FUTURE LOGIC:
Categorical and Conditional Deduction and Induction of the Natural, Temporal, Extensional, and Logical Modalities.

By **Avi Sion** PH.D.

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Future Logic can be freely read online at The Logician.net and in various other locations.
It can be purchased in Amazon.com, Lulu.com and many other online booksellers.

The present document contains **excerpts** from this book, namely: The Abstract; the Contents in brief; Sample text (Chapters 54-56); and the References.

Avi Sion (Ph.D. Philosophy) is a researcher and writer in logic, philosophy, and spirituality. He has, since 1990, published original writings on the theory and practice of inductive and deductive logic, phenomenology, epistemology, aetiology, psychology, meditation, ethics, and much more. Over a period of some 28 years, he has published 27 books. He resides in Geneva, Switzerland.

It is very difficult to briefly summarize Avi Sion's philosophy, because it is so wide-ranging. He has labeled it '**Logical Philosophy**', because it is firmly grounded in formal logic, inductive as well as deductive. This original philosophy is dedicated to demonstrating the efficacy of human reason by detailing its actual means; and to show that the epistemological and ethical skepticism which has been increasingly fashionable and destructive since the Enlightenment was (contrary to appearances) quite illogical – the product of ignorant, incompetent and dishonest thinking.

Abstract

Future Logic is an original, and wide-ranging treatise of formal logic. It deals with deduction and induction, of categorical and conditional propositions, involving the natural, temporal, extensional, and logical modalities.

> (Simply put, deduction and induction are inferences of more or less certainty; propositions refer to relations between things; modalities are attributes of relations like necessity, actuality or possibility.)

Traditional and Modern logic have covered in detail only formal deduction from actual categoricals, or from logical conditionals (conjunctives, hypotheticals, and disjunctives). Deduction from modal categoricals has also been considered, though very vaguely and roughly; whereas deduction from natural, temporal and extensional forms of conditioning has been all but totally ignored. As for induction, apart from the elucidation of adductive processes (the scientific method), almost no formal work has been done.

This is *the first work ever* **to strictly formalize the inductive processes of generalization and particularization**, through the novel methods of factorial analysis, factor selection and formula revision.

This is *the first work ever* **to develop a formal logic of the natural, temporal and extensional types of conditioning** (as distinct from logical conditioning), including their production from modal categorical premises.

Future Logic contains a great many other new discoveries, organized into a unified, consistent and empirical system, with precise definitions of the various categories and types of modality (including logical modality), and full awareness of the epistemological and ontological issues involved. Though strictly formal, it uses ordinary language, wherever symbols can be avoided.

Among its other contributions: a full list of **the valid modal syllogisms** (which is more restrictive than previous lists); the main formalities of **the logic of change** (which introduces a dynamic instead of merely static approach to classification); the first formal definitions of **the modal types of causality**; a new **theory of class logic**, free of the Russell Paradox; as well as a critical review of modern metalogic.

But it is impossible to list briefly all the innovations in logical science -- and therefore, epistemology and ontology -- this book presents; it has to be read for its scope to be appreciated.

Contents in brief

Sample text (chapters 54-56)

54. Modal Induction

1. Knowability

Some skeptical philosophers have attempted to write-off natural necessity, and potentiality, as unknowable, if not meaningless. We have shown the meaningfulness and importance of these concepts, in the preceding pages. Here, we will begin to show systematically how they may be induced.

At the outset, let us note that to assert that natural necessity cannot be known, is to claim knowledge of a naturally necessary phenomenon; this is implicit in the use of 'cannot' in such assertion. If the assertion were merely put as 'man does not know natural necessity', in an attempt to be consistent, we see that the statement would have no force; we could still ask 'but can he?' Thus, this concept is undeniable, and its attempted rejection untenable.

Furthermore, the formal link between natural necessity and potentiality, makes the latter also inevitable. They are two sides of the same coin, if either is admitted then the other logically follows by systematization: every concept must have a contradictory. The potentiality of something is merely negation of the natural necessity of its absence. Thus, the intrinsically concealed and invisible aspect of unactualized potentiality, is not a valid argument against its existence.

The induction of natural modality, and for that matter the more readily recognized temporal modality, follows the same patterns as those involved in the process of induction of extensional modality.

How are universal propositions induced? By a process of generalization, moderated by particularization. We consider it legitimate to move from empirically encountered instances to cases we have not yet come across, until the facts suggest otherwise. We do not regard our universal statements to cover no more than the perceived phenomena; but normally move beyond them into prediction.

Likewise, with constancy of conjunction, in the sense of temporal modality; this too involves an extrapolation from the known to the unknown, as everyone admits.

So 'all' and 'always' involve just as much assumption as 'necessarily' (in the sense of natural modality). They are all just as hard to establish. Why should we recognize the former and not the latter?

Further, the concepts of universality and constancy are ultimately just as mysterious, ontologically hard to define, as that of natural necessity, so the latter's elusiveness cannot be a legitimate reason for singling it out.

If natural necessity is understood as one level higher (or deeper) than constancy, subject to all the usual laws of logic, generalization and particularization, it is seen to be equally empirical and pragmatic.

While the denial of natural necessity as such is unjustified, with regard to specific applications of the concept, we may of course in a given instance be wrong in our assumption that it is there. It is up to Logic to teach us proper procedures of induction and deduction, concerning such relationships. There is no problem in this viewpoint; belief in natural necessity as such does not obligate us to accept every eventual appearance of it as final.

As with any generalization, the movement from always to must, or from never to cannot, is legitimate, so long as it remains confirmed by experience. If ever a contradictory instance occurs, obviously our assumption is put in doubt and we correct our data-base accordingly, in the way of particularization.

2. Equality of Status

We saw, in chapter 50, on induction of actuals, that induced particulars are based on the observation of singulars. Similarly, induction of temporaries or potentials is based on the observation of actuals. The same can be said of the bipolar particular fractions, which involve temporary or potential elements: they can be established by observation of the same instance of the subject being actually related to the predicate in different ways at different times or in different circumstances.

And just as not all particular actuals are induced, but some are arrived at by deductive means, so also temporary or potential knowledge is in practice not invariably inductive, but may derive from reasoning processes. Though ultimately, of course, some empirical basis is needed, in any case.

