot both be true, but may both be false. If either is true the other is false, but if either is false the truth or falsehood of the other is possible. They are said to be contraries. Their affirmations are incompatible, but not their denials. P implies nonQ, but nonP does not imply Q; and, Q implies nonP, but nonQ does not imply P.

e. **Subcontrariety**: occurs when two propositions cannot be both false, but may be both true. If either is false, the other is true; but the truth of either leaves that of the other indeterminate. They are said to be subcontraries. Their denials are incompatible, but not their affirmations. nonP implies Q, but P does not imply nonQ; and, nonQ implies P, but Q does not imply nonP.

f. **Unconnectedness** (or neutrality): two propositions are 'opposed' in this way, if neither formally implies the other, and they are not incompatible, and they are not exhaustive. Note that this definition does not exclude that unconnecteds may, under certain conditions, become connected (or remain unconnected under all conditions).

Note that these seven types of opposition define both directions of the relations concerned, in contrast to the basic logical relations. For this reason they may be called 'full' relations: they leave no question marks. They are logically exhaustive, allowing us to classify the relation of any pair of propositions.

There are other kinds of compound logical relations, besides the above mentioned seven. These concern paradoxical propositions, which imply even their own contradictory, or some contradiction. For example, 'X is not X' formally implies both that 'something, called X, exists' (by the law of identity), and that 'there is no such thing as X' (by the law of contradiction).

However, paradoxes are very rare in formal logic; rather they occur, only a bit less rarely, with specific contents. Formal logic is mainly interested in the oppositions between normal propositions, which are in principle consistent in form. More will be said about paradoxes later, when we look into the logic of logic.

The official terminology for the various kinds of opposition, here suggested, may not always accord with common usage. Especially note that in practice the word 'contradiction' is very often taken as equivalent to 'incompatibility', signifying (in official parlance) 'either contradiction or contrariety'; thus, for instance, with the expression 'law of contradiction'; we mean incompatibility. Also, the word 'opposite' is sometimes used to mean contradictory.

It is curious to note, too, that the words 'subaltern' and 'subcontrary', though quite old, are rarely used in practice; I have only seen them used by logicians. Such failures of words or meanings to enter the mainstream of language, are sad testimonies to the popular disinterest in studying logic.

The following table summarizes the above through analysis of the possibilities of combination of the affirmations and denials of two propositions, P and Q, which are given as being related by a certain opposition, specified in the left column. '**Yes**' indicates possible combinations, '**no**' impossible ones.

Table 6.1 Definitions of Full Oppositions.

POSSIBILITY OF:	P+Q	P+nonQ	nonP+Q	nonP+nonQ
Implicance	yes	no	no	yes
Subalternating	yes	no	yes	yes
Being Subalternated	yes	yes	no	yes
Contradiction	no	yes	yes	no
Contrariety	no	yes	yes	yes
Subcontrariety	yes	yes	yes	no
Unconnectedness	yes	yes	yes	yes

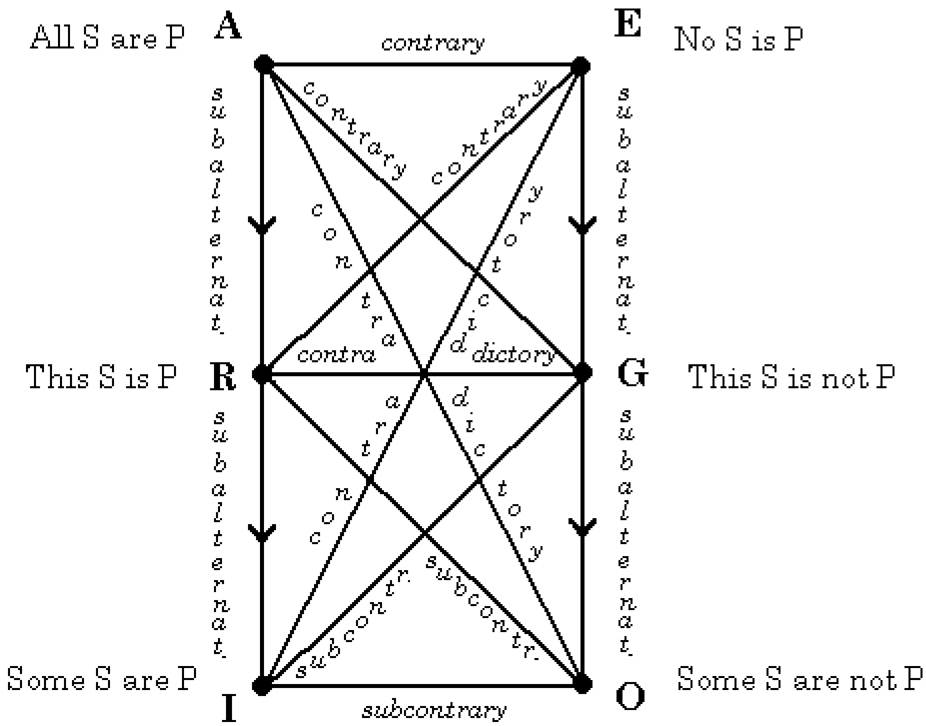
Note that incompatibles are either contradictory or contrary, while exhaustives are either contradictory or subcontrary. Also worth noting, compatibles may be either implicant, or subalternative (in one or the other direction), or subcontrary, or unconnected. The seven definite oppositional relations are mutually exclusive (i.e. contrary, to be exact), but one of the seven must hold.

2. Applications.

The doctrine of opposition arose out of the need to apply the laws of thought to propositions more complex than the initial forms 'S is P' and 'S is not P'. The concepts of equality, conflict, and limitation, had to be expanded upon, to reflect the more qualified relations found to exist between forms once they are quantified.

We know that two singular propositions differing only in polarity (viz. **R**, **G**) are contradictory, for at any given instant This-S cannot both be P and not-be P, and must be one or the other. But what of the plural versions of these forms? The following diagram shows their interrelationships.

Diagram 6.1 Rectangle of Opposition.



We note, to begin with, that for each polarity, the universal (all) subalternates the singular (any specific individual), which in turn subalternates the particular (some is an indefinite quantity, meaning one or more). Next, A universal and particular of opposite polarity (**A** and **O**, or **E** and **I**) are contradictory, just as two singulars (**R** and **G**) concerning one and the same individual are contradictory. Lastly, universals are contrary to universals or singulars of opposite polarity (**A** and **E**, **A** and **G**, **R** and **E**), and particulars are subcontrary to particulars or singulars of opposite polarity (**I** and **O**, **I** and **G**, **R** and **O**).

We may summarize these findings in the form of a 'truth-table'. This tells us which other propositions must be true (**T**) or false (**F**), or may be either (**.**), in the context of each form given on the left under heading **T** being true, or each form given on the right under **F** being false.

Table 6.2 Truth-table.

T	A	R	I	E	G	O	F
A	T	T	T	F	F	F	O
R	.	T	T	F	F	.	G
I	.	.	T	F	.	.	E
E	F	F	F	T	T	T	I
G	F	F	.	.	T	T	R
O	F	.	.	.	.	T	A

The conjunction of **I** and **O** may be viewed as a form of proposition in its own right, though composite. If we oppose this to the above standard forms, we obtain the following. Since '**I + O**' subalternates **I** and **O** (considered separately), it is contrary to **A** and **E**. It is unconnected to **R** and **G**, since either may be true or false without affecting it.

Also note in passing the position of forms quantified by 'most' or 'few', which we mentioned earlier. See **Appendix 2** for remarks on this topic.

Note that two propositions with the same subject, but with different predicates, may be considered opposites, if the predicates are well known to be antithetical. Thus, 'S is P' and 'S is Q' may implicitly intend 'S is P (but not Q)' and 'S is Q (but not P)', respectively. In such case, the forms may of course be treated as effective contradictories.

3. Validations.

These oppositions are proved as follows. Remember that each of the plural propositions can be defined by a series of singular propositions of the same polarity. Thus, **A** and **I** are reducible to a series S1 is P, S2 is P, S3 is P, etc., differing in that All-S covers the whole class of S, whereas Some-S covers only part of the same class. Likewise in the case of negatives, **E** and **O**. Thus the subalternation of singular or particular, to a generality of like polarity, is simply the inclusion by the whole of the class of any part thereof. This relation is unidirectional in that if the whole is affirmed or denied so is every part of it, whereas if some part is affirmed or denied it does not follow that other parts are.

Similarly, the contradictions of **A** and **O**, or **E** and **I**, are proven by consideration of their subsumptions. If all the members of a class are included in a predication, then any which is declared excluded would be found to be both P and nonP, an impossibility. The same can be argued in the negative case: if all are excluded, then none can be included without inconsistency.

With regard to **I** and **O** (or **I** and **G**, or **R** and **O**), they are subcontrary insofar as conflicting predicates can consistently be applied to different parts of the same subject-class, although it is impossible to evade either affirming or denying any predicate of a subject, i.e. one must be true. The contrariety of **A** and **E** (or **A** and **G**, or **R** and **E**) is due to the observation that, while they cannot be both true without implying some singular case(s) of inconsistency, they could be both false without antinomy, as occurs in the case of **I** and **O** being both true.

The concepts of inclusion and exclusion are geometrically evident. They were implicit in the original formulation of the laws of thought, when we referred to the whole or part of a singular phenomenon. In this logical discipline, we broaden the laws of thought, by treating individual instances as parts of a larger phenomenon we call a class or universal, and then applying our laws to this new whole. Essentially, no information has been added, we have merely in fact elucidated inherent data.

To conclude, let us point out that 'opposition' can be viewed as a kind of immediate inference, like eduction. This is especially obvious when we draw out an implicant or subaltern, but can also be said about affirming a proposition on the basis of another's falsehood, or denying one on the basis of another's truth or falsehood. Opposition is not a mere theoretical construct for logicians, but of practical value to the layman.

7. EDUCTION.

1. Definitions.

Immediate inference is the process of discovering another proposition implicit in a given proposition, without use of additional information. It differs from syllogistic reasoning, in that the latter draws a new proposition from two or more previous ones. We have come across one sample of such inference in the foregoing text, namely opposition. Here we will deal systematically with another, which may be called eduction.

What eduction does is to change the position and/or polarity of the terms; this often results in a proposition of different polarity or quantity. The original proposition is the premise, the educed proposition an implication of it.

Often, to fully understand a proposition, we have to restate it in another way, some hidden character of it is thereby revealed, facilitating further thought. The structural change we effect in the given form yields new information, although a simple process.

Starting from an S-P format, we may be able to obtain propositions through transposition and/or negation of terms, in the following ways.

Table 7.1 Eductive Processes.

Process:	From S-P to:
Obversion	S-nonP
Conversion	P-S
Obverted Conversion	P-nonS
Conversion by Negation	nonP-S
Contraposition	nonP-nonS
Inversion	nonS-nonP
Obverted Inversion	nonS-P

The source proposition is then called obvertend, convertend, contraponent, invertend, and so on, while the target proposition is called obverse, converse, contraposite, inverse, as the case may be.

Whereas such processes are generally possible with one or both of the universals, they are not always feasible in the case of singulars or particulars, as we shall see. Note also that some processes are *reversible*, and some are not: only in some cases may the source proposition be educed again from its implication (by the same or any other eductive process).

2. Applications.

We shall now list the implications of the various plural forms, and then validate the processes involved. Although these may be tedious details, they do constitute an important training for the mind.

a. Obversions (S-P to S-nonP).

A	All S are P	implies	E	No S is nonP
E	No S is P	implies	A	All S are nonP
R	This S is P	implies	G	This S is not nonP

G	This S is not P	implies	R	This S is nonP
I	Some S are P	implies	O	Some S are not nonP
O	Some S are not P	implies	I	Some S are nonP

Thus, all forms are obvertible, and so reversibly.

b. Conversions (S-P to P-S).

A	All S are P	implies	I	Some P are S
E	No S is P	implies	E	No P is S
R	This S is P	implies	I	Some P are S
I	Some S are P	implies	I	Some P are S

Thus, affirmatives yield a particular. Only **I**'s and **E**'s conversions are reversible. **G** and **O** propositions are not convertible.

c. Obverted Conversions (S-P to P-nonS).

A	All S are P	implies	O	Some P are not nonS
E	No S is P	implies	A	All P are nonS
R	This S is P	implies	O	Some P are not nonS
I	Some S are P	implies	O	Some P are not nonS

Thus, affirmatives yield a particular. Only **I**'s and **E**'s obverted conversions are reversible. **G** and **O** propositions lack an obverted converse.

d. Conversions by Negation (S-P to nonP-S).

A	All S are P	implies	E	No nonP is S
E	No S is P	implies	I	Some nonP are S
G	This S is not P	implies	I	Some nonP are S
O	Some S are not P	implies	I	Some nonP are S

Thus, negatives yield a particular. Only **A**'s and **O**'s conversions by negation are reversible. **R** and **I** propositions have no converse by negation.

e. Contrapositions (S-P to nonP-nonS).

A	All S are P	implies	A	All nonP are nonS
E	No S is P	implies	O	Some nonP are not nonS
G	This S is not P	implies	O	Some nonP are not nonS
O	Some S are not P	implies	O	Some nonP are not nonS

Thus, negatives yield a particular. Only **A**'s and **O**'s contrapositions are reversible. **R** and **I** propositions are not contraposable.

f. Inversions (S-P to nonS-nonP).

A	All S are P	implies	I	Some nonS are nonP
E	No S is P	implies	O	Some nonS are not nonP

Only universals are invertible, and that irreversibly, to particular form. **R**, **G**, **I**, and **O** propositions are not invertible.

g. Obverted Inversions (S-P to nonS-P).

A	All S are P	implies	O	Some nonS are not P
E	No S is P	implies	I	Some nonS are P

Only universals may be subjected to obverted inversion, and that irreversibly, to particular form. Process not applicable to **R**, **G**, **I**, and **O** propositions.

We note at the outset that while quantity may be lost, it cannot be gained. A universal or singular proposition may yield a particular, but a singular or particular cannot produce a universal. It is also clear that, with the exception of obversion, the processes applicable to singulars are so only by virtue of the corresponding particulars implicit in them by opposition. This is true also of **A** in conversion and obverted conversion, **E** in conversion by negation and contraposition. Universality plays an active role only in conversion and obverted conversion of **E**, in conversion by negation and contraposition of **A**, and in inversion and obverted inversion.

3. Validations.

We can validate all these processes by working on two: obversion and conversion, for the others follow.

a. ***Obversion***. 'S is P' to 'S is not nonP'. The negation of a term normally signifies the absence of some phenomenon. In the absence of a phenomenon, other phenomena necessarily exist: there is a world out there, be it real or illusory; appearances constantly occur. Furthermore, by the law of contradiction, a phenomenon S cannot both be and not-be something called P. Thus, the phenomenon P cannot be both present and absent in the thing called S. Just as is and is-not are mutually exclusive, so are the affirmation and negation inconsistent.

To say S is P posits that P is found in S; to say S is-not nonP means that the absence of P is absent from S. Which is not to imply, in either case, that S is not simultaneously other things than P or nonP — Q, R, etc. So, S can be P and something other than P, although it cannot both exhibit and not-exhibit P. These arguments thus define the copula is-not and the term nonP more precisely.

What is true here in the case of singular propositions, can be argued equally for plural propositions, since the latter subsume the former. That is, they collect them together as a unit while at the same time dealing with them each one singly; so that the statement does not concern them as either a count of individuals or as a collective unity, but is merely an abbreviated statement being distributed out to its instances equivalently.

Thus S-P merely means S1-P1, S2-P2, S3-P3, etc. Here again, this doctrine provides an opportunity to more precisely define formal concepts.

b. ***Conversion***. Here each quantitative is considered separately.

(i) For **I**: 'Some S are P' and 'Some P are S' each means 'Some things are both S and P'; we are seeing S and P together, we may attribute either to the other; this defines the generality of our copula is, and proceeds from the law of Identity.

(ii) For **E**: likewise, 'No S is P' and 'No P is S' each means 'Nothing is both S and P'; S and P never appear together, have no instances in common. This clarifies our copula is-not, telling us that S is-not P is the same as P is-not S; and also reminding us that 'No X is Y' means 'All X are-not Y'.

(iii) For **A**: 'All S are P' by subsumption implies that 'Some S are P', and therefore also that 'Some P are S' as shown above. However, it could-not imply 'All P are S', although in some cases such mutual inclusion occurs, because there are cases where it does not. Here again, we are better defining our form, in accord with its common usage.

(iv)For **O**: from 'Some S are not P' we cannot infer 'Some P are not S', for it happens that 'All P are S'; that is, it happens that only S are P, i.e. that P does not occur elsewhere; our form is intended as that broad and inclusive of possible circumstance.

Other approaches to these validations are possible. But the intent here was to show that these need not be viewed as 'proofs', so much as focusing more precisely on the forms our consciousness naturally uses, and inspecting every aspect of their selected meanings to delimit the extent of their application to phenomena as they appear to us.

With regard to the *other types* of eduction, they can be reduced to combinations of the above two processes, and thus validated. Thus:

c. **Obverted Conversion**. Convert, then obvert.

d. **Conversion by Negation**. Obvert, then convert.

e. **Contraposition**. Obvert, then convert, then obvert.

f. **Inversion**. For **A**: contrapose, then convert. For **E**: convert, then contrapose.

g. **Obverted Inversion**. Invert, then obvert.

Note lastly, one can say 'some nonS are nonP' (or the converse) for just about any S and P chosen at random, with the exception of certain very broad terms, like 'existence', which have no real negatives. So processes which yield such conclusions are not very informative.

Addendum: "No S is P" means that S and P are incompatible – if one of them is present, the other one cannot also be present. "No nonS is nonP" means that S and P are exhaustive - if one of them is absent the other cannot also be absent. To affirm both these propositions is to say the two terms S and P (or nonS and nonP) are contradictory. To affirm the first and deny the second is to say S and P are contrary. To deny the first and affirm the second is to say S and P are subcontrary. To deny both is to say some S are P and some nonS are nonP – i.e. they are compatible and inexhaustive.

8. SYLLOGISM: DEFINITIONS.

1. Generalities.

We call inference the mental process of becoming aware of information implicit in given information, be it concrete or abstract. When we draw ideas from experience or generalities from particulars, we are involved in induction; otherwise, it is deduction. In any case, the original data is called the premises, and the logically derived proposition, the conclusion.

When the conclusion is already known to us, and we are considering its validity in the context of other knowledge, we are said to argue. Furthermore, if the motive of our argument was to arrive at the conclusion for its own sake, we are said to be proving it; if on the other hand our motive was to show the contradictory or a contrary of our conclusion to be false, we are said to be engaged in a process of refutation.

The difference in connotation between inference and argument is merely one of sequence: what was posited first, premise(s) or conclusion? The distinction between proof and refutation lies in our motive. But the logical form of all these processes is the same, so their names are used interchangeably here.

The term deduction is sometimes used in a restricted sense which excludes eduction. Eduction has already been defined as eliciting information from one proposition (granting that the logical principles involved in this are not regarded as premises too). The deductive process which concerns us here, in contrast, is drawing information implicit in two or more propositions together, and not separately. P and Q are true, ergo R is true.

This is called mediate inference, because it is found that the premises must have some factor in common, which serves as the medium of inference, making possible the eliciting of a conclusion. This might be thought of as ‘conduction’. The technical name for it is syllogism, from Greek, the language of Aristotle.

It can be shown that arguments involving more than two categorical propositions are reducible to a series of syllogisms and eductions. In this analysis, we will concentrate on categorical syllogism, that involving only categorical propositions. Argument involving noncategorical propositions will be dealt with later.

2. Valid/Invalid.

Now, an argument may be valid or invalid. The science of Logic shows that the validity of the method is independent of the truth or falsehood of the premises or conclusion. A formal argument only claims that if the premises are true, the conclusion must be true; if the conclusion is found false, then one or more of the premises must be false. It may happen that the premises are false, yet the conclusion is independently true; rejection of the premises does not necessarily put the conclusion in doubt. The validity or invalidity of an argument is a formal issue, irrespective of the content of the propositions involved.

Logic analyses the variety of forms possible, and distinguishes the valid from the invalid, by reference to the Laws of Thought. The results are analyzed, in the search for general rules. Strictly speaking, only valid syllogisms are ultimately so called; invalid syllogisms are mere fallacies. But at the outset, Logic lists all possible combinations of propositions on an equal footing, to ensure the exhaustiveness of its treatment; then it finds out which are good and which bad.

Its ultimate aim is of course to draw the maximum consequent information from any data. This allows us to correlate the different aspects of our experience, and improve our knowledge of the world. By comparing and connecting together all our beliefs, we can through logic discover inconsistencies, which cause us to reassess our assumptions at some level, and correct our data banks. In this way our beliefs are 'proved'; at least until there is good reason to think otherwise.

Scientific proof always depends on the context of knowledge. It is always conceivable that some aspect of knowledge turns out to be open to doubt, even after seeming fundamental and unassailable for ages. For instance, certain axioms of Euclidean geometry. So proof never entirely frees a conclusion from review, given some new motive. Finding an inconsistency does not in itself guarantee that we will succeed in finding the source of the error, i.e. some false premise. In such cases, we register that there is some doubt yet to resolve, and either wait for new experience or search for an answer imaginatively.

3. Figures.

A syllogism, then, involves three propositions, two premises and a conclusion. These together involve three, and only three, terms. They are: the middle term, one common to both premises, but absent in the conclusion; the minor term, which is the subject of the conclusion, and present in one of the premises; and the major term, which is the predicate of the conclusion, and present in the other premise. The minor and major term are also called the extremes; the middle term acts as intermediary between them, to yield the conclusion. The premise involving the minor term is called the minor premise, that with the major term the major premise.

The position of the middle term in the premises, that is, whether it is subject or predicate in each, determines what is called the 'figure' of the syllogism. (The colloquial expression for thought, 'to figure' or 'to figure out' may derive from this usage.) There are four possible figures of the syllogism. They are shown in the following table, with S, M, P symbolizing the minor, middle and major terms, respectively:

Table 8.1 Figures of the Syllogism.

Figure	First	Second	Third	Fourth
Major premise:	M-P	P-M	M-P	P-M
Minor premise:	S-M	S-M	M-S	M-S
Conclusion:	S-P	S-P	S-P	S-P

Note well the variety in the position of the terms. The order of the propositions in Logic is conventionally set as major-minor-conclusion, so that symbolic references can always be understood. But of course in actual thought any order of appearance may occur. Thus it is seen that syllogism is mediate inference; from their respective relationships to a middle term, a relationship may be found to follow between the extremes.

Each figure of the syllogism reflects a structure of our thinking. In practice, the Fourth figure is not regarded by many logicians as very significant. Aristotle, though aware of its existence, had this viewpoint. Galen, however, introduced it as a formal alternative for the sake of completeness.

4. Moods.

We previously identified six categorical forms, **A**, **E**. **I**, **O**, **R** and **G**, which can be involved in such syllogism. Each of the propositions in each figure might at first glance have any of these six forms. So there are 6X6X6 = 216 possibilities per group of proposition in each figure. Each of these combinations is called a mood of the syllogism. Altogether, in the four figures, there are 216X4 = 864 imaginable syllogistic forms. Each such form can be designated clearly by mentioning its figure and mood; for example, 'mood **EAA** in the first figure', or more briefly, '**1/EAA**'.

Our task is differentiate the valid from the invalid, in this multiplicity of theoretical constructs. It will be seen that very few actually pass the test. The valid moods per figure should be justified, and the invalid ones shown wrong. This will enable us to know when a conclusion can be drawn from given premises, and when not.

Note that each of the propositions may be positive (**+**) or negative (**-**), so that there are 2X2X2 = 8 possible combinations of polarity in each figure; they are: **+++**, **++-**, **+-+**,**+--**, **-++**, **-+-**, **--+**, **---**. Likewise, as three quantities exist, viz. universal (**u**), particular (**p**), and singular (**s**), there are 3X3X3 = 27 possible combinations of quantity in each figure; which are: **uuu**, **uup**, **uus**, **upu**, **upp**, **ups**, **puu**, **pup**, **pus**, **ppu**, **ppp**, **pps**, and so on. It will be seen that many of these combinations are nonsensical, and rules concerning polarity and quantity can be formulated. Some rules are general to all figures, some are specific to each. In any case, the conclusion sought is always the maximal one; if a universal can be concluded, the subaltern conclusion is not of interest, though it follows *a-fortiori*.

A more traditional way to express the task of logic with respect to syllogism is as follows. In each figure, which of the 6X6 = 36 combination(s) of premises yield a conclusion? Or which of the 2X2 = 4 combination(s) of polarity: **++**, **+-**, **-+**, **--**? And which 3X3 = 9 combination(s) of quantity: **uu**, **up**, **pu**, **pp**, **us**, **su**, **sp**, **ps**, **ss**?

5. Psychology.

Some critics of Logic have accused it of puerility, arguing that the syllogism is too simple in form, and yields no new information, whereas actual thinking is somehow a more creative and complex process. But the ‘event’ of syllogistic reasoning is not as mechanical and automatic as it is made to appear on paper. Logic presents a static picture of what is psychologically a very dynamic and often difficult process.

There is a mental effort in bringing together the concepts which form the separate propositions involved; this requires complex differential perceptions and insights. We also have to think of bringing together the propositions which constitute our premises; they are not always joined and compared automatically, sometimes a veritable inspiration is required to achieve this. And even then, actual drawing of the conclusion is not mechanically inevitable; honesty, will, and intelligence are needed.

Thus, Logic merely establishes standards of proper reasoning, identifying common aspects of thought and justifying its sequences. But mentally, in practice, the processes are complexes of differentiation and integration. Sometimes such events are easy to produce, but often years of study and even genius are necessary to produce even a single result. Virtues such as open-mindedness, reality-orientation, perceptiveness, intuition, will-power are involved.

9. SYLLOGISM: APPLICATIONS.

In this chapter we will list the valid moods of the syllogism, and make some generalizations and comments, so as to acquaint the reader with the central subject of our discussion. Thereafter, validation will be dealt with in a separate chapter. Please remember that we are dealing here specifically with one type of proposition, the actual, classificatory, categorical. Other types of proposition require eventual treatment, of course.

Our main concern here is classical logic in all its beauty, the showpiece of the science, which we owe to Aristotle and subsequent masters. There are related topics of lesser importance, these will be mentioned in the course of development.

1. The Main Moods

Syllogism is inference from two propositions of a third whose truth follows from the given two. In categorical syllogism, we deduce a relation between two terms by virtue of their being each related to a third term. According to the direction of their relationship to the third term, the syllogism is said to form different figures, or "movements of thought" (Joseph). The polarities and quantities of the premises, because of their diverse ways of distributing their terms, generally affect the character and validity of the conclusion. These differences are used to distinguish moods of the syllogism in each figure, which may reflect a variety of approaches through which our minds analyze a subject to attain understanding of it.

In this section, we will list the principal valid moods of plural syllogism, that is, of syllogism both of whose premises are plural. They are the most important in this doctrine. Valid moods involving one or two singular premises will be listed in the next section. Derivatively valid syllogisms, of an artificial or subaltern nature, or involving atypical conclusions, will be discussed separately. Moods not included in these listings of valid moods are to be regarded as paralogisms, they are either *non-sequitur*s ('it does not follow' in Latin) or self-contradictory.

a. First Figure.

Form:

Major premise	M-P
Minor premise	S-M
Conclusion	S-P.

AAA	AII
All M are P	All M are P
All S are M	Some S are M
∴ All S are P	∴ Some S are P
EAE	EIO
No M is P	No M is P
All S are M	Some S are M
∴ No S is P	∴ Some S are not P

We may observe that the major premise is always universal, and the minor premise always affirmative, here. The principle of such reasoning, called the first canon of logic, could be expressed as 'Whatever satisfies fully the condition of a rule, falls under the rule'. The condition here means 'being M', and the rule means 'being P' or 'not being P'.

b. Second Figure.

Form:

Major premise	P-M
Minor premise	S-M
Conclusion	S-P.

AEE
All P are M
No S is M
∴ No S is P

AOO
All P are M
Some S are not M
∴ Some S are not P

EAE
No P is M
All S are M
∴ No S is P

EIO
No P is M
Some S are M
∴ Some S are not P

We observe that the major premise is always universal, and the conclusion always negative. The second canon of logic, implicit in these moods, can be stated as 'Whatever does not fall under a rule, does not satisfy any full condition to the rule'. The condition here meaning 'being P' and the rule 'being, or not-being, M'.

c. Third Figure.

Form:

Major premise	M-P
Minor premise	M-S
Conclusion	S-P.

AII
All M are P
Some M are S
∴ Some S are P

EIO
No M is P
Some M are S
∴ Some S are not P

IAI
Some M are P
All M are S
∴ Some S are P

OAO
Some M are not P
All M are S
∴ Some S are not P

We observe that the minor premise is always affirmative, and the conclusion is always particular. Two more moods, **AAI** and **EAO**, are normally included by logicians with the above; but these are true only by virtue of the truth of **AII** and **EIO**, respectively, whose minor premises theirs imply; I have therefore chosen to exclude them. The principle here, our third canon, is expressed as 'Rules following from the same condition are in that instance at least compatible'. The common condition being instances of subsumed M in both premises, and the rules being their relations to S and P.

d. The Fourth Figure.

Form:

Major premise	P-M
Minor premise	M-S
Conclusion	S-P.

EIO
No P is M
Some M are S
∴ Some S are not P

We note that the major premise is a negative universal, the minor is affirmative, and the conclusion a negative particular one. (The mood **EAO** might also have been included here, but its validity is only due to its minor premise implying that of **EIO**.) This figure is rather controversial. It formally has three more valid moods, **AEE**, **IAI** and **AAI**, but these are left out as too insignificant for such central exposure. This topic will be further discussed. No canon is normally formulated for this figure.

There are therefore a total of 4+4+4+1 = 13 moods of the plural syllogism which are valid, nonderivative, and significant.

2. On the Fourth Figure.

If we consider the second and third figures, we see that transposition of the premises does not change the figure, although the conclusion if any will have transposed terms; the middle term remains common subject or predicate, as the case may be, of the premises. But in the first figure, if the major and minor premises are transposed, not only are the major and minor terms transposed in the conclusion, but a new figure emerges, the fourth. The reverse is also true, shifting from fourth to first. Yet, the order of appearances of the premises is essentially conventional, and should not matter.

It is doubtful whether anyone ever thinks in fourth figure terms, probably because of the double complication it involves. The minor term shifts from being a predicate in its premise to being a subject in the conclusion, and the major term switches from subject in its premise to predicate in the conclusion. While each of these changes does occur in the third and second figures respectively, in the fourth figure both of these mental acrobatics are required. We have difficulty in reasoning thus, whereas the process should be obvious enough for the mind to concentrate on content.

Some logicians have opted for ignoring the fourth figure altogether, on such grounds. Others have insisted on including it as a formal possibility, arguing that the science of logic should be exhaustive and systematic, and show us all the information we can draw from any given data.

My own position is a compromise one. The valid moods **AEE**, **IAI**, and **AAI** (which is implicit in **IAI**, incidentally), clearly do not present us with information not available in the first figure (after transposition of premises). Given the two premises, we are sure to process them mentally in the first figure, and then, if we need to, convert their conclusions as a separate act of thought. In the case of valid mood **EIO** (and likewise **EAO**, which is implicit in it), however, the conclusion ‘Some S are not P’ would not be inferable in the first figure, since **O**-propositions have no converse. It follows that it must be retained to achieve a complete analysis of possibilities, even if rarely used in practice.

This position can be further justified by observing the lack of uniformity in these five moods. They do not have clear common attributes like the valid moods of other figures; they rather seem to form three distinct groups when we consider their polarities and quantities. **EIO** (and **EAO**) make up one group; **AEE**, another; **IAI** (and **AAI**), yet another.

3. Subaltern Moods.

Under this heading we may firstly include the two third figure moods, **AAI** and **EAO**, and the fourth figure mood, **EAO**, which were mentioned earlier as mere derivatives. The reason why logicians have traditionally counted them among the principal moods, was that they inform us that in the cases concerned, only a particular conclusion is obtainable from universal premises; but I have chosen to stress rather their implicitness in the corresponding moods with a particular minor premise, so that from this perspective they give us no added information. They do not constitute an independent process, but are reducible to an eduction followed by a deduction, or vice versa. Note in passing that the insignificant mood **AAI** in the fourth figure is such a derivative of **IAI**, also insignificant.

We can also call subaltern, moods which simply contain the subaltern conclusion to any higher conclusion found valid. Thus, though valid, they are regarded as products of eduction after the main deduction. They are: in the first figure, **AAI** and **EAO**; in the second figure, **AEO** and **EAO**; the third figure has none; in the fourth figure, **AEO**.

Thus, there are altogether of 2+2+2+3 = 9 plural moods which, though valid, are subaltern, in the four figures.

4. Singular Moods.

These contain one or more singular propositions. The valid ones are as follows.

In the first figure, **ARR** and **ERG**; in the second figure, **AGG** and **ERG**. In these figures, we have singular conclusions, higher than in the corresponding valid particular moods (since singulars are not implied by particulars), and so novel syllogisms. They are worth listing.

First Figure:

ARR	ERG
All M are P	No M is P
This S is M	This S is M
∴ This S is P	∴ This S is not P

Second Figure:

AGG	ERG
All P are M	No P is M
This S is not M	This S is M
∴ This S is not P	∴ This S is not P

I would not regard the moods **AAR** and **EAG** in the first figure as valid, in spite of their apparent subalternation by **ARR** and **ERG**, respectively, because they introduce a ‘this’ in the conclusion which was not in the premises (so that there is an implicit third premise ‘this is S’). Likewise in the second figure for **AEG** and **EAG**, they are not true derivatives of **AGG** and **ERG**. This issue will be confronted more deeply later.

The subalterns of these valid moods, viz. in first figure, **ARI** and **ERO**, and in the second figure, **AGO** and **ERO**, are of course also valid, but not of interest.

In the third figure, the two moods **RRI** and **GRI** are worthy of attention. Each exceptionally draws a conclusion from two singular premises, without involvement of a universal premise; this is of course due to the position of the middle term as individual subject of both premises. This reflects the fact that one instance often suffices to make a particular point (and is sometimes enough to disprove a general postulate). Note that the conclusion is particular, and not singular, because the 'this' cannot be passed on from a subject to a predicate.

Third Figure.

RRI	GRO
This M is P	This M is not P
This M is S	This M is S
∴ Some S are P	∴ Some S are not P

Also valid in the third figure, are **ARI**, **ERO**, **RAI**, and **GAO**. But in these cases the conclusions from singular premise moods are no more powerful than those from their particular premise equivalents, so that we have mere subaltern forms.

In the fourth figure, **ERO** and **RAI** are valid, but as they offer no new conclusion, they may be ignored as subaltern. Because in this figure validation occurs through the first figure, after conversion of premises or conclusion, and a singular proposition converts only to a particular, there cannot be any special valid singular syllogisms.

The total number of valid singular moods, which are not subaltern, is thus 2+2+2+0 = 6. Additionally, we mentioned 2+2+4+2 = 10 subalterns.

Regarding syllogisms involving propositions which concern a majority or minority of a class, we get results similar to those obtained with singular moods.

Thus, in the first figure, there are four main valid moods, their form being: 'If All M are (or are-not) P, and Most (or Few) S are M, then Most (or Few) S are (or are-not) P'. In the second figure, there are four main valid moods, too, with the form: 'If All P are (or are-not) M, and Most (or Few) S are-not (or are) M, then Most (or Few) S are not P'.

In the third figure, we have only two main valid moods. They are especially noteworthy in that they manage without a universal premise. Their form is: 'Most M are (or are-not) P, Most M are S, therefore Some S are (or are-not) P'. Note that the two premises are majoritive, and the conclusion is only particular. The validity of these is due to the assumption that 'most' includes more than half of the middle term class, so that there is overlap in some instances.

There are no nonsubaltern valid moods in the fourth figure. Subaltern versions of the above listed syllogisms, involving majoritive or minoritive premises, exist, but will not be listed here.

5. Summary.

The following table lists the 19 moods of the syllogism in the four figures, which were found valid, nonsubaltern, and sufficiently significant. These may be called the primary valid moods, because of their relative independence and originality. Another 25 moods are valid, but are either subaltern to the primary syllogisms or insignificant fourth figure moods. These may be grouped together under the name of secondary valid moods.

Table 9.1 Valid Moods in Each Figure.

Figure	First	Second	Third	Fourth
Primary Moods	AAA EAE AII EIO ARR ERG	AEE EAE AOO EIO AGG ERG	AII EIO IAI OAO RRI GRO	EIO
Secondary Moods	AAI EAO ARI ERO	AEO EAO AGO ERO	AAI EAO ARI ERO RAI GAO	EAO ERO AEE AEO AAI IAI RAI

The count of primary valid moods is thus (secondaries in brackets): 6 (+4) in figure one, 6 (+4) in figure two, 6 (+6) in figure three, 1 (+7) in figure four. Thus out of 864 imaginable moods, barely 2.2% are valid and significant. A further 2.9% are logically possible, but of comparatively little interest, for reasons already given. These calculations show the need for a science of Logic. If there is a 95% chance of our thought-processes being in error, it is very wise to study the matter, and not leave it to instinct.

6. Common Attributes.

We may observe some characteristics the valid moods have in common, relating to polarity or quantity.

a. Polarity.

- One premise is always affirmative. Two negative premises are inconclusive.
- If both premises are affirmative, so is the conclusion.
- If either premise is negative, so is the conclusion.

b. Quantity.

- Only when both premises are universal, may the conclusion be so; though in some cases two universals only yield a particular.
- One premise is always universal. Two particular premises are inconclusive. (Exceptions occur in Figure Three, if both premises are singular or majoritive; the conclusion is in such cases particular.)
- If either premise is particular, so is the conclusion. (To note, additionally, a singular conclusion may sometimes be drawn from a singular premise, in Figures One and Two. Likewise for majoritives and minoritives.)

Comparing these, it is interesting to note how polarity relations are almost similar to quantity relations. Positive is a connection superior in force to negative, much like as universal is to stronger than particular.

These half-dozen 'general rules of the syllogism' (as they are called), together with the couple of specific rules mentioned above within each individual figure, are intended to be sufficient, if memorized, to allow us to reject moods which do not fit into any one of them. They apply to the main forms under discussion, though some exceptions occur in a wider context, as will be seen.

c. ***Distribution***. Additional rules have been formulated, which focus on the distribution of terms. These rules help explain the generalities encountered in the previous approach. They are:

- The middle term must be distributive once at least. That is, there must be common instances between the members of the middle term class subsumed in the two premises; this explains the general need of a universal, as well as the mentioned exceptions.
- A minor or major term which was not distributive in its premise, cannot become distributive in the conclusion. That is, we cannot elicit more information concerning a class than was implicit in the given data.

Euler diagrams are very helpful in this context. Through drawing the extensions of the three classes, we can observe on paper the transition from minor to major via the middle.

7. Imperfect Syllogisms.

If logic is viewed as having the task of drawing the most information from given data, then certain additional formal possibilities of deduction from some pairs of categorical premises should be mentioned. We have seen that normal syllogisms always yield a conclusion with the minor term as subject and the major as predicate, ‘S-P’. We may ask if there are cases where such a typical conclusion may not be drawn, but the deduction of some other form of conclusion, at least, is still possible.

It is found that indeed this occurs in certain cases. The conclusion involved always has the form ‘Some nonS are nonP’, a particular proposition connecting as subject and predicate the negations of the minor and major terms, instead of the terms themselves. The list of such imperfect moods is as follows. In the first figure, **EE**, **OE** (and **GE**); in the second figure, **AA**, **EE**; in the third figure, **EE**, **EO**, **OE** (and **EG**, **GE**); in the fourth figure, **EE**.

These syllogisms are of course very artificial, and will not be discussed further.

10. SYLLOGISM: VALIDATIONS.

1. Function.

Validation of a syllogism consists in showing its consistency with the axioms of logic. If it is shown that the conclusion follows from the premises, the form of thought is justified. When we encounter a syllogism which results in some antinomy, we obviously reject it; when we reject a sequence of premises and conclusion, we automatically validate that sequence of premises with a contradictory conclusion. Only thus is the balance of consistency restored. This defines validation.

Note well that the conclusion must follow from the premises; mere compatibility between the propositions is not sufficient to imply a connection between them. Thus, invalid syllogisms will display either a conclusion incompatible with the premises somehow; or a conclusion which, though compatible with them, is not more compatible than its contradictory is. Thus, validation could be viewed as the discovery of those forms of thought which satisfy a precondition set by Logic, namely that the premises be shown to imply the conclusion. Invalidity signifies failure of the syllogism to fall under the class so defined.

The validation process itself uses logic; but this circularity does not logically put it in doubt. This apparent paradox can easily be explained as follows. The science of Logic is merely a verbalization of observed fundamental phenomena (identity, inclusion, the need for consistency); these phenomena are out-there and in accordance with themselves; our science's task is to apprehend the exact extent and limit of their manifestations. If the use by logic, for the validation of its processes, resulted in an inconsistency with any of the apparent controlling principles of our world, we would be justified in questioning it. But so long as no inconsistency is found, it must be trusted. For to say that the validation processes depend on their own conclusions to work, merely confirms how basic this science is. Whereas, the attempt to cast doubt on logic itself appeals to our logical instincts for its credibility, and therefore constitutes an inconsistency, that between the primary denial and hidden dependence on logic. Of the two theses only the former, then, is self-consistent. We conclude that validation is meaningful as a process of clarifying the consistency of valid logical processes with the axiomatic basics.

2. Methods.

Many approaches to validation have been developed by logicians. From the start, Aristotle was aware that each figure had its own character, and was able to identify the method of validation most appropriate to each. However, other methods are always worth exploring, to obtain further confirmation, to be exhaustive, and to train the mind.

a. The First Figure.

This is the most basic figure, and essentially defines the nature of subsumption and inclusion. It is validated by 'exposition'. Aristotle formulated the Law of Identity as "What is, is what it is". If a thing exists, it has certain attributes. If according to our perceptions and insights it appears that anything which is X is Y, then anything which appears to be X must appear Y. Our suppositions are justified, until otherwise proven, and we must submit to the reality of our experiences and to the meaning of our words. This reflects the self-evidence of the world around us, and attaches our words to their intention.

Thus, if one says 'All M are P', then indeed anything which is M, is likewise P, so that the (all or some) S which are M must also be P. If any S were not P, this would signify that some M are not P, and contradict our original assumption that all M are P. Therefore, granting the two premises, the conclusion follows, and **AAA**, **AII**, and **ARR**, are valid. The same can be argued in the case of a negative major, or we can reduce the forms **EAE**, **EIO**, and **ERG**, to the affirmative form, by obverting the major.

Subaltern forms of course follow by eduction. Syllogism with the contradictory conclusions to the above, are proved invalid, by opposition. No conclusion logically follows from all the remaining pairs of premises, so they have no validity as syllogistic processes.

b. The Second Figure.

Reduction consists in demonstrating that the validity of one inference proceeds from that of another, already established. Reductio ad absurdum shows that a major (A) and minor (B) premise together imply a conclusion (C), because if A were asserted together with the negation of C, they would together imply the negation of B, through an already validated process. This method, with the major premise kept constant while the rest is tested, is used to validate the second figure by reference to the first.

Thus, 'All P are M and No S is M' imply 'No S is P', for granting that all P are M, if some S were P, then some S would be M, which contradicts our original minor premise that no S is M. In this way, **AEE** in the second figure is reduced to **AII** in the first figure, through a syllogism involving the original major term as middle term. We can proceed likewise to validate the other valid moods of the second figure.

Although so provable, the second figure should also be viewed as reasonable on its own merit, by exposition. Because essentially it defines for us the mechanics of exclusion, just as the first figure reflected more those of inclusion. If we consider two things one of which is excluded and another is included in a third, they cannot reasonably be visualized as contiguous.

c. The Third Figure.

Validation by exposition seems to be the method most suited for the third figure, although again other approaches are possible, because the conclusion is always particular. We proceed by showing that in certain instances under scrutiny, two events are contiguous (because the whole includes the part), so that the conclusion holds. This is a positive approach, which can be buttressed by reductions.

Thus, we could take **AII** in the third figure, and reduce it ad absurdum through **EIO** in the first figure. We test the effect of contradicting the conclusion while holding on to the minor premise, this time. The resulting syllogism has the original subject as its middle term, and its conclusion contradicts the original major premise. We can likewise validate the other valid moods of the third figure.

Of course, we could use similar methods to reduce the third figure to the second, or vice versa by changing our constant.

d. The Fourth Figure.

Here, direct reduction is the most natural treatment. The two premises of **EIO** in the fourth figure are each converted, to yield **EIO** in the first figure, which results in the same conclusion. Alternatively, convert the minor premise only, and reduce to the second figure; or convert the major only, to obtain the third figure.

Reductio ad absurdum is an indirect form of reduction, which we use quite often in everyday thinking. Another approach, just sampled, is direct reduction. This is more formal minded, in that one or both premises are subjected to an eductive process to reduce the syllogism to a first figure mood with the same conclusion or one implying it. This method is not restricted to the fourth figure, but can equally be practiced in the second and third. For example, for **AEE** second figure, the major is converted by negation, and the minor obverted, to obtain **EAE** in the first figure. Again, for **EIO** in the third figure, the minor is converted, to yield **EIO** in the first. The full list of such processes is easy to develop and well established, and available in most logic text books, so we will not here belabor the reader with excessive detail.

e. Secondary Syllogisms.

Though subalterns could be analyzed independently, once a subalternant syllogism is established, its subalterns are easily seen to follow by eduction.

With regard to the imperfect syllogism, combinations of premise which do not yield a normal S-P conclusion, but nevertheless can be wrung-dry to yield a nonS-nonP conclusion, they are dealt with by direct reduction through valid third figure arguments. For the first figure, use obversion of the major and obverted conversion of the minor; for the second, contrapose the two premises if positive, or use obverted conversion if negative; for the third figure, obvert both premises; for the fourth figure, draw the obverted converse of the major and obvert the minor.

f. Rejection.

As already indicated, this is the process of invalidation, and of course should be applied to each and every invalid mood systematically. The method is similar, since a mood which concludes something contradictory or contrary to our valid forms must be rejected. More broadly, forms which are not established as valid somehow, are automatically kept apart: the onus of proof can be left to them, as it were.

3. In Practice.

The science of Logic has, as above, analyzed validation and invalidation processes used to establish the general truth of the reasoning processes described in the previous chapter. Whereas it works in formal terms, we normally do not refer to formal logic in practice to verify our thinking or spot fallacies in it. We repeat the expository or reductive processes, every time we need to understand or convince ourselves of an argument, with the specific contents of our propositions. Going through such a process serves to integrate our knowledge, comparing its elements and checking their consistency.

Once, however, one is trained in logic, one may well refer to the science's findings to unravel some argument. In this context, the rules and the canons of Logic may be appealed to intellectually. Analysis of the quantities and polarities involved, consideration of the distribution of terms, are then valuable tools, if one has them well in mind.

A popular way to verify that arguments are kept in accord with logical rigor, is through application of the fallacy tests developed by Aristotle and logicians since. These warn of common pitfalls which one may encounter. They reveal how one may, through hidden equivocation (the Four Terms), confusing suggestions (as in the Many Questions), self-contradiction (Begging the Question), or other such devices, befuddle ourselves or others. Study of these, found in most text books, is of course valuable training.

4. Derivative Arguments.

We have stated that syllogism involves three, and only three, propositions; and likewise three and, only three, terms. In practice, it may seem that other possibilities exist. But logic shows that such atypical argument is actually either abridged or compound syllogism, which can be reduced to the standard formats.

a. ***Enthymemes*** are syllogism a premise or the conclusion of which is left unstated, but which is clearly taken to be understood or implied. This artifice is common in normal discourse, as when we rely on context, and can only be formally validated by bringing the suppressed proposition out in the open, and checking that the argument obeys the rules of logic.

An ***epicheirema*** is an argument in which one or both of the premises is supported by a reason. This simply means that the explained premise is itself the result of a prior syllogism.

b. We often have trains of thought: these may be reduced to chains of two or more syllogisms, of any kind. Such an entangling of argumentation is called a ***sorites***. The name is more traditionally applied specifically to certain regular chains of argument in the first figure, which suppress intermediate conclusions. These are as follows:

All (or Some) A are B,
All B are C,
All C are D,
All D are E,
All (or No) E are F,
therefore All (or Some) A are (or are not) F'.

We move from a universal or particular, but always affirmative, minor premise, through one or more intermediate universal affirmative premises, to a final affirmative or negative, but always universal, major premise, to obtain a conclusion with the quantity and subject of the minor premise and the polarity and predicate of the major.

There are thus four valid moods. **AAAA**, **AAEE**, **IAAI**, **IAEO**, for each set of three or more premises. The validation of these is achieved by listing a series of syllogism with the same result. For instance:

A is B and B is C, therefore A is C;
A is C and C is D, therefore A is D;
A is D and D is E, therefore A is E;
A is E and E is F, therefore A is F.

The conclusion of each syllogism is used as premise in the next, if any. Clearly, the middle terms must all be distributive.

The name 'sorites' could equally be applied to any complex of arguments, in any combination of figures, instead of just to such a regular series of first figure syllogism. Irregular sorites takes the conclusion of any unit of argument, and transfers it to another argument where it serves as a premise.

Thus, sorites in the widest sense is simply the multiple branching of thought in all directions. Each unit argument within this network may be indicated by only a highlight — a premise or two, or a conclusion — the most significant or controversial part. A sorites is a collection of such highlights, an abridged argument.

c. Certain arguments called immediate inference by **added determinants** or by complex conception, seem like immediate inference, but are really mediate inference. This refers to arguments like 'since X is Y, then ZX is ZY'. If the qualifying Z is an adjective, the argument is valid, since if some X are Y, and all X are Z, we may infer, in a third figure syllogism, that some Y (those which are X) are indeed Z. But if the Z clause does not fit in such a valid syllogism, it in some cases cannot be passed on.

In practice, such argument can easily be fallacious, as a result of double meanings (as in 'science is fun, so scientists are funny'), or the use of terms in inappropriate ways (as in 'horses are fast, so the head of a horse is the head of a fast').

Such rough logic is not very reliable, and should not be considered a part of formal logic. It is better to insist on strict conformity to formal processes. If a specific kind of content allows for special logical rules, then these may be clarified explicitly in a small field of logic all of their own.

PART II. MODAL CATEGORICALS.

11. MODALITY: CATEGORIES AND TYPES.

1. Seeds of Growth.

Aristotelean Logic, we have seen, deals with categorical propositions of the form 'S is P'. The copula 'is' is often conceived as having an absolute or timeless quality; it is viewed as the essential relationship between things in a scientific body of knowledge. Although this knowledge may involve particular statements, their role is merely that of either stepping stones towards eventual general statements or tools for denying general statements. Science's goal is mainly to discover universals. For this reason, time, change, and causality were not given formal attention in the traditional approach.

But if Logic as a science is to be universal in scope, it must go into deeper detail, and analyze the full range of existing phenomena reflected in language and everyday thought processes. This is painstaking, perhaps never-ending, work. Logic is not to be confused with grammar; it is not primarily concerned with the structure of sentences, which may vary from one language to another, and indeed sometimes seem illogical. But Logic can observe commonplace statements to identify possible areas of interest for treatment in its peculiar way. In any case, its ultimate goal is to say some general things about reality, and about how we may properly think about it.

In this perspective, then, classical Logic is but a beginning, a specialized investigation which needs to be pushed further gradually. In this chapter, we will indicate some of the possible areas of expansion for our discipline.

The concept of modality is extremely interesting, because its detailed development has a powerful systematizing effect on logical science. From the seeds of thought provided by a few insights, like postulates, a large chunk of knowledge can be organized into a formal whole. A relentless progression of problem solutions and predictions is put in motion, providing us with exciting tools for the growth of knowledge.

A theory is ultimately judged not only by its consistency and truth, but also by its fruitfulness. The distinctions and classifications, the understandings and guidance, which modality generates, show its importance to advanced logic, and thereby to the broader concerns of philosophy.

Modalities are certain qualifications of relations, expressing the frequency of events, within some framework. In the deepest sense, modality is concerned with the differing and varying *levels of being*; hence its central place in both ontology and epistemology. The study of modality could be called 'Tropology': it is a broad field.

The term modality may be used in the sense of a 'category of modality', or in the sense of a 'type of modality'. Category refers to the frequency aspect, type defines the framework. When referring to the modality of a relation, we may mean either of these senses, or their intersection. For we find, within each type of modality, the same categories of modality, only with a somewhat different meaning.

The types and categories define the multiplicity of ways in which anything may said to 'exist'.

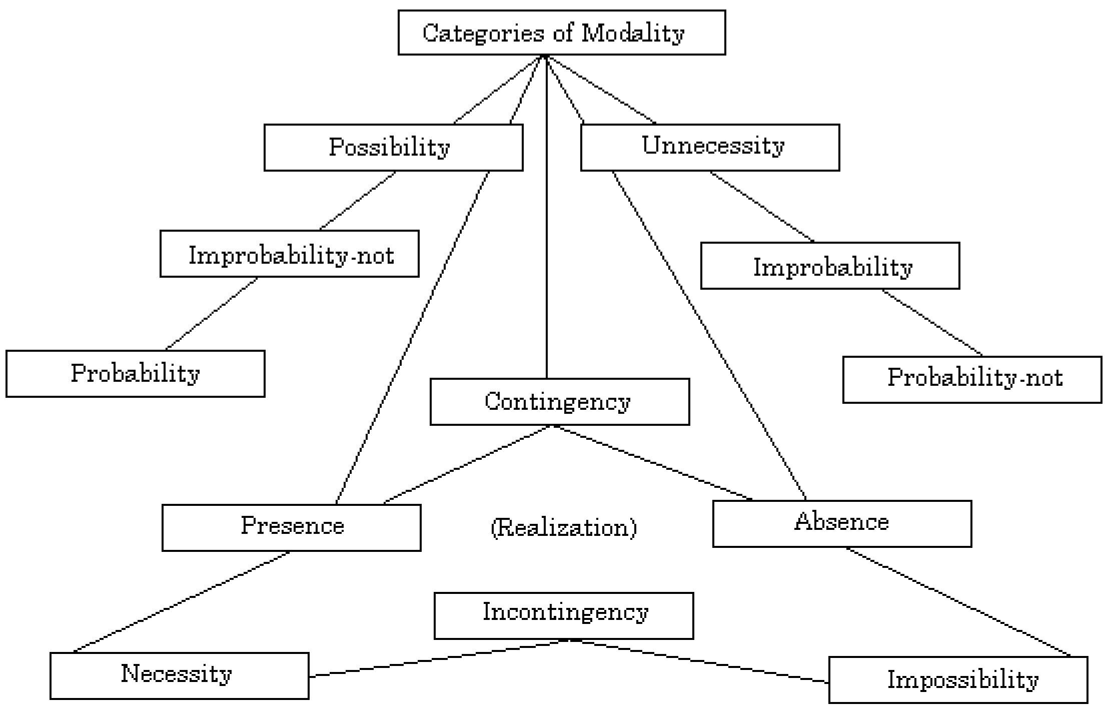
2. Categories of Modality.

We call categories of modality the concepts of possibility or necessity, impossibility or unnecessity, contingency or incontingency, probability or improbability and their degrees — as well as presence or absence.

These terms will all be more fully defined further on. Meanwhile, let us note that they are interrelated in various ways. The following tree illustrates some aspects of their interrelationships.

Presence signifies the occurrence of an ostensible individual phenomenon, a unit clearly defined in time and place; and absence is the negation of this. Presence is a class standing under possibility and above necessity; absence, between unnecessity and impossibility. Presence or absence occur either because of incontingency, or through the realization of contingency.

Diagram 11.1 Tree of Modalities.



Possibility may be viewed as a generic concept which embraces either contingency or necessity. Likewise, contingency and impossibility may be viewed as mutually exclusive species of unnecessity. Contingency signifies possibility and unnecessity taken together. Incontingency a genus for necessity or impossibility. The various degrees of probability are subcategories of possibility or unnecessity.

In practice, these concepts are expressed in sentences by words like ‘in some cases’, ‘sometimes’, ‘can’, ‘may’, ‘might’, ‘possibly’, ‘potentially’, ‘permissibly’, ‘perhaps’, and all their related terms. The differences between these modal expressions are not merely verbal.

Indeed, in normal discourse, we tend to interchange terminology indiscriminately. For instances, in some cases we say ‘always’ to mean ‘all’; in some cases, ‘can always’ means ‘all can’. This is not our concern as logicians: we identify the connotations closest to what we are trying to discuss, and henceforth adopt restrictions which serve our purposes.

3. Types of Modality.

I have identified *five* main types of modality, five senses in which the various categories of modality may be understood. Within each type, all the categories occur, but with other meanings than in the other types. The categories have similar interrelationships and properties within each type. These uniformities allow us to abstract them, but ultimately each type needs to be considered separately. The interactions between types must also be analyzed.

Quantity, or extensional modality, is the primary type of modality, and is the one which was thoroughly dealt with by Aristotle. Two more, temporal modality and natural modality, will presently be analyzed in detail; they interact intimately with quantity. The last two types, logical modality and ethical modality, are each sui generis, and require independent treatment.

It will be soon be evident that the temporal and natural modalities have characteristics in common with quantity. They represent different ways the subject and predicate might be related. They can be combined in certain ways with quantity, to form complex propositions. They are mutually related, in fact form a continuum, although they cannot be compounded together as they can be with quantity. They are subject to rules resembling those found for quantity, because they derive from the same geometric fundamentals.

Each type of modality has its own character. Quantity refers to the proportion of a whole class that is subject to a certain relation to a predicate. Temporal modality refers to the proportion of its whole existence in time that any individual subject happens to have a certain relation to a predicate. Natural modality expresses the degree of causal conditionality concerning such relation.

Extensional modality recognizes the variations which can be found to exist between instances of similar phenomena, be they static or dynamic. Temporal modality proceeds from the occurrence of change in individual things during their existence. Natural modality stems from the belief that 'laws' guide events. Our world is diverse in all these senses. There is thus an ontological basis for such distinctions.

Furthermore, Logic must investigate the differences and similarities in behavior of such phenomena, and the results of their interplay. Here, then, is a possible area of new activity for Logic, clarifying the meanings of forms involving modality, and analyzing their oppositions, the eductions possible from them, and the syllogistic arguments involving them. This topic will be dealt with in considerable detail in this treatise.

The two types of modality we are introducing here are effectively qualifications of terms similar to distribution, although strictly speaking they apply to the relationships of terms. Such propositions are complex variations of the standard forms researched by Aristotle, involving an additional factor, modality, which can be subjected to whole-and-part, inclusion-exclusion type analyses, as was done with quantity.

4. Extensional Modality.

Consider the ways in which we use expressions of possibility or necessity. As stated previously, we are in everyday discourse not consistent in our use of terms like 'sometimes', 'can', 'may', 'might', 'must', and so on. Ultimately these are semantic issues, not important to us, though they need pointing out. Logic simply establishes conventions for terminology, and focuses on the material issues.

Now, it happens, for instance, that when we say 'S may be P' we mean 'some S are P'. This use of a modal-looking qualification to express quantity is not accidental. When considering a specimen of S, we may want to note that the fact that other S have been found P suggests that this one also could fall in that group for all we know.

But is there an ontological basis for considering quantity a type of modality, or are we just dealing with a mode of thinking, a useful artifice? Quantity is essentially a qualification of universals, which we suppose to have some kind of reality, although we cannot yet understand their nature adequately. When we say that some S are P, we are not merely intending to express a quantitative fact, but to affirm the compatibility between 'S-ness' and 'P-ness'.

This is traditionally known as the distinction between viewing a concept in its extension (the units it applies to) and its intension (its meaning). A universal may be viewed as a 'substance' (or stuff) which is scattered in the world. When two universals, S and P, coincide in some entities, we learn more than simply the fact of contiguity; we learn that the natures of the two universals do not intrinsically prevent such occurrences, and this is for us significant information.

Similar argument is possible for the other quantities. They tell us of the compatibility or incompatibility, necessity or contingency, high or low probability of coincidence between universals. The numerical aspect of quantity is incidental, though Logic develops by concentrating on it because of its manageability.

Social statistics, for example, are mostly based on this approach. The information we obtain concerning a social group is applied to each individual in the group, with the corresponding degree of probability. The mere fact that most individuals in a sample behave in a certain way, should not imply that there is any possibility that individuals who did not behave in that way at all could have. And yet we do feel justified in so reasoning, because we believe that reality functions through the forces inherent in universals.

As stated before, although many skeptical philosophers have denied validity to such modes of thinking, my position is pragmatic optimism. This is the position of science: that even if an appearance is not fully understood, it is received with an open mind, provided or so long as no inconsistency arises from the belief.

Humans inevitably conceive the world in terms of universals; therefore it appears that they exist. That they are difficult to fully grasp does not mean they are untrue. Only if they were logically contradictory to evidence, would doubt be reasonable. But no credible cause for doubt has arisen. Indeed, most importantly, to deny universals through some speech, is using universals to deny them: that position is the inconsistent one of the two, and therefore absolutely false.

So when we say that ‘some S are P and some are not P’, we still believe that there was a ‘possibility’ even for S which are not P, to have been P (or vice versa), although they did not happen to concretize in this way. We think this, because the universals S and P (or nonP) have displayed compatibility in some cases of their existence.

Thus, we mentally distribute not only generals, but even particulars to each of all the individuals involved, via the universals, while remaining aware that the factual concretization of the universals in that contingent way is final. In this sense, quantity can legitimately be viewed as a type of modality.

It must be stressed, however, that extensional modality differs radically from temporal and natural modalities, in that it can be combined with either of them, whereas they cannot be superimposed with each other. That is because they are really part of the same modal continuum of individual capabilities, whereas quantity remains essentially a factor concerning groups of phenomena.

Temporal and natural modality may be called ‘intrinsic’ modalities, because they concern the properties of concrete individuals; extensional modality is comparatively ‘extrinsic’, in that it focuses on abstract universals.

5. Temporal Modality.

While it is true that often the copula ‘is’ is intended in a timeless sense, we sometimes use the word with a more restrictive connotation involving temporal limits.

The temporal equivalent of what is a singular instance in extension, is a momentary occurrence; this is the unit under consideration here. When we say ‘S is P’ we may mean either that S is always P, or that S is now P, or even that S is sometimes P. This ambiguity must be taken into consideration by Logic explicitly. A possible modification of standard propositions is therefore through the factor of temporal frequency.

We can say of an individual S that it is now or not-at-this-time P, or sometimes or always, or sometimes-not or never P, or usually or rarely P. We recognize that a thing can vary in attributes during time, and often use such forms to express such experiences. Such propositions can in turn be quantified, so that complex combinations emerge.

According to the traditional approach, we are supposed to deal with these forms simply by attaching the frequency qualification to the predicate, to obtain a new predicate. This process is called permutation; we encountered it previously, in the context of changing propositions into the "is" form, and obversion is a sample of it, too. Tradition has assumed that once permuted, such propositions can be processed in the normal way, through Aristotelean syllogism.

But this first impression was wrong; the device is misleading where modality is concerned, for three reasons. Firstly, it fails to account for a large number of practical inference, whose validity can only be established through analysis of the propositions in their original forms. With such propositions in their permuted forms, syllogisms would contain a middle term which is not identical in the two premises (for example, 'S is sometimes-M, M are always-P'), or a minor or major term not identical in premises and conclusion (for example, the conclusion 'S is sometimes-P' from the said premises).

Secondly, and even more importantly, permutation can result in erroneous inference. For, in fact, as analysis shows, we cannot always transmit a frequency unchanged from premises to conclusion (for example, as in 'S is M, M are always-P, therefore S is always-P'), and sometimes not at all (for example as in 'S is M, M are sometimes-P, therefore S is sometimes P').

Thirdly, in some cases, we can deduce from a given frequency, not capable of being itself simply transmitted, another, lower frequency; if we merely relied on permutation, the conclusion would not be formally valid. (For example, in 'S is M, M are always-P, therefore S is P'). So we have no choice but to demand special treatment; the issues are more complex than we are led to believe by the permutation theory.

All this will become clearer by and by. It will be seen, as the analysis of modal forms proceeds in full detail, that, although our method of analysis is similar to Aristotle's, we cannot mechanically reduce temporal modality arguments to traditional forms. Such situations must be investigated systematically, and special principles must be formulated to guide our reasoning in relation to them. The results obtained are often unexpected and instructive, and justify our research effort.

Temporal modality is especially useful, when reporting the behavior patterns of organisms; this is especially true for animals, who have powers of volition, and even more so for humans, who we consider as having free will. For, with regard to certain actions or states of such subjects, we cannot say that they 'must' or 'cannot' do or have them, in the sense of natural determinism, but only that they always or occasionally or never do so.

Thus, for instance, we can study the psychology of people, and predict their reactions to some extent, without having to postulate a more rigid degree of necessity than mere constancy, and before being able to explain volition or free will.

6. Tense and Duration.

We have indicated that the unit considered by temporal modality is a moment of existence. But 'now' is not the only individual moment we can refer to. The individual moment involved may be located anywhere in time, past, present, or future; and that location may be expressed precisely, by date and time o'clock, or roughly. This issue is known to grammar as tense, and we may adopt the same name for it in logic.

Also, the individual moments we speak of vary in size. The segment of time involved may be a fleeting moment, or an extended period of time; it may be expressed vaguely, or precisely, as a year, week, hour, or microsecond. This is an issue of duration.

These different units in the continuum of time, defined by the tense and duration of existence, of the subject and predicate relation under scrutiny, are the instances of the 'class' under consideration in the context of temporal modality, in analogy to the cases of a universal in the context of quantity.

We can in principle thus develop an infinite list of possible tense/duration characterizations for propositions, according to where in the time continuum the event is projected, and for how long. Thus ‘things S and P’ could mean: things now S and P, or which were or had earlier been S and P, or which will be or are later going to be S and P; and the time locations and periods tacitly intended could be specified explicitly.

Here again, following the permutation idea, we would suppose it possible to merge the tense into a term so related, to form a new term capable of timeless treatment; for example, ‘S was P’ would become “S is a ‘was-P’”. This presupposes that, provided no equivocation was involved, a proposition so altered could then enter into a syllogism without causing problems.

However, in fact, this artifice does not work; it conceals the validity of certain arguments which it assumes false, and it causes us to assume certain arguments correct, which closer inspection reveals false. So a specific analysis is required. These claims will be seen evident as formal treatment proceeds.

Tense is not in itself a distinct type of modality qualification; but an integral part of the doctrine of frequency. It simply defines the possible variety of locations in time, besides the elementary ‘now’; without awareness of them, we might make logical mistakes.

However, apart from these general guidelines, the topic of tense will not be developed in full detail in this paper. It is enough for our present purposes to make the reader aware that our use of the expression ‘now’ is intended to include past and future nows, and nows of any size. So long as the now involved in any argument is one and the same, the rules we will establish for such arguments will work. The possible interactions of different nows will not be covered, however.

7. Natural Modality.

The most significant type of modality is what I call natural modality. This refers to propositions such as ‘S can be P’, ‘S cannot be P’, ‘S can not-be P’, and ‘S must be P’, with the sense of real, out-there potential or necessity. These relations were effectively recognized by Aristotle in his philosophical discussions, but were not systematically dealt with in the framework of his logic works.

Note in passing that often, when people write ‘S can not be P’, they mean ‘cannot be’ rather than ‘can not-be’; in the former case, the ‘not’ negates ‘can be’ (it means ‘not-can be’ in spite of its position in the phrase), whereas in the latter, the ‘not’ only negates the ‘be’.

Such modality differs radically from temporal modality. We do not here merely recognize that something may be sometimes one thing and sometimes another, or always or never so and so. We tend to go a step further, and regard that there is a character intrinsic to the object which makes it able to behave in this way or that, or incapable of doing so or forced to do so. Thus, temporal and natural modalities represent distinct outlooks, which cannot be freely interchanged.

We can infer from S being sometimes P, the implication that it can be P, arguing that otherwise it would never be P; likewise that S is sometimes not P, implies that it can not-be P, or else it would always be P. But when we say that S can be P or nonP, we mean something deeper than merely an observed conjunction. We often claim, through indirect discovery, to know that S can be P (or nonP), even though this potentiality is never actualized. Whereas, with S is sometimes P (or nonP), we are making a statement that requires the relation of S and P to be actualized at least once.

Similarly, we may induce, in the way of a generalization from experience, from S always being P (or never being P), that it must (or cannot) be P. But when we say that S must (or cannot) be P, we intend a more profound relationship than mere constant recurrence (or nonoccurrence, as the case may be). We claim knowledge of the inner nature of the object (whence my choice of the term ‘natural modality’, by the way); we claim to be explaining why the observed constancy took place. We may thereafter discover indirectly that S can be and can not-be P; we would then conclude that, although S is always (or never) P, this is not a case constancy due to necessity, but just the way a contingency was actualized.

The indications here given should be enough to clarify ostensibly what phenomenon we are trying to refer to. Before discussing the concept of natural modality further, on a more philosophical plane, a pragmatic definition, sufficient for the needs of logical science, will be proposed.

An event is said to be potential if it occurs *in some circumstances*; it is said to be naturally necessary if it occurs *in all circumstances*. Unnecessity is, then, nonoccurrence under some circumstances, and impossibility occurrence under no circumstances.

This concept of circumstance refers us, then, not to time as did temporal modality, but to the assumption that, scattered in the environment of an event, are certain causative factors, be they known or unknown, specified or unspecified.

'S can be P' thus means 'When certain causes occur, S is P', 'S can not-be P' means 'Under certain conditions, S is nonP', 'S must be P' means 'In all situations, S remains P', 'S cannot be P' means 'Whatever the surrounding circumstances, S remains nonP'.

That definition justifies our calling this phenomenon a type of modality, because, like the previous types of modality (temporal and extensional), it is reducible to an issue of enumeration: we use the same ideas of whole and part, inclusion and exclusion, all/this/some, frequency.

In the case of extensional modality, we are dealing with instances of a universal; in that of temporal modality, with moments of an existence; in natural modality, with causal conditions. All these implicit concepts are admittedly inscrutable in their essences, but their applications are numerical and so capable of systematic treatment by logical science.

We can argue, as we did for temporal modality, that natural modality is not permutable. I will not repeat the arguments here, especially since this truth becomes so obvious once we start dealing with formal issues.

8. Other Types.

Two other main types of modality, the logical and the ethical, need to be indicated to complete our introductory synopsis of the topic. As previously stated, these types are each sui generis, and worthy of thorough treatment on their own. Logical modality will be dealt with later in this work, but ethical modality is left to some future volume.

What distinguishes these types from those previously considered, is their object of attention. Extensional, temporal and natural modalities tell us something concerning the subject and predicate related themselves. Logical and ethical modalities, in contrast, either report about the state of our knowledge, or make recommendations for action, in connection to those objects.

a. ***Logical Modality.*** This expresses the compatibility or otherwise of a proposed assumption with the general framework of our knowledge to date. Logical modality makes use of terms such as 'might' (or perhaps) and 'surely'(or certainly), for possibility and necessity. Remember that we defined truth and falsehood as contextual, so this definition fits in consistently.

To the extent that such an evaluation is scientific, based on rigorous process, thorough, common public knowledge, and so on, it is objective information. To the extent that thought is deficient in its methodology, such modality is subjective.

Whereas the extensional, temporal and natural types of modality may be called 'materialistic', in that they refer directly to the world out there, which is mainly material or in any case substantial, logical modality may be called 'formalistic', because it operates on a more abstract plane.

b. ***Ethical Modality.*** Ethical statements tacitly refer to some value to be safeguarded or pursued, and consider the compatibility or otherwise of some proposed event with that given standard. We use terms such 'may' (for permissibles) and 'should' (for imperatives), to indicate ethical possibility or necessity.

Ethical modality is of course relative to standards of value. The complex issue of how to establish absolute standards, or whether we are able to, will not be discussed here. Suffices to say that, within a given framework, an ethical statement can in principle be judged true or false like any other.

Subjectivity comes into play here, not only in the matter of selecting basic values, but also to the extent that, in this field more than any other, factual knowledge is often very private.

Logic must, of course, eventually analyze such modality types in detail. But for our present purposes, let us note only that, in either case, the resemblance to the other types of modality is the aspect of conditionality. They are defined through the conditions for their realization.

Their distinction is that they do not concern the object in itself (i.e. the S-P relationship as such) like the others, but involve an additional relation to man the knower of that object, or man the eventual agent of such object. The latter relation is thus a new object, which includes the former, but is not identical with it. Such modalities, then, are not essentially subjective, though they can degenerate into subjectivity, but rather concern another object.

The reader should beware of the various ways the words 'modality' or 'modal' will be used in this volume. In its broadest sense, 'modality' applies to any type and category of modality, which details should be specified, and every proposition is 'modal'.

In practice, we sometimes use the word 'modality' to refer specifically to the natural, temporal or extensional types of modality, to the exclusion of the logical. Sometimes, the sense is restricted to only natural and temporal modality, as distinct from quantity. Likewise, we may in some cases call a proposition 'modal', to signify that it is other than actual or singular or factual.

The context should always make the intent clear.

12. SOURCES OF MODALITY.

1. Diversity.

Underlying the existence and concept of modality, are the phenomena of difference and change. At any given time, the world appears as a multiplicity of distinguishable phenomena, distributed in space; and across time, the world reappears before us, comparatively differently constituted and deployed.

a. The concept of difference implies that of similarity. If everything was absolutely different from everything else, things would not coexist: they would have nothing in common, neither existence nor space nor time nor any character, each would have to be a 'world' by itself; and only one such 'world' could exist, which would have but one point of space and time. Thus, paradoxically, to deny similarity is to deny difference; to posit a world consisting only of diversity logically implies that we believe the world to contain no diversity at all, not even dimensions.

While it seems obvious that the phenomenon of motion requires that we postulate a time dimension, a universe devoid of motion but extended in time seems conceivable. While a universe of only two, or even only one, space dimensions (with or without motion) seems conceivable — a universe devoid of space or time (an unextended point and instant, rather than a minuscule and short-lived one) seems unthinkable, impossible not to measure up against infinity and eternity.

Diversity is of various kinds. A thing may for instance be both green and flat, at the same time and in the same place; this is diversity of character in the purest sense, a coincidence of 'incomparables'. A thing may be both green and red, but only in different places of it or times in its existence; or it may partly or wholly move, and occupy two different places, though only at different times; these are diversity of comparable characters or of location, made possible by the existence of space and time dimensions.

b. With regard to diversity in time, there is no escape from the fact that change appears to be happening; this phenomenon is alone sufficient to demand from us a recognition and concept of change, independently of whether we evaluate given cases as specifically real or illusory.

Some change is indeed illusory, meaning that the newly perceived difference was already there, but was previously unperceived. The 'change' is due to the movement of the spotlight of our consciousness, rather than to an event in the object itself. The concept of illusion is built on such 'changes of mind'; we are not omniscient, our knowledge evolves; every appearance is assumed real, unless or until it fails to be consistent with the mass of other appearances, in which case it is reclassified as illusory.

But some of the change has to be real: to deny this is logically untenable, because even an 'illusion' is in itself a specific kind of object. What we call 'illusory change' is more precisely a real change from one illusory appearance to another illusion or a reality. The form of change is real enough, it is its particular content which is illusory.

Even if, in an attempt to explain away time and change, all apparent mobility in the material world were attributed to the travels of consciousness, we would still be left with the need to understand the latter movements as themselves changes dependent on time. Therefore, nothing radical is to be gained from such an attempt.

c. It may well be that, at some higher plane of being and consciousness, the world merges into undifferentiated and immobile oneness, which is somehow more 'real' than our ordinary, sublunary experiences, because it allegedly unifies and explains them. However, this idea does not logically deny the side by side existence, in some respect, of the variegated and dynamic, illusory lower world.

The appearances of difference and change may be illusory, may be inferior on some spiritual scale, but even so, they have to have a sort of existence. It is a phenomenon presented to our consciousness, which we cannot avoid admitting to exist as such, even if we believe it to be a warped image of an otherwise uniform and static reality. It is conceivable that at some past or future time the world was or will become One; but this in no way excludes the current existence of some form of difference and change, as appears, if only as appearance.

d. In conclusion, the world must stand somewhere between the extremes of absolute diversity and absolute unity, which are incidentally one and the same idea. The mere experience of difference and change, whether real or illusory, is enough to guarantee this fence-sitting position. We cannot logically evade or wipe out this given phenomenon; we can only at most delimit it to some narrower domain and relegate it to some lower status.

Note that mystics of many traditions claim that the conflict of dualism and monism is itself illusory, and that at some higher level the contradiction disappears convincingly. While keeping an open mind toward such a special experience, we may plod on with a logic designed for our commonplace world.

e. Once difference and change are admitted, concepts such as polarity, similarity and quantity, time, modality and causality, are inevitable and needed. If the world was, against experience, without diversity or change, there would be but one polarity, one entity, one character, no space or time, no contingency, only necessity, and no need for causal explanations.

2. Time and Change.

Time and change appear to be extremely fundamental phenomena in our world and experience, and simultaneously very mysterious and difficult to analyze conceptually. At first sight, they seem evident and obvious, but once we try to understand them in a deep way we uncover a mass of difficulties and complexities.

As far as I am concerned, no satisfactory solutions to the crucial problems involved have been found. Some of these ontological issues will be touched upon in later discussion, but for the most part we will bypass them and concern ourselves with formal logical issues.

With regard to Time, let us pragmatically accept, as appears intuitively, the existence of past, present and future, and that events somehow occur in measurable relative locations in this continuum, which we imagine to be a dimension similar to the three of space.

As for Change, it may be pragmatically defined as occurring when something has one property at one time, and not at another time; or lacks it, and later has it; or, compositely, exchanges one attribute for another. Things change across time, losing properties, acquiring new properties, or changing in degree of some otherwise enduring property, or replacing properties. They may change place (that is motion), or qualitative attributes (alteration), or even change pattern of movement (acceleration) or vary in uniformity of qualitative change. Some changes are irregular, some cyclical, and so on.

3. Causality.

An assumption that man regularly makes in his cognition of the world, is that objects behave in the way they do, not merely by happenstance, but because this is somehow programmed into what they are, as part of their identity, their nature. This reference to the inner nature of things is a reference to causality, in its widest sense. We believe not merely in the coincidence of the thing and its attributes, but that the particular identity of that thing has caused it to display this particular behavior rather than any other.

Some philosophers deny this assumption, and claim that all we can say is that things just occur, not that they somehow had to. As with all insights, it is a function of the rules of logic to resolve the debate. I accept the common sense viewpoint, because I have found it consistent and useful.

It seems to us to be so, that there is such a thing as causality, it is one of the appearances in the world, we instinctively think in such terms. If there were some solid reason to deny the concept, we would have to, but no convincing argument has been presented by the skeptics so far. Doubt on the mere basis of difficulty of precise definition or explanation, is not logically sufficient. Logically, some things are bound to be irreducible; why not causality? We can know that it is there somehow, while admitting our inability to adduce its essence, in view of its fundamental nature. There is no self-contradiction in this position.

Fundamental phenomena, like universals and difference, time and change, causality and necessity, are inevitably difficult to fully describe and understand, and perhaps even ultimately, in principle, undefinable and incomprehensible. They are so radical to our world that they cannot be reduced to something else. But this in itself is not a reason to altogether reject them. And indeed, even if some philosophers choose to reject them gratuitously, it changes nothing. People will rightly continue to think in these terms, trusting appearance, unintimidated. It works.

The concept of causality is indeed extremely difficult, if not impossible, to define. It is, like attribution, something we intuitively understand, but which is so fundamental that, although we can discuss it to some extent, we can never pin it down. What we can do with relative ease, however, is identify its varieties.

In the widest sense, any event signifies causality; the nature of an object is viewed as the underlying cause of its 'behavior'. In this sense, any attribute or change, be it permanent or inevitable, or transient or accidental, is caused by the thing being what it innately is.

But more specifically, causality is limited to the suggestion of necessity. It is most often related by philosophers to time and change, or the explanation of movements. But in fact, in practice, even in the empirical sciences, we conceive it as a force explaining static, as well as dynamic, connections.

Indeed, it will be shown further on that there is one type of causality corresponding to each type of modality. 'Extensional' causality concerns uniformities or differences relating to universals. 'Temporal' causality concerns constancies or changes across time. 'Natural' causality relates to necessity or contingency on a deeper level. 'Logical' causality concerns the relationships of ideas.

The pragmatic definition of causality by David Hume as merely "constant conjunction", simply does not adequately capture what we intend by this concept. J. S. Mill equated natural to temporal modality, in an attempt to bypass philosophical problems relating to the former's definition. He defined the way we induce causality by generalization, as a substitute for telling us what it is.

13. MODAL PROPOSITIONS.

1. Categories and Types.

Let us review some of the modal concepts introduced thus far, before examining them in more detail.

Modality in its widest sense is an attribute of relationships. The paradigm of modality is the quantity attribute of (the terms of) propositions. When phenomena are observed to be alike in some way, they may be grouped into a class, and be regarded as instances of that class.

We may refer to such units in various ways. The units intended by a reference are said to be included in it; those not so, excluded. When a unit is focused on individually and specifically (if only through a pointing to it), the reference is singular; otherwise, our focus is plural.

When we refer to a fraction of the class, it is particular; when to its totality, it is general. The greater division of a class is a majority; the smaller, a minority. Singular and particular frequencies concern mere incidence; the other plurals — generality, or majority or minority — are relative frequencies, and describe prevalence.

Quantity is one type of modality, namely the extensional. Other types of concern to us here are temporal modality and natural Modality. These have in common with quantity the mode of analysis defined above. However, the classes under consideration are not the terms of propositions, but respectively the temporal existence or the causal conditions of the connection between the terms.

Just as quantity concerns the application of a term to one, some, all, most or few of its instances; so temporal modality analyses the application of the predicate to one, some, all, most or few of the moments of its given subject's existence; and natural modality concerns the application of the subject and predicate relation to one, some, all, most or few, of the circumstances surrounding such happening.

These common factors may be called the categories of modality. They are: presence (unitary event), possibility (partial reference to the events-class), necessity (complete reference to it), high or low probability (inclusive of more or less than half the units). Derivative concepts are: absence (presence of negation, or negation of presence), possibility-not (possibility of negation, or negation of necessity), contingency (sum of possibility and possibility-not), impossibility (negation of possibility, or necessity of negation), and incontingency (either necessity or impossibility). These general categories may be given specialized names when applied to each type of modality.

In extensional modality, the main ones are, as we have seen, singularity, particularity, generality (or universality). In temporal modality, we will use the words momentariness, temporariness, constancy, for the corresponding concepts. In natural modality, actuality, potentiality, necessity.

Note that the sub-categories of possibility should not be taken to imply contingency, as often the case in everyday discourse; they are compatible with necessity. Also note our double use of words such as necessity for both abstract categories and especially natural modality sub-categories.

Additionally, let us point out that presence may be usefully viewed either as stemming from necessity or as an occasion of contingency. This way of viewing presence, as the realization of a deeper phenomenon of necessity or contingency, follows from the oppositional relations between these concepts, which will be analyzed below. Accordingly a singular instance may be viewed as the concretization, of either a generality or a distinction. A momentary event may be viewed as the eventualization, of either a constancy or a variability. An actual occurrence may be viewed as the actualization of either a (natural) necessity or a (natural) contingency. Similarly, on the negative side.

We reserve the following terminologies in formal treatment of these three types. This, some, all, most, few, will express quantity. Now, sometimes, always, usually, rarely, will be used to express temporal modality. Is, can be, must be, is likely to be, is unlikely to be, will express natural modality. In ordinary discourse, these various expressions of frequency, quantifiers and modifiers, are of course often interchanged.

It is stressed that all plural such expressions are intended to include the units they subsume on a one by one basis. That is, ‘in some or all cases’ means ‘in each and every one of the cases in the part or whole of the group under consideration’. It is not a collective reference to the units considered together. This quality applies equally to all three types of modality, each in its own domain (extension, time, circumstances).

Every proposition has quantity (implicitly if not explicitly); and every proposition has either temporal or natural modality. The unitary forms of these latter two modalities coincide; but their plural forms cannot be combined, being factors in one and the same continuum. That is, when we colloquially say ‘X can always be Y’, for instance, we may mean formally-speaking ‘All X can be Y’, but it is not possible to combine ‘can’ or ‘must’ with ‘sometimes’ or ‘always’ in the reserved senses of words, because, strictly, must implies always implies sometimes implies can, i.e. these concepts are related in specific ways, as will be seen.

2 List and Notation.

Aristotelean logic recognized six main propositional forms, as we have seen, labeled **A**, **E**, **I**, **O** and **R**, **G**. Actually, classical logic is usually developed in terms of the first four of these, i.e. the universal and particular. I added on the last two, i.e. the singular, to complete the picture systematically; they were not unknown to Aristotle, anyway. The labeling above mentioned is of course mere convention. Another notation could have been devised, using the letters **u**, **p**, **s** for quantity specification, and **+**, **-** for polarity. In that case, **A=u+**, **E=u-**, **I=p+**, **O=p-**, **R=s+**, **G=s-**. Generally, I have found it practical to continue using the letters **A**, **E**, **R**, **G**, **I**, **O**, in most work, though the separate labeling of quantity and polarity are sometimes valuable.

The value of this alternative notation becomes more evident once modality is introduced, because the laws of inference in Aristotelean logic can thereby be brought out more clearly. (Note how I often use the term modality in a restrictive sense excluding quantity.) By analogy to **u**, **p**, **s**, we may introduce the symbols **c**, **t**, **m**, for constant, temporary and momentary propositions, respectively; and **n**, **p**, **a** for naturally necessary, potential and actual propositions, respectively. (The equivocal use of ‘**p**’ for particularity and potentiality is perhaps unfortunate, but context will always make clear which of the two is meant, so it is not serious). The modality symbols may be used as subscripts to the standard six letters. The following is a list of all the categorical forms under consideration in this study.

a. Propositions involving natural modality. These, for the purposes of definition, could equally be expressed in the form ‘In all/this/some circumstance(s), all/this/some S is/is-not P’ (Or, ‘Under any/the given/certain conditions, all/this/some S is/is-not P’.) Note well the difference between ‘cannot be’ (which should have been written ‘not-can be’, to signify negation of potentiality) and ‘can not-be’ (signifying potentiality of negation).

An	All S must be P	En	No S can be P
Rn	This S must be P	Gn	This S cannot be P
In	Some S must be P	On	Some S cannot be P
A	All S are P	E	No S is P
R	This S is P	G	This S is not P
I	Some S are P	O	Some S are not P
Ap	All S can be P	Ep	All S can not-be P
Rp	This S can be P	Gp	This S can not-be P
Ip	Some S can be P	Op	Some S can not-be P

b. Propositions characterized by temporal modality. These can be defined by the overall form ‘At all/this/some time(s), all/this/some S is/is-not P’. Note that we here use the word ‘now’ equivalently to ‘at this time’, to avoid getting involved with issues of tense in this context.

Ac	All S are always P	Ec	No S is ever P
Rc	This S is always P	Gc	This S is never P
Ic	Some S are always P	Oc	Some S are never P
A	All S are now P	E	No S is now P
R	This S is now P	G	This S is not now P
I	Some S are now P	O	Some S are not now P
At	All S are sometimes P	Et	All S are sometimes not P
Rt	This S is sometimes P	Gt	This S is sometimes not P
It	Some S are sometimes P	Ot	Some S are sometimes not P

It will be observed that, in the above listing, we left out subscription of actual propositions with an ‘**a**’, and momentary propositions with an ‘**m**’. This was an intentional ambiguity, which will now be explained. If we analyze common usage of the form ‘S is P’, we find that it is really very vague and capable of many interpretations. This is not said as a criticism of Aristotle’s logic; in a way it has been one of its strengths, the reason why he seemed to have succeeded in describing human thought processes fully. But logic requires that ambiguities be brought out in the open, to ensure that nothing is left to chance. That is precisely why I have taken the trouble to develop a theory of modal logic, and researched it in such detail.

In its broadest sense, ‘S is P’ could be understood to mean any of the following: ‘S must be P’ (an absolute sense, often though not exclusively encountered in theoretical sciences), or ‘S is always P’ (a timeless sense, often found in empirical sciences), or ‘S is in the present circumstances P’ or ‘S is at the present time P’ (such meanings are usually intended in everyday descriptions of social events), or even no more than ‘S can be P’ or ‘S is sometimes P’ (with the qualification left tacit for purposes of stress). We are sometimes not aware of just how high or low on this scale our thoughts or statements fall; sometimes, though aware, we allow our meaning to be suggested by the context, or regard the distinction as not important enough to call for explicit expression. Sometimes, of course, our intention is not left tacit, and we say exactly what we mean.

To further complicate matters, the ‘S is P’ form is sometimes used in a likewise indefinite, but more restricted sense; that is, one not including natural necessity or potentiality, but broad enough to include any temporal modality. In this sense, ‘S is P’ signifies a generic actuality, capable of embracing either constancy or momentariness or temporariness.

As far as formal logic is concerned, the ‘broadest sense’ described above, means no more than ‘S can be P’, which is its least assuming interpretation. Likewise, the ‘more restricted sense’ next described, must be taken by formal logic at its minimal power, meaning ‘S is sometimes P’. Thus, paradoxically, the broader the possible meaning, the lower is its logical value; that is, given a more or less indefinite ‘S is P’ statement, without further specification, we are forced to adopt its most all-inclusive interpretation. Logical science therefore ignores such vague references, and prefers to deal in fully specified forms.

This leaves us with one more ambiguity. If an ‘S is P’ statement is not intended in the above vague senses, is it intended in the sense of actuality (in this circumstance) or in that of momentariness (at the present time)? Are these parallel but different, or are they essentially one and the same? I suggest that the latter answer is ultimately to be preferred. The concepts of ‘present circumstance’ and ‘present time’ indeed have somewhat different conceptual roots, namely causality and time; but they represent the point of intersection of these two frameworks.

Just as a singular proposition points to ‘this’ instance and not merely ‘an’ unspecified instance of the subject-concept, so in natural and temporal modality, there is an mentally understood environment to the event under scrutiny (i.e. S being P). In a natural modality perspective, we view this vague environment as the surrounding disposition or layout of other objects, constituting an undefined set of causal conditions, which may have given rise to our event. In a temporal modality perspective, we merely locate the event in time, but it is taken for granted that the underlying circumstances, however unclear precisely which, may be involved somehow in our event.

Thus, the difference between a-forms and m-forms, in their most definite senses, is merely one of perspective, but they both point to the same factual material. We may therefore regard them as identical, when the interactions of natural and temporal modal propositions are analyzed.

We thus have 18 natural modality forms and 18 frequency forms, or a total of only 30 forms, according to our perspective. We may deal with the two modalities as separate phenomena, or as part of the same continuum of modality. The interrelationships between these various forms will be much clarified by oppositional analysis.

3. Distributions.

The concept of distribution of terms, which was developed in the context of Aristotelean logic, can be broadened to apply to modality. It has been found a useful doctrine, often aided by pictorial representations, for understanding the workings of arguments, and its utility would be increased. We defined a term as being distributive if, as a result of the structure of the proposition, it was found to be referring to all the instances of the class concerned; otherwise, the term was being used undistributively. Now, this concerns quantity, the extensional type of modality, and could be called extensional distribution.

We could then by analogy consider a term as naturally distributive if it was being referred to under all conditions, and naturally undistributive if the reference was dependent on circumstance. Likewise, temporal distribution would indicate reference to all or some of the times concerning a term. The following properties can then be formulated.

a. Whatever the polarity, concerning the subject: universals are extensionally distributive; but particulars are not; necessaries are naturally distributive, but not potentials; constants are temporally distributive, but not so temporaries.

b. The predicates of negatives are distributive in all three senses, whereas those of affirmatives are in all senses undistributive.

Thus, a given proposition may be distributive of this or that term in one sense, but not in another. In this way, we can explain why a certain inference is possible, or why another is not. This is not a very important doctrine, but, as already stated, a useful tool.

14. MODAL OPPOSITIONS AND EDUCTIONS.

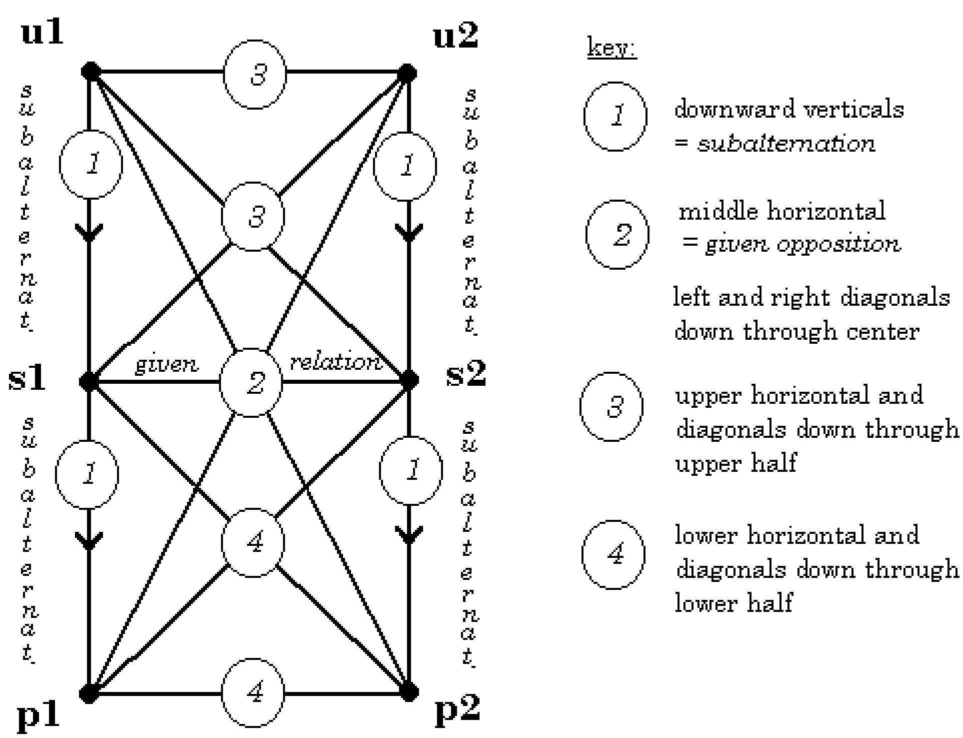
We have already encountered the oppositions of actuals or momentaries in classical logic. There is subalternation from **A** to **R**, to **I**; and from **E** to **G**, to **O**. **A** and **E**, **A** and **G**, **R** and **E**, are pairs of contraries; **A** and **O**, **I** and **E**, **R** and **G**, are pairs of contradictories, **R** and **O**, **I** and **G**, **I** and **O**, are pairs of subcontraries. These relationships were shown to proceed from analysis of the forms' meanings and application of the laws of thought. In the wider context of modal logic, we are concerned with the oppositions of, not only these six forms, but 24 more.

Remember that subalternation is one-way implication, contradictories can neither be both true nor both false, contraries cannot be both true but may be both false, subcontraries may be both true but cannot be both false, and unconnecteds do not affect each others' truth or falsehood.

1. Quantification of Oppositions.

At this point, I would like show how, given a certain oppositional relation to exist between two singular propositions (**s1**, **s2**), referring to the same instance of the same subject-concept, we can systematically predict the oppositions involving one or two of the corresponding universal (**u1**, **u2**) and particular (**p1**, **p2**) forms. This doctrine may be called quantification of oppositions, meaning more precisely opposition of quantified forms. It allows us to introduce quantity into basic figures of opposition, such as that between the categories or types of modality which will presented in the next sections. Consider the following general-model figure of opposition.

Diagram 14.1 Quantification of Oppositions.



Grant that we already know the subalternations, labeled (1), to be true, since universality includes singularity, which includes particularity. For any given opposition between singulars, labeled (2) horizontal, we need to discover the remaining lines of oppositions, namely (2) diagonal, (3), and (4). The following results are obtained.

If the singulars are *implicants*, then all horizontal lines signify implicance, and all diagonals signify subalternation, downward. Proof for the horizontals: since it is given any pair of singular forms **s1**, **s2** mutually imply each other, then any full or partial enumeration of such pairs, as in **u1**, **u2**, or **p1**, **p2**, will likewise mutually imply each other, provided the extensions involved are the same. For the diagonals: since **u1** implies **s1**, and **s1** implies **s2**, then **u1** implies **s2**. Since **u1**, **s1**, imply **s2**, and **s2** implies **p2**, then they also imply **p2**. Likewise, **u2**, **s2** can be shown to imply **s1**, **p1**.

If the singulars are *subalternative*, left implying right, then all horizontal or left down to right diagonals signify subalternation in that direction, and all right down to left diagonals signify unconnectedness. Proof: similar to previous case, though the relations involved here are unidirectional. Unconnectedness, of course, applies when no more finite opposition can be established.

If the singulars are *contradictory*, then all lines labeled (2) signify contradiction, all lines labeled (3) contrariety, all lines labeled (4) subcontrariety. Proof for the upper square: given that **s1** and **s2** cannot both be true, then any enumerations which include them both, such as **u1** + **s2**, **s1** + **u2**, or **u1** + **u2**, cannot be both true (so, for instance, if **u1**=T, then **u2**=F; i.e. if **u1**, then not-**u2**). Proof for the lower square: given that **s1** and **s2** cannot both be false, then any enumerations which exclude them both such as not-**p1** + not-**s2**, not-**s1** + not-**p2**, or not-**p1** + not-**p2**, cannot both be true (so, for instance, if not-**p1** = true, then not-**p2** = false; i.e. if not-**p1**, then **p2**). So far, we have proven the claimed contrarieties and subcontrarieties. But what of the contradictions of **u1** + **p2**, or **p1** + **u2**? If we affirm such a pair, we do not necessarily thereby affirm a specific **s1** + **s2** pair true, but we do imply that some unspecified pair(s) of **s1** and **s2**, referring to one and the same individual, would be posited together; this shows the incompatibility of **u1** + **p2**, or **p1** + **u2**. Likewise, for the incompatibility of not-**u1** + not-**p2**, or not-**p1** + not-**u2**, there is bound to be some unspecified case(s) of not-**s1** + not-**s2** subsumed, against our given information.

If the singulars are *contrary*, then all lines labeled (2) or (3) signify contrariety, and all lines labeled (4) unconnectedness. Proof: see the relevant ('not both true') parts of the arguments above for contradiction.

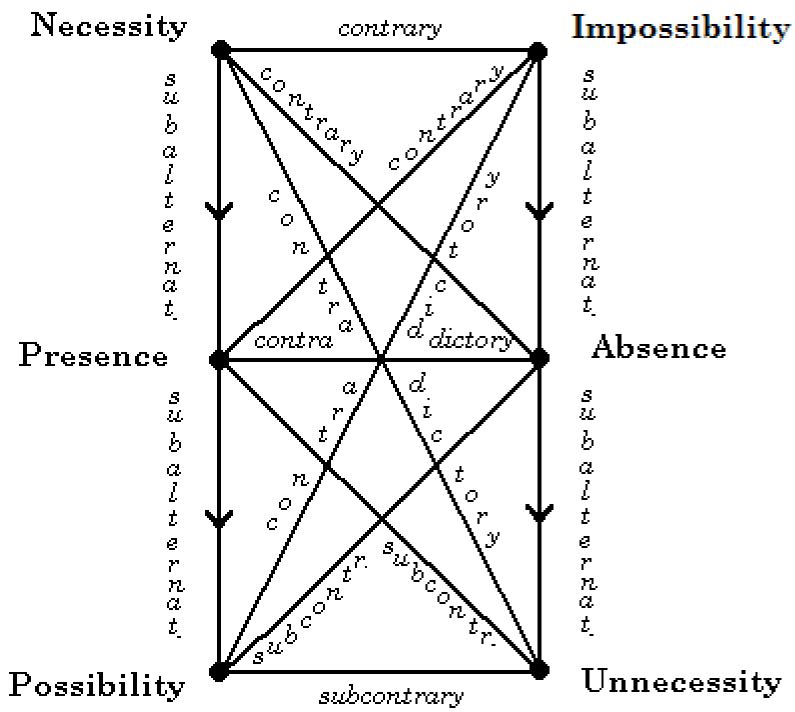
If the singulars are *subcontrary*, then all lines labeled (2) or (4) signify subcontrariety, and all lines labeled (3) unconnectedness. Proof: see the relevant ('not both false') parts of the arguments above for contradiction.

These general rules of opposition can now be used in any context, saving us from having to deal with each case of quantification anew.

2. Basic Intramodal Oppositions.

The following diagram concerns singular propositions only, and is designed to illustrate the relationships of the different categories of modality, whether of the natural type or of the temporal type (each type separately).

Diagram 14.2 Oppositions of Main Categories of Modality.



The above is equivalent to the figure of oppositions of the six quantities of Aristotelean propositions, and may be established by similar argument. The vertical, downward subalternations proceed from the definitions of the concepts involved; 'all' the circumstances or times includes any 'this one' we pick, and any specific 'this one' implies 'some' unspecified number.

The horizontal contradiction is simply the axiomatic presence and absence incompatibility. The diagonal contradictions between necessity and unnecessity, or impossibility and possibility, follow, on the basis that there would otherwise be individual circumstance(s) or time(s) which contained both presence and absence, or neither.

For the rest, the proofs are very mechanical consequences of the above. For example, using the symbols **n**, **a**, **p**, with subscripts **+** and **-**, we can say: **n+** implies **a+** implies not{**a-**}, whereas not{**n+**} does not imply not{**a+**}, nor therefore **a-**, so that **n+** and **a-** are contrary; or again, not{**p+**} implies not{**a+**} implies **a-**, whereas **p+** does not imply **a+**, nor therefore not{**a-**}, so that **p+** and **a-** are subcontraries.

With regard to contingency; being defined as the sum of possibility and unnecessity, it subalternates **p+** and **p-**, and is contrary to **n+** and **n-**. Incontingency, its negation, therefore means either necessity or impossibility, and is subalternated by **n+** and **n-**, and subcontrary to **p+** and **p-**. Contingency and incontingency are both oppositionally unconnected to presence and absence. These relationships could be represented in a wedge-shaped diagram.

As for the oppositions of probability forms See **Appendix 2** for remarks on this topic.

Furthermore, necessity implies probability, and impossibility implies probability-not. Improbability implies unnecessity, and improbability-not implies possibility. It follows that high or low probability are contrary to necessity of opposite polarity, and subcontrary to possibility of opposite polarity. These wider relations are easily established.

We can view necessity as the highest form of probability. Also, probability, whether high or low, is merely a more defined form of possibility. If we express a more specific proportion of cases (e.g. 75% or 33%), we obtain sub-categories of probability. Lastly, of course, none of the probability forms are connected oppositionally to the presence/absence forms. Nevertheless, the whole idea of probability thinking is to try and predict the chances of realization of presence or absence.

3. Quantified Intramodal Oppositions.

If we take each of the oppositional relations between singulars of natural modality and quantify them with the general rules, we obtain the following table of opposition for all the forms of natural modality.

Table 14.1 Table of Oppositions in Natural Modality.

Key to symbols:		Unconnected	☯
Implicant	⌘	Contradictory	💣
Subalternating	⇗	Contrary	⇘
Subalternated	⇙	Subcontrary	⇖

	An	A	Ap	Rn	R	Rp	In	I	Ip	En	E	Ep	Gn	G	Gp	On	O	Op
An	⌘	⇗	⇗	⇗	⇗	⇗	⇗	⇗	⇗	⇘	⇘	⇘	⇘	⇘	⇘	⇘	⇘	💣
A	⇙	⌘	⇗	☯	⇗	⇗	☯	⇗	⇗	⇘	⇘	☯	⇘	⇘	☯	⇘	💣	⇖
Ap	⇙	⇙	⌘	☯	☯	⇗	☯	☯	⇗	⇘	☯	☯	⇘	☯	☯	💣	⇖	⇖
Rn	⇙	☯	☯	⌘	⇗	⇗	⇗	⇗	⇗	⇘	⇘	⇘	⇘	⇘	💣	☯	☯	⇖
R	⇙	⇙	☯	⇙	⌘	⇗	☯	⇗	⇗	⇘	⇘	☯	⇘	💣	⇖	☯	⇖	⇖
Rp	⇙	⇙	⇙	⇙	⇙	⌘	☯	☯	⇗	⇘	☯	☯	💣	⇖	⇖	⇖	⇖	⇖
In	⇙	☯	☯	⇙	☯	☯	⌘	⇗	⇗	⇘	⇘	💣	☯	☯	⇖	☯	☯	⇖
I	⇙	⇙	☯	⇙	⇙	☯	⇙	⌘	⇗	⇘	💣	⇖	☯	⇖	⇖	☯	⇖	⇖
Ip	⇙	⇙	⇙	⇙	⇙	⇙	⇙	⇙	⌘	💣	⇖	⇖	⇖	⇖	⇖	⇖	⇖	⇖
En	⇘	⇘	⇘	⇘	⇘	⇘	⇘	⇘	💣	⌘	⇗	⇗	⇗	⇗	⇗	⇗	⇗	⇗
E	⇘	⇘	☯	⇘	⇘	☯	⇘	💣	⇖	⇙	⌘	⇗	☯	⇗	⇗	☯	⇗	⇗
Ep	⇘	☯	☯	⇘	☯	☯	💣	⇖	⇖	⇙	⇙	⌘	☯	☯	⇗	☯	☯	⇗
Gn	⇘	⇘	⇘	⇘	⇘	💣	☯	☯	⇖	⇙	☯	☯	⌘	⇗	⇗	⇗	⇗	⇗
G	⇘	⇘	☯	⇘	💣	⇖	☯	⇖	⇖	⇙	⇙	☯	⇙	⌘	⇗	☯	⇗	⇗
Gp	⇘	☯	☯	💣	⇖	⇖	⇖	⇖	⇖	⇙	⇙	⇙	⇙	⇙	⌘	☯	☯	⇗
On	⇘	⇘	💣	☯	☯	⇖	☯	☯	⇖	⇙	☯	☯	⇙	☯	☯	⌘	⇗	⇗
O	⇘	💣	⇖	☯	⇖	⇖	☯	⇖	⇖	⇙	⇙	☯	⇙	⇙	☯	⇙	⌘	⇗
Op	💣	⇖	⇖	⇖	⇖	⇖	⇖	⇖	⇖	⇙	⇙	⇙	⇙	⇙	⇙	⇙	⇙	⌘

These relationships may be clarified by use of a truth-table. In the table below, given the truth of a proposition listed under column heading **T**, or the falsehood of one under **F**, we see the reactions, along the same row, of all other propositions, listed as column headings. This data follows from the preceding table.

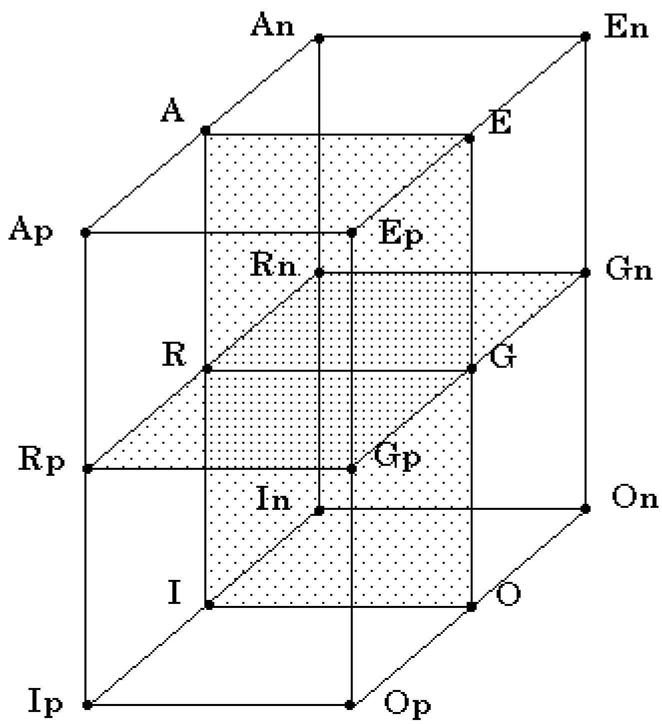
Table 14.2 Truth-Table for Natural Modality.

(key: **T** = true, **F** = false, **.** = undetermined.)

T	An	A	Ap	Rn	R	Rp	In	I	Ip	En	E	Ep	Gn	G	Gp	On	O	Op	F
An	T	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F	F	Op
A	.	T	T	.	T	T	.	T	T	F	F	.	F	F	.	F	F	.	O
Ap	.	.	T	.	.	T	.	.	T	F	.	.	F	.	.	F	.	.	On
Rn	.	.	.	T	T	T	T	T	T	F	F	F	F	F	F	.	.	.	Gp
R	.	.	.	.	T	T	.	T	T	F	F	.	F	F	.	.	.	.	G
Rp	.	.	.	.	.	T	.	.	T	F	.	.	F	.	.	.	.	.	Gn
In	.	.	.	.	.	.	T	T	T	F	F	F	.	.	.	.	.	.	Ep
I	.	.	.	.	.	.	.	T	T	F	F	.	.	.	.	.	.	.	E
Ip	.	.	.	.	.	.	.	.	T	F	.	.	.	.	.	.	.	.	En
En	F	F	F	F	F	F	F	F	F	T	T	T	T	T	T	T	T	T	Ip
E	F	F	.	F	F	.	F	F	.	.	T	T	.	T	T	.	T	T	I
Ep	F	.	.	F	.	.	F	.	.	.	.	T	.	.	T	.	.	T	In
Gn	F	F	F	F	F	F	.	.	.	.	.	.	T	T	T	T	T	T	Rp
G	F	F	.	F	F	.	.	.	.	.	.	.	.	T	T	.	T	T	R
Gp	F	.	.	F	.	.	.	.	.	.	.	.	.	.	T	.	.	T	Rn
On	F	F	F	.	.	.	.	.	.	.	.	.	.	.	.	T	T	T	Ap
O	F	F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	T	T	A
Op	F	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	T	An

Needless to say, the easiest way to visualize and transmit all the above information is by means of a figure of opposition. However, since in this context the required diagram is three-dimensional, it is rather difficult to present on paper. Below is a sketch of it, but without the various lines of opposition. Note that the shaded planes have already been presented earlier, with all their lines of opposition shown. The reader can work out the remaining planes, with reference to the above two tables, or more radically to the principles of 'quantification of oppositions' developed earlier.

Diagram 14.3 Figure of Oppositions of Natural Propositions.

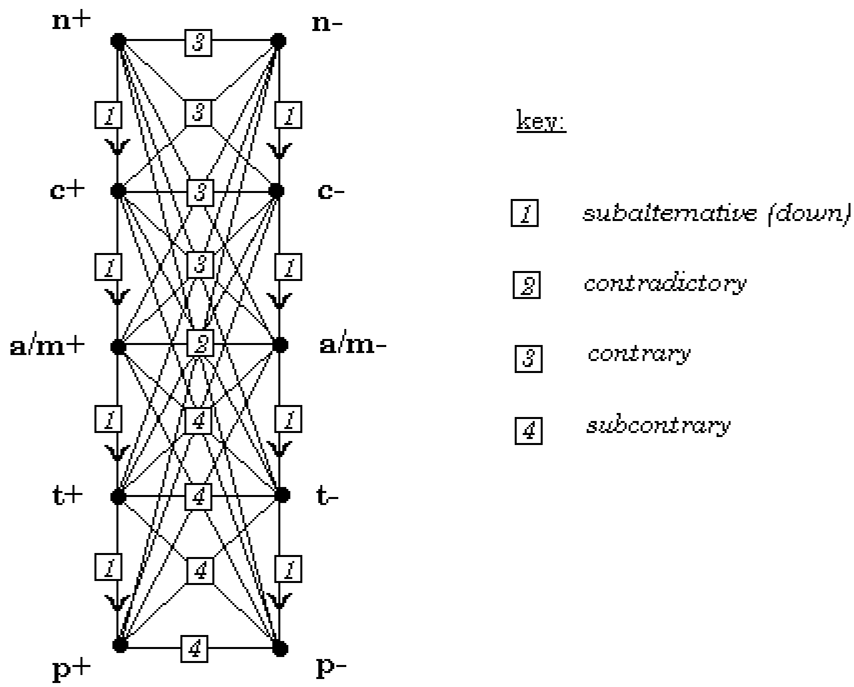


Identical results are obtainable for temporal modality, substituting **c** for **n**, and **t** for **p**, throughout.

4. Intermodal Oppositions.

Lastly, needing elucidation, is the inter-opposition of natural and temporal modalities. The following diagram shows the continuum of modality, including both natural and temporal types together. Parts of this diagram have already been presented, when we dealt with each modal type separately. But it is interesting to have an overview, anyway.

Diagram 14.4 Oppositions between Modality Types.



This diagram concerns singulars. We know from our analysis of modality that **n** implies **c**, which implies **a** or **m**, which implies **t**, which implies **p**, for either polarity; that is, the illustrated subalternations proceed from the meanings of the concepts involved. From these, and the already established intramodal oppositions, it is easy to infer the contrariety between **n** and **c**, or **n** and **t**, forms of opposite polarity (upper diagonals), and the subcontrariety between **c** and **p**, or **t** and **p**, forms of opposite polarity (lower diagonals).

These relationships between singulars can now be quantified by reference to the general rules of opposition, and the results tabulated as follows.

Table 14.3 Table of Oppositions between Natural and Temporal Modalities.

Key to symbols:		Unconnected	☯
Subalternating	⇗	Contrary	⇘
Subalternated	⇙	Subcontrary	⇖

	Ac	At	Rc	Rt	Ic	It	Ec	Et	Gc	Gt	Oc	Ot
An	⇗	⇗	⇗	⇗	⇗	⇗	⇘	⇘	⇘	⇘	⇘	⇘
Rn	☯	☯	⇗	⇗	⇗	⇗	⇘	⇘	⇘	⇘	☯	☯
In	☯	☯	☯	☯	⇗	⇗	⇘	⇘	☯	☯	☯	☯
Ap	⇙	⇙	☯	☯	☯	☯	☯	☯	☯	☯	⇖	⇖
Rp	⇙	⇙	⇙	⇙	☯	☯	☯	☯	⇖	⇖	⇖	⇖
Ip	⇙	⇙	⇙	⇙	⇙	⇙	⇖	⇖	⇖	⇖	⇖	⇖
En	⇘	⇘	⇘	⇘	⇘	⇘	⇗	⇗	⇗	⇗	⇗	⇗
Gn	⇘	⇘	⇘	⇘	☯	☯	☯	☯	⇗	⇗	⇗	⇗
On	⇘	⇘	☯	☯	☯	☯	☯	☯	☯	☯	⇗	⇗
Ep	☯	☯	☯	☯	⇖	⇖	⇙	⇙	☯	☯	☯	☯
Gp	☯	☯	⇖	⇖	⇖	⇖	⇙	⇙	⇙	⇙	☯	☯
Op	⇖	⇖	⇖	⇖	⇖	⇖	⇙	⇙	⇙	⇙	⇙	⇙

These relationships may be clarified by use of a truth-table. This data follows from the preceding table.

Table 14.4 Truth-Table for Intermodal Oppositions.

(key: **T** = true, **F =** false, **. =** undetermined.)

T	Ac	At	Rc	Rt	Ic	It	Ec	Et	Gc	Gt	Oc	Ot	F
An	T	T	T	T	T	T	F	F	F	F	F	F	Op
Rn	.	.	T	T	T	T	F	F	F	F	.	.	Gp
In	.	.	.	.	T	T	F	F	.	.	.	.	Ep
Ap	.	.	.	.	.	.	.	.	.	.	.	.	On
Rp	.	.	.	.	.	.	.	.	.	.	.	.	Gn
Ip	.	.	.	.	.	.	.	.	.	.	.	.	En
En	F	F	F	F	F	F	T	T	T	T	T	T	Ip
Gn	F	F	F	F	.	.	.	.	T	T	T	T	Rp
On	F	F	.	.	.	.	.	.	.	.	T	T	Ap
Ep	.	.	.	.	.	.	.	.	.	.	.	.	In
Gp	.	.	.	.	.	.	.	.	.	.	.	.	Rn
Op	.	.	.	.	.	.	.	.	.	.	.	.	An

In the table above, given the truth of a proposition listed under column heading T, or the falsehood of one under F, we see the reactions, along the same row, of all other propositions, listed as column headings.

5. Eductions.

The following is a list of the eductions possible from propositions with natural modality. The methods of validation used for these are similar to those developed for Aristotelean forms. That is, conceptual analyses and appeal to the laws of thought, in the cases of obversion and conversion; and reduction to these first two process, in the other cases.

a. ***Obversion*** (S-P to S-nonP).

S must be P implies S cannot be nonP; S cannot be P implies S must be nonP; S can be P implies S can not-be nonP; S can not-be P implies S can be nonP. These are true irrespective of the quantity (all/this/ some) involved; and the obverse has in all cases the same quantity as the obvertend. These results follow from the definitions of the concepts involved and the law of contradiction.

b. ***Conversion*** (S-P to P-S).

Affirmatives, be they necessary or potential, general or particular, all convert to a particular potential, Some P can be S, but no better. In the case of negatives, only No S can be P (**En**) is convertible, and that fully to No P can be S; **Ep**, **Gn**, **Gp**, **On**, **Op** are not convertible. These results can be established by consideration of the subsumptions of circumstance involved.

c. ***Obverted Conversion*** (S-P to P-nonS).

This process is applicable only to convertibles, which are then all obvertible. Thus, affirmatives all yield Some P can not-be nonS; and **En** yields All P must be nonS.

d. Conversion by Negation (S-P to nonP-S).

This is obversion, followed by conversion. Thus, all originally negative propositions can be converted by negation, to yield Some nonP can be S. But of originally affirmative propositions, only All S must be P (**An**) can be so processed, to yield No nonP can be S; **Ap**, **Rn**, **Rp**, **In**, **Ip** are not convertible by negation.

e. ***Contraposition*** (S-P to nonP-nonS).

This requires conversion by negation, followed by obversion. Therefore, all negatives are contraposable, and that to Some nonP can not-be nonS. Whereas, in the case of affirmatives, only **An** can be so processed, yielding All nonP must be nonS.

f. ***Inversion*** (S-P to nonS-nonP).

Of affirmatives, only **An** can be so treated, by contraposing then converting it, to obtain Some nonS can be nonP. Of negatives, only **En** is invertible, by converting then contraposing it, with the result Some nonS can not-be nonP.

g. ***Obverted Inversion*** (S-P to nonS-P).

This being inversion followed by obversion is applicable only to universal necessaries, **An** yielding Some nonS can not-be P, and **En** yielding Some nonS can be P.

We note, in conclusion, that only **An** and **En** (as well as **A** and **E**) can be subjected to all six of these processes.

Similar results can easily be established regarding propositions with temporal modality.

15. MAIN MODAL SYLLOGISMS.

1. Valid Modes.

We called a mood of syllogism, a combination of formally fully specified premises and conclusion in a given figure (e.g. **1/AAA**). We will call mode, any combination of symbols which does not by itself fully specify a syllogistic form, but which abstracts a specific aspect of such, in a given figure (e.g. **1/uuu**). It was shown, in Aristotelean logic, that the primary valid modes of polarity and quantity are as in the following table.

Table 15.1 Valid modes of Polarity and Quantity.

Figure	First	Second	Third	Fourth
Polarities	+++ -+-	+-- -+-	+++ -+-	-+-
Quantities	uuu upp uss	uuu upp uss	upp pup ssp	upp

We can at the outset, prior to systematic validation, predict that the valid modes for natural and temporal modality will be the following, by analogy to the results obtained for extensional modality.

Table 15.2 Valid Modes of Natural and Temporal Modalities.

Figure	First	Second	Third	Fourth
Natural Modality	aaa nnn npp	aaa nnn npp	aaa npp pnp	aaa npp
Temporal Modality	mmm ccc ctt	mmm ccc ctt	mmm ctt tct	mmm ctt

Note the slight difference between quantity modes and modality modes. The modes **aaa** and **mmm** are valid in all figures, whereas **sss** is not (**3/ssp** is exceptional, and anyway does not yield an **s** conclusion). This is due to modality standing outside the relationship between the terms, whereas quantity concerns the subject more directly.

Natural and temporal modality being essentially analogous, we can concentrate on developing the theory of syllogism for the former, and then generalize the results to the latter. Apart from the above we will need to investigate the valid modes of mixed, natural and temporal syllogism.

In the broadest sense, of course, all syllogism is modal. But for the sake of convenience we will often find it useful to call nonmodal, syllogism both of whose premises are actual or momentary (**aaa** or **mmm**); so that syllogism with one or both premises necessary or possible, can be called modal. Aristotelean logic can then be said to have concerned nonmodal syllogism, while this thesis concerns modal syllogism.

2. Valid Moods.

If we combine together the valid modes of polarity and quantity for a given valid mode of modality, in each of the figures, we should obtain the valid moods of syllogism. Let us now do so, using the valid natural modality modes, to develop a full list of natural syllogism, including both the nonmodal (Aristotle's achievement) and the modal (the new contribution). This is the principal goal of our whole formal research. The notation system used for this, consists in applying modality subscripts (**n**, **p**, **a**) to the six standard symbols, **A**, **E**, **I**, **O**, **R**, **G**.

We see in the list below that only 56 primary moods emerge as logically valid, not counting derivative syllogism. There are 18 valid moods in each of the first three figures, and 2 in the fourth. Since 19 of the above moods are actual, only 37 are original forms.

Table 15.3 Primary Valid Moods of Natural Syllogism.

Mode/Figure	First	Second	Third	Fourth
aaa	AAA EAE AII EIO ARR ERG	AEE EAE AOO EIO AGG ERG	AII EIO IAI OAO RRI GRO	EIO
nnn	AnAnAn EnAnEn AnInIn EnInOn AnRnRn EnRnGn	AnEnEn EnAnEn AnOnOn EnInOn AnGnGn EnRnGn		
npp	AnApAp EnApEp AnIpIp EnIpOp AnRpRp EnRpGp	AnEpEp EnApEp AnOpOp EnIpOp AnGpGp EnRpGp	AnIpIp EnIpOp InApIp OnApOp RnRpIp GnRpOp	EnIpOp
pnp			ApInIp EpInOp IpAnIp OpAnOp RpRnIp GpRnOp	

We will now present these 37 valuable new forms in full, for the record.

a. *First Figure*. Form: M-P, S-M, S-P.

AnAnAn	EnAnEn
All M must be P	No M can be P
All S must be M	All S must be M
All S must be P	No S can be P

AnInIn All M must be P Some S must be M Some S must be P	EnInOn No M can be P Some S must be M Some S cannot be P
AnRnRn All M must be P This S must be M This S must be P	EnRnGn No M can be P This S must be M This S cannot be P
AnApAp All M must be P All S can be M All S can be P	EnApEp No M can be P All S can be M All S can not-be P
AnIpIp All M must be P Some S can be M Some S can be P	EnIpOp No M can be P Some S can be M Some S can not-be P
AnRpRp All M must be P This S can be M This S can be P	EnRpGp No M can be P This S can be M This S can not-be P

b. *Second Figure*. Form: P-M, S-M, S-P.

AnEnEn All P must be M No S can be M No S can be P	EnAnEn No P can be M All S must be M No S can be P
AnOnOn All P must be M Some S cannot be M Some S cannot be P	EnInOn No P can be M Some S must be M Some S cannot be P
AnGnGn All P must be M This S cannot be M This S cannot be P	EnRnGn No P can be M This S must be M This S cannot be P
AnEpEp All P must be M All S can not-be M All S can not-be P	EnApEp No P can be M All S can be M All S can not-be P
AnOpOp All P must be M Some S can not-be M Some S can not-be P	EnIpOp No P can be M Some S can be M Some S can not-be P

AnGpGp	EnRpGp
All P must be M	No P can be M
This S can not-be M	This S can be M
This S can not-be P	This S can not-be P

c. *Third Figure*. Form: M-P, M-S, S-P.

AnIpIp	EnIpOp
All M must be P	No M can be P
Some M can be S	Some M can be S
Some S can be P	Some S can not-be P
InApIp	OnApOp
Some M must be P	Some M cannot be P
All M can be S	All M can be S
Some S can be P	Some S can not-be P
RnRpIp	GnRpOp
This M must be P	This M cannot be P
This M can be S	This M can be S
Some S can be P	Some S can not-be P
ApInIp	EpInOp
All M can be P	All M can not-be P
Some M must be S	Some M must be S
Some S can be P	Some S can not-be P
IpAnIp	OpAnOp
Some M can be P	Some M can not-be P
All M must be S	All M must be S
Some S can be P	Some S can not-be P
RpRnIp	GpRnOp
This M can be P	This M can not-be P
This M must be S	This M must be S
Some S can be P	Some S can not-be P

d. *Fourth Figure*. Form: P-M, M-S, S-P.

EnIpOp
No P can be M
Some M can be S
Some S can not-be P

A similar listing would be obtained for temporal syllogism. Secondary modes, valid derivatively and of lesser significance, will discussed later. Mixed syllogism will also be dealt with separately.

3. Validations.

We have seen that each figure has a method of validation most appropriate to it. Aristotelean syllogism being identical with our nonmodal (actual or momentary) forms, the task of validation of modal syllogisms is much facilitated. Similar approaches can be used with regard to modal syllogism; and moreover we can appeal, if we need to, to correct nonmodal argument, in the process. The following description of validation and rejection processes for natural modal syllogism, can all be repeated for temporal modes.

a. First figure.

We previously defended Aristotle's valid moods in the first figure, on the basis of the principle that one must mean what one says. Some phenomena have been observed, perceptually and/or conceptually; within a complex of appearances, certain aspects have been distinguished; names have been assigned to their various components; thereby, a framework is established which we are logically required to adhere to; such recognition guarantees the accord between thought and reality (that is, long-term, overall, appearance.)

Now, granting the six valid actual moods of this figure, the corresponding moods in the modes nnn and npp, are to be demonstrated valid.

A necessary proposition 'X must be Y' may be viewed as merely a collection of actual propositions 'In circumstance 1, X is Y', 'in circumstance 2, X is Y', 'in circumstance 3, X is Y', and so on; it says 'Whatever the surrounding circumstances, X is Y'. Likewise, a potential proposition may be viewed as a partial enumeration of circumstances in which the stated relationship of X and Y is actualized. An actual proposition indicates a specific single circumstance in which the event occurs.

Now, let us consider a group of three propositions which, in the actual mode **aaa**, constitute a valid syllogism, e.g. **AAA**. In the case of **AnAnAn**, the **nnn** equivalent, we can predict that in each and any circumstance we may select, we will find the two premises **AA** actual, and yielding the conclusion **A**. It follows that, given the premises **AnAn**, we can say, 'Whatever the circumstances, the conclusion **A** occurs'; which means that the **An** conclusion is valid. Thus, any mood valid in **aaa** mode is equally valid in **nnn** mode. With similar reasoning, we can demonstrate the validity of **npp**, since the 'all circumstances' in the major premise includes the 'some circumstances' in the minor premise, which are in turn posited as framing the conclusion, too.

With regard to invalidation of invalid modal modes. Although the onus of proof is on anyone who wants to defend them, as it were, it is important to give special attention to the mode **pnp**, which might at first sight seem reasonable. We might think that the 'in all circumstances' of the minor premise, includes the 'some circumstances' of the major premise, so that a potential conclusion can be drawn. However, in any modal proposition, the circumstances under consideration apply primarily to the subject of the proposition. When we refer to all the circumstances surrounding the subject's existence, we do not claim these to be the only circumstances which can coincide with the predicate's existence, or any other subject's existence.

In the case of **npp**, the 'some circumstances' under consideration, are implied for the middle term since the minor premise is affirmative, and concern the same subject in minor premise and conclusion. But in the case of **pnp**, the specific conditions under which the middle term is addressed in the major premise do not necessarily coincide with any condition concerning the minor term in the minor premise, and so cannot be transferred to it in the conclusion. The change of subject being qualified makes the process illicit. The invalidity of **ppp** is all the more obvious, since it has no misleading unconditional premise. Thus the analogy of valid modal modes to valid quantitative modes is complete.

What of **aaa**, which is posited as valid, although we reject **sss**? Here too, one could argue that the unitary circumstance referred to by each of the two premises may not coincide, since their subjects differ. In truth, this argument against **aaa** is justified, and serves to warn us that the **aaa** mode is valid only on the condition that we know the unitary circumstance involved in the two premises to be one and the same. However, actual propositions by definition concern an ostensible circumstance (which though left tacit is understood). So **aaa** is a valid mode, when we know the *sous-entendu* circumstance to be common.

Although we might attach a similar proviso for the validity of **sss**, we in fact cannot, because of a structural difference between actuality and singularity. The 'this' in a singular proposition is more firmly attached to the subject; it identifies the subject itself, and not a circumstance surrounding it. Comparing one subject's 'this' to another's is nonsensical, as is the idea of moving our mental finger from one to the other; because all they have in common is 'this-ness', not this one 'this-ness'. In actuals, on the other hand, the focus of the 'this' is a circumstance standing outside the subject of the proposition, though bounded by its existence; it is not the subject as such which is focused on by that 'this'. It follows that, here, the two 'this' occurrences in the premises may be compared, and the specification transferred to their conclusion.

b. Other figures.

The valid moods of the second figure are established by reduction ad absurdum through the first figure. We attach the denial of the conclusion to the major premise, and see that the result would be denial of our original minor premise. Thus, **2/nnn** is reduced through **1/npp**, and **2/npp** follows from **1/nnn**; always of course provided the underlying actual mood has valid polarity and quantity properties. Invalid modes in the second figure are dealt with similarly, by showing that the combination of the major premise with the suggested conclusion results in a contradiction or a non-sequitur, through the first figure.

The third figure modes could be reduced ad absurdum to the first figure for systematic validation; the denial of the conclusion would be combined with the minor premise and result in denial of the original major premise. Rejection of invalid modes could be achieved similarly. However, exposition reflects more accurately the way we deal with this figure in practice. We can reproduce our arguments for the first figure, showing that the circumstances in the premises intersect, and are passed on to the conclusion. This is facilitated by the fact that, in this figure, the two premises have the same subject (the middle term).

For the fourth figure, direct reduction is the appropriate approach. There is, furthermore, only one primary valid mood to consider. The premises **EnIp** are both converted, allowing us to process them in the first figure, and obtain the desired **Op** conclusion. Other validations, and invalidations, are likewise easy to deal with.

16. OTHER MODAL SYLLOGISMS.

1. Secondary Modes.

Concerning subaltern valid syllogism. Any combination of premises not included in the above list of primary valid moods, but implying one which is included, can obviously be listed as a derivatively valid mood. Likewise, propositions implied by one of the conclusions to the valid moods form subaltern moods with the same premises. There are also fourth figure moods to take into account, which though valid are insignificant.

We have two stages to consider. To begin with, applying the primary valid modal modes to the secondary valid actual (or momentary) moods. And then, listing the secondary valid modal modes, which can be applied to both primary and secondary valid actual (or momentary) moods. These two lists together make up the full list of secondary valid modal moods. We shall for a start deal with the first three figures, before turning our attention to the less regular fourth figure.

Table 16.1 Secondary Modes of the Regular Figures.

Figure	First	Second	Third
Quantity	uup usp	uup usp	uup usp sup
Natural Modality	nna naa ana nnp nap anp aap	nna naa ana nnp nap anp aap	nna naa ana nnp nap anp aap

Similarly for Temporal Modality.

The subaltern quantity modes were implicit in the list of secondary moods established previously, when considering Aristotelean syllogism. Note that the subaltern modality modes are the same in the three figures. They could be mostly predicted by analogy; but let us derive them quickly from the primary modes, at least in the case of natural modality. In these three figures, **nna**, **naa** or **ana** are derived from **aaa**, whose premises theirs imply; and **nnp**, **nap**, **anp**, and **aap** follow from these by virtue of their subaltern conclusions. Temporal modality modes can similarly be dealt with.

Now let us count the number of subaltern moods which we can expect to encounter in these three figures. The full list will not be drawn up, being too large and relatively unimportant; the numbers are interesting, however, as will be seen. Each of these figures has 2 valid polarity modes. These are combinable with 3 valid primary quantity modes in each figure; plus 2 valid subaltern quantity modes each, in the first and second figure, and 3 of them, in the third. These are in turn combinable, in any of these figures, with 3 valid primary natural modality modes, plus 7 valid subaltern natural modality modes.

We thus obtain, in the first figure, a total of 2X(3+2)X(3+7) = 100 valid moods. In the second figure, we have the same results. In the third figure, our total is 2X(3+3)X(3+7) = 120. In each of the three figures, there are 2X3X3 = 18 primary moods, and the rest are secondary. Identical results are of course obtainable for temporal modality.

Now, whereas in the first three figures any valid polarity mode can be correctly combined with any valid quantity or modality modes, in the fourth figure only specific combinations are permissible. The fourth figure, as earlier indicated, lacks uniformity, and seems to in effect contain three different sets of valid moods, which we may call **4a**, **4b**, **4c**, for the sake of convenience.

Figure **4a**, whose polarity mode is **-+-**, is the significant one; and we saw that it contained two valid primary moods, **EIO** and **EnIpOp**. Figures **4b** and **4c** are insignificant, being mere derivatives of the first figure by transposition of premises and conversion of conclusion. The prototype of **4b** is **AEE**, with polarity **+–**; and that of **4c** is **IAI** with polarity **+++**. The corresponding modal forms are **AnEnEn** and **IpAnIp**. We shall now list the implicit quantity and modality modes of these forms and their subalterns.

Table 16.2 Secondary modes of the Fourth Figure.

Sub-figure	4a.	4b.	4c.
Polarity	(-+-)	+--	+++
Quantity	(upp) uup usp	uuu uup	pup uup sup
Natural Modality	(npp) (aaa) nna naa ana nnp nap anp aap	nnn aaa nna naa ana nnp nap anp aap	pnp aaa nna naa ana nnp nap anp aap

Similarly for Temporal Modality.

The primary valid modes of the fourth figure are included in the above table in brackets to facilitate reading of the sources of their subaltern modes; these are in **4a**. With regard to **4b** and **4c**, the first quantity and the first two modalities listed for each are the sources of the others, but all are viewed as secondary, as well as the corresponding polarities.

Let us now count the number of moods implied valid. For **4a**, 1X(1+2)X(2+7) = 27, of which only two are primary. For **4b**, 1X2X9 = 18, and for **4c**, 1X3X9 = 27; all these being secondary in one way or another. Thus, figure four consists of two primary valid moods, and another 70 secondary valid moods. This is said for natural modality, and can be repeated for temporal modality, as usual.

Thus, to conclude this section, there are a total of 100+100+120+72 = 392 valid moods in each type of modality, of which 56 are primary, and 336 are secondary. The full list of secondary moods is easily developed given the above lists of modes.

2. Mixed Modes.

To systematically cover all possibilities of combination, we now need to investigate mixed syllogism, that is, syllogism involving a mixture of natural modalities (**n**, **p**, **a**) and temporal modalities (**c**, **t**, **m**), in their premises and/or conclusions. It will be seen that valid such combinations are entirely derivable from syllogisms previously encountered under this or that type of modality separately, so that we can say that mixed modes are in fact all secondary.

Analysis shows that the mixed modes we seek are all derivable from the primary modes of temporal modality of each figure. We have seen that these are: in figures **1** and **2**, **mmm**, **ccc**, **ctt**; in figure **3**, **mmm**, **ctt**, **tct**; in figure **4a**, **mmm**, **ctt**; in figure **4b**, **mmm**, **ccc**; in figure **4c**, **mmm**, **tct**. If we analyze the subalternations (through premises and/or conclusion) possible for each of these four modes, we get the following results. From **mmm**: **nnt**, **ncm**, **nct**, **ncp**, **cnm**, **cnt**, **cnp**, **nmt**, **mnt**, **ccp**, **cmp**, **mcp** (12 modes, in common to all figures). From **ccc**: **nnc**, **ncc**, **cnc** (3 modes, applicable to figures **1**, **2**, and **4**b). From **ctt**: **ntt**, **ntp**, **ctp** (3 modes, applicable to figures **1**, **2**, **3**, and **4a**). From **tct**: **tnt**, **tnp**, **tcp** (3 modes, applicable to figures **3** and **4c**).

We thus obtain, for the first three figures, 18 valid mixed modes each; for each subset of figure four, 15 valid modes. Other mixed modes are found not to follow from any valid nonmixed modes, or to be only apparently mixed because of the different symbolization of actual and momentary propositions.

The valid mixed modes may each be combined with the polarities and quantities existing in their respective figures. It follows that the number of valid mixed moods are as follows: 2X5X18 = 180, for each of figures **1** and **2**; 2X6X18 = 216 for figure **3**; and 3X15 = 45 for figure **4a**, 2X15 = 30 for figure **4b**, 3X15 = 45 for figure **4c**. The total number of valid mixed moods is therefore 696. These valid moods, to repeat, count as secondary. Most may be practically useless, but they had indicated for completeness.

The above listed mixed modes, you will observe, do not include combinations involving the a-form and certain combinations involving the **m**-form. The reason for this is simply that the actual and momentary forms are essentially identical, although they appear different. Their distinction is one of perspective, and a verbal one sometimes, but their logical value is the same. It follows, not only that **aaa** and **mmm** are equivalent, but also that other combinations of **a** and **m**, namely **amm**, **ama**, **mam**, **maa**, are all identical. Furthermore, combinations involving modal propositions together with one or both these, are also redundant; this includes groups such as **nnm**, **cca**, **ncm**, **nca**, **nmm**, **nma**, **nam**, and so on.

We must of course avoid the duplicate listing as mixed modes, of syllogism which have already been presented as nonmixed modes, merely because they superficially appear different through the use of different notation, so combinations such as those just mentioned must be left out of our accounts. On the other hand, some combinations involving nonmodal proposition(s) mixed with modal(s), are noteworthy, even though subaltern, because they provide additional logical information. We need only to select either of the symbols **a** or **m**, to represent nonmodal forms, and work with that exclusively. I selected m as more appropriate, after finding that all valid mixed modes could be derived from solely temporal syllogism. But this is strictly-speaking mere convention; modes such as **ncm** or **nmt** could equally have been written **nca** or **nat**. The underlying meaning is the same.

3. Summation.

If we examine the principal 37 new syllogism introduced in this paper, it is clear that they are not at first sight obviously valid. An effort of thought is needed to see their truth. This shows that our enterprise, the development of a modal logic, was a worthwhile endeavor, a valuable addition to human knowledge. The justification is still greater, if we analyze our work in this chapter statistically, and sum-up the number of new syllogistic forms introduced.

We saw earlier that there are 2X3X5 = 30 possible categorical forms of the kind under study, a proposition may have one of two polarities (**+** or **-**), one of three quantities (**s**, **u**, or **p**), and one of five modalities (**a** or **m**, or **n**, **c**, **t**, or **p**). A syllogism contains three propositions, in any of four possible figures; therefore the total number of imaginable combinations is (30 cubed)X4 = 108,000 moods, whether valid or invalid. Of these, ((2X3)cubed)X4 = 864 would be wholly nonmodal moods; (((2X3X3)cubed)X4)-864 = 22,464 would be natural modal moods; and again 22,464 would be temporal modal moods; the remainder 62,208 moods would be of mixed modal type.

Now, let us calculate how many out of this theoretical total of possibilities, are in fact valid. We saw that in nonmodal logic, there are 44 valid moods, of which 19 are primary and 25 are secondary. Next, in modal logic, we established 37 primary moods for natural modality and 37 for temporal modality; and we found these to have 336 and 336 secondary moods, respectively. Lastly, we identified 696 mixed moods as valid, and pronounced them all secondary. Thus, the total number of valid moods obtained is 1486, of which only 93 are primary, and the remaining 1393 are secondary.

Thus, only 1486 out of 108,000 = 1.4% of possible combinations are logically valid; versus 98.6% chances of erroneous reasoning. This shows the importance of our thesis, that modality needed to be considered and systematically analyzed by logical science. The number 1486 is of course quite large in itself; this shows the value of the notation system I invented, which made it possible for me to analyze so many combinations with a certainty of exhaustiveness and in a minimum of space.

Of the valid moods, only 6.3% are primary and 93.7% are secondary. Primary moods are the most significant and independent forms of reasoning; secondary moods are relatively less significant and more derivative. This does not mean, however, that secondary moods are necessarily less commonly used in practice; although many of them occur rather rarely, many may nonetheless be as important as primary moods. For example, **naa** moods in figure **1** or **2** are quite valuable, although technically subaltern to **aaa**.

It must be stressed, also, that the recognition of invalid modes of thought is as important as the knowledge of valid modes. We indicated, for example, how at first sight one might suppose moods such as **1/ApAnAp** valid; an analytical effort is required to understand the error involved. Some invalid moods are of course instantly seen to be wrong; but some contain pitfalls for the logically untrained mind.

Further research, which I will not develop in detail in this paper, shows that some invalid moods may be made to yield imperfect conclusions, of the types 'Some nonS can be nonP' or 'Some nonS are sometimes nonP'. Similar cases arose in nonmodal syllogism, with 'Some nonS are nonP' conclusions, the reader will recall.

Another kind of atypical conclusion is drawable in some cases; for example, **1/ApAp** (All M can be P and All S can be M) does not conclude **Ap** (All S can be P), but does allow us to infer that 'All S either can be or can become P'. Indeed, this may be viewed as an explanation why the simple 'All S can be P' cannot be inferred.

Syllogism which mix copula in this way might be characterized as mixed-form. They involve another kind of copula (becoming, instead of being). The logic of becoming is a large field on its own, which will be touched upon further in the next chapter.

4. General Principles.

To conclude, general rules of the syllogism, which summarize, or explain, the valid moods, may be presented. They reveal the underlying principles of such reasoning, or the common attributes.

a. ***Rules of Polarity***. If the extreme terms are positively connected to the middle term in the premises, they will be connected in the conclusion, if any. But if either extreme term is unconnected to the middle term in its premise, it will be unconnected to the other extreme term in the conclusion, if any. If neither extreme term is positively connected to the middle term in the premises, no conclusion can be drawn.

b. ***Rules of Quantity***. At least some instance(s) of the middle term must be found in common to both premises, for a conclusion to be conceivable. And no instance of either extreme term may be found referred to in a suggested conclusion which was not covered in the premise. However, instances found in a premise may not-reappear in the conclusion.

c. ***Rules of Modality***. There must be found some circumstance(s) and time(s) in common to both premises, for a conclusion to be capable of being drawn from them. And no circumstance or time may make its appearance in the conclusion, which was not already mentioned in a premise. Though, of course, the conclusion may be circumstantially or temporally more narrow than the premises.

These are common sense conditions; we can see at a glance that they are reasonable. They tell us that the information in our proposed conclusion must have been implicit in our premises, if we are to claim that it was drawn from them. These rules can be expressed graphically or in the language of 'distribution'.

a. The middle term must occur in both premises, and be distributive once at least, extensionally, naturally and temporally, for any conclusion to be possible. Breach of this rule is labeled fallacy of the undistributive middle term.

b. One of the terms of each premise must be found in the conclusion, and if such an extreme term was undistributive in its premise, extensionally, naturally, or temporally, it must remain likewise undistributive in the conclusion, if any. Breach of this rule is labeled fallacy of illicit process of the major or minor term, as the case may be.

