THE LAWS OF THOUGHT

A Thematic Compilation

Avi Sion, Ph. D.

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Abstract

The Laws of Thought is an exploration of the deductive and inductive foundations of rational thought. The author here clarifies and defends Aristotle's Three Laws of Thought, called the Laws of Identity, Non-contradiction and Exclusion of the Middle – and introduces two more, which are implicit in and crucial to them: the Fourth Law of Thought, called the Principle of Induction, and the Fifth Law of Thought, called the Principle of Deduction.

This book is a thematic compilation drawn from past works* by the author over a period of twenty-three years.

^{*} Note: Some chapters have been merged here. Also, some chapters have been split up into smaller sections.

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1. CHAPTER ONE

Drawn from *Future Logic* (1990), Chapter 2.

THE FOUNDATIONS OF LOGIC

Logic is founded on certain 'laws of thought', which were first formulated by Aristotle, an ancient Greek philosopher. We shall describe them separately here, and later consider their collective significance.

1. The Law of Identity

The Law of Identity is an imperative that we consider all evidence at its face value, to begin with. Aristotle expressed this first law of thought by saying 'A is A', meaning 'whatever is, is whatever it is'.

There are three ways we look upon phenomena, the things which appear before us, however they happen to do so: at their face value, and as real or illusory.

We can be sure of every appearance, that it is, and is what it is. (i) *Something* has presented itself to us, whether we thereafter judge it real or illusory, and (ii) this something displays *a certain configuration*, whether we thereafter describe and interpret it rightly or wrongly. The present is present, the absent is absent.

Every appearance as such is objectively given and has a certain content or specificity. We can and should and commonly do initially regard it with a simple attitude of receptiveness and attention to detail. Every appearance is in itself neutral; the qualification of an appearance (thus broadly defined) as a 'reality' or an 'illusion', is a subsequent issue.

That statement is only an admission any minimally phenomenon exists and has characteristics, without making claims about the source and significance of this existence or these characteristics. The moment we manage to but think of something, it is already at least 'apparent'. No assumption need be made at this stage about the nature of being and knowledge in general, nor any detailed categorizations, descriptions or explanations of them.

Regarded in this way, at their face value, *all* phenomena are evident data, to be at least taken into consideration. The world of appearances thus offers us *something to work with, some reliable data* with which we can build the edifice of knowledge, a starting point of sorts. We need make no distinctions such as those between the physical/material and the mental, or sense-data and hallucinations, or concrete percepts and abstract concepts; these are later developments.

The law of identity is thus merely an acknowledgement of the world of appearances, without prejudice as to its ultimate value. It defines 'the world' so broadly, that there is no way to counter it with any other 'world'. When we lay claim to another 'world', we merely expand this one. All we can ever do is subdivide the world of appearances into two domains, one of 'reality' and one of 'illusion'; but these domains can never abolish each other's existence and content.

What needs to be grasped here is that every judgment implies the acceptance, at some stage, of some sort of appearance as real. There is no escape from that; to claim that nothing is real, is to claim that the appearance that 'everything is illusory' is real. We are first of all observers, and only thereafter can we be judges.

Reality and illusion are simply terms more loaded with meaning than appearance or phenomenon — they imply an evaluation to have taken place. This value-judgment is a final characterization of the object, requiring a more complex process, a reflection. It implies we went beyond the immediately apparent. It implies a broader perspective, more empirical research, more rigorous reasoning. But what we finally have is still 'appearance', though in a less pejorative sense than initially.

Thus, 'real' or 'illusory' are themselves always, ultimately, just appearances. They are themselves, like the objects of consciousness which they evaluate, distinct objects of consciousness. We could say that, there is a bit of the real in the illusory and a bit of the illusory in the real; what they have in common is appearance. However, these terms lose their meaning if we try to equate them too seriously.

On what basis an appearance may or should be classified as real or illusory is of course a big issue, which needs to be addressed. That is the overall task of logic, to set precise guidelines for such classification. But the first step is to admit the available evidence, the phenomenal world as such: this gives us a data-base.

2. The Law of Contradiction

The Law of Contradiction (or Non-contradiction) is an imperative to reject as illusory and not real, any apparent presence together of contradictories. This second law of

thought could be stated as 'Nothing is both A and not-A', or 'whatever is, is not whatever it is not'.

We cannot say of anything that it is *both* present and absent at once: *what is present, is not absent*. If the world of appearance displays some content with an identity, then it has effectively failed to display nothing. Contradictory appearances cannot coexist, concur, overlap: they are 'incompatible extremes'.

We can say of something that it 'is' something else, in the sense of having a certain relation to something distinct from itself, but we cannot say of it that it both has and lacks that relation, in one and the same respect, at one and the same place and time.

It is evident, and therefore incontrovertible (by the previous law), that appearances are variegated, changing, and diverse. Phenomena have a variety of aspects and are usually composed of different elements, they often change, and differ from each other in many ways. However, for any respect, place and time, we pinpoint, the appearance as such is, and is whatever it is — and not at once otherwise.

The law of contradiction is not a mere rephrasing of the law of identity, note well, but goes one step further: it sets a standard for relegating some appearances to the status of illusions; in a sense, it begins to define what we mean by 'illusion'. It does not, however, thereby claim that all what is leftover in the field of appearance is real with finality; nor does it deny that some of the leftovers are real (as is assured us by the law of identity).

By the law of identity, whatever appears is *given some credence*: therefore, one might suggest, the coexistence of opposites has some credence. The law of contradiction interposes itself at this point, and says: no, such events

carry no conviction for us, once clearly discerned. The first law continues to function as a recognition that there is an apparent contradiction; but the second law imposes on us the need to resolve that contradiction somehow.

The law of contradiction is itself, like anything else, an appearance among others, but it strikes us as an *especially credible* one, capable of overriding the initial credibility of all other considerations. It does not conflict with the message of the law of identity, since the latter is open to any event, including the event that some appearances be more forceful than others. The law of contradiction is precisely one such forceful appearance, an extremely forceful one.

Thus, though the world of appearances presents itself to us with some seeming contradictions, they appear *as* incredible puzzles — their unacceptability is inherent to them, obvious to us. We may verbally speculate about a world with real contradictions, and say that this position is consistent with itself even if inconsistent with itself. But the fact remains that whenever we are face to face with a specific contradiction (including that one) we are unavoidably skeptical — something seems 'wrong'.

The way we understand the apparent existence of contradictions is by viewing the world of appearances as *layered*, or stratified. Our first impressions of things are often superficial; as our experience grows, our consciousness penetrates *more deeply* into them. Thus, though each level is what it is (law of identity), parallel levels may be in contradiction; when a contradiction occurs, it is because we are superimposing different layers (law of contradiction). In this way, we resolve the 'general contradiction' of contradiction as such — we separate the conflicting elements from each other.

(Note in passing, as an alternative to the metaphor of 'depth', which likens consciousness to a beam of light, we also sometimes refer to 'height'. Here, the suggestion is that the essence of things is more elevated, and we have to raise ourselves up to make contact with it.)

That resolution of contradiction refers to the diversity and change in the world of appearance as due to the perspectives of consciousness. Thus, the appearance of the phenomena we classified as 'illusory' is due to the limitations of ordinary consciousness, its failure to know everything. This restriction in the power of consciousness may be viewed as a 'fault' of our minds, and in that sense 'illusion' is a 'product' of our minds. For that reason, we regard the illusory as in some sense 'imaginary' — this is our explanation of it.

On a more objective plane, we may of course accept diversity and change as real enough, and explain them with reference to the space and time dimensions, or to uniform and unchanging essences. In such cases, we are able to meet the demands of the law of contradiction without using the concept of 'illusion'; only when space, time, and respect, are clearly specified, does a contradiction signify illusion.

3. The Law of the Excluded Middle

The Law of the Excluded Middle is an imperative to reject as illusory and not real, any apparent absence together of contradictories. This third law of thought could be stated as 'nothing is neither A nor not-A', or 'whatever is, either is some thing or is-not that thing'.

We cannot say of anything that it is at once neither present nor absent: what is not present, is absent. If the

world of appearance fails to display some content with an identity, then it has effectively displayed nothing. There is no third alternative to these two events (whence the expression 'excluded middle'): they are exhaustive.

We may well say that some parts or aspects of the world are inaccessible to our limited faculties, but (as pointed out in the discussion of identity) we cannot claim a world beyond that of appearances: the moment we mention it, we include it.

It may be that we neither know that something is so and so, nor know that it is not so and so, but this concerns knowledge only, and in reality that thing either is or isnot so and so. Whatever we consider must either be there or not-there, in the specified respect, place and time, even if we cannot discern things enough to tell at this time or ever. There is an answer to every meaningful question; uncertainty is a 'state of mind', without 'objective' equivalent.

Moreover, *strictly speaking*, 'questions' are artificial attempts to anticipate undisplayed layers of appearance. As things appear now, if nothing is being displayed, *that* is the (current) 'answer' of the world of appearances; in the world of appearances there are no 'questions'. 'Questions' merely express our resolve to pursue the matter further, and try to uncover other layers of appearance; they are not statements about reality.

If we choose to, *loosely speaking*, regard doubts as kinds of assertions, the law of the excluded middle enjoins us to class them at the outset as illusory, and admit that in reality things are definite. Problematic statements like 'it might or might not be thus' are not intended to affirm that 'neither thus nor not-thus' *appeared*, but that what did appear (whether it was 'thus' or 'not-thus' — one of

them did, for sure) was not sufficiently forceful to satisfy our curiosity.

Even if no phenomenon is encountered which confirms or discredits an idea, there must be a phenomenon capable of doing so, in the world somewhere, sometime. We have to focus on the evidence, and try and distinguish the appearance or nonappearance of that imagined phenomenon.