We additionally pointed out how, in the formation of particular propositions, there is also a large share of conceptual work. The same is true of other types of possibility. All statements involve concepts (the terms, the copula, the polarity, the qualifications of quantity or modality). They presuppose a mass of tacit understandings, relating to logical structure and mechanisms. Furthermore, there is always an evaluation process, placing the proposal in the broad context of current knowledge, to determine its fit and realism.

Thus, although pure observation is instrumental in the process, other mental efforts are involved. Abstraction and verbalization of possibility are not automatic consequences of awareness of singular actual events, and error is always a risk. This is equally true in all types of modality, whether extensional, temporal or natural. Thus, actual particulars cannot be claimed more plausible than temporaries or potentials.

And indeed, just as particularity is not superior in status to generality, so are the other types of possibility not intrinsically more credible than their corresponding necessaries. If we consider the controversies among philosophers to be resolved, and view the whole of Logic in perspective, we can say that all forms involve only some degree of observation, and a great deal of thought. Although the degree of empiricism admittedly varies, the amount of conceptualization is essentially identical.

This insight must not be construed to put knowledge in general in doubt, however. Such skepticism would be self-contradictory, being itself the pronouncement of a principle. That there is a process does not imply that its outcome is false. The process merely transports the data from its source to its destination, as it were; the data need not be affected on the way.

Rather, its significance is to put all forms on an equal plane, with regard to their initial logical value. Particulars are no better than universals; particulars are no better than temporaries, which in turn are no better than potentials; and the latter are no better than constants or natural necessaries. Every statement, whatever its form, has at the outset an equal chance of being true or false, and has to be judged as carefully.

3. Stages of Induction

The classical theory of induction, we saw, describes two processes, generalization and particularization, as fundamental. If all we know is a particular proposition, **I** or **O**, we may assume the corresponding general proposition, **A** or **E**, true; unless or until we are forced by contradictory evidence to retract; and acknowledge the contingency **IO**.

Now, this description of the inductive process is adequate, when dealing with the closed system of actual propositions, because of the small number of forms it involves. In a broader context, when modal propositions of one or both types are taken into consideration, the need arises for a more refined description of the process.

This more complex theory brings out into the open, stages in or aspects of the process which were previously concealed. The ideas of generalization and particularization were basically correct, but their application under the more complicated conditions found in modal logic require further clarifications, which make reference to factorial analysis.

Needless to say, the new theory should be, and is, consistent in all its results with the old theory. It should be, and is, capable of embracing actual induction as a special case within a broader perspective which similarly guides, validates, and explains modal induction.

Our modified theory of induction, in the broadest sense, recognizes the following stages:

a. ***Preparation***. The summary of current data in gross formulas, and their factorization. This is in itself a purely deductive process.
b. ***Generalization***. Selection of the strongest factor in a factorial formula.
c. Drawing consequences, empirical testing, and comparing results to wider context. These include deductive work and observation.
d. ***Particularization***. Revision of current formulas in the light of new data. This may necessitate weighting of information. Also, certain conflicts are resolved by factor selection, as in generalization.
e. Repeat previous steps as required.

Each of these processes requires detailed examination. The tasks of listing all conceivable gross formulas, and analyzing them factorially, as well as the tasks relating to deductive inference and comparison, have previously been dealt with. We now need to deal with the processes of factor selection and formula revision, which are the most characteristically inductive.

4. Generalization *vs*. Particularization

We call generalization, those thought processes whose conclusions are higher than their premises; and we call particularization, those whose conclusions are lower. This refers to expansions and contractions on the scales of quantity and modality, essentially. As we move beyond the given, or its strictly deductive implications, into prediction, we are involved in induction of one kind or another.

The problem of generalization, which way and how far to advance and on what basis, is solved entirely by the method of factor selection. The problem of particularization, which way and how far to retreat and on what basis, is solved by the methods of formula revision, which may involve factor selection.

It will be seen that factor selection has a static component, which consists of the uniformity principle, which tells us which factor to select, and an active component, the practical carrying out of that decision. The act and basis of factor selection is technically identical, whether applied to generalization or to particularization.

The theory of factor selection makes clear that these processes do not consist of wild guesses, but proceed in a structured manner, requiring skill and precision.

We may view generalization as the positive force in induction, and particularization as the negative side. Generalization would often be too sweeping, if not kept in check by particularization. The function of the latter is to control the excesses of the former. Only the interplay of these two vectors results in proper induction. Induction is valid to the extent that it is a holistic application of both factor selection and formula revision.

In the pursuit of knowledge, laziness leads to error. An idea must be analyzed to the full, because its faults are sometimes concealed far down that course. The uncovering of a fault is a boon, allowing us to alter our idea, or take up a new one, and gain increased understanding and confidence.

The processes of generalization and particularization are going on in tandem all the time, in an active mind. Induction is not linear or pedestrian. Thoughts extend out tentatively, momentarily, like trial balloons, products of the imagination. But at the same time, verification is going on, unraveling the consequences of a suggestion, bringing other facts into focus from memory, or making new empirical inquiries, for comparison to the proposals made, and construction of a consistent idea. The wider the context brought into play, the greater the certainty that our course is realistic.

The role of Logic as a science is to provide the tools, which enable us to play this mental game with maximum efficiency and success. It is an art, but training and experience improve our performance of it.

5. The Paradigm of Induction

Let us reconsider the paradigm of induction given by actual induction. By reviewing the closed system of actual propositions using factorial concepts, we can gain some insights into the stages and guiding assumptions of induction within any system.

There are only four plural actual forms: **A**, **E**, **I**, **O**. These are also the system's fractions: (**A**), (**E**), (**I**), and (**O**). These in turn constitute three integers: (**A**), (**E**), and (**I**)(**O**), which are mutually exclusive and exhaustive. The 4 forms allow for 5 gross formulas: **A**, **E**, **I**, **O**, **IO**. These can be analyzed factorially using the integers: **A** = (**A**), **E** = (**E**), **I** = '(**A**) or (**I**)(**O**)', **O** = '(**E**) or (**I**)(**O**)', **IO** = (**I**)(**O**). But two disjunctions of factors remain unexpressed, namely: '(**A**) or (**E**)', signifying incontingency, and '(**A**) or (**E**) or (**I**)(**O**)', signifying no concrete information.