Such breaches of logic are essentially commissions of the fallacy of four terms. A syllogism has a valid structure only if these rules are obeyed; otherwise it is a paralogism. If the middle term is really different somehow in the two premises, it is as if there were no middle term, and therefore no basis for deduction. Likewise, if either term in the conclusion has a meaning or scope different from that given in the premises, it is as if a new term has been introduced in the equation, so that it cannot be called deduction.

Lastly, note the possibility of **sorites** with modal syllogisms, as with actual ones. A regular sorites, consisting entirely of first figure arguments, would look as follows:

All (or Some) A must (or can) be B,
All B must be C, All C must be D, All D must be E,
All E must be F (or No E can be F)
therefore, All (or Some) A must (or can) be F (or the negative equivalent).

The rules of quantity or polarity are the same as before: only the first premise (the most minor) may be particular, only the last one (the most major) may be negative, and the conclusion follows accordingly. Here, we may add the rule of modality, that only the first premise may be potential, and the conclusion follows accordingly.

17. TRANSITIVE CATEGORICALS.

1. Being and Becoming.

Consider the wide form of actual proposition 'This thing, which was/is/will be S at some time, was/is/will be P at some (same or other) time'.

Note that the primary subject is 'this thing', an existent which is being mentally or physically being pointed to; however, that designation is further specified by the thing's characterization as S at some (explicitly specified or tacitly intended) time, say t1. Next, note that the time at which P applies may be the same or a different time, call it t2. This form is uncommitted as to whether, at time t1, this thing is P or nonP, as well as S; nor is it specific as to whether, at time t2, this thing is S or nonS, as well as P.

a. We normally understand the form 'this S is P' to mean 'this thing, which is now S, is now P', implying S and P to be simultaneous. It is a static event, not implying process. The past and future tenses of this likewise only give a still snapshot of the situation at the given time. Such propositions may be called ***attributive***, since their original function is to record the attributes of things.

The copula of 'being' is the most fundamental copula, but it is not the only kind of relation Logic needs to consider. The way categorical propositions of this kind are structured limits their applicability to certain phenomena (the majority, no doubt); certain other phenomena seem best dealt with using categorical propositions with other relational features, defined using attributives.

One such alternative copula is, let us say, 'S changes to P' (intended in the most generic sense of 'something, which initially is S, later is P'). It is close to fundamental, though derived from 'is' (and less used). Such propositions might be called ***transitive***, since they suggest a process across time. We have to be careful to distinguish between the subject-definition time, and the predicate-activation time.

b. However, this verb 'to change to' is colloquially used in many senses, and we should first isolate the ones which concern us most here. When we say 'S has or will, change to, or is changing to, P', what do we mean? A convention is needed, so we can develop two somewhat more specific forms describing change. Let us agree that:

'This S will get to be P' signifies a process starting at S (with or without P) at some earlier time, and ending at P with S at some later time.

'This S will become P' signifies a process starting at S (with or without P) at some earlier time, and ending at P without S at some later time.

We have dichotomized change into two kinds, each covering a segment of the phenomena we come across. These forms represent two species of transitive propositions. Sub-classes of these, or also classifications otherwise defined, can be developed as needed, of course. For example, 'S ceases to be P', for change from SP to nonP.

Note that in both cases, we start as S, without specifying whether P or nonP applies at the same initial time; and we end at P. The copula 'will get to be' is reserved for 'S and P' finales, and the copula 'will become' is reserved for 'nonS and P' finales. If we are not sure whether the finale contains S or nonS, we simply say 'S will get to be or become P', to specify our knowledge of P at least; this is equivalent to the generic 'change to' copula.

These two copula are enough to cover all processes. With nonP as predicate, we say 'will get to be nonP' or 'will become nonP'; with nonS as subject, we say 'this nonS will get to be' or 'this nonS will become'. To deny processes, we may negate the copula, using 'won't' (or will-not) get to be or become. If we wish to specify whether the initial state includes or excludes P, we may do so separately, by saying 'this S is at first P (or not P)'.

This distinction allows us to express whether the change in question is: an ***alteration***, with the starting point S remaining or reappearing (= 'eventual being') at the end, when P arises;

or a ***mutation***, with the original subject S disappearing (= 'becoming') at the end, when P arises. Thus, we may speak of alterative and mutative propositions.

c. Note in passing that there are other 'types of change'. The above types are based on natural or temporal modality; they concern real changes in objects, transitions through which individual things go, over time. However, there is also 'extensional change', as when we say that a genus 'changes' differentiae from one species to another; this refers to relatively static differences, and treats a universal as if it was an individual going through changes (because the mind works serially). More broadly, we have 'logical change', which traces the mental realization of a previously unknown yet existing truth, as if its to us novel appearance was equivalent to its coming into being.

2. Various Features.

a. In the interim, between the beginning at S and the result at P, any state or combination of states may take place (S and P, S and nonP, nonS and P, nonS and nonP), or the change may be abrupt and without intervening state. The forms adopted are left open in that respect. If indeed complications arise in the interim, they can be specified separately, in additional statements.

The transition itself may be gradual or sudden; also, there may be a slow build up of surrounding forces before the process in question is put in motion, or it may take off immediately.

b. In the past tense, 'got to be' and 'became' may be used. They report the culmination of processes. In the present continuous tense, 'is getting to be' (implying 'will get to be') and 'is becoming' (implying 'will become') may be used. Note that these 'presents' do not in fact describe the present stage (i.e. whether S or nonS, P or nonP, are at this time active), but merely define the beginning and end of a process and tell us that it is in progress.

In the cases of 'will change' or 'is changing', if 'must change' is not meant, there is not a real actuality, like for 'has changed', but a certain probability of success, assuming no extraordinary interference, without totally excluding failure. A specific example of such future is of course the expression of intention, of resolve of will-power, including free-will.

The negatives of those actual forms are interpreted accordingly. 'Hasn't changed' denies completion of process so far; 'won't change' denies it for the future, or some part thereof; and 'is not changing' denies a process is taking place.

However, remember that 'will' and 'won't' may strictly-speaking be both wrong as predictions, even though it is, by the law of contradiction, *ex-post-facto* true in general that the event in question must ultimately either happen or not-happen. Modalities are also useable here, to specify more exact statistics of past culminations of process, giving the degree of 'will' or 'won't'.

c. These forms may, of course, all be quantified and naturally or temporally modalized. Thus, quantity and modality, like polarity, are characteristics of relations which transcend the specific copula involved. The plurals 'all' and 'some' used for these purposes are, as usual, intended subsumptively, addressing each unit singly; the meaning is more rarely collectional or collective.

Note that in plural forms, the times at which the implied individual events happen, and the relative times of start and finish of each event, fade in importance, in comparison to singular propositions; the various events span across time, indefinitely, 'whenever they happen'.

Potentiality, as in 'S can get to be or become P', means that in some circumstances, S does get to be or become P'. Natural necessity here signifies inevitability of the alteration or mutation, in contrast to the suggestion of invariability in the static 'S must be P'. If 'can' is combined with 'can not-' (get to be or become), we have contingency. 'Cannot' of course means 'in no circumstances' does the event occur.

Note that potential alteration and potential mutation are compatible, although they cannot actually happen together, since the one finally keeps its subject while the other loses it. It is also conceivable that 'S cannot get to be P' and 'S cannot become P' are both true: together they mean that P is entirely closed to S, by whatever process.

Likewise, with temporal modalities.

d. The oppositions between alterative or mutative propositions, and also those of these two groups with each other, and with attributives, need to all be worked out in detail, of course. But this job will not be carried out here, to avoid expanding this paper more than necessary.

Simply, beginning at the singular level, examine each form's definition for implications, and compare pairs of forms for points of agreement or disagreement; once singular oppositions are established, they can be quantified following the rules developed for attributives.

Similarly for eductive arguments.

3. Various Contrasts.

a. Consider the following examples:

- *Attributive*: This egg is soft (or hard).
- *Alterative*: This egg has hardened (gotten to be hard).
- *Mutative*: This soft egg has become hard (or a hard egg).

A soft egg can't be or get to be a hard egg, but only become one; however, soft and hard eggs are both still eggs. The transition from soft to hard is gradual, depending on heat supply; and the terms soft and hard may be viewed as extremes separated by unnamed in-between states, or we could say that all states before hard are degrees of soft, depending on whether we define soft as raw or uncooked.

In terms of class-thinking, each of these copulae plays a distinct role. The first merely classifies. The second indicates a process leading to a classification. The third declassifies on one side and reclassifies on the other. The logic of classification would be incomplete if we did not consider the more dynamic relations of change.

Note that, whereas 'This S is or gets to be nonS' are self-contradictory, 'This S becomes nonS' is not so, but rather implied whenever an S becomes anything (P or nonP); almost every S eventually becomes a nonS in this world. On the other hand, whereas 'This S is or gets to be S' are formally acceptable, 'This S becomes S' is indeed self-contradictory, unless we add the word 'again', to mean: 'This S becomes nonS, which in turn becomes S'.

b. The similarities and differences between attributives and alteratives should be noticed. 'This S will be P' and 'This S will get to be P' are obviously very close in meaning; in fact, if you look at the definitions, the former is a special case of the latter, and is implied by it. The difference is merely in the time-definition of the subject; when quantity or modality are introduced, the difference blurs, because that time is less definite.

Still, the one is static and the other dynamic. This is more noticeable in the past tense; compare 'This S was P' to 'This S got to be P'. Similarly, in the present continuous sense, 'is' and 'is getting to be' are obviously different. For this reason, two distinct copulae are needed.

c. We should henceforth not confuse 'getting to be' and 'becoming', with each other or with the more general 'changing to'. We make this convention, although these expressions, and others like them (end up as, turn out to be), are colloquially interchangeable, because logic needs fixed terminologies to proceed.

In alteration, the initial term, defining the subject, effectively remains in force at the end, underlying the predication.

Note that, normally, 'S gets to be P' implies 'S is not initially P', suggesting a switch of predicates. But this issue is best left open, since the form is then more widely useable for cases starting and ending at SP, yet having intermediate stages, of nonS and/or nonP, or even for cases involving no change. Often, the terms S and P are two extremes in a range, and the motion from the one to the other is a gradual transition whose intermediate stages are irrelevant, and often unnamed. If the process starts and ends at SP, yet passes through other states, we would say 'S gets to be again P '.

In mutation, there is a radical exchange; both terms are effectively subjects, and we swap one for the other; the initial subject is gone at the end, replaced by another. There may indeed be an implied substratum to the mutation, an underlying constancy; the changing thing remains always at least a 'thing', but often some narrower genus is understood. The paradigm of mutation is of course biological metamorphosis; but such change is found also in physics or other areas.

Note that, 'This S can or must become P' do not imply that 'it cannot become nonP', because the subject is not defined by P or nonP. There is no obversion here; the polarity of the copula (become) and of the predicate (P) are not interchangeable.

4. Some Syllogisms.

We will only concern ourselves here with the some of the more significant moods of syllogism. Our purpose is only to show how introducing transitive categoricals allows us to draw conclusions from arguments which would otherwise be sterile.

The various copula discussed above are in practice often used in tandem, to specify a situation with more precision. We might for instance say the conjunction 'This S cannot be P, but can become it'. In syllogism, a conclusion in the form of a logical disjunction like 'This S can (get to) be or become P' is often possible where otherwise no other conclusion can be drawn.

Systematic treatment of syllogism involving transitives would proceed as follows. We are dealing with four families of proposition: attributives, transitives, alteratives and mutatives; the second of these being a genus for the last two. Each of these classes covers a number of forms varying in polarity, quantity, modality. We must consider, for each figure of the syllogism, every combination of premise, however mixed, and find out what conclusions, of whatever family, can be drawn from them.

However, for now, let us concentrate on a more limited task. We will look at the first figure only, and review only moods whose premises are attributive and/or mutative, and modal. The conclusions obtained have the generic transitive form 'S can get to be or become P (or nonP)'.

a. When both premises are attributive, an attributive conclusion can only be drawn (if at all) from a necessary major premise; if the major is potential, we cannot draw an attributive conclusion. However, a transitive conclusion can be drawn, showing that the modality may have an impact on the copula. The following moods are valid:

All M can be P (or nonP),
All/This/Some S can (or must) be M,
ergo, All/This/Some S can get to be or become P (or nonP).

We know from the premises that what started as S, will in some circumstances be S and M; and that whatever is M, will in some further circumstances be M and P (or nonP); but we cannot predict whether the end result of this process includes or excludes S. It is conceivable that S stays on with P (or nonP), but it is also conceivable that S disappears prior to the arrival at P (or nonP). For this reason, our conclusion cannot be merely 'S can be P (or nonP), but must be open to the 'S can become P (or nonP)' outcome.

b. The same can be argued with a mutative major premise, whatever the modalities involved. Thus, the following are valid:

All M can (or must) become P (or nonP),
All/This/Some S can (or must) be M,
ergo, All/This/Some S can get to be or become P (or nonP).

If one or both premises are potential, so is the conclusion, as above; but if both premises are necessary, a necessary conclusion can be drawn, as below:

All M must become P (or nonP),
All/This/Some S must be M,
ergo, All/This/Some S must get to be or become P (or nonP).

c. In cases where the minor premise is mutative, whether the major is attributive or mutative, similarly disjunctive conclusions may be drawn. We know from the minor premise that S will disappear to become M, but we cannot be sure whether, in the circumstances when M is or becomes P, S reappears or stays away.

Note that in all the cases so far considered, we could view the conclusion as a logical disjunction as we did, or we could say that a more specific conclusion can be drawn if we know one or the other alternative to be excluded. This would be equivalent to having a third premise, viz. 'S cannot get to be P' or 'S cannot become P'. But formally speaking, this constitutes an additional argument (apodosis) after the syllogism as such.

d. All the above only concerns cases with both premises affirmative (whether the predicate be P or nonP). Now, the minor cannot be negative in the first figure, but what if the major premise is negative (in the sense of negating the copula, not merely the predicate)? In such case, we cannot draw a likewise negative conclusion, because we can construct a syllogism with compatible affirmative premises yielding a conflicting affirmative conclusion. Thus, for example, the following mood is invalid:

No M can become P,
This S must be M,
ergo, This S cannot get to be or become P.

This is invalid, because it is conceivable that, though no M can become P, all M can nonetheless be P, in which case the following syllogism could be constructed, as earlier established:

all M can be P,
This S must be M,
ergo, This S can get to be or become P.

We could interpret this to mean that, a (compound) negative conclusion is possible, if the negative major is compound, as in the following mood. Note that major premise and conclusion are conjunctions, not disjunctions, of negatives. The result is due to the attachment of S to M.

No M can be or become P,
This S must be M,
ergo, This S cannot get to be or become P.

If either or both of these premises were potential instead of necessary, a potential conclusion would be drawn.

I shall not, however, develop the matter further, but remain content with having shown some of the impact of transitives on syllogistic reasoning. A full theory should of course list all the valid moods, and do so in all figures. Also, all the above concerns natural modality, but similar arguments can be presented for temporal modality.

18. PERMUTATION.

We need to determine the limits of applicability of classificatory forms by looking more closely at the matter of everyday reasoning. We also need to observe common sense practices, and find out if any forms of argument, other than those dealt with previously, are instinctively used by us.

The ultimate goal of Logic is, of course, to bring out into the open for scrutiny all the details of everyday reasoning. We want to encompass into the sphere of Logic, any forms or processes capable or worthy of formal treatment.

1. Two Senses of 'Is'.

As previously pointed out, the form 'S is P', as commonly understood in Logic, serves the function of classification. Such use of the copula 'to be' is rather abstract and specialized. In this sense, 'S is P' tells us that S is an individual, or some or all members of a class, which also count(s) as among the units of some other class. S and P have to some extent the roles of species and genus.

Most often, in practice, 'S is P' means that P is an attribute of S; we are describing an object (most typically, an entity) in terms of its qualities. We mean that P somehow is in S, something which S has, part of the being of S, one of the many phenomena which all together add up to the thing we call S, and which are distinguishable within it.

This sense of 'is' as attribution is strictly-speaking quite distinct from the use of 'is' for classifying; there is an ambiguity, the same word is used for two different relations. The common ground of the two is the information that the 'universals' S and P, in some degree intersect. Two domains of reality are compared for overlap, in their instances, in space and time, in causal contexts.

Thus, 'S is P', may mean 'S has P-ness' or 'the unit(s) of S is/are unit(s) of P'. The attributive sense may be 'permuted' to the classificatory, by saying 'S is P-ness having'. This puts the specific copula 'has' into the predicate, in a proposition with more numerical intent.

But the possessive relation remains; it has value and meaning quite apart from such permutation; otherwise, we would have no need for the concept. Classification cannot occur without prior attribution. Before one can put a unit under a class, the class-defining quality has to be attributed to the unit. Permutation merely conceals or bypasses the attribution, it does not erase it.

The classificatory 'Ss are Ps' contains attributive propositions within its terms, since it means 'things which are S are things which are P'. The 'are' in the two terms are attributive, while the 'are' of their relation to each other is classificatory.

Attribution concentrates on the substantive (i.e. in terms of universals), qualitative, so-called *'connotative'* aspect, of this relation of coincidence. Classification focuses on the enumerative, quantitative, so-called *'denotative'* aspect. Both aspects exist out there (at whatever level the phenomenon might be); we mentally isolate the one from the other somewhat, to stress each in turn.

Although we regard these as two aspects of the same term — we say that in one case, it is taken 'in its denotation', and in the other case, it is taken 'in its connotation' — it is more accurate to say that these are two kinds of term, which have a close relationship, both objective and verbal. For example, dogs may be viewed as things which have dog-ness (or in better English, caninity), and dog-ness may be viewed as that which dogs have in common and distinctively.

In truth, dog-ness cannot be called the meaning of dogs, as some have suggested; nor is the reverse correct. Each of these terms subsumes slightly different referents: the former refers to every corporeal dog, and the latter (though we regard it as effectively singular) refers to every occurrence of the qualities which make up dog-ness. These two are indeed equal in number, and have no existence apart from each other, but the intention is not identical.

2. Other Permutations.

Other propositions are permutable, besides ‘S is (or has) P’. Some propositions are geometrical: they locate the subject in space, by reference to the location of the predicate; thus, ‘S is at or in, next-to, above or below, near or far-from P’. Some involve placement in time, using relatives like ‘earlier’, ‘later’, ‘at the same time’. One can say that ‘S is at P’ implies ‘S is an at-P thing’, but one cannot say that they are equivalent and identical.

Some propositions describe actions, implying change and/or causality. To do, is to change or move in a certain way, or to cause something else to be or to change somehow. The verb may be simple or continuous; ‘S does or is doing P’. Here again, permutation is possible, to ‘S is a P-doer or one of the P-doing things’. We permute when it is useful, but the original form does not lose its utility and disappear.

The logical properties of the classificatory forms are only generic properties, applicable to all permutables. We may well expect that each of the original, more specific, forms, has its own special logical properties. It is part of the long-term task of logical science to gradually intercept all such forms, and confront them for analysis of their peculiar properties. To establish the implications, oppositions, and arguments, which are peculiar to each form.

While modality can be permuted, e.g. ‘S can be P’ to ‘S is one of the things which can be P’, this easily leads to error. The reason is that, normally, when we say of a unit that it belongs to predicate P, we mean this to apply to times and circumstances when it is actually P. This is not equivalent to reference to the class of things only potentially P. As a result, as we have seen, if this process is used inattentively, one may draw wrong conclusions in syllogism.

Similarly with transitive copulae, like becoming. They are too fundamental, too much part of the structure of things, to be permuted with any more than superficial interest. To process ‘S becomes P’ to ‘S is one of the P-becoming things’ merely puts the need to investigate the logic of becoming one step removed, since it still there, but now hidden in a more detailed predicate. Likewise, with regard to causality. To find the specific properties of such a copula, we have to keep it intact, as a relation in its own right.

(Note that any goals pursued by such permutations can, as we shall later see, be fulfilled by using ‘extensional conditional’ propositions.)

Copulae represent relations. Each relation has its own nature. ‘Is’ in the sense of class-thinking, is a broad relation, with a number of rules; ‘is’ (in the attributive), ‘becomes’, ‘causes’, are all perhaps narrower relational concepts, but still worthy of special attention, in search of the rules applicable to them specifically.

Relations like the verbs ‘sings’ or ‘digs’ also no doubt have their own characteristics, and are legitimate topics of inquiry; but their limited scope, assigns them to a secondary position in logical science, while they may well be of primary importance to other sciences. We might make a distinction between formal logic and material logic, on this basis.

3. Verbs.

Let us look briefly at propositions with a verb of possession or action. Their general form is ‘X / does-Y / Z’. There is a logical subject (X), a verb (here written ‘does-Y’), and some or no appendages (the ‘Z’ part).

‘Does’ is here meant to include agency and passion, doing and being done to. But not every action implies a patient, note. The agency, by the subject, of the act (verb/relation), may range from an act of free-will to a fixed absolute. An act may be static or dynamic, it may signify a posture, a movement, a forcing. In this context, even 'has' is an act.

Note that quantity and modality are applicable to any such proposition, as usual. The simple and continuous present tenses are not interchangeable, of course. The modality involved is often left implicit, and should always be clarified. For instance, 'animals sleep' implies 'animals are sleeping some of the time' without implying 'all the time' or 'at this time'.

The appendages concern parameters such as: who, what, where, when, why, whence, how, what for, how much. They serve to further specify the relation, and delimit it. Prepositions like 'by', 'at', 'to', 'for', are definable in this context.

We may mention any combination of the following: the patient of the action, or agent of the passion (e.g. electrons repel each other); the locale of the incident in time and/or space (e.g. he went west at sunrise); the conditions or causes (e.g. water boils at 100 deg.C, at s.t.p.); the effects or consequences (e.g. it drove him to work harder); the means or ends (e.g. the water was boiled for tea).

Also, generally, any measurement, qualitative or quantitative, of the above, may be mentioned (e.g. she sang beautifully and softly). The 'measure' may concern the relationship itself (e.g. he gave liberally), or an appendage of it (e.g. he gave lots of money). Attaching expressions of number, magnitude or degree to a statement (e.g. everyone has some virtue), should not be confused with the attempt of some logicians to 'quantify' the predicates of classificatory propositions (as in 'all X are some Y').

Often, the proposition can be restructured from categorical form to subjunctive form, and so the logic of subjunction comes into play (e.g. 'She sings when happy' is more subjunctive than categorical).

Causality is often concealed in statements which do not mention it. For instance, 'her beauty attracted him' implies that the agent caused the patient to act or move in a certain way. Causality of course is of various degrees, ranging from mere influence on a voluntary act or probabilistic tendency, to compulsion or mechanical force.

Thus, we see that many material statements can be analyzed for more formal components, and thereby be subjected to certain rules of formal logic. They may nonetheless yet contain further logical properties, not found in the formal components.

4. 'As Such' Subjects

We have looked at permutation which encloses a nonclassificatory copula and its appendages into the predicate of a classificatory proposition. But we also need to consider permutation of the subject.

Many propositions have a 'universal' as their subject. This may be a quality as such (e.g. turquoise is a bright color), or any act as such (e.g. 'love is a nice feeling', 'running is good for you'). 'As such' subjects like these are being focused on, not so much in their capacity as classes embracing the various appearances of the 'universal' in the world, but as if the 'universal' is an individual thing (a whole, whose various manifestations in the world are its parts).

A class concept regards the individual manifestations of a universal, or of a complex intersection of universals, as separate entities, which happen to display that simple or complex distinct character in common. An 'as such' concept, instead, views the particular manifestations as merely the segments of a single, continuous thread, and refers to that more or less uniform whole as a distinct entity.

Before a proposition can be processed according to the logic of classificatories, its subject may need permuting. (Thus, for our examples above, we would say 'turquoise things are bright-colored', 'people in love feel nice', or 'runners are healthier').

5. Commutation.

Concerning verbs, the process of commutation should be mentioned.

Relations are generally directional, so that when something is true in one direction, then something else is true in the other direction. The 'total' relation between the terms includes both directions, though only one of the components might be named, leaving its correlative implicit. The correlative copulae may be identical (e.g. A=B and B=A) or quite different (e.g. owns and belongs-to, or shot and was-shot).

We may call 'commutation' the inference from one direction of a relation to the other. The correlative of 'is' is 'is', as conversion shows. The proposition 'X causes Y' is commutable to 'Y is caused by X'. 'He bought IBM shares' implies and is implied by 'IBM shares were sold to him'. A lot of our reasoning consists in rewording statements like that, changing the perspective to improve our understanding.

Commutation may apply not only to the main copula of a proposition, but to copulae implicit in its terms. For example: 'the bodies attract with a force of 10 dynes: the force, with which the bodies attract, is 10 dynes'.

19. MORE ABOUT QUANTITY.

In this chapter, we will look into various topics which involve quantitative considerations.

1. Substitution.

Substitution is a widely used, yet little noticed logical process, which is open to formal treatment of sorts. It consists in replacing a term with another which has the same units, but views them in a somewhat different perspective. The entity referred to remains the same, only its label changes (*qua* what it is referred to); the substitution is thus justifiable.

We may substitute a generic term for a species, if we keep the same quantity, or a species for an individual. For instance: 'X has (some number of) Y, All Y are Z, so X has (that many) Z', or 'X has this Y, this Y is Z, so X has a Z'. Example: 'Man has a mind, a mind is an organ, so man has an organ (at least one)'.

Note that this is not a normal, first-figure, classificatory syllogism. Here, the major premise must be or be made affirmative and classificatory; but the minor premise and conclusion are possessive (in this case). Needless to say, verbs other than 'to have' are open to substitution, too. Example: 'Bill hit Joan: Bill hit a woman'.

If the major premise is negative, it should be obverted before the substitution. Thus, 'X has Y, No Y is Z' conclude 'X has nonZ', rather than 'X doesn't have Z'. For example, 'Tom has a dog, a dog is not a cat'. Likewise, if the major premise is not classificatory, it should first be permuted. Thus, in 'X has Y, all Y have Z', the term to substitute would be 'something which has Z', rather than just 'Z'. Example: 'Tom has a dog, dogs have fleas'. We are said to commit the 'fallacy of accident' when we make errors of this kind.

If the minor premise is negative, the conclusion must be formulated very carefully, if at all. One is forced to keep the middle term in the conclusion, only qualifying it with the major term, to ensure we do not change the implicit quantity of units referred to. Examples: 'Tom doesn't have a dog: Tom doesn't have an animal of the dog kind (though he may have some other animal)', 'Bill did not hit Joan: Bill did not hit this woman (though perhaps another)'.

We often profit by substituting a subject. This process takes the third figure form: 'Y is X, Y has Z, so an X has Z'. Here, the major premise may have any copula or polarity (in this case we used 'has'), while the minor should be affirmative and classificatory. Example: 'Joe is a man, Joe runs 40 miles a day, so (there's) one man (which) runs 40 miles a day'. Substitution of a pronoun would take this form.

Even the verb may be substituted, using an exact description of some necessary aspect of it. Examples: 'the magnet was repelled 3 feet: the magnet was caused to move 3 feet' or 'he sprinted to the finish line: he ran to the finish line'.

What logicians call immediate inference by added determinants (e.g. 'horses are animals: therefore, the heads of horses are heads of animals') or complex conception (e.g. 'Physics is a science: therefore, physical treatises are scientific treatises'), involve substitutive syllogism, with a tacit minor premise (e.g. 'horses have heads' or 'some treatises are about physics'), which enables the conclusion to be drawn. These processes are illicit when the rules of substitution are not properly obeyed (e.g. 'horses are animals: the majority of horses are the majority of animals' or 'physics is fun, physical treatises are funny').

2. Comparatives.

Logic interfaces with mathematics, whenever we compare the number, position, magnitude or degree of something, relative to something similar; or of two things measured by reference to a third. We may call such propositions ‘comparative’.

This concerns forms like ‘X is more Y than Z’, ‘X is less Y than Z’, ‘X is as Y as Z’. They affirm X to be greater, smaller or equal to Z, in some respect Y (e.g. this metal is as strong as steel). It is implied that X and Z are each Y, though to different extents.

Sub-categories of these measures may be defined by inserting more precise quantities, like ‘much more’ or ‘30% more’, say. Further complications are often introduced, through the concepts of ‘enough’ and ‘too much’, which evaluate the measurements in relationship to some goals.

Copulae other than ‘is’ may be involved, and the comparison may concern the verb or an appendage (e.g. ‘he ran faster than her’ or ‘they ordered more food’). Often the comparative aspect is verbally concealed (e.g. ‘she was happier today’ or ‘that ball is the closest’).

The corresponding negative forms are defined as follows. ‘X is not more Y than Z’ means ‘either X is not Y and/or Z is not Y or X is less or equally Y compared to Z’. Similarly for ‘not less’ (= not at all, or more or as much), and similarly for ‘not as much’.

Since only one of the three affirmative measures may be true, and, granting that Y is applicable to X and Z, one of them must be true, they are contrary to each other. It follows that the negatives which contradict them are subcontrary to each other.

Comparative propositions can be commuted. If X is (or is not) more/as much/less Y in comparison to Z, then Z is (or is not) less/as much/more Y in comparison to X, respectively. Example: ‘he left just before sunrise’ to ‘the sun rose soon after he left’.

Syllogistic style arguments can be constructed. For instances, using the symbols of mathematics (>, =, <; and / for their negations), to signify the possession or lack, of some common character, we can predict that ‘If A>B and B>C, then A>C’ or that ‘If A≠B and B=C, then A≠C’. In some cases, no deduction is possible; as for instances in ‘A>B and B<C’ or ‘A≠B and B≠C’. These relations are generally well known, and need not be pursued further here.

Comparative propositions are significant in so-called *a-fortiori* arguments. These arguments are quite important and very commonly used in everyday reasoning, but apart from a brief mention of them in a later chapter, they will not be analyzed in detail in this volume. I hope to deal with them in a later work.

3. Collectives and Collectionals.

A proposition is ‘dispensive’, or ‘collective’, or ‘collectional’, according to the way its subject subsumes its units for the predication.

a. Most propositions are ***dispensive*** (many authors prefer the name ‘*distributive*’ instead), and this is the way of subsumption we have dealt with so far, in detail. A plural dispensive refers us to its class members severally, each one singly; it is simply a conjunction of a number of mutually independent singulars. Thus, ‘all or some S are (or are not) P’, means ‘this S is P; that S is P;… and so on’, until all the S intended to be included under the all or some quantifier have been enumerated.

b. In contrast, some propositions are ***collective***, applying a predicate to the units included by their subject only if they are taken together, and not separately. Such propositions are effectively singular; they conjoin the units into a group, rather than a class. A collective has the form ‘these S together are P’, meaning ‘this S and that S and…so on, taken as one, are P’. Note that, unlike with dispensives, ‘all S together are P’ does not imply that ‘some S together are P’.

The group may have a summational property, which sums up the lesser measures or degrees of the same property displayed by the parts (e.g. we each have ten dollars, but both of us together have twenty). Or the group may have a composite property, due to the causal interaction of the parts, which is not found in any measure or degree in the parts themselves (e.g. as individual cells together make up a human being, the whole having various powers the parts lack). In the latter case, a certain arrangement of the parts may be tacitly required for the predication to work, so that a statement more descriptive than mere conjunction may be needed for accuracy.

The logical subject here is 'these S together'; we may, if we so wish, form a new collective term from it (like 'crowd' or 'society'). Note that, in some propositions, the intention is a dispensive summary of collectives (e.g. 'fifty books form a big pile' means any set of fifty books). This may be formalized as follows (where '*n*' signifies some number): 'Any *n*S together are P' or 'Certain *n*S together are P'; here, '*n*S together' refers to a class of collectives.

c. Some propositions are ***collectional***. These differ from dispensives and collectives, in that, although they refer to events each one singly, they also tell us whether these can or cannot, are bound to or may not, be actual jointly — simultaneously, at the same definite or indefinite instant or period of time. This is usually signified by stressing the quantity (by the tone of voice or italics).

Thus, here, '*All* S can be P' means 'the conjunction of this S as P and that S as P and… so on — is potential': this does imply the dispensive 'all S can be P', but further reveals that the actualization of these potentials can take place all at once. We would use '*All* S can be nonP', if we want to say 'it can happen that all S are simultaneously nonP'.

Accordingly, '*All* S cannot be P' denies the potential for simultaneous actualization, the 'not' being directed at the 'all' (rather than at the 'can be'): it is formally compatible with 'all S can be P' in a dispensive sense; though usually used in such context, it does not imply it. We would use '*All* S cannot be nonP', if we want to say 'it cannot happen that all S are nonP at once'.

(In contrast, the form '*All* S must be P' would be interpreted as 'it cannot happen that some S are not P: if any S are P, then all are P'; similarly with '*All* S must not-be P'; to deny these statements, we would say 'it can happen that some S… etc'.)

The particular versions of such statements, '*Some* S… etc'. may be similarly analyzed. There are also singular versions, like 'this S can be P, alone', which tells us about the potential for actualization of 'this S is P' when all *other* S are nonP. More broadly, any quantity '*n*' may be specified: thus, for instance, '*n*S can be P' informs us that this number of S can be P at the same time; in some cases, we additionally specify 'at least' or 'no more than' to open or limit the statement.

The above concerns natural modality, but equivalent statements involving temporal modality are conceivable: '*All* S are sometimes P', '*All* S are never P', and so forth. Note that collectionality is used in a modal context; the actual proposition '*All* S are not P' (meaning that not-all S are P, meaning that some are not, though some are), is not really collectional.

Collectional intent is often encountered in the antecedent or consequent of conditional propositions (for examples, 'when *all* the cog-wheels are aligned, the key is able to turn' or 'when the button is pressed, *all* the lights come on').

I will not here work out the logics of collective and collectional forms in detail. Each form needs to be analyzed for its exact implications, then the interactions of all the forms with each other and with dispensives (including all immediate and mediate inferences) must be looked into.

4. Quantification of Predicate.

The forms people currently use, and accordingly adopted by the science of Logic, are so designed that we can specify alternate quantities for the predicate, if necessary, simply by making another, additional statement in which the original predicate is subject and the original subject is predicate, with the appropriate distributions.

However, as an offshoot of the distribution doctrine, there have been attempts to invent forms which explicitly 'quantify the predicate' of classificatory propositions. Let us look into them briefly.

a. On a singular level, the basic form would be ‘this S is this P’. The contradictory ‘this S is not this P’ would be compatible with ‘this S is that (meaning, some other) P’.

Normally, we need to know, say, ‘whether the girl is or is not (at all) pretty’, rather than ‘whether she is or not that pretty thing’. We may of course say ‘her dress was this shade of brown’; but here the indicative only specifies a kind of color, not an individual qualitative phenomenon. Someone may tell me ‘the girl I mean is the one we met last week’; but here the predicate is intrinsically a one-member class.

Normally, we use indicatives in the subject, rather than the predicate. The indicative is used to ‘hold down’ a first appearance, as our initial designation of the object: once, that is settled, we are only interested in discovering its further attributes as such.

Suppose I see a green and blue object, I may say ‘this green thing is blue’ (or vice versa), but I would have no need to establish class correspondence, since the object is already one and the same right before my eyes. It is not inconceivable that I perceive a green object and later a blue object, and then equate (or distinguish) them, saying ‘this green thing is (or is not) the same as that blue thing’; but this is a rare exception, and is it really classification?

When we say ‘this S is P’ we first intend to qualify the subject by the predicate (e.g. that baby was rather cute). We cannot transfer the designation ‘this’ from the subject to the predicate without missing the point, which is attribution. Also, we normally use ‘this’ to refer to entities, rather than qualities (though we can say ‘this green is rather dark’).

Still, theoretically, ‘this thing’ under discussion is indeed theoretically an instance of P as well as S, so that permutation to classificatory form is feasible. We have to remain formally open in this issue, since we do regard ‘all S’ as implying ‘this S’.

b. With regard to plurals, ‘quantification of the predicate’ would give rise to the following forms: ‘all (or some) S are all P’ (both implying that all P are S); ‘No S are certain P’ (implying some P are not S) and ‘some S are not certain P’ (the latter two not excluding that all S be P — i.e. other instances of P).

The forms: ‘all S are some P’, ‘some S are some P’, ‘no S are any P’ and ‘some S are not any P’, would be equivalent to the established **A**, **I**, **E**, **O**. The rest would be relatively new.

Only the form ‘some S are not certain P’ contains information we cannot express in natural language: but that may be simply because we never need to make such a statement of partial exclusion in practice.

These forms have not aroused much interest, because they are artificial to our normal ways of thinking. If we have so far managed very well without them, why complicate things and try to introduce something no one will ever use?

However, to be fair, such statements are indeed used by logicians, if not by laymen, to clarify the distributions of terms. We would speak in that way to explain Euler diagrams, mentioning the one-one correspondence of individual members of distinct classes, or the overlap or separation of segments of classes. Thus, we may view them as specialized, rarely used — but still legitimate.

Quantification of the predicate could also be viewed as a special case of substitution.

PART III. LOGICAL CONDITIONING.

20. CREDIBILITY.

1. Laws of Thought.

We began our study by presenting the laws of thought — the Laws of Identity, of Contradiction, and of the Excluded Middle — as the foundations of logic. We can see, as we proceed, that these first principles are repeatedly appealed to in reasoning and validation processes. But in what sense are they 'laws'?

a. Many logicians have been tempted to compare these laws to the *axioms* of geometry, or the top postulates of natural sciences. According to this view, they are self-consistent hypotheses, which however are incapable of ultimate proof, from which all other propositions of logic are derived.

There is some truth to this view, but it is inaccurate on many counts. The whole concept of 'systematization' of knowledge, ordering it into axioms and derivatives, is itself a device developed and validated by the science of logic. It is only *ex post facto* that we can order the information provided by logic in this way; we cannot appeal to it without circularity. If logic was based on so tenuous a foundation, we could design alternative logics (and some indeed have tried), just as Euclidean geometry or Newtonian mechanics were replaced by others.

Logic is prior to methodology. The idea that something may be 'derived' from something else, depends for its credibility on the insights provided by the 'laws of thought'. The 'laws of thought' ought not to be viewed as general principles which are *applied* to particular cases, because the process of application itself depends on them.

Rather, we must view *every particular occurrence* of identity, contradiction, and excluded-middle, as *by itself compelling*, an irreducible primary independently of any appeal to large principles. The principles are then merely statements to remind us *that* this compulsion occurs; they are not its source. This means that the 'laws of thought' are not general principles in the normal sense, but recognitions that 'there are such events'. The science of logic is, then, not a systematic application of certain axioms, but *a record of the kind of events which have this compelling character* for us.

Note this well. *Each* occurrence of such events is self-sufficiently evident; it is only thereafter that we can formulate statements about 'all' these events. We do not know what to include under the 'all' beforehand, so how could we 'apply' the laws to anything? These laws cannot be strictly-speaking 'generalizations', since generalization presupposes that you have some prior data *to* generalize.

Thus, we must admit that *first* comes specific events of identity, contradiction and excluded-middle, with a force of their own, then we can say 'these and those are *the kinds of* situations' where we experience that utter certainty, and only lastly can we *loosely-speaking* format the information in the way of axioms and derivatives.

Nevertheless, it remains true that the laws of thought have a compelling character on their own. There is no way to put these laws in doubt, without implicitly arousing doubt in one's own claim. Sophisms always conceal their own implications, and tacitly appeal to the laws of thought for support, to gain our credulity. We could, therefore, equally say that the principles as units in themselves are entirely convincing, with utter finality — provided we *also* say that every act of their 'application' is likewise indubitable. It comes to the same.

However, the previous position is more accurate, because it explains how people unversed in the laws of thought, can nonetheless think quite logically — and also how we can understand the arguments here made about the laws of thought. The inconsistency of denials of the laws of thought is one instance of those laws, and not their whole basis.

b. What, then, is this 'compulsion' that we have mentioned? It is evident that people are not forced to think logically, say like physical bodies are forced to behave in certain ways. This is given: we do make errors, and these sometimes seem 'voluntary', and sometimes accidental. In any case, if thought was a mechanistic phenomenon, we would have no need of logical guidelines. We may only at best claim that we *can and should, and sometimes do*, think in perfect accord with these laws.

The answer to this question was implicit in the above discussion. It is or seems evident that things do present themselves and that they do have certain contents (identity), and that these presentations are distinct from their absences (contradiction), and that there is nothing else to refer to (excluded-middle). Because these statements concern appearances as such, it is irrelevant whether we say 'it *is* evident' or 'it *seems* evident'.

The concepts of reality and appearance are identical, with regard to the phenomenal; the concept of illusion is only meaningful as a subdivision of the phenomenal. These laws are therefore *always* evident, whether we are dealing with realities or illusions. We can wrongly interpret or deliberately lie about what we 'see' (if anything), but *that* we 'saw' and *just what* we 'saw' is pure data. Thus, the 'compulsion' is presented to us an intrinsic component of the phenomenal world we face.

The practical significance of this can be brought out with reference to the law of contradiction. We are saying, in effect, that whatever seems contradictory, is so. This statement may surprise, since we sometimes 'change our minds' about contradictions.

To understand it, consider two phenomena, say P1 and P2, in apparent contradiction, call this C1. One way to resolve C1, is to say that one or both of P1 and P2 are illusory. But we might find, upon closer inspection, that the two phenomena are not in contradiction; call this noncontradiction C2. So we now have two new phenomena, C1 and C2, in apparent contradiction; call this new contradiction C3.

The question is, does C3 imply that one or both of C1 and C2 are illusory? The answer is, no — what happened 'upon closer inspection' was not a revision of C1, but a revision of P1 and/or P2. So that in fact C2 does not concern exactly the same phenomena P1 and P2, but a slightly different pair of phenomena with the same names.

Thus, C1 and C2 could never be called illusory (except loosely speaking), because they were never in conflict, because they do not relate the same pair of phenomena. Nor for that matter may C3 be viewed as now erroneous, because the pair of phenomena it, in turn, related have changed.

Which means that our 'intuition' of contradiction is invariably correct, *for exactly the data provided* to it. A similar argument can be made with regard to other logical relations. The phenomena related may be unclear and we may confuse phenomena (thinking them the same when they are different) — but, at any level of appearance, the logical relation between phenomena is 'compulsively evident', inflexibly fixed, *given*.

In other words, *among phenomena, logical relations are one kind which are always real*; in their case, appearance and reality are one and the same, and there are no illusions. The laws of thought are presented as imperatives, to urge us to focus on and carefully scrutinize *the phenomena related*, and not to suggest that the *logical* intuitions of thought are fallible, once the effort is made to discern the relation.

This is not a claim to any prior omniscience, but a case by case accuracy. As each situation arises, its logical aspects are manifest to the degree that we inspect things clearly. Note well, we do not need to know *how* the intuition functions, to be able to know and prove *that* it functions well. We have called it 'intuition' to suggest that it is a direct kind of consciousness, which may well be conceptual rather than perceptual, but these descriptive issues are secondary.

Thus, with regard to the laws of thought, we have no ground for wondering whether they are animal instincts imposed by the structure of the mind, or for wondering whether they control the events external to it as well. In either case, we would be suggesting that there is a chance that they might be illusory and not real. If we claim that the mind is distortive, one way or the other, we put that very claim in doubt.

The mind is doubtless limited. It is common knowledge that mental conditions, structural or psychological or voluntary, can *inhibit* us from comparing phenomena with a view to their logical relation — but that does not mean that when the elements *are* brought together, the comparison may fail.

Nervous system malfunctions, personality disorders, drunkenness, fatigue — such things can only arrest, never alter these intuitions. As for evasions and lies, we may delude ourselves or others, to justify some behavior or through attachment to a dogma — but these are after the fact interventions.

2. Functions.

The laws of thought relate to the credibility, or trustworthiness, of phenomena. They clarify things in three stages. At the identity level, appearances are acknowledged and taken as a data base. At the contradiction level, we learn to discriminate clearly between real and illusory appearances. At the excluded-middle level, we introduce a more tempered outlook, without however ignoring the previous lessons. More specifically, their functions are as follows:

The first law assigns a minimal credibility to any thought whatsoever, if only momentarily; the evidence, such as it is, is considered. If, however, the ‘thought’ is found to consist of meaningless words, or is overly vague or obscure — it is as if nothing has appeared, and credibility disappears (until and unless some improvement is made). To the extent that a thought has some meaning, precision, and clarity, it retains some credibility.

The second law puts in doubt any thoughts which somehow give rise to contradictions, and thereby somewhat enhances the credibility of all thoughts which pass this test. In the case of a thought which is self-inconsistent (whether as a whole or through the conflicts of its parts), its credibility falls to zero, and the credibility of denial becomes extreme. In the case of two or more thoughts, each of which is self-consistent, but which are incompatible with each other, the loss of credibility is collective, and so individually less final.

The third law sets bounds for any leftover thoughts (those with more than zero and less than total credibility, according to the previous two laws): if special ways be found to increase or decrease their credibilities, the overall results cannot in any case be such as to transgress the excluded-middle requirement (as well as the no-contradiction requirement, of course). As we shall see, the processes of confirmation and discrediting of hypotheses are ways logic uses to further specify credibilities.

We see that, essentially, the law of identity gives credence to *experience*, in the widest sense, including concrete perceptions and abstract conceptual insights. The law of contradiction essentially justifies the logical intuitions of *reason*. The law of the excluded-middle is essentially directed at the projections of the *imagination*. This division of labor is not exclusive — all three laws come into play at every stage — but it has some pertinence.

The credibility of a phenomenon is, then, a measure of how well it fits into the total picture presented by the world of appearances; it is a component of phenomena, like bodies have weight. This property is in some cases fixed; but in most cases, variable — an outcome of the interactions of phenomena as such.

The laws of thought are, however, only the first steps in a study of credibility. The enterprise called logic is a continual search for additional or subsidiary norms. Logic theory develops, as we shall see, by considering various kinds of situations, and predicting the sorts of inferences which are feasible in each setting.

More broadly the whole of philosophy and science may be viewed as providing us with more or less rough and ready, practical yardsticks for determining the relative credibility of phenomena. However, such norms are not of direct interest to the logician, and are for him (relatively speaking) specific world views. Logic has to make do with the two broadest categories of reality and illusion — at least, to begin with.

3. More on Credibility.

Every phenomenon appears to us with some degree of *'credibility'*, as an inherent component of its appearance; this is an expression of the law of identity. That initially intuitive credibility may be annulled or made extreme, through the law of contradiction; or it may be incrementally increased or decreased, by various techniques (yet to be shown), within the confines of the laws of contradiction and of the excluded middle.

Thus, credibility is primarily an aspect of the phenomenal world, and a specific phenomenon's degree of credibility is a function of what other phenomena are present in the world of appearances at that stage in its development. Because phenomena interact in this way, and affect each other's credibilities, credibility may be viewed as a measure of how well or badly any phenomenon 'fits in' with the rest.

'Reality' and 'illusion' are just the extremes of credibility and incredibility, respectively; they are phenomena with that special character of total or zero force of conviction. We cannot refer to a domain beyond that of appearances, for good or bad, without thereby including it within the world of appearances.

How do we know that all appearances must ultimately be real or illusory? How do we know that *median* credibility cannot be a permanent state of affairs in some cases, on a par with the extremes of credibility and incredibility? We answered this question, in broad terms, in our discussions on the laws of thought, as follows. More will be said about it as we proceed.

Reality and illusion are a dichotomy of actual appearances: for them, whatever is inconsistent is illusory, and everything else is real enough. Median credibility only comes into play when we try to anticipate future appearances, but has no equivalent in the given world. In the actual field of concrete and abstract experiences, things have either no credibility or effectively total credibility; it is only through the artificial dimension of mental projections that intermediate credibility arises.

Knowledge is merely consciousness of appearances; the flip-side, as it were, of the event of appearance. Viewed in this perspective, without making claims to anything but the phenomenal, knowledge is always a faithful rendering of the way things appear. We may speak of knowledge itself as being realistic or as unrealistic or as hypothetical, only insofar as we understand that this refers to *the kind of* appearance it reflects. These characterizations refer primarily, not to knowledge, but to its objects.

The difference between knowledge (in its narrower sense of, knowledge of reality) and opinion (in the sense of, the practically known), is thus merely one of degree of credibility *manifested by their objects* (at that time); we cannot point to any essential, structural difference between them. However, this distinction is still significant: it matters a lot that the objects carry different weights of conviction.

Changes or differences in appearances and opinion are to some extent *explained* by reference to variations in our perspective, and breadth and depth of consciousness. But this explanation does not annul the primacy of phenomena, in all their aspects.

In practice, median credibility is often not patiently accepted, but we use our 'wisdom' to lean one way or the other a bit, according to which idea seems to 'hang together' the best. But a contrary function of wisdom is the ability to see alternatives, or the remote possibility of suggested alternatives, and thus keep an open mind. The intelligent man is able to take positions where others dither, and also to see problems where others see certainties.

4. Opinion and Knowledge.

I would like to here mention in passing, without going into details, that work has been done by some logicians, in clarifying the logical properties of belief (or opinion) and knowing.

The logic of belief is concerned with the implications of propositions such as 'S believes that P', where S is a subject of consciousness and P is any proposition. Belief, disbelief, and uncertainty are subjectively given: they are facts immediately accessible to the subject, though they may be wrongly remembered or dishonestly reported. There are also iterative forms to consider, like ‘X believes that Y believes that Z’.

The following are some of the formal issues in this field. Mutual oppositions: believing something does not imply disbelieving its contradictory, since people sometimes do (however ‘illogically’) believe both a thesis and its contradictory; therefore, disbelieving and not-believing are not identical. Also, relationships to ‘alethic’ propositions: believing something does not imply that it is true, and the true may be disbelieved.

The logic of knowing may similarly be investigated, for forms like ‘S knows that P’. These topics are not unrelated, since knowing is taken to imply believing (ordinarily, though sometimes we resist), even if we believe some things without the degree of certainty which we qualify as knowing (or perhaps with reference to other standards of judgment). The distinction between conscious awareness and ‘tacit knowledge’ has to be considered.

Knowing is often regarded as based on more rigorous methodology than belief, and hence effectively implying (at least contextual) truth; whereas belief may be groundless or even contrary to reason. Knowing implies some effort of review and control of belief, with reference to logical standards of some sort. If one has trained oneself in logic, avoided all laziness, and tried to be honest, then as far as that subject is concerned his or her belief has become knowledge. For that subject now, though not necessarily at some other time or for other subjects, this is equivalent to the ideal of truth.

Belief, knowing and alethic truth are three parts of the same curve: truth is the ‘vanishing point’ toward which belief and knowing tend. Belief is more inertial, more affected by emotional forces, like peer group pressure or psychological factors. Knowing involves willfully freeing one’s mind of such prejudices and influences, but is still a function of one’s intelligence, logical skills, research efforts, the limitations of one’s cognitive faculties.

21. LOGICAL MODALITY.

1. The Singular Modalities.

I do not claim that my theory of logical modality as it stands solves all issues, but I think you will find it very productive, an impressive integrative force.

The concepts of 'logical modality' enable us to predict systematically all the ways credibility may arise in knowledge over the long-term. Credibility itself is not a type of modality, but the ground and outcome of logical modality. We shall immediately define the primary categories of logical modality, and thereafter discuss their development, their significance, and their justification:

Truth is the character of a proposition which seems more convincing than its negation, in a given context of knowledge. In the case of any proposition implied by its own negation, its credibility is extreme.

Falsehood is the character of a proposition which seems less convincing than its negation, in a given context of knowledge. In the case of any proposition implying its own negation, its incredibility is extreme.

A proposition is ***'problematic'***, with regard to its truth or falsehood, if it seems to carry neither more nor less conviction than its negation, in the given context of knowledge. This is indicated by such expressions as 'might or not be' or 'perhaps is and perhaps is not'.

In practical terms, the ***degree of credibility***, whether high, low, or median, of a proposition is a measure of the amount of evidence or counterevidence put forward on its behalf or against it. This refers to the weighting of information by confirmation or undermining, which topic will be dealt with more fully under the heading of adduction.

By (logical) ***context*** is meant, the accumulated experiences and conceptual insights of the knower (a person or society) at the time concerned.

The context-specific concepts of logical modality are built on the awareness that: at every stage of knowledge, some things somehow seem 'true', other things somehow seem 'false', yet others seem 'problematic'; and that these attributes often vary with the growth of experience and reasoning.

These observations suggest that, although every appearance is accompanied by some such characterization, the characterization is not in all cases firmly attached to the object, but is often a function of the experience and reasoning which have preceded them.

The concepts are thus formed, to begin with, only in recognition that such events occur, and that they are distinguishable by our consciousness, and that they each display such and such properties. Then we say: 'Let us call this truth or falsehood or problemacy, as the case may be....'

It must be stressed that underlying the foregoing definitions of truth, falsehood, and problemacy, is the assumption that a sincere effort of awareness took place. It is difficult to insert such technical specifications in our definitions explicitly, without engaging in circularity, but there is no doubt that the definitions would lose all their value and significance without this tacit understanding.

A true or false proposition is called *'assertoric'*, because it makes a definite claim. A problematic proposition is not assertoric: it presents an appearance with equal tendency in both directions, and therefore devoid of tendency; it calls upon us to consider a hypothesis.

Problemacy signifies a suspension of judgment. It does not signify the existence of 'real' indeterminacy, but only recognizes the appearance of indeterminacy in contexts less than complete. In reality, we believe, every issue is settled, once the event takes place; in omniscience, there would accordingly be no problemacy — it only arises in more limited viewpoints.

Problemacy has no equivalent outside logical modality; being freely open to change as knowledge evolves, there is no error in saying that any proposition we choose to formulate is at first encounter problematic.

Note that meaningful, precise, and clear, propositions may be true, false or problematic. Meaningless propositions are classified as false. Vague or obscure propositions, as at best problematic, if not false.

Factual assertorics of less than extreme credibility and problematics, give a semblance of co-presence or co-absence of opposites. The laws of contradiction and of the excluded middle are our reminders that that impression is transient; ultimately, everything is either totally credible or completely incredible. In other words, so long as we make no attempt to at once apply both truth and falsehood, or both untruth and unfalsehood, no law is broken; but as soon as we lay claim to more than the propositions suggest, we err.

For this reason, we can effectively discard nonextreme assertions and problems, and say of any proposition: it cannot be both true and false, and cannot be neither true nor false. There is ultimately no mixing or in-between of these attributes; our goal is to arrive to the extremes, not to linger on intermediate stages. There would be no point in constructing a logical system with reference to the finer gradations of credibility: it would be immobile.

2. The Plural Modalities.

Truth and falsehood are the categories of logical modality *with a single, given context* as their frame of reference.

Truth is a category of logical modality lying between logical necessity and possibility. Falsehood is the exact contradictory of truth, lying between logical impossibility and unnecessity. Truth is fact and falsehood is fiction, ideally. So we may call them the *'factual'* level of logical modality; in analogy to the actual level of natural or temporal modality, or the singular level of extensional modality; but this is only an analogy, not an equation.

The categories of logical modality referring to *a plurality of unspecified contexts*:

Logical ***necessity*** characterizes a proposition which is true in every context, and in that sense is true irrespective of any given context.

Logical ***impossibility*** characterizes a proposition which is false in every context, and in that sense is false irrespective of any given context.

Logical ***contingency*** characterizes a proposition which has neither the attribute of necessity nor that of impossibility, as they are above defined, so that it is true in some contexts and false in others.

Logical ***incontingency*** is the negation of contingency, the common attribute of necessary and impossible propositions. Logical *possibility* is the negation of impossibility, the common attribute of necessary and contingent propositions: truth in some contexts. Logical *unnecessity* is the negation of necessity, the common attribute of impossible and contingent propositions: falsehood in some contexts.

With regard to corresponding concepts of logical ***probability*** or ***improbability***.

We can say that, in this system, truth or falsehood correspond to mere incidence or nonincidence; necessity or impossibility signify the extremes (100%) of probability or improbability, and contingency concerns intermediate degrees (less than 100%) of these. Thus, to be consistent, we must define the logically probable as what would be true in most contexts (or false in a minority of contexts), and the logically improbable as what would be true in few contexts (or false in a majority of contexts).

These concepts would then enable us to specify our breadth of vision — effectively, how many eventual changes of context we have taken into consideration in making a prediction. The practical feasibility of this, with some precision, and the relation of logical probability and credibility, will be explored when we deal with adduction.

Thus, in summary, ***logical modality*** may be defined as a qualification of propositions as such, informing us as to whether each is true or false, in this (i.e. a given) context, only some (unspecified) contexts, or all contexts, or somewhere in between these main categories.

Here again, it must be emphasized that 'is true' (meaning, seems more convincing than not) and 'is false' (seems less convincing than its contradictory), depend for their plausibility on our having sought out and scrutinized the available information with integrity. This issue is discussed in more detail in the next chapter.

I want to emphasize here that the concepts of logical modality, as here defined, are prior to concepts of logical relation, like implication, which (as we shall see) they are used to define.

The former are built on the vague, notion of a proposition being variously credible *'in'* some context(s). Although this 'in' suggests that a kind of causality is taking place, it is not yet at the stage where specific relations like implication may be discussed. There is only a mental image of items 'pushing' others into existence; a very sensory notion.

Likewise, our first encounter with 'credibility' is very intuitive, something intrinsic to our every consciousness. The later systematic understanding of credibility, with reference to adduction, is merely a report on when it occurs, not a substitute for that primitive, inner notion.

It is interesting that, in Hebrew, the word for 'with' is *'im'* (spelt ayin-mem), and that for 'if' is *'im'* (spelt aleph-mem). In that language, if I am not mistaken, when verbal roots are that close, it signifies that the thoughts underlying them are also close. I wonder if the English words 'in' and 'if' have similar origins, rather than those most philologists assume.

Incidentally, also similar in Hebrew, are the words *'az'* (spelt alef-zayin), meaning 'then' in time or logic, and *'oz'* (spelt ayin-zayin), meaning 'strength'. This confirms what I said above, that the notion of logical causality is rooted in an intuitive analogy to physical force.

3. Analogies and Contrasts.

Various analogies and contrasts between the singular and plural modalities are worthy of note. The former measure credibilities in any one context. The latter take a broader perspective, and compare credibilities in a variety of contexts. Thus, true, false, and problematic are comparable to necessary, impossible, and contingent — but they are not identical.

Contingent truth and falsehood are contextual, whereas necessity and impossibility (incontingent truth and falsehood) effectively transcend context. What holds in every context, holds no matter what the context, whereas the contextual is tied to context and in principle liable to revision (though that may never happen).

Note that it is the *realization* of contingency as truth or falsehood, which is relative to context, but the contingency in itself is no less absolute (with respect to context) than necessity or impossibility.

A careful distinction must be made between the truth, falsehood, or problemacy, of a proposition whose logical necessity, contingency, or impossibility is unspecified — and the truth, falsehood, or problemacy, of any proposed modal specification for that proposition. Failure to distinguish between these perspectives can be very confusing.

A proposition may be problematic to the extent that, not only do we not know whether it is true or false, but we do not even know whether it is logically necessary, contingent, or impossible.

Less extremely, we may know the proposition to be true or false (and thus, possible or unnecessary), yet not know whether it is logically necessary, contingent (possible *and* unnecessary), or impossible. In such case, the singular modality (the proposition per se) is assertoric, but the plural modality is still to some extent problematic.

If a proposition is known to be logically necessary or impossible, then it is assertoric with regard to both its plural modality (the incontingency) and to its singular modality (accordingly, true or false).

If a proposition is known to be logically contingent, it is assertoric with respect to its plural modality (the contingency). We may additionally know that the proposition per se is true or false, in which case it is also assertoric with respect to its singular modality. Or we may still be at a loss as to whether it is true or false, so that it is problematic with respect to its singular modality.

In any case, here again, problemacy does not signify real indeterminacy, but merely absence of sufficient knowledge, remember.

Our definitions make clear that problemacy should not be confused with logical contingency. A proposition may be definitely true or false, and so unproblematic, and still contingent; and a problematic proposition may after serious consideration be found to be necessary or impossible, whereas a properly contingent proposition should not thus change status.

Yet problemacy and contingency have marked technical analogies, which allow us to treat any problematic proposition (and therefore any proposition whatever, at first encounter) as *effectively* contingent in logical properties. Logic repeatedly makes use of this valuable principle. As will be seen, if the proposition is not indeed contingent, it will be automatically revealed so eventually through dilemmatic argument, so that no permanent damage ensues from our assumption.

Note that the definitions of the logical modalities are very similar to those of extensional, natural and temporal modalities. There is a marked quantitative analogy (this, some, all), so that we can refer to them as 'categories of modality'; and there is a broad qualitative analogy (inclusion or exclusion in a wider perspective), yet with enough difference that we can refer to them as distinct 'types of modality'.

Logical modality puts more emphasis on epistemology than ontology, in comparison to the other types. It primarily qualifies knowledge, rather than the objects of knowledge. Whereas natural modality refers to the objective circumstantial environment of events, temporal modality to surrounding times, and extensional modality to cognate instances — logical modality looks at the informational setting.

With regard to technical properties, logical modality is often similar to the other types, but some notable differences also occur, as we shall see as we go along.

4. Apodictic Knowledge.

The many-contexts concepts of logical modality are formed by reference to the awareness that there are items of knowledge which somehow would seem to be true or false no matter what developments in knowledge may conceivably take shape, while others seem somehow more dependent on empirical evidence for their acceptance or rejection. The former are often called 'a priori' or *'apodictic'*, and the latter 'a posteriori'.

At first sight, apodictic statements present a difficulty. They seem inaccessible to anyone with less than total knowledge. Only the fully omniscient could know what is necessary or impossible in the widest context. A normally limited mind like ours cannot have foreknowledge of any final verities. Indeed, even if we ever reached omniscience, how could we be sure we have reached it?

However, these skeptical arguments can be rebutted on several grounds. To begin with, they are self-defeating in that they themselves claim knowledge about the capabilities of omniscience, and they do so in no uncertain terms: therefore, they are intrinsically conceptually flawed. Logically, then, it is conceivable for a limited mind to acquire apodictic knowledge, somehow.

Secondly, it is noteworthy that our minds, though admittedly less than omniscient, are not rigidly limited in their powers of imagination. We are able to construct innumerable hypotheses even with a limited amount of factual data to play with. Thus, we are never limited to one context, the present one, but can manipulate ideas which go beyond it. Of course, this does not mean that our imagination is able to foresee all contexts. The more factual data we have to feed on, the more our imagination can stretch out — but we never have all the seeds.

Thirdly, the skeptical arguments misconstrue the issues. We defined the necessary as true, and the impossible as false — 'in every context'. We did not say, the necessary is what is true, and the impossible is what is false — 'to the omniscient'. Our definition does not exclude that the quality of necessity or impossibility be *given as such within any single context*, as an inherent component of the appearance. It does not logically mean that we have to foretell what goes on in other contexts besides our own.

And indeed, we find within common knowledge many instances of manifest necessity or impossibility, without need of further investigations. Such events constitute the experiential basis for these concepts.

The primary examples of this are Aristotle's laws of thought. They strike us as intrinsically overwhelming, as in themselves capable of overriding any other consideration of knowledge. We can only ever deny them reflectively, by obscuring their impact; but the moment we encounter them plainly, their practical force is felt. When we are face to face with a specific contradiction, we see that it is nonsense and that something, somewhere must be amiss. That is why the laws of identity, of contradiction, and the excluded middle are naturally adopted as the axioms of logical science.

But other examples abound. More generally, as we shall see, a proposition is self-evident, if it is implied by its own negation, or implied by any contradictories; and a proposition is self-contradictory, if its affirmation implies its own negation, or implies any contradictories. It will be shown that a self-evident proposition displays the consequent property of being implied by any conceivable proposition, and a self-contradictory proposition that of implying any conceivable proposition. 'Any' here means 'every' — so that these are cases of logical necessity or impossibility.

This may occur *formally*, for all propositions of a certain kind whatever values be assigned to their variables. Indeed, the science of logic itself may be viewed as a record of all such occurrences. Or it may occur *contentually* (or 'materially'), in the sense: not for all propositions of a certain kind, but only with certain specific contents. Note that this distinction is somewhat relative, depending on what we hold fixed and what we allow to vary.

Another way apodictic knowledge (or, for that matter, any knowledge) might conceivably be made available to a limited mind is through *revelation*, a communication from an omniscient mind. This is the logical premise of religion. *Faith* might be defined as the conviction that the information does indeed come from an infallible source, G-d. This topic is too vast to be discussed in this treatise, but I merely wanted to indicate the entry point.

Now, if logical necessity or impossibility are somehow given as components of the appearance of things in any context of knowledge, what is their difference from (contingent) truth or falsehood, which are also given?

Theoretically, once a proposition has been seriously scrutinized and found not to be necessary or impossible, it henceforth remains permanently contingent — just as once a proposition is seen to be necessary or impossible, its status is thenceforth established. In practice a mistake might conceivably be made, but this does not affect the principle.

The essence of necessity or impossibility is their property of self-evidence or self-contradiction; it is not their permanence, which is only incidental. Contingent truths or falsehoods may also be permanent; a proposition may happen to remain true or false without change as knowledge evolves, and yet never lose its contingent status. That some contingent truths or falsehoods do change over time, is irrelevant. Even in a total knowledge context, truths or falsehoods may be characterized as contingent.

Thus, we do not regard an obvious empirical truth like 'it is now raining', or a well-established law of nature like 'the amount of matter and energy in the universe are constant', as logically necessary, even though we believe them to happen to be fixed truths (each in its own way), because they do not seem self-evident; they are both therefore intrinsically logically contingent. The raw, factual finality of the former or the natural necessity of the latter do not affect their common logical status.

On this basis, we can also say that logical contingency is conceptually distinct from problemacy. In omniscience, problemacy disappears, but not logical contingency. The latter remains as a further qualification of certain truths and falsehoods, distinguishing them from logical necessities and impossibilities, respectively. It follows that contingency as such is not a lower status than necessity or impossibility.

Lastly, note, a necessity or impossibility may be immediately apparent to anyone, or we may need to go though a long or complicated reasoning process to make it apparent. But in either case, the sense of obviousness is given within the appearance itself, so that the ease or difficulty with which we were brought to the insight are irrelevant to its finality.

It is hard to distinguish a priori and a posteriori knowledge by reference to the concepts of reason and experience. The former is indeed more purely analytical, but it cannot occur without the minimum of experience on the basis of which the concepts involved are meaningful and clear. Likewise, the latter is indeed more likely to be affected by changes in experience, but its conceptualization and logical evaluation involve a great deal of rational activity.

22. CONTEXTUALITY.

1. Statics.

We defined logical modalities with reference to the relative credibilities of appearances ‘within contexts’. We will here try to clarify what constitutes a context, and its role.

In a very narrow, ‘logical’ sense, one might refer to the context of a proposition as any arbitrary set of propositions. In this sense, a proposition could be taken in isolation and constitute its own context. It might still appear to us as true (if in itself reasonable looking) or false (if obviously internally inconsistent) or even problematic (if of uncertain meaning). Likewise for any larger set of propositions we choose to focus on exclusively. But this leads to a very restricted sense of truth or falsehood.

In practice, there is no such animal. A more ‘epistemological’ understanding of context is called for. The effective context of any proposition is not arbitrarily delimitable, but is a very wide body of information, which, whether we are conscious of it or not, impinges on our judgement concerning the proposition. It is the ‘status quo’ of knowledge at a given time, for a given individual or group.

A proposition is not just a string of words or symbols written on a piece of paper; it has to mean something to become an object of logical discussion. We cannot consider it in isolation, because our consciousness is, like it or not, always determined by a mass of present or remembered perceptual and conceptual data. This periphery is bound to affect our reaction to the proposition at hand.

It is in acknowledgement of this dependency that our definitions of logical modality must be constructed. The context of a proposition is thus all the things we are experiencing or thinking, or remember or forgot having experienced and thought — which happen to color the proposition at hand as credible or not, to whatever degree.

This is not intended as a psychological observation, suggesting that our judgment is being warped by structural or emotional factors; in some cases it indeed is, in others not. Nor is the issue what we consciously take into consideration; that may have no effect, and there may be unconscious influences anyway.

It is merely a recognition that the appearance of realism or unrealism of any proposition is always a function of a great amount of data, besides it and any artificially selected framework. The contextual data generating such a result include: perceptions, direct conceptual insights, and indirect inductions and deductions. Hence the concept of a context, as here used. It refers to the actual surrounding conditions of our knowledge.

It is hard to pinpoint precisely and with unfailing accuracy just which of the peripheral information impinges on a given proposition’s evaluation. Innumerable wordless sensations, mental images, and intuitions, are involved, and merely having had logically relevant experiences or thoughts, does not entail that they played any effective role in the present result. All we can say with certitude is that a lot of data is involved in the final display of some quality of credibility by a proposition.

The whole of logical science may be viewed as an ongoing attempt to investigate this aetiology. Its job is to find just what causes propositions to carry conviction or fail to do so, and how the totality of knowledge can be gradually perfected. We have seen its work in the domain of deduction with certain categorical propositions; now other forms are about to be analyzed. The solution to the problem of knowledge is not found in simplistic and vague pontifications, nor in a step-by-step linear guidebook, but in a vast tapestry of interlocking considerations.

2. Dynamics.

The concepts of truth, falsehood, and problemacy, refer to the deployments of credibility in a static context, the ‘state of affairs’ in knowledge at a given stage. The concepts of necessity, impossibility, and contingency, refer to the changes of credibility: they consider knowledge more dynamically.

Knowledge is an evolving thing. We, human beings, are none of us ever omniscient or infallible. If our consciousness was unlimited by space, time, and structural resources, like Gd’s, there would be no problematic knowledge: every proposition would be true or false with finality. Just as reality is one, knowledge would be one and complete.

But reality is opened to our consciousness piecemeal, over time. We are obliged to repeatedly adapt to new factual input. Indeed, we have to actively dig into reality, if we want to approach that ultimate goal of total consciousness of everything.

We know we cannot reach that goal, since we have already missed out on enormous tracts of reality in the distant past, and the whole future is ahead of us, unexplored. We know that innumerable phenomena are happening all around us and within us, all the time, at every level (from the sub-atomic to the astronomical, from the material and physiological to the mental and spiritual); and we cannot keep track of all that. Thus, the data available to us is inevitably restricted.

Furthermore, our faculties of knowledge can play tricks on us, and draw us away from the goal. Our eyes may be myopic, our memory may fail, our reasoning may be muddled, we may be too imaginative, our mind may be moved by very subjective, emotional, considerations. We have to somehow make-do, in spite of all such imperfections in our make-up.

Our response to these limitations, if we are intent on knowing reality, is staying aware of our mental processes, and unflagging reevaluation of what and how much we know or ignore. This is where logical modality comes into play. It provides us with labels we can attach to each and every proposition, which assign it a rank, as we proceed.

Theoretically, we take the full body of everything we have experienced or thought thus far, and order the present information in a hierarchy. Tools may be invented to increase our certainties: eyeglasses, the written word, a science of logic. The sources of information are considered: we distinguish between the fictions of our imagination and the facts of sense data, between vague and clear concepts, between fallacious and rigorous argumentation.

In practice, things are more dynamic than that. We may take some part of our data base, and hold it still long enough to evaluate it with the proper amount of reflection. But, on the whole, the process is on-going, an ad hoc response to the flux of information. Logical modalities allows us to register our value-judgments of this kind as we proceed, like a running commentary.

3. Time-Frames.

Now, there are three ways for knowledge to evolve, and credibility to change. We may associate the word ‘context’ to the sum total of knowledge, the whole environment — or, more restrictively, to a given body of fundamental axioms and raw data, a *framework*. Here, let us use it in the latter sense.

We may not have drawn all the possible lessons from these primary givens; the process is not automatic, but has a time dimension. A proposition may be logically implicit in knowledge I already have, but it may take me time and effort to discover it.

There is always a great deal of undigested, unexploited information in our memory banks, and accessing it and assessing it demand time and skill. I mean, Philosophy, for example, requires relatively little raw data to develop considerably, because it pursues facts implicit in every existent. This is internal development, or context *intensifying*.

Or we may receive new input of rational axioms and empirical data to consider. Here, two alternatives exist: either the new facts already existed out there, but unbeknown to us; or some change occurred in these external objects themselves, which we accordingly now absorb as new existents. These are developments fed from the outside, or context *extending*.

Thus, we may distinguish between three time-frames for modality change: the external time in which objects change into new objects; the interfacial time of turning our attention and sensors towards pre-existing objects — to extend context; and the internal time of mental assimilation of memory (analyzing, comparing, checking consistency) — the work of intensifying context.

The first of these essentially pertains to natural and temporal modality; the second, extensional modality; the third, is the time-frame of logical modality. But all of them, if only incidentally, concern logical modality.

4. Context Comparisons.

That our definitions of truth and falsehood do not specify the context taken as being final and ideal, is not a relativistic position. It is merely intended as a statement that every proposition's credibility is conditioned by a totality.

The given context is pragmatically accepted as a starting point for further inquiry, without thereby being regarded as 'the best of all possible contexts'. It is subject to change, to improvement. Some contexts are to be favored over others — the exact grounds just need to be elucidated.

We might refer to the overall credibility of a context. We could perhaps consider any given context as a whole, and (of course, very roughly) sum-up and average the credibilities of its constituents, and thus get an estimate of its finality or staying-power. But, quite apart from the issue of practical feasibility, I do not think this would be of any use. The relative credibilities given within each context pertain to that context alone, and have no bearing on the relative credibilities in other contexts.

The general principle for comparing contexts seems obvious enough. Contexts are of varying scope and intensity, and it is clear that *the deeper and wider the context, the closer to final* will the impressions of truth or falsehood concerning any proposition in it be; and the less numerous will doubtful cases be. Thus, the bigger and more cohesive the context, the better.

The ideal context of omniscience is beyond man's power, we can only gradually approach it. But we can say that in that ultimate, limiting case, the impressions of truth or falsehood would be final, subject to no further change or appeal; and furthermore, there would be no in-between impressions of a doubtful kind, since reality once established is determinate. Here, knowledge and reality would correspond entirely.

When we apply the above principle to one person over time, it is relatively easy to say which context is to be preferred. The more information at his or her disposal, the more this information has been carefully sifted for hidden messages, the more certain may that person be. For the individual, improvement is almost inevitable over time, because his or her context is a widening circle.

We always refer to appearance, though we can distinguish between prima-facie impressions and well-tested impressions. The two kinds of impression are essentially the same in nature, but they have different positions in a continuum stretching from subjectivity and mere belief (which still however contain seeds of objectivity and knowledge) to ultimate realism and certainty.

When, however, we compare the contexts of two (or more) people, it is not so easy to say which is better or worse. Each may have data the other lacks, and each may have thought about any item of data they have in common more thoroughly than the other. Thus, they may disagree in their conclusions, and yet both be 'right' for their respective contexts. And since their contexts overlap in only some respects, so that neither embraces the other as a whole, the contexts cannot be rated better or worse.

All we can do is focus on specific areas of knowledge, and consider the relative expertise of each individual in that area. If someone is a specialist in some field, we may well assign greater credibility to his or her pronouncements on the subject. On this basis, we may even trust a person we know to be generally very wise, without committing the fallacy of 'ad hominem'.

5. Personal and Social.

We must distinguish, here, between personal and social knowledge.

At the lowest level, is 'personal knowledge'. Some people are better at knowing than others, because of their healthier faculties, or because they are endowed with more intelligence and insight, or because they are more interested, more careful, and make more of an effort, in this domain. Also, individuals inevitably have different quantities of information at their disposal, both inner and outer.

'Social knowledge' is an ideal. We collectively, across cultural boundaries and the generations, gradually compile a record of common knowledge, agreed upon methods, information and conclusions. It is the human heritage, our shared data bank.

An individual may admittedly have more knowledge of some field than everyone else at a given time; he may get to share it, or it may disappear with him. There may be specific disagreements at any time between groups of individuals. It may even happen that the majority of the peer group wrongly rejects an individual's valuable contribution.

Yet, over time, the collective enterprise we call Science develops, a pool of knowledge greater and truer than any which individuals can fully match, based on a methodological consensus.

Since credibilities depend on context, individuals may assign different credibilities to the same proposition. To that extent, truth and falsehood are often 'subjective', since they reflects the mental abilities and dispositions of people.

Still, I may take all the premises of another person and demonstrate that his evaluations are logically incorrect even for his context. In a sense, I start off with the same context as him, and end up with a slightly different version; but in another sense, I have merely clarified the given context, brought out its full potential, without significantly altering it. If he is intelligent and honest enough, he normally bows to the evidence.

Thus, the contextuality of credibility need not imply its utter subjectivity. The evaluation can only ultimately be viewed as subjective in the pejorative sense, if it is contextually wrong.

And even then, such accusation can only be leveled fairly if the individual allowed psychological forces to sway his judgment. He may be intellectually negligent through laziness, or dishonestly evade unpleasant or frightening data or thoughts, or insincerely report his conclusions. If the error was honest, merely due to a failure to notice a connection, we can hardly criticize him, only correct him.

We get around these problems of personal weakness through the institution of social knowledge, science. This allows us to collectively 'average-out' the subjective vector. We mutually scrutinize and criticize each other's contributions, until we are of one mind. There may still be collective delusion, but that at least eliminates personal deviations from logical norms.

We presume that the influence of our collective mind-sets will gradually wither away as knowledge develops further. This assumption is justified by previous developments: we have seen historical examples of liberation from ideas which seemed immovable. The notion that science is inevitably subjective, is derived from such liberations, and cannot be used to denigrate them.

23. CONJUNCTION.

1. Factual Forms.

In this chapter, we begin to analyze the various ways two or more propositions, or sets of propositions, of any kind, may be correlated. A proposition so considered, in relation to other propositions, is called a ***thesis***; we symbolize theses by using letters such as P, Q, R,…. The negation of any thesis is called its ***antithesis***; that is its exact logical contradictory: the antithesis of thesis 'P' is 'nonP', and vice versa.

The primary form of correlation is conjunction; this is expressed by means of the operator 'and', or its negation. On a factual level, the conjunction (or positive conjunction) of two theses typically takes the form **'P and Q'**. The contradictory of 'P and Q' would be ***'Not*-{P and Q}'**, where the 'not' negates the 'and'; this may be called nonconjunction (or negative conjunction). In the context of conjunction, a thesis may be called, more specifically, a conjunct.

Most simply, the theses are categorical propositions of any form, so that their conjunction may be viewed as a compound categorical. But, by extension, a thesis may itself be a conjunction of two or more categorical propositions; or it may consist of any other, more complex, kind of proposition, or any mix of various kinds of propositions conjoined together. Thus, a thesis may ultimately be a whole, intricate theory.

Logical conjunction of two theses simply affirms both of them as true, implying that they are true separately as well as together. Thus, 'P and Q' (or 'P with Q') may be read as '{P and Q} is true', implying '{P is true} and {Q is true}'. The 'is true' segment may be left tacit or made explicit, as with categorical affirmations.

The contradictory form, 'not-{P and Q}' simply denies that the two theses are *both* true, without asserting that they are *both* false. All it tells us is that at least one of the two theses is false, without excluding that an unspecified one of them be true, nor excluding that both be false. Thus, 'not-{P and Q}' may be read as '{P and Q} is false', which does not imply that '{P is false} and {Q is false}'.

Thus, whereas the 'and' relation is fully assertoric, with regard to the parts as well as the whole, the 'not-and' relation is much more indefinite. It gives us limited information: it is assertoric with regard to the whole, but leaves the parts problematic. This problemacy should not even be interpreted as a logical contingency: not only do we not know of each thesis in isolation whether it is true or false, we do not even know whether it is contingent or incontingent. Keep that well in mind.

By definition, 'P and Q' and 'Q and P' are equivalent: the relation is reversible; also, 'P and P' is equivalent to 'P' alone: repetition of a thesis does not affect it. Likewise, by definition, 'not-{P and Q}' and 'not-{Q and P}' are equivalent: the relation is reversible; note however that 'not-{P and P}' is equivalent to 'not-{P}' alone, since 'P and P' means 'P'.

The three forms 'P and nonQ' (or, 'P without Q'), 'nonP and Q' (or 'Q without P'), 'nonP and nonQ' (or 'neither P nor Q'), are derivative forms of positive conjunction, obtained by substituting antitheses for theses in the original formula. Likewise, the three forms 'not-{P and nonQ}', 'not-{nonP and Q}', 'not-{nonP and nonQ}', are derivative forms of negative conjunction, obtained by substituting antitheses for theses in the original formula. We thus have a grand total of eight forms.

Note the we have used the word 'conjunction' in two senses. In a wider sense, it includes both the positive and negative forms. In a narrower sense, it includes only the former, the latter being called 'nonconjunction'. Note that a positive conjunction is denied by negating any one, or any set, or all, of its parts, which means that one of the remaining alternative positive conjunctions must be true; thus, nonconjunction may be viewed as an abridged reference to the outstanding conjunctions.

Conjunction may of course involve more than two theses, as in 'P and Q and R and..'., signifying that they are all true individually as well as collectively. Conjoining an additional thesis to a conjunction of two or more other theses, just results in a conjunction of all the theses, in a normal string: '{P and Q} and {R}' simply means 'P and Q and R'. Knowledge as a whole may be viewed as a conjunction of all the propositions in our minds.

Nonconjunction of more than two theses, as in 'not-{P and Q and R and…}' accordingly signifies that the theses are not *all* true, without implying any further information concerning each thesis alone. Any combination of theses and antitheses other than the one denied, would be acceptable. We shall develop the theory of conjunction with reference to two-theses forms, but the results can be extended with appropriate carefulness to forms with more than two theses.

2. Oppositions of Factuals.

The following table lists the various forms of conjunction (or positive conjunction), and shows the truths (**T**) and falsehoods (**F**) of theses and antitheses they imply. We see that, in contrast, nonconjunctions (or negative conjunctions) leave the individual theses and antitheses problematic (**?**): their information is purely collective. I have labeled these forms **K1**-**K4** and **H1**-**H4**, as shown, for convenience.

Table 23.1 Truth-Table for Factual Conjunctions.

Symb.	Conjunction	P	Q	nonP	nonQ
K1	P and Q	T	T	F	F
H1	not-{P and Q}	?	?	?	?
K2	P and nonQ	T	F	F	T
H2	not-{P and nonQ}	?	?	?	?
K3	nonP and Q	F	T	T	F
H3	not-{nonP and Q}	?	?	?	?
K4	nonP and nonQ	F	F	T	T
H4	not-{nonP and nonQ}	?	?	?	?

The four positive conjunctions exhaust the possible ways two theses and their antitheses may be positively conjoined, and are mutually exclusive. That is, *one of them must be true, and three of them must be false*. If any one is true, the other three must be false; but if one of them is false, the status of each the others is undetermined. Thus, the oppositional relation of any pair of positive conjunctions is contrariety.

The oppositions of the four negative versions relative to each other is: *three of them must be true, and one of them must be false*. If one of them false, the other three must be true; but if one of them is true, it is uncertain what the status of each of the others is. This follows from the interrelations of the positive versions. Thus, the oppositional relation of any pair of negative conjunctions is subcontrariety.

The opposition of any pair of positive and negative conjunctions, other than a pair of formal contradictories, is therefore subalternation. Proof: consider any positive conjunction, its truth implies the three others to be false, and therefore implies their contradictories to be true; on the other hand, its falsehood does not have further implications.

Thus, we could present the eight forms of conjunction in a cube of opposition, with the four positive forms in the upper corners and the four negative forms in the lower corners. The top plane involves contrariety, the bottom plane involves subcontrariety, the diagonals through the cube involve contradiction, and the four remaining faces involve subalternation in a downward direction.

3. Modal Forms.

The eight factual forms of conjunction are the singular level of logical modality. Let us now investigate the corresponding plural levels of logical modality. Each of the factual conjunctions has a possible equivalent below it and a necessary equivalent above it. Thus, we have to consider 2X8 = 16 modal conjunctions, in addition to the 8 factual ones. They are (always referring to logical modality, needless to repeat):

Table 23.2 List of Modal Conjunctions.

Positives	Negatives
{P and Q} is necessary	{P and Q} is impossible
{nonP and Q} is necessary	{nonP and Q} is impossible
{P and nonQ} is necessary	{P and nonQ} is impossible
{nonP and nonQ} is necessary	{nonP and nonQ} is impossible
{P and Q} is possible	{P and Q} is unnecessary
{nonP and Q} is possible	{nonP and Q} is unnecessary
{P and nonQ} is possible	{P and nonQ} is unnecessary
{nonP and nonQ} is possible	{nonP and nonQ} is unnecessary

The factuals fit in between these two levels of modality, of course; they are less than necessary, but more than possible.

Now, just as the factual positives implied that their respective theses are not only collectively true, but individually true — so the necessary positives imply that their theses are each (as well as all) necessary, and the possible positives imply that their theses are each (as well as all) possible. However, in the latter case, it does not follow that the antitheses are equally possible, note well.

In contrast, none of the negatives tell us anything about the logical modalities of their respective theses. In all cases, the statuses of the individual theses are left entirely problematic; all we have is collective information. Not only are we left in the dark as to whether any thesis is true or false, but there is no specification as to whether it is necessary or possible or unnecessary or impossible.

Thus, for examples. 'P and Q are necessary' implies that P is necessary (and nonP is impossible); and likewise for Q. 'P and Q are possible' implies that P is possible, not impossible (and nonP is unnecessary, not necessary) — but without excluding that P be necessary or contingent: both are acceptable; and likewise for Q.

'P and Q are impossible' (meaning: 'not-{P and Q} is necessary') does not imply that P and Q are each impossible, but is equally compatible with each of them being contingent or necessary — except that in the latter case, if one theses is necessary, the other would needs be impossible, to satisfy the overall requirement of the form. 'P and Q are unnecessary' (meaning: 'not-{P and Q} is possible') allows for each of the theses to be necessary, contingent or impossible — provided they are not both necessary at once.

Similarly, for the remaining forms. Thus, we see that each form delimits some collective property of the theses, in some cases implying some individual properties; but in most cases, the form leaves some open questions, some areas of doubt, which would require additional statement(s) to specify in full.

Only the necessary positives fully define the factual and modal status of the theses (they are equally necessary). The factual positives establish the factuality and possibility of the theses, but leave their exact modal status (necessary or contingent) undetermined. The possible positives establish the possibility of the theses, but leave their factual and exact modal status untold.

The negatives are even less committed with regard to their theses. It is very significant to note that although a negative conjunction makes mention of a proposition as one of its theses, it does not thereby imply it as even logically possible. One might think that the mere mention of a proposition is always an admission of its possible truth; but here we learn that such assumption is unjustified.

The value of such indeterminacy is that it allows us to verbally capture just precisely those relational details which are of interest to us, without being forced to know more than we do at that point in time. If we could only make statements where every issue is already resolved, we would be left wordless until we had all the requisite details.

Be careful not to confuse problemacy and logical contingency. A proposition may be so problematic, that we do not even know whether it is logically contingent or incontingent, let alone whether it is true or false; or it may be only problematic to the extent that, though we know it to be contingent, we do not know whether this contingency is realized as truth or falsehood on the factual level.

4. Oppositions of Modals.

The following table lists the various forms of modal conjunction, and shows the necessity (**N**), impossibility (**M**), possibility (**P**), unnecessity (**U**), or problemacy (**?**), of individual theses and antitheses, implied by each modality (cum polarity) of conjunction, in accordance with our previous comments. Note the labels assigned, namely **K1**-**K4**, **H1**-**H4**, with suffix **n** or **p**, as the case may be, for convenience.

Table 23.3 Truth-Table for Modal Conjunctions.

Symb.	Conjunction	Modality	P	Q	nonP	nonQ
K1n	P and Q	necessary	N	N	M	M
K1p	P and Q	possible	P	P	U	U
H1n	P and Q	impossible	?	?	?	?
H1p	P and Q	unnecessary	?	?	?	?
K2n	P and nonQ	necessary	N	M	M	N
K2p	P and nonQ	possible	P	U	U	P
H2n	P and nonQ	impossible	?	?	?	?
H2p	P and nonQ	unnecessary	?	?	?	?
K3n	nonP and Q	necessary	M	N	N	M
K3p	nonP and Q	possible	U	P	P	U
H3n	nonP and Q	impossible	?	?	?	?
H3p	nonP and Q	unnecessary	?	?	?	?
K4n	nonP and nonQ	necessary	M	M	N	N
K4p	nonP and nonQ	possible	U	U	P	P
H4n	nonP and nonQ	impossible	?	?	?	?
H4p	nonP and nonQ	unnecessary	?	?	?	?

Since the categories of logical modality are by definition distinguished with reference to a quantity of contexts, the oppositions of the various modalities of conjunction among themselves, can be deduced from the oppositions between the corresponding factual conjunctions, given in an earlier section of this chapter, and the general doctrine of ‘quantification of oppositions’, which we worked out in an earlier chapter (14.1) with reference to categoricals.

Thus, since the forms **K1** and **H1** are contradictory, the oppositions between **K1n**, **K1**, **K1p**, **H1n**, **H1**, **H1p**, are the same of those between the categoricals **A**, **R**, **I**, **E**, **G**, **O**. Likewise for similar sets.

Since the forms **K1**, **K2**, **K3**, **K4**, are contrary to each other, it follows that: the forms which subalternate these factuals, **K1n**, **K2n**, **K3n**, **K4n**, are contrary to each other, and to them;

and the forms which these factuals in turn subalternate, **K1p**, **K2p**, **K3p**, **K4p**, are neutral to each other, and to them.

Since the forms **H1**, **H2**, **H3**, **H4**, are subcontrary to each other, it follows that: the forms which subalternate these factuals, **H1n**, **H2n**, **H3n**, **H4n**, are neutral to each other, and to them; and the forms which these factuals in turn subalternate, **H1p**, **H2p**, **H3p**, **H4p**, are subcontrary to each other, and to them.

Since the form **K1** subalternates the forms **H2**, **H3**, **H4**, it follows that: **K1n** subalternates **H2n**, **H3n**, **H4n**, and therefore **H2**, **H3**, **H4**, and **H2p**, **H3p**, **H4p**; but **K1** is neutral to **H2n**, **H3n**, **H4n**, though it subalternates **H2p**, **H3p**, **H4p**; and **K1p** is neutral to **H2**, **H3**, **H4**, though it subalternates **H2p**, **H3p**, **H4p**. Likewise, for similar sets.

24. HYPOTHETICAL PROPOSITIONS.

1. Kinds of Conditioning.

We saw in the previous chapter that two or more propositions may be correlated in various ways, with reference to conjunctions (involving the operator 'and') of various polarities and logical modalities.

Implicit in certain conjunctive forms are relationships of 'conditioning'; they signify a certain amount of interdependence between the truths and/or falsehoods of the theses involved. These relationships are definable entirely with reference to modal conjunction, so that we may fairly view all forms of conjunction, and all forms which may be derived from them, as one large family of propositions called 'conditionals'.

However, in a narrower sense, and usually, we restrict the name conditional to the derivative forms which employ operators like 'if'. The remaining derivative forms, which employ operators like 'or', are called disjunctive.

These issues of terminology are of course of minor import. What counts is that conjunctive, conditional, and disjunctive propositions are ultimately all different ways of saying the same things, as far as logic is concerned. Nevertheless, because each of these formats reflects a quite distinct turn of thought, they are worthy of separate analyses.

We are in this part of our study concerned with conditioning in the framework of logical modality. But as we shall see eventually, each other type of modality also gives rise to a distinct type of conditioning.

Logical conditionals are more commonly known as 'hypothetical' propositions — this more easily distinguishes them from non-logical (not meaning illogical) conditionals, meaning natural, temporal or extensional conditionals, which may therefore simply be called 'conditionals', in a narrower sense.

Hypothetical propositions are essentially concerned with the logical relations between propositions, or sets of propositions. This area of Logic is therefore quite important, as it constitutes a self-analysis of the science, to a great extent — the 'logic of logic'. But it is also a specific investigation, like any other area of Logic, for the purposes of everyday reasoning.

The sequence in hypotheticals, the ordering of their theses, is what we call 'logical'. It is not essentially temporal, though the mental sequence is of course temporal, one thought preceding the next — we can be aware of only so much at a time, beyond that we function linearly, in trains of thoughts. Some thoughts are linked into chains by precise relational expressions, but their sequence should not be viewed as to do with natural causation between mental phenomena per se. Thought processes are sometimes apparently involuntary, but for the most part there plainly seems to be a volitional element involved; indeed, if thought was automatic, there would be no call for logic.

Logical sequence has rather to do with conceptual breadth. The wider proposition is viewed as including, or implying, its consequences, in a timeless manner. The exclusive proposition 'Only if P, then Q', though formally identical to the reciprocal relation 'If P, then Q, and if Q, then P', suggests that P and Q are not logically quite interchangeable, but that P has a certain conceptual primacy over Q, that their order matters. The suggested order is not merely in the time of arrival of the thoughts about P and Q, but more deeply concerns the hierarchy of their factual contents.

2. Defining Hypotheticals.

a. The paradigmatic form of hypothetical proposition is *'If P, then Q'*, where P and Q are any theses. The former, P, is known as the antecedent, and the latter, Q, as the consequent. The relation between them is minimally defined by saying that *the conjunction of P and nonQ is impossible*. This means that the affirmation of P and the denial of Q are incompatible; given that P is true, Q cannot be false, and it follows that Q must also be true. We can also say: P implies Q.

Note the correspondence of this proposition to the negative modal conjunction labeled **H2n** in the previous chapter; as we saw, this leaves the individual theses P and Q entirely problematic at the outset: they need not even be logically possible. Note well also that the unmentioned conjunctions 'P and Q', 'nonP and nonQ', and 'nonP and Q' are all left equally problematic; one should not surmise, from the allusion to P being followed by Q, that the conjunction of P and Q is given as logically possible.

The expression 'if' normally suggests that the truth of the antecedent 'P', and thereby of its consequent 'Q', are not established yet; they are still in doubt. Note that the 'if' effectively colors both the theses.

The expression 'then' (which in practice is often left out, but tacitly understood) informs us that, in the event that the truth of the antecedent is established, the truth of the consequent will logically follow. The form 'if P, then Q' does not specify whether P is likewise implied by Q, or not; it takes an additional statement to express a reverse relation.

A note on terminology: officially, in logical science, the whole relation 'if P, then Q' is called a hypothetical proposition, in the sense that it includes one thesis in another. The proposition as a whole is assertoric, not problematic (unless we specify uncertainty about it, of course); it is the two theses in it which are normally problematic. But colloquially, we understand the expression 'hypothetical' as signifying problemacy, so confusion is possible.

Etymologically, the word 'hypothesis' could suggest a thesis which is placed under another, and so might be applied to the consequent; here, the sense is that it is 'conditioned' upon the truth of the antecedent (which, however, is normally in turn conditioned by other theses). But, again in practice, we often look upon the antecedent as the 'hypothesis', because it is qualified by an 'if' and underlies the other thesis; here the sense is that our thesis is placed before the consequent (which, however, is normally more or less equally 'iffy').

Be all that as it may, logical science has frozen the various expressions in the special senses described.

b. The contradictory of the 'if P, then Q' form is *'If P, not-then Q'*. This merely informs that *the conjunction of P and nonQ is not impossible*. It tells us that: if P is true, it does not follow that Q is true; Q may or not be true for all we know, given only that P is true. We can also say, P does not imply Q.

Note the correspondence to the positive modal conjunction labeled **K2p** in the previous chapter; as we saw, this implies that P is logically possible and Q is logically unnecessary, though both individual theses are of course left problematic with regard to their factual status. One should not surmise, from the allusion to Q rather than nonQ, that Q is given as logically possible. Note well also that the unmentioned conjunctions 'P and Q', 'nonP and nonQ', and 'nonP and Q' are all left equally problematic.

It is not excluded that P and Q have some other positive relation; for instances, that P together with some additional conditions imply Q, or that Q implies P. It is also conceivable that P is not only compatible with the negation of Q, but implies it; or at the other extreme, that P and Q are totally unrelated to each other. In any case, here again, the theses P, Q are normally problematic, though the proposition as a whole is assertoric.

The name 'hypothetical' may be retained for such negative forms insofar as the prefix 'if' is equally involved; likewise, the name 'antecedent' for P remains correct; but for Q, the name 'inconsequent' would be more accurate here. For, whereas the positive form 'If P, then Q' suggests that Q is a logical consequence of hypothesizing P, the negative form 'If P, not-then Q' denies such connection (for this reason it is called the 'nonsequitur' form, the Latin for 'it does not follow'). We may use the word 'subsequent' (without chronological connotations) to mean 'consequent or inconsequent'; or we may simply use the word 'consequent' in an expanded sense.

The form 'if P, not-then Q' should not be confused with the form 'If P, then nonQ', which means that the conjunction of P and Q are impossible; sometimes we say the latter with the intent to mean the former. There is a world of difference between 'P does not imply Q' and 'P implies nonQ'. To make matters worse, we sometimes leave out the 'then', and just say 'if P, not Q', which can be interpreted either way.

It is important to note that we commonly assume that 'if P, not-then Q' is true, whenever we have searched and found no reason to think that 'if P, then Q' is true. This is effectively an inductive principle for negative hypotheticals: strong relations like 'if P, then Q' require specific proof, whereas weak relations like 'if P, not-then Q' may usually be taken for granted, so long as their contradictory has eluded us.

c. The following table clarifies the relations between the antecedent and consequent and their antitheses, in positive and negative hypotheticals. It shows what follows as true (**T**), false (**F**), or undetermined (**?**), from the truth of any of them. Note well that the table is an outcome of the hypothetical relations, but does not constitute their definition.

Table 24.1 Truth-Table for Hypotheticals.

Proposition	Given	P	Q	NonP	NonQ
If P, then Q					
	P	T	T	F	F
	Q	?	T	?	F
	NonP	F	?	T	?
	NonQ	F	F	T	T
If P, not-then Q					
	P	T	?	F	?
	Q	?	T	?	F
	NonP	F	?	T	?
	NonQ	?	F	?	T

d. Hypotheticals are not only used in everyday reasoning, but also to develop logical theory; they express the formal connections between theses. The hypothetical relations validated by formal logic are not defined by mere denial of the occurrence of this or that conjunction in a specific instance, but by claiming the logical impossibility of it with any content.

We use them to indicate the oppositional relations between any propositional forms, or the inferences which can be drawn from one, or a conjunction of two or more, propositional forms. Premises are antecedents, valid conclusions are consequents; an argument is valid if the premises imply the conclusion, invalid if they do not. Likewise, when we speak of assumptions and predictions, we refer to such logical relations.

The psychology of assumption consists in mentally imagining as true a proposition not yet so established, or even which is already known false. In the latter case, we phrase our hypothetical as 'If this had been true, that would have been true'. Because of logic's ability to deal with form irrespective of content, even untrue contents may be considered and analyzed.

As will be seen, hypothetical relations are established through a process of 'production'. Most, if not all, of the logical relations we intuit in everyday reasoning processes are in fact expressions of formal connections.

3. Strict or Material Implication.

Note well that the definitions of both the positive and negative hypothetical forms involve two essential factors. First, they refer to a conjunction of two theses, symbolized by 'P' and 'nonQ' (meaning, the negation of Q). Secondly, hypotheticals are essentially modal propositions; they refer to the logical impossibility or possibility of such a conjunction.

Many logicians have defined the 'if P, then Q' form as identical with the negative conjunction 'not-{P and nonQ}'. They have called this 'material' implication to distinguish it from the above 'strict' implication. The suggestion being that implication is a relation which ranges from singular contextuality or actuality (material), to all contexts or necessity (strict).

It is true that we often for practical purposes, intend an implicative statement as merely applicable to the present context. However, since the 'present context' is notoriously difficult to identify precisely, this is a practice which cannot be subjected to formal treatment. Two propositions cannot be compared or combined, if it is unclear what parts of the ever-changing context they depend on. The unstated conditions may be different enough that their fluxes are not in harmony.

My position is therefore that the idea of 'material implication' is mistaken. There is no such thing as nonmodal implication, in the sense they intended. All implication is inherently modal, 'strict'. The realization of implication is not a more restrictive implication, but simply a factual conjunction or nonconjunction.

One mere denial of the bracketed conjunction is not implication: such definition only seems to work because it conceals a repetitive denial, coming into force whenever we bring the definition to mind.

The reason why the error arose, is because negative conjunction, even on a factual level, is intrinsically indefinite. When we say 'not-{P and nonQ}', we think: 'well, if P, then not nonQ, and if nonQ, then not P'. However, these seemingly implicit hypotheticals are not themselves assertoric: they are preconditioned by a tacit '*if not-{P and nonQ}, then*: if P, then not nonQ, and if nonQ, then not P'. There is a hidden nesting involved. The consequent hypothetical proposition is in fact quite modal; it only appears nonmodal, because the antecedent nonconjunction is taken for granted.

I very much doubt that the form 'not-{P and nonQ} ever occurs in practice, except insofar as it is logically implied by a factual conjunction like 'nonP and nonQ' or 'P and Q' or 'nonP and Q', or by the modal form 'if P, then Q' (in the sense of '{P and nonQ} is impossible'). For example, even though the conjunction '{chickens have teeth} and {squares are round}' is indeed false, we do not interpret this to mean that these two happenstances are at all linked; the proposition as a whole can only be constructed *as a result* of our foreknowledge (in this case) that both clauses are separately false, and would not be otherwise arrived at.

This misconception has caused the logicians in question to ignore the contradictory 'if P, not-then Q' form altogether, since that would be equivalent to the positive conjunction 'P and nonQ', according to that theory. Yet, we commonly reason in such terms, saying 'it does not follow that' or 'it does not imply that', without intending to affirm the theses categorically thereby [as in negation of conjunction].

The antecedent does not merely happen to precede the subsequent, as that theory suggests. In the 'if P, then Q' case, the consequent follows it as a logical necessity; it means effectively, 'if P, necessarily Q'. In the 'if P, not-then Q' case, the inconsequent is denied such necessary subsequence, without affirming or denying that it may possibly happen to be conjoined; it means effectively 'if P, possibly not Q'.

If we compare the truth-tables of 'P strictly implies Q' and 'P materially implies Q', we may be misled by the identity of the positive side (see the 'if P, then Q' half of table 24.1). But when we look at the negative side (i.e. the denial of 'if P, then Q'), the difference between the two cases is glaring (for strict implication, see the second half of table 24.1; and for material implication, see row 'K2' of table 23.1).

That is to say, though strict and material implication seem to have the same truth-table, their negations have very different truth-tables, so their logical behaviors will be different. Moreover, the former is permanent (i.e. true for all time if true), whereas the latter (except when it is true by implication from the former) is temporary (i.e. true for a limited time if true).
Of course, we can invent any forms we please; but logical theory should reflect practice, and not be allowed to degenerate into an arbitrary game. What the proponents of material implication were looking for, the seed of truth they were trying to express, was, I suggest, the analogues of implication found in other types of modality — the natural, temporal or extensional. I will discuss these in detail later, and the truth of this statement will become more apparent then.

For all these reasons, I have not followed suit. I ignore so-called material implication (though not factual negative conjunction, of course), and limit hypotheticals to strict implication.

4. Full List of Forms.

Now, the forms ‘If P is true, then (or not-then) Q is true’ are paradigms. If we substitute in place of P and/or Q, their respective contradictories, that is, the antitheses nonP (P is false) and/or nonQ (Q is false), we obtain the following full list of eight possible relations. The symmetries involved ensure the completeness of our list of hypotheticals. Each hypothetical is defined by a modal conjunction, as shown, on the basis of our original definitions.

Table 24.2 List of Hypotheticals and their Definitions.

Form	Equivalent Modal Conjunction	Symb.
If P, then Q	{P and nonQ} is impossible	H2n
If P, not-then Q	{P and nonQ} is possible	K2p
If P, then nonQ	{P and Q} is impossible	H1n
If P, not-then nonQ	{P and Q} is possible	K1p
If nonP, then Q	{nonP and nonQ} is impossible	H4n
If nonP, not-then Q	{nonP and nonQ} is possible	K4p
If nonP, then nonQ	{nonP and Q} is impossible	H3n
If nonP, not-then nonQ	{nonP and Q} is possible	K3p

a. As earlier decided, hypotheticals with the ‘if, then’ operator, which posit a consequence, are classified as ‘positive’; these are fully defined by reference to the logical impossibility of a conjunction. Hypotheticals with the ‘if, not-then’ operator, which negate a consequence, are classified as ‘negative’; these are fully defined by the logical possibility of a conjunction. The unmentioned conjunctions in each case are of undetermined status; this means problematic, and should not be taken to mean logically contingent.

The oppositions between hypotheticals and factual conjunctives follow accordingly. Given the truth of a positive hypothetical, it follows that the conjunction which it by definition denies as possible is false,; and vice versa: so these are contraries. Given the falsehood of a negative hypothetical, *the negation of* the conjunction which it by definition admits as possible is true; so these are subcontraries. With regard to all other factual conjunctions, hypotheticals are neutral.

b. There is another respect in which polar expressions might be applied to hypotheticals. We will reserve the labels ***‘affirmative’*** and ***‘negatory’*** for this new division; here, unlike with categoricals, the terms must not be confused.

Thus, ‘If P, not-then not Q’, involving a double negation, is essentially as positive as ‘If P, then Q’ towards the subsequent Q; these forms, and their equivalents with nonP as antecedent, will therefore be classified as affirmative. Whereas ‘If P, then not Q’ or ‘If P, not-then Q’, which involve only one negation, effectively negate the subsequent thesis Q; so that they, and likewise the corresponding forms with nonP as antecedent will be said to be negatory hypotheticals. Such polarity considerations, also, as will be seen, clarify the basis of validity of certain hypothetical syllogisms.

c. Although the hypotheticals included in our initial list of forms are all tenable and useful, half of them are somewhat artificial as they stand.

Forms involving a thesis 'P' as antecedent can be regarded as ***'perfect'*** in comparison to those involving an antithesis 'not P' as antecedent, labeled ***'imperfect'***, whether the forms are positive or negative, and whether the consequent or inconsequent is 'Q' or 'not Q'.

These characterizations are relative, and not of great importance, but they are useful. The significance of this division of hypotheticals will become more apparent in due course, when we deal with hypothetical inference. But the following are some explanations.

Those with the antecedent P are most 'true to form' and express a normal 'movement of thought', and may therefore be called perfect, whether P be in itself a thesis with an affirmative or negative content. But those with the antecedent notP, *qua* antithesis (and not because it may present a negative content), are not as such representative of a natural way of thinking. If notP is taken up as a thesis in itself (be it intrinsically affirmative or negative in form or content), rather than by virtue of its being the antithesis of P, the form is quite normally hypothetical, proceeding from a *posited* antecedent, which may happen to be of negative polarity, to some consequences or inconsequences. But if the focus or stress is on the anti-P aspect of our 'nonP', the form is relatively artificial, and so 'imperfect'.

d. The use of ***'substitution'***, putting an antithesis in place of a thesis, or vice versa, is a theoretical device of the science of formal logic, rather than a process in the practical art of logic. The science of logic is built as a conceptual algebra, with 'variables' open to any content, related by selected 'constants'. In categoricals, the variables are terms, the constants, the copula, the polarity, the quantity, and so on. In hypotheticals, the variables are propositions, the constants, the relational factors peculiar to them.

But the use of substitution, in the sense of putting specific 'values' in the place of logic's variables, is a practical, rather than theoretical, process, and should be counted as a form, or at least stage, of inference. Here, the thinker is applying logical principles to a given situation, appealing to generally established processes to justify a particular act of thought. Such movement from knowledge of logical science to practical application, is in itself a reasoning process.

25. HYPOTHETICALS: OPPOSITIONS AND EDUCTIONS.

1. Connection and Basis.

We defined positive and negative hypothetical propositions in terms of the logical impossibility or possibility, respectively of a certain conjunction. This phenomenon refers to the logical *connection* between the theses concerned. Taken by itself, such a relation does not require that the theses be more than problematic; we need not know whether each of them is contingent, necessary or impossible.

However, in everyday discourse, we commonly regard the logical modality of the theses as tacitly, mutually understood. That is, we take for granted that the respondent has the same idea as the speaker with regard to the contingency, necessity or impossibility of each of the theses. This phenomenon refers to the logical *base(s)* of the theses, or the *basis* of a hypothetical proposition.

Normally, in most cases we ordinarily encounter, this underlying modality is logical contingency, for both the theses. ***Abnormally***, in rare cases of a usually philosophical nature, the modality of one or both of the theses is found to be logical necessity or impossibility. For this reason, we may refer to two broad classes of hypotheticals, the normal or the abnormal.

As we shall see, hypotheticals behave according to different logics. 'Baseless' hypotheticals, those with a problematic basis, representing only various connections, without specifying the logical modality of the theses — display what may be called the general or absolute or unconditional behavior patterns. Normal hypotheticals, which have contingent bases, and abnormal hypotheticals, which have one or both theses incontingent, each display slightly different patterns, their own particular or relative or conditional patterns.

Thus, we could develop considerably different logics for each variety of hypothetical. In this volume, we will try to highlight the main features of hypothetical logic, sometimes for unspecified basis, sometimes for specified bases, normal (fully contingent) or abnormal (partly or fully incontingent), as appropriate.

Note that we could similarly regard conjunctions as having a variety of bases. The logics would parallel those of hypotheticals of specified bases.

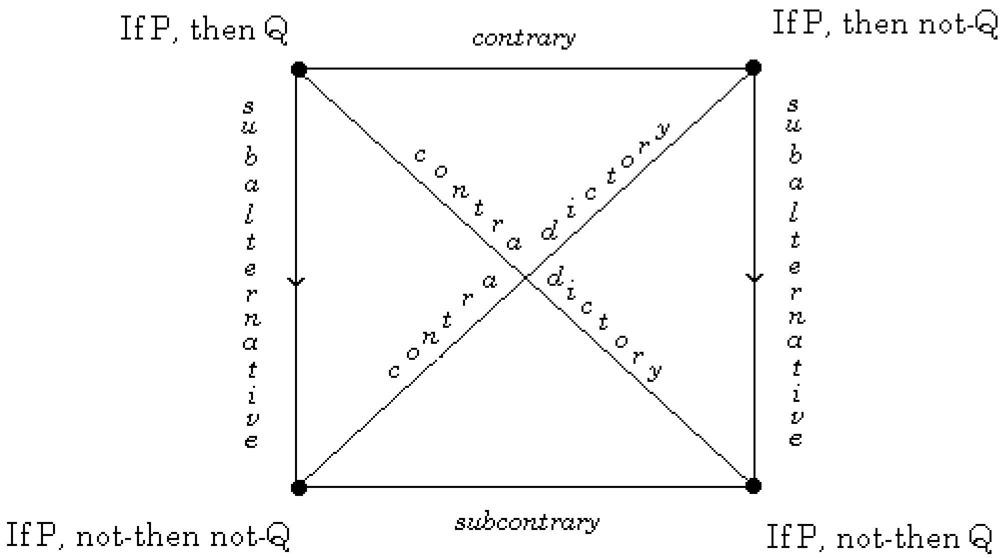
2. Oppositions.

a. The absolute oppositions, between the forms of hypothetical proposition whose bases are unspecified, proceed from the definitions of connections as modal conjunctions. They are identical to the oppositions between the conjunctives **H1n**, **H2n**, **H3n**, **H4n**, **K1p**, **K2p**, **K3p**, **K4p**, which we discussed in a previous chapter.

Here, our purpose is to identify the oppositions between hypotheticals, especially in cases where the logical modality of the theses is more specifically known. We will first deal with merely connective and/or normal hypotheticals, for which the theses may be assumed both contingent, and thereafter consider some of the differences in oppositional properties for abnormal hypotheticals.

b. Normal hypotheticals are opposed as follows. Note well the unstated condition that the theses are logically contingent. Let us consider, to begin with, the four forms with a common antecedent P.

Diagram 25.1 Square of Opposition for Hypotheticals with Common Antecedent.



Since ‘If P, then Q’ and ‘If P, not-then Q’ inform that the conjunction ‘P and nonQ’ is, in the former case, impossible, and, in the latter case, possible, they are contradictory. Likewise for the other diagonal.

The contrariety of ‘If P, then Q’ and ‘If P, then nonQ’ is obtained by supposing them both true; in that case, if P was true, Q and nonQ would be both true; therefore, these hypotheticals are incompatible; on the other hand, supposing them both false yields no impossible result.

The subcontrariety of ‘If P, not-then Q’ and ‘If P, not-then nonQ’ follows, since if they were both false, their contradictories would be both true, though incompatible; on the other hand, supposing them both true yields no impossible result.

Finally, if ‘If P, then Q’ is true, then ‘If P, then nonQ’ is false, by contrariety; then ‘If P, not-then nonQ’ is true, by contradiction; whereas nothing can be shown concerning the latter if ‘If P, then Q’ is false; so their subalternative relation (downward) holds. The other subalternation can be likewise shown.

A similar square of opposition can be demonstrated for the forms with a common antecedent nonP, namely, ‘If nonP, then (or not-then) Q (or nonQ)’. We can show that hypotheticals with a common consequent Q, but different antecedents, P or nonP, fall into such a square of opposition, by contraposing the forms (see next section on eduction). Likewise, if the common consequent is nonQ, of course.

However, concerning propositions whose antecedents and consequents are both different, namely, ‘If P, then (or not-then) Q’ and ‘If nonP, then (or not-then) nonQ’, the same cannot be said. For their definitions as impossibility (or possibility) of the conjunctions ‘P and nonQ’ and ‘nonP and Q’, respectively, leave them quite compatible, and unconnected. Likewise, for opposite pairs of the forms ‘If P, then (or not-then) nonQ’ and ‘If nonP, then (or not-then) Q’

The oppositions of the eight forms of hypothetical could be illustrated by means of a cube. However, the following tables summarize all these results for us, just as well. (The numbering of forms and symbols for oppositions used in these tables is arbitrary.)

Table 25.1 Table of Oppositions between Hypotheticals.

Key to symbols:		Unconnected	☯
Implicant	⌘	Contradictory	💣
Subalternating	⇗	Contrary	⇘
Subalternated	⇙	Subcontrary	⇖

Form	No.	1	2	3	4	5	6	7	8
If P, then Q	1	⌘	⇘	⇘	☯	☯	⇗	⇗	💣
If P, then nonQ	2	⇘	⌘	☯	⇘	⇗	☯	💣	⇗
If nonP, then Q	3	⇘	☯	⌘	⇘	⇗	💣	☯	⇗
If nonP, then nonQ	4	☯	⇘	⇘	⌘	💣	⇗	⇗	☯
If nonP, not-then nonQ	5	☯	⇙	⇙	💣	⌘	⇖	⇖	☯
If nonP, not-then Q	6	⇙	☯	💣	⇙	⇖	⌘	☯	⇖
If P, not-then nonQ	7	⇙	💣	☯	⇙	⇖	☯	⌘	⇖
If P, not-then Q	8	💣	⇙	⇙	☯	☯	⇖	⇖	⌘

These relationships may be clarified by means of a truth-table, in which given the truth of a form under heading **T**, or the falsehood of one under heading **F**, the status of the others along the same row is revealed.

Table 25.2 Truth-Table for Opposing Hypotheticals.

(key: T = true, F = false, . = undetermined.)

Form	T	1	2	3	4	5	6	7	8	F
If P, then Q	1	T	F	F	.	.	T	T	F	8
If P, then nonQ	2	F	T	.	F	T	.	F	T	7
If nonP, then Q	3	F	.	T	F	T	F	.	T	6
If nonP, then nonQ	4	.	F	F	T	F	T	T	.	5
If nonP, not-then nonQ	5	.	.	.	F	T	.	.	.	4
If nonP, not-then Q	6	.	.	F	.	.	T	.	.	3
If P, not-then nonQ	7	.	F	.	.	.	.	T	.	2
If P, not-then Q	8	F	.	.	.	.	.	.	T	1

3. Hierarchy.

The square of opposition shown in the previous section, you will notice, is the familiar one encountered for the categorical propositions **A**, **E**, **I**, **O**. The analogy is not accidental. The contrariety between 'If P, then Q' and 'If P, then nonQ' is obviously similar in meaning to that between 'All S are P' and 'All S are nonP', and the diagonal contradictions can also obviously be likened.

This analogy suggests that normal positive and negative hypotheticals constitute a hierarchy, the former being '***uppercase***' forms similar to general propositions and the latter '***lowercase***' forms similar to particulars. Indeed, this is implicit in the definitions of hypotheticals.

Thus, 'If P, not-then notQ' (note the double negation) is the lowercase form corresponding to the uppercase 'If P, then Q'; likewise, 'If P, not-then Q' is the subaltern form of 'If P, then notQ', 'If notP, not-then notQ' is the subaltern form of 'If notP, then Q', and 'If notP, not-then Q' is the subaltern form of 'If notP, then notQ'. Each positive hypothetical includes the negative hypothetical with like antecedent and unlike subsequent (i.e. consequent or inconsequent).

This uppercase/lowercase classification will be found useful in understanding of much hypothetical inference. By expressing the form 'If P, then Q' as a generality 'All P occurrences are Q occurrences', and the form 'If P, not-then notQ' as a particular 'Some P occurrences are Q occurrences', we will be able to understand why, for instance, the major premise in first figure hypothetical syllogism must be uppercase, and cannot be lowercase.

Now, what of the oppositions between the eight hypotheticals and the four factual conjunctions referred to in their definitions? First, we note that any pair of the four conjunctions are opposed to each other in the way of contraries; that is, they cannot be both true, but may be both false.

Secondly, we know that each uppercase hypothetical form is contrary to the conjunction which it denies as possible by definition; it is oppositionally neutral to the remaining three conjunctions, since, taken as a pair with any one of them, they may be both true or both false without problem. Thirdly, each lowercase form is subaltern to (implied by) the conjunction which it affirms as possible by definition; and unconnected oppositionally to the other conjunctions.

From this we may conclude that while, for example, 'P and Q' implies 'If P, not-then notQ', in the same way as a singular categorical implies a particular, the analogy stops there. For 'P and Q' is not in turn implied by 'If P, then Q', as analogy would require. That is, the conjunctions are not exactly 'middle case' forms, between the upper and lower cases.

This discussion of course serves to clarify the inter-relationships of the categories of logical modality. Uppercase is logical incontingency, lowercase is logical possibility or unnecessity; and conjunction is plain fact, lying in between. It concerns, of course, contingency-based hypotheticals, rather than hypotheticals with one or both theses incontingent. It applies to normal logic, rather than abnormal or general-case forms.

4. Eductions.

Here again, we will first consider normal hypotheticals, and then mention merely-connective hypotheticals and abnormals.

We need only, to begin with, deal with the primary hypothetical forms, 'If P, then Q' and 'If P, not-then Q', as our source propositions, to elucidate the processes. What is found valid for these, is mutatis mutandis applicable to forms involving 'nonP' and/or 'nonQ' as the source theses. The educed hypotheticals may have different polarity ('not-then' instead of 'then', the reverse never occurs), may involve the antithesis of one or both of the original theses as a new thesis, and may switch the positions of the theses. The valid processes are:

a. Obversion. From P-Q to P-nonQ.
If P, then Q implies If P, not-then nonQ.

b. Conversion. From P-Q to Q-P.
Not applicable.

c. Obverted Conversion. From P-Q to Q-nonP.
If P, then Q implies If Q, not-then nonP.

d. Conversion by Negation. From P-Q to nonQ-P.
If P, then Q implies If nonQ, not-then P.

e. Contraposition. From P-Q to nonQ-nonP.
If P, then Q implies If nonQ, then nonP.
If P, not-then Q implies If nonQ, not-then nonP.

f. Inversion. From P-Q to nonP-nonQ.
Not applicable.

g. Obverted Inversion. From P-Q to nonP-Q.
If P, then Q implies If nonP, not-then Q.

The primary process here is (e) contraposition. These eductions are validated by reference to the forms' definitions. Since 'If P, then Q' means that the conjunction 'P and nonQ' is impossible, and 'If nonQ, then nonP' that 'nonQ and not-nonP' is impossible, and these two conjunctions are equivalent, it follows that the two hypotheticals involved are also equivalent.

The same can be said with regard to the negative forms: they are defined by the same possibility of conjunction, and therefore equal. Contraposition is therefore a reversible process, and applicable as described to all hypotheticals without loss of power.

This process applies to unspecific hypotheticals and abnormals, as well as to contingency-based normals, because it only requires for its validity the connection implied by the defining modal conjunction.

The other processes, however, are only applicable to normal positive hypotheticals, if at all, and always yield a weaker, negative result. These processes are only applicable to normal hypotheticals, because they presume that the theses are to be understood as both logically contingent.

They are proved by reductio ad absurdum, combining the source proposition with the contradictory of the target proposition, to yield an inconsistency, in some cases after some contraposition(s).

Thus, given 'If P, then Q' to be true, (a) if 'If P, then nonQ' was true, it would follow that P implied both Q and nonQ, an absurdity, therefore the stated obverse must be valid; (c) if 'If Q, then nonP' was true, we could contrapose it and obtain the same absurdity, therefore the stated obverted converse must be valid; (d) if 'If nonQ, then P' was true, it would follow, after contraposing 'If P, then Q', that nonQ implied both P and nonP, an absurdity, therefore the stated converse by negation must be valid; and (g) if 'If nonP, then Q' was true, we could contrapose both it and the source proposition, and obtain the same absurdity, therefore the stated obverted inverse must be valid.

All processes with theses P, Q in the source propositions, excluded from the above list, cannot be likewise validated, and so are invalid.

By substituting the antitheses of P and/or Q in the above validated processes, we get the following full list of possible eductions, which is useful for reference purposes.

a.	Obversions.		
	If P, then Q	implies	If P, not-then nonQ.
	If P, then nonQ	implies	If P, not-then Q.
	If nonP, then Q	implies	If nonP, not-then nonQ.
	If nonP, then nonQ	implies	If nonP, not-then Q.
b.	<u>Conversion.</u> Not applicable.		
c.	Obverted Conversions.		
	If P, then Q	implies	If Q, not-then nonP.
	If P, then nonQ	implies	If nonQ, not-then nonP.
	If nonP, then Q	implies	If Q, not-then P.
	If nonP, then nonQ	implies	If nonQ, not-then P.
d.	Conversion by Negations.		
	If P, then Q	implies	If nonQ, not-then P.
	If P, then nonQ	implies	If Q, not-then P.
	If nonP, then Q	implies	If nonQ, not-then nonP.
	If nonP, then nonQ	implies	If Q, not-then nonP.
e.	Contrapositions.		
	If P, then Q	implies	If nonQ, then nonP.
	If P, then nonQ	implies	If Q, then nonP.
	If nonP, then Q	implies	If nonQ, then P.
	If nonP, then nonQ	implies	If Q, then P.
	If P, not-then Q	implies	If nonQ, not-then nonP.
	If P, not-then nonQ	implies	If Q, not-then nonP.

If nonP, not-then Q	implies	If nonQ, not-then P.
If nonP, not-then nonQ	implies	If Q, not-then P.

f. Inversion. Not applicable.

g. Obverted Inversions.

If P, then Q	implies	If nonP, not-then Q.
If P, then nonQ	implies	If nonP, not-then nonQ.
If nonP, then Q	implies	If P, not-then Q.
If nonP, then nonQ	implies	If P, not-then nonQ.

A final comment. We may observe in the above that obversion of uppercase hypotheticals merely yields the corresponding lowercase form, so such eduction yields no more than the oppositional inference of a subaltern.

We could have regarded the obverse of 'If P, then Q' to be 'If P, then not-nonQ', rather than merely 'If P, not-then nonQ'. This would obviously be correct, and analogous to the obversion of 'All S are P' to 'No S are nonP'. Effectively, we would be introducing a relational operator 'then-not' (and its negation, 'not-then-not'), to complement 'then' (and 'not-then'). But I think such multiplication of 'nots' is without value.

26. DISJUNCTION.

One way to introduce the topic of 'disjunction', is to view it in contradistinction to 'subjunction'. According to this approach, we may divide hypotheticals into two groups, with reference to the emphasis they put on their theses and antitheses.

1. Subjunction.

Hypotheticals which relate two theses as such, or two antitheses as such, may be called 'subjunctive'. The reason these two sets are grouped into one class becomes clearer when their definitions are considered.

The primary form of subjunction is 'If P, then Q', which tells us that '{P and nonQ} is logically impossible' (**H2n**). This is known as implication. Its negation is 'if P, not-then Q', meaning '{P and nonQ} is possible' (**K2p**).

The other form of subjunction, 'If nonP, then nonQ', tells us that '{nonP and Q} is logically impossible' (**H3n**), and so is equivalent to the statement 'If Q, then P', which has a similar meaning to 'If P, then Q', but in the anti-parallel direction. This could therefore be called reverse implication. The corresponding negative form is 'if nonP, not-then nonQ', meaning '{nonP and Q} is possible' (**K3p**).

We may view implication and its reverse as forms of subjunction, and their contradictories as forms of nonsubjunction. Or we may conventionally broaden the sense of the word subjunction, and speak of positive and negative subjunction, respectively.

Now, taken individually, these various logical relations are indefinite. Hypotheticals are elementary forms, capable of various combinations, called compounds, which define relationships more definitely. The forms are intentionally left open, to allow expression of the maximum number of combinations using a minimum number of building blocks. These effects have already been encountered in the context of opposition theory, and will only be briefly reviewed here for the sake of thoroughness.

Implication and its reverse are oppositionally neutral to each other (likewise, therefore, their contradictories). They are therefore capable of four combinations: they may be both true, or one true and the other false, or both false. The hypotheticals conjoined in such combinations are called complementary, in that they together serve to define the relationship between the theses in both directions.

In such case as 'If P, then Q' and 'If nonP, then nonQ' are both true, the resulting relation is one of mutual or reciprocal implication of P and Q (or nonP and nonQ). This may be called implicance, and viewed as asserting the logical equivalence of these two theses (or of their antitheses).

In such case as 'If P, then Q' and 'If nonP, not-then nonQ' are both true, P is said to subalternate Q; in such case as 'If P, not-then Q' and 'If nonP, then nonQ' are both true, P is said to be subalternated by Q. Thus, subalternation, in contrast to implicance, is one-way subjunction, and not reversible.

In such case as 'If P, not-then Q' and 'If nonP, not-then nonQ' are both true, we are left with a relation which might be called 'unsubjunction'. This is not a fully defining combination, unlike the preceding three compounds, in that it allows the possibility of disjunction.

2. Manners of Disjunction.

In contrast, we call 'disjunctive' those hypotheticals which relate a thesis with an antithesis, or an antithesis with a thesis. We usually express such relationship by means of the word 'or'. Rephrasing a hypothetical in disjunctive form allows us to conceal the negative polarity of the antitheses involved, so that the statement is made purely in terms of theses. The two theses are known as the 'alternatives' (or disjuncts).

Two essential *manners* of disjunction may be distinguished. As usual in logic, we must adopt some clear-cut differences in terminology to facilitate treatment; but, although the underlying distinctions of meaning are indeed intended in practice, they are not always verbalized so exclusively.

(i) 'P and/or Q' (or 'P or also Q') signifying simply 'If nonP, then Q' (or 'if nonQ, then P'), in other words, '{nonP and nonQ} is logically impossible' (**H4n**). This is known as inclusive disjunction, and expresses the *exhaustiveness* of P and Q: one of them must be true. This is the more commonly intended sense of 'P or Q'; it stresses the theses (P, Q), rather than the 'or' operator.

The negation of this form 'not-{P and/or Q}' (which could be written 'P not-{and/or} Q') means 'If nonP, not-then Q' (or 'if nonQ, not-then P'); in other words '{nonP and nonQ} is not logically impossible' (**K4p**). This of course signifies inexhaustiveness.

(ii) 'P or else Q' (or 'P otherwise Q') signifying simply 'If P, then nonQ' (or 'if Q, then nonP'); in other words, '{P and Q} is logically impossible' (**H1n**), suggesting a difference. This is known as exclusive disjunction, and expresses the *incompatibility* of P and Q: one of them must be false. This is a rarer sense of 'P or Q'; it stresses the separation of the theses (P, Q), the 'or' operator.

The negation of this form 'not-{P or else Q}' (which could be written 'P not-{or-else} Q') means 'If P, not-then nonQ' (or 'if Q, not-then nonP'); in other words, '{P and Q} is not logically impossible' (**K1p**). This of course signifies compatibility.

We may view exhaustiveness and incompatibility as forms of disjunction, and their contradictories as forms of nondisjunction. Or we may conventionally broaden the sense of the word disjunction, and speak of positive and negative disjunction, respectively.

Note, sometimes when we say 'P and/or Q', we intend to admit of only two alternatives, 'P and Q' or 'nonP and Q', in advance excluding or not meaning to include 'P and nonQ', as well as 'nonP and nonQ'. Sometimes, this is what we intend when we say 'P or else Q', for that matter; meaning, 'at least Q, whether or not P'. Likewise, 'P or also Q' may be intended to mean: 'P and Q' or 'P and nonQ'; that is, 'at least P, possibly without Q but also possibly with it'. Sometimes, 'P or Q' is understood to mean 'P and nonQ' or 'P and Q'.

Such implications are often obvious to us by virtue of the subject involved; the subject-content is well known to everyone to exclude certain alternatives, so that these exclusions are virtually formal. The logic of such forms can easily be derived from the logic of the forms here considered, so they will be ignored.

The recasting of a hypothetical form into disjunctive form, or vice versa, may be called 'transformation'. This may be viewed as a form of inference, or of elucidation, insofar as the mind may favor such process to more fully understand the relationship under consideration.

Note that disjunctives, like hypotheticals, may each be dissected into their implicit connection and basis. The general case comprises only the 'connective' (a modal conjunction) for its definition, whereas normal and abnormal disjunctions specify the logical modalities of the theses in various ways. Many processes are only valid for contingency-based disjunctions.

Needless to say, the theses of disjunctions may be any kind or complex of proposition(s): categoricals, conjunctives, hypotheticals, or also disjunctive clauses. The logic involved becomes progressively more intricate and complicated, accordingly. Some such logical 'compositions' will be analyzed in the next two chapters.

Each of the forms of disjunction is, we note, nondirectional, unlike the forms of subjunction. By reference to their definitions, it is easy to see that: 'If P, then nonQ' is equivalent to 'If Q, then nonP'; 'If P, not-then nonQ' is equivalent to 'If Q, not-then nonP'; 'If nonP, then Q' is equivalent to 'If nonQ, then P'; and 'If nonP, not-then Q' is equivalent to 'If nonQ, not-then P'. These equations have already been encountered under the heading of contraposition.

The forms of elementary disjunction are complementary; any pair of them, other than contradictories of course, may be used in conjunction to define a compound relationship, as follows. Note that each of these relations is reversible.

Contradiction combines 'If P, then nonQ' and 'If nonP, then Q'. We could assign to the disjunctive form 'Either P or Q' this specific meaning, comprising both incompatibility and exhaustiveness of P and Q. The proposition 'Either nonP or nonQ' is equivalent, note well.

Contrariety combines 'If P, then nonQ' and 'If nonP, not-then Q'. Thus, contrariety means incompatibility without exhaustiveness.

Subcontrariety combines 'If nonP, then Q' and 'If P, not-then nonQ'. Thus, subcontrariety means exhaustiveness without incompatibility.

'Undisjunction' might be used to label the combination of 'If P, not-then nonQ' and 'If nonP, not-then Q', which means inexhaustive and compatible. This is not a fully defining combination, unlike the preceding three compounds, in that it allows the possibility of subjunction.

The oppositions of all forms of subjunction and disjunction, elementary or compound, to each other, and the eductions feasible from each of them, are all easily inferred from the findings for the corresponding hypotheticals. I will not list them all, to avoid repetition, but a couple are worth highlighting.

Thus, note that 'P and/or Q' and 'nonP or else nonQ' are equivalent, and likewise, 'P or else Q' and 'nonP and/or nonQ' are equivalent. Also, 'either P or Q' and 'either nonP or nonQ' are identical.

3. Broadening the Perspective.

a. Interface of Subjunction and Disjunction.

Since the conjunctive roots of subjunctions and disjunctions, namely **H2n**, **H3n**, and **H4n**, **H1n**, are neutral to each other, they are in principle combinable together. However, normally, subjunctions and disjunctions are contrary to each other and not combinable; this applies to formal logic, where the theses and antitheses are all granted the status of logical contingency, as in the theory of opposition. This further justifies their division into two classes.

In contrast, nonsubjunctions and nondisjunctions, namely **K2p**, **K3p**, and **K4p**, **K1p**, are generally combinable, since they are compatible both in absolute terms (neutral) and in formal situations (subcontrary).

In opposition theory (ch. 6), we identified seven fully defining logical relations. The six main ones — implicance, subalternating, being-subalternated, contradiction, contrariety, and subcontrariety — have been reviewed in the previous sections of the present chapter. The remaining one was, you will recall called 'unconnectedness' or 'neutrality', in formal logic discussions. This may be defined as a combination of 'unsubjunction' and 'undisjunction'. Although each, taken alone, is still an indefinite compound, taken together they form a fully defining and reversible relationship.

In formal logic contexts, these 7 fully defining compounds are all mutually exclusive and constitute an exhaustive list of possibilities; if any one holds, the other six are out, and if any six are rejected, the remaining one must stand. The negation of any one of them means one or more of its constituent hypotheticals is false, without specification as to which one(s); so we must be careful not to make errors here.

In particular, note that the expression 'neither P nor Q' is normally equivalent to 'both nonP and nonQ', and should not be thought to be the logical negation of 'either — or —' in the above suggested sense, though it is sometimes so intended.

Beyond these definitions, we will not further discuss compound forms, so as not to complicate matters further. The inferences possible from them are all implicit in those concerning the constituent elementary forms, and can easily be derived.

b. Vague Disjunctions.

The important thing is not to confuse the elementary forms with their compounds, and to be aware of the reducibility of compound forms to their elementary positive and negative constituent hypotheticals. Especially, disjunctive propositions are in practice often notoriously ambiguous.

Sometimes, when we say, ‘P and/or Q’ we only intend ‘if nonP, then Q’, sometimes an additional ‘if P, not-then nonQ’ is *sous-entendu*. The elementary case merely forbids ‘nonP and nonQ’, without specifically allowing or forbidding ‘P and Q’, whereas the compound case specifically allows for the latter. Similarly, mutatis mutandis, with regard to ‘P or else Q’.

The difficulty is due to the previously mentioned inductive rule for weak relations in logical modality: here, there is little distinction between the ‘open’ and the ‘possible’. Ultimately, a conjunction which is neither specifically allowed nor specifically forbidden, is effectively allowed. The difference is merely one of degree. If the open turns out to be impossible, it is just eliminated from the list of alternatives as a matter of course, without affecting the overall truth of the disjunctive proposition.

In practice, we often use a vague form of disjunction, ‘P or Q’, which might mean anything from an elementary inclusive or exclusive disjunction, to a compound like subcontrariety, contrariety, or even contradiction. It is thus relatively uninformative; nevertheless, it shows why we can class all these relations under the common heading of disjunctions.

The forms ‘P and/or Q’ and ‘P or else Q’ and ‘either P or Q’ all suggest that ‘P or Q’, though for different reasons. The form ‘P or Q’ in its broadest sense recognizes at least ‘P and nonQ’ or ‘Q and nonP’ as conceivable outcomes, without telling us at the outset whether ‘P and Q’ or ‘nonP and nonQ’ are allowed or forbidden, though it is understood that at least one of them (if not both) is forbidden.

The implicit questions are left open, unless the relation is further specified by ‘and’ or ‘else’ or ‘either’, in which case the additional allowance is made more firm (given a greater degree of eventuality) by what is specifically forbidden. If both the open questions are answered negatively, then ‘or’ means ‘either-or’.

The vague form ‘P or Q’ may thus be defined by the disjunction of all the clearer forms of disjunction. The following table shows the common ground between these forms. Note that the ‘allowances’ here should be interpreted minimally, as problemacies, though they are often in practice meant to be logical possibilities in the stricter sense.

Table 26.1 Common Ground of Disjunctions.

Conjunction	P+Q	P+nonQ	nonP+Q	nonP+nonQ
P and/or Q	allow	allow	allow	forbid
P or-else Q	forbid	allow	allow	allow
Either P or Q	forbid	allow	allow	forbid

The negation of ‘P or Q’ may be stated as ‘not-{P or Q}’ (or ‘P not-or Q’). What we mean by that of course depends on what we intend by ‘P or Q’.

c. Involving Antitheses.

We presented subjunction and disjunction as subdivisions of hypotheticals. But unlike subjunction, disjunction involves a distinct set of operators, ‘or’ and its derivatives. So disjunction deserves to be viewed as a logical relation in its own right. We can see from its name that we intend this relation as conceptually opposed to conjunction.

What this means is that, in addition to ‘P or Q’, we should consider ‘P or nonQ’, ‘nonP or Q’, ‘nonP or nonQ’. Similarly for the less vague operators ‘and/or’, ‘or-else’, and their compounds, including ‘either-or’: we can insert one or both antitheses, in place of the original theses, to obtain other forms, as we did for hypotheticals. And of course, all these have contradictories.

It is very easy to determine the conjunctive definition for each form, and then compare it to all the others. Since each operator gives rise to four impossible conjunctions and four possible ones, and these eight conjunctions are ubiquitous, there is bound to be a corresponding number of equations.

I will not go into this domain in any detail, so as not to expand this treatise unnecessarily. The reader is invited to explore it for him or her self.

27. INTRICATE LOGIC.

1. Organic Knowledge.

People think that logic is a linear enterprise, antithetical to the curvatures of poetic knowledge. But, viewed holistically, knowledge is not essentially a mechanical activity and product, but more akin to a living organism.

Just as any living organism functions on many levels, from the physical-chemical, through the biochemical and cellular, to the gross level of our sensory perceptions, and beyond that, as an intricate part of the natural environment as a whole, through the intellectual and spiritual dimension, the whole being sustained by the Creator — so knowledge ought to be viewed.

Logic is the way we establish the chemical bonds between the different data elements of our knowledge. These bonds vary in kind and effect, and can occur cooperatively in any number and complication of combinations. The result is a network, from the microscopic level of precise logical relations, to the less-magnified level of clusters of information, to the organic whole, to the cultural context.

This knowledge network is not stationary, but like an organism, pulses and glows with life, growing, ordering, clarifying, strengthening. This life has a mechanical level, a vegetative level, and a conscious and volitional level, which is animal and human, and therefore spiritual.

So viewed, logic and the poetic side of us are not in conflict, but in easy, friendly, fruitful togetherness. A balanced, healthy mind, requires some degree of rigor in observation and thought, and also some degree of freedom to move, some room to maneuver. Because knowledge is always in flux, and there is always some inconsistency involved.

One has to be able to flow with the tides of information, the momentary waves, and even the momentary storms, and remain patient and awake at one's center. Logic maps for us the wide terrain of the mind, improving our research skills. Thus, ultimately, logic is an aspect of wisdom, knowing to navigate smoothly in the changing sea of information.

Logic teaches us to clarify information, by *engineering* tools for this purpose. Especially multiple-theses, mixed-form, modal logic provides us with ways to express ideas precisely, and thus construct and check them more rigorously. Let us look at some of the possible intricacies of logical relations between items of knowledge.

2. Conjunctives.

Let us now broaden our understanding of conjunctive logic, in different directions. Note that we refer to any specialized field of logic as 'a logic'.

a. ***Multiple-Theses Logic.*** We have talked of conjunction with reference to two theses, because the logic of conjunctions of more than two theses is derivable from it.

Thus, we may inspect the theses of a proposition of the form 'P and Q', and find that, say, Q is itself composed of two theses 'Q1 and Q2'; from this we conclude that 'P and Q1 and Q2' is also true. Likewise, though with diminishing statistical probability, any of the theses P, Q1, Q2, may in turn be found subdividable. Thus, conjunctives may have any number of theses.

It follows that a conjunctive clause within a conjunction, is equivalent to a larger conjunctive proposition, so that we need not think in terms of clauses. This process may be viewed, in analogy to mathematics, as 'addition of conjuncts'; or we may refer to it as 'logical composition', the formation of composites out of elements or other components. For example:

'P and {Q and R}' is identical to 'P and Q and R'.

A corollary of this is that we can isolate part of a conjunction as a clause, at will. All that should be obvious, since 'P and Q..'. simply informs us that the theses are individually, as well as together, true. Since the order of the theses is irrelevant in the case of two-theses conjunction, meaning 'P and Q' equals 'Q and P', it can likewise be shown that order does not affect the logical relation of any number of conjuncts.

This begins for us the topic of multiple-theses logic.

b. ***Matrix Logic.*** It is well, as we shall see, to think of multiple conjunctions as forming a continuum. The number of conjuncts (ands) is one less than the number of theses. Here, a single thesis is the limiting case of the continuum, a conjunction without conjunct, as it were. Clearly, the more theses are conjoined, the more overall information we have. Thus:

P (one thesis)
P and Q (two theses)
P and Q and R (three theses)
P and Q and R and… (and so on).

The logic of nonconjunction should follow, though it is more complex. Thus, the negative conjunction 'not-{P and Q}', where the theses are entirely problematic, signifies that any of the positive conjunctions 'P and nonQ', 'nonP and Q', or 'nonP and nonQ' might be true, since they are formally the only conceivable alternatives to the negated one.

We can therefore think of negative conjunctions with reference to positive ones entirely. The existence of a negative is expressed only through positives; negation is a lesser, derivative expression of existence. It is useful therefore, to view negative conjunctions as equivalent to *'matrixes'* of positive alternatives, as follows:

Table 27.1 The Matrixes of Negative Conjunctions.

not-{P+Q}	not-{P+nonQ}	not-{nonP+Q}	not-{nonP+nonQ}
P+nonQ	P+Q	P+Q	P+Q
nonP+Q	nonP+Q	P+nonQ	P+nonQ
nonP+nonQ	nonP+nonQ	nonP+nonQ	nonP+Q

We may call each of the alternatives in a matrix, a *'root'* conjunction of the nonconjunction. Such matrixes are very useful in clarifying the logic of negative conjunction, since we need only find the common ground of the positive alternatives (the roots) in each matrix, to know the properties of the corresponding negative. Thus, for instance, we can here too prove that the order of the theses is irrelevant, since all the alternatives of the matrix can be reversed.

It can thus be shown that clauses may be inserted or removed arbitrarily, with negative conjunctions as well. We can accordingly develop a logic of negations of multiple conjunctions, again thinking of all such conjunctions as forming a continuum. The difference here being that the more alternatives there are the wider, vaguer, and weaker, is their overall negation. That is, the more theses are involved in a negative conjunction, the less information we have; negation of one thesis being the most definite, limiting case. Thus, we have an upside-down continuum:

not-{P and Q and R and…} (and so on).
not-{P and Q and R} (three theses)
not-{P and Q} (two theses)
not-{P}…or P (one thesis)

The one-thesis case may be P, as well as nonP, if we understand these negations of conjunctions as effectively disjunctions, meaning 'P or Q or..'., for then P is one of the ways the disjunction can be resolved (since Q or R may be negated instead).