Thus, the law of the excluded middle serves to create a breach of sorts between the 'objective world' and the 'world of ideas', and establishes the pre-eminence of the former over the latter. The breach is not an unbridgeable gap, but allows us to expand our language, in such a way that we can discuss eventual layers of appearance besides those so far encountered, even while we admit of the evidence at hand.

Such an artifice is made possible by our general awareness from past experience that appearances do change in *some* cases, but should not be taken to mean that any given appearance *will* change. It is only the expression of a (commendable) 'open-mindedness' *in principle*, with no specific justification in any given case. What we have done, effectively, is to expand what we mean by 'appearance', so as to include future appearance, in addition to appearances until now in evidence. Thus far, our implicit understanding was that appearance was *actual*, including present realities and present illusions. Now, we reflect further, and decide to embrace our anticipations of 'possible' appearances as a kind of actuality, too.

Such hypothetical projections are also, in a sense, 'apparent'. But they are clearly imaginary, inventions of the mind. Their status as appearances is therefore

immediately that of 'illusions'; that is their present status, whatever their future outcome. However, they are illusory with less finality than the phenomena so labeled by the law of contradiction; they retain some degree of credibility.

2. CHAPTER TWO

Drawn from *Future Logic* (1990), Chapter 3 (part of section 3 omitted).

LOGICAL RELATIONS

1. True or False

Reality and illusion are attributes of phenomena. When we turn our attention to the implicit 'consciousness' of these phenomena, we correspondingly regard the consciousness as realistic or unrealistic. The consciousness, as a sort of peculiar relation between a Subject (us) and an Object (a phenomenon), is essentially the same; only, in one case the appearance falls in the reality class, in the other it falls in the illusion class.

Why some thoughts turn out to be illusory, when considered in a broader context, varies. For example, I may see a shape in the distance, and assume it that of a man, but as I approach it, it turns out to be a tree stump; this latter conclusion is preferred because the appearance withstands inspection, it is firmer, more often confirmed. A phenomenon always exists as such, but it may 'exist' in the realms of illusion, rather than in that of reality. The fact that I saw some shape is undeniable: the only question is whether the associations I made in relation to it are valid or not.

'Propositions' are statements depicting how things appear to us. Understood as mere considerations (or

'hypothetically'), they contain no judgment as to the reality or illusion of the appearance. Understood as *assertions* (or 'assertorically'), they contain a judgment of the appearance as real or illusory.

Assertoric propositions must either be 'true' or 'false'. If we affirm a proposition, we mean that it is true; if we deny a proposition, we mean that it is false. Our definitions of truth and falsehood must be such that they are mutually exclusive and together exhaustive: what is true, is not false; what is false, is not true; what is not true, is false; what is not false, is true.

Strictly speaking, we call an assertion *true*, if it verbally depicts something which appears to us as real; and *false*, if it verbally depicts something which appears to us as illusory. In this ideal, absolute sense, true and false signify total or zero credibility, respectively, and allow of no degrees.

However, the expressions true and false are also used in *less stringent senses*, with reference to less than extreme degrees of credibility. Here, we call a proposition (relatively, practically) true if the appearance is more credible than any conflicting appearance; and (effectively) false, if the appearance is not the most credible of a set of conflicting appearances. Here, we can speak of more or less true or false.

The ultimate goal of logic is knowledge of reality, and avoidance of illusion. Logic is only incidentally interested in the less than extreme degrees of credibility. The reference to intermediate credibility merely allows us to gauge tendencies: how close we approach toward realism, or how far from it we stray. Note that the second versions of truth and falsehood are simply wider; they include the first versions as special, limiting cases.

Propositions which cannot be classed as true or false right now are said to be 'problematic'. Both sets of definitions of truth and falsehood leave us with gaps. The first system fails to address all propositions of intermediate credibility; the second system disregards situations where all the conflicting appearances are equally credible.

If we indeed cannot tip the scales one way or the other, we are in a quandary: if the alternatives are all labeled true, we violate the law of contradiction; if they are all labeled false, we violate the law of the excluded middle. Thus, we must remain with a suspended judgment, and though we have a proposition to consider, we lack an assertion.

2. Branches of Logic

The concepts of truth and falsehood will be clarified more and more as we proceed. In a sense, the whole of the science of logic constitutes a definition of what we mean by them — what they are and how they are arrived at. We shall also learn how to treat problematic propositions, and gradually turn them into assertions.

The task of sorting out truth from falsehood, case by case, is precisely what logic is all about. What is sure, however, is that that is in principle feasible.

If thought was regarded as not intimately bound with the phenomena it is intended to refer to, it would be from the start disqualified. In that case, the skeptical statement in question itself would be meaningless and self-contradictory. The only way to resolve this conflict and paradox is to admit the opposite thesis, viz. that some

thoughts are valid; that thesis, being the only internally consistent of the two, therefore stands as proven.

This is a very important first principle, supplied to us by logic, for all discussion of knowledge. We cannot consistently deny the ultimate realism of (some) knowledge. We cannot logically accept a theory of knowledge which in effect invalidates knowledge. That we know is unquestionable; how we know is another question.

Now, logical processes are called *deductive* (or analytic) to the extent that they yield indisputable results of zero or total credibility; and *inductive* (or synthetic) insofar as their results are more qualified, and of intermediate credibility. Deductive logic is conceived as concerned with truth and falsehood in their strict senses; inductive logic is content to deal with truth and falsehood in their not so strict senses.

This distinction is initially of some convenience, but it ultimately blurs. Logical theory begins by considering deductive processes, because they seem easier; but as it develops, its results are found extendible to lesser truths. Likewise, inductive logic begins with humble goals, but is eventually found to embrace deduction as a limiting case.

As we shall see, both these branches of logic require intuition of logical relations, and both presuppose some reliance on other phenomena. Both concern both concrete percepts and abstract concepts. Both involve the three faculties of experience, reason and imagination; only their emphasis differs somewhat. There is, at the end, no clear line of demarcation between them.

3. Tools of Logic

The following are three logical relations which we will often refer to in this study: implication, incompatibility, and exhaustiveness. We symbolize propositions by letters like P or Q for the sake of brevity; their negations are referred to as notP (or nonP) and notQ, respectively.

a. *Implication*. One proposition (P) is said to imply another (Q) if it cannot happen that the former is true and the latter false. Thus, if P is true, so must Q be; and if Q is false, so must P be — by definition. It does not follow that P is in turn implied by Q, nor is this possibility excluded. This relationship may be expressed as "If P, then Q", or equally as "If nonQ, then nonP". We can deny that Q is implicit in P by the formula "If P, not-then Q", or "If nonQ, not-then nonP".

When we use expressions like 'it follows that', 'then', 'therefore', 'hence', 'thence', 'so that', 'consequently', 'it presupposes that' — we are suggesting a relation of implication.

b. *Incompatibility* (or inconsistency or mutual exclusion). Two propositions (P, Q) are said to be incompatible if they cannot both be true. This relation is also called 'exclusive disjunction', and expressed by the formula 'P or else Q'. Thus, if either is true, the other is false. The possibility that both be false is not excluded, nor is it affirmed. This relation can be formulated as "If P, then nonQ", or equally as "If Q, then nonP". The denial of such a relation would be stated as "If P, not-then nonQ"., or "If Q, not-then nonP".

We can also say of more than two propositions that they are incompatible; meaning, if any one of them is true, all

the others must be false (though they might well all be false).

c. **Exhaustiveness**. Two propositions (P, Q) are said to be exhaustive if they cannot both be false. This relation is also called 'inclusive disjunction', and expressed by the formula 'P and/or Q'. Thus, if either is false, the other is true. The possibility that both be true is not excluded, nor is it affirmed. This relation can be formulated as "If nonP, then Q", or equally as "If nonQ, then P". The denial of such a relation would be stated as "If nonP, not-then Q", or "If nonQ, not-then P".

We can also say of more than two propositions that they are exhaustive; meaning, if all but one of them is false, the remaining one must be true (though they might well be all true).

We note that whereas implication and its denial are directional relations, incompatibility and exhaustiveness and their denials are symmetrical relations.

Also, underlying them all is the concept of 'conjunction', whether or not one can say one thing with or without the other. Consequently, these expressions are interconnected; we could rephrase any one in terms of any other. For example, 'P implies Q' could be restated as 'P is incompatible with notQ' or as 'notP and Q are exhaustive'.

4. Axioms of Logic

We can now re-state the laws of thought with regard to the truth or falsehood of (assertoric) propositions as follows. These principles (or the most primary among them) may be viewed as the axioms of logic, while however keeping in mind our later comments (ch. 20) on the issue of their development.

a. *The law of identity*: Every assertion implies itself as 'true'. However, this self-implication is only a claim, and does not by itself prove the statement.

More broadly, whatever is implied by a true proposition is also true; and whatever implies a false proposition is also false. (However, a proposition may well be implied by a false one, and still be true; and a proposition may well imply a true one, and still be false.)

b. **The law of contradiction**: If an affirmation is true, then its denial is false; if the denial is true, then the affirmation is false. They cannot be both true. (It follows that if two assertions are indeed both true, they are consistent.)

A special case is: any assertion which implies itself to be false, is false (this is called self-contradiction, and disproves the assertion; not all false assertions have this property, however).

More broadly, if two propositions are mutually exclusive, the truth of either implies the falsehood of the other, and furthermore implies that any proposition which implies that other is also false

c. The law of the excluded middle: If an affirmation is false, then its denial is true; if the denial is false, then the affirmation is true. They cannot both be false. (It follows that if two assertions are indeed both false, they are not exhaustive).

A special case is: any assertion whose negation implies itself to be false, is true (this is called self-evidence, and proves the assertion; not all true assertions have this property, however).

More broadly, if two propositions are together exhaustive, the falsehood of either implies the truth of the other, and furthermore implies that any proposition which that other implies is also true (though propositions which imply that other may still be false).