In this framework of factorial analysis, we can understand the induction of **A** from **I**, or of **E** from **O**, as a process involving factor selection, rather than solely as one of increase in quantity from some to all. The reverse process, of decrease in quantity, would also here be regarded differently, as primarily focusing on a new factorial situation.

Given **I** alone, we prefer the alternative outcome (**A**) to the deductively equally conceivable alternative (**I**)(**O**). Or, given **O** alone, we inductively anticipate the factor (**E**) as more likely than its alternative (**I**)(**O**). Our selection of one factor out of the available two is the dynamic aspect of the process. That we have specifically preferred the general alternative to the contingent one, is a second aspect; here, we take note of a principle that statically determines which of the alternative factors is selected.

If thereafter we find that our position must shift to **IO**, so well and good; in that case, only one integer is conceivable: (**I**)(**O**). In this case, we believed **A** to be true, then discovered **O**, or we assumed **E** then found **I**: the only available resolution of this conflict is by the compromise compound proposition **IO**; this is formula revision per se. Now, we analyze **IO** and find that it has only one factor (**I**)(**O**), so we can select it without doubt. However, had there been more than one conflict resolution or more than one factor (as occurs in wider systems), we would have had to again engage in factor selection.

Such an outlook seems somewhat forced and redundant within the closed system of actuals, but in the wider systems of modal propositions it becomes essential. It is only applied to actuals here for initial illustration purposes. For whereas with actuals, our choices are very limited numerically, when modality is introduced, they are much more complicated, as will be seen.

In the wider systems, induction can usually take many paths, and has various possible limits. For instance, from **Ip** should we generalize in the direction of **Ap** or to **In**? Or again, from **An** should we particularize to **Ap** or to **In**? And how far up or down the scale may we go? Obviously, this depends on context, so when may **Ip** ascend to **An**, and when must it stop earlier, and when must **An** descend to **Ip**, and when may it stop earlier?

Such questions can only be answered scientifically and systematically by resorting to factorial analysis and related processes. This brief review of actual induction in such terms points the way to the solution of the problem.

6. The Pursuit of Integers

The factor selection theory suggests that the goal of induction is to diminish the areas of doubt involved in deficient states of knowledge. Selecting a factor means eliminating a number of other factors, which, though they are formally logically conceivable alternatives, are intuitively thought to be less likely.

The ultimate result pursued by all induction is knowledge of integers, which does not necessarily mean a generality. Without integers, too many questions arise, and the mind cannot proceed. It is better to take up a working hypothesis, and keep testing it, than to passively await for an in any case unattainable absolute certainty. Knowledge is fed by action; it involves choices, decision-making.

The whole point of induction is to decide what integral proposition is most suggested by a given statement of deficient knowledge. We are to scrutinize its factorial equivalent and, on the basis of precise principle, select one factor as our inductive conclusion, or at least reduce the number of factors considerably. Deductively, all factors are equally likely outcomes, but inductively they can be narrowed down.

In certain cases, as factorial analysis showed, there is only one factor anyway; in such case, the conclusion is deductive, not inductive, and contextually certain. But in most cases, there are more than one factor, and selection is necessary. In some cases, we may for some purpose be satisfied with eliminating only some of the excess factors; and be left with a formula of two or more factors; the conclusion is not a single integer, but, still, less vague than previously, and might be expressed as a gross formula.

55. Factor Selection

1. Prediction

We indicated in the previous chapter that induction depends on factorial analysis of our knowledge context. Once this is done, we are usually faced with a number of factors to choose from, which represent the various outcomes our knowledge may move towards.

But reality can only exist in terms of integers; it is only the deficiencies of knowledge which make possible the indefinite situation of integers in disjunction. On this basis, we know for sure that one, and only one, of the factors of a formula can be factually correct. The other alternatives, if any, are a sign of doubt; they do not represent a fact of reality.

There is no recognition of an ‘Uncertainty Principle’ in this logic. Uncertainty is a phenomenon of consciousness, with no equivalent in the Object. It is perhaps conceivable that certain motions of matter occur indeterministically, without order or cause, as modern Physics suggests. But, according to Logic, whatever has occurred, once it has occurred, is firmly fixed, be it discernible or not.

The inductive process of factor selection consists in anticipating reality, trying to predict, from the available knowledge of contextually allowed factors, which of the factors is most likely to emerge as the right one. In some cases, while such a definite result is inaccessible, we try to at least approach it, by diminishing the number of factors. In other cases, the given formula has only one factor, anyway, so there is no problem, and the result is deductive.

The question arises, how do we know which factor is most likely? Formally speaking, they are all equally possible; this is the verdict of deductive logic. But induction has less strict standards of judgment.

2. The Uniformity Principle

The principle involved in factor selection may be glimpsed in the paradigm of generalization from actual particulars. We will call it the uniformity principle, understanding by this term a broad, loose reference to repetitiveness of appearances, coherence, continuity, symmetry, simplicity.

Consider for example generalization from **I**. The general alternative (**A**) is more likely then the contingent one (**I**)(**O**), ***because the former involves no unjustified presumption of***

variety in polarity like the latter. We are not so much inventing information, as refraining from baseless innovation and maintaining continuity.

Thus, the qualitative inertia of the first factor is more significant than the quantitative change (from some to all) it introduces. In contrast, the second factor introduces just as much quantitative change (through the **O**), so that it is no better in that respect; and additionally, to its detriment, a novel fragmentation of the extension, absent in the original data and the preferred factor.

We obviously select the factor most resembling the given data, as its most likely outcome. Unless or until we have reason to believe otherwise, we assume the given information to be reproduced as far as it will go. We can thus express the principle that, in factor selection, the most uniform factor is to be accorded priority.

Ontologically, this signifies the assumption of maximum uniformity in the world, in preference to an expectation of diversity. Events are believed representative, rather than unique. The world seems to tend in the direction of economy.

On a pragmatic level, the reason for it is that a generality is easier to test than a particular statement, since deductive logic, through which the consequences of assumptions are inferred, requires general statements. Thus, the preference for uniformity also has an epistemological basis. In the long run, it assures us of consistency.