In this way, this here continuum of negative conjunctions, can be attached to the previously described positive conjunctions continuum, resulting in a larger continuum, stretching from the negative forms with the most theses, through the central one-thesis case (the 'P' common to both positive and negative conjunction), up to the positive forms with the most theses.

The negative, say left, side is a virtual kind of knowledge, getting ever vaguer, a storehouse of possibilities. The positive, say right, side is a growing categorical knowledge, ever more precise. As we move from left to right, our knowledge becomes more specific; we have more information, a higher, wider, deeper view.

It is interesting to note, in passing, that in Hebrew the word for 'and' is *'oo'* (spelt, vav; also pronounced as 've'), and the word for 'or' is *'o'* (spelt, alef-vav). This similarity confirms that the notions conjunction and disjunction are intuitively conceived as continuous, different degrees of the same thing.

Each of the multiple-theses negative conjunctions may be dissected into a matrix of positive conjunctions, the alternatives to the one negated. Negation of one thesis, P, leaves us with only one alternative, nonP. Effectively, every theses should be viewed as including the denial of its contradictory; P, say, may be taken as implying 'P and not-{nonP}'; nonP likewise becomes 'nonP and not-{P}'.

Negation of two theses, as we saw above, leaves us with three alternatives. Since three theses and their negations are combinable in eight ways, negation of a conjunction of three theses, leaves us with seven positive alternatives out of the eight:

P + Q + R	nonP + Q + R
P + Q + nonR	nonP + Q + nonR
P + nonQ + R	nonP + nonQ + R
P + nonQ + nonR	nonP + nonQ + nonR

Beyond that, the general formula is clearly, for n theses, there are: two to the nth power combinations, and therefore that number minus one positive alternatives to the negation of any root. For instance, for 5 theses, there are 2X2X2X2X2 = 32 possible combinations.

Accordingly, the positive conjunction of two (or more) negative conjunctive clauses, may be also expressed by reference to the leftover positive combinations. We can thus develop a general logic of conjunction; that is, any complex of positive and negative conjunctions can be interpreted in positive terms. I will not go into such detail here, however.

c. ***Modal Logic.*** The above concerns factual conjunction; modal conjunction has yet to be considered. To say that a conjunction is logically necessary means that it holds, no matter what the surrounding conditions. In contrast, a logically contingent conjunction depends for its eventual realization on certain conditions. If those conditions are unspecified, we have a nonhypothetical modal proposition; if sufficient conditions are specified, we have a precise hypothetical relation. All this applies to positive and negative conjunctions.

It follows that modal conjunctions can always be understood in terms of factual ones, whether the latter are framed by conditions, specified or unspecified, or unconditional. In conjoining modal conjunctions, we must however be careful, and consider whether the conditions under which each clause is realized are compatible with the conditions for the other clause(s) to become factual.

Consider, for instance, the following complex: {P and Q} is possible and {Q and R} is possible. Does it follow that: {P and Q and R} is possible? The answer is clearly, No! It is conceivable that, though these possibilities are compatible as modal propositions, they are incompatible in their factual embodiments. That is, it may be that: {P and Q and R} is impossible, and the given possibilities can only be realized separately, through {P and Q and nonR} or {nonP and Q and R}, respectively.

In this way, by focusing on the underlying factual conjunctions, we can develop a detailed logic of modal conjunction. In formal logic, using variables for terms or propositions, whatever is conceivable is logically possible. But in practice, when dealing with specific terms and specific relations, we must be careful to distinguish between problemacy and logical contingency.

In the above example, for instance, if the given two 'possibilities' are mere problemacies, then any combination is conceivable; and we can say (also problematically) that {P and Q and R} might well be true. But if the premises are logical possibilities, we cannot conclude that the {P and Q and R} conjunction is also logically possible.

d. Thus, a complete logic of conjunction, whether positive or negative, factual or modal, evolves entirely from the logic of positive conjunctions.

Since hypothetical and disjunctive propositions are in turn defined with reference to conjunctions, the logic of all mixtures of logical relations is likewise reducible to the logic of positive conjunctions.

Any statement, whatever its mix of logical relations — of whatever modalities and polarities — can thus be analyzed through matrixes, and compared to any other statement similarly analyzed.

3. Hypotheticals.

a. ***The form of argument.*** We present an argument by listing its premises and conclusions as follows. There are of course arguments with one premise (eductions), and arguments with more than two premises (as in sorites), and some with more than one conclusion, but the typical unit of deduction is two premises, one conclusion.

P,
and Q,
therefore R.

A valid categorical, Aristotelean syllogism, for instance, may be regarded as establishing a hypothetical link between premises and conclusion, *by way of* the common terms in these propositions, in specific figures and with precise polarity, quantity and modality specifications.

Thus, although we cannot say generally of any group of propositions that P and Q imply R, we do know that under specific conditions (where for instance P means 'X is Y', Q means 'Y is Z', and R means 'X is Z'), such a bond can be established, for all cases of that form. Thus, categorical syllogism may be viewed as one condition under which the form 'if P and Q, then R' may be viewed as universally true.

Now, this can be interpreted as a hypothetical proposition with a conjunctive antecedent: (a) 'If {P and Q}, then R'. Alternatively, we tend to interpret it as a hypothetical proposition with a hypothetical clause as its consequent: (b) 'if P, then {if Q, then R}', meaning that, under the condition P, Q implies R. The former states that '{P and Q and nonR} is impossible', whereas the latter states that '{P and possibly[Q and nonR]} is impossible'.

At first sight, the two statements may seem significantly different, yet if we analyze them with reference to the underlying positive conjunctions, it is seen that they make identical allowances. The form '{P and Q and nonR} is impossible' obviously allows for seven alternative positive conjunctions. The form '{P and possibly[Q and nonR]} is impossible' allows for:

(i) 'P and impossibly{Q and nonR}', implying the factual 'P and not-{Q and nonR}', which might be realized as 'P and {Q and R}', 'P and {nonQ and R}', or 'P and {nonQ and nonR}';

(ii) 'nonP and possibly{Q and nonR}', which grants the realizability of 'nonP and {Q and nonR}';

(iii) 'nonP and impossibly{Q and nonR}' implying the factual 'nonP and not-{Q and nonR}', which might be realized as 'nonP and {Q and R}', 'nonP and {nonQ and R}', or 'nonP and {nonQ and nonR}'.

Clearly, here again all seven alternatives to 'P and {Q and nonR}' are eventually permitted. Thus, the two expressions compared are equal: they have the same root conjunctions. This is an important finding for hypothetical logic.

The allowances in all cases are of course problemacies. In purely formal contexts, these problemacies do ordinarily signify that there are unspecified contents fitting the various alternatives. But in contexts of specified content, these problemacies should not be taken as formally logical possibilities, since some of the alternatives may well be excluded by additional statements.

b. ***Nesting.*** The definition of hypotheticals accurately reflects our formation of such thoughts. Assuming the antecedent clause allows us to hold it mentally in place, so that we can be free to deal with other matters, namely the relative status of the subsequent clause. This process may be called 'nesting', or 'framing'. It is similar to the technique of control in the experimental sciences, where, while keeping all other things equal, we observe the effects on our subject, of a precise change in the single remaining factor.

In the case of two theses, appropriately related, we frame the one by means of the other, in a simple hypothetical proposition, 'If P, then Q'. In the case of three theses, we can say 'If P, then {if Q, then R}', meaning that P is a context or framework for Q implying R. Likewise, for four theses, 'If P and Q and R, then S' can be reformed as 'If P, then {if Q, then [if R, then S]}'.

We can in this manner nest any number of hypotheticals within each other. In practice, much of the framework is often left tacit, note. Such multiple-theses hypotheticals serve to express partial or conditional antecedence. They may be viewed as forming a continuum, ranging from a single, unconditional thesis, to one framed by more and more difficult demands.

The value of such successive framing by hypotheticals can be seen in analysis of the process of *reductio ad absurdum* used in validation of syllogisms. To prove that 'If {P and Q}, then R'; we infer 'If P, then {if Q, then R}' by framing; then we contrapose the inner hypothetical to obtain 'If P, then {if nonR, then nonQ}'; then we remove the frame to obtain 'If {P and nonR}, then nonQ'; thus showing that denial of the conclusion leads to denial of a premise.

c. ***Mixed-Form Logic.*** Just as the antecedent of a hypothetical may be composite, so may the consequent be, as in 'if P, then {Q and R}'; this is equivalent to the conjunction of 'if P, then Q' and 'if P, then R'. Just as the consequent may be hypothetical, so may the antecedent be, as in 'if {if P, then Q}, then R'; this is not equivalent to 'if {P and Q}, then R', note well.

We can also use the disjunctive format in complicated propositions, which present alternative antecedents and/or consequents. For example, 'If {P or Q}, then R' (which is ordinarily taken to imply 'if P, then R' and 'if Q, then R'); or again, 'if P, then {Q or R}', which, though not incompatible with 'if P, then Q' or 'if P, then R', does not imply them. Those methods are used to find alternate conclusions from weaker premises (as seen in transitive syllogism), or weaker conclusions from alternate premises (as we shall see with 'double syllogism').

More broadly still, any kind of conjunction, hypothetical, or disjunction, positive or negative, may be involved with any other(s), in countless, intricate relations. Of course, it is wise not to get too carried away, it must be possible for the mind to unravel the meaning with relative ease. Going into the mechanics of all these relations in detail is beyond the scope of this book, but it can be expected to be an interesting field.

4. Disjunctives.

a. ***Multiple Disjunctions.*** Disjunctions may involve more than two alternatives, as in 'P or Q or R or..'.. We tend to use the general operator 'or', rather than the more specific 'and/or', 'or else', and 'either-or', because with three or more alternatives, disjunction has more nuances in meaning. Indeed, we need not specify any disjunctive operator at all, but could just list the theses under consideration (P, Q, R, etc.) and verbally specify their collective relations (as explained below).

Usually, of course, the inclusive form 'P and/or Q and/or R and/or…' may be supposed to mean 'at least one of P, Q, R, etc. must be true' (leaving open whether each of the others can or must be true or false). Similarly, the exclusive form 'P or else Q or else R or else…' may be supposed to mean 'all but one of P, Q, R, etc. must be false' (implying only one can be true, but leaving open whether it can be false or must be true; note too that any pair of theses are incompatible). If both these disjunctions are affirmed, the two or more theses involved may be said to be both exhaustive and incompatible.

More generally conceived, a multiple disjunction depends for its definition on *how many theses, out of the total number listed, must be true, and/or how many must be false.* These components specify the degrees of exhaustiveness and/or incompatibility of the alternatives. In some cases they are independent variables, in others, they affect each other, according to the total number of theses available.

Strictly, we should specify the definitions of our disjunction parenthetically; though in practice they are often left unsaid, when we do not know them precisely, or when we consider them as obvious in the context. Note well that the definitions do not tell us exactly which of the theses are true and which false; they only tell us that some stated number are this or that.

With two theses, as already seen, 'one must be true' signifies that both cannot be false, 'one must be false' signifies that both cannot be true, and those two specifications may occur without each other or together.

With three theses, the specifications 'one must be true', 'two must be true', 'one must be false', 'two must be false', can be combined every which way, except for 'two must be true and two must be false' together, normally (though in abnormal logic, this is not excluded).

With four (or more) theses, likewise, we can specify that one to three (or more) of the theses must be true and/or that one to three (or more) of the theses must be false, though the total number of theses so specified should not normally exceed the total number of theses available.

Whatever the number of theses, it is clear that the more of them are specified as having to be true or false, the firmer the implied bond between them. For instance, 'two must be true' is a more forceful relation than 'one must be true'. The more definite the bond, the more restrictive the relation, but also the more informative.

In the maximalist case, where we are given that all the theses must be true, or all must be false, or exactly which are true and which false is specified, we are left with no degree of freedom, and no ignorance. In the minimalist case, where any number may be true or false, in any combination, there is no link between the theses, and all the issues remain unresolved. The various degrees of disjunction lie in between these extremes.

Inversely, the greater the total number of theses listed in a disjunction, the looser the bond implied by the 'or' operators in it. For instance, 'one thesis must be true' represents a weaker relation with reference to a total of three or four theses than to a total of two theses. The more alternatives are available, the more of them we have to eventually eliminate to arrive at categorical knowledge, therefore the less we know so far.

Thus, the operator 'or' has many gradations of meaning, depending on various factors. However, we can think of all disjunctives as aligned in a continuum, ranging from one to any number of theses *in toto*, and from one to any number among them specified as having to be true or false. In some cases the inclusive and exclusive specifications diverge, in some cases they converge. Ultimately, all disjunctives are part of the same continuum as conjunctives.

b. ***Matrixes.*** It is best, when faced with such multiplicity of alternatives, to think in terms of the underlying possible outcomes of positive conjunction. For example, 'one of {P or Q or R} must be true, and two must be false' may be interpreted as '{P and nonQ and nonR}, {nonP and Q and nonR}, and {nonP and nonQ and R}, are possible (that is, at least problematic) conjunctions of the given theses'.

This format is least ambiguous, because we may on formal grounds understand the disjunction of the factual conjunctions listed to be formally of the 'one must be true and all the others must be false' degree, without having to say so, no matter what the original number of theses. We earlier referred to this as matrix logic.

Note that any of the underlying positive conjunctions involving a negative thesis, may themselves conceal an internal disjunction. For negation is often a shorthand expression of a number of positive alternatives; thus, nonP might mean 'P1 or P2', if it so happens that P, P1, and P3 are exhaustive. This is applicable even to elements, and all the more so to compounds and all composites.

Thus, we may find disjunctions within disjunctions within disjunctions; these may be referred to as different levels of disjunction. This phenomenon is interesting, because it illustrates the complexities of stratification which occur among propositions. There is an enormous wealth of possible relations among propositions.

Disjunctions may also may be expressed in hypothetical form, and vice versa. For instance, 'P or Q or R' (as defined in the above example) can be reformulated as: 'If nonP and nonQ, then R, and if nonP and nonR, then Q, and if nonQ and nonR, then P' (the 'one thesis is true' component), and 'If P, then nonQ and nonR, and if Q, then nonP and nonR, and if R, then nonP and nonQ' (the 'two theses are false' component). But such formulas can get pretty intricate and confusing. This is what justifies disjunction as a valuable form in itself.

But it follows anyway that the laws of intricate logic for hypotheticals may be used to obtain analogous laws for disjunctives; and vice versa. Thus, for instance, the case of a disjunctive proposition with a disjunctive clause as one of its theses, corresponds to the case of premise nesting we encountered in an earlier section.

We found that a modal conjunction within a larger modal conjunction, is equivalent to a factual clause. That is, since 'nonP and {nonQ and nonR} is impossible', and 'nonP and possibly{nonQ and nonR} is impossible', yield the same matrix of seven alternative conjunctions, they have the same logical properties. It follows that the corresponding disjunctives 'P or Q or R' and 'P or {Q or R}', intended in the 'one thesis must be true' sense, are equivalent.

A disjunction may be taken as a gross unit, as well as with reference to the alternatives it lists. We may focus on the whole or the parts, and determine the one or the others as our clause(s).

Such intricacies will not be covered in any great detail in this work, though interesting. All this is part of a yet broader field of research. The nesting case concerns a possible conjunction within impossible conjunction. But other combinations of 'modality within modality' can also be worked out.

c. Another direction of development for disjunctive logic, is the introduction of ***modalities of disjunction***. The concepts of connection and basis are applicable to disjunction. Purely connective disjunction has entirely problematic bases; if the base of each thesis is specified, whether as logical contingency (normally) or as incontingency (abnormally), special logics may apply.

The 'connection' of the disjunction is the impossibility of the conjunction(s) which are excluded from the underlying matrix. Here, the law of contradiction is that at least one of all the possible conjunctions in a matrix, for the given number of theses, must be true; the law of the excluded middle is that all but one of them must be false. Thus, connection is inherently incontingent.

One could argue that, since we can place a disjunction as the consequent of a hypothetical statement, we can think of conditional levels of disjunction as well. In that event, the connection may be logically contingent, valid in some specific (though not always specified) context(s). It follows that we can also think of a factual level of disjunction (loosely speaking), signifying that it is operative in the presently held context.

A more modal logic of disjunction may accordingly be developed, and here again basis may come into play. Possible disjunction implies that the disjunction is consequent to certain conditions, and therefore can be made factual by revealing the implicit antecedent. Problematic or logically possible disjunctives, underlie hypothetical propositions with a disjunction as antecedent or consequent. Disjunctions may of course also appear within larger disjunctions.

However, factual (contextual) versus incontingent (unconditional) disjunction, may be compared to material versus strict implication. So these concepts may be used to some extent, if we remain conscious of their main pitfall — namely, the difficulty of pin-pointing precisely just which parts of the overall context frame our propositions, making up our effective so-called 'context'. In practice, we wordlessly 'know' the intended context, but in formal work this vague knowledge is not a useable capital.

In conclusion, the concept of modality provides us with a means of clarifying thoughts to a much greater degree than purely factual logic, giving us a new/improved tool of analysis of data. I leave it to you, to explore this field more thoroughly; this may be compared to presenting you with an object for inspection under your own microscope, using the techniques developed in this treatise.

28. LOGICAL COMPOSITIONS.

1. Symbolic Logic.

This chapter very briefly describes various processes having to do with the logical composition of conjunctions and disjunctions. The equations developed here, are selected to enable us to deal efficiently, in later chapters, with factorial formulas, especially. They are only a tip of an enormous iceberg, comprising hypothetical relations as well, consideration of all modalities of connections and bases, and full analysis of the negative side.

Although I personally avoid symbolic logic as much as possible, so as not to obscure for myself and others the meaning of what I am doing — in the case of the theorems below, I find that symbolization of logical relations does indeed bring out the processes more clearly. I will use a nomenclature and symbolic representation, which I personally find more comfortable, but which differs slightly from that adopted by modern logic (Copi, 319). It is, of course, to modern logicians that we owe these valuable clarifying formulas.

Let **p**, **q**, **r**, **s** be any theses, which will be conjoined or disjoined; their antitheses **not**p, **not**q, etc., may be symbolized by a so-called 'curl' (a curved minus sign like this: ~), as in ~p, ~q, etc. We may write: 'p **and** q' symbolically as 'p **+** q' (with plus sign) or as 'pq' (with no separation) or as 'p**.**q' (the dot suggesting a product). Also, we may write 'p **or** q', taken in the weakest 'and/or' sense, that 'one of the theses must be true', as 'p **v** q' (v for versus, supposedly) or as 'p**,**q' (note the comma); note that some computer programming languages use a vertical bar (like this: |), instead. Brackets '{}' are used to signify a clause within a larger sentence.

Note that the results for other forms of disjunction, like 'or else' or 'either-or', are often different; these will not be discussed here.

There are two sets of logical composition processes for us to consider:

a. ***'Addition'***, which is merging of conjunctions, or of disjunctions, with others of the same sign; the reverse process of separation, where it is feasible, might be called 'subtraction' (symbolized by a minus sign: -); and

b. ***'Multiplication'***, which is merging a mix of conjunctions and disjunctions with each other; the reverse direction, where it is feasible, might be called 'division'.

Two or more propositions which are added together are said to form a single, composite proposition. A proposition which is not itself a composite of others is called elementary. Specifically, a proposition consisting of a conjunction of others is called a compound; this is one form of composition.

Note well that this logic is limited to the one sense of disjunction, and to fully problematic bases. Other manners of disjunction, such as 'two (or more) theses must be true' and 'one (or more) theses must be false', and more specific bases, each have their own logic. Also, we are here going to deal with disjunction with minimal reference to modality, although a more modal approach would be more precise and interesting. Consideration of conditional disjunction would cause the study to spill over into the interplay of hypotheticals with the processes here considered.

So the present research is limited, because its purpose is utilitarian. A fuller theory of logical composition requires additional work. However, the material dealt with here has indirect applications. Many theorems can be derived from those here described. We can, for instance, change the polarity of theses or logical relations in various ways. We can also expect that some of the laws of 'at least one thesis must be true' logic, carry over into 'more than one thesis must be true' logic.

The theorems below are presented as usually reversible eductions, meaning that given the form on the left, the form on the right follows at will, and usually vice versa. However, many of them can also be classed as deductive processes, and will reappear in later chapters in that guise.

Another way to view them is, as rules of transformation, like changes in what we regard as clauses.

The analogies between mathematics and logic should not be overrated; they only go so far. Logic may be regarded as the manipulation of concepts of any kind, whereas mathematics concerns specifically numerical concepts. Although there is some intervention of mathematics in logic, for the resolution of quantitative issues, and we may be said to think logically when engaged in mathematics, these sciences are very different fields of interest. They have rationalism in common, but their scopes are different and neither is really a subsidiary of the other.

2. Addition.

a. Addition of conjuncts:

p + {q + r} = p + q + r
{p + q} + {r + s} = p + q + r + s

…and so on for any number of conjuncts. Proof is that p, q, r, s are all independently true, anyway, on both sides of the equations. The elimination of repetitives is a special case of addition: since p + p = p, it follows that p + {p + q} = p + q.

b. Addition of alternatives:

p v {q v r} = p v q v r
{p v q} v {r v s} = p v q v r v s

…and so on for any number of disjuncts. Proof is in such cases best sought by matrixual analysis; that is, *testing* each and every eventual combination of theses and antitheses, to see whether or not it obeys the demands of the given composite, then comparing the results on the two sides of the equation. Thus, with three theses:

p v {q v r}	p v q v r
p + q + r	p + q + r
p + q + notr	p + q + notr
p + notq + r	p + notq + r
p + notq + notr	p + notq + notr
notp + q + r	notp + q + r
notp + q + notr	notp + q + notr
notp + notq + r	notp + notq + r
(notp + notq + notr)	(notp + notq + notr)

The conjunctions shown in brackets are those which, having been tried out on the given disjunctions, failed the test. Both sides evidently mean that, so far, any conjunction of p, q, r and their antitheses is a conceivable outcome, to the exception of 'notp and notq and notr'. Equations involving more theses are similarly dealt with.

As with conjuncts, the elimination of repetitives is a special case of addition. Since p v p = p (meaning, p will be affirmed in either case), it follows that p v {p v q} = p v q. Note that in the case of 'p or else p', notp would follow; the form 'either p or p' is of course inconceivable.

c. With regard to subtraction, the equations above are to be reread from right to left. Addition followed by subtraction is useful to remove a common factor from two brackets:

{p + q} + {p + r} = p + p + q + r = p + {q + r} {p v q} v {p v r} = p v p v q v r = p v {q v r}

…or to reshuffle brackets:

{p + q} + {r + s} = p + q + r + s = {p + r} + {r + s}
{p v q} v {r v s} = p v q v r v s = {p v r} v {r v s}

d. But the idea of 'subtraction' more precisely fits the equation 'p + {~q} = p - q', of course. This suggests implications like the following, which shall be seen again in the context of 'logical apodosis':

{p v q} - q implies p
{p v q v r} - r implies p v q

…and so on for any number of theses. Note that these implications are valid only in one direction. For instance, in the first case, p alone cannot tell us whether q or notq is true, and therefore cannot yield the conclusion that '{p v q} and notq' are both true.

Nor may one push the analogy to mathematics so far as to move q to the other side of the implication and claim that 'p v q' and 'p + q' are equal. It only follows that ~p + ~q together imply ~{p v q}, and ~p + {p v q} together imply 'q'.

3. Multiplication.

The significance of multiplication in practice, is to clarify the logically possible combinations of theses, into compounds or other composites, which are implied by various interplays of conjunction and disjunction. This is mixed-form logic. Any impossible combinations are put aside.

a. Conjunctive multiplication:

p + {q v r} = {p + q} v {p + r} = pq v pr
{p v q} + {r v s} = pr v ps v qr v qs

Proof is best sought by matrixual analysis. Thus, with three theses, we find that only three of the conjunctions in each matrix are allowed so far, and those three are the same on both sides. Each side excludes the five bracketed conjunctions. So the two statements are equivalent.

p + {q v r}	{p + q} v {p + r}
p + q + r p + q + notr p + notq + r	p + q + r p + q + notr p + notq + r
(p + notq + notr) (notp + q + r) (notp + q + notr) (notp + notq + r) (notp + notq + notr)	(p + notq + notr) (notp + q + r) (notp + q + notr) (notp + notq + r) (notp + notq + notr)

Equations involving more theses are similarly dealt with. Note well that if the result of such a multiplication contains an inconsistent clause, it is simply canceled out; for instance, if q = ~p, then the 'p + q + r' and 'p + q + notr' combinations are automatically eliminated, leaving only 'p + notq + r' as possible.

Also note that ‘p + {p v q}’ implies p (with the status of q left open), since p is already affirmed independently; this incidentally limits the ‘p v q’ clause to the two roots ‘p + q’ and ‘p + ~q’. As for ‘{p v q} + {p v r}’, it does not imply p, since notp may concur with q and r, without disobeying the premise.

b. Disjunctive multiplication:

p v {q + r} = {p v q} + {p v r} = p,q + p,r
{p + q} v {r + s} = p,r + p,s + q,r + q,s

Proof is again best sought by matrix logic. The two sides of the equation yield five identical allowances, and three identical exclusions (in brackets).

p v {q + r}	{p v q} + {p v r}
p + q + r	p + q + r
p + q + notr	p + q + notr
p + notq + r	p + notq + r
p + notq + notr	p + notq + notr
notp + q + r	notp + q + r
(notp + q + notr)	(notp + q + notr)
(notp + notq + r)	(notp + notq + r)
(notp + notq + notr)	(notp + notq + notr)

Note that the special composite ‘p v {p + q}’ implies p (with the status of q left open), since p v p = p. In contrast, the special composite ‘{p v q} + {p v r}’ does not imply p, since notp may concur with q and r,

More complex cases are proved similarly, by testing the various roots, by exposing implied possibilities of conjunctions between all the theses and antitheses, and seeing if they correspond on both sides of the equations. In practice, multiplication of more than two clauses is best dealt with by successive multiplication of pairs of clauses.

Observe, incidentally, that the matrix of ‘p v {q + r}’ includes the three roots of ‘p + {q v r}’, and an additional two alternatives.

c. With regard to division, the equations above are to be reread from right to left. The idea of division lies in our seeming to take the common factor p out of the brackets, as in mathematics.

4. Expansions.

a. The various equations developed thus far can be used to analyze more complex mixtures of conjunction and disjunction. Processes like the following may be called ‘expansions’:

p = p + {q v ~q} = {p.q} v {p.~q}

This equation teaches us that any proposition p may be logically composed, in the way of conjunctive multiplication, with any other meaningful proposition q and its negation ~q, since the proposition ‘q or notq’ is always true by the law of the excluded middle. We can repeat the process as often as we wish, as in:

p = p + {q v ~q} + {r v ~r} = {p.q.r} v {p.q.~r} v {p.~q.r} v {p.~q.~r}

b. The purpose of logical composition, is *to reduce any given formula to a disjunction of conjunctions*. It appears that our faculty of understanding requires such reduction, to fully grasp the significance of any complex formula. This means that the results we obtained earlier for multiplication are not final, because they do not satisfy the mind's requirement.

Expressions like 'pq v pr' or 'p,q + p,r' are not satisfactory, because they do not specify the truth or falsehood of every proposition involved. They must be expanded further, as follows:

(i) Conjunctive multiplication.

p + {q v r} = {p + q} v {p + r} = pq v pr

but, pq = {p.q.r} v {p.q.~r}
pr = {p.q.r} v {p.~q.r}

therefore, p + {q v r} = {p.q.r} v {p.q.~r} v {p.~q.r}

(ii) Disjunctive multiplication.

p v {q + r} = {p v q} + {p v r} = pp v pr v qp v qr = p v pq v pr v qr

but, p = {p.q.r} v {p.q.~r} v {p.~q.r} v {p.~q.~r}
pq = {p.q.r} v {p.q.~r}
pr = {p.q.r} v {p.~q.r}
qr = {p.q.r} v {~p.q.r}

therefore, p v {q + r} = {p.q.r} v {p.q.~r} v {p.~q.r} v {p.~q.~r} v {~p.q.r}

Notice the elimination of repetitive conjuncts or disjuncts, and the reordering of clauses, in accordance with the principles of addition earlier established. (That is, since xx = x and x,x = x and xy = yx and x,y = y,x.)

The above expansions are the ultimate solutions of the problems of multiplication: the most informative interpretations. The object of such process is to express the original formula in less ambiguous form. The preceding results do not clearly define the status of each of the propositions p, q, r. The components have to always be fully expanded to become comprehensible.

We see that the final equations for multiplications, are simply restatements of *the matrixes* of {p + [q v r]}, and {p v [q + r]}, respectively. They provide us with a set of roots, like 'p and q and r' or 'p and q and notr' or 'p and notq and r'.

The various conjunctions in disjunction represent all the possible outcomes of the original formula. They tell us the various ways it can be read, making a list of its alternative meanings. These are the eventual inferences which can be drawn from it. Any combination which is not mentioned in the conclusion, is not inferable from the premise.

Thus, just as ordinary disjunction is best understood with reference to a matrix, so in more complex situations we must reassemble the components of our proposition into more mentally accessible results. Two formulas with the same matrix, are logically equal.

c. These findings allow us to deal with still more intricate combinations of addition and multiplication. Consider, for instance, the puzzle: What does 'p and q or r' mean? Using the symbolic techniques introduced thus far, we can 'expand' that proposition as follows. q is in an ambiguous position, between an 'and' and an 'or', so:

p + q v r may mean p + {q v r}, or may mean {p + q} v r
that is, conjunctive or disjunctive multiplication.

so, {p + q v r} = {p + [q v r]} v {[p + q] v r}

but, {p + [q v r]} = {p.q.r} v {p.q.~r} v {p.~q.r}
and, {[p + q] v r} = {p.q.r} v {p.q.~r} v {p.~q.r} v {~p.q.r} v {~p.~q.r}

then, by addition of alternatives and elimination of repetitives, it follows that:

{p + q v r} = {p.q.r} v {p.q.~r} v {p.~q.r} v {~p.q.r} v {~p.~q.r}

This teaches us incidentally that, since the roots of {p + q v r} are all among the roots of {[p + q] v r}, and vice versa, these two composites are no more nor less informative than each other. That is, the following equation is valid:

p + q v r = {p + q} v r

On the other hand, the composite {p + [q v r]} is more specific and restrictive than either of the composites {p + q v r} or {[p + q] v r}, because it makes allowance for less possibilities. It writes off the alternative outcomes ‘notp and q and r’ and ‘notp and notq and r’, at the outset.

d. Still more complex puzzles can be resolved. These are interesting training exercises, like ladders. The easiest course is to apply already known and simpler processes, *successively*.

For instance, to expand the formula: {p v q} + {r v s}, let {p v q} = x, say. Then, by substitution and conjunctive multiplication, x + {r v s} = xr v xs. This means {[p v q] + r} v {[p v q] + s}. These clauses can now be expanded, and the resulting alternatives added together. Similarly, we can clarify the formula: {p + q} v {r + s} in stages. Try doing it.

5. Utility.

In conclusion, we see that symbolic logic can be a valuable tool for untangling perplexing statements. Modern logicians have also developed similar techniques for compositions involving hypothetical relations, as already mentioned.

However, it should also be apparent that the more intricate the formula, the less likely are we to come across it in practice. This is why modern, symbolic logic tends to degenerate into irrelevancy, and give logic as a whole a bad name.

The value of the main equations, is to show us that our sentences should be clearly formulated, so that the phrases we intend as our clauses are apparent to all. Otherwise, we might be misunderstood. This is especially important when drawing up legal documents, or making scientific statements. Perhaps the best practical applications are in computer and robot programming.

However, beyond a certain point, there is no utility in studying complex formulas, because they are sure to be misinterpreted by the uninitiated, anyway. Likewise, when interpreting texts written by other people, we cannot always be sure that *they* formulated them with expert knowledge and total awareness of their logical significance.[3]

Even if overly intricate logic is of limited practical utility, it is an important enough doctrine. It is a part of the grand enterprise of pursuit of consistency in Knowledge as a whole. It describes for us, how to make peace within or between large bodies of information.

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3 Perhaps we may use such logic to understand religious precepts, their extents and limits, since what is of Divine origin is theoretically bound to be consistent. Though our motive may be purely to implement these precepts, there is always a danger here of not-knowing all the rules of exegesis G-d intended.

29. HYPOTHETICAL SYLLOGISM AND PRODUCTION.

There are several kinds of deductive argument involving hypothetical propositions or their derivatives. They are distinguished according to whether they involve only hypotheticals, or hypotheticals mixed with categorical forms. The main kinds are syllogism, production, apodosis and dilemma. Note that the valid moods are not here listed in symbolic terms, as we did with categoricals, to avoid obscuring their impact.

1. Syllogism.

Hypothetical syllogism is argument whose premises and conclusion are all hypotheticals. It is mediate inference, with minor (symbol P), middle (M), and major (Q) theses, deployed in figures, as was the case in categorical syllogism.

Its *most primary valid mood*, from which *all* others may be derived by direct or indirect reduction, is as follows. It tells us, as for the analogue in categorical syllogism, that, as H.W.B. Joseph would say, 'whatever falls under the condition of a rule, follows the rule'.

This primary mood is valid irrespective of whether the hypotheticals involved are of unspecified base, normal (contingency-based), or abnormal. That is generally true for its primary derivatives, too; but subaltern derivatives are only applicable in cases where both theses are known to be logically contingent (and not just problematic), because the subalterns require eductive processes which depend on this condition for their validity.

If M, then Q
if P, then M
so if P, then Q

This is a first figure syllogism. Its validity obviously follows from the meaning of the operator 'if-then' involved. Although the connection in hypotheticality is expressed by modal conjunctive statements, 'if-then' underscores an additional, not-tautologous, sense, occurring on a finer level. This teaches us a purely conjunctive argument, from which many laws for the logic of conjunction may be inferred, that:

The premises: {M and nonQ} is impossible,
and {P and nonM} is impossible, together
yield the conclusion: {P and nonQ} is impossible.

This could be written symbolically as **1/H2nH2nH2n**, note.

a. Figure One.

(i) From the primary valid mood, we can draw up the following full list of valid, **uppercase, perfect** moods, in first figure, by substituting antitheses for theses in every possible combination.

If M, then Q if P, then M so, if P, then Q	If nonM, then Q if P, then nonM so, if P, then Q
If M, then nonQ if P, then M so, if P, then nonQ	If nonM, then nonQ if P, then nonM so, if P, then nonQ
If M, then Q if nonP, then M so, if nonP, then Q	If nonM, then Q if nonP, then nonM so, if nonP, then Q
If M, then nonQ if nonP, then M so, if nonP, then nonQ	If nonM, then nonQ if nonP, then nonM so, if nonP, then nonQ

(ii) Next, from one of the valid, uppercase, perfect moods, we derive the primary, valid, **lowercase, perfect** mood, by reductio ad absurdum, as follows. Note that the major premise is uppercase, and the minor premise and conclusion are lowercase.

If M, then Q	contrapose major:	If nonQ, then nonM
if P, not-then nonM	deny conclusion:	if P, then nonQ
so, if P, not-then nonQ	get anti-minor	if P, then nonM

From this primary mood, we can draw up the following full list of valid, lowercase, perfect moods, in the first figure, by substituting antitheses for theses in every possible combination.

If M, then Q if P, not-then nonM so, if P, not-then nonQ	If nonM, then Q if P, not-then M so, if P, not-then nonQ
If M, then nonQ if P, not-then nonM so, if P, not-then Q	If nonM, then nonQ if P, not-then M so, if P, not-then Q
If M, then Q if nonP, not-then nonM so, if nonP, not-then nonQ	If nonM, then Q if nonP, not-then M so, if nonP, not-then nonQ
If M, then nonQ if nonP, not-then nonM so, if nonP, not-then Q	If nonM, then nonQ if nonP, not-then M so, if nonP, not-then Q

(iii) Next, from one of the valid, uppercase, perfect moods, we derive the primary, valid, **imperfect** mood, by reductio ad absurdum, as follows. Note the change in polarity of the minor thesis in the conclusion, which defines the moods as imperfect, and the distinct mixed polarity of the middle thesis in the two premises. Note also that the minor premise is uppercase, and the major premise and conclusion are lowercase.

If M, not-then Q	deny conclusion:	If nonP, then Q
if P, then nonM	contrapose minor:	if M, then nonP
so, if nonP, not-then Q	get anti-major:	if M, then Q

From this primary mood, we can draw up the following full list of valid, imperfect moods, in the first figure, by substituting antitheses for theses in every possible combination.

If M, not-then Q if P, then nonM so, if nonP, not-then Q	If nonM, not-then Q if P, then M so, if nonP, not-then Q
If M, not-then nonQ if P, then nonM so, if nonP, not-then nonQ	If nonM, not-then nonQ if P, then M so, if nonP, not-then nonQ
If M, not-then Q if nonP, then nonM so, if P, not-then Q	If nonM, not-then Q if nonP, then M so, if P, not-then Q
If M, not-then nonQ if nonP, then nonM so, if P, not-then nonQ	If nonM, not-then nonQ if nonP, then M so, if P, not-then nonQ

(iv) ***Subaltern moods***. These are valid only with normal hypotheticals, unlike the preceding, because they are derived from the latter by subalternating a lowercase premise or being subalternated by an uppercase conclusion. Their premises are always both uppercase, and their conclusion lowercase.

The following sample can be derived from moods of type (i) by obverting the conclusion, or equally well from moods of type (ii) by replacing the minor premise with its obvertend. On this basis, 8 subaltern moods can be derived in the usual manner. These are perfect in nature.

If M, then Q
if P, then M
so, if P, not-then nonQ.

The following sample can be derived from moods of type (i) by obvert-inverting the conclusion, or equally well from moods of type (iii) by replacing the major premise with its obvertend. On this basis, 8 subaltern moods can be derived in the usual manner. These are imperfect, since the minor thesis changes polarity in the conclusion.

If M, then Q
if P, then M
so, if nonP, not-then Q.

In summary, we thus have a total of 3X8 = 24 primary valid moods in the first figure, plus 2X8 = 16 subaltern valid moods. Or a total of 40 valid moods, out of 8X8X8 = 512 possibilities.

b. Figure Two.

(i) From one of the valid, lowercase, perfect moods, of the first figure, we derive the primary, valid, **uppercase, perfect** mood, of the second figure, by reductio ad absurdum, as follows. Alternatively, we could have used direct reduction, by contraposing the major premise, through a valid, uppercase, perfect mood, of the first figure.

If Q, then M	with same major:	If Q, then M
if P, then nonM	deny conclusion:	if P, not-then nonQ
so, if P, then nonQ	get anti-minor:	so, if P, not-then nonM

From this primary, valid mood, we can draw up the following full list of valid, uppercase, perfect moods, in the second figure, by substituting antitheses for theses in every possible combination.

If Q, then M
if P, then nonM
so, if P, then nonQ

If Q, then nonM
if P, then M
so, if P, then nonQ

If nonQ, then M
if P, then nonM
so, if P, then Q

If nonQ, then nonM
if P, then M
so, if P, then Q

If Q, then M
if nonP, then nonM
so, if nonP, then nonQ

If Q, then nonM
if nonP, then M
so, if nonP, then nonQ

If nonQ, then M
if nonP, then nonM
so, if nonP, then Q

If nonQ, then nonM
if nonP, then M
so, if nonP, then Q

(ii) Next, from one of the valid, uppercase, perfect moods, of the first figure, we derive the primary, valid, **lowercase, perfect** mood, of the second figure, by reductio ad absurdum, as follows. Alternatively, we could have used direct reduction, by contraposing the major premise, through a valid, lowercase, perfect mood, of the first figure. Note that the major premise is uppercase, and the minor premise and conclusion are lowercase.

If Q, then M	with same major:	If Q, then M
if P, not-then M	deny conclusion:	if P, then Q
so, if P, not-then Q	get anti-minor:	if P, then M

From this primary mood, we can draw up the following full list of valid, lowercase, perfect moods, in the second figure, by substituting antitheses for theses in every possible combination.

If Q, then M
if P, not-then M
so, if P, not-then Q

If Q, then nonM
if P, not-then nonM
so, if P, not-then Q

If nonQ, then M
if P, not-then M
so, if P, not-then nonQ

If nonQ, then nonM
if P, not-then nonM
so, if P, not-then nonQ

If Q, then M
if nonP, not-then M
so, if nonP, not-then Q

If Q, then nonM
if nonP, not-then nonM
so, if nonP, not-then Q

If nonQ, then M
if nonP, not-then M
so, if nonP, not-then nonQ

If nonQ, then nonM
if nonP, not-then nonM
so, if nonP, not-then nonQ

(iii) ***Subaltern moods***. These are valid only with normal hypotheticals, unlike the preceding, because they are derived from the latter by subalternating a lowercase premise or being subalternated by an uppercase conclusion. Their premises are always both uppercase, and their conclusion lowercase.

The following sample can be derived from moods of type (i) by obverting the conclusion, or equally well from moods of type (ii) by replacing the minor premise with its obvertend. On this basis, 8 subaltern moods can be derived in the usual manner. These are perfect in nature.

If Q, then M
if P, then nonM
so, if P, not-then Q.

The following sample can be derived from moods of type (i) by obvert-inverting the conclusion. On this basis, 8 subaltern moods can be derived in the usual manner. These are imperfect, since the minor thesis changes polarity in the conclusion.

If Q, then M
if P, then nonM
so, if nonP, not-then nonQ.

The following sample can be derived from moods of type (ii) by replacing the minor premise with its obvert-invertend. On this basis, 8 subaltern moods can be derived in the usual manner. These are imperfect, since the minor thesis changes polarity in the conclusion. Note the distinct uniform polarity of the middle thesis in the two premises.

If Q, then M
if P, then M
so, if nonP, not-then Q.

In summary, we thus have a total of 2X8 = 16 primary valid moods in the second figure, plus 3X8 = 24 subaltern valid moods. Or a total of 40 valid moods, out of 8X8X8 = 512 possibilities.

c. Figure Three.

(i) From one of the valid, uppercase, perfect moods, of the first figure, we derive the primary, valid, **perfect** mood, with **lowercase major** premise, of the third figure, by reductio ad absurdum, as follows. Alternatively, we could have used direct reduction, by contraposing the major premise, and transposing, through a valid, lowercase, perfect mood, of the first figure. The conclusion is of course lowercase.

If M, not-then nonQ	deny conclusion:	If P, then nonQ
if M, then P	with same minor:	if M, then P
so, if P, not-then nonQ	get anti-major:	if M, then nonQ

From this primary, valid mood, we can draw up the following full list of valid, perfect moods, with lowercase major premise, in the third figure, by substituting antitheses for theses in every possible combination.

If M, not-then nonQ if M, then P so, if P, not-then nonQ	If nonM, not-then nonQ if nonM, then P so, if P, not-then nonQ
If M, not-then Q if M, then P so, if P, not-then Q	If nonM, not-then Q if nonM, then P so, if P, not-then Q

If M, not-then nonQ	If nonM, not-then nonQ
if M, then nonP	if nonM, then nonP
so, if nonP, not-then nonQ	so, if nonP, not-then nonQ
If M, not-then Q	If nonM, not-then Q
if M, then nonP	if nonM, then nonP
so, if nonP, not-then Q	so, if nonP, not-then Q

(ii) Next, from one of the valid, lowercase, perfect moods, of the first figure, we derive the primary, valid, **perfect** mood, with **lowercase minor** premise, of the third figure, by reductio ad absurdum, as follows. Alternatively, we could have used direct reduction, by contraposing the minor premise, through a valid, lowercase, perfect mood, of the first figure. The conclusion is of course lowercase.

If M, then Q	deny conclusion:	If P, then nonQ
if M, not-then nonP	with same minor:	if M, not-then nonP
so, if P, not-then nonQ	get anti-major:	if M, not-then Q

From this primary, valid mood, we can draw up the following full list of valid, perfect moods, with lowercase minor premise, in the third figure, by substituting antitheses for theses in every possible combination.

If M, then Q	If nonM, then Q
if M, not-then nonP	if nonM, not-then nonP
so, if P, not-then nonQ	so, if P, not-then nonQ
If M, then nonQ	If nonM, then nonQ
if M, not-then nonP	if nonM, not-then nonP
so, if P, not-then Q	so, if P, not-then Q
If M, then Q	If nonM, then Q
if M, not-then P	if nonM, not-then P
so, if nonP, not-then nonQ	so, if nonP, not-then nonQ
If M, then nonQ	If nonM, then nonQ
if M, not-then P	if nonM, not-then P
so, if nonP, not-then Q	so, if nonP, not-then Q

(iii) Next, from one of the valid, lowercase, perfect moods, of the first figure, we derive the primary, valid, **imperfect** mood, of the third figure, by direct reduction, as follows. Note the change in polarity of the minor thesis in the conclusion, which defines the mood as imperfect, and the distinct mixed polarity of the middle thesis in the two premises. Note also that both premises and the conclusion are uppercase.

If M, then Q	with same major:	If M, then Q
if nonM, then P	contrapose minor:	if nonP, then M
so, if nonP, then Q	get conclusion:	so, if nonP, then Q

From this primary mood, we can draw up the following full list of valid, imperfect moods, in the third figure, by substituting antitheses for theses in every possible combination.

If M, then Q	If nonM, then Q
if nonM, then P	if M, then P
so, if nonP, then Q	so, if nonP, then Q

If M, then nonQ
if nonM, then P
so, if nonP, then nonQ

If nonM, then nonQ
if M, then P
so, if nonP, then nonQ

If M, then Q
if nonM, then nonP
so, if P, then Q

If nonM, then Q
if M, then nonP
so, if P, then Q

If M, then nonQ
if nonM, then nonP
so, if P, then nonQ

If nonM, then nonQ
if M, then nonP
so, if P, then nonQ

(iv) ***Subaltern moods***. These are valid only with normal hypotheticals, unlike the preceding, because they are derived from the latter by subalternating a lowercase premise or being subalternated by an uppercase conclusion. Their premises are always both uppercase, and their conclusion lowercase.

The following sample can be derived from moods of type (i) by replacing the major premise with its obvertend, or equally well from moods of type (ii) by replacing the minor premise with its obvertend. On this basis, 8 subaltern moods can be derived in the usual manner. These are perfect in nature.

If M, then Q
if M, then P
so, if P, not-then nonQ.

The following sample can be derived from moods of type (i) by replacing the major premise with its obvert-invertend, or equally well from moods of type (iii) by obvert-inverting the conclusion. On this basis, 8 subaltern moods can be derived in the usual manner. These are perfect in nature, but note the distinct mixed polarity of the middle thesis in the two premises.

If M, then Q
if nonM, then P
so, if P, not-then Q.

The following sample can be derived from moods of type (ii) by replacing the minor premise with its obvert-invertend, or equally well from moods of type (iii) by obverting the conclusion. On this basis, 8 subaltern moods can be derived in the usual manner. These are imperfect, since the minor thesis changes polarity in the conclusion. Note the distinct mixed polarity of the middle thesis in the two premises.

If M, then Q
if nonM, then P
so, if nonP, not-then nonQ.

In summary, we thus have a total of 3X8 = 24 primary valid moods in the third figure, plus 3X8 = 24 subaltern valid moods. Or a total of 48 valid moods, out of 8X8X8 = 512 possibilities.

d. With regard to the *fourth figure*, it can be ignored in hypothetical syllogism. Since the first figure here (unlike with categorical syllogism) includes imperfect moods, the fourth figure here would introduce no new valid moods for us. Its valid moods can of course all be reduced directly to the first figure, by transposing or contraposing the premises, but they do not represent a movement of thought of practical value.

We therefore have, in the three significant figures taken together, a total of 24+16+24 = 64 primary valid moods, plus 16+24+24 = 64 subaltern valid moods. Or a total of 128 valid moods, out of 3X512 = 1536 possibilities; meaning a validity rate of 8.33%.

2. Other Derivatives.

The chaining of syllogisms into a series forming a *sorites* is possible with hypothetical syllogism, similarly to categorical syllogism. This is used in practice, of course, and applies irrespective of basis. The typical sorites looks as follows:

If A, then B
if B, then C
…
if G, then H
therefore, if A, then H.

Note that we are in the figure one, and we state the most minor premise first, and successively work up to the most major premise, and lastly the conclusion. A sorites should be reducible to valid syllogisms to be valid.

Of course, sorites is only the most regular form of continuous argument, the easiest to think without aid of paper and pencil. More broadly, any succession of premises, in any combination of figures, yielding a valid final conclusion, may be viewed as continuous, even though we have to think out the intermediate conclusions, zigzagging from figure to figure, to reach the result.

We can readily reformulate all the above syllogisms using derivative forms, such as simple disjunctions. For examples, the following arguments, taken at random, are easily validated by transforming the disjunctives into standard hypotheticals:

M and/or Q	Q or else M
P or else M	P not and/or nonM
P or else nonQ	P not and/or Q.

Here again, I would not regard these as distinct valid moods. Even if they are used in practice, we are mentally required to restate them in 'If/then' form to understand them. It will however be seen, in the context of dilemma, that there are certain arguments, which mix 'If/then' forms with disjunctives, which are comprehensible on their own merit, and used in everyday discourse.

Such arguments may also be regarded as 'logical compositions'. With multiple alternatives, the possible number of arguments increases and so does the mental confusion. When translating the given disjunctions into 'If-then' statements causes us as much confusion, the best course is to express each proposition in terms of the conjunctions is allows and forbids; then we can best see what conclusion, if any, may be drawn.

We can also, it is noted, appeal to the above valid moods of the syllogism to clarify reasoning involving compound forms. That is, when one or both premises signifies implicance or subalternation or contradiction or contrariety or subcontrariety, we may be able to fuse the results of two or more simple syllogisms, and get a compound conclusion.

Lastly, arguments may be fashioned in conditional frameworks, so that we have nested hypotheticals for premise(s) and conclusion. This may be viewed as a wider logic, concerning composite antecedents or consequents, conjunctive or even disjunctive ones. Researching the mechanics of partial or alternative theses is an area that deserves eventual attention, but presumably it can be reduced to the findings of unconditional logic.

Subaltern moods are implicitly conditional; they have as hidden premises, the categorical propositions that the theses are logically contingent, rather than merely problematic or partly or wholly incontingent. The tacitly understood premises are: ‘P (and nonP) is contingent, and Q (and nonQ) is contingent’. I have made no effort to develop subaltern moods with abnormal bases, because once a thesis is known to be incontingent it is rarely thereafter used in hypothetical propositions.

3. Production.

How are hypothetical propositions produced? By their very nature they do not presuppose the reality of their theses, so how do we know that the antecedent does (or does not) engage the consequence? This question will be answered in this section.

Hypothetical propositions signify a logical connection between the theses, so that any argument which is logically valid may be recast in hypothetical form.

The theses involved may of course have any form, including themselves hypothetical. The term ‘connection’ here is to be understood in its widest sense, including any logical relationship, positive or negative, normal or abnormal. Thus, all oppositions, eductions, deductions, are included here; overall, a valid inference of any kind produces a positive hypothetical, an invalid inference produces a negative hypothetical.

Also, the expression ‘logically valid’ should be taken as comprehensive of the known and the unknown; there is no presumption here that the science of logic as we know it to date is complete. It is important to stress this; while all established logical truths are capable of producing hypotheticals, it does not follow that hypotheticals cannot be produced by means not yet clarified by this science. No claim to omniscience is required.

An example of production would be recasting a categorical syllogism in hypothetical form: e.g. ‘If all S are M and all M are P, then all S are P’. This is a conclusion, whose premises are the process of validation of that mood of the syllogism via the laws of logic.

If we instead produced the briefer conclusion ‘If all S are M, all S are P’, the process to be valid must have included, after the above, a nesting (to ‘If all M are P, then if all S are M, all S are P’) and an apodosis (with minor premise ‘All M are P’). Thus enthymeme need not be viewed as merely syllogism with a suppressed (tacit) premise, but as the end product of a series of definite arguments.

However, production is not limited to relationships in terms of variables, but is especially useful for application to specific values. Using a formal relationship as major premise, we may, through the act of substitution as minor premise, produce a hypothetical with particular contents as conclusion. Continuing the above example, we might for instance produce, ‘If all men were wise, they would not make war’.

In short, any logical series which is incomplete, may be made to at least yield a hypothetical conclusion, and thus constitute a productive process.

The missing information may simply be the exact quantity involved. Thus, if in the above example we do not know whether all or only some S are M, we can still conclude from ‘All M are P’ that ‘If any S is M, it is P’. This produces a hypothetical proposition which seems general, but in fact only suggests that some S may be M. Incidentally, the expression ‘whether’ may itself be viewed as a derivative form of hypothetical, concealing a dilemma.

Similarly, a negative hypothetical would express a nonsequitur. For example, ‘If no S are M and all M are P, it does not follow that no S are P’. Likewise, with particular contents or indefinite quantities, as above.

Clearly, the possibilities are virtually infinite. Any formal or informal sequence permitted or forbidden by the laws of logic constitutes a productive process. Ordinarily, a hypothetical would not be formed, unless information was missing or already known wrong, and only problematic elements would be included in it as theses; but there is nothing illicit in forming one even with definite theses of known truth.

30. LOGICAL APODOSIS AND DILEMMA.

1. Apodosis.

Apodosis is argument involving a hypothetical proposition as major premise, and the affirmation or denial, of one thesis as minor premise, and of the other as conclusion. Needless to say, the two premises must be true, for the conclusion to follow, as in all argument. There are essentially two valid moods, as follows:

If P, then Q and P hence, Q	If P, then Q but not Q hence, not P

We see that the major premise has to be a positive hypothetical, the minor premise must either affirm the antecedent or deny the consequent, and the conclusion can only, accordingly, affirm the consequent or deny the antecedent. Such argument merely activates, as it were, the dormant power of the hypothetical, when the minor premise is independently found true.

In the first case, the conclusion follows directly; the second case could be reduced to the first, by contraposition of the major premise. The validity of these moods can be demonstrated by reference to the definition of the major premise as 'the conjunction of P and nonQ is impossible'; it follows that, if P was true without Q being so, or Q was false without P being so, this impossibility fail to be upheld.

We can also present as valid moods, and in like manner validate, moods involving the remaining positive hypothetical forms. These are to some extent interesting in themselves, showing us still different ways otherwise inaccessible information might be indirectly arrived at, or that assumptions might be eliminated. But they are also valuable for the validation of certain arguments involving disjunctive or compound, derivative forms.

If P, then not Q and P hence, not Q	If P, then not Q but Q hence, not P
If not P, then Q and not P hence, Q	If not P, then Q but not Q hence, P
If not P, then not Q and not P hence, not Q	If not P, then not Q but Q hence, P

Other moods of apodosis, including those with a lowercase major premise, such as 'If P, not-then Q', have no demonstrable basis or lead to inconsistency, and so are invalid. The following two are specially noteworthy, because they represent oft-made errors of judgement.

If P, then Q and Q hence, P.	If P, then Q but not P hence, not Q.

These are, to repeat, formally invalid. However, note that they are used for purposes of 'adduction', the provision of evidence or counterevidence to support or discredit theories. The positive mood tells us that the more a theory P makes successful predictions, such as Q, the more credible it becomes (though it is not proved); the negative mood tells us that when the presuppositions, such as P, of a theory Q collapse, it is undermined (though it is not disproved). This topic is dealt with in detail in a later chapter.

We can interpret the following expectative propositions as abridged, 'failed' apodoses of this kind. 'Even if P were true, Q would be true' implies 'if P, then Q' and 'P is false, and Q is true'. 'Though P is true, Q is true' implies 'if not P, then Q' and 'P is true and Q is true'.

The following invalid moods also typify fallacious attempts at apodosis; note the negative hypothetical major premises, lowercase forms implying a link yet too weak to yield any definite conclusion. People often fail to first establish a bond between the theses, because they hurriedly assume that no contrary hypotheticals can be put forward.

If P, not-then Q	If P, not-then Q
and P	but not Q
hence, not Q.	hence, P.

In summary, there are 8 valid moods of apodosis. That being out of 8X4X4 = 128 possible moods, the validity rate is 6.25%. Compounds of these premises would yield compound conclusions.

The above described valid moods of logical apodosis involve a factual minor premise: P or Q is true or false. The concept can be broadened to involve minor premises which are modal (this refers of course to logical modality). The following four valid moods are derivable from the basic two by exposition, as usual:

If P, then Q	If P, then Q
and necessarily P	but impossibly Q
so, necessarily Q	so, impossibly P
If P, then Q	If P, then Q
and possibly P	but unnecessarily Q
so, possibly Q	so, unnecessarily P

Think of the major premises as modal conjunctions. For example, in the first of these moods, '{P and nonQ} is impossible' plus 'P is necessary' imply that 'Q is necessary', for if nonQ ever occurred in any context, we would be faced with a contradiction.

The remaining six factual moods can likewise be used to construct another twelve valid modal moods. The significance of these arguments is of course the transmissibility of logical modality across the hypothetical relation. Hypotheticals per se have problematic theses; under the appropriate conditions, this merely prima facie thinkability is up or down graded to a more definite logical status.

Modal apodosis suggests that 'if P, then Q' implies 'if P**n**, then Q**n**' and 'if P**p**, then Q**p**', where the suffixes **n** and **p** refer to logical necessity and possibility respectively. However, note well that the reverse does not follow; the version with nonmodal theses cannot be inferred from either of the modal-theses versions, since the relation might conceivably only apply in the collective case or in the indefinite case without implying a singular and specific equivalent.

Moreover, I see no point in treating hypotheticals with modal theses independently, and I do not think that we ever do so in practice. Since an ordinary hypothetical, with nonmodal theses, contains within it all the requisite information for the solution of problems of a modal nature, we have no need of these implicit forms, and introducing them would be very artificial.

As already mentioned, the valid moods of apodosis can be reformulated to describe arguments involving derivative forms. In particular, note the following examples:

P and/or Q but not P hence, Q	P and/or Q but not Q hence, P
P or else Q and P hence, not Q	P or else Q and Q hence, not P

I would not count the rewriting of valid moods in derivative forms as yielding additional valid moods. But they become more significant with multiple disjunctions, which yield more complex conclusions, so long as more than one alternative have not been eliminated.

P and/or Q and/or R and/or S but not P so, Q and/or R and/or S.	P or else Q or else R or else S and P so, not Q and not R and not S.

In the mood with inclusive disjunction (left), we are given that at least one of the theses listed must be true (i.e. they cannot all be false); so if one is found false, we can conclude that at least one of the remaining ones must be true. In the mood with exclusive disjunction (right), we are given that all but one of the theses listed must be false (i.e. only one can be true); so if one is found true, we can conclude that all the remaining ones must be false. Note that if both major premises are true, i.e. if the theses are both 'exhaustive' and 'mutually exclusive', then a conclusion is possible from the truth or falsehood of any of the theses, as shown in these two moods.

Most of what we have said about apodosis concerns all hypotheticals, whether of unknown logical basis, normal or abnormal. However, apodosis with a necessary or impossible minor premise and conclusion (as shown earlier) obviously concerns abnormal hypotheticals in particular, because the basis is implied to be not contingent at all. In contrast, apodosis with a possibility or unnecessity as its minor premise, teaches us the logic specific to normal hypotheticals, which are contingency-based.

Thus, we have here a foundation for the specialized study of normal or abnormal hypotheticals, an entry point into the topic; I will not however here pursue the matter further. The same can be said for disjunctives.

2. Dilemma.

Colloquially, we call a 'dilemma', any impossible choice. 'If I do this, I've had it; if I do that, I've had it — so I've had it anyway (and it is no use my doing this or that)'. This is indeed a case of dilemma, but in logic the expression is understood more broadly, to cover more positive situations. Thus, often, in action contexts, when we are faced with a choice of means to get to a goal, we might resolve the dilemma by using all available means, even at the cost of redundancies, so as to ensure that the goal is attained one way or the other.

Although dilemmatic argument may be derived from apodosis and syllogism, it has a certain autonomy of cogency and is commonly used in practice, so it deserves some analysis. Note well first that the disjunction used in dilemma is the 'and/or' type (not the 'or else' type), even if in practice this is not always made clear.

The hypotheticals which constitute the major premise of a dilemma are called its 'horns'; they give an impression of presenting us with a predicament. The minor premise is a disjunction; it is said to 'take the dilemma by its horns'. The conclusion is said to 'resolve' the dilemma.

a. *Simple* dilemma consists of a conjunction of subjunctives as major premise, a disjunctive as minor premise, and a (relative) categorical as conclusion. It normally involves three theses. Tradition has identified two valid moods.

(i) The simple *constructive* dilemma.

If M, then P — and — if N, then P
but M and/or N
hence, P

This is proved by reduction ad absurdum through two negative apodoses, as follows:

If M, then P — and — if N, then P (original major premise)
and not P (denial of conclusion)
so, not M and not N (contrary of minor).

Alternatively, we could regard the simple constructive dilemma as summarizing a number of positive apodoses, with reference to the matrix of alternative conjunctions underlying the minor premise:

If M, then P — and — if N, then P			(common major)
but 'M (and not N)'	or 'N (and not M)'	or 'M and N'	(alternative minors)
whence, P	whence, P	whence, P and P	(common conclusion).

This shows the essential continuity between the concepts of apodosis and dilemma, note.

(ii) The simple *destructive* dilemma.

If P, then M — and — if P, then N
but not M and/or not N
hence, not P

This is proved by reduction ad absurdum through two apodoses, as follows:

If P, then M — and — if P, then N (original major premise)
and P (denial of conclusion)
so, M and N (contrary of minor).

In contrast, the following two arguments would be fallacious:

If M, then P — and — if N, then P	If P, then M — and — if P, then N
but not M and/or not N	but M and/or N
hence, not P	hence, P

b. *Complex* dilemma consists of a conjunction of subjunctives as major premise, and disjunctives as minor premise and conclusion. Tradition has identified two valid moods. It normally involves four theses, though two are occasionally merely mutual antitheses.

(i) The complex *constructive* dilemma.

If M, then P — and — if N, then Q
but M and/or N
hence, P and/or Q

This can be proved by reductio ad absurdum, as in simple dilemma. Alternatively, we may analyze it through a sorites, as follows:

If not P, then not M (contrapose left horn)
if not M, then N (from minor)
if N, then Q (right horn)
therefore, if not P, then Q (transform to conclusion).
(ii) The complex *destructive* dilemma.

If P, then M — and — if Q, then N
but not M and/or not N
hence, not P and/or not Q

This can be proved by reductio ad absurdum, as in simple dilemma. Alternatively, we may analyze it through a sorites, as follows:

If not not P, then P (axiomatic)
if P, then M (left horn)
if M, then not not M (axiomatic)
if not not M, then not N, (from minor)
if not N, then not Q (contrapose right horn)
therefore, if not not P, then not Q (transform to conclusion).

In contrast, the following two arguments would be fallacious:

If M, then P — and — if N, then Q
but not M and/or not N
hence, not P and/or not Q

If P, then M — and — if Q, then N
but M and/or N
hence, P and/or Q

c. Concerning both the simple and complex valid moods, note that, formally speaking, we could use as minor premises the equivalent forms ‘not M or else not N’ and ‘M or else N’, respectively, in the valid constructive and destructive moods. But this would not reflect the true format of dilemma. The goal here is only to describe actual thought processes, not to accumulate useless formulas. However, in view of the similarity in appearance between these valid substitutes, and the minor premises of the invalid moods, it is well to be aware of the possibility of confusion.

A *special case* of complex constructive dilemma is worthy of note, because people sometimes argue in that way. Its form is:

If M, then {P and nonQ} — and — if N, then {nonP and Q}
but M and/or N
hence, either P or Q.

We may understand this argument as follows: contrapose the left horn to ‘if not-{P and nonQ}, then nonM’; the minor premise means ‘if nonM, then N’; these propositions, together with the right horn, form a sorites whose conclusion is ‘if not-{P and nonQ}, then {nonP and Q}’. But we know on formal grounds, for any two propositions, that ‘if {P and nonQ}, then not-{nonP and Q}’. Therefore, ‘either {P and nonQ} or {nonP and Q}’ is true, which can in turn be rephrased as ‘either P or Q’.

Thus, what this argument achieves is the elimination of the remaining two formal alternatives, {P and Q} and {nonP and nonQ}; the combinations {P and nonQ} and {nonP and Q} become not merely incompatible, but also exhaustive. There is no destructive version of this argument, because its result would only be ‘if {P and nonQ}, then not-{nonP and Q}’, which is formally given anyway.

There is also no equivalent argument in simple dilemma. But note that if we substitute nonM for N in the one above, we obtain something akin to it: if M, then {P and nonQ}, and if nonM, then {nonP and Q}; but either M or nonM; hence, either P or Q. This is not really simple dilemma because the antecedents are not identical; but there is a resemblance, in that only three theses are involved. Also, the minor premise here is redundant, since formally true, so the conclusion may be viewed as an eduction from the compound major premise.

Also note, simple and complex dilemmas may consist of more than two horns. The following are examples of *multi-horned* simple dilemma:

Constructive:
If B and/or C and/or D… is/are true, then A is true
but B and/or C and/or D…etc. is/are true
therefore A is true.

Destructive:
If A is true, then B and C and D …etc. are true
but B and/or C and/or D…etc. is/are false
therefore A is false'.

Similarly with other sorts of arrays. This shows that we can view the horns of dilemmas as forming a single hypothetical proposition whose antecedent and/or consequent is/are conjunctive or disjunctive. It follows that simple and complex dilemma should not be viewed as essentially distinct forms of argument; rather, simple dilemma is a limiting case of complex dilemma, the process involved being essentially one of purging our knowledge of extraneous alternatives.

The commonly employed form *'Whether P or Q, R'* is normally understood as an abridged simple constructive dilemma, meaning 'If P, then R, and if Q, then R, but P and/or Q, hence R anyway'. However, we should be careful with it, because in some cases we intend it to dissociate R from P and Q, meaning 'If P not-then R, and if Q not-then R, but R'. Note well the difference. In the former case, the independence is an outcome of multiple dependence; in the latter case, the independence signals lack of connection.

Dilemma, especially its ultimate, simple version, is a very significant form of reasoning, in that *it is capable of yielding factual results from purely problematic theses* (implicit in hypotheticals or disjunctives). Like the philosopher's stone of the alchemist, it turns lead into gold. Without this device, knowledge would ever be conjectural, a mass of logically related but unresolvable problems.

Note however that the conclusion of a simple dilemma is still, logically, only factual in status. A thesis only acquires the status of logical necessity or impossibility, when it is implied or denied by all eventualities; this means, in dilemma, when the exhaustiveness of the alternatives in the premises is itself logically incontingent (rather than a function of the present context of knowledge). The significance of this will become more transparent as we proceed further, and deal with paradoxical logic.

3. Rebuttal.

The so-called '*equally cogent* rebuttal' is a special case of dilemma, worthy of analysis in this context. It happens in debate that a seemingly cogent dilemma may be rebutted by a seemingly equally cogent dilemma.

a. With regard to complex dilemma, though the arguments are indeed equally cogent, the impression of 'rebuttal' is illusory, due to a misconception of the opposition between the conclusions.

If M, then P — and — if N, then Q

but M and/or N
hence, P and/or Q

If P, then M — and — if Q, then N
but not M and/or not N
hence, not P and/or not Q

Clearly, the major premises are compatible; taken together, they signify two reciprocal subjunctions. The minor premises are also compatible, since they mean, respectively, 'if nonM, then N' and 'if not nonM, then nonN (i.e. if M, then nonN)'; taken together, they signify contradictive disjunction between M and N.

Likewise, for the conclusions: they are not inconsistent with each other, but taken together mean that P and Q are contradictory. So in fact the two dilemma do not exclude each other, it is formally quite possible for them to be both true. If indeed all the propositions involved are true, they merely together constitute a compound dilemma which is quite valid.

We seem to be faced with equally cogent arguments yielding conflicting conclusions, but this is an erroneous impression, because in fact the conclusions are consistent. They may seem to conflict, because they refer to contradictory theses, P and Q, nonP and nonQ; but the disjunctive way in which these theses are connected, makes the conclusions complementary, rather than inconsistent.

Restating the entire arguments in standard hypothetical syllogisms can be helpful. The conclusions should be viewed as 'If nonP, then Q' and 'If P, then nonQ', respectively, to avoid confusion. If the result persists in seeming unintelligible, the wording may be misleading or there may be a factually erroneous premise.

The frustration underlying such arguments, why they are experienced as somehow in conflict — is due to the fact that each party assumed the contradictory of the other's assumption to be tacitly included in his or her own premises. Thus, it is the compound each implicitly assumed, rather than the explicit elements, which each finds rightly denied by the other.

In some cases, the presumptions are inductively legitimate for the context each has at hand, following the principle that what is not found connected may be assumed unconnected, so that the face-off with the rebuttal view indeed intimates a possible error somewhere in one's own views. Someone with an open mind does not feel threatened by such an eventuality, but may give some attention to the problem without resentment, if the issue is sufficiently interesting.

b. With regard to simple dilemma, the rebuttal is, on formal grounds, *never* 'equally cogent', so it should not surprise us that the conclusions are contradictory.

If M, then P — and — if N, then P
but M and/or N
hence, P

If P, then M — and — if P, then N
but not M and/or not N
hence, not P.

Although the two major premises are formally compatible with each other, and the two minor premises are formally compatible with each other, the conclusions are indubitably incompatible with each other. What this tells us is that the premises, though severally consistent, are taken together inconsistent. They are not, therefore, equally cogent dilemmas; one or both must contain a factual error.

In other words, a simple dilemma is not logically valid, if the horns of the major premise are reversible hypotheticals and the minor premise is a contradictive disjunctive. The compound propositions 'Only if M, then P — and — only if N, then P' and 'Either M or N' cannot coexist. This may be shown as follows:

The first minor 'M and/or N' taken alone allows for the conjunction 'M and N', while excluding 'nonM and nonN'. The second minor 'not M and/or not N' taken alone allows for the conjunction 'nonM and nonN', while excluding 'M and N'. When these disjunctions are conjoined together, they mean 'either M or N' which still allows for 'M and nonN' or 'nonM and N', but now formally excludes both 'M and N' and 'nonM and nonN'.

Yet, in the case of M and nonN being both true, the left horn of the first major and the right horn of the second, would yield conflicting conclusions: P and nonP; and, in the case of nonM and N being both true, the left horn of the second major and the right horn of the first, would yield conflicting conclusions: P and nonP.

Thus, rebuttal of simple dilemma is formally unfeasible with contingency-based hypotheticals. With an incontingent theses P or nonP, this paradox is acceptable, because if P is necessary or impossible, the arrival at its negation does not cause a serious conflict, since then the necessary theses is implied by its impossible antithesis. Equally cogent simple dilemmas are therefore feasible in abnormal logic specifically, even though they cannot arise in normal logic. It follows that in the logic of unspecified-basis hypotheticals, these are conditionally possible.

The foregoing means that the valid moods of simple dilemma given initially were not as fully defined and unconditional as they should have been, in other respects, besides.

For a simple dilemma to be valid, one or both of the horns in the major premise must be implicitly a subalternation, rather than an implicance (whereas we left them open as implications); and/or the minor premise must be implicitly a subcontrariety (if constructive) or contrariety (if destructive) between the theses in question, rather than a contradiction (whereas we left it open as a not fully defined disjunction).

31. PARADOXES.

A very important field of logic is that dealing with paradox, for it provides us with a powerful tool for establishing some of the most fundamental certainties of this science. It allows us to claim for epistemology and ontology the status of true sciences, instead of mere speculative digressions. This elegant doctrine may be viewed as part of the study of axioms.

1. Internal Inconsistency.

Consider the hypothetical form 'If P, then Q', which is an essential part of the language of logic. It was defined as 'P and nonQ is an impossible conjunction'.

It is axiomatic that the conjunction of any proposition P and its negation nonP is impossible; thus, a proposition P and its negation nonP cannot be both true. An obvious corollary of this, obtained by regarding nonP as the proposition under consideration instead of P, is that the conjunction of any proposition nonP and its negation not-nonP is impossible; thus, a proposition P and its negation nonP cannot be both false.

So, the Law of Identity could be formulated as, "For any proposition, 'If P, then P' is true, and 'If nonP, then nonP' is true". The Laws of Contradiction and of the Excluded Middle could be stated: "For any proposition, 'If P, then not-nonP' is true (P and nonP are incompatible), and 'If not-nonP, then P' is true (nonP and P are exhaustive)".

Now, consider the paradoxical propositions 'If P, then nonP' or 'If nonP, then P'. Such propositions appear at first sight to be obviously impossible, necessarily false, antinomies.

But let us inspect their meanings more closely. The former states 'P and (not not)P is impossible', which simply means 'P is impossible'. The latter states 'nonP and not P is impossible', which simply means 'nonP is impossible'. Put in this defining format, these statements no longer seem antinomial! They merely inform us that the proposition P, or nonP, as the case may be, contains an intrinsic flaw, an internal contradiction, a property of self-denial.

From this we see that there may be propositions which are logically self-destructive, and which logically support their own negations. Let us then put forward the following definitions. A proposition is ***self-contradictory*** if it denies itself, i.e. implies its own negation. A proposition is therefore ***self-evident*** if its negation is self-contradictory, i.e. if it is implied by its own negation.

Thus, the proposition 'If P, then nonP' informs us that P is self-contradictory (and so logically impossible), and that nonP is self-evident (and so logically necessary). Likewise, the proposition 'If nonP, then P' informs us that nonP is self-contradictory, and that P is self-evident.

The existence of paradoxes is not in any way indicative of a formal flaw. The *paradox*, the hypothetical proposition itself, is not antinomial. It may be true or false, like any other proposition. Granting its truth, it is its antecedent thesis which is antinomial, and false, as it denies itself; the consequent thesis is then true.

If the paradoxical proposition 'If P, then nonP' is true, then its contradictory 'If P, not-then nonP', meaning 'P is not impossible', is false; and if the latter is true, the former is false. Likewise, 'If nonP, then P' may be contradicted by 'If nonP, not-then P', meaning 'nonP is not impossible'.

The two paradoxes 'If P, then nonP' and 'If nonP, then P' are contrary to each other, since they imply the necessity of incompatibles, respectively nonP and P. Thus, although such propositions taken singly are not antinomial, double paradox, a situation where both of these paradoxical propositions are true at once, is unacceptable to logic.

In contrast to positive hypotheticals, negative hypotheticals do not have the capability of expressing paradoxes. The propositions 'If P, not-then P' and 'If nonP, not-then nonP' are *not* meaningful or logically conceivable or ever true. Note this well, such propositions are formally false. Since a form like 'If P, not-then Q' is defined with reference to a positive conjunction as '{P and nonQ} is possible', we cannot without antinomy substitute P for Q here (to say '{P and nonP} is possible'), or nonP for P and Q (to say '{nonP and not-nonP} is possible').

It follows that the proposition 'if P, then nonP' does not imply the lowercase form 'if P, not-then P', and the proposition 'if nonP, then P' does not imply the lowercase form 'if nonP, not-then nonP'. That is, in the context of paradox, hypothetical propositions behave abnormally, and not like contingency-based forms.

This should not surprise us, since the self-contradictory is logically impossible and the self-evident is logically necessary. Since paradoxical propositions involve incontingent theses and antitheses, they are subject to the laws specific to such basis.

The implications and consistency of all this will be looked into presently.

2. The Stolen Concept Fallacy.

Paradoxical propositions actually occur in practice; moreover, they provide us with some highly significant results. Here are some examples:

- denial, or even doubt, of the laws of logic conceals an appeal to those very axioms, implying that the denial rather than the assertion is to be believed;
- denial of man's ability to know any reality objectively, itself constitutes a claim to knowledge of a fact of reality;
- denial of validity to man's perception, or his conceptual power, or reasoning, all such skeptical claims presuppose the utilization of and trust in the very faculties put in doubt;
- denial on principle of all generalization, necessity, or absolutes, is itself a claim to a general, necessary, and absolute, truth.
- denial of the existence of 'universals', does not itself bypass the problem of universals, since it appeals to some itself, namely, 'universals', 'do not', and 'exist'.

More details on these and other paradoxes, may be found scattered throughout the text. Thus, the uncovering of paradox is an oft-used and important logical technique. The writer Ayn Rand laid great emphasis on this method of rejecting skeptical philosophies, by showing that *they implicitly appeal to concepts which they try to explicitly deny*; she called this 'the fallacy of the Stolen Concept'.

A way to understand the workings of paradox, is to view it in the context of dilemma. A self-evident proposition P could be stated as 'Whether P is affirmed or denied, it is true'; an absolute truth is something which turns out to be true whatever our initial assumptions.

This can be written as a constructive argument whose left horn is the axiomatic proposition of P's identity with itself, and whose right horn is the paradox of nonP's self-contradiction; the minor premise is the axiom of thorough contradiction between the antecedents P and nonP; and the conclusion, the consequent P's absolute truth.

If P, then P — and — if nonP, then P
but either P or nonP
hence, P.

A destructive version can equally well be formulated, using the contraposite form of identity, 'If nonP, then nonP', as left horn, with the same result.

If nonP, then nonP — and — if nonP, then P
but either not-nonP or nonP
hence, not-nonP, that is, P.

The conclusion 'P' here, signifies that P is logically necessary, not merely that P is true, note well; this follows from the formal necessity of the minor premise, the disjunction of P and nonP, assuming the right horn to be well established.

Another way to understand paradox is to view it in terms of knowledge contexts. Reading the paradox 'if nonP, then P' as 'all contexts with nonP are contexts with P', and the identity 'if P, then P' as 'all contexts with P are contexts with P', we can infer that 'all contexts are with P', meaning that P is logically necessary.

We can in similar ways deal with the paradox 'if P, then nonP', to obtain the conclusion 'nonP', or better still: P is impossible. The process of resolving a paradox, by drawing out its implicit categorical conclusions, may be called *dialectic*.

Note in passing that the abridged expression of simple dilemma, in a single proposition, now becomes more comprehensible. The compound proposition 'If P, then {Q and nonQ}' simply means 'nonP'; 'If nonP, then {Q and nonQ}' means 'P'; 'If (or whether) P or nonP, then Q' means 'Q'; and 'If (or whether) P or nonP, then nonQ' means 'nonQ'. Such propositions could also be categorized as paradoxical, even though the contradiction generated concerns another thesis.

However, remember, the above two forms should not be confused with the lesser, negative hypothetical, relations 'Whether P or nonP, (not-then not) Q' or 'Whether P or nonP, (not-then not) nonQ', respectively, which are not paradoxical, unless there are conditions under which they rise to the level of positive hypotheticals.

3. Systematization.

Normally, we presume our information already free of self-evident or self-contradictory theses, whereas in abnormal situations, as with paradox, necessary or impossible theses are formally acceptable eventualities.

A hypothetical of the primary form 'If P, then Q' was defined as 'P and nonQ are impossible together'. But there are several ways in which this situation might arise. Either (i) both the theses, P and nonQ, are individually contingent, and only their conjunction is impossible — this is the normal situation. Or (ii) the conjunction is impossible because one or the other of the theses is individually impossible, while the remaining one is individually possible, i.e. contingent or necessary; or because both are individually impossible — these situations engender paradox.

Likewise, a hypothetical of the contradictory primary form 'If P, not-then Q' was defined as 'P and nonQ are possible together'. But there are several ways this situation might arise. Either (i) both the theses, P and nonQ, and also their conjunction, are all contingent — this is the normal situation. Or (ii) one or the other of them is individually not only possible but necessary, while the remaining one is individually contingent, so that their conjunction remains contingent; or both are individually necessary, so that their conjunction is also not only possible but necessary — these situations engender paradox.

These alternatives are clarified by the following tables, for these primary forms, and also for their derivatives involving one or both antitheses. The term 'possible' of course means 'contingent or necessary', it is the common ground between the two. We will here use the symbols '**N**' for necessary, '**C**' for contingent (meaning possible but unnecessary), and '**M**' for impossible. The combinations are numbered for ease of reference. The symmetries in these tables ensure their completeness.

Table 31.1 Modalities of Theses and Conjunctions.

No.	Theses				Conjunctions			
	P	nonP	Q	nonQ	P and Q	P and nonQ	nonP and Q	nonP and nonQ
Normal (P,Q both contingent)								
1.	C	C	C	C	C	C	C	C
2.	C	C	C	C	M	C	C	C
3.	C	C	C	C	C	M	C	C
4.	C	C	C	C	C	C	M	C
5.	C	C	C	C	C	C	C	M
6.	C	C	C	C	C	M	M	C
7.	C	C	C	C	M	C	C	M
Abnormal (one or both of P, Q not contingent)								
8.	M	N	C	C	M	M	C	C
9.	N	M	C	C	C	C	M	M
10.	C	C	M	N	M	C	M	C
11.	C	C	N	M	C	M	C	M
12.	N	M	N	M	N	M	M	M
13.	N	M	M	N	M	N	M	M
14.	M	N	N	M	M	M	N	M
15.	M	N	M	N	M	M	M	N

The following table follows from the preceding. '**Yes**' indicates that an implication and its contraposite are implicit in the form concerned, while '**no**' indicates that they are excluded from it. '→' here means implies, and '←' means is implied by.

Table 31.2 Corresponding Definite Hypotheticals.

No.	Name	Implications (→) and Contraposites (←)			
		P→nonQ	P→Q	nonP→nonQ	nonP→Q
		nonP←Q	nonP←nonQ	P←Q	P←nonQ
	Normal (P,Q both contingent)				
1.	Neutral	no	no	no	no
2.	Contrary	yes	no	no	no
3.	Subalternating	no	yes	no	no
4.	Subalternated	no	no	yes	no
5.	Subcontrary	no	no	no	yes
6.	Implicant	no	yes	yes	no
7.	Contradictory	yes	no	no	yes

	Abnormal (one or both of P, Q not contingent)				
8.	P impossible, Q contingent	yes	yes	no	no
9.	P necessary, Q contingent	no	no	yes	yes
10.	P contingent, Q impossible	yes	no	yes	no
11.	P contingent, Q necessary	no	yes	no	yes
12.	P, Q both necessary	no	yes	yes	yes
13.	P necessary, Q impossible	yes	no	yes	yes
14.	P impossible, Q necessary	yes	yes	no	yes
15.	P, Q both impossible	yes	yes	yes	no

Normal hypothetical logic thus assumes the theses of hypotheticals always both contingent, and so limits itself to cases Nos. **1** to **7** in the above tables. However, the abnormal cases Nos. **8** to **15**, in which one or both theses are not contingent (that is, are self-evident or self-contradictory), should also be considered, to develop a complete logic of hypotheticals.

The definition of the primary positive form 'If P, then Q', while remaining unchanged as 'P plus nonQ is not possible', is now seen to more precisely comprise the following situations: Nos. **3**, **6**, **8**, **11**, **12**, **14**, or **15**, that is, all the cases where 'P and nonQ' is impossible ('M'), or 'P implies Q' is marked 'yes'.

The definition of the primary negative form 'If P, not-then Q', while remaining unchanged as 'P plus nonQ is not impossible', is now seen to more precisely comprise the following situations: Nos. **1**, **2**, **4**, **5**, **7**, **9**, **10**, or **13**, that is, all the cases where 'P and nonQ' is contingent (C), or 'P implies Q' is marked 'no'.

The other six hypothetical forms, involving the antitheses of P and/or Q, can likewise be given improved definitions, by reference to the above tables.

Notice the symmetries in these tables. In case No. **1**, all conjunctions are 'C' and all implications are 'no'. In cases Nos. **2**-**5**, one conjunction is 'M', and one implication is 'yes'. In cases **6**-**11**, two conjunctions are 'M', and two implications are 'yes'. In cases Nos. **12**-**15**, three conjunctions are 'M', and three implications are 'yes'. Note the corresponding statuses of individual theses in each case.

The process of contraposition is universally applicable to all hypotheticals, positive or negative, normal or abnormal, for it proceeds directly from the definitions. For this reason, in the above tables, each implication is firmly coupled with a contraposite. Likewise, the negation of any implication engenders the negation of its contraposite, so that the above tables also indirectly concern negative hypotheticals, note well.

We must be careful, in developing our theory of hypothetical propositions, to clearly formulate the breadth and limits of application of any process under consideration, and specify the exceptions if any to its rules. The validity or invalidity of logical processes often depends on whether we are focusing on normal or abnormal forms, though in some cases these two classes of proposition behave in the same way. If these distinctions are not kept in mind, we can easily become guilty of formal inconsistencies.

4. Properties.

Paradoxical propositions obey the laws of logic which happen to be applicable to all hypotheticals, that is, to hypotheticals of unspecified basis. But paradoxicals, being incontingency-based hypotheticals, have properties which normal hypotheticals lack, or lack properties which normal hypotheticals have. In such situations, where differences in logical properties occur, general hypothetical logic follows the weaker case.

The similarities and differences in formal behavior have already been dealt with in appropriate detail in the relevant chapters, but some are reviewed here in order to underscore the role played by paradox.

a. Opposition.

In the doctrine of opposition, we claimed that 'If P, then Q' and 'If P, then nonQ' must be contrary, because if P was true, Q and nonQ would both be true, an absurdity. However, had we placed these propositions in a destructive dilemma, as below, we would have obtained a legitimate argument:

If P, then Q — and — if P, then nonQ
but either nonQ or Q
hence nonP

Likewise, 'If P, then Q' and 'If nonP, then Q' could be fitted in a valid simple constructive dilemma, yielding Q, instead of arguing as we did that they must be contrary because their contrapositions result in the absurdity of nonQ implying nonP and P.

It follows that these contrarieties are only valid conditionally, for contingency-based hypotheticals. There are exceptional circumstances in which they do not hold, namely relative to abnormal hypotheticals (including paradoxicals).

This is also independently clear from the observation of 'yes' marks standing parallel, in cases Nos. **8**, **14**, **15** (allowing for both 'P implies nonQ' and 'P implies Q', where P is impossible), and in cases Nos. **11**, **12**, **14** (allowing for both 'P implies Q' and 'nonP implies Q', where Q is necessary).

Similar restrictions follow automatically for the subcontrariety between 'If P, not-then nonQ' and 'If P, not-then Q', and likewise for the subalternation by the uppercase 'If P, then Q' of the lowercase 'If P, not-then nonQ' (which corresponds to obversion). These oppositions only hold true for normal hypotheticals; when dealing with abnormal hypotheticals (and therefore in general logic), we must for the sake of consistency regard the said propositions as neutral to each other.

b. Eduction.

Similarly with the derivative eductions. The primary process of contraposition is unconditional, applicable to all hypotheticals, but the other processes can be criticized in the same way as above, by forming valid simple dilemmas, using the source proposition and the denial of the proposed target, or the contraposite(s) of one or the other or both, as horns.

Alternatively, these propositions can be combined in a syllogism, yielding a paradoxical conclusion. Thus:

In the case of obversion or obverted conversion (in the former, negate contraposite of target):

If Q, then nonP (negation of target)
if P, then Q (source)
so, if P, then nonP (paradox = nonP)

In the case of conversion by negation or obverted inversion (in the latter, negate contraposite of target):

If P, then Q (source)
if nonQ, then P (negation of target)
so, if nonQ, then Q (paradox = Q)

Thus, eductive processes other than contraposition are only good for contingency-based hypotheticals, and may not be imitated in the abnormal logic of paradoxes. This is made clear in the above tables, as follows.

Consider the paradigmatic form ‘If P, then Q’. If we limit our attention to cases Nos. 1-7, then it occurs in only two situations, subalternating (**3**) or implicance (**6**). In these two situations, ‘P implies nonQ’ is uniformly ‘no’, so the obverse, ‘If P, not-then nonQ’ is true; and the contraposite ‘Q implies nonP’ is also ‘no’, so the obverted converse, ‘If Q, not-then nonP’ is true; ‘nonP implies Q’ is uniformly ‘no’, so the obverted inverse ‘If nonP, not-then Q’ is true; and the contraposite ‘nonQ implies P’ is also ‘no’, so the converse by negation ‘If nonQ, not-then P’ is true. With regard to inversion and conversion, they are not applicable, because ‘nonP implies nonQ’ and ‘Q implies P’ are ‘no’ in one case, but ‘yes’ in the other.

However, if now we expand our attention to include cases Nos. **8**-**15**, we see that ‘If P, then Q’ occurs additionally if P is self-contradictory and Q is contingent (**8**) or P is contingent and Q is self-evident (**11**) or P,Q are each self-evident (**12**) or P is self-contradictory and Q is self-evident (**14**) or P,Q are each self-contradictory (**15**). The above mentioned uniformities, which made the stated eductions feasible, now no longer hold. There is a mix of ‘no’ and ‘yes’ in the available alternatives which inhibits such eductions.

c. Deduction.

With regard to syllogism, the *nonsubaltern moods*, validated by reductio ad absurdum, remain universally valid, since such indirect reduction is essentially contraposition, and no other eductive process was assumed. But the *subaltern moods* in all three figures, are only valid for normal hypotheticals. Since these moods presuppose subalternations for their validation, i.e. depend on direct reductions through obversion or obverted inversion, they are not valid for abnormal hypotheticals.

With regard to apodosis, the moods with a modal minor premise provide us with the entry-point into abnormal logic. As for dilemma, it is the instrument *par excellence* for unearthing paradoxes in the course of everyday reasoning. If we put *any* simple dilemma, constructive (as below) or destructive (mutatis mutandis), in syllogistic form, we obtain a paradoxical conclusion:

If P, then R — and — if Q, then R
but P and/or Q
hence, R

This implies the sorites:
If nonR, then nonP (contrapose left horn)
if nonP, then Q (minor)
if Q, then R (right horn)
hence, if nonR, then R (paradoxical conclusion = R)

Thus, paradoxical propositions are an integral part of general hypothetical logic, not some weird appendix. They highlight the essential continuity between syllogism and simple dilemma, the latter being reducible to the former.

It follows incidentally that, since (as earlier seen) apodosis may be viewed as a special, limiting case of simple dilemma, and simple dilemma as a special, limiting case of complex dilemma — all the inferential processes relating to hypotheticals are closely related.

The paradox generated by simple dilemma of course depends for its truth on the truth of the premises. We should not hurriedly infer, from the paradox inherent in every simple dilemma, that all truths are ultimately self-evident, and all falsehoods ultimately self-contradictory. Knowledge is not a purely rational enterprise, but depends largely on empirical findings.

As already pointed out, simple dilemma yields a categorical necessity or impossibility as its conclusion, only if all its premises are themselves indubitably incontingent. Should there be tacit conditions for, or any doubt regarding the unconditionality of, the hypotheticals (the horns) and/or the disjunction (the minor premise), then the conclusion would be proportionately weakened with regard to its logical modality.

Thus, with reference to the foregoing example, *granting the horns* of the major premise: in the specific case where our minor premise is a formally given disjunction — if, say, P and Q are contradictory to each other (P = nonQ, Q = nonP) — then the R conclusion is indeed necessary. But usually, the listed alternatives P and Q are only contextually exhaustive, so that the R conclusion is only factually true.

So, although every logical necessity is self-evident, and every logical impossibility is self-contradictory, formally speaking, according to our definitions, we might be wise to say that these predications are not in practice reciprocal, and make a distinction between apodictic and factual paradox. The former is independently obvious; the latter derives from more empirical data, and therefore, though contextually trustworthy, has a bit less weight and finality.

Note lastly, the inconsistency of two ‘equally cogent’ simple dilemmas can now be better understood, as due to their implying contrary paradoxes.

32. DOUBLE PARADOXES.

1. Definition.

We have seen that logical propositions of the form 'if P, then nonP' (which equals to 'nonP') or 'if nonP, then P' (which equals to 'P'), are perfectly legal. They signify that the antecedent is self-contradictory and logically impossible, and that the consequent is self-evident and logically necessary. As propositions in themselves, they are in no way antinomial; it is one of their constituents which is absurd.

Although either of those propositions, occurring alone, is formally quite acceptable and capable of truth, they can never be both true: they are irreconcilable contraries and their conjunction is formally impossible. For if they were ever both true, then both P and nonP would be implied true.

We must therefore distinguish between *single paradox*, which has (more precisely than previously suggested) the form 'if P, then nonP; but if nonP, not-then P; whence nonP', or the form 'if nonP, then P; but if P, not-then nonP; whence P' — and *double paradox*, which has the form 'if P, then nonP, *and* if nonP, then P'.

Single paradox is, to repeat, within the bounds of logic, whereas double paradox is beyond those bounds. The former may well be true; the latter always signifies an error of reasoning. Yet, one might interject, double paradox occurs often enough in practice! However, that does not make it right, anymore than the occurrence of other kinds of error in practice make them true.

Double paradox is made possible, as we shall see, by a hidden *misuse of concepts*. It is sophistry par excellence, in that we get the superficial illusion of a meaningful statement yielding results contrary to reason. But upon further scrutiny, we can detect that some fallacy was involved, such as ambiguity or equivocation, which means that in fact the seeming contradiction never occurred.

Logic demands that *either or both* of the hypothetical propositions which constituted the double paradox, or paradox upon paradox, *be false*. Whereas single paradox is *resolved*, by concluding the consequent categorically, without denying the antecedent-consequent connection — double paradox is *dissolved*, by showing that one or both of the single paradoxes involved are untrue, nonexistent. Note well the difference in problem solution: resolution 'explains' the single paradox, whereas dissolution 'explains away' the double paradox.

The double paradox *serves to show* that we are making a mistake of some kind; the fact that we have come to a contradiction, is our index and proof enough that we have made a wrong assumption of sorts. Our ability to intuit logical connections correctly is not put in doubt, because the initial judgment was too rushed, without pondering the terms involved. Once the concepts involved are clarified, it is the rational faculty itself which pronounces the judgment against its previous impression of connection.

It must be understood that every double paradox (as indeed every single paradox), is *teaching us something*. Such events must not be regarded as threats to reason, which put logic as a whole in doubt; but simply as lessons. They are sources of information, they reveal to us certain logical rules of concept formation, which we would otherwise not have noticed. They show us the outer limits of linguistic propriety.

We shall consider two classical examples of double paradox to illustrate the ways they are dissolved. Each one requires special treatment. They are excellent exercises.

2. The Liar Paradox.

An ancient example of double paradox is the well-known 'Liar Paradox', discovered by Eubulides, a 4th cent. BCE Greek of the Megarian School. It goes: 'does a man who says that he is now lying speak truly?' The implications seem to be that if he is lying, he speaks truly, and if he is not lying, he speaks truly.

Here, the conceptual mistake underlying the difficulty is that the proposition is *defined by reference to itself*. The liar paradox is how we discover that such concepts are not allowed.

The word 'now' (which defines the proposition itself as its own subject) is being used with reference to something which is not yet in existence, whose seeming existence is only made possible by it. Thus, in fact, the word is empty of specific referents in the case at hand. The word 'now' is indeed usually meaningful, in that in other situations it has precise referents; but in this case it is used before we have anything to point to as a subject of discourse. It looks and sounds like a word, but it is no more than that.

A more modern and clearer version of this paradox is 'this proposition is false', because it brings out the indicative function of the word 'now' in the word 'this'.

The word 'this' accompanies our pointings and presupposes that there is something to point to already there. It cannot create a referent for itself out of nothing. This is the useful lesson taught us by the liar paradox. We may well use the word 'this' to point to another word 'this'; but not to itself. Thus, I can say to you 'this "this", which is in the proposition "this proposition is false"', without difficulty, because my 'this' has a referent, albeit an empty symbol; but the original 'this' is meaningless.

Furthermore, the implications of this version seem to be that 'if the proposition is true, it is false, and if it is false, it is true'. However, upon closer inspection we see that the expression 'the proposition' or 'it' has a different meaning in antecedents and consequent.

If, for the sake of argument, we understand those implications as: if this proposition is false, then this proposition is true; and if this proposition is true, then this proposition is false — taking the 'this' in the sense of *self*-reference by every thesis — then we see that the theses do not in fact have one and the same subject, and are only presumed to be in contradiction.

They are not formally so, any more than, for any P1 and P2, 'P1 is true' and 'P2 is false' are in contradiction. The implications are not logically required, and thus the two paradoxes are dissolved. There is no self-contradiction, neither in 'this proposition is false' nor of course in 'this proposition is true'; they are simply meaningless, because the indicatives they use are without reference.

Let us, alternatively, try to read these implications as: if 'this proposition is false' is true, then that proposition is false; and if that proposition is false, then that proposition is true' — taking the first 'this' as self-reference and the 'thats' thereafter as all pointing us backwards to the original proposition and not to the later theses themselves. In other words, we mean: if 'this proposition is false' is true, then 'this proposition is false' is false, and if 'this proposition is false' is false, then 'this proposition is false' is true.

Here, the subjects of the theses are one and the same, but the implications no longer seem called for, as is made clear if we substitute the symbol P for 'this proposition is false'. The flavor of paradox has disappeared: it only existed so long as 'this proposition is false' seemed to be implied by or to imply 'this proposition is true'; as soon as the subject is unified, both the paradoxes break down.

We cannot avoid the issue by formulating the liar paradox as a generality. The proposition 'I always lie' can simply be countered by 'you lie sometimes (as in the case 'I always lie'), but sometimes you speak truly'; it only gives rise to double paradox in indicative form. Likewise, the proposition 'all propositions are false' can be countered by 'some, some not', without difficulty.

However, note well, both the said general propositions are indeed self-contradictory; they do produce single paradoxes. It follows that both are false: one cannot claim to 'always lie', nor that 'there are no true propositions'. This is ordinary logical inference, and quite legitimate, since there are logical alternatives.

With regard to those alternatives. The proposition 'I never lie' is not in itself inconsistent, except for the person who said 'I always lie' intentionally. The proposition 'all propositions are true' is likewise not inconsistent in itself, but is inconsistent with the logical knowledge that some propositions are inconsistent, and therefore it is false; so in this case only the contingent 'some propositions are true, some false' can be upheld.

3. The Barber Paradox.

The Barber Paradox may be stated as: 'If a barber shaves everyone in his town who does not shave himself, does he or does he not shave himself? If he does, he does not; if he does not, he does'.

This double paradox arises through confusion of the expressions 'does not shave himself' and 'is shaved by someone other than himself'.

We can divide the people in any town into three broad groups: (a) people who do not shave themselves, but are shaved by others; (b) people who do not shave themselves, and are not shaved by others; (c) people who shave themselves, and are not shaved by others. The given premise is that our barber shaves all the people who fall in group (a). It is tacitly suggested, but not formally implied, that no one is in group (b), so that no one grows a beard or is not in need of shaving. But, in any case, the premise in fact tells us nothing about group (c).

Next, let us subdivide each of the preceding groups into two subgroups: (i) people who shave others, and (ii) people who do not shave others. It is clear that each of the six resulting combinations is logically acceptable, since who shaves me has no bearing on whom I can shave. Obviously, only group (i) concerns barbers, and our premise may be taken to mean that our barber is the only barber in town.

Now, we can deal with the question posed. Our barber cannot fall in group (a)(i), because he is not shaved by others. He might fall in group (b)(i), if he were allowed to grow a beard or he was hairless; but let us suppose not, for the sake of argument. This still allows him to fall in group (c)(i), meaning that he shaves himself (rather than being shaved by others), though he shaves others too.

Thus, there is no double paradox. The double paradox only arose because we wrongly assumed that 'he shaves all those who *do not* shave themselves' excludes 'he shaves some (such as himself) who *do* shave themselves'. But *'X shaves Y' does not formally contradict 'X shaves nonY'*; there is no basis for assuming that the copula 'to shave' is obvertible, so that 'X shaves Y' implies 'X does not shave nonY'.

If the premise was restated as 'he shaves all those *and only* those who do not shave themselves' (so as to exclude 'he shaves himself'), we would still have an out by saying 'he does not shave at all'. If the premise was further expanded and restricted by insisting that 'he somehow shaves or is shaved', it would simply be self-contradictory (in the way of a single paradox).

Further embellishments could be made to the above, such as considering people who shave in other towns, or making distinctions between always, sometimes/sometimes-not, and never. But I think the point is made. The lesson learned from the barber 'paradox' is that without clear categorizations, equivocations can emerge (such as that between 'shaves' and 'is shaved'), which give the illusion of double paradox.

PART IV. DE RE CONDITIONING.

33. CONDITIONAL PROPOSITIONS.

1. *De-Re* Conditioning.

Logic has traditionally been focused on two types of proposition, the actual categorical of Aristotle, and the logical hypothetical or disjunctive of later logicians. Categorical propositions (including their factual, positive conjunctions) were seen as essentially '*de-re*', telling us about things in themselves. Hypothetical and disjunctive propositions (essentially, modal or negative conjunctions) were seen as essentially '*de-dicto*', telling us about connections between thoughts.

However, we will now develop a more accurate, broader theory of conditioning, which acknowledges not only logical conditioning, but also '*de-re* conditioning', constructed with reference to other *types* of modality. (The reader is referred to all our previous definitions of the different types of modality.)

This does not mean to imply that logical conditioning is any less 'real' than *de-re* conditioning. But rather, only that the type of modality qualifying the connection and basis is different in each case. As we shall see, each type of conditioning has to do with a distinct type of causality.

The following should serve to illustrate the distinction between types of conditioning:

Logical: 'if this, then that', meaning: in such context as this is true, that is also true.

Natural: 'when this, that', meaning: in such natural circumstance as this is actual, that is also actual.

Temporal: 'when this, that', meaning: at such times as this is actual, that is also actual.

Extensional: 'where this, that', meaning: in such cases as this occurs, that also occurs. (By 'cases' we here refer to instances of a universal.)

Whereas every *de-re* conditional implies some kind of *de-dicto* conditional, the reverse does not always hold. This is because *de-re* propositions are formally more demanding than logical statements; we need more information to be able to formulate them.

For example, I can formulate an argument like 'if nothing is knowable, then…' without thereby suggesting that I acknowledge the antecedent as even logically possible, whereas with other types of conditioning such speculative freedom is lacking. However, note, the rephrasing of *de-re* into *de-dicto* will not be studied in detail here.

Natural, temporal and extensional conditionals and disjunctives, are essentially as *de-re* as two-term, single categoricals, even though they may tell us about connective relationships between three or more terms, or two or more categoricals.

Conditionals have many forms, but we will give most of our attention to those with three terms: the subject and the antecedent and consequent predicate, which best highlight the nature and properties of this family of propositions.

I do not intend to analyze natural, temporal, and extensional conditioning, in as much detail as categorical propositions were and will be treated. I will especially not attempt to develop theories of factorial analysis, and induction by factor selection and formula revision, relating to conditionals. The work done in later chapters on categoricals should be viewed as prototypical, a model for future investigations of the same kind in the field of conditionals.

Each type of modality has its own specific disjunctive propositions, distinguished by their bases and connectives. Relatively little attention will be devoted in this volume to disjunction, although it is in itself valuable, because its logic is derivable from that of conditionals. But some introductory comments will be made in their proper place.

Incidentally, one of the utilities of studying disjunction, is that it clarifies the logic of *degrees*. The various degrees or measures of any thing X may be viewed as standing in a disjunction 'X1 or X2 or X3 or…', of whatever modal type is appropriate. Each degree is a logical, natural, temporal or extensional alternative, and they usually range from some maximum to some minimum.

Disjunctive logic teaches us, for instance, not to confuse the affirmation of X as such (which is indefinite as to degree) with the affirmation of its extreme or most typical manifestation (a specific degree or range, say X1). Likewise, denial of X should mean negation of all its degrees (X1, X2, X3,…), and not mere negation of the more extreme or typical degree or range (as we often intend in practice). The fallaciousness of many an argument is explained with reference to such confusions.

2. Types of Causality.

Our expansion of the theory of conditioning is the gateway through which Logic enters into the field of 'material' causality.

Hypotheticals are concerned with logical causes; they show us the 'reasons why' of items of knowledge, with reference to the contextuality of information. Non-logical conditionals are concerned with more 'substantive' causation, occurring in the objective realms of matter or mind, irrespective of the stage of development of our knowledge.

Whereas hypotheticals tell us that 'In all or this or some knowledge contexts, two theses P and Q both logically arise', other conditionals tell us that 'In all or this or some circumstances or times or cases, two events SP and SQ both really happen'.

The various types of conditioning are differentiated by the type of modality intended, in the connection (which qualifies the whole relation of antecedent and consequent), and in the basis (the underlying possibilities), which they respectively imply.

In typical hypotheticals, of the form 'if P, then Q', the connection is a logical incontingency and the basis is a problemacy or logical possibility of truth.

In contrast, typically, for natural conditionals like 'when P is, Q must be', the connection is a natural necessity and the basis is a potentiality of actualization in some circumstances.

For temporal conditionals like 'when X is, Y always is', the connection is a temporal constancy and the basis is the sometime occurrence of the events concerned.

For extensional conditionals 'where X applies, Y applies', the connection is a generality and the basis is applicability to part of the subject's instances.

Through such formal analysis of conditioning, using the tools of modal logic, we can begin to understand and seriously examine the concept of causality.

Causality is of various types, in parallel to the types of modality. We can talk of logical causality, natural causality, temporal causality, and extensional causality. These are distinct, yet not unrelated, types of determinism. Making this distinction allows for more accurate and efficient reasoning processes.

Each type of causality orders reality in a special way. Logic determines reality in accordance with the order of development of knowledge; nature and time order individual external events as such; extension refers to the classification of universals. These represent distinct methods of explanation.

When we say that X causes Y, or Y is caused by X, we must first establish the type of causality intended Expressions like 'because of' or 'as a result of' or 'depends on', and such, are in everyday discourse used indiscriminately, without awareness of the modality type involved. Yet, epistemologically and ontologically the difference is important.

The various types of causality display both some similarities and some differences in structure and in logical behavior patterns. The common properties of all types of causality may be seen as the general laws of causality. The distinctive uniformities within each type give rise to a special logic for that specific modality. Thus, both the similarities and differences are significant.

The field of aetiology, the study of causality, is not intended to be within the scope of this dissertation. I have personally already done the needed logical work, so I know how vast and interesting it is. But these results belong in a separate volume. My purpose here is to give one more justification for my theory of modality, to highlight how useful to logic and all areas of knowledge this tool is. My policy here will therefore be to focus on information most relevant to this purpose, the bare essentials.

The differentiation of modality into types and categories allows for hitherto unmatched clarity and precision in the development of conceptual knowledge. Not that modality is something new to human thinking, but its systematic study greatly improves our understanding of it and our reasoning processes.

For the concept of modality, as indeed that of causality, transcends any specific content of knowledge, and is equally valuable in physical sciences, psychology, politics, religious discourse, or personal deliberations. It is not attached to any particular theory of the universe, or of any domain within it. It is grounded in common overall experience and logical consistency.

3. Laws of Causality.

I should perhaps, however, say a word or two about the so-called 'Laws of Causality' which some philosophers have advanced.

a. Some claim that 'cause and effect must be substantially the same'. Thus, they deny that G-d could have created the world, 'because' a purely spiritual entity (G-d) cannot generate a material and mental one (the world we commonly experience). But there is no formal justification for such an argument, for spirit and matter still have in common one thing, namely *existence*, so that the conclusion is not inferable from the premise. In any case, we commonly regard material and mental phenomena, though substantially different, as having mutual causal relations, whether in acts of will or in more reactive psychological situations, so that even within the empirical world such a 'law of causality' is untenable.

b. Some claim that 'something static cannot cause a motion', in order to prove that G-d, who is unchanging, cannot have created a world of change, or to deny that human volition is initiation of motion by an unmoved soul. But this is contrary to common-sense intuition, so that aetiology may not *ab-initio* reject this from formal possibility. One may seek to prove it eventually, but not posit it as a logical principle from the start.

c. Some claim that 'everything must have a cause, *ad infinitum*'. They say that there are no prime movers, that everything is mechanistically determined, and from thence argue that G-d, and likewise human action, must also have a cause. Here again, there is no formal basis for such a claim. As we proceed, it will become clear that causality is quite definable without reference to such 'laws'. We might posit such infinite regression as a generalization, an *inductive* principle, but there is no conceptual necessity in it.

34. NATURAL CONDITIONALS: FEATURES.

1. Basis and Connection.

There are six singular forms of natural conditionals with three terms, as follows. These forms are so structured that we can analyze the behavior of individual subjects, the relationships between their predicates, independently of other individuals. Note the three categories of modality and two polarities they feature.

(We could if need be use the same symbolic conventions as we did for categoricals, only perhaps prefix them with, say, a paragraph (**§**), to remind us of the differences.)

§**Rn**: When this S is P, it must be Q
§**Gn**: When this S is P, it cannot be Q
§**R**: This S is P and Q
§**G**: This S is P and not Q
§**Rp**: When this S is P, it can be Q
§**Gp**: When this S is P, it can not-be Q

Let us examine the structure of these forms in more detail:

a. **The expression *'when'*** used here signifies a conditionality of the type *'in such circumstances as'*; and it is intended to imply that the condition 'this S is P' is potential. Note well that the reference here is to *natural* circumstances; we are dealing with a real, objective type of causality.

'When' suggests that the underlying 'this S can be P' is an established fact, and not merely something logically conceivable. Thus, it is not equivalent to the 'if' of hypothetical propositions, which only signifies that the condition might turn out to be true, not being so far inconsistent with the context of knowledge.

Needless to say, by now, we are not always careful, in everyday discourse, to use 'when' (instead of 'if' or similar expressions) wherever natural conditioning is intended, or 'must' (instead of 'is') wherever necessity is intended. There is no harm in confusing words in practice, provided we know what we mean.

The S being P condition is called the antecedent; it is only operative when actual, and needs be at least potential to fit in this formal position. The S being or not-being Q conjunction is called the consequent; here too, the relevant modality is actuality, and potentiality is formally implied. These two actualities may be called 'events'.

The implied potential of the events and their conjunction is called the 'basis', and the natural modality qualifying the conjunction as a whole specifies the 'connection' involved.

b. ***Basis.*** Every natural conditional proposition may be said to be 'based on' the natural possibility, the potentiality, of the antecedent's eventual actualization. Each of the six forms introduced above logically implies the categorical proposition 'this S can be P'.

Likewise, since when the condition is actualized, the consequence will also be actualized, whether unconditionally or under certain unspecified additional conditions, it follows that the consequent is also logically implied to be potential. That is, 'this S can be (or can not-be) Q' may be educed from these same forms (with the appropriate polarity).

More precisely, the full basis of these forms is the conjunctive categorical 'this S can be both P and Q (or can be both P and nonQ, in negative cases)', which incidentally implies the two above-mentioned separate potentialities. The conditional proposition implicitly guarantees that the said base potentiality exists. This joint potentiality underlying every natural conditional is the foundation on which the subjunction is built.

Potentiality signifies that certain unspecified surrounding circumstances, may underlie the specified event. This refers to the various postures of the real world, the situation of the rest of the material, mental, and even spiritual world. Since potentiality is compatible with both necessity and contingency, items in the wider environment may or not be responsible for triggering the reaction or inhibiting it.

Although the original function of the form is to capture actualization of naturally contingent phenomena, it is so engineered that one or both of the events could in fact be naturally necessary. The formal basis of any natural conditional is the potentiality of the events, not their natural contingency.

The precise function of a natural conditional is thus only to point out to us the *intersections, inclusions or exclusions, between the circumstances surrounding the two events.* This may be compared to the doctrine of 'distribution of terms', in categorical propositions.

That is, though each form is based on the potentiality of antecedent and consequent and their conjunction, this does not logically necessitate that both the events be conditional, but admits as logically possible that one or both of the events exist(s) under all natural circumstances.

Thus, though for instance the necessary form 'When this S is P, it must be Q' implies 'This S can be P and Q', it is still logically compatible with any of the conjunctions 'This S can be nonP and nonQ' (double contingency), or 'This S must be P and must be Q' (double necessity), or 'This S can not-be P, and must be Q' (contingency with necessity).

However, that necessary form is logically incompatible with the conjunction 'This S must be P, and can not-be Q' (necessity with contingency), because of the connection, as we shall see. Similarly, with a negative consequent (substitute nonQ for Q throughout).

In contrast, the corresponding actual and potential forms, allow for all those eventual modal conjunctions, though only the said basic joint potentiality is formally implied.

If one or both of the events is necessary, the conditioning is admittedly effectively redundant, since a necessary event exists independently, it is 'already there'; but the relationship is still formally true.

c. ***Connection.*** Although the modal qualification (the 'must', 'cannot', 'can', or 'can not' modifier) is placed on the side of the consequent — it is not part of the consequent, but properly concerns *the relation* between it and the preliminary condition, that is, the subjunction as a whole. This should be grasped clearly: the antecedent and consequent of natural conditional propositions can only be actualities or actualizations.

That is, 'When this S is P, it must be Q' does not say that the phenomenon 'this S is P' will be followed by the phenomenon of natural necessity 'this S must be Q', for it admits that 'this S can not-be Q' might be true. Rather 'this S is P' will, *whatever the surrounding circumstances*, be followed by the phenomenon of actuality 'this S is Q'. Likewise for a negative consequence.

Similarly, 'When this S is P, it can be Q' does not say that the phenomenon 'this S is P' will be followed by the phenomenon of potentiality 'this S can be Q', for that is already given as part of the basis. Rather 'this S is P' will, *in some unspecified surrounding circumstances*, be followed by the phenomenon of actuality 'this S is Q'. Likewise for a negative consequence.

It is thus very appropriate to regard the antecedent actuality and the consequent actuality, as the two 'events' referred to by the proposition. The modality merely acts as a bridge between them.

Note well that, even in the case of necessary conditioning, the natural circumstances in which the antecedent is actualized are not specified. What is specified, is that the conditions which suffice to actualize the antecedent, whatever they be, will also be sufficient to actualize the consequent.

That directional link between the events is formally expressed by saying that 'When this S is P, it must be Q' implies 'This S cannot be {P and nonQ}'; and 'When this S is P, it cannot be Q' implies 'This S cannot be {P and Q}'. These implications, in the form of naturally impossible conjunctions, are the connections between the events.

Thus, to define a necessary conditional, we must specify two categorical conjunctions (with appropriate polarities): the basis 'this S can be both P and Q (or nonQ)', and the connection 'this S cannot be both P and nonQ (or Q)'. We cannot, with such natural conditioning (unlike with logical conditioning), ignore one or the other of these specifications; both must be kept in mind.

In the case of potential conditioning, the link between the events is formally expressed by contradicting the above necessary connections, and saying that 'When this S is P, it can not-be Q' implies 'This S can be {P and nonQ}'; and 'When this S is P, it can be Q' implies 'This S can be {P and Q}'. We see that, here, the implied basis and connection are one and the same naturally possible conjunction.

In merely potential conditionals, the (unspecified) conditions for actualization of the antecedent will not be enough to bring about the consequent; some additional (also unspecified) conditions are required for that. Clearly, these propositions enable us to express cases of partial, instead of complete, causality of natural phenomena; their subjunctive form is not artificial.

It is understood that there are some sets of circumstances, like say R, which in conjunction with P will suffice to cause Q (or nonQ, as the case may be) in this S. That is, for instance, 'When this S is P, it can be Q, and when it is not P, it can not-be Q' minimally implies 'When this S is P and R, it must be Q', for at least one (known or unknown) 'R'.

However, that specifically concerns fully *deterministic* systems, and does not take *free will* into account. Indeed, denying such implication altogether is the way we can begin to formally develop the topic of spontaneous events. For this reason, I will not go into these issues in greater detail in the present study.

d. ***Definitions.*** In summary, we can define modal natural conditionals entirely through categorical conjunctions, but all the implied categoricals must be specified.

Thus, 'When this S is P, it must be Q' means 'This S can be both P and Q, but cannot be P without being Q'; similarly, 'When this S is P, it cannot be Q' means 'This S can be P without being Q, but cannot be both P and Q'. In contrast, 'When this S is P, it can be Q' means no more than 'This S can be both P and Q'; and 'When this S is P, it can not-be Q' only means 'This S can be both P and nonQ'.

It follows from these understandings that each of the necessary forms subalternates the potential form of like polarity (identical with their basis). Natural conditionals thus constitute a modal continuum, as did categoricals.

The *actual* forms, 'This S is P and Q' and 'This S is P and not Q', refer to conjunctions of events existing *'in the present natural circumstances'*. They obviously imply, as their bases, the propositions 'This S can be both P and Q' and 'This S can be P and nonQ', respectively, since what is true of 'one specified circumstance' is equally true of 'some unspecified circumstance(s)'.

The position of these actual conjunctions in the modal hierarchy of conditionals, to some extent parallels the position of single actuals among modal categoricals, since they are the way the potential conjunctions, which are the basis of all modal conditionals, are actualized. However, the analogy is limited, because in the field of conditionals, natural necessity does not imply actuality, though both necessity and actuality do imply potentiality.

This is obvious from the greater complexity of the necessary forms. A connective like 'This S cannot be both P and Q' remains *problematic* with respect to which of the alternative positive conjunctions 'P and nonQ', 'nonP and Q', 'nonP and nonQ' will actually take the place of the excluded 'P and Q'. We cannot even be sure that all these conjunctions are even potential; the only one formally given as potential is the one serving as basis, namely 'P and nonQ', the others may or not be so. Similarly, with appropriate polarity changes, for 'This S cannot be both P and nonQ'.

Thus, a naturally impossible conjunction involves a certain amount of leeway, like a logically impossible conjunction. It does not by itself formally fully determine any actuality or even all potentialities. However, to repeat, a natural connective is not by itself ground enough to form a conditional proposition; an adequate basis is also required for that (whereas in the case of hypotheticals, logical basis is varied and optional).

Note, however, in exceptional cases, our use of the expression 'when and if' to suggest that we know the natural connection to apply (as suggested by the 'when'), but we do not know the natural basis to be applicable (whence the 'if' proviso). But this expression may have other meanings (see section 3b further on).

e. Note well that **actual** 'conditionals' are in fact conjunctions, and cannot meaningfully be written in conditional form, with a 'when'. With regard to the seemingly nonmodal conditional form 'When this S is P, it *is* Q', which we commonly use to describe habitual, voluntary actions or events, the following may be said:

A proposition such as 'When she is happy, she sings', should not be regarded as an actual conditional, but rather as a form vaguely expressing a degree of natural *probability* below necessity. It means, in ordinary circumstances, so and so is very likely, but in extraordinary circumstances, it is less to be expected. Alternatively, the intention may be to express a temporal modality, as in 'When this S is P, it is always (or usually or sometimes) Q'; in which case the form properly belongs under the heading of temporal conditionals.

Ultimately, volitional conditioning involves a type of modality different from natural conditioning. Note that the antecedent of a natural conditional proposition may be voluntary, since even something freely willed may have naturally necessary consequences. What distinguishes volitional conditioning is that, whether the antecedent is emerges naturally or voluntarily, the consequence is voluntarily chosen and brought about. For example, 'If you do this, I will do that' involves two voluntary actions.

Volitional conditionals are thus statements of conditional intention. The 'will' involved, concerns another type of causality, than the 'must' of naturals. Volition is a special domain within Nature (in the broadest sense), where otherwise common relations (those of natural modality) do not all apply. Volition denotes a greater than usual degree of agency.

However, this type of modality will not be dealt with in this treatise, but belongs in a work on aetiology.

2. Quantification.

Quantification of the six prototypes expands the list of such natural conditional propositions to 18.

§**An**: When any S is P, it must be Q
§**En**: When any S is P, it cannot be Q
§**In**: When certain S are P, they must be Q
§**On**: When certain S are P, they cannot be Q
§**A**: All S are P and Q
§**E**: No S is P and not Q
§**I**: Some S are P and Q
§**O**: Some S are P and not Q
§**Ap**: When any S is P, it can be Q
§**Ep**: When any S is P, it can not-be Q
§**Ip**: When certain S are P, they can be Q
§**Op**: When certain S are P, they can not-be Q

We have already analyzed the expression 'when', signifying the natural conditionality, and the features of polarity of consequent (is or is not Q), and modality (in all, the given, or some circumstances). Here, we introduce plural quantity (any, certain), in place of the singular indicative (this).

The first thing to note is that the quantifiers are here intended as dispensive, and not collective or collectional. They refer to the instances of the subject severally, each one singly, so that the plural forms are merely a shorthand rendition of a number of singular propositions. The all or some units of the subject-concept do not have to simultaneously fulfill the condition for the consequence to follow, and the two predicates apply to the individual units, and not to a group of such units as a whole.

For this reason, the words 'any' and 'certain' are preferably used in this context, less misleading (however, the word 'certain' should understood as meaning 'at least some', and not 'only some').

Secondly, the basis of the general conditional propositions should be a categorical generality. For instance, 'When any S is P, it must be Q' implies that 'all S can be P and Q'. In practice, we tend to confuse or mix the methodology of natural and extensional modality (see the discussion of the latter, in a later chapter), and often intend only a particular basis for a seemingly general natural conditional; however, here, the stated generality will be regarded as genuine. The corresponding particular conditionals only have particular bases, obviously.

Note that, although we have dealt with forms with a negative consequent, we did not so far mention forms with a *negative antecedent*, like 'When this S is not P,….' Obviously, we could construct another 18 forms (6 singulars and 12 plurals; or 6 actuals and 12 modals), with this added feature in mind.

I will not devote much attention to these extra forms, because their logic is easily derived. With reference to the eductions feasible from forms with positive antecedents, we can infer their oppositions to those with negative antecedents. And all the inferences feasible with the former can be duplicated with the latter, by simply substituting 'nonP' for 'P' throughout.

I do not here mean to underrate negative antecedents. Taking the antecedent as a whole, its polarity is of course logically irrelevant. Undeniably, forms with antithetical antecedents are important, because they complement each other.

For instance, a form like 'When this S is P, it must be Q' does not by itself communicate change, but combined with 'When this S is not P, it cannot be Q', we get a sense of the dynamics involved. That is, not merely is the static actuality of P accompanied by that of Q (or nonP by nonQ), but the actualization of P brings about that of Q (or nonP, nonQ).

We may view this as a formal implication, by certain combinations of conditionals involving actualities and inactualities, of similar conditionals concerning the triggering or prevention of actualizations (using the transitive copula, 'gets to be').

Besides natural conditionals with three terms, there are other varieties: those with four terms, such as 'When this S1 is P, that S2 is Q'. Quite often used and important, is the case: 'When any S1 is P, *the corresponding (or some unspecified)* S2 is Q'. Here, the mediation provided by an explicit common subject is lacking, though some hidden thread links the two events. For examples, 'When a car runs out of fuel, its motor stops' or 'When evil is let loose, somebody somewhere suffers'.

Often, of course, we use still more complex versions, involving composite antecedent and/or consequent, such as 'When {S is P1 and P2} and {S2 is P3}, {S3 is Q1 and Q2}', say.

The logical mechanisms applicable to these more complex varieties should be similar to those for the standard three-term forms we are focusing on. So long as we clearly understand which individual subjects are denoted, so that we know precisely which one affects which, there should be no logical confusion.

3. Other Features.

The forms mentioned thus far deal with most natural conditioning situations. In this section, we will mention various notable departures from these norms.

a. The order of ***sequence***, or chronology, of the antecedent and consequent events must be kept in mind to avoid errors of judgment with regard to natural conditionals. Here, I assume that a consequence takes place as soon as and so long as its antecedent. More broadly:

The antecedent may accompany the consequent immediately (and thus be simultaneous), or later in time, or earlier in time; and the time lapse between them may be mentioned explicitly, or tacitly understood (as we do here).

In the case of simultaneity, the events may happen at the same time, and yet not be contemporaneous, that is, not last for equal lengths of time. All the more, in cases of nonsimultaneity, the lasting power of the events may be very different; for instances, a flash of lightning may cause permanent damage, a long burning fuse may end in a sudden explosion. These issues should be taken into consideration in reasoning from natural conditionals.

In real causality, the cause is immediately or after some time followed by the effect; if we place the effect temporally before the cause, we are considering it as an 'index', 'sign' or 'symptom' of the cause's presence or absence. Natural conditionals mirror reality if expressed in the right sequence, otherwise they are a logical artifice (wherein, instead of cause causing effect, the knowledge of the effect 'causes' the knowledge of the cause).

In any case, the temporal qualification of the events is usually relative; the time of one event is defined as before or after the other's time, by so much. In some cases, we refer to 'absolute' time — that is, date and o'clock; the relative time follows by inference.

Note also the different ways time may be specified: we can say that an event does or does not happen at some (stated or undefined) point or segment of time; or permanently, in past and/or future areas of time.

b. ***Modalities of Actualization.*** The 'when' should be taken in its weakest sense, as suggested in the expression 'when and if' or 'if ever', and not as implying the inevitability of actualization of the condition. This sense of 'it can happen, but is not bound to happen' is to be preferred as our standard because it is broader, more generally applicable.

We could work out a specialized logic for inevitable antecedents. A complex proposition like 'When S gets to be P, and it is bound to be P eventually, it must be Q', would have as its first base that 'although this S can be and can not-be P, sooner or later it is must change from nonP to P', and imply the inevitability of Q too (unless already actual through other causes).

Note well that inevitability of actualization signifies an underlying natural contingency of actuality, it is only the transition from nonP to P which is naturally necessary. Obviously, this should not be confused with the more static natural necessity of actuality, which we are usually concerned with, which is the antithesis of contingency.

All this brings to mind the wider field of natural conditionals for transitive events, incidentally. No one has researched it.

However, these are relatively narrow topics, and will not be discussed further here.

c. Within natural modality, we also need to recognize the phenomenon of ***acquisition or loss of 'powers'***. The concept of a power is rather difficult to define. By a power, we mean a potential close to actualization; something readily available, without too many preparatory measures. But this definition is too vague for formal work.

Anyway, something may remain outside the powers of a subject for a part of its existence and then eventually appear (e.g. through the maturing of an organism); or a subject may initially have a power and then lose it irrecoverably (e.g. the use of a hand which is cut off). Thus, we can talk of actualization of the presence or absence of powers.

Obviously, an 'acquired power' was always potential, even before it became more accessible; so the concept of a 'acquired power' is subsidiary to the concept of a potentiality, and included in it as a special case. However, a 'lost power' is something previously potential and henceforth naturally impossible; so this concept introduces a serious complication into modal logic, namely the logical possibility for changes in bases and connections.

Thus, in some cases, the modality given within a natural conditional, may be intended to be an intrinsic part of the antecedent or consequent. Such modal specifications are effectively actualities, as far as the conditional proposition as a whole is concerned, and should not be confused with the modality of the relation between them.

Powers may be indicated by use of modal expressions like ‘is able to be’ (which is less demanding than ‘is’, but more specific than ‘can be’) or ‘is unable to be’ (which lies between ‘cannot be’ and ‘is not’); or more dynamically, ‘is henceforth able to be’ or ‘is no longer able to be’ (more explicitly implying a change in powers). Likewise for ‘not-be’.

Thus, ‘When this S is able to be P, it is Q’ would mean ‘when this S has the (actual) power to be P, it is Q’. Likewise, ‘When this S is P, it is able to be Q’ would mean ‘When this S is P, it has the (actual) power to be Q’. More precisely, the latter statement should be modal, like all conditionals; that is, we mean ‘it must be able to be Q’ or ‘it can be able to be Q’, where ‘must’ or ‘can’ define the connection, while ‘is able to be’ signifies an actuality of power. Similarly for the interpretation of negatives.

This topic requires further study, but will not be pursued further here.

d. Note that in practice if one finds natural modality expressions, like ‘can’ or ‘must’ (or their negative equivalents), appearing in the antecedent or being intended as an intrinsic feature of the consequent — it does not follow that the conditioning is of the natural type.

On the contrary, this usually signifies that the conditional proposition is of the logical or extensional type. For examples, ‘If S must be P, then it can be Q’ is supposedly a hypothetical, and ‘In cases where S can be P, it must be Q’ is supposedly an extensional conditional, even though the antecedents and consequents are in natural modality.

As earlier pointed out, in practice the words we use are not always consistent with the intended modality of conditioning. One should therefore be careful to identify just what type of conditioning is intended, because their logics are considerably different.

4. Natural Disjunction.

Disjunction has traditionally been approached as an essentially logical relation. But our analysis of the types of modality shows clearly that disjunction also exists in nature. It can be understood with reference to natural conditioning.

a. There are various **modalities and polarities** of natural disjunction. Consider the simplest case of three terms, in the singular:

The necessary form ‘This S must be P or Q’, can be taken to mean that ‘When this S is not P, it must be Q, and when it is not Q, it must be P’, it follows that the implied connection is that ‘This S cannot be both nonP and nonQ’, and the implied basis is that ‘This S can be nonP and Q, and it can be P and nonQ’, which in turn imply that ‘This S can be and can not-be P, and can be and can not-be Q’. Note well the implied natural contingency of the individual events.

The corresponding potential form ‘This S can be P or Q’ accordingly means ‘When this S is not P, it can be Q, and when it is not Q, it can be P’ (same as the above basis).

As for the parallel negative forms: ‘This S can not-be P or Q’ has to mean ‘This S can be both nonP and nonQ’ (contradicting the above connection), and ‘This S cannot be P or Q’ may therefore be understood as ‘This S must be both nonP and nonQ’ (subalternating the preceding).

These various forms can of course be quantified.

b. **Other manners** of disjunction may also be used:

To describe a specifically ‘P and/or Q’ situation, we would have to add to the said ‘This S must be P or Q’ definitions, that ‘This S can be both P and Q’.

The natural disjunction ‘This S must be nonP or nonQ’ can be similarly interpreted, by substituting antitheses for theses throughout; briefly put, it means ‘When P, nonQ; when Q, nonP’. To describe a specifically ‘P or else Q’ situation, we would have to add to the said ‘nonP or nonQ’ definitions, that ‘This S can be both nonP and nonQ’.

An ‘either-or’ situation would be represented by a compound of the two disjunctions ‘P or Q’ and ‘nonP or nonQ’, meaning four natural conditional propositions.

c. Also, analogous forms involving **more than three terms** can be constructed, constituting multiple natural disjunctions. Their connections can be defined like multiple logical disjunctions, except with reference to numbers of actualities or inactualities, instead of truths or falsehoods.

However, here, note well, every one of the alternatives must be, taken individually, naturally contingent, as the two-alternative paradigm makes clear. Otherwise, the basis of disjunction is not properly, entirely natural, but closer to merely logical. Natural disjunction has a very different basis from logical disjunction; much more information is demanded of us, before we can formulate a natural one.

Note in any case that a logical 'cannot' implies, but is not implied, by a natural 'cannot'; and therefore potentiality implies, but is not implied by, logical possibility.

After thus defining the various types of natural disjunction through naturally modal, categorical and conjunctive propositions, their logical interrelationships and processes can be worked out with little difficulty. The reader is invited to do this work.

In practice, it is not always clear whether we intend a disjunctive proposition looking like the above as natural or as logical. For instance, even though there is no such thing as actual natural disjunction, a proposition of the form 'S is P or Q' might be intended to mean 'S must be P or Q', rather than imply mere logical disjunction. But such ambiguities need not deter us from investigating the respective logical properties of these two types, and learning their differences. Some more comments will be made on this topic, in the chapter on condensed propositions.

35. NATURAL CONDITIONALS: OPPOSITIONS AND EDUCTIONS.

1. Translations.

We may call 'translation' the reformulation of a proposition in other form, such as the change from conditional to categorical, or vice versa.

Natural conditionals are reducible to their categorical definitions, their implicit bases and connections, of course. Thus, for instance, 'When any S is P, it must be Q' implies 'All S can be P' and 'No S can be P and not Q', and therefore 'All S can be P and Q' and 'All S can be Q'. These implications could be viewed as distinct immediate inferences, which are collectively though not individually reversible.

Another way to translate such natural conditionals into categoricals would be by joining the antecedent predicate to the subject, to form a new, narrower, subject. Thus, for instance, 'When any S is P, it must be Q' would become 'All SP must be Q'. However, the new class 'SP' would have to be actual, or such a necessary categorical must be regarded as not implying actuality and so tacitly still conditional.

Modern logicians tend to regard all categoricals as involving a conditional subject, and so would regard such translation of conditionals into categoricals as formally true. However, I beg to differ with current opinion on this point. My contention is that, logically, there has got to be categoricals which are genuinely so, before we can build up conditional forms; categoricals are logically prior to conditionals, since the latter correlate the former.

Cases where the subject is not actual are only artificially categorical; they are made to seem so, but in fact are still conditional. (This argument also holds for imaginary subjects, where there is a hidden hypothesis 'Though the subject is nonexistent, if it existed, so and so would follow'.)

Thus, the hidden conditionality in some categoricals is an exception, rather than the rule. The position taken by certain logicians to the contrary is not logically tenable, in my view. This issue is further discussed in the chapter on modalities of subsumption.

2. Oppositions.

The form 'When this S is P, it must be Q' means 'this S can be P, but it cannot be P without being Q', which implies that 'this S can be P and Q'. It follows that the logical contradictory of this form is 'This S cannot be P, *or* it can be P without being Q', and not merely 'This S can be P without being Q'. That is, 'When this S is P, it can not-be Q' is not formally contradictory, but only contrary; it is contradictory only if we take for granted that 'This S can be P'.

Similarly, 'When this S is P, it cannot be Q' is on an absolute level merely contraried by 'When this S is P, it can be Q', and becomes contradicted only in such case as 'This S can be P' is already given.

On the other hand, the form 'When this S is P, it must be Q' implies that 'When this S is P, it can be Q', since the latter means no more than 'This S can be P and Q', which is the tacit basis of the former. Likewise, 'When this S is P, it cannot be Q' implies 'When this S is P, it can not-be Q'.

It follows that 'When this S is P, it must be Q' and 'When this S is P, it cannot be Q' are invariably contrary to each other, since they imply each other's contraries.

As for 'When this S is P, it can be Q' and 'When this S is P, it can not-be Q', they may be both be true, since 'This S can be P, with or without Q' occurs in some cases; and they may both be false, since it is conceivable that 'this S neither can be P and Q, nor can be P and not Q', as occurs in the case of 'this S cannot be P' being true. Thus, these two bases are normally neutral to each other, though if 'This S can be P' is granted, they become subcontrary.

With regard to actuality, 'When this S is P, it must be Q' does not imply, nor exclude, that 'this S is P (and thereby Q)', although 'This S is P and Q' does imply that 'when this S is P, it can be Q'. Thus, the necessary form is ontologically a relationship which exists potentially, even when not actually operative. It is, of course, conceivable that 'This S is P and Q' in the actual circumstance but not in all circumstances, or in some circumstance(s) but not the actual one. The same can be said about the forms negating the consequent.

As for the parallel forms which negate the antecedent, their basis is different, namely 'This S can not-be P and be (or not-be) Q at once'.

Therefore, 'When this S is not P, it must be Q' is compatible with 'When this S is P, it must be Q' (these together would imply that 'this S must be Q'), and likewise with 'When this S is P, it cannot be Q' (in which case, we have a *sine-qua-non* situation every which way). All the more, the potential versions are all compatible. We need not, for our present purposes, go beyond this degree of detail.

These oppositions concern singular forms, note well; the corresponding oppositions for plural forms follow automatically, in accordance with the general rules of 'quantification of oppositions', which we dealt with in the chapter on opposition of modal categoricals. Thus, for example, 'When any S is P, it must be Q' is ordinarily contrary to 'When certain S are P, they can not-be Q'; but if it is established that 'All S can be P', they become contradictory.

3. Eductions.

Eduction from conditionals consists in changing the position and/or polarity of antecedent and consequent.

a. With regard to ***actuals***, suffices to say that 'This S is P and Q' and 'This S is Q and P' are, from our point of view, equivalent. For the rest:

Obversion obviously applies to all the forms, without loss of modality. Thus 'When this S is P, it can be (or must be) Q' imply 'When this S is P, it can not-be (or cannot be) nonQ'; likewise, 'When this S is P, it can not-be (or cannot be) P' imply 'When this S is P, it can be (or must be) nonQ'.

'When this S is P, it can be or must be Q' *convert* to 'When this S is Q, it can be P', since 'This S can be P and Q' is implicit basis of the source.

'When this S is P, it can not-be or cannot be Q' *convert by negation* to 'When this S is not Q, it can be P', since the latter target means 'This S can be P and not Q', which is given in the original proposition; note well, they are not convertible to 'When this S is Q, it can not-be P', since the source contains no basis for 'This S can be Q and not P'.

These results are of course in turn obvertible.

We note that these simple eductions, other than obversion, yield a potential conclusion, even from a necessary premise.

b. However, a necessary conclusion may be drawn, if we are granted that the negation of the consequent is potential. This process may be called ***complex contraposition***, and viewed either as a deduction from two premises, or as an eduction from a compound premise. The following is the primary valid mood:

When this S is P, it must be Q
and This S can not-be Q

hence, When this S is not Q, it cannot be P

The proof of this argument is by reduction ad absurdum. The denial of the conclusion implies either that 'This S must be Q' (base denied) or that 'This S can be nonQ and P' (connection denied); but either way this results in the denial of the minor or major premises; therefore, the conclusion is valid.

From this mood we may derive the following, by obversion:

When this S is P, it cannot be Q
and This S can be Q
hence, When this S is Q, it cannot be P

Thus, full contraposition is feasible, but only on the proviso that the basis of the conclusion is in advance given as true; without this additional information, it is not permissible. The reason for this is that the original conditional is in principle compatible with the categorical necessity of its consequent.

Note that the above arguments incidentally yield the conclusion that 'This S can not-be P'. This may be viewed as modal apodosis from the given premises.

c. When ***quantity*** is introduced into all these equations, it is important to note that it is unaffected, unlike the modality. That is, a general natural conditional, is general for both the antecedent and consequent, implying that 'all S can be Q' as well as 'all S can be P'.

So 'When any S is P, it can be or must be Q' converts to 'When any S is Q, it can be P', and 'When any S is P, it can not-be or cannot be Q' converts by negation to 'When any S is not Q, it can be P'. Similarly, a particular premise is convertible, though to a particular conclusion.

Likewise, given that 'All S can not-be Q', the necessary 'When any S is P, it must be Q' contraposes to 'When any S is not Q, it cannot be P'. Also, note well, when only one of the premises is general, whichever one — that is, given 'When any S is P, it must be Q' and 'Some S can not-be Q', or given that 'When certain S are P, they must be Q' and 'All S can not-be Q' — we can still infer that 'When certain S are not Q, they cannot be P' (and so that 'Some S can not-be P'). However, if both premises are particular, contraposition is not permitted. Similarly, throughout, for propositions with negative predicates.

Derivative processes behave accordingly. For instance, inversion, being contraposition followed by conversion or vice-versa, requires two premises at least one of which is general, and always results in a potential conclusion.

Lastly note, these changes all essentially concern the predicates of natural conditionals. We might additionally have considered changes affecting the subject, such as conversions within the antecedent or consequent clause. But the idea seems somewhat artificial in this context, unlike in hypotheticals.

36. NATURAL CONDITIONAL SYLLOGISM AND PRODUCTION.

1. Syllogism.

Syllogism in this context involves three natural conditional propositions, all having a common subject, and whose three predicates are positioned in figures analogous to those found in categorical syllogism. Although the rules of modality, polarity, and quantity are essentially similar, there are interesting differences of detail in the results obtained.

a. The premier valid mood of syllogism involving natural conditionals is the following ***first figure*** singular necessary argument, where M is the middle term. From this mood all others are derivable.

1/nnn
When this S is M, it must be Q
When this S is P, it must be M
so, When this S is P, it must be Q.

This is validated by exposition: consider any random circumstance in which this S is actually P; then, by apodosis from the minor premise, it is also M; and, by apodosis with that from the major premise, it is also Q.
By substituting nonQ for Q, we derive a similar negative-consequent version:

When this S is M, it cannot be Q
When this S is P, it must be M
so, When this S is P, it cannot be Q.

Next, a potential version may be constructed:

1/npp
When this S is M, it must be Q
When this S is P, it can be M
so, When this S is P, it can be Q.

This mood can be validated by reductio ad absurdum to the previous. If the conclusion were denied, then 'this S cannot be P and Q' would be true; but the original major premise implies as its basis that 'this S can be Q'; it follows that:

When this S is Q, it cannot be P;
but When this S is M, it must be Q,
therefore, When this S is M, it cannot be P.

The connection implied by this result, being 'this S cannot be M and P', causes the original minor premise to be denied. Ergo, the original conclusion is undeniable.
The negative-consequent version of this mood is the following:

When this S is M, it cannot be Q
When this S is P, it can be M
so, When this S is P, it can not-be Q.

Needless to say, any modes subaltern to the above are also valid. Thus, **nnp** is implied valid, by **nnn** or **npp**.

Syllogism in this figure with a potential major premise are not valid. Consider, for example, the mood below:

1/pnp
When this S is M, it can be Q
When this S is P, it must be M
so, When this S is P, it can be Q.

Although this S is M in all the circumstances relating to this S being P (minor premise), it remains conceivable that there be circumstances in which this S is M without being P (as conversion attests); these latter circumstances may be precisely among the only ones in which this S is Q, as well as M (major premise); so there is no guarantee that this S can be P and Q together (as in the attempted conclusion), indeed it may well be that this S must cease to be P before it is allowed to be Q (in which case, when this S is P, it *becomes* Q).

A-fortiori, this invalidation also applies to the mode **1/ppp**. The argument is essentially that denying the attempted conclusion, by saying 'This S cannot be P and Q', does not result in the inconsistency of a denied major or minor premise. Analogous negative-consequent versions are equally spurious, of course.

We can also construct parallel actual moods. But, the following one might be regarded as more akin to apodosis than syllogism, though valid:

1/naa
When this S is M, it must (or cannot) be Q
This S is P and M
so, This S is P and Q (or nonQ).

As for the mood below, it concerns the mechanics of categorical conjunction, and hardly any longer qualifies as conditional argument in the narrow sense.

1/aaa
This S is M and Q (or nonQ), in actual circumstance,
This S is P and M, in the same circumstance,
so, This S is P and Q (or nonQ).

What we have here, of course, are interface situations, where different domains of logic meet.

Note that the mode **naa** is subaltern to **aaa** (even though necessity does not imply actuality here), because we can also infer that 'This S is M and Q (or not Q)' from the combination of major and minor premise. However, an actual conclusion from a necessary minor premise (as in **1/nna** or **1/ana**), and modes involving a mix of actual and potential premises (**ap** or **pa**), are invalid. This is easily demonstrated.

So much for the first figure. The parallels to categorical syllogism should be obvious; and indeed, categorical syllogism can be viewed as a special case of conditional syllogism, where the subject is 'thing' instead of a specific 'S'.

Note in passing that sorites are possible with natural conditionals, as with categoricals.

b. The valid singular moods of the ***other figures*** can easily be derived from those given so far, using the methods of reduction developed in other contexts. The primary ones are listed below, for the record, without little further discussion, for the sake of brevity.

For the *second* figure:

2/nnn

When this S is Q, it must be M
When this S is P, it cannot be M
so, When this S is P, it cannot be Q.

When this S is Q, it cannot be M
When this S is P, it must be M
so, When this S is P, it cannot be Q.

2/npp

When this S is Q, it must be M
When this S is P, it can not-be M
so, When this S is P, it can not-be Q.

When this S is Q, it cannot be M
When this S is P, it can be M
so, When this S is P, it can not-be Q.

Note the change of polarity of the major event, in this figure. Mode **nnp** is subaltern to **nnn** or **npp**; but **pnp** is not valid. Also valid, in the fig. 2, is mode **2/naa**; though not **nna**, **ana**. Two actual premises (**aa**), with the polarities of the events as shown above, are naturally impossible, since the middle term would have mixed polarity; however, if the middle event has exceptionally the same polarity in the two premises, **aaa** becomes feasible, though the minor premise is useless to the inference. Also invalid, as before, are **ap**, **pa** or **pp**.