Thus, in summary, every statement implies itself true and its negation false; it must be either true or false: it cannot be both and it cannot be neither. In special cases, as we shall see, a statement may additionally be self-contradictory or self-evident.

Some of these principles are obvious, others require more reflection and will be justified later. They are hopefully at least easy enough to understand; that suffices for our immediate needs.

Note in passing that each of the laws exemplifies one of the logical relations earlier introduced. Identity illustrates implication, contradiction illustrates incompatibility, excluded-middle illustrates exhaustiveness.

Although we introduced the logical relations before the laws of thought, here (for the sake of clarity and since we speak the same language), it should be obvious that, conceptually, the reverse order would be more accurate.

First, come the intuitions of identity, contradiction, and excluded-middle, with the underlying notions (visual images, with velleities of movement), of equality ('to go together'), conflict ('to keep apart'), and limitation ('to circumscribe'). Thereafter, with these given instances in mind, we construct the more definite ideas of implication, incompatibility, and exhaustion.

3. CHAPTER THREE

Drawn from *Future Logic* (1990), Chapter 20 (sections 1-3).

CREDIBILITY

1. Ground of the Laws

We began our study by presenting the laws of thought—the Laws of Identity, of Contradiction, and of the Excluded Middle—as the foundations of logic. We can see, as we proceed, that these first principles are repeatedly appealed to in reasoning and validation processes. But in what sense are they 'laws'?

a. Many logicians have been tempted to compare these laws to the *axioms* of geometry, or the top postulates of natural sciences. According to this view, they are self-consistent hypotheses, which however are incapable of ultimate proof, from which all other propositions of logic are derived.

There is some truth to this view, but it is inaccurate on many counts. The whole concept of 'systematization' of knowledge, ordering it into axioms and derivatives, is itself a device developed and validated by the science of logic. It is only *ex post facto* that we can order the information provided by logic in this way; we cannot appeal to it without circularity. If logic was based on so tenuous a foundation, we could design alternative logics

(and some indeed have tried), just as Euclidean geometry or Newtonian mechanics were replaced by others.

Logic is prior to methodology. The idea that something may be 'derived' from something else, depends for its credibility on the insights provided by the 'laws of thought'. The 'laws of thought' ought not to be viewed as general principles which are *applied* to particular cases, because the process of application itself depends on them.

Rather, we must view *every particular occurrence* of identity, contradiction, and excluded-middle, as *by itself compelling*, an irreducible primary independently of any appeal to large principles. The principles are then merely statements to remind us *that* this compulsion occurs; they are not its source. This means that the 'laws of thought' are not general principles in the normal sense, but recognitions that 'there are such events'. The science of logic is, then, not a systematic application of certain axioms, but *a record of the kind of events which have this compelling character* for us.

Note this well. *Each* occurrence of such events is self-sufficiently evident; it is only thereafter that we can formulate statements about 'all' these events. We do not know what to include under the 'all' beforehand, so how could we 'apply' the laws to anything? These laws cannot be strictly-speaking 'generalizations', since generalization presupposes that you have some prior data *to* generalize.

Thus, we must admit that *first* comes specific events of identity, contradiction and excluded-middle, with a force of their own, then we can say 'these and those are *the kinds of* situations' where we experience that utter

certainty, and only lastly can we *loosely-speaking* format the information in the way of axioms and derivatives.

Nevertheless, it remains true that the laws of thought have a compelling character on their own. There is no way to put these laws in doubt, without implicitly arousing doubt in one's own claim. Sophisms always conceal their own implications, and tacitly appeal to the laws of thought for support, to gain our credulity. We could, therefore, equally say that the principles as units in themselves are entirely convincing, with utter finality — provided we *also* say that every act of their 'application' is likewise indubitable. It comes to the same.

However, the previous position is more accurate, because it explains how people unversed in the laws of thought, can nonetheless think quite logically — and also how we can understand the arguments here made about the laws of thought. The inconsistency of denials of the laws of thought is one instance of those laws, and not their whole basis.

b. What, then, is this 'compulsion' that we have mentioned? It is evident that people are not forced to think logically, say like physical bodies are forced to behave in certain ways. This is given: we do make errors, and these sometimes seem 'voluntary', and sometimes accidental. In any case, if thought was a mechanistic phenomenon, we would have no need of logical guidelines. We may only at best claim that we *can and should, and sometimes do*, think in perfect accord with these laws.

The answer to this question was implicit in the above discussion. It is or seems evident that things do present themselves and that they do have certain contents (identity), and that these presentations are distinct from their absences (contradiction), and that there is nothing else to refer to (excluded-middle). Because these statements concern appearances as such, it is irrelevant whether we say 'it *is* evident' or 'it *seems* evident'.

The concepts of reality and appearance are identical, with regard to the phenomenal; the concept of illusion is only meaningful as a subdivision of the phenomenal. These laws are therefore *always* evident, whether we are dealing with realities or illusions. We can wrongly interpret or deliberately lie about what we 'see' (if anything), but *that* we 'saw' and *just what* we 'saw' is pure data. Thus, the 'compulsion' is presented to us an intrinsic component of the phenomenal world we face.

The practical significance of this can be brought out with reference to the law of contradiction. We are saying, in effect, that whatever seems contradictory, is so. This statement may surprise, since we sometimes 'change our minds' about contradictions.

To understand it, consider two phenomena, say P1 and P2, in apparent contradiction, call this C1. One way to resolve C1, is to say that one or both of P1 and P2 are illusory. But we might find, upon closer inspection, that the two phenomena are not in contradiction; call this noncontradiction C2. So we now have two new phenomena, C1 and C2, in apparent contradiction; call this new contradiction C3.

The question is, does C3 imply that one or both of C1 and C2 are illusory? The answer is, no — what happened 'upon closer inspection' was not a revision of C1, but a revision of P1 and/or P2. So that in fact C2 does not concern exactly the same phenomena P1 and P2, but a slightly different pair of phenomena with the same names.

Thus, C1 and C2 could never be called illusory (except loosely speaking), because they were never in conflict, because they do not relate the same pair of phenomena. Nor for that matter may C3 be viewed as now erroneous, because the pair of phenomena it, in turn, related have changed.

Which means that our 'intuition' of contradiction is invariably correct, for exactly the data provided to it. A similar argument can be made with regard to other logical relations. The phenomena related may be unclear and we may confuse phenomena (thinking them the same when they are different) — but, at any level of appearance, the logical relation between phenomena is 'compulsively evident', inflexibly fixed, given.

In other words, among phenomena, logical relations are one kind which are always real; in their case, appearance and reality are one and the same, and there are no illusions. The laws of thought are presented as imperatives, to urge us to focus on and carefully scrutinize the phenomena related, and not to suggest that the logical intuitions of thought are fallible, once the effort is made to discern the relation.

This is not a claim to any prior omniscience, but a case by case accuracy. As each situation arises, its logical aspects are manifest to the degree that we inspect things clearly. Note well, we do not need to know *how* the intuition functions, to be able to know and prove *that* it functions well. We have called it 'intuition' to suggest that it is a direct kind of consciousness, which may well be conceptual rather than perceptual, but these descriptive issues are secondary.

Thus, with regard to the laws of thought, we have no ground for wondering whether they are animal instincts

imposed by the structure of the mind, or for wondering whether they control the events external to it as well. In either case, we would be suggesting that there is a chance that they might be illusory and not real. If we claim that the mind is distortive, one way or the other, we put that very claim in doubt.

The mind is doubtless limited. It is common knowledge that mental conditions, structural or psychological or voluntary, can *inhibit* us from comparing phenomena with a view to their logical relation — but that does not mean that when the elements *are* brought together, the comparison may fail.

Nervous system malfunctions, personality disorders, drunkenness, fatigue — such things can only arrest, never alter these intuitions. As for evasions and lies, we may delude ourselves or others, to justify some behavior or through attachment to a dogma — but these are after the fact interventions.

2. Functions of the Laws

The laws of thought relate to the credibility, or trustworthiness, of phenomena. They clarify things in three stages. At the identity level, appearances are acknowledged and taken as a data base. At the contradiction level, we learn to discriminate clearly between real and illusory appearances. At the excluded-middle level, we introduce a more tempered outlook, without however ignoring the previous lessons. More specifically, their functions are as follows:

The first law assigns a minimal credibility to any thought whatsoever, if only momentarily; the evidence, such as it is, is considered. If, however, the 'thought' is found to consist of meaningless words, or is overly vague or obscure — it is as if nothing has appeared, and credibility disappears (until and unless some improvement is made). To the extent that a thought has some meaning, precision, and clarity, it retains some credibility.

The second law puts in doubt any thoughts which somehow give rise to contradictions, and thereby somewhat enhances the credibility of all thoughts which pass this test. In the case of a thought which is self-inconsistent (whether as a whole or through the conflicts of its parts), its credibility falls to zero, and the credibility of denial becomes extreme. In the case of two or more thoughts, each of which is self-consistent, but which are incompatible with each other, the loss of credibility is collective, and so individually less final.

The third law sets bounds for any leftover thoughts (those with more than zero and less than total credibility, according to the previous two laws): if special ways be found to increase or decrease their credibilities, the overall results cannot in any case be such as to transgress the excluded-middle requirement (as well as the nocontradiction requirement, of course). As we shall see, the processes of confirmation and discrediting of hypotheses are ways logic uses to further specify credibilities.

We see that, essentially, the law of identity gives credence to *experience*, in the widest sense, including concrete perceptions and abstract conceptual insights. The law of contradiction essentially justifies the logical intuitions of *reason*. The law of the excluded-middle is essentially directed at the projections of the *imagination*. This division of labor is not exclusive — all three laws

come into play at every stage — but it has some pertinence.

The credibility of a phenomenon is, then, a measure of how well it fits into the total picture presented by the world of appearances; it is a component of phenomena, like bodies have weight. This property is in some cases fixed; but in most cases, variable — an outcome of the interactions of phenomena as such.