The uniformity principle, then, is a philosophical insight and posture, which sets an order of priority among the factors of a formula.

But, it is important to stress that this principle is merely a utilitarian guideline to factor selection, it does not in this format have the binding force or precision found in the laws of deductive logic. Inductive logic merely tries to foresee the different situations which may arise in the pursuit of knowledge, and to suggest seemingly reasonable decisions one might make.

Choices other than those proposed remain conceivable, and might be intuitively preferred in specific cases. There is an artistic side to induction, to be sure. Our general recommendations, however, have the advantage of having been thought out in an ivory tower, and of forming a systematic whole.

3. The Law of Generalization

Fortunately, we can neatly summarize the results, obtained by application of the uniformity principle, in a single, precise law for generalization. This has greater practical value.

The reader will recall that when the integers were defined, they were organized, in order of the number of their fractions. Those with the least fractions came first, then those with two fractions, then those with three, and so on. Within each such group, comparable integers of opposite polarity were paired off, with the more positive one preceding the more negative. Also, they were ordered according to their level of modality in the continuum concerned.

Thus, in the closed systems of natural or temporal modality, the 15 integers **F1-F15**, and in the open system of mixed modality, the 63 integers **F1-F63**, are ordered in such a way that their numbers reflect their degree of 'strength'. The lower the ordinal number, the stronger the factor.

A stronger factor is less fragmented (i.e. has less fractions, out of a possible 4 in the closed systems, and 6 in the open). It is closer to universal (in the closed systems, **F1-F4** are universal; in the open system, **F1-F6**). It has higher modality; for instances, (**An**) is higher than (**AEp**), (**In**)(**On**) is higher than (**IOp**)(**IpO**).

Thus, in any factorial formula, the factors in the series are already numerically ordered according to their relative strengths. This was not done with factor selection in mind, but because of the clarity it generated in the doctrine of factorial analysis. As detailed work will presently reveal, it turns out that:

In any factor selection, the strongest factor is the one to prefer.
This is the law of generalization.

In a few exceptional cases, the first two factors must be selected, in disjunction, for reasons that we shall see. But, on the whole, this law holds firm, and successfully sums up all our findings.

This law is a summary of results. In point of fact, it only emerged at the end of painstaking analysis of a large number of specific inductive arguments, attempting to make sense of them, case by case, through the intuited uniformity principle. However, once arrived at, it seems obvious. But the true justification of it all, is the consistency and cogency of the totality of the theory, with all its details, of course.

Note well, incidentally, that henceforth, to avoid neologisms, the term ‘generalization’ is used in a general sense not limited to quantity. It is applied to either increase in quantity, from some to all; this is extensional generalization. And/or to increase in modality from possibility to actuality to necessity; this being modality generalization, (natural and/or temporal, as the case may be). Likewise, the term ‘particularization’ may be used for any such type of decrease.

But most precisely, generalization may now be defined as inductive selection of the strongest factor(s) of a formula, by suppression of weaker factor(s). Particularization will be dealt with under the heading of formula revision.

Generalization can, therefore, be applied to deficient states of knowledge not expressible in gross formulas. We saw in the chapter on factorial analysis that, while all disjunctions of integers represent deficient states of knowledge, some of them do not correspond to any gross formula. In other words, gross formulas with two or more factors are not all the possible states of relative ignorance, other combinations of factors are conceivable.

The law of generalization makes selection of the strongest factor legitimate in such already factorial formulas, too.

56. Applied Factor Selection

1. Closed Systems Results

We will, to begin with, deal with the closed system of natural modality, first listing the results of factor selection, then analyzing and justifying our proposals. As usual, all the results obtained can by analogy be replicated for the closed system of temporal modality. The corresponding results for the bulkier open system of mixed modality will be presented later.

The following table shows the proposed preferred (natural) factors for natural gross formulas, selected on the basis of the uniformity principle. Deductive cases, those with a single factor on formal grounds, are included for completeness.

The information in the elementary or compound premise is always assumed to be all available data on the subject to predicate relation concerned. If more data makes its appearance, then we are faced with another premise, and the conclusion may accordingly be different.

The column '**NF**' indicates the original number of factors, the next column lists them in sequence, and the column '**SF**' shows the selected factor among them, which is our proposed conclusion.

Table 0.1 Factor Selection in Natural Modality.

Premises	NF	Factors	SF	Conclusion
Group F1				
An	1	F1	F1	(An)
AIn	2	F1, F6	F1	(An)
A	3	F1, F3, F6	F1	(An)
ApIn	4	F1, F6, F8, F13	F1	(An)
ApI	6	F1, F3, F6, F8, F10, F13	F1	(An)
Ap	7	F1, F3, F4, F6, F8, F10, F13	F1	(An)
In	8	F1, F5-F6, F8, F11-F13, F15	F1	(An)
I	12	F1, F3, F5-F6, F8-F15	F1	(An)
Ip	14	F1, F3-F15	F1	(An)
Group F2				
En	1	F2	F2	(En)
EOn	2	F2, F7	F2	(En)
E	3	F2, F4, F7	F2	(En)
EpOn	4	F2, F7, F9, F14	F2	(En)
EpO	6	F2, F4, F7, F9-F10, F14	F2	(En)
Ep	7	F2-F4, F7, F9-F10, F14	F2	(En)
On	8	F2, F5, F7, F9, F11-F12, F14, F15	F2	(En)
O	12	F2, F4-F5, F7-F15	F2	(En)
Op	14	F2-F15	F2	(En)