For the *third* figure:

3/npp

When this S is M, it must be Q
When this S is M, it can be P
so, When this S is P, it can be Q.

When this S is M, it cannot be Q
When this S is M, it can be P
so, When this S is P, it can not-be Q.

3/pnp

When this S is M, it can be Q
When this S is M, it must be P
so, When this S is P, it can be Q.

When this S is M, it can not-be Q
When this S is M, it must be P
so, When this S is P, it can not-be Q.

Subaltern to **npp** or **pnp**, is mode **3/nnp**; but mode **nnn** is invalid. Also valid, in the fig. 3, is mode **aaa**; and its subalterns **naa** and **ana**, though not **nna**. Also invalid, are **ap**, **pa** or **pp**, as always.

For the *fourth* figure (significant mood):

4/npp

When this S is Q, it cannot be M
When this S is M, it can be P
so, When this S is P, it can not-be Q.

Note the change of polarity of the major event, in this figure; also, the mixed polarity of the middle event. Mode **nnp** is subaltern to **npp**; but **nnn** or **pnp** are not valid. Also valid, in the fig. 4, is mode **4/naa**; though not **nna**, **ana**. Two actual premises (**aa**) are naturally impossible, unless the middle event has exceptionally the same polarity in the two premises. Also invalid, are **ap**, **pa** or **pp**.

c. In addition to all the above, we could construct an equal number of valid moods, whose premises and/or conclusions involve a ***negative antecedent***, obviously. Such moods are easily validated by substituting the negation of a term for a term, in various ways. Some interesting results emerge, as the samples below show.

In figure one, all the primary moods can be reiterated, with a negative middle term (as in the sample below) and/or a negative minor term.

1/nnn
When this S is not M, it must be Q
When this S is P, it cannot be M
so, When this S is P, it must be Q.

In figure two, all the primary moods can be reiterated, with a negative major term (as in the sample below) and/or a negative minor term.

2/nnn
When this S is not Q, it must be M
When this S is P, it cannot be M
so, When this S is P, it must be Q.

In figure three, all the primary moods can be reiterated, with a negative minor term (as in the sample below) and/or a negative middle term.

3/npp
When this S is M, it must be Q
When this S is M, it can not-be P
so, When this S is not P, it can be Q.

In the fourth figure, we may switch the (mixed) polarities of the middle term, and/or of the major term, and/or insert a negative minor term. We thus have a total of 8 valid modes of polarity in each of the 4 figures.

These random examples demonstrate that the rules of polarity may seemingly be by-passed. Thus, for examples, we seem to process a negative minor premise in the first figure, or to obtain a positive conclusion in the second figure, or to draw a positive conclusion from a negative premise in the third figure. But of course, the rules of polarity are still essentially operative, the changes are illusory.

Still, such moods have practical significance. Without their clarification, we might miss out on possible inferences from data, or make errors. The reader is therefore advised to develop a full list of such syllogisms, as an exercise.

2. Summary and Quantities.

The following table neatly summarizes the results obtained in the previous section. Note the similarities and differences between the modes of modality here, and those for categorical syllogism.

Table 36.1 Natural Conditional Syllogisms.

Figure One.

	Polarities				Valid	Subaltern	Invalid
MQ	++	+-	-+	- -	nnn	nnp	pnp
PM	++	++	+-	+-	npp	naa	nna, ana
PQ	++	+-	++	+-	aaa		ap, pa, pp

plus 4 with negative minor term.

Figure Two.

	Polarities				Valid	Subaltern	Invalid
QM	++	+-	-+	- -	nnn	nnp	pnp, (aaa)
PM	+-	++	+-	++	npp		nna, ana
PQ	+-	+-	++	++	naa (aaa)		ap, pa, pp

plus 4 with negative minor term.

Figure Three.

MQ	++	+-	++	+-	npp	nnp	nnn
MP	++	++	+-	+-	pnp	naa	nna
PQ	++	+-	-+	- -	aaa	ana	ap, pa, pp

plus 4 with negative middle term.

Figure Four.

QM	+-	++	--	-+	npp	nnp	nnn, pnp
MP	++	-+	++	-+	naa		(aaa)
PQ	+-	+-	++	++	(aaa)		nna, ana ap, pa, pp

plus 4 with negative minor term.

In the first figure, 2 modal modes, and 1 actual mode, are valid (and these have 2 subalterns). For 8 polarity modes, this means a total of 24 (+16) valid moods. Similarly, in fig. 2, there are at least 24 (+8) valid moods, not counting the special cases of **aaa**. In fig. 3, the total is 24 (+24). In fig. 4, it is at least 16 (+8), not counting the special cases of **aaa**.

The grand total of primary moods is thus 88 (not counting specials alluded to in parentheses), of which 56 are modal and 32 are actual; plus 56 subalterns.

All the valid moods listed above are in the singular mode of quantity '**sss**', but they may of course be ***quantified***. However, the rules of quantity are less stringent for conditional syllogism than with categorical syllogism.

This is due to **sss** being here valid throughout, because an individual instance of the subject, indicated by 'this S', effectively stands outside the syllogistic procedure as such, and remains recognizable independently of the three predicates, P, Q, and M which are being manipulated.

It follows that, *so long as one premise is universal*, a conclusion can be drawn, having the same quantity as the other premise; but no conclusion is possible from two particular premises, and the conclusion cannot be higher than the lower of the two premises.

In other words: **uuu**, **upp**, **pup**, **uss**, **sus**, are all valid, in all the figures, for all the moods established in **sss**. The only invalid inferences with regard to quantity, are therefore **upu**, **ups**, **puu**, **pus**, **ppp**, **ppu**, **pps**, **usu**, **suu**, obviously.

Below are the modes of quantity for each figure, with a minimum of examples, to illustrate some of the deviations from previous rules.

Thus, in the first and second figures, while **uuu**, **upp**, and **uss**, remain valid, we have additionally **pup** and **sus**. For examples,

1/sus
When this S is M, it must be Q
When any S is P, it must be M
so, When this S are P, it must be Q.

2/pup
When certain S are Q, they must be M
When any S is P, it can not-be M
so, When certain S are P, they can not-be Q.

In the third figure, in addition to **upp** and **pup**, the modes **uuu**, **uss** and **sus** are valid. For example,

3/uuu
When any S is M, it must be Q
When any S is M, it must be P
so, When any S is P, it can be Q.

In the fourth figure, for the significant mood listed above, instead of just **upp**, we also have **uuu**, **pup**, **uss**, **sus**. For example,

4/pup
When certain S are Q, they cannot be M
When any S is M, it can be P
so, When certain S are P, they can not-be Q.

The reader is invited to develop a full list of plural syllogisms, as an exercise.

3. Production.

Production of natural conditionals is their inference from categorical propositions. This shows us how to construct natural conditionals deductively, rather than empirically. The structure of the premises follows the model of categorical syllogism, while the conclusion encompasses all the original terms.

a. The chief mood of such argument is in the ***first figure***; it involves a necessary major, a potential minor, and a necessary conclusion, as follows:

All P must be Q
This S can be P
therefore, When this S is P, it must be Q.

We manage, exceptionally, to reason in the **npn** mode, note, because the conclusion, though stronger than the minor premise, concerns a narrower set of circumstances (SP instead of just S).

This argument can be validated by exposition; for any circumstance in which this S is actually P, we know that it will also be Q according to the categorical syllogism **1/AnRR**. Note well that we are exceptionally drawing a necessary, though conditional, conclusion from a merely potential minor premise.

Alternatively, we can use reduction ad absurdum. Denying the conclusion means either that 'this S cannot be P', which contradicts the minor premise, or that 'this S can be P and not Q', which implies that, for this S at least, some P can not-be Q, in contradiction to the major premise. Thus, the conclusion is indubitable.

Note well that 'When this S is P, it must be Q' does not imply 'All P must be Q'. Although natural conditionals may be inferred from categorical premises, it does not follow that that is the only way we can get to reach such conclusions. Natural conditionals can also be known by induction; so, they do not logically imply categoricals other than their bases and connections.

The negative version of the above mood is:

No P can be Q
This S can be P
therefore, When this S is P, it cannot be Q.

Note that if the major premise is necessary, and the minor premise is the actual or necessary 'This S is or must be P', then the conditional conclusion as such is unaffected; so these are subaltern moods of production.

If both premises are actual, concerning the same circumstances, the conclusion is a categorical conjunction of all three terms, which represents the actual form of natural conditional. The positive and negative versions of this **aaa** mode, still in the first figure, are:

All P are Q
This S is P
therefore, This S is P and Q.

No P is Q
This S is P
therefore, This S is P and not Q.

We may also have, with the same actual major, a necessary minor 'This S must be P', without change of conclusion (mode, **ana**).

Note that the **nnn** mode is also valid, by subalternation from **npn**. It is interesting to note, however, that given the premises 'This S must be P and all P must be Q' we would rather draw the categorical conclusion 'This S must be Q', than the inferior conditional 'When this S is P, it must be Q'. It shows the essential continuity between categorical and conditional syllogism. Given that 'Some S can be P' (which is the base of the minor premise) the conditional conclusion is a subaltern of the categorical one.

Also, two necessary categorical premises, with adequate modality of subsumption, may also be used to draw an actual conjunctive conclusion (**nna**). All the above conclusions of course further imply that 'When this S is P, it can be Q' or '… nonQ', respectively (as in the subaltern **aap** mode).

However, although **npn** and **nnn** are valid, the modes **npa** or **nna** are invalid, since a necessary conditional does not imply an actual conjunction. Also, the major premise could not be merely potential, since the middle term P would then not be distributive in respect of modality, even if the minor premise were necessary (**pnp**, or **ppp**). For the same reason, an actual major cannot be combined with a potential minor (**ap**), or vice versa (**pa**).

With regard to quantity, the rules of categorical syllogism remain applicable here, so that the major premise must be universal, while the minor may be universal or particular, as well as singular; the conclusion has the same quantity as the minor.

b. So much for the first figure; the valid moods of the ***other figures*** follow from these, using the usual methods. Below is a quick overview, ignoring actuals and subalterns, which are obvious.

In figure *two*, the model moods are in the **npn** mode:

No Q can be P
This S can be P
therefore, When this S is P, it cannot be Q.

All Q must be P
This S can not-be P
therefore, When this S is not P, it cannot be Q.

Observe, in the latter case, the production of a natural conditional with negative antecedent, exceptionally.

We can in both cases introduce different modalities and quantities, as we did in figure one. Note that only the minor premise may be potential or particular.

In figure *three*, the process seems rather contrived, though formally supportable, because of the change of position of the minor term. The model moods are:

This P must be Q
This P can be S
therefore, When certain S are P, they must be Q.

This P cannot be Q
This P can be S
therefore, When certain S are P, they cannot be Q.

This P can be Q
This P must be S
therefore, When certain S are P, they can be Q.

This P can not-be Q
This P must be S
therefore, When certain S are P, they can not-be Q.

Note the necessary conditional conclusion from a necessary major coupled with a merely potential minor, in contrast to the conclusion being no better than potential if the major is only potential, even though the minor is necessary. Thus, though modes **npn** and **pnp** are valid, the **pnn** mode is invalid.

With regard to other modalities and quantities, the rules of categorical syllogism apply here. Only the major premise may be negative; one of the premises must be necessary (or both actual); one of the premises must be particular (or both singular); and the conclusion is in any case particular.

For the *fourth* figure, again the impression of artificiality, but here is the significant model mood (mode, **npn**) for the record, anyway, without further comment:

No Q can be P
This P can be S
therefore, When certain S are P, they cannot be Q.

c. Lastly, note that ***the combination of syllogism and production*** allows us to form arguments involving four terms, in a categorical major premise and a natural conditional minor premise and conclusion. For example,

All M must be Q
When this S is P, it must be M
therefore, When this S is P, it must be Q.

Such argument need not be considered as a distinct process. We draw the proposition 'This S can be M' from the minor premise, and use this with the major premise to produce 'When this S is M, it must be Q', which is then coupled with the minor in a syllogism with the said conclusion.

d. Some additional comments on production. Consider the first figure valid mood,

All P must be Q
All S can be P
therefore, When any S is P, it must be Q.

Note well the difference between this production of a natural conditional, and the production of a logical hypothetical from the same premises: in the latter case, the conclusion would be 'If all P must be Q and all S can be P, then all S can be Q', or even 'If all P must be Q and all S can be P, then when any S is P, it must be Q'. The focus in natural production is on concrete actualities, whereas logical production is concerned with formal truths.

It is also well, in this context, to keep in mind the difference between a dispensive natural conditional, 'When any S is P, it must be Q', which implies a number of independent singulars; and a collectional one, 'When <u>all</u> S are P, they are Q', which refers to the conjunction of singulars as the required condition.

In the former case, we mean: 'When this S is P, it is Q, and when that S is P, it is Q, and …'; whereas in the latter, 'When this S is P <u>and</u> that S is P <u>and</u> …, they are Q'. The same can be said about particulars.

37. NATURAL APODOSIS AND DILEMMA.

1. Apodosis.

Natural apodosis is deductive argument mainly involving (i) a necessary natural conditional as major premise, and (ii) an actual categorical corresponding to the antecedent or to the negation of the consequent as minor premise, with (iii) an actual categorical corresponding to the consequent or to the negation of the antecedent, respectively, as conclusion. Other modalities are less typical, though derivable.

a. Actual Moods.

The premier valid mood, from which all others may be derived, consists in 'affirming the antecedent' (*modus ponens*), as follows. Note well that the conclusion is not 'This S must be Q', in spite of our placing the necessary modality of the conditional proposition in the consequent; however, the conclusion 'this S is P', although not naturally necessary, is of course logically necessary given the premises; the mode is **naa**.

When this S is P, it must be Q
and This S is P,
hence, This S is Q.

The major premise informs us that, whatever the surrounding circumstances for this S, its being P is accompanied by its being Q. The apodosis merely takes it at its word, and applies it to the actual circumstance given by the minor premise, to obtain the conclusion.

The following mood follows from this by obversion:

When this S is P, it cannot be Q
and This S is P,
hence, This S is not Q.

The following mood, which consists in 'denying the consequent' (*modus tollens*), may be reduced to the primary one above, ad absurdum: deny the conclusion, while retaining the major premise, and the minor premise is contradicted.

When this S is P, it must be Q
and This S is not Q,
hence, This S is not P.

A complex contraposition underlies this argument, of course. The major premise does not by itself imply the contraposite 'When this S is not Q, it cannot be P'; but when the major is combined with 'This S can not-be Q', as implied by the minor premise, the contraposite is inferable, as we saw in the chapter on eduction. With the contraposite, this mood becomes identical to the one before.

The next follows by obversion from this:

When this S is P, it cannot be Q
and This S is Q,
hence, This S is not P.

The actual moods draw an actual conclusion from a necessary major premise and an actual minor premise, in mode **naa**. The mode **nna** is accordingly valid, granting the actuality of the subject. Modes **aaa** or **ana** are valid for *modus ponens*, but their minor premise is redundant; in modus tollens, they are invalid, because the premises are incompatible.

b. Modal Moods.

Modal moods are those with a modal conclusion from modal premises.

Moods with a necessary major and minor premise, affirming the antecedent, yield a necessary conclusion. These moods can be viewed as repetitive applications of the corresponding actual moods, since natural necessity means actuality in all circumstances, and they teach us that if the antecedent is naturally necessary, so must the consequent be. The following are **nnn** moods valid:

When this S is P, it must be Q
and This S must be P,
hence, This S must be Q.

When this S is P, it cannot be Q
and This S must be P,
hence, This S cannot be Q.

On the other hand, moods with a necessary major and minor premise, denying the consequent (and thus depending on our switching the positions of the events), are not valid, because their conclusion would contrary the major premise. Thus, the following **nnn** moods are invalid, note well:

When this S is P, it must be Q
and This S cannot be Q,
hence, This S cannot be P.

When this S is P, it cannot be Q
and This S must be Q,
hence, This S cannot be P.

Such apodosis is invalid, even with an unnecessary conclusion (as in **nnp**), since the major premise requires 'this S can be P and Q (or nonQ)' as its basis, and thus formally excludes the logical possibility of the attempted minor premise, let alone any conclusion.

As for mode **npp**, necessary major premise combined with a potential affirming minor and conclusion, form a valid mood, since as soon as the minor actualizes, so will the conclusion. For instances:

When this S is P, it must be Q
and This S can be P,
hence, This S can be Q.

When this S is P, it cannot be Q
and This S can be P,
hence, This S can not-be Q.

This mood teaches us that the connection together with the base of the antecedent suffice to define a natural conditional, since the base of the consequent (and also the compound basis) follow anyway. But we could also view this mood as redundant, granting that we already know that both the minor premise and conclusion are formally implicit in the major premise.

On the other hand, a necessary major premise combined with a potential denying minor and conclusion, form a more significant, as well as valid, mood of apodosis (mode **npp**). These arguments have already been encountered in the context of complex contraposition:

When this S is P, it must be Q
and This S can not-be Q,
hence, This S can not-be P.

When this S is P, it cannot be Q
and This S can be Q,
hence, This S can not-be P.

What of moods with a potential, instead of necessary, major premise? *Modus ponens* cases are redundant, and modus tollens cases are invalid, as shown below:

If the associated minor premise affirms the antecedent, whether necessarily or potentially, a necessary conclusion (mode **pnn** or **ppn**) is of course out of the question. Drawing a potential conclusion (in **pnp** or **ppp**) would teach us nothing new, since that is already implied in the basis of the major, anyway. For instance:

When this S is P, it can be Q
and This S can or must be P,
hence, This S can be Q.

If the associated minor premise denies the consequent necessarily, we cannot draw a conclusion, because the two premises are anyway contrary to each other. Thus, for instance, the following is invalid (mode **pnp**):

When this S is P, it can be Q
and This S cannot be Q,
hence, This S can not-be P.

If the associated minor premise denies the consequent potentially, we cannot draw a conclusion, because we have no guarantee that the some circumstances referred to by the major overlap with those referred to by the minor. Thus, for instance, the following is invalid (mode **ppp**):

When this S is P, it can be Q
and This S can not-be Q,
hence, This S can not-be P.

c. We can construct further valid moods, analogous and equal in number to the above described, by substituting a ***negative antecedent***, 'When this S is not P,…' in all the majors. The polarity of the corresponding minor or conclusion must of course be changed to match, in every case.

d. Also, we can ***quantify*** all the valid moods. One of the premises must be general, to guarantee overlap; the quantity of the conclusion then follows that of the other premise. Thus, we have two sets of quantified moods, with some overlapping cases (both premises general).

Those with a general major premise, and any quantity in the minor, like:

When any S is P, it must be Q
and All/This/Some S is/are P,
hence, All/This/Some S is/are Q.

When any S is P, it must be Q
and All/This/Some S is/are not Q,
hence, All/This/Some S is/are not P.

And those with any quantity in the major premise, and a general minor, like:

When any/this/some S is/are P, it/they must be Q
and All S are P,
hence, All/This/Some S is/are Q.

When any/this/some S is/are P, it/they must be Q
and No S is Q,
hence, All/This/Some S is/are not P.

Similarly with the allowable changes in modality, and with negative consequents and/or antecedents, of course. Clearly, the rules of quantity here are less restrictive than those of modality; this is because the quantity of antecedent and consequent is one and the same, whereas the modality concerns their relationship.

Moods such as those below are, of course, not valid, because they go beyond the brief of the forms concerned. However, if we regard the minor premise as an adduction of evidence or counterevidence, we may view the suggested conclusion as tending to be confirmed.

When this S is P, it must be Q
and This S is Q (is given as evidence),
hence, This S is P (is somewhat confirmed).

When this S is P, it must be Q
and This S is not P (is given as counterevidence),
hence, This S is not Q (is somewhat confirmed).

Compare such natural adduction to logical adduction. Here, we are assuming that the actual set of circumstances surrounding the minor premise, is among the sets of natural circumstances in which the major premise holds, namely all the circumstances when this S is P or all the circumstances when this S is not Q.

2. Dilemma.

Natural *disjunctive* arguments are reducible to natural conditional processes, at least in the case of two alternatives. For example, the following apodosis could be validated by inferring ‘When this S is not P, it must be Q’ from the major premise.

This S must be P or Q
This S is not P
so, This S is Q.

Or again, the following sample of ‘syllogistic’ argument, admittedly somewhat forced and not likely to be used as such in practice, could be validated in a similar way.

This S must be M or Q
This S must be P or not M
so, This S must be P or Q

Likewise, we can develop arguments for production of natural disjunctives.

These are of course only the simplest samples. Other polarities, other modalities, other manners of disjunction, and multiple disjunction, would need be considered for full treatment of the field. But these topics will not be analyzed further, here.

Natural dilemma, however, deserves some attention, because of the improved insight into the meaning of natural necessity which it provides, and to stress its distinction from logical dilemma.

a. ***Simple constructive*** natural dilemma consists, as shown below, of premises and conclusion all of which are necessary; the major premise consists of a conditional whose antecedent is a natural disjunction (or, alternatively, of the equivalent conditionals in conjunction), the minor premise is disjunctive, and the conclusion is categorical.

When this S is M or N, it must be P,
but, This S must be M or N,
hence, This S must be P.

Whereas in apodosis to draw such a necessary conclusion, the minor premise had to be a categorical necessity, here we are taught that a necessary conclusion may still be drawn from a slightly less demanding minor premise, namely a disjunctive necessity — provided, of course, that the conditional major premise(s) is/are necessary.

We learn from this that if some event P is 'bound to' follow each of circumstances M, N, etc. (however many there be), and the set of circumstances M, N, etc. is exhaustive, then the event P is immovable and effectively independent of any circumstance. Thus, the dilemma as a whole tells us 'Whether this S is M or N, it must be P'.

Note well that the minor premise and conclusion could not have been actual, as in apodosis. There is no actual form of natural disjunction; the proposition 'This S is M or N', taken literally, is a logical disjunctive, based on a doubt as to whether 'This S is M' or 'This S is N' is true, without implying that both these actualities are potential in the real world in the present circumstances.

b. Note well the *special case* of simple constructive natural dilemma:

'When this S is M, it must be P, and when it is not M, it must be P'.
but, This S must be M or not M,
hence, This S must be P.

Or, more briefly, 'Whether this S is or is not M, it must be P'. There is nothing in the structure of natural conditionals preventing contradictory antecedents from having the same necessary consequent. In such case, the consequent is absolutely, and not just relatively, necessary, so that the antecedents are redundant.

(This is the nearest thing to logical paradox, which we find in natural conditioning; there is of course no exact equivalent, since 'When this S is not P, it must be P' would imply that 'This S both can not-be P, and must be P', a natural impossibility.)

It is with this phenomenon in mind that we developed our original definition of natural necessity as 'actuality *in every* circumstance, *whatever* the actual circumstance'. Strictly-speaking the concept of 'in' is more primitive than of 'when' or 'or'; but the above dilemma serves as a clarification, anyway.

(Any seeming circularity is due to the fundamentality of the concepts involved; there is no inconsistency in that; nor is it redundant, because it aids our understanding, and the development of a formal logic of modality.)

c. In contrast, the ***simple destructive*** natural dilemma has to consist of conditional major premise with a disjunctive consequent, which combined with an actual minor premise, yields a categorical actual conclusion, as shown below.

When this S is P, it must be M or N
but, This S is not M and not N;
hence, This S is not P.

Why so? Because the other alternatives are meaningless. Had we formulated destructive dilemma as follows:

When this S is P, it must be M, and when this S is P, it must be N;
but, This S is not M and not N;
hence, This S is not P.
…we would be faced with two ordinary apodoses, each one of which would suffice to obtain the required conclusion.

If, on the other hand, we had formulated it as follows:

When this S is P, it must be M, and when this S is P, it must be N;
but, This S cannot be M or N
or even, This S must be not M or not N;

…we would be faced with incompatible premises, since the majors imply that 'This S can be M and N', and yet the minors deny that. Also, concluding that 'This S cannot be P' would deny the implication of the major that 'This S can be P'.

(If the minor premise said 'This S can no longer be M or N', instead of 'cannot', then we might assume a similar loss of power for the antecedent, and conclude 'This S can no longer be P'; however, that interpretation is far from certain: for it is conceivable that the major premise relationships are entirely different in such eventuality.)

d. With regard to ***complex*** natural dilemma, it takes the following constructive and destructive forms, for similar reasons.

When this S is M, it must be P, and when this S is N, it must be Q;
but, This S must be M or N,
hence, This S must be P or Q.

When this S is P, it must be M, and when this S is Q, it must be N;
but, This S must be not M and not N;
hence, This S must be not P or not Q.

Note that, in complex natural dilemma, there is a destructive form which is an exact analogue of the constructive, i.e. having a necessary disjunctive minor premise and conclusion. We can reduce the destructive to the constructive, by contraposing its major premise's horns, on the basis of its minor premise.

e. Lastly note, ***rebuttal*** of a natural dilemma, by a seemingly 'equally cogent' dilemma involving antithetical terms, is in no case logically possible, in view of the formal incompatibility between the needed minor premises. Try it and see.

38. TEMPORAL CONDITIONALS.

1. Structure and Properties.

a. ***Structure***. The forms of conditional proposition of temporal modality, are very similar to those of natural modality. I will therefore analyze them only very briefly. They are presented below without quantifier, but of course should be used with a singular or plural quantifier.

When S is P, it is always Q
When S is P, it is never Q
S is P and Q
S is P and not Q
When S is P, it is sometimes Q
When S is P, it is sometimes not Q

(The symbolic notation for temporal conditionals could be similar to that used for naturals, except with the suffixes **c**, **t** instead of **n**, **p**; **m** and **a** are of course identical.)

Temporal conditional propositions have structures and properties very similar to their natural analogues. There is no need, therefore, to reiterate everything here, since only the modal type differs, while the categories of modality involved remain unchanged.

Temporal conditionals signify that at all, this given, or some time(s), within the bounds of any, the indicated, or certain S being P, it/each is also Q (or: nonQ), as the case may be. (Similarly, it goes without saying, with a negative antecedent, nonP.)

Here, 'when' means '*at such times as*'. The actuals (momentaries) exist 'at the time tacitly or explicitly under consideration', the modals (constants or temporaries) concern a plurality of (unspecified) times.

The antecedent and consequent events are actualities. The modal basis of their relationship is the temporal possibility: 'this/those S is/are sometimes both P and Q (or: nonQ)'. The connection between them is expressed by a temporal modifier placed in the consequent; for constants, it is 'this/those S is/are never both P and nonQ (or: Q)', for temporaries, it is identical with the basis. The quantifier specifies the instances of S concerned.

The order of sequence of the events, though often left unsaid, should be understood. Each has a relative duration, as well as location in time. Expressions like 'while', 'at the same time as', 'before', 'thereafter', 'whenever', are used to specify such details.

b. ***Properties***. With regard to opposition, constant conditionals (like 'Whenever S is P, it is Q') do not formally imply the corresponding momentaries ('S is now P and Q', for example), although both the former and the latter do imply temporaries (their common basis, 'S is sometimes P and Q', here).

A constant like 'When this S is P, it is always Q', is contradicted by denial of either its basis or connection; that is, by saying 'This S is never P' or, 'This S is sometimes both P and nonQ'. A temporary like 'When this S is P, it is sometimes Q', is contradicted by denying the base of either or both events; that is, by saying 'This S is never both P and Q'.

Other oppositional relations follow from these automatically, and the same may be repeated for negative events. Momentaries are identical to, and behave like, actuals, of course.

The processes of translation, eduction, apodosis, syllogism, production, and dilemma, likewise all follow the same patterns for temporals as for naturals.

Temporal disjunction is also very similar to natural disjunction, and its logic can be derived from that of temporal subjunction.

2. Relationships to Naturals.

Although temporal and natural conditionals have analogous structure and properties, each within its own system, the continuity between the two systems is here somewhat more broken than it was in the context of categoricals.

In conditionals, natural necessity does not imply constancy. Compare, for instance, ‘When this S is P, it must be Q’ and ‘When this S is P, it is always Q’. Although the natural connection ‘This S cannot be P and nonQ’ implies the temporal connection ‘This S is never P and nonQ’ — the natural basis ‘this S can be P and Q’ does not imply (but is implied by) the temporal basis ‘this S is sometimes P and Q’.

Since the higher connection is coupled with an inferior basis, while the lower connection is coupled with a superior basis, the ‘must’ conditional as a whole is unable to subalternate the ‘always’ version. This is easy to understand, if we remember that even within natural conditioning, ‘must be’ does not imply ‘is’; it follows that ‘must be’ cannot imply ‘is always’, which is essentially a subcategory of ‘is’ (though it too does not imply ‘is’, as already mentioned).

This breach in modal continuity, in the context of conditionals, further justifies our regarding natural and temporal modal categories, as belonging to distinct systems of modality. In categorical relationships, these two types of modality differ merely in the frame of reference of their definitions (circumstances or times); but a more marked divergence between them takes shape when they are applied to conditioning.

For similar reasons, natural necessity does not even imply temporariness. On the other hand, temporariness does imply potentiality, since, for instance, ‘When this S is P, it is sometimes Q’ implies ‘When this S is P, it can be Q’. Here, the categorical continuity is still operative.

Also, the actualities for both types coincide: ‘in the present circumstances’ and ‘at the present time’ mean the same thing. ‘Circumstances’ refers to the existential layout of the world, how all the substantial causes are positioned in the dimensions of space; while ‘time’ focuses on the positioning of these various circumstances along the dimension of time; at any given present, these two aspects of a single happening are bound to correspond, like two sides of the same coin.

These first principles allow us to work out the valid processes which correlate natural and temporal conditionals in detail.

3. Mixed Modality Arguments.

I will not explore deductive arguments which mix natural and temporal modalities, in any great detail, but only enough to make the reader aware of their existence.

In syllogism, we should note valid arguments such as the following (which follow from **1/naa** by exposition):

1/ncc
When this S is M, it must be Q (or: cannot be Q)
When this S is P, it is always M
so, When this S is P, it is always Q (or: is never Q).

1/ntt
When this S is M, it must be Q (or: cannot be Q)
When this S is P, it is sometimes M
so, When this S is P, it is sometimes Q (or: nonQ).

However, an argument like the following would be invalid, because there is no guarantee that the circumstances for this S to be P are compatible with those for it to be Q (or, nonQ, as the case may be).

1/cnp
When this S is M, it is always Q (or: is never Q)
When this S is P, it must be M
so, When this S is P, it can be Q (or: nonQ).

This mode is invalid, note well. Although **1/ccc**, **1/cmm** and **1/ctt** are valid, the temporal conditionals **c**, **m**, or **t** are not subalterns of the natural conditional **n**.

In production, modes of mixed modal type are subalterns of modes of uniform type, in accordance with the rules of categorical syllogism. This may result in compound conclusions, as in the following case:

All P must be Q (implying, is always P)
This S is sometimes P (implying, can be P)
therefore, When this S is P, it must be Q (**1/npn**)
and, When this S is P, it is always Q (**1/ctc**)
(likewise with a negative major term.)

In apodosis, mixed-type '*modus ponens*', like the following ones in **ncc** or **ntt**, are valid (since they can be reduced to a number of **naa** arguments):

When this S is P, it must be Q (or: nonQ)
and This S is sometimes, or always, P
hence, This S is sometimes or always Q (or: nonQ).

And also, note well, mixed-type '*modus tollens*', like the following ones in **ncc** or **ntt**, are valid (since they can be reduced to a number of **naa** arguments):

When this S is P, it must be Q (or: nonQ)
and This S is sometimes not, or never, Q (or: nonQ)
hence, This S is sometimes not, or never, P.

This result is interesting, if we remember that the arguments below are not valid, since they involve inconsistent premises (the minor contradicts a base of the major):

When this S is P, it must be Q (or: nonQ)
and This S cannot be Q (or: nonQ)
hence, This S cannot be P.

When this S is P, it always be Q (or: nonQ)
and This S is never Q (or: nonQ)
hence, This S is never P.

Additionally, note, a constant major premise coupled with a naturally necessary minor premise, yield a conclusion, granting that for categoricals **n** implies **c**. Thus, **cnc** is valid, as a subaltern of **ccc**. But since **ccc** is invalid in cases of denial of the consequent, **cnc** only applies to cases of affirmation of the antecedent:

When this S is P, it is always Q (or: is never Q)
and This S must be P (implying, is always P)
hence, This S is always Q (or: is never Q).

We can similarly investigate disjunctive arguments of mixed modal type, and dilemma.

39. EXTENSIONALS: FEATURES, OPPOSITIONS, EDUCTIONS.

Very different from naturals and temporals, are the conditionals built on extensional modality. These are quite important, because they broaden the theory of classification, providing us with the formal means for more complex thinking processes.

1. Main Features.

a. ***Actual forms***. The following are prototypical forms of extensional conditional, those with three terms. The antecedent and consequent might in this context be called 'occurrences'. We will first consider forms with actual occurrences, and thereafter deal with those with modal ones.

(The forms, if need be, could be symbolized like their categorical analogues, except for, say, an ampersand '**&**' as prefix, to distinguish them also from natural or temporal conditionals.)

&**A**: Any S which is P, is Q
&**E**: No S which is P, is Q
&**R**: This S is P and Q
&**G**: This S is P and not Q
&**I**: Some S which are P, are Q
&**O**: Some S which are P are not Q

b. Basis and Connection.

The basis of all these forms is a particular proposition of the form 'Some S are P and Q (or nonQ)', which incidentally implies that 'some S are P' and 'Some S are Q (or nonQ)'. The basis is a particular conjunction of the same modality as the occurrences.

Note well, the difference between such extensional basis, and the basis 'All/this/some S can be, or sometimes is/are, P and Q (or nonQ)' of natural or temporal conditionals, which is a potential or temporary conjunction of the same quantity as the events. Contrast also to the basis of hypotheticals.

The connection implicit in 'Any S which is P, is Q' is the general proposition 'No S are both P and not Q'; and that in 'No S which is P, is Q' (meaning, 'Any S which is P, is not Q') is 'No S is P and Q'. Note that 'Any S…' can be expressed in many ways, like 'In any case that S…', or 'Whatever S…', or 'Where S….' In the forms 'Some S which are P, are Q (or nonQ)', the connection is identical with the basis.

Thus, to define the general forms of extensional conditional, we must mention both the connection and basis; the connection alone provides us only with a sort of logical conditional — an adequate basis is additionally required to form an extensional conditional. For the particular forms, the basis is all we need to define them. For singulars, we must present a specific case which fits the description; the basis follows incidentally.

The ***modal qualification*** of the relation as a whole, here, is the quantity. Note that in practice we often say 'In such case as S is P, it must or may be Q (or nonQ)', with the intent to mean an extensional conditional; here, 'must' signifies generality, and 'may' particularity. What matters, is that we mean the relationship here discussed, however we choose to verbalize it.

In extensional conditionals, it is the (general, singular or particular) quantity which expresses the (extensional) necessity, existence or possibility of the relationship, so that it is essential to the relation. In contrast, in natural or temporal conditionals, the quantity is merely incidental, allowing us to summarize many individual events in one statement.

The forms 'This S is P and Q (or nonQ)' signify that we have found an instance of the subject-concept which displays the said conjunction. An 'extensional possibility' concerning the universal S, has been found 'realized', in this pointed-to instance of S. We could have written 'In this case, S is P and Q (or nonQ)'.

The singular versions are also often expressed as 'There is (or this is) an S which is P, which is Q (or nonQ)', or 'This S is a P, which is Q (or nonQ)', to emphasize the mediative role played (which is more evident in plurals). These forms inform us, with reference to the sample of S, of the factual relationship between P and Q.

The expression ***'which'*** is interesting. It strings together two extreme terms, through the medium of a merely particular middle term. Because extensional conditionals have three terms, we do not need the distributive middle term of categorical syllogism to express the passage from minor to major term. The syllogism 'This S is P and some P are Q, so this S is Q' is invalid — unless we have inside information assuring us that the middle term is known to overlap in this case. That assurance is given us by the 'which'.

Note lastly that the consequent may be positive or negative. Needless to say, the antecedent in the above forms may equally be negative: 'In such case as S is not P,….'

c. Function.

Extensional conditionals describe 'cases' of correspondences between the manifestations of distinct universals. Though their quantity is dispensive, as in categoricals, their focus is not so much the behavior of cases as that of universals.

Note that the antecedent and consequent occurrences may coincide in time, or be unequal or separate, like any two events. They may be transient, or permanent; they may be qualitative or concern action. But the message of such forms is not primarily these dynamic details, but the extensional relations between them.

It is as if the universal involved is regarded as an individual, something in itself, which changes over time. In fact, no actual, objective change needs be taking place. The time lapse involved may be subjective, relating merely to the observer, as he or she focuses on one instance after another of the unchanging universal. In extensional modality, opposites may happen simultaneously in objective time, because they happen in different instances.

Extensional conditional propositions differ from naturals and temporals, in that they study (record, report) the behaviors of universals, instead of individuals, *as if the various manifestations of a universal are like the various states of an individual*. Extensional contingency is diversity; incontingency is positive or negative universality.

Extensional modality is concerned with instances of the subject-concept; instances are its 'modal units', instead of surrounding circumstances or times. The effective subject of such a proposition is S-ness as such. The varying cases of S, signal varying hidden (extensional) conditions, and thus serve a function analogous to the various circumstances or times in the existence of an individual thing, which are natural or temporal conditions. This explains why all these modal types have many similar characteristics.

We see here an important underlying assumption concerning universals, that they are ruled by a kind of static and plural causality, similar to and yet distinct from the mobile causality relating individual events. For natural or temporal conditioning, real change is implied; for extensional conditioning, only real difference is implied.

Here, we are still concerned with real-world causality, but it is of a clearly different type. Natural and temporal causality essentially concern the changes within individual things stretching across time and the links between them (this is true for quantified forms as well as singulars, by subsumption). Whereas, extensional causality refers to the differences and ties affecting universals as such.

The logic of conditioning for this type of modality, investigates more intricate relationships, than those dealt with by Aristotle's categorical propositions. These relationships have analogies to those found in natural and temporal conditioning, and even in logical conditioning, but they also have their own peculiar attributes and properties. We must therefore study them separately.

This research results in a better understanding of quantity and universals, and a powerful verbal and conceptual tool. The clarity of language it offers, will become apparent when we look into class-logic.

The main function of extensional conditionals is *classification, ordering of data*. These forms record the impacts of universals on each other, with reference to some or all of their instances. Extensionals are thus useful in explaining differences in structure or behavior patterns by reference to certain characteristics of the species.

For example, **in biology**. Suppose the species S1, S2, S3, display the attributes or properties {P1, Q1, R1}, {P2, Q2, R2}, {P3, Q3, R3}, respectively; we might infer that they stand in a hierarchy, proportional to the differences of degree between P1, P2, P3, or Q1, Q2, Q3, or R1, R2, R3. In this way, we conclude that, say, birds are related to reptiles, or men to monkeys. Although we have no film footage of natural and temporal transitions, we presume common ancestries (***theory of evolution***) with reference to character continuities.

But of course, strictly speaking, as our analysis of the definitional features of the various types of conditioning show, ***extensional comparisons are not proof of natural or temporal causation***. Awareness of the type of modality involved is therefore very important.

2. Modal and Other Forms.

a. Modal Forms.

The antecedent and consequent of an extensional need not be both actual propositions (as above), but may involve any combination of natural and temporal modalities. I use the actuals as standard forms, because they suffice to analyze the main logical properties of extensional conditioning, but any natural or temporal category is a fitting occurrence.

To begin with, consider an extensional conditional of the form 'This S can be P and can be Q'. Its intent is only to record that these two potentialities are each consistent with the subject-concept in the given case. The form does not insist that this S can be both P and Q at once. If we wanted to specify the latter, we would have to elaborate with a natural conditional of the form 'When this S is P, it can be Q'. Note well the difference.

Thus, the said extensional is a wider, vaguer conjunction of two categoricals: 'This S can be P, and this S can be Q', whereas the corresponding natural presents the special case: 'This S can be {P and Q}'. The natural form therefore subalternates the extensional form.

The purpose of the extensional is to specifically inform us of the identity of the indication 'this S' in the two potential occurrences, leaving open the issue as to whether or not their potentials can actualize in tandem. The purpose of the natural is to inform us of the concurrence of actual events, and not merely their potentialities, in the indicated instance, in some circumstances.

If the form 'There is an S which can be P, which can be Q' was taken to imply that that S, *as a P*, can be Q, then in cases where a P cannot be Q we would have to say 'There is an S which can be P, which can *become* Q'. It follows that in cases of uncertainty about the compatibility of P and Q, we would say: 'There is an S which can be P, which can be or become Q'.

It is therefore better to admit the extensional form in its widest sense, only implying that S can be Q, without determining whether SP can be or become Q. An extensional is concerned specifically with the extensional aspects of the relation (the coincidence of modal occurrences), and leaves the issue of circumstantial compatibility of the actual events to a natural proposition. Their functions are distinct.

The basis and connection of the corresponding general form 'Any S which can be P, can be Q' are: 'Some S can be P, and (at least) these S can be Q' and 'No S both can be P and cannot be Q', respectively. In every case, the implied basis is a positive conjunction of *particular* propositions (of equal extension), each of which has *the same* natural or temporal modality as the occurrence it underlies, note well. The connection, for general conditionals, is a general denial of the conjunction of the antecedent modality with the negation of the consequent modality. The basis and connection of the corresponding particular form 'Some S which can be P, can be Q', are one and the same proposition 'Some S can be P, and these S can be Q'

It may be mentioned here, that the colloquialism 'S can or can not be P', does not disjoin 'can' and 'can not', but rather (redundantly) disjoins 'P' and 'nonP'; it should more strictly be expressed as 'S can and can not be P' (the antinomy between P and nonP being given by the law of contradiction, anyway).

Modal extensionals, one or both of whose occurrences is/are of natural necessity, have different basis and connection. Thus, 'an S which must be P, can be Q' is based on 'Some S must be P, and these can be Q', whereas 'an S which can be P, must be Q' is based on 'Some S can be P, and these must be Q'; and similarly with two natural necessities. Although such forms happen to imply that 'these S can (or even must) be {P and Q}' (and therefore that 'some P can be Q'), that is not the primary message, and they are still very different from the natural conditionals with the same implications.

The reader is encouraged to always mentally compare, as we proceed with our study, the logical behavior of extensionals, with that of natural and temporal conditionals and hypotheticals of similar appearance. The evident differences in attributes and properties, serve to justify our making a distinction between these various forms.

We can similarly analyze other combinations of natural and/or temporal modalities, of whatever polarities and quantities. In all cases, the natural or temporal modality is effectively a part of the occurrence it appears in, and does not qualify the relation as a whole; it is the quantity which performs the task of modalizing the relation. (In that large sense, all plurals are 'modal', be their internal components actual or modal — in contrast to singulars which are 'nonmodal' with respect to extensional modality.)

Some random examples of occurrences of mixed modality are: 'Any S which must be P, is Q', or 'Some S are sometimes P and always Q', or 'There are S which can be P, yet are never Q'.

In this text, we shall of course try to use a uniform terminology, at least in strictly formal presentations. But in practice, people are not always consistent in their choice of words to express the modal type of a conditional proposition. We may for example say 'If or When S are P, they must be or are always Q' and yet mean 'All S which are P, are Q'.

To complicate matters further, we sometimes intend conditioning of mixed modal type — in structure, not just content. We may say 'when any S is P, it must be Q', and mean both that 'All S can be both P and Q' and that 'Some S are both P and Q'; here, the extensional 'Any S which is P, is Q' is tacitly understood. Effectively, we are constructing a distinct type of conditioning, using *a compound type of modality*, which expresses a two-edged probability argument.

(Note, concerning symbolization: the seeming actuality of the symbols &**A**, &**E**, &**R**, &**G**, &**I**, &**O**, is irrelevant, what matters is that they specify the polarity and extensional modality concisely. If we insist on a symbolic notation to indicate the natural or temporal modalities in antecedent and consequent, we could insert two suffixes of modality, as in &**Anp** for example. But it is better to avoid complications; if we need to, we can always write a proposition in full.)

b. Other Forms.

Extensional conditional propositions may also have more than three terms, which may be related in noncategorical ways.

The subject may remain the same in antecedent and consequent, while its predicates are more complex. For examples: 'Any S which is P1 and P2, is Q' has a conjunction of categoricals as antecedent; 'Any S among those which 'when they are P1, must be P2', is Q' has a natural conditional as antecedent. Likewise, the consequent may be more complex.

Also, the antecedent and consequent may conceivably concern different subjects. Since a 'one for one' correspondence is usually involved, though we can expect some common substratum to underlie them, and make possible their linkage somehow. For example, 'For all S1 which are P, there is an S2 which is Q' would occur if S1 and S2 are both, say, aspects of the same entity S, or are caused to occur together by some third thing S.

Extensional *disjunction* may be understood with reference to extensional conditionals. It is quite distinct in its implications from other modal types of disjunction.

With three terms and actual predications, the general form is 'S are all P or Q', meaning 'Any S which is not P, is Q, and any which is not Q, is P'. This implies that 'Some S are P and some not, and some S are Q, and some not' (bases) and that 'No S is {both nonP and nonQ}' (connective). It does not imply that all S can be P, nor that all S can be Q, note well.

Here again, the different senses of 'or' would need to be considered, as well as the corresponding particular form, 'S may be P or Q', and the parallel negative forms, 'No S is P or Q' and 'Some S are not P or Q'. More broadly, multiple disjunctions can be defined, with reference to the number of predicates which are found to occur together or apart, in any instances of the subject.

Disjunction of modal predications is also feasible, of course. For example, in 'S all must be P or can not-be Q', which means 'Any S which can not-be P, must be Q, and any S which must be Q, can not-be P, though some S must be P, and some S can not-be Q'.

Note well that the natural modalities are parts of the occurrences, and have nothing to do with the conditioning as such, which is itself extensional. Also, do not confuse the above extensional interpretation, from that of a similarly worded logical disjunction, meaning '{All S must be P} or {All S can not-be Q}'.

Similarly, with any other internal polarities and modal categories and types, in any combinations. We can also construct forms with more than three terms, like 'In all cases, an S1 is P or an S2 is Q'.

However, detailed analysis of these various forms will not be attempted here. Our treatment of the analogous forms in other types of modality, should serve as a model for further research in this area. The reader is invited to do the job.

3. Oppositions.

I shall only here sketch with a broad pen, the oppositions between extensional conditionals, among each other and in relation to categoricals. The reader should draw three-dimensional diagrams, to clarify all implications.

The singular form 'This S is P and Q' is contradicted by 'This S is nonP and/or nonQ', in the sense of a logical disjunction.

The general form 'Any S which is P, is Q' means 'Some S are P, and these S are Q, and no S is both P and nonQ'; it may therefore be contradicted by saying 'No S is both P and Q, or some S are both P and nonQ'. But each of these alternatives, whether denying the basis or denying the connection, taken by itself, is only contrary to the form as a whole.

The particular form 'Some S which are P, are Q' is contradicted by saying 'No S is both P and Q'. This may arise because 'No S is P' or 'No S is Q', but it is also compatible with 'Some S are P, and some (other) S are Q'. It follows that general denial of the antecedent or of the consequent, only contraries the basis.

Note that a proposition like 'Any S which is P, is Q', or its particular version, does not exclude the logical possibility that 'All S are P' and/or that 'All S are Q'.

In extensional conditioning, a general proposition subalternates a particular one, since the latter is identical with the basis of the former, if they are alike in polarities and modalities. But (here, unlike in natural or temporal conditioning) *a general proposition does not subalternate a singular one*; saying that 'any S which is P, is Q' does not imply that this given S is among those which are P (and therefore Q). However, a singular proposition subalternates a particular one; saying that 'this S is P and Q' does imply that there are at least some cases of S (if only this one) which are P and Q.

Comparing forms with consequents of opposite polarity, the singulars 'This S is P and Q' and 'This S is P and nonQ' are merely contrary, since they may both be false, as in cases where 'This S is not P'.

The generals 'Any S which is P, is Q' and 'No S which is P, is Q' (meaning, 'Any S which is P, is not Q') share the same partial basis 'Some S are P'; but their connectives are respectively 'No

S is both P and nonQ' and 'No S is both P and Q'; thus, they disagree on whether the S which are P, are or are not Q, and are contrary.

Note well that 'No S which is P, is Q' means more than 'No S is both P and Q' (its connective); the former has as basis 'Some S are P and nonQ', whereas the latter does not have that implication, since it may be true *because* 'No S is P' and/or 'No S is Q'.

As for 'Some S which are P, are Q' and 'Some S which are P, are not Q', they are compatible, but neither implies the other, since they may be referring to distinct cases of S. They are not subcontrary, since if 'No S is P' is true, both are false; they are therefore neutral to each other.

The parallel forms negating the antecedent can similarly be dealt with. Their antecedent is of course based on 'Some S are not P', instead of 'Some S are P', so they are bound to be compatible, with forms which imply the latter base. That is, for instance, 'Any S which is P, is Q' and 'Any S which is not P, is Q' may both be true, implying that 'Some S are P, some not, but all S are Q'. Likewise for a negative consequent.

The four particular forms 'Some S are P and Q', 'Some S are P and nonQ', 'Some S are nonP and Q', 'Some S are nonP and nonQ', are together exhaustive: one of them must be true, though up to four of them may be true.

We can also find the oppositions between extensional conditionals whose occurrences have natural or temporal modalities other than actualities. The oppositions between categoricals obviously affect this issue. For example, *provided* 'some S must be P' is given, 'Any S which is P, is Q' implies 'Any S which must be P, is Q' (note well that the modality of Q is unaffected); but it may equally be of course that 'only those S which are P and can not-be P, are Q', in which case we must say so.

Still needing to be dealt with are the oppositions between extensional conditionals, and natural and temporal conditionals. Samples of such relationships have been hinted at throughout this chapter. A fuller picture is left to the reader to try and work out.

We will not go into further detail here. Once the similarities between extensional conditioning, and natural or temporal conditioning, are understood, all their attributes and properties can be predicted by analogy, if only we switch our focus to the appropriate modal type.

4. Translations and Eductions.

Extensional conditionals may be translated into the form of conjunctions of categoricals, by eliciting their defining basis and connection. One can also abridge them without error, by forming a narrowed subject out of the original subject and antecedent predication, as in 'All/this/some SP is/are Q', since it is given that 'some S are P'.

With regard to eduction. For singulars, note the following: 'This S is both P and Q' is equivalent to 'This S is both Q and P', and implies 'This S is neither {P and nonQ}, nor {nonP and Q}, nor {nonP and nonQ}'.

For plurals, obversion is always possible, i.e. 'Any SP is Q' implies 'No SP is nonQ', and 'Some SP are Q' implies 'Some SP are not-nonQ', obviously, and vice versa.

'Any S which is P, is Q', like 'Some S which are P, are Q', converts only to 'Some S which are Q, are P'; 'No S which is P, is Q', like 'Some S which are P, are not Q', only convert by negation, to 'Some S which are not Q, are P'.

The polarities may not be changed, without their extensional possibility being first given. Thus, only knowing that 'Some S are not Q' could we contrapose 'Any S which is P, is Q' to 'Any S which is not Q, is not P'; likewise, only knowing that 'Some S are Q' could we contrapose 'No S which is P, is Q' to 'No S which is Q, is P'.

Similarly, with more complex forms involving natural or temporal necessity or possibility. For example, 'Any S which can be P, must be Q' converts to 'Some S which must be Q, can be P' without proviso, but contraposes to 'Any S which can not-be Q, cannot be P' only if we are additionally given that 'Some S can not-be Q'.

The subject S has remained the same throughout, note. Note well the differences between all these immediate inferences, and those applicable to similar looking natural or temporal conditionals.

40. EXTENSIONAL CONDITIONAL DEDUCTION.

1. Syllogism.

We can expect the valid quantity modes of extensional conditional syllogism to be analogous to the valid modality (not quantity) modes of natural or temporal conditional syllogism. The valid polarity modes are bound to be the same in all types of conditioning.

a. Extensional conditional syllogism in the *first* figure, has the valid plural modes **uuu**, **upp**. These may be validated by exposition, or we may reduce the particular version to the general ad absurdum (using the major premise). Negative moods may be derived from positive ones by obversion. The following moods are typical:

1/uuu.

Any S which is M, is Q
Any S which is P, is M
so, Any S which is P, is Q.

No S which is M, is Q
Any S which is P, is M
so, No S which is P, is Q.

1/upp.

Any S which is M, is Q
Some S which are P, are M
so, Some S which are P, are Q.

No S which is M, is Q
Some S which are P, are M
so, Some S which are P, are not Q.

Additionally, the mode **1/uss** is valid; that is, the minor premise and conclusion could equally well have been singular (though that would be closer to apodosis than syllogism). Providing the indicated instance of the subject is one and the same, the mode **1/sss** is also valid (though more to do with conjunction than conditioning), and indeed is the argument we appeal to repeatedly in exposition.

Subaltern modes are **uup**, **usp**, **ssp**. But the modes **uus**, **sus** are invalid, because here **u** does not imply **s** (unless we are additionally given that 'This S is P'). Also, **pup**, **ppp**, **psp**, **spp**, are not valid: the major premise cannot be particular.

Also note, though the major premise consequent may be negative, the minor premise consequent has to be positive, unless the middle term is negative in both premises. We can further design an equal number of valid moods with a negative minor term, by substituting nonP for P.

Lastly, we could introduce other natural or temporal categories of modality in the premises. It goes without saying that the conclusion must be altered accordingly, in each case. For example:

Any S which can be M, is Q
Any S which must P, can be M

so, Any S which must be P, is Q.

Sorites can be formed with extensional conditionals, as with categoricals.

Note that *formal relations* are often left tacit in arguments. For instance, in the last example, if the minor premise consequent had been 'must be M', we could still draw the same conclusion, since 'Any S which must be M, can be M' is formally true (it being already given that 'Some S must be M').

b. The remaining figures follow, using the appropriate methods of reduction. Typical examples of each are given below, without further ado. As in the first figure, many variations on these themes are workable:

In the *second* figure, the mode **2/uuu** is valid:

Any S which is Q, is M
No S which is P, is M
so, No S which is P, is Q.

No S which is Q, is M
Any S which is P, is M
so, No S which is P, is Q.

Similarly, with particular or singular minor premise and conclusion; that is, the modes **upp** and **uss** are valid. Subaltern modes are **uup**, **usp**; but the modes **uus**, **sus** do not work. The mode **sss** is valid, only if the middle term has the same polarity in both premises (contrary to the habitual configuration for this figure). The modes **pup**, **ppp**, **psp**, and **spp** are not valid, as before.

In the *third* figure, we have arguments like:

Any S which is M, is Q
Any S which is M, is P
so, Some S which are P, are Q.

No S which is M, is Q
Any S which is M, is P
so, Some S which are P, are not Q.

These **uup** moods (note the particular conclusion) are of course subaltern to those in **upp** or **pup**, in which one or the other premise is particular. Also valid, are moods with two singular premises, in **sss**; subaltern to this mode, are modes **uss** and **sus**, since the middle term is antecedent in both premises. But note that **uuu**, **uus**, and **ppp**, **psp**, **spp**, are all invalid modes.

In the *fourth* figure, we have (the significant mood):

No S which is Q, is M
Some S which is M, is P
so, Some S which are P, are not Q.

This argument is in mode **upp**; similarly valid is the mode **uss**, with a singular minor premise. Subaltern to these are **uup**, **usp**. All other modes are invalid, namely: **uuu**, **pup**, **sss**, **uus**, **sus**, **spp**, **psp**, **ppp**. Note that **sss** would require a contradictory middle term.

For all these figures, as in the first, other combinations of polarities may be introduced; see our treatment of this issue in the context of natural modality for full details. Likewise, as in the first figure, the occurrences may have any combinations of natural and/or temporal modalities.

c. Note well, in all the figures, the analogies between the valid modes of extensional syllogism in quantitative issues (with **u**, **s**, **p** symbols), and the valid modes of modality in natural (**n**, **a**, **p** symbols) or temporal (**c**, **m**, **t** symbols) conditional syllogism. These uniformities facilitate remembering.

However, note also, the differences between their respective treatments of quantity and modality. The valid quantity modes for extensionals differ from the valid quantity modes for naturals or temporals. And likewise, modality inferences differ. It is therefore important to be aware of the modal type of any conditional proposition.

2. Production.

The situations and results for extensional production are again clearly different from those concerning natural or temporal production.

a. In the *first* figure, production of extensional conditionals from categorical premises proceeds as in the following samples, mode **1/upu**:

All P are Q
Some S are P
therefore, Any S which is P, is Q.

No P is Q
Some S are P
therefore, No S which is P, is Q.

We thus are able to infer, given a universal major premise, a conditional universal from a particular minor; also of course inferable is the categorical 'Some S are Q (or not Q)'. In thinking of the natural or temporal type, our conclusion would have been 'Some S are P and Q (or nonQ)', instead.

The above minor premise could equally be universal, with the same conditional conclusion, in the subaltern mode **uuu**; but here a better conclusion could be drawn, the categorical 'All S are Q (or nonQ)', which subalternates the extensional conditional. This again shows us the essential continuity between categorical and conditional argument.

With a singular minor premise, a singular conclusion can be drawn, the conjunction 'This S is P and Q (or nonQ)', so the mode **1/uss** is valid.

When the premises can have modalities besides actuality, the conclusion is also modal, but it must reflect a valid categorical syllogism, as in the following samples with natural modalities. Note how, in some cases, the conclusion retains similar occurrences, whereas in other cases the conclusion may alter modality and even copula, in accordance with earlier findings.

All P must be Q
Some S must be P
(whence, Some S must be Q)
so, Any S which must be P, must be Q.

All P must be Q
Some S can be P
(whence, Some S can be Q)
so, Any S which can be P, can be Q.

All P can be Q
Some S must be P
(whence, Some S can be or become Q)
so, Any S which must be P, can be or become Q.

All P can be Q
Some S can be P
(whence, Some S can be or become Q)
so, Any S which can be P, can be or become Q.

Contrast the extensional conditional conclusion in the second of these samples, to the natural conditional conclusion which could also be drawn from the same premises, namely ‘When certain S are P, they must be Q’). Their concerns are clearly distinct.

Similarly, for moods with a negative major term. And again similarly with temporal modalities, or with mixtures of modal and actual premises, or premises of mixed modal type. In every case, the rules of modal categorical must be respected, to produce a valid extensional conditional.

b. The valid moods of the other figures follow from those of the first figure, as usual.

In figure *two*, we have mainly (mode **2/upu**):

No Q is P
Some S are P,
therefore, No S which is P, is Q.

All Q are P
Some S are not P,
therefore, No S which is not P, is Q.

Note the polarity of the antecedent of the conclusion, in the latter case. With a singular minor premise, a singular conclusion could also be drawn (mode **2/uss**), of the form ‘This S is P (or nonP), and Q’.

In the *third* figure, we can draw a general extensional, if the major premise is general and the minor particular, singular or general (**3/upu**, **usu**, **uuu**); but we can draw only a particular extensional, if the major is particular or singular, and the minor premise is general (**3/pup**, **sup**). The main moods are thus:

All P are Q (or nonQ)
Some P are S,
therefore, Any S which is P, is Q (or nonQ).

Some P are Q (or nonQ)
All P are S,
therefore, Some S which are P, are Q (or nonQ).

In this figure, a singular premise does not yield a singular conclusion, because of the inappropriate positions of the terms.

In the *fourth* figure, we have:

No Q is P
Some P are S,
so, Any S which is P, is not Q.

Likewise with a singular or general minor premise. Again, a singular premise does not yield a singular conclusion, due to the position of terms.

Production of modal extensional conditionals in these figures, is also feasible — keeping in mind the rules of categorical syllogism, as well as the above models for each figure. The reader should explore some examples.

c. Lastly, note that we can combine syllogism and production to form arguments involving a categorical major premise and a conditional minor premise and conclusion, as in the following example:

All M are Q
Any S which is P, is M,
so, Any S which is P, is Q.

The minor implies that 'Some S are M'; this, together with the major premise, produces 'Any S which is M, is Q'; which, in a syllogism with the original minor premise, in turn yields the required conclusion.

Similarly with modals, as for instance in:

All M must be Q
Any S which must be P, can be M,
so, Any S which must be P, can be Q.

Note also the following derivative argument, involving a categorical minor premise and a conditional major premise and conclusion. The fact that necessity implies possibility, because necessity is one of the species of possibility, gives us the hidden premise in parentheses, provided the categorical minor is true.

Any S which can be P, can be Q,
and Some S must be P
(whence, any S which must be P, can be P),
therefore, Any S which must be P, can be Q.

Here, we 'produce' a new, narrower, conditional from a given conditional, instead of the categorical 'All P must be Q'; or this process could be viewed as 'eduction' complicated by a proviso.

Note that extensional conditionals can also be arrived at by inductive means (observation and generalization); they do not have to be deduced by syllogism or production.

3. Apodosis.

a. Extensional apodosis follows the pattern set by the primary moods presented below. These are ***modus ponens*** (affirming the antecedent) arguments, in mode **uss**; they simply apply the principle expressed in the major premise to a singular case:

Any S which is P, is Q,	No S which is P, is Q,
and This S is P,	and This S is P,
hence, This S is Q.	hence, This S is not Q.

The major premise cannot be particular. But the minor can be universal or particular, and the conclusion will have the same quantity. The plural moods are:

Any S which is P, is Q,	No S which is P, is Q,
and All S are P,	and All S are P,
hence, All S are Q.	hence, No S is Q.

Note, with regard to the mode **uuu**, *modus ponens*, that the major premise is compatible with the minor and conclusion; general extensional conditionals only imply particular bases, and particularity means contingency or generality. Also:

Any S which is P, is Q,	No S which is P, is Q,
and Some S are P,	and Some S are P,
hence, Some S are Q.	hence, Some S are not Q.

The mode **upp**, *modus ponens*, may be regarded in two ways: (i) it teaches us that all you need for definition of a general conditional is the base of the antecedent plus the connection, because the base of the consequent follows by such apodosis; or (ii) since we know that the two bases are formally implicit, such argument is in practice redundant.

However, the latter viewpoint is incorrect, because not all conditionals are formulated from knowledge of the basis and connection, but some are arrived at obliquely, as by syllogism, so that *modus ponens* in **upp** is informative, it aids understanding of the data in hand.

b. The following moods are ***modus tollens*** (denying the consequent) arguments, in mode **uss**. These may be validated directly, by contraposition of the major premise on the basis of the minor; the conclusion is new information, emerging from the contraposite and the base of its antecedent in a *modus ponens* apodosis. Or we may validate them by reductio ad absurdum, contradicting the conclusion results in denial of the minor premise, by *modus ponens*.

Any S which is P, is Q,	No S which is P, is Q,
and This S is not Q,	and This S is Q,
hence, This S is not P.	hence, This S is not P.

The major premise again cannot be particular. The minor and conclusion can be particular; but note well that they cannot be general, since they would contradict the bases of the major. The valid plural moods of *modus tollens* are, therefore, only the following:

Any S which is P, is Q,	No S which is P, is Q,
and Some S are not Q,	and Some S are Q,
hence, Some S are not P.	hence, Some S are not P.

Thus, modes **uss** and **upp** are valid in both ponens and tollens extensional apodosis. But the mode **uuu** is only valid in ponens; in tollens, it is invalid, note well:

Any S which is P, is Q,	No S which is P, is Q,
and No S is Q,	and All S are Q,
hence, No S is P.	hence, No S is P.

c. We may of course introduce a negative antecedent into any of the arguments above or below; just replace P with nonP throughout. For examples:

Any S which is not P, is Q,	No S which is not P, is Q,
and This S is not P,	and This S is not P,
hence, This S is Q.	hence, This S is not Q.

Also, any natural or temporal modality, or mixture of them, may be involved, provided we adhere to the set interpretations of extensional conditionals. The rules of quantity of the extensional apodosis process are the same with modals, as with actuals.

The following are some examples of modal *modus ponens*. Note the faithful transmission of natural modality from consequent to conclusion. If the antecedent is necessary, nothing less than a necessary minor will activate it.

Any S which can be P, can be Q,
and This S can be (or is or must be) P,
hence, This S can be Q.

Any S which must be P, can be Q,
and This S must be P
hence, This S can be Q.

Any S which can be P, must be Q,
and This S can be (or is or must be) P,
hence, This S must be Q.

Any S which must be P, must be Q,
and This S must be P,
hence, This S must be Q.

The following are some examples of modal *modus tollens*. It is interesting how, granting the premises, we are able to draw a conclusion of opposite natural modality, as well as polarity, to the antecedent. If the consequent is potential, nothing less than a necessary minor will activate it.

Any S which can be P, can be Q,
and This S cannot be Q,
hence, This S cannot be P.

Any S which must be P, can be Q,
and This S cannot be Q,
hence, This S can not-be P.

Any S which can be P, must be Q,
and This S can not-be (or is not or cannot be) Q,
hence, This S cannot be P.

Any S which must be P, must be Q,
and This S can not-be (or is not or cannot be) Q,
hence, This S can not-be P.

d. Note well in all the above arguments, the differences between extensional apodosis, and natural or temporal such arguments.

Thus, in natural (or temporal) apodosis the major premise may be particular if the minor is general; but not here: in extensional apodosis the major must be general. On the other hand, in natural (or temporal) apodosis the consequent cannot be potential (or temporary), whereas here it can.

Such differences in process are due to the switched *roles* of the features of quantity and modality, from one type of conditioning to the next. In naturals or temporals, the conditioning is defined by the modality, and the quantity is incidental. In extensionals, the conditioning is defined by the quantity, and the modalities involved are incidental.

Lastly, note the existence here too of adductive arguments, which merely suggest a result, with some degree of probability, though not certainty:

Any S which is P, is Q

and This S is Q (is given as evidence)
hence, This S is P (is somewhat confirmed).

Any S which is P, is Q
and This S is not P (is given as counter-evidence)
hence, This S is not Q (is somewhat confirmed).

Compare extensional adduction, to logical, natural or temporal adduction. Here, we are expressing a likelihood that the indicated instance of the subject, is indeed one of the instances of the subject covered by the major premise. Note that though the conditional is general, it may be based on a very limited number of cases.

4. Extensional Dilemma.

Extensional *disjunctive* arguments are reducible to extensional conditional processes. It is important to always clarify just what we intend by the disjunction, because often different interpretations are feasible.

An example of an extensional disjunctive *apodosis*

All S are P or Q (implying Any S which is not P, is Q)
This S is not P
hence, This S is Q.

An example of disjunctive *syllogism* (reduced to two conditional syllogisms):

All S are M or Q (all S-nonM are Q, all S-nonQ are M)
All S are P or nonM (all S-nonP are nonM, all S-M are P)
hence, All S are P or Q (all S-nonP are Q, all S-nonQ are P)

Production of extensional disjunctives may likewise be achieved by production and reconstruction of extensional conditionals.

Extensional dilemma is more complicated, and worth exploring more deeply. The reader should compare it to logical, natural and temporal dilemma, to see the analogies and differences.

a. The *simple* dilemmas look as follows (taking ‘or’ to mean that one of the alternatives has to be applicable):

The simple *constructive* form:

Any S which is M or N, is P
but, All S are M or N
therefore, All S are P.

In this argument, the major premise tells us that those S which are not M, are N and P; and those S which are not N, are M and P; and none of all these S are both nonM and nonN. The minor premise tells us that all S fit the preconditions expressed in the major, yielding the conclusion that all S are also subsumed by the consequent, categorically.

The simple *destructive* form:

Any S which is P, is M or N
but, Some S are not M and not N
therefore, Some S are not P.

The major premise informs us that of all the S which are P, none is also both nonM and nonN. The minor premise presents us with some cases of S which are indeed both nonM and nonN. The conclusion is, therefore the latter S cannot be counted among the former, and there must be some S which are not P.

Note that the constructive version has a general minor premise and conclusion (mode, **uuu**), whereas the destructive version only works with a particular minor premise and conclusion (mode, **upp**). A constructive dilemma with a merely particular minor (**upp**) would yield a conclusion already known, since it is a base of the major; a destructive dilemma with a universal minor (**uuu**) would yield a conclusion contradictory to a base of the major.

In the singular, **uss** mode, the minor disjunction cannot be meant extensionally; where it happens in ordinary discourse, we intend a logical basis; the conclusion would still be valid on that basis, however. Logical basis disjunction is of course also sometimes intended within universals or particulars. More on this topic in the chapter on condensed propositions.

b. If we look at the special case of antithetical antecedent predicates:

Any S which is M or not M, is P,
but All S are M or not M,
whence All S are P.

…we see this means that '*Whether* any S is M or not M, it is P' implies 'All S are P'. This reflects the compatibility of the propositions 'Any S which is M, is P' and 'Any S which is not M, is P', provided 'All S are P' to prevent their contraposability.

We can reword it as 'Though some S are M and some others not M, all S are P'. The 'though' stresses the independence of the general consequent from the contingent antecedent: their 'link' is so strong, that it is effectively absent. This model allows us to understand universality as a type of necessity; what is found in all the cases of a subject is viewed as more ingrained in their nature, than attributes which differ from similar case to case.

c. The *complex* constructive and destructive extensional dilemmas, respectively, look as follows.

Any S which is M is P, and any S which is N is P,
but All S are M or N,
therefore, All S are P.

In this constructive version, the extensional disjunction in the minor premise ensures that all S fit the preconditions of one or the other of the horns of the major premise, and so make the antecedents exhaustive, and their common consequent general.

Any S which is P is M, and any S which is P is N,
but Some S are not M or not N,
therefore, Some S are not P.

In this destructive version, notice that the minor premise is disjunctive. It could have been, more narrowly, 'Some S are not M and not N'; but since the conclusion may be drawn by apodosis from either of these negatives without the other, we can broaden the applicability of the argument by saying 'or'. However, this disjunction may be intended as merely logical.

Note that the valid quantity modes here are **uuu** for the constructive, and **upp** for the destructive; as for simple dilemma, a constructive **upp** is uninformative, and a destructive **uuu** is logically impossible.

The singular **uss** mode is conceivable with a logical, rather than extensional, disjunctive minor, constructively or destructively; it is also conceivable in a destructive mood with 'This S is not M and not N' as the minor premise.

e. All the above forms of dilemma may of course involve antecedents and consequents of other polarities, and of natural or temporal modalities other than actuality. The rules of modality here are similar to those of modal extensional apodosis. The reader should construct some examples of modal dilemma, to get acquainted with it.

Lastly note, there is no argument in extensional dilemma, equivalent to *rebuttal* of a logical dilemma by an 'equally cogent' dilemma. The minor premises required for that would be contradictory. The reader should experiment, and find out if this statement is correct.

41. MODALITIES OF SUBSUMPTION.

We need to analyze our presuppositions regarding the modalities of subsumption by the terms of categoricals, as distinct from the copulative modalities.

1. Formal Review.

In formulating the logic of modal categoricals so far, we have taken for granted certain ideal assumptions, which will now be reviewed.

a. ***Singular subsumption***. We granted that 'All S are P' implies 'This S is P'. However, closer inspection suggests the truth of such subalternation, only on the proviso that we have directed our attention to something, which we designate by 'this', and have discerned that 'this is S'.

For, whereas 'all S' can be talked about without needing to be attentive any one S, the indicative 'this' requires a definite act of focusing on one thing, and judging whether or not it is S. This psychological requirement also means for logic that 'this S is P' and 'this S is not P' are both deniable at once, by saying 'but this is not an S'.

Thus, **A** and **R**, or **E** and **O**, are only relatively subalternative, since this relation only works conditionally; absolutely speaking they are neutral to each other. Likewise, although **R** and **G** are relatively contradictory, they are absolutely only contrary. When the preliminary judgements regarding subsumption are settled, the relative opposition comes into effect; otherwise, the absolute opposition is operative. Similarly for modal singulars.

b. ***Actual subsumption***. We granted that 'All S must be P' implies 'All S are P'. However, closer inspection suggests the truth of such subalternation, only on the proviso that there be Ss in the present actuality. We have to consider the two modalities of 'all S'.

Normally, we understand **An** to refer to 'all S, ever' (i.e. past, present, or future); although it could refer more restrictively to 'all now S'. In the timeless (i.e. across time) case, there is no guarantee that any S exist in the present actuality, taken at random. In contrast, **A** is normally understood to refer to 'all S now', since any absent S are out of the present picture; although, if we view all the scattered actualities as one actuality, then we could say that the implication holds in the timeless case.

Thus, **An** and **A** may have distinct extensions. If they both mean the same 'all S', the subalternation holds. But if **An** means 'all S at all times' and **A** only means 'all S at this time', then **An** ceases to imply **A**, unless we have already established that 'some S are actual'.

We can argue in the same way that 'All S are P' implies 'All S can be P', provided they have the same extension; if **A** means 'all S now' and **Ap** is understood to mean 'all S ever', the inference is illicit, and we can only accept that 'Some S can be P'.

Thus, **An** implies **A**, and likewise for **En** and **E**, only conditionally. Also, **A** implies **Ap**, and likewise for **E** and **Ep**, only conditionally, though they still respectively imply **Ip** and **Op** unconditionally.

For the same reason, **A** and **O**, or **E** and **I**, may both be false, if it happens that 'No S are actual'. Their contradictions apply at such times, but there are times when both can be denied. And likewise, the subcontrariety of **I** and **O** is only relative to there being actual Ss.

With regard to the interrelationships of modal propositions, since normally the subsumption of 'all S' has the same modality for all of them, such problems do not arise. They all imply, and presuppose, that Ss are potential. (It is true that if Ss do not exist even potentially, then modals behave like actuals without actual subjects; but this is another issue, dealt with later.)

Just as singulars like 'This S is P' presuppose 'this is S', so with any actual propositions we have to assume that 'there are Ss at this time': these are separate, preliminary judgements, which affect the logical properties of the propositions that conceal them.

c. ***Subsumption by the predicate***. The above concerns subsumption by the subject. With regard to the predicate, it seems obvious that, if the subject of an affirmative, actual or necessary, proposition is actual, then so is the predicate, for the same extension. On this basis, we can convert 'all or some S are P' to 'some P are S'. Also, since necessity implies actuality when the subject is actual, **An** and **In** can be converted to **I**, under those conditions; otherwise, only to **Ip**.

In the case of the corresponding negatives, it would at first sight not be thought that the predicate needs be actual. However, if 'No S is P' is to be converted, there has to be actual Ps to support the actuality of the inference; if this precondition is not met the eduction is invalid. If only some P are actual, then **E** is convertible, but only to 'Some P are not S'; if all P are actual, **E** is convertible fully to 'No P are S'; if no P are actual, nothing actual about Ps may be denied or affirmed.

Also, since **En** implies **E**, conversion of 'No S can be P' to 'No P is S' is only conditionally feasible, even though that to 'No P can be S' is independent of actuality. (Note however, in passing, that the conversion of **En** to **En** does presuppose the potentiality of the predicate.) For **On** and **O**, such problem does not arise, since they are inconvertible in any case.

d. All the above can be repeated with reference to temporal modality.

2. Impact.

Thus, the singular 'this' and the plural 'all' or 'some' are more weakly related than previously intimated. Also, actual copulae require at least actual subsumption by the terms, whereas modal copulae (whether necessary or possible) need only possible subsumption by the terms. The type of modality subsuming the terms corresponds to the type affecting the copula. In natural modal propositions, the subsumptions are potential; in temporals, they are temporary.

Thus, we have seen that many processes adopted as standard by both actual and modal logic, are only conditionally true. Some other logical processes, which depend on those considered above for their validity, may be expected to in turn become equally conditional. For example, if **E** is only conditionally convertible, then obverted conversion or inversion of **E** is likewise restricted.

Even syllogism may be affected. We have to look at the results of arguments, to make sure they are unconditional with reference to modal subsumption. For example, the mood **4/EIO** does convert both its premises unconditionally, because the middle term in the minor premise, allows conversion of the middle term in the major premise. In contrast, the mood **3/RRR** was rejected, essentially because the degree of specificity of the middle term could not be transferred to the minor term; but we could equally view this mood as conditionally valid, if we can indicate the subject.

We had made some ideal assumptions, to better emphasize the essential natures of the forms under consideration. These assumptions are reasonable — one would not normally formulate a proposition unless its subsumptive conditions seemed fulfilled; it is only in further ratiocination that an illicit process may occur, which yields a presumptive subsumption. However, we must be made aware of the exceptions and provisos, so that the system as a whole remain unassailable.

Thus, an avenue for further logical analysis is to check the unconditionality or conditionality of all our validations or rejections of logical processes. That investigation is left to the reader.

Note well that these theoretical requirements are not necessarily fulfilled in practice. There is a difference between 'common parlance', which is more flexible and approximate, and the ideal language of formal logic, which must needs have fixed and precise meanings.

For example, when in practice we say ‘All S are P’, we often mean **A**, but may also mean **An** or **Ac** or **Ap** or **At**, or even sometimes just **I** or **Ip** or **It**. Also, we may mean ‘all now’ or ‘all ever’. We may even misrepresent the terms. This is all harmless, if our thought is clear enough to oneself and successfully conveyed to others. One can reason logically with the rough sentences of everyday language, but there is less likelihood of error using formal language.

So long as the normative system is capable of verbalizing all situations encountered in practice, it is successful and sufficient. Thus, the science of Logic must extend its tentacles as far as necessary, enough to make possible the verbalization of any intention we may encounter in practice.

In that case, all casual statements must be carefully reformulated, to fit the standard forms provided by Logic, before they can be subjected to its rigid analysis. It is impossible to develop a system of Logic which parallels common practice exactly, because the variations in it are too arbitrary and too subjective.

Obviously, if the standard forms are not properly used, if the translation picks the wrong forms to express our pre-verbal intention, the results are likely to go awry. The process of forming a clarified thought is by no means automatic and guaranteed.

3. Primitives.

Completely categorical propositions may be called primitives. They vary in degree of *specificity*, but conceal no conditions.

Indication is the instrument of full specification. Only something which is precisely indicated — extensionally, naturally and temporally — is fully specified.

The indicative, singular and actual: ‘this thing, at this time, and in these circumstances, is so and so’, refers to an unnamed, pointed-to thing, existing in a pointed at time and set of circumstances. This form is specific extensionally, and temporally and naturally.

‘This’ (or that or these or those) is a sui generis term, which is meaningless without the presence in front of one of what is being referred to. One can say that ‘this is not so and so’ (to deny a statement starting with ‘this so and so is…’), *but one cannot say “this is not a ‘this’”*.

The next level of specificity is the indefinite, particular, actual: ‘there are, at this time and in these circumstances, some things which are so and so’, which informs us that, out-there somewhere in the world, ‘some things are so and so’. This form is unspecific extensionally, though still specific with regard to time or circumstance.

Further down the scale, the indicative singular modal ‘this thing is possibly so and so’, and the particular modal ‘some things are possibly so and so’, are indeed categorical, but unspecific. Note well that ‘this thing’ in singular modals is less specific than ‘this thing’ in singular actuals; because the latter concerns an actual relation, whereas the former concerns a modal one. The indicative is less demanding, here. Likewise for ‘some things’, the modality of subsumption depends on the modality of the copula.

The above mentioned primitive forms are the only absolutely categorical propositions. All other ‘categorical’ forms used by formal logic are more complex, and thus implicitly conditional. Their categorical format is somewhat conventional, artificial — hiding their compositeness.

The singular actual ‘This S is P (or not P)’ presupposes that ‘this thing is indeed S’, which may be said to specify the subject under discussion. As well, all actuals require and imply that the units subsumed by their terms be as actual as the copula between them (else, how would the relationship be viewed as actual?). Here, natural circumstances or times are being tacitly specified.

Plural actuals ‘All or Some S are P’ presuppose that ‘some things are indeed S’, which just means ‘there are actually unspecified Ss out there’. The specific actuality involved is supposed to be clearly understood.

Modals only require and imply that ‘this or some thing(s)’ — ‘are in some circumstances S’ (in the case of natural modality) or ‘are at some times S’ (in the case of temporal modality). Here, the circumstances or times for S remain unspecified, implying mere potentiality or temporariness of the subject, rather than a specified 'this now'.

Similarly for the predicate, whatever the polarity of the copula, if conversion is accepted. We could alternatively, consistently, say that conversion of a universal negative is a valid process, only if the predicate is specific; in which case, the predicate of negative propositions does not need to be formally specific.

We cannot consistently say that all propositions are conditional, because then we would have no way to express categorically that the conditions have been met (as in apodosis). But it is logically permissible to regard the primitive statements 'This thing is actually S' and 'There are actual Ss' (= some things are S), as the only truly categorical forms, while all others as only relatively categorical.

4. Transformations.

Let us, therefore, reword the more complex categorical forms, in such a way that their implicit assumptions are brought out in the open, using primitives. We may call this 'transformation'; it is done below, for actuals, then potentials, then naturally necessary propositions. A parallel listing can be made for temporal modality. We see that they all concern conjunctions involving the two terms, with varying degrees of specificity and complexity.

R: 'This thing is now S and P'
G: 'This thing is now S and not P'
I: 'Some things are now S and P'
O: 'Some things are now S and not P'
A: 'Some things are now S and P, but nothing is now S and not P'
E: 'Some things are now S and not P, but nothing is now S and P'

Rp: 'This thing can be S and P'
Gp: 'This thing can be S and not P'
Ip: 'Some things can be S and P'
Op: 'Some things can be S and not P'
Ap: 'Some things can be S and P, and no other things can be S and P'.
Ep: 'Some things can be S and not P, and no other things can be S and not P'.

Rn: 'This thing can be S and P, but cannot be S and not P'
Gn: 'This thing can be S and not P, but cannot be S and P'
In: 'Some things can be S and P, but none of these things can be S and not P'
On: 'Some things can be S and not P, but none of these things can be S and P'
An: 'Some things can be S and P, but nothing can be S and not P'
En: 'Some things can be S and not P, but nothing can be S and P'

Thus, the forms 'All S are P' or 'Some S cannot be P', and such, are really abbreviations, shorthand versions, of the above, more descriptive, forms. Their full definition shows many of them to be conjunctive, of two or more primitive categorical propositions.

Notice that the implicit conditionalities, may be a mix of extensional and natural, modal subjunctions. Plurals may be reworded in extensional conditional form, and modals in natural conditional form; so plural modals will involve both types of subjunction, one inside the other. Thus, we may have an extensional conditional, whose antecedent and consequent are two natural conditional propositions, involving different polarities.

For examples. **Ap** means: 'For some things: in some circumstances, S and P coincide; but for other things: in no circumstances do S and P coincide'. **An** means 'For some things: in some circumstances, S and P coincide; but for all things: in no circumstances do S and nonP coincide'.

Similarly with temporal modality, instead of natural, throughout.

I will not here analyze such forms further, although this is the obvious next step in the logical development of a complete system of modal logic. We would want to verify that the oppositions, eductions and syllogistic arguments, which were developed for complex categoricals, remain in force, when the later are transformed into their clearer, subjunctive versions. (If any inconsistencies in properties are uncovered, the transformations would have to be further perfected, until consistency is indeed achieved.)

5. Imaginary Terms.

Another issue relating to modality of subsumption is, how to view imaginary terms. This is a further complication, concerning logical modality.

An imaginary term may be built up out of certain suppositions and/or assumptions. 'Supposition' concerns what is already granted to be true in some cases, and/or in some circumstances or times, is singularized in an indicated instance and/or actualized in an indicated circumstance or time; whereas assumption concerns the granting of such particular, and/or potential or temporary subsumptions, to begin with.

Thus, supposition is based on given extensional, and/or natural or temporal possibility, and only presumes applicability to the specific instance or actuality; whereas assumption involves hypothetical constructs, it presumes the realization of what is merely logically conceivable. They differ in audacity, the former having more empirical grounds than the latter; but ultimately, they are both presumptive, bringing together certain events or characteristics in novel conjunctions, *with more specificity than contextually justified*.

Just as, with regard to extensional, natural or temporal modality, the modalities of the copula and terms affect each other — so, with regard to logical modality, the modalities of the copula and terms, are proportional. If a proposition involves some term of less than established status, then its truth is correspondingly no more than conceivable.

A concept which is believed to involve no presumptions may be viewed as realistic, while a concept is imaginary to the degree that it involves suppositions and assumptions. If we are at a stage where the projected parameters are still conceivable, then our concept tends towards realism with varying success. If we know already that the projections are not realizable, then our concept will remain imaginary.

In science, we construct imaginary concepts in the hope of eventually establishing them as realistic. But literature allows for pure imagination, whether it is in the form of a novel built on the suppositions that certain particulars, potentials or temporaries are in effect, or in the form of science fiction or fantasy built on unrealistic assumptions. The latter kind of imagination has no pretensions of literal truth, it is mere entertainment or example setting.

Thus, we can say that, apart from deliberate fictions, the difference between imaginary and realistic concepts is one of degree of contextual credibility. The degree is greatest, if no presumption was involved; intermediate, if only supposition was involved (the less supposition, the more realistic); and least, if assumption was involved (the more assumption, the more imaginary).

Our belief of a proposition is a function of our belief in its terms. If a term is imaginary, then we do not in the fullest sense accept the proposition as true, even if the formula makes internal sense. As the chances that our term be realistic increase, so accordingly does the proposition as a whole become closer to 'true' in the ultimate sense.

Thus, the hypotheticality of a term influences the degree of truth in the proposition. But such conditioning must be the exception, rather than the rule. We cannot consider knowledge as hypothetical ad infinitum: there has to be some definite knowledge.

Some propositions must be admitted as categorically true; the proof is that, if we claim all knowledge hypothetical, we thereby posit that claim as unconditionally true, and thus contradict ourselves. Because some propositions are unconditionally true, then these at least must involve realistic terms: ergo, some concepts must be admitted as realistic.

In practice, we commonly call even propositions with fictional terms 'true' — this is in the sense of internal consistency within a narrow framework, without regard to the unrealizability of the terms. For example: 'Dragons are lizard-like' has a mythical subject and yet is in a sense 'true'. Here, 'truth' merely signifies an accurate description of a mental image known to be fictional; effectively, there is a tacit bracket saying 'all this is imaginary'.

Closer scrutiny reveals that our example really means (should be rephrased as) 'We have formed a fantasy, to be called a dragon, with an arbitrary description including the shape of a lizard': so formulated, the proposition is factual. The format 'Dragons are lizard-like' is merely an abbreviation of that true statement; but taken literally, it is false (since there are no dragons).

In practice, we often have a fictional predicate in a negative proposition, as in 'Lizards are not dragons'. This is formally more justifiable, since we can regard the convertibility of universal negatives as conditional on the factuality of the predicate. We could then demand that all subjects be factual, since nothing can really be said about nonexistents, without insisting on the same requirement for predicates. If so, 'X does not exist' would have to be worded 'No existents are X'.

In conclusion, just as, with regard to the extensional, natural and temporal subsumption, we said that, whatever the polarity the copula, the terms must be specified in primitive form (indicatively for singulars or actuals, through the corresponding possibility for plurals or modals) — so, with regard to logical modality, subsumption in fully true propositions must be factual (or necessary), whereas subsumption in logically modal propositions need only be logical possibility (of varying degree: from mere notion, through relevant and consistent, up to logically necessary).

The rules of subsumption are essentially the same for all types of modality. In logical modality, a proposition has to be conceivable at some level, however low. The minimum requirement is that the words involved all mean something. That something may be any kind of appearance: one either rightly believed in, or realistic but disbelieved or unsure, or wrongly believed in, or unrealistic and disbelieved or unsure. But there must in any case be some kind of appearance, whether empirical, conceptually arrived at, or imaginary, which serves as the intent of the word.

These restrictions concern any proposition presented as having some possibility of truth. False propositions are not subject to law; they can even be meaningless or self-contradictory. Likewise, the antithesis, 'nonX', of a meaningful and consistent term, need not itself be so conceivable.

The various types of modality should not be viewed as making up a hierarchy along one line. Rather, each is like a dimension, at right-angles to the others, with analogous categories of modality. They thus are capable of combining together, while remaining mutually independent continua.

42. CONDENSED PROPOSITIONS.

Conditional propositions provide us with a powerful formal language, enabling us to elucidate a large variety of derivative forms we commonly use.

1. Forms with Complex Terms.

We are now in a position to consider 'condensed' propositions, which have a conjunctive or disjunctive subject and/or predicate. These propositions are made to appear like single categoricals, through this device of complex terms, but in fact conceal two or more standard propositions.

a. The 'subject' may be a conjunction of subjects. Thus, 'Every S1 and S2 is P' normally means that whatever is {both S1 and S2}, is P, without implying (nor denying) that something S1 but not S2, or something S2 but not S1, satisfies the condition for being P.

Note in passing that complex propositions like 'all ABC are XYZ' are often used in practice, because we try to abbreviate a multiplicity of relations in a minimum of words. Such a form implies smaller statements like 'some A are B' or 'some A are X' or 'some X are Y'. Such implicits may be the precise premises of categorical syllogisms, rather than the more complex form which is presented as a premise. This explains why theoretical logic may seem so much more bare than practical examples. For instance, 'my computer sounds like a duck' contains many smaller statements like: 'I have a computer' or 'my computer makes sounds'.

b. The 'subject' may be a disjunction of subjects. Thus, 'Every S1 or S2 is P' normally means that every S1 is P and every S2 is P, without telling us whether anything may or may not be S1 but not S2, or S2 but not S1.

c. The 'predicate' may be a conjunction of predicates. Thus, 'Every S is P1 and P2' means that every S is P1 and every S is P2.

d. The 'predicate' may be a disjunction of predicates. Thus, 'Every S is P1 or P2' normally means that some S are P1 and some S are P2, without telling us whether or not any S is both P1 and P2.

However, other interpretations of some of these forms are feasible. Firstly, our interpretation depends on whether the 'or' is understood as an 'either-or' or as an 'and/or' or as an 'or-else', and on how definite these disjunctions are. Secondly, our interpretation depends on the type of modality intended: is the 'or' intended to convey an extensional disjunction, or a natural or temporal one, or even a problemacy?

Form (a) admits of singular or particular versions 'this/some {S1 and S2} is/are P'. Note that we often in practice intend the conjunction more loosely, so that we really mean the same as form (b).

Form (b) was above understood extensionally, so that the predicate was dispensed to both subjects generally. In that case, there are no corresponding singular or particular versions. However, we can say 'this/some S is/are {P1 or P2}', if we regard the disjunctive clause as a whole, rather than the disjuncts, as the subject. In this case, how the 'or' is understood, and the type of modality involved, becomes more variable.

Form (c) is the least ambiguous of them all, and readily admits of singular or particular versions 'this/some S is/are P1 and P2'.

Form (d) was above understood extensionally, so that both predicates were dispensed to the subject particularly. In that case, there are no corresponding singular or particular versions. However, we can say 'this/some S is/are {P1 or P2}', if we regard the disjunctive clause as a whole, rather than the disjuncts, as the predicate. In this case, how the 'or' is understood, and the type of modality involved, becomes more variable.

To illustrate alternative interpretation, consider the form 'Every S is P1 or P2' again. If we wanted our 'or' to suggest that S may be split into two groups S1, S2, such that no S1 are S2, and all S1 are P1 and all S2 are P2, it would not suffice for us to say that 'some S are P1, and all other S are P2'. We would have to make use of extensional conditionals, as follows: 'any S which is P1, is not P2; and any S which is P2, is not P1'.

This last form is important because it introduces fractionating of a subject, which topic will be dealt with in more detail later.

Similarly, natural conditionals may be used to express other interpretations, such as 'when any S is not P1, it is P2; and when any S is not P2, it is P1'. And likewise with temporal modality. We can even understand the 'or' in a logical sense, even as a mere problemacy; for instance, 'if all S are P1, no S is P2; and if all S are P2, no S is P1'.

Thus, we see that the ambiguities of such condensed forms are dealt with through the instrument of our more precise conditional forms in each type of modality, and we do not need to develop a new logic for them (except as an exercise).

The condensed forms presented above were all affirmative and actual. We may similarly analyze negative forms, like 'No S1 and S2 is P', or modal forms, like 'this S must be P1 or P2'. Note in passing that exceptive propositions, like 'all S but S1 are P', can be similarly analyzed. Note also that conditional propositions may also involve complex terms, and are similarly analyzable in a multitude of ways.

I will not go into such detail, however: I must move on; the job is left as an exercise for the reader, and for other logicians. It is clear, in any case, that the same principles apply.

In conclusion, it should have become obvious by now that the issue of complex terms, involving a conjunction or disjunction of subjects or predicates, is ultimately an enlargement of the issue of modalities of subsumption. Also, it shows clearly that the distinction between categorical and conditional forms, is ultimately somewhat arbitrary; there is a continuum of forms running into each other.

2. Making Possible or Necessary.

Another, unrelated, family of forms which condense conditionals, can be mentioned at this juncture: making possible or making necessary. These relate to causality. Here again, the concepts involved can be applied to any type of modality.

'P makes Q possible' signifies, in logical modality, that if nonP, then nonQ (nonP and nonQ are possible, and 'nonP and Q' is impossible), whereas if P, not-then nonQ (P and Q are possible). This commutes to 'Q is made possible by P'. Clearly, only on the condition of P being true, does the possibility of Q being true have an effective chance of arising; P is thus said to be an *exclusive condition* for Q, a *sine-qua-non*.

'P makes Q necessary' signifies, in logical modality, that if P, then Q (P and Q are possible, and 'P and nonQ' is impossible), whereas if nonP, not-then Q (nonP and nonQ are possible). This commutes to 'Q is made necessary by P'. Clearly, the truth of P alone would raise the mere possibility (within a contingency) of Q's truth to an effective necessity (more precisely, it is Q's realization, rather than Q itself, which becomes necessary); P is thus said to be a *sufficient condition* for Q.

These concepts 'making possible' and 'making necessary' are obviously correlative. If P makes Q possible, then nonP makes nonQ necessary; and if P makes Q necessary, then nonP makes nonQ possible. Also, if P makes Q possible, then Q makes P necessary; likewise, if P makes Q necessary, then Q makes P possible.

We have other concepts of a similar nature. Thus, 'P is *necessary for* (or to) Q' means that without P, Q would not be true; which is equivalent to 'P makes Q possible'. Again 'P *necessitates* Q' means that in order for P to be true, Q would be required to be true; which is equivalent to 'P makes Q necessary'.

We can similarly analyze the concepts of 'making impossible' (implying prevention or inhibition) and 'making unnecessary'.

All this can be duplicated in other types of modality. Thus, in natural modality, 'When this S is not P, it cannot be Q, and when this S is P, it can be Q' implies that P makes Q potential in this S; also, 'When this S is P, it must be Q, and when this S is not P, it can not-be Q' implies that P makes Q (that is, Q's actualization) a natural necessity in this S. Similarly with regard to temporal modality. In extensional modality, 'Any S which is not P, is not Q, and some S which are P, are Q' implies that P makes Q possible for S's; also, 'Any S which is P, is Q, and some S which are not P, are not Q' implies that P makes Q necessary for S's.

When issues of sequence arise, the possibility or necessity involved may be, if not simultaneous, precedent or subsequent in time.

These concepts obviously refer us to the various types and categories of *aetiology*. They allow us to begin a classification of causes. Within each modality, some causes are both exclusive (or necessary) and sufficient (or necessitating); some are only the one or the other; some are neither of these, but rather *occasional* (or contingent), meaning that they depend on additional partial conditions (a conjunctive antecedent) to effect the consequence. Many more subdivisions of causality are of course possible.

Clearly, a *formal logic of causality* can be derived from the logic of conditioning. Forms like 'A being B causes C to be D' are commonly used, and capable of precise analyses, by specifying the category and type of modality involved. I will not here develop this field of logic, since it is derivable; but it is important, and should be eventually done.

Many statements conceal this sort of form. In some cases, syllables like 'en-' or '-fy', meaning 'to make', are used to signal causality, as in 'he verified the statement', meaning 'he (did something which) caused the statement to be (accepted as) true'. In some cases, the verb entirely hides the causal aspect, as for instance in 'water dissolves these crystals', the verb 'dissolves' means 'causes to dissolve', and we can rephrase the whole more precisely as 'when some water is mixed with these crystals, a solution is obtained'.

We enter here, also, into the subordinate realm of *teleology*, the study of needs in the context of given goals or purposes. For examples, with making possible. In logical pursuits: what must I prove first, before I can prove so and so? In natural or temporal causation: what should I do, in order to achieve so and so? In extensional choices: which of these things should I choose, to obtain so and so?

These forms play an important role in the formal logic of *ethical modality*. Granting certain standards of value, all the ways and means follow, with reference to objective aetiological (and teleological) relations. Something is permissible if compatible with *all* our ends, impermissible if incompatible with some of them; something is imperative if a *sine-qua-non* of some of our final causes, unimperative if not a *sine-qua-non* of any of them. A full study of ethical modality would have to analyze volition, and discuss the source of our ultimate norms. These issues are of course beyond the scope of the present treatise.

PART V(a). CLASS LOGIC.

43. THE LOGIC OF CLASSES.

1. Subsumptive or Nominal.

We have to distinguish between the *subsumptive* use of a word, and its (say) *'nominal'* use. In the former case, the word has an only incidental role, serving to direct our minds to the objects we label by it; in the latter case, the word itself is the object of our attention, while the things it refers to are incidental. Thus, for example, when we speak of dogs, we think of our tail-wagging and barking friends; but when we speak of "dogs", we mean the word enclosed by inverted commas itself.

This should not be confused with the distinction between denotative and connotative terms, which we made in discussing permutation. We can take denotative terms subsumptively or nominally (as we did with above, comparing dogs and "dogs"), and we can take connotative terms likewise subsumptively or nominally (for example, caninity and "caninity"). That is not what is at issue here. What is at issue is, whether our focus is purely objective (as in, dogs and caninity), or we are focusing on the instrument (as in, "dogs" and "caninity").

Thus, in subsumptive intent, we mean what the word refers to, and the instrument is transparent; whereas in nominal intent, the instrument itself is what we mean, and what the word refers to specifically is of lesser moment. Normally, our intent is subsumptive (let us symbolize this as an X); but sometimes, especially in epistemological discussions, our intent is nominal (let us symbolize this as "X").

Nomenclatural propositions have the primary forms: "X" is the name of all X; or: all X are the referents of "X".

We may extend the distinction between subsumptive and nominal intent to other aspects of our instruments of thought, not only to the verbal. An 'idea' or a 'class', or any similar construct, may like a word be considered 'nominally', in contrast to subsumptively. This does not mean to imply that words, ideas, classes, and such, are all one and the same thing, but only that they have in common the property we mentioned. There is no doubt that they are significantly different concepts, yet also somehow related.

The precise relation between these various concepts is not the topic of this chapter. Rather, we shall specifically explore some of the mechanics of classification, in an effort to better understand the logical relations between things and our concepts of them. This research into *the way knowledge is organized*, has been of great interest to logicians of this century, under the heading of 'the logic of classes'.

2. Classes.

We think of a class and its members, as having a similar relation to that of a receptacle and the things it contains. The container, an elastic and permeable wrapping, is a figment of our imagination; yet, its shape and size are determined by the contents. This visual analogy is not perfect, but is a starting point.

The subsumptive outlook is directed at the contents, labeling each member as X; this is the only kind of classificatory relationship we have dealt with so far: it is the concern of Aristotelean logic. The nominal outlook is directed at the container, labeling the class as "X"; this gives rise to a new field of logic.

a. Definitions.

For any thing X, we can invent a corresponding thing "X", such that:

- whatever is X, is a member of "X"; and whatever is not X, is not a member of "X".

Conversely, we say:

- "X" is the class of (all) X; and "X" is a class of anything which is X, and not a class of anything which is not X.

These two sets of statements mean the same thing, they are just two sides of the same coin, they commute each to the other; we call the whole relationship 'class-membership'.

The above begin to formally define the difference between what we mean by X and "X", relating them through a new pair of copulas, which are different from the copula 'is'. In one direction, the copula is labeled 'is a member of', and has a subsumptive as subject and a nominal as predicate; in the other direction, the copula is labeled 'is a class of', and has a nominal as subject and a subsumptive as predicate.

Since in speech, unlike writing, we have no way to display inverted commas, we merge the two and say: this thing is a member of the class of X; the latter expression, class of X, is equivalent to "X", in relation to X. We understand "X", or the class of X, as a mental construct of some sort, which we intend to bear a certain relation to the things we have labeled as X. We assign the virtually same label to the construct as we did to the original things, except for a small distinguishing mark ("" or the class of) to keep their distinction in mind.

The plain name is subsumptive, referring directly to the things concerned, the marked name is nominal, referring rather to the invented correspondent of the things. Note that, although the member to class relation has some similarity to the relation of an individual to a group, they are not identical. The subsumptive versus nominal distinction, should not be confused with the dispensive versus collective (or even collectional) distinction, which we made earlier (in the discussion of quantity).

Thus far, what we have done is to point to a set of phenomena, which we commonly encounter in our current ways of thinking, and sorted them out somewhat, and named the various factors. But all we have achieved is at best a technical definition; a fuller definition requires some further understanding of the distinctive properties of these factors. That is what we will look into now.

Consider the following example, which accords with our normal manner of speaking. Dogs are dogs, and are members of "dogs" (or the class of dogs). But, dogs are not "dogs", only members of "dogs"; and "dogs" is not a dog, and not a member of "dogs". Note that there is no self-contradiction in saying that dogs are not "dogs", or that "dogs" is not a dog, even though the statement that dogs are not dogs is of course absurd.

Such examples suggest the following features and processes. (Note that I concentrate mainly on the properties of 'is a member of'; those of 'is a class of' follow obviously, I do not highlight them, to avoid repetitions.)

b. Features.

Whereas in a proposition of the form 'this thing is X', the subject and predicate are both subsumptive — in a proposition of the form 'this thing is a member of "X" (or the class of X)', ***the predicate is nominal***. This principle is necessary, because the whole concept of membership was built with the intent to study that special kind of term we call nominal. Membership by definition relates any kind of thing to one kind of thing specifically: mental constructs.

With regard to the subject of membership, the above definition concerns only subsumptive subjects, but we shall presently consider nominal ones.

What is X, is not "X", but only a member of "X". The copula 'is a member of' positively relates two things, X and "X", which the copula 'is' negatively relates, at least in examples like ours (dogs are not "dogs", but only members of "dogs").

"X" is not an X, nor a member of "X". A class is not a member of itself: the relation of membership is not reversible, at least not in examples like ours ("dogs" is not a member of "dogs", since it is not a dog).

With regard to the latter two principles, the examples only prove that they hold in some instances; however, we may generalize from such cases, if we find no examples to the contrary.

c. Immediate inferences.

Obviously, by definition, since all X are X, all X are members of “X”; and since no nonX are X, no nonX are members of “X”. The class of X includes all things which are X, and excludes all things which are not X. Similar eductions apply for the class of nonX, or “nonX”. It follows that membership in “X” and membership in “nonX” are exact contradictories.

More broadly, we can infer from the above definition of membership that: ***if any X is Y, that X is a member of “Y”; and if any X is not Y, that X is not a member of “Y”***. Any thing which is X and also Y, is an X which is a member of “Y”; any thing which is X but not Y, is an X which is not a member of “Y”. That is, “X” is *the* class of all X, but not *the only* class for any X; there are normally other classes like “Y”, of which we can say that it is *a* class of some or all X. For examples, retrievers are members of the class of dogs, but not members of the class of cats.

It follows that: if all X are Y, then all X are members of “Y”; if only some X are Y, then only some X are members of “Y”; if no X is Y, no X is a member of “Y”. (Note in passing, in the middle case, we regard the membership of some X in “Y”, as ‘accidental’, or ‘incidental’ to their being X, since not all X fall in this category; or we say that these X are Y, but not *‘qua’* X or not as X ‘*per se*’, not by virtue of being X.)

These statements are reversible: if all X are members of “Y”, then all X are Y; if some X are members of “Y”, then some X are Y; if some X are not members of “Y”, then some X are not Y; if no X are members of “Y”, then no X are Y.

d. Deductive arguments.

We thus can construct the following syllogisms for the copulas ‘is (or is not) a member of’, on the basis of Aristotelean syllogisms for the copulas ‘is (or is not)’.

Figure 1.
All Y are members of “Z”,
all/this/some X is/are member(s) of “Y”,
so, all/this/some X is/are member(s) of “Z”.
Likewise, with negative major and conclusion.

Figure 2.
No Z are members of “Y”,
all/this/some X is/are member(s) of “Y”,
so, all/this/some X is/are not member(s) of “Z”.
Likewise, with positive major and negative minor.

Figure 3.
All/this/some Y are members of “Z”,
some/this/all Y is/are member(s) of “X”,
so, some X are members of “Z”.
Likewise, with negative major and conclusion.

Figure 4.
No Z are members of “Y”,
some Y are member(s) of “X”,
so, some X is/are not members of “Z”.

Such deductions are easily validated, by translating them into their customary forms. Note that a term may be subsumptive in one proposition and nominal in another, according to its position by virtue of the figure.

e. Modal class-logic.

The definition of class-membership is easily modalized, if we wish to work out a more modal class logic.

Thus, for natural modalities: if something can be X, then it can be a member of "X"; and if something cannot be X, it cannot be a member of "X"; if something must be X, then it must be a member of "X"; and if something can not-be X, it can not-be a member of "X". Similarly for temporal modalities. The quantification of these singular forms represents extensional modality.

Note that these definitions are in the form of extensional conditionals. The logical properties of their consequent forms are easily derived from the modal logic of their antecedent forms, which are ordinary categoricals. That includes: oppositions, eductions, and deductions.

3. Classes of Classes.

In the previous section, we defined and analyzed the membership of a non-class (subsumptive) in a class; now, we need to look into what we mean when we say of a class (nominal) that is a member of another class.

a. Definitions.

We propose that, for any X and Y:

- if all X are Y, then "X" (or the class of X) is a class of Y, and therefore is a member of "classes of Y", (or the class of classes of Y).

Conversely:

- if less than all or no X are Y, then "X" (or the class of X) is not a class of Y, and therefore not a member of "classes of Y" (or the class of classes of Y).

Note the variety in wording; we also often abbreviate 'class(es) of Y' to '*Y-class(es)*'.

This definition of so-called classes of classes reflects our common practice. For examples, since all dogs are animals, "dogs" is an animal-class, or a member of "animal-classes"; but since some dogs are not black animals, "dogs" is not a class of black animals, or a member of "classes of black animals".

Now, this is an artifice. The reason why we construct this new concept is that we want to be able to talk about classes in the same way as we talk about things. We build up a parallel domain, in which classes bear relations to each other, somewhat similar to the relations between their ultimate referents. Thus far, the stratification of things had no equivalent in the realm of classes, since nominal terms were defined as predicates of the 'is a member of' copula. In order to place classes as subjects of similar propositions, we introduce appropriate special predicates: classes of classes. A class of classes is a subsumptive whose referents are specifically nominal.

Note that an ordinary class (that is, one which is not a class of classes) stands as subject of membership when the predicate is a class of classes; there are no grounds for assuming that an ordinary class can ever be a member of another ordinary class. We cannot, for instance, say "dogs" is a member of "animals", but only, dogs are members of "animals", or "dogs" is a member of "animal-classes".

This was already suggested in the previous section, in the claim that "X" is not a member of "X"; now, we can generalize further, and say that "X" cannot be a member of any "Y", granting that these terms are not classes of classes of anything. Other than the above defined case, ***there are no known conditions regarding any X and Y, under which we could conclude that "X" is a member of "Y"***.

Similarly, ***there are no known conditions under which propositions of the form: "X-classes" is a member of "Y", may arise***. However, as we shall presently see, propositions of the form: all/some X-classes are (are not) members of "Y-classes", do indeed arise, directly out of the definition of classes of classes. However, note that the subject is subsumptive here, not nominal.

Let us now investigate how successful our above definition of classes of classes is, some of the logical properties it implies.

b. Features.

"X" is an X-class, and a member of "X-classes" (or the class of X-classes), since all X are X, and even though "X" is not an X, nor a member of "X". This principle proceeds deductively from the definition, by substituting X where we find Y. It means that every class is a member of the class of classes bearing its name. It does not mean that it is a member of itself, however; we should not confuse a class with a class of classes; thus far, we have no cause to doubt the earlier postulate that classes cannot be members of themselves. For example, "dogs" is a dog-class, and a member of "dog-classes".

However, ***no X is an X-class, nor a member of "X-classes"***, even though all X are members of "X", and "X" is a member of "X-classes". The definition of a class of classes refers to a nominal "X" as its subject, not a subsumptive X. The relationship of membership is not passed on all the way down the chain to the individuals subsumed by X; the only individuals subsumed by a class like "X-classes" are classes like "X". For example, dogs are members of the class of dogs, but not of the class of dog-classes.

Similarly, ***no X is a Y-class, nor a member of "Y-classes"***, even if all X are Y, and therefore members of "Y". Contrast those statements to saying that "X" is a Y-class (or a class of Y), and therefore a member of "Y-classes" (or the class of Y-classes, or the class of classes of Y). Keep the distinctions clear.

We might strengthen these insights by calling ordinary classes, classes *'of the first order'*, and classes of classes, classes *'of the second order'*; then we can say: ***members of a class of the first order cannot be members of a class of the second order***; at best, they might be said to be members of a member of a class of the second order. This may be referred to as the *principle of intransmissibility* of membership across orders of classification.

c. Immediate Inferences.

Obviously, by definition, if "X" is a Y-class, then all X are Y; and if "X" is not a Y-class, at least some X are not Y. Likewise, with any of the alternative wordings.

The following theorems are important, because they construct propositions in which a class of classes is the subject, a novelty; thus far, classes of classes only appeared as predicates.

If all X are Y, then all X-classes (including "X" itself) are Y-classes, or members of "Y-classes", the class of Y-classes. Proof is by exposition: consider any class "W" which fits the definition of an X-class, so that all W are X, then (since all X are Y) all W are Y, and it will follow that "W" is a Y-class; this can be repeated for any "W", and even "X" fits in (since all X are X). For example, all dog-classes (such as "retrievers") are animal-classes.

A corollary is: ***if "X" is a Y-class, then all (other) X-classes are (also) Y-classes***; the conclusion follows, since the premise implies that all X are Y.

If some X are Y, then some X-classes are Y-classes. Proof: those things which are both X and Y can be said to be XY, and self-evidently all XY are X and all XY are Y; thus we have, in the case of "XY" at least, an X-class which is a Y-class.

If some X are not Y, then some X-classes are not Y-classes. Proof: those things which are X but not Y can be said to be XnonY, and self-evidently all XnonY are X and no XnonY are Y; thus we have, in the case of "XnonY" at least, an X-class which is not a Y-class.

If no X is Y, then no X-classes are Y-classes. For if, say, "W" is an X-class, then all W are X; and since no X is Y, it follows that no W is Y, which means that "W" is not a Y-class.

Thus, note well, if some X are Y, it follows only that some X-classes are Y-classes, for we may find a class "W" (other than "X") for which all W are X and yet no W is Y. Likewise, if some X are not Y, it follows only that some X-classes are not Y-classes, for we may find a class "W" (other than "X") for which all W are X and also all W are Y.

Conversely, if all X-classes are Y-classes, then all X are Y; if some X-classes are Y-classes, then some X are Y; if some X-classes are not Y-classes, then some X are not Y; and if no X-classes are Y-classes, no X is Y.

d. Deductive arguments.

It is important to note that syllogistic reasoning with the copula 'is a member of' depends for its validity on the manner of reference of its terms.

We saw that, if any X is a member of "Y", and "Y" is a member of "Z", it follows that that X is a member of "Z". The proof being, since that X is Y, and all Y are Z, then that X is Z.

However, if even all X are members of "Y", and "Y" is a member of "Z-classes", it does not follow that any X is a member of "Z-classes". For, even though it be implied that all X are Z, this only signifies, as already pointed out, that "X" is a member of "Z-classes", not that any X is a Z-class.

Thus, we have the same arrangement of premises, with the copula 'is a member of' in both cases, yet the conclusions are of fundamentally different form. In the former case, subsumptives are members of an ordinary class; in the latter case, a nominal is member of a class of classes. This of course illustrates the earlier mentioned principle of intransmissibility of membership.

The following arguments may be validated with reference to the indicated Aristotelean syllogisms.

Figure 1 (from 1/AAA).
"Y" is a member of "Z-classes",
and "X" is a member of "Y-classes",
therefore, "X" is a member of "Z-classes".

Figure 2 (from 2/AOO).
"Z" is a member of "Y-classes",
and "X" is not a member of "Y-classes",
therefore, "X" is not a member of "Z-classes".

Figure 3 (from 3/OAO).
"Y" is not a member of "Z-classes",
and "Y" is a member of "X-classes",
therefore, "X" is not a member of "Z-classes".

However, no other arguments of that sort are possible. In the first figure, a negative major premise, "Y" is not a member of "Z-classes", would only imply that some Y are not Z, from which no conclusion can be drawn; and as for **1/AII**, it has no equivalent here, since "X" is a member of "Y-classes" would require that all X be Y. In the second figure, likewise with regard to a negative major premise; and as for **2/AEE**, it has no equivalent here, since "X" is not a member of "Y-classes" only implies that some X are not Y. We can similarly write off the remaining moods of the third figure. The fourth figure has no equivalent here, since the minor premise of **4/EIO** is not enough to imply membership of a class in a class of classes.

Thus, we only have three valid moods for propositions of this kind; no other moods are valid. The first is used for including a class in a class of classes, the other two for purposes of exclusion. These can be restated as follows, in accordance with the theorems of immediate inference:

Figure 1 (1/AAA)
"Y" is a Z-class (or, all Y-classes are Z-classes),
"X" is a Y-class (or, all X-classes are Y-classes),
so, "X" is a Z-class (or, all X-classes are Z-classes).

Figure 2 (2/AOO).
"Z" is a Y-class (or, all Z-classes are Y-classes),
"X" is not a Y-class (or, some X-classes are not Y-classes),
so, "X" is not a Z-class (or, some X-classes are not Z-classes).

Figure 3 (3/OAO)
"Y" is not a Z-class (or, some Y-classes are not Z-classes),
"Y" is an X-class (or, all Y-classes are X-classes),
so, "X" is not a Z-class (or, some X-classes are not Z-classes).

For examples. (i) The class of retrievers is a class of dogs, and the class of dogs is a class of animals, therefore "retrievers" is an animals-class. (ii) "Roses" is a class of plants, but "dogs" is not a class of plants, therefore "dogs" is not a member of "classes of roses". (iii) "roses" is not a class of animals, but "roses" is a class of plants, therefore "plants" is not a member of the class of classes of animals.

Although the subsumptive relation between classes and classes of classes allows for only these three valid moods, it is clear that the subsumptive relation of classes of classes with each other allows for a fuller range of syllogistic argument. The three arguments indicated in brackets are obviously not all the valid moods for such terms, but any valid Aristotelean syllogism might be applied here. For example: some X-classes are Y-classes, no Y-classes are Z-classes, therefore some X-classes are not Z-classes. The explanation is simply that first order classes are effectively singular, whereas second order class subsume many such singulars.

e. Modal class-of-classes logic.

To modalize the concept of classes of classes, we would have to appeal to a *collectional* proposition, of the form '*all* X can be Y. This, you may recall, signifies, not only that for each X there are some circumstances in which it is Y, but also that there is at least one set of circumstances in which all X *at the same time* are Y.

The modal definitions are: for any X and Y, if *all* X simultaneously can be Y, then "X" can be a class of Y; but if some X cannot be Y, or all X can be Y, but not *all* at once, then "X" cannot be a class of Y; and if all X must be Y, then "X" must be a class of Y; but if some X can not-be Y, then "X" can not-be a class of Y.

From these definitions, the entire modal logic of classes of classes is easily derived, with reference to the logic of ordinary modal categoricals and collectionals. Note that the defining propositions are all intended as extensional conditionals, but two of them are special in that they contain a collectional antecedent.

44. HIERARCHIES AND ORDERS.

1. First Order Hierarchies.

a. Reconsider the definition: if all X are Y, then "X" is a class of Y (or member of "Y-classes"). The condition only implies that some Y are X.

In the case where all Y are X, they are coextensive and their relation is reciprocal; then "Y" is also a class of X (or member of "X-classes"), and "X" and "Y" are members of each other's group of classes (which does not mean that they are members of each other, note well); such classes may be called equal. "X" is an equal-class of "Y", signifies that X-ness and Y-ness are two 'aspects' of the same ultimate referents.

But in the case where some Y are not X, they cover a different extension and their relation is uneven. "X" is a member of Y-classes, but "Y" is not a member of X-classes. In such case we say that "X" is a ***subclass*** of "Y" and that "Y" is an ***overclass*** of "X". Alternatively, we say that "X" is a lower class than "Y", and "Y" is a higher class than "X"; or again, we speak of species and genus.

Note in passing, we often define a species by stating its genus (or one of its genera) together with a differentia; the latter is that character in the ultimate referents of a species, which distinguishes them from the ultimate referents of other species of same genus; the referents of all the species have in common the generic character.

Thus, we here introduce three new copulas, one of which is reversible, and two of which are relative to each other. These of course may be denied, making six altogether. These copulas differ from those previously defined, in that *the subject and predicate are both nominal*. Their function is to establish, or more precisely express, the *hierarchies* among classes. These various relations have their own logic, which can be analyzed in detail as we did for previous ones; I will not get into that here, however (the reader is invited to do the job).

b. We call ***'division'***, listing the subclasses of a class; If the subclasses of the latter are in turn listed, we call the process 'subdivision'. We represent these relations on paper by means of (upside down) *'trees'*, in which the highest class (or summum genus) is placed at the top, and successively divided into lower classes, like a downward branching.

Since all classes ultimately fall under the heading of "things", there is only one big tree; however, we may speak of branch systems as trees, too. Note that we must have at least one general positive proposition 'all X are Y' and/or 'all Y are X', to be able to say that "X" and "Y" are in the same tree, or branch of a tree. Otherwise, they are neither equal, nor lower, nor higher classes, in relation to each other, but are in separate trees, or branches of a tree.

If we stand back and consider all possible classes, we see that, though they form a single tree, it is not flat. We have a multitude of hierarchies, all stemming down from "things", in three dimensions. Hierarchies with entirely different referents, have no intersecting branch lines; hierarchies with some but not all referents in common, have some intersecting branch lines; hierarchies with all the same referents have the same branch lines.

The latter occurs when we have two sets of equal classes: they run along the same branch lines, but they signify different 'principles of division', different aspects of the same referents. Thus, for example, humans can be divided into those with male sex-organs and those with female sex-organs, or alternatively, into those without bosoms and those with bosoms: yet these two divisions overlap exactly (ignoring exceptions).

The ultimate referents of all these classes are at the very bottom, in a 'horizontal' plane (representing the space-time continuum). There is, as it were, a fanning-out below the lowest classes, to cover the ultimate referents. The relation of referents to lower or higher classes is the same (membership), but it is not the same as the relation of lower classes to higher classes (hierarchy), note well.

2. Second Order Hierarchies.

a. With all this in mind, we see that what a class of classes does is refer us to all the subclasses of a class, plus the class itself. Thus, we should not confuse a class of classes with a first-order overclass, which stands higher up in the continuum of classes.

Whereas an upper first-order class is nominal, and bears certain hierarchical relations to others like it — a class of classes subsumes a class and its subclasses, without thereby becoming part of the same hierarchy, and thus constitutes a second order. Thus, 'hierarchy' and 'order' are two distinct ways we can stratify classes, and should not be confused.

The two orders of class, "X" and "X-classes", for any X, are *not comparable*. The former refers to all things which are X as its members, the latter refers to all (mental) groupings of things which are X as its members. The one concerns numerous individual things, the other untold collectives (in every which way) of these very same things. Their world of reference is one and the same in size, so it is hard to say which is 'bigger'. The number of referents each has is different, but (in most cases) incalculable.

b. If we apply the definition of classes of classes to classes of classes, we obtain the following result: *if all X-classes are Y-classes, then "X-classes" is a class of Y-classes, or a member of "classes of Y-classes"*. Here, now, we have classes of classes of classes. We can repeat the process, and obtain an infinity of levels upon levels. But it does not seem to mean anything more than "Y-classes", to me at least.

The basis on which we form various classes about anything, is in the things they concern. For example, the different kinds of dogs differ in sizes, colors, and so on. Beyond that, the 'containers' as such are uniform, there is nothing to distinguish them from each other, other than the differences observed in their 'contents'. Thus, to pile up level upon level, over and above classes of things and classes of classes of things, is a meaningless redundancy. We may reasonably conclude that *there is no order of classification above the two already considered*.

c. *We may, however, organize second order classes into hierarchies among themselves*, on the basis of statements like 'all X-classes are Y-classes'. In that case, "X-classes" is an equal-class or subclass of "Y-classes"; and similarly in other cases, in accord with the above definitions of hierarchical relations.

Obviously, the hierarchies in the second order *reflect* those in the first order, on the basis of inferences like 'if all X are Y, then all X-classes are Y-classes'. This just signifies that formal eductions are feasible from one system to the other.

However, the relationship of second-order to first-order classes is not hierarchical, but simply subsumptive. It is like the relation of first-order classes to their ultimate referents — namely, a relationship of inclusion as members; it is not like the relation of higher classes to lower classes of one and the same order.

For first-order classes, as we pointed out, the theater of reference is the space-time continuum, represented as a horizontal plane. For second-order classes, the theater of reference is the vertical dimension in which the tree of first-order classes evolves. However, the tree of second order classes need not be viewed as implying yet another dimension; we can view it as a distinct branch system within the same vertical dimension. The two orders of classes are layered in neat harmony with each other.

What distinguishes the second-order classes is that their members are first-order classes, but not the members of first order-classes. Thus, the lowest second-order classes 'fan-out' to first-order classes, but stop short of similarly relating to the members of first-order classes.

d. In conclusion, it is important to keep in mind that the concept of 'inclusion' has many meanings. It can mean inclusion of things in a first order class, or inclusion of first order classes in a second order class, or inclusion of a subclass in an overclass. These relations are not one and the same, though we call them all 'inclusion'.

In practice, we are not always clear about the exact distinctions between subsumptives and nominals; first order classes (or simply, classes) and second order classes (or, classes of classes); equal-classes, subclasses and overclasses. But we have to be careful, because as we saw, their logical properties vary considerably.

3. Extreme Cases.

It is important to understand that the concepts of classes, or classes of classes, are *purely relational*. Although we colloquially use these expressions as if they were terms, there is no such thing as a 'class' which is not a class *of* certain things, or a class of classes *of* certain things. The word 'of' is operative here, and should not be ignored. It follows that we cannot say that classes are classes, or that "classes" is a class, or make similar statements, except very loosely speaking; we can only strictly say that such and such are classes *of* so and so.

Our habit of speaking of 'classes' or 'classes of classes' without awareness of the subtext, causes us to think that 'classes' is a collection of all classes, supposedly including all 'classes of classes' together with all 'classes not of classes', and even 'classes' itself and also 'non-classes'. Similar ambiguity is generated by 'classes of classes'. It is all very confusing, and due to the above mentioned imprecision.

If we want to think at once of *the events* of class-relating-to-its-members, we of course may do so. This is a class of all the 'lines' joining classes to their members (whether first-order classes to ultimate referents, or second-order classes to first-order classes). The resulting concept is, however, what we call 'subsumption' (or 'membership', in the reverse direction). If we want to think of hierarchical relations, we again may do so; but the resulting concept is again a copula.

If we want to speak of *the terms* of such relations, say, all classes *indefinitely* — that is, without having to specify what they are classes of — we strictly should say 'the classes of anything', where 'anything' is understood like a variable 'X', standing for any kind of thing we choose to substitute in its place. Likewise, for classes of classes (of anything), or with reference to hierarchies.

The *largest* possible class, is the class *of* all things (including real and illusory things), or simply "things"; it is not just 'classes'. From our definition, since every thing is a thing, every thing is a member of "things" or the class of things. The largest possible class of classes, is the class of all classes *of* things, or simply the "things-class"; it is not just 'classes of classes'. This means, again by definition, since all things are things, "things" (or the class of things) is a class of things, or a member of the "things-class", or the class of classes of things.

Since a nominal (the class of anything) is itself a thing, it follows that the classes "things" and "things-classes" are both things, and so members of "things". Additionally, since for any X, "X" is an X-class, it follows that "things" is a member of "things-classes". Thus, exceptionally, the classes "things" and "things-classes" seem to be equal to each other and, somehow, members of themselves. They are (it is) the summum genus of all hierarchies.

When this summum genus branches out into species like "dogs", "machines", and such, it is preferably called "things"; when we focus on its subsumption, not of the ultimate referents, but of the ideational instruments standing between it and them, we call it "things-classes"; alternatively, we may embrace both these categories.

45. ILLICIT PROCESSES IN CLASS LOGIC.

1. Self-membership.

With regard to the issue of self-membership, more needs to be said. Intuitively, to me at least, the suggestion that something can be both container and contained is hard to swallow.

Now, self-membership signifies that a nominal is a member of an exactly identical nominal. Thus, that all X are X, and therefore members of "X", does not constitute self-membership; this is merely the definition of membership in a first order class by a non-class.

We saw that, empirically, at least with ordinary examples, "X" (or the class of X) is never itself an X, nor therefore a member of "X". For example, "dogs" is not a dog, nor therefore a member of "dogs".

I suggested that this could be generalized into an inductive postulate, if no examples to the contrary were forthcoming. My purpose here is to show that all apparent cases of self-membership are illusory, due only to imprecision of language.

That "X" is an X-class, and so a member of "X-classes", is not self-membership in a literal sense, but is merely the definition of membership in a second order class by a first order class. For example, "dogs" is a class of dogs, or a member of "classes of dogs", or member of the class of classes of dogs.

Nor does the formal inference, from all X are X, that all X-classes are X-classes, and so members of "X-classes" (or the class of classes of X), give us an instance of what we strictly mean by self-membership; it is just tautology. For example, all dog-classes are members of "classes of dogs".

Claiming that an X-class may be X, and therefore a member of "X", is simply a wider statement than claiming that "X" may be X, and not only seems equally silly and without empirical ground, but would in any case not formally constitute self-membership. For example, claiming "retrievers" is a dog.

As for saying of any X that it *is* "X", rather than a member of "X"; or saying that it *is* some other X-class, and therefore a member of "X-classes" — such statements simply do not seem to be in accord with the intents of the definitions of classes and classes of classes, and in any case are not self-membership.

The question then arises, is "X-classes" itself a member of "X-classes"? The answer is, no, even here there is no self-membership. The impression that "X-classes" might be a member of itself is due to the fact that it concerns X, albeit less directly so than "X" does. For example, dog-classes refers to "retrievers", "terriers", and even "dogs"; and thus, though only indirectly, concerns dogs.

However, more formally, "X-classes" does not satisfy the defining condition for being a member of "X-classes", which would be that 'all X-classes are X' — just as: "X" is a member of "X-classes", is founded on 'all X are X'. As will now be shown, this means that the above impression cannot be upheld as a formal generality, but only at best as a contingent truth in some cases; as a result, all its force and credibility disappears.

If we say that *for any and every* X, all X-classes are X, we imply that for all X, "X" (which is one X-class) is X; but we have already adduced empirical cases to the contrary; so the connection cannot be general and formal. Thus, we can only claim that perhaps *for some* X, all X-classes are X; but with regard to that eventuality, no examples have been adduced.

Since we have no solid grounds (specific examples) for assuming that "X" or "X-classes" is ever a member of itself, and the suggestion is fraught with difficulty; and we only found credible examples where they were not members of themselves — we are justified in presuming, by generalization, that: *no class of anything, or class of classes of anything, is ever a member of itself.*

I can only think of one possible exception to this postulate, namely: “things” (or “things-classes”). But I suspect that, in this case, rather than saying that the class is a member of itself, we should regard the definition of membership as failing. That is, though this summum genus *is* a thing, it is not ‘a member of’ anything.

2. The Russell Paradox.

The Russell Paradox is modern example of double paradox, discovered by British logician Bertrand Russell.

He asked whether the class of “all classes which are not members of themselves” is or not a member of itself. If “classes not members of themselves” is not a member of “classes not members of themselves”, then it is indeed a member of “classes not members of themselves”; and if “classes not members of themselves” is a member of “classes not members of themselves”, then it is also a member of “classes which are members of themselves”. Thus, we face a contradiction either way.

In contrast, the class of “all classes which are members of themselves” does not yield a similar difficulty. If “self-member classes” is not a member of “self-member classes”, then it is a member of “classes not members of themselves”; but if “self-member classes” is a member of “self-member classes”, no antinomy follows. Hence, here we have a single paradox coupled with a consistent position, and a definite conclusion can be drawn: “self-member classes” is a member of itself.

Now, every absurdity which arises in knowledge should be regarded as an opportunity for advancement, a spur to research and discovery of some previously unknown detail. So what is the hidden lesson of this puzzle?

As I will show, the Russell Paradox proceeds essentially from an equivocation; it is more akin to the sophism of the Barber paradox, than to that of the Liar paradox. For *whether self-membership is possible or not, is not the issue*. Russell believed that some classes, like “classes” include themselves; though I disagree with that, my disagreement is not my basis for dissolving the Russell paradox. For it is not the concept of self-membership which results in a two-way inconsistency. It is the concept of non-self-membership which does so; and everyone agrees that at least some (if not all, as I believe) classes do not include themselves: for instance, “dogs” is not a dog.

What has stumped so many logicians with regard to the Russell paradox, was the assumption that we can form concepts at will, if we but formulate a verbal definition. But this viewpoint is without justification. The words must have a demonstrable meaning; in most cases, they do; but in some cases, they are isolated or pieced together without attention to their intrinsic structural requirements. We cannot, for instance, use the word ‘greater’ without specifying ‘than what?’; many words are attached, and cannot be reshuffled at random. The fact that we commonly, in everyday discourse, use words loosely, to avoid boring constructions, does not give logicians the same license.

3. Impermutability.

The solution to the problem is so easy, it is funny, though I must admit I was quite perplexed for a while. It is simply that: ***propositions of the form ‘X (or “X”) is (or is not) a member of “Y” (or “Y-classes”)’ cannot be permuted***. *The process of permutation is applicable to some forms, but not to all forms.*

a. In some cases, where we are dealing with relatively simple relations, the relation can be attached to the original predicate, to make up a new predicate, in an ‘S is P’ form of proposition, in which 'is' has a strictly classificatory meaning. Thus, 'X is-not Y' is permutable to 'X is nonY', or 'X is something which is not Y'; 'X has (or lacks) Y-ness' is permutable to 'X is a Y-ness having (or lacking) thing'; 'X does (or does not do) Y' is permutable to 'X is a Y-doing (or Y-not-doing) thing'. In such cases, no error arises from this artifice.

But in other cases, permutation is not feasible, because it falsifies the logical properties of the relation involved. We saw clear and indubitable examples of this in the study of modalities.

For instance, the form 'X can be Y' is not permutable to 'X is something capable of being Y', for the reason that we thereby change the subject of the relation 'can be' from 'X' to 'something', and also we change a potential 'can be' into an actual 'is (capable of being)'. As a result of such verbal shenanigans, formal errors arise. Thus, 'X is Y, and all Y are capable of being Z' is thought to conclude 'X is capable of being Z', whereas in fact the premises are quite compatible with the contradictory 'X cannot be Z', since 'X can become Z' is a valid alternative conclusion, as we saw earlier.

It can likewise be demonstrated that 'X can become Y' is not permutable to 'X is something which can become Y', because then the syllogism 'X is Y, all Y are things which can become Z, therefore X is something which can become Z' would seem valid, whereas its correct conclusion is 'X can be or become Z', as earlier seen. Thus, modality is one kind of relational factor which is not permutable. Even though we commonly say 'X is capable or incapable of Y', that 'is' does not have the same logical properties as the 'is' in a normal 'S is P' proposition.

b. The Russell Paradox reveals to us the valuable information that the copula 'is a member (or not a member) of' is likewise not open to permutation to 'is something which is a member (or not a member) of'.

The original 'is' is an integral part of the relation, and does not have the same meaning as a solitary 'is'. The relation 'is or is not a member of' is an indivisible whole; you cannot just cut it off where you please. The fact that it consists of a string of words, instead of a single word, is an accident of language; just because you can separate its verbal constituents does not mean that the objective relation itself can similarly be split up.

Permutation is a process we use, when possible, to bypass the difficulties inherent in a special relation; in this case, however, we cannot get around the peculiar demands of the membership relations by this artifice. The Russell paradox locks us into the inferential processes previously outlined; it tells us that there are no other legitimate ones, it forbids conceptual short-cuts.

The impermutability of 'is (or is not) a member of' signifies that you cannot form a class of 'self-member classes' or a class of 'non-self-member classes'. These are not terms, they are relations. Thus, the Russell paradox is fully dissolved by denying the conceptual legitimacy of its terms. There is no way for us to form such concepts; they involve an illicit permutation. The connections between the terms are therefore purely verbal and illusory.

The definition of membership is 'if something *is* X, then it is a member of "X"' or 'if all X *are* Y, then "X" is a member of "Y-classes"'. The Russell paradox makes us aware that the 'is' in the condition has to be a normal, solitary 'is', it cannot be an 'is' isolated from a string of words like 'is (or is not) a member of'. If this antecedent condition is not met, the consequent rule cannot be applied. In our case, the condition *is not met*, and so the rule does not apply.

c. Here, then, is how the Russell paradox formally arises, step by step. We will signal permutations by brackets like this: {}.

Let "X" signify any class, of any order:
(i) If "X" is a member of "X", then "X" is {a member of itself}. Call the enclosed portion Y; then "X" is Y, defines self-membership.
(ii) If "X" is not a member of "X", then "X" is {not a member of itself}. Call the enclosed portion nonY; then "X" is nonY, defines non-self-membership.

Next, apply the general definitions of membership and non-membership to the concepts of Y and nonY we just formed:

(iii) whatever is not Y, is nonY, and so is a member of "nonY".

(iv) whatever is Y, is not a member of "nonY", since only things which are nonY, are members of "nonY".

Now, the double paradox:

(v) if "nonY" is not a member of "nonY":

— then, by putting "nonY" in place of "X" in (ii), "nonY" is {not a member of itself}, which means it is nonY;

— then, by (iii), "nonY" is a member of "nonY", which contradicts the starting premise.

(vi) if "nonY" is a member of "nonY":

— then, by putting "nonY" in place of "X" in (i), "nonY" is {a member of itself}, which means it is Y;

— then, by (iv), "nonY" is not a member of "nonY", which contradicts the starting premise.

Of all the processes used in developing these arguments, only one is of uncertain (unestablished) validity: namely, permutation of 'is a member of itself' to 'is {a member of itself}', or of 'is not a member of itself' to 'is {not a member of itself}'. Since all the other processes are valid, the source of antinomy has to be such permutation. Q.E.D.

d. The existence of impermutable relations suggests that we cannot regard all relations as somehow residing *within* the things related, as an indwelling component of their identities. We are pushed to regard some relations, like modality or membership, as bonds standing outside the terms, which are not actual parts of their being.

Thus, for example, that 'this S can be P' does not have an ontological implication that there is some actual 'mark' programmed in the actual identity of this S, which records that it 'can be P'. For this reason, the verbal clause {can be P} cannot be presumed to be a unit; there is nothing corresponding to it in the actuality of this S, the potential relation does not cast an actual shadow.

Thus, there must be a reality to 'potential existence', outside of 'actual existence'. When we say that 'this S can be P', we consider this potentiality to be P as somehow part of the 'nature' of this S. But the S we mean, itself stretches in time, past, 'present', and future; it also has 'potential' existence, and is wider than the actual S.

The same can be argued for can not, or must or cannot. Thus, natural (and likewise temporal) modalities refer to different degrees, or levels, of existence.

Similarly, the impermutability of membership relations, signifies that they stand external to their terms, leaving no mark on them, even when actual.

It seems like a reasonable position, because if every relation of something to everything else, implied some corresponding trait inside that thing, then each thing in the world would have to contain an infinite number of messages, one message for its relations to each other thing. Much simpler, is to regard relations (at least, those which are impermutable) as having a separate existence from their terms, as other contents of the universe.

PART V(b). ADDUCTION.

46. ADDUCTION.

1. Logical Probability.

Induction, in the widest sense, is concerned with finding the probable implications of theses. Deduction may then be viewed as the ideal or limiting case of induction, when the probability is maximal or 100%, so that the conclusion is necessary. In a narrower sense, induction concerns all probabilities below necessity, when a deductive inference is not feasible.

a. All this refers to logical probability. A thesis is logically possible if there is some chance, any chance, of it being found true, rather than false. 'Probability' signifies more defined possibility, to degrees of possibility, as it were.

Thus, we understand that low probability means fewer chances of truth as against falsehood; high probability signifies greater chances of such outcome; even probability implies that the chances are equal. High and low probability are also called probability (in a narrower sense) and improbability (with the im- prefix suggesting 'not very'), respectively. Necessity and impossibility are then the utter extremes of probability and improbability, respectively.

There are levels of possibility, delimited by the context, the logical environment. This can be said even with regard even to formal propositions. Taken by itself, any proposition of (say) the form 'S is P', is possible. But, for instance, in the given context 'S is M and M is P', that proposition becomes (relatively) necessary: its level of possibility has been formally raised. Alternatively, in the given context 'S is M and M is not P', that proposition becomes (relatively) impossible: its level of possibility has been formally lowered.

The same applies with specific contents. At first sight, every statement about anything seems logically 'possible'. This just means that the form is acceptable, there exist other contents for it of known value — a well-guarded stamp of approval.

As we analyze it further, however, we find the statement tending either toward truth or toward falsehood. We express this judgement by introducing a modality of probability into the statement. We place the statement in a logical continuum from nil to total credibility.

In any case, we know from experience that such probabilities are rarely permanent. They may increase or decrease; they may first rise, then decline, then rise again. They vary with context changes. Keeping track of these probabilities is the function of induction. For example, when a contradiction arises between two or more propositions, they are all put in doubt somewhat, and their negations are all raised in our esteem to some extent, until we can pinpoint the fault more precisely.

b. In the chapter on credibility, we described degrees of credibility as impressions seemingly immediately apparent in any phenomenon. Thus, credibility is a point-blank, intuitive notion. In the chapter on logical modality, on the other hand, we showed that the definitions of unspecific plural modalities coerced us into the definition of logical probabilities with reference to a majority or minority of contexts. Thus, knowledge of logical probability presupposes a certain effort and sophistication of thought, a greater awareness of context.

Here, we must inquire into the relation between credibility and logical probability.

Every proposition has, *ab-initio*, *some* credibility, if only by virtue of our being able to formulate it with any meaning. This intuitive credibility is undifferentiated, in the sense that, so long as it is unchallenged, it is virtually, effectively, total. But at the same time, this credibility is not very informative or decisive, because the opposite thesis may have been ignored or may be found to have equal credibility.

As we begin to consider the proposition in its immediate context, and we find contradictions (or even sense some unspecified cause for doubt), the credibility becomes more comparative, and it is certified or annulled, or seen as more or less than extreme one way or the other, or as problematic (equally balanced).

As our perspective is broadened, and we project changes in context, the problematic credibilities become more qualified — that is, they are quantified by some specific logical probability, so that they shift more decidedly in either direction. Thus, problemacy (median credibility) may be viewed as the very minimum, the beginning, of probability.

In this way, all the plural logical modalities may be viewed as 'filtering down' to the single-context level of truth or falsehood. This transmission of modality, from the high level of many-contexts to the low level of the present context, may be immediately apparent (as in the case of necessities and impossibilities), or may gradually develop over time (as with all contingent probabilities).

As probabilities vary, through new inputs of raw data into the actual context, so that more alternative contexts are imaginable, and through closer scrutiny of available data — the credibilities under their influence also and proportionally change.

Logical probability, as formally defined, is impossible to know with finality. The exception is in the extreme cases of logical necessity or impossibility, which can be known even without access to all conceivable contexts, through the one-time discovery of self-evidence or self-contradiction (in paradoxical propositions); these modalities are permanent.

But in all cases of logical probability based on contingency, there is no way to make a sure statement of the form 'In most contexts,….' All we can refer to are: most of the contexts *considered so far*; these may in reality be a minority of all possible contexts, for all we know. Such modal statements are therefore not static, never entirely final.

We have shifted the concept of logical probability from its rigid formal definition as 'true in most contexts', to a more practical version: 'true in most *known* contexts'. It thus is no longer implied to be static; but it is now flexible, and suggests comparison of credibilities with a reasonable degree of purpose.

Thus, the concepts of (comparative) credibility and logical probability ultimately blur, and can to some extent be used interchangeably. However, if we understand logical probability in its strictest sense, as *based on and implying* logical possibility, then it should not be confused with credibility, which is even applicable to logically impossible propositions (until their self-contradiction is discovered). Here, I use 'probability' in an indeterminate sense, so as to avoid the issue.

The main purpose of induction is to lead us to facts, to hopefully true specific contents. How we know their logical probabilities is not a separate or additional goal for inductive research; it is one and the same issue with that of knowing their truths. In the process of pursuit of facts, by evaluating our current distance from the establishment of truth, *we are incidentally also finding their logical probabilities*.

Ultimately, we would like to construct a clear, step-by-step, *model* of human knowledge, showing precisely how each proposition in it is arrived at; but in the meantime, the processes involved can be broadly defined. How exactly do we get to know these logical gradations? They are not arbitrary, not expressions of subjective preference, not intuitive guesses; there is a system to such evaluations.

2. Providing Evidence.

The investigation of this problem in general terms, that is, without reference to specific forms, may be called *'adduction'*. Adduction provides us with the rules of evidence and counterevidence, which allow us to weight the varying probabilities of theses.

The more evidence we adduce for our proposed thesis, the more it is *confirmed* (strengthened); the more evidence we adduce to a contrary thesis, the more is ours *undermined* (weakened). These valuations should not be confused with proof and refutation, which refer to the ideal, extreme powers of evidence.

Adduction is performed by means of the logical relations described by hypothetical and disjunctive propositions. These, we saw, are normally based on the separate logical possibility of two theses, and inform us about the logical modalities of their conjunctions, together or with each other's antitheses. They establish connections of varying degree, direction, and polarity.

Now, 'If P, then Q' represents necessary connection, the highest level; it could be stated as 'if P, necessarily Q'. Accordingly, 'if P, then nonQ', incompatibility, could be stated 'if P, impossibly Q'. The contradictories of these would be 'if P, possibly Q' (= 'if P, not-then nonQ') and 'if P, possibly not Q' (= 'if P, not-then Q'). We can, following this pattern, think in terms of probabilities of connection.

a. Adductive argument evolves out of apodosis. It most typically takes the forms:

If P, then Q	If P, then Q
and Q	but not P
hence, probably P.	hence, probably not Q.

These conclusions, so far, do not express the precise degree of probability; they do indicate that the possible result has *increased in probability*. The possibility of the result is already implicit in the major premise to some extent. A deductive, necessary, conclusion would not be justified. But we are one step ahead, in that it is conceivable that the minor premise is true *because* the proposed conclusion was true.

We argue backwards, from the consequent to the antecedent, or from the denial of the antecedent to the denial of the consequent. As apodosis, this is of course invalid; but here we view the minor premise as an *index to*, rather than proof of, the conclusion.