The laws of thought are, however, only the first steps in a study of credibility. The enterprise called logic is a continual search for additional or subsidiary norms. Logic theory develops, as we shall see, by considering various kinds of situations, and predicting the sorts of inferences which are feasible in each setting.

More broadly the whole of philosophy and science may be viewed as providing us with more or less rough and ready, practical yardsticks for determining the relative credibility of phenomena. However, such norms are not of direct interest to the logician, and are for him (relatively speaking) specific world views. Logic has to make do with the two broadest categories of reality and illusion — at least, to begin with.

3. More on Credibility

Every phenomenon appears to us with some degree of 'credibility', as an inherent component of its appearance; this is an expression of the law of identity. That initially intuitive credibility may be annulled or made extreme, through the law of contradiction; or it may be incrementally increased or decreased, by various techniques (yet to be shown), within the confines of the laws of contradiction and of the excluded middle.

Thus, credibility is primarily an aspect of the phenomenal world, and a specific phenomenon's degree of credibility is a function of what other phenomena are present in the world of appearances at that stage in its development. Because phenomena interact in this way, and affect each other's credibilities, credibility may be viewed as a measure of how well or badly any phenomenon 'fits in' with the rest.

'Reality' and 'illusion' are just the extremes of credibility and incredibility, respectively; they are phenomena with that special character of total or zero force of conviction. We cannot refer to a domain beyond that of appearances, for good or bad, without thereby including it within the world of appearances.

How do we know that all appearances must ultimately be real or illusory? How do we know that *median* credibility cannot be a permanent state of affairs in some cases, on a par with the extremes of credibility and incredibility? We answered this question, in broad terms, in our discussions on the laws of thought, as follows. More will be said about it as we proceed.

Reality and illusion are a dichotomy of actual appearances: for them, whatever is inconsistent is illusory, and everything else is real enough. Median credibility only comes into play when we try to anticipate future appearances, but has no equivalent in the given world. In the actual field of concrete and abstract experiences, things have either no credibility or effectively total credibility; it is only through the artificial dimension of mental projections that intermediate credibility arises.

Knowledge is merely consciousness of appearances; the flip-side, as it were, of the event of appearance. Viewed

in this perspective, without making claims to anything but the phenomenal, knowledge is always a faithful rendering of the way things appear. We may speak of knowledge itself as being realistic or as unrealistic or as hypothetical, only insofar as we understand that this refers to *the kind of* appearance it reflects. These characterizations refer primarily, not to knowledge, but to its objects.

The difference between knowledge (in its narrower sense of, knowledge of reality) and opinion (in the sense of, the practically known), is thus merely one of degree of credibility *manifested by their objects* (at that time); we cannot point to any essential, structural difference between them. However, this distinction is still significant: it matters a lot that the objects carry different weights of conviction.

Changes or differences in appearances and opinion are to some extent *explained* by reference to variations in our perspective, and breadth and depth of consciousness. But this explanation does not annul the primacy of phenomena, in all their aspects.

In practice, median credibility is often not patiently accepted, but we use our 'wisdom' to lean one way or the other a bit, according to which idea seems to 'hang together' the best. But a contrary function of wisdom is the ability to see alternatives, or the remote possibility of suggested alternatives, and thus keep an open mind. The intelligent man is able to take positions where others dither, and also to see problems where others see certainties.

4. CHAPTER FOUR

Drawn from *Future Logic* (1990), Chapter 31 (sections 1 & 2).

PARADOXES

A very important field of logic is that dealing with paradox, for it provides us with a powerful tool for establishing some of the most fundamental certainties of this science. It allows us to claim for epistemology and ontology the status of true sciences, instead of mere speculative digressions. This elegant doctrine may be viewed as part of the study of axioms.

1. Internal Inconsistency

Consider the hypothetical form 'If P, then Q', which is an essential part of the language of logic. It was defined as 'P and nonQ is an impossible conjunction'.

It is axiomatic that the conjunction of any proposition P and its negation nonP is impossible; thus, a proposition P and its negation nonP cannot be both true. An obvious corollary of this, obtained by regarding nonP as the proposition under consideration instead of P, is that the conjunction of any proposition nonP and its negation not-nonP is impossible; thus, a proposition P and its negation nonP cannot be both false.

So, the Law of Identity could be formulated as, "For any proposition, 'If P, then P' is true, and 'If nonP, then nonP' is true". The Laws of Contradiction and of the Excluded Middle could be stated: "For any proposition, 'If P, then not-nonP' is true (P and nonP are incompatible), and 'If not-nonP, then P' is true (nonP and P are exhaustive)".

Now, consider the paradoxical propositions 'If P, then nonP' or 'If nonP, then P'. Such propositions appear at first sight to be obviously impossible, necessarily false, antinomies.

But let us inspect their meanings more closely. The former states 'P and (not not)P is impossible', which simply means 'P is impossible'. The latter states 'nonP and not P is impossible', which simply means 'nonP is impossible'. Put in this defining format, these statements no longer seem antinomial! They merely inform us that the proposition P, or nonP, as the case may be, contains an intrinsic flaw, an internal contradiction, a property of self-denial.

From this we see that there may be propositions which are logically self-destructive, and which logically support their own negations. Let us then put forward the following definitions. A proposition is *self-contradictory* if it denies itself, i.e. implies its own negation. A proposition is therefore *self-evident* if its negation is self-contradictory, i.e. if it is implied by its own negation.

Thus, the proposition 'If P, then nonP' informs us that P is self-contradictory (and so logically impossible), and that nonP is self-evident (and so logically necessary). Likewise, the proposition 'If nonP, then P' informs us that nonP is self-contradictory, and that P is self-evident.

The existence of paradoxes is not in any way indicative of a formal flaw. The *paradox*, the hypothetical proposition itself, is not antinomial. It may be true or false, like any other proposition. Granting its truth, it is its antecedent thesis which is antinomial, and false, as it denies itself; the consequent thesis is then true.

If the paradoxical proposition 'If P, then nonP' is true, then its contradictory 'If P, not-then nonP', meaning 'P is not impossible', is false; and if the latter is true, the former is false. Likewise, 'If nonP, then P' may be contradicted by 'If nonP, not-then P', meaning 'nonP is not impossible'.

The two paradoxes 'If P, then nonP' and 'If nonP, then P' are contrary to each other, since they imply the necessity of incompatibles, respectively nonP and P. Thus, although such propositions taken singly are not antinomial, double paradox, a situation where both of these paradoxical propositions are true at once, is unacceptable to logic.

In contrast to positive hypotheticals, negative hypotheticals do not have the capability of expressing paradoxes. The propositions 'If P, not-then P' and 'If nonP, not-then nonP' are *not* meaningful or logically conceivable or ever true. Note this well, such propositions are formally false. Since a form like 'If P, not-then Q' is defined with reference to a positive conjunction as '{P and nonQ} is possible', we cannot without antinomy substitute P for Q here (to say '{P and nonP} is possible'), or nonP for P and Q (to say '{nonP and not-nonP} is possible').

It follows that the proposition 'if P, then nonP' does not imply the lowercase form 'if P, not-then P', and the proposition 'if nonP, then P' does not imply the

lowercase form 'if nonP, not-then nonP'. That is, in the context of paradox, hypothetical propositions behave abnormally, and not like contingency-based forms.

This should not surprise us, since the self-contradictory is logically impossible and the self-evident is logically necessary. Since paradoxical propositions involve incontingent theses and antitheses, they are subject to the laws specific to such basis.

The implications and consistency of all this will be looked into presently.

2. The Stolen Concept Fallacy

Paradoxical propositions actually occur in practice; moreover, they provide us with some highly significant results. Here are some examples:

- denial, or even doubt, of the laws of logic conceals an appeal to those very axioms, implying that the denial rather than the assertion is to be believed;
- denial of man's ability to know any reality objectively, itself constitutes a claim to knowledge of a fact of reality;
- denial of validity to man's perception, or his conceptual power, or reasoning, all such skeptical claims presuppose the utilization of and trust in the very faculties put in doubt;
- denial on principle of all generalization, necessity, or absolutes, is itself a claim to a general, necessary, and absolute, truth.

• denial of the existence of 'universals', does not itself bypass the problem of universals, since it appeals to some itself, namely, 'universals', 'do not', and 'exist'.

More details on these and other paradoxes, may be found scattered throughout the text. Thus, the uncovering of paradox is an oft-used and important logical technique. The writer Ayn Rand laid great emphasis on this method of rejecting skeptical philosophies, by showing that *they implicitly appeal to concepts which they try to explicitly deny*; she called this 'the fallacy of the Stolen Concept'.

A way to understand the workings of paradox, is to view it in the context of dilemma. A self-evident proposition P could be stated as 'Whether P is affirmed or denied, it is true'; an absolute truth is something which turns out to be true whatever our initial assumptions.

This can be written as a constructive argument whose left horn is the axiomatic proposition of P's identity with itself, and whose right horn is the paradox of nonP's selfcontradiction; the minor premise is the axiom of thorough contradiction between the antecedents P and nonP; and the conclusion, the consequent P's absolute truth

If P, then P — and — if nonP, then P but either P or nonP hence, P.

A destructive version can equally well be formulated, using the contraposite form of identity, 'If nonP, then nonP', as left horn, with the same result.

If nonP, then nonP — and — if nonP, then P but either not-nonP or nonP hence, not-nonP, that is, P.

The conclusion 'P' here, signifies that P is logically necessary, not merely that P is true, note well; this follows from the formal necessity of the minor premise, the disjunction of P and nonP, assuming the right horn to be well established.

Another way to understand paradox is to view it in terms of knowledge contexts. Reading the paradox 'if nonP, then P' as 'all contexts with nonP are contexts with P', and the identity 'if P, then P' as 'all contexts with P are contexts with P', we can infer that 'all contexts are with P', meaning that P is logically necessary.