Premises	NF	Factors	SF	Conclusion
Group F3				
AEp	1	F3	F3	(AEp)
ApIEp	2	F3, F10	F3	(AEp)
AOp	2	F3, F6	F3	(AEp)
IEp	4	F3, F9-F10, F14	F3	(AEp)
ApIOp	5	F3, F6, F8, F10, F13	F3	(AEp)
(IOp)	8	F3, F6, F9-F11, F13-F15	F3	(AEp)
IOp	11	F3, F5-F6, F8-F15	F3	(AEp)
Group F4				
ApE	1	F4	F4	(ApE)
ApEpO	2	F4, F10	F4	(ApE)
IpE	2	F4, F7	F4	(ApE)
ApO	4	F4, F8, F10, F13	F4	(ApE)
IpEpO	5	F4, F7, F9-F10, F14	F4	(ApE)
(IpO)	8	F4, F7-F8, F10, F12-F15	F4	(ApE)
IpO	11	F4-F5, F7-F15	F4	(ApE)
Group F3-4				
ApEp	3	F3-F4, F10	F3-4	(AEp) or (ApE)
ApOp	6	F3-F4, F6, F8, F10, F13	F3-4	(AEp) or (ApE)
IpEp	6	F3-F4, F7, F9-F10, F14	F3-4	(AEp) or (ApE)
IpOp	13	F3-F15	F3-4	(AEp) or (ApE)
Group F5				
InOn	4	F5, F11-F12, F15	F5	(In)(On)
InO	6	F5, F8, F11-F13, F15	F5	(In)(On)
IOn	6	F5, F9, F11-F12, F14-F15	F5	(In)(On)
InOp	7	F5-F6, F8, F11-F13, F15	F5	(In)(On)
IpOn	7	F5, F7, F9, F11-F12, F14-F15	F5	(In)(On)
IO	9	F5, F8-F15	F5	(In)(On)
Group F6				
AInOp	1	F6	F6	(In)(IOp)
ApInOp	3	F6, F8, F13	F6	(In)(IOp)
Group F7				
IpEOn	1	F7	F7	(On)(IpO)
IpEpOn	3	F7, F9, F14	F7	(On)(IpO)

Premises	NF	Factors	SF	Conclusion
Group F8				
ApInO	2	F8, F13	F8	(In)(IpO)
ApIO	3	F8, F10, F13	F8	(In)(IpO)
Group F9				
IEpOn	2	F9, F14	F9	(On)(IOp)
IEpO	3	F9-F10, F14	F9	(On)(IOp)
Group F10				
ApIEpO	1	F10	F10	(IOp)(IpO)
(IOp)(IpO)	4	F10, F13-F15	F10	(IOp)(IpO)

A similar table can be drawn up for temporal modality, substituting the suffixes **c**, **t** for **n**, **p**.

2. Some Overall Comments

The above table shows that, given a particular and/or potential (or even actual) proposition, we are unable to decide which way and how far to generalize it, without reference to the whole gross formula. If the gross formula consists of a single element, the conclusion is easy; it is the universal necessary proposition of like polarity. But if the gross formula is a compound, then the inductive path of any element in it depends on which other elements are involved. This is important to keep in mind.

We see that in some cases a particular proposition has become general, without change of modality; in other cases, the modality is raised, without change of quantity; in others still, both quantity and modality are affected. Also, two particular elements of a gross premise may emerge in the factorial conclusion as overlapping, or they may be separated.

Effectively, we have obtained the valid moods of natural modal induction (and by extension, those for temporal modality). They are not as numerous as appears, for we can distinguish 11 groups of valid moods among them, each defined by the best conclusion yielded. The conclusions being **F1**-**F10** and **F3-4**.

The groupings together include 13 primary valid moods, each of which has a number of subalterns. A primary mood in any group is one yielding the highest conclusion from the lowest premise. Subaltern moods are of two kinds.

The secondary premise may be higher than the primary one, yet yield a no-better conclusion, so that in effect the induction proper occurs after eduction of the lower premise. For example, **ApI** first implies **Ip**, from which **An** is thereafter induced. Or the secondary conclusion may be lower than the primary one, in which case it is in effect educed from the higher conclusion after the induction proper. For example, **Ip** yields **An** by induction, and then **AIn**, say, is inferred, since implied by **An**.

However, note well that subaltern moods are more certain than their corresponding primaries, because the number of factors they eliminate is lesser. Thus, for instance, **In** to **An** only eliminates 7 factors, whereas **Ip** to **An** eliminates 13 factors. The movement is more cautious, and therefore more likely to turn out to be correct in the long run.

The generalization from **I** to **A**, or from **O** to **E**, found in the closed system of actuals, can in this wider system of modal induction be viewed as a partial generalization. We move from a formula of 12 factors to one of 3 factors. We have not narrowed our position down to a single integer, but have nevertheless diminished the area of doubt considerably. Such limited generalizations are always permissible, of course, if they suffice for the needs of a specific inquiry.

3. Rules of Generalization

The rules of generalization clarify the various aspects of the uniformity principle. They are presented here, prior to detailed analysis of the valid moods, to facilitate the reader's understanding of the discussion, but in fact they simply summarize the insights accumulated in the course of case by case examination.

The uniformity principle for factor selection, has a variety of implications. Some of these emerge in the paradigm of actual induction, but others become apparent only in modal logic. The rules of generalization serve to expose the variety of considerations which arise, and provide us with more specific guidelines than the basic principle.

The various vectors of uniformity often interfere with each other, in such a way that satisfying the requirements of the one, frustrates the demands of the other. This is because different factors stress different things. For instance, one factor may stress quantitative generalization, another may stress modality generalization. Case study of such conflicts of interest gradually clarified the order of importance of the different tendencies. The rules of generalization thus have an order of priority.

a. ***Polarity***. First in line is the requirement that the conclusion resemble the premise in polarity. If there is but one polarity in the premise, the same will remain solitary in the conclusion. If the premise is a bipolar compound, so must the conclusion be. One cannot induce a different or supplementary polarity. Such innovation has no basis in the uniformity principle, and can only occur with factual justification. Many factor selections, seeming to involve change of quantity or modality, rather stem from this inertia of polarity.

b. ***Quantity***. Next in line is increase in quantity, as far as consistent. This is the prime change induction seeks to effect. This is because a universal proposition is most open to testing, by drawing its consequences through deductive logic. Maximal extensional generalization is to be favored over improvements in modality or other uniformities, wherever possible. It is the paradigm of the uniformity principle, an assumption that properties tend to relate to classes, rather than being scattered accidentally.

c. ***Modality***. Uniformity implies an overall preference, not only for the more general alternative, but also for the factor of higher modality. However, modality generalization is only next in importance to that of quantity. But it is still this high on the list, for similar reasons: practically, because the higher the category, the more testable the result; metaphysically, because we assume a stable substratum beneath the changes we perceive.