The more hypotheses suggest a conclusion, the more probably will it turn out to be true. The less hypotheses suggest a conclusion, the more probably will it turn out to be false. Thus, ***'evidence'*** may be defined as whatever increases the logical probability of a thesis by any amount, and ***'counterevidence'*** refers to sources of decrease.

Through adduction, we mentally shift from incipient credibility and problemacy, to a more pondered logical probability.

Note that the first mood, the affirmative one, is strictly more correct than the second, negative, mood. For, in the negative case, we presuppose the major premise not to be complemented by 'if nonP, then Q', even though the latter is a formally conceivable adjunct. That is, we are presuming that 'nonQ' is logically possible, without prior justification, since this is not always part of the basis of the major premise. Whereas, in the positive case, if 'if nonP, then Q' were also given, the additional conclusion 'probably not P' would balance but not strictly contradict 'probably P', and also allow Q to be logically necessary.

It follows that the conclusion of the negative mood is more precisely, 'if nonQ is at all possible, then it is now more probable'. But since, as earlier pointed out, every proposition is at first encounter logically possible, this is not a very significant distinction. The issue of basis is more serious for natural, temporal or extensional conditionals than for logical conditionals.

We can simply say that if 'nonQ' turns out to be logically impossible for other reasons, then of course the initial possibility is thenceforth annulled. Such an eventuality is not excluded by the negative adductive argument, just as the positive version allows for the eventual denial of P, anyway.

Note then that the loose sense of logical probability here intended does not imply that 'P is logically possible' (in the first mood) or that 'nonQ is logically possible' (in the second mood), unless these possibilities were part of the tacit basis of the major premise. Logical possibility must still be strictly understood as signifying an established necessity or contingency.

b. Other moods of adduction follow by changing the polarities of theses. These represent other valuable approaches to provision of evidence or counterevidence, confirmation or undermining.

If P, then nonQ and not Q hence, probably P.	If P, then nonQ but not P hence, probably Q.
If nonP, then Q and Q hence, probably not P.	If nonP, then Q but P hence, probably not Q.
If nonP, then nonQ and not Q hence, probably not P.	If nonP, then nonQ but P hence, probably Q.

Note that if the major premise is contraposed, the conclusion remains the same. This shows that the listed moods constitute a consistent system.

We can also form disjunctive adductive arguments, like the following, with any number of theses:

P or else Q but not P hence, probably Q	P and/or Q but P hence, probably not Q

c. It is clear that if the major and/or minor premise in all these arguments were probabilistic, instead of fully necessary or factual, some probability would still be transmitted down to the conclusion, albeit a proportionately more tenuous one.

This principle of 'transmissibility' of credibility, let us call it, is very important to logic, because it means that, although deductive logic was designed with absolutely true premises in mind, its results are still applicable to premises of only relative truth. Thus, deductive processes also have some inductive utility.

We previously made a clear distinction between the 'uppercase' forms of hypothetical, like 'if P, then nonQ', which involve a logically necessary connection, with the lowercase forms, like 'if P, not-then Q', which merely establish a compatibility. This distinction is especially important in deductive argument, such as apodosis.

We can conceive of less than necessary major premises, having forms like 'if P, possibly or probably Q'. Some probability is still transmitted down to the conclusion, though of course again much more tentatively and insignificantly. We can regard thus arguments like the following as also adductive; in fact, they are the most comprehensive formats of adductive argument.

If P, probably Q, and probably P, hence, probably Q.	If P, probably Q, and probably Q, hence, probably P.
If P, probably Q, and probably not P, hence, probably not Q.	If P, probably Q, and probably not Q, hence, probably not P.

In such argument, the probabilities involved may have any degree. Also, the premises may have very different probabilities; and the probability of the conclusion depends on the overlap, if any, of the conditions for realization of the premises, so that it is generally far inferior. It is normally very difficult to quantify such probabilities precisely; but, when we can estimate the degrees of the premises, we can accordingly calculate the degree of the conclusion (which may be zero, if there is no overlap).

We could thus expand our definitions of apodosis and adduction, so that they are equivocal. In that case apodosis and adduction (in the narrow senses we adopted) would respectively be: forward and backward apodosis (in the larger sense), or necessary/deductive and merely-probable/inductive adduction (in the larger sense). This is mentioned only to show the continuity of the two processes.

Note that when we formulate hypothetical propositions, we often order the theses according to their probabilities. ‘If P, then Q’ may intend to implicitly suggest, that P is so far more probable than Q, and may be used deductively to improve the probability of Q; or that Q is so far more probable, and may be used to inductively to raise the probability of P. Tacitly, this signifies an argument with a necessary major premise, and a probabilistic minor premise and conclusion.

Similarly, by the way, for disjunctive argument. Premises and conclusion may have any degrees of logical probability. Also, the minor premise may be implicit in the major, by virtue of our ordering the alternatives, from the most likely (mentioned first to attract our attention) to the least (relegated to the periphery of our attention); or from the least likely (because easiest to eliminate) to the most (the leftover alternative, when we reach the end of the sentence).

3. Weighting Evidence.

We have thus far described adductive argument, but have not yet validated it. We have to explain why the probable conclusion is justified, and clarify by how much the logical probability is increased. The answer to this question is found in the hidden structure of such argument, the pattern of thought which underlies it.

a. Let us suppose that P1, P2,… Pn are the full list of all the conceivable theses, each of which is separately capable of implying Q, so that the denial of all of them at once results in denial of Q. This means:

If P1, then Q; and if P2, then Q; etc.
or, more succinctly,
If P1 or P2 or… Pn, then Q.
And, since the list is exhaustive,
If not-P1 and not-P2 …and not-Pn, then notQ.

(i) In that ideal situation, we can say that if Q is found true, then each of P1, P2,… Pn has prima facie an equal chance of having anteceded that truth. We know at least one of them must be true (since otherwise Q would be false), but not precisely which. Each carries an nth part of the total probability which this necessity embraces. Thus, the degree of probability is in principle knowable, and the process justifiable.

If one of the alternative antecedents is thereafter found false, the number of alternatives is decreased, and so the probability of each of the remainder is proportionately increased. Where only one alternative remains it becomes maximally probable, that is, necessary; and the conclusion is deductive rather than adductive or inductive (in the narrow sense).

In practice, we do not always know or consider all the alternatives; even when we think we are aware of them all, it may only be an assumption, a generalization. Still, the principle remains, even if the degree of probability we assign to the conclusion turns out to be inexact. This is because we are here dealing with logical probability, which is intrinsically tentative and open to change. That is just the function and raison-d’être of logical probability, to monitor the current status of propositions in an evolving body of knowledge.

(ii) If, not yet knowing whether Q is true or false, we find one of the alternatives, say P1, false, we can say that we are one step closer to the eventuality that all are false, from which the falsehood of Q would follow. In that case, the probability of Q being false has increased by an increment of 1/nth.

If thereafter say P2 is also found false, the chances of Q being false are further increased. When all the conditions of that event are fulfilled, the probability becomes maximal — a necessity.

b. In formal terms, what the above means is that 'If P, necessarily Q' is convertible to 'If Q, *(a bit more)* probably P'. Similarly, 'If P, necessarily Q' is invertible to 'If not P, *(a bit more)* probably not Q'. Even if we do not know what, and how many, are the other shareholders of the overall probability, these inferences retain their value.

In aetiological terms, we thus have two sources of probability increase. A thesis (here, P1 for instance) may be rendered more probable by the truth of another (viz., here, Q), of which it is an alternative contingent cause. Or a thesis (here, nonQ) may be rendered more probable by the truth of another (viz., here, not-P1 for instance), which is a component of a necessary cause of it.

Thus, more broadly, probability is transmitted across the logical relationship signified by hypotheticals: in both directions, from antecedents to consequents and vice versa, and to varying degrees, reflecting the intensity of the link.

Each such probability change is relative: it applies within that limited environment which we projected. In practice, the degree of probability we assign to a thesis is a complex result of innumerable such incremental changes. Needless to say, when a thesis is strengthened, its contraries are proportionately weakened; and vice versa.

A thesis may be increasingly confirmed for a variety of reasons, and at the same time increasingly undermined for a variety of other reasons. What matters is its resultant probability, its overall rating, the sum and average of all the affirming and denying forces impinging upon it, at the present stage of knowledge development.

If follows that, though the alternative theses are, to begin with, of equal weight, they may, in a broader context, be found of unequal weight. In that case, we select the relatively most weighty, the logically most probable, as our preferred thesis at any stage of the proceedings.

All the above can be repeated with respect to disjunctions. Consider two or more theses, each with some degree of credibility from other sources. If they are found to be contrary, their credibilities are all proportionately lowered, since we know they cannot all be true. If they are found to be subcontrary, their credibilities are all proportionately raised, since we know they cannot all be false. However, in the case of exact contradictories, their independent credibilities are unaffected, since their mutual exclusion and exhaustiveness offset each other.

c. Lastly, note that we have to clearly discriminate between: exhausting the known possibilities, on the one hand, and open-mindedness to the eventual possibility that new alternatives be found one day, on the other hand.

At any given stage in the development of knowledge we have to bow to all the apparent finalities; this does not prevent us from accepting the principle that some correction might later be called upon. On the other hand, that attitude of receptiveness to change should not be allowed to belittle our trust in acquired certainties.

When all but one of the known theories concerning some phenomena have been eliminated, or one theory is shown to be their only conceivable explanation, we must accept our conclusion as final and unassailable, provided no inconsistency or specific cause for doubt remains. The truth that some such certainties have in the past been overturned, does not logically imply that this particular certainty will ever be overturned.

There is a formal difference between the status of logical possibility within a context, and the general admission that context does change, which stands outside of any context. They are not identical in power: the former affects contextual reasoning, the latter plays no active part in deliberations, being only an open-ended philosophical truth without specific applicability.

We ordinarily think assertorically, in terms of statements like 'if P, then Q', meaning 'if P is established, then Q may be claimed to be known'. But sometimes we remain dubious, and say 'if perhaps P, then perhaps Q'. Some people reason in this manner more often than others, hanging on to uncertainties so insistently that they inhibit the forward motion of their knowledge.

But such reasoning, which may be called 'problematic logic', is essentially no different from assertoric logic. Its inferences are exactly parallel, the only difference is the explicit emphasis it puts on the probabilities of the theses.

Perhaps the legitimate context for such statements would be whenever we inquire into eventual developments of knowledge. Right now, say, P is to all appearances true; but there is always an off-chance that it might turn out not to be true, after all; in that case, we ask, *what would happen if* P was not true. We look ahead, even though we are without strict justification, in order *to be prepared* for eventual alternatives to 'established fact'.

4. Other Types of Probability.

As we saw in the discussion of *de-re* conditioning, adduction is also feasible using natural, temporal or extensional conditionals, but it must be stressed that the emergent probability is essentially in logical modality. We might call it para-logical probability, meaning not *purely* logical, if we wish to underline the faint difference, which relates to source of judgment.

a. A categorical proposition always has adductive implications. 'Most (or Few) S are P' is taken to imply 'This S is probably (or improbably) P'; that is, for any random S, the logical probability is high (or low) that it will be P, in proportion to the quantity. We consider the likelihood that the given case of S happens to be one of those which are P.

Likewise, 'This S is P in most (or few) circumstances' implies 'This S is probably (or improbably) P' that, for any randomly chosen circumstance, there is a logical probability that this S will be P in it, commensurate with the number of natural circumstances favoring such event. We consider the likelihood that the given circumstance surrounding this S happens to be one of those in which this S is P. Similarly with temporal modality.

When two or more of the extensional, and natural or temporal, modalities are involved in a proposition, the logical effect is compounded. The logical probability is increased (or decreased) to some extent by each of the *de-re* modalities, and the resultant is whatever it happens to be.

b. Such transmission of logical probability, from a plural *de-re* proposition down *to a single-unit case* for the type of modality concerned, on the ground of a majority or minority of instances, circumstances or times — is also to be found with conditionals. The following are some typifying examples:

a. In **extensional** adduction:
Any S which is P, is Q,
and this S is Q — therefore, this S is probably P;
or: and this S is not P — so, this S is probably not Q.

b. In **natural** adduction:
When this S is P, it must be Q,
and this S is Q — therefore, it is probably P;
or: and this S is not P — so, it is probably not Q.

c. In **temporal** adduction:
When this S is P, it is always Q,
and this S is Q — therefore, it is probably P;
or: and this S is not P — so, it is probably not Q.

These concepts can be further broadened by reference to majoritive or minoritive conditionals, in arguments like the *de-re* adductions here shown, and likewise for corresponding apodoses. Some logical probability is still transmitted down from premises to conclusion.

Thus, if the major premises in such arguments had been the extensional 'Most (or few) S which are P, are Q', or the natural 'When this S is P, it is in most (or few) circumstances Q', or the equivalent temporal conditional — the conclusion would still have some degree of logical probability, proportionately to the numbers of instances, circumstances or times involved. Likewise, in cases of compound modal type.

If the minor premises were respectively of the form 'Most S are Q' (or 'Most S aren't P'), or 'This S is in most circumstances Q' (or 'This S is in most circumstances not P'), or the equivalent temporal categorical — a probable conclusion can likewise be drawn. Note, however, that if the minor premise is of low *de-re* probability, it does not follow that the conclusion is likewise of low probability; all we can say is that the conclusion has very slightly increased in probability. Likewise, in cases of compound modal type.

A probabilistic major premise, of any modal type or combination of modal types, together with a probabilistic minor premise, of any modal type or combination of modal types, yield a conclusion of some, though much diminished, degree of logical probability.

More broadly still, such conditional major premises, and indeed the minor premises, may have varying degrees of purely logical probabilities as propositions in a knowledge context, quite apart from the inherent 'para-logical' (*de-re*) probabilities just discussed. In that case, the resultant logical probability is still further diminished.

We can similarly adduce evidence through *de-re* disjunctive adduction, in each or any combination of these types of modality.

47. THEORY FORMATION.

1. Theorizing.

Every theory involves an act of imagination. We go beyond the given data, and try to mentally construct a new image of reality capable of embracing the empirical facts. The more nimble our imagination, the greater our chances of reaching truth. Think how many people were stumped by the constancy of the velocity of light discovered by the Michelson-Morley experiment, until an Einstein was able to conceive a solution!

Without creativity our understanding would be very limited. We need it both to construct hypotheses, and to uncover their implications. Neither of these achievements is automatic. Conceiving alternatives and prevision both involve work of imagination.

In practice, no theory is devoid of hidden assumptions, besides its stated postulates. We may try to be as explicit as possible, but often later discover new dependencies. Thus, with Newton's assumption of Euclidean geometry, which was much later discarded in the General Relativity theory.

Thus, our theorizing is always to some extent limited by our ability to make mental projections, and the depth and breadth of our conceptual insight.

These faculties of course depend very much on the mind being fed by new empirical input. Creativity depends on the ideas provided us by new experience, and revision of fundamentals depends on the stimulus of discovered difficulties.

Each individual has his own limits. People often remain attached to preconceptions, and are unable or refuse to consider alternatives. This can be a weakness or vice, but it is also a normal part of the way the mind works.

We have to hold something steady while considering the impact of new perspectives. We cannot re-invent the wheel all the time, without justification. We review our presuppositions, only when the need arises, when some empirical problem presents itself.

This does not exclude 'art for art's sake'. The pursuit of theoretical improvements is always permissible. But it is anyway serial. We are mentally unable to change all our knowledge at once, but are forced to proceed in an orderly, structured manner, gradually focusing on this or that proposed change while the rest is taken for granted.

Logical and mathematical skills also count for much in the development of theories. Many a wild speculation is built on unsound reasoning. These skills include, among many others: clarifying inter-relationships, finding analogies and implications, distinctions and contradictions, ordering information.

A good grasp of the methodology of adduction is very important. It opens minds to the ever-present possibility of alternative explanations and further testing. Adduction is essentially a process of trial and error.

The tentative, and often transient, nature of theories, as well as their ability to make impressive predictions, has been exemplified in some stunning scientific revolutions in the past few centuries. Even seemingly unshakable theories have been known to fall, and some of the discoveries occasioned by the new perspectives would have seemed unthinkable previously.

There is much to learn by observing the 'life' of theories, their historical courses, the ways they have augmented or displaced, complemented or contested, each other, their dynamics.

2. Structure of Theories.

Any one general proposition can of course be viewed as in itself a theory, and the processes of generalization and particularization are samples of adduction. The relation of a general proposition to particular observations, is logically one of antecedent and consequent, though the chronological order may be the reverse.

However, we normally use the term 'theory' in larger, more complex, situations. We think of a rational system for understanding some subject-matter. The sciences of course consist of theories, which attempt to explain the empirical phenomena facing them. But we also build small personal theories about events in our lives of concern only to ourselves.

Let us examine the structure of theories. A ***theory*** (say, **T**) consists of a number of conceptual and/or mathematical propositions. Among these propositions, some cannot be derived from the others: they may be called primary; the others, being of a derivative nature may be called secondary. The derivation, of course, is supposed to be logically or mathematically flawless.

Among the primary propositions, some are distinctive to that theory: they are called its ***postulates*** (label these **p1**, **p2**, **p3**, etc.). Postulates should be as limited in number, as simple in conception and broad-based, as we can make them. Though postulates may be particular (as for instance in a theory concerning historical events), the postulates of sciences are normally general propositions. These are usually obtained by generalization from directly observable particulars, but not always (consider, for instance, the idea of curved space).

If a primary proposition is not distinctive to that theory, but found in all other theories of the subject under investigation, then it is not essentially part of that specific theory, but stands outside it to some extent. Such external primaries may be transcendent axioms, or they may be borrowed from some adjacent or wider field of investigation, taken for granted so long as that other theory holds.

The secondary propositions are called the theory's ***predictions*** (label these **q1**, **q2**, **q3**, etc.), even if not distinctive to that theory.

Some predictions are testable, open to empirical observation, perhaps through experiment; some predictions are intrinsically difficult to test. To the extent that a theory offers untestable predictions, it tends to be viewed as speculative. Among the testable predictions, some are normally already tested: they provided the raw data around which the theory was built; others may be novel items, which anticipate yet unobserved phenomena, providing us with opportunity for further testing.

Predictions are derived from the postulates by a process of production, mediated by the relatively external primaries. We regard the external primaries as categorical, as far as our theory is concerned, so that they may remain tacit, though they underlie the connections between our postulates and predictions.

Thus, postulates are hypothetically linked to predictions, in the way of antecedent to consequent. The antecedent need not include the external primaries, since the latter are considered as affirmed anyway, and were used to establish the connection. For example, Newton's laws of motion were the postulates distinguishing his mechanics, while his epistemological, ontological, algebraic and geometrical assumptions lay outside the scope of his theory as such.

Theories often draw on findings in other domains outside their direct concern, and may have powerful repercussions in other domains. Thus, Newton had to develop calculus for his mechanics; this mathematical tool might well have been researched independently, as indeed it was by Liebnitz, but it was also stimulated or given added meaning when its value to physics became apparent.

A theory, then, may be described as follows, formally:

T = If p1 and p2 and… , then q1 and q2 and…

Note that this overall relation may in some cases be supplemented by narrower ones. It may be that all the postulates are required to make all the predictions; or it may be that some of the postulates are alone sufficient to make some of the predictions.

3. Criteria.

Theories serve both to explain (unify, systematize, interpret) known data, and to foresee the yet unknown, and thus guide us in further research, and in action. The criteria for upholding a theory are many and complex; they fall under three headings:

a. Criteria of ***relevance***. A theory may be upheld as possibly true, so long as it is meaningful, internally consistent, applicable to (i.e. indeed implying) the phenomena under investigation, and consistent with all other observation to date.

This possibility of truth signifies no more than that the theory is conceivable, and has some initial degree of probability. This may be called relevance.

b. Criteria of ***competitiveness***. But the work of induction is not complete until the theory has been compared to others, which may be equally thinkable and defensible in the given context. Induction depends on critically pitting theories against each other.

Two or more theories may each fulfill the conditions of relevance, and yet be incompatible with each other. They might converge in some respects, having some postulates and/or predictions in common, but found divergent in other respects.

It might be possible to reconcile them, finding postulates which succeed in encompassing the ones in conflict, while retaining the same uniform predictions. Or we may have to find exclusive predictions for each, which can be tested empirically to help us make a choice between postulates.

This is where adduction comes into play. It is the process used to evaluate, compare, and select theories through their predictions. It is the main tool for the induction of theories, commonly known as 'the scientific method'.

c. ***Utilitarian*** criteria. Although utility is a relatively 'subjective' standard for evaluating theories, being man-centered, it plays a considerable role. For us, knowledge is not a purely theoretical enterprise, but a practical necessity for survival. We use it to support and improve our lives.

We judge a theory to some extent by how accessible it is to our minds, by virtue of its simplicity, or the elegance of its ordering of information. All other things being equal, we would choose the theory which approaches this ideal most closely, on the general grounds that the world is somehow simple and beautiful. The onus of proof is on the more complex, the more *'far-fetched'*, theories: avoidable complications need additional justification.

However, simplicity should not be confused with *superficiality*. People often opt for overly simplistic viewpoints, which only take the most obvious data into consideration, and ignore deeper issues. A theory should preferably be simple, but not at the expense of accuracy; it must cover more known phenomena and answer more questions, than any other, to be credible. The easy solution often has a limited data base, and reveals a *naive* outlook.

Apart from such rationalistic and esthetic bias, we also look at the implementation value of a theory. Even if a theory or group of theories is/are known to contain some contradictions, we may hang on to them, in the absence of a viable substitute. We assume that the problem will eventually be resolved; meanwhile, we need a tool for prediction, decision-making and action, however flawed. Thus, for example, with the particle-wave dichotomy in physics.

We will look at some of the dynamics of theory selection in more formal terms, in the next chapter.

4. Control.

It must be stressed that the primary problem in theorizing is producing a theory in the first place. It is all very well to know in general how a theory is structured, but that does not guarantee we are able to even think of an interpretation of the facts. All too often, we lack a hypothesis capable of embracing all the available data.

Very often, theories regarded as being 'in conflict', are in fact not strictly so. One may address itself to part of the data, while the other manages to deal with another segment of the data; but neither of them faces all the data. Their apparent conflict is due to their implicit ambition to fit all the facts and problems, but in reality we have no all-embracing theories before us.

However, quite often, we do easily think up a number of alternative theories. In that case, we are wise to resort to *structured theorizing and testing*, to more clearly pose the problems and more speedily arrive at their solutions.

This is known to scientists as 'controlled experiment', which consists in changing (by small alterations or thorough replacement) one of the variables involved, while 'keeping all other things equal'. The method is applicable equally to forming theories and to testing them (by simple observation or experiment).

Structuring consists in ordering one's ideas in a hierarchy, so as to systematically try them out, and narrow down the alternatives.

a. List the independent *issues*. A subject-matter may raise several questions, which do not seemingly affect each other; these various domains of concern must first be identified. For example, in geometry, whether or not space is continuous, and whether or not parallels meet, seem to be two separate issues.

b. For each issue, list the alternative postulates, which might provide an answer. Combine the various postulates of each issue, with the various postulates of all other issues involved, to yield a number of theories (equal to the product of the numbers of postulates in the various issues). Some of these combinations may be logically inconsistent, and eliminatable immediately; in other words, there may be some partial or conditional dependencies between the issues.

c. Within each issue, distinguish between alternative postulates which are radically different, and between postulates which may be viewed as minor alterations of one common assumption. In the former case, we may expect to eventually find some radically different predictions from the alternative postulates. In the latter case, varying the main postulate may merely cause small variations in the predictions, and the work involved is more one of fine tuning our theory.

d. The best way to test ideas is to organize them in terms of successive specific theses and antitheses, as follows:

Starting with the seemingly broadest, most independent issue, focus on one postulate p1, and find for it a prediction q1, which is denied by the denial of that postulate, thus:

If p1, then q1, but if not p1, then not q1.

Next, suppose that p1 wins that contest, and concentrate on the next issue; within that issue, consider one postulate p2, and again look for some exclusive prediction q2 for it:

If p2, then q2, but if not p2, then not q2.

Proceeding in this manner, we can gradually foresee the course of all possible events, and eventually of course test our results experientially. This is an ideal pattern, in that it is not always easy to find such distinctive implications; but it often works.

The trick, throughout the process of theorizing and testing is to structure one's thoughts, so as to advance efficiently to the solutions of problems. A purposeful, constructive, orderly approach, is obviously preferable to a hesitant, vague, muddled one. It often helps to use paper and pencil, or computer, and draw flow-charts; it generates new ideas. Sometimes, of course, it is wise not to insist, and to let the mind find its way intuitively.

I would like to here praise the inventors and developers of the modern personal computer, and all software. Imagination and verbal memory greatly improve the mind's ability to formulate and test thoughts. The invention of the written word, and pen and paper to draw and write with, provided us with an enormous expansion in these capabilities.

The word-processing and other computer applications increase our mental powers still further, by an enormous amount. A patient person can keep improving ideas on a screen, again and again, to degrees which were previously beyond reach. This has and will make possible tremendous advances in human thinking.

48. THEORY SELECTION.

1. The Scientific Method.

The 'scientific method' consists in trying out every conceivable imaginary construct, and seeing which of them keep fitting all new facts, and which do not. Those which cease to fit, must be eliminated (or at least corrected). Those which continue to fit, are to that extent increasingly probable, until they in turn cease to fit. Whatever theory alone survives this eliminative process, is effectively proved, since all the shares of probability have been inherited by it.

In practice, the construction of alternative postulates, and the discovery of the full implications of each, are both gradual processes. We do not know these things immediately. Also, the given context is not static, but itself grows and changes as we go along. This feeds our imagination and insight, helping theory developments, and stimulating further research.

We may start with one or two partially developed theories, and slowly find additional alternatives and make further predictions, as events unfold and the need arises. The extent of our creative and rational powers affects the exhaustiveness of our treatment.

Several theories concerning some group of phenomena may, at any stage in the development of knowledge, simultaneously equally fulfill the criteria of relevance; namely, conceptual meaningfulness, internal consistency, ability to explain the phenomena in question, and compatibility with all other empirical givens so far.

In formal terms, this simply means that competing theories T1, T2, T3,… may, while being contrary to each other, each still logically imply the already experienced phenomena Q. That is, the hypotheticals 'if T1, then Q', 'if T2, then Q', etc., are formally compatible, even though 'T1 or else T2 or else T3…' is true.

The statement that our list of theories for Q is exhaustive, has the form 'If T1 or T2 or T3… , then Q', plus 'one of T1, T2, T3… must be true'. Although it may be hard to prove that our list is exhaustive, we may contextually assume it to be so, if every effort has been expended in finding the alternative explanations.

Each theory contains a number of postulates: T1 = p11 + p12 + p13 +…, T2 = p21 + p22 + p23 +…, and so on. Some of these postulates might well be found in more than one theory; it may be, for instance, that p13 = p29 = p36. But each theory must have at least one distinctive postulate or a distinctive combination of postulates, which makes it differentiable from all the others.

Also, the phenomenon or group of phenomena labeled Q are already known empirically, and supposed to be equally embraced by the various theories put forward. But each theory may have other implications, if we can determine them through reason, open to empirical testing, though not yet tested.

Each theory has a set of predictions: T1 = q11 + q12 + q13 +…, T2 = q21 + q22 + q23 +…, and so on. Some of these must be in common, constituting the given phenomena Q which gave rise to our theorizing in the first place. That is, say, Q = q15 = q27 = q31.

The rest may likewise be all identical, one for one; or some overlaps may occur here and there, while some predictions found here are missing there; or, additionally, some conflicting predictions may occur, so that one or more theories affirm some prediction that certain other(s) deny.

In principle, it is conceivable that the various theories all make only the same predictions, in which case they are factually indistinguishable, and we cannot choose between them on an empirical basis, though we may still refer to utilitarian criteria.

Most often, however, we may eventually find distinctive further predictions for each theory, or at least some which are not common to all. A difference in postulates usually signifies a difference in predictions. Here, we must be careful to differentiate between:

a. a prediction implied by, say, T1, but neither implied nor excluded by T2, T3, etc. — if such a prediction passes the test of experience, T1 is confirmed, but T2, T3,… are neither confirmed nor rejected, though their probabilities are diminished by the increased probability of T1; whereas if such a prediction fails the test of experience, T1 is rejected, while T2, T3,… become more probable by virtue of being less numerous than before; and:

b. a prediction implied by, say, T1, and logically excluded by T2, T3, etc. — if such a prediction turns out empirically successful, T2, T3… are rejected, and (if only T1 is leftover) T1 is proved; whereas if such a prediction turns out empirically unsuccessful, T2, T3,… are confirmed by their anticipation of the negative event, while T1 is rejected.

Thus, theory selection depends on finding distinctive predictions, which can be used in adductive argument or apodosis. These should be empirically testable predictions, of course.

If one or more theories have an implication which the others lack, though are compatible with, or if one or more theories have an implication which the others are incompatible with — we have at least an eventual source of divergent probabilities, allowing us to prefer some theories over others, even if we cannot eliminate any of them; and in some cases, we may be able to eliminate some of them, and maybe ultimately all but one of them.

These methods are of course well known to scientists today. But all this concerns not only scientists at work, but the development of opinions by individuals in every domain. It is the 'trial and error' process through which we all learn and improve our knowledge.

Even if at a later stage we might manage to validate some of our beliefs more deductively and systematically, this is the method we usually use to initially feel our way to them and develop them. Knowing the 'scientific method' explicitly and clearly can help individuals to make their personal thinking on topics remote from abstract science more scientific.

2. Compromises.

We have described the ideal pattern of scientific evaluation of theories; but, in practice things are not always so neat, and we often have to make do with less than perfect intellectual situations.

a. For a start, the coexistence of conflicting theories may be viewed less generously as a source of doubt for all of them; they may each be corroborated by the delimited data they explain, but their mutual incompatibility is a significant inconsistency in itself.

We may remain for years with equally cogent, yet irreconcilable theories, which we are unable to decide between. Our minds are often forced to function with a baggage of unresolved contradictions.

In such case, we suspend judgment, and make use of each theory for pragmatic purposes, without considering any as ultimately true as a theoretical image of reality.

Even as we may give more credence to one theory as the more all-embracing and most-confirmed, or as the simplest and most-elegant, we may still withhold final judgment, and not regard that theory as our definite choice, because the evidence does not seem to carry enough conviction.

b. Sometimes the available theories only partially explain the given data. They may embrace some details in common, with comparable credibility, but one may be more useful than the others in some areas, while another is more thorough in other respects.

Although this suggests that the theories have distinct implications, they are each supportable on different grounds, perhaps with the same overall probabilities. We may not find a way to choose between them empirically, or to unify them somehow.

In such case, narrowing the field by elimination of alternatives is hardly our main concern; rather, we are still at a stage where we need a unifying principle, we effectively do not have a theory in the full sense of the term. An example of this is the particle-wave dichotomy, and the search for a unified field theory to resolve it.

Sometimes, we know our list of available theories is faulty, because their connections to the data are not entirely satisfactory and convincing. In that case, our 'if-then-' statements are themselves probabilistic, rather than necessary. Our ideas then had better be called notions or speculations.

c. Sometimes, no theory at all can be found for the phenomena at hand, for years. There may be seemingly insurmountable antinomies. We are forced to wait for an inspiration, a new idea, a new insight, a new observation, which might lead us to a satisfactory solution.

Because it is in some domains very difficult to develop a meaningful and consistent conceptual framework, we may be forced to accept one which is conceptually or logically flawed, as a working hypothesis.

Sometimes, the problem may be shelved, because its impact lies elsewhere, creating doubts and questions in distant disciplines. For example, Heisenberg's Uncertainty Principle seems to assault our common-sense conceptions of determinism for inanimate matter: this might later be resolved by Physics itself, or might remain an issue for Philosophy to deal with.

In practice, an imperfect tool of knowledge is often better than none at all. We prefer to have a theory formulated in terms of vague or seemingly contradictory concepts, with practical value, than to remain paralyzed by a dogmatic insistence on an elusive ideal.

d. Thus, sometimes, although a theory may apparently be strictly speaking felled by hard evidence, and we are unable to pinpoint its mistakes, we may nonetheless pragmatically hang on to it, if there is no other to replace it. We simply mentally attach a reservation to it, retain an awareness of its limitations, and move on cautiously to practical applications.

This is especially justifiable when the reason for its empirical rejection was an extreme situation, or 'boundary case', not encountered in the normal course of events. We then recognize the need to specify some limiting conditions to the theory, without being able to fulfill this need more precisely at the present stage.

3. Theory Changes.

Even when a theory is found empirically wrong, yet has alternatives, we may avoid outright rejection, and rather first seek to rectify it somehow, limiting it in scope or shifting some of its postulates slightly. This is feasible on the ground that there must have been some grain of truth in the original insight, and we may be able to tailor our assumptions to fit the new data.

Even if we cannot immediately conceive a correction, we may still choose to hang on to the original idea in the hope of its eventual redemption. We all carry a baggage of beliefs through life, which we know lead to contradictions or have been apparently disproved or rendered very improbable; we keep them in mind for further verification, anyway. This attitude taken to an extreme is of course contrary to logic, but within reasonable bounds it has some utility.

The pursuit of truth is not cold and vengeful, as it were, towards flawed theories, intent on rarefying the alternatives at all costs. Rather, it is a process of flexible adaptation to changing logical conditions. Our goal is, after all, to indeed arrive at truth, and not merely to give the impression that we did.

If we manage to modify a theory well enough to fit the new facts, then effectively we have developed a new theory. It may be a new version of the old, but still merits consideration as a theory in its own right.

We defined a theory as a number of distinctive postulates together implying a number of predictions. More loosely, the range of applicability of a theory might be varied, without radically affecting the substance of its proposals or its details.

Also, we may distinguish between essential postulates and postulates open to change. The former may be generic proposals, the latter specifics within them which we have not yet resolved — postulates within postulates, as it were. Likewise, we might distinguish between generic predictions, which are necessary consequences, and their specifics, which may be less firmly bound to the postulates.

With these thoughts in mind, we can talk of a theory 'changing', while remaining essentially the same theory. This may refer to changes in scope or changes in detail which do not affect the main thrust of a hypothesis. In other words, a theory may involve logical conditional propositions, as well as categoricals, leaving room for variations.

Denial of a postulate may mean: either denial of the broadness of the postulate, without excluding the possibility that a more moderate formulation is acceptable, or denial of a specific position, which can be replaced by another specific position with the same generic impact, or radical denial of a generic position, in the sense that all its possible embodiments are consequently denied.

Denial of a prediction may accordingly either merely cause us to regard the theory as having a more limited applicability than originally thought, or to make relatively small corrections in our assumptions, or force us to formulate a completely new theory.

Thus denial of a postulate or prediction does not necessarily mean rejection of the whole theory as such, it may be only partly discredited, requiring a less ambitious or a slightly altered formulation.

Accordingly, a new theory may totally replace an old one, or it may embrace it as a special case. For example, Einstein's Relativity resulted in our particularization of Newtonian mechanics to commonplace physical levels; it was thenceforth seen as inapplicable to more extreme astronomical or sub-atomic situations, but retained much of its usefulness.

4. Exclusive Relationships.

We know from apodosis that affirmation of a postulate implies acceptance of all its necessary predictions (even those untestable empirically), and denial of a prediction obliges us to reject (or at least change) the postulates which necessitate it.

Denial of a postulate does not engender denial of its still untested predictions; it only diminishes their probability. However, empirically untestable predictions can still be discarded, if we can show them to be logically exclusive to some empirically rejected postulate(s). The argument is a valid apodosis:

Only if postulates p, then predictions q
(implying: if notp, then notq),
but not p,
hence, not q.

Doubt may remain, depending on how sure we are of the postulate's denial, and especially on the strength of the exclusiveness. Also, what has been said does not prevent the possibility that a slightly different version of the predictions still hold.

Likewise, affirmation of a prediction does not in itself prove any of the postulates giving rise to it, but only confirms them. However, theoretical postulates can still be established, if we can show them to make some logically exclusive empirically tested prediction(s).

Only if postulates p, then predictions q
(implying: if notp, then notq),
but q,
hence, p.

This too is a valid apodotic argument. Again, such exclusiveness may often be hard to determine indubitably, but the principle remains valid.

It is not always easy or even possible to find such exclusive relationships. In such case, we are of course limited to the adductive approach. Note that, just as necessity is the extreme of probability, so apodosis is the limiting case of adduction: they differ in degree, not in essence.

Thus, it is not permissible to regard, as some philosophers seem to have intimated, science as incapable of certitude in disproof of empirical matters, or of certitude in proof of theoretical constructs. Admittedly, a good deal of theory selection is based on the processes of adduction and elimination; but this is only one arrow in the arsenal of the scientific method.

If we regard science as capable of establishing logical (or mathematical) connections for the purposes of mere confirmation or undermining of theories, then it is equally capable in principle of establishing exclusive connections which can be used for the above described demonstration purposes.

All the hypothetical forms are structurally identical, irrespective of the polarities of their theses. If any one of them is recognized as accessible to science, then they are all equally so. If we can rely on the ‘if p, then q’ of adduction, then we can just as well rely on the ‘if notp, then notq’ of exclusive apodoses.

There is no intent, here, to underrate the importance of competitive induction, only to point out that other, more certain, means are *sometimes* available to us, though not always. What is at issue here is the suggestion that we only have a choice of a-priori, axiomatic knowledge versus a posteriori, probabilistic knowledge.

There is an in-between alternative: knowledge which is at once theoretical, and certifiable, and empirical. It is arrived at through the logical discovery of exclusive relationships between postulates and predictions. This methodology has the stamp of approval of logical science, and is perfectly reliable.

Indeed, all our so-called mind-set concepts, even the axioms of logic, have such exclusive-empirical grounding, as well as self-evidence (i.e. self-contradiction of their contradictories). Every particular proposition, for example, appeals to this reasoning. More generally, any concept which appears as sole available interpretation or explanation of the experienced phenomena is justifiable on that basis.

49. SYNTHETIC LOGIC.[4]

1. Synthesis.

Knowledge requires inquisitiveness and creativity. It cannot advance far inertially. The role of the knower is to actively ask questions and look for answers, not to sit back passively and assume all is well. Knowledge is a *constructive* activity.

In forming one's opinions, one has to *think things through*, and not unfocus one's stare and avoid the effort. One should not rely excessively on generally-held opinion, though of course its general acceptance is in most cases well-earned. One is duty-bound to verify, repair, and contribute, if one can.

Knowing is not mere maintenance work, 'when something goes wrong, fix it', but involves searching for flaws or improvements even without apparent cause. Speculation, the attitude of 'what if things are otherwise than they now seem or are said to be?', has considerable value in the pursuit of truth.

In forming our world-view, we all make use of some prejudicial ideas, or preconceptions. We take for granted many basic assumptions, often unconsciously, without awareness of having made them, without ever having analyzed them to any great extent, without having tried the alternative assumptions.

Some such assumptions become deeply ingrained in a sub-culture, a culture, a period of history, or all human thinking. If such a philosophical prejudice is institutionalized, it is called a dogma. But our concern here is also with unconscious dogmas. My purpose in this chapter is to show informally how such ideas can be brought out into the open and evaluated.

The first thing is always a willingness to face the issue explicitly, and confront the possibly unpleasant results. Next, try to reconcile the apparent opposites, find a *synthesis* of some sort. Look for the ultimate premises, and even if speculatively, consider alternative conceptions which are capable of fitting the known facts.

The synthesis of knowledge is an attempt to 'wrap it all up', or at least take stock of the situation as a whole thus far. You lay out the data you have, and you firmly evaluate their significance on your current opinions:

- Where are you at?
- What do you know, what don't you know?
- What do you need to know?
- What can you know, what can't you know?

An inventory and a summation, to the best of one's ability.

2. Self-Criticism.

Thus far, one's logic may have been lenient. One perhaps wanted to get ahead, to cover ground. There was no time for scrupulous analysis of the degrees of logical probability in one's information and inferences. Now, the whole must be reviewed, each part considered in the light of

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4 In the present (1995) edition of the book, the present chapter (49) is somewhat abridged. When I wrote it in 1990, I unfortunately included in it a number of comments which can only be classed as religious polemics. Having since then considerably evolved in my views concerning religion, I felt it wise to tone down the essay and restore it to a purely methodological function.

all the others. One must disengage oneself, and become a neutral referee between contending ideas.

One must challenge one's previous viewpoints. One must look at things *more critically*, less intent on the object than on the process which led us to our viewpoint. It is time to linger on detail, digress a little, consider the full impact of what one is saying.

This may mean taking-off in all directions, even to the point of looking into metaphysical implications. One should not limit one's vision to one field, but range as far and wide as necessary to prove a point. One may appeal to epistemological reasons, or consider ontological outcomes.

Initially, we accept our deductions and inductions with fair-minded tolerance. But, in the final analysis, the limits of one's certainties must be emphasized. There are different degrees of strictness of outlook; different modalities of implication. There is a 'take it for granted', working level; and there is a more severe, philosophical level.

Within philosophy, 'anything goes', and even doubts about logic, about the laws of thought or the trustworthiness of experience, have some legitimacy. At this strict level, it is healthy to give skepticism some rein, to enable us to judge with honest detachment (though total skepticism remains invalid, since paradoxical).

For instance, an adductive argument is ordinarily allowed; it is acknowledged to increase the probability of the conclusion. But viewed deductively, its inference is worthless. Synthetic logic probes into theories by considering, not only their internal consistency and continuing confirmation, but more fully and deeply:

- What are the ultimate assumptions?
- What are the implied conclusions?
- Are there alternative premises or inferences?
- How do they compare and contrast, how much do they agree or disagree?
- How reliable are the apparent consistencies and how serious are the seeming inconsistencies?
- How solid are the logical connections between postulates and predictions, and what are they based on?
- What is the data, and how empirical is it?

The enterprise of science is an open pursuit of knowledge. If it is objective, as it wants to be, then it should have no prejudice as to what the object presented to it is, or how it got there. The process of adduction, we saw, has the form:

If Theory, then Predictions:
Yes to any of these predictions,
therefore, possibly yes to the theory.
(but if No to any prediction, no to the theory.)

This may be countered by the equally valid adduction:

If Other Theory, then Same or Other Predictions:
Yes to any of those predictions,
therefore, possibly yes to the other theory.
(but if No to any prediction, no to the theory.)

Now, note the following methodological implications, according to strict logic. Here, the emphasis is more on the criteria of relevance and competitiveness. Utilitarian or esthetic criteria are not granted much weight, so that a far-fetched theory may be as respectable as a more obvious one.

(i) If the two theories make predictions which coincide exactly, or if none of their predictions logically impinge on each other, there is no way to choose between them. They are effectively undifferentiated, or irrelevant to each other.
(ii) If the two theories have some different prediction(s), but these differences are in practice or in principle untestable, again there is no ground for preferring the one to the other. But we may not regard untestable predictions as strictly logically equivalent to non-predictions.
(iii) If the two theories have been confirmed by adduction to an equal degree of logical probability — that is, as many times, by equally firmly-implied and credible phenomena, whether these phenomena be the same or different — no conclusion is permissible. The logical modality is the same.

All this applies as well to theories with mutually exclusive postulates, and to theories with postulates which are independent of each other.

3. Fairness.

Clearly, the mere fact that someone takes up a theory of his own, and keeps testing it, and finds it repeatedly confirmed, does *not* in itself make his work fully scientific, and in accord with the neutral demands of logic.

The scientific approach, under the terms set by epistemology (*not* ontology, mind you), is to consider all other available theories, and busy oneself to an equal extent in testing and confirming *them too*. If difficulties arise, we are duty-bound to try to repair *all* the known theories with equal zeal, and not just the one we hope will win, for whatever personal reasons.

The same methodological demands should be made for one's own pet theory, as one makes for others'; and the same leniency should be granted to others' theories, as one grants to one's own.

Similarly, one should refrain from negative pronouncements on sectors of human inquiry about which one is not adequately informed. In other words, one may regard oneself as a specialist, advancing a limited domain of the inquiry, without laying claim to any authority beyond those limits.

To be professional in the pursuit of knowledge, completely objective and neutral, without prejudice, one must proceed in accord with the rules of argument set by logic. The scientist who merely works on one theory at a time, without regard to the inadequacy of his methodology, is kidding himself and everyone else; he has ignored the alternatives, his conclusions are strictly invalid.

Of course, one can only do one thing at a time; but one must always keep the global perspective in mind, or refrain from comment.[5]

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5 We can use the story of Galileo (as I was taught it at school) to give an example of synthesis. Until Galileo's time, people believed that our planet was the center of the Universe (comprising all the heavenly bodies - Sun, Moon, Planets and Stars); then, various observational and theoretical discoveries changed our picture of things, and the Sun became central (to our solar system, at least). This was initially received very harshly by a certain religious establishment; everyone knows the story. Today of course, after the Relativity theory, the issue is irrelevant to astronomy.

Now, I have no personal attachment to the pre-Galilean thesis, nor does my religion advocate it — the spiritual centrality of mankind has nothing to do with the physical position of planet Earth. However, it seems to me that the argument was in any case fallacious. For the new theories only posited that the mathematical formula describing the movement of the Planets around the Sun was much simpler than the formula which placed Earth at the center of things — but that did not prove that the latter more complex equations could not be formulated.

If I am not mistaken, every trajectory can in principle be 'turned on its head,' and described mathematically from any point of origin. Simplicity is an inductive criterion, but it is never ontologically unassailable. Thus, it is ironic Galileo was in fact not even a threat to the world-view of the Inquisitors. For me, this example illustrates the need to always clarify the precise degree of conflict between theses.

PART VI. FACTORIAL INDUCTION.

50. ACTUAL INDUCTION.

1. The Problem.

Induction is the branch of Logic concerned with determining how general propositions — and, more broadly, how necessary propositions — are established as true, from particular or potential data.

By ‘actual induction’, I mean induction of actual propositions; by ‘modal induction’, I mean induction of modal propositions (referring to *de-re* modality).

We saw, in the analysis of Deductive processes, that although we can infer a general or particular proposition from other general propositions, through opposition, eduction or syllogism, it seems impossible to deductively infer general truths from particular ones only.

Indeed, it is even, according to the rules of syllogism, just about impossible to deduce a particular proposition from particular premises only: there has to be a general premise; the only exceptions to this rule are found in eduction, and in a limited number of third figure syllogisms, which allow us to obtain particular conclusions without use of a general premise: but these are too special to be claimed as important sources.

If, then, virtually all deduction presupposes the prior possession of general premises, where do these first general premises originate, or more precisely, how are they themselves shown to be true? Obviously, if such first premises, whatever their content, are open to doubt and of little credibility, then all subsequent deduction from them, however formally trustworthy, may be looked upon with healthy skepticism. As computer programmers say, “Garbage in, garbage out”. Conclusions drawn from spurious premises could nonetheless be true, but it would be mere chance, not proof.

Furthermore, these ‘first general premises’ we mentioned are not few in number. We are not talking here of a few First Principles, like the axioms of logic, from which exclusively all knowledge is to be derived. We require an extremely large number of first general premises, with all sorts of contents, to be able to develop a faithful image of our actual knowledge base. While mathematical sciences, like arithmetic, algebra or geometry, can seemingly be reduced to a very limited number of axioms, this is a feat not easy to duplicate in sciences like physics or psychology, or in everyday thinking.

If, now, we introspect, and observe our actual thinking processes as individuals, and analyze the actual historical development of Science, the accumulation of knowledge by humankind as a whole, we see clearly that, although deduction plays a large and important role, it is not our only source of knowledge. Even axioms in mathematics have been identified over time, and been subject to improvement or change. In practice, however faultless our deductions, our knowledge is clearly an evolving, flexible, thing. Ideas previously ignored, eventually make their appearance in our body of knowledge; thoughts once considered certain, turn out to be incorrect, and are modified or abandoned.

The primary source of knowledge is not deduction, but *observation*. This term is to be understood here is its broadest, and most neutral, sense, including both passive experiences and those experimentally generated.

Observation is to be understood as in itself a neutral event. It is consciousness, awareness, of appearances, phenomena, such as they present themselves, without judgement as to their ultimate meaning or value in the full scheme of things. Observation concerns the given, in its most brutal, unordered, unprocessed form.

Any *interpretation* that we attach to an observation, is to be regarded as a separate phenomenon; the distinction between these two is not always easy to make, nevertheless. Interpretation, in contrast to observation, attempts to relate phenomena, to place them in a supposed order of things, to evaluate their credibility and real significance in the widest possible context. It is a relatively complex mental process, and more subject to error. Its purpose is to tell us whether, all things considered, an experience was illusory or real.

2. Induction of Particulars.

In this treatise, I will evolve an original theory of induction, in considerable detail, with reference to categorical propositions: first for actuals, then more broadly for modals. I will not here deal with natural, temporal, or extensional conditionals, at all, but it will become obvious that the same methods and principles can be extended to those forms as well, though the formulas involved are bound to be enormously more complex; I leave the task to future logicians with my compliments!

The first step in induction is formulation of particular propositions on the basis of observation. This is a more complicated process than we might at first sight suppose. It does not merely consist in observation of a perceptible phenomenon, but includes the conceptual factor of abstraction of 'universals', the similarities on which we base our verbalization of terms, copula, and particular quantity. Pure observation forms no judgement; it is meditation on, simple consciousness of, the object at hand. The moment a thought is expressed, even a particular proposition, we have interpretation, conceptual correlation. The question of truth or falsehood is yet a separate judgement.

It follows, in passing, that a particular proposition based on observation of concrete phenomena, cannot be viewed as extremely superior in value to one based on observation of abstract phenomena. Both involve abstraction of sorts and verbalization. Their difference is only in the qualitative character of object involved, in the relative accessibility of the evidence.

Now, all observation concerns primarily individual instances. We have seen that singular propositions point to a single specific individual under consideration (referred to by 'this'), whereas particular propositions are quantitatively indefinite and need not specify the individuals they concern (we just say 'some'). A plural but specific proposition, involving the quantity 'these', is essentially singular in nature, or a conjunction of singulars; it differs from a genuine particular, which is more broadly intended. We have seen, too, that singulars imply particulars, by formal opposition.

Normally, unless the subject is a namable individual person or animal, a uniquely complex entity we deal with on a regular basis, our singular propositions are only temporary furniture in our knowledge base. I may say to you "look, this rose, unlike the others in my garden, is blue' or "this particle swerved to the left in our experiment", but ultimately, the individual is ignored or forgotten, and only an indefinite particular proposition is retained in the record. Furthermore, although a particular can be inferred from one singular, it is more often based on a plurality of observations.

In any case, induction of a particular proposition is free of generalization. It is observed that some S are P, so we say 'Some S are P'. If some S are scrutinized and observed not to be P, we say 'Some S are not P'. If no observation has been carried out, our faculties being shut off to the question, or the objects concerned being inaccessible to direct observation, or indirect observation (experiment through instruments), no inductive conclusion is drawn. We may still infer this or that particular deductively, of course.

3. Generalization.

The induction of general propositions, however, occurs by generalization. This obviously does not concern special cases where full enumeration is possible, as in 'all these S are P', or in cases where the subject class is very prescribed so that 'all S' is an accessible number of instances; here, the general proposition can be viewed as effectively singular in nature. Normally, a general proposition is open-ended, and the number of instances involved extremely large (e.g. all the insects in the world), and inaccessible to observation (for example, having existed in the past, or yet to be born). Here, we tend to extrapolate from known instances, to the unknown. We predict many other phenomena, from a limited number of observed phenomena.

The basic principle of generalization is to assume observed, particular uniformities to be applicable generally, until and unless we have reason to think otherwise. A particular proposition arrived at by deductive means can also of course be used as a basis for generalization. The reliability of a generalization is variable, depending on certain factors.

Observation is itself not always a simple process of perception. It may involve research or experiment with certain prior assumptions, methodological or factual, which may require review and testing. The validity of the final generalization depends on the reliability of such prior factors. As well, if a research or experiment process is easily duplicated by other people, socially accessible, it is granted more credence, than a one-time, esoteric observation. Even so, ad hominem arguments count in this domain; a person of known honesty and intelligence may be allowed considerable leeway, in comparison to a habitual liar or scatterbrain.

The degree of effort and ingenuity involved in making the observations in question, also affects the reliability of the generalization. If we observe a limited number of instances and then generalize, and thereafter make no effort to, periodically or in new situations, check our result, it is less reliable obviously than if we remain open-minded, vigilant, and actively research possible deviations from our initial assumption.

The generalization should be reviewed whenever the surrounding context of knowledge has been modified in any way which might conceivably affect it. Comparison of the assumed generality to new information as it comes up, serves not only to verify it but to further confirm it if it stands the test. Here, deductive logic plays its crucial role, guiding us in verifying consistency, by opposition or uncovering implications, helping us to interconnect all our knowledge.

The more alike in nature, the simpler, the phenomena in question are known to be, the more credible and trustworthy our generalization. A generalization concerning, say, gold nuggets, is more reliable than one concerning living cells, because the instances of the former differ in little more than time and space, whereas instances of the latter, though exhibiting some considerable uniformities, are more often found to have individual differences.

The following might be presented as the valid moods of generalization from particular propositions, whether obtained by induction or deduction, to illustrate its basic method.

I → A
Knowing that some S are P,
and not having found any S which are not P,
we may induce that ‘All S are P’.

O → E
Knowing that some S are not P,
and not having found any S which are P,
we may induce that ‘No S is P’.

I + O, knowing some S to be P and some not to be P, inhibits generalization.

Lastly, not having found any S which are P or any S which are not P, strictly leaves us with nothing to say.

However, in practice, if research was made, we might tentatively induce that ‘No S are P’ or ‘All S are P’, preferring the **E** conclusion if P is in content a positive quality, or the **A** conclusion if P is in content a negative quality. A distinction is here made between presence and absence of something, which cannot be expressed in formal terms, but is comprehensible. Such generalization concerns, not so much the subject-matter of our propositions, but the process of observation itself.

4. Particularization.

The reverse process of particularization, is also noteworthy. We start with a general proposition, obtained by generalization or deduction, and a new observation which contradicts it; granting that the latter and its sources more credible than the former, we scale it down for consistency. Thus:

A + O → IO
Having supposed that all S are P,
but finding some S not to be P,
we conclude that ‘only some S are P’.

E + I → IO
Having supposed that no S are P,
but finding some S to be P,
we conclude that ‘only some S are not P’.

In practice, faced with such a situation, we might try to mitigate the result, by reformulating the original general thesis, so that we retain a generality. In the above, this would mean altering the subject, by delineating exceptions to it or substituting a narrower subcategory of it, and/or altering the predicate, by widening it (in positive cases) or narrowing it (in negative cases). Thus, suppose S1 and S2 are subspecies of S, and suppose P’ is a genus embracing P among others, and that P1 and P2 are subspecies of P, then:

in **A + O → IO**, we may review the initial All S are P, to:
- All S1 are P (and No S2 is P), or to:
- All S are P’ (though only some S are P).

Here, we narrow the subject or widen the predicate.

in **E + I → IO**: we may review the initial No S is P, to:
- No S1 is P (and All S2 are P), or to:
- No S is P1 (though some S are P2).

Here, we narrow the subject or narrow the predicate.

A pitfall in generalization is selection of too broad a subject-concept, or too wide or narrow a predicate-concept, when formulating the initial observation.

When particular entities are observed as having a certain property, the question arises are they so *qua* being of some species classification (like crocus, say), or *qua* belonging to some genus (like flowers, say). If we are tempted at the outset to adopt the genus as our subject, we may soon be disappointed, and have to later retract, and particularize the property down to the species, as above. Alternatively, we may be cautious, and adopt the species as subject, and later, finding the wider statement true, would generalize as follows:

All S1 and all S2 are P,
S1 and S2 are all the species of S,
therefore, All S are P.
Here, we broaden the subject.

Likewise, we may initially select a too limited predicate (e.g. blue) or a too vague one (e.g. colored), and later be obliged to qualify our assumption, as shown above.

Either way, in the long run, the correct subject and predicate should impose themselves, assuming the pursuit of knowledge is continued. So the process is not in itself flawed, but induction proceeds by gradual evolution.

5. Validation.

It should be obvious that the above ‘inductive arguments’, and those presented further on, involve a premises-conclusion relationship, of a logical modality other than that found in ‘deductive argument’. Here, we are concerned with inductive implication, which boasts a connection only of logical probability; it is less binding than the logical necessity which characterizes deductive implication.

The validity of man’s inductions, his observations and generalizations, as such, cannot be consistently denied. One can deny this or that specific case to be justified, by adducing evidence to the contrary, but the processes themselves cannot be in principle doubted. For the simple reason that, in so doing, the skeptic is himself formulating a general statement, and so bringing about its own demise. A self-contradictory statement simply has no logical standing. It is automatically and irretrievably false. There are no loop-holes in this reasoning.

The fact that knowledge is contextual, does not imply that it is entirely problematic. The appearances involved in observation and generalization must be taken at their face value, and recognized as indubitably valid, until and unless some specific cause for doubt is brought to the fore, which itself stands the tests of inductive and deductive logic. If that doubt turns out to be indeed justified, the initial observation or generalization is admitted, *ex-post-facto*, to have been mistaken, and modified or abandoned to restore consistency.

Our ignorance of a great variety of epistemological and ontological descriptive facts, such as the nature of consciousness, the workings of our sensory perception or conceptualization, the nature of universals, and all related issues, in no way constitutes a credible reason for doubt. We are well protected by the axioms of logic. We may be humbly aware of our limitations, know with certainty that some of the beliefs we even now may cherish most are bound to turn out to be spurious as the adventure of knowledge progresses, but we may rest assured that not all will be overturned. It is logically impossible, inconceivable to suppose otherwise.

Man does not need to be omniscient to know. Our faculties are effective instruments of knowledge. Knowledge is a continuously evolving, flexible entity. Like a living organism, it changes and shifts, but somehow endures. We have not been endowed with a finished product, but we have been blessed with the means to gradually progress towards that distant goal. Knowledge is essentially functional, a biological tool of survival; as the need for information presents itself, so normally does the opportunity for its procurement. Knowledge is also a spiritual value, one to be attained by effort.

The important thing is to tailor one’s judgements to fit the facts. So long as one’s assumptions and beliefs are up to date, and continuously updated by new data as it appears, they remain reliable and useful.

To trust in one’s judgements does not abrogate one’s right to investigate alternatives and implications; indeed, it is responsible behavior. Certainty and open-mindedness, certainty and verification, are quite compatible. However, there is also a limit to how much one may toy with new ideas, without good reason and rigorous thought.

More broadly, we can say that cognition, like volition, has an ethic, including virtues and vices. Among the virtues are: reasonableness, honesty, making an effort, facing facts, courage, willingness to debate an idea one considers outrageous. Among the vices are: irrationalism, dishonesty, lethargy, evasion, fear of opposition or change, autism. This topic borders on psychology, and could be the subject of a whole treatise by itself.

51. ELEMENTS AND COMPOUNDS.

Our inquiry must now turn to a new doctrine, which may be called factorial analysis. This doctrine is to some extent an offshoot of that of opposition, and interesting for its own sake. Its essential value, however, is to prepare us for the investigation of modal induction, although some information of relevance to deduction is to be found in it. This doctrine is new, because modal logic involves a lot more forms than the traditional logic, and so an issue which was obvious and minor now looms large.

1. Elements and Compounds.

The various categorical propositions, **A**, **E**, **I**, **O**, and their modal counterparts, were presented as the building blocks or elements of knowledge. Elementary propositions are relatively abstract items of knowledge, which intersect in various combinations.

The conjunctions of two or more such elementary propositions, concerning the same subject and predicate, may be referred to as compounds. A compound is in a sense a unit of information too, although it is expressed by us as a sum of elements. Knowledge could conceivably have been constructed by giving each compound a distinct form, but then the elements of data they contain in common would have remained hidden. We wisely, even if instinctively, chose to limit the number of forms in our thoughts, and deal with compounds in terms of their constituent elements.

Not all elementary propositions may be conjoined, of course; some are incompatible, for example '**A** and **O**'. Some conjunctions are redundant, as when a proposition is conjoined with another which is in any case implicit in it; for instance, **A** and **I** together mean no more than **A** alone. However, some compounds are significant, and our task will now be to identify these.

2. Gross Formulas.

At any given stage in the development of knowledge we may have no or partial or complete information, concerning the relation between a specific subject and predicate pair. The sum of information available may be called a formula. A formula may consist of one or more elementary propositions. The elementary propositions taken individually may all be formulas, if they happen to summarize the state of knowledge at that point. Their combinations in distinct consistent compounds, summing up the known without redundancies, are also possible formulas.

Any information not included in a formula is to be considered unavailable in the context of knowledge; thus a formula must contain all known data concerning the two terms in question.

We will express compound formulas in the briefest way, e.g. '**AIn**' signifying '**A** and **In**', without use of extraneous words or symbols for conjunction; it being understood that propositions so fused concern the same subject and predicate, of course.

This study will concern itself only with plural propositions, although some comments about singulars will be made when useful. This is done for the sake of simplicity and clarity, but also in recognition that science is primarily interested in broad statements, and only incidentally in minutiae.

Within the closed system of actual propositions, that covered effectively by classical logic, only five formulas were conceivable: **A**, **I**, **E**, **O**, and **IO**. This in a sense resolves the issue of formulas with regard to extensional modality taken in isolation.

When the other types of modality are introduced, the issue becomes less obvious and more complex. The following table shows methodically what combinations, of the 20 elementary propositions (singulars ignored), can occur consistently and without redundancy. It results that there are a total of 195 distinct formulas, 20 of which are of course elementary, and the rest compounds of up to 4 propositions (75X2 + 79X3 + 21X4).

Note that compounds are expressed in their most compressed form (e.g. **AI** is included in **A**). In practice, we do not always compress compound statements; sometimes we prefer to stress an implication. For instances, 'can never be' stresses that 'cannot be' implies 'is never'; 'can sometimes be' suggests a compound of 'is sometimes' and 'can'.

Table 51.1 Consistent Conjunctions of Categoricals.

Formula	An	Ac	A	At	Ap	In	Ic	I	It	Ip	En	Ec	E	Et	Ep	On	Oc	O	Ot	Op
An	An	Ac	A	At	Ap	In	Ic	I	It	Ip										
AcIn		Ac	A	At	Ap	In	Ic	I	It	Ip										
Ac		Ac	A	At	Ap		Ic	I	It	Ip										
AIn			A	At	Ap	In	Ic	I	It	Ip										
AIc			A	At	Ap		Ic	I	It	Ip										
A			A	At	Ap			I	It	Ip										
AtIn				At	Ap	In	Ic	I	It	Ip										
AtIc				At	Ap		Ic	I	It	Ip										
AtI				At	Ap			I	It	Ip										
At				At	Ap				It	Ip										
ApIn					Ap	In	Ic	I	It	Ip										
ApIc					Ap		Ic	I	It	Ip										
ApI					Ap			I	It	Ip										
ApIt					Ap				It	Ip										
Ap					Ap					Ip										
In						In	Ic	I	It	Ip										
Ic							Ic	I	It	Ip										
I								I	It	Ip										
It									It	Ip										
Ip										Ip										
AcInOp		Ac	A	At	Ap	In	Ic	I	It	Ip										Op
AcOp		Ac	A	At	Ap		Ic	I	It	Ip										Op
AInOp			A	At	Ap	In	Ic	I	It	Ip										Op
AIcOp			A	At	Ap		Ic	I	It	Ip										Op
AOp			A	At	Ap			I	It	Ip										Op
AtInOp				At	Ap	In	Ic	I	It	Ip										Op
AtIcOp				At	Ap		Ic	I	It	Ip										Op
AtIOp				At	Ap			I	It	Ip										Op
AtOp				At	Ap				It	Ip										Op
ApInOp					Ap	In	Ic	I	It	Ip										Op
ApIcOp					Ap		Ic	I	It	Ip										Op
ApIOp					Ap			I	It	Ip										Op
ApItOp					Ap				It	Ip										Op
ApOp					Ap					Ip										Op
InOp						In	Ic	I	It	Ip										Op
IcOp							Ic	I	It	Ip										Op
IOp								I	It	Ip										Op
ItOp									It	Ip										Op
IpOp										Ip										Op
Op																				Op

AInOt			A	At	Ap	In	Ic	I	It	Ip									Ot	Op
AIcOt			A	At	Ap		Ic	I	It	Ip									Ot	Op
AOt			A	At	Ap			I	It	Ip									Ot	Op
AtInOt				At	Ap	In	Ic	I	It	Ip									Ot	Op
AtIcOt				At	Ap		Ic	I	It	Ip									Ot	Op
AtIOt				At	Ap			I	It	Ip									Ot	Op
AtOt				At	Ap				It	Ip									Ot	Op
ApInOt					Ap	In	Ic	I	It	Ip									Ot	Op
ApIcOt					Ap		Ic	I	It	Ip									Ot	Op
ApIOt					Ap			I	It	Ip									Ot	Op
ApItOt					Ap				It	Ip									Ot	Op
ApOt					Ap					Ip									Ot	Op
InOt						In	Ic	I	It	Ip									Ot	Op
IcOt							Ic	I	It	Ip									Ot	Op
IOt								I	It	Ip									Ot	Op
ItOt									It	Ip									Ot	Op
IpOt										Ip									Ot	Op
Ot																			Ot	Op
AtInO				At	Ap	In	Ic	I	It	Ip								O	Ot	Op
AtIcO				At	Ap		Ic	I	It	Ip								O	Ot	Op
AtIO				At	Ap			I	It	Ip								O	Ot	Op
AtO				At	Ap				It	Ip								O	Ot	Op
ApInO					Ap	In	Ic	I	It	Ip								O	Ot	Op
ApIcO					Ap		Ic	I	It	Ip								O	Ot	Op
ApIO					Ap			I	It	Ip								O	Ot	Op
ApItO					Ap				It	Ip								O	Ot	Op
ApO					Ap					Ip								O	Ot	Op
InO						In	Ic	I	It	Ip								O	Ot	Op
IcO							Ic	I	It	Ip								O	Ot	Op
IO								I	It	Ip								O	Ot	Op
ItO									It	Ip								O	Ot	Op
IpO										Ip								O	Ot	Op
O																		O	Ot	Op
ApInOc					Ap	In	Ic	I	It	Ip							Oc	O	Ot	Op
ApIcOc					Ap		Ic	I	It	Ip							Oc	O	Ot	Op
ApIOc					Ap			I	It	Ip							Oc	O	Ot	Op
ApItOc					Ap				It	Ip							Oc	O	Ot	Op
ApOc					Ap					Ip							Oc	O	Ot	Op
InOc						In	Ic	I	It	Ip							Oc	O	Ot	Op
IcOc							Ic	I	It	Ip							Oc	O	Ot	Op
IOc								I	It	Ip							Oc	O	Ot	Op
ItOc									It	Ip							Oc	O	Ot	Op
IpOc										Ip							Oc	O	Ot	Op
Oc																	Oc	O	Ot	Op
InOn						In	Ic	I	It	Ip						On	Oc	O	Ot	Op
IcOn							Ic	I	It	Ip						On	Oc	O	Ot	Op
IOn								I	It	Ip						On	Oc	O	Ot	Op
ItOn									It	Ip						On	Oc	O	Ot	Op
IpOn										Ip						On	Oc	O	Ot	Op
On																On	Oc	O	Ot	Op

AcEp		Ac	A	At	Ap		Ic	I	It	Ip					Ep					Op
AIcEp			A	At	Ap		Ic	I	It	Ip					Ep					Op
AEp			A	At	Ap			I	It	Ip					Ep					Op
AtIcEp				At	Ap		Ic	I	It	Ip					Ep					Op
AtIEp				At	Ap			I	It	Ip					Ep					Op
AtEp				At	Ap				It	Ip					Ep					Op
ApIcEp					Ap		Ic	I	It	Ip					Ep					Op
ApIEp					Ap			I	It	Ip					Ep					Op
ApItEp					Ap				It	Ip					Ep					Op
ApEp					Ap					Ip					Ep					Op
IcEp							Ic	I	It	Ip					Ep					Op
IEp								I	It	Ip					Ep					Op
ItEp									It	Ip					Ep					Op
IpEp										Ip					Ep					Op
Ep															Ep					Op
AIcEpOt			A	At	Ap		Ic	I	It	Ip					Ep				Ot	Op
AEpOt			A	At	Ap			I	It	Ip					Ep				Ot	Op
AtIcEpOt				At	Ap		Ic	I	It	Ip					Ep				Ot	Op
AtIEpOt				At	Ap			I	It	Ip					Ep				Ot	Op
AtEpOt				At	Ap				It	Ip					Ep				Ot	Op
ApIcEpOt					Ap		Ic	I	It	Ip					Ep				Ot	Op
ApIEpOt					Ap			I	It	Ip					Ep				Ot	Op
ApItEpOt					Ap				It	Ip					Ep				Ot	Op
ApEpOt					Ap					Ip					Ep				Ot	Op
IcEpOt							Ic	I	It	Ip					Ep				Ot	Op
IEpOt								I	It	Ip					Ep				Ot	Op
ItEpOt									It	Ip					Ep				Ot	Op
IpEpOt										Ip					Ep				Ot	Op
EpOt															Ep				Ot	Op
AtIcEpO				At	Ap		Ic	I	It	Ip					Ep			O	Ot	Op
AtIEpO				At	Ap			I	It	Ip					Ep			O	Ot	Op
AtEpO				At	Ap				It	Ip					Ep			O	Ot	Op
ApIcEpO					Ap		Ic	I	It	Ip					Ep			O	Ot	Op
ApIEpO					Ap			I	It	Ip					Ep			O	Ot	Op
ApItEpO					Ap				It	Ip					Ep			O	Ot	Op
ApEpO					Ap					Ip					Ep			O	Ot	Op
IcEpO							Ic	I	It	Ip					Ep			O	Ot	Op
IEpO								I	It	Ip					Ep			O	Ot	Op
ItEpO									It	Ip					Ep			O	Ot	Op
IpEpO										Ip					Ep			O	Ot	Op
EpO															Ep			O	Ot	Op
ApIcEpOc					Ap		Ic	I	It	Ip					Ep		Oc	O	Ot	Op
ApIEpOc					Ap			I	It	Ip					Ep		Oc	O	Ot	Op
ApItEpOc					Ap				It	Ip					Ep		Oc	O	Ot	Op
ApEpOc					Ap					Ip					Ep		Oc	O	Ot	Op
IcEpOc							Ic	I	It	Ip					Ep		Oc	O	Ot	Op
IEpOc								I	It	Ip					Ep		Oc	O	Ot	Op
ItEpOc									It	Ip					Ep		Oc	O	Ot	Op
IpEpOc										Ip					Ep		Oc	O	Ot	Op
EpOc															Ep		Oc	O	Ot	Op

IcEpOn							Ic	I	It	Ip					Ep	On	Oc	O	Ot	Op
IEpOn								I	It	Ip					Ep	On	Oc	O	Ot	Op
ItEpOn									It	Ip					Ep	On	Oc	O	Ot	Op
IpEpOn										Ip					Ep	On	Oc	O	Ot	Op
EpOn															Ep	On	Oc	O	Ot	Op
AEt			A	At	Ap			I	It	Ip				Et	Ep				Ot	Op
AtIEt				At	Ap			I	It	Ip				Et	Ep				Ot	Op
AtEt				At	Ap				It	Ip				Et	Ep				Ot	Op
ApIEt					Ap			I	It	Ip				Et	Ep				Ot	Op
ApItEt					Ap				It	Ip				Et	Ep				Ot	Op
ApEt					Ap					Ip				Et	Ep				Ot	Op
IEt								I	It	Ip				Et	Ep				Ot	Op
ItEt									It	Ip				Et	Ep				Ot	Op
IpEt										Ip				Et	Ep				Ot	Op
Et														Et	Ep				Ot	Op
AtIEtO				At	Ap			I	It	Ip				Et	Ep			O	Ot	Op
AtEtO				At	Ap				It	Ip				Et	Ep			O	Ot	Op
ApIEtO					Ap			I	It	Ip				Et	Ep			O	Ot	Op
ApItEtO					Ap				It	Ip				Et	Ep			O	Ot	Op
ApEtO					Ap					Ip				Et	Ep			O	Ot	Op
IEtO								I	It	Ip				Et	Ep			O	Ot	Op
ItEtO									It	Ip				Et	Ep			O	Ot	Op
IpEtO										Ip				Et	Ep			O	Ot	Op
EtO														Et	Ep			O	Ot	Op
ApIEtOc					Ap			I	It	Ip				Et	Ep		Oc	O	Ot	Op
ApItEtOc					Ap				It	Ip				Et	Ep		Oc	O	Ot	Op
ApEtOc					Ap					Ip				Et	Ep		Oc	O	Ot	Op
IEtOc								I	It	Ip				Et	Ep		Oc	O	Ot	Op
ItEtOc									It	Ip				Et	Ep		Oc	O	Ot	Op
IpEtOc										Ip				Et	Ep		Oc	O	Ot	Op
EtOc														Et	Ep		Oc	O	Ot	Op
IEtOn								I	It	Ip				Et	Ep	On	Oc	O	Ot	Op
ItEtOn									It	Ip				Et	Ep	On	Oc	O	Ot	Op
IpEtOn										Ip				Et	Ep	On	Oc	O	Ot	Op
EtOn														Et	Ep	On	Oc	O	Ot	Op
AtE				At	Ap				It	Ip			E	Et	Ep			O	Ot	Op
ApItE					Ap				It	Ip			E	Et	Ep			O	Ot	Op
ApE					Ap					Ip			E	Et	Ep			O	Ot	Op
ItE									It	Ip			E	Et	Ep			O	Ot	Op
IpE										Ip			E	Et	Ep			O	Ot	Op
E													E	Et	Ep			O	Ot	Op
ApItEOc					Ap				It	Ip			E	Et	Ep		Oc	O	Ot	Op
ApEOc					Ap					Ip			E	Et	Ep		Oc	O	Ot	Op
ItEOc									It	Ip			E	Et	Ep		Oc	O	Ot	Op
IpEOc										Ip			E	Et	Ep		Oc	O	Ot	Op
EOc													E	Et	Ep		Oc	O	Ot	Op
ItEOn									It	Ip			E	Et	Ep	On	Oc	O	Ot	Op
IpEOn										Ip			E	Et	Ep	On	Oc	O	Ot	Op
EOn													E	Et	Ep	On	Oc	O	Ot	Op
ApEc					Ap					Ip		Ec	E	Et	Ep		Oc	O	Ot	Op

IpEc										Ip		Ec	E	Et	Ep		Oc	O	Ot	Op
Ec												Ec	E	Et	Ep		Oc	O	Ot	Op
IpEcOn										Ip		Ec	E	Et	Ep	On	Oc	O	Ot	Op
EcOn												Ec	E	Et	Ep	On	Oc	O	Ot	Op
En											En	Ec	E	Et	Ep	On	Oc	O	Ot	Op

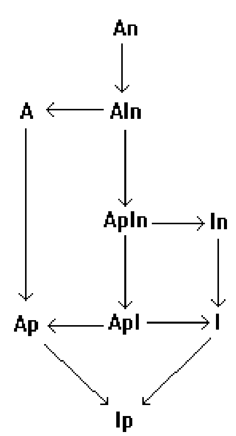
Note in passing that if we considered either natural or temporal modality as a closed system, we would find ourselves in each case with a total of 49 formulas, 12 of which were elementaries, and the remaining 37 were compounds of up to 4 propositions. Formulas involving actual propositions only are 5 in number, and formulas which mix modality types number 102.

Now although this list of formulas is complete in itself, it will become apparent that it does not in fact exhaust the possible states of knowledge. We shall see that formulas of this kind are gross assertions, which do not clarify all the issues involved.

3. Oppositions.

Once we view a compound as a unit, one complex proposition, we may ask what oppositional relations exist between compounds. Consider, for example, the affirmative compounds **AIn**, **ApIn**, **ApI**. They may be placed in a hierarchy relative to each other and to the cognate elements, as follows:

Diagram 51.1 Hierarchy of Compounds.



Looking at the arrows of subalternation, we see a gradual softening of position, ranging from **An** to **Ip**. There is a continuum of affirmative statements, in which temporal modality could also be inserted. A similar hierarchy may be developed for the analogous negatives. More complex, bipolar compounds also have their inter-oppositions, including many such subalternations.

The contradictory of any compound is a disjunctive proposition, note well; it disjoins the contradictories of the various elements involved, in an 'and/or' manner. Thus, for examples:

AIn is contradicted by '**O** and/or **Ep**',
ApIn is contradicted by '**On** and/or **Ep**',

ApI is contradicted by '**On** and/or **E**'.

If **AIn** is false, then one of **O** or **EpO** or **Ep** must be true; each of the latter is by itself only contrary to **AIn**: it is the disjunction as a whole which is contradictory.

Similarly for all other compounds. Note that some 'ands' yield impossible combinations; these are as such automatically eliminated. For example: the contradictory of **ApIOc** is '**On** and/or **E** and/or **At**', in which any combination of **On** with **At** is rejectable at once, meaning that only the alternatives '**On** or **EOn** or **E** or **AtE** or **At**' are viable.

There is no need for us to work out all the interrelationships in advance. The work can be done ad hoc, as specific need arises.

4. Double Syllogisms.

Once we regard a compound proposition as a single whole in its own right, we are enticed to ask whether there are corresponding compound syllogisms. Consider, for example, the closed system of actuals. Here, we have one conjunctive formula, '**I** and **O**'; its contradictory is '**A** or **E**', since not-{**I** and **O**} means not**I** and/or not**O**, which means **E** and **O** (= **E**) or **I** and **A** (= **A**) or **E** and **A** (impossible).

With regard to the conjunctive compound '**I** and **O**'. Compound syllogism is impossible in the first figure, since we would need both an **A** and an **E** major premise with the same terms. It is also impossible in the second figure, since this figure only yields negative conclusions. However, in the third figure, we have the following valid double syllogism, merging **3/IAI** and **3/OAO**:

Some M are P and some M are not P,
and All M are S,
therefore, Some S are P and some S are not P.

It must follow that the disjunctive compound '**A** or **E**' (which contradicts **IO**) also has a valid mood of the syllogism. It must be in the first figure, disjoining **1/AAA** and **1/EAE**, so that denial of its conclusion causes denial of its major premise, by reductio ad absurdum to the above one:

All M are P or No M is P,
and All S are M,
therefore, All S are P or No S is P.

This shows that the compound **IO**, and its contradictory, have a deductive life of their own. These are the only Aristotelean syllogisms capable of processing compounds.

The same can be done with modal compounds. I will not go into detail but simply give a pair of examples:

Some M can be P and some M can not-be P,
and All M must be S,
therefore, Some S can be P and some S can not-be P.

All M must be P or No M can be P,
and All S must be M,
therefore, All S must be P or No S can be P.

Other quantities and modalities than these can similarly be processed. The reader is encouraged to try and evolve a full list of compound syllogisms, as an exercise, with reference to the full list of compound propositions given earlier. Are there tandems involving triple or quadruple compounds?

5. Complements.

To fully understand how any two terms, S and P, are related, we must know their relations in both directions: from S to P and from P to S. These may be called the front and reverse side of the overall relation. The S-P side alone can only provide us with a 'flat' picture of the intersection of the terms; the reality is 'stereoscopic', and to express it entirely we need to specify the P-S side as well.

The S to P and P to S relations may be called complementary. The possible complements of any S to P relation are the propositions compatible with its converse. Thus, the doctrine of complements is an offshoot of the doctrines of eduction and opposition.

Consider actual categoricals. Since 'All/This/Some S are P' are convertible to 'Some P are S' only, the possible complements of **A**, **R**, or **I** (in S-P), are **A** or **O** (in P-S). Since 'No S is P' is convertible fully to 'No P is S', the latter is the only possible complement of the former. Lastly, since **G** and **O** are not at all convertible, they are compatible with any of **A**, **E**, or **IO**, on the reverse side.

Similarly for modals. Since **An**, **Rn**, **In**, **Ap**, **Rp**, **Ip**, are all convertible to **Ip** only, their possible complements are all the propositions compatible with this converse, namely any form but **En**. For **En**, which converts fully to **En**, the only possible complement is **En**. Lastly, since **Gn**, **On**, **Ep**, **Gp**, **Op**, are none of them at all convertible, any form may complement them. As with naturals, so with temporals.

Just as we developed a list of possible gross formulas for the S to P relationship, we could additionally work out the compatible P to S gross formulas for each S to P gross formula. This would provide us with more complex, 'two-way' gross formulas, yielding a fuller picture of reality than heretofore available.

(If the complement is identical in form to the original proposition, then the relation may be said to be reciprocal; otherwise, it is nonreciprocal. Thus, for instance, if 'all S are P' and 'all P are S' are both true, the relation of S and P is reciprocal; in contrast, **A** complemented by **IO** is nonreciprocal.)

Note in passing that we could go a step further, and consider not only P-S relations as complements to S-P, but also relations involving the antitheses of one or both of the terms. In that case, obversion, obverted conversion, conversion by negation, contraposition, inversion, and obverted inversion, all become significant, telling us more about the possible combinations of S and P in all their facets.

To get deeper still, we would perhaps have to take transitives into consideration, looking into their possible conjunctions, as 'supplements', on the S-P and P-S sides, and indeed, on every other side.

However, all these complications will be ignored in this treatise.

52. FRACTIONS AND INTEGERS.

1. Fractions.

To achieve a fuller analysis of the states of knowledge, we must introduce certain tools, which we will call fractions and integers. These concepts relate, not primarily to **states of knowledge**, but to **states of being**. Whereas knowledge can be deficient, being must be definite, so that the possibilities it involves are more limited in number.

In view of the large amounts of data involved, we will develop these concepts in two stages. First, we will consider natural modality in isolation, as a **closed system**. Whatever results are obtained for this type, can be obtained by analogy for temporal modality taken by itself, by substituting the subscripts **c** and **t** for **n** and **p** throughout, as usual. Thereafter, we will broaden the perspective, and deal with both types of modality together, as a continuous, **open system**.

a. To begin with, consider *singulars*. Within natural modality, an individual subject's relation to a predicate has only 4 possible states of being, whatever the state of our knowledge concerning it. Two of these are elementary, and two are compound. They are as follows. The significance of the brackets, which are not really needed for singulars, will become apparent as we proceed.

(**Rn**)	This S must be P
(**Gn**)	This S cannot be P
(RGp)	This S is P, though it can not-be P
(RpG)	This S is not P, though it can be P.

All four of these imply actuality. The latter two are of course singular extensional conditional propositions. They have in common the fact of contingency, but the compound **RpGp** is not a state of being since it does not tell us which of the two possibilities is in fact actualized. Still, **RGp** and **RpG** are close relatives to each other, insofar as the individual may switch from the one to the other state of being, whereas the two necessaries and contingency as such are immutable and may not replace each other over time.

Logically, then, these four states are not only exhaustive (one of them must be true, of any individual), but also mutually exclusive (only one may be true, at least at the same time).

Similarly within a closed system of temporal modality, an individual must have one of the following 4 states of being:

(**Rc**) (**Gc**) (RGt) (RtG)

In the mixed modality system, an individual has 6 alternative states of being:

(**Rn**) (**Gn**) (RcGp) (RpGc) (RGt) (RtG)

Note that only two states are carried over from each of the closed systems, and two are new contributions by the open system. Thus, note well, although (**RGp**) and (**RpG**) are recognized as states of being within natural modality, they lose this status in the wider perspective; likewise, (**Rc**) and (**Gc**), though so recognized within temporal modality, they are found deficient in full context.

Nevertheless, in practice we often do limit our thinking to one system or the other, so it is worth considering them also in isolation. The precise correlation between these closed system fractions, and open system fractions, is as follows (no other alternatives than those listed being applicable):

(**RGp**)	=	(**RcGp**) or (**RGt**),	since both imply **R** and **Gp**.
(**RpG**)	=	(**RpGc**) or (**RtG**),	since both imply **Rp** and **G**.
(**Rc**)	=	(**Rn**) or (**RcGp**),	since both imply **Rc**.
(**Gc**)	=	(**Gn**) or (**RpGc**),	since both imply **Gc**.

The states of being may be called integers, because they are fully defining of the actual relationship of subject to predicate. They are whole units of information, involving no ambiguity, vagueness or remaining questions. The concept of fractions becomes useful when we turn to plural propositions; with regard to singulars it is identical with that of integers.

b. Let us now *quantify* the above ideas. We will henceforth ignore singular propositions, so as to simplify treatment.

Within the closed system of natural modality, we may by analogy recognize 8 plural fractions. These could be assigned the symbols **f1**-**f8**, contextually. They are:

f1	f2	f3	f4	f5	f6	f7	f8
(An)	(En)	(AEp)	(ApE)	(In)	(On)	(IOp)	(IpO)

Note the similarity between the two quartets, **f1-f4** and **f5-f8**, as well as their correspondence to the earlier mentioned singulars. As will be seen, the 4 universal fractions are also integers; but the 4 particular fractions are not integers, they are merely building blocks of integers.

Brackets are not really needed for the universal fractions, though used to maintain a uniform notation, since universals cover an identifiable extension. But in the case of particular fractions, they are essential, because the instances involved are not formally designated or enumerated.

We will adopt the convention that two elements enclosed in brackets, such as **I** and **Op** in (**IOp**), subsume exactly the same extension: for every instance of the subject in the one, there corresponds an instance in the other. Thus, (**IOp**) means ‘Some S are P, though these same S also can not-be P’, or more briefly, as ‘Some S both are P and can not-be P’; similarly for (**IpO**). Such propositions are of course particular extensional conditionals.

Brackets serve as well to stress separation between two parts of an extension. Thus for example, the conjunction of two fractions (**In**)(**IpO**) indicates that ‘Some S must be P, while some other S both can be but are not P’. Such relationships can be expressed in practice through the language of extensional conditioning.

Obviously, two fractions may be identical in appearance, but in fact concern distinct or only partly overlapping segments of the whole extension of the subject. Thus for example, though (**On**) and (**On**) are outwardly the same, they may happen to refer to different instances.

Nevertheless, in such case, they can be merged into one fraction, which simply covers a wider extension equal to the sum of the original two. The conjunction of two similar fractions results in one similar fraction. For example, (**On**)(**On**) equals (**On**).

Within the closed system of temporal modality, there are similarly 8 plural fractions:

f1	f2	f3	f4	f5	f6	f7	f8
(Ac)	(Ec)	(AEt)	(AtE)	(Ic)	(Oc)	(IOt)	(ItO)

In the open system, viewing natural and temporal modalities as a continuum, we may recognize the following 12 fractions. These could be assigned the symbols **f1**-**f12**, contextually.

f1	f2	f3	f4	f5	f6	f7	f8	f9	f10	f11	f12
(An)	(En)	(AcEp)	(ApEc)	(AEt)	(AtE)	(In)	(On)	(IcOp)	(IpOc)	(IOt)	(ItO)

Here again, note the differences between the open system and the two systems limited to one modality type. The open system is the factually true system, the others being artificial constructs to some extent.

2. Double Syllogisms.

Aristotelean logic considered syllogism as a deductive process applicable to elementary propositions. But we saw in the previous chapter that compound propositions have a logic of their own, so that there are also (derivative) valid moods which function by compounding premises and conclusions.

The following are the most significant samples of this in the present context. They involve a bipolar fractional premise and conclusion, and thus show the transmissibility of particular fractional subsumption.

a. With appropriate complementary major premise (**A** and **An**), the fraction (**IOp**) or (**IpO**) for the minor and middle terms, yields a similar conclusion for the minor and major terms, through a mix of first and second figure syllogism.

1/AII and 2/AnOpOp:
All M are P, and all P must be M,
Some S are M, though these S can not-be M,
so, Some S are P, though these S can not-be P.

2/AOO and 1/AnIpIp:
All P are M, and all M must be P,
Some S are not M, though these S can be M,
so, Some S are not P, though these S can be P.

b. With appropriate necessary minor premise (**An**, which implies **A**), the fraction (**IOp**) or (**IpO**) for the middle and minor terms, yields a similar conclusion for the minor and major terms, through a double third figure syllogism.

3/IAI and 3/OpAnOp:
Some M are P, though these M can not-be P,
All M are S, indeed all M must be S,
so, Some S are P, though these S can not-be P.

3/OAO and 3/IpAnIp:
Some M are not P, though these M can be P,
All M are S, indeed all M must be S,
so, Some S are not P, though these S can be P.

We note that, since the fractional premise (and conclusion) are bipolar, the other premise compound must be all affirmative. Similar valid moods can be obtained for temporal and mixed modality fractions, with the appropriate changes in the accompanying premise.

3. Integers.

Integers represent the possible states of being. As we pointed out, reality has to materialize in some fully definite way, though knowledge of it may be lacking, only partial or complete. The integers are thus, as states of knowledge, the few cases of complete information. Knowing these clearly, we can use them as factors to predict the many and various possibilities of incomplete information.

Plural integers consist of fractions, either universal fractions taken individually, or some combination of particular fractions. In reality, of course, the integers are monoliths and come first, and the fractions are abstractions we draw out of them by observing their common characters; but we move in the reverse direction as we construct a logical system to represent them.

a. In the *closed system of natural modality*, there are exhaustively 15 integers; these are mutually exclusive by the laws of opposition. Four of them consist of universal fractions, and eleven consist of conjunctions of two to four particular fractions.

We shall adopt, when useful, the symbols **F1**-**F15** for the 15 integers; note that the numbering of these symbols is applicable within the modal framework under discussion, the same ones may be used with different meaning in another context. But it is well not to get overly symbolic, and remain conscious of the underlying significance in terms of standard **A**, **E**, **I**, **O** notation.

The following table lists the 15 integers and shows what conjunctions of fractions constitute them. Cells marked '**yes**' signify which fractions are included in the corresponding integer. This list must be complete since, mathematically, the 4 particular fractions can only combine in 2 to the 4th power - 1 = 15 ways, 4 of which are the universal fractions.

<u>Table 52.1 The Integers of Natural Modality.</u>

		Fractions							
		(An)	(En)	(AEp)	(ApE)	(In)	(On)	(IOp)	(IpO)
		f1	f2	f3	f4	f5	f6	f7	f8
Integers									
(An)	F1	yes				yes			
(En)	F2		yes				yes		
(AEp)	F3			yes				yes	
(ApE)	F4				yes				yes
(In)(On)	F5					yes	yes		
(In)(IOp)	F6					yes		yes	
(On)(IpO)	F7						yes		yes
(In)(IpO)	F8					yes			yes
(On)(IOp)	F9						yes	yes	
(IOp)(IpO)	F10							yes	yes
(In)(On)(IOp)	F11					yes	yes	yes	
(In)(On)(IpO)	F12					yes	yes		yes
(In)(IOp)(IpO)	F13					yes		yes	yes
(On)(IOp)(IpO)	F14						yes	yes	yes
(In)(On)(IOp)(IpO)	F15					yes	yes	yes	yes

We may in passing mention the relations of the 4 singular integers to plurals. We can say that (**Rn**) = (**An**) or (**In**)(); (**Gn**) = (**En**) or (**On**)(); (**RGp**) = (**AEp**) or (**IOp**)(); and (**RpG**) = (**ApE**) or (**IpO**)(), with the empty brackets signifying any combination of other particular fractions.

A similar list of integers can be drawn up for the closed system of *temporal* modality.

b. With regard to the *mixed modality system*, since this involves 12 fractions, of which 6 are universal and 6 particular, we can expect it to yield 2 to the 6th power - 1 = 63 integers. These are listed below (fractions conjoined into integers being marked **'yes'**). Remember, not only is this list exhaustive, but the integers are mutually exclusive. These will be assigned the contextual symbols **F1**-**F63**, when useful.

Table 52.2 The Integers of Mixed Modality.

		Fractions					
		(In)	(On)	(IcOp)	(IpOc)	(IOt)	(ItO)
		f7	f8	f9	f10	f11	f12
Integers							
(An)	F1	yes					
(En)	F2		yes				
(AcEp)	F3			yes			
(ApEc)	F4				yes		
(AEt)	F5					yes	
(AtE)	F6						yes
(In)(On)	F7	yes	yes				
(In)(IcOp)	F8	yes		yes			
(On)(IpOc)	F9		yes		yes		
(In)(IpOc)	F10	yes			yes		
(On)(IcOp)	F11		yes	yes			
(In)(IOt)	F12	yes				yes	
(On)(ItO)	F13		yes				yes
(In)(ItO)	F14	yes					yes
(On)(IOt)	F15		yes			yes	
(IcOp)(IpOc)	F16			yes	yes		
(IcOp)(IOt)	F17			yes		yes	
(IpOc)(ItO)	F18				yes		yes
(IcOp)(ItO)	F19			yes			yes
(IpOc)(IOt)	F20				yes	yes	
(IOt)(ItO)	F21					yes	yes
(In)(On)(IcOp)	F22	yes	yes	yes			
(In)(On)(IpOc)	F23	yes	yes		yes		
(In)(On)(IOt)	F24	yes	yes			yes	
(In)(On)(ItO)	F25	yes	yes				yes
(In)(IcOp)(IpOc)	F26	yes		yes	yes		
(On)(IcOp)(IpOc)	F27		yes	yes	yes		
(In)(IcOp)(IOt)	F28	yes		yes		yes	
(On)(IpOc)(ItO)	F29		yes		yes		yes
(In)(IcOp)(ItO)	F30	yes		yes			yes
(On)(IpOc)(IOt)	F31		yes		yes	yes	
(In)(IpOc)(IOt)	F32	yes			yes	yes	
(On)(IcOp)(ItO)	F33		yes	yes			yes
(In)(IpOc)(ItO)	F34	yes			yes		yes
(On)(IcOp)(IOt)	F35		yes	yes		yes	
(In)(IOt)(ItO)	F36	yes				yes	yes
(On)(IOt)(ItO)	F37		yes			yes	yes
(IcOp)(IpOc)(IOt)	F38			yes	yes	yes	
(IcOp)(IpOc)(ItO)	F39			yes	yes		yes

		Fractions					
		(In)	(On)	(IcOp)	(IpOc)	(IOt)	(ItO)
		f7	f8	f9	f10	f11	f12
Integers							
(IcOp)(IOt)(ItO)	F40			yes		yes	yes
(IpOc)(IOt)(ItO)	F41				yes	yes	yes
(In)(On)(IcOp)(IpOc)	F42	yes	yes	yes	yes		
(In)(On)(IcOp)(IOt)	F43	yes	yes	yes		yes	
(In)(On)(IpOc)(ItO)	F44	yes	yes		yes		yes
(In)(On)(IcOp)(ItO)	F45	yes	yes	yes			yes
(In)(On)(IpOc)(IOt)	F46	yes	yes		yes	yes	
(In)(On)(IOt)(ItO)	F47	yes	yes			yes	yes
(In)(IcOp)(IpOc)(IOt)	F48	yes		yes	yes	yes	
(On)(IcOp)(IpOc)(ItO)	F49		yes	yes	yes		yes
(In)(IcOp)(IpOc)(ItO)	F50	yes		yes	yes		yes
(On)(IcOp)(IpOc)(IOt)	F51		yes	yes	yes	yes	
(In)(IcOp)(IOt)(ItO)	F52	yes		yes		yes	yes
(On)(IpOc)(IOt)(ItO)	F53		yes		yes	yes	yes
(In)(IpOc)(IOt)(ItO)	F54	yes			yes	yes	yes
(On)(IcOp)(IOt)(ItO)	F55		yes	yes		yes	yes
(IcOp)(IpOc)(IOt)(ItO)	F56			yes	yes	yes	yes
(In)(On)(IcOp)(IpOc)(IOt)	F57	yes	yes	yes	yes	yes	
(In)(On)(IcOp)(IpOc)(ItO)	F58	yes	yes	yes	yes		yes
(In)(On)(IcOp)(IOt)(ItO)	F59	yes	yes	yes		yes	yes
(In)(On)(IpOc)(IOt)(ItO)	F60	yes	yes		yes	yes	yes
(In)(IcOp)(IpOc)(IOt)(ItO)	F61	yes		yes	yes	yes	yes
(On)(IcOp)(IpOc)(IOt)(ItO)	F62		yes	yes	yes	yes	yes
(In)(On)(IcOp)(IpOc)(IOt)(ItO)	F63	yes	yes	yes	yes	yes	yes

Note that 6 of the integers are universals, and 57 are particulars. For reasons of space, the universal fractions **f1**-**f6** are not shown here, but it should be clear that they coincide with the integers **F1**-**F6**; these incidentally imply the lone fractions **f7**-**f12**, respectively.

Any subject and predicate must be related in one of these ways, and only one. If any fraction involved contains an actual proposition, the applicable integer may change over time, though only one will be applicable at any moment. If none of the fraction(s) involved consist of actual propositions, the integer is immutable.

These are all the possible states of being, in the open system including all modal types, but they do not cover all states of knowledge. We may to different degrees be ignorant as to which of these full realities to apply in a given case, having only partial or no information concerning it.

In passing, let us mention that, here again, singular integers can be reduced to a disjunction of the universal and particular integers which resemble them.

4. Further Developments.

Just as in reality S-P gross formulas are incomplete, without specification of the reverse P-S side, so likewise the integers we have so far considered are deficient pictures of reality. Integers which are solely defined by S to P relations, are ‘flat’ — in the real world, every S and P relation also has a P to S facet. Thus, only ‘stereoscopic’ integers are really ‘integers’, in the ultimate sense of full expressions of a relationship.

The combination of flat integers into stereo integers resembles the combination of fractions into integers. For example, the S to P relation is ‘All S must be P’ and the P to S relationship is ‘some P must be S and some cannot be S’. This could be written symbolically as, say, SP:(**An**) + PS:(**In**)(**On**). Many such two-way conjunctions of integers are possible; but of course, some combinations are interdicted by the laws of conversion.

I will not, in the following chapters, develop a logic for such complex integers, because the topic is just too vast. I think that the innovations in inductive logic, presented in this work, will be best served by avoiding such further complications. The factorial approach is what I want to highlight, and it will be more clearly put across using the simpler medium which I have adopted.

The reader is asked, nevertheless, to keep in mind the avenues of further development here hinted at. The logic covered here concerns ‘flat integers’: that for ‘stereo integers’ is yet to be dealt with, in some future work or by other logicians. The truth of what is said in this treatise is not affected, it is only made more partial a truth than implied. A flat integer may be viewed as a genus including a number of possible stereo integers.

Incidentally, we can further expand the whole study by considering transitive relations, like ‘S can or must become P’, in various combinations with each other or with static subsumptives. I will not venture into this field here.

Another direction of development to take note of, is consideration of conditional propositions, to the same extent as categoricals are dealt with in this treatise.

I pointed out, in the part on *de-re* conditioning, that categoricals and conditionals are all particles of a large continuum of modal propositions.

They share many hierarchies of implication, like for instance: ‘All S must be P’ (a categorical) implies, among other things, that ‘When certain things are S, they must be P’ (a natural conditional), and that ‘Anything which can be S, can be P’ (an extensional conditional).

Also for instance, the premises ‘All S must be P, and all P must be Q’ may be viewed as forming a categorical syllogism yielding ‘All S must Q’, or a productive argument for ‘When any S is P, it must be Q’ (natural) or ‘Any S which must be P, must be Q’ (extensional). This shows that a one-predicate (categorical) form is the top of both natural and extensional hierarchies, of (conditional) forms with two predicates (or eventually more).

Similarly with temporal modality. And, in a still larger picture, all manners of disjunctive propositions (with any number of terms) can be included. The basis of any form used should always be kept in mind, of course.

It is clear that the whole doctrine of fractions and integers (including stereo as well as flat fractions and integers), and likewise all other aspects of factorial analysis and induction, can be expanded to include all *de-re* conditioning, of any form and modal type. Various and numerous compounds emerge from the combinations of all such propositions, whether of the same type and form, or of mixed type and/or form.

Particular fractions of categorical propositions may, as already mentioned, be expressed in conditional language. Similarly, compounds of such fractions, forming integers, may be clarified by conditionals.

For example: (**In**)(**IOp**) means {Some S must be P} and {Some S which are P, can not-be P}, but also takes for granted the formal truth that {No S which must be P, can not-be P}, and its contraposite {No S which can not-be P, must be P}’. As more fractions are conjoined, the interrelative statements become more complex, but are in any case expressible through extensional conditionals.

We may also interpose natural (or temporal conditionals) to express formal truths applicable within brackets, like (**IOp**) tacitly appeals to {When certain S are (as now) P, they cannot be nonP} and its contraposite {When certain S are not P, they cannot be P}, since the required bases are given in the categorical premises.

Thus, all types of conditioning are involved to some extent even in categorical fractionating and integration, fulfilling the role played by the brackets in symbolic descriptions. Our brackets are not artificial constructs, but shorthand notation for such implied conditionals, delimiting and separating extensions, and circumstances or times.

These examples are just some of the intersections of the different formal continua. A complete theory would have to be more systematic than that, and consider all conceivable conditionals rather than the few implied by categorical compounds.

53. FACTORIAL ANALYSIS.

1. Factorization.

We are now in a position to analyze the precise content of gross formulas, restating them as factorial formulas. These consist in complex propositions identifying the alternative integers with which the given gross formula is consistent. In this context, integers will be referred to as factors, whether one or more of them are involved. The process may then be viewed as factorization or factorial analysis of information.

By restructuring information in factorial terms, we are able to recognize more clearly how close to, or far from, full knowledge we are with regard to the subject and predicate in question. We know that reality must fall under one or the other of the various integers in any case. Full knowledge implies that we can pinpoint one integer as the right one. Total ignorance implies that all the integers are equally likely outcomes for us. In between lies a mass of possibilities, where we know that certain of the integers are excluded, but we are still left with more than one integer to choose from.

Even in situations where we do have full knowledge, restating a gross formula as an integer composed of fractions, permits us to trace or express more precisely the way the extension is fragmented into different particular relations. But let us proceed, and the importance of this approach will become clear.

Note in passing that since singular statements are reducible to disjunctions of plurals, they can also be factorized, though this is not done below.

2. Applications.

Let us concentrate again on natural modality as a closed system. Whatever is found true for natural modality can as usual be duplicated for temporal modality. Mixed modality will be dealt with further on.

The following table interprets the 49 gross formulas in factorial terms. A factorial formula is expressed as a <u>disjunction</u> of one or more of the 15 integers (those marked '**yes**': the number of factors = 'NF'); such disjunctives exclude all other integers (those left blank).

For example, as the table reveals, **F1**, **F3**, and **F6** are the factors of '**A**'. '**A**' is thus to be read as '**F1** or **F3** or **F6**', i.e. as '(**An**) or (**AEp**) or (**In**)(**IOp**)': these are the only 3 eventualities conceivable given that '**A**' is true, the rest being impossible outcomes. The disjunctive proposition is the factorial equivalent of '**A**', then.

<u>Table 53.1 Factorial Analysis of Natural Gross Formulas.</u>

NF		F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
	Elements															
1	An	yes														
1	En		yes													
3	A	yes		yes			yes									
3	E		yes		yes			yes								
7	Ap	yes		yes	yes		yes		yes		yes			yes		
7	Ep		yes	yes	yes			yes		yes	yes				yes	
8	In	yes				yes	yes		yes			yes	yes	yes		yes
8	On		yes			yes		yes		yes		yes	yes		yes	yes

NF		F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
	Elements (continued)															
12	I	yes		yes		yes	yes		yes	yes	yes	yes	yes	yes	yes	yes
12	O		yes		yes	yes		yes	yes	yes	yes	yes	yes	yes	yes	yes
14	Ip	yes		yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
14	Op		yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes

NF		F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
	Compounds															
1	AEp			yes												
1	ApE				yes											
1	AInOp						yes									
1	EOnIp							yes								
1	ApEpIO										yes					
2	AIn	yes					yes									
2	EOn		yes					yes								
2	AOp			yes			yes									
2	EIp				yes			yes								
2	ApIEp			yes							yes					
2	EpOAp				yes						yes					
2	ApInO								yes					yes		
2	EpOnI									yes					yes	
3	ApEp			yes	yes						yes					
3	ApInOp						yes		yes					yes		
3	EpOnIp							yes		yes					yes	
3	ApIO								yes		yes			yes		
3	EpOI									yes	yes				yes	
4	ApIn	yes					yes		yes					yes		
4	EpOn		yes					yes		yes					yes	
4	IEp			yes						yes	yes				yes	
4	OAp				yes				yes		yes			yes		
4	InOn					yes						yes	yes			yes
5	ApIOp			yes			yes		yes		yes			yes		
5	EpOIp				yes			yes		yes	yes				yes	
6	ApI	yes		yes			yes		yes		yes			yes		
6	EpO		yes		yes			yes		yes	yes				yes	
6	ApOp			yes	yes		yes		yes		yes			yes		
6	EpIp			yes	yes			yes		yes	yes				yes	
6	InO					yes			yes			yes	yes	yes		yes
6	OnI					yes				yes		yes	yes		yes	yes
7	InOp					yes	yes		yes			yes	yes	yes		yes
7	OnIp					yes		yes		yes		yes	yes		yes	yes
9	IO					yes			yes	yes	yes	yes	yes	yes	yes	yes
11	IOp			yes		yes	yes		yes	yes	yes	yes	yes	yes	yes	yes
11	OIp				yes	yes		yes	yes	yes	yes	yes	yes	yes	yes	yes
13	IpOp			yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes

This table is drawn up as follows. First we deal with elementary propositions, identifying which of them are logically implied by each factor (downward). Thus for instances, **F5** = (**In**)(**On**) implies **In**, **On**, and all their subalterns; **F6** = (**In**)(**IOp**) implies **A**, **In**, **Op**, and all their subalterns. Note well that if a particular element is implicit in all the fractions, a universal element is implied by them together.

Then we derive the factorization of compounds from that of their elements, as follows. When two or more disjunctions involving incompatible alternatives are conjoined, the result is a disjunction of their common alternatives (if they have only one in common, that is the result; if they have none in common they are not conjoinable). Thus, to factorize a compound gross formula, we simply find the common factors of its elements. (See chapter 28.3, on ‘conjunctive multiplication’.)

For example: Since **E** = **F2** or **F4** or **F7**, and **Ip** = all factors but **F2**, then **EIp** = **F4** or **F7**. This is because no two distinct factors, being mutually exclusive integers, can consistently be conjoined; while a factor conjoined with itself, yields itself as result.

3. Overlap Issues.

Table 52.1 used to define the 15 integers as conjunctions of fractions, was read across. If it is read downward, it serves to factorize the 8 fractions; the fractions being then viewed as disjunctions of the integers which include them. If we compare the results so obtained to those in table 53.1, we see that they are the same in all cases except two, note well.

Whereas (**IOp**) = the fraction **f7** has factors **F3**, **F6**, **F9**-**F11**, **F13**-**F15**, **IOp** = the gross formula has additionally factors **F5**, **F8**, **F12**. This is because in (**IOp**) the extensions of **I** and **Op** are known to fully coincide, while in **IOp** we do not know whether the extensions of **I** and **Op** overlap wholly, partly, or not at all. Similarly, the fraction (**IpO**) and the gross formula **IpO** are not factorially the same; the former being a disjunction of **F4**, **F7**-**F8**, **F10**, **F12**-**F15**, whereas the latter further allows the alternatives **F5**, **F9**, **F11**.

This issue of overlap of particular elements is found in other gross formulas. Thus, most obviously, in **IpOp**, it is not clear whether **Ip** and **Op** overlap, or if they do, to what degree. In the case of **IO**, it would at first sight seem that they cannot overlap at all, since no instance of their extension can simultaneously fall under both; but as the factorial analysis of **IO** as **F5**, **F8**-**F15** makes clear, overlap may indeed occur involving subalterns of **I** and/or **O**. That is, the gross formula **IO** is merely an abbreviation of **I+Ip+O+Op**, so that (**IpO**) or (**IOp**) are conceivable fractions within it, though concealed.

The same issue can be raised for most other gross formulas involving particular elements. Only in 3 cases, mentioned in the next section, is the issue resolved unambiguously. This shows the inadequacy of gross formulas, their capacity to mislead, and it shows the value of factorial analysis.

When a gross formula involves a mix of universal element(s) and particular one(s), the ambiguity concerning overlap is lessened, though rarely removed. Thus for instance, in **AOp**, we can be sure that part of the extension of **A** is the whole extension of **Op**, and so we could make a ‘partial factorization’ to **I**+(**IOp**). But though this approach may improve our understanding of the situation somewhat, factorially speaking information is lost. For whereas **A+Op** = **F3** or **F6**, the formula **I**+(**IOp**) equals no more than (**IOp**) alone, namely **F3** or **F6** or **F9**-**F11** or **F13**-**F15**. The result is vaguer and lesser, because (**IOp**) implies I in any case.

When a gross formula involves only universal elements, total overlap is of course assured. But factorial analysis is still relevant, since without it we might remain unaware that **ApEp** not only has the possible outcomes (**AEp**) or (**ApE**), but also has (**IOp**)(**IpO**) as an alternative.

4. More Factorial Formulas.

Notice in table 53.1 our separation of elements and compounds, and also the classification of formulas by number of factors. The less factors a formula has, the closer it is to full definition, and the more knowledge we have. Ignorance would be disjunction of all 15 factors.

It is interesting to note that only 7 gross formulas have a single factor, i.e. result in an integer as their factorial equivalent. They are: **An**, **En**, **AEp**, **ApE**, **AInOp**, **EOnIp**, and **ApEpIO**. The latter three cases are worth stressing, since they provide us with novel immediate inferences. Without the above systematic approach, it might not seem evident that the whole extension of the universal element(s) of the gross formulas are necessarily covered by only the particular fractions shown in these corresponding factorials.

AInOp = **F6** = (**In**)(**IOp**), meaning: 'All S are P and some S must be P and some S can not-be P' equals 'Some S must be P, and some other S are P but can not-be P'.

EOnIp = **F7** = (**On**)(**IpO**), meaning: 'No S is P and some S cannot be P and some S can be P' equals 'Some S cannot be P, and some other S are not P but can be P'.

ApEpIO = **F10** = (**IOp**)(**IpO**), meaning: 'All S can be P and all S can not-be P, and some S are P and some S are not P' equals 'Some S are P, but can not-be P, whereas some other S are not P, but can be P'

All other gross formulas yield factorial formulas with more than one alternative; they are deficient stages of knowledge. Nevertheless, they are very valuable in this form, as will be seen when we deal with modal induction.

It also interesting to note that the 49 gross formulas, which are an exhaustive list (within natural modality) as we showed, are able to express only 7 of the 15 integral states of being (within this closed system). More than half the integers are inaccessible to this gross manner of formulation, namely **F5**, **F8**-**F9**, **F11**-**F15**!

This shows the importance of the concepts we have introduced in this study. Without them, Logic cannot fulfill its task. And indeed, in everyday thought and discourse we commonly use such complicated statements as 'Some S are P, and <u>these</u> can not-be P, while some <u>others</u> are always P'. Such statements try to clarify the deployment of the extension, and are efforts towards factorial analysis.

There are in fact a total of 2 to the 15th power, minus 1 = 32,767 ways to disjoin the 15 factors **F1**-**F15**. Any combination of one, two, three… up to 15 of them, is conceivable. Fifteen of these ways are the integers in isolation: they represent full knowledge. One way, the disjunction of all 15 factors, represents total ignorance (equivalent to saying nothing, except the formal truth that one of the factors must be applicable eventually). In between, we are left with 32,751 states of knowledge involving varying degrees of ignorance.

The 49 gross formulas thus represent only a very small number of the possible states of knowledge. As we said, they include only 7 of the 15 integers, and therefore only 42 of the 32,751 intermediate states. They may be the most common states of knowledge, being simple of expression; but as a system, gross formulation is very defective. It is incapable of expressing the finer gradations, the nuances, of information accessible to factorial analysis.

As knowledge unfolds, we move from one formula to the other. There is a dynamic process. One may first discover, whether deductively or inductively, that **A** is true, say; then **Op** is found true, and our knowledge adds up to **AOp**; then perhaps **O** is seen true, contradicting **A**, so we may suppose **IO** true instead; and so on. As our opinions shift around, reflecting new observations, new insights, new efforts of reasoning, we adopt an alternative formula to express our contextual position.

Factorial formulas are simply more precise tools than gross formulas, for handling such changes in data. Knowing exactly how many and which integers are still open, and how many and which integers are already excluded, the pursuit of full knowledge becomes more efficient. The goal is to eliminate all alternative integers from the list of possibilities, until only one remains: then our knowledge concerning the subject-predicate in question is clear. We may at a later stage acquire new information, which once again puts us in doubt, but in any given context factorial analysis lets us know where we stand.

Note that it may be — I am just guessing — that some of the deficient states of knowledge are equivalent to fractions or integers composed of conditional propositions. This is uncharted territory, I have not looked into it; but it seems likely, since categoricals and conditionals are particles of intersecting continua.

5. Open System Analysis.

The above concerns, remember, the closed system of natural modality; or, by analogy (substituting **c**, **t** suffixes for **n**, **p**), that of temporal modality. But these, though used independently, are relatively artificial. It is only when we consider the two types together, in an open system, that our results are absolute, realistic. This will now be done.

We saw that in mixed modality (limiting ourselves to consideration of plurals), there are 12 fractions, from which 63 integers can be constructed. It follows that, here, there are 2 to the 63rd power, minus 1 = 9.2233 X 10 to the 18th power factorial formulas. That is, 9 million million million distinct states of knowledge, concerning any given subject to predicate relation! Of these 63, the one-factor formulas, are full knowledge; one, the 63-factor formula, is total ignorance; and the remainder are intermediate states, involving 2 to 62 factors in different disjunctive combinations.

We identified 20 elementary propositions, yielding 175 compounds, a total of 195 gross formulas. These are the most common states of knowledge, worthy of special attention, but still only a minute part of the total picture. These are analyzed factorially in **Appendix 1**, for the record. Though the table is parked at the end of the text because of its bulk, it is important. It is split into several segments for reasons of space, but they should be read as one.

The method used to develop it is the same as for the closed systems. First, we identify the elements implied by each of the 63 integers, viewing its fractions (if more than one) both individually and collectively. The factorial formula corresponding to an elementary gross formula is then a disjunction of all the integer(s) which imply it, since the list of integers is complete. Thereafter, we derive the factor(s) of a compound, simply by identifying the common factor(s) of its constituent elements.

Similar comments to those made previously apply here, concerning overlap. Especially note the difference between the factors of the fractions (**IOt**) and (**ItO**), given in table 52.2 and numbering 32 each, and the factors of the gross formulas **IOt** and **ItO**, given in the appendix and numbering 53 each (including the 32 of the corresponding fraction).

Note that here we have 11 gross formulas resulting in a single factor; namely the 6 universal factors (**An**), (**En**), (**AcEp**), (**ApEc**), (**AEt**), (**AtE**), plus the following 5 compounds:

AcInOp	=	F8	=	(In)(IcOp)
EcOnIp	=	F9	=	(On)(IpOc)
AIcEpOt	=	F17	=	(IcOp)(IOt)
EOcApIt	=	F18	=	(IpOc)(ItO)
AtIEtO	=	F21	=	(IOt)(ItO).

These equations can be interpreted like the corresponding closed system equations were. The reader should recite or write their explicit meaning, to see how unexpected these results are.

Without factorial analysis, we would have great difficulty finding, understanding or proving these inferences.

Thus 52 integers remain unexpressed by gross formula statements, and can only be specified through fractional notation. All gross formulas other than these 11 have more than one factor; the maximum number of factors for elements being 62, for **Ip** and **Op**, and for compounds being 61, for **IpOp**.

Note also that the fractions and integers specific to natural or temporal modality could be factorized in the open system. This is not done here, to avoid further complications and because the results are not needed in subsequent discussion. But the reader is invited to do it, as a significant exercise, with a warning not to ignore overlap issues.

Other possible developments include: factorial analysis of stereo integers, of transitive categoricals and fractions and integers of them, and of all types of conditionals.

Many of the factorial formulas resulting from such analysis, may intersect in meaning with those considered here; I mention this as a speculation, to suggest to future logicians that they look for eventual equations.

54. MODAL INDUCTION.

1. Knowability.

Some skeptical philosophers have attempted to write-off natural necessity, and potentiality, as unknowable, if not meaningless. We have shown the meaningfulness and importance of these concepts, in the preceding pages. Here, we will begin to show systematically how they may be induced.

At the outset, let us note that to assert that natural necessity cannot be known, is to claim knowledge of a naturally necessary phenomenon; this is implicit in the use of 'cannot' in such assertion. If the assertion were merely put as 'man does not know natural necessity', in an attempt to be consistent, we see that the statement would have no force; we could still ask 'but can he?' Thus, this concept is undeniable, and its attempted rejection untenable.

Furthermore, the formal link between natural necessity and potentiality, makes the latter also inevitable. They are two sides of the same coin, if either is admitted then the other logically follows by systematization: every concept must have a contradictory. The potentiality of something is merely negation of the natural necessity of its absence. Thus, the intrinsically concealed and invisible aspect of inactualized potentiality, is not an valid argument against its existence.

The induction of natural modality, and for that matter the more readily recognized temporal modality, follows the same patterns as those involved in the process of induction of extensional modality.

How are universal propositions induced? By a process of generalization, moderated by particularization. We consider it legitimate to move from empirically encountered instances to cases we have not yet come across, until the facts suggest otherwise. We do not regard our universal statements to cover no more than the perceived phenomena; but normally move beyond them into prediction.

Likewise, with constancy of conjunction, in the sense of temporal modality; this too involves an extrapolation from the known to the unknown, as everyone admits.

So 'all' and 'always' involve just as much assumption as 'necessarily' (in the sense of natural modality). They are all just as hard to establish. Why should we recognize the former and not the latter?

Further, the concepts of universality and constancy are ultimately just as mysterious, ontologically hard to define, as that of natural necessity, so the latter's elusiveness cannot be a legitimate reason for singling it out.

If natural necessity is understood as one level higher (or deeper) than constancy, subject to all the usual laws of logic, generalization and particularization, it is seen to be equally empirical and pragmatic.

While the denial of natural necessity as such is unjustified, with regard to specific applications of the concept, we may of course in a given instance be wrong in our assumption that it is there. It is up to Logic to teach us proper procedures of induction and deduction, concerning such relationships. There is no problem in this viewpoint; belief in natural necessity as such does not obligate us to accept every eventual appearance of it as final.

As with any generalization, the movement from always to must, or from never to cannot, is legitimate, so long as it remains confirmed by experience. If ever a contradictory instance occurs, obviously our assumption is put in doubt and we correct our data-base accordingly, in the way of particularization.

2. Equality of Status.

We saw, in chapter 50, on induction of actuals, that induced particulars are based on the observation of singulars. Similarly, induction of temporaries or potentials is based on the observation of actuals. The same can be said of the bipolar particular fractions, which involve temporary or potential elements: they can be established by observation of the same instance of the subject being actually related to the predicate in different ways at different times or in different circumstances.

And just as not all particular actuals are induced, but some are arrived at by deductive means, so also temporary or potential knowledge is in practice not invariably inductive, but may derive from reasoning processes. Though ultimately, of course, some empirical basis is needed, in any case.

We additionally pointed out how, in the formation of particular propositions, there is also a large share of conceptual work. The same is true of other types of possibility. All statements involve concepts (the terms, the copula, the polarity, the qualifications of quantity or modality). They presuppose a mass of tacit understandings, relating to logical structure and mechanisms. Furthermore, there is always an evaluation process, placing the proposal in the broad context of current knowledge, to determine its fit and realism.

Thus, although pure observation is instrumental in the process, other mental efforts are involved. Abstraction and verbalization of possibility are not automatic consequences of awareness of singular actual events, and error is always a risk. This is equally true in all types of modality, whether extensional, temporal or natural. Thus, actual particulars cannot be claimed more plausible than temporaries or potentials.

And indeed, just as particularity is not superior in status to generality, so are the other types of possibility not intrinsically more credible than their corresponding necessaries. If we consider the controversies among philosophers to be resolved, and view the whole of Logic in perspective, we can say that all forms involve only some degree of observation, and a great deal of thought. Although the degree of empiricism admittedly varies, the amount of conceptualization is essentially identical.

This insight must not be construed to put knowledge in general in doubt, however. Such skepticism would be self-contradictory, being itself the pronouncement of a principle. That there is a process does not imply that its outcome is false. The process merely transports the data from its source to its destination, as it were; the data need not be affected on the way.

Rather, its significance is to put all forms on an equal plane, with regard to their initial logical value. Particulars are no better than universals; particulars are no better than temporaries, which in turn are no better than potentials; and the latter are no better than constants or natural necessaries. Every statement, whatever its form, has at the outset an equal chance of being true or false, and has to be judged as carefully.

3. Stages of Induction.

The classical theory of induction, we saw, describes two processes, generalization and particularization, as fundamental. If all we know is a particular proposition, **I** or **O**, we may assume the corresponding general proposition, **A** or **E**, true; unless or until we are forced by contradictory evidence to retract, and acknowledge the contingency **IO**.

Now, this description of the inductive process is adequate, when dealing with the closed system of actual propositions, because of the small number of forms it involves. In a broader context, when modal propositions of one or both types are taken into consideration, the need arises for a more refined description of the process.

This more complex theory brings out into the open, stages in or aspects of the process which were previously concealed. The ideas of generalization and particularization were basically correct, but their application under the more complicated conditions found in modal logic require further clarifications, which make reference to factorial analysis.

Needless to say, the new theory should be, and is, consistent in all its results with the old theory. It should be, and is, capable of embracing actual induction as a special case within a broader perspective which similarly guides, validates, and explains modal induction.

Our modified theory of induction, in the broadest sense, recognizes the following stages:

a. ***Preparation***. The summary of current data in gross formulas, and their factorization. This is in itself a purely deductive process.
b. ***Generalization***. Selection of the strongest factor in a factorial formula.
c. Drawing consequences, empirical testing, and comparing results to wider context. These include deductive work and observation.
d. ***Particularization***. Revision of current formulas in the light of new data. This may necessitate weighting of information. Also, certain conflicts are resolved by factor selection, as in generalization.
e. Repeat previous steps as required.

Each of these processes requires detailed examination. The tasks of listing all conceivable gross formulas, and analyzing them factorially, as well as the tasks relating to deductive inference and comparison, have previously been dealt with. We now need to deal with the processes of factor selection and formula revision, which are the most characteristically inductive.

4. Generalization *vs*. Particularization.

We call generalization, those thought processes whose conclusions are higher than their premises; and we call particularization, those whose conclusions are lower. This refers to expansions and contractions on the scales of quantity and modality, essentially. As we move beyond the given, or its strictly deductive implications, into prediction, we are involved in induction of one kind or another.

The problem of generalization, which way and how far to advance and on what basis, is solved entirely by the method of factor selection. The problem of particularization, which way and how far to retreat and on what basis, is solved by the methods of formula revision, which may involve factor selection.

It will be seen that factor selection has a static component, which consists of the uniformity principle, which tells us which factor to select, and an active component, the practical carrying out of that decision. The act and basis of factor selection is technically identical, whether applied to generalization or to particularization.

The theory of factor selection makes clear that these processes do not consist of wild guesses, but proceed in a structured manner, requiring skill and precision.

We may view generalization as the positive force in induction, and particularization as the negative side. Generalization would often be too sweeping, if not kept in check by particularization. The function of the latter is to control the excesses of the former. Only the interplay of these two vectors results in proper induction. Induction is valid to the extent that it is a holistic application of both factor selection and formula revision.

In the pursuit of knowledge, laziness leads to error. An idea must be analyzed to the full, because its faults are sometimes concealed far down that course. The uncovering of a fault is a boon, allowing us to alter our idea, or take up a new one, and gain increased understanding and confidence.

The processes of generalization and particularization are going on in tandem all the time, in an active mind. Induction is not linear or pedestrian. Thoughts extend out tentatively, momentarily, like trial balloons, products of the imagination. But at the same time, verification is going on, unraveling the consequences of a suggestion, bringing other facts into focus from memory, or making new empirical inquiries, for comparison to the proposals made, and construction of a consistent idea. The wider the context brought into play, the greater the certainty that our course is realistic.

The role of Logic as a science is to provide the tools, which enable us to play this mental game with maximum efficiency and success. It is an art, but training and experience improve our performance of it.

5. The Paradigm of Induction.

Let us reconsider the paradigm of induction given by actual induction. By reviewing the closed system of actual propositions using factorial concepts, we can gain some insights into the stages and guiding assumptions of induction within any system.

There are only four plural actual forms: **A**, **E**, **I**, **O**. These are also the system's fractions: (**A**), (**E**), (**I**), and (**O**). These in turn constitute three integers: (**A**), (**E**), and (**I**)(**O**), which are mutually exclusive and exhaustive. The 4 forms allow for 5 gross formulas: **A**, **E**, **I**, **O**, **IO**. These can be analyzed factorially using the integers: **A** = (**A**), **E** = (**E**), **I** = '(**A**) or (**I**)(**O**)', **O** = '(**E**) or (**I**)(**O**)', **IO** = (**I**)(**O**). But two disjunctions of factors remain unexpressed, namely: '(**A**) or (**E**)', signifying incontingency, and '(**A**) or (**E**) or (**I**)(**O**)', signifying no concrete information.

In this framework of factorial analysis, we can understand the induction of **A** from **I**, or of **E** from **O**, as a process involving factor selection, rather than solely as one of increase in quantity from some to all. The reverse process, of decrease in quantity, would also here be regarded differently, as primarily focusing on a new factorial situation.

Given **I** alone, we prefer the alternative outcome (**A**) to the deductively equally conceivable alternative (**I**)(**O**). Or, given **O** alone, we inductively anticipate the factor (**E**) as more likely than its alternative (**I**)(**O**). Our selection of one factor out of the available two is the dynamic aspect of the process. That we have specifically preferred the general alternative to the contingent one, is a second aspect; here, we take note of a principle that statically determines which of the alternative factors is selected.

If thereafter we find that our position must shift to **IO**, so well and good; in that case, only one integer is conceivable: (**I**)(**O**). In this case, we believed **A** to be true, then discovered **O**, or we assumed **E** then found **I**: the only available resolution of this conflict is by the compromise compound proposition **IO**; this is formula revision per se. Now, we analyze **IO** and find that it has only one factor (**I**)(**O**), so we can select it without doubt. However, had there been more than one conflict resolution or more than one factor (as occurs in wider systems), we would have had to again engage in factor selection.

Such an outlook seems somewhat forced and redundant within the closed system of actuals, but in the wider systems of modal propositions it becomes essential. It is only applied to actuals here for initial illustration purposes. For whereas with actuals, our choices are very limited numerically, when modality is introduced they are much more complicated, as will be seen.

In the wider systems, induction can usually take many paths, and has various possible limits. For instance, from **Ip** should we generalize in the direction of **Ap** or to **In**? Or again, from **An** should we particularize to **Ap** or to **In**? And how far up or down the scale may we go? Obviously, this depends on context, so when may **Ip** ascend to **An**, and when must it stop earlier, and when must **An** descend to **Ip**, and when may it stop earlier?

Such questions can only be answered scientifically and systematically by resorting to factorial analysis and related processes. This brief review of actual induction in such terms points the way to the solution of the problem.

6. The Pursuit of Integers.

The factor selection theory suggests that the goal of induction is to diminish the areas of doubt involved in deficient states of knowledge. Selecting a factor means eliminating a number of other factors, which, though they are formally logically conceivable alternatives, are intuitively thought to be less likely.

The ultimate result pursued by all induction is knowledge of integers, which does not necessarily mean a generality. Without integers, too many questions arise, and the mind cannot proceed. It is better to take up a working hypothesis, and keep testing it, than to passively await for an in any case unattainable absolute certainty. Knowledge is fed by action; it involves choices, decision-making.

The whole point of induction is to decide what integral proposition is most suggested by a given statement of deficient knowledge. We are to scrutinize its factorial equivalent and, on the basis of precise principle, select one factor as our inductive conclusion, or at least reduce the number of factors considerably. Deductively, all factors are equally likely outcomes, but inductively they can be narrowed down.

In certain cases, as factorial analysis showed, there is only one factor anyway; in such case, the conclusion is deductive, not inductive, and contextually certain. But in most cases, there are more than one factor, and selection is necessary. In some cases, we may for some purpose be satisfied with eliminating only some of the excess factors, and be left with a formula of two or more factors; the conclusion is not a single integer, but, still, less vague than previously, and might be expressed as a gross formula.

55. FACTOR SELECTION.

1. Prediction.

We indicated in the previous chapter that induction depends on factorial analysis of our knowledge context. Once this is done, we are usually faced with a number of factors to choose from, which represent the various outcomes our knowledge may move towards.

But reality can only exist in terms of integers; it is only the deficiencies of knowledge which make possible the indefinite situation of integers in disjunction. On this basis, we know for sure that one, and only one, of the factors of a formula can be factually correct. The other alternatives, if any, are a sign of doubt; they do not represent a fact of reality.

There is no recognition of an 'Uncertainty Principle' in this logic. Uncertainty is a phenomenon of consciousness, with no equivalent in the Object. It is perhaps conceivable that certain motions of matter occur indeterministically, without order or cause, as modern Physics suggests. But, according to Logic, whatever has occurred, once it has occurred, is firmly fixed, be it discernible or not.

The inductive process of factor selection consists in anticipating reality, trying to predict, from the available knowledge of contextually allowed factors, which of the factors is most likely to emerge as the right one. In some cases, while such a definite result is inaccessible, we try to at least approach it, by diminishing the number of factors. In other cases, the given formula has only one factor, anyway, so there is no problem, and the result is deductive.

The question arises, how do we know which factor is most likely? Formally speaking, they are all equally possible; this is the verdict of deductive logic. But induction has less strict standards of judgment.

2. The Uniformity Principle.

The principle involved in factor selection may be glimpsed in the paradigm of generalization from actual particulars. We will call it the uniformity principle, understanding by this term a broad, loose reference to repetitiveness of appearances, coherence, continuity, symmetry, simplicity.

Consider for example generalization from **I**. The general alternative (**A**) is more likely then the contingent one (**I**)(**O**), ***because the former involves no unjustified presumption of variety in polarity like the latter***. We are not so much inventing information, as refraining from baseless innovation and maintaining continuity.

Thus, the qualitative inertia of the first factor is more significant than the quantitative change (from some to all) it introduces. In contrast, the second factor introduces just as much quantitative change (through the **O**), so that it is no better in that respect; and additionally, to its detriment, a novel fragmentation of the extension, absent in the original data and the preferred factor.

We obviously select the factor most resembling the given data, as its most likely outcome. Unless or until we have reason to believe otherwise, we assume the given information to be reproduced as far as it will go. We can thus express the principle that, in factor selection, the most uniform factor is to be accorded priority.

Ontologically, this signifies the assumption of maximum uniformity in the world, in preference to an expectation of diversity. Events are believed representative, rather than unique. The world seems to tend in the direction of economy.

On a pragmatic level, the reason for it is that a generality is easier to test than a particular statement, since deductive logic, through which the consequences of assumptions are inferred, requires general statements. Thus, the preference for uniformity also has an epistemological basis. In the long run, it assures us of consistency.

The uniformity principle, then, is a philosophical insight and posture, which sets an order of priority among the factors of a formula.

But, it is important to stress that this principle is merely a utilitarian guideline to factor selection, it does not in this format have the binding force or precision found in the laws of deductive logic. Inductive logic merely tries to foresee the different situations which may arise in the pursuit of knowledge, and to suggest seemingly reasonable decisions one might make.

Choices other than those proposed remain conceivable, and might be intuitively preferred in specific cases. There is an artistic side to induction, to be sure. Our general recommendations, however, have the advantage of having been thought out in an ivory tower, and of forming a systematic whole.

3. The Law of Generalization.

Fortunately, we can neatly summarize the results, obtained by application of the uniformity principle, in a single, precise law for generalization. This has greater practical value.

The reader will recall that when the integers were defined, they were organized, in order of the number of their fractions. Those with the least fractions came first, then those with two fractions, then those with three, and so on. Within each such group, comparable integers of opposite polarity were paired off, with the more positive one preceding the more negative. Also, they were ordered according to their level of modality in the continuum concerned.

Thus, in the closed systems of natural or temporal modality, the 15 integers **F1**-**F15**, and in the open system of mixed modality, the 63 integers **F1**-**F63**, are ordered in such a way that their numbers reflect their degree of 'strength'. The lower the ordinal number, the stronger the factor.

A stronger factor is less fragmented (i.e. has less fractions, out of a possible 4 in the closed systems, and 6 in the open). It is closer to universal (in the closed systems, **F1**-**F4** are universal; in the open system, **F1**-**F6**). It has higher modality; for instances, (**An**) is higher than (**AEp**), (**In**)(**On**) is higher than (**IOp**)(**IpO**).

Thus, in any factorial formula, the factors in the series are already numerically ordered according to their relative strengths. This was not done with factor selection in mind, but because of the clarity it generated in the doctrine of factorial analysis. As detailed work will presently reveal, it turns out that:

In any factor selection, the strongest factor is the one to prefer.
This is the law of generalization.

In a few exceptional cases, the first two factors must be selected, in disjunction, for reasons that we shall see. But, on the whole, this law holds firm, and successfully sums up all our findings.

This law is a summary of results. In point of fact, it only emerged at the end of painstaking analysis of a large number of specific inductive arguments, attempting to make sense of them, case by case, through the intuited uniformity principle. However, once arrived at, it seems obvious. But the true justification of it all, is the consistency and cogency of the totality of the theory, with all its details, of course.

Note well, incidentally, that henceforth, to avoid neologisms, the term 'generalization' is used in a general sense not limited to quantity. It is applied to either increase in quantity, from some to all; this is extensional generalization. And/or to increase in modality from possibility to actuality to necessity; this being modality generalization, (natural and/or temporal, as the case may be). Likewise, the term 'particularization' may be used for any such type of decrease.

But most precisely, generalization may now be defined as inductive selection of the strongest factor(s) of a formula, by suppression of weaker factor(s). Particularization will be dealt with under the heading of formula revision.

Generalization can, therefore, be applied to deficient states of knowledge not expressible in gross formulas. We saw in the chapter on factorial analysis that, while all disjunctions of integers represent deficient states of knowledge, some of them do not correspond to any gross formula. In other words, gross formulas with two or more factors are not all the possible states of relative ignorance, other combinations of factors are conceivable.

The law of generalization makes selection of the strongest factor legitimate in such already factorial formulas, too.

56. APPLIED FACTOR SELECTION.

1. Closed Systems Results.

We will, to begin with, deal with the closed system of natural modality, first listing the results of factor selection, then analyzing and justifying our proposals. As usual, all the results obtained can by analogy be replicated for the closed system of temporal modality. The corresponding results for the more bulky open system of mixed modality will be presented later.

The following table shows the proposed preferred (natural) factors for natural gross formulas, selected on the basis of the uniformity principle. Deductive cases, those with a single factor on formal grounds, are included for completeness.

The information in the elementary or compound premise is always assumed to be all available data on the subject to predicate relation concerned. If more data makes its appearance, then we are faced with another premise, and the conclusion may accordingly be different.

The column '**NF**' indicates the original number of factors, the next column lists them in sequence, and the column '**SF**' shows the selected factor among them, which is our proposed conclusion..

Table 56.1 Factor Selection in Natural Modality.

Premises	NF	Factors	SF	Conclusion
Group F1				
An	1	F1	F1	(An)
AIn	2	F1, F6	F1	(An)
A	3	F1, F3, F6	F1	(An)
ApIn	4	F1, F6, F8, F13	F1	(An)
ApI	6	F1, F3, F6, F8, F10, F13	F1	(An)
Ap	7	F1, F3, F4, F6, F8, F10, F13	F1	(An)
In	8	F1, F5-F6, F8, F11-F13, F15	F1	(An)
I	12	F1, F3, F5-F6, F8-F15	F1	(An)
Ip	14	F1, F3-F15	F1	(An)
Group F2				
En	1	F2	F2	(En)
EOn	2	F2, F7	F2	(En)
E	3	F2, F4, F7	F2	(En)
EpOn	4	F2, F7, F9, F14	F2	(En)
EpO	6	F2, F4, F7, F9-F10, F14	F2	(En)
Ep	7	F2-F4, F7, F9-F10, F14	F2	(En)
On	8	F2, F5, F7, F9, F11-F12, F14, F15	F2	(En)
O	12	F2, F4-F5, F7-F15	F2	(En)
Op	14	F2-F15	F2	(En)

Premises	NF	Factors	SF	Conclusion
Group F3				
AEp	1	F3	F3	(AEp)
ApIEp	2	F3, F10	F3	(AEp)
AOp	2	F3, F6	F3	(AEp)
IEp	4	F3, F9-F10, F14	F3	(AEp)
ApIOp	5	F3, F6, F8, F10, F13	F3	(AEp)
(IOp)	8	F3, F6, F9-F11, F13-F15	F3	(AEp)
IOp	11	F3, F5-F6, F8-F15	F3	(AEp)
Group F4				
ApE	1	F4	F4	(ApE)
ApEpO	2	F4, F10	F4	(ApE)
IpE	2	F4, F7	F4	(ApE)
ApO	4	F4, F8, F10, F13	F4	(ApE)
IpEpO	5	F4, F7, F9-F10, F14	F4	(ApE)
(IpO)	8	F4, F7-F8, F10, F12-F15	F4	(ApE)
IpO	11	F4-F5, F7-F15	F4	(ApE)
Group F3-4				
ApEp	3	F3-F4, F10	F3-4	(AEp) or (ApE)
ApOp	6	F3-F4, F6, F8, F10, F13	F3-4	(AEp) or (ApE)
IpEp	6	F3-F4, F7, F9-F10, F14	F3-4	(AEp) or (ApE)
IpOp	13	F3-F15	F3-4	(AEp) or (ApE)
Group F5				
InOn	4	F5, F11-F12, F15	F5	(In)(On)
InO	6	F5, F8, F11-F13, F15	F5	(In)(On)
IOn	6	F5, F9, F11-F12, F14-F15	F5	(In)(On)
InOp	7	F5-F6, F8, F11-F13, F15	F5	(In)(On)
IpOn	7	F5, F7, F9, F11-F12, F14-F15	F5	(In)(On)
IO	9	F5, F8-F15	F5	(In)(On)
Group F6				
AInOp	1	F6	F6	(In)(IOp)
ApInOp	3	F6, F8, F13	F6	(In)(IOp)
Group F7				
IpEOn	1	F7	F7	(On)(IpO)
IpEpOn	3	F7, F9, F14	F7	(On)(IpO)
Group F8				
ApInO	2	F8, F13	F8	(In)(IpO)
ApIO	3	F8, F10, F13	F8	(In)(IpO)
Group F9				
IEpOn	2	F9, F14	F9	(On)(IOp)
IEpO	3	F9-F10, F14	F9	(On)(IOp)
Group F10				
ApIEpO	1	F10	F10	(IOp)(IpO)
(IOp)(IpO)	4	F10, F13-F15	F10	(IOp)(IpO)

A similar table can be drawn up for temporal modality, substituting the suffixes **c**, **t** for **n**, **p**.

2. Some Overall Comments.

The above table shows that, given a particular and/or potential (or even actual) proposition, we are unable to decide which way and how far to generalize it, without reference to the whole gross formula. If the gross formula consists of a single element, the conclusion is easy; it is the universal necessary proposition of like polarity. But if the gross formula is a compound, then the inductive path of any element in it depends on which other elements are involved. This is important to keep in mind.

We see that in some cases a particular proposition has become general, without change of modality; in other cases, the modality is raised, without change of quantity; in others still, both quantity and modality are affected. Also, two particular elements of a gross premise may emerge in the factorial conclusion as overlapping, or they may be separated.

Effectively, we have obtained the valid moods of natural modal induction (and by extension, those for temporal modality). They are not as numerous as appears, for we can distinguish 11 groups of valid moods among them, each defined by the best conclusion yielded. The conclusions being **F1**-**F10** and **F3-4**.

The groupings together include 13 primary valid moods, each of which has a number of subalterns. A primary mood in any group is one yielding the highest conclusion from the lowest premise. Subaltern moods are of two kinds.

The secondary premise may be higher than the primary one, yet yield a no-better conclusion, so that in effect the induction proper occurs after eduction of the lower premise. For example, **ApI** first implies **Ip**, from which **An** is thereafter induced. Or the secondary conclusion may be lower than the primary one, in which case it is in effect educed from the higher conclusion after the induction proper. For example, **Ip** yields **An** by induction, and then **AIn**, say, is inferred, since implied by **An**.

However, note well that subaltern moods are more certain than their corresponding primaries, because the number of factors they eliminate is lesser. Thus, for instance, **In** to **An** only eliminates 7 factors, whereas **Ip** to **An** eliminates 13 factors. The movement is more cautious, and therefore more likely to turn out to be correct in the long run.

The generalization from **I** to **A**, or from **O** to **E**, found in the closed system of actuals, can in this wider system of modal induction be viewed as a partial generalization. We move from a formula of 12 factors to one of 3 factors. We have not narrowed our position down to a single integer, but have nevertheless diminished the area of doubt considerably. Such limited generalizations are always permissible, of course, if they suffice for the needs of a specific inquiry.

3. Rules of Generalization.

The rules of generalization clarify the various aspects of the uniformity principle. They are presented here, prior to detailed analysis of the valid moods, to facilitate the reader's understanding of the discussion, but in fact they simply summarize the insights accumulated in the course of case by case examination.

The uniformity principle for factor selection, has a variety of implications. Some of these emerge in the paradigm of actual induction, but others become apparent only in modal logic. The rules of generalization serve to expose the variety of considerations which arise, and provide us with more specific guidelines than the basic principle.

The various vectors of uniformity often interfere with each other, in such a way that satisfying the requirements of the one, frustrates the demands of the other. This is because different factors stress different things. For instance, one factor may stress quantitative generalization, another may stress modality generalization. Case study of such conflicts of interest gradually clarified the order of importance of the different tendencies. The rules of generalization thus have an order of priority.

a. ***Polarity***. First in line is the requirement that the conclusion resemble the premise in polarity. If there is but one polarity in the premise, the same will remain solitary in the conclusion. If the premise is a bipolar compound, so must the conclusion be. One cannot induce a different or supplementary polarity. Such innovation has no basis in the uniformity principle, and can only occur with factual justification. Many factor selections, seeming to involve change of quantity or modality, rather stem from this inertia of polarity.

b. ***Quantity***. Next in line is increase in quantity, as far as consistent. This is the prime change induction seeks to effect. This is because a universal proposition is most open to testing, by drawing its consequences through deductive logic. Maximal extensional generalization is to be favored over improvements in modality or other uniformities, wherever possible. It is the paradigm of the uniformity principle, an assumption that properties tend to relate to classes, rather than being scattered accidentally.

c. ***Modality***. Uniformity implies an overall preference, not only for the more general alternative, but also for the factor of higher modality. However, modality generalization is only next in importance to that of quantity. But it is still this high on the list, for similar reasons: practically, because the higher the category, the more testable the result; metaphysically, because we assume a stable substratum beneath the changes we perceive.