We can in similar ways deal with the paradox 'if P, then nonP', to obtain the conclusion 'nonP', or better still: P is impossible. The process of resolving a paradox, by drawing out its implicit categorical conclusions, may be called *dialectic*.

Note in passing that the abridged expression of simple dilemma, in a single proposition, now becomes more comprehensible. The compound proposition 'If P, then {Q and nonQ}' simply means 'nonP'; 'If nonP, then {Q and nonQ}' means 'P'; 'If (or whether) P or nonP, then Q' means 'Q'; and 'If (or whether) P or nonP, then nonQ' means 'nonQ'. Such propositions could also be categorized as paradoxical, even though the contradiction generated concerns another thesis.

However, remember, the above two forms should not be confused with the lesser, negative hypothetical, relations 'Whether P or nonP, (not-then not) Q' or 'Whether P or nonP, (not-then not) nonQ', respectively, which are not

paradoxical, unless there are conditions under which they rise to the level of positive hypotheticals.

5. CHAPTER FIVE

Drawn from *Future Logic* (1990), Chapter 32.

DOUBLE PARADOXES

1. Definition

We have seen that logical propositions of the form 'if P, then nonP' (which equals to 'nonP') or 'if nonP, then P' (which equals to 'P'), are perfectly legal. They signify that the antecedent is self-contradictory and logically impossible, and that the consequent is self-evident and logically necessary. As propositions in themselves, they are in no way antinomial; it is one of their constituents which is absurd.

Although either of those propositions, occurring alone, is formally quite acceptable and capable of truth, they can never be both true: they are irreconcilable contraries and their conjunction is formally impossible. For if they were ever both true, then both P and nonP would be implied true.

We must therefore distinguish between *single paradox*, which has (more precisely than previously suggested) the form 'if P, then nonP; but if nonP, not-then P; whence nonP', or the form 'if nonP, then P; but if P, not-then nonP; whence P' — and *double paradox*, which has the form 'if P, then nonP, *and* if nonP, then P'.

Single paradox is, to repeat, within the bounds of logic, whereas double paradox is beyond those bounds. The former may well be true; the latter always signifies an error of reasoning. Yet, one might interject, double paradox occurs often enough in practice! However, that does not make it right, anymore than the occurrence of other kinds of error in practice make them true.

Double paradox is made possible, as we shall see, by a hidden *misuse of concepts*. It is sophistry par excellence, in that we get the superficial illusion of a meaningful statement yielding results contrary to reason. But upon further scrutiny, we can detect that some fallacy was involved, such as ambiguity or equivocation, which means that in fact the seeming contradiction never occurred.

Logic demands that either or both of the hypothetical propositions which constituted the double paradox, or paradox upon paradox, be false. Whereas single paradox is resolved, by concluding the consequent categorically, without denying the antecedent-consequent connection — double paradox is *dissolved*, by showing that one or both of the single paradoxes involved are untrue, nonexistent. Note well the difference in problem solution: resolution 'explains' the single paradox, whereas dissolution 'explains away' the double paradox. The double paradox serves to show that we are making a mistake of some kind: the fact that we have come to a contradiction, is our index and proof enough that we have made a wrong assumption of sorts. Our ability to intuit logical connections correctly is not put in doubt, because the initial judgment was too rushed, without pondering the terms involved. Once the concepts involved are clarified, it is the rational faculty itself

which pronounces the judgment against its previous impression of connection.

It must be understood that every double paradox (as indeed every single paradox), is *teaching us something*. Such events must not be regarded as threats to reason, which put logic as a whole in doubt; but simply as lessons. They are sources of information, they reveal to us certain logical rules of concept formation, which we would otherwise not have noticed. They show us the outer limits of linguistic propriety.

We shall consider two classical examples of double paradox to illustrate the ways they are dissolved. Each one requires special treatment. They are excellent exercises.

2. The Liar Paradox

An ancient example of double paradox is the well-known 'Liar Paradox', discovered by Eubulides, a 4th cent. BCE Greek of the Megarian School. It goes: 'does a man who says that he is now lying speak truly?' The implications seem to be that if he is lying, he speaks truly, and if he is not lying, he speaks truly.

Here, the conceptual mistake underlying the difficulty is that the proposition is *defined by reference to itself*. The liar paradox is how we discover that such concepts are not allowed.

The word 'now' (which defines the proposition itself as its own subject) is being used with reference to something which is not yet in existence, whose seeming existence is only made possible by it. Thus, in fact, the word is empty of specific referents in the case at hand. The word 'now' is indeed usually meaningful, in that in

other situations it has precise referents; but in this case it is used before we have anything to point to as a subject of discourse. It looks and sounds like a word, but it is no more than that.

A more modern and clearer version of this paradox is 'this proposition is false', because it brings out the indicative function of the word 'now' in the word 'this'. 'this' accompanies our pointings and The word presupposes that there is something to point to already there. It cannot create a referent for itself out of nothing. This is the useful lesson taught us by the liar paradox. We may well use the word 'this' to point to another word 'this'; but not to itself. Thus, I can say to you 'this "this", which is in the proposition "this proposition is false", without difficulty, because my 'this' has a referent, albeit an empty symbol; but the original 'this' is meaningless. Furthermore, the implications of this version seem to be that 'if the proposition is true, it is false, and if it is false, it is true'. However, upon closer inspection we see that the expression 'the proposition' or 'it' has a different meaning in antecedents and consequent.

If, for the sake of argument, we understand those implications as: if this proposition is false, then this proposition is true; and if this proposition is true, then this proposition is false — taking the 'this' in the sense of *self*-reference by every thesis — then we see that the theses do not in fact have one and the same subject, and are only presumed to be in contradiction.

They are not formally so, any more than, for any P1 and P2, 'P1 is true' and 'P2 is false' are in contradiction. The implications are not logically required, and thus the two paradoxes are dissolved. There is no self-contradiction, neither in 'this proposition is false' nor of course in 'this

proposition is true'; they are simply meaningless, because the indicatives they use are without reference.

Let us, alternatively, try to read these implications as: if 'this proposition is false' is true, then that proposition is false; and if that proposition is false, then that proposition is true' — taking the first 'this' as self-reference and the 'thats' thereafter as all pointing us backwards to the original proposition and not to the later theses themselves. In other words, we mean: if 'this proposition is false' is true, then 'this proposition is false, and if 'this proposition is false' is false, then 'this proposition is false, then 'this proposition is false' is true.

Here, the subjects of the theses are one and the same, but the implications no longer seem called for, as is made clear if we substitute the symbol P for 'this proposition is false'. The flavor of paradox has disappeared: it only existed so long as 'this proposition is false' seemed to be implied by or to imply 'this proposition is true'; as soon as the subject is unified, both the paradoxes break down.

We cannot avoid the issue by formulating the liar paradox as a generality. The proposition 'I always lie' can simply be countered by 'you lie sometimes (as in the case 'I always lie'), but sometimes you speak truly'; it only gives rise to double paradox in indicative form. Likewise, the proposition 'all propositions are false' can be countered by 'some, some not', without difficulty.

However, note well, both the said general propositions are indeed self-contradictory; they do produce single paradoxes. It follows that both are false: one cannot claim to 'always lie', nor that 'there are no true propositions'. This is ordinary logical inference, and quite legitimate, since there are logical alternatives.

With regard to those alternatives. The proposition 'I never lie' is not in itself inconsistent, except for the person who said 'I always lie' intentionally. The proposition 'all propositions are true' is likewise not inconsistent in itself, but is inconsistent with the logical knowledge that some propositions are inconsistent, and therefore it is false; so in this case only the contingent 'some propositions are true, some false' can be upheld.

3. The Barber Paradox

The Barber Paradox may be stated as: 'If a barber shaves everyone in his town who does not shave himself, does he or does he not shave himself? If he does, he does not; if he does not, he does'.

This double paradox arises through confusion of the expressions 'does not shave himself' and 'is shaved by someone other than himself'.

We can divide the people in any town into three broad groups: (a) people who do not shave themselves, but are shaved by others; (b) people who do not shave themselves, and are not shaved by others; (c) people who shave themselves, and are not shaved by others. The given premise is that our barber shaves all the people who fall in group (a). It is tacitly suggested, but not formally implied, that no one is in group (b), so that no one grows a beard or is not in need of shaving. But, in any case, the premise in fact tells us nothing about group (c).

Next, let us subdivide each of the preceding groups into two subgroups: (i) people who shave others, and (ii) people who do not shave others. It is clear that each of the six resulting combinations is logically acceptable, since who shaves me has no bearing on whom I can shave. Obviously, only group (i) concerns barbers, and our premise may be taken to mean that our barber is the only barber in town.

Now, we can deal with the question posed. Our barber cannot fall in group (a)(i), because he is not shaved by others. He might fall in group (b)(i), if he were allowed to grow a beard or he was hairless; but let us suppose not, for the sake of argument. This still allows him to fall in group (c)(i), meaning that he shaves himself (rather than being shaved by others), though he shaves others too.

Thus, there is no double paradox. The double paradox only arose because we wrongly assumed that 'he shaves all those who *do not* shave themselves' excludes 'he shaves some (such as himself) who *do* shave themselves'. But 'X shaves Y' does not formally contradict 'X shaves nonY'; there is no basis for assuming that the copula 'to shave' is obvertible, so that 'X shaves Y' implies 'X does not shave nonY'.

If the premise was restated as 'he shaves all those *and* only those who do not shave themselves' (so as to exclude 'he shaves himself'), we would still have an out by saying 'he does not shave at all'. If the premise was further expanded and restricted by insisting that 'he somehow shaves or is shaved', it would simply be self-contradictory (in the way of a single paradox).

Further embellishments could be made to the above, such as considering people who shave in other towns, or making distinctions between always, sometimes/sometimes-not, and never. But I think the point is made. The lesson learned from the barber 'paradox' is that without clear categorizations, equivocations can emerge (such as that between 'shaves'

and 'is shaved'), which give the illusion of double paradox.