Within either closed system, necessity is preferred to actuality, and actuality to possibility. In the open system, mixing modality types, natural necessity should be favored over constancy, and temporariness over potentiality, whenever the prior guidelines allow it. This is obvious from the relative positions of these various categories on the modality continuum.

d. ***Symmetry***. If the premise consists of elements of opposite polarity which are identical in both quantity and modality, the conclusion must have the same evenness. There would have to be factual basis for one side or the other to grow in quantity or modality more than the other; the uniformity principle does not justify such loss of symmetry. This is why the conclusion in a few cases cannot be a single factor, but a disjunction of two.

If on the other hand, the compound premise gives one or the other polarity a higher quantity or modality, the conclusion may or may not favor the one over the other: it depends on other considerations. Many subaltern moods have the unevenness of their premise in this way removed by the conclusion.

e. ***Overlap***. If some elements of a compound premise are known to converge, at least that same degree of overlap must reappear in the conclusion. Overlap cannot be lost by induction.

On the other hand, it may be gained. If overlap is not at all assured originally, it may be assumed, provided no prior considerations are put in jeopardy. Where there is a question as to whether two separately discovered particulars overlap or not, the uniformity principle would seem to suggest that they be applied to each other's extensions, so that both be maximally generalized.

However, if overlap is open to doubt, and making its assumption would cause problems in other respects, the adoption of the divergence hypothesis is acceptable. Overlap is of less importance than other issues, because it is conceptually derived from them.

f. ***Simplicity***. Lastly, but still significant, is the concern with fragmentation. In a choice between a factor with few fractions and another with many, both of which satisfy the prior guidelines, the former is preferable. We should not fragment the extension beyond the minimum feasible, always preferring the simplest alternative. This is an aspect of uniformity, in that it opposes diversity between the members of the class concerned. If indeed the more complex alternative is true, it will eventually impose itself through particularization.

The applications of these rules of generalization will now be seen through specific examples.

4. Review of Valid Moods

Let us now review each primary valid mood of natural induction in some detail. In every case, to repeat, the gross premise, be it elementary or compound, is assumed to represent all available information on the subject to predicate relation concerned.

a. From any premise of single polarity, may be induced a universal necessary of same polarity. This is the most obvious application of the uniformity principle: there is no basis for presuming the other polarity at all possible. The primary moods in this group involve increase in both quantity and modality. They are:

Ip → An

Given solely that Some S are P,

we may induce that All S must be P.

Op → En

Given solely that Some S are not P,

we may induce that No S can be P.

The subaltern premises to **Ip → An** are: **I**, **In**, **Ap**, **ApI**, **ApIn**, **A**, **AIn**, **An**. The case **An** to **An** is of course deductive, even tautologous, and only listed to show the continuity. The subaltern (elementary) conclusions to **Ip** are: **A**, **Ap**, **In**, **I**; to **I**: **A**, **Ap**, **In**; to **In**: **A**, **Ap**; to **Ap**: **A**, **In**, **I**; to **ApI**: **A**, **In**; to **ApIn**: **A**; to **A**: **In**; and to **AIn**: none. Similarly, **Op → En** has some 16 subaltern inductive moods (not counting compound conclusions).

b. From a conjunction of particular premises of different polarity, one of which is actual and the other potential, the best inductive conclusion is a similar conjunction of universal premises. Here, the uniformity principle leads us to assume the particulars to fully overlap, and to generalize quantity only (not modality), to obtain a result with the original bipolarity.

IOp → AEp

Given solely that Some S are P and some S can not-be P,

we may induce that All S are P though all can not-be P.

IpO → ApE

Given solely that Some S are not P and some S can P,

we may induce that No S are P though all can be P.

It is clear that this induction occurs in stages. Consider the mood **IOp → AEp**. First the elements of **IOp** are made to converge into the fraction (**IOp**), dropping 3 factors, then this particular fraction is generalized into its universal equivalent (**AEp**), dropping a further 7 factors. Effectively, **I** has been generalized to **A**, and **Op** to **Ep**.

Alternative conclusions, though formally conceivable, seem less justifiable. For instance, (**In**)(**On**), by assuming nonoverlap, would cause baseless fragmentation of the extension, and result in a modal equality between the poles which was originally lacking. Whereas, say, (**In**)(**IOp**), while granting partial overlap and uneven modality, would fragment the extension without specific reason. Furthermore, a general conclusion is always to be preferred to a particular one, even one of stronger modality, because it is more readily tested.

The 4 subaltern premises **ApIOp**, **IEp**, **AOp**, **ApIEp** yield the same result. In their case, a partial overlap, meaning the fraction (**IOp**), is already implied, since one of the elements of the compound is universal already. In each case, consequently, less generalization is involved than in the primary mood, and the result is somewhat more trustworthy.

All the same comments can be made concerning the mood **IpO → ApE** and its subalterns.

c. When the premise is a compound of two particular potentials of different polarity, an imperfect conclusion may be drawn, diminishing the number of factors to two universal compounds in disjunction. Here, the original modal symmetry inhibits a more definite result, which would strengthen one side more than the other. But there is still an improvement in specificity, a guarantee of overlap and generalization of quantity having been achieved. The disjunctive result can be used in dilemmatic arguments.

IpOp → '(AEp) or (ApE)'
Given that Some S can be P and some S can not-be P,
we may induce that
either 'All S are P, though all can not-be P'
or 'All S are notP, though all can be P'.

The subaltern premises **IpEp**, **ApOp**, and **ApEp** have the same result. Note that the conclusion is not simply **ApEp**, which would allow the factor (**IOp**)(**IpO**) as an alternative. Precisely for this reason, **ApEp** → '(**AEp**) or (**ApE**)' is not a deductive inference, as those from **AEp** to (**AEp**) or from **ApE** to (**ApE**) were, but an induction diminishing the number of factors from 3 to 2. Even eliminating the fragmentation inherent in (**IOp**)(**IpO**) makes the effort worthwhile.