Within either closed system, necessity is preferred to actuality, and actuality to possibility. In the open system, mixing modality types, natural necessity should be favored over constancy, and temporariness over potentiality, whenever the prior guidelines allow it. This is obvious from the relative positions of these various categories on the modality continuum.

d. ***Symmetry***. If the premise consists of elements of opposite polarity which are identical in both quantity and modality, the conclusion must have the same evenness. There would have to be factual basis for one side or the other to grow in quantity or modality more than the other; the uniformity principle does not justify such loss of symmetry. This is why the conclusion in a few cases cannot be a single factor, but a disjunction of two.

If on the other hand, the compound premise gives one or the other polarity a higher quantity or modality, the conclusion may or may not favor the one over the other: it depends on other considerations. Many subaltern moods have the unevenness of their premise in this way removed by the conclusion.

e. ***Overlap***. If some elements of a compound premise are known to converge, at least that same degree of overlap must reappear in the conclusion. Overlap cannot be lost by induction.

On the other hand, it may be gained. If overlap is not at all assured originally, it may be assumed, provided no prior considerations are put in jeopardy. Where there is a question as to whether two separately discovered particulars overlap or not, the uniformity principle would seem to suggest that they be applied to each other's extensions, so that both be maximally generalized.

However, if overlap is open to doubt, and making its assumption would cause problems in other respects, the adoption of the divergence hypothesis is acceptable. Overlap is of less importance than other issues, because it is conceptually derived from them.

f. ***Simplicity***. Lastly, but still significant, is the concern with fragmentation. In a choice between a factor with few fractions and another with many, both of which satisfy the prior guidelines, the former is preferable. We should not fragment the extension beyond the minimum feasible, always preferring the simplest alternative. This is an aspect of uniformity, in that it opposes diversity between the members of the class concerned. If indeed the more complex alternative is true, it will eventually impose itself through particularization.

The applications of these rules of generalization will now be seen through specific examples.

4. Review of Valid Moods.

Let us now review each primary valid mood of natural induction in some detail. In every case, to repeat, the gross premise, be it elementary or compound, is assumed to represent all available information on the subject to predicate relation concerned.

a. From any premise of single polarity, may be induced a universal necessary of same polarity. This is the most obvious application of the uniformity principle: there is no basis for presuming the other polarity at all possible. The primary moods in this group involve increase in both quantity and modality. They are:

Ip → An
Given solely that Some S are P,
we may induce that All S must be P.

Op → En
Given solely that Some S are not P,
we may induce that No S can be P.

The subaltern premises to **Ip → An** are: **I**, **In**, **Ap**, **ApI**, **ApIn**, **A**, **AIn**, **An**. The case **An** to **An** is of course deductive, even tautologous, and only listed to show the continuity. The subaltern (elementary) conclusions to **Ip** are: **A**, **Ap**, **In**, **I**; to **I**: **A**, **Ap**, **In**; to **In**: **A**, **Ap**; to **Ap**: **A**, **In**, **I**; to **ApI**: **A**, **In**; to **ApIn**: **A**; to **A**: **In**; and to **AIn**: none. Similarly, **Op → En** has some 16 subaltern inductive moods (not counting compound conclusions).

b. From a conjunction of particular premises of different polarity, one of which is actual and the other potential, the best inductive conclusion is a similar conjunction of universal premises. Here, the uniformity principle leads us to assume the particulars to fully overlap, and to generalize quantity only (not modality), to obtain a result with the original bipolarity.

IOp → AEp
Given solely that Some S are P and some S can not-be P,
we may induce that All S are P though all can not-be P.

IpO → ApE
Given solely that Some S are not P and some S can P,
we may induce that No S are P though all can be P.

It is clear that this induction occurs in stages. Consider the mood **IOp → AEp**. First the elements of **IOp** are made to converge into the fraction (**IOp**), dropping 3 factors, then this particular fraction is generalized into its universal equivalent (**AEp**), dropping a further 7 factors. Effectively, **I** has been generalized to **A**, and **Op** to **Ep**.

Alternative conclusions, though formally conceivable, seem less justifiable. For instance, (**In**)(**On**), by assuming nonoverlap, would cause baseless fragmentation of the extension, and result in a modal equality between the poles which was originally lacking. Whereas, say, (**In**)(**IOp**), while granting partial overlap and uneven modality, would fragment the extension without specific reason. Furthermore, a general conclusion is always to be preferred to a particular one, even one of stronger modality, because it is more readily tested.

The 4 subaltern premises **ApIOp**, **IEp**, **AOp**, **ApIEp** yield the same result. In their case, a partial overlap, meaning the fraction (**IOp**), is already implied, since one of the elements of the compound is universal already. In each case, consequently, less generalization is involved than in the primary mood, and the result is somewhat more trustworthy.

All the same comments can be made concerning the mood **IpO → ApE** and its subalterns.

c. When the premise is a compound of two particular potentials of different polarity, an imperfect conclusion may be drawn, diminishing the number of factors to two universal compounds in disjunction. Here, the original modal symmetry inhibits a more definite result, which would strengthen one side more than the other. But there is still an improvement in specificity, a guarantee of overlap and generalization of quantity having been achieved. The disjunctive result can be used in dilemmatic arguments.

IpOp → '(AEp) or (ApE)'
Given that Some S can be P and some S can not-be P,
we may induce that
either 'All S are P, though all can not-be P'
or 'All S are notP, though all can be P'.

The subaltern premises **IpEp**, **ApOp**, and **ApEp** have the same result. Note that the conclusion is not simply **ApEp**, which would allow the factor (**IOp**)(**IpO**) as an alternative. Precisely for this reason, **ApEp → '(AEp)** or (**ApE**)' is not a deductive inference, as those from **AEp** to (**AEp**) or from **ApE** to (**ApE**) were, but an induction diminishing the number of factors from 3 to 2. Even eliminating the fragmentation inherent in (**IOp**)(**IpO**) makes the effort worthwhile.

These moods may be viewed as to some extent subsidiary to the preceding group, tending toward the same sort of conclusion, but not quite succeeding. The elimination of particularistic alternatives, such as (**In**)(**On**), is based on similar argument.

d. From two particular actuals of opposite polarity, we induce two particular necessaries with corresponding polarities. Here, we may not in any case generalize quantity, for the four universal factors are deductively inconceivable, anyway; none of them would be compatible with the premise; they are not among the available factors. Thus, only modality, the next best thing, is increased as far as it goes, up to necessary; thusly, for both poles, to retain the original symmetry.

IO → (In)(On)
Given solely that Some S are P and some S are not P,
we may induce that Some S must be P and some cannot.

Note that in this special case, the uniformity principle causes divergence, rather than overlap, for the sake of obtaining a higher modality, while retaining the original evenness in modality. Although the compound **IO** implies **I+Ip+O+Op**, so that we might induce (**IOp**)(**IpO**) to achieve maximum overlap, the proposed conclusion is preferable, because it involves necessity instead of mere actuality and effectively no greater fragmentation of the extension. As for (**In**)(**On**)(**IOp**)(**IpO**), though equally conceivable in principle, and involving both necessity and overlap advantages to some extent, it is rejected, because it introduces an excessive fragmentation, for which no argument is forthcoming.

The premises **IOn**, **InO**, and **InOn** may be viewed as subalterns to **IO**, as well as to the primaries considered next.

e. From the conjunction of two particulars of opposite polarity, one of which is necessary and the other potential, a conjunction of two particular necessaries of opposite polarity is induced. Here, the original asymmetry and the conceivable partial overlap, are sacrificed to improvement in modality. Any universal conclusion is again out of the question, on formal grounds.

InOp → (In)(On)
Given solely that Some S must be P and some can not-be,
we may induce that Some S must be P and some cannot be.

IpOn → (In)(On)
Given solely that Some S can be P and some cannot be P,
we may induce that Some S must be P and some cannot be.

These two moods are independent primaries, and not subalterns to **IO** → (**In**)(**On**), note well, since neither **InOp** nor **IpOn** formally implies **IO**. They are, however, closely related, having in common the same conclusion, and the same subaltern premises **IOn**, **InO**, **InOn**.

Note well, incidentally, that **InOn** → (**In**)(**On**) is indeed an inductive argument, and not a deductive one, since InOn has 4 factors originally, 3 of which are then eliminated, for reasons of asymmetry or excessive fragmentation, as our table shows.

f. Two more groups of valid moods are distinguished by their more complex primary premises and conclusions. They are the following.

ApInOp → (In)(IOp)
Given that All S can be P, some S being necessarily P,
and others potentially not P,
we may induce that the latter S are actually P.

IpEpOn → (On)(IpO)
Given that All S can not-be P, some S being necessarily not P,
and others potentially P,
we may induce that the latter S are actually not P.

In the positive case, we first separate the (**In**) fraction from the remainder **IpOp**, which we know to overlap since **Ap** is general and given; then we favor the (**IOp**) outcome, generalizing **Ip** to **I**, on the basis that **I** is already implicit in **In**. In comparison, the (**IpO**) eventuality, though conceivable, would require a move from **Op** to **O**, for which no specific basis is found, so that it may be inductively eliminated. The mood **AInOp** yields the (**In**)(**IOp**) conclusion deductively, not inductively, since this is its only factor. Similar comments can be made with regard to the parallel negative cases.

ApIO → (In)(IpO)
Given that All S can be P, some S being actually not P,
and others being actually P,
we may induce that the latter S must be P.

IEpO → (On)(IOp)
Given that All S can not-be P, some S being actually P,
and others being actually not P,
we may induce that the latter S cannot be P.

Here again, in the positive case, we first separate the (**IpO**) fraction, on the grounds that **Ap** is general and that **I** and **O** cannot overlap; then we generalize the remaining **I** segment of the extension to **In**. The (**IOp**) eventuality, though conceivable since **O** implies **Op**, is rejected on the basis that it involves a weaker category of modality compared to (**In**); as for the conjunction of both (**In**) and (**IOp**), this would introduce a needless additional fragmentation into the equation. The subaltern premise **AInO** yields the same inductive conclusion, by elimination of only the latter eventuality, for the same reason. Similar comments can be made with regard to the parallel negative cases.

g. The inference from **ApIEpO** to (**IOp**)(**IpO**) is deductive, as we saw in factorial analysis.

On the other hand the move from the gross conjunction of the two particular fractions (**IOp**) and (**IpO**) as a premise, to the integer (**IOp**)(**IpO**) is inductive, not deductive. For the common factors of the fractions are not only **F10**, but also **F13**, **F14**, **F15**. The latter three, which involve the conjunction of (**In**) or (**On**) or (**In**)(**On**) to (**IOp**)(**IpO**), are formally conceivable, but in this context rejected, on the basis that they introduce new fragments without specific justification.

The other gross conjunctions of fractions, in twos or threes, similarly yield their integral counterparts, **F11**-**F14**, by induction. In the case of four fractions, the **F15** conclusion is deductive.

All that has been said for natural factor selection, could be repeated for temporal factor selection. The two closed systems behave identically.

5. Open System Results.

We shall now list the valid moods of open system induction, with a minimum of comments, for the record. The reader is encouraged to review the valid moods, with reference to the rules of generalization, to justify the selection of this or that factor rather than any other, in each case.

We saw, in earlier chapters, that when natural and temporal modality are considered together, 63 integers (see table 52.2) and 195 gross formulas (see table 51.1) may be generated. In an appendix, we developed a table showing the factorial analysis of all gross formulas. The factorial analysis of the particular fractions, on the other hand, may be found in table 52.2 (reading it downward).

The valid moods of open system induction, are easily extracted from these sources of information. In accordance with the law of generalization, the factor to select in induction is usually the first, the one with the lowest ordinal number; though, in a few cases, we must select the first two factors in disjunction to maintain symmetry. This is so, simply because I numbered the factors that way, in order of generality, necessity, and simplicity.

Be careful not to confuse the closed system factors with the open system factors; the symbols **F1**-**F15** have mostly different meanings in each context. Also remember not to equate the four compound particular fractions, (**IcOp**), (**IpOc**), (**IOt**), (**ItO**) to their gross equivalents. Each of the former has 32 factors, whereas **IcOp** and **IpOc** have 47 each, and **IOt** and **ItO** 53 each.

The table below shows the selected factors for all gross formulas in the mixed modality system. Premises with the same inductive conclusion are grouped together, and their common result is given. The number of factors for each formula is listed under the heading '**NF**'.

There are, we see, 23 groups of valid moods, with numbers lying between **F1** and **F21**. A total of 33 of the moods are primary; these are indicated by 3 asterixes (*******). The remaining moods are subalterns of these.

Note that 11 moods are in fact deductive, rather than inductive, since they were found to have only one factor when analyzed; one of these is the sole listed representative of **Group F21**. These are included for completeness.

While the individual fractions are also included in our table, the various gross conjunctions of two to six particular fractions have been ignored, to avoid excessive volume; these obviously yield their integral counterparts, **F7**-**F63**, as inductive results.

Table 56.2 Factor Selection in the Open System.

Group F1	Group F2	
Premise(s)	Premise(s)	NF
An	En	1
AcIn	EcOn	2
Ac	Ec	3
AIn	EOn	4
AIc	EOc	6
A	E	7
AtIn	EtOn	8
AtIc	EtOc	12
AtI	EtO	14
At	Et	15
ApIn	EpOn	16
ApIc	EpOc	24
ApI	EpO	28
ApIt	EpOt	30
Ap	Ep	31
In	On	32
Ic	Oc	48
I	O	56
It	Ot	60
Ip ***	Op ***	62
Conclusion (An)	Conclusion (En)	1

Group F3	Group F4	
Premises	Premises	NF
AcEp	ApEc	1
AcOp	ApEOc	2
AIcEp	IpEc	2
AEp	ApE	3
AtIcEp	ApEtOc	4
AIcOp	IpEOc	5
AOp	ApEtO	6
AtIEp	IpE	6
AtEp	ApEt	7
ApIcEp	ApEpOc	8
AtIcOp	IpEtOc	11
ApIEp	ApEpO	12
AtIOp	IpEtO	13
AtOp	ApEpOt	14
ApItEp	IpEt	14
IcEp	ApOc	16
ApIcOp	IpEpOc	23
IEp	ApO	24
ApIOp	IpEpO	27
ItEp	ApOt	28
ApItOp	IpEpOt	29
(IcOp)	(IpOc)	32
IcOp	IpOc	47
IOp	IpO	55
ItOp ***	IpOt ***	59
Conclusion (AcEp)	Conclusion (ApEc)	1

Group F3-4	
Premises	NF
ApEp	15
ApOp	30
IpEp	30
IpOp ***	61
Conclusion (AcEp) or (ApEc)	2

Group F5	Group F6	
Premises	Premises	NF
AEt	AtE	1
AEpOt	AtEtO	2
AtIEt	ApItE	2
AOt	AtEpO	4
ApIEt	ItE	4
AtIEpOt	ApItEtO	5
IEt	AtO	8
AtIOt	ApItEpO	11
ApIEpOt	ItEtO	11
IEpOt	ApItO	23
ApIOt	ItEpO	25
(IOt)	(ItO)	32
IOt ***	ItO ***	53
Conclusion (AEt)	Conclusion (AtE)	1

Group F5-6	
Premises	NF
AtEt	3
AtEpOt	6
ApItEt	6
AtOt	12
ItEt	12
ApItEpOt	13
ApItOt	27
ItEpOt	27
ItOt ***	57
Conclusion (AEt) or (AtE)	2

Groups F7	
Premises	NF
InOn	16
InOc	24
IcOn	24
InO	28
IOn	28
InOt	30
ItOn	30
InOp ***	31
IpOn ***	31
IcOc	36
IcO	42
IOc	42
IcOt ***	45
ItOc ***	45
IO ***	49
Conclusion (In)(On)	1

Group F8	Group F9	
Premises	Premises	NF
AcInOp	IpEcOn	1
AInOp	IpEOn	3
AtInOp	IpEtOn	7
ApInOp ***	IpEpOn ***	15
Conclusion (In)(IcOp)	Conclusion (On)(IpOc)	1

Groups F10	Groups F11	
Premises	Premises	NF
ApInOc	IcEpOn	8
ApInO	IEpOn	12
ApIcOc	IcEpOc	12
ApInOt	ItEpOn	14
ApIOc	IcEpO	14
ApItOc ***	IcEpOt ***	15
ApIcO	IEpOc	18
ApIcOt ***	ItEpOc ***	21
ApIO ***	IEpO ***	21
Conclusion (In)(IpOc)	Conclusion (On)(IcOp)	1

Group F12	Group F13	
Premises	Premises	NF
AInOt	ItEOn	2
AIcOt	ItEOc	3
AtInOt	ItEtOn	6
AtIcOt ***	ItEtOc ***	9
Conclusion (In)(IOt)	Conclusion (On)(ItO)	1

Group F14	Group F15	
Premises	Premises	NF
AtInO	IEtOn	4
AtIcO	IEtOc	6
AtIO ***	IEtO ***	7
Conclusion (In)(ItO)	Conclusion (On)(IOt)	1

Groups F16	
Premises	NF
ApIcEpOc	4
ApIcEpO	6
ApIEpOc	6
ApIcEpOt ***	7
ApItEpOc ***	7
ApIEpO ***	9
Conclusion (IcOp)(IpOc)	1

Group F17	Group F18	
Premises	Premises	NF
AIcEpOt	ApItEOc	1
AtIcEpOt ***	ApItEtOc ***	3
Conclusion (IcOp)(IOt)	Conclusion (IpOc)(ItO)	1

Group F19	Group F20	
Premises	Premises	NF
AtIcEpO	ApIEtOc	2
AtIEpO ***	ApIEtO ***	3
Conclusion (IcOp)(ItO)	Conclusion (IpOc)(IOt)	1

Group F21	
Premises	NF
AtIEtO ***	1
Conclusion (IOt)(ItO)	1

57. FORMULA REVISION.

1. Context Changes.

As knowledge evolves, our position shifts from one set of givens to another, and the inductive or deductive conclusion concerning any subject to predicate relation must be adapted to the new situation. All knowledge is contextual and tentative, anyway, in principle. Changes in context are to be taken in stride, as normal and to be expected. The current formula is revised, reformulating our state of knowledge in the light of new input, and then induction and deduction proceed as usual.

There are two kinds of context change. Starting with some formula, we discover new data, concerning the same subject to predicate relation. The new input may either be compatible with the preceding context, and be implicit in it and so without effect on it, or add to it, making it more specific. Or the new input may be incompatible with previous positions, in which case some conflict resolution is required.

We may discover such factual or logical errors in our beliefs by deductive or inductive means, from whatever sources.

Some new line of thought or generalization or observation may have taken place, which shows our preceding belief to be too limited or too vague or over-extended. Or the novelty involved may be relative: we may have come across this additional data before the data under consideration, but simply did not instantly make the conceptual connection; here, the novelty lies in our only now becoming aware of its impact.

The old and new information may have the same or different form: each may be positive, negative or bipolar; it may be particular, singular or general; it may have any modality; it may be elementary or compound; it may be a fraction, an integer, or even already in factorial form.

Whatever the case, formula revision is needed. We must step back and reconsider our situation in the light of the new data, formulating a new gross statement of our position to fit it, and drawing a new inductive conclusion from that.

Nevertheless, we want to retreat from previous positions as conservatively as possible. We do not want to radically revise our ideas or beliefs every time we face new material, though in some cases we may have to do just that. We do not want to overreact and lose valuable information, unless we have to. So we must learn to evaluate the seriousness of our predicament, and develop techniques for handling the various kinds of problems.

Formula revision, like factor selection, is largely an art, rather than an exact science. In some cases, the result is clear-cut; but in many situations, we are faced with a variety of paths which may seem equally credible, and the choice among them is intuitive and esthetic to a great degree. The task of logical theory is to facilitate decision making in such cases, by clarifying the options and their significances. It provides the artist with the tools, without rigidly prescribing their use.

2. Kinds of Revision.

We may distinguish two kinds of formula revision: amplification and harmonization.

Amplification occurs when the additional information is consistent with the original givens, and so can be simply conjoined to them. Note the connotation of growth. (Perhaps the name ‘apposition’ would have been more appropriate, but I settled on the latter because of its musical analogies.)

Amplification is of two kinds. It may narrow down the potential scope of a proposition; we call this process ‘specification’. Or it may broaden the actual scope of a proposition; we may call this ‘elaboration’. For example, given first that some S are P — if we thereafter find that some other S are not P, the initial proposition is further specified, whereas if we find that all other S are P, it is broadened. The logical possibility of the particular proposition to become general, is stifled in specification, but confirmed in elaboration.

Harmonization occurs when merging the two formulas would yield an inconsistent conjunction, so that some decision or compromise between them must be sought. We often call this process ‘reconciliation’.

Amplification may occur between propositions of similar or different polarity, provided they are not contrary or contradictory. Harmonization, in contrast, always concerns propositions of somehow opposite polarity, which are wholly or partly in conflict.

The premises and conclusions of these operations may be of similar strength, or weaker, or stronger, depending on our point of view.

Amplification of a formula is straightforward enough, formally speaking. Still, having assumed the original formula complete, in the sense of summarizing available knowledge, we may have made a generalization, and then deductions from this, which must now be reconsidered: they are now open to doubt, though not deserving of outright rejection. For the new, amplified formula will very likely suggest other inferences. Such review of the wider context is very often difficult; sometimes it is impossible to retrace our past course, and we must hope that inconsistencies will eventually arise, allowing us to streamline our knowledge base.

With regard to harmonization, or conflict resolution, one or both of the clashing, or adverse, theses must be changed to remove the problem and harmonize our knowledge. If one or the other is dominant, because of the greater credibility of its foundations, the other will be downgraded alone, or even totally eliminated if required; the latter may then be said to have conceded or yielded to the former. If they are of equal weight, for lack of a reason to prefer the one over the other, the common ground between them is sought: they in principle have to both be downgraded (though in certain cases it is permissible and sufficient to downgrade only one). Whatever the conflict, questions arise as to how deep a correction is called for, and in what direction it should be effected. Obviously, the retreat in quantity and/or modality should be the minimal permissible.

Here again, the consequences on the wider context of knowledge must be considered, to the extent possible, and these may in turn boomerang on the propositions under consideration, through successive formula revisions.

If a premise was itself obtained by deduction, and has been denied or downgraded for the purposes of conflict resolution, those prior sources are now known to certainly contain some error, and some or all of them must in turn be revised. Also, if either or both of the two original theses were generalized, before our becoming aware of their conflict, we can expect the inductive conclusion from their harmonization to disagree with one or both of these anterior inductions. If any deductions were made from a premise or its generalization, they are now put in some doubt, even if not automatically to be rejected.

Formula revision always means the conjunction of an old and new thesis. They may both be gross formulas (elementary or compound), or both be fractional formulas (isolated fractions or seeming to make up an integer). Or we may be dealing with the interactions between these various kinds of formula. Even deficient formulas not expressible as gross formulas may be involved. We have to look into all the possibilities.

All these issues will become clearer as we proceed with applications.

While the pursuit of consistency is recognized as in the logical domain by tradition, it has been dealt with in relatively vague terms. Effectively, we were given the tables of opposition as tools, but no step by step tactical instruction. We were told that in the event of inconsistency we should review our assumptions, but we were not provided with more specific guidance. The reason for this is that the classical model, where categorical propositions are all actual, is too limited and simplistic. The modal system provides us with a larger field of activity, complex enough to suggest the kind of difficulties which occur in practice.

3. Particularization.

Formula revision involves two initial theses, to be somehow fused in the conclusion. Formula revision occurs because of time lags between the emergence of items of knowledge, which may be consistent or inconsistent. But at the moment of revision, the time ingredient becomes irrelevant, and the theses are logically at the same level. One may not be regarded as more of a premise than the other.

Since formula revision involves two theses as premises, our understanding of each operation depends on which premise we compare to the conclusion. Looking at the one, we will notice this or that change has been effected on it by the process; looking at the other, the process has a different character. Both must be looked at, rather than subjectively focusing on either as ‘the premise’, to avoid misinterpreting the process.

Also, we may be tempted to compare the possible generalizations from the premises to the anticipated generalization from the conclusion. Or the one as-is to the generalization of the other. Inquiry of this sort is not without value, but should be done consciously, without confusion as to what precisely are the starting points and end result of the formula revision per se.

We should view formula revision as only including the work of amplification or harmonization as such. The generalizations which might have been made from the premises, or the generalization which normally follows the conclusion, are in principle optional and independent operations. Although, as we shall see, these may play a central role in the direction the formula revision takes.

Now, we would characterize as ‘particularization’ any process whose result is weaker than (or at best equal in strength to) the givens. This refers to decreases quantity and/or modality, essentially. Such contraction can be expressed as an increase in the number of weak factors, or as disappearance of stronger factors.

While formula revision does indeed usually involve particularization of the elements involved, there are certain special cases where it in fact yields a stronger conclusion. Sometimes there is a particularizing effect in one respect and a generalizing effect in another. The term ‘formula revision’ therefore has a more neutral connotation than the term ‘particularization’, and they may not always be equated, though they are often loosely-speaking confused.

Amplification of gross formulas is purely deductive revision, and only the subsequent generalization from its conclusions may be called inductive. But amplification of fractional formulas is itself inductive, quite apart from any subsequent generalization.

Harmonization, on the other hand, is only deductive in its application of the laws of opposition; with regard to its evaluations of credibilities, and its choices between alternative conflict resolutions, it is inductive, as much so as subsequent generalizations from its results.

We saw that generalization starts from a consistent body of knowledge, which, viewed simultaneously, has been summarized and factorized; thereafter, the strongest factor among those available is selected, so that the conclusion is generally superior to the premise.

Formula revision does not exactly refer to a mirror image of this process. It has a different structure and goal, the marriage of two premises. Particularization is not its essential goal, and not always its result. Furthermore, as we shall see, formula revision often solves problems by factor selection under the law of generalization.

Particularization is not a distinct process, but refers to certain specific applications of processes already defined. Consequently, it has no clear-cut ‘law’ or ‘rules’ analogous to those for generalization. We cannot simply convert the latter to predict the former. For instance, we cannot say that, since the latter prescribes that we favor quantity over modality, the former will affect modality before quantity. As will be seen, in some cases the result is one way, in other cases, the other way.

58. GROSS FORMULA REVISION.

1. Amplification.

In gross formula amplification, we merely add the new data to the old, usually to obtain a more complex gross formula, of greater informational specificity. The tables of opposition tell us what elements are compatible; so that, as long as all the elements involved may coexist, they can be conjoined together consistently. The revised formula should then be analyzed factorially, and its strongest factor selected in accordance with the law of generalization. The new result may or may not differ from the original. Let us consider some examples.

The easiest case is when the additional data is already implicit in the original, as in **A**+**I** = **A** or **InOp**+**I** = **InOp**. Here there has effectively been no change of formula, though one can say that the original formula is further confirmed. Any generalization made from the original remains the same.

Next, we have the case of a formula of single polarity amplified by data of the same polarity not implicit in it, as in **A**+**In** = **AIn**. Here, there is a change of formula, but this has still no effect on eventual generalization, except to render it more certain by diminishing the number of factors dropped by it.

Thirdly, we have the case of a formula being so affected by additional, though consistent, data, that its inductive results may change. For instance, take **ApIn**+**Op** = **ApInOp**: the original conclusion from **ApIn** would be (**An**); after addition of **Op**, the gross formula becomes **ApInOp**, and its conclusion is (**In**)(**IOp**), quite different. Or again, take **ApInOp**+**O** = **ApInO**: here, instead of concluding (**In**)(**IOp**), we are now forced to conclude (**In**)(**IpO**).

There is no need to list all possible cases of gross formula amplification, for they are already effectively spelled out in the chapter on factor selection. Such situations may occur either prior to generalization, when we are still dealing with gross formulas, or at a more advanced stage of the proceedings, provided we are able to retrace our past course.

In the event of a conflict between two formulas, if we are able to remember the consistent gross formulas which gave rise to these generalizations, or these deductive consequences of generalization, which are now in conflict, we can resolve the conflict by merging the root propositions into a new, amplified formula, which henceforth will serve as our inductive premise. But such radical revision is not always possible.

All that has been said so far is applicable equally to naturals, to temporals, and to mixed modality gross formulas.

2. Harmonization.

Harmonization, or reconciliation, concerns situations where the original and additional data are in conflict. In a looser sense, we might view even amplification as involving a kind of ‘conflict’: that between the assumption that the original thesis told the whole story, and the discovery through the new input that it did not. But conflict here means specifically oppositional incompatibility.

Some of the elements are contradictory or contrary, and the issue is how to resolve such discrepancy once it is noticed. Something has to ‘give’, on one side or the other or on both sides, for the controversy to be defused. An adaptation of sorts is required.

Harmonization follows the familiar ***dialectical*** pattern: *thesis, antithesis, and synthesis*.

Conflicts arise because, somewhere along the line, there has been an over-generalization. We exaggerated the impact of certain observations, insights, or narrow inferences, and, sooner or later, this was bound to lead to inconsistency, whether with these same terms, or in consequent propositions involving other terms. However, when the inconsistency arises, it may not be immediately clear which side to blame for the error.

The way conflicts are to be resolved depends, firstly, on the relative weights of the original and new data.

The degree of credibility of a thesis is a function of how clearly it is formulated, how tightly it is argued, how much connection it has to the rest of knowledge, how well established its conceptual and logical sources are, how empirical it is, how often it is confirmed, how dependent we are on it for practical purposes. Credibility is thus a variable appearance, a phenomenon by which the many forces affecting our trust in a thesis are summed up at any given point in the development of knowledge.

The credibility of a thesis is also a function of the existence of alternative ideas or theories, and their relative credibility. A thesis may be credible on its own, yet, when viewed in perspective, in comparison to others, it may seem less likely than some other. When two or more theses are in conflict, an evaluation of their relative weights must be made, and an order of priority assigned to them.

Note that very often, we assign greater weight to an older thesis, one to which we are more accustomed; such conservatism is somewhat justified on the basis that the older thesis is time tested. But it can happen that a new thesis may be shown, by thorough examination, to be in all respects of equal weight or better; from a scientific point of view, age is ultimately accidental, and not an over-riding cause for inertia.

3. Unequal Gross Formulas.

The simplest case of conflict resolution is when either the original or the new data, whichever, has greater weight. In such case, the dominant thesis remains untouched by the conflict, and its submissive antithesis alone must bear the burden of adjustment.

Each thesis may be an element or compound; and it may be positive, negative, or of mixed polarity. Obviously, however, only elements of opposite polarity may be inconsistent with each other; though of course, if elements of opposite polarity are low enough on the scale of quantity and modality, they may not create a conflict.

The general rule of harmonization of theses of unequal weight is, any element(s) explicit or implicit in the submissive thesis, which are contradictory or contrary to some element(s) in the dominant thesis, must be diminished in quantity and modality until compatible, or even, if needful, totally denied.

Some examples. If the dominant is **An**, then all negative theses from **Op** on up must be rejected. If the dominant is **AIn**, then only **Op** may coexist, since **Ep** would contradict **In**, and **O** would contradict **A**, and perforce any statement still stronger than those would be contrary. If the dominant is **ApI**, then only **O** and/or **Ep** are acceptable, since **On** or more would conflict with **Ap**, and **E** or more would clash with **I**. If the dominant is **ApInO**, then **A**, **Ep**, **On**, and all their subalternants are impossible.

Thus, when one thesis is given more weight, the outcome is easy enough to predict, using the rules of opposition. However, harmonization does not end there. We must now consider the significance of our rejection of certain elements. If, say, the submissive thesis was **An**, and we were forced to particularize it to **ApI**, then we must find out if any of our other beliefs include or imply **An** or **A**, and correct these too in turn; also, if any deductive inferences were drawn from **An** or **A**, they are now rendered less sure (though not automatically deniable).

Furthermore, such weighted harmonization usually, though not always, leaves a remainder. If, for instance, **An** is pitted against **En**, with the latter dominant, the result is simply **En**, since even **Ip** would maintain conflict. But if **A** is pitted against **E**, again with the latter dominant, only **A** is rejected, while its subaltern **Ap** remains in force, since there is no reason to surrender it, and is to be added to **E**, forming the compound **ApE**.

In this way, we do not push the harmonization beyond the necessary minimum, but subtract only those levels of quantity or modality which cause the interference. Such conjunction of remainder (of the submissive) to the dominant follows the pattern already described under the heading of amplification.

Harmonization may change our original inductive conclusion, or leave it unaffected. For instance, if we start with thesis **ApIOp**, and find **E** from other sources, and deciding the former stronger, resolve the conflict to **ApIEpO**, the inductive conclusion is changed from (**AEp**) to (**IOp**)(**IpO**). However, if **AOp** was the original thesis, and **E** arose in submissive conflict with it, the resolution **AEp** does not affect our initial induction of (**AEp**).

Again, all that has been said so far is applicable as well to naturals, to temporals, and to mixed modality gross formulas.

4. Equal Gross Formulas.

By far more complex, and interesting, is harmonization between gross formulas of equal weight which are in contention. We know of no reason to favor one thesis over the other, so that their credibilities are in equilibrium, and we seek their common grounds, their points of agreement. We are supposedly unable to retrace the course which led up to them, or pin-point where we may have erred, and so must deal with them as we find them, narrowing their scopes to acceptable levels, down to where they can coexist.

More often than not, there may be more than one way to resolve the conflict. This is where factor selection comes into play, providing us with a convincing basis for arbitration. Without this beautiful instrument, such formula revision would have been pure guesswork.

Conflicts between gross formulas of equal weight are resolved step by step, as follows:

a. Fuse the original two gross formulas, whatever they be, into one *inconsistent* compound of mixed polarity; we can do this, because all elements have equal weight. Then, separate the positive and negative elements or compounds from each other, to form two gross formulas each of which has uniform polarity.

b. Now, try out the hypothesis of harmonization of quantity. This means, firstly, diminish the quantity of all the elements in both formulas to particular; we shall call the resulting compound the ‘quota’. Secondly, find the ‘remainder’ of this operation, if any; that is, what explicit or implicit elements in the two formulas are compatible with the quota. Thirdly, fuse quota and remainder into one compound, and identify its strongest factor.

c. Next, similarly try out the hypothesis of harmonization of modality. Thus, firstly, diminish the modality of all the elements in both formulas to potential; the resulting compound being the ‘quota’. Secondly, find the ‘remainder’ of this operation, if any; that is, what explicit or implicit elements in the two formulas are compatible with the quota. Thirdly, fuse quota and remainder into one compound, and identify its strongest factor.

d. Lastly, compare the strongest factors of these two alternative conclusions. Whichever has the stronger strongest factor is the revised gross formula. The selected revised formula is, alone, the conclusion of the harmonization process, the resolution of the original conflict, the goal we pursued. Its strongest factor is accordingly the optional inductive conclusion, if we choose to generalize at this point.

Thus, the decision as to whether to revise the formulas by reference to quantity or to modality is made for us by factor selection. It is not subjective, but systematic, controlled by what would be the inductive result in either case. There is but one universal law for both inductive processes.

Our results demonstrate that, had we sought guidance through some separate ‘law of particularization’, we would have made many mistakes, as will be seen. Some conflicts are resolved on the side of contraction of quantity, others are resolved through modality contraction, others still yield the same result on either side; there is no general rule in that respect.

Particularization has no special basis, but is entirely determined by the same law as generalization: namely, selection of the strongest factor.

5. Applications.

The following table shows the results of conflict resolution obtained, by the method described above, for gross formulas in the closed system of natural modality. As usual, similar results could be worked out for temporal modality.

The positive and negative elements of the conflicting theses are grouped separately, and listed under the heading 'given conflict'. The resolutions by quantity and modality, and their respective strongest factor (**SF**) are listed in adjacent columns. In each case, the quota is displayed first, and the remainder if any is added on to it with a '**+**' sign. The corresponding strongest factor, note well, concerns the complete compound of quota and remainder (this compressed version is not shown in the table, being easily constructed). We know the selected factor in each case from our prior inquiry on this topic (the reader is referred to the chapter on factor selection).

Finally, the inductive conclusion is given: notice that this is the selected factor with the lowest ordinal number of the two. The data has been classified by similarity of inductive conclusions, to show the inherent continuities. The range is **F3**-**F10**. The revised gross formula is the one whose strongest factor is selected as the inductive conclusion. Observe the uniformities found in each grouping.

Note the inclusion in this table, for the sake of completeness, of cases where the positive and negative sides are not in conflict (these are marked 'okay'). Excluding these, there are in all 50 valid moods. Among them, 18 have been earmarked with an asterisk (*****): these may be viewed as the main moods, in that they present the strongest conflicts for each distinct revised formula. Examples are given after the table.

Table 58.1 Conflict Resolutions for Equal Gross Naturals.

Given Conflict		Quantity		Modality		Inductive Conclusion	
Posi-tive	Nega-tive	Resolve	SF	Resolve	SF		
An	Ep	InOp+A	F6	ApEp+A	F3	(AEp)	F3 *
AIn	Ep	InOp+A	F6	ApEp+A	F3	(AEp)	F3
ApIn	Ep	InOp+Ap	F6	ApEp+I	F3	(AEp)	F3 *
In	Ep	InOp	F5	IpEp+I	F3	(AEp)	F3 *
Ap	En	IpOn+E	F7	ApEp+E	F4	(ApE)	F4 *
Ap	EOn	IpOn+E	F7	ApEp+E	F4	(ApE)	F4
Ap	EpOn	IpOn+Ep	F7	ApEp+O	F4	(ApE)	F4 *
Ap	On	IpOn	F5	ApOp+O	F4	(ApE)	F4 *
An	En	InOn	F5	ApEp+IO	F10	(In)(On)	F5 *
An	EOn	InOn	F5	ApEp+IO	F10	(In)(On)	F5
AIn	En	InOn	F5	ApEp+IO	F10	(In)(On)	F5
An	EpOn	InOn	F5	ApEp+IO	F10	(In)(On)	F5
ApIn	En	InOn	F5	ApEp+IO	F10	(In)(On)	F5
AIn	EOn	InOn	F5	ApEp+IO	F10	(In)(On)	F5
AIn	EpOn	InOn	F5	ApEp+IO	F10	(In)(On)	F5
ApIn	EOn	InOn	F5	ApEp+IO	F10	(In)(On)	F5
ApIn	EpOn	InOn	F5	ApEp+IO	F10	(In)(On)	F5

Given Conflict		Quantity		Modality		Inductive Conclusion	
Posi-tive	Nega-tive	Resolve	SF	Resolve	SF		
An	On	InOn	F5	ApOp+InO	F8	(In)(On)	F5
AIn	On	InOn	F5	ApOp+InO	F8	(In)(On)	F5
ApIn	On	InOn	F5	ApOp+InO	F8	(In)(On)	F5
A	On	IOn	F5	ApOp+IO	F8	(In)(On)	F5 *
ApI	On	IOn	F5	ApOp+IO	F8	(In)(On)	F5
In	En	InOn	F5	IpEp+IOn	F9	(In)(On)	F5
In	EOn	InOn	F5	IpEp+IOn	F9	(In)(On)	F5
In	EpOn	InOn	F5	IpEp+IOn	F9	(In)(On)	F5
In	E	InO	F5	IpEp+IO	F9	(In)(On)	F5 *
In	EpO	InO	F5	IpEp+IO	F9	(In)(On)	F5
In	On	okay	F5	okay	F5	(In)(On)	F5
In	O	okay	F5	okay	F5	(In)(On)	F5
I	On	okay	F5	okay	F5	(In)(On)	F5
In	Op	okay	F5	okay	F5	(In)(On)	F5
Ip	On	okay	F5	okay	F5	(In)(On)	F5
I	O	okay	F5	okay	F5	(In)(On)	F5
An	O	InO+Ap	F8	ApOp+In	F6	(In)(IOp)	F6 *
AIn	O	InO+Ap	F8	ApOp+In	F6	(In)(IOp)	F6
An	Op	InOp+A	F6	ApOp+AIn	F6	(In)(IOp)	F6 *
AIn	Op	okay	F6	okay	F6	(In)(IOp)	F6
ApIn	Op	okay	F6	okay	F6	(In)(IOp)	F6
I	En	IOn+Ep	F9	IpEp+On	F7	(On)(IpO)	F7 *
I	EOn	IOn+Ep	F9	IpEp+On	F7	(On)(IpO)	F7
Ip	En	IpOn+E	F7	IpEp+EOn	F7	(On)(IpO)	F7 *
Ip	EOn	okay	F7	okay	F7	(On)(IpO)	F7
Ip	EpOn	okay	F7	okay	F7	(On)(IpO)	F7
An	E	InO+Ap	F8	ApEp+IO	F10	(In)(IpO)	F8 *
An	EpO	InO+Ap	F8	ApEp+IO	F10	(In)(IpO)	F8
AIn	E	InO+Ap	F8	ApEp+IO	F10	(In)(IpO)	F8
AIn	EpO	InO+Ap	F8	ApEp+IO	F10	(In)(IpO)	F8
ApIn	E	InO+Ap	F8	ApEp+IO	F10	(In)(IpO)	F8
ApIn	EpO	InO+Ap	F8	ApEp+IO	F10	(In)(IpO)	F8
A	O	IO+Ap	F8	ApOp+IO	F8	(In)(IpO)	F8 *
ApIn	O	okay	F8	okay	F8	(In)(IpO)	F8
ApI	O	okay	F8	okay	F8	(In)(IpO)	F8

Given Conflict		Quantity		Modality		Inductive Conclusion	
Posi-tive	Nega-tive	Resolve	SF	Resolve	SF		
A	En	IOn+Ep	F9	ApEp+IO	F10	(On)(IOp)	F9 *
A	EOn	IOn+Ep	F9	ApEp+IO	F10	(On)(IOp)	F9
A	EpOn	IOn+Ep	F9	ApEp+IO	F10	(On)(IOp)	F9
ApI	En	IOn+Ep	F9	ApEp+IO	F10	(On)(IOp)	F9
ApI	EOn	IOn+Ep	F9	ApEp+IO	F10	(On)(IOp)	F9
ApI	EpOn	IOn+Ep	F9	ApEp+IO	F10	(On)(IOp)	F9
I	E	IO+Ep	F9	IpEp+IO	F9	(On)(IOp)	F9 *
I	EpOn	okay	F9	okay	F9	(On)(IOp)	F9
I	EpO	okay	F9	okay	F9	(On)(IOp)	F9
A	E	IO+ApEp	F10	ApEp+IO	F10	(IOp)(IpO)	F10 *
A	EpO	IO+ApEp	F10	ApEp+IO	F10	(IOp)(IpO)	F10
ApI	E	IO+ApEp	F10	ApEp+IO	F10	(IOp)(IpO)	F10
ApI	EpO	okay	F10	okay	F10	(IOp)(IpO)	F10

Some examples are in order.

- **ApIn** versus **Ep**. Contracting their quantities, we obtain **InOp**; we can still keep the **Ap** explicit in **ApIn**, since it is not in conflict with **Ep**; hence, we get **ApInOp** (strongest factor **F6**), under this hypothesis. Alternatively, contract their modalities, to obtain **ApEp**; this allows for **I**; so, **ApIEp** (strongest factor **F3**) is yielded by this hypothesis. Comparing the two, the latter is found stronger, and so the revised gross formula is **ApIEp**, and **F3** is its eventual generalization. Effectively, the harmonization particularized **In** to **I**, necessity to actuality, in this case.
- **An** versus **En**. Quantity harmonization is **InOn**; there is no remainder since even **Ap** is excluded by **On** and **Ep** by **In**. Modality harmonization is **ApEp** (**AE** being impossible); with remainder **IO** (rather than **A** or **E**, which would be asymmetrical). **InOn** has **F5** as its strongest factor, whereas **ApIEpO** has **F10**, so the former wins. In this case, the harmonization particularized **An** to **In** and **En** to **On**, that is, quantity instead of modality.
- **An** versus **Op**. This is a case where both directions lead to the same conclusion. Whether our quota and remainder are **InOp** and **A**, or **ApOp** and **AIn**, the result is still **AInOp** (SF = **F6**). Here, particularization has downgraded both the quantity (**An** to **In**) and the modality (**An** to **A**).

An especially interesting case is that of **A** versus **E**. We saw that, within the closed system of actual propositions, this conflict yields conclusion **IO**, or (**I**)(**O**). However, here, in the wider context of natural modality, we see that the conclusion by any means is, more fully, **ApIEpO**, or (**IOp**)(**IpO**). Thus, we agree concerning **IO**, but we note that **ApEp** were previously hidden in the traditional system. In any case, this shows that our broadened theory of induction is consistent and continuous with the classical.

A similar approach may be used for conflict resolution among gross formulas in the open system of mixed modality. Since the results stem directly from tables of opposition and factor selection, they will not be listed in this treatise, to avoid excessive volume. The processes involved are now firmly established and clearly exemplified, and that is sufficient for our present purposes.

59. FACTORIAL FORMULA REVISION.

1. Adding Fractions to Integers.

We saw that, in natural (or temporal) modality, only 10 factors out of 15 are induced by factor selection from gross elements or compounds. Likewise, in the wider context of mixed modality, only 21 out of 63 factors emerge out of factor selection from gross formulas. Since all gross formulas were considered, the question arises, what of the remaining 5 or 42 factors, how do they ever occur in knowledge?

There is clearly a gap to fill; our theory of induction is not so far exhaustive, and is in need of further refinement. The answer has to be that we add fractions to induced integers, in analogy to gross formula amplification. Let us examine this idea.

As a state of being, an integer is final (if only for the time or circumstances concerned), it is a whole, complete as it is. This means that any fraction not included in it, is factually incompatible with it, and may not be joined to it.

But as a state of knowledge, an apparent integer may turn out to be incomplete. The conjunction of two or more particular fractions may have been thought to be complete, a correct image of reality, but then we find that the truth was more complex. In this perspective, adding fractions on to others is quite conceivable.

As we saw in the chapter on factor selection, a mere conjunction of particular fractions always has more than one factor, except for the conjunction of all the fractions in the system under consideration which has only one. If we select the strongest factor available, we obtain, as our inductive conclusion, an integer which looks identical to the original conjunction of fractions, differing only in its being defined as having only one factor.

The movement from mere-conjunction-of-fractions to integer-by-definition is an inductive one, since it involves elimination of weaker factors. When the conjunction involves all the fractions in the system concerned it exceptionally yields the corresponding integer deductively, because no other fractions are available, so there can only be one factor, anyway.

The example given in natural factor selection was (**IOp**)(**IpO**) → (**IOp**)(**IpO**). Whereas the sum of the natural fractions **f7+f8** has four factors, **F10**, **F13**-**F15**, the identical-looking integer **F10** has by definition only one factor, **F10**. Similar examples could be provided for temporal modality, or the open system.

This signifies that we may amplify presumed integers by addition of (particular) fractions, without our having to return to the gross formula level. This simply involves removing the presumption that the original fractions constituted an integer, whenever there is reason to believe that a further fraction should be added on to them. The presumption of integrity is removed by restoring the factors we previously selected out. This operation might be called *regression*.

In that case, addition of any further fraction(s) to two or more fractions is logically demonstrable, as a straightforward conjunction between the disjunctions of factors involved. The result of such an operation, as we saw in factorial analysis, is simply the common factors of the merged strings. If we thereafter select the strongest of these common factors, we obtain a new inductive integer as conclusion, which is descriptively identical to the fractions conjoined.

For example, if (**In**)(**IOp**) + (**IpO**) = (**In**)(**IOp**)(**IpO**) is to be proved, we say: (**In**)(**IOp**) has only factor **F6** as an integer, but restored to the fractional form **f5f7**, its factors are **F6**, **F11**, **F13**, **F15**. The fraction (**IOp**), or **f8**, has the factors **F4**, **F7-8**, **F10**, **F12-15**. The only factors these have in common are **F13**, **F14**. These equal the fractional formula **f5f7f8**, or inductively the integer **F13**, as was required to prove.

The following table displays the results of adding one fraction to a given fractional formula, for the whole closed system of natural (or, similarly, temporal) modality. When two fractions are to be added, proceed by adding one at a time. The two columns on the left show the given formula, in integral and fractional form. The headings of the next four columns show the particular fraction to be added. And the cells where they intersect show the concluding integers. Such conclusions are inductive, except in the cases where F15 results, since no further additions are then possible. Note that the addition of a fraction to a formula already containing it leaves the formula unchanged.

Table 59.1 Adding Fractions in Closed Systems.

Integer	Fractions	+f5=	+f6=	+f7=	+f8=
F1	f5f5	F1	F5	F6	F8
F2	f6f6	F5	F2	F9	F7
F3	f7f7	F6	F9	F3	F10
F4	f8f8	F8	F7	F10	F4
F5	f5f6	F5	F5	F11	F12
F6	f5f7	F6	F11	F6	F13
F7	f6f8	F12	F7	F14	F7
F8	f5f8	F8	F12	F13	F8
F9	f6f7	F11	F9	F9	F14
F10	f7f8	F13	F14	F10	F10
F11	f5f6f7	F11	F11	F11	F15
F12	f5f6f8	F12	F12	F15	F12
F13	f5f7f8	F13	F15	F13	F13
F14	f6f7f8	F15	F14	F14	F14
F15	f5f6f7f8	F15	F15	F15	F15

Note that the universal integers **F1**-**F4** are translated into their corresponding particular fractions **f5**-**f8**, respectively. A universal fraction may be viewed as a sum of two particular fractions, identical in all but extension, and the latter may then be fused into one particular extension, e.g. **F1 = f5+f5 = f5**.

But, let us now examine more closely the conditions of validity of this process. The fractions (**IOp**) and (**IpO**) can be directly observed, by experience of the same part of the extension under different circumstances; but not so the fractions (**In**) and (**On**): they depends on generalization. As well, all four may derive from deductive arguments. While these avenues are individually easy to take for granted, it is more difficult to rest assured that had we had all the data at our fingertips at once, we would have interpreted it as we do when we receive it piecemeal. We are very much assuming that the new additions do not affect the original positions.

This criticism only goes to show that such formula revision is an inductive process, however neatly mechanical it looks. It admittedly involves ambitious assumptions, but so do all inductive processes. We are free to return to the gross formula level, and generalize from that. All logic does here is to provide us with a cogent shortcut, which may just as well turn out to be accurate.

The underlying justification is that, since the various fractions have so far held their ground, we have no specific empirical or logical cause for complaint; we remain protected by the conviction that, if we are wrong, an inconsistency will sooner or later arise to awaken us to the fact. Returning to the gross formula level would only make us lose already confirmed information; we are free to try it and, if the results are found more reliable, to choose that course, in any case.

Furthermore, the fact that this process of adding fractions to integers is the only way we can conceivably arrive at the missing integers, namely **F11**-**F15** for the closed systems as earlier mentioned, is a supplementary justification for it. If this method of induction were not valid, the missing integers would be unknowable, since neither factorial analysis, nor generalization, nor revision of gross formulas are able to yield such conclusions, as was seen.

A table similar to the above can easily be drawn up for the open system. The various combinations of the particular fractions **f7**-**f12**, taken from 2 to 6 at a time, are amplified by one of these six fractions, to yield an integer in the range **F7**-**F63**. Since the results are implicit in our initial definitions of the integers by reference to the fractions, and in the law of generalization, we may avoid taking up more space here.

2. Reconciliation of Integers.

Now let us consider how conflicts between induced integers might be resolved, again in analogy to the doctrine of harmonization between gross formulas.

a. If two induced integers appear in the course of knowledge development, and they are judged as having unequal weights, the resolution their conflict is obviously to keep the more weighty one as it stands, and entirely reject the lighter one.

Note well that such rejection does not mean simply labeling the unfortunate integer as ‘canceled out’. The denial of an integer signifies that one or more of its implications is false. That is, some implicit element(s) and/or overlap(s) must be false, to cause the downfall of the integer as a whole. However, there is no need to seek for this precise cause of downfall: it is fully defined by the leftover heavier integer.

Effectively, such harmonization between unequals is a special case of factor selection, guided by ad hoc considerations of credibility, instead of the regular appeal to the uniformity principle.

If any two (or more) integers make their appearance, then we are faced with a deficient formula disjoining them, and whichever one is declared more likely, on whatever basis, the other(s) is/are eliminated. For example, if **F3** and **F11** both emerge, then ‘**F3** or **F11**’ is true; if now say **F11** is judged more weighty, then by apodosis **F3** is false.

b. If the conflicting integers are of equal weight, we could accordingly, simply select the stronger of the two as in any generalization. Thus, in the example just given, lacking any other reason for preference, we would choose **F3**.

However, another solution to the problem seems more satisfactory. Instead of reacting to such a situation in an extremist, either-or, manner, we could seek to fuse the fractions implicit in the presumed integers, into a new integer comprising all original data other than their implicit characterization as exclusive.

The justification of such synthesis is that we thus avoid loss of significant information, which has otherwise so far found confirmation (since it has made its appearance here). Also, the repercussions on the wider context are minimized, until and unless we are forced to be more decisive.

This topic is clearly an corollary of that of integer amplification by fractions, and all that has been said in the previous section continues to be relevant here. A presumed integer can always be made to regress into a mere conjunction of fractions, so that it is associated with more factors, and thus be made compatible with further fractions, with factors in common. Thus, though real integers, having but one factor each, are mutually exclusive, their fractional equivalents may be merged.

The following table shows how conflicts between presumed integers may be resolved by merger, in the closed systems. The integers in the column on the left are added to the integers heading the subsequent columns, and the results are given in the cells of intersection. More precisely, of course, the fractions (not shown here) corresponding to the original integers are merged, and the result is then generalized into the new integer.

Note that the previously inaccessible closed-systems fractions **F11**-**F15** are made possible by harmonization, as by amplification, of integers.

Table 59.2 Harmonization of Equal Closed-Systems Integers.

Int.	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
F1	F1														
F2	F5	F2													
F3	F6	F9	F3												
F4	F8	F7	F10	F4											
F5	F5	F5	F11	F12	F5										
F6	F6	F11	F6	F13	F11	F6									
F7	F12	F7	F14	F7	F12	F15	F7								
F8	F8	F12	F13	F8	F12	F13	F12	F8							
F9	F11	F9	F9	F14	F11	F11	F14	F15	F9						
F10	F13	F14	F10	F10	F15	F13	F14	F13	F14	F10					
F11	F11	F11	F11	F15	F11	F11	F15	F15	F11	F15	F11				
F12	F12	F12	F15	F12	F12	F15	F12	F12	F15	F15	F15	F12			
F13	F13	F15	F13	F13	F15	F13	F15	F13	F15	F13	F15	F15	F13		
F14	F15	F14	F14	F14	F15	F15	F14	F15	F14	F14	F15	F15	F15	F14	
F15	F15	F15	F15	F15	F15	F15	F15	F15	F15	F15	F15	F15	F15	F15	F15

These results may be validated by factorial analysis; or more simply by appeal to the previous table, adding on one fraction at a time. An example of validation: the presumed integers **F5** and **F9**, say, make their appearance. Regressing them to their fractional equivalents, we get **f5f6** and **f6f7**. Factorization of these yields **F5**, **F11-12**, **F15** and **F9**, **F11**, **F14-15**. The common factors in these two series are **F11**, **F15**, and these are the factorial formula of **f5f6f7**. By selection of the strongest factor, we conclude **F11**, as tabulated.

We may here raise objections similar to those raised with regard to amplification of integers, and they may be similarly countered.

A table similar to the above can easily be drawn up for the open system. Since the results are implicit in our initial definitions of the integers by reference to the fractions, and in the law of generalization, we may avoid taking up more space here.

3. Indefinite Denial of Integers.

It is conceivable that we may discover an induced integer to be wrong, without knowing precisely what is wrong with it. This situation would arise if we had drawn deductive inferences from the integer, and perhaps again from its implications in turn, and found some such consequence factually in error. We then know that some implied element(s) or overlap(s) in the integer must be false, but we may have no way to pin-point the culprit. This may be referred to as indefinite denial of an integer.

a. If we remember the factorial formula from which we selected this integer, because it was the strongest available, then a solution to the problem is forthcoming. If the initially selected factor has turned out to be incorrect, it is simply eliminated from the series of alternatives, and the next strongest factor in line is selected in its stead. Thus, eventually, by successive elimination, any of the unused factors in the series might conceivably appear as inductive conclusions.

b. If, however, we are unable to recapture the original factorial formula, then we are forced to backtrack to gross formula levels, in successively more radical retreats. Each further retreat should be tested, to find out if it is sufficient to remove the inconsistency we set out to combat. This means, generalizing from the attempted gross formula, and checking whether the new integer still implies the inconsistency.

The two tables below show the main results of this approach.

The first retreat would consist in abandoning all fractionating, and considering only the gross formula implied by the doubted integer. The results for the natural integers, listed in the column on the left, are shown under the heading 'implied gross'.

This gross formula is then again generalized. This changes nothing in the simpler integers, which are the only or strongest factor of the implied gross, anyway; but it does affect the more fragmented integers. Thus, in natural modality, **F1**-**F10** are unchanged, but **F11**, **F12**, **F15** become **F5**, and **F13** and **F14** become **F8** and **F9**, respectively, as shown in the column labeled 'first generalization'. More radical revision is thus always required for **F1**-**F10**, and sometimes for **F11**-**F15** (when the first proposal fails to solve the problem).

The next step is to consider all the possible gross denials of each implied gross formula. Then, in a preliminary wave of generalization, determine the strongest factor of each these possible gross denials, by referring to table 58.1. Then, in a final wave of generalization, in accordance with the same law, select the strongest of these strongest factors, as suggested integral denial.

This tells us both, what the generalization of each of the alternative gross denials is, and which of them to prefer as integral denial. If this first option of denials fails to solve the problem, then the next strongest is chosen; and so forth, till the inconsistency disappears.

If none of the strongest integral denials solves the problem, there is in fact still the possibility that one of the weaker factors of the various alternative gross denials, which we eliminated in the preliminary wave of generalization, manages to solve it. These hidden factors should therefore be reactivated, ordered, and tried successively.

If we grant the beginning assumption that the problem inconsistency was indeed to be attributed to our integer, then we are eventually bound to arrive at a solution by this method, since our treatment is exhaustive. If none of all the possible factors gets rid of the inconsistency, then we can be sure that it must have been due to another source. The assumption we started with, that the other sources were all more trustworthy than our integer, must be wrong, and each of the other sources now must be subjected to the same kind of scrutiny.

For example, the denial of **InOn** is '**Ep** and/or **Ap**', which translates into three possible formulas: **ApInO**, **IEpOn**, **ApIEpO**, whose factorial counterparts are, respectively, **F8**, **F13**, and **F9**, **F14**, and **F10**. The strongest factor in these three sets are **F8**, **F9**, and **F10**. The strongest of these three in turn is **F8**. The first solution to test is therefore **F8**; if the problem remains, **F9** is tried, and if this fails, **F10** should succeed; lastly, we can try **F13**, then **F14**. If none of these work, the problem must lie elsewhere.

Table 59.3 Indefinite Denial of Natural Integers.

Bad Int.	Implied Gross	1st Gen.	Denials (and/or)	Strongest Gross	SF	Out of
F1	An	F1	Op	AInOp	F6	1
F2	En	F2	Ip	IpEOn	F7	1
F3	AEp	F3	O, In	AInOp	F6	3
F4	ApE	F4	On, I	IpEOn	F7	3
F5	InOn	F5	Ep, Ap	ApInO	F8	3
F6	AInOp	F6	O, Ep, An	An	F1	4
F7	IpEOn	F7	En, I, Ap	En	F2	4
F8	ApInO	F8	On, Ep, A	AEp	F3	5
F9	IEpOn	F9	E, In, Ap	ApE	F4	5
F10	ApIEpO	F10	On, E, In, A	AEp	F3	7
F11	InOn	F5	Ep, Ap	ApInO	F8	3
F12	InOn	F5	Ep, Ap	ApInO	F8	3
F13	ApInO	F8	On, Ep, A	AEp	F3	5
F14	IEpOn	F9	E, In, Ap	ApE	F4	5
F15	InOn	F5	Ep, Ap	ApInO	F8	3

Table 59.4 Other Possible Strong Denials.

Int.	Gross	2nd	SF	3rd	SF	4th	SF	5th	SF	6th	SF	7th	SF
F1	An												
F2	En												
F3	AEp	ApInO	F8	ApIEpO	F10								
F4	ApE	IEpOn	F9	ApIEpO	F10								
F5	InOn	IEpOn	F9	ApIEpO	F10								
F6	AInOp	AEp	F3	ApInO	F8	ApIEpO	F10						
F7	IpEOn	ApE	F4	IEpOn	F9	ApIEpO	F10						
F8	ApInO	InOn	F5	AInOp	F6	IEpOn	F9	ApIEpO	F10				
F9	IEpOn	InOn	F5	IpEOn	F7	ApInO	F8	ApIEpO	F10				
F10	ApIEpO	ApE	F4	InOn	F5	AInOp	F6	IpEOn	F7	ApInO	F8	IEpOn	F9
F11	InOn	IEpOn	F9	ApIEpO	F10								
F12	InOn	IEpOn	F9	ApIEpO	F10								
F13	ApInO	InOn	F5	AInOp	F6	IEpOn	F9	ApIEpO	F10				
F14	IEpOn	InOn	F5	IpEOn	F7	ApInO	F8	ApIEpO	F10				
F15	InOn	IEpOn	F9	ApIEpO	F10								

Note well that all the information in these tables is derived from previous work. Similar tables can be prepared for temporal modality, and mixed modality.

4. Other Formula Revisions.

To achieve a comprehensive treatment of formula revision we would still need to consider the situations listed below. Though Logic must eventually deal with all such situations in detail, I will not do it all in the present work, so as not to swamp my main findings in excessive minutiae. Also, in any case the results are derivative, and already effectively contained in the situations already covered (viz. interactions between gross formulas, and interactions between integers).

The method used to solve the problem of indefinite denial of integers, can be applied to two other situations:

a. Indefinite denial of any gross compound.

b. Any interaction between integers and gross formulas.

Indefinite denial of integers is at the confluence of these two larger issues, a specific case of each, and an example for both.

a. Effectively, this example shows us how to deal with denial of certain compounds, those which integers imply. In natural modality, these are **AEp**, **ApE**, **InOn**, **AInOp**, **IpEOn**, **ApInO**, **IEpOn**, **ApIEpO**. But we know that there are quite a few more possible compounds. In natural modality, there are another 29, to be exact. Each of these is deniable by the contradictories of its two or more elements, and any consistent conjunction(s) of these. And, therefore, each is deniable by two or more contrary compounds.

The method established for indefinite denial of the compounds implied by integers may be applied to all other compounds. Namely, we select the strongest factor of each alternative compound, and then in turn opt for the strongest of these as our first solution; if this is not satisfactory, we go on, successively choosing the factors in descending order of strength. This method is nothing new: it is a special application of the law of generalization.

For example, if **AIn** is false, then **O** and/or **Ep** must be true, then **AEp** or **InO** or **IEpO** follow. The strongest factors of these are **F3**, **F5**, and **F9**, respectively, so our first solution will be (**AEp**), our second (**In**)(**On**), and our third (**On**)(**IOp**). If these fail to work, we try the hidden weaker factors of **InO** and **IEpO**, namely **F8**, **F1O-15**, successively.

b. Indefinite denial of integers is one of the concerns in the larger issue of interaction between integers and gross formulas. It deals with the specific case of integers versus the contradictories of their implied elements. This still leaves us with the cases of amplification of integers by the compatibles of their implied elements, and the cases of harmonization between integers and the contraries of their implied elements.

Only when all combinations of integers and gross elements and compounds are considered, is the topic of formula revision exhausted. However, we have already established the overall method, there is only a need to apply it further. The method here again is to simply find the gross formula implied by each integer, and amplify it with the various compatible elements or compounds, or harmonize it with the various contrary elements or compounds, then generalize the results and select the strongest of all available factors.

This should be done for the closed systems of natural and temporal modality, and the open system of mixed modality. In this manner, using the methods of factorial analysis and factor selection, we can cover the whole field of formula revision.

5. Revision of Deficient Formulas.

Lastly, we should mention, for the record, the issue of revision of deficient formulas, consisting of the various disjunctions of two or more factors, not embraced by gross formulas. We saw that these rare, though conceivable, stages of knowledge fall under the law of generalization, like any other factorial formula. With regard to their interactions, likewise, the methods are the same.

If two such deficient formulas with common factors are conjoined, the result is a formula containing only their common factor(s), which may be integral, gross, or deficient. This amplification may in turn be generalized, by selection of the strongest factor, as usual.

If two such deficient have no common factor, their conjunction represents a conflict to resolve. If they are of unequal weight, the weightier is preferred. If they are of equal weight, their harmonization is worked out as follows.

We know that the conjunction of two disjunctions, say ‘p or q’ and ‘r or s’, results in a disjunction of all the pairings-off of alternatives, as in ‘p and r’ or ‘p and s’ or ‘q and r’ or ‘q and s’. Since we are here concerned with factors, which are mutually exclusive, then in the case of factorial formulas without common factors, each pairing will be inconsistent. Each pair, therefore, should be treated as a case of harmonization between equal integers, and the result will be an integer embracing all their fractions.

When all the individual pairs have been so harmonized, we are left with disjunction of one or more factors, which is our revised formula. This, whether integral, gross, or again deficient, may now be generalized by selection of the strongest factor, as usual.

Thus, amplification or harmonization of such deficient formulas present no special problem, but can be readily reduced to previously dealt with issues. Likewise, the interactions between deficients and integers or gross formulas are easily handled.

PART VII. PERSPECTIVES.

60. PHENOMENA.

This chapter confronts certain ontological issues.

1. Empirical or Hypothetical.

A basic principle of science is that ***we may rely on empirical evidence, and indeed that all our hypotheses must ultimately be grounded in experience***. This means that we attach special credibility to the empirical, from which the credibility of the hypothetical is to be derived. The former is raw data, the latter involves processing of data.

This is all well and nice, but just what do we mean by 'the empirical', and how do we distinguish it from 'the hypothetical'? The question is exceedingly difficult to answer with precision, as we shall see. For example, I may look out of my window, and see rain, but then discover that all I saw was a shower of water from the roof. The 'rain' seemed empirical enough at first, but then had to be declassified as a failed hypothesis.

In a large sense, anything appearing before us is 'empirical' — it is in itself given, whether we interpret it correctly or not; in this sense, even the 'hypothetical' is empirical, and it is so even if misleading. In a narrower sense, not all appearances count as 'empirical'; those we label 'hypothetical' are either excluded from this heading, or only included under certain inductive conditions.

In any case, we cannot refer to the concepts of 'reality' and 'illusion' for the distinction, except after the fact. If we try to refer to divisions we commonly make, like the physical versus the mental, or the concrete versus the abstract, we still encounter difficulty. Still, these dichotomies play a role of sorts, so we should explore them in more detail.

We shall see that, ultimately, all phenomena are *in themselves* empirical; the characterization of certain phenomena as hypothetical only arises insofar as they are taken as *representing* something other than themselves.

2. Physical or Mental.

It is very difficult to define the difference between physical (or material) and mental (or imaginary) phenomena.

a. Most evident to us are what we call *'physical or material'* phenomena. This at the outset includes the experience of sights, sounds, feelings, smells, and tastes, of various kinds and intensity (for example, sights vary in shape, color and intensity of light).

However, some of the sights, sounds, feelings, smells, and tastes we commonly experience, those in thinking or dreaming, for instances, somehow do not seem physical to us; so we call these *'mental or imaginary'* phenomena, to differentiate them.

Thus, the various phenomena we primarily associate with the physical domain, are on second thought found not to be exclusive to that world, but also found in the mental domain. This means that we must refer to some other or additional factor(s), to define what we intend by the words 'physical' or 'mental'.

b. In the field of physical phenomena, each of us experiences a group of phenomena as being peculiarly close to self: we call this our personal body.

Briefly put, within this body, we distinguish various organs, to which we assign different functions. Some of these organs, which we call the sense-organs, seem especially related to the above-mentioned physical qualities: the eyes to experience of sights, the ears to experience of sounds, and so forth; further scrutiny by biologists has shown us more precisely how each of these operates.

Our position at that stage is that phenomena like sights and sounds are physical if (seemingly) experienced 'through' the sense-organs, and otherwise they are mental. The body and sense-organs are themselves physical, being visible with our own eyes, audible to our own ears, and so forth.

c. Incidentally, we may now take note of another group of phenomena: the bodily pleasures and pains of different sorts, which we commonly experience.

These phenomena are not sights, sounds, smells, tastes, nor quite like other touch-feelings. Yet they seem to take place inside our personal body, in the head, heart, digestive tract, sex organs, members, and indeed further study by biologists has uncovered relevant sense-organs within the body.

Later, we consider that some of these bodily phenomena have physical causes, some mental causes. But their common location in the body establishes them as in themselves physical (specifically, physiological) phenomena, and we are led to expand the definition accordingly.

It is not clear to me whether the bodily pleasures and pains experienced during dreams, say, are occurring within the dream itself (i.e. are themselves dreamed), or are merely triggered in the physical body in conjunction with the dream. For this reason, I am not sure whether these feelings have mental equivalents, as the other physical phenomena do.

Note that modern biology, according to Curtis and Barnes (440-466), groups the sensory receptors as follows:

> Like most animals we have mechanoreceptors (touch, hearing, position), chemoreceptors (taste and smell), photoreceptors (vision), temperature receptors, and receptors for the sensation we recognize as pain. We do not have electroreceptors or magnetoreceptors, but some animals do (458).

I do not know why pleasure is not mentioned here, incidentally[6]. In any case, the similarities of operation among some receptors does not imply that the sense-modalities, the phenomena apprehended by the perceiver, are qualitatively the same, note well.

d. We are tempted to define the physical domain with reference to space and time. The body and what lies beyond it seem extended in a continuum. However, this presents a difficulty, in that mental phenomena like thoughts and dreams are obviously also extended — certainly in time, and in a mental parallel of space, if not physical space.

With regard to time, we can say that physical phenomena are on the whole more persistent, and mental phenomena on the whole more ephemeral. However, this distinction is more statistical, than applicable to individual phenomena. Many physical phenomena are fleeting, and even those that are assumed permanent are not constantly experienced by us.

With regard to space, the issue is further complicated when we take into consideration various illusions:

Some physical illusions are explained with reference to physical causes other than the sense-organ through which they appeared. For example, optical illusions like the moon seeming in the lake due to reflection; we learn that the moon is not in the lake by diving in and trying to touch it. Other examples: an echo, or a lingering odor due to the continued presence of certain molecules in the nose.

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6 Furthermore, I wonder if we are truly unable to perceive electromagnetic waves. I have rather often had the following experience: I spontaneously 'hear' a musical piece or song 'inside my head', then turn on the radio, and perhaps tune it, and discover that precisely that music or song is being aired. Coincidence? Or did I somehow 'receive' the radio program directly?

Some physical illusions are explained with reference to the experienced or inferred malfunctions of the sense-organ through which they appeared. For example, if I cross my eyes and see double, or plug my ears and hear nothing. Or again, I see certain threads in front of my eyes, and assume they might be projections of scars in the lenses of my eyes.

Such physical illusions are judged unreal through alternative sense-organs, and relatively easy to explain. For instance, what I precisely see, when I seem to see the moon, say, is the light from the moon; there is always an extrapolation from the sensory interface. However, we thus learn that not all physical appearances are 'real' — some of the things which appear in physical space cannot be taken at face value, but must be regarded in a wider context. Such phenomena are said to be *virtual*.

More difficult to understand, are *hallucinations*:

Some mental projections seem to occur 'in the head': as when I experience my verbal thoughts, or I close my eyes and visualize certain vague shapes or clearly remember the scene in a movie. Some mental projections are much larger and more vivid, but still seem to take place in an 'inner space': thus with strong dreams (we all occasionally have them), or in certain meditative experiences or prophetic visions, or under the influence of psychotropic drugs or of psychosis.

But *some mental projections seem to go right out into physical space*: through powerful memories, like a beloved face, or a frightening or disgusting animal we came across long ago; or again, in certain meditative experiences and prophetic visions, or through psychotropic drugs or psychosis.

Such phenomena, which we call hallucinations, are judged illusory by appeal to a wider context: because they are relatively intangible and fleeting, or by the verdict of one's other sense-organs or other people's, or with reference to their having been preceded by meditation or drug-ingestion. (There are, of course, many kinds of meditation.) However, as far as I know, they are not attributable to lingering or artificial impressions at the sense-receptors.

The fact remains that hallucinations seem to inhabit physical space although not regarded as physical phenomena. You can truthfully say of such an apparition: 'it seems to be placed out there, to my left, between the table and the chair' — even if it does seem unpalpable, and more transparent and transient (because these attributes are not exclusive to hallucinations). For these reasons, we cannot make a clear, spatial distinction between the physical and mental domains.

e. Another difference we might point to is that physical phenomena are public knowledge, whereas mental phenomena, however vivid, are private.

Not all the physical events which one experiences are also experienced by other people, but that kind of event is often agreed upon by two or more people.

In contrast (to my knowledge), mental phenomena are never, in that sense, shared. We can only report to others what we intimately see, hear, feel, smell or taste, and we presume others have more or less similar experiences under the same circumstances, but there is no way we know of to intimately test and confirm each other's individual mental experiences simultaneously. Scientists can detect and measure their physiological accompaniments, but to date cannot 'photograph' figments of the imagination.

However, the reference to publicity or privacy does not provide us with a clear differentia. For a start, it involves a circularity: the anterior anti-solipsistic assumption that 'other people' are not themselves chimera, but as physical and conscious as they seem, and that our languages are coherent. Secondly, physical events are not invariably public, though we assume them to be potentially so, and perspectives differ anyway. Thirdly, even if you could see my fantasies and I yours, we would still (I daresay) agree that they somehow differ from physical events.[7]

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7 An acquaintance of mine, Robert Fox, has pointed out to me, after reading the above remarks, that people seem to be able to have collective delusions, of a perceptual as well as conceptual kind; in such case, the irony is that someone without this delusion, or with a different one, would be the one judged 'deluded', because singular.

f. In conclusion, all we can say to distinguish physical and mental phenomena is that they are, assumably, in some significant respect, *'substantially' different — distinct stuffs*.

The controversy as to which of the two domains is more 'real' than the other (some cultures favor the physical, some the mental), is not relevant to defining their difference. Whether this difference is profoundly radical; or the stuff of mental phenomena is only a peculiar kind of matter; or physical events are themselves but dreams — we do not know the answer to this question, and it is seen as not immensely important to philosophy once clearly posed. We can admit of a noticeable difference, without having to be able to explain it.

The two domains have, evidently, much in common: the experienced 'qualities' of sights, sounds, feelings, smells, and tastes. But our intuition tells us that they are somehow at odds; and a difference of some sort has to be assumed, because it helps us to resolve perceptual contradictions, such as (to take an extreme example) a hallucination being and not being in the same place as a table and chair. So we take it for granted.

Logic is quite able to deal with these issues in formal terms, through special kinds of conditionings, which may be called *'domain specification'* propositions. We can thus say: 'It physically appears that X is Y', whereas 'It mentally appears that X is not Y'. So conditioned, the statements 'X is Y' and 'X is not Y' may be both factually true and yet not in contradiction to each other.

A statement about physical appearance may be further delimited by specifying the sense organ(s) on which it is based. Likewise, a statement about mental appearance may be more precisely specified as an imagination in an awake state, or while asleep, or as a hallucination induced in such or such a way.

These forms are not new inventions, but are used in everyday practice. For examples: 'I dreamed she had left me, but when I woke up I found her still there' or 'The surface seemed smooth (visually), but when I touched it I realized it was rough'.

Clearly, there is still a need for reconciliation of sorts, but it is not as pressing as it would be without domain specification. The reconciliation may consist in granting preponderance to one of the statements (for instance, awake experience is more credible than asleep), or making a compromise statement (for instance, the surface is smooth or rough to a limited degree).

3. Concrete and Abstract.

a. We experience the world as an enormous space, in which are a multiplicity of individual entities, which have a diversity of attributes and mutual relations; in time, these come or go, move, alter or remain, and interact, in innumerable ways. (This brief description makes no pretensions to completeness.) These things, be they real or illusory, are apparent. They are experienced *in both* the physical and mental domains. But what are they?

Concretely, all we can point to in either domain are individual phenomena like: blobs of green or blue, noises, odors, bitter-sweet, penetrability, texture, temperature — in short, the perceptible qualities. Everything else we ever discuss is *'abstract'*.

The distinction of the concrete is its conspicuousness, it stands out; but the abstract is also somehow apparent, even though not manifestly so. The concrete as such seems more obvious, so we regard it as less open to discussion about how 'real' it is. The abstract as such is invisible, inaudible, you cannot touch it, smell it, taste it, it has in itself no perceptible quality — so we wonder how 'real' it is, if at all.

We all implicitly believe that the physical domain consists not only of concretes, but also and mostly of abstracts. Ironically, the concrete aspects of the physical domain are regarded by us as the least 'real' expressions of matter. Space, time, numbers, particles, waves, movements, forces, are all essentially abstract aspects.

Likewise, the mental domain is not limited to its perceptible qualities, but includes invisible, inaudible, and in no way perceptible, components. Furthermore, just as many of the physical domain's concrete aspects have equivalents in the mental domain, so they have many abstract aspects in common.

b. Now, the questions arise, what are abstracts and how do we know them? If we cannot perceive them, can they be said to have any existence or reality, and how can they be in any way described or discussed?

Most people and many philosophers, focusing primarily on the material world, try to answer this question with reference to mental images. The abstract 'squareness' of concrete physical squares, they say, is a mental image we call up and match against them. However, this proposal does not solve the problem.

In the example given, all we are doing is comparing a concrete mental square to a concrete physical one: the abstract squareness they have in common, on the basis of which a match is made, remains unexplained. Furthermore, the example given is a relatively concrete one; the suggestion becomes irrelevant in more abstract cases. For examples, in 'possession' or 'action' or 'force' or 'causality' or 'entropy' or 'relativity' — there is not only no physical concrete to point to, there is no mental concrete to point to (though concrete and abstract factors may be allied).

Relegating physical abstracts to the mental domain is a useless exercise, for the simple reason that mental abstracts are equally imperceptible there. The problem is only *once removed*, swept under the carpet, it is in no way solved. (The same argument can be made, incidentally, with regard to any other domain, like Plato's transcendental world or Kant's noumenal world.)

At this point we might be tempted to regard all abstracts as unreal, no more than meaningless words, for both the physical and mental domain. Some philosophers have attempted this.

But here again, logic intervenes. How can such a claim have any credence, if it consists of meaningless words. Either the statement is presented as meaningful and true, in which case it tacitly admits what it tries to deny, being filled with implicit references to abstracts, or the statement itself is meaningless and unrelated to reality, in which case how can we even consider it.

c. This leaves us with only one alternative. Namely, that some abstracts do really exist, even though they are imperceptible, and we are able to 'experience' them somehow, even though we do not know quite how. This position is logically tenable — it is relevant and consistent, unlike the other two.

We could still say that all abstracts are mental, but there would be no basis for such a discrimination. Once the experience of imperceptibles is accepted for the mental domain, there is no reason to exclude it in principle from the physical one. It would be an arbitrary complication, without specific justification.

I suggest, therefore, that ***there are abstracts in both the physical and mental domains***, as we presume in common-sense. This means that abstracts are immanent within things; just other components of things, besides their concrete aspects, and as real as them.

This is not a claim that whatever abstract we assign to something is indeed there, but only an admission that *some* abstracts are there and somehow known to be there. Some abstracts are *not* there, even though believed to be there. We cannot make sweeping generalizations either way.

With regard to the *physical* domain, concretes are as a rule perceived through the senses. As for abstracts, some are known directly; they are on the surface of things together with their concrete aspects (e.g. the squareness of two squares). Other abstracts are known indirectly, and more fallibly, by imagination and inference from concretes and directly known abstracts (e.g. the chemical composition of water). Likewise, with regard to the mental domain, except that perception is inner.

4. Presentative or Representative.

An appearance may be merely *presentative*, a phenomenon without pretensions (we say, 'in itself'), or it may be *representative*, a phenomenon which seems to signify something beyond itself. In the former case, it is merely 'an appearance', in the latter, it is 'an appearance *of*' something. For example, a piece of paper is just that, whereas the words or drawings on it are intended to refer us to other things as well as just being what they are.

A daydream or dream, a psychic mood, a word or sentence uttered as mere sounds or written as mere shapes, taken in themselves, are just givens; they become questionable only provided we assign some interpretation to them.

We commonly err in judging a mental experience to be physical, or to 'represent' an unexperienced physical one; taken as they appear, these mental phenomena are empirical; what is hypothetical is their characterization as specifically physical. If we imagine a 'talking horse' without making claims that it does or can exist in the physical world, all we have is an 'empirical' mental phenomenon; this phenomenon becomes 'hypothetical' only as soon as we propose, for instance, that it has an analogue in the physical domain.

Some of our ideas are formed by idle manipulations of images or words, in the way of *experiments* to be tested. We try out new reshufflings of elements which were originally given in specific combinations, to find out whether these inventions also have or can be made to have a similar existence. Thus, for instance, we may wonder whether any 'talking horses' exist or can be genetically engineered.

Many of our ideas are formed *by analogy*. There is a use of analogy whenever we classify things together, under a vague impression that they are in some way alike, and assign them a common name. Often the argument by analogy consists in extrapolating some phenomenon from one domain to the other, or from some field within a domain to another.

Thus, for instance, whereas physical pleasure and pain are concretely manifest in the body, mental pleasure and pain are more elusively abstract; our idea of the latter may be formed by saying that they are 'something like' the former, 'except in the mental instead of physical domain'. Likewise, psychological sickness may initially be merely an analogue of physiological sickness. Or again, I assume that other people's minds are very similar to my own.

In some cases, we have doubts concerning some aspect(s) of empirical phenomena. Most phenomena seem to us to be clearly physical or mental. But some are not obviously the one or the other: an optical illusion or a hallucination may require an effort to categorize. The empirical force of concretes seems greater to us than that of abstracts. More broadly, some insights seem evident from the start; others require considerable work to convince us.

Taken neutrally, any impression we have, any idea that comes to mind, whether concrete or abstract, whether concerning the physical or mental domain, has the status of a presentation; it is in itself 'empirical' (in the largest sense), the moment it but appears to our awareness. What qualifies it as 'hypothetical' (as against 'empirical' in a more stringent sense) is any suggestion it might inherently be making (implicitly or explicitly), that it relates in a certain way to something else.

Although 'hypotheticality' always entails some mental construct, it is not the mental aspect, as such, of its existence which makes it 'hypothetical', but rather that it refers the mind from one thing to another. The act of hypothesizing is indeed mental; but the contents of the hypothesis may be physical or mental, concrete or abstract, whatever.

It is not *what* kind of thing we focus on, which determines our empiricism — but rather, *how* we regard that thing. Whatever we perceive or conceive is always given and 'real', provided it is taken 'in itself'. To the extent that we draw inferences from it, it is of course fallible. An inference may be true or false; it is true, to the extent that predictions fit the evidence, and false, to the extent that they fail to.

61. CONSCIOUSNESS AND THE MIND.

My purpose here is to propose a consistent framework and terminology for epistemology.

1. A Relation.

Consciousness is a specific, peculiar kind of *relation* between an entity like ourselves (called the Subject); and any ‘appearance’, ‘phenomenon’, ‘thing’ which presents itself to us (called the Object). One can figuratively view consciousness as a line stretching between subject and object. (Capital letters are sometimes used for these terms, to avoid confusion with the use of the same words in other contexts, note.)

Consciousness is itself, of course, a phenomenon — one very difficult to grasp and define, because it is such a fundamentally unique and distinctive part of the world. We are here merely indicating it, without presuming to know what it is much more precisely, or just how it works.

The point made here is just that it is primarily a relational phenomenon, a placid ‘seeing’; it is not itself an activity, though many activities surround it. The ‘effort’ of attention or the ‘state’ of being aware or the ‘activity’ of thought, are secondary aspects of this phenomenon, which depend on the relational definition for their understanding.

The reason why consciousness is best described as ‘a relation’, is that we cannot consistently claim that consciousness is ‘subjective’, because that claim is itself an event of consciousness which has pretensions of being ‘objective’. This means that *the subject and object must be related by consciousness in such a way that neither affects the other when they are so related*.

Consciousness, then, is a relation which is neither passive nor active. Consciousness cannot be said to consist of changes of or within the subject caused by the object, because such changes would not guarantee the existence of an object, let alone that the same object would always cause the same change or that different objects would never cause the same change. And consciousness cannot be said to consist in a creation by the subject of an object, because we would still have to explain how the object is apprehended once produced.

The ***Subject*** is itself also a phenomenon — again, one very difficult to grasp and define, because it is such a fundamentally unique and distinctive part of the world. We can say that it remains unaffected by consciousness or its Object. If consciousness was passive or active (as above defined), the Subject would be unable to be conscious of itself, not even hypothetically.

The ***Object*** is, note well, whatever presents itself to us, as it stands — without initial concern as to whether it is to be regarded as ‘real’ or ‘illusory’: these are later judgments about the object. The Object, likewise, remains unmoved by consciousness or by the Subject as such.

What matters here is that ultimately all consciousness is essentially observation, by someone, of something. The nature or type or source or status, of observer, consciousness, and observed, are other issues, which philosophy indeed has to discuss at length and try to resolve, but which need not concern us at this stage.

Whether the object is faced by the subject with detachment, dispassionately, objectively — or the subject is unwilling or unable to ‘distance’ himself from the object — these are attitudinal aspects, which pertain to reaction and do not affect the essentially ‘observatory’ nature of consciousness.

The existence of the object is immediately given in its appearance as a phenomenon. However we interpret what has appeared, we can be sure that *something* has appeared. If nothing had appeared, there would be nothing to discuss. The existences of subject and consciousness are not so obvious, a reflection of sorts is required to notice them.

Objects seem to be of various substance: some seem 'materially concrete' (e.g. a stone), some 'mentally concrete' (e.g. a dream); some seem 'abstract' (e.g. entropy or humaneness). Subjects are believed to be of a substance other than such material or mental entities: we view them as 'spiritual entities' or 'souls'. Consciousness also seems a very special component of the world.

We sometimes label our awareness of subject and consciousness jointly as '*self-consciousness*'. For us humans at least, that awareness seems to peripherally accompany our every cognition of other objects, if only we make a minimal effort to activate it. This direct impression is further confirmed indirectly, by observation of other apparent people and higher animals. The extrapolation from object to consciousness and subject seems obvious to us.

We know very little about what constitutes a Subject, what gives some existents the power of cognition. Judging by their behavior, humans and higher animals have it (animists believe that all things have consciousness to some degree).

One cannot postulate that consciousness is bound to be distortive, without thereby putting one's own skeptical principle in doubt. It would not, however, be inconsistent to claim that *consciousness is occasionally distortive*. The power of our consciousness is evidently more or less limited; only G-d is viewed as omniscient.

2. Kinds of Consciousness.

The term consciousness is to be understood generically. In common to all *kinds* of consciousness, is the central fact of consciousness, seemingly always one and the same Subject-Object relation.

a. Consciousness is called by different names, with reference to **the kind of phenomenon which is its object**. But this does not imply that the consciousness as such as structurally different in each of its subdivisions.

Thus, we call ***perception***, consciousness with a concrete phenomenon as its object; and ***conception*** (or ***conceptual insight***), that with an abstract phenomenon or a phenomenon mixing concrete and abstract components.

Identification is consciousness of the identities between parts of a phenomenon or between two or more phenomena. ***Distinction*** is consciousness of the differences between parts of a phenomenon or between two or more phenomena. Since similarity and dissimilarity are in themselves abstract aspects of phenomena, such *comparisons and contrasts* are conceptual. These insights allow us to *discern* the various constituents or aspects of individual phenomena, and to *classify* several phenomena together or separately.

Understanding refers to consciousness of the causality (in the largest sense) of phenomena — the natural causes of material or mental phenomena as such, or the meanings or explanations of ideas. Understanding is primarily a consciousness of the order of things; it is conceptual, since causality is an abstract phenomenon. The reaction of fulfillment or satisfaction which follows such insight is secondary.

b. Consciousness is classified variously, with reference to **the location in space or time of object**.

Thus, we label consciousness as ***introspective*** (or inner) or ***extrospective*** (or outer), according to whether its object is placed inside or outside of us (the terms are ambiguous, depending on how much we consider as being 'us' — our minds, our bodies, or even our segment of society).

The objects of perception are ordinarily temporally located in the present. Direct perception of long past or future events seems impossible to us — though prophets are said to have this power. ***Remembering*** concrete events seems to be perception of present mental images of past events, rather than of the events themselves.

Conception, however, does not seem equally bound by time, in the sense that we can more or less ***predict*** past events from their present effects, or future events from their present causes, or either of them from general laws. Such predictions are conceptual insights, even when they concern concrete events, in that the premises of the conclusion are abstract relations. Still, the result is a consciousness of past or future, so we are justified in saying that (predictive) conception as such transcends time: the subject and object are related by it across time.

c. Many subdivisions of 'consciousness' refer to **the attendant processes**, as well as to the location and kind of phenomenon. But that different processes lead up to an event of consciousness, does not in itself mean that their result is essentially different; once consciousness is aroused it may be one and the same.

Thus, perception mediated by activity of the sense-organs is called ***sensory*** perception (or sensation). It is called seeing, hearing, tasting, smelling, touch-feeling, according to whether the eye, ears, mouth, nose, or touch-organs, were involved. The perceptions of various pleasures and pains in one's own body, and of movements or stillness in or of one's own body, are also sensory, and called feelings (sentiments, if to be distinguished from touch-feelings).

Perception of mental images could be called ***'intimate'*** perception. (I adopt this label for lack of a better one; the colloquial expression 'mind's eye' might be more fitting were it not for its limiting suggestion of visual images.) It is hard to classify this as sensory perception, in that the usual sense-organs do not seem to be involved (though the brain supposedly plays an analogous role of some sort). But it is still a form of perception, insofar as its objects are as 'concrete' as material ones, though mental.

Some people claim, correctly or not, powers of ***extra-sensory*** perception (ESP). That is, the ability to perceive events which are outside one's own mind and body, and beyond the normal range of the sense-organs. We might distinguish ESP of purely material phenomena, *clairvoyance* (say), from ESP of mental phenomena or material phenomena linked to mental ones, *telepathy* (say).

I cannot personally claim to have ever experienced clairvoyance, but I have had the impression of telepathy (for example, thinking of someone and almost immediately getting a phone call from that person) often enough to discount coincidence. I remain open to the idea, without insisting on it, on the grounds that thought-transmission (awake or even in dreams) could be too fragile to withstand the stress of scientific probing. In any case, I mention ESP here, only for purposes of taxonomy.

Conceptual insight may be ***intuitive,*** immediate and direct, as when we 'see' as obvious that two entities are in some way alike or that two statements are contradictory. Or it may be ***reflective***, final and indirect, occurring at the end of a long and tangled effort of thought, comprising sensory and imaginary experiences, and inductive and deductive reasonings — a complex of perceptions and conceptual insights.

The immediate and final insight are essentially the same in character; the process leading up to the latter may be regarded as only ***a preparatory positioning of self, faculties of cognition, and objects***. The process merely 'shows' us the object, presents it to us, but we still need to 'see' it.

Conception is considered less immediate, direct and spontaneous, than perception, but there is no reason to think so. Both usually involve a process, an alignment of self, faculties, and objects, plus an effort of attention. We may or not be conscious of the preliminaries. What counts is the terminal event of perception or conception as such. That singular event has a certain, specific character, whatever its own causes or the nature of its objects.

Imagination is not in itself a kind of consciousness. It is a complex of three factors: the ('voluntary' or 'spontaneous') *act* of projecting a concrete mental image or abstract mental construct, the image or construct projected as an *entity* in itself, and the eventual consciousness of that finished product. The precedent projection is merely a creative activity of the will or nervous system; only the subsequent observation of its result properly qualifies as consciousness. The source of the object is irrelevant here, just as we would not regard the making of a table as part of seeing the table.

The images formed by imagination exist without doubt; we experience them daily. Some obvious instances: our thoughts are expressed as imaginary sounds; our dreams may clearly depict people we know. Such images are, however, considered as made of *a substance distinct from common matter, which we label 'mental'*. This mental substance, like common matter, has both concrete and abstract components.

Concrete imagination, or '**perceptualization**' is projection of concrete mental images of any kind. This includes not only visualization (visual imagination), but also its equivalents in the other sense phenomena (auditory, olfactory, gustatory, tactual, emotional). Abstract imagination, or '**conceptualization**' is projection of abstract mental constructs of any kind.

The expression 'projection of images' suggests the existence of a mental *'matrix'* (let us call it) in which the images are formed or imbedded. This might be viewed as a multidimensional screen, capable of displaying visible, audible, and other phenomena. I find this idea occasionally convenient (to replace the broader word 'mind'), but it need not be taken literally, because the images might be 'holographs', of a common substance but without a substratum.

The words ***percept*** and ***concept*** may here be explicated. We often intend them in the sense of 'thought-units', but I prefer to stress their alternative sense of *objects* of perception or conception. A concrete object of perception should be called a percept, like the green we perceive; an abstract object of conception, should be called a concept, like the greenness we conceive.

A percept is always concrete (meaning, it has perceptible qualities); it may be physical (ordinarily implying sensory perception) or mental (the object of intimate perception). In the latter case, it may have been actively fashioned by us or have arisen involuntarily: perceptualization is implied. Exactly likewise, a concept is always abstract; may be physical or mental; and in the latter case, may have been willed (reflective conception) or passively experienced (intuited): conceptualization is implied.

In practice, because concrete and abstract factors are intertwined in the objects we commonly face, we sometimes broaden the word 'concept' to include percepts as well as concepts. Alternatively, we apply the word 'percept' to all physical phenomena, whether concrete or abstract, and 'concept' to all mental phenomena, whether concrete or abstract: this reflects an understanding that there is no essential difference between perception and conception.

All these, however called, are in themselves objects. But besides this characterization, mental objects may additionally have a representative intent, as we saw in the previous chapter: they may make claim to some analogy to physical objects, or other mental objects. In themselves, all objects are empirical facts; the characterization as fiction only concerns claims of representation, whether the imagined object was perceptualized or conceptualized.

Lastly, note, consciousness may be ***verbal*** or ***wordless***. The role of words has been discussed in an earlier chapter. They help us to think and communicate, and play a role in remembering. Wordless consciousness is sometimes called 'subconscious' — we learn or imagine, decide or intend, but without comment.

But all use of words implies an underlying consciousness of the meaning intended (meaningless sounds or written symbols do not strictly qualify as 'words'). Words in themselves are just objects; they play no role if we are not conscious of them, and if we are only conscious of them they have no meaning. They should not be confused with the underlying consciousness of what they are intended to refer to.

Words may refer to percepts as well as concepts, or to complexes of both. Words facilitate imagination, especially conceptualization. In the latter case, words are very valuable, because they are concrete, and concrete objects are easier to manipulate and hold on to than the abstract objects they are standing in for. However, even then, for the verbal construct to have meaning, there has to been an underlying reshuffling of abstract elements. Needless to say, the resulting fiction may or may not have a factual equivalent. Either way, it is not strictly the word combination itself which is fact or fiction, but the construction that they propose.

3. The Mind.

What we call ‘the mind’ is a grab-bag of many things. It collects together: the self or **soul**; our faculties of **cognition** and **volition**, and **imagination** and **affection**; and the various states and motions of those faculties, and entities produced by or through them.

The *soul*, the spiritual entity which is our self in the deepest sense, is the unaffected Subject of consciousness and Agent of will.

The soul occupies a central position, surrounded by certain faculties. By a *faculty* we mean, the structures underlying an ability to perform a certain function. These infrastructures are specific arrangements of physical entities, which make possible the sort of event referred to. They are known to biology as the nervous system, and include our brains and sense and motor organs.

These biological faculties, then, constitute the physical conditions under which cognition and volition can operate. As earlier posited, cognition is essentially a relational phenomenon; likewise, volition. The states and motions which surround cognition and volition, and the entities these may result in, concern the underlying structures, and are not to be confused with cognition and volition as such. Their role is merely to provide supporting services to these functions.

Different animal species and individuals have differently structured faculties, and therefore varying powers of cognition and volition. Machines and computers are assumed to lack souls, and therefore can never be Subjects or Agents which engage in cognition or volition; they are at best as passive and mechanistic as nervous systems.

The soul is viewed as substantially different from the nervous system; they are not a part of each other, though contiguous or inhabiting the same place. The soul is *in no way internally altered* by cognitive or volitional or surrounding physiological and physical events; only the nervous system undergoes alterations, whether by the soul’s apprehensions and actions or by events in the rest of the body or beyond it.

However, the *sphere of influence* of the soul may be maximized or minimized, according to the structural condition, and present states and motions, of its allied nervous system. This means that the soul’s previous cognitions and volitions, or even external events, may — through their alterations of the nervous system — make *more easy* (facilitate) or render *more difficult*, or even permanently arrest (in the case of irreparable damage to the nervous system), the soul’s later powers of cognition and volition. It may have to go through A, B, C to get to D; or it may have D immediately available.

Thus, the soul can be said to be an ‘unmoved mover’, without thereby implying that its powers of cognition and volition are unlimited by physical conditions. The ethical doctrine of freedom of the human soul is simply that certain powers of cognition and volition remain inalienable, even when much complicated, so long as life goes on and the relevant organs are undamaged.

The faculties of imagination and affection are merely tributary aspects of cognition and volition. *Affections* (ranging from love to hate), for instance, are inferred from the attitudes (positionings) and expressions (actual directions) of the will, and from the content and intensity of correlative passions — bodily pleasures and pains (sentiments), and mental ones (emotions), before or after action.

Thus, to summarize, what we call ‘the mind’ is a grouping of disparate things: a central soul (with Subject and Agent capabilities); surrounding faculties (biological infrastructures, organs) which enable, delimit, and assist its cognitive and volitional relations to other things; and a power of the soul and nervous system to produce the special entities we call mental images.

The mental entities we imagine are evidently such that they can be formed either ‘spontaneously’ by the nervous system or ‘voluntarily’ by the soul. These are intimate experiences we all have. I suspect that in the latter case, the soul produces mental phenomena by acting on the nervous system, rather than directly (this would be the simplest hypothesis, since it adds no extra assumptions).

The *interactive properties* of soul, matter (the nervous system and the physical world around), and mental images might, in conclusion, be described as follows (I go into such detail to show the theory's precision):

a. the soul itself cannot be altered by matter or mental phenomena, though (i) it can seemingly be pushed around space by matter, (ii) the sphere of influence of its will can be increased or diminished by the states of matter, and (iii) it is sometimes 'incited' to acts of will by mental images;

b. the soul can, through its will, alter matter (only through the nervous system — unless we grant telekinesis), though this power of volition has precise bounds;

c. the soul can, through its will, produce mental images (the latter probably only via the nervous system), though this power of imagination has precise bounds; if we grant telepathy, a soul can transmit mental images to other souls, or be presented with mental images transmitted to it by others (it is doubtful that this would occur via matter);

d. the nervous system can directly produce mental phenomena — but other matter (and probably the soul) cannot do so, except through the nervous system;

e. as for whether mental phenomena as such can directly affect matter — I see no reason to suppose so, since indirect explanations seem sufficient: (i) in the case of imagination by the soul, the soul acts on the nervous system with that intention, but the nervous system may yield unintended side-effects in the rest of the body (and thence beyond); (ii) in the case of involuntary imagination, the nervous-system events which produced the image may simultaneously have other effects in the rest of the body (and thence beyond); (iii) alternatively, the soul's perception of (voluntary or involuntary) images may incite it to act (or act again) on the nervous system (and thereby beyond);

f. I doubt that mental phenomena can affect each other directly, in the way that physical ones do; this may be the most telling distinction between the two domains.

Note lastly that I do not intend the statements made here concerning the soul as dogmatically perfect and final. My concern has been to specify the logical requirements of a coherent theory of the consciousness and volition relations: what is sure is that the subject or agent must be unaffected, *within that relation*. But I do not exclude offhand the possibility that souls may undergo change as a result of other relations, or spiritual events.

It is noteworthy that religion suggests, and many believe, that souls (as well as having been created and being perhaps in some cases permanently destroyed) may be 'purified' or 'sullied' by their thoughts or actions. However, such improvement or deterioration of a soul is explained as a subtraction or addition of coatings of 'impurity' *around* the in itself clean soul, rather than as an intrinsic qualitative change. The 'impurity' interferes with clarity of insight and freedom of action; it 'weighs down' the soul, causing it to descend on the spiritual scale, and thus distancing it from G-d.

4. Popular Psychology.

Some philosophers exclude the soul from the description of mind, arguing that the self is merely the sum total of the other elements. But that view is logically untenable, because it raises the specter of 'subjectivity'. As earlier pointed out, the Subject of consciousness must be such that it is unaltered by events of consciousness; if we equate self to the altered elements of mind, we transgress this logical requirement. The reason why the soul-less hypothesis seems at first sight to have some credibility, is as follows.

Many people have a vague notion of the mind, regarding it as a sort of psychical organ over and above the brain, with parallel functions and mutual influence. Here, the mind is regarded as a sort of cupboard, made of some nonphysical substance, in which we store entities like 'ideas' and 'emotions'. When these are placed in the lower shelves, they are held 'unconsciously', in the middle shelves, 'subconsciously', and in the upper shelves, 'consciously'. Thought is accordingly viewed as the production, alteration and movement of such entities.

Some versions of this hypothesis explicitly or tacitly admit of a soul above, next-to, or within the 'mental cupboard', which to varying degrees experiences and to some extent manipulates ideas and emotions. Other versions effectively identify the soul with the 'mental cupboard', to admit that someone is doing the seeing, feeling, and manipulating. Still others, effectively deny the existence of a Subject and Agent, and view these events as physically-caused or relatively causeless.

However, this 'mental cupboard' postulate of popular psychology is simplistic. There is no basis for considering ideas and emotions as persisting, continuing to exist as mental entities somewhere, beyond the time when they are actually experienced. It is much simpler to regard them as merely occasional 'peri-phenomena' of the physical organs.

It is sufficient to say that to each idea or emotion there corresponds a specific chemistry in the brain cells. When the appropriate molecules are constructed and properly positioned, the mental entity is created; when thereafter the circuit is cut off, the mental entity ceases to exist. What is stored are the molecules, not the idea or emotion; the latter is recreated every time the former is re-activated.

In that case, the 'mental storage cupboard' is an extraneous construct. If we postulate it, the role of the brain becomes incomprehensible. There is no point in our assuming duplicate functions; it is a needless complication. Thus, actual ideas and emotions are mental phenomena, but their potentiality is a physical phenomenon.

In conclusion, then, *there is no such thing as a mind, in the sense of a mental structure or 'psyche'*. There is only a uniform, unchanging soul, which experiences and wills as its way of relating to other things, a nervous system serving as physical infrastructure, and *from time to time* the production by these of transient mental apparitions. This scenario is by far simpler, more logical, and more empirical.

62. PERCEPTION AND RECOGNITION.

In this chapter, I want to specify some of the logical preconditions for any theory of knowledge. Some such criteria have of course been developed throughout the present treatise, here my concern is with issues relating to the role of the nervous system.

The intent is not to present a complete and definitive model of knowledge, but merely to demonstrate how a theory of cognition and memory must be tailored around certain fundamental insights of logic. Proposals falling short of these specifications may be rejected at the outset as without credibility.

1. The Immediacy of Sense-Perception.

There is a very important first principle for all philosophy, all ontology, all epistemology, all science, supplied to us by logic. It is that ***we cannot consistently deny the ultimate objectivity of (some) knowledge***. We cannot logically accept a theory of knowledge which in effect invalidates knowledge. (I personally learned this insight from Ayn Rand, though I seem to recall that she attributed it to Aristotle, in spirit at least.)

This means that ***the currently popular view that sense-perception is no more than a production of mental images — is logically untenable***. Such a statement might at first sight seem ridiculous, since it denies something universally accepted as common sense, not only by most lay people, but even by some major philosophers and many scientists; however, bear with me, and we shall see its logic.

Ask anyone to express the work performed by our senses, and they are likely to reply: 'light or sound or whatever impinges upon the corresponding sense-organ, and produces an electrochemical message, which is transmitted to the brain, where it is somehow translated into a mental image — which is what we in fact perceive in sense-perception (rather than the external physical phenomenon itself)'. To evaluate that position, we must make a distinction between its descriptive and interpretative aspects.

The description is given us by common experience and research by biologists. Sense-organs (from sense receptors to brain centers) play some crucial role in perception, since if they are blocked or damaged it is affected, and they have such and such a physiological configuration and manner of functioning. That is the empirically evident data underlying the above statement, and I am not contesting it.

On the other hand, the interpretative element is the belief that what we perceive, at the tail end of the described processes, are 'mental images', psychological phenomena which hopefully 'resemble' the original physical phenomena, produced in the brain somehow. This is a theory, which is open to question on purely logical grounds: if all what we perceive are 'mental images', then how can we know that these are images *of* anything, and even if they are, how can we know they in any way *resemble* their physical causes?

More specifically, our descriptive knowledge of the sense-organs and their processes, becomes no longer empirical but a mere postulate, which therefore cannot be used to confirm the theory. If our apparent perception of our body and the physical surrounds may itself only be a day-dream, as the theory suggests, it cannot be used as empirical evidence that there is a body surrounded by a physical world which together produce mental images.

Thus, though the theory in question begins with a presumption that there is a material world (including the sense-organs and external stimuli), it ends with a possibly contradictory logical conclusion that there may well not be such a world, precisely *because* our knowledge of it is mediated by the senses. On the one hand, it views its data on the pathways of sensory messages as *physical* evidence for itself; on the other hand, it goes on to possibly deny the reality of such physical evidence.

Had we not begun with a presumption that there is a material world (radically distinct somehow from the immediately knowable mental world), there would have been no need to construct a theory relating certain perceptions to the sense-organs. All objects, whether mental or physical, including the sense organs, would be of the same, essentially fantasmic, stuff — and thus all equally directly knowable.

An issue only arises when we take for granted the common sense view that there is a physical (as against mental) world, from which the perceiver is separated by a body with sense-organs. This view is credible, since mental and physical phenomena do experientially seem to us somehow substantially different. It follows that the theory in question intrinsically presupposes (logically implies) that the descriptive data is specifically physical.

We thus have two modal hypothetical propositions in contradiction: 'if the theory, then the data may not-be physical' and 'if the theory, then the data had-to be physical'. The antinomy involved is not of the form 'if P, then nonP', but of the form 'if P, then both 'possibly not Q and necessarily Q', which implies 'possibly {nonQ and Q}'. A theory which denies its own starting point has no logical standing.

The objects of sense-perceptions cannot be claimed to be mental images of otherwise inaccessible physical phenomena: because, if they are *inaccessible*, how can the proponents of that theory claim to have *access* to them and know anything about them?

They cannot logically lay claim to any underlying physical events; and even if they do, what guarantee have they that the mental images perceived have any *resemblance* whatsoever to any presumed physical causes? An effect need not resemble its cause. Thus, it may well be that, say, the mental image of 'green' is invariably caused in everyone's mind by a physical event of 'square': no one could tell. No claim of 'truth' could be made by anyone — not even by the proponents of that theory (it is intrinsically unconfirmable).

But in any case, there would be no justification in regarding perception of 'externally generated' mental images as in any wise more mysterious than perception of 'inwardly produced' mental images: all phenomena would have the same status. The subjectivity theory constructs redundant 'duplications', with the group of mental phenomena labeled 'physical' needing repetition as mental phenomena labeled 'nonphysical'.

The problem has baffled philosophers, but I do not see why. If a position leads to paradox, logic demands that we simply reject it and find another. Here, although it is obvious that the apparent sense organs *indeed must play a significant role of some sort* in physical perceptions (since without them, it is lacking), the initial assumption that this role is production of mental images turns out to be inconsistent. Ergo, that assumption is nonsense, and some other explanation of the function of these organs must be sought.

To resolve the paradox, while maintaining that the data is indeed physical, we are logically forced to conclude that sense-perception (no matter what many people believe), is a *direct, unmediated* relation of consciousness, between the physical objects and the perceiver. We must accept that when we perceive an external object, *it is the object itself and not some 'representation' of it that we in fact perceive*. We must from the outset admit the *objectivity* of sense-perception.

We must accept this primary logical requirement, and build our theory of knowledge around it. There is no escape from the logic. Thus, the light from a material object, its activity in the retina of the eye, the messages sent on to the brain — all these physically evident intermediaries of sight must be regarded as *mere causal preliminaries*, preparing us somehow for the actual act of seeing, which however is an unhindered Subject-Object relation. Likewise for the other senses.

(The computer provides us with an interesting analogy, though a partial one. The Subject keys in a statement he is reading on his table. The keyboard and CPU/Disk of the machine represent the sense-receptors and brain; the changes produced in the machine correspond to the nervous impulses and imprints; the partial display on-screen are analogous to mental images. But note well that the Subject sees both the external object and the on-screen copy if any, and is not himself to be confused with the machine.)

Philosophy is still left with the task of proposing alternative explanations for the role of the sense organs and their processes in physical perception. Starting with the admission of the common-sense view that there are physical, as distinct from mental, phenomena (including our bodies), manifold functions may be suggested offhand:

a. Some have recently suggested that the senses may serve to *filter out* impressions *other than* the ones focused upon. It may well be that the senses produce a preliminary, relatively rough, mental image which allows us to decide whether we are sufficiently interested in the underlying object to awaken and invest a further effort of (more direct) consciousness towards it. Indeed, analysis of sensory messages seems to indicate that their content is relatively skeletal.

b. Perhaps the sense organs serve to somehow pin-point a consciousness which would otherwise be too general. It may be that the awareness of a disembodied soul would be too dilute to be effective in this world, like the state of mind called 'enlightenment' pursued by mystics. The senses may provide a material framework, a set of physiological conditions, in which an adequate 'line of relation' between perceiver and physical percept can be established, a 'pipe' though which a ray of consciousness can be sufficiently intensified.

c. It also seems likely that mental images are indeed simultaneously produced by sensory messages (always or often, automatically or by choice), *as an incidental side-benefit, for purposes of future recall*. Whether the sensory-messages produce both the nervous-imprint and the mental image, or (less likely) it is the actual perception which produces the mental image, independently of the nervous-imprint, I do not know.

These are just suggestions which come to mind; there may be better explanations. But in any case, the suggestion that the function of the senses is exclusively the production of mental images, which are all that we perceive, is logically unacceptable, and therefore wrong without a doubt.

Whatever the biological processes involved, then, at the moment of sensory perception (and, it seems to me, lingering on for a brief time thereafter, at least in some cases), the perception is direct. That the physical perception is thus direct, does not guarantee that it is complete, nor that it is pure of additional projections of an interpretative nature, note well. But if we are properly attentive, we can focus on the given exclusively.

An image may indeed incidentally be formed in the brain, for purposes of preliminary filtering and/or future recall. Such an image may be a clear and faithful mental 'photograph' (or 'audiograph' or whatever, as appropriate), or a vague and distortive one, but the initial perception relates to the object itself, not this image. This, to repeat, is a logical necessity.

It may be that we have some difficulty in accepting sensory perception as direct, because we tend nowadays to regard the soul as localized 'in the head', contiguous with the brain. This creates a physical *distance* between the perceiver and the things perceived, which are located at the other extremities of the sense organs. But it may well be that the soul is more extended than we assume, permeating the whole body; in that case, the issue of distance would be resolved.

With regard to introspective perceptions, they are generally of course accepted without question as irreducible primaries. This refers to concrete mental phenomena which are not, at the time they arise, stimulated by sensory stimuli, though they may well in the past have been, wholly or partly, given initial existence and form by sensory stimuli.

2. Logical Conditions of Recognition.

But the main function of the nervous impulses generated by sensation (and similarly for nervous impulses underlying intimate perception), is production of biological imprints which are necessary to *recognition*. Not mental images, note well, but codes of a physical (meaning *nonmental*) nature in the cells of the brain.

When we perceive two objects *at the same time*, we can immediately 'see' (in the largest sense of intuitive insight) that they are 'similar' or 'different' in various respects. These direct comparisons may not at once reveal all the similarities and differences, and some of them may later be disagreed with and judged illusory — but in any case, these acts of consciousness are the primary building blocks of what we call 'conception'.

Comparing simultaneous percepts seems simple enough, but what of comparison of percepts which are *separated by time*? It is hard to say that in such case we 'evoke' a mental image of the past percept and match it with the present percept, because introspection shows that in most cases we are able to construct only a very imperfect analogue of the initial impression, if any.

Indeed, even as we call the image up, we know the image *itself* to be (usually) only a rough copy of the original direct percept, which implies that we are able to compare the present mental image to the past object by some means *other than* with reference to a mental image. In other words, the image itself is liable to some judgment regarding its correctness.

It follows that our decision as to whether the perceptual object now facing us is or is not the same as some past manifestation, is not (or not exclusively) made through the intermediary of a stored mental image. How, then, are we able to *'recognize'* anything, how can we claim that we have seen anything before?

This is a logical problem, as well as a more broadly epistemological one, in that logical science is based on the assumption that similarities and differences are recognizable *across time*.

Our goal here is not to debunk human knowledge, for as we have seen such a reaction is logically untenable. Our goal is more humbly to determine the logical conditions for objectivity of knowledge. That knowledge is objective is indubitable, since the premise of subjectivity is self-contradictory. Two solutions to the problem may be proposed.

One, is to suppose that direct perception of past concrete objects is feasible; that long after an event is over, we may transcend time and space somehow, and sometimes 'see' it, the past event itself (not its present repetition or continuation), extra-sensorily. But this solution seems very far-fetched, even though not impossible to conceive. I have made attempts in that direction, but they are too speculative to include here.

Two, is to suppose that the distinguishable components of each concrete object (whether physical or mental) we perceive produce a certain *'nervous imprint'* (let us call it) in the brain, which is substantially a physical (rather than mental) phenomenon of some sort. Such an 'imprint' may be a certain electrochemistry of the nervous cells — a molecular arrangement, a location or orientation of certain molecules, a specific combination of electrical charges — perhaps including a distinct synaptic network; whatever it is (it is for biologists to determine just what), we here predict it on logical grounds.

Thus, what happens in recognition is not comparison of the new percept, to the mental image of the old percept, but comparison of the nervous imprint of the new percept, to the nervous imprint of the old percept. If they match perfectly, the objects 'seem' identical; to the extent that the nervous imprints do not entirely fit each other, the objects are 'experienced as' dissimilar. This idea admits that not only sense-percepts produce imprints, but even mental images we construct voluntarily or otherwise may do so, note well.

In this way, even the mental image of an old percept can be judged as rough or accurate, according to whether the nervous imprint of the image is in all respects the same or only partly so, to the nervous imprint of the object it claims to reproduce. We may well suppose that the mental image is often a projection caused by the nervous imprint; this would explain why images which we normally find difficulty evoking clearly at will, may suddenly appear with force in dreams or under the influence of drugs, say.

The 'matching' of nervous imprints should not be viewed as a conscious comparison, but rather as a *subliminal* process whose end-product is a signal, directly perceived or intuited by the conscious Subject, that the objects in question, be they physical or mental phenomena, match to a greater or lesser degree. Note well, it is not the imprints themselves that we 'see', but some signal from them. Uncertainties may be explained by supposing that nervous imprints sometimes decay, or are lost; likewise, distorted memories may be due to deterioration of imprints.

In this way, past and present physical and/or mental objects are comparable. This theory frees us of the problems associated with the idea that mental images are the intermediaries of recognition across time. However, it contains logical difficulties of its own! What guarantee is there that an old nervous imprint has not been distorted, so that we 'recognize' a new object which is in fact unlike the old, or fail to 'recognize' a new object which is in fact like the old?

The only solution I can think of is 'holistic'. To claim that such confusions invariably occur would be logically inconsistent, since such a statement (again) would be invalidating itself. Therefore, logic demands that at least some such comparisons have to be admitted to be correct. The question of 'which?' can only be answered by a broad consideration of all experience and logical insight.

That is, over time, if such errors have crept in, inconsistencies will eventually arise, which will signal to us that something, somewhere, went wrong, and we will accordingly modify our outlook in an attempt to resolve the contradictions. In other words, the experiences of similarity or difference are phenomenal, and are taken at face value until and unless otherwise proven, like all other experiences.

3. Other Applications.

a. Once we come to the realization that perception of physical objects (sensory perception) logically has to be as direct as perception of mental objects (intimate perception), it is much easier to accept the statement made in the previous chapter that conceptual insight also may be a direct Subject-Object relation, when its object is external as well as when its object is internal. Such immediate conception has been called intuition, in contrast to reflective conception.

The only difference between perception and conception is that the former is directed at concrete objects, and the latter at abstract objects. The only difference between sensory and intimate perception is that the former is directed at physical objects, and the latter at mental objects. A similar division can be made with regard to conceptual insight, whether intuitive or reflective, by reference to the physicality or mentality of the objects concerned. But in all these cases, the consciousness relation is one and the same phenomenon.

In conclusion, physical as well as mental concretes, as well as certain intuited abstracts, may be known directly and immediately. These, whether rooted in externally or internally directed acts of consciousness, serve as the raw givens of knowledge, and are *in themselves* indubitable. However, beyond these 'received' primaries, most knowledge is constructive, and open to doubt and review.

By 'constructive' is meant, that concrete or abstract mental images may be reshuffled in any number of ways, forming innovative, hypothetical entities. The 'building-blocks' of such imagination are given from previous, 'received' concrete or abstract experiences; but these may be separated from each other and combined again together in new ways. Such fictions are often effected by manipulation of allied words (see ch. 4), but they may also be made wordlessly.

These constructs are to begin with imaginary, but some of them may eventually be supposed, with varying degrees of logical probability, to have equivalents which are not imaginary. Inductive work is of course required to confirm such suppositions. Fictions are of course not always deliberate imaginings for research purposes; they may be unintentional misperceptions or misconceptions, or only intended for entertainment or more obscure ends.

b. It should be obvious that what has in this text been referred to as a 'nervous imprint', is simply one of the senses of the word 'memory'. I avoided that word, because it is variously used, also in senses which suggest renewed consciousness, or the reviewed objects, or actual images of previous objects — whereas I wanted to stress the subliminal aspect of memory, its material substratum.

Now, let us consider recognition more broadly. We suggested that when a percept (a concrete object of perception) is recognized, each of its many concrete attributes is encoded in the brain in some way, and matched against previous such nervous imprints.

Thus, there is supposedly a peculiar code for 'red', another for 'hot', and so forth, as well as for the various measures or degrees of such characteristics. We may similarly suppose that there is a special code for each of an object's abstract attributes — that is, for each concept (abstract object of conception).

You may remember, we distinguished between two kinds of imagination: 'perceptualization', the projection of any concrete mental image, and 'conceptualization', the projection any abstract mental image. Whether such projections are voluntary or not, their recognition is effected in the same way.

Note well also that a mental image, whether concrete or abstract, may be recognized as resembling a physical phenomenon, in any respect other than the substantial one (obviously they will remain differentiated as mental and physical, respectively).

However, concrete and abstract phenomena, whatever their substance, cannot be equated to each other, though they may of course be in some way causally associated. In practice, of course, almost everything we consider is a mix of concrete and abstract components, so some comparison usually does occur.

Thus, recognition, in its widest sense, concerns any kind of object. Anything distinguishable in some way, be it physical or mental, concrete or abstract, is supposedly recognizable. Thus, recognition is ultimately recognition of what we call 'universals', the various components of things, which bundle together into what we call 'particulars' (more precisely, we mean 'individuals').

That is not to say that there is nothing more to a 'universal' than a distinct code, for the codes themselves are in fact just 'individuals' — but it is merely an observation as to what we may reasonably expect the nervous imprints, which we earlier posited, to correspond to. The point is that there is no essential difference between recognition of concretes and abstracts, be they physical or mental, however they were generated.

63. PAST LOGIC.

The next few chapters, 63-67, contain material which is perhaps more of interest to academics (teachers and students of philosophy and logic, mainly), than to the ordinary reader. I would prefer the latter to skip this segment of the book and just go to the final chapter. The 'flavor' of Logic as it appears thus far, is I believe very clear and pleasant; one is also left with a practical tool. Those who read on, will experience a change of taste, as we enter into concerns and disputes, which fatigue the mind rather unnecessarily (at least, that was my experience). There can be too much of a good thing, as the saying goes.

Nevertheless, it was of course a scholarly duty for me to write this segment, and of course theoreticians are well-advised to study it closely. It may be viewed as a philosophical commentary on all the preceding chapters. As well as re-evaluating the work of past logicians, it deals with broader issues, like metalogic, and induction from logical possibility. The smallness and the errors of many doctrines of modern logic are demonstrated. Historical logic is also touched upon here and there.

1. Historical Judgment.

As stated from the beginning, a detailed history of logic is beyond the scope of this work. My concern here has been substantive — to develop a wide-ranging system of logic with an emphasis on the role played by modality in its main senses. My approach has been very independent, a research in the sense of *fresh thought*, rather than one based on scrutiny of past achievements.

It was inevitable that there would be overlap between my own discoveries, and those of past logicians. However, the prospect did not bother me, as I felt that enough of my findings would be completely original, so that I could afford not to lay claim to all of them. One can never, in any case, be sure that one was not influenced indirectly — or directly, in some forgotten reading — by the work of anterior researchers.

Thus, to be fair, one must acknowledge the authorship of any idea which even remotely resembles one's own. I will here try to minimally satisfy this standard of judgment, even knowing that without very extensive scholarly research into all the original sources, I can never hope to properly do so. I have only so far attempted to be a logic theorist; the label of logic historian is still well beyond my grasp.

I therefore hereby make a general disclaimer for the benefit of historians of logic: any idea which they find to have been put forward by previous logicians, they may attribute to them without any argument from me! The true scientist, after all, is not so much interested in personal aggrandizement (though one naturally wants one's own contributions recognized), as in getting the job done and the science of one's choice moved forward.

However, even without having read all that has been written on a subject, one can estimate what is generally known and accepted by its experts, by observing not only what they say about it, but also what they *omit* to say, directly or indirectly. This is an important point, to keep in mind. For example, if no mention is made in current presentations or histories of logic of the various modal-types of conditioning, or of something resembling factorial analysis, one can fairly assume that these insights have not been previously arrived at.

For it is inconceivable that certain doctrines of such high importance should remain without mention, even in elementary presentations or histories intended for the general public. It is of course conceivable that the discoveries in question may have been made independently by different researchers too recently to be widely known and accepted.

Note that the issue is never one of terminology, but of essences. Different theorists may give different names to the same insights, and one of the tasks of the historian is to recognize the similarities in essence irrespective of terminology used. Of course, one should not dilute any significant differences. But naming or renaming an already known concept can hardly be called a notable advance.

Also, the historian has to distinguish seminal, pivotal, or comprehensive contributions, from derivative or trivial ones. I mean, there has to be some perspective in evaluating material, a sense of proportion. One may of course be ungenerously critical, and ignore valuable points. But to linger overmuch on artificial or superficial schemes is a disservice to logical science, making it appear quaint and redundant, and diminishing its respectability.

The reader is now referred to *The New Encyclopaedia Britannica: Macropaedia* (henceforth occasionally referred to as *NEB*), to the article 'History of Logic' by **Czeslaw Lejewski** of Manchester University (23:235-250). I chose that historical overview of logic, because it is very recent (1989) and readily available to a reader wishing to consult it, in most public libraries. It will be used as our first, and to begin with virtually our sole, source of information. That is, we will assume that:

a. It presents a reasonably complete picture of the 'state of the science', in its main lines, to date;

b. It accurately reflects the opinions of current historians of logic, with regard to authorship of ideas;

c. It accurately reflects the attitudes of current theoreticians in the field, with regard to substance.

These assumptions are no doubt debatable. We may well suppose that many significant doctrines have already been published, which are not taken into consideration in that article, and that there are divergent historical viewpoints and theoretical perspectives. But, for now, let us look on this one prestigious source as the academic mainstream; we can later broaden the picture somewhat.

My purpose here is not only to recount the salient facts and features of logic as it is currently perceived, but **to reevaluate these current perceptions in the light of the insights arrived at within my own work**. Thus, I am to some extent reviewing the article critically, and suggesting how the history of logic might be rewritten.

Unless otherwise stipulated, all quotations are from the said article. References to previous chapters and sections of my own work, will be distinguished by using the abbreviation 'ch'. — for instance, this is ch. 63.1.

2. Aristotle, and Hellenic Logic.

The ancient Greeks gave birth to logical science as we know it, discussing both categorical and conditional logic in considerable detail (*NEB* 23:235-238).

a. The logic of *categorical* propositions is generally attributed to **Aristotle** (c. 350 BCE), the great Greek philosopher, the pioneer of Logic. He is credited with founding the science, through several major breakthroughs, among which: his description of the laws of thought, and introduction of symbols for terms, as well as the ensuing formalities for this class of proposition. Aristotle also discussed common fallacies of argumentation.

Aristotle's model of formal logic included: analysis of the structure of categorical propositions: terms and copula; polarity, quantity (and, to some extent, modality); and systematic treatment of opposition, eduction, and most importantly syllogistic deduction. He discovered the figures and moods of syllogism, and methods of validation like exposition, and *reductio ad absurdum*. These doctrines were developed in several treatises, collectively known as the *Organon*.

Aristotle's main concern was with *actual* categoricals, though he 'initiated the development of modal logic', through philosophical exploration of concepts like potentiality, change and causality in his *Metaphysics* (which is not counted as part of the *Organon*), as well as in more formal discussion of the categories of modality and the inferences which may drawn from them in other works.

In everyday discourse, we commonly use the indefinite form 'X is Y', which tacitly intends but does not specify a quantity. It can be taken to mean minimally that 'at least one X is Y' (a particular proposition); but often it is meant universally as 'all X are Y'. By formalizing these alternative interpretations through quantification of the subject, Aristotle made possible the systematic development of categorical logic.

However, with regard to modality, Aristotle does not seem to have decisively opted for a similarly hard and fast distinction. As earlier mentioned (ch. 11.1), he seems to have on the whole intended, as we all often do, the copula 'is' in a timeless sense, more akin to the 'must be' of natural laws, than to the mere 'here and now' sense of temporal events. But this ambiguity and hesitation retarded formal development of modal logic.

There is no doubt in my mind that Aristotle did have a rich philosophical understanding of the various types of modality. For instance, he pointed out that, in a certain sense, things are 'possible prior to the event, actual then, and necessary thereafter, so that their modal status is not omnitemporal… but changes in time'; also, that futuristic statements, like 'there will be a sea-battle tomorrow', often cannot in advance be judged true or false (thus implying admission of some indeterminism).

I am only saying that he did not fully exploit his knowledge in formal logic. His modal syllogism seems to refer specifically to logical, rather than *de-re*, modality; or perhaps his intent was to develop general principles applicable equally to all types of modality — but then, he was presuming that there are no significant technical differences between them.

In any case, he seems to permit drawing a possible conclusion from a possible major premise in the first figure (see for instance his *Prior Anylytics*, book 1, chapter 14) — a serious error (see ch. 15.3, 17, 36.3), which more precise definition of the modalities would have allowed him to avoid, since he had already discovered the invalidity of such argument with a particular (extensional possibility).

Likewise, his profound conceptual analyses of change and causality were never translated into formal logical doctrines, concerning transitive categoricals and conditioning of various types. I guess Aristotle had so much to say about so many things, that he could not find time, in the few years of his writing career, to say everything within his grasp.

It is safe to say, anyway, that the contribution to logic by Aristotle is incomparable, in its breadth and depth. He is the grand master of Western logic. All anterior and subsequent work in the field seems like mere footnote or embellishment in comparison, because after all the initial impetus, the *idea* of a *formal* logic, is an unrepeatable feat.

I say this, because it seems to me that many modern logicians (best not named) try to put down and belittle Aristotelean logic, seemingly in an attempt to raise the value of their own contributions. Logicians also should acknowledge their teachers, and behave without ego. It is just a matter of respect for the effort and work of others. Criticism and improvement are of course not excluded; only, the boast of revolution is unnecessary.

There has of course been other great logicians and philosophers, before and since Aristotle. Earlier, the Sophists made various distinctions between sentences. **Socrates** searched heuristically for definitions. **Plato** encouraged axiomatization as 'the best method to use in presenting and codifying knowledge', and addressed the philosophical problems of universals. There was a broad cultural heritage to draw from; a practice, at least among intellectuals, and especially lawyers, of argumentation.

Later, among the ancients, **Theophrastus** of Eresus, a Peripatetic, indeed a direct pupil of Aristotle, and his successor as head of the Lyceum, is reported to have emphasized doctrines like the fourth figure (see ch. 9.2); also the (not entirely accurate) principle that the conclusion follows the modality of the weakest premise; and 'prosleptic syllogism', or the process of substitution (see ch. 19.1).

Diodorus Cronus of Megara (4th century BCE) explored modality with reference to time, under 'the influence of Socrates and the Eleatics'. Tense, the before-after aspect of chronology, was discussed. Temporal modalities were defined; 'the actual is that which is realized now, the possible… at some time or other, the necessary… at all times' (see ch. 11).

Porphyry, a Neoplatonist, clarified classification by the use of 'trees' dividing genera into species. **Ariston** of Alexandria is said to have introduced the subaltern moods. **Galen** contributed 'compound syllogism' (ch. 10.4).

Although many of these contributions were no doubt original, many of them were also or already known to Aristotle, but considered by him to deserve relatively little attention. Work of the latter sort may be viewed as a process of digesting the received information, making it more explicit.

b. The logic of *hypothetical* propositions is rooted in Aristotle to a much lesser extent. As a logician he reasoned clearly in hypothetical form, giving later theoreticians an example of and opportunity for their theories. He effectively was the first to formally *produce* hypotheticals, since all his inferences had the form 'if the premise(s), then the conclusion'; and skillfully used the *reductio ad absurdum* method of validation. But apparently, apart from mentioning 'syllogism from hypothesis', to show his self-awareness as a thinker and perhaps hint at a possible line of further inquiry, he did not go into this area of logic in detail.

The dilemma, and rebuttal, were forms of hypothetical argument used long before Aristotle, as evidenced by the case of Protagoras vs. Euathlus in the 5th century BCE (Copi, 258-259).

Paradoxical hypotheticals made their appearance early in the history of logic (5th-4th century BCE), though they were not formally understood. The paradoxes of **Zeno**, an Eleatic, gave rise to analytic conclusions seemingly contrary to experience and common-sense (but these were single paradoxes). **Eulibedes**, a Megarian, developed paradoxes like that of The Liar (a double paradox), which revealed purely conceptual contradictions (ch. 32). But the significance and logical acceptability in principle of single paradoxes, and the dialectic of self-inconsistency, is a modern realization (ch. 31).

Theophrastus is credited with the development of hypothetical syllogism, in figures similar to those of categorical syllogism (ch. 29.1). The hypothetical propositions were positive logical relations (if/then), with positive or negative theses; negative hypotheticals (if/not-then) were apparently ignored, then and since (ch. 24.2).

Diodorus defined implication by pointing out that the antecedent is always followed by the consequent (note the modal definition, albeit with reference only to what seems like temporal modality). His pupil, **Philo** (called the Megarian), considered it enough to just deny conjunction of antecedent and negation of consequent. He thus defined what is today called 'material' implication and is credited with the 'truth-functional' analysis of positive hypothetical propositions (see ch. 24.3). Some hundred years later, **Chrysippus** of Soli, a Stoic philosopher, developed logical apodosis (ch. 30.1), and considered hypotheticals with conjunctive theses and nesting (see ch. 27.3).

All the above seems to have concerned only logical conditionals. Perhaps, however, the conditional relation was unconsciously intended (as we commonly do) as not only logical, but generic, with the (incorrect) presumption that all types behave identically. But on the whole, natural, temporal or extensional conditionals were not clearly distinguished or treated in any detail. Diodorus did, it is true, analyze temporal conditioning, through the form 'all times after {the sun has risen} are times when {it is daytime}' (see ch. 33-40).

Let us continue now, and consider logic in later periods, or in some other cultures.

3. Roman, Arab, and Medieval European Logic.

The article now deals with the ensuing centuries, between the end of antiquity and the end of the Middle Ages (*NEB* 23:238-240).

After Greek logic, there was a period of some five centuries during which 'little or nothing happened in the field of logic'. There was 'little creative work'. Some compendiums and scholarly commentaries on Aristotle's logic were prepared, many of them later to be lost. There were, 'on occasion, improvements in the minutiae', but no major contributions. However, we 'are indebted to [these authors] for salvaging numerous fragments from the lost writings of earlier logicians'.

Some works were translated into Latin. Roman authors, like **Cicero** (106-43 BCE) and **Boethius** (d. 524/5 CE), 'transmitted the achievements of the Greek logicians to logicians of the middle ages'. But by the 4th century of the common era, 'logic was treated as a subsidiary subject, providing useful training for students of law and theology'.

In the ensuing centuries, interest in logic seems to have been more lively, with the translation into Latin of some more Aristotelean works. Textbooks were written. Logical studies spread deeper into Europe, as far as England. The Scholastic period, as it is called, was more active, in that theoretical innovations were attempted. Ancient logic was brought into focus, clarified, systematized, extended and improved upon in various ways.[8]

Among the new inputs were: the distinction between subsumptive and nominal use of terms (see ch. 43.1); the concept of distribution of terms (ch. 5.3); recognition of the fourth figure; and, I might add, the use of symbols **A**, **E**, **I**, **O** for the four actual, plural categorical propositions, which in my view was an important novelty.

There was a heightened understanding of logical relations: premises and conclusions were seen to be theses of a hypothetical form; hypotheticals could have conjunctive or disjunctive theses; a distinction was apparently made between formal and material implication. The latter concept was, in my view, an unfortunate development, which may have helped them to grasp some properties of implication simplistically, but also produced some at best trivial if not downright misleading results.

More impressive, however, was the interest in modal logic, by logicians like '**Pseudo-Scotus**' and **William of Ockham** (14th century). They seem to have concentrated their efforts on logical modality, since their concern was with concepts like truth, falsehood, knowledge, opinion.

They are credited with defining the oppositional relations between the categories of modality, including actuality; for instances, that necessity implies actuality and actuality implies possibility, or that possibility of negation contradicts necessity (see ch. 11.2, 13.1, 14.2). They also

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8 An example is the traditional doctrine of the Fallacies. The 'fallacies' have been of interest since the time of Aristotle and before, but their study as a body rather dates from the Scholastic period. The whole approach is somewhat antiquated; and, though it still has didactic value, it has become less relevant, as strictly-formal logic has evolved. Common errors of reasoning and rhetoric, were named, listed, described and classified — traditionally as follows (according to NEB, 23:280-281). I do not consider this analysis perfect, but proposing an alternative has not been high on my list of priorities.

'Material' fallacies: making erroneous analogies (secundum quid, accidens and its converse), errors in causal judgment (non causa pro causa, post hoc ergo propter hoc, non sequitur), circularity in definition or argument, (petitio principii, begging the question, a vicious circle), as well as various kinds of intimidation or appeal (ad hominem, ad populum, ad misericordiam, ad verecundiam, ad ignorantiam, ad baculum, and also bribery).

'Verbal' fallacies: equivocation, figures of speech taken literally, ambiguity, distortive accentuation, confusing collective and dispensive senses of terms.

'Formal' fallacies: like errors of syllogism ('the four terms', 'illicit processes'), or apodosis ('denying the antecedent' or 'affirming the consequent').

discovered that (in logical modality) an impossible proposition implies all other propositions and a necessary proposition is implied by all others (ch. 21.2).

Scholastics also studied fallacies, and ancient paradoxes were confronted. The Liar paradox was seen as a meaningless proposition, in an anonymous manuscript of the 14th century. It is clear that logicians of this period were quite creative.

There was also a transmission of data by way of Byzantium, and the Arab lands. Arab logicians ‘played an important role in reviving the interest of Western scholars’ in Aristotle and other Greek logicians. Included are **Avicenna** (of Persia, 11th century) and **Averroes** (of Muslim Spain, 12th century), both of whose work on ‘temporal’ modality (by which is meant a mixture of natural and temporal modalities, to be precise) was of some value. But overall, according to the article, there does not seem to be work of any great originality or importance emerging from these logicians.

Incidentally, Jewish residents of those countries also played a part in this process, some of them as translators bridging the Moslem and Christian cultures. One may also mention logician **Isaac Albalag** (13th century).[9]

4. Oriental Logic.

With regard to the Indian and Chinese logics, here are some of the findings of research mentioned in the article under review (*NEB* 23:240-242):

Indian logic dates from as early as the 5th century BCE, with grammatical investigations. Later on, it also evolved in the framework of religious studies. Thinkers were ‘interested in methods of philosophical discussion’, although ‘logical topics were not always separated from metaphysical and epistemological topics’.

In a Hindu text of the 1st century CE, we encounter sophisticated philosophical concepts, among which some of a very logical character, like ‘separateness, conjunction and disjunction, priority and posteriority, …motion, …genus, ultimate difference, …inherence, …absence’. In the 2nd century, examples of arguments akin to syllogism appear, which enjoin that generalities be applied to specific cases. In the 7th-8th century, the various ways statements can be negated are explored.

A Buddhist text of the 5th century teaches that a mark found exclusively in a certain kind of subject may be used to infer that subject. Another, appears to describe some properties of implication and logical apodosis: an if-then statement is presented, and it is pointed out that admission of the antecedent coupled with rejection of the consequent is wrong; although the if-then statement has a specific content, its elucidation uses logical terminology.

But variables were not consciously used. The article concludes that, though Indian logic ‘developed independently of Greek thought’, its achievements were comparatively ‘not very impressive’. I may add that I find it curious that Greek-Indian contacts, at least after Alexander the Great, did not result in transmission of logical science.

With regard to China, in the 5th to 3rd century BCE, during ‘the controversies between the major philosophies of Confucianism, Taoism, and Moism’, there was some logical activity, especially by the latter school. However, once Neo-Confucianism became well-established, in the 11th century CE, the subject was virtually abandoned; the authority of that philosophy was so overriding, that there was nothing to argue about[10]. Though Taoism survived to some extent, it was not a philosophy of a kind inclined to intellectual argumentation.

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9 The NEB treatment of logic history is, in my view, defective, in that it makes virtually no mention of Judaic logic. Yet there is evidence of logic use in the Jewish Bible and the Talmud, and later Rabbinical writings are replete with logical discourse, including theoretical statements; and these manifestations of logic are bound to have had some influence on Western logic. But the subject is too vast for me to try and deal with it here. See my work Judaic Logic.

10 This explanation was suggested to me by a Japanese acquaintance, Matski Masutani.

The Moists (followers of **Motzu**) made distinctions, like those between personal and common names, the various senses of philosophical terms, or absolute identity and sameness in a specified respect; they even explored inferences by added determinants or complex conception. But, 'in developing logic, the Chinese thinkers did not advance beyond the stage of preliminaries, a stage that was reached in Greece by the Sophists in the 5th century BCE'.

I would like to add that this assessment may be somewhat harsh. My own minimal acquaintance with Asian philosophy (I read many books and received some practical training, years ago), including some aspects of Yoga, Tai Chi Ch'uan, and various meditations of Hindu, Buddhist and Taoist origin, incite me to greater respect for its achievements[11]. I am not prepared to go into this issue in detail here, but here are two examples which come to mind.

The *Tao Teh Ching* of **Laotzu** (China, 7th century BCE; some regard the work as a 2nd century BCE compilation), may be viewed as a treatise on holistic logic. One need only consider the opening sentence to see the truth of this claim:

> Existence is beyond the power of words to define:
> terms may be used, but none of them are absolute.

…and there are many more such profound insights in it. The lesson of the limits of verbalization, in particular, should be taken to heart by modern axiomatic logicians.

If we view one of the practical functions of logic to be the efficient completion of one's everyday tasks, by the most direct and least entangling route, then we may well regard the art of *Tai Chi* as a teaching of logic in action. The ideas of Yin and Yang may be considered as logical tools, akin to polarity and modality. Yin is the potential but not quite actual, hence the receptive; Yang is the necessarily actualizing, the willed to be; actuality is a certain balance between these two components of being.

Ultimately, no civilization can take shape without logic. One cannot plant a field, build a house, develop a language and laws, or produce the marvels of contemporary Japanese technology, without some sort of logical culture. We are sure to find some kind of logical knowledge, in the native cultures of Africa, America, and Australasia, as well as in Asia.

All the more, any philosophy or religion is bound to involve logical presuppositions or implications, at least within its epistemological and ontological pronouncements, or in its practical guidelines. The use of formal variables, or explicitly logical principles, are just possible ways for logic to find expression; the Orientals used other, more abstract or practical ways to achieve the same educational ends.

For instance, Zen Buddhism's belief in the efficacy of meditation or in spontaneity, in the pursuit of mystical 'Illumination', is very obviously a logical doctrine, since it prescribes *a way of knowledge*. Ontologically, it posits ultimate reality to be a unity and of a spiritual nature; epistemologically, it to varying degrees opposes structured knowledge, enjoining wordlessness and unselfconsciousness.

This logic is in contrast to the Occidental, which posits a more step-by-step and intellectual procedure, but whose ultimate goal or result may yet be the same. It may well be that both ways, and still others, are equally efficacious (that is the premise of multiculturalism).

Buddhism has it of course, as a basic epistemological and ontological premise, that the world is ultimately created by the (solipsistic) individual living being. This idea is contrary to Judaism, which acknowledges that we are humble creatures of a single universal Creator. But it may be that Buddhism by this thesis refers to an ultimate return of all souls to their Maker….

In any case, even by Western standards, Oriental philosophies, particularly Buddhism, are clearly internally consistent world-views. They are large-scale systems of epistemology and

11 Note that this is not intended as a blanket endorsement; there are doubtless invalid forms of reasoning in these philosophies.

ontology, which if not explicitly, at least implicitly, demonstrate the logical-mindedness of those who constructed them.[12]

5. Modern Tendencies.

I continue passing on data from the 'History of Logic' article previously mentioned, interspersing comments emerging from my own perspective (*NEB* 23:242-247). It tells us: 'the logical tradition of the Middle Ages survived for about three centuries after it had reached its maturity in the 14th century'. Thereafter, 'the advent of the Renaissance and Humanism did not enhance logical studies'.

Petrus Ramus discussed concepts and judgments, and introduced singular syllogism (see ch. 9.4). The Port-Royal logicians formulated in 1662 'rules of the syllogism', summarizing the common attributes of valid arguments in each of the figures; this was an important development, in my view, because it showed that logic could be expressed conceptually instead of formally: using ordinary language without need of symbolic terms (ch. 9.6). This approach 'continued to be popular to the mid-19th century'; it did not yield new moods of the syllogism, but offered a system of explanation.

The great French philosopher **René Descartes** (early 17th century), whose epistemology is summarized by the statement '*cogito, ergo sum*', ('I think, therefore I am') and who founded coordinate geometry, insisted on the definition and ordering of scientific knowledge, in accordance with the model of Euclidean geometry: clear and precise terminology, ('self-evident') axioms, and (not so 'self-evident') corollaries.

The German philosopher **Gottfried Leibniz** (late 17th century), who discovered mathematical calculus independently of Isaac Newton, conceived of logic as 'a general calculus of reasoning'; it would be an algebra for all thoughts, with 'unanalyzable notions' expressed as numerals and signs, from which more complex notions would be derived.