6. CHAPTER SIX

Drawn from *Buddhist Illogic* (2002), Chapters 1 & 2.

1. The Tetralemma

Western philosophical and scientific thought is based on Aristotelian logic, whose founding principles are the three "Laws of Thought". These can be briefly stated as "A is A" (Identity), "Nothing is both A and non-A" (Non-contradiction) and "Nothing is neither A nor non-A" (Exclusion of the Middle). These are not claimed as mere hypotheses, note well, but as incontrovertible premises of all rational human thought¹.

Religions like Judaism, Christianity and Islam, even while adhering to these laws in much of their discourse and paying lip-service to them, in their bids to interpret their own sacred texts and to make their doctrines seem reasonable to their converts, have often ignored these same laws. This is especially true of mystical trends within these religions, but many examples could be given from mainstream writings. The same can be said of some aspects of Buddhist philosophy.

See my *Future Logic* (Geneva: Author, 1996. Rev. ed.), ch. 2 and 20, and later essays on the subject (published on my website www.thelogician.net).

The *tetralemma*² is a derivative of the laws of thought, with reference to any two terms or propositions, labeled A and B, and their opposites non-A and non-B. Four combinations of these four terms are conceivable, namely "A and B" (both), "non-A and non-B" (neither), "A and non-B" and "non-A and B" (one or the other only). According to Aristotelian logic, these four statements are incompatible with each other (*only one of them can be true*, because if two or more were affirmed then "A and non-A" or "B and non-B" or both would be true, and the latter implications are self-contradictory) and exhaustive (*at least one of them must be true*, since if they were all denied then "not A and not non-A" or "not B and not non-B" or both would be true, and the latter implications go against the excluded middle).

Now, what Nagarjuna does is insert the term A in place of B (i.e. he takes the case of B = A), and effectively claim that the above four logical possibilities of combination apply in that special case – so that "A and A (=B)", "non-A and non-A (=non-B)", "A and non-A (=non-B)", "non-A and A (=B)" seem logically acceptable. He then goes on to argue that there are four existential possibilities: affirmation of A (A + A = A), denial of A (non-A + non-A = non-A), both affirmation and denial of A (A and non-A) and neither affirmation nor denial of A (not A and not non-A). He is thus apparently using the principles and terminology of common logic to arrive at a very opposite result. This gives him and readers the impression that it is

See Cheng, pp. 36-38, on this topic. He there refers to MT opening statement, as well as XVII:12a and XXIII:1a. Etym. Gk. *tetra* = four, *lemma* = alternatives. Term coined in contrast to the dilemma "A or non-A".

quite reasonable to both affirm and deny or to neither affirm nor deny.

But in Aristotelian logic, the latter two alternatives are at the outset excluded - "both A and non-A" by the Law of Non-contradiction and "neither A nor non-A" by the Law of the Excluded-Middle – and the only logical possibilities left are "A" or "non-A". The anti-Aristotelian position may be viewed, in a positive light, as an anti-Nominalist position, reminding us that things are never quite what they seem or that things cannot be precisely classified or labeled. But ultimately, they intend the death of Logic; for without the laws of thought, how are we to distinguish between true and false judgments? The law of identity "A is A" is a conviction that things have some identity (whatever it specifically be) rather than another, or than no identity at all. It is an affirmation that knowledge is ultimately possible, and a rejection of sheer relativism or obscurantism. Nagarjuna's goal is to deny identity.

It should be noted here that Aristotle is very precise in his formulation of the law of contradiction, stating in his *Metaphysics* "The same attribute cannot at the same time belong and not belong to the same subject in the same respect" (italics mine). Thus, an alternative statement of the laws of thought would be the 'trilemma' (let us so call it) "either wholly A, or wholly non-A, or both partly A and partly non-A", which excludes the fourth alternative "both wholly A and wholly non-A". The Buddhist attack on the laws of thought draws some of its credibility from the fact that people subconsciously refer to this 'trilemma', thinking superficially that indeed opposite things may occur in the same place at different times or at the same time in different places or in various

respects, without thereby giving rise to logical difficulty incapable of resolution. But it should be clear that the Buddhist position is much more radical than that, accepting thoroughgoing antinomy.

Similarly with regard to the law of the excluded middle, which affirms the situation "neither A nor non-A" to be impossible *in fact*. People are misled by the possibility of uncertainty *in knowledge*, as to whether A or non-A is the case in fact, into believing that this law of thought is open to debate. But it must be understood that the thrust of this logical rule is inductive, rather than deductive; i.e. it is a statement that *at the end* of the knowledge acquisition process, either "A" or "non-A" will result, and no third alternative can be expected. It does not exclude that *in the interim*, a situation of uncertainty may occur. Nagarjuna's position exploits this confusion in people's minds.

2. Nagarjuna's Misinterpretation

Nagarjuna interprets the limitation implied by the dilemma "A or non-A" as an arbitrary 'dualism' on the part of ordinary thinkers³. It only goes to show that he

It is misleading to call this a 'duality' or 'dichotomy', as Buddhists are wont to do, because it suggests that a unitary thing was arbitrarily cut into two – and incidentally, that it might just as well have been cut into four. But, on a perceptual level, there is no choice involved, and no 'cutting-up' of anything. A phenomenon appearing is one single thing, call it 'a' (a proper name, or an indicative 'this'), and not a disjunction. The issue of 'dichotomy' arises only on a conceptual level. Negation is a rational act, i.e. we can only speak of 'non-a', of what does not appear, by first bringing to mind something 'a', which previously appeared (in sensation or imagination). In initial

misunderstands formalization (or he pretends to, in an attempt to confuse gullible readers). When logicians use a variable like "B" and allow that "non-A and B" and "A and non-B" are both in principle possible, they do not intend that as a generality applicable to *all* values of B (such as "A"), but only as a generic statement applicable to *any consistent* values of B. In the specific case where B = A, the said two combinations have to be eliminated because they are illegal (i.e. breach two of the laws of thought).

above-stated property of symbols, i.e. their The applicability only conditionally within the constraints of consistency, is evident throughout the science of formal logic, and it is here totally ignored by Nagarjuna. His motive of course was to verbalize and rationalize the Buddha's doctrine that the ultimate truth is beyond *nama* and rupa, name and form (i.e. discrimination and discourse). knowable only by transcendental a consciousness (the Twofold Truth doctrine). More precisely, as Cheng emphasizes, Nagarjuna's intent was to show that logic is inherently inconsistent and thus that reason is confused madness to be rejected. That is, he was (here and throughout) not ultimately trying to defend

conceptualization, two phenomena are compared and contrasted, to each other and to other things, in some respect(s); the issue is then, are they similar enough to each other and different enough from other things to be judged 'same' and labeled by a general term (say 'A'), or should they be judged 'different' or is there an uncertainty. At the later stage of recognition, we have to decide whether a third phenomenon fits in the class formed for the previous two (i.e. falls under 'A') or does not fit in (i.e. falls under 'non-A') or remains in doubt. In the latter case, we wonder whether it is 'A' or 'non-A', and forewarn that it cannot be both or neither.

a tetralemma with B equal to A – or even to affirm that things are both A and non-A, or neither A nor non-A – but wished to get us to look altogether beyond the distinctions of conceptualization and the judgments of logic.

But as above shown he does not succeed in this quest. For his critique depends on a misrepresentation of logical science. He claims to show that logic is confused and self-contradictory, but in truth what he presents as the thesis of logical science is not what it claims for itself but precisely what it explicitly forbids. Furthermore, suppose logical theory did lead to contradictions as he claims, this fact would not lead us to its rejection were there not also a tacit appeal to our preference for the logical in practice. If logic were false, contradictions would be acceptable. Thus, funnily enough, Nagarjuna appeals to our logical habit in his very recommendation to us to ignore logic. In sum, though he gives the illusion that it is reasonable to abandon reason, it is easy to see that his conclusion is foregone and his means are faulty.

3. Neither Real Nor Unreal

But Nagarjuna also conceives ultimate reality ("emptiness"⁴) as a "middle way" ⁵ – so that the world of experience is neither to be regarded as real, nor to be regarded as unreal ("there is nothing, neither mental nor non-mental, which is real" and it "cannot be conceived as

Beyond consciousness of "Shunyata" is a more vivid awareness called "Mahamudra", according to Chögyam Trungpa,. But such refinements need not concern us here.

See Cheng, pp. 38-39, on this topic. He there refers to MT XIII:9a and XVIII:7.

unreal," reports Cheng). In this context, Nagarjuna is clearly relying on one of the above-mentioned logically impossible disjuncts, namely "neither A nor non-A" (be it said in passing). I want to now show why Nagarjuna's statement seems superficially reasonable and true.

As I have often clarified and explained⁶, knowledge has to be regarded or approached phenomenologically (that is the only consistent epistemological thesis). We have to start by acknowledging and observing *appearances*, as such, without initial judgment as to their reality or illusion. At first sight all appearances seem *real* enough. But after a while, we have to recognize that some appearances conflict with other appearances, and judge such appearances (i.e. one or more of those in conflict) as *illusory*. Since there is nothing in our 'world' but appearances, all remaining appearances not judged as illusions (i.e. so long as they are not logically invalidated by conflicts with other appearances) maintain their initial status as realities.

That is, the distinction between appearances as realities or illusions emerges within the world of appearances itself, merely classifying some this way and the rest that way. We have no concept of reality or illusion other than with reference to appearance. To use the category of reality with reference to something beyond appearance is concept stealing, a misuse of the concept, an extrapolation which ignores the concept's actual genesis in the context of appearance. To apply the concept of illusion to all appearances, on the basis that some appearances are illusions, is an unjustified generalization ignoring how this concept arises with reference to a

See my *Future Logic*, ch. 60-62, and later essays on the subject.

specific event (namely, inconsistency between certain appearances and resulting diminishment of their innate credibilities). Moreover, to claim that no appearances are real or that all are illusions is self-defeating, since such claim itself logically falls under the category of appearance.