These moods may be viewed as to some extent subsidiary to the preceding group, tending toward the same sort of conclusion, but not quite succeeding. The elimination of particularistic alternatives, such as (**In**)(**On**), is based on similar argument.

d. From two particular actuals of opposite polarity, we induce two particular necessaries with corresponding polarities. Here, we may not in any case generalize quantity, for the four universal factors are deductively inconceivable, anyway; none of them would be compatible with the premise; they are not among the available factors. Thus, only modality, the next best thing, is increased as far as it goes, up to necessary; thusly, for both poles, to retain the original symmetry.

IO → (In)(On)
Given solely that Some S are P and some S are not P,
we may induce that Some S must be P and some cannot.

Note that in this special case, the uniformity principle causes divergence, rather than overlap, for the sake of obtaining a higher modality, while retaining the original evenness in modality. Although the compound **IO** implies **I**+**Ip**+**O**+**Op**, so that we might induce (**IOp**)(**IpO**) to achieve maximum overlap, the proposed conclusion is preferable, because it involves necessity instead of mere actuality and effectively no greater fragmentation of the extension. As for (**In**)(**On**)(**IOp**)(**IpO**), though equally conceivable in principle, and involving both necessity and overlap advantages to some extent, it is rejected, because it introduces an excessive fragmentation, for which no argument is forthcoming.

The premises **IOn**, **InO**, and **InOn** may be viewed as subalterns to **IO**, as well as to the primaries considered next.

e. From the conjunction of two particulars of opposite polarity, one of which is necessary and the other potential, a conjunction of two particular necessaries of opposite polarity is induced. Here, the original asymmetry and the conceivable partial overlap, are sacrificed to improvement in modality. Any universal conclusion is again out of the question, on formal grounds.

InOp → (In)(On)
Given solely that Some S must be P and some can not-be,
we may induce that Some S must be P and some cannot be.

IpOn → (In)(On)
Given solely that Some S can be P and some cannot be P,
we may induce that Some S must be P and some cannot be.

These two moods are independent primaries, and not subalterns to **IO** → (**In**)(**On**), note well, since neither **InOp** nor **IpOn** formally implies **IO**. They are, however, closely related, having in common the same conclusion, and the same subaltern premises **IOn**, **InO**, **InOn**.

Note well, incidentally, that **InOn** → (**In**)(**On**) is indeed an inductive argument, and not a deductive one, since InOn has 4 factors originally, 3 of which are then eliminated, for reasons of asymmetry or excessive fragmentation, as our table shows.

f. Two more groups of valid moods are distinguished by their more complex primary premises and conclusions. They are the following.

ApInOp → (In)(IOp)
Given that All S can be P, some S being necessarily P,
and others potentially not P,
we may induce that the latter S are actually P.

IpEpOn → (On)(IpO)
Given that All S can not-be P, some S being necessarily not P,
and others potentially P,
we may induce that the latter S are actually not P.

In the positive case, we first separate the (**In**) fraction from the remainder **IpOp**, which we know to overlap since **Ap** is general and given; then we favor the (**IOp**) outcome, generalizing **Ip** to **I**, on the basis that **I** is already implicit in **In**. In comparison, the (**IpO**) eventuality, though conceivable, would require a move from **Op** to **O**, for which no specific basis is found, so that it may be inductively eliminated. The mood **AInOp** yields the (**In**)(**IOp**) conclusion deductively, not inductively, since this is its only factor. Similar comments can be made with regard to the parallel negative cases.

ApIO → (In)(IpO)
Given that All S can be P, some S being actually not P,

and others being actually P,
we may induce that the latter S must be P.

IEpO → (On)(IOp)
Given that All S can not-be P, some S being actually P,
and others being actually not P,
we may induce that the latter S cannot be P.

Here again, in the positive case, we first separate the (**IpO**) fraction, on the grounds that **Ap** is general and that **I** and **O** cannot overlap; then we generalize the remaining **I** segment of the extension to **In**. The (**IOp**) eventuality, though conceivable since **O** implies **Op**, is rejected on the basis that it involves a weaker category of modality compared to (**In**); as for the conjunction of both (**In**) and (**IOp**), this would introduce a needless additional fragmentation into the equation. The subaltern premise **AInO** yields the same inductive conclusion, by elimination of only the latter eventuality, for the same reason. Similar comments can be made with regard to the parallel negative cases.

g. The inference from **ApIEpO** to (**IOp**)(**IpO**) is deductive, as we saw in factorial analysis.

On the other hand, the move from the gross conjunction of the two particular fractions (**IOp**) and (**IpO**) as a premise, to the integer (**IOp**)(**IpO**) is inductive, not deductive. For the common factors of the fractions are not only **F10**, but also **F13**, **F14**, **F15**. The latter three, which involve the conjunction of (**In**) or (**On**) or (**In**)(**On**) to (**IOp**)(**IpO**), are formally conceivable, but in this context rejected, on the basis that they introduce new fragments without specific justification.

The other gross conjunctions of fractions, in twos or threes, similarly yield their integral counterparts, **F11**-**F14**, by induction. In the case of four fractions, the **F15** conclusion is deductive.

All that has been said for natural factor selection, could be repeated for temporal factor selection. The two closed systems behave identically.

5. Open System Results

We shall now list the valid moods of open system induction, with a minimum of comments, for the record. The reader is encouraged to review the valid moods, with reference to the rules of generalization, to justify the selection of this or that factor rather than any other, in each case.

We saw, in earlier chapters, that when natural and temporal modality are considered together, 63 integers (see table 52.2) and 195 gross formulas (see table 51.1) may be generated. In an appendix, we developed a table showing the factorial analysis of all gross formulas. The factorial analysis of the particular fractions, on the other hand, may be found in table 52.2 (reading it downward).

The valid moods of open system induction are easily extracted from these sources of information. In accordance with the law of generalization, the factor to select in induction is usually the first, the one with the lowest ordinal number; though, in a few cases, we must select the first two factors in disjunction to maintain symmetry. This is so, simply because I numbered the factors that way, in order of generality, necessity, and simplicity.

Be careful not to confuse the closed system factors with the open system factors; the symbols **F1**-**F15** have mostly different meanings in each context. Also remember not to equate the four compound particular fractions, (**IcOp**), (**IpOc**), (**IOt**), (**ItO**) to their gross equivalents. Each of the former has 32 factors, whereas **IcOp** and **IpOc** have 47 each, and **IOt** and **ItO** 53 each.