While both the Cartesian method and the 'universal mathematics' of Leibniz were valuable contributions, of course, they were from our point of view overly rationalistic. They seemed to regard knowledge as essentially given, needing only to be manipulated; they did not yet quite grasp the gradual and empirical apprehension of most data. A limited number of words in a limited number of combinations, of obscure origin, would somehow suffice to define and prove all others.

Although Leibniz' ideas of 'an artificial language' and of 'reducing reasoning to computing' led to mathematical logic and computing science, I must say that his specific logical insights (those known to me) seem trivial to me; as far as I can see, he was just repeating the known in other words — at best, it was work of clarification.

Leibniz also did research on the whole-part relation. The use of diagrams to represent categorical propositions, though 'already in evidence in the 16th century' and after, 'has come to be associated with the name of **Leonhard Euler**, an 18th century Swiss mathematician'. I find these to be useful learning instruments, though they can be misleading at times (see ch. 5.3).

In the 19th century, **Joseph Gergonne** (French) analyzed these diagrams through concepts of co-extension, inclusion, intersection and mutual exclusion. In Britain, quantification of the predicate was proposed by **George Bentham** and Sir **William Hamilton** (ch. 19.4); **Augustus de Morgan** focused on complementary propositions using antithetical terms, like 'all X are Y' and 'all nonX are nonY' (ch. 51.5), and the interactions of eduction and opposition, as in 'the contradictory of the converse is the converse of the contradictory' (this only applies to **E** and **I**).

These efforts were in my view far from remarkable. They were accompanied by elaborate symbolic languages, but added little to Aristotle's findings. I see them as assimilation of received information, but in a manner which *more and more divorced logical science from logical practice*. Only logicians have occasion to wonder about the quantity of the predicate or to compare the

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12 I unfortunately no longer remember the names of the books this judgment was based on.

outcomes of immediate inference; these issues are one step removed from ordinary thought processes.

The movement toward symbolic logic, and the preoccupation with extensional issues, was further accentuated by **George Boole**, who constructed logical formulas using symbols like those of mathematics. ‘Boolean algebra was subsequently improved by various researchers’, including **William Jevons** (who ‘constructed a “logical piano”, a forerunner of the modern computer’) and American philosopher **Charles Pierce**. I will not go into the details (see ch. 28 for one version of such algebras); much of this was essentially just rewording known things, as far as I am concerned.

These logicians began to see the common grounds between categorical and hypothetical logic, and Pierce also became aware of ‘the notion of a proposition that implies any other’. Within the system of the present treatise, the explanation is modal: quantity expresses the portion of the extension of the subject addressed by a categorical proposition (see ch. 11.4), and implication has a similarly quantitative aspect with reference to logical modality, namely the contexts applicable to the antecedent (ch. 21).

In the late 19th century, in Germany, **Ernst Schroeder** developed an algebra of logic, with reference to the notion of inclusion; this was a more systematic system, using axioms like ‘a non-empty class contains at least one individual’. The notion of inclusion, by the way, can be rather ambiguous: a proposition which implies another is said to include it, and a class is said to include its subclasses or its members; yet, these senses are antiparallel, since we may also say that the presence of a member or a subclass implies that of the classes above it.

At the end of the 19th century, we find **Guiseppe Peano** (Italian) sought ‘to base arithmetic on axiomatic foundations’. This enterprise may be viewed as an application of logic to mathematics, rather than a work on logic as such.

6. In The 20th Century.

The *New Encyclopaedia Britannica* article goes on to describe developments in the 20th century (*NEB* 23:247-250). I am sorry if my evaluation of modern logic seems at times overly critical; it is not my intention to put anyone down. More will be said about modern logic in the succeeding chapters; for now, I will only make brief comments.

I acknowledge the advances made, the refinements in definition and the more rigorous systematization, but I must take the long view for the centuries to come. What counts for me, the bottom-line or *tachlis* (as they say in Yiddish) is: is there anything really new in it from the point of view of logic (like new moods of syllogism, say)? Philosophical or mathematical findings are all very well, but our concern here is with logic as such.

Georg Cantor (1845-1918) introduced a theory of ‘sets’ or classes; this was ‘just another version of’ the logic dealt with by Aristotle, except for an emphasis on the denotative rather than connotative aspect of terms (ch. 18.1). These systems were later found to contain double paradoxes; but their goals were not abandoned. **Gottlob Frege** (1848-1925) worked to systematically reduce arithmetic, which concerns natural numbers, to logic; as evidenced by axioms like ‘for all p and q, if p then if q then p’, the hypothetical relation was taken in the petty sense of ‘material implication’ (see ch. 24.3).

The British philosopher **Bertrand Russell** (1872-1970) set out ‘to show that arithmetic is an extension of logic’, with the publication in 1903 of his *The Principles of Mathematics*. Later, he together with **Alfred Whitehead**, ‘produced the monumental *Principia Mathematica*, 3 vol. (1910-13), which has become a classic of logic’. A ‘work of impressive scope’, including ‘topics such as the logic of propositions, and the theories of quantification, of classes, and of relations’. It ‘marked a climax of the researches in logic and the foundations of mathematics’, and ‘provided a starting point for… development… in the 20th century’.

Several double paradoxes had been discovered in the 15 years prior, including those in the systems of Frege and Cantor, and those of **Burali-Forti**, **Berry**, **Richard**, and **Grelling**, and the ancient Liar paradox was still of interest. These antinomies 'cast doubt upon men's logical intuitions', and Russell wanted to resolve them. I quote at length, to leave as-is the language used in the article, with an emphasis on 'the paradox of the class of all classes not members of themselves':

Russell argued that they result from a "vicious circle" that consists in assuming illegitimate totalities. A totality is illegitimate when it is supposed to involve *all* of a collection but is itself *one* of the same collection… such totalities cannot be generated because of… the theory of logical types, [which] demands that… a class belongs to a higher logical type than that to which its elements belong. (Similarly, a predicate belongs to a higher logical type than the object of which it is predicated.) Consequently, to say that a class is an element of itself is neither true nor false but simply meaningless…. Although the theory of types obviated the paradox…, it raised certain problems of its own. It was by no means clear whether the theory was a kind of ontology that classified extra-linguistic entities or a kind of grammar that classified expressions of a logical language. Moreover, some critics charged that it was an ad hoc palliative… and that the ramifications… were unduly complicated.

Although I found myself in agreement with some of Russell's viewpoints and findings with regard to class logic, my sense was that he confused many of its concepts. Rather than try to define our differences point by point, it seemed more effective to develop a consistent system of my own from scratch. This work took me about a month, and the results are to be found in ch. 43-45. Even though this is intended as a reply to Russell, including a definitive solution of his paradox, I recognize he is to be credited with initiating such research. My conclusion is that, although the logic of classes qualifies as an important field in its own right, it is merely a derivative of Aristotle's logic.

The view that mathematics is an offshoot or segment of logic (known as 'logicism'), was countered by **L. Brouwer** and **Arend Heyting**, of Holland, who regarded it as an independent field. These 'intuitionists' (as they are called) felt that Aristotle's Law of the Excluded Middle tended to be overemphasized, and doubted that every problem is soluble; they rejected the 'elimination of double negation', which 'allows its proponents erroneously to infer the provability of a proposition from the unprovability of its negation'.

Though I agree to some extent with these views, I can also see that there is a confusion, here, between deductive and inductive logic. The third law of thought is an ultimate goal, set by deductive logic, which is not always easy to apply in the interim, during induction. According to strict logic, a double negative is equivalent to a positive; whereas the extrapolation from unprovable to absent is just a generalization.

Next, **David Hilbert** came up with the claim that 'freedom from contradiction in an arbitrarily posited axiom system is the guarantor of the truth of the axioms', less concerned than his predecessors 'with the meaning of the axioms' (a position known as 'formalism'). In 1931, **Kurt Godel** reportedly argued that no theory can be both complete and consistent, at least in mathematics.

As I see it, these logicians had become so concerned with the problems of systematization of mathematics, that their view of logic was very narrow, intent on deduction, and ignoring the inductive aspect of concept formation on empirical grounds. Consistency is only one of the tests of truth; and no theory is 'complete', anyway. Needless to say, principles proved with mathematical terms do not necessarily apply to other, more conceptual, terms.

Deeper down, their 'philosophy of logic' was faulty. There was a misapprehension of the 'laws of thought' as axioms, the ordering of logic on the model of geometry; as I have argued, that model is inapplicable to logic, whose grounding has to be much more subtle, since one of its roles is to justify that very model (see ch. 2, 20).

Continuing, efforts were made to investigate all the varieties of logical relations, 'the logic of propositions' (see ch. 23-27). The method of 'truth tables' was developed by Pierce, **Jan Lukasiewicz** (Polish, d. 1956), **Emil Post** (U.S.), **Ludwig Wittgenstein** (Austrian-British, d. 1951), and others, with the aims of defining logical relations and evaluating intricate formulas. Since relations like implication and disjunction were interdefinable, the question arose as to which came first. Pierce, and later **Henry Sheffer** (of Harvard), and **J.-G.-P. Nicod** (French), considered disjunction to be the most fundamental.

In my view, the T-F values of truth tables do not define, but are merely implied by, logical relations; they are effects, not causes. The primary relations in logic are plain conjunction and the negation of such conjunction; both implication and disjunction are thereafter derived with reference to logical modalities, so that their order is irrelevant. Intuitively, they are different angles: implication suggests forced appearance of a thesis, and disjunction suggests mutual replacement of theses without regard to underlying forces.

Logicians like Lukasiewicz, of the Warsaw school, and others in Dublin, traced the order of derivation of logic from a few well defined axioms. Others with an interest in systematization included Hilbert, **Ingebright Johansson** (Oslo), and Heyting. Conscious of the weakness of the current concept of implication, so-called Philonian implication, **Wilhelm Ackermann** talked of 'rigorous implication' and **Alan Anderson** (Pittsburgh) of 'entailment', to bind the theses together more firmly.

Logicians like the Poles **Stanislaw Lesniewski** (d. 1939), and later Lukasiewicz and **Alfred Tarski**, attempted to express logical relations in terms of quantifiers; but they do not seem to have had a clear definition of logical modality, which would explain why quantifiers work (ch. 21).

Much more impressive, was the work of **Clarence I. Lewis** in 1918, and later as 'author (with **C.H. Langford**) of *Symbolic Logic* (1932), a classic of modal logic'. He revived the definition of implication in its 'strict' sense [by Diodorus], in contrast to the pretensions of 'material' implication [by Philo], with reference to the modal category of possibility; this seemingly refers to the logical type of modality, though the intention may have been generic.

However, since he apparently had no clear definition of the concept of possibility, but used it as his undefined starting point, he developed 'several different systems of strict implication (S1-S5), and many more have been constructed since'. The word 'system' is here used in a limited sense, not in the grand sense of an overall, methodical presentation of the science of logic.

Godel built an alternative system, 'with necessity as the primitive notion', and **E. Lemmon** (of Oxford) constructed one 'with the notion of contingency'. The same criticism can be repeated: these categories of modality are interrelated, so that which one takes as one's starting point is arbitrary; these different systems just reshuffle the data. What matters, is to define modality first, with reference to nonmodal notions; the rest follows.

Lukasiewicz analyzed Aristotle's notions of modality, and suggested a 'many-valued logic', which in addition to truth and falsehood, would recognize 'intermediate' states of knowledge. His intent may have been to formalize 'indeterminism', but he seems to have been unaware of the distinction between logical contingency and problemacy. I have not taken a similar approach, because I regard it as redundant. There is nothing it can do which cannot be done through two-valued logic, so why complicate things? And in any case, intermediate concepts are only definable with reference to the extremes (see ch. 20, 21, 46).

However, we are told, 'many-valued systems that have no such defects have been developed by **Boleslaw Sobocinski**… and… **Jercy Slupecki**' (both of Poland). Post constructed 'purely formal systems of many-valued logics… independently'. But not all logicians have approved of these tendencies. Note, my own position is not that this threatens two-valued logic, but merely that we do not need it; I see no harm in building multi-valued logics in the way of exercise or for cybernetic purposes.

Other notable developments were, an interest in 'deontic' logic (see ethical modality, ch. 11.8), and the work of **A. Prior** in tense logic (ch. 11.6).

The work done after Russell with regard to categorical propositions, seems mostly paltry to me. Lukasiewicz rehashed the syllogism. **'Edmund Husserl**, the founder of Phenomenology, …was concerned with the grammar of logical language and not with any extra-linguistic entities'. Lesniewski attempted to define the copula 'is' (while, I notice, using the word 'is' in his definition), claiming to use a language which 'implies no existential assertions'.

He also worked on 'the theory of part-whole relations', (which he labeled 'mereology'). Work in this field was also done, independently, by **J. Woodger**, **Henry Leonard**, and **Nelson Goodman**.

In the U.S. **Ernst Zermelo**, of Germany, 'axiomatized the theory [of sets] and succeeded in avoiding the paradoxes without resort to the theory of types'. This work was continued by **Abraham Fraenkel**, of Israel, and **John Von Neuman**, a Hungarian-American 'pioneer in computer mathematics'. The latter 'distinguished between sets, which can be members of other sets, and classes which have no such property'.

More recently, **Paul Bernays**, of Switzerland, 'made important contributions' to this field. Harvard's **W. Quine** replaced the theory of types 'by the much simpler theory of stratification'. However, the article points out, these 'axioms have scarcely made the notion of a set any clearer. If [they] describe some aspect of reality, it is not known what these aspects are. Thus, the theory is becoming detached from any interpretation'.

These ideas are cast in obscure and esoteric language, which make them hard to judge by common mortals. Presumably, if they were used in computer work, they have been found to have some practical value. They seem to have concepts in common with my own theory, but my sense is that we differ, and the said hermeneutic complaint confirms it. The question of interpretation is answered by my theory at the outset, and a comprehensible and comprehensive system is easily derived. I leave the comparison to others. In any case, more detailed critiques are presented in later chapters.

'Once systems of logic were axiomatized, …logicians began to examine the systems themselves', in search of 'problems of consistency, completeness, and decidability, …as well as… the independence of the axioms… the definability of primitive terms'. These studies were called 'metalogic', and among the people engaging in them were **Alessandro Padoa** (d. 1938), Post, Lukasiewicz, **Alonzo Church** (U.S.), Godel, **Paul Cohen** (Stanford).

Among the disciplines of metalogic were syntax and semantics, and work in those areas was done by **Rudolph Carnap** (German-American), Lesniewski, **Kazimierz Ajdukiewicz** (Polish), Godel, Tarski, **Karl Popper** (Austrian-British). Also mentioned are **Gerhard Gentzen** and **Stanislaw Jaskowski**, for their 'systems of natural deduction'.

These issues are dealt with in more detail in later chapters. However, for now, my overall impression is that these logicians had a very a priori view of logic. My view is that the role of intuition and empirical data is much stronger, at all levels of the enterprise. It is vain to pursue consistency and order in logical science, in a purely rationalistic manner, without a broader epistemological and ontological outlook.

The copula 'is', the early notions of 'conditioning', and many other concepts, are irreducible primaries, which however do not exist in some detached plane, but are continuously reinforced by intuitive and empirical input. Likewise, identity, incompatibility, and exhaustiveness, are not strictly abstract 'laws of thought', but repeatedly 'experienced' concepts, given within phenomena. There is no such thing as deduction without induction; the human mind does not function like a computer, merely processing bits of information, but involves an ongoing *consciousness* which gives meaning to the data.

64. CRITIQUE OF MODERN LOGIC.

1. Formalization and Symbolization.

Let us first clarify the important difference between formalization of logic, and its symbolization, because these ideas are often confused with each other, as well as with the ideas of systematization and axiomatization.

A categorical proposition like ‘S is P’ or like ‘All S can become P’, or a conditioning proposition like ‘if P, then Q’ or like ‘either P or Q’, is called ‘a form’ or characterized as ‘formal’, because its terms or theses are *variables*. The expressions S, P or P, Q are meant to stand for any set of appropriate ‘contents’ which may arise, as in ‘the sky is blue’ or ‘if we don’t stop pollution, many people will suffer’.

The discovery of ‘formal logic’ by Aristotle, consisted in the realization that processes like opposition, eduction, or syllogism, could be validated without reference to specific terms. He succeeded in showing us that such inferences depended for their validity only on the *constants* involved in the forms: the copula, the polarity, the quantity, and eventually the modality.

Other Greek logicians, like his pupil Theophrastus and Philo of Megara, broadened the idea of formalization, by applying it to hypothetical propositions, in which the variables were propositions instead of terms, and the constant was the ‘if-then-’ relation. And since then, many other applications have been developed.

In this way, the (contentual) art of logic was refined into a (formal) science. The intuitions of logic were demonstrated to display certain regularities. The expressions S, P or P, Q were effective instruments, not because they were isolated letters, but because they were generalities open to any particular manifestation.

Aristotle could equally have used *whole words* like ‘the subject’ and ‘the predicate’ for the terms, and indeed did. Likewise, the antecedent and predicate were originally referred to verbally, for instance by the Stoic Chrysippus, as ‘the first’ and ‘the second’ theses. The use of letters was quite incidental; what mattered was the underlying concept of position and function in certain kinds of statements.

Or again, Aristotle could just as well, and did, teach us *by way of examples*. Consider any inference, say the conversion of ‘All X are Y’ to ‘Some Y are X’. It suffices to say: ‘all trees are plants’ implies (forces us to accept to avoid contradiction) ‘some plants are trees’, and *thusly* (similarly) for any other things, other than ‘trees’ and ‘plants’ (leaving us with only ‘all’, ‘some’ and ‘are’ to focus on). Such specimen or paradigmatic schemata would also qualify as ‘formal logic’, in the full sense. We do not absolutely need X’s and Y’s *to communicate* this; they are just neat tools to facilitate the process.

The achievement of those logicians was *conceptual*; it had very little to do with ‘symbolization’. Formalization was a major advance, by virtue of making us *aware* of the various components of propositions, and the role each played in determining inference. They were *pointed out* to us. The variables were clearly *distinguished* from the constants, and the latter from each other, and it was found that the impacts of two or more propositions on each other *depended only on* their forms, not on their contents.

Changes in the values of the variables, without changes in constants, did not affect the form of the result; whereas changes in the constants, without changes in the values of the variables, could yield different formal results. This is the extraordinary significance of formal logic, and its beauty and magic.

Also qualifying as ‘formal logic’, though it makes no use of symbols, is *the descriptive method*. A case in point is the ‘rules of distribution’, which are used to explain inferential processes by simply telling us about their inner workings in plain words. In that sense, all discussions on logic qualify as formal, including the symbolic. Note in any case that there is no *purely* symbolic communication, we always have to explain the symbols to each other first, in ordinary language, to give them some meaning.

What makes logic formal, in conclusion, is its *goal* of somehow describing our thinking processes about the world, with a view to prescribing some of them rather than others. The means for carrying that out may vary. There are different research and teaching strategies and techniques. Symbolization, to whatever degree, is just one of many such strategies, which may or may not be the best.

Let us now contrast modern symbolic logic. All it did was to substitute certain typographies (isolated Latin or Greek *letters*, or newly invented doodles), for pre-existing words. These 'symbols' did not serve as variables, but merely as stand-ins for constants: they performed no function that the original verbal expressions did not or could not fulfill. They implied no new conceptual insight whatsoever; they added exactly zilch to human knowledge, *nada, rien*.

Thus, for examples (say), the copula 'is' might be symbolized by a sign of equality, and the polarity 'not' written as a minus sign (or a curl); the quantifiers 'all' and 'some' (or 'there exists') are replaced by an upside-down capital-A and a laterally-inverted capital-E; necessity and possibility are signaled by square and diamond shapes; a dot plays the role of 'and', 'if-then-' becomes an arrow (or a horseshoe or a fishhook), and 'v' stands for the inclusive 'or'. Many other such symbolizations have been introduced; we need not go into the details.

All the concepts underlying the symbols had already been discovered, and previously named. All modern logicians did was introduce *a smaller string of letters*. Even if that was a harmless exercise, it cannot by any standards be viewed as an earth-shattering feat. *A symbol is just another word*, only more brief (and less widely understood) than the corresponding word of ordinary language.

The medieval symbolization of whole (actual categorical) propositions, as **A**, **E**, **I**, **O** (I have added **R** and **G**, for the singulars) was a valuable achievement, simply because it gave us a way to refer to them briefly: longer words take up more attention span of their own. Likewise, my introduction of modal symbols like **n**, **a**, **p**, which can be used as suffixes to **A**, **E**, **I**, **O**, as in **An** or **Ap**, were very useful to me, as tools in clarifying modal argument.

I am not putting down symbolism as such, but only warning against certain excesses and mistakes it can engender. Since symbolization as such was no novelty (it had long been used in mathematics), applying it to all the components of propositions can only be regarded as an argument for *a language of shorter words*; that was surely a very minor contribution.

But, in any case, there is a world of difference between symbolizing a variable and symbolizing a constant. The various values of a variable are very different from each other; their only common ground is having a certain position in the proposition concerned. Whereas a constant is uniform in all its manifestations; for instance, every instance of the 'is' relation is thought to be identical in all respects, except for being another individual instance. Thus, symbolization does not play the same role in both cases; for variables, it signifies formalization, but for constants, it has no such effect.

The only way to formalize the constants of categorical propositions would be as follows. Let **Q** mean 'whatever the quantity, be it *all* or *some*'; and **P** mean 'whatever the polarity, positive or negative'; and even **R** mean 'whatever the relation, whether the copula *is* or some other' (and eventually '**M**' mean 'whatever the modality', if you want to get fancy). Then, if x and y are variable terms, the general form of categorical is QxPRy (or QxMPRy).

The trouble with this idea, is that *we know of no logical processes which apply to a form so vague as this: so there would be no point in constructing it.* If you lump together disparate propositions, or worse whole families of propositions, there is virtually nothing left to say about them. Logic develops by differentiating one form from another.

Similarly, for an all-embracing form of logical relation: the relations of conjunction, implication, and disjunction, *have no common properties*, or none of any significance. It is for that reason that no such form has made its appearance in human language, and it would be useless to try and invent one.

Modern symbolic logic gives *the impression* that it is extending the scope of formalization, by its symbolizations, but it is in fact doing nothing of the kind. It is just abbreviating language. I admit that such abbreviation is occasionally not without utility: it allows us to display a lot of data in a small space, and thus more readily perceive its patterns. But it can also be counterproductive, in many ways (see ch. 1).

For a start, few people have the inclination to learn and memorize the meanings of symbols. Doubly fatiguing are shapes which are not letters in our alphabet. The result of this is that a large segment of the general public, even intellectuals, are discouraged from studying logic; and it becomes the reserve of only a handful of academics. Yet the point of science is to educate people, not scare them away. Logic is a valuable segment of the human cultural heritage; and, properly presented, can be very enjoyable to study.

Logical science has to resort to some artifice, because ordinary language is not always 'consistent': idioms vary, there are synonyms (same meaning, other word) and homonyms (same word, other meaning). Logic selects and 'freezes' certain senses of the words it uses, so as to avoid all ambiguity or equivocation. It symbolizes some of these words, to shorten them. This does not affect the data which concerns it.

But even for logicians, symbolization can be very misleading, and should only be carried as a final embellishment to a pre-existing system. If anything, symbols are likely to only conceal the weaknesses of a system; they may give it an appearance of finality it may not deserve, and arrest development.

The truth of this warning is evident in the many errors of logic congealed and enshrined by various modern theoreticians. If the underlying concepts are wrong, or too narrow in scope, then the system built upon them are bound to be confusing.

A case in point is the belief that implication can be defined nonmodally as 'It is not true that {the antecedent is true and the consequent is false}', or (inclusive) disjunction as 'It is not true that {both theses are false}'. These definitions refer to actual negation of actual conjunction, but they are misinterpretations of what we actually mean when we say 'if-then-' or '-or-' (see ch. 23).

Of course, one can use words as one wishes, as long as they are clearly defined; but why misrepresent one's achievements? Why not just admit that one is dealing with the limited field of negative conjunction, and avoid all ambiguity? To define 'if p, then q' as 'not-{p and notq}' results in ignorance of the contradictory form 'if p, not-then q', since its interpretation would then have to be simply 'p and q'. Likewise, defining 'p or q' as 'not-{notp and notq}' forces us to interpret 'not-{p or q}' as 'notp and notq'.

In any case, such 'Philonian' implication or disjunction is of little use to formal logic, which even when it compares 'material' relations does so in terms of 'strict' relations. For example, the middle implication in "'p materially implies q' *implies* 'notq materially implies notp'" is a strict one: it is this that gives it significance, and that is how it is understood.

Moreover, although modern logic has lately become more aware of modal definitions of implication and disjunction, these concepts are still understood in a very limited sense of logical relations. *De-re* forms of conditioning have as yet been very rarely discussed, and so far as I know there is no integrated formal system which includes them, other than the present one. Concepts like the 'basis' of conditioning, or like modal induction, seem to be totally ignored.

Even if logicians themselves are aware of the limited intent of their vocabulary, others may easily be misled, especially since the whole is cast in esoteric symbols. But in fact, the hasty departure from ordinary language has misled the logicians themselves. Ordinary language is rich in still unexplored meanings, whose elucidation is an ongoing process: they have locked themselves out of that process to a great extent.

Perhaps the best way to express my misgivings is through the following principle:

If a theory of logic cannot be comprehensively and convincingly constructed in ordinary language, it cannot be built any more solidly by using a symbolic treatment.

While on the topic of purely verbal changes, I would like to make an additional comment, concerning changes in terminology.

A process may be known to previous logicians, but they have not seen fit to assign it a special label, preferring for the sake of brevity to refer to it merely by a defining phrase. Likewise, past logicians may have noticed a fine distinction between very similar processes, but refrained from naming them distinctively, to avoid unnecessary pomposity. If later logicians come along, and rename something already named, or name something not previously named, for reasons like those, it does not signify that they discovered anything.

Making new, finer distinctions is sometimes valuable, of course; but cosmetic changes should not be regarded as momentous contributions. It depends on how useful they are found to be by the society using them. Language has to change over time. But it is well to remember Ockham's *Razor*, which in this context would be that names are not to be needlessly multiplied.

2. Systematization and Axiomatization.

Now, let us consider the other processes which are much touted by modern logicians — systematization and axiomatization. These terms are often used interchangeably, with each other and with formalization and symbolization, but they are not the same.

We would label a theory of logic, or concerning some specific field of logic, as 'systematic' to the extent that it is broad-based, and its various findings are well-integrated with each other, cohesively bound together by common threads, so that their relations to each other are made evident. We expect some degree of comprehensiveness; and a consistent and enlightening ordering of the data.

By *broad-based*, is meant that all seemingly relevant information has been compiled and taken into consideration, so that we can be confident that our judgment proceeds from a sufficiently large context. To the extent that some of the data is unconsciously or willfully ignored, it is reasonable to suppose that some hidden difficulties exist that have not been taken into account in formulating the theory; out in the open, such difficulties might alter or cause rejection of the theory.

What makes a theory systematic, once some database is gathered, is primarily that it 'structures' the information, through comparisons and contrasts. Integration presupposes differentiation; but analysis and synthesis proceed together, feeding on each other. Structuring both unifies and divides, making the *patterns* of sameness and difference involved apparent. We want to arrange the data in such a way, that its full message is apparent to all, with ease.

The purpose of this exercise is to consolidate all items of data, to show how each sits comfortably in the whole context. It obliges us to review the data methodically, again clarifying and checking our methods. Issues and themes are highlighted. The apparatus holding the data together is made transparent. Some arguments are seen to be 'reasonable', others more 'forced'. New ideas emerge, strengthening the whole, or enabling growth. The crucial factors become more visible.

An example of *integration* would be the definition of the various types of plural modality, through similar statistical formulas, differing only in their focal singularity. Thus: 'in all or some or no or not-all cases, circumstances, times, instances, contexts' — the focal determinant varies, but the quantitative aspect and the understood meaning of 'in-ness' are similar.

We can then differentiate the various categories of modality, within each type, though the horizontal analogies remain apparent. The vertical development of modal propositions, from the categorical forms to the forms of conditioning, is also integration: the threads of continuity are made evident. The significant relation here, is not a 'logical' one (in the sense of implication, or the like), but more accurately the highlighted *similarities and differences*: these are much more primary.

A badly, or not at all, integrated theory, is one which lists information without rhyme or reason, incoherently. The value and purpose of integration, is its ability to clarify things for us and make them more 'understandable'. The best way to achieve that goal, is open to debate, at least *ab-initio*. The axiomatic model favored by modern logicians is one possible answer, which must be evaluated; it is not automatically the best. Furthermore, it is not as 'primary' as it is claimed to be.

There are two ways data may be *ordered*. One, is the 'historical' order, the exact order in which the author or all authors collectively actually developed the research; this is commonly avoided because of all the trial and error involved in making theories: taken to an extreme it would be a waste of time and confusing, akin to 'stream of consciousness' literature. Two, is some sort of 'logical' order.

If we ask what constitutes a logical order, our first impulse may be to suggest that it is 'deductive', proceeding from the general to the particular. This is the model used, for instance, by geometrical science; and it is effectively prescribed by modern logicians for all science, including logic. We are to highlight the independent postulates and show what may be derived from them, including prediction of empirical data, 'if any'.

However, upon reflection, the issue is not essentially one of what is written down before what; one could equally well present things in an 'inductive' order, from the particular to the general, provided the same empirical data is taken into consideration and all consistencies are maintained. The choice is only esthetic, or related to didactic value.

But, in the last analysis, neither of these characterizations of logical order is accurate. The truth is that there are both inductive and deductive elements in all knowledge, and all sciences. There is *no such thing* as an 'a priori' purely deductive science: it would consist of meaningless words. And indeed there is no such thing as an 'a posteriori' purely inductive science: it would consist of nothing more of the impressions of the moment, unrelated to each other and unevaluated.

The so-called *axiomatic* model proposed by modern logicians for logical science, is claimed to be a purely deductive process, whose 'axioms' are therefore at best exactly identical with the empirical data base, or at worse devoid of any empirical reference. If the 'axioms' are intended as postulates related to certain other data in the way of adduction (by prediction and confirmation), as in Physics, then they cannot be properly regarded as 'axioms' in a purely 'a priori' system.

Realistic systematization, a strictly-speaking logical order, must include an awareness of the *conceptual* under-currents of knowledge. Even geometry, whether Euclidean or otherwise, involves empirical data: if we did not have a perception of physical or mental space, we would be unable to formulate postulates like those about parallelism at all. The same applies to logical science: without a mass of experiences, like (say) that things 'have properties' or that many things may have 'similar' properties, there would be no forms *to* put in any order.

The first condition for accurate systematization is, not 'axiomatization' of mysterious entities, but awareness of the genesis of the entities to be ordered. Inquiry into the *'genealogy'* of the concepts involved, their precise concrete and abstract referents, logically precedes any generalizing propositions concerning their interrelations, and the selection of some of these propositions as postulates (or axioms, in the proper sense) 'standing before' and 'encompassing' the others.

Ordering presupposes having something to order; ordering meaningless 'symbols' does not give them meaning. If they symbolize nothing, we are ordering nothing. If they do indeed represent something, then the ordering is referring to certain perceptions and conceptions, and is not an arbitrary construct. 'Axiomatization', taken in its extreme sense, has nothing to do with scientific systematization; it is just convoluted artwork.

Let us now consider the actual order of things for logical science:

a. First, it looks at some *categorical* propositions, discerning their various parts, taking note of their positions and relative variations, and (to begin with) *intuitively* estimating their functions in the whole. The most constant part is 'is', but others may take its place, like 'becomes' for instance; somewhat less constant are 'all', 'some', 'not', and the like; most variable are the so-called terms, their roles seem to be such and such.

Next, the logical relations (oppositions, eductions, syllogisms) between such propositions are, again, (to begin with) *intuitively* estimated. What seems 'contradictory' is rejected, what does not is called 'consistent', apparent 'implications' are noted, and so forth.

All such intuitions are taken at their face values, unless further intuitions interfere, and impose a selection. For us, intuitions are pure data.

b. Second, the logician confronts conditional propositions. Until now, we have been aware peripherally of relations which involve more than two terms, more than one categorical proposition; in particular, we noticed our intuitive access to 'logical' relations. Now, we repeat the same intuitive analysis for conditionals, as we used for categoricals.

We dissect them, and note the similarities and differences and the apparent functions. We distinguish varieties like incompatibility, conjunction, implication, disjunction of various sorts. We discover that there are not only 'logical' such conditionings, but also other types, and we try (to begin with) to intuit their analogies and distinctions. Then we (again, to begin with) intuit the logical relations (oppositions, eductions, syllogisms, apodoses, and so forth) among the various forms of logical conditionings, and likewise for other types of conditional propositions.

c. Only now, *at the last,* can we propose to *identify* (equate) our logical intuitions of various sorts, with the constructs designed to explain them. This is where we order the data *on a grand scale* (some more limited ordering has been performed in the course of the two preceding stages, now we consider the whole).

This grand ordering *itself*, is done by means of *intuitions* (of similarities and differences, and of apparent logical relations). Its role is only to test and confirm, by sifting through the data with that goal in mind, that our constructs are indeed able to reproduce the *preceding* logical and other intuitions.

The stage of overall systematization, which includes efforts of 'axiomatization' (in a proper, limited sense), is a final effort. There can be no pretensions that this final ordering *justifies* the intuitions which were used before it and in performing it. Its only intention is to confirm the theory that our constructions of 'logical relations' *correspond to* the intuitions we have labeled 'logical', that they predict the same results.

It is the particular intuitions (including those used in the final ordering) which justify the proposed postulates, by showing that, yes indeed, they (are intuited to) make the correct predictions, and everything fits together without apparent contradictions. Ergo, our constructs for logical relations are 'valid'.

That, briefly put, is the *logical* order of things in logic, in the strictest sense, an order which acknowledges the empirical antecedents (both perceptual and conceptual, and including all intuitions of sameness and difference, as well as those of conflict and forced inference). And that this genealogical order is displayed in practice, in personal intellectual development and in the history of logic, is no accident.

'Axiomatization' advocates (in the extreme sense) completely reverse this order, and posit the forms as preceding the contents, and logical constructs as more dependable than logical intuitions. Those premises are fundamentally wrong; they are inattentive to actual conceptual developments.

3. Modern Attitudes.

With these preliminaries in mind, I want to now criticize certain broader attitudes which have shaped the direction of modern logic. I do not wish to appear overly critical, but too often, looking at the work of modern logicians, I am reminded of H.C. Anderson's story of the emperor's new clothes. In less ironic moods, the words a 'guide for the perplexed' float in my mind.

Let us look into some more specific statements, reflecting the attitudes of modern logic, with reference to the *New Encyclopaedia Britannica* articles on the kinds of logic (23:234-235, 250-272, 279-289), namely the introduction, the section on 'Formal Logic'(authored by **G.E. Hughes** of Wellington University, in New Zealand), and the section on 'Applied Logic' (authored by **N. Rescher** of Pittsburgh University). I deal with the section on 'Metalogic' in a later chapter.

The introductory article at the outset declares the traditional division between deductive and inductive logic 'obsolete', on the grounds that induction is nowadays dealt with under heading of 'methodology of the natural sciences', so that the logic is henceforth to be taken to refer specifically to deduction.

I do not agree with this position, for a start. As I think has been clearly shown throughout my work, deductive and inductive logic make up a continuum; they are not radically distinct. Deduction is a limiting case of induction, where logical probability hits the ideal 100% mark; and induction is in a sense a form of deduction, in that it also prescribes a specific conclusion from given premises, albeit one of lesser logical probability.

In practice, there is no clear demarcation line between the two; how we categorize a judgment is very relative. If all the conditions are specified, we call it deductive — but that label ultimately depends on how well established the premises are inductively. If the conclusion is the most probable thesis in a disjunction of conclusions of varying probability, we call it inductive — but in the context of the law of generalization it is effectively deductive.

Furthermore, to limit the concept of induction to that of methodology of the 'natural sciences' is a very narrow viewpoint. Inductive logic, as we have seen, is not of value only to the institution of science, but to all human thought; and it does not apply only to material phenomena (the usual connotation of 'nature'), but to any phenomena, without prejudice as to their nature (which may be mental or even spiritual).

In any case, let us note that the descriptions and histories of 'Logic' here considered refer specifically to deductive logic in isolation, and move on. Inductive logic is dealt with in a later chapter.

Next, the article on 'Formal Logic' effectively distinguishes between the logic of categorical propositions and that of conditioning (using other terminologies: 'logic of noun expressions' and 'logic of propositions'), but regards the latter as *in principle* 'logically prior' to the former, although historically in fact a later development.

Again, I am not in total agreement with this modern viewpoint. Although it is true that, say, categorical syllogism is *ex-post-facto* visibly *a case of* hypothetical proposition (the *production of* 'if these premises, then that conclusion'), it does not follow that one can think of logical relations before one has developed some concept of categorical propositions.

The historical order is not accidental, but reflects the fact that the concept of a 'proposition' must have *some meaningful content* (a categorical, non-logical relation) before we can think of 'propositions about propositions'. The attempt to cast thought processes into an artificial 'logical order', without regard to the constraints of conceptual development, is not justifiable.

I dealt with this issue in more detail during the discussion of the laws of thought (ch. 20), where I showed that:

> The 'geometrical' model of axioms and inferences is in any case not applicable to the science of logic as a whole (though it can be used fragmentally), since it is a function of logic to justify it (for all other sciences).
>
> The attempt of some logicians to reduce logic to a concept like implication signals an unawareness of the conceptual order. *We can only construct such complex concepts out of simpler, more empirical and intuitive, notions*; they cannot be taken as primaries (see ch. 21).

A 'logical order' which is not conceptually feasible does not reflect any practical reality; it is a meaningless exercise.

Further on, the article describes 'formal' logic in the following terms.

It abstracts from their content the structures or logical forms that they embody… to enable manipulations and tests of validity. Formal logic is an a priori, not an empirical study.

The logician is concerned only with [the deductive necessity of the conclusion]… the determination of the truth or falsity of the premises, is the task of some special discipline or of common observation, etc., appropriate to the subject matter of the argument. [Inferences which] differ in subject matter… require different procedures to check… their premises.

In my view, I am sorry to say, this is utter nonsense. Even the process of abstracting form from content is an inductive one, presupposing certain perceptual and conceptual experiences. The morphologies which are studied by the logician do not come out of the blue; nor are they arbitrary constructs which later find interpretations.

At every moment in the enterprise of knowledge, including formal logical science, *a complex interplay of inductive and deductive forces* is involved. These ingredients continuously check and correct each other, in a holistic way. Deduction without induction or induction without deduction are impossible.

The intuition of contradiction is itself an inductive act, and the logical necessity of a deductive conclusion very much depends on the logical probability of the premises. Likewise, empirical observation cannot ultimately be separated from its deductive evaluation, in the broadest possible context.

Next, the work of logicians is described as follows. I am quoting a length so as to display the ideas of modern logicians in their own words.

The construction of a system of logic… consists in setting up… a set of symbols, rules for stringing these together into formulas, and rules for manipulating these formulas. Attaching certain meanings to these symbols and formulas [is viewed as a separate process, and it is claimed that] systems of logic turn out to have certain properties quite independently of any interpretations that may be placed on them.

Certain unproved formulas, known as axioms, are taken as starting points, and further formulas [theorems] are proved on the strength of these… the question whether a proof of a given theorem in an axiomatic system is a sound one or not depends solely on which formulas are taken as axioms and on what the rules are for deriving theorems from axioms, and not at all on what the theorems or axioms mean.

Moreover, a given uninterpreted system is in general capable of being interpreted equally well in a number of different ways; hence in studying an uninterpreted system, one is studying the structure that is common to a variety of interpreted systems. Normally a logician who constructs a purely formal system does not have a particular interpretation in mind, and his motive for constructing it is the belief that when this interpretation is given to it the formulas of the system will be able to express sound or true principles in some field of thought.

Again, a very negative reaction from this here logician. The traditional division between inductive and deductive logic is an artificial measure, allowing theoreticians to develop and teach the subject gradually. But once all the processes involved in knowledge acquisition have been adequately described and validated, it must be recognized that they are common aspects of all actual thought, with no isolated existence in practice.

In reality, there are no relations without things related, nor any things without some sort of (positive or negative) relations to others. For these reasons, a 'purely formal system' of logic is a pretension: either it is totally meaningless, or it naively or dishonestly conceals its intended meaning(s). A ‘symbol’ which is not a symbol *of something*, has no right to the name, which is only applicable to something representing something else.

It is no wonder that modern logicians have been able to develop so many ‘alternative systems’, if one considers the fact that more often than not they proceeded too impatiently, without bothering or sufficiently trying to first analyze the concepts they were using. With so many issues left tacit and unresolved, it was inevitable that situations would arise where the outcomes were uncertain. A case in point is the treatment of modality.

On the surface, it is of course quite reasonable that a word or symbol may have generic value, so that its properties are those in common to various specific applications, while each variety may additionally display special properties of its own. We have encountered this often in the course of our research, and that is not the point at issue.

But note in passing, that in any case the generic properties of the different types of modality and relations can only be obtained by generalization, *after* separately analyzing each type; we summarize what is in common to the special logics, we do not discover them without specific studies.

Quite different, is the claim that any ‘system’ of logic can be developed relativistically, without even a hidden reference to any experienced phenomena.

‘Symbols’ have no magical powers. Any system can of course be symbolized: this is just a change of language, which may be more convenient. But there has to be some underlying significance to what is being said. One can intuitively grasp logical and other relations without words or symbols, but the latter are nonsensical without the former.

Needless to say, *all the new findings in the present work can equally well be expressed in a ‘symbolic system*’, and no doubt someone will chose to do it sooner or later. But had I begun with meaningless symbols, instead of first approaching the subject-matter conceptually and with reference to pregnant ordinary language, there would be no data to systematize, no system to symbolize. There is no magic formula, no way to avoid the need for empirical and intuitive consciousness.

To top it all, modern logicians present their theories in a manner they find esthetically pleasing, and call ‘rigorous’, and then make grandiose claims, like:

> Aristotle’s methods of reduction achieved something approaching an informal axiomatic system of syllogistic… [but] a formalized axiomatic system of syllogistic has, in fact, been made possible only by the methods of modern logic.

Excuse my frankness, but these modern contributions are comparatively trivial. Aristotle taught us all about the syllogism; and codified the whole, by declaring the three laws of thought to be fundamental. He provided the tools; it is of little relevance that he did not take the time to use them fully.

The modern pretension at ‘systematization’ could not exist, without Aristotle’s lessons of rigorous formally-validated reasoning (with the example of categorical syllogism), and the model he presented us with, of a science based on three main general ‘axioms’ capable of controlling all subsequent processes. He taught them how it was done; how can they now claim to be one up on him? They are just carrying on his programme!

Re-ordering data in as structured a way as possible, so as to actualize all their potential, is of course valuable work. But it is surely not revolutionary. For instance, the ‘axiomatization of syllogistic’ by Jan Lukasiewicz was perhaps a clarification, an interesting rewording and classification of the components of such reasoning; but it was a relatively easy thing to do, *ex-post-facto*, after Aristotle’s work in the field.

One can hardly claim that ‘the derivations… depend entirely on rules for the manipulation of concatenations of signs, and not any intuitive insights or knowledge of the meaning of these signs’. Why this desire for magical mumbo-jumbo? Why not view it simply as an effort to grasp all facets of traditional logic, using some more or less novel ‘techniques’ or emphases?

There is no great expansion of substance, nor any great review of fundamentals. To put down Aristotle's contribution as 'informal', and to suggest that 'formal systematization' was *'made possible'* by modern 'methods', is surely hyperbolic. It suffices to say that improvements were made.

More will be said on these issues in the chapter on metalogic (ch. 66).

4. Improvements and Innovations.

Let us now look at modern logic with a more positive emphasis, and consider certain theoretical details in the context of the theories developed in the present treatise. The reader is again referred to the encyclopedic article on 'Formal Logic', and to that on 'Applied Logic'. Examples of specific improvements are many.

More attention was paid to invalidation of invalid syllogism, whereas previously validation of valid syllogisms had been the center of attention (ch. 10.2f). Various properties of axioms were stipulated: they must together be complete, consistent, interpretable, nonredundant, independent.

An improved notation for categorical propositions was introduced by Lukasiewicz, in which 'lower case Latin letters… are employed to stand for general terms', as in **A**ba for 'all b are a' (I personally would have transposed the letters a and b). Diagrams were designed, such as those of **John Venn** (in 1880), to illustrate quantitative aspects of relationships (but some of these, in my opinion are too artificial to reflect any easygoing functioning of the mind).

The logical properties of categoricals with singular or negative terms were further analyzed (for instance, their oppositions). Forms with complex terms were considered (ch. 42.1). The intricacies of logic were worked out to an extreme degree (ch. 27).

In that context, modern logicians learned 'to draw hypothetical conclusions from categorical premises', a process which was called 'conditionalization'. Though the concept arose in the context of 'material implication' (p implies {q implies p}), and is invalid in strict implication (unless p is necessary, see ch. 31), it may be seen as a precursor of productive argument (ch. 29.3).

It is also interesting to note that **John Neville Keynes**, in 1887, stressed the difference between (*de-re*) 'conditional' propositions, which concern 'implication between terms', and (logical) hypotheticals, which concern 'implication between propositions' (see ch. 33). **Georg Von Wright** (of Finland, b. 1916), should be mentioned for his work in the logic of preference and deontic logic.

There was also a lot of work done in the field of *modal logic*. Distinction was made between logical necessity (as in '2 + 2 = 4') and contingent truth (as in 'France is a Republic'), and similarly on the side of falsehood. The oppositional relationships between the various categories of modality of this type were clarified, on 'intuitive' grounds. Strict implication was identified with necessity of 'material implication'. Symbols were introduced for modal notions, such as L for necessity or M for possibility.

Propositions like 'if p is necessary, then p is true' (subalternation) and 'if it is necessary that p implies q, then the necessity of p would imply the necessity of q' (an apodosis, see ch. 30.1), were posited as axioms, which 'appear to have a high degree of intuitive plausibility'. Others, like 'if p is true, then p is possible', and 'if {p and q} is necessary, then p is necessary and q is necessary' were viewed as derivative theorems. The superimposition of modal categories, as in 'a necessity is necessarily-necessary', was investigated (seemingly, in an attempt to distinguish relative and absolute contextuality).

But many conclusions, like 'the [material] disjunction of p is necessary and q is necessary, [materially] implies the strict disjunction of p and q', were of doubtful validity; and some, like 'if p is true, then p is necessarily-possible', were trivial or misleading. This 'modal system' was labeled T; others were constructed with more or less similar premises, such as S4, S5, and B (for Brouwer), which added superimposition propositions as axioms instead of mere side-issues.

At the root of these difficulties, as already mentioned, lay a failure to prepare for these purely technical discussions of modality, by a preliminary philosophical analysis of the concepts involved. An adverb like possibly 'is taken as the fundamental undefined modality in terms of which the other modalities are constructed'. The attempt to work 'intuitively', without first clearly defining the categories of modality within the framework of each modal type, was bound to result in confusions and disagreements.

So-called systems were worked out first, and only thereafter were their properties (allegedly) found to match this or that type of modality. Given the truth values of the components of a modal formula, it was 'not obvious how one should set about calculating the truth value of the whole'. It was suggested that 'necessity is truth in every "possible world" or "conceivable state of affairs"' — a valuable statement which can be likened to my definitions of necessity (see ch. 11, 21).

But that definition seems circular and vague, without the stratification between primitive, intuitive notions, and their more constructed, conceptual derivatives. As far as I can see, there was no major effort to understand modality ontologically and epistemologically; no precise explanations for the similarities and differences in behavior of the various types of modality, no theory as to how modalities are or are-to-be induced day by day by common humans.

More will be said on the developments in modal logic in the next chapter (ch. 65).

There was a strong interest in developing a theory of *class membership*. This concerns the relationship between 'collections (finite or infinite)… of objects of any kind' and the individual objects themselves. Membership and its absence, having been identified as relations, were assigned special symbols. A class was to be defined 'either by listing all of its members or by stating some condition of membership'. Classes with all the same members, yet different specifying conditions, were 'extensionally identical'.

A class without members (like 'Chinese popes') was referred to as a null (or empty) class, and it was realized that 'there is only one null class'. The 'complement' (negative version) of the latter would be 'the universal class… of which everything is a member'. The technical impact of the concept of an empty class on Aristotelean categoricals was considered, and its 'existential import' was discussed (My own treatment of such issues appears under the heading of 'modalities of subsumption', ch. 41).

A distinction was made between classes which are themselves members of classes (called 'sets' by some logicians), and those which are not so. It was understood that 'class membership is… not a transitive relation', that is, a member of some class is not thereby a member of the classes of classes of which that class is a member.

However, the seemingly reasonable assumption (known as 'the principle of comprehension') that 'for every statable condition there is a class (null or otherwise) of objects that satisfy that condition' was 'found to lead to inconsistencies' — specifically, Russell's paradox concerning 'the class of all classes that are not members of themselves'. This was indeed an important finding.

Russell tried to resolve the difficulty by suggesting that statements like 'X is (or is not) a member of X' may be 'ill-formed' and only applicable to classes at different levels in some 'hierarchy'. But no explanation was apparent as to why such restriction should occur, why all predicates do not behave in the same way.

Other solutions were therefore proposed. For instance, Lesniewski suggested, in the context of his theory of whole and part (called 'mereology'), that a confusion between 'the distributive and the collective interpretations of class expressions' was involved (but I fail to see the relevance of this suggestion: which of the terms in the paradox are collective?). In any case, of all the solutions proposed so far, 'none has won universal acceptance'.

My own solution is not with reference to issues of self-membership, or to any hierarchy or stratification of classes (though I have clarified more traditional and legitimate senses of that kind of concept), but with reference to the known process of *permutation*. The principle of comprehension is too loosely formulated, in that the expression 'statable condition' seems to refer to any arbitrary verbal utterance, without regard for the conceptual feasibility of grouping and isolating the words used (see ch. 43-45).

More will be said about class logic in later chapters (see ch. 66.3 and 67.1).

There has been notable progress, in this century, in *extending the scope of logic theory*. The logical properties of new forms, of more limited applicability, were researched. This field was called ‘applied logic’, because it dealt with more specific relations than those traditionally studied by logicians. It was viewed as one step closer to ‘material logic’ than current ‘formal logic’.

Parenthetically, let me state that I do not agree with this division, or its underlying beliefs. Ultimately, the whole science of logic is ‘formal’, whether it is expressed to some degree or other symbolically or not at all, whether its principles are expressed in detailed schemata or in broad statements (though the former method is clearer), and whether its subject-matter is of broad applicability and manifold in properties (like ‘X is Y’), or less often useful (like ‘X sings to Y’).

When we notice that in practice a certain inference seems ‘logical’, but we cannot place this event in the list of thought forms already assimilated by the current science of logic, we classify the event as ‘material logic’. But this is a mere convenience; it should not be taken to imply that there is some unformalizable force at work. There is still usually formal aspects to the event, except in the extreme case where the combination of features (copulative or modal) is unique. The latter case is conceivable (this would be ‘material logic’ properly speaking), but I suspect very unlikely: a solitary, unrepeatable universal.

In any case, however we call it, ‘the applications envisaged in applied logic cover a vast range, relating to reasoning in the sciences and in philosophy, as well as in everyday discourse’. This is a constructive programme, with which I whole-heartedly agree: an all-out effort to uncover the formal aspects of as much of human reasoning as possible, till each inference which seems logical in practice is fully understood and justified in theoretical terms.

The exact dividing line between logic and more specific sciences, like psychology, ethics, and natural sciences (including biology), is ultimately artificial and irrelevant. All knowledge is one enterprise, and specialization should not be excessive, insular. Everyone, of course, is agreed on this point.

Subjects arousing interest, apart from those mentioned above (like the logic of logic, the logic of modality, and the logic of classes), included:

(i) ‘epistemic’ logic (knowing) and ‘doxastic’ logic (belief), as well as the logic of questions;

(ii) ‘practical’ logic, comprising the logics of preferences, command, obligation (‘deontic logic’); Rescher elsewhere mentions ‘boulomaic logic’ (concerning hopes and fears, ‘bulimic’ would be more accurate English) and ‘evaluative logic’ (concerning good and bad);

(iii) ‘physical’ logic, concerning time and space; Rescher elsewhere mentions causality (see White, 168).

I have noted some of the work done in some of these fields in earlier chapters. With regard to ‘practical logic’, this classification mixes psychological (desire and aversion) and ethical (teleological) phenomena rather indiscriminately, as far as I am concerned. With regard to ‘physical logic’ (a misnomer, in my view, since these concepts are not limited to the physical domain), I would include the logic of change as of fundamental interest, (see ch. 17 for an introduction); the logic of causality is of course very important too (see ch. 42.2 for an introduction).

5. The Cutting Edge.

I would like to first mention Aristotle’s ‘inductive syllogism’, which establishes a general rule by making an assuredly complete examination of all cases (one by one) and showing it to be true for every one’. This is also known as argument by enumeration, and looks as follows:

Each of S1, S2, S3,… is S,
and Nothing other (than these pointed-to things) is S,
but Each of S1, S2, S3,… is P,
therefore, All S are P.

This is of course the ideal case of 'induction' (effectively, a 'deduction'), where all the objects of a certain kind can be pointed to, and the conclusion merely states in abbreviated form what is already given in the premises. The implicit singular propositions are supposedly given by observation. For each of the objects, there is an underlying third figure singular syllogism, which establishes the link between the attributes S and P.

In practice, induction is of course more of a problem, because most of our concepts are open-ended, and we cannot enumerate every instance. But with this model, Aristotle may be said to have founded the science of formal inductive logic. We will later consider in more detail more recent developments in this field, but for now let us take note of some related issues which are also mentioned in the article on 'applied logic'.

Some modern logicians have made a distinction among hypotheticals, with 'reference to the status of the antecedent'. It could be 'problematic (unknown), or known-to-be-true, or known-to-be-false', so that three kinds of conditional emerge: the 'problematic… "Should it be the case that p — which it may or may not be — then q"; the factual… "Since p, then q"; and the counterfactual… "If it were the case that p — which it is not — then q"'. Such 'contrary-to-fact' theses were seen to have 'a special importance in the area of thought experiments in history as well as elsewhere'. It was recognized that this had to concern strict, rather than 'material', implication.

This idea may be viewed as a precursor of my fuller theory of 'bases' for logical (see ch. 25.1, 31.3) and other types (ch. 34.1, 38.1, 39.1) of conditionals, and also for disjunctions. However, note that they are referring to problemacy and knowledge, rather than as I do to logical modalities (for hypotheticals). Such propositions may of course also be viewed as compounds, of a generic conditional and a statement about the antecedent, with the latter left tacit or expressed in parentheses or made explicit.

With reference to counterfactuals, a Polish linguistic theorist **Henry Hiz** (b. 1917) suggested (effectively) that they were constructed by applying a general law (unstated but implied as underlying accepted fact), to a specific case (the known to be false antecedent). Such laws might be 'all or part of the corpus of scientific laws'. 'This approach has been endorsed by **Roderick Chisholm** of Brown University… [and] many recent writers'.

This was an interesting development. First, however, I would like to point out that this is just enthymeme, argument with a suppressed premise or a hypothetical with part of its antecedent left tacit — known since antiquity. Also, although we do indeed in practice draw in this way on a reservoir of strongly accepted 'laws', we must admit that in a more holistic epistemological context such 'laws' (unless strictly self-evident) are not distinguishable from other propositions: they are subject to eventual review like any others.

But, apparently, Hiz was formulating a (first figure) syllogism, with a categorical (general) major premise and a categorical (specific) minor premise, whose conclusion was to be a (counterfactual) hypothetical. I am therefore inclined to say that Hiz was struggling with the notion of 'productive' argument, which I have developed considerably (see ch. 29.3, 36.3, 40.2).

Also, the above objection to a special status for 'laws' seems to have been grasped, because researchers proceeded to analyze the problem without necessarily taking the major premise for granted. Instead, the falsehood of the conclusion was seen to put in doubt both the premises, so that a more open evaluation is required: 'a contradiction obviously ensues. How can this situation be repaired?' They went on to enumerate the various alternative combinations of premises and conclusions (or their negations) which would overcome the contradiction.

They go on to discuss whether it is better in such cases to 'sacrifice… a particular fact in favour of a general law… [or] a law to a purely hypothetical fact', and suggest that 'in actual cases one makes laws give way to facts, but in hypothetical cases one makes the facts yield to laws'.

However, there still ‘remains… a choice between laws’, cases where ‘the distinction between facts and laws does not resolve the issue’, so that ‘some more sophisticated mechanism for a preferential choice among laws is necessary’.

I see these contemporary ideas as akin in concept to my theory of ‘revision’, where conflicts are resolved with reference to alternative outcomes. The harmonization depends on the relative credibilities of the theses, and we may interpret the suggestion that actual facts are superior to laws, and laws in turn to hypothetical facts, as a principle for instant decision-making. However, that formulation is inadequate, not only because of its failure to reconcile disagreements among laws, but also because it presumes that we can always distinguish actual from hypothetical facts.

It is for these reasons that I developed my whole theory of ‘factorial induction’, which is still much broader in scope than these contemporary stirrings (see part VI). I believe that this theory qualifies as the ‘more sophisticated mechanism’ that is being called for. It is interesting that these issues conclude the encyclopedic articles on Logic, because I view them as being at the cutting edge of logical science, which is why I have called my work *Future Logic*.

More will be said on these issues in the chapter on inductive logic (ch. 67).

65. DEVELOPMENTS IN TROPOLOGY.

1. Tropology.

The study of modality on a philosophical plane may be called Tropology (from the Greek for 'figure', *tropos*); it is a broad field, with Ontological and Epistemological ramifications, and a direct relevance to fields like Aetiology, the study of causality (and thence Ethics), as well as to the likes of Physics, Biology and Psychology. Modal logic is a branch and accessory of Tropology, clarifying the formal aspects and processes of modality.

We have seen in this treatise, that modalities are attributes of relations (or of any things, in relation to existence). Modalities are distinguished first with reference to their 'types', and within each type with reference to their 'categories' (which are similar from type to type). We dealt in detail with four major types of modality (which may also be more briefly called 'modes'), which are defined with reference to their 'fulcrums' — namely, the *de-re* modalities, natural, temporal and extensional, and *de-dicto* or logical modality. Within each type, we distinguished the categories of necessity, actuality, possibility, and their negative versions (all of which have special names assigned to them in the various modes).

The combinations of category with type, yield specific modalities within each type; but additionally, there are modalities of compound categories and of mixed types, so that the list of specific modalities is quite long. Other types of modality exist, like the volitional (a subset of natural/temporal modality), and the teleological (from which the ethical is derived, at least in part); but we have not studied them closely in the present work (though I have myself studied them, and can assert that they fit into the general scheme here presented).

The fulcrums (or fulcra) of the four modes which here concern us are: natural circumstances, times, instances of a universal, and logical contexts; the generic fulcrum is labeled 'cases'. The general definition of modalities is that they are 'attributes of things or relations which *exist in a number of cases*', the number specified determines the 'category' and the 'type' of cases involved is specified by the fulcrum. For example, potentiality, or natural possibility) is defined as 'the modality of what is actual in some circumstances'. All other modalities are defined by similar statements, *mutatis mutandis*.

The fulcra are the focal points, the themes, the cruxes, the frames-of-reference, which distinguish one mode from another. The common relation is referred to by means of the tiny word 'in' — in some circumstances, at all times (in all segments of time), in most instances (in most manifestations of the universal), or in a few contexts, for examples. The fulcrum frequently has 'boundaries', which delimit the applicability of the relation; for example, the times or circumstances involved may be those in which the subject is actual or potential, to the exclusion of those in which it is not.

The relation of 'in-ness' plays an important role, though it is notional and intuitive. Its quantitative aspect is explicated spatially, by analogy to a dot within a larger area (or a point in a circle, say). Its qualitative aspect is the insight that things are in many cases 'affected' by their surrounds, that things somehow 'interact' in their environments. Thus, a wordless reference to causality is involved; but 'causality' at a very vague and intuitive stage. Later, this notion of causality, which is used to build up modality from a notion to a scientific concept, is in turn built up into a scientific concept, by the concept of modality, as we saw.

After such preliminaries, we proceeded with an analysis of the interdependencies and interactions of the various modalities, with reference to the oppositions, eductions, and syllogisms between different modalities. At a subsequent stage, different copulas were also considered. (See part II.)

All this concerns 'categorical' relations, but the concept of modality gives rise to parallel 'conditional' relations, which in turn further clarify what we mean by modality. Here, too, we found, we must distinguish between types of conditioning — the natural, the temporal, the extensional, as well as the logical (and not only the latter).

For each of the four modes, there exists a variety of conditionings, distinguished with reference to the 'connection' and the 'basis' intended, as well as to issues of polarity. There is also a distinction between implicative conditioning, and various manners of disjunctive conditioning. The connections and bases were defined with reference to modal (and polar) concepts, and the logic of their compounds was analyzed in detail. The concept of basis is a newly discovered one, which refers to the possibilities *underlying* a connective actuality. We thus gradually arrived at a greater understanding of causality. Each type of modality and category of conditioning gives rise to a distinct concept of causality. (See parts III, IV.)

Lastly, the issue of how modalities are (or are to be) known in practice, in specific cases arose (which should not be confused with the more philosophical issue, just mentioned, of how the concepts of modality as such were constructed). There is of course deduction of modal propositions from previously identified modal propositions; but ultimately, some modal propositions have to be induced somehow. We discovered precisely how modal induction works, in strictly formal terms, through the novel theories of factorial analysis, factor selection and formula revision (see part VI).

In this way, we developed a pretty thorough theory of modality, which set terms and methodological standards for Tropology. As far as I can see, this treatment is original on many crucial counts. Philosophers and logicians have of course over time done much work in this field. But the present situation seems to be as follows:

a. the four modes are by now more or less known, and it is known that they have analogous categories, which are somehow determined by quantitative issues — but no one so far has arrived at clear definitions and devised a classificatory understanding of these phenomena;

b. some work has been done since antiquity on modal syllogism, but errors were made, through failure to take into consideration the phenomena of change; also, subsidiary matters like productive argument seem to have been altogether ignored;

c. although logical conditioning has been analyzed in detail, especially in the modern era, the *de-re* forms of conditioning, and of course their respective and interactive logics, remain essentially unknown to this day;

d. the whole issue of modal induction has never till now been raised, and therefore of course factorial methods are totally unheard of.

This is the situation as I found it. A complete history of modality theory, is beyond the scope of this work. As we have seen, the topic dates at least from ancient Greece, and crops up thereafter again and again. Here, my purpose is rather to trace, in a very random-sample and fragmentary manner, but in somewhat more detail than thus far attempted, more recent developments, with a view to a fair evaluation of where my colleagues stand today. Also, comparing and contrasting my thesis, serves not only to defend it, but to further define it.

My methodology is far from exhaustive: it consists in gleaning information from a number of works, found in the library of the University of British Columbia and the Vancouver central public library, having to do with modal logic. True scholarship would demand a more thorough approach, with a special concentration on the Big Names in the field: the work of many years. My spot-checks are not sure to be representative. Such a method may paint an inaccurate patchwork: it estimates the shape of the whole from points on a graph.

2. Roots.

As earlier stated, I consider Aristotle's understanding of modality to be broad and profound. The evidence is to be found both in his logical works, like *De Interpretatione* and the *Prior and Posterior Analytics*, and in his discussions of change, coming into being and passing away, potentiality, actuality, and natural necessity, and causality, in works like the *Metaphysics*, the *Physics*, and *De Generatione et Corruptione*.

Quotations of all his direct or indirect references to issues of interest to modality theory, would no doubt fill a volume. In any case, what is relevant to note here is that Aristotle was aware of both the ontological and epistemological variants and dimensions of modality.

Rescher's *Temporal Modalities in Arabic Logic* describes the treatments of natural and temporal modalities found in Arabic texts, such as Avicenna's (Ibn Sina's) *Kitab al Isharat wa-'l-tanbihat* and Averroes' *In I De Caelo*, which 'are unquestionably of Greek provenience'. It is well known that the historic value of Islamic Middle-Eastern logic, lay in its bridging the gap of centuries between the worlds of Antiquity and Christian Europe.

The latter, in its firm will to overthrow paganism, had as it were 'thrown out the baby with the bath water', and indiscriminately rejected some of the more positive achievements of the old world. Arabic logic, judging from the said source, concentrated on some of the ontological aspects of modality, which the Medieval scholars dubbed *de re*. Over time, after that, the emphasis (at least in formal theory) shifted to more epistemological faces of modality, the *de-dicto* aspects.

Let us to begin with scrutinize Arab contributions more closely, in the context of our own theories. First, I would like to say that Arabic modalities are misnamed when they are called 'temporal', for it is clear that *they are more precisely mixtures of natural as well as temporal modality*.

The Arabs had a respectable concept of the various *categories* of modality (such as necessity, actuality, possibility, impossibility, inactuality, possibility-not, contingency), as well as of their interrelations (a square of opposition was presented by Averroes); but confusions arose when dealing with the *types* of modality. Although they were clearly conscious of the complexities involved, they were not entirely successful in separating the various issues from each other.

a. Avicenna distinguishes "absolute" (meaning, unmodalized) propositions from those which are modalized. However, strictly speaking, this should be viewed as a grammatical rather logical distinction. In common discourse, we admittedly do not always explicitly qualify our statements modally, but from the logical point of view, every proposition has some at least implicit modality; if the modality is not apparent, then the lowest possible modality may be assumed, just as an unquantified statement is considered as particular rather than as a distinct kind of quantity.

Similarly, for so-called "categorical necessity" and "general possibility", which refer to these categories without explicit qualification as regards type. Likewise, also, for 'impossibility in the primary and general sense' and for "special possibility" (by which is meant, contingency). These were erroneously considered as categories of a distinct type ("modes"); but they are simply generic concepts, of unspecified type.

b. Avicenna does not make clear initial distinctions between natural and temporal modality, nor between different modalities of subsumption, nor between the categorical and (*de-re*) conditional manifestations of modality, nor between different modalities of actualization. Instead, the significances of various *compounds* of these elements are discussed, as the following definitions make evident:

Thus, "absolute necessity" refers to what is 'in essence capable of' being predicated of a subject, throughout its duration as such (even if only in most instances of it or most of the time or in most circumstances, even if statistically there may be a few rare exceptions). In contrast, "general conditional necessity" refers to predications applicable to a subject constantly, while it is in a certain state or it is surrounded by certain conditions; whereas, "special conditional necessity" refers to similarly conditional, but temporary events, which may in turn be "temporal" ('as with an eclipse') or "spread" ('as with respiration').

Again, "general absolute possibility" refers to events which are 'not perpetual', yet 'necessary some of the time'. Alternatively, "general possibility" refers to events which are 'actual at some times but not others' without being 'necessary at all — neither at a given time' ("non-perpetual existential") 'nor under certain circumstances' ("non-necessary existential"). Underlying this distinction is a concern with the inevitability of actualization, or its absence, obviously.

We can also see the mixture of considerations in the subdivisions proposed by Avicenna for actuality: it may be, 'as long as the subject really exists', "absolute perpetual" (always there, 'but without necessity') or "general conventional" (there, 'always under certain definite circumstances') or "special conventional" (there 'at certain times,… though not perpetually'). Note the appeal to a modality of subsumption. I will not belabor the topic further: the point is made.

However, one more thing is worthy of note. It is evident (in Rescher's Table X) that Arabic logicians, if not all past logicians, regarded first-figure syllogism with a merely possible major premise, whether the minor premise is necessary or itself possible, as yielding a valid possible conclusion. Thus, according to them, *the modes **1/pnp** and **1/ppp*** (however possibility be interpreted) are valid. This is of course, as I have shown, a historic error (see ch. 15-17).

Lest the impression have been given that discussions of that period centered exclusively on *de-re* modality, I should briefly mention as an example the doctrine of the Mu'tazilite school of Arab philosophers, who according to A.Y. Heschel 'rejected the idea of causality and taught: What seems like a law to us is merely a "habit of nature"….' Thus they 'followed the principle that no heed to is be taken of reality, since it also rests on a habit whose opposite is equally conceivable'.

To these ideas, the Jewish philosopher **Maimonides** replied: 'Reality is not contingent on opinions, opinions are contingent on reality"' (117). Clearly, what was at issue in these discussions was the precise relation between *de-dicto* modality (the conceivable) and *de-re* modality (the real, the natural).

3. Shifts in Emphasis.

We find in Aristotle an interest in both the *de-re* and the *de-dicto* senses of modality. In Arabic logic, as we just saw, the emphasis was more on the former. But thereafter, as we shall now see, European thinkers put more emphasis on the latter sense. This is already evident in Ockham's discussion of modal propositions, in the early 14th century. Of course, some logicians, like **J.S. Mill** in the mid-19th century, in the context of his study of causality, continued to focus on modality in a more objective sense.

The tone was perhaps set by the great, 18th century German philosopher **Immanuel Kant** who, at the turn of the 19th century, defined modality in a more subjective sense, as determining 'the relation of [an] entire judgment to the faculty of cognition'. He distinguished between problematic, assertoric and apodeictic judgments; defining these, respectively, as 'accompanied with the consciousness of the mere possibility,… actuality,… [and] necessity of judging' (115). Incidentally, Kant's definitions seem circular to me, unless possibility, actuality and necessity are defined elsewhere, or considered obvious notions.

Other influential distinctions were suggested by Kant, among them that between analytical and synthetic judgements. In the former, the predicate is 'contained [though covertly] in the conception' of the subject; in the latter, the predicate 'lies completely out of the conception' of the subject (Joseph, 207). Again, these definitions are open to technical criticism, since the terms used in them are very ambiguous, but that need not concern us here. Kant was apparently trying to distinguish between the self-evident (which he considered purely 'a priori') and the empirical.

But Kant's understanding of self-evidence was very naive. For him, a proposition like 'cats are animals' would be analytical, because 'the definition' of cats includes that they are animals. But this is an error: such a statement is synthetic.

When we perceive an object, we distinguish various attributes in it; perceiving many objects, we find that they have some attribute(s) in common, and others not in common; lastly, we assign a different name to each distinct uniformity. Thus, *a statement like '(all or some) X are (or are not) Y' signifies:*

(All or some of) the things which had the resemblance(s) we labeled "X",
also have (or lack) the distinct resemblance(s) we labeled "Y"

— which may or not also be found in things not having the resemblance(s) we labeled "X". That 'cats are animals' is therefore an empirical finding. That 'cats' is a subclass of 'animals', simply means that all cats are animals, but not all animals are cats (see ch. 44). The selection of the animal attribute of cats, as essential, as *the* defining genus although it is one of many overclasses, may be due to its being obviously very 'striking' (the automobility of animals is impressively different from the growth of plants, for instance), or to some logical intuition based on wider considerations (adductive processes).

In the absence of any deciding factor, the decision may indeed, as a last resort, be very conventional. Some definitions are admittedly mere word equations, like 'Bachelors are unmarried men', but even then we always draw on some underlying experience (in this case, the fact that some men are not bound to a woman by public vows). Kant evidently focused on a very minor contingency at the tail of the conceptual process. The same can be said concerning many later philosophers (and certain modern logicians), who took the more extreme position that definition is linguistic and conventional. Reasoning is impossible without some sort of empirical data behind it.

The situation at the turn of the 20th century may be illustrated in the writings of **H.W.B. Joseph**, an Oxford logician. His virtue lay in unfettered discussion of issues, gently bringing many examples to bear, without the compulsion to quickly and rigidly systematize; I personally learned logic from his work. With regard to modality, Joseph evidently echoes Kant in focusing on the distinction between assertoric, problematic, and apodeictic judgments.

Incidentally, he points out that the word *'tropos'* (Greek for 'modality') first occurs in the *Commentary of Ammonius*, where it is taken as 'signifying how the predicate belongs to the subject'. This might be interpreted as widely applying to any adverb; but logic is more concerned with adverbs which 'determine the connexion' (according to **Michael Psellus**), or more precisely which 'attach *to the copula*, and not to the subject or predicate' (according to **Buridanus**).

Discussing the modal qualifications, Joseph pronounces them 'clearly logical', but adds that it is not 'the act of judging' nor 'the matter judged' as such which they concern; rather, they somehow 'mark the distinction between knowledge and opinion… and differences in certainty'. Judgements not modally qualified, he calls *'pure'*; these as **Bain** suggested express mere 'primitive credulity', they are 'assertions… made without reflection; we do not ask whether they are consistent with others'.

Apodeictic judgment is such that its 'ground… is seen to lie within the nature of' the terms involved: it is 'self-evident', but that does not mean that 'it is evident without need for understanding'. Mathematical statements are not commonly explicitly expressed as necessary, but they all (those proven) fall in this category. Empiricists 'rightly insisting that there is no knowledge without experience, wrongly suppose that we cannot by thinking discover the nature of anything that we have not perceived'. Others, including **G.W. Hegel**, **F.H. Bradley**, **H.H. Joachim**, counter that 'only in apprehending everything could we know anything as it is'.

Problematic judgment signifies 'our belief of certain facts which are not sufficient ground for the judgement… though we believe that along with other facts they would be'. It is 'provoked by knowledge', yet it so qualified 'because of ignorance' (**Bosanquet** is cited in connection with these insights). Lastly, assertoric judgment is distinguished from pure, in 'being not a bare unreflective assertion, but expressing besides our mental attitude towards a suggested doubt'.

Joseph goes on to say 'these distinctions of modality do not then express differences in the necessity with which the elements connected in reality are connected'. He also introduces the concept of probability, as being related. He concludes: 'what gave modality to a judgment was the presence of the thought of grounds for what is alleged… the grounds [being] given in other judgements', so that 'a modal judgement expresses reflection upon the question of the truth of what is judged or suggested'.

We see here that the initially almost *psychological* definitions by Kant, have subtly shifted over into explanations which have more to do with *logic* as such. Modalities (of the type under consideration) are neither entirely subjective, nor entirely objective, but somehow relate to causal and inductive reasoning (in the largest sense). We thus return full circle to the 'old distinction between *ratio essendi* and *ratio cognoscendi*, a reason for being of a fact, and a reason for acknowledging its being'.

In my own theory of modality, I attempt to contain all these trends. There are various types of *de-re* modality, and there is *de-dicto* modality, and the interrelations of the two groups are probed by formal methods. *De-dicto* modality is founded in epistemic notions akin to those of Kant, but the concepts of logical modality as such are more complex constructs. They depend on an interplay of rational insights, empirical data, and a holistic approach.

A couple of final comments. First, I want to say that, though I respect Kant highly, as an imaginative and stimulating philosopher, I view him as a not always very powerful formal logician or practitioner. Secondly, with regard to Joseph, although his main treatment of modality (188-207) ostensibly revolves around the *de-dicto* concepts, he should not be construed to have at all discarded *de-re* concepts.

That is evident for instance in his distinction between disjunctions which 'express the state of our knowledge' from disjunctions 'in the facts' (giving as examples 'Plato was born either in 429 or 427 B.C'. and 'Number is either odd or even'). He was of course in this case referring to the difference between logical and extensional disjunction. I do not recall whether he made a similar distinction with regard to implication. But he also points out that 'X may be Y' in some cases signifies a particular proposition 'Some X are Y', or 'that under certain conditions, not specified, though perhaps known, X is Y' — here again, the interpretation is factual (extensional, natural) rather than pertaining to knowledge.

It is also interesting to note Joseph's comment that hypothetical propositions are often used even 'when we do not see the consequent to be necessarily involved with the antecedent'. He thus anticipated what I have called 'lower case' hypotheticals, which only describe a possible consequence of an antecedent. He also mentions the use of possible disjunctions, as in 'a G may be either S1, S2, or S3'. It is a pity that other logicians did not follow up on these observations.

4. Setting the Stage.

Now to the current century. (The reader is referred once and for all to Part III of this book, for a fuller discussion of the topics raised here.)

An interesting development was that of 'many-valued' logics, like the one proposed by Lukasiewicz in 1917. The 'values' under scrutiny were truth and falsehood — again, purely *de-dicto* concepts — and the suggestion was that intermediate values, any number of them, were conceivable. An example given was 'I shall be in Warsaw in a year's time', which 'is neither true nor false'; this of course resembles Aristotle's 'There will be a sea-battle tomorrow'. (**Bochenski**, 405.)

As I have said before, I have no argument in principle with such an idea: it is applicable, to the different degrees of credibility or of logical probability, or even to 'partial truths' (propositional compounds some of whose elements are true and some false), as well as specifically to future events conditioned by voluntary factors. My only objection has been that, ultimately, such logics have to be reducible to the two-valued kind: they do not displace the latter kind, because it is used to judge their proposals.

That is, *we still have to decide, with regard to any proposition of many-valued logic, whether it is true or false.* Modal nuances are not primaries, but merely quantifications of certain primaries. In any case, many-valued logic was interesting, as an effort to formally recognize the philosophical truth that reality and illusion are extremes between which lie uncertain cases.

'Up to 1918 all mathematical logicians — unlike the Megarians, Stoics and Scholastics — used only one notion of implication, the Philonian or material' [ignoring the earlier understanding of implication proposed by Philo's teacher Diodorus]. At that time, C.I. Lewis 'introduced a new notion of implication and with it a modal logic'. (Bochenski, 403). This refers to 'strict' implication, which was definitely a welcome improvement [or rather, revival] in my view. Lewis clearly brought out the interrelatedness of modality and logical relations.

Note that Frege, not long before, considered modal qualifications to have 'no place in pure logic'; but **H. MacColl** 'had included some suggestions for modal logic' in his work (Kneale, 548-549).

However, Lewis' modality was only of a logical kind, since its focal points were the alethic concepts of truth and falsehood; with few exceptions, this seems to be the main concern of contemporary logic. More importantly, the relation between these concepts and more stringent concepts such as 'impossibility' was left undefined, with the latter taken as an irreducible primary. This failure to define the logical modalities, whether impossibility or any other category is taken as the starting point, plagued modern logic with manifold problems.

Attempts to effectively define logical modality were woefully weak. Consider for instance that of Rudolph Carnap in his intricate 1947 symbolic study of logical modalities, with reference to semantics. His system centered on necessity; and the 'explicata' he gives for it, which he finds 'clear and exact' unlike 'the vague concepts… used in common language and in traditional logic', is as follows:

> it applies to a proposition [whose truth] is based on purely logical reasons and is not dependent upon the contingency of facts; in other words, if the assumption [of its negation] would lead to a logical contradiction, independent of facts (174).

Look at the description: it is technically no better, indeed much worse, than most common or traditional 'understandings'. Most of the words used (like 'applies', 'based', 'logical', etc.) refer to complex modal and logical concepts, which themselves need to be defined; indeed, many of these concepts require for their definitions prior definition of modality, so they can hardly be used to define a modality. Not only is the philosophical background vague, but the author fails to make an obvious self-test for a *petitio principii*.

The impression of rigor given by subsequent symbolic manipulations, however admittedly 'conventional', is entirely illusory, since no formal rigor was exercised with regard to the starting points. Apart from such criticisms, let us anyway note that Carnap was interested, at least primarily, in logical modality, rather than any *de-re* concept of modality.

Instead of seeking definitions, which would conceptually explain the accumulated intuitions of logical science, in the way of a theory to cover the facts — Lewis and those after him used certain known logical phenomena as *axioms* from which the others were to be derived, following the model of the *Principia Mathematica*. Consider, for instance, the axioms of Lewis' modal system S1.

One 'axiom' was that a proposition implies the negation of its own negation, or that 'P' implies 'not nonP'. For me this is not an axiom. The law of contradiction, as I have argued at length, cannot strictly-speaking be construed as a general first premise from which others are deduced; rather, it must be viewed as an inductive summary of countless specific logical insights. A fact and its negation never occur together in our experience; or if they seem to, that event itself is experienced as somehow faulty and needing some sort of correction. The word 'not' merely labels such phenomena; it does not invent or create them.

Likewise, some of the 'axioms' relate to the logical relation of mere conjunction, for instance that 'P and Q' implies 'Q and P' or even just 'P' and just 'Q'. For me these are not axioms. We commonly 'experience' situations where two or more propositions all seem true in a context.

The word 'and' is used to refer our attention to such situations. It is evident *from the experiences* we intend by it, that the 'togetherness' is a nondirectional relation and does not exclude separate existence (had we experienced something else, we would have said so).

Such underlying experiences can be, and often are, *wordless*; the words (or symbols) cannot therefore be regarded as conventional determinants. The relation of implication cannot in any case be used to define 'not' and 'and', because it is itself much more complex than them, and anyway (as it turns out) they are required to define *it*. All that these so-called axioms do is *report* what we commonly and invariably all intuit: they do not serve to *justify* these intuitions, which are primary givens. If they are neither 'conventions' nor definitions nor first premises, then it is misleading to call them axioms.

All the more reason, propositions like 'if P implies Q and Q implies R, then P implies R' (the primary mood of hypothetical syllogism) and 'if P implies Q, and P is true, then Q is true' (the primary mood of logical apodosis), cannot be characterized as axioms. They cannot be placed, as Lewis and others have done, at the fountainhead of logical science. They are only *at all meaningful* provided we first come to an understanding of logical modality, which would allow us to define implication in such a way that, indeed, these properties emerge.

Similarly, that 'possibly{P and Q} implies possibly P and possibly Q', is a very derivative propositions, which depends for its recognition on a preceding understanding of logical modality. Once we know that possibly means 'in some contexts', it is easily *seen* that if 'there are contexts where both P and Q seem true', then 'there are contexts where P seems true and contexts where Q seems true, and some of those contexts at least are the same'. Likewise, that 'there exists cases where P neither implies Q nor implies nonQ' is a common observation: some propositions seem unrelated to each other.

In each case, we have an appeal to the idea of *subsumption*; or, if you wish, to the topological principle that the whole is made up of and includes the part. But even here, the relation involved is repeatedly intuited as 'logically forceful'; our statement of it is *a mere verbalization of information, and not the source of our conviction.*

Additionally, we must distinguish the intuitive *notion* of implication, reflecting our experiences of one thing seeming to lead to another, from the more conceptual *construct* of implication, defined with reference to modality. For this reason, we can make statements like the above, reporting common logical experiences, even before we have proposed a theory as to the definition of implication (with reference to modality). Common intuitive knowledge (what we call 'common sense') is used to test and tailor the eventual definitions.

All this to say: such 'axiomatic systems' *grossly oversimplify* the conceptual complexities involved in logical concepts. They fail to pay attention to what might be called the *genealogy* of the ideas involved. One cannot avoid having to define the logical modalities, and they are not definable arbitrarily. There is a step by step process, layer upon layer:

a. first, we have specific intuitions and experiences of the kind we label 'logical', about various things;
b. notions are formed about these logical phenomena: they are pointed to, distinguished from each other, and variously named;
c. regularities in behavior are faithfully observed and duly recorded: these will serve as the database for subsequent theorizing;
d. concepts can now be formed, which are capable of embracing the said regularities: such construction itself involves reliance on ad hoc logical insights;
e. only finally, do we have complex logical principles, to play around with symbolically, and order into axioms and theorems: and even these actions depend on the intuitive experience of their logical 'rightness' or lack of it.

The issues are further complicated by the fact that the progressive developments of different ideas impinge on each other at different stages of the proceedings. The logical intuitions and notions concerning them *all* come into play *at all* stages of each concept's development; and additionally the concepts are tiered relative to each other. We have therefore to zigzag from one idea to the other; there is no way to hierarchize *whole* sequences.

Thus, at the lowest level, we have: appearances, their apparent credibilities, their apparent impacts on each other, their apparent contextuality. Next, the concepts of truth and falsehood are defined, by comparing the seeming credibilities of seemingly opposite appearances in a given context. Next, modal concepts, like necessity or possibility, are defined with reference to numbers of contexts in which truth or falsehood are found. Next, logical relations, like implication, are defined, by modalizing conjunctions and their negations. At the highest level, probabilities can be analyzed, with reference to all the preceding.

Let us now consider the kind of proposition modern logicians considered as *optional axioms or as theorems worth deriving*; specifically, we shall consider the doctrine of 'orders of modality'.

Many different 'modal systems' were proposed by Lewis and others, according to which established logical principles were taken as primary: if p implies q and q implies p, then either of p and q can be taken as more primary. That assumption ignored conceptual considerations, as already made clear. Also, certain logical relations are *ab-initio* of undetermined value: therefore, different systems could be constructed, by arbitrarily assuming an additional proposition or its negation as one of the axioms. Thus, what is an axiom in one system might be denied in another, or it might be derived in the way of a theorem.

A great fascination arose with one kind of question especially: that of 'superimposition of modalities'. Starting with **O. Becker** (according to **Feys**), logicians like Lewis, Carnap, Godel, Von Wright, **McKinsey**, **Parry**, debated it with the utmost seriousness.

They called logical categories like necessity and possibility 'first order' modalities; their reiterations were called 'second (or higher) order' modalities. This refers to verbal constructs like 'necessity of necessity' or 'possibility of necessity', and similarly in other combinations. The questions they asked were: Does necessity imply necessity of necessity, or what? Does possibility imply necessity of possibility, or perhaps possibility of necessity, or maybe possibility of possibility? (Why not necessity of necessity, for that matter?) These were called 'reduction principles'.

Carnap, for instance, claimed to demonstrate, on purely semantical grounds, that necessity implies necessity of necessity (174-175). In view of the controversies, the **Kneale**s, in 1962, wondered 'if it is not possible [to resolve such queries] how shall we ever be able to settle the question? What sort of evidence should we seek and where?' (556).

But, I say, once a quantitative definition of the logical modalities has been constructed, these questions appear utterly puerile. If the reiterated categories in question are of uniform modal type, that is, all *de-dicto*, logical concepts — the answers easily proceed from purely quantitative considerations. 'All of all the contexts' is equivalent to 'all the contexts' (necessity). 'All of some', 'some of all' and 'some of some' of the contexts, all signify 'some of the contexts' (possibility), although their precise extensions may well vary.

Admittedly we commonly do repeat modal qualifications. But often, the intent is only to emphasize, a mere linguistic flourish: I am sure that I am sure, I am unsure that I am sure, and so forth. Sometimes, perhaps, we intend a sequence: I am still sure, I am no longer sure, and so on. Such statements tell us whether a verification has or has not taken place, and whether further research is or is not called for.

Knowledge and opinion, as we have seen, vary over time, in the transition from context to context; thus, assumed (that is, contextually imposed) modalities do change, and logical science may have an interest in analyzing such changes in precise detail. But in its essence, logical modality is a static phenomenon: *we are not so much interested in the history of our present modal qualifications, but rather in specifying how the present context is determining them.*

Thus, the logical modality of a logical modality is not in practice meaningful: the weakest component determines the whole. In any case, such issues cannot be construed as having so much importance in modal logic that they may play an axiomatic role, even optionally.

The only significant way such nestings of categories of modality occur in practice, is when the categories are of *mixed modal type*. Concepts like 'the logical necessity of a natural necessity' are quite legitimate. In this example, we are asking whether the proposition concerned is not only naturally necessarily but logically so; I believe, in this case, the reply to be that logical necessity implies natural necessity, though a natural necessity may well not be logically necessary.

Any mix of logical, natural, temporal, and extensional modalities can similarly be considered. My analysis of compound, fractional and integral modal propositions is intended as *an exhaustion of all the combinations of natural, temporal and extensional modalities with each other, for categorical propositions.* Logical modality is not included, because the other modalities are viewed as the ultimate objects of study; they are the goal, logical modality is only a means (ch. 51, 52).

But in practice, we should not rush to judgment, for the intent is often more complex than it seems. Thus, taken simply, 'X must always be Y' is redundant, since 'must' implies 'always'. But the intent may rather be that, each time 'X is Y' *actualizes*, it does so *inevitably (rather than spontaneously or voluntarily).* Likewise, 'X can sometimes be Y' may be simply viewed as an abbreviation of the compound 'X can and sometimes is Y', or in more clever ways. Such statements may also be intended to refer to *acquisitions or losses of powers*. (See ch. 34.3, 51 for introductory comments on these topics).

The issue is never verbal or grammatical or symbolic. Words do not affect the issue: what matters to logic is what we intend by them. It makes no difference whether we say 'It is necessary that X is Y' or 'X is necessarily Y' or 'That X is Y is of necessity true', contrary to what the Kneales suggest (553). It makes no difference whether 'must' refers to 'logically must' or 'naturally must' or 'always' or 'all', so long as we agree on a terminology: we all have access to the underlying concepts, anyway.

In any case, note, logicians cannot analyze such mixed-type stacks of modalities in a *generic* way, because for all we know to start with, different combinations may have different explanations. Some general rules might emerge as a final conclusion; but they should not be predicted offhand. The issue is complicated by the interrelations of modal types, which are not entirely continuous (see for instance ch. 38.2).

Logical modality differs radically from the *de-re* types, in that it concerns a different domain, that of 'contexts', instead of that of 'circumstances' or 'times', or again that of 'instances'. Yet logical necessity implies the natural and temporal, and natural and temporal possibility implies the logical, in categorical forms (ignoring issues of modality of subsumption). However, in conditioning, these continuities are inhibited, because logical forms have been designed as mere connectives, whereas *de-re* forms must have a proper basis.

Natural modality likewise surrounds the temporal, *but their categorical continuity is broken in conditional frameworks*, because of their different bases. Again, extensional modality stands somewhat apart from the natural and temporal, since it concerns groups rather than individuals. (See ch. 25, 34, 38, 39).

5. Contemporary Currents.

We find reference to the 'resemblance to quantity' of logical modality, in a 1962 book by A.N. Prior. Again, the focus is on that specific type of modality — 'assertoric', 'apodeictic' and 'problematic' are the words used for its categories (185). But in any case this shows that the analogy of logical to extensional modality, which was obvious since antiquity (with reference to the oppositions of modal categories and to modal syllogism), was acknowledged in modern times.

However, this analogy is useless without a significant explanation: the quantitative aspect in the logical modalities can only be brought out by defining them. The *given datum* of similar logical behavior between modality and quantity, should have been seen as *a clue to* some essential similarity: it was a missed opportunity for constructing a fitting definition of the modalities.

Robert Feys suggests that, already in the 19th century, 'it would have been rather natural that the calculus of propositions be conceived as… a modal one' in analogy to the 'calculus of classes'. Because, 'propositions were conceived as applicable to (verified in) various "cases", "circumstances"", "moments of time", "states of affairs"', so that 'an implication was a proposition asserting that all cases in which p is verified are also cases in which q is verified' (3).

This statement shows that, at least at the time it was written, in 1965, logicians did indeed come very close to a precise, quantitative definition of modality and conditioning, at least in a generic sense (if we take the latter use of the expression "cases" at its broadest). But, on second thoughts, the statement seems more intended to define *the form-content relation*, rather than the modal underpinning of implication.

In any event, as far as I know, modern logicians did not explicitly work out distinct modal and conditional logics for each of the types of modality implied by the words they used. They should have taken, as I did, "cases" (in a narrow sense) as the focus for an extensional logic, "circumstances" as that for a natural one, "moments of time" for a temporal one, and "states of affairs" (in the sense of knowledge contexts) for a logical one. For, when one does so, it becomes clear that these various logics have distinct (though parallel) properties, which justify their separate developments. My impression is that they lumped all types together, and considered the logical sense to be generic.

This impression is not entirely offset by, for instance, Rescher's listing of many types of modality, including the 'alethic' and 'likelihood' (logical), the causal (natural), the temporal, the deontic and evaluative (ethical). For, though it is clear that he is aware that there are varieties, he also lumps such intentional relations as 'believing' and 'hoping' into the list, showing that he is not aware of the characteristic pattern which defines a relation as modal. One may well argue that such attitudes are determined by modal judgments, but they are not themselves modal (White, 168).

Questions posed by Feys concerning modality reveal the state of knowledge of current logicians; he asks if it can be used 'for the description of the physical world', or 'perhaps… in the analysis of causality'. I infer that they had not yet formally treated the relations between logical and *de-re* modality, and had not yet developed the logic of *de-re* conditionings.

It is also interesting to note that modern logicians seem still disturbed by the existence of paradoxical propositions (like p implies notp, or notp implies p). Thus we find the Kneales complaining about the lack of success of logicians 'in excluding these so-called paradoxes without also excluding at the same time arguments which everyone regards as valid' (549).

Again, had they had good definitions in mind, they would have seen that there is no antinomy in such statements provided they occur singly, not in pairs. On the contrary, *precisely the existence of paradoxical forms allows us to define the concepts of logical necessity and impossibility,* as self-evidence (a proposition implied by its negation) and self-contradiction (a proposition implying its negation), respectively.

Let us now look at how a recent (1976) *Dictionary of Philosophy* discusses modalities. It focuses on 'alethic' modalities; these include the necessity or possibility 'of something being true'; the 'factual' is defined as neither necessary nor impossible nor merely possible. But such a definition is admitted to be circular: 'it is hard to define modal terms without begging the question'.

A statement without *explicit* modal qualification is called 'assertoric'; only if *the word* 'necessary' or 'possible' appears in it may the statement be called 'apodictic' or 'problematic'. The author, **A.R. Lacey**, admits that 'Kant uses "apodictic", etc. slightly differently to indicate how judgments are thought, not expressed'.

But this difference is far from 'slight', it is a still more massive confusion of the issues. Whether or not certain words are used, the logical status of the proposition is not affected; moderns do not seem to understand that. We could say that the words used indicate whether the maker of a statement is aware or not (or wants to stress or ignore) its logical modality. But, as far as logical science is concerned, *within the context of knowledge* of that person, the logical modality of the proposition is determinate, whether known and stated or not.

Admitting that 'modal logic… is not always limited to the alethic modalities', the article goes on: 'a difficult and controversial distinction, of medieval origin, is that between de re and de dicta modality', applying the former 'to the possession of an attribute by a subject' and the latter 'to a proposition'. Some 'view that de re modality is intelligible, and that there are cases of it, even if ultimately they must be analyzed in terms of de dicta modality', while others deny this view in some way.

Further on: 'the nature of physical necessity and possibility has been disputed for centuries, especially since Hume. Are they independent of logical necessity and possibility, or ultimately reducible to them, or merely illusory?' Also: 'can there be possibilities which remain possibilities throughout all time but are never actualized? Aristotle and **Thomas Hobbes**, among others, said no'.

It is suggested that the logically or physically necessary may be 'what happens… in all conceivable worlds or all worlds compatible with certain laws', though admittedly the words '"conceivable" and "compatible" have modal endings'. (Incidentally, **Bradley** and **Swartz** seem to attribute this suggestion to Wittgenstein (7), but as I recall the phrase 'all possible worlds' dates from Leibniz; in any case, such a phrase obviously cannot be used as a definition of possibility.)

I think that the results presented in my book adequately answer all these questions. The fact that they (and others like them) are still asked, in so recent an article, tells me that similar results have not been obtained by others. *The objectivity and scientific knowability of natural modality is demonstrated by my formal theory of modal induction.*

Natural necessity may be deduced from logical necessity (or of course from other natural necessities), *or induced by generalization* (according to strict rules) from actuality or potentiality, *or even logical possibility (by adduction)*; actuality is observable, as well as inferable deductively or inductively; potentiality may be deduced from actuality or necessity, *or be discovered indirectly by syllogistic means* (from other potentialities), or even induced from logical possibility (as a last resort, by adduction). And so forth: these issues are easily resolved, very formally, once the categories and types of modality are clearly defined.

It might be contended that I am being too picky, in evaluating modern understanding of modality. Are my definitions of the modalities so far different from the current ones? For instance, **Paul Snyder** of Temple University, in 1971, writes: 'Alethic modality is concerned with what must be the case in *every* possible state of affairs (necessity) or in *some* possible state of affairs (possibility)'. Is that so different from my own proposals in ch. 21?

The point is well-taken. The quantitative aspect of logical modality is by now, in Snyder, clearly brought out. The choice of words '*must* be the case that' and '*possible* state of affairs', may be excused, as not a circularity but a parenthetical emphasis. 'State of affairs' is equivalent in intent to what I call 'context'. It is sufficient to remark that 'while contexts exist (mentally or perceivably), they are *actual*'; there is no need to say that they are 'possible'. I am open, but still at least insist on my own wording.

The necessary is that given in all contexts of knowledge, the possible is that given in some. We do not need to specify that some of the contexts are merely 'possible', because that is understood from the awareness that contexts change across time and from person to person, that is, that there 'are' (in the widest, timeless sense) actual contexts other than here and now. And of course, it is important to stress that the environments concerned are (in the widest senses) empirical and logical data, that is, items of 'knowledge'.

Indeed, sticking with Snyder, we can regard the idea that there are many 'systems' of (modal) logic with more generosity. He says, provocatively, 'There is not precisely one correct system of logic. There are many'. But later he makes clear that by that he means alethic modal logic, temporal logic, deontic logic, and so forth. Some rules hold '*generically* for all the senses of "necessary" and "possible"', while 'some features… will not be shared' for instance by logical and physical modality.

Again, I cannot but congratulate and agree. I also subdivide modal logic into various 'types' (the extensional, the natural and temporal, the logical, the ethical), each of which has to be treated as a separate field, because of their distinct properties, though eventual parallelisms do emerge. Note again that I do not regard tense as such, nor knowing and believing, nor intending, willing or preferring, or the like, as 'modalities' in a strict sense, but as considerations which may underlie modal concepts (which are distinguished by quantification of certain phenomena).

One example of distinct properties of modal types is paradox. We have seen that logical conditioning need not be based on logical possibility; here, a *de-dicto* connection without basis, that is, based only on problemacy (a mere mental consideration of the theses) is quite thinkable; and paradox may therefore arise. In contrast, *de-re* conditioning must be based on the corresponding *de-re* possibility; a *de-re* connection without a corresponding basis does not exist (or more precisely, is too formal), and problemacy does not suffice; for this reason, there is no equivalent of paradox in *de-re* logic. You could say that we design our forms of conditioning, in such a way as to avoid such embarrassments on a *de-re* level, and keep them on a logical level. But in any case, the logics of *de-dicto* and *de-re* conditionings end up looking rather different.

In my view, in any case, it is misleading to call these fields 'systems', because that would suggest relativistically that there are many Truths. That each type of modality has distinct properties implies nothing of the kind, anymore than saying that 'cats and dogs behave in disparate ways' would do so. However, Snyder's statement does in fact proceed from the modern approach we have already noted, that according to the 'axioms' we more or less arbitrarily adopt, radically different complete systems of logic emerge, which may or not find practical application. For this reason, he adds that there are 'literally hundreds of distinct systems of formal logic'. With such attitudes, I cannot agree.

In my view, generic logic sets the common 'axioms'; the subsidiary 'axioms' serve for purposes of specification. There are no systems which qualify as logical, outside the general framework of laws like non-contradiction; special fields of logic merely add additional laws of their own. The perverse delight of relativism is not serious, and should be avoided; the logician is dedicated to strengthening common sense, not to try and debunk it (since that is in fact impossible, as repeatedly shown).

I also cannot accept the modern view, which is a direct consequence of such extreme axiomatism, and more deeply of conventionalist interpretations of language — that, once the axioms are declared, the theorems follow relentlessly, to quote Snyder: as 'a straight-forward mechanical procedure… that could just as well be done by a computer (and, as a matter of fact, *has* been done by an IBM 7074)' (1-12).

The work of logicians can never be divorced from philosophical considerations. Logic is inextricably interwoven with epistemology and ontology; the three evolve in tandem, stimulating inquiries in each other, mutually informed and informing. The directions taken by logical science result from a mass of insights into the world and our knowledge of it.

The totality of primary propositions required to develop a mechanical model of logic, is far greater than a few limited 'axioms': it is an innumerable number of experiences and intuitions related in very complex ways. The attempt to ignore that subtext is a sad falsification.

At every stage in the development of any theory of logic, one is called upon to consider countless, interrelated philosophical issues. The success of the theorist depends mainly on his ability to resolve such issues with vision, with the broadest regard for available data. There are *some points* in our progress, from which a series of developments follow more or less mechanically. But even then, the logician must be present, to determine what is relevant to human experience; to test, modify or reject. All a computer, which has no consciousness, only symbols of data (whether fed by humans or robotically acquired), can do, is duplicate the said mechanical segments of logic's growth.

The validation of logic is a function of a large number of insights. The reordering of the propositions *formed from* these insights, in accordance with the model of axioms and derived theorems, is perhaps an interesting and worthy research, but it is an auxiliary and *ex-post-facto* development. To suggest, as modern logicians do, that a dozen or a score of 'axioms' suffice to construct a logical system from scratch, is ridiculous.

Apart from that, we may severely doubt that the 'axioms' they propose are all indeed primary propositions. Most are certainly *not conceptually primitive*, but in practice and in theory the *end-products* of very significant preliminary, philosophical positionings, whether conscious or effective. This does not mean that logic is a derivative science, but only that its mental separation from philosophy is an artifice; the two are part of and depend on each other.

The *more geometrico* concept is itself an outcrop of logic and cannot be regarded as validating it; rather, logic confirms it for us, and encourages us in its use thenceforth. Logic develops from innumerable *individual* experiences (in the largest sense), including perceptual and conceptual insights, and *intuitions* of logical correlation, that we call consistency, conflict, implication or alternation. Intellectual validity is merely a subset of notional validity.

The geometrical model emerges from the theory of adduction, not from purely deductive motives. The perceptual and conceptual insights may be taken for what they are, or eventually grouped into forms; the logical intuitions may be taken ad hoc, or generalized into logical science. The 'common sense' art of logic, is the parent, not a poor cousin of the science. As the patterns of our inductions emerge, we come to see the value and importance of deductive ordering of the information into 'axioms and theorems', as a final step and a test. But the validity of the whole stems from the roots.

The relation of logical science to logical practice is not merely one of consistent one-way implication — in the manner, say, that physical theories relate to their predictions. That adductive relation is indeed visible, after the fact; but it is two directional. Not only is logical practice generalized into logical science, but logical intuition demands that we admit logic to be *the only* framework suitable to our experiences. It is more than just *a* theory with fitting predictions: it is the sole theory acceptable to logical intuition. The deductive inference of theorems from axioms depends on this notion, and cannot therefore be used against it.

6. Philosophical Discussions.

I find myself in closer agreement with the views of **Alan White**, of Hull University, expressed in his 1975 study of modal thinking. He analyzes common usage of modal qualifications, discussing what we ordinarily mean by them with reference to examples, and arrives at very balanced and intelligent conclusions. His style reminds me much of Joseph's: perhaps there is a British way; in the case of White, three quarters of a century of modern subjectivism and linguisticism have intervened, but the good sense he brings to bear is characteristic (165-179).

The views held by some, that modalities express 'some subjective feeling or mental experience… of compulsion… or confidence', or 'the attitude or mood' qualifying our assertions, are rejected by White. His argument is that the alternatives are not either some such thesis, or a naive objectivism, according to which the modalities are grossly physical — but there is a position in between, a more nuanced objectivism.

Cases in point are, the argument of the great, 18th century British philosopher **David Hume**, with regard to logical necessity and also to 'causality' (meaning natural necessity), to the effect that 'since there is no ostensive quality or object called "necessity",… [it] must be the name of something subjective', and similarly that of Ludwig Wittgentein, in the present century, that '"essential" is never the property of an object, but the mark of a concept'. Note that in the latter case, the relevance of the statement proceeds from the Kantian position that necessity is a property of definitional predicates; but of course Wittgenstein's emphasis is more linguistic.

For White, 'if anything is modally characterizable, it is so because in a certain situation in terms of alternatives relative to a given end… and viewed under a certain aspect (e.g. physically, logically…), it is open (can, may, possibly)… or is the only one (necessary, must…). In short, modal concepts do not signify particular items either in the world or in our minds, but the relation of one item to others in a situation'. The 'relative nature of the modals' (he also uses the word 'contextual'), allows for 'an objectivity free from traditional objections'. What is so nice to see, is White's firm conviction that *logicians who do not arrive at some sort of objectivist conclusion, have simply failed to answer the question.*

This is my view, too, that modality is a relational qualification, which is objective, not in the sense that it is concretely perceptible, but in the way of an abstract existent which is apprehended by conceptual means. The choice is not merely between mental or verbal inventions, and sensory phenomena; we may also, through direct intuitions (on a notional level), or indirectly by the accumulated constructions of such intuitions (on an intellectual level), arrive at an insight of realities, which are just as objective as sense data. The abstract is simply another kind of phenomenon than the concrete, but equally 'empirical'. I have shown justifications for this view in ch. 60-62; I need not repeat them.

White is also concerned about the modern interpretation of the old *de-dicto/de-re* distinction. For logicians like Russell, Rescher, Von Wright, all types of modality qualify whole propositions, and are 'therefore' *de-dicto* (suggesting that they are not objective, as already said). White rejects the conclusion, saying 'there is no such thing as modality de dicta… different modals… can all be classed as de re'. (in the sense that they are objective, in the way of descriptions of relational phenomena, as just seen); but he goes on to infer that they do not qualify propositions as such.

Here, I slightly disagree with him, but the differences are inconsequential and verbal, rather than fundamental. I agree that even logical modality, let alone the natural other such types, is essentially objective. It does not, however, follow that we may not regard modal concepts as qualifying propositions, in the sense of their contents (rather than as utterances). He is contraposing an incorrect hypothetical.

I favor retaining the *de-dicto/de-re* names, to stress an important difference. Logical modality appeals to a maximal context, which includes constructs *granted as fictional*, as well as presentative data of a perceptual or conceptual kind; whereas modalities like the natural, arise in *more limited frameworks*, to the exclusion of known fantasies. Thus, we have good reason to make a differentiation, even while acknowledging the equal objectivity of results obtained in all these fields.

For me, the concept of modality refers essentially to *degrees of being*. At first, the 'is' copula appears to have a single, straightforward sense: this is the 'is' of *indicative* propositions, referring to singular, actual, now, true, events. But as we proceed with the enterprise of knowledge, we grasp that there are nuances in this concept. In some cases, the relation is very *finite*, in others, it has a broader frame of reference.

It may be more *'forceful'*: general, independent of circumstance, timeless, free of the influences of phenomenal changes; or it may be more *'open'*: particular, potential, temporary, imaginable. In this manner, the need for concepts of necessity and possibility emerge, as stronger or weaker *versions of* the primitive concept of mere presence. Likewise, needless to say for the negative equivalents (and similarly also, incidentally, for ethical value judgements, except that they involve still more complex relational issues).

These (pre-scientific) notions of other levels of being, are eventually formalized, in such a way that their hierarchies are made evident, and rules for their induction are developed. This additional layer of intellectualization (the science of logic) is what gives the modalities an appearance of being propositional qualifications, but they essentially concern the content. Mental processes leading to their acceptance as justified knowledge are incidental; what counts is the final status they deserve.

Incidentally, in reply to Hume: it is clear that the causal notions of force or openness are pre-verbally implicit in the notions of modality, but their formal elucidation is a later stage. First, modality is applied to single predications; then, to their conjunctions; finally, after the various types of conditioning have been studied, we can begin to study causality, in its various senses (see ch. 33.2, 42.2). Also, in reply to Wittgenstein: the fact that we are free to *focus* or not, on different aspects of things, does not imply that what we have any choice over what we thereby become aware of; the contingency of consciousness does not signify contingency for its objects.

A contemporary writer on issues relating to modality, who seems to be quite prolific and distinguished, is **Jaakko Hintikka**, of Helsinki and Stanford Universities. He also, like White, is I would say a philosopher, rather than a mere logician, in his approach to modality. I do not agree with every detail of his views, but his overall attitude that the battleground for modality is in the wider neighborhood of epistemology and ontology, is refreshing.

Thus, we find Hintikka saying, in a 1969 essay on epistemic logic, 'the usual straightforward axiomatization of the logic of philosophically interesting concepts is likely to be a worthless enterprise unless it is backed by a deeper analysis of the situation'. A similar guideline has always controlled my own theorizing.

Hintikka criticizes not only symbolic logicians, but also those 'ordinary language analysts' who engage in 'mere description of the raw data' of ordinary language; he calls for a 'genuine theory of the meaning of the words and expressions involved'. Of course, that criticism is not applicable to all ordinary language analysis; Hintikka engages in some himself (also, incidentally, White's analysis does not have this fault, in my view.) I will use this essay as a springboard for certain remarks (3-19).

When we scrutinize examples (or 'paradigms'), with a view to understanding both their epistemological determinants and their ontological content, our aim is not a mere 'summary of the data' of common practice, a descriptive generalization. If logical science consisted merely of summaries of our linguistic habits, it could not have any prescriptive role.

Rather, we select those cases which seem most significant, and by adduction formulate a theory around them, which is then worked out in formal detail. Such seminal and pivotal concepts acquire a normative character, not in the way of an artificial imposition (or 'regimentation'), but because they *all together* suffice to eventually construct and explain all less important cases.

Thus, when we reserve the word 'can' to one of its senses evident in common practice (namely, potentiality), we do not thereby exclude all the other senses, but intend to later deal with them, either as subsidiary senses or using other words. If an ordinary use of 'can' is not included by our rigid definition of the word, we are free to assign another word to that other sense (like 'sometimes' for temporal possibility, or 'may' for the extensional version). The choice of words is to some extent arbitrary, though we try to keep close to the more frequent idiom; but the underlying conceptual distinctions are in any case not open to choice.

This also seems to be Hintikka's thrust. He contrasts 'basic meanings', which are used as 'explanatory models', and 'residual meanings', which are modifications of the former by various factors. He mentions as an example, how a double negation may in practice convey hesitation or uncertainty, or signal diffidence, or express irony. I will not review his discussion in more detail: the purpose is served.

However, I want to object parenthetically, once again, to the consideration of 'epistemic' relations as types of modality. For me, knowledge and belief are *not* the primary parameters underlying modality (logical or otherwise): it is *phenomenal appearance and their seeming credibilities*, which structure our logical intuitions and modal concepts. Epistemic relations are legitimate objects of study for logical science, but they are not modalities: they do not fit the 'statistical model' which differentiate modalities, nor serve as singular cases for the plural constructs.

They may indeed take the place of modalities in everyday discourse, but their meanings are not equivalent. 'I know so and so' may well imply that so and so has been found true in my context or absolutely necessary, but it is not a qualification of the being of the appearances at hand; an extraneous relation of them to their observer is juxtaposed. The additional information may well be valuable, but it is incidental; the focus is on the Subject and his consciousness, instead of only on their Object.

The role of epistemic logic, in logic as a whole, is therefore not as fundamental as that of proper modalities; it is a tributary field, and it is no accident that it has arisen in our science rather late in the proceedings and with great difficulties.

The attempt to bypass philosophical issues, and deal with modality in a limited framework of 'just logic', is either naive or a self-imposed tunnel vision. What becomes clear, is that *the issues of modality pervade philosophy,* since its inception. Most philosophical issues have something to do with modality, one way or the other. This is well brought out in a 1976 paper by Hintikka, with the tantalizing title of *Gaps in the Great Chain of Being* (Knuuttila, 1-17).

In it, in the way of a commentary on a book by Arthur Lovejoy (who coined the key phrase), he traces the history of 'the idea that all possibilities are eventually realized', which is known as 'the Principle of Plenitude', showing how it has taken on different meanings in different cultural and philosophical contexts. So doing, Hintikka demonstrates his pretty clear grasp of the various types of modality, and how the emphasis has shifted from one type to the other across time.

According to Hintikka, apparently with reference to the famous sea-fight problem, Aristotle subscribed to this principle, at least ontologically. *In natural modality*, plenitude would mean that all potentialities either have been, are or will sooner or later be actualized: a general inevitability of actualization. (I am personally not convinced that Aristotle had such deterministic opinions, but we will let it pass.) Among later philosophers, Hobbes affirmed such a view, Liebniz denied it.

But the argument behind the belief in plenitude being 'how can a possibility prove its mettle — its reality — except by being realized in time?' — its significance is especially epistemological. *In logical modality*, plenitude would signify that whatever we but conceive, has a past, present or future correspondent in reality. That is how, we are told, **Thomas Aquinas** interpreted the argument, as a result of his 'kind of empiricist epistemology', which required that 'in order for us to receive "true" concepts, they must… already be exemplified in antecedent reality'.

Additionally, René Descartes' statement that 'matter takes on, successively, all the forms it is capable [of]', is interpreted by Hintikka as an *extensional* version of plenitude. He argues, 'if all possible species are always realized, we have a doctrine of the permanence of species'; or alternatively, 'if all potential kinds of beings will sooner or later emerge… we have a doctrine of infinite evolution'. With reference **G.E. Moore**'s discussion, in this century, of the idea that the 'goodness' in some things is identical with the things themselves — Hintikka also proposes a *temporal* interpretation of plenitude: 'whatever happens always, happens necessarily'. I will comment on these topics presently.

Hintikka's main purpose in these and other examples, is to show how *an issue* traverses history, taking on different shapes as the 'conceptual environment' changes. His thesis is that the 'hidden ambiguities' in ideas like the principle of plenitude, make them excellent reflectors of the varieties of philosophical outlooks, serving as analytical vehicles for historians.

Thus, he points out how, as of the Renaissance, the 'gradual widening of what counts as possible' caused a shift in the concept of compulsory realization of all possibilities. At the time of Aristotle, and up until the Middle Ages, the realms of the naturally possible and the logically possible were more or less equal in extent, so that these concepts could be confused. However, as of the Renaissance, when social and technological developments and intellectual imagination expanded tremendously, 'the richness of the range of possibilities' (in every sense) was highlighted.

In one respect, the new optimism could be interpreted as meaning that, not only was more feasible and therefore more was conceivable, but also because more was conceivable, more was feasible. But, on another level, the rates of change were different, so that in the last analysis, more was conceivable than was feasible. For this reason, I say (and Hintikka seems to have this view), the question of whether possibilities are bound to be realized, changed sense: from a nature-oriented one, to a more extreme imagination-oriented one.

Kant's position on the topic is ambivalent, according to Hintikka; but he argues that if it is the mind's structure which ultimately limits human experience, as Kant suggests, then a different structure could well 'in principle' reveal a different range of possibilities, and no structure, unlimited possibilities. He points out that there is no 'independently defined range of possibilities'.

In any case, this argument (though somewhat circular) is interesting, not only because it highlights the epistemological subtext of possibility, but because it shows that if the possibilities are infinite, logical and natural modality issues fuse into one. Enough of history. Hintikka's paper clearly shows that modality issues may be found at the center of many of the most fundamental issues of philosophy.

Today, the principle of plenitude is supposedly not taken seriously by most thinkers. But anyway, let me make my own positions on some of the issues raised above clear.

First, logical science cannot presume to make any extrapolations like ‘all possibilities are eventually realized’. *Ab-initio*, as is evidenced by the fact that some people believe otherwise, we must if possible make *formal allowance* for the eventuality of the opposite thesis, otherwise philosophers would be deprived of a language with which to discuss the issue at all. Thus, the issue is not of concern to formal logic as such; for us, the concept of inevitability of realization is a legitimate object of study, which may or may not eventually be found to be a null-class.

Personally, I have no doubt in the existence of some noninevitable actualizations (referring to natural modality), since I believe in the existence of free will (though not in purely material indeterminism, incidentally). Discussion of free will, or spontaneity more broadly, belongs in a work on aetiology, but I have already mentioned how it can be formally introduced into logic (see ch. 34.1). Similarly, for me, the realm of the logically possible is in principle greater than the realm of the naturally possible (though there may well be potentialities that we never get to even conceive), since we seem able to imagine fictions, and if everything was true, contradictions would arise. These are also the views which seem balanced and sensible to most people.

In a purely deterministic world, a given piece of clay would either be bound to become a pot sometimes or be bound to never become one; but in a world with free will, a piece of clay may remain forever only potentially a pot. In formal modal logic, we express this in the doctrines of opposition and compounding of modal propositions. For instance, a never actualized potential is signified by the compatibility and conjunctions of **Ip** and **Ec** (that is, ‘some X can be Y’ and ‘no X is ever Y’). Similarly, with reference to imagination.

Secondly, note the intrusion of the concept of time in the generic statement ‘all possibilities are eventually realized’. In natural and temporal modality, the time involved is that framing the events *themselves*; whereas, in logical modality, the time-frame is *how soon we become conscious of* the events, which may themselves be perfectly static. Extensional modality has *no* inherent time-frame, though we tend to apply a chronology to it, with reference to the sequence in which we become aware of different instances (see ch. 22.3).

The plenitude principle can indeed be conceptually applied to natural and logical forms of conditioning, but it has no equivalent in temporal or extensional forms of conditioning. ‘This X is sometimes Y’ already *formally implies* the temporal actualization of some ‘this X is Y’, and, likewise ‘some X are Y’ formally implies the extensional instancing of some ‘this X is Y’. Since the implied singular is in either case an actuality, it is a redundancy to raise the plenitude issue concerning them.

We can indeed ask, *ex-post-facto,* whether these implied actualities or instances arose with natural inevitability or otherwise, but there is no equivalent concept of temporal or extensional ‘inevitability’. In contrast, ‘this X can be Y’ or ‘this X might be Y’ do not formally require that ‘this X is Y’ be ever actual or true (or, at least, in their case, the point is at issue). But even with regard to logical plenitude, the ‘inevitability’ of realization involved is not itself strictly a logical modality, but a natural one: it is the natural necessity that we *come to know* the facts concerned.

With regard to the Cartesian claim mentioned earlier. It could be interpreted naturally: the potentialities in any individual (and therefore for the species as a whole) must all eventually be actualized; or logically: the conceivable properties can in principle all be assumed to exist objectively, and perhaps will inevitably all be directly known by someone someday. But in any case, these are *not* purely extensional propositions, contrary to Hintikka’s analysis.

Likewise, the interpretation of G.E. Moore’s discussion by Hintikka, in terms of ‘what happens always, happens necessarily’, is neither relevant to the Moore issue, nor does it formally constitute a temporal modality application of the plenitude principle. The relation of goodness as such to the entities, states, or motions, of matter or mind, which it characterizes, is a form-content relation. The things which produce goodness as such in the world, do not have to themselves be instances of it; that is, ‘X *causes* Y’ does not imply ‘X *is* Y’. As for the unrelated extrapolation from always to necessarily, it does not fit the bill of ‘inevitable actualization’; taken dynamically to mean determinism, it would be a natural modality proposition, and taken statically to mean implication, it would be a logical modality proposition.

Thus, in conclusion, there is in any case no generic principle of plenitude. There are only specialized situations in which such a principle might be discussed. One additional remark: none of the above discussions focus on the logic of 'acquisition and loss of powers', which, like the logic of 'modalities of actualization', represents a dynamic complication of static modal logic. I have not attempted to work these logics out in detail in the present treatise, but it should not be too difficult.

Thirdly, the historic discussions of modality confuse two issues. One, is defining modality and its forms, and working out their deductive properties. Two, is determining how we *induce* modality for specific contents. The argument 'how can possibility be known to exist, except through its actual manifestations?' at the outset limits us to perception, and ignores the very conceptual methods which allow its formulation.

The inductive problem, of predicting possibilities *indirectly*, has never clearly been differentiated. As far as I can see, my effort to formalize the complexities of modal induction is completely original, not only in its specific findings, but *in at all having raised the issue*. For this reason, many of the queries raised by past philosophers have become obsolete: they henceforth belong to pure logic.

Enough said. The wide range of mutual dependencies between modal logic and philosophy has been amply demonstrated. I do not propose to try answering all the question raised (and why not solve the world's problems while at it!). In any case, the literature research effected in this chapter has served to show the level of knowledge (and the blank areas) other authors have attained, with regard to tropology.

66. METALOGIC.

1. Language and Meaning.

It is a truism that ‘there is a grain of truth in all falsehood’. Always keep this in mind when evaluating theories. No idea would ‘make it’ in the world, if it did not have some appearance of plausibility. The trick is to remove the husk of confusions, to go past the surface appearances of things.

Some individuals are dishonest, and deliberately try to fool people; though as we all know, ‘you can fool some of the people some of the time, but you cannot fool all of the people all of the time’. But mostly, of course, errors arise inadvertently, because people are careless in their thinking, lulled into security by familiar words and superficial consistencies. Mistakes snowball, with people uncritically accepting what others have done, especially if those others have acquired prestige and fame.

Deep inside, many people fear ridicule, and are often tempted to gloss over what they cannot understand, rather than dare challenge current authorities. On the other hand, of course, every nincompoop with a crazy theory claims to be a misunderstood and cruelly rejected Galileo. The point I am trying to make is this: I ask the reader to be both open-minded and critical; to think anew for him or her self.

The ideas of modern logic have become so accepted by the academic establishment that it does take a special effort of independent thinking to overcome their power.

The reader is now referred to the article entitled ‘Metalogic’, by **Hao Wang** of Rockefeller University in New York, in the *New Encyclopaedia Britannica* (23:272-279).

In my view, the term ‘metalogic’ may be taken in a broad sense, to refer to *the study of the perceptual and conceptual foundations of the science of logic*. But the term is of modern coinage, and is currently associated with the specifically modern view of what these foundations are. For this reason, it is defined in the article as ‘the study of the syntax and semantics of formal languages and formal systems’.

The reader is asked to keep in mind the distinction between the open sense of metalogic, and any partisan position about its application. What is at issue, here, is not the legitimacy of such a study, but the correctness of the current view of its content. I have already pointed out certain confusions which lie at the root of the modern position, such as the confusions between formalization, symbolization, systematization and axiomatization.

A fundamental issue for logical science, is *the relation between words and things* — meaning, intention, reference, or significance. The study of this relation is known as *semiotics* (with or without the final *s*); it is **John Locke** who first applied the term to this context, in the 17th century. Essentially, this is a theoretical study of the function of language as such in knowledge; it is a branch of philosophy, with both epistemological and ontological components. Language, as a medium of thought, memory, and communication, whatever its actual embodiment.

Obviously, the study of language*s* which exist or have existed, is a helpful empirical accessory. This is the role of *linguistics*, and branches of it, like philology and etymology. Linguistics studies word and sentence formations; the sounds, shapes of alphabets; the varieties and changing meanings of vocabularies; the structures, uniformities and differences, of grammars; the historical development and geographical varieties, of past and present languages everywhere, looking at all their literary, cultural, or social manifestations. It may even tie in with ‘physiological, psychological, ethnological, sociological’, and similar researches.

Now, more recently, **Charles Morris** distinguished three branches for semiotics (which is not to be confused with linguistics, note well): ‘syntax’ (or ‘syntactics’), which studies words and their patterns of arrangement, without reference to their meanings; ‘semantics’, which studies the meanings of words, without reference to their users; and ‘pragmatics’, which considers the users as well. These distinctions were endorsed by Rudolph Carnap, and became commonplace in modern metalogic.

On the surface, these distinctions might seem reasonable enough, and indeed their having been made by modern logicians would seem to belie my contention that modern logicians lack a clear understanding of the relation of words and things: after all, do they not by means of these distinctions acknowledge the three components of ‘meaning’? However, reflect.

We might list existing words; we might describe how they happen to come together in actual languages; or we might discuss words collectively. But, without reference to any actual or proposed meanings, there is nothing more to say about specific words. So, in fact, *‘syntax’ and ‘semantics’ are inseparable*. Similarly, we might well view ‘pragmatics’ as a branch of linguistics; but it would be artificial to isolate within the theoretical study of semiotics all the propositions which mention the medium between words and things, and label them as ‘pragmatics’.

The relation of words and things is *mediated by a conscious being*. Symbols used by a computer or robot cannot be said to be representative of anything, except insofar as *we* humans assign them some external object. A machine cannot intend anything, only we can. We may indeed think of specific words and their meanings *in abstraction from* ourselves, the ‘users’; but the equation between them is inextricably tied to us. In short, all theoretical discussion of semiotics involves mention of *all three* concepts: words, their meanings, and those to whom they are meaningful; it is not possible to meaningfully divide semiotics.

The tie between semiotics (the philosophy of language) and linguistics (research into actual languages) has been pointed out: the latter provides a database of examples for the former to take into consideration in its theoretical investigation. However, one important difference need be pointed out, and that is: *the essential difference between a grammatical sentence and a logical proposition*.

A sentence need not be meaningful: if it consists of *separately* meaningful words which are strung together in accordance with the *usual* patterns of our tongue, it is grammatically okay. A proposition has to be a logically tenable construct: logic may *upon reflection* declare a sentence, albeit its apparent meaningfulness (in the sense just described), to be in fact meaningless. This is a conceptual judgment, with reference to the extraordinary *collective* impact of the words used.

In some respects, as we have seen repeatedly, logic is not bound by language. In principle, it could be wordless: it needs only consciousness, words are only instruments for it; any language will do, an existing one or an invented one, provided we all know what we mean by it. In practice, logical science uses a modified version of ordinary language (to avoid ambiguities and equivocations), or even a symbolic equivalent. But, in other respects, logic is more restrictive than grammar: it pays more attention to the end result of word constructions, their overall meaning.

Logic performs this additional selection with reference to internal consistency. For grammar, anything goes, if the parts are understandable; for logic, the result, the whole, too, must be understandable. You may say that grammar is syntactical and logic is semantical, but that would not be an accurate rendition of what modern logicians understand by these terms.

For example, the Liar Paradox. We saw that, although ‘This statement is false’ is a grammatically meaningful sentence, it results in a double paradox. In trying to make sense of this logically impossible result, we noticed that we used the indicative ‘this’ to refer to a construct which included it. Not finding any other explanation for our predicament, we inferred that it was caused by the artifice of self-reference. And indeed, upon reflection, we realized that the very idea of something pointing to itself (its whole self, not just a finger pointing to a chest), is so convoluted as to be unconscionable.

We thus came to the conclusion that such a sentence is logically meaningless: it is a non-proposition, it is as if nothing had been uttered. It is perhaps not the inherent self-contradiction alone which implies meaninglessness: it may merely serve to reveal a conceptual problem, or to confirm its seriousness. Perhaps all self-contradictions are ultimately meaningless, but in this case the concept involved was already unsound-looking.

In any case, *once we become aware of the illusoriness of such sentences, we may no longer even say of them that they are false* (let alone true). To say of them that they are false, already elevates them to a status of conceivability: but they do not even qualify for that. They just have no place in the universe of logic.

For classical logic, a meaningless sentence like the liar paradox is neither true nor false. That is, both ‘this statement is false’ *and* ‘this statement is true’ are neither true nor false; once we grasp that the former statement is meaningless, it follows that the latter is too. The meaninglessness of self-reference is in either case intuitively obvious: the double paradox arising from the first statement only serves to further highlight and confirm that meaninglessness, and the second statement is just as meaningless even though it leads to no overt contradiction. The paradox is not the conceptual source, but a symptom, of the inherent meaninglessness.

Now, classical logic, as we have seen, is concerned only with meaningful sentences — propositions — and its purpose is to determine how they are to be judged true or false. For us, *the meaningless has no logic*: once a statement is uncovered as intrinsically unconscionable, it ceases to be a topic of discussion; no sense can be made of it, there is no profit in looking for its ‘logical properties’, only confusion and contradiction may be expected to result.

In contrast, it seems to me, modern logic would like to be so *‘generic’*, as to be a formal study over and above meaning, and therefore applicable to meaningless sentences, as well as the meaningful. It is to such a broad formal inquiry that moderns seem to apply the term ‘metalogic’. For this reason, logic is for them primarily ‘syntactic’ (purely symbolic, entirely a priori and deductive), and only thereafter, more or less optionally, ‘semantic’ (applicable to meaningful systems, ‘satisfiable’).

2. Definition and Proof.

Modern logic is built on the idea that everything in a system must be defined and proved. On the surface, this seems like a perfectly reasonable demand, and indeed it has been a driving force of classical logic, and for that matter science in general (substituting ‘as much as possible’ for ‘everything’). However, logicians like Carnap have attempted to push this demand to *an impossible extreme*, by giving the words ‘define’ and ‘prove’ very narrow interpretations.

They argue, effectively: We have to ‘define’ every word by a previously defined *word*. If so, the regression is bound to be infinite. If it is not to be infinite, then there must be some arbitrary first words. Ergo, knowledge is as conventional as language (this is of course a *non-sequitur*).

They argue, similarly: We have to ‘prove’ to be true every sentence we claim. If so, the regression is bound to be infinite. If it is not infinite, then there must be some arbitrary first sentences. Ergo, knowledge is an axiomatic system (again a *non-sequitur*, since as we shall see an alternative position is possible).

Furthermore, they argue: ‘truth’ itself is a word that has to be ‘defined’, and that sentence in turn has to be ‘proved’ as a theorem within the system, or be an axiom of the system. So, truth too is arbitrary, in both those respects. Thus we read, ‘there is a definite sense in which it is impossible to define the truth of a language in itself’.

Their ideal of logic was therefore to construct a calculus which would consist of the barest minimum of ‘axioms’ from which all subsidiary ‘theorems’ would be derivable mechanically — say, by a calculator such as **Alan Turing** imagined — with reference to precise ‘formation rules’ and ‘rules of inference’.

It is interesting to note that they nevertheless introduce and discuss their theories, not in the ideal language they are presenting, but in ordinary language. Evidently, this implies that their language and logic is a subset of ordinary knowledge; that is, that they depend for their understanding and conviction on the knowledge subsumed by ordinary discourse and methodology.

If their 'formal languages' and 'formal systems', so-called, are so ideal, then they should be able to present them *entirely* in their own gibberish. In that case, would they or anyone understand or believe anything they say, do you think? Clearly (to use a Randian phrase), they are 'stealing the concepts', and they fail to fulfill their own metalogical ambitions. To be perfectly independent, an ideal language and logic would have to be comprehensible and convincing without any use whatever of ordinary language. If they wrote such a 'purely symbolic' book, would anyone go for it?

Thus, to continue, what was not definable and provable by and within a system, seemingly had to be referred over to something outside the system, presumably a larger or antecedent system. Ultimately, as we saw, that implied the reliance on some arbitrary system.

Within that framework, it is no wonder that Kurt Godel's theorems of consistency and completeness are labeled as 'fundamental discoveries'. In a celebrated 1931 paper, Godel argued as follows, in reply to the said 'paradoxes' (I am paraphrasing the philosophical thrust of his theorem; I am not concerned with its mathematical ramifications). Note that logicians call a system syntactically 'complete' if 'there is in [it] any sentence having a definite truth-value in the intended interpretation such that neither that sentence nor its negation is a theorem'.

If a system asserted itself as 'complete' (entirely self-contained), it would be admitting of itself: 'I am not definable and provable by myself, within myself (or ultimately, not in a nonarbitrary way)'. But Godel apparently interpreted that sentence as equivalent to, or giving rise to, the liar paradox. Hence, he inferred, such a system would be 'inconsistent'.

Contrapositely, if the system was 'consistent', the statement of its consistency would need be internal to the system (which would be arbitrary), or external to the system so that the statement 'I am not definable and provable etc'. would hold, and that would be an admission of 'incompleteness'.

Thus, Godel concluded that a system cannot be both complete and consistent. This principle, however puerile it may seem, was welcomed as a crucial defense of reason, because it seemingly put a limit on the excessive arbitrariness of the purely linguistic programme, like perhaps the Logical Positivism of Wittgenstein. It showed that *some* limits exist; it suggested that there were rules of behavior even on the purely syntactic level, over and above any semantic model. It gave more specific shape to, and justified, the whole idea of a 'formal metalogic'.

I think that is a fair assessment of the views in question, at least in the context of my knowledge of them. Let me now try and answer them. All these arguments contain 'grains of truth' which make them seem credible and perpetuate them in logical circles; but all of them are dead wrong in my view. *The whole enterprise of seeking 'fully and internally defined and proved' or at least 'openly limited' knowledge is fallacious.*

As one acquaints oneself with modern logic, one notices that it comprises a number of normative concepts which are not found in classical logic, or at least not with quite the same significance. They misunderstand concepts like definition, truth, proof, and validity. Classical logic uses words to that effect with the utmost caution, whereas modern logicians indulge in them freely, as if they have some clear and absolute knowledge of these things. Their theories effectively deny the power of knowledge, but apparently they except themselves from such judgment.

a. Definition.

In classical logic, definition does not consist in equation of words. The concept of 'definition', as a process, is gradual sharpening of our focus on an object; we select as 'definite' that manifestation of an object which has the sharpest focus. Thus, an object viewed through a microscope or telescope is accepted as 'at its best' when it seems at its most solid and colorful. Definition is essentially an act of seeing, perceiving, paying attention to, an object, and selecting one of its enduring manifestations (whether concrete or abstract, whether simple or complex) as its 'defining' aspect.

Definition involves two (compound) propositions: first, that certain phenomena have appeared to us, and that they had such and such configuration; second, that those certain aspects of those phenomena are their most enduring, and (in the light of all previous experiences, perceptual or conceptual) somehow intuitively most 'interesting' to us.

Both these propositions are empirical in the widest sense: both are based on a mass of perceptions, conceptual insights, and logical intuitions. Note that the relational concept 'is', is itself very abstract; so one can in no wise claim any proposition to be entirely concrete in content. Every act of perception is allied with acts of conception (in the simplest sense).

Also noteworthy: all these propositions are quite thinkable without any use of words whatsoever. In fact, a large part of our everyday cogitation is completely wordless: we perceive, we conceive, we mentally imagine, without reference to words. All that is necessary for 'definition' is our consciousness and something to be conscious of. The 'defining' aspect is first of all an aspect *of the object* itself.

We may choose to represent something thus seen by a word or symbol, but we do not thereby create anything. We give meaning to the word, merely by (mentally) relating it to the experience, in the way of a token for it. But we do not thereby invent the meaning itself, the object, the aspect of the object, its seeming exclusiveness and import. The only arbitrary thing we do is choosing a certain combination of sounds and/or shapes, as the one we will attach to that object. But *that* the object exists, *that* it has such and such a configuration, and so forth, *and that* we honestly (rightly from the start, or ultimately wrongly) experienced these events is indubitable.

Also to be kept in mind, the ontological notion of *predication* is fundamental to definition. This refers to a sense of 'S *is* P' more elusive and yet deeper, than the mere numerical classification of S in P, in the sense that 'something S' *is one and the same as* 'something P' (what is called 'extensionality' in class logic). The latter is a permutation of the former, one of its implications; they are not 'equal'. The meaning of the copula 'is' is richer than its quantitative aspect; it has a qualitative aspect which should not be totally ignored by logic.

b. Truth, Proof, Validity.

Similarly, in classical logic, some material proposition is called 'true', if it appears to have more intuitive credibility than its contradictory, or all its contraries, in the light of all our accumulated perceptual and conceptual experiences, and all our logical insights of both inductive and deductive kinds. A formal proposition may also be called 'true', insofar as it is (in part) a material statement, concerning its constants specifically, which seems entirely unaffected by the content or status of its variables.

When we use words like 'intuitively true' or 'logically true', it does not mean that we believe them to be true in different senses, but merely to point out the kind of proposition involved (how full or bare it is). There is only one kind of truth; all truths are both intuitive and logical. All insights, including the logical, require an act of consciousness; it is impossible to think without thinking *of* something, there is no thought without some *content*.

Our abstract knowledge of the 'laws of thought' of Aristotle is not the predominant source of our conviction that some particular intuited contradiction is an 'unacceptable' phenomenon. These 'laws' *add to* our conviction (if we have learned them), because they remind us that *similar events have occurred before*, and that events of that kind are 'unacceptable' and to be dealt with in some way or other. But in each case, the particular intuition still has an independent force of its own.

Thus, these 'laws' are not 'axioms' in the modern sense. Indeed, if one reflects, it is obvious that to *apply* a principle to a particular situation presupposes an ability to *recognize* that situation as a case in point. With regard to a situation of contradiction, it is precisely the *insight* of an inherent flaw in the given conjunction which allows such recognition. It follows that we do not need the 'laws' (in the way of modern 'axioms'), since their application can only be a later event, and is just as particularly intuitive.

The practical value of Aristotle's principles, is that they *remind us* to consider the data in an orderly fashion, so that any contradictions which might in fact exist are made *visible*. They encourage us to *look out for* certain kinds of problems; that is all. The normative ingredient of 'unacceptability' is inherent in the phenomena themselves; awareness of the 'laws' is not the *cause* but an effect of our disbelief in contradictory situations. The 'logical necessity' involved is the sense of self-sufficiency of such experience.

In classical logic, the word 'proof' is preferably used with reference to material propositions, and its value is in the overwhelming majority of cases contextual; only in very rare cases do we encounter extremely unassailable logical necessity. Proof is generally a mix of inductive as well as deductive processes; there is no such thing as purely deductive proof. For moderns, in contrast, 'proof' is a mechanical process.

Classical logic prefers to use the word 'validation' when dealing with formal propositions, because their dependence on empirical developments is comparatively minimal; once their constants are induced, there is almost no expectation that the logical insights made in relation to them will ever need revision (though it does happen: witness the historical error concerning first figure syllogism with two potential premises). For us, validity is not something applicable to 'all possible worlds', as the moderns say; it concerns all the possibilities in *this here* world (without prejudice as to its dimensions, physical or mental): only *it* concerns us and is accessible to us.

Modern logicians like Godel yearn to 'prove consistency'. But in classical logic, a proposition is internally 'consistent', or a set of propositions are mutually 'consistent', if we have had no logical insight of contradiction concerning it or them. Consistency is not something that is demonstrable deductively, it is only an inductive conclusion, based on our not having come across any inconsistency albeit having carried out a diligent search for one. If we thus presume something to be logically possible, and are thereafter confronted with a clear intuition of contradiction, and no alternative explanation be found, all other considerations must yield to the overriding claim of logical *im*possibility.

Another concept which recurs often in modern logic is 'decidability'. This refers to the degree of dependence of the truth or falsehood of a logical relation on the truth of falsehood of its clauses. For instance, in 'material implication' (that is, negative conjunction), if the antecedent is false or the consequent is true, then the whole implication (which just means 'not-{p and notq}', remember) is also true — thus, to that extent 'decidable' (in the context of p or of notq, it is of course not 'decidable').

But in modal logic, most logical forms are, to varying degrees undecidable. For instance, strict implication cannot be equated to [the positive side of] the truth-table it shares with 'material implication', but only subalternates such a table [because their negative sides are quite different]. It is therefore surprising to read that Turing, in 1936, made the 'discovery… that every complete formal system (though not every logical calculus) is decidable'. Classical logic was built on the very idea of forms, having some degree of undecidability (a fully decidable form would be useless); as an extreme case, the form 'if —, not-then —' is entirely undecidable. To establish that there are partly or fully undecidable forms, one need only point them out: there is nothing to 'prove' about it.

Pursuing further, 'proof', as just explained, is a process depending on a mass of experiences: all the experiences which gave rise to the terms, copulas, and other features of the *de-re* propositions involved, plus all the logical intuitions (which are also experiences) concerning the relations between the *de-re* propositions involved. Our verbalization of such logical intuitions into a formal logic, in no way justifies our viewing the principles of formal logic as 'axioms' in a verbal 'system'. Our conviction is not due to any preferred ordering of words, but to the fact that we had a certain complex of wordless experiences.

Furthermore, the words that I have just written in the preceding paragraph, which are incidentally 'metalogical' in a much richer sense, are not themselves to be viewed as 'axioms'. What counts is their underlying meaning, and the conviction it carries. The words merely make it miraculously and wonderfully possible for me to communicate with you, by *drawing your attention* in certain directions, towards the same objects as I was looking at as I was writing them. 'Turning your attention' to certain objects, does not imply determining *the content* of what you thereby see (except by temporary exclusion, in that *during that time* you will to some extent not be aware of other things).

Thus, yes, it follows that *no* verbal knowledge is complete, or self-contained: neither in its 'definition', since meaning is not other words, but certain objects we have experienced some way or other; nor in its 'proof', since all proof is ultimately inductive: even seemingly pure deductions are with reference to intuitions as to what seems contradictory, not to mention the perceptual and conceptual sources of the formal premises. Let us say even more: no wordless knowledge is ever complete, either; our world is in constant flux and forever revealing new things to us!

The very pursuit of a 'complete formal system' is thus flawed from its inception. The inference that incompleteness implies arbitrariness is without justification: all we can say is that the given world we experience, in every which way, is 'arbitrary': we have no other to refer to. But surely that is not cause for concern: it suffices that we *have* a world, the world of appearances is all we need to have knowledge. So long as there is *something* to know, we have knowledge (however phenomenal): there is no basis for a 'logical' demand for more or other things to know. Something may be incomplete and yet sufficient in itself.

Returning parenthetically specifically to Godel's theorem, I am not at all convinced that his proposed opposition between completeness and consistency is justified. The statement 'P is not provable by P' (that is, 'P implies P' does not imply 'P') is perfectly consistent, and is not equivalent to the statement 'P disproves P' (that is, 'P implies nonP'). No liar paradox is implied, the analogy is most superficial. The self-assertion of a system is not its justification (every proposition asserts itself); its justification is consistency with all the data of our experience, including the absence of logical intuitions of inconsistency. In contrast, the self-reference of the liar's indicative is meaningless precisely because it has nothing outside itself to refer to.

That a closed system cannot 'prove' itself, indeed implies that it cannot 'disprove' its own negation, if we understand 'proof' in some overwhelming sense, since in the plain sense of implication, the system does of course both imply itself and deny its own negation. All that means, is that we must indeed refer our system outward — not to other words, but to appearances, objects of consciousness. It is they that 'prove' or 'disprove' anything, and scrutiny and reflection shows that they do so through the complex relations of cognition, recognition, distinguishing, naming/meaning, and gradual adduction (rather than simple implication).

As for the appearances themselves, they do not in turn need proof: they are not words, they are given objects, they are *all* the objects we actually have and may justifiably appeal to and discuss, at the stage of the proceedings we happen to be in (this is said, obviously, without intent to prejudicially exclude creationism and divine inspiration from the eventual scope of our world of appearances).

It is our experiences (concrete, abstract, and logical) which are in the truest sense the 'axioms' (the ultimate logical antecedents) of knowledge; and these are ultimately particular propositions, and not generalities as modern logicians desire in vain. There is no inconsistency or difficulty whatsoever in such a position, and it is taken for granted by all people of common sense.

Let us now consider certain terminologies and statements. One should always look at the plain *meaning* of what one reads; and not be intimidated and assume that there is some other, deeper meaning, clear to a select few, but not to oneself. Surely, if the authors meant more than what they are saying, they would say *that* in plain English; surely, if they are so highbrow, they can formulate a clear sentence. Therefore, one may presume them to be saying just what they seem to mean, and no more (unless of course, it is taken out of context).

All the following statements are claimed to be 'stable and exact conceptions… that explicate the intuitive concept' we have of their subject-matter. Many of the 'proofs' presented are apparently worked out in relation specifically to numerical concepts, to mathematics, but are mostly understood as having a larger impact, since they provide specimens for 'axiomatic theory'.

We are concerned here with what Hilbert called 'proof theory'. In this context, 'proof' is taken as a 'carry[ing over]' of truth from a given item to some non-given item. We are told an axiom is 'valid' if it is 'a tautology… a sentence true in all possible worlds'; and that not only can this be ‘checked’, but ‘only valid sentences are provable’. Completeness is taken to apply to a system (like the ‘propositional calculus’), if ‘every valid sentence in it… is a theorem’.

The ‘decision’ of validity can be ‘tested mechanically’ by showing that whatever combinations of truth and falsehood are assumed for the letters in the sentence, the sentence as a whole will always ‘come out true’. In a many-valued logic, the ‘independence of the axioms is proved by using more than two truth-values’, although those values may be ‘divided into two classes: the desired and the undesired’; here, an axiom is independent if decidedly desirable, and otherwise it is not.