The illusory exists even though it is not reality – it exists as appearance. The real is also apparent – some of it, at least. Therefore, appearance per se is neither to be understood as reality (since some appearances are illusory), nor can it be equated to illusion (since not all appearances have been or can be found illusory). Appearance is thus the *common ground* of realities and illusions, their common characteristic, the dialectical synthesis of those theses and antitheses. It is a genus, they are mutually exclusive species of it. (The difference between appearance and existence is another issue, I have dealt with elsewhere – briefly put, existence is a genus of appearance and non-appearance, the latter concepts being relative to that of consciousness whereas the former is assumed independent.)

None of these insights allows the conclusion that appearances are "neither real nor unreal" (granting that 'unreal' is understood to mean 'non-real'). All we can say is that some appearances are real and some unreal. Formally, the correct logical relation between the three concepts is as follows. *Deductively*, appearance is implied by reality and illusion, but does not imply them; for reality and illusion are contradictory, so that they cannot both be true and they cannot both be false. Moreover, *inductively*, appearance implies reality, until and unless it is judged to be illusion (by virtue of some inconsistency being discovered).

More precisely, all appearances are initially classed as real. Any appearance found self-contradictory (deductively) illusory, and contradictory its is consequently self-evident and (deductively) real. All remaining appearances remain classed as real, so long as uncontested. Those that are contested have to be evaluated dynamically. When one appearance is belied by another, they are both put in doubt by the conflict between them, and so both become initially problematic. Thereafter, their relative credibilities have to be tentatively weighed in the overall context of available empirical and rational knowledge – and repeatedly reassessed thereafter, as that context develops and evolves. On this basis, one of these appearances may be judged more credible than the other, so that the former is labeled probable (close to real) and the latter relatively improbable (close to illusory). In the limit, they may be characterized as respectively effectively (inductively) real or illusory. Thus, reality and illusion are the extremes (respectively, 100% and 0%) in a broad range of probabilities with many intermediate (including problemacy at the mid-point).

To be still more precise, *pure percepts* (i.e. concrete appearances, phenomena) are never illusory. The value-judgment of 'illusory' properly concerns concepts (i.e. abstract appearances, 'universals') only. When we say of a percept that it was illusory, we just mean that we misinterpreted it. That is, what we initially considered as a pure percept, had in fact *an admixture of concept*, which as it turned out was erroneous. For example, I see certain shapes and colors in the distance and think 'here comes a girl on a bike', but as I get closer I realize that all I saw was a pile of rubbish by the roadside. The pure

percept is the shapes and colors I see; the false interpretation is 'girl on bike', the truer interpretation is 'pile of rubbish'. The initial percept has not changed, but my greater proximity has added perceptual details to it. My first impression was correct, only my initial judgment was wrong. I revise the latter concept, not through some superior means to knowledge, but simply by means of *further perception and conception*.

Strictly speaking, then, perception is never at issue; it is our conceptions that we evaluate. It is in practice, admittedly, often very difficult to isolate a percept from its interpretation, i.e. from conceptual appendages to it. Our perception of things is, indeed, to a great extent 'eidetic'. This fact need not, however, cause us to reject any perception (as many Western philosophers, as well as Buddhists, quickly do), or even all conception. The conceptual 'impurities' in percepts are not necessarily wrong. We know them to have been wrong, when we discover a specific cause for complaint - namely, a logical or experiential contradiction. So long as we find no such specific fault with them, they may be considered right. This just means that we have to apply the rules of adduction⁷ to our immediate interpretations of individual percepts, just as we do to complex theories relative to masses of percepts. These rules are universal: no judgment is exempt from the requirement of careful scrutiny and reevaluation.

Adduction treats all conceptual knowledge as hypothetical, to be tested repeatedly – in competition with all conceivable alternative hypotheses – with reference to all available logic and experience.

4. Common Way vs. Middle Way

Now, judging by Cheng's account and certain quotations of Nagarjuna therein, we could interpret the latter as having been trying to say just what I have said. For instance, Cheng writes⁸: "What Nagarjuna wanted to deny is that empirical phenomena... are absolutely real.... However, [this] does not mean that nothing exists. *It does not nullify anything in the world*" (my italics). I interpret this non-nullification as an acknowledgment of appearance as the minimum basis of knowledge. Nagarjuna may have had difficulties developing an appropriate terminology (distinguishing existence, appearance and reality, as I do above), influenced no doubt by his penchant for paradoxical statements seeming to express and confirm Buddhist mystical doctrine.

But if that is what he meant, then he has not succeeded to arrive at a "middle way" (a denial of the Law of the Excluded Middle), but only at a "common way" (a granted common ground). As far as I am concerned, that is not a meager achievement — the philosophical discovery of phenomenology! But for him that would be trivial, if not counterproductive — for what he seeks is to deny ordinary consciousness and its inhibiting rationales, and to thereby leap into a different, higher consciousness capable of reaching transcendental truth or ultimate reality.

It is interesting to note that the Madhyamika school's effective denial of reality to all appearance was not accepted by a later school of Mahayana philosophy, the

P. 42.

Yogachara (7th-8th cent. CE). Cheng describes the latter's position as follows9: "Every object, both mental and nonmental, may be logically or dialectically proven illusory. But in order to be illusory, there must be a certain thought that suffers from illusion. The very fact of illusion itself proves the existence and reality of a certain consciousness or mind. To say that everything mental and non-mental is unreal is intellectually suicidal. The reality of something should at least be admitted in order to make sense of talking about illusion" (italics mine). That is the tenor of the phenomenological argument I present above, although my final conclusion is clearly not like Yogachara's, that everything is consciousness or mind (a type of Idealism), but leaves open the possibility of judging and classifying appearances as matter or mind with reference to various considerations.

The Madhyamika rejection of 'dualism' goes so far as to imply that "emptiness" is not to be found in nirvana, the antithesis of samsara (according to the earlier Buddhist viewpoint), but in 'neither samsara nor nirvana'. In truth, similar statements may be found in the Pali Canon, i.e. in the much earlier Theravada schools, so that it is not a distinctly Mahayana construct. The difference is one of emphasis, such statements, relatively rare in the earlier period, are the norm and frequently repeated in the later period. An example may be found in the *Dhammapada*, a sutra dating from the 3rd cent. BCE¹⁰, i.e. four or five

⁹ P. 25.

London: Penguin, 1973. This is supposedly the date of composition, though the translator, Juan Mascaro, in his Introduction, states "compiled" at that time, thus seeming to imply an earlier composition. It is not clear in that commentary when the sutra is estimated to have been first written down. And if it was much later, say in the period of crystallization of

hundred years before Nagarjuna. Here, samsara is likened to a stream or this shore of it, and nirvana to the further shore; and we are told to get beyond the two.

When you have crossed the stream of Samsara, you will reach Nirvana... He has reached the other shore, then he attains the supreme vision and all his fetters are broken. He for whom there is neither this nor the further shore, nor both....

Such a formula is legitimate if taken as a warning that *pursuing* nirvana (enlightenment and liberation) is an obstacle to achieving it, just a subtle form of samsara (ignorance and attachment); there is no contradiction in saying that *the thought of* nirvana as a goal of action keeps us in samsara — this is an ordinary causal statement. The formula is also logically acceptable if taken as a reminder that no word or concept — not even 'samsara' or 'nirvana' — can capture or transmit the full meanings intended (i.e. 'not' here should more precisely be stated as 'not quite'). There is also no contradiction in saying that one who has attained nirvana does not need to leave the world of those locked in samsara, but can continue to exist and act in it though distinctively in a way free of attachment.

But it would be a contradiction in terms to speak of 'emptiness' as 'neither samsara nor nirvana', given that nirvana as a concept is originally defined as non-samsara; the truth cannot be a third alternative. At best, one could say that emptiness is a higher level of nirvana (in an enlarged sense), which is not to be confused with the lower level intended by the original term nirvana, nor

Mahayana thought, say in 100 BCE to 100 CE, the latter may have influenced the monks who did the writing down. See ch. 26 (383-5) for the quotation.

of course with samsara. In that case, nirvana (in a generic sense of the term, meaning literally non-samsara) includes both a higher species and a lower one; and the statement 'neither samsara nor lower-nirvana' is then compatible with the statement 'higher nirvana'. There is a big difference between rough, poetic, dramatic language, and literal interpretation thereof.

7. CHAPTER SEVEN

Drawn from *Phenomenology* (2003), Chapter 1 (sections 1 & 2).

1. Phenomenology

Phenomenology may be defined as the study of appearances as such. By an 'appearance' is meant any existent which impinges on consciousness, anything cognized, irrespective of any judgment as to whether it be 'real' or 'illusory.' The evaluation of a particular appearance (an existent within the field of consciousness) as an illusion (existing *only in* consciousness) or a reality (existing *not merely in* consciousness, but also before it, after it, without it or beyond its range) is a complex process, involving inductive and deductive logical principles and activities. Opinion has to earn the status of strict knowledge. To begin with, appearance must be taken neutrally, at face value, as the common ground of reality and illusion (i.e. one of a triad).

An appearance *is* whatever it *seems to be*. At this level of consideration, the verbs 'to seem' and 'to be' are one and the same. It is only at the next level, where an assessment of status is involved, that they have to be separated.

Since appearing is being known, phenomenology can be regarded as a branch of both Ontology (the study of being as such; or more restrictively, of real being) and Epistemology (the study of knowledge as such; or more restrictively, of true knowledge). Phenomenology differs

from ontology in being less presumptive as to the nature or status of the object dealt with, and it is for this reason a study essential to epistemology. The basic insight or premise of phenomenology is that knowledge develops from neutral appearance. The common-sense view of knowledge would seem to be that knowledge develops from data considered *at the outset* as 'sensory,' but as we shall see this view involves logical difficulties. The phenomenological approach is an attempt to overcome these difficulties, and propose a more coherent order of development.