The table below shows the selected factors for all gross formulas in the mixed modality system. Premises with the same inductive conclusion are grouped together, and their common result is given. The number of factors for each formula is listed under the heading '**NF**'.

There are, we see, 23 groups of valid moods, with numbers lying between **F1** and **F21**. A total of 33 of the moods are primary; these are indicated by 3 asterixes (***). The remaining moods are subalterns of these.

Note that 11 moods are in fact deductive, rather than inductive, since they were found to have only one factor when analyzed; one of these is the sole listed representative of **Group F21**. These are included for completeness.

While the individual fractions are also included in our table, the various gross conjunctions of two to six particular fractions have been ignored, to avoid excessive volume; these obviously yield their integral counterparts, **F7**-**F63**, as inductive results.

<u>Table 0.2 Factor Selection in the Open System.</u>

(See the tables below.)

Group F1	Group F2	
Premise(s)	Premise(s)	NF
An	En	1
AcIn	EcOn	2
Ac	Ec	3
AIn	EOn	4
AIc	EOc	6
A	E	7
AtIn	EtOn	8
AtIc	EtOc	12
AtI	EtO	14
At	Et	15
ApIn	EpOn	16
ApIc	EpOc	24
ApI	EpO	28
ApIt	EpOt	30
Ap	Ep	31
In	On	32
Ic	Oc	48
I	O	56
It	Ot	60
Ip ***	Op ***	62
Conclusion **(An)**	Conclusion **(En)**	**1**

Group F3	Group F4	
Premises	Premises	NF
AcEp	ApEc	1
AcOp	ApEOc	2
AIcEp	IpEc	2
AEp	ApE	3
AtIcEp	ApEtOc	4
AIcOp	IpEOc	5
AOp	ApEtO	6
AtIEp	IpE	6
AtEp	ApEt	7
ApIcEp	ApEpOc	8
AtIcOp	IpEtOc	11
ApIEp	ApEpO	12
AtIOp	IpEtO	13
AtOp	ApEpOt	14
ApItEp	IpEt	14
IcEp	ApOc	16
ApIcOp	IpEpOc	23
IEp	ApO	24
ApIOp	IpEpO	27
ItEp	ApOt	28
ApItOp	IpEpOt	29
(IcOp)	(IpOc)	32
IcOp	IpOc	47
IOp	IpO	55
ItOp ***	IpOt ***	59
Conclusion **(AcEp)**	Conclusion **(ApEc)**	**1**

Group F3-4	
Premises	NF
ApEp	15
ApOp	30
IpEp	30
IpOp ***	61
Conclusion **(AcEp) or (ApEc)**	**2**

Group F5	Group F6	
Premises	Premises	NF
AEt	AtE	1
AEpOt	AtEtO	2
AtIEt	ApItE	2
AOt	AtEpO	4
ApIEt	ItE	4
AtIEpOt	ApItEtO	5
IEt	AtO	8
AtIOt	ApItEpO	11
ApIEpOt	ItEtO	11
IEpOt	ApItO	23
ApIOt	ItEpO	25
(IOt)	(ItO)	32
IOt ***	ItO ***	53
Conclusion **(AEt)**	Conclusion **(AtE)**	**1**

Group F5-6	
Premises	NF
AtEt	3
AtEpOt	6
ApItEt	6
AtOt	12
ItEt	12
ApItEpOt	13
ApItOt	27
ItEpOt	27
ItOt ***	57
Conclusion **(AEt) or (AtE)**	**2**

Groups F7	
Premises	NF
InOn	16
InOc	24
IcOn	24
InO	28
IOn	28
InOt	30
ItOn	30
InOp ***	31
IpOn ***	31
IcOc	36
IcO	42
IOc	42
IcOt ***	45
ItOc ***	45
IO ***	49
Conclusion **(In)(On)**	**1**

Group F8	Group F9	
Premises	Premises	NF
AcInOp	IpEcOn	1
AInOp	IpEOn	3
AtInOp	IpEtOn	7
ApInOp ***	IpEpOn ***	15
Conclusion **(In)(IcOp)**	Conclusion **(On)(IpOc)**	**1**

Groups F10	Groups F11	
Premises	Premises	NF
ApInOc	IcEpOn	8
ApInO	IEpOn	12
ApIcOc	IcEpOc	12
ApInOt	ItEpOn	14
ApIOc	IcEpO	14
ApItOc ***	IcEpOt ***	15
ApIcO	IEpOc	18
ApIcOt ***	ItEpOc ***	21
ApIO ***	IEpO ***	21
Conclusion **(In)(IpOc)**	Conclusion **(On)(IcOp)**	**1**

Group F12	Group F13	
Premises	Premises	NF
AInOt	ItEOn	2
AIcOt	ItEOc	3
AtInOt	ItEtOn	6
AtIcOt ***	ItEtOc ***	9
Conclusion **(In)(IOt)**	Conclusion **(On)(ItO)**	**1**

Group F14	Group F15	
Premises	Premises	NF
AtInO	IEtOn	4
AtIcO	IEtOc	6
AtIO ***	IEtO ***	7
Conclusion **(In)(ItO)**	Conclusion **(On)(IOt)**	**1**

Groups F16	
Premises	NF
ApIcEpOc	4
ApIcEpO	6
ApIEpOc	6
ApIcEpOt ***	7
ApItEpOc ***	7
ApIEpO ***	9
Conclusion **(IcOp)(IpOc)**	**1**

Group F17	Group F18	
Premises	Premises	NF
AIcEpOt	ApItEOc	1
AtIcEpOt ***	ApItEtOc ***	3
Conclusion **(IcOp)(IOt)**	Conclusion **(IpOc)(ItO)**	**1**

Group F19	Group F20	
Premises	Premises	NF
AtIcEpO	ApIEtOc	2
AtIEpO ***	ApIEtO ***	3
Conclusion **(IcOp)(ItO)**	Conclusion **(IpOc)(IOt)**	**1**

Group F21	
Premises	NF
AtIEtO ***	1
Conclusion	
(IOt)(ItO)	**1**

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