‘Functions mechanically computable by a finite series of purely combinatorial steps’ were called ‘recursive’ by Godel. ‘Recursion theory’ is now able to ‘prove not only that certain classes of problems are mechanically solvable (which could be done without the theory) but also that certain others are mechanically unsolvable (or absolutely unsolvable)’. In the latter case, we have ‘no algorithm, or rule of repetitive procedure for solving’ them.

Note the ‘grains of truth’ in many of these statements; but their proponents remain unconscious of the ‘husks’ of their circularities: they do not test them on themselves. For instance: ‘a world may be assumed in which there is only one object *a*’, so that ‘all quantifiers can be eliminated’ and we can ‘reduce [that world] to the simple sentence *A{a}*’; in this way, ‘all theorems of the (predicate) calculus become tautologies (i.e. theorems in the propositional calculus)’.

I ask you: if ‘A{a}’ can be said about ‘a’, do not the symbols ‘A{ }’ also exist? In that case, how can ‘a’ be claimed to be a solitary existent, and all quantifiers eliminated? That is surely a serious inconsistency! Also, on what basis does the author of such statements at all trust his or her intuitions as to what ‘follows’ what, when so and so is ‘assumed’? Surely that constitutes an appeal to something outside the projected framework — another inconsistency!

Lastly, if I tell you tautologies in the Hokan-Coalhuiltecan language, will you grasp them? What a *poor* view of logic these people have, who ‘reduce’ everything to ‘axioms’ which they themselves claim to be nothing more than repetitive nonsense! Clouds circling on and on.

They claim by these and similar ‘methods’ to ‘prove… that the calculus is consistent [and] also that all its theorems are valid’. Or again: ‘its completeness was proved by Godel in 1930; its undecidability by… Church and Turing in 1936’. Look for instance at the following argument, please (it is drawn from the same source, almost word for word). Completeness is taken to mean that ‘for every closed sentence in the language of the theory, either that sentence or its negation belongs to the theory’.

a. if a calculus is complete, then:
either X or nonX belongs to the theory
and ‘all valid sentences are theorems’

b. whence (still for a complete calculus):
if X is consistent, then nonX is not a theorem
if nonX is not a theorem, then nonX is not valid
if nonX is not valid, then X is satisfiable
hence, if X is consistent, then X is satisfiable
(that is, X has an interpretation or model)

Comments: (a) How can it be *known to start with* that the calculus is ‘complete’? If X and nonX are meaningless, then surely *neither* of them ‘belongs’ to the theory. (b) How is X *known* to be ‘consistent’ in the first place? Why cannot nonX *also* be presumed ‘consistent’? Surely the disjunction in (a) of X and nonX is intended to mean that the theory does not *ab-initio* imply either of them, but is compatible with both; in which case, both may be consistent. Otherwise, the argument is entirely circular: that X is consistent, and that nonX is neither a theorem nor valid, are tacitly granted in (a) and then claimed to be ‘derived’ in (b).

Lastly, with regard to (b): why should consistency imply ‘satisfiability’? X may well seem in itself free of contradictions and *still* be meaningless (for instance, ‘This sentence is true’). My impression is that by words like ‘if-then’, ‘consistent’, ‘valid’, they refer to a logic *so elementary and* *nonmodal*, that they are all exactly equivalent: the ultimate in particularity and triviality in theorizing — and they are therefore bound to lead to over-generalizations concerning metalogic.

Thus, the argument as a whole consists of a tangled web of equivocations and ambiguities, of *quid-pro-quo* and inane tautologies, and alternately *non-sequitur* or *petitio principii* sophisms. From such an argument the following grand conclusion is drawn: 'therefore, the semantic concepts of validity and satisfiability are seen to coincide with the syntactic concepts of derivability and consistency'. What does it all mean? Nothing — or whatever we choose it to mean.

These people have completely confused themselves and each other, with a multiplication of different words for the same things, and words with borrowed but not admitted connotations. They do not consider how their starting points might or might not be arrived at, or how the links between the theses of their hypotheticals are to be established. The cart is put before the horse, and worse still the horse may be a goat, and they travel round and round! This is not logical *science*, by any stretch of the imagination.

3. Infinity in Logic.

Meaning is not a relationship between two sets of words, but between words and things. No 'model theory' would be communicable, if the words used by those who describe 'uninterpreted systems' to us were not plain English, which means something to us even if ultimately nonverbally. You can go around in circles till you are blue in the face, and you will still only have circles.

Systems with a limited plurality of interpretations are conceivable, but systems totally devoid of interpretation are simply meaningless, as are systems with *an infinity* of interpretations. This brings us to another trend in modern metalogic: the attempt to evade the issue by relying on infinite formulas. A system has to ultimately be 'satisfied' by nonverbal information; it cannot be 'satisfied' with reference to an infinite chain of other verbal constructs (called 'models').

This issue is not to be confused with the 'open-endedness' of the quantifiers 'all' and 'some'. They have an element of indefiniteness, referring usually to a not-fully-enumerated series of individuals; but each individual, as it presents itself, is self-sufficient in its existence, though it is classified with reference to its evident similarities to preceding ones. Whereas in the modern infinities here criticized, each case is defined by *the next* case.

Logicians have no basis for a belief that an infinity of purely symbolic constructs will acquire meaning and truth at some hazy infinity, like in mathematics, when a curve tends to some vanishing point, and may be presumed to actually cross the line 'at infinity'. Logic cannot ignore the Zeno Paradoxes. The infinite tape of a Turing machine is not physically possible, so why discuss it at all?

We may call this *the Anchor Principle*: A relation relates something *to* something; forms do not exist without contents, they have to be eventually pinned down. An infinite nesting of relations within relations within relations remains meaningless, until and unless *a term* finally consummates it. Infinity is unfathomable, and cannot be treated by logical science as by itself capable of zeroing in on some actuality. 'The buck has to stop somewhere'.

To be conscious of myself being conscious, I must first be conscious of something else, and then, after that first consciousness is aroused, I can take note of it in a supplementary act of consciousness. If a statement has an infinity of meanings, then it has effectively no meaning, because infinity includes everything, all opposites: that is the ultimate in ambiguity and indefiniteness; there has to be some limit in number of meanings, for the statement to have some specificity.

Modern logicians have, for instance, suggested a study of 'infinitary logic', which would 'include functions or relations with infinitely many arguments, infinitely long conjunctions and disjunctions, or infinite strings of quantifiers'. **William Hanf** of the U.S. is mentioned in this regard. I must admit that I do not, without knowing any more about it, see how such a study is conceivable, or could bear any fruit.

In fact, knowledge evolves as follows. Our systems are always *somewhat* meaningful and *contextually* true. They mostly grow, but sometimes they are modified (they give up some of their assumptions, lose extraneous fat or old skin); as they grow, they also become more defined and proven (or less so, if big inconsistencies or doubts make their appearance). This process tends towards infinity (again, we find that 'grain of truth'), where omniscience of the world limits the alternative experiences and interpretations to just one (when knowledge will be whole and perfect), where everything has full meaning and final truth — but there is no logical need to presume that this goal is reachable.

The justification for active formal studies, as with scientific experiment, is that they hopefully accelerate that ongoing process of knowledge growth, by strengthening our faculties of consciousness, concentrating our awareness, and orienting us more purposefully towards existing phenomena of many kinds. The role of such studies is not ultimately to vindicate our experiences, but to describe their processes, so as to yet more efficiently, more broadly and deeply, more fully get to know. The blueprints must eventually fit the experiences; this is a test for them, not for the experiences.

The experiences are given, though gradually. We only need to know what the apparent patterns they exhibit are, and whether one apparent pattern is to be preferred *to another* which is also apparent. Ultimately, we believe — this is the Law of the Excluded-Middle, note (much maligned by 'intuitionist' logicians) — that *some one* of those patterns will remain unchallenged. Just as, viewing with a magnifying glass, some positions are more blurred than others, more ambiguous, and we try to find the most sharply defined position among them, the one with the least ambiguity.

It may well be that none of the patterns discerned thus far will be that special one, but what is sure for each one is that it either will or will not be it: there is no third alternative. The *inductive value* of this principle is that, when we encounter a contradiction, a seeming coexistence of both an 'is' and an 'is not' with the same terms exactly, we can be sure that the solution is not something other than these two.

The role of logic is to elect, as being 'real and not illusory', one *subset* of our experiences, *rather than* any other subset of them; one pattern which appears and which we discern, rather than any other. Experience, appearance, as such, as a whole, is already self-sufficiently credible. Logic *itself* is but a subset of it, and therefore cannot in any wise ever be construed to somehow stand as judge and jury of it.

The meanings and truths of knowledge (including logical science), are an ongoing dynamic product of a syndrome of perceptions, conceptual insights, logical intuitions, ingenuity, and many other interactive factors (including our physiological and psychological makeup). The only consistent position is such a holistic and open one.

Infinity is one of the misconceptions at the very root of modern metalogic. To understand how it arose, we must refer to certain crucial errors modern logicians made in their formulations of class-logic, or set-theory (the reader is referred to ch. 43-45, to avoid repetitions).

a. ***Modern logicians confuse subsumptive and nominal terms***. That is, for instance, dogs and "dogs" are not clearly distinguished by them. But dogs is a subsumptive term, it is *not* a class at all, it is a non-class. Only *nominal* terms (expressed distinctively, in inverted commas or with the preamble 'the class of..'.) qualify as classes; and of those, "dogs" is a class (or first-order class), and "dog-classes" is a class of classes (or second-order class). The relationships between these various kinds of terms are precisely formally definable, as we saw, and they may not be equated.

b. ***Consequently, also, they (often, though not always) confuse classes with classes of classes, and hierarchies with orders***. Note that modern logicians are of course aware that there is a difference between classes and classes of classes, and that membership of individuals in a class does not qualify them for membership in classes of that class — that is, that membership is not transmissible (they say, 'transitive') from one order to the next. It is after all they who discovered this field of logic!

But confusion still arises (especially in symbolic contexts, and in some examples they give) between, say, a genus or overclass of "dogs", like "animals", and an upper-order class, like "dog-classes". And the root of this confusion is the said confusion between the roles of subsumptive and nominal terms.

c. ***Modern logicians consequently assume that there are orders of classes higher than the second***. For them, 'classes of classes of classes' is a meaningful concept, different from 'classes of classes'. Just because we can say 'the parent of the parent of the parent of..'., it does not follow that we can say 'the class of the classes of the classes of….' Try to think of an example of the latter, if you can.

As I explained, the sub-classes of "dogs" (like "retrievers") form a hierarchy, of the first order, and the sub-classes of "dog-classes" (like "retriever-classes") form another hierarchy, of the second order; and these hierarchies are indeed distinct, though exactly parallel, and they may contain *any number of* classes (of the appropriate order). But that does not imply that there exists an infinite number of orders: the concept of orders is quite distinct from that of hierarchies.

We can form a concept like "dog-classes" because dogs *differ from each other*: retrievers differ from bulldogs, and so on; whereas a concept like "classes of dog-classes" has no differences to refer to other than those *already encapsulated* by the concept of "dog-classes": it is therefore exactly identical to it, in intent and extent.

Orders higher than the second are therefore just *verbal* figments of the imagination: they refer to nothing new. All they do is keep reproducing the first and second orders with new names. Their infinite manipulations will not add an iota to the science of logic.

The impression that there exists any number of orders is due the existence of multiple hierarchies within each of the first two orders (this is the 'grain of truth' behind that fallacy); the inference drawn that there are more than two orders, merely serves to display that modern logicians confuse orders with hierarchies.

d. Modern logicians tend to give credit to the idea of self-membership, because they conceive of "classes" or "classes of classes" as themselves classes, and more deeply because they do not make a clear distinction between nominal and subsumptive terms.

But as we have seen, classification is a relational concept; the class of all classes is "things" and that of all classes of classes is "things-classes". There is no indubitable example of self-membership, except in the case of "things" and "things-classes", and these summum genera can be definitionally excluded. All other alleged examples can be explained away, so that we may inductively write-off the whole idea of self-membership, which is anyway conceptually unconscionable (how can a container contain itself?).

Parenthetically, with regard to *the Russell Paradox*, we have seen that it is resolvable, *not* by means of this rejection of self-membership, but with reference to the process of *permutation* it involves. The mere presence of the word 'is' in the string 'is a member of', does not allow us to split the latter relation into a subsumptive 'is' plus a predicate 'a member of….' Modal syllogism provides us with a clear independent confirmation that such limits to permutation exist, since interpretation of 'is capable of' (a colloquial for 'can') as equivalent to 'is {capable of…}' results in an invalid syllogism (see ch. 17).

I submit therefore that, at every fork in the road of the logic of class membership, modern logicians have taken the wrong turn. Their choices have been repeatedly improbable and contrary to reason, seemingly with a view to innovate at all costs. One is consequently highly tempted to wonder, in some cases, whether there is not a subconscious urge of nihilism — to deny common sense, to shock and bewilder students, to dominate. Instead of logical science being our vehicle to understanding of reality, it has been turned into a pit of confusion.

I certainly do not want to give the impression that I believe myself all-knowing. My knowledge of modern logic is admittedly patchy and limited; what you see is what you get: all I know is mentioned in these pages. It is not much, because my personal interest in these matters has never been highly stimulated; my interest is in a logic which is of daily utility. Notwithstanding, I believe that the judgments made here are essentially correct, because I find that the deeper I dig, the more I disagree. The extrapolation may be wrong. Okay: I can live with that thought.

4. Conceptual Logic.

In assessing modern logic, we must ask: what is logic, what is its purpose? The mathematically inclined study of logic is a very narrow field, in the grand domain of logic. Our ambition is to develop a *conceptual* logic, eventually capable of understanding the wealth of *qualitative* relationships in this wonderful world. Within that larger enterprise, logicians have found it necessary for a while to concentrate their efforts on the quantitative aspects of these relationships; but these are merely effects of, not identical with, the qualitative aspects.

The approach of modern logic is thus very specialized. It was to some extent necessary; it was valuable; but enough is enough. There are deeper and more important issues to look into. Look at the enormous arena of the physical universe; then look at the size of a man's brain, or the total volume of all the brains on earth. That is exactly the relative importance between conceptual logic, and logic of the first and second classes.

Conceptual logic is the 'zero' order of subsumptive relations, of all predications before any permutation. Class logics of the first and second orders (there are no more orders, as already argued) are merely additional layers over and above the zero order, and of much narrower scope. The logic of classes is also entirely derivable from the logic of subsumption (as we saw); it is only new in the sense of having only recently been explicitly considered. It is an interesting field, but it is not all of logic.

Modern metalogic cannot claim to be beyond meaning; it is always tied to some meaning. But that some meaning is only a fraction of the total meaning. If the full meaning is not taken into consideration, our abstractions are bound to give us a distorted image of things. We must range far and wide to get a proper perspective on things; openly, humbly, flexibly, with respect for the complexities of the issues, their many facets and their depth. It is pointless to rigidly simplify, to reject whatever we are unable to assimilate thus far, to write off whatever befuddles our intelligence.

There is need of a more profound 'philosophy of logic'. Let us now refer to a *New Encyclopaedia Britannica* article on this topic, written by K.J.J. Hintikka while at Florida State U. in Tallahassee (25:719-723). We are told that logic is 'the study of truths based completely on the meanings of the terms they contain'. This is a traditional view, and I agree with it in essence. Indeed, the article goes on 'the meanings in question may have to be understood as embodying insights into the essences of the entities denoted by the terms, not merely codifications of customary linguistic usage'. Again, a very sensible position.

However, Kant effectively took 'the meanings of the terms' to signify 'the verbal definitions of the words', in the sense of tautology. This interpretation has strongly influenced and pervaded modern logic, witness Carnap's 'syntactic language' for instance. But another interpretation is feasible: we know the 'truth' of anything, because we are *aware of* that thing, to whatever degree; the 'meanings' of our terms (words) are the things they refer to, *of which we are conscious*. The equation between words is called true, *because* those words represent for us such and such objects we have perceived and/or conceived, and these objects were seen to behave in the way asserted. The verbal aspect of judgment is incidental.

Logical science looks at certain specific aspects of the total picture, and attempts to discern certain patterns. For instance, Aristotle's argument 'if sight is perception, the objects of sight are the objects of perception' may at first glance seem obvious, a materially evident inference. But the logician says, 'no, be careful; sometimes such arguments commit the fallacy of composition, confusing the parts of things with the whole'. He then goes on and tries to isolate the distinctive factors of correct such arguments.

In this case, there seems to be a productive substitutive syllogism, of the following form. In place of the specific relation of 'seeing', we put a genus of it, 'perceiving', without touching the terms (subject and object) of the relation. It is in effect a change of copula:

All seeing is perceiving
I see a certain object
therefore, I perceive that object
or, in more conditional form,
all seeing of an object is perceiving of that object.

Thus, the argument consists simply in a *deeper insight* into a case of the 'seeing' relation, and discerning its 'perceiving' aspect (by comparison to hearing, and so on). The terms 'I' and 'the object' remain unaffected, because they are also found in the other species of 'perception', so that they are accepted as conforming to 'the perception relation' in general.

In contrast, 'all seeing is enlightening' would lead to an illicit process, for the reason that the relation of 'enlightening' refers to some other sorts of terms — in this case, as the object enlightens me, the Subject (instead of vice versa); thus, we must be careful (and preferably specify who is being enlightened). Thus, before making a general statement about any process, we must *inductively* find the distinct 'isomorphisms' of apparently legitimate cases.

In this way, logical science generalizes and formalizes. It just reports the general aspects and conditions of right-seeming arguments, and distinguishes them from the general aspects and conditions of wrong-seeming arguments. The meanings of the words involved are determining, because words refer us to certain preceding conceptual processes (in our example, the awareness of similarity between seeing and say hearing, and their difference from say enlightening, in the ways their subject and object are placed).

Note the primary importance in formalization of comparison and contrast — the intuitions of sameness and difference. Logical concepts, just like all concepts, are built up by identifying and distinguishing phenomena. That different people and peoples evolve common languages or languages with common structures and meanings, is due to the uniformities in the Objects, more so than to physiological uniformities in the Subjects. Not that the latter are irrelevant, of course. Worthy of mention in this connection, is the research by Swiss psychologist **Jean Piaget**, into 'the developmental stages of a child's thought by reference to the logical structures he can master'.

Deductive logic always depends on a certain amount of induction. The intuitions of the logical practitioner are no less trustworthy in principle that the intuitions of the logical theoretician; the latter is just more deliberately careful than the former, he compares and contrasts more. 'Form' is itself a content of the world; it is merely considered in isolation from other contents, by the logician. If he is lucky and perspicacious, he will from the beginning make assumptions of lasting value; but there is no guarantee that centuries later someone else will not find fault with his work.

We must precisely understand the stratifications involved in our enterprise. The logic practitioner intuits the material logical relations between material *de-re* relations (let us call this the 'zero order' of logic). The theoretical logician intuits the common and distinctive aspects (or 'forms') of material *de-re* relations having such logical relations, as well as the common and distinctive aspects (or 'forms') of material logical relations, and records the material (*informal*) logical relations between the formal *de-re* relations (this is 'first order' logic), and even between the formal logical relations (this is logic of the 'second order', and there are no still 'higher' orders).

The latter two levels (logical science) are *performed by* an exercise of logical art, the ground level. They are not somehow removed and superior, mere linguistic pronouncements. The language of logic is itself an object, a part of the world, to be explained within that world. It cannot be studied in total abstraction from the world. How could Ludwig Wittgenstein believe that 'language-games' can 'give the expressions of language their meanings', or Willard Van Quine, of Harvard, consider that 'relations of synonymy' — presumably, he means similarity — 'cannot be fully determined by empirical means'?

It is indeed impossible, as Godel asserted, to completely and consistently axiomatize logic — but why should that surprise? All knowledge is and must be empirically based; words can never on their own acquire meaning or truth. Only a very small part of thinking is 'mechanical'. Formal logic can to some extent be made 'recursive', *only because of* the preceding intuitions of informal logic.

Even lower animals have some degree of consciousness; but computers (sorry, trusty Old Pal) and robots do not and never will; I cannot speak about 'androids' (not having met any lately!). The role of will in consciousness is in *awakening and directing* it (switching and scanning functions). In lower animals this power is supposedly less 'free' than in humans. But consciousness itself is a unique phenomenon, however it is moved. No amount of manipulations of 'data symbols' will give a machine consciousness of what they mean; the concept of 'artificial intelligence' is misnamed, a gross exaggeration.

Quine's objections, around 1950, to 'the non-empirical character of analytic truth (logical truth in the wider sense… arising from meanings only)', might seem to class him as a defender of empiricism. But to me his position only serves to reveal his failure to trace the empirical roots and development of logic. Logical Positivists, who believe 'that logical truths are really tautologies', might seem like pragmatic realists. But I wonder how they lay claim to this 'really' of theirs, and how come their words have some communicable content. It seems clear to me that these people have passed all their lives making the trivial manipulations of modern symbolic logic.

Happily, some modern philosophers still do believe that 'logical… truths are informative', and not trivial. The issue of 'cross-identification' — recognition of individuals, as well as of the uniformities among individuals — is correctly pinpointed as crucial, ontologically and epistemologically. The age-old problem of 'universals' is an ongoing challenge for logicians. One cannot quantify, without first having something (qualitative) to quantify. There is no simple solution; *the complexities of induction have to be analyzed one by one, specimen by specimen, in excruciating detail*. One thing is sure, as **Ayn Rand** has eloquently said (rephrasing Aristotle's Law of Identity), 'existence *exists*' (942).

67. INDUCTIVE LOGIC.

1. Degrees of Being.

Before determining where the philosophy of science stands today, I would like to highlight and review some of the crucial findings of our own research in this volume.

The first thing to note are the implications of certain of our findings in modal logic. We saw (ch. 17) that, contrary to what has been assumed throughout the history of logic, the premises:

All M can be P
This S can be P (or: This S is P, or: must be P)

…do not yield the conclusion 'therefore, This S can *be* P', but a more disjunctive result, namely:

therefore, This S can (get to) be *or become* P.

Thus, the mode **ppp** is valid, but only provided we take transitive propositions into consideration. Past logicians, including moderns, failed to take the existence of *change* into account, in their analysis of modal logic, and for this reason did not spot this important alternative conclusion from a merely potential first-figure major premise. It is true that Aristotle analyzed change with great perspicacity in his ontological works — and indeed, my own formalization of change is based on his insights — but even he did not integrate this relation into his formal logic.

The immediate formal significance of this finding is that ***natural modality is not permutable***. Although in common discourse we rephrase 'S can be P' as 'S *is* {capable of being P}' or as 'S *is* {potentially P}', in strict terms, we may not do so — we may not enclose the modality within the predicate, and consider these 'is' copulae as having the same meaning as that in an actual 'S is P'. If this is true of potentiality, it has to be equally true of natural necessity, since the oppositional relations between modal forms have to be maintained. By similar argument, we can show that temporal modality is impermutible.

These *formal* findings force upon us certain *ontological* inferences of the highest import. I was myself surprised by the conclusions; I had not intentionally 'built them into' my system. The implication is, that we may not regard a potential relation as signifying the presence of an *actual* 'mark' in the subject; the subject contains, within its 'identity', the potentiality *as such*, and not by virtue of some actuality. Thus, ***there really are 'degrees of being'***. We may not reduce all being to the actual; there are lesser degrees of being, called potentialities, and (by extension) higher degrees of being called natural necessities.

In between these extremes, therefore, the degrees of natural probability are also different degrees of being. And likewise, temporal modalities have to be so interpreted. Note well, none of this is speculative: these positions are imposed upon us by formal logic, by the requirement of impermutability (which, incidentally, was also useful in understanding the Russell Paradox — see ch. 45.3). Thus, we are not making a vague metaphysical statement, but referring to precise technical properties which reveal and demonstrate the 'self-evidence' (in the formal sense, of logical necessity) of the concept of degrees of being.

Thus, although the concepts of modality are at first presented as purely *statistical* characterizations of relations, we come to the final conclusion (on formal grounds) that this numerical aspect is *merely a symptom* of a real ontological variation in the meaning of 'is'. Aristotle left us with a limited vision of the scope of the copula 'is', because of the restrictions of his nonmodal logic; but now we see that there are *real nuances* in the sense of that copula, which only a modal logic can bring out into the open for our consideration.

We see, in this way, the impact modal deductive logic may have on ontology. But, as we shall see, the ramifications in modal inductive logic are even more significant, for epistemology. However, beforehand, I would like to make some incidental remarks.

Until now, the formal theory of classification, or class logic, has been notoriously simplistic. No one can deny how valuable it has been to science: for instance, Aristotle, and in modern times the Swedish biologist Carolus Linnaeus, have used it extensively in constructing their taxonomies of plant and animal life, and indeed every systematization involves reference to genus-species-individual relations. However, this approach has always seemed somewhat rigid and static.

Our world is conspicuously a world of change. Things come and go, there is generation and corruption, alteration, development, and evolution. What was yesterday a member of one class, may tomorrow be a member of another instead. Something may belong to a class only conditionally. And so forth. Only a *modal* class logic can assimilate such dynamic relations. Science needs this methodological tool, to fully depict the world of flux it faces.

Instant 'state of affairs' pictures are not enough; there is need to specify the avenues and modalities of *transition* (or absence of transition) from one state to another, as well as the causal relations involved. It is not enough to say vaguely what things 'are': we have to specify what they 'must be', what they 'can be', and from what to what and via what, and in which circumstances, they go: only thus can science fulfill its responsibilities.

For this reason, formal logic is obligated to study transitive categoricals and *de-re* conditioning of all types, in great detail. Without such a prolegomenon, many philosophical and scientific controversies will remain alive indefinitely. Right now, there is no formal logic (other than the one here proposed) which *provides a language and neutral standards of judgment* for, say, Darwin's evolutionary theory or Hegel's dialectic of history.

It is just so obvious that someone who is aware of the complexities of dynamic relations, is more likely to construct interesting and coherent theories on whatever subject-matter.

Returning now to modality. You will recall that we distinguished between types of modality and categories of modality, and we said that a modality is 'fully' specified only when both its type and its category are specified. Upon reflection, now, we can say that even then, the modality is not quite fully specified: to do so, we would still need to pinpoint the exact compound of modality it is an expression of, and indeed, we must do this in both directions of the categorical relations (see ch. 51, 52). Furthermore, to complete our description of the relation, we would need to specify the precise *de-re* conditions of its actualization (see part IV).

Now, just as natural necessity, actuality, and potentiality form a continuum of 'degrees of being', and likewise for temporal modalities — so all the subdivisions of these modalities implied in the previous paragraph clarify the various degrees of being. That is, once we grasp the ontological significance of modality, as we did, then by extrapolation *all the other* formal distinctions, which occur within the types of modality in question, acquire a real dimension (of which we were originally unaware).

Moreover, the very concept of 'degrees of being' can be carried over into the field of extensional modality, in view of the powerful analogies which exist between it and the natural and temporal fields. This is not a mere generalization, because we from the start understood extensional modality as more than mere statistics; it relates to the possibilities inherent in 'universals' as units. Thus, 'Some S are P' and 'All S are P' are different degrees in which S-ness as such may 'be' related to P-ness as such. Thus, the quantifier is not essentially something standing outside the relation, but is ultimately a modification of the copula of being.

Going yet further, the valid *modes* of the syllogism, and indeed all argument, like **nnn** or **npp** for instances — they too may be viewed as informing us of the inherent complexities of modal relations. That 'All S must be P' implies only 'some P can be S' tells us something about being 'in rotation', as it were. That premises **np** yield conclusion **p** (rather than **n** or **a**) tells us something about the causal interactions of these different degrees of being. Likewise, for all types and mixtures of modality. All these so-called processes, therefore, serve to define for us the properties of different types and measures of being, giving us a fuller sense of their connotations.

Which brings us, at last, to the most radical extrapolation of all, and the most relevant to induction theory. Since, as we saw, in principle, logical necessity implies (though it is not implied by) natural necessity, and logical possibility is implied by (though it does not imply) potentiality — we may interpret these *logical* modalities as, in turn, themselves stronger or weaker *degrees of being*. The inference is not as far-fetched as it may at first seem. That something is such that its negation is 'inconceivable' or such as to be itself 'conceivable' is a measure of its belonging in the world as a whole (including the 'mental' aspects thereof).

Between minimal logical possibility (which simply means, you will recall, having at all *appeared* in the way of a phenomenon, with any degree of credibility) and logical necessity (which means that the negation has *not even* a fictional, imaginary place in the world), are any number of different degrees of logical probability. If our extrapolation is accepted, then high and low logical probability are measures of 'being', not merely in a loose epistemological sense, but in a frankly ontological one. This continuum *overlaps with but is not limited to* the continua of being in a natural, temporal or extensional sense.

'Truth', the *de-dicto* sense of 'realization', and 'singular actuality' in the natural/temporal and extensional sense, become one and the same in (concrete or abstract) phenomena. The really here and now is the level of experience of phenomenal appearances (in the most open senses of those terms); we might even say of concrete and abstracts that they are also different degrees of presence, in their own way. Beyond that level of the present in every respect, 'existence' fans out into various ways of stronger and weaker being. Thus, logical probability may be viewed as *in itself informative concerning the object*, and not merely a somehow 'external' characterization of the object.

This suggestion is ultimately made to us by formal logic itself, remember; it is rooted in the concept of impermutability. Thus, the contention by some that Werner Heisenberg's Principle of Uncertainty signifies an objective indeterminism, rather than merely an impossibility to measure — may well have significance. I am myself surprised by this possible conclusion, but suddenly find it no longer unthinkable and shocking: once one accepts that there are 'degrees of being' in a real sense, then anything goes.

Thus, we may also view the mental and the physical, the conceptual and the perceptual, the 'universal' and the individual, the ideal and the real, knowledge and fact, and why not even the absolute and the relative — as different types and degrees of being. Being extends into a large variety of intersecting continua. In this way, all the distinct, and seemingly dichotomous, domains of our world-view are reconcilable.

2. Induction from Logical Possibility.

Let us now return to the main topic, that of induction, and consider the impact of what has been so far said. We acquainted ourselves with two major processes of induction: adduction (see ch. 20-22 and 46-48) and factorial induction (see part VI).

Adduction concerns theory formation and selection. The logical relation between postulates and predictions, consists of a probabilistic implication of some degree, conditioned by the whole context of available information. The postulates logically imply, with more or less probability (hopefully, lots of it) the predictions; and the latter in turn logically imply with more or less probability (anything from minimal possibility, even to logical necessity) the postulates. The logical relations note well are *mutual*, though to different degrees, and *in flux*, since they depend on a mass of surrounding data.

Thus, the adduced probability, in any given context, of any single proposition, be it frankly theoretical *or seemingly empirical*, is the present result of a large syndrome of forces, which impact on each other too. Theories are formed (appear to us), and are selected (by comparison of their overall-considered probabilities, to those of any modifications or alternative theories), with reference to the totality of our experiences.

Concrete experience, note, is by itself informing, even when it is not understood; abstract theories are also in a sense experiences, to be taken into account. Empirical phenomena determine our theories, and they in turn may affect our particular interpretations of empirical phenomena. There is a symbiotic give and take between them, which follows from the holistic, organic, nature of their logical relation.

Thus, adduction may be viewed as the way we generally identify *the degree of being of any object, relative to the database present to our consciousness.* Within the domain delimited by our attention, each object has a certain degree of being; and this degree is *objective*, in the sense that from the present perspective the object indeed appears thus and thus. The appearance may not be the central 'essence' of that object, but it is in a real sense a facet of it, a projection of it at level concerned. In that way, we see that *logical probabilities, and logical modality in general, ultimately have a de-re status too*: their way of 'being' may be more remote, but it is still a measure of existence.

Deduction is merely one tool, within the larger arsenal of adductive techniques. Deductive processes are, apart from very rare exceptions of self-evidence (in the formal sense), always contextual, always subject to adductive control in a wider perspective. Modern logicians, so-called Rationalists, who attempt to reduce knowledge to deductive processes, fail to grasp the aspect of holistic probability. Our knowledge is not, and can never be made to be, a static finality; the empirical reality of process must be taken into account for a truly broad-based logic. Likewise, the opposite extreme of Empiricism is untenable, because fails to explain how it allowed *itself* to be formulated in a way that was clearly far from purely empirical terms.

Now, factorial induction is another major tool at our disposal in the overall process of induction. In fact, we may view all induction as essentially adductive, and say that deduction and factorial induction are specific forms or methods of adduction. Essentially, factorial induction is built on the adductive method of listing all the alternative 'explanations' about a 'given datum' — in our case, the given datum is the gross element or compound, and the list of eventual explanations is the factorial formula; that is, the formally exhaustive series of integers compatible with the gross formula, and therefore constituting logically possible outcomes of it.

In the general adductive relation, the hypothetical proposition 'these predictions *probably imply* those postulates (and thus the theory as a whole)', *the terms* of the antecedent categorical need not be the same as *the terms* of the consequent categorical. Thus, the terms of the hypothesis may be *mere constructs*, of broader meaning and application than the more singular, actual and real terms of the allegedly empirical ground. That there are degrees of being, implies not only that there are degrees of truth (as explained, logical modality has a *de-re* status too), but also that there are degrees of *meaning* (again, in the objective sense that something has at least *appeared*).

The terms of a theory may be at first vague, almost meaningless concepts, but gradually solidify, gaining more and more definition, as well as credence. This evolution of meaning and credibility, as we look at the apparent object every which way, may be viewed a change in the degree of 'being'; as long as the apparent object does not dissolve under scrutiny, it carries some weight, some 'reality', however weak. It remains true that any alternative with apparently more weight of credibility and meaning, has a 'fuller' reality, more 'being'. Thus, even though 'truth' is a comparative status, it may still be regarded as an objective rendering of the 'world' of our context.

In contrast, factorial induction deals with generalization and particularization of information. What distinguishes it from adduction (in a generic sense) is *the uniformity of the terms in its processes.* Factorial induction concerns the selection and revision of 'laws'. We generalize 'this S is P' to 'all S must be P' or some less powerful compound (some other integer), *with reference to precise rules*. Here, note well, the terms are the same. This sameness is at least nominal; for it is true that by generalizing the singular actual to a general natural necessity (or whatever), we *modify* the degree of being and meaning of the terms somewhat. This modification is not arbitrary, but is determined by the whole context, including the rules followed.

But anyway, factorial induction is obviously *a case of adduction* (though a special case because of the continuity of terms). That means that the terms themselves may well be more or less theoretical, in the sense of having lower degrees of meaning. Also, the seeming empiricism of their singular actual relation may or may not be true; that is, it too has degrees of credibility and truth, determined by the overall context. At all levels, from the seemingly empirical, through factorial induction, to the adduction of overt constructs — there is some interactive reference to overall context.

Thus, the rules of factorial induction *remain the same*, however meaningful or true the terms appear at a given stage: they are formal rules, which continue to apply *all along* the development of knowledge. At each stage, they determine a certain answer, or a range of answers, depending on how definite and credible the terms and relations involved appear to us at the time, taking into consideration all available information. The factorial approach to induction is distinguished by its utter formalism, and independence from specific contents.

I want to stress here the profound importance of such an integrated theory of *modal* induction. Through it we see graphically that there is no essential discontinuity between logical (*de-dicto*) modality and the *de-re* modalities. The modality of a thing's being, is the meeting point of all these aspects: on the outer edge, its logical meaning and truth, ranging from logical necessity to extremely dilute conceivability; closer to the center, the *de-re* modalities at play; at the very center, the empirical realization of the essence, towards which we try to tend.

Truth and full definition are approached in a spiral motion, as it were. We can tell that we are closer, but there is always some amount of extrapolation toward some presumed center. Our position at any stage, however composed of theoretical constructs and generalizations, always has some reality, some credibility, some meaning — it just may not be as advanced as that which someone else has encountered or which we will ourselves encounter later. But it is still *a product of* the Object, the whole world of appearances, and as such may well be acknowledged to have some degree of objective being in any case.

Another way to view inductive processes is as follows. Since logical possibility is a subaltern of natural possibility (potential), we can generalize (subject to appropriate rules of corrective particularization) from logical possibility to natural possibility, just exactly as we generalize (under particularizing restrictions) from, say, natural possibility (potentiality) to temporal possibility (temporariness). This means that adduction in general (that is, even with imaginary terms) is a species of factorial induction.

We have already developed a definitive inductive logic for the *de-re* modalities (with the example of categoricals — *de-re* conditionals can similarly be dealt with, almost entirely by a computer: we know the way). This *de-re* inductive logic can now be extended further to *de-dicto* aspects, simply by introducing *more factors* into our formulas. We saw that the combinations of the natural and extensional types of modality gave rise to 12 plural elements, and thence to 15 factors. When temporal modality is additionally taken into consideration, the result is 20 plural elements and 63 factors. It is easy (though a big job) to extend the analysis further, with reference to the fourth type of modality, namely the logical.

Roughly speaking (I have not worked out all the details), we proceed as follows. Each previously considered element becomes three elements: a logically necessary version (say, prefixed by an **N**), a just-true version (unprefixed), and a logically possible version (say, prefix **P**). These more complex elements are then combined into fractions, and thence into integers; the resulting number of integers is the new maximum number of factors a formula may consist of.

Every gross formula is then given a factorial interpretation, comprising a disjunction of one to all the available factors. The factors must of course be ordered by modal 'strength', to allow for easy application of the law of generalization. Logical necessity or impossibility are 'stronger' than logical contingency coupled with truth or falsehood. *The overall factorial formula for any event is accordingly much longer, but with the factors ordered by 'strength', factor selection or formula revision proceeds in accordance with exactly the same unique law of generalization.*

Thus, our manifesto for modal induction is not limited to the special field of *de-re* categoricals (and eventually *de-re* conditionals), but is capable of coherently and cohesively encompassing even logical modalities (applied categorically, or eventually hypothetically). We have therefore discovered *the* precise mechanics of *all* adduction. At any stage in knowledge, it should henceforth therefore be possible to characterize any apparent proposition with reference to a precise integer, the strongest allowed by the context.

This refers, not only to simple generalization of 'laws' (observed regularities), but to determining the status as well as scope of any complex 'theory' whatever (however abstract or even constructed be its terms, even if their definitions are still notional and their truths still hypothetical). Of course, the terms still have to be at least minimally intuitively meaningful and credible. But the selection (subject to revision) of the strongest available factor *precisely determines* a proposition (or its negation) as true. There is no appeal to some rough extrapolation on vague grounds, toward the central 'truth'; we now have a formal depiction of the process of pin-pointing the truth at any time.

3. History of Inductive Logic.

I want to now refer the reader to the article on philosophy of science in the *New Encyclopaedia Britannica* (25:660-678). Written by **Stephen Toulmin**, of Chicago University, this paper is the most refreshingly balanced of all those referred to so far. The impression it gives is that current understanding in inductive logic, is by far superior in quality to modern trends in deductive logic. This is no doubt to a great extent the author's achievement, his ability to avoid extreme positions, his awareness of all the nuances in the matter at hand.

My task is therefore much facilitated. It is to follow the history of inductive logic, and determine where I agree or disagree, or what I may add in the way of comment. By comparison and contrast, the distinctive and original aspects of my own contributions will be highlighted, and further defined and defended.

(Although I do not here review them, the interested reader might consider studying, in addition to the said article, the rest of the entry on 'Philosophies of the Branches of Knowledge' of which it is part, as well as the *NEB* article 'Epistemology' (18:601-623)).

One thing is clear at the outset: *no one has to date formulated any theory remotely resembling factorial induction.* Adduction is well known — it is the hypothetico-deductive method, attributed to **Bacon** and **Newton**; actual induction may, I believe, be attributed to Aristotle (I certainly learned it from his work); but *factorization, factor selection and formula revision* (not to mention the prior logics of transition and of *de-re* modal conditioning) are completely without precedent.

These constitute, I am happy to report, a quantum leap in formal logic. I stress this not to boast, but to draw attention to it. It was the most difficult piece of intellectual problem-solving (it took 2 or 3 months) this logician has been faced with, and the most rewarding. *The problem was finding a systematic way to predict and interpret all consistent compounds of (categorical) modal propositions*; many solutions were unsuccessfully attempted, until the ideas *of fractions and integers, and of factorial analysis*, presented themselves, thanks G-d.

The historical absence of a *formal* approach to induction, or *even the idea of searching for* such an approach, is the source of many enduring controversies, as we shall see. Once a formal logic of induction exists, as it now does, many doubts and differences become *passé*. Just as formal deductive logic set standards which precluded certain views from the realm of the seriously debatable, so precisely the formal inductive logic made possible by factorial analysis of modal propositions simply changes the whole ball game.

Toulmin discusses inductive logic under the name of Philosophy of Science. This reflects the fact that it is currently with reference to the examples provided by modern science that philosophers try to understand induction. Which is as it should be, but implicit in the name of the research is the lack of a sufficiently formal approach. Induction is first of all an issue for Logic to sort out. However, Toulmin does mention ‘the formal study of inductive logic (which reasons from facts to general principles)’.

In any case, the research in question has both ontological and epistemological ‘preoccupations’, reflecting larger subject-object issues. ‘Any hard-and-fast distinction between the knower and the known or between the observer and his observation’ is alleged to be ‘discredited’ by modern discoveries in Physics, like relativity and quantum mechanics (that is to some extent true, but not itself as hard-and-fast as suggested, in my opinion).

‘Ontological preoccupations… have frequently overlapped into the substantive areas of the sciences’, with reference to ‘the existence and reality’ of their theoretical entities. For example, the atom was debated early in this century by the likes of physicists Ernst Mach and Ludwig Boltzmann; similarly also, in biology and sociology. ‘Epistemic concerns’ have also somewhat been affected by psychological research into cognitive processes.

In any case, philosophy of science has tried to analyze and evaluate ‘both the general concepts and methods characteristic of all scientific inquiries and also the more particular ones that distinguish the subject matters and problems of different special sciences’. I agree that all input from special sciences is valuable, and helps to define and test any formal theory of induction. Toulmin, as already said, surveys the field very openly, with ‘no effort to prejudge’.

What becomes apparent is an enduring division, across the centuries, into roughly three camps: the first two are opposite extremes, in a spectrum of proposed answers to any question; and in between them, in the middle, lies any number of attempts at *reconciliation* between the extremes. These are all known historical divisions. They do not form a uniform vertical continuum, because the problems shifted in emphasis across time. Thus, for the main periods of Ancient Greece we have, briefly put:

Pre-Socratic	Monism Parmenides	Pluralism Heraclitus
Classical	Idealism Plato	Materialism (say) Aristotle
Later Antiquity	Stoicism Zeno	Epicureanism Democritus

Parmenides and Heraclitus were concerned with the issue of unity and reality, versus plurality and transitoriness, of appearances. Plato and Aristotle were more focused on the issue of transcendence versus immanence of ‘universals’, both more or less acknowledging particulars. The Stoics and Epicureans, in contrast, functioned in the more limited domain of the material world, debating the regularity or spontaneity of its bodies’ movements.

Let us note that Plato was methodologically more committed to axiomatization and less empirical-minded, whereas Aristotle was both a biologist and also the founder of formal logic; so with regard to rationalism, they differed only in degree. Similarly, throughout history, the common ground is as significant as the differences. Therefore, in spite of seeming repetitiveness in the divisions, their frames of references do change a bit and become more defined.

‘The ensuing Hellenistic, Islamic, and medieval periods added little to the understanding of scientific methodology and explanation’.

As of the Renaissance, Empiricists faced-off with Rationalists. **Francis Bacon** insisted on use of ‘empirically observed fact’ (similarly, Locke, Hume), from which theoretical propositions would be ‘formally deduced’ (by ‘exhaustive enumeration’) or eliminated;. René Descartes, in contrast, looked to the model of Euclidean geometry, and considered that comprehensive scientific principles should be deducible from a structured set of ‘self-evident axioms [and] definitions’; similarly, Liebniz, Bishop **George Berkeley**.

Both these tendencies found expression in the practical scientific work of **Isaac Newton**, which referred both to observation and experiment, and to theoretical tools like the mathematical calculus. He thus gave birth to the *'hypothetico-deductive method'*: a working hypothesis is assumed, its specific implications are deduced, and these are compared to empirical evidence; so long as harmony prevails well and good, otherwise another hypothesis must be found. Thus, Newton was neither as 'enumerative' as Bacon (though they agreed on 'elimination'), nor as 'self-evident' as Descartes, but managed to find a harmonizing middle way, satisfying the concerns of both sides to some extent.

In 1733 and 1766, consistent alternative geometries to Euclid's were developed, showing that the latter 'could no longer claim a formal uniqueness'. Also, since all the alternatives were presumably compatible with empirical evidence, a decision between them became seemingly impossible. It therefore became imperative to justify our preference for one of them, in some (perhaps new) way.

Immanuel Kant initially subscribed to the Cartesian ideal, believing that 'Newton's physical principles would eventually be put on a fully demonstrative… basis', but later developed a more 'critical philosophy'. He advocated a 'transcendental method', which would refer to the mind's structure to explain our adherence to certain categories and axioms of knowledge. Effectively, Kant was saying, we think in such and such a way (for instance, with respect to space and time), because our minds are so structured that we *have* to.

In my view, funnily enough, that interposition was totally irrelevant and inconsistent. It claimed for itself a transcendental status, and tried to skirt the issue as to whether it had *itself* been cognized rationally or empirically or in a combination of both ways. It seems ingenious, only *because* it is laden with paradox. On the one hand, its intent was to impose some certainties into knowledge; but on the other hand, the implication was that our knowledge is rather accidental (that is, it could have been otherwise, were the mind differently structured), and therefore conceivably incorrect (and therefore uncertain).

However, historically, Kant's influence has been grave, because he effectively equated the concept of rational 'self-evidence' with mere tautology, making it lose all content. Simultaneously, Kant put even experience in doubt, since there was a possibility of it having been conditioned (read: distorted) by the mental prism. We can also view this influence in a more positive light: Kant revealed for us the weaknesses of extreme rationalism and extreme empiricism, and forced us to take these problems into account when formulating any subsequent theory.

But in my view, the solution of Kant's dilemmas is simply to apply Newton's adductive method to *the whole* enterprise of knowledge, *including* philosophy itself. Every item must be equally self-consistent, and consistent with experience — or at least seemingly so. The preferred alternative is that with more such cumulative credibility than all its rivals. These tests apply equally to epistemological and ontological theories: they cannot exempt themselves from the same scrutiny as they apply to the special sciences.

In that case, Kant's insinuations that the self-evident is contentless and that the mind distorts experience, are merely internal difficulties *within his theories*, and only serve to prove that they themselves were not thoughtfully constructed. The only self-consistent position is that the intuitively evident *has* meaning and credibility (subject to ongoing confirmation), and that the mind's conditioning of experience need *not* be distortive (though in specific cases it might be so judged, on the basis of other experiences or logical considerations). It suffices to develop a broad-based theory consistent with these prime logical requirements. Kant's simply does not fit the bill; he did not understand Newton's method.

Nevertheless, the intervention of Kant's Idealism, suggesting that consciousness imposes (rather than merely discerns) a structure on its objects, was historically valuable. It stimulated a healthy and fruitful interest in the mechanisms of sensation, stretching from research by **Hermann Von Helmholtz** in the mid-19th century to wide-ranging present-day efforts by biologists and psychologists. It also helped, *by negation*, to better define some of the conditions for a rival theory of knowledge. Thus arose, for instance, what Toulmin calls the 'epiphenomenal view of experience — as a kind of mental froth without causal influence on the underlying physical mechanisms'.

With regard to physiological aspects, the old debate between vitalists and mechanists, as to whether life processes were or were not radically different from other physical phenomena, gained relevance in epistemological discussion. Note however that Kantianism damns you if you do and damns you if you don't; for free will seems to imply intentional arbitrariness, and mechanical determinism or causelessness seems to imply accidental deviations. As far as I am concerned, objective consciousness is conceivable whatever our aetiological presuppositions.

Continuing our survey, in the 19th century, William Whewell made the important contribution of stressing *the temporal dimension* of Newton's method: 'it was only by a progressive approach that physicists arrived at the most coherent and comprehensive systems'. John Stuart Mill's practical rules for experimental inquiry and *causal reasoning* (the methods of agreement, difference, residues, and concomitant variations) were also crucial contributions to scientific method (as well as formal aetiology). It seems to me that, in spite of their rivalry, these two men were essentially on compatible courses.

At the turn of the 20th century, a modern 'critical reanalysis' of science and its philosophy began. As science appealed to more and more abstract, and indirectly arrived at, concepts — 'Kant's lesson about the constructive character of formal theories' gained credence. Consecutively, modern science remained of course deeply committed to referral to empirical data. Thus, **Ernst Mach**, **Richard Avenarius**, saw 'theoretical concepts [as] intellectual fictions, introduced to achieve economy' in the 'organization of sensory impressions'; such constructs could be useful tools without needing to be claimed to correspond to any reality.

'As against this instrumentalist or reductionist position, **Max Planck**, author of the quantum theory, defended a qualified Realism'. **Henri Poincaré**, **Pierre Duhem** adopted 'intermediate, so-called conventionalist positions'. These 'attempted to do justice to the arbitrary elements in theory construction while avoiding… radical doubt about the ontal status of theoretical entities'. **Norman Campbell** responded by 'sharpening the distinction between laws and theories'; the former are concerned with 'cataloging and describing', the latter with 'making intelligible… compact[ing] and organiz[ing]'.

My own position on the issue at hand is simply *open and formal* (noncommittal, without contentual prejudice): there is a generalization from logical possibility to natural actuality; so long as no empirical finding or logical insight arises which effectively, by the rules of induction, requires us to revise our position (either totally abandoning the proposed integer or granting equal credence to an alternative integer), it remains *true*. 'Particular observations', 'laws' and 'theories', all fall under the same rules; there is no pressing need to distinguish them.

It is only *ex-post-facto*, with regard to demoted ideas, that we can credibly say 'ah, yes, that one turned out to be a fiction'. If two or more ideas are equally conceivable, we might well adopt one as a mere 'working hypothesis' (which may turn out to be fictional). But if only one is predominant, and so long as it stays that way, it cannot *consistently* be characterized in a skeptical fashion, but must be acknowledged as a reality. To claim *all* concepts fictional, implies that very claim itself, which is also conceptual, to be fictional (that is, *false*).

The imaginariness or remoteness of a construct may affect our assessment of credibility in specific cases, but cannot be viewed as having any relevance *in principle*, since induction is not arbitrary but subject to rules. Any claim that a specific construct is fictional, implies a claim to knowing that there is something else which is real and different from that construct; a general accusation is disqualified in advance. The distinction between fiction and reality presupposes some standards of judgment; it cannot therefore be meaningfully applied without tacit appeal to and acknowledgement of those standards.

However, it must be admitted that just as Kant's insights, though logically untenable, had a creative influence on subsequent philosophy, Mach's view of scientific theories as mere flights of fancy, in spite of its internal inconsistency, had a positive effect on scientific thinking. What it did was to psychologically liberate scientists, to give more rein to their imaginations, at a time when science was in full expansion and needed new ideas, new constructs with which to assimilate new empirical findings.

Philosophy had come to a clearer realization of *the crucial role of imagination in theorizing*. It called for a less pedestrian, richer science. It is noteworthy that this new found freedom was explicitly used and hailed by the likes of **Albert Einstein**, who talked of scientific theories as 'free creations of the human mind'. Relativity and **Heisenberg**'s Indeterminacy were distinguished by their philosophical daring.

I also agree with Mach that 'submicroscopic atoms… derived their scientific meaning entirely from the macroscopic sense experiences that they are used to explain'. For me, this is an important point, because it illustrates how our theoretical constructs often refer to mental images of physical objects and events. The conclusion to draw is not however that they are all fictions (that is for inductive logic to determine, case by case); rather, we should notice that this gives initial meaning to the words used, and it is significant that it refers back to causally related experiences. In this context, the Liebniz idea of worlds within worlds, reflecting each other at all levels to some extent, is very pertinent.

In the period between the two World Wars, Mach's attempt 'to reduce all knowledge to statements about sensations', and the modern symbolic logic of Russell and Whitehead, and other similar strands, coalesced in the Vienna Circle of Logical Positivists, which still has a considerable influence today (though less than then).

It is interesting to note that this philosophy was composed of two somewhat contradictory extremes. On the one hand, a neo-Humean focus on only the most concrete of sense impressions; on the other hand (as we saw in the previous chapter, with reference to Carnap), a narrowly 'linguistic' analysis of conceptual knowledge. Therefore, in traditional philosophical terminology, they were both extreme empiricists *and* extreme rationalists. The method advocated by logical positivists was thus, strictly speaking, 'hypothetico-deductive' only in name.

They were hedging their bets: pursuing the Cartesian programme of an axiomatic system of science, derived from some most-general postulates 'posited without proof', yet at the same time claiming for those first principles, 'by comparing them with actual experience', a measure of 'substantiation'. Still, in that mixed-up context, valuable concepts like *'verification, confirmation, or corroboration'* (and their negative equivalents), became more common currency and were better understood.

Concurrently, a school of Neo-Kantians 'questioned the very possibility of identifying the pool of theoretically neutral observations'. **Heinrich Hertz** advanced the idea that, in a theory like Newton's dynamics, the logical relations linking postulates and phenomena were *themselves too* part of the theory. Wittgenstein developed this further with reference to a philosophy of language, and his successors joined, to the concern with the 'structure' of theories, a concern with 'the manner in which [they] succeed one another'.

These issues are dealt with in my own theories, as follows. Regarding primary observations, it does not matter, within a formalized inductive logic, how 'neutral' they are, because they are as themselves propositions subject to the same controls and rectifications as more abstract components of theories. With regard to Hertz' contention, it effectively denies the existence of a deductive logic and mathematics which is truly formal, that is, independent of any particular terms; it has an appearance of credibility, only because it is true that the contents of conclusions depend on the contents of premises, but there remains nevertheless a formal continuity. As for the issues of theory structure and changes, they are discussed in the chapters on theory formation and selection (ch. 47-48).

Toulmin goes on to describe controversies which developed among scientists. For instance, 'about the legitimacy of extending the methods and categories of physical science to the sphere of the higher, distinctively human mental processes'. He mentions **B.F. Skinner**, who rejected 'any distinctive class of mental laws and processes', and **Noam Chomsky**, who argued that 'linguistic activities are creative and rule-conforming in respects that no behaviorist can explain'. Or again, conflicts in sociology and anthropology 'to do with the significance of history in the explanation of collective human behavior'. Marxists emphasized the 'dynamic, developing character of social structures and relationships'.

There is still today 'deep disagreement' about 'the relation of theory and observation'. For the very Empiricist, 'general theoretical principles have authentic scientific content only when interpreted as empirical generalizations about directly grasped empirical data'. For the rest, they 'suggest that theory construction is totally arbitrary or unconstrained' — surely, Toulmin says, an exaggeration. (I have shown generalizations, whether from the particular to the general, from the potential to the necessary, and from conceivability to existence, are all *identical* in formal process.)

At the other end of the spectrum, the very Rationalist 'reject the idea that raw empirical facts… display any intelligible or law-governed relationships whatever — and still less any necessary ones'. Thus, they ask 'can one, after all, speak of natural events themselves as happening "of necessity"?' Carnap even criticized 'empirical generalizations'. (It is interesting to note that this position is crypto-Heraclitean; it is, of course, logically untenable, since it purports to formulate just such a lawful and even necessary relationship, *itself*. For me, once we have clearly defined necessity, and determined the rules for its induction, the question loses its credibility; note also that to deny necessity implies denial of possibility, too.)

Toulmin very reasonably points out that all the above approaches 'emphasize valid and important points; but, in their extreme forms, they give rise to difficulties'. The task 'is, accordingly, to find an acceptable middle way'. The philosopher has to 'come to grips with the full complexity of the scientific enterprise', without however 'taking too dogmatic a stand'. The philosophy of science has certain recurring themes and issues to deal with, notably (following Toulmin) the procedural, the structural, and the developmental.

a. ***Procedure***. There have been efforts of 'careful analysis of the procedures by which empirical data are actually handled' by science. These include observation, design of experiments, measurement, statistical analysis to deal with large numbers of variables, and systematic classification. These procedures, as well as being 'necessary preconditions for effective theorizing' are 'themselves, in turn, subject to revision and refinement in the light of subsequent theoretical considerations'.

It is true that the scientist is often selective in his observations and that the experiments he designs are expressions of his theoretical assumptions. Kant called it 'putting Nature to the question'. But this is only a reflection of our limits in time and financial resources, not to mention intelligence. We are obliged to search for short-cuts, but we must also be careful. Selectivity often enough leads to erroneous inductions and narrow views, and many experiments fail or give distorted results.

b. ***Structure***. 'The formal structure of science' has been studied. This refers to 'the straightforward extension of methods already familiar in deductive logic', and the more inductive goals of finding 'rigorous formal definitions of… probability, degree of confirmation, and all the other evidential relations'. This is precisely what I have tried to do, through my theory of modal induction.

Modern logicians, Toulmin suggests, are tempted 'to play down important differences between mere descriptive generalizations… and the explanatory theories', I have shown the difference to be as follows: for the former, there is a movement within *de-re* modality; for the latter, the movement is from logical (*de-dicto*) to *de-re* modality. However, I have also shown the structural similarity, and single common source of certification (the law of generalization; see ch. 55).

More important, in my view, is modern logic's confusions concerning the relations between deductive and inductive logic. The former is formally recognized by moderns, *ad nauseam*; but the latter is only discussed by them in very nonformal ways. 'It has not been easy', Toulmin admits, 'to analyze the formal structure of the sciences' and give them a 'working language'. An attempt was made by **R.G. Collingwood** in 1940, with reference to 'mutual presuppositions between more or less general concepts' instead of 'direct entailments'.

I think the best way to overcome the difficulty, is to view the task as one of developing a formal logic of 'inductive implication', as an extension of the concept of 'deductive implication'. There has, supposedly, been some work done in this direction; perhaps **Hans Reichenbach**'s 'analysis of probabilistic argument' falls in this category. But the dilemma presented by **Carl Hemper**, who found it hard to understand the 'logical link' between hypotheses and phenomena, seems to belie this supposition.

There is always, admittedly, some 'reinterpreting' of nature — and the terms of all propositions, as well as the relations between terms, are to varying extents hypothetical. But the thing is to keep in mind the fine thread of referral involved, which gives meaning to the whole; all constructs, however abstract, have some concrete building blocks. Interpretation presupposes *something* to interpret and something to interpret *with*, and therefore cannot be wholly divorced from reality. Abstract theories are just *more* general and theoretical than concrete generalizations, but not essentially different.

Toulmin very responsibly rejects excessive 'relativism', which would destroy 'the objectivity of scientific knowledge', and give 'the impression that the conceptual structures of science are imposed on phenomena by the arbitrary choice of the scientific theorist himself'.

I too vote against sheer Relativism, of course; but I do also recognize that there is some relativity in existence. We have to admit there are relative appearances, in the simple sense that an object is different-looking from different angles or at different times; this is not in itself a major threat to objectivism, but merely an acknowledgement of the complexity of our phenomenal world. Relativity is one of the relations found in our world. But admitting this relation does not prevent us from making distinctions. It just does not follow that all imaginations are realistic or create realities, or that all appearances have equal status so that contradictions may exist, or anything of the sort. Only through a both holistic and case-by-case consideration, can such judgements be made.

c. ***Development***. The reaction to relativism in the late 1960s took the form 'of questioning… that the entire intellectual content of a science can be captured in a propositional or presuppositional system'. Charles Pierce noted that 'the logical status of the theoretical terms and statements in a science is… subject to historical change'. More recently, Quine rejected 'any attempt to classify statements… using the traditional hard-and-fast dichotomies — contingent-necessary and synthetic-analytic — as fallacious and dogmatic'. Thus, a shift developed, away from 'analyzing a science in static logical terms' towards 'analyzing the dynamic processes'.

For me, abandoning the goal of a formal inductive logic is an excessive and defeatist reaction. It is indeed very important to keep in mind, like Pierce, the changing and adaptive character of theorizing and scientific belief. However, that is part of the challenge: to develop a formal logic which is sensitive to the flux of knowledge. I believe the modal inductive logic presented in this volume fulfills these conditions. Quine's rejection of modality is not self-consistent, and therefore it is without credibility; factorial induction shows clearly that formalism and flexibility are not at odds.

'The crucial question … "What is a concept?"… had been largely disregarded' by Logical Empiricists. Viennese Positivists, following Frege, viewed it as 'a matter for psychologists' — with reference, for instance, to the equation of the concept of 'force' with 'a feeling of effort or a mental image'. It is clear that the symbolism of modern deductive logic has had a devastating effect on such thinkers. It produced in them a rigidity, filled with preconceptions. There is no reason why the notion of force should not serve as a springboard for a more defined concept of it; why a closed-minded prejudice against intuition?

Reality is infinitely nuanced and varied, and should be categorized only with a very open and nimble attitude. For instance, consciousness and volition range widely in stature, from the insect's level to the much broader and freer genius and heroism possible to humans. Even inanimate matter and plants may, for all we know, be to varying extents less mechanistic and mindless than we presume. We must remain aware of both the continuities and the differences in degree within that broad range. If one starts with rigidly limited definitions of those concepts, one is bound to disbelieve any manifestations which do not match our simplistic expectations.

The beautiful *mystery* of existence is the mutual reflection and interconnection of everything, and this must be taken in stride. An honestly universal logic is one which is capable of handling, not only the 'square' outlook of science, but the full range of thought, from the notional and vague to very clear concepts. Precisely the role of logic is to help us to gradually move from the former to the latter. A logic which is only capable of dealing with the end-product of this process is useless, since we are ever far from that ideal. A purely 'linguistic' and non-'substantive' logic is meaningless, and is in any case impossible to build without secretly using and trusting intuition.

There are in fact *no* propositions without concepts, and no concepts which do not appeal to intuitive notions. This is not a problem, it is a solution. What matters is to take as much as we can of *the whole* of experience, concrete, abstract and logical, into consideration in constructing both our methodological 'standards' and the substantive 'interpretations' of the sciences. Toulmin rightly points out how 'methodological clarification' and 'creative advance in science itself', develop hand in hand. They have a symbiotic relationship, implying a dynamic give-and-take or feedback. 'It is questionable whether any change, however drastic… is ever as discontinuous or revolutionary' as rigid logicians or scientists would have us believe.

Toulmin describes the ideal inductive logic. It acknowledges 'the parts played by intuition, guesswork and chance in scientific investigation', which **Michael Polanyi** and **Arthur Koestler** emphasized; the 'creativity' of intellection. It avoids the 'pedestrian desire to clip the wings of imagination and confine the scientist to stereotyped procedures', and to a 'barren… accountancy'. But it also avoids 'a romantic anti-rationalism'. 'Chance', he remarks, 'favors the prepared mind'; the 'best trained mind' is 'best qualified to appraise… current problems and recognize significant clues [and] promising lines of analysis'.

There is, in my view, a 'logic of discovery' which satisfies those criteria. It is, first of all, *modal* — it acknowledges the gradual clarification of meanings, the gradual certification of truths. Because it is modal, it avoids the sweeping rationalistic and empiricist generalizations concerning the content and validity of knowledge, which narrow-based modern logic has engendered in legions. Raw data and its interpretation form a *continuum*; logical modality is itself a *de-re* aspect of the world, an extension and manifestation of the central object. The chasm between them is merely an illusion produced by naive and rigid symbolism and axiomatism.

Without compromising the 'to be or not to be' and quantitative requirements of two-valued logic, a multi-valued logic emerges, in which things 'are' in some or all similar instances, sometimes or always, in some or all circumstances, in some or all perspectives. Logical necessity claims the very core of being, the *esse*, the essence. Natural necessity is a slightly broader sphere around that, and temporal necessity yet broader. Still further removed and superficial are the spheres of the temporary, the potential and the conceivable. Extensional modality operates at all these levels, strengthening or weakening the intensity of the other modalities.

The further from the center, the lesser the degree of being. Logical possibility is the most outer wave of an emanation from the core Object, which is *part of* the Object in its fullest sense. It is not 'in the mind' but just closest to the Subject. In some cases, all the Subject is able to penetrate with his consciousness is that superficial level of being; in other cases, it goes deeper. Some appearances are empty shells of possibility, illusions; others are more strongly affirmed, closer to a central reality, more necessary.

The Subject's position relative to the Object affects his insight in some cases; in some cases, the Subject 'makes waves', in the Ether as it were, which blur the Object. Only through a global perspective, by a consideration of the whole field of experience, can these specific relativities and contingencies be assessed for what they are; and that is an ongoing process. There is no case for *ab-initio* rejecting the appearance of any facet of being, and like the empiricists accepting *only* concrete surface impressions or like the rationalists *only* the most enduring abstractions.

A truly broad-based theory of knowledge accepts both that not everything is contingent, and that not everything is necessary. We may, within limits, aspire to a science which is 'an accurate, objective mirror' of reality — for phenomenalist (in the sense above described) reasons. What Toulmin calls 'the strict Realist position' is an impatient or conceited claim for only absolutes; the 'strict conventionalist' or 'constructivist' position claims everything relative. None of them recognizes the *full range* of probabilities, and it is for this reason that they are formally biased. We may grant ontal credibility to *some* theoretical entities, without having to grant it to *all*. The test of 'truth' is always particular to a given proposition and context; it is a vain prejudice to lay claim to a single, sweeping qualification, which ignores all nuances.

Kant's 'attack on things-in-themselves', and Mach's later 'operationalist' dismissal of 'all debates about reality and objectivity as inescapably barren and empty', suffer incurably of self-negation. How do *they* know enough about that 'external' world *to* be able to deny it? Is not that very denial *itself* a claim to having information which is in every sense real and absolute?

If the meaning and truth of a proposition derive *only* from the 'scientific operations' surrounding it, and 'scientists are not to be understood as claiming or disclaiming anything' — then what about these very statements themselves? Are they meaningful or not? Are they true or not? Are they purely 'operational' too, and are they 'claiming or disclaiming' nothing? How are those very 'operations' known (known to have occurred as described, known to be real or valid), let alone anything else?

These philosophers did not ask themselves such obvious questions. Worse still, their positions are still today considered respectable by many. But in each case, we find them to be limited in perspective and unalert to the variegated nature of being and knowing. Most of all they are mostly inconsistent with themselves, when applied to themselves.

In this context, I think philosopher Ayn Rand deserves attention and respect. Her writings in the sixties and seventies, including *Atlas Shrugged* and *The Objectivist Newsletter*, were apparently received with an embarrassed silence by most of the academic community. But, in view of the confusions reigning in epistemology and ontology, I do not see why. One may well not endorse all her pronouncements on every subject — I certainly do not[13] — but one is obliged to recognize what is evidently a valuable contribution to these discussions. She wrote:

An axiom is [not] a matter of arbitrary choice… An axiom is a statement that identifies the base of knowledge and of any further statement pertaining to that knowledge, a statement necessarily contained in all others, whether any particular speaker chooses to identify it or not. An axiom is a proposition that defeats its opponents by the fact that they have to accept it and use it in the process of any attempt to deny it (965).

As the saying goes, 'one cannot have one's cake and eat it too'. She thus proposed a radical standard of judgement for all epistemological and ontological theorizing: *philosophers must test their pronouncements on themselves.* A simple test: if the philosopher is effectively denying his or her structural ability to make that very pronouncement, or that it has truth or meaning, then that statement is false, null and void, untenable. End of discussion. There is no escape from this logic, no convoluted way to claim a transcendent insight, which bypasses this obvious test.

Note the new twist. It is not contradiction between the terms of a categorical, or the elements of a compound ('self-contradiction' in the more Aristotelean sense); nor is it simply a proposition logically implying an opposite proposition or a self-contradiction ('internal inconsistency' in the more modern sense). It refers more specifically to the ramifications of *the act* of formulating the proposition: the acknowledgment of the act implies certain strictures on the content (she called it 'concept-stealing').

As a logician, I have found this ingenious test repeatedly valuable; I acknowledge the debt. In all fairness, this contribution by itself classes Rand as a major player, a logician of the highest order. Certainly, this does not solve every problem, but it considerably narrows down the field as to what is acceptable.

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13 I dearly hope my mention of Ayn Rand in this volume does not cause me to be viewed or labeled as a 'disciple' of hers, or Randite. It would be quite unfair. While I freely admit having been influenced by her writings in my youth, I have long ago dissociated myself from the large majority of her ideas (except for those mentioned herein in her name, out of honesty). Her approach to most issues is far too loosely-reasoned and doctrinaire for my taste.

Toulmin's article contains many other valuable insights. I will now very briefly note some of these, and point out the parallelisms in my own work.

He raises a question concerning the significance of the 'statistical character' of scientific probabilities; I have described modalities as degrees of being, signifying different tendencies towards full realization. He suggests a 'reappraisal of traditional taxonomy — in the light of evolution theory, genetics, and population dynamics'; this is dealt with in my theory of transitive propositions and modal classification.

He calls for a 'framing of authentically empirical questions about perception and cognition'; my direct-relation logical criterion is, I believe, very relevant to any such investigation. He points out the 'variety of perceptual systems'; I have described some features of a logic of the sense-modalities, taken separately and in their interactions. (See ch. 60-62.)

Discussing the relationship of natural science to ethics and religion, he wisely gives credit to theists who 'deliberately limit the claims of science so as to preserve a freedom of maneuver for ethics, for example, or theology'. Many thinkers agree that scientists must be socially responsible, and learn to balance 'a whole range of diverse considerations — economic and aesthetic, environmental and human, as well as merely technical'. In some cases, 'a moratorium on further scientific research' may be called for.

I whole-heartedly agree with such views. Science is not an end in itself; it is only justifiable as an instrument of human welfare. If science expands in ways which harm or destroy mankind, who will be left to know anything? Knowledge presupposes someone alive enough, and even healthy and happy enough, *to* know. These issues are particularly important in this age of genetic engineering, nuclear weapons and industries which endanger our whole environment. Morality is the mainstay of all science.

68. FUTURE LOGIC.

1. Summary of Findings.

Let us now, finally, try and summarize the information presented in this book on logic, part by part and chapter by chapter.

I. ***Actual Categorical Logic***. This is classical, Aristotelean logic, embellished somewhat over the centuries.

1. We distinguished between the art of logic, and the science of it. The former is commonly practiced, the latter is intended to guide practice, as well as serve to provide theoretical grounds for human knowledge.
2. We discussed the three Laws of Thought instituted by Aristotle, the founder of logical science as we know it. They are our principal equipment in sorting out the phenomena appearing before us.
3. The basic tools of logic were introduced: the concepts of truth and falsehood, and logical relations like implication, incompatibility and exhaustiveness.
4. We discussed how words are related to the things they refer to, the concepts of sameness and difference, and the role of definition.
5. The features of the propositional forms called actual categorical were described, distinguishing the terms and copula, and the polarity and quantity. The traditional notation for these various propositions was introduced.
6. The various oppositional relations of propositions were defined, and these concepts were applied to actual categoricals, by means of diagrams and tables, and the findings were validated.
7. The various eductive processes which propositions may be subjected to were defined, and these concepts were applied to actual categoricals, and the findings were validated.
8. Syllogistic deduction was defined, its figures and moods, and the discrimination between valid and invalid such arguments.
9. The main valid moods, plural and singular, of actual categorical syllogism were listed; and the less significant moods of the fourth figure and the subaltern moods, as well as imperfect syllogism were also mentioned. The common attributes of the valid moods were noted.
10. Finally, why and how syllogism are validated was considered. Also, derivative arguments, like sorites, were mentioned.

II. ***Modal Categorical Logic***. This is a broadening of Aristotelean logic, with the addition of modality. Though to some extent known since antiquity, this field has never been properly and fully developed as here done.

11. We distinguished the categories of modality — necessity, presence, possibility, and their negations, as well as other degrees of probability. We distinguished various types of modality, concentrating to begin with on three — the extensional, the temporal and the natural. Tense and duration were discussed incidentally.
12. We discussed certain phenomena underlying these concepts of modality, namely diversity, time and change, and causality.
13. A full list of propositions involving the various categories and types of modality under discussion was presented, and a new notation facilitating our reference to them was introduced.
14. We devised a general theory for predicting the oppositions between plural and modal forms, from the known oppositions of singular and actual forms. We applied these findings

to the forms previously listed, and determined all their interrelations methodically, in enlarged diagrams and tables. We also investigated the eductions feasible from modal propositions.

15. The main valid moods of modal syllogism were listed for each type of modality, and their validations were described.

16. Valid moods of lesser or derivative significance were also listed, including moods of mixed modal types. The statistics of validity were looked into, and general principles formulated.

17. The concepts of being and becoming were analyzed, and new propositional forms concerning change (transitives) were introduced. Some important syllogistic arguments involving them were pointed out.

18. Permutation was discussed in more detail, and other copulae than those thus far considered were mentioned.

19. We looked into substitutive processes, comparative propositions, and the differences between dispensive, collective and collectional quantities; also, the doctrine of quantification of the predicate was discussed.

III. ***The Logic of Logical Conditioning***. This is a closer inspection of the logical relations used in practice, a field which may also be described as the self-analysis of logic. Its beginnings date from Aristotle and Philo in Ancient Greece, but it has been especially developed formally in modern times. However, our own treatment of the subject is considerably novel, on many counts.

20. We discussed the genesis and role of the three Laws of Thought in logic, and the distinctive functions of each of them. The notion of phenomenal credibility was further highlighted.

21. We made original formal definitions of the categories of logical modality, and justified the knowability of these concepts.

22. We discussed the various aspects of contextuality, which affect logical modality.

23. The formal treatment of logical relations begins with the concepts of conjunction. The factual forms of conjunction were distinguished with reference to polar considerations, and their oppositions to each other were identified. The modal forms followed accordingly, and so did their oppositions, by means of the general theory of opposition earlier presented.

24. Conjunction gives rise to various kinds of conditioning. We began by analyzing logical conditionals, known as hypothetical propositions or 'strict' implications. Novel negative forms were also considered, which in the next chapter led us to a hierarchy of forms.

25. We discerned that hypotheticals are defined not only by the connection they signify, but often also with reference to certain logical bases. Thus, we distinguished between hypotheticals with unspecified bases, and those with normal or abnormal specified bases. The oppositions between hypotheticals, and the eductions from hypotheticals, were described and validated.

26. Disjunctive, as distinct from subjunctive, conditioning was considered. Various manners of disjunction were described and interrelated.

27. Then we looked into various intricacies of logic, expanding on what had so far been presented. Conjunctive, hypothetical and disjunctive propositions form a broad continuum of relations, affected by the number of theses involved, their respective polarities, the polarities and modalities of their relations. Nesting and mixed-form relations were looked into, and we evolved the unifying method of matrixual logic.

28. Next, using some modern symbolic techniques, we investigated the principal interactions between logical relations. We did so with reference to matrices, and thus demonstrated the precise intellectual goal of all such manipulations, and the limits of their practical utility.

29. We developed a full list of hypothetical syllogisms, including those with negative forms, showing how they are derived from the most obvious case. We also introduced the novel process of production, drawing conditional conclusions from unconditional premises.

30. Logical apodosis was dealt with, including its modal forms. Dilemmas and their rebuttals were analyzed.

31. Paradoxical propositions were considered. They allowed us to formally define self-contradiction and self-evidence, and some important philosophical applications were pointed out. We also evolved a more thorough theory of hypotheticals, listing all the normal and abnormal forms which may arise, and investigating their distinct properties in opposition, eduction and deduction.

32. We considered apparent double paradoxes, which are not as legitimate as single paradoxes, showing their logical function and how they are to be dissolved. The examples of the Liar Paradox and the Barber Paradox were dealt with.

IV. ***The Logic of De-re Conditioning***. This field may be viewed as an expansion of the modal categorical logic presented in part II (by considering propositions with more than two terms), or as a broadening of the logic of conditioning presented in part III (by considering non-logical types of modality); in either case, it seems to be entirely original. The value of this field to all future science and ontology is inestimable — it sets new, very high standards of precision for any discussion of causal relations.

33. Just as logical modality gives rise to logical conditioning, so the natural, temporal and extensional modalities give rise to their own quite distinct types of (*de-re*) conditioning — and thence in turn to various types of causal relation.

34. We began our analysis with reference to natural conditioning, proceeding much as we had done for logical conditioning. The features of natural conditionals were distinguished, including their bases and connections and their quantities. We also noted the issues of sequence of the theses, modalities of actualization, and acquisition and loss of powers. Natural disjunction was also mentioned.

35. We looked into the relations between natural conditionals and categoricals; and we investigated the oppositions among, and eductions from, these new forms.

36. We developed the main valid moods of natural conditional syllogism, and also listed the subaltern and invalid moods. We described the productive arguments, which enable us to infer, and thus form, natural conditionals from modal categorical premises.

37. Natural apodosis, both actual and modal, was analyzed; and so was natural dilemma.

38. We similarly investigated the structure and properties of temporal conditionals, showing their exact relation to naturals (especially noteworthy, was their discontinuity), and various arguments of mixed modal-type were presented.

39. Then we considered extensional conditionals — their (analogous yet distinct) features, oppositions, and eductions.

40. And we analyzed, for extensional conditionals, syllogism, production, apodosis and dilemma in some detail.

41. Development of the logics of various types of conditionals allowed us to review certain issues concerning categorical propositions, with more powerful formal tools. Until then, the modalities of subsumption by the terms of categoricals had been skimmed over; now, a more nuanced approach became possible. Included here was discussion of imaginary terms.

42. We were also now able to analyze certain condensed forms, which are thought of as categorical but involve some kind of conditioning, including forms with complex terms, and those having to do with aetiology, teleology and ethical modality.

V. ***The Logics of Classification and of Adduction***. These two topics were lumped together, without intent to imply a close relation between them. They are fields of logic which derive from the previous, though important in themselves. There are many significant innovations in our treatments of class-logic. The novelty in our treatment of adduction lies in its modal orientation.

43. We saw that class-logic takes terms ‘nominally’, in a way distinct from the subsumptive approach of Aristotelean forms. However, classes and classes of classes are easily defined with reference to Aristotelean forms; and the features, immediate inferences, and deductive arguments of forms with such terms are readily derivable from these definitions.

44. We distinguished classes and classes of classes as two separate ‘orders’ of classes, each with its own though parallel ‘hierarchy’ of classes. The relational aspect of these concepts was stressed, when we sought to clarify their extreme manifestations.

45. We analyzed the concept of self-membership both conceptually and with reference to examples, and found it wanting. We then considered the famous Russell Paradox, and demonstrated that the solution of the problem lay in the concept of permutation (rather than in issues of membership), whose ontological significance was also clarified.

46. Adduction is the general method by which we induce the logical probability of any information. We discussed its well-known form of argument, which is similar to apodosis, only with less established premises and/or conclusions. We showed how it provides and weights evidence, and thus validated it. We also discussed *de-re* adduction.

47. We looked into the psychology of theorizing, described the structures of theories and various criteria we use in making them, and we suggested ways theories may be more purposefully formed and tested.

48. We described in formal terms the scientific method of judging between theories, but also indicated the pragmatic compromises that are often called for, and how theories may gradually be changed. Theories with exclusive empirically-tested predictions were granted formal certainty.

49. Under the heading of Synthetic Logic, we advocated a healthy skepticism and flexibility, which transcends rigidly formal standards of theory-evaluation — an open-mindedness to more far-fetched hypotheses which are not definitely disproved.

VI. ***The Logic of Factorial Induction***. This is a completely new field of logic — the first genuinely formal theory of generalization and particularization in history. Again, this sets entirely new, extremely precise standards for all future science, and answers some of the most fundamental questions of epistemology (if I may be forgiven for sounding such a loud trumpet).

50. The problem of induction was to begin with posed with regard to actual categorical propositions — how are they known, whether particular or general? The solution is so simple, with relation to actuals, that it seems puerile; but as we later see, when modal propositions are considered, the solution appears much more interesting.

51. In order to deal with modal induction, we first had to develop a precise theory of all the logically possible combinations of modal propositions. The forms considered thus far were mere elements, that may be compounded in a certain number of ways, according to their oppositions. We noted that compounds give rise to special arguments; also, that directional issues may be raised.

52. Next, we introduced the concepts of fractions and integers, which describe states of being more definitely than elements or compounds can do. The former are parts of the latter. These concepts and the resulting formulas depend on the logics of *de-re* conditionals, and different systems evolve according to the mixtures of *de-re* modality we choose to consider, and whether we ignore or include directional issues.

53. These preliminaries led us to a formal theory of factorial analysis of elements and compounds, and indeed of all states of knowledge concerning anything. A factorial formula consists of all the alternative integers which logically may come out of any given item of knowledge. In some cases, only one alternative is formally acceptable, so that an unexpected deductive situation occurs.

54. Thereafter, we proceeded to formally demonstrate the knowability of all types of necessity, whether extensional (generality), natural or temporal, or any combination of these. We described the stages of induction, and defined the central issue of induction as a pursuit of solitary integers.

55. After discussing the philosophical aspects of induction, we proposed an exact ‘Law of Generalization’ in formal, factorial terms — one which precisely determines the factor selection from any given datum whatsoever. As later shown, this same Law also controls the formula revisions we call particularization.

56. We applied the Law of Generalization to all possible elementary and compound forms, and showed the predicted valid inductive conclusion in each and every case to be rationally credible, thus also demonstrating the correctness, value and validity of the Law as a whole.

57. We then considered context changes, which require us to amplify previous conclusions or harmonize them with new data. This was called formula revision, and the difficulties it involves were clearly described, in order to show the power of their formal resolution.

58. We applied the Law of Generalization to all inconsistent conjunctions of elements or compounds, and obtained precise inductive conclusions from all of them.

59. Lastly, we applied the Law of Generalization to other situations requiring formula revision, like adding fractions to integers, reconciliation of integers, indefinite denial of integers, among others.

In this way, we demonstrated that we have evolved a single, uniform, consistent tool for dealing with all knowledge contexts. We used the specific example of categorical propositions involving different mixtures of *de-re* modalities; but we also indicated how expanded application to still more complex situations is to be effected.

VII. ***Perspectives***. In this final part of the book, we dealt with wider, more philosophical or historic issues. Chapters 60-62 together (with reference of course to all previous chapters) sketch my theory of cognition. Chapters 63-67 could be viewed as a separate volume, called *For Future Logicians*; it is a small-print commentary mostly intended for academics rather than general readers.

60. We looked into various ontological issues. What do we mean by phenomena? How do we distinguish the empirical from the hypothetical, the physical from the mental, the concrete from the abstract? Representation and analogy were briefly discussed.

61. Next, we pointed out the primarily relational nature of consciousness, and on this basis evolved a novel systematic classification of the kinds of consciousness, defining many of the epistemological terms used in the course of our logical treatise. We thereby proposed a more logically consistent theory of the mind, than that suggested by popular psychology and many philosophers and biologists.

62. We also considered some important logical issues surrounding sense-perception, and recognition and memory. These insights, together with those made previously, about the various kinds of phenomena and consciousness, and about imagination, allowed us to arrive at some understanding of 'universals'.

63. We reviewed a history of logic, from Ancient Greece to the present, making positive or negative evaluations as we proceeded.

64. We analyzed concepts like formalization, symbolization, systematization, and axiomatization; and thus we began our critique of modern logic, mentioning also the more constructive contributions.

65. Continuing, with reference to literature on the subject, we tried to estimate the level of knowledge in current modal logic, its achievements and its weaknesses and blank areas.

66. We looked more closely into current views on metalogic (and incidentally class-logic), countering them with our own views of language and meaning, definition and proof, and indeed of the foundations and role of logic as a whole.

67. We then deepened our understanding of all modality, as signifying different kinds and degrees of being; and we indicated how our theory of factorial induction can be expanded to include logical modality — to precisely solve the problem of induction from logical possibility, and thus explain the essential continuity between this mode and the *de-re* types. Then we looked into the current state of knowledge in inductive logic, endorsing or disagreeing as appropriate, and pointed out a methodological standard.

68. The whole was finally summarized here, and in the next section I mention some possible areas of research for future logicians.

2. Gaps to Fill.

This treatise, though evidently somewhat encyclopedic in scope, makes no claim of exhaustiveness. We have pointed out, as we proceeded, various directions of possible further research, often spelling out the major parameters for it. The general reader was invited to personally try and do the job; one does not have to be a certified expert, one learns by doing. The value of such research is not merely that it takes the science of logic forward, but especially the exercise in the art of logic that it provides the researcher.

Let us now, therefore, mention some of the major opportunities for further theoretical inquiry. The field of logic is still wide open, the task is still enormous; no one person can do it all. *The goal is nothing less than clarifying the formalities of all human knowledge, starting on the nonmodal, categorical level, then fanning out into modal and conditional considerations; and moving from deductive to inductive issues.*

a. Categorical Logic.

The main focus of logicians through the ages has been the 'is' copula, which was gradually understood to be intended in a purely subsumptive sense. The sense of 'X is Y' before permutation may however contain formal properties which have yet to be thoroughly investigated. We have to go beyond the simplistic, merely quantitative aspects of logic, into its more conceptual, qualitative aspects.

The possessive 'X has Y', the active 'X does Y', and other similar verbs, are all copulae of broad applicability which are open to further analysis, including their oppositions, their eductions, their syllogisms, and so forth. Substitutive arguments, the logic of comparatives, the logic of collectives and collectionals (as distinct from dispensives), all still require work.

The class-membership copula has received a lot of attention in modern times, but as I have shown some serious conceptual errors were made. The whole field needs to be reconstructed, in the framework of my new definitions and initial analyses, avoiding the flights of fantasy which have characterized previous incursions. It is also important to keep a humble perspective on things; this field is not as crucial as has been construed, but a very limited and derivative one.

Issues of tense and duration have to be more systematically clarified (though much work has admittedly been done by some moderns in this field). My work in the formal logic of change is just a beginning, a sketch of some of the main components; much is still to be done, to cover all eventualities. For all categoricals, once the actual logic is dealt with, the corresponding modal logics must also be developed.

b. The Logic of Conditioning.

Once the categorical manifestations of any family of forms has been thoroughly formalized, its conditional versions must in turn be investigated. Each type of modality requires separate treatment, first on a categorical level, and then in the distinct type of conditioning it gives rise to. Mixtures of modes must also be dealt with. With regard to the more generic conditional logics considered in the present work, many details have been left out.

Even in logical conditioning, which moderns have investigated extensively using symbolic techniques, and which I have handled in some detail in ordinary language, there are still opportunities. Context comparisons could be elucidated in more formal terms. The dynamics of changes in logical modality can be further investigated. Also, although the various forms of modal conjunctives, hypotheticals, and disjunctives, may strictly be equated to each other, so that analyzing their interactions is somewhat repetitive — they nevertheless inform thought in different ways, and therefore it would be valuable to treat them as if distinct; interesting lessons might transpire.

I have developed the logics of *de-re* conditioning in some detail, but not fully. In view of the novelty of these doctrines, my prime concern was to put the concepts across clearly, avoiding fatiguing minutiae; but logical science is eventually obligated to consider every little thing, however outwardly repetitive it seems. Thus, for instance, temporal conditioning might be looked into more thoroughly.

De-re connections devoid of basis might be profitably investigated, or again unusual *de-re* connections of one modal-type combined with bases of another modal-type. But especially important, in any case, is a thorough investigation of the oppositions and syllogistic mutual-impacts of forms of entirely different modal-types. Productive argument is a new field, which may be open to further expansion.

More work needs to be done on the modalities of subsumption in categorical propositions. *De-re* conditioning involving more than three terms, or other special constructions, is worth pursuing further. The temporal sequences of *de-re* conditionals which we considered were very simple; more complex ones should be looked into. The issue of modalities of actualization is still wide open to formal treatment, as is that of acquisition and loss of powers.

Condensed propositions involving complex terms may be subjected to more systematic treatment (working out their separate and combined oppositions, eductions, deductions). Other forms deriving from the various types of conditional logic, like 'making possible' or 'causing' or 'requiring', are important in themselves, and should be exhaustively dealt with. A complete taxonomy of causal relations has to be developed.

The field of causal logic cannot of course be completed without reference to volition — but, though I have indicated how this type of modality is to be formally defined with reference to natural modality (by denial of all deterministic antecedents), I have here avoided the big job of developing this field. The auxiliary field of influences on volition (including habit), which is definable with reference to temporal modality, has also been purposely ignored. I have many preparatory notes on aetiology which I may one day use, but the reader is welcome to try and do these jobs independently.

Ethical modality has been all but ignored here, and I find the treatments of it by modern logicians to be very simplistic. Yet this field of formal logic follows very naturally and systematically from that of aetiology (via teleology), and the job is not overly difficult. My old notes on the subject show very interesting ways that absolute standards can be formally defined and logically induced, and I may one day publicize them. Meanwhile, go ahead, try and do the job.

In any case, it is my conviction that we can formally demonstrate the harmony of Ethics and Science. That is, I consider illogical, the view that scientists may pursue any research, however harmful to human welfare; the view that the pursuit of scientific truth sometimes necessitates ethical compromises. Ethical modality is ultimately just as 'factual' as logical or *de-re* modality, and the findings in all fields are bound by logic to be harmonious. Thus, if ethical logic concludes 'do not research this matter further', science might be slowed by obedience, but in the long run can discover as much if not more than it would by disobedience.

c. Inductive Logic.

My work in this area, though extensive, merely opens the door to a host of new possibilities. The science is now founded, the model is sharply drawn and part of the edifice is built; but it is far from complete. Especially here will the enterprising future logician find rich rewards.

Logic until now has focused on elementary forms, but compound forms are also propositions in their own right, though describing more specific relations. I have indicated some of the oppositions between compounds, but a more thorough treatment is required. Compounds involving singulars should be researched further. I have dealt with one-directional ('flat') compounds, but two-directional ('stereo') compounds may also be looked into.

Fractions and integers are of course still more definite compositions than gross compounds, and their formal interrelations may be looked into further. But in any case, those I have presented and dealt with here are flat; the field of stereo fractionating and integration is a large and important one, which has yet to be developed; patience, imaginativeness, and a big-CPU computer and powerful software are required for this work. Once stereo factors have been developed, all gross compounds should be factorized in their terms.

Another avenue of expansion for factorial analysis, as we mentioned, is to take logical modality into consideration, as well as the *de-re* types. The value of this is more theoretical than practical, since logical modalities are inherently implicit in the concept of 'strength' of factors; nevertheless, this work would serve to demonstrate the formal character of induction from logical possibility to *de-re* modalities. Here again, the volumes of data involved require appropriate computing equipment, as well as intellectual capacities.

In any case, all such expansions of factorial analysis will engender enormous growth in the fields of factor selection and formula revision. However, the Law of Generalization we introduced remains the sole operative principle, so the job is reasonably straightforward. Even without going into such expansions, I have left quite a bit work for future researchers to do in the field of formula revision; for instances, solving indefinite denial of gross compounds, or interactions between integers and gross formulas.

But all this is only the beginning: it concerns categorical propositions (with the standard copula). A greater challenge still is the inductive logic of conditional propositions of all types and forms. These can, as we have shown, be effectively deduced from categoricals, through the process of production. However, that does not mean that they cannot be and are not also induced.

Thus, a factorial logic for all conditionals is also required, which will be much more complicated than that for categoricals, yet may obviously be developed along the same lines. First consider compounds, then fractions, then integers, then factorization, then inductive decisions, as before. Again, better have a big machine at your disposal!

As for changes in copula, they are only significant at the initial, experiential stage of induction; subsequent processes of generalization and particularization are as far as I can see exactly identical.

d. Logical Philosophy.

With regard to ontology and epistemology, we have seen some of the issues and conditions logic implies for them. Above all, logic demands of any philosophical suggestion concerning the presuppositions or implications of human knowledge that it be self-consistent, that it include itself in its considerations, that it explain itself. Beyond what has been said, many questions of course arise, which are relevant to logical science, but were outside the chosen scope of our treatise.

In the process of clarifying the history of logic, in order to determine what was old and what was new in our own findings, we incidentally came across issues relating to the methodology of historical judgment. This is an interesting field in my view, which yet requires more pondered and systematic consideration, at least in my case. I learned from this discovery that indeed, as modern thinkers maintain, each science has its own methodological issues, and logicians may be called upon to set specialized standards.

In any case, logicians, philosophers, historians, and special scientists, indeed all of us, should of course aim for as wide a perspective as possible in theories. By this I mean, that it is not enough to be very knowledgeable in one field, like Western Philosophy or Modern Physics say, but to keep in mind (as far as possible) the beliefs and insights of all peoples and all periods of history and all disciplines (including religion). There is no profit in moving from one closed-minded dogma to another; open-mindedness is of the essence of knowledge.

I do not of course advocate that we pay attention to obviously deranged or perverse ideas, or clearly discredited ones; but just that unfashionable ideas, which are less probable in our eyes, be kept in mind. Our criteria in theory selection are not all binding; many theories are effectively discarded, not because they are intrinsically inconsistent or discordant with empirical data, but because they are more far-fetched (they require more confirmation than their alternatives) or because their practical value is relatively minimal. Such theories are not strictly-speaking out of the running, though of course we need not be committed to them or endorse them.

With regard to the philosophy of logic, I have argued vehemently — and I hope very convincingly — against modern extreme symbolism and axiomatism. These trends are confused and futile. Logicians must be able to build their case essentially in ordinary language terms, which do not obscure what they are saying if anything; they must consider whether the epistemologies and ontologies they imply are internally consistent and in accord with common experience; and they must strive for a more genetic, conceptual understanding of the development of logic.

Historians of logic, for their part, should keep in mind the distinction between the art of logic and the science of logic, between implicit logical skills and overt (formal or informal) pronouncements about logic. These are separate issues, which do not always travel in tandem. There is still much work to do in this field, as indeed in that of philology. The self-centeredness of many Western evaluations have to be guarded against, without of course going to the opposite extreme of belittling evident differences and imposing interpretations which were never intended.

3. Concluding Words.

No human can claim omniscience, and no human achievement can be claimed to be perfect. I am of course human, more ignorant than a great many people, and of limited intelligence and virtue. Still, I believe this work on logic has considerably revived a virtually moribund enterprise. Important advances have been made in traditional fields, major new fields have been pioneered and developed, and the science as a whole has been set on much stronger foundations. Much work remains to be done, and it is my fond hope that many will take up the challenge, for I consider logic to be of high importance to human knowledge, personal improvement, and social development.

Logic is of course not everything, but it helps. It will not make a person or society virtuous, but it will facilitate such pursuits. Many people avoid logic precisely because they are afraid of the imperatives it may impose on them; they would rather not know, than find themselves forced to accept what they refuse to believe, or to behave in demanding ways. Also, logic requires an effort of thought, just what the mentally lazy wish to avoid. But the important thing is to realize its benefits; knowledge is enlightening, it is pleasant, it saves us from wasteful or irrevocable mistakes, it improves one's life.

I ask myself now, what is the specific message of this book? To the general reader, it is: *think more clearly*. Consider the alternatives, the possibilities, the strictures, the necessities, the conditions under which things take place or are true, and those in which other things come to the fore. Consider the sources or bases of your beliefs, opinions or knowledge; and evaluate them.

Educators are to be encouraged to *teach children, youths, and adults formal logic* — the classical, and the new modal, conditional, and inductive doctrines. Using ordinary language, not esoteric symbols. Each age group should of course be addressed at its own level; children can learn simple syllogisms, Ph.D. students can research factorial formulas.

The goal is not to produce thinking machines, people enslaved to verbal mental processes, but merely to strengthen the natural faculty of logical intuition we all share. If we teach mathematics, the ability to reason with numbers, should we not also teach logic, the ability to reason with concepts? The latter would surely seem more important, since we are more than merely economic beings.

Even economic thinking is of course improved by logic. A house is better planned, a business better run, by a person better trained in the art of thought. But furthermore, in this era of democracy and ecology, people are called on to make all sorts of complex judgments, for which it is to everyone's advantage that they be well-equipped. The habit of reasoning, thinking things through, is also of moral value — teaching people to resolve their differences in rational ways, to judge each other more objectively and dispassionately (which does not of course mean cruelly or without regard for feelings).

With regard to special scientists, I am here calling for *a more modality-conscious level of Science*. It is time for the sciences to specify the precise types and categories of *de-re* modality of their statements. I believe this is the next step in the evolution of the scientific enterprise, one which will greatly enhance its capabilities, generating new ideas and solving many outstanding and future problems. Thus far, science has proceeded with the limited tools of thought put at its disposal by Aristotle and Philo, and their successors to this day. It seems obvious that with more sensitive cognitive equipment it successes will be still more impressive.

To give an example. Everyone will agree that the more rigorously we reason, the more likely are our conclusions to be correct. It should therefore be obvious that if we distinguish, say, between a natural causal relation and an extensional one, we save ourselves from error. For if these two types of conditioning are lumped together in a generic if-then statement we may assume them to be in disagreement while they are in fact compatible, or harmonious while they conflict. Only by specifying the types involved (mentally if not explicitly), and knowing the precise formal oppositions between forms of these types, can a proper judgment be made.

Where our thinking is not modality-conscious, we may thus assume a theory to be consistent when it is not, and lose the opportunity to improve it or reject it. Or we may assume a theory to be inconsistent when it is not, and adhere to an unproved alternative in its stead. Clearly, the more perspicacious, the less naive, our logical tools are, the more intelligent and interesting are our theories. Knowing faults in one's theories stimulates the imagination; and knowing that alternatives are still viable also expands one's consciousness.

Furthermore, I challenge scientists to make the effort to specify the exact logical modalities of their statements, with reference to inductive logic. I believe I have made available to them unequivocal tools for this purpose, namely the processes of factorial analysis, factor selection and formula revision. My vision is of a science which knows precisely where it stands logically, which can precisely trace and honestly displays its inductive as well as deductive justifications. Again, such efforts can only be rewarding, because they promote open-mindedness, imagination, awareness of extra problems, awareness of alternative solutions.

Scientists no longer have any philosophical reason to doubt causality, or to feel embarrassed in referring to it. We have here shown in clear and indubitable formal terms its definability, its varieties, and just how it is to be established case by case. Likewise, the Cartesian ideal of deductive science, which was so far from the empirical standards of practising scientists, has been demonstrated to be absurd; instead formal logic now demands the organization of knowledge in accord with the model set by factorial induction. It is possible, and it is necessary.

All this concerns not only natural scientists, like physicists, biologists, psychologists, but also social scientists, like historians or political scientists (though in their case considerations of ethical logic impinge more directly). Needless to say, it also applies to less officially theoretical professionals, like doctors, business leaders, politicians or journalists, who are to some extent — like all people, though perhaps more than most — making daily theoretical judgments of their own.

Philosophers too are subject to the same confines of inductive and deductive logic. Speculations are of course often valuable, and unavoidable. Sometimes even, to be sure, paradoxical insights, like some of Kant's or some mystical philosophies, have enormous creative potential, generating ideas in diverse fields. We do not advocate naive and impatient over-simplifications. But still, ultimately, the goal is to make genuine sciences out of epistemology and ontology, and in order to achieve this philosophers must make an effort *to keep track of their methodological grounds*.

My concluding message to future logicians, the theoreticians of logical science, is to keep in mind above all *their role and responsibility as teachers*. It is useless to write something most or all other people will never understand. Our goal is not to impress others with our exclusive knowledge. The logician is not supposed to be a solipsistic manipulator of virtually meaningless symbols, divorced from reality and from society.

The logician is a scientist and communicator. As a scientist, he or she must refer to the widest possible perspective on things, and find non-naive and neutral methodological information. As a communicator, he or she must find ways to pass this information on to anyone open to it. The goal of logic is to clarify and improve knowledge, not to obscure it and effectively leave people without methodological ways and means. Thus, symbolism must be eschewed so far as possible (it is admittedly not always easy); today, symbolization has become almost synonymous with academic respectability and proof — but our attitude should be the very opposite.

Furthermore, the logician must be philosophically aware, and never range out too far from common-sense ideas in epistemology and ontology. There must after all be reasons why the human psyche has cumulatively come to these conclusions, these views; the job is to understand what these reasons might be. The logician must in any case learn to apply logic to his or her own thinking, and thus avoid arriving at ridiculous conclusions like those of certain 'linguistic' schools. Only ideas which somehow or other vindicate the human faculties of knowledge, admitting their essential effectiveness and objectivism, have any logical credibility.

It is my hope that the present manifesto for future logic will change things for the good, G-d willing.

Completed[14] on Denman Island, B.C., Canada,
on 26 June, 1990 (the 3rd of Tammuz, 5750),
with G-d's help. *Baruch HaShem*.

Here, in gratitude for the knowledge received, is a beautiful psalm of praise by David, lovely king of Israel:

The heavens declare the glory of G-d,
And the firmament tells of His handiwork.
Day unto day utters the tale,
Night unto night unfolds knowledge.
There is no word, no speech,
Their voice is not heard,
Yet their course extends through all the earth,
And their theme to the end of the world….
The symbols of the L-rd are faithful,
Teaching the simple man wisdom.

Dedicated to my beloved Parents, z"l.

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14 Writing started in late 1988, but was based to some extent on notes made in 1968-74.

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List of Tables

List of Diagrams

Appendix 1. On Factorial Analysis

The 12-page table shown next is an appendix to chapter 53 (in particular, section 5).

It shows the factorial analysis of all **195 gross formulas** (listed in the column labeled **Formulas**), in terms of the **63 factors** (columns **F1-F63**) in the 'open system' of mixed (natural and temporal) modality.

Due to the size of the table, it is split into three 4-page segments. The first concerns factors **F1-F21**; the second, factors **F22-F42**; the third, factors **F43-F63**. Thus, to see the factors allowed for by any gross formula, it is necessary to look along the row corresponding to it in all three segments.

The factors of any gross formula are signaled by a '**1**' in the cell concerned (where row and column cross); if a cell is blank, it means that the factor heading the column is not a possible outcome of the gross formula heading the row.

The gross formulas are first sorted into elements and compounds. Then the 20 elementary (plural) propositions and the remaining compound formulas (consisting of two or more elements) are, respectively, sorted according to the **number of factors** they each have (indicated in the column labeled **NF**).

The factors of the 20 elements are already known (see ch. 52). The factors of the remaining 175 gross formulas (compounds), follow automatically from them. We need only do the following:

1. Split the compound into its component elements (which number two, three or four, as the case may be).
2. Look and see which, if any, of these elements have the factor concerned.
3. If *all* have it, the compound in question also has it; otherwise, not.

(For example, the compound **AcInOp** has factor **F8**, because its three component elements **Ac**, **In** and **Op**, have only this one factor **F8** *in common*.)

The value of this table is, as we have seen (ch. 54-59), to guide us in generalization and particularization, by indicating successive inductive preferences. In some cases (the eleven cases with a single factor, to be specific), it even indicates deductive inferences.

Thus, to repeat: the following large table shows the factorial analysis of all 195 gross formulas in terms of the 63 factors (F1-F63) in the open system, mixing modality types. Elements and Compounds are separated, then sorted by number of factors (NF). The factors of any gross formula are those marked '1'; non-factors are left blank (instead of '0') for clarity. The factors for each gross formula are split up into three sets (F1-F21, F22-F42, F43-F63) by reason of the great width of the table.

Factors F1-F21 pages 452-455
Factors F22-F42 pages 456-459
Factors F43-F63 pages 460-463

FACTORIAL ANALYSIS OF GROSS FORMULAS										FACTORS F1-F21												
NF	Formulas	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20	F21
	Elements																					
1	An	1																				
1	En		1																			
3	Ac	1		1					1													
3	Ec		1		1					1												
7	A	1		1		1			1				1					1				
7	E		1		1		1			1				1					1			
15	At	1		1		1	1		1				1		1			1		1		1
15	Et		1		1	1	1			1				1		1			1		1	1
31	Ap	1		1	1	1	1		1		1		1		1		1	1	1	1	1	1
31	Ep		1	1	1	1	1			1		1		1		1	1	1	1	1	1	1
32	In	1						1	1		1		1		1							
32	On		1					1		1		1		1		1						
48	Ic	1		1				1	1		1	1	1		1		1	1		1		
48	Oc		1		1			1		1	1	1		1		1	1		1		1	
56	I	1		1		1		1	1		1	1	1		1	1	1	1		1	1	1
56	O		1		1		1	1		1	1	1		1	1	1	1		1	1	1	1
60	It	1		1		1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1
60	Ot		1		1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1
62	Ip	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
62	Op		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Compounds																					
1	AcInOp								1													
1	AcEp			1																		
1	AIcEpOt																	1				
1	AEt					1																
1	AtIEtO																					1
1	AtE						1															
1	ApItEOc																		1			
1	ApEc				1																	
1	IpEcOn									1												
2	AcIn	1							1													
2	AcOp			1					1													
2	AInOt												1									
2	AIcEp			1														1				
2	AEpOt					1												1				
2	AtIcEpO																			1		
2	AtIEt					1																1
2	AtEtO						1															1
2	ApIEtOc																				1	
2	ApItE						1												1			
2	ApEOc				1														1			
2	ItEOn													1								
2	IpEc				1					1												
2	EcOn		1							1												
3	AInOp								1				1									
3	AIcOt												1					1				
3	AEp			1		1												1				

NF	Formulas	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20	F21
3	AtIcEpOt																	1		1		
3	AtIEpO																			1		1
3	AtEt					1	1															1
3	ApIEtO																				1	1
3	ApItEtOc																		1		1	
3	ApE				1		1												1			
3	ItEOc													1					1			
3	IpEOn									1				1								
4	AIn	1							1				1									
4	AOt					1							1					1				
4	AtInO														1							
4	AtIcEp			1														1		1		
4	AtEpO						1													1		1
4	ApIcEpOc																1					
4	ApIEt					1															1	1
4	ApEtOc				1														1		1	
4	IEtOn															1						
4	ItE						1							1					1			
4	EOn		1							1				1								
5	AIcOp			1					1				1					1				
5	AtIEpOt					1												1		1		1
5	ApItEtO						1												1		1	1
5	IpEOc				1					1				1					1			
6	AIc	1		1					1				1					1				
6	AOp			1		1			1				1					1				
6	AtInOt												1		1							
6	AtIcO														1					1		
6	AtIEp			1		1												1		1		1
6	AtEpOt					1	1											1		1		1
6	ApIcEpO																1			1		
6	ApIEpOc																1				1	
6	ApItEt					1	1												1		1	1
6	ApEtO				1		1												1		1	1
6	IEtOc															1					1	
6	ItEtOn													1		1						
6	IpE				1		1			1				1					1			
6	EOc		1		1					1				1					1			
7	AtInOp								1				1		1							
7	AtIO														1					1		1
7	AtEp			1		1	1											1		1		1
7	ApIcEpOt																1	1		1		
7	ApItEpOc																1		1		1	
7	ApEt				1	1	1												1		1	1
7	IEtO															1					1	1
7	IpEtOn									1				1		1						
8	AtIn	1							1				1		1							
8	AtO						1								1					1		1
8	ApInOc										1											
8	ApIcEp			1													1	1		1		
8	ApEpOc				1												1		1		1	

NF	Formulas	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20	F21
8	IcEpOn											1										
8	IEt					1										1					1	1
8	EtOn		1							1				1		1						
9	AtIcOt												1		1			1		1		
9	ApIEpO																1			1	1	1
9	ItEtOc													1		1			1		1	
11	AtIcOp			1					1				1		1			1		1		
11	AtIOt					1							1		1			1		1		1
11	ApIEpOt					1											1	1		1	1	1
11	ApItEpO						1										1		1	1	1	1
11	ItEtO						1							1		1			1		1	1
11	IpEtOc				1					1				1		1			1		1	
12	AtIc	1		1					1				1		1			1		1		
12	AtOt					1	1						1		1			1		1		1
12	ApInO										1				1							
12	ApIcOc										1						1					
12	ApIEp			1		1											1	1		1	1	1
12	ApEpO				1		1										1		1	1	1	1
12	IcEpOc											1					1					
12	IEpOn											1				1						
12	ItEt					1	1							1		1			1		1	1
12	EtOc		1		1					1				1		1			1		1	
13	AtIOp			1		1			1				1		1			1		1		1
13	ApItEpOt					1	1										1	1	1	1	1	1
13	IpEtO				1		1			1				1		1			1		1	1
14	AtI	1		1		1			1				1		1			1		1		1
14	AtOp			1		1	1		1				1		1			1		1		1
14	ApInOt										1		1		1							
14	ApIOc										1						1				1	
14	ApItEp			1		1	1										1	1	1	1	1	1
14	ApEpOt				1	1	1										1	1	1	1	1	1
14	IcEpO											1					1			1		
14	ItEpOn											1		1		1						
14	IpEt				1	1	1			1				1		1			1		1	1
14	EtO		1		1		1			1				1		1			1		1	1
15	ApInOp								1		1		1		1							
15	ApItOc										1						1		1		1	
15	ApEp			1	1	1	1										1	1	1	1	1	1
15	IcEpOt											1					1	1		1		
15	IpEpOn									1		1		1		1						
16	ApIn	1							1		1		1		1							
16	ApOc				1						1						1		1		1	
16	InOn							1														
16	IcEp			1								1					1	1		1		
16	EpOn		1							1		1		1		1						
18	ApIcO										1				1		1			1		
18	IEpOc											1				1	1				1	
21	ApIcOt										1		1		1		1	1		1		
21	ApIO										1				1		1			1	1	1

NF	Formulas	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20	F21
21	IEpO											1				1	1			1	1	1
21	ItEpOc											1		1		1	1		1		1	
23	ApIcOp			1					1		1		1		1		1	1		1		
23	ApItO						1				1				1		1		1	1	1	1
23	IEpOt					1						1				1	1	1		1	1	1
23	IpEpOc				1					1		1		1		1	1		1		1	
24	ApIc	1		1					1		1		1		1		1	1		1		
24	ApO				1		1				1				1		1		1	1	1	1
24	InOc							1			1											
24	IcOn							1				1										
24	IEp			1		1						1				1	1	1		1	1	1
24	EpOc		1		1					1		1		1		1	1		1		1	
25	ApIOt					1					1		1		1		1	1		1	1	1
25	ItEpO						1					1		1		1	1		1	1	1	1
27	ApIOp			1		1			1		1		1		1		1	1		1	1	1
27	ApItOt					1	1				1		1		1		1	1	1	1	1	1
27	ItEpOt					1	1					1		1		1	1	1	1	1	1	1
27	IpEpO				1		1			1		1		1		1	1		1	1	1	1
28	ApI	1		1		1			1		1		1		1		1	1		1	1	1
28	ApOt				1	1	1				1		1		1		1	1	1	1	1	1
28	InO							1			1				1							
28	IOn							1				1				1						
28	ItEp			1		1	1					1		1		1	1	1	1	1	1	1
28	EpO		1		1		1			1		1		1		1	1		1	1	1	1
29	ApItOp			1		1	1		1		1		1		1		1	1	1	1	1	1
29	IpEpOt				1	1	1			1		1		1		1	1	1	1	1	1	1
30	ApIt	1		1		1	1		1		1		1		1		1	1	1	1	1	1
30	ApOp			1	1	1	1		1		1		1		1		1	1	1	1	1	1
30	InOt							1			1		1		1							
30	ItOn							1				1		1		1						
30	IpEp			1	1	1	1			1		1		1		1	1	1	1	1	1	1
30	EpOt		1		1	1	1			1		1		1		1	1	1	1	1	1	1
31	InOp							1	1		1		1		1							
31	IpOn							1		1		1		1		1						
36	IcOc							1			1	1					1					
42	IcO							1			1	1			1		1			1		
42	IOc							1			1	1				1	1				1	
45	IcOt							1			1	1	1		1		1	1		1		
45	ItOc							1			1	1		1		1	1		1		1	
47	IcOp			1				1	1		1	1	1		1		1	1		1		
47	IpOc				1			1		1	1	1		1		1	1		1		1	
49	IO							1			1	1			1	1	1			1	1	1
53	IOt					1		1			1	1	1		1	1	1	1		1	1	1
53	ItO						1	1			1	1		1	1	1	1		1	1	1	1
55	IOp			1		1		1	1		1	1	1		1	1	1	1		1	1	1
55	IpO				1		1	1		1	1	1		1	1	1	1		1	1	1	1
57	ItOt					1	1	1			1	1	1	1	1	1	1	1	1	1	1	1
59	ItOp			1		1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1
59	IpOt				1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1
61	IpOp			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

FACTORIAL ANALYSIS OF GROSS FORMULAS									FACTORS F22-F42													
NF	Formulas	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41	F42
	Elements																					
1	An																					
1	En																					
3	Ac																					
3	Ec																					
7	A							1														
7	E								1													
15	At							1		1						1				1		
15	Et								1		1						1				1	
31	Ap					1		1		1		1		1		1		1	1	1	1	
31	Ep						1		1		1		1		1		1	1	1	1	1	
32	In	1	1	1	1	1		1		1		1		1		1						1
32	On	1	1	1	1		1		1		1		1		1		1					1
48	Ic	1	1	1	1	1	1	1		1		1	1	1	1	1		1	1	1		1
48	Oc	1	1	1	1	1	1		1		1	1	1	1	1		1	1	1		1	1
56	I	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1
56	O	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	It	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	Ot	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
62	Ip	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
62	Op	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Compounds																					
1	AcInOp																					
1	AcEp																					
1	AIcEpOt																					
1	AEt																					
1	AtIEtO																					
1	AtE																					
1	ApItEOc																					
1	ApEc																					
1	IpEcOn																					
2	AcIn																					
2	AcOp																					
2	AInOt							1														
2	AIcEp																					
2	AEpOt																					
2	AtIcEpO																			1		
2	AtIEt																					
2	AtEtO																					
2	ApIEtOc																				1	
2	ApItE																					
2	ApEOc																					
2	ItEOn								1													
2	IpEc																					
2	EcOn																					
3	AInOp							1														
3	AIcOt							1														
3	AEp																					
3	AtIcEpOt																			1		
3	AtIEpO																			1		

NF	Formulas	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41	F42
3	AtEt																					
3	ApIEtO																				1	
3	ApItEtOc																				1	
3	ApE																					
3	ItEOc								1													
3	IpEOn								1													
4	AIn							1														
4	AOt							1														
4	AtInO									1						1						
4	AtIcEp																			1		
4	AtEpO																			1		
4	ApIcEpOc																	1	1			
4	ApIEt																				1	
4	ApEtOc																				1	
4	IEtOn										1						1					
4	ItE								1													
4	EOn								1													
5	AIcOp							1														
5	AtIEpOt																			1		
5	ApItEtO																				1	
5	IpEOc								1													
6	AIc							1														
6	AOp							1														
6	AtInOt							1		1						1						
6	AtIcO									1						1				1		
6	AtIEp																			1		
6	AtEpOt																			1		
6	ApIcEpO																	1	1	1		
6	ApIEpOc																	1	1		1	
6	ApItEt																				1	
6	ApEtO																				1	
6	IEtOc										1						1				1	
6	ItEtOn								1		1						1					
6	IpE								1													
6	EOc								1													
7	AtInOp							1		1						1						
7	AtIO									1						1				1		
7	AtEp																			1		
7	ApIcEpOt																	1	1	1		
7	ApItEpOc																	1	1		1	
7	ApEt																				1	
7	IEtO										1						1				1	
7	IpEtOn								1		1						1					
8	AtIn							1		1						1						
8	AtO									1						1				1		
8	ApInOc					1						1		1								
8	ApIcEp																	1	1	1		
8	ApEpOc																	1	1		1	
8	IcEpOn						1						1		1							

NF	Formulas	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41	F42
8	IEt										1						1				1	
8	EtOn								1		1						1					
9	AtIcOt							1		1						1				1		
9	ApIEpO																	1	1	1	1	
9	ItEtOc								1		1						1				1	
11	AtIcOp							1		1						1				1		
11	AtIOt							1		1						1				1		
11	ApIEpOt																	1	1	1	1	
11	ApItEpO																	1	1	1	1	
11	ItEtO								1		1						1				1	
11	IpEtOc								1		1						1				1	
12	AtIc							1		1						1				1		
12	AtOt							1		1						1				1		
12	ApInO					1				1		1		1		1						
12	ApIcOc					1						1		1				1	1			
12	ApIEp																	1	1	1	1	
12	ApEpO																	1	1	1	1	
12	IcEpOc						1						1		1			1	1			
12	IEpOn						1				1		1		1		1					
12	ItEt								1		1						1				1	
12	EtOc								1		1						1				1	
13	AtIOp							1		1						1				1		
13	ApItEpOt																	1	1	1	1	
13	IpEtO								1		1						1				1	
14	AtI							1		1						1				1		
14	AtOp							1		1						1				1		
14	ApInOt					1		1		1		1		1		1						
14	ApIOc					1						1		1				1	1		1	
14	ApItEp																	1	1	1	1	
14	ApEpOt																	1	1	1	1	
14	IcEpO						1						1		1			1	1	1		
14	ItEpOn						1		1		1		1		1		1					
14	IpEt								1		1						1				1	
14	EtO								1		1						1				1	
15	ApInOp					1		1		1		1		1		1						
15	ApItOc					1						1		1				1	1		1	
15	ApEp																	1	1	1	1	
15	IcEpOt						1						1		1			1	1	1		
15	IpEpOn						1		1		1		1		1		1					
16	ApIn					1		1		1		1		1		1						
16	ApOc					1						1		1				1	1		1	
16	InOn	1	1	1	1																	1
16	IcEp						1						1		1			1	1	1		
16	EpOn						1		1		1		1		1		1					
18	ApIcO					1				1		1		1		1		1	1	1		
18	IEpOc						1				1		1		1		1	1	1		1	
21	ApIcOt					1		1		1		1		1		1		1	1	1		
21	ApIO					1				1		1		1		1		1	1	1	1	
21	IEpO						1				1		1		1		1	1	1	1	1	
21	ItEpOc						1		1		1		1		1		1	1	1		1	

NF	Formulas	F22	F23	F24	F25	F26	F27	F28	F29	F30	F31	F32	F33	F34	F35	F36	F37	F38	F39	F40	F41	F42
23	ApIcOp					1		1		1		1		1		1		1	1	1		
23	ApItO					1				1		1		1		1		1	1	1	1	
23	IEpOt						1				1		1		1		1	1	1	1	1	
23	IpEpOc						1		1		1		1		1		1	1	1		1	
24	ApIc					1		1		1		1		1		1		1	1	1		
24	ApO					1				1		1		1		1		1	1	1	1	
24	InOc	1	1	1	1	1						1		1								1
24	IcOn	1	1	1	1		1						1		1							1
24	IEp						1				1		1		1		1	1	1	1	1	
24	EpOc						1		1		1		1		1		1	1	1		1	
25	ApIOt					1		1		1		1		1		1		1	1	1	1	
25	ItEpO						1		1		1		1		1		1	1	1	1	1	
27	ApIOp					1		1		1		1		1		1		1	1	1	1	
27	ApItOt					1		1		1		1		1		1		1	1	1	1	
27	ItEpOt						1		1		1		1		1		1	1	1	1	1	
27	IpEpO						1		1		1		1		1		1	1	1	1	1	
28	ApI					1		1		1		1		1		1		1	1	1	1	
28	ApOt					1		1		1		1		1		1		1	1	1	1	
28	InO	1	1	1	1	1				1		1		1		1						1
28	IOn	1	1	1	1		1				1		1		1		1					1
28	ItEp						1		1		1		1		1		1	1	1	1	1	
28	EpO						1		1		1		1		1		1	1	1	1	1	
29	ApItOp					1		1		1		1		1		1		1	1	1	1	
29	IpEpOt						1		1		1		1		1		1	1	1	1	1	
30	ApIt					1		1		1		1		1		1		1	1	1	1	
30	ApOp					1		1		1		1		1		1		1	1	1	1	
30	InOt	1	1	1	1	1		1		1		1		1		1						1
30	ItOn	1	1	1	1		1		1		1		1		1		1					1
30	IpEp						1		1		1		1		1		1	1	1	1	1	
30	EpOt						1		1		1		1		1		1	1	1	1	1	
31	InOp	1	1	1	1	1		1		1		1		1		1						1
31	IpOn	1	1	1	1		1		1		1		1		1		1					1
36	IcOc	1	1	1	1	1	1					1	1	1	1			1	1			1
42	IcO	1	1	1	1	1	1			1		1	1	1	1	1		1	1	1		1
42	IOc	1	1	1	1	1	1				1	1	1	1	1		1	1	1		1	1
45	IcOt	1	1	1	1	1	1	1		1		1	1	1	1	1		1	1	1		1
45	ItOc	1	1	1	1	1	1		1		1	1	1	1	1		1	1	1		1	1
47	IcOp	1	1	1	1	1	1	1		1		1	1	1	1	1		1	1	1		1
47	IpOc	1	1	1	1	1	1		1		1	1	1	1	1		1	1	1		1	1
49	IO	1	1	1	1	1	1			1	1	1	1	1	1	1	1	1	1	1	1	1
53	IOt	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1
53	ItO	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1
55	IOp	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1
55	IpO	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1
57	ItOt	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
59	ItOp	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
59	IpOt	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
61	IpOp	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

FACTORIAL ANALYSIS OF GROSS FORMULAS		FACTORS F43-F63																				
NF	Formulas	F43	F44	F45	F46	F47	F48	F49	F50	F51	F52	F53	F54	F55	F56	F57	F58	F59	F60	F61	F62	F63
	Elements																					
1	An																					
1	En																					
3	Ac																					
3	Ec																					
7	A																					
7	E																					
15	At										1											
15	Et											1										
31	Ap						1		1		1		1		1					1		
31	Ep							1		1		1		1	1						1	
32	In	1	1	1	1	1	1		1		1		1			1	1	1	1	1		1
32	On	1	1	1	1	1		1		1		1		1		1	1	1	1		1	1
48	Ic	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1
48	Oc	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1
56	I	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
56	O	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	It	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	Ot	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
62	Ip	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
62	Op	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Compounds																					
1	AcInOp																					
1	AcEp																					
1	AIcEpOt																					
1	AEt																					
1	AtIEtO																					
1	AtE																					
1	ApItEOc																					
1	ApEc																					
1	IpEcOn																					
2	AcIn																					
2	AcOp																					
2	AInOt																					
2	AIcEp																					
2	AEpOt																					
2	AtIcEpO																					
2	AtIEt																					
2	AtEtO																					
2	ApIEtOc																					
2	ApItE																					
2	ApEOc																					
2	ItEOn																					
2	IpEc																					
2	EcOn																					
3	AInOp																					
3	AIcOt																					
3	AEp																					
3	AtIcEpOt																					
3	AtIEpO																					

NF	Formulas	F43	F44	F45	F46	F47	F48	F49	F50	F51	F52	F53	F54	F55	F56	F57	F58	F59	F60	F61	F62	F63
3	AtEt																					
3	ApIEtO																					
3	ApItEtOc																					
3	ApE																					
3	ItEOc																					
3	IpEOn																					
4	AIn																					
4	AOt																					
4	AtInO										1											
4	AtIcEp																					
4	AtEpO																					
4	ApIcEpOc														1							
4	ApIEt																					
4	ApEtOc																					
4	IEtOn											1										
4	ItE																					
4	EOn																					
5	AIcOp																					
5	AtIEpOt																					
5	ApItEtO																					
5	IpEOc																					
6	AIc																					
6	AOp																					
6	AtInOt										1											
6	AtIcO										1											
6	AtIEp																					
6	AtEpOt																					
6	ApIcEpO														1							
6	ApIEpOc														1							
6	ApItEt																					
6	ApEtO																					
6	IEtOc											1										
6	ItEtOn											1										
6	IpE																					
6	EOc																					
7	AtInOp										1											
7	AtIO										1											
7	AtEp																					
7	ApIcEpOt														1							
7	ApItEpOc														1							
7	ApEt																					
7	IEtO											1										
7	IpEtOn											1										
8	AtIn										1											
8	AtO										1											
8	ApInOc						1		1				1							1		
8	ApIcEp														1							
8	ApEpOc														1							
8	IcEpOn							1		1				1							1	

NF	Formulas	F43	F44	F45	F46	F47	F48	F49	F50	F51	F52	F53	F54	F55	F56	F57	F58	F59	F60	F61	F62	F63
8	IEt											1										
8	EtOn											1										
9	AtIcOt										1											
9	ApIEpO														1							
9	ItEtOc											1										
11	AtIcOp										1											
11	AtIOt										1											
11	ApIEpOt														1							
11	ApItEpO														1							
11	ItEtO											1										
11	IpEtOc											1										
12	AtIc										1											
12	AtOt										1											
12	ApInO						1		1		1		1							1		
12	ApIcOc						1		1				1		1					1		
12	ApIEp														1							
12	ApEpO														1							
12	IcEpOc							1		1				1	1						1	
12	IEpOn							1		1		1		1							1	
12	ItEt											1										
12	EtOc											1										
13	AtIOp										1											
13	ApItEpOt														1							
13	IpEtO											1										
14	AtI										1											
14	AtOp										1											
14	ApInOt						1		1		1		1							1		
14	ApIOc						1		1				1		1					1		
14	ApItEp														1							
14	ApEpOt														1							
14	IcEpO							1		1				1	1						1	
14	ItEpOn							1		1		1		1							1	
14	IpEt											1										
14	EtO											1										
15	ApInOp						1		1		1		1							1		
15	ApItOc						1		1				1		1					1		
15	ApEp														1							
15	IcEpOt							1		1				1	1						1	
15	IpEpOn							1		1		1		1							1	
16	ApIn						1		1		1		1							1		
16	ApOc						1		1				1		1					1		
16	InOn	1	1	1	1	1										1	1	1	1			1
16	IcEp							1		1				1	1						1	
16	EpOn							1		1		1		1							1	
18	ApIcO						1		1		1		1		1					1		
18	IEpOc							1		1		1		1	1						1	
21	ApIcOt						1		1		1		1		1					1		
21	ApIO						1		1		1		1		1					1		
21	IEpO							1		1		1		1	1						1	
21	ItEpOc							1		1		1		1	1						1	

NF	Formulas	F43	F44	F45	F46	F47	F48	F49	F50	F51	F52	F53	F54	F55	F56	F57	F58	F59	F60	F61	F62	F63
23	ApIcOp						1		1		1		1		1					1		
23	ApItO						1		1		1		1		1					1		
23	IEpOt							1		1		1		1	1						1	
23	IpEpOc							1		1		1		1	1						1	
24	ApIc						1		1		1		1		1					1		
24	ApO						1		1		1		1		1					1		
24	InOc	1	1	1	1	1	1		1				1			1	1	1	1	1		1
24	IcOn	1	1	1	1	1		1		1				1		1	1	1	1		1	1
24	IEp							1		1		1		1	1						1	
24	EpOc							1		1		1		1	1						1	
25	ApIOt						1		1		1		1		1					1		
25	ItEpO							1		1		1		1	1						1	
27	ApIOp						1		1		1		1		1					1		
27	ApItOt						1		1		1		1		1					1		
27	ItEpOt							1		1		1		1	1						1	
27	IpEpO							1		1		1		1	1						1	
28	ApI						1		1		1		1		1					1		
28	ApOt						1		1		1		1		1					1		
28	InO	1	1	1	1	1	1		1		1		1			1	1	1	1	1		1
28	IOn	1	1	1	1	1		1		1		1		1		1	1	1	1		1	1
28	ItEp							1		1		1		1	1						1	
28	EpO							1		1		1		1	1						1	
29	ApItOp						1		1		1		1		1					1		
29	IpEpOt							1		1		1		1	1						1	
30	ApIt						1		1		1		1		1					1		
30	ApOp						1		1		1		1		1					1		
30	InOt	1	1	1	1	1	1		1		1		1			1	1	1	1	1		1
30	ItOn	1	1	1	1	1		1		1		1		1		1	1	1	1		1	1
30	IpEp							1		1		1		1	1						1	
30	EpOt							1		1		1		1	1						1	
31	InOp	1	1	1	1	1	1		1		1		1			1	1	1	1	1		1
31	IpOn	1	1	1	1	1		1		1		1		1		1	1	1	1		1	1
36	IcOc	1	1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1	1	1
42	IcO	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1
42	IOc	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1
45	IcOt	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1
45	ItOc	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1
47	IcOp	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1
47	IpOc	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1
49	IO	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
53	IOt	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
53	ItO	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
55	IOp	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
55	IpO	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
57	ItOt	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
59	ItOp	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
59	IpOt	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
61	IpOp	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Appendix 2. On Majority & Minority

Redefining Majority and Minority in *Future Logic*

This essay was written end March 2011 to correct certain errors spotted in Future Logic.

Introduction

In chapter 5.2, about Propositions, I introduce the quantities ‘most’ and ‘few’ as follows:

Other quantifiers define ‘some’ more precisely. Thus, ‘a few’ or ‘many’ mean, a small or large number; ‘few’ or ‘most’ mean, a minority or majority, a small or large proportion. These for most purposes have the same logical properties as particulars, though the latter two sometimes require special treatment.

This statement is correct, though rather vague. In chapter 6.2, concerning Oppositions, I define them more precisely as follows:

Also note in passing the position of forms quantified by ‘most’ or ‘few’, which we mentioned earlier. For a given polarity, the former includes the latter. Further, these quantities are intermediates between ‘all’ (which includes them) and ‘some’ (which they include). If their polarity is different, most (over 50%) and few (defined as 50% or less) are contradictory to each other. So that majority and minority are both contrary to the universal of opposite polarity, and both subcontrary to the particular of opposite polarity. They are unconnected to the singular forms.

The paragraph just quoted is unfortunately *filled with errors*, which we shall return to further on. The problem was brought to my attention a couple of days ago by a loyal reader and perspicacious critic from Trinidad and Tobago with the alias “zahc” – whom I warmly thank. I can only plead in my defense that this book, like all others, was written in a race against time.

As regards Eductions, I apparently said nothing. But there is more on these quantities in chapter 9.4 on Syllogisms: Applications, specifically:

Regarding syllogisms involving propositions which concern a majority or minority of a class, we get results similar to those obtained with singular moods. Thus, in the first figure, there are four main valid moods, their form being: ‘If All M are (or are-not) P, and Most (or Few) S are M, then Most (or Few) S are (or are-not) P’. In the second figure, there are four main valid moods, too, with the form: ‘If All P are (or are-not) M, and Most (or Few) S are-not (or are) M, then Most (or Few) S are not P’. In the third figure, we have only two main valid moods. They are especially noteworthy in that they manage without a universal premise. Their form is: ‘Most M are (or are-not) P, Most M are S, therefore Some S are (or are-not) P’. Note that the two premises are majoritive, and the conclusion is only particular. The validity of these is due to the assumption that ‘most’ includes more than half of the middle term class, so that there is overlap in some instances.

The validity of these moods is not affected by the above mentioned problem; i.e. what is said here is okay.

Definitions

Let us try now to construct an accurate theory of the quantities ‘most’ and ‘few’ (not to confuse with ‘a few’, which is contrasted with ‘many’). The table below clarifies them enormously. We see from this table that, to deal with all eventualities, we need no less than *five* specifications of quantity, viz. **all, most, half, few,** and **none**. ‘Most’ and ‘few’ can also be stated as ‘in the majority of cases’ and ‘in the minority of cases’.

If we have ‘all’ on the positive side, we have ‘none’ on the negative side; and vice versa. If we have ‘most’ on the positive side, we have ‘few’ on the negative side; and vice versa. If we have ‘half’ on the positive side, we have ‘half’ on the negative side, too; and vice versa. The positive and negative sides always add up to 100%, covering all possible cases.

Thus, ‘all’ and ‘none’ mean, of course, 100 and 0 percent respectively; ‘half’ must be defined as *exactly* 50 percent, no more and no less; ‘most’ must be *defined* as less than ‘all’ and more than ‘half’ (meaning, roughly, from 99 to 51 percent); ‘few’ must be *defined* as less than ‘half’ and more than ‘none’ (meaning, roughly, from 49 to 1 percent).

Quantity of S that are P	% of instances of S that are P	% of instances of S that are not P	Quantity of S that are not P	Sum total of %
all	**100**	**0**	**none**	**100**
most	99	1	few	100
most	98	2	few	100
most	97	3	few	100
And so on…				
most	53	47	few	100
most	52	48	few	100
most	51	49	few	100
half	**50**	**50**	**half**	**100**
few	49	51	most	100
few	48	52	most	100
few	47	53	most	100
And so on…				
few	3	97	most	100
few	2	98	most	100
few	1	99	most	100
none	**0**	**100**	**all**	**100**

Note that, in the past, I did not separately account for the ‘half-half’ situation, but stuffed it under ‘few’. I tried to define ‘few’ as ‘50% or less’ and ‘most’ as ‘over 50%’. These definitions in terms of open ranges (‘or less’, ‘over’) caused me some confusion and led me into error. The definitions now proposed are much clearer, because they are made with reference to exact quantities only, viz. 100%, 50% and 0%, even though rough ranges can be inferred from them. It should be said that the exact quantity ‘half’ does not always exist; for example, if the number of voters in an election is odd, there will be no possibility of a tie.

We can from these various considerations now propose precise eductions and oppositions.

Eductions

- ‘Most S are P’ implies ‘Few S are not P’, and vice versa.
- ‘Half S are P’ implies ‘Half S are not P’, and vice versa.
- ‘Few S are P’ implies ‘Most S are not P’, and vice versa.

Similarly, any more precise quantity on one side implies another on the other side. The quantities always come in pairs adding up to 100%, i.e. 99+1, 98+2, 97+3, etc. to 3+97, 2+98, 1+99, so that if one is known so is the other.

Each specific value between 100 and 0 implies either ‘most’ or ‘half’ or ‘few’. For instance, if 70% of S are P and (as is implied anyway) 30% of S are not P, then it is true to say that ‘Most S are P’ and (as is implied anyway) ‘Few S are not P’.

Note well that ‘Most S are P’ does *not* imply ‘Few S are P’ – they are mutually exclusive. Likewise, ‘All S are P’ does not include the lesser quantities ‘Most S are P’, ‘Half S are P’, and ‘Few S are P’.

Also, ‘Most S are P’, ‘Half S are P’, and ‘Few S are P’ tell us nothing about individual cases (i.e. do not imply ‘This S is P’).

Oppositions
'Most S are P', 'Half S are P', 'Few S are P', as well as 'all S are P', each implies 'Some S are P' and denies 'No S is P'.
'Most S are P', 'Half S are P', and 'Few S are P', each denies 'all S are P'.
Indeed, 'Most S are P', 'Half S are P', and 'Few S are P', deny each other.
Similarly, of course, on the negative side (i.e. regarding 'S are not P').
Granting that 'Some S are P', it follows that these Some must be All or Most or Half or Few; i.e. these four quantities are exhaustive. But of course, 'No S is P' is an alternative to 'Some S are P' and its four subalterns.
Thus, we see that 'Most S are P' and 'Few S are P' are contrary to each other, and not contradictory as previously suggested. That is, they are incompatible but not exhaustive.

Syllogism
What has been said earlier regarding syllogisms with quantities 'Most' and 'Few' remains unchanged. But what of the quantity 'Half'? In the first and second figures, if the minor premise concerns 'Half S', so will the conclusion. In the third figure, we still have overlap for the middle term if one premise concerns 'Most M' (>50%) and the other concerns 'Half M' (=50%), so a particular conclusion (for some S) is valid. However, if both premises concern 'Half M', there is no overlap and no conclusion can be drawn.

Evaluation of my past treatment
When I wrote chapter 6.2 of *Future Logic*, I defined 'most' and 'few' respectively as 'over 50%' and '50% or less'. Let us now temporarily label these two concepts '**mmost**' and '**ffew**', for the sake of the present discussion. If we compare these concepts to those here defined, here is what we get:

- 'mmost' means 'all or most'.
- 'ffew' means 'half or few or none'.

I was clearly wrong to say in the past that "For a given polarity, the former includes the latter." They complement each other, but exclude each other. I was also clearly wrong to depict 'mmost' and 'ffew' as "intermediates between 'all' (which includes them) and 'some' (which they include)." From the definitions just proposed, it is clear that 'all' is included in 'mmost' and excluded from 'ffew'; and moreover, 'some' includes 'mmost' and somewhat conflicts with 'ffew' (since the later includes the possibility of 'none'). Conversely, 'all' implies 'mmost' and denies 'ffew'; and 'some' is implied by 'mmost' but not implied by 'ffew'.
Moreover, since the five quantities 'all or most or half or few or none' are together exhaustive and contrary to each other, it follows that the two quantities 'mmost' and 'ffew' are also both exhaustive and incompatible, i.e. they are contradictory. However, this is true for a given polarity, and (contrary to what I said in the past) not for *opposite* polarities. What is the opposition between, say, 'mmost S are P' and 'ffew S are *not* P'? Well, 'mmost S are P' means 'all or most S are P', and this is equivalent to 'few or no S are not P'; and, 'ffew S are *not* P' means 'half or few or no S are not P'. Since the former disjunction (viz. 'few or no S are not P') is included in the latter one (viz. 'half or few or no S are not P'), it follows that 'mmost S are P' (being more specific) logically implies 'ffew S are *not* P' (which is more generic). Thus, their opposition is one of subalternation (and not contradiction, obviously).
No need to say more; it is evident that there were indeed errors in my past treatment.

Probability
'Most', 'Half', and 'Few' are first of all quantities, but of course, more broadly, *modalities* and degrees of probability. As quantities of the subject, they are extensional modalities. But they can also qualify the context of knowledge (logical modalities), the circumstances (natural modalities), the times and places (temporal and spatial modalities, respectively), and so forth. What we have said above must therefore be carried over to these other fields.
This concerns, principally, chapter 14.2 on Modal Oppositions and Eductions, where I wrote:

As for the oppositions of probability forms. Probability (most cases) subalternates improbability-not (few cases), and probability-not subalternates improbability; which is why we speak of degrees or levels of probability. By definition, probability (covering over half the times or circumstances) and improbability (half or less of them, let us say) are contradictory; likewise probability of negation and improbability of negation. Therefore, probability and probability-not are contrary, and their negations are subcontrary. These relations could be illustrated by a square.

This paragraph contains errors, as we shall now show. To avoid confusion here, let us use the above redefined quantities again. Let us use the term "cases" to refer to individual contexts, instances, circumstances, times, places, whatever, in accord with the mode of modality concerned. Then:

- Probably X means "In most cases, X exists."
- Probably not X means "In most cases, X does not exist."
- Improbably X means "In few cases, X exists."
- Improbably not X means "In few cases, X does not exist."

We can also introduce the idea of 'Fifty-fifty', say, where the number of cases is the same either way. Now, the main eductions and oppositions will be as follows:

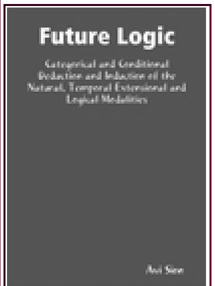
- Probably X implies and is implied by Improbably not X.
- Fifty-fifty X implies and is implied by Fifty-fifty not X.
- Improbably X implies and is implied by Probably not X.
- Probably X, Fifty-fifty X, and Improbably X, all imply (but are not implied by) Possibly X, and are all contrary to each other and to Necessarily X and Impossibly X.
- Probably not X, Fifty-fifty not X, and Improbably not X, all imply (but are not implied by) Possibly not X, and are all contrary to each other and to Necessarily not X and Impossibly not X.

This should set the record straight.

Avi Sion

Works by Avi Sion include:

The author's ten principal works:



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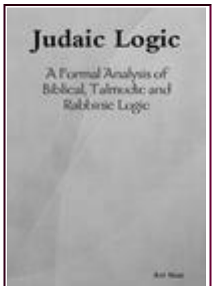

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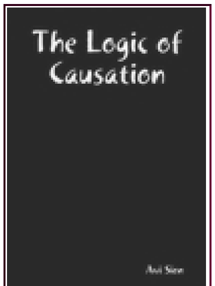

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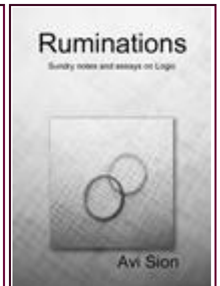

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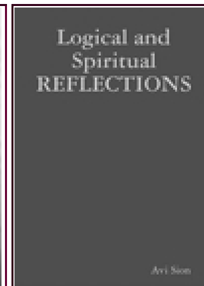

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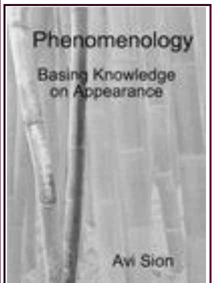

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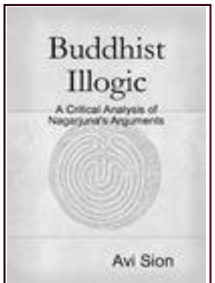

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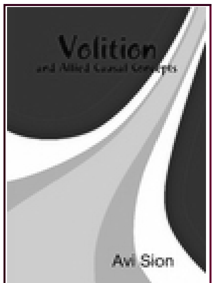

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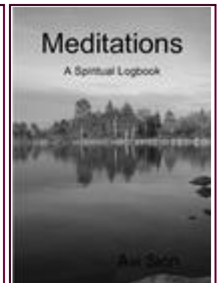

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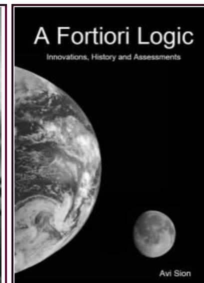

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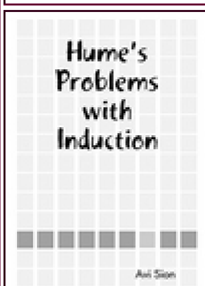

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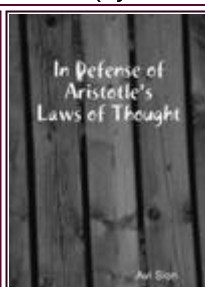

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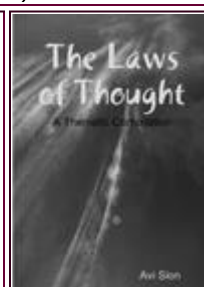

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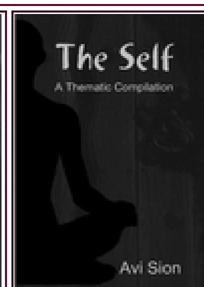

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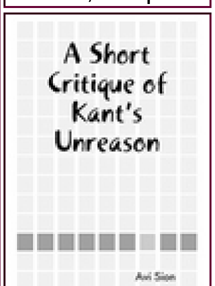

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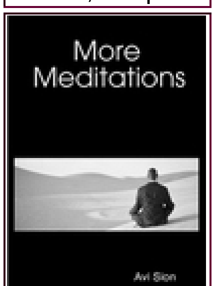

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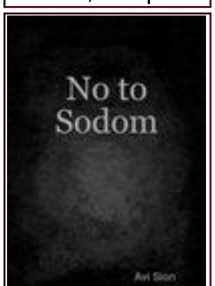

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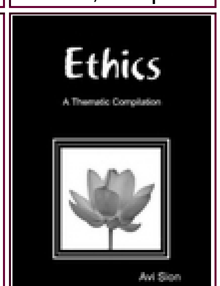

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A5, 174p

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