As I have shown in my work *Future Logic*, no item of apparent knowledge, not even a percept, is ever immediately and definitively 'true' all by itself. An item may initially *seem* to be true, or contain some truth; but it is only in relation to all other items, which likewise *seem* to be true, that the judgment as to whether it is *really* or entirely true can be made. Even the various criteria and tests involved in such terminal judgments are themselves to start with merely seemingly true. The science of phenomenology is built on the same basic insight.

volume, we shall understand 'appearance' very broadly as including: (a) objects of perception, i.e. concretes or phenomena in the physical or mental domains; (b) objects of intuition, i.e. one's subjective self, cognitions, volitions and valuations; and/or (c) objects of conception, i.e. simple or complex abstracts of preceding appearances. Abstraction relies on apprehensions of sameness and difference between appearances (including projected received or appearances, and projected negations of appearances). Abstracts are firstly simply summaries of information, and at a later stage more complex hypothetical entities.

Coherence in knowledge (perceptual, intuitive and conceptual) is maintained by apprehensions of compatibility or incompatibility.

With regard to terminology, the reader is advised to keep in mind that in philosophy, and in this particular philosophical treatise, we use words somewhat differently or more specifically than in common parlance. Contrary to the impression given by the term 'phenomenology,' it should be understood as a study not merely of 'phenomena,' but of all appearances, including intuited particulars and abstract data¹¹. The word 'appearance' is often confused with 'illusion,' but here includes 'reality.' It is about equivalent in scope to the term 'object' (content of consciousness) or 'thing' in logic (anything existing or thought of). Note well that here 'experiences' refers not only to the phenomena of physical perception, but includes mental percepts, and even intuited data. In common parlance, the term can be more restrictive (limited to sensory inputs) or even with 'appearances' coextensive (e.g. 'my experiences' includes my abstract thoughts). And so forth - all terms will be made clear in due course. See *Illustrations* at the end of the book.

Phenomenology is a science based primarily on attentive detailed observation of one's own experience and discursive behavior, and only secondarily on

There is no point in coining a new term, even though the term phenomenon is in the present volume used in its primary sense of material or mental concrete particular, in contradistinction to intuited objects or abstracts. But note that in practice the term is often used more loosely with reference to complex appearances like 'a social phenomenon' – which include not only concretes, but also intuitive experiences and even abstracts.

careful logical analysis and ordering of such Thus, practice of observations. meditation prerequisite to development of this philosophical discipline, and our success in the latter depends on our skills in the former. Although philosophical awareness and thinking are ultimately obstacles to meditation (which rises above intellectual pursuits), the former can in the interim still draw significant lessons from the latter. Labeling phenomena as "phenomena", or making distinctions between them, or distinguishing them from intuitive experiences or from abstractions – such acts are all non-meditative; but they may well occur and be remembered in the course of meditation.

2. Knowledge is Based On Appearance

Our primary consideration ought to be just what is apparent to our awareness at each and every moment. Nothing can be granted offhand except this first given. Appearance is immediately granted – because there is nothing else to discuss or refer to, because discourse arises solely in reaction and in relation to it. Thereafter, we may stage by stage show how knowledge in general, including our alleged knowledge of those stages, develops.

The core thesis of phenomenology, thus, is that knowledge is based on appearance. This is in stark contrast to other approaches to epistemology, which propose that knowledge is based on 'external reality' or on 'subjective truth' or some such premature thesis. Moreover, phenomenology regards as essential that the sequence in which knowledge arises and develops out of appearance be clarified. A notion or suggestion may be

appropriate if intelligently placed in the 'order of things,' but very misleading if misplaced.

- Consider. for instance. Realism Naïve Materialism or Objectivism)¹². This philosophy proposes that we have a body with sense-organs, that when these come in contact with external objects sensations are produced, which in turn produce primary ideas (images) in the mind, which are what we experience and build more complex ideas (abstract concepts) from. At first glance, this thesis may appear obvious and worthy of universal belief. But upon reflection, we see that it leads to serious logical problems. If, as it suggests, ideas 'represent' external reality, how do we know that they indeed 'correspond' to it? If, as this theory implies, all we know are ideas (sense-data and their combinations). how can we even get to know that there is an external reality at all, let alone a body with sense organs in which our minds reside? Thus, surprisingly enough, this approach to knowledge is internally inconsistent.
- In reaction to this conundrum, some philosophers have opted for the opposite extreme, a **Mentalism** (or Idealism or Subjectivism)¹³. They have, in fact,

Historically, at least in its modern version in the West, we owe this philosophy to John Locke (English, 1632-1704). The difficulties inherent in it were noticed implicitly by his predecessor René Descartes (French, 1596-1650), and later by the likes of David Hume (Scottish, 1711-76) and Immanuel Kant (German, 1724-1804). Notwithstanding, Naïve Realism has remained a basic belief, and a source of considerable confusion, for many people, including philosophers and scientists.

¹³ For example, the Yogachara school of Buddhist philosophy.

accepted the core tenet of Naïve Realism that what we perceive and build knowledge on are mental substances called ideas, while simply dropping its thesis that these ideas originate in physical sensations in response to stimuli from external objects. The trouble with this thesis is that it involves a stolen concept, since it would be hard put to define mentality after having done away with that of materiality. Moreover, it does not really explain the mass of data at hand – it merely explains it away as illusory happenstance. It does not elucidate why there would appear to be an enormous universe of matter 15 billion years old, composed of innumerable galaxies, stars, atoms, quarks, including on a small planet called Earth apparent human beings, with apparent bodies, with apparent sense organs. Mentalism just ignores all this, or discards it as sheer fantasy; it does not make it comprehensible. It is therefore incomplete.

Having grasped the problem inherent in the former theory, we might be tempted to opt for the latter, however imperfect, were it not for the possibility of another approach, that of **Phenomenology**, which presents neither the flaw of internal inconsistency nor that of incompleteness. Phenomenology brings together the best in both those theories, while weeding out their faulty elements.

• Phenomenology starts like Mentalism with the *given* content of consciousness, but identifies that content neutrally as 'appearance,' instead of taking up the prejudice that it is something mental (idea). For it must be realized that the concept of mind was built in contrast to that of matter; it has no meaning by itself,

and would not have arisen were it not for the concept of matter. Phenomenology therefore posits a concept of appearance, which leaves the question of mind or matter open to begin with, a question to be answered in a larger context.

• Phenomenology ends like Naïve Realism with a belief in matter as well as mind, but it does not get to that thesis in the same manner. The error of Naïve Realism is not essentially its notion of a physical body having sensations that generate ideas, but the fact that it takes this notion for immediately granted, treating it effectively as a mere observation. Phenomenology avoids this error by understanding the notion in question as a hypothetical model, through which we manage to organize appearances into an orderly and consistent whole called knowledge.

Our premise is that the starting point of epistemology is never a blank mind in a social vacuum, but the belief framework of ordinary persons in a given historical and cultural Researchers geographical context. epistemology are themselves such ordinary persons in a given societal climate, with their particular viewpoints, though hopefully outstanding intellectual capacities. Any theory such researchers propose must ultimately convincingly explain the genesis of the ordinary frameworks. Whether the latter are thus wholly justified, or demonstrated to be aberrant to some extent, they can neither be ignored nor entirely rejected without logical absurdity.

It is worth making a comment here, parenthetically, about the cultural context. A man like me, born in the 20th Century and educated in the West, normally takes

the Realist viewpoint for granted, and assumes that everyone else in the world naturally does too. People with an opposite perspective seem at first unnatural (philosophical nitpickers or weirdo mystics), if not nonexistent. But it must be kept in mind that in other regions of the world and in other periods of history, there have been humans who sincerely held very different worldviews (consider animism or shamanism, for instances). One should remain open minded.

8. CHAPTER EIGHT

Drawn from *Phenomenology* (2003), Chapter 2 (sections 2 & 3).

1. Appearance and Other Large Concepts

By 'appearance' is meant, first of all, anything and everything — but upon reflection, more specifically anything which 'comes to mind,' by whatever means. This is not a definition, but an indication. The term appearance is too fundamental to be definable without circularity, we can only 'point to' its instances; indeed, whatever we can point to, in any sense of the term (physically with a finger, mentally by projecting a boundary, verbally by defining or intentionally by focusing on), is an appearance. Thus, 'appearance' refers to any object — of consciousness (but of course, 'consciousness' is itself too basic to be definable — see further on).

The concept of appearance differs from that of 'existence' as of when we assume that things exist before or after we are aware of them, and therefore by extrapolation that things exist that we are never aware of. This assumption that there are things (existents) we are not conscious of, serves to explain or integrate, among others, the appearance that things disappear and reappear (signifying continuity of existence in the interim — granting reliability to memory). It also expresses our belief that other selves beside oneself exist

(as opposed to solipsism), each of which is aware of (and reports) some things one is not aware of, or unaware of some things one is aware of.

Thus, although the two concepts may initially coincide, at some stage we come to regard appearance as a subcategory of existence, implying that whereas all appearances exist, some existents are not apparent. Non-apparent existents are, note well, hypothetical; i.e. 'nonappearance' is a word whose content is by definition unknown but not in principle unknowable. Non-existents do not, of course, exist; which means that the word 'nonexistence' has no ideational content, but is just a verbal construct by negation (an artifice we use as a sort of garbage can for incoherent hypothetical concepts or propositions).

We may here also mention, in passing, the subsidiary concept of *actuality*, or 'present existence,' which arises in the specific context of natural modality, to distinguish between potentiality *with* present existence and that *without* present existence.

The concept of appearance likewise to begin with coincides with that of 'reality.' But as of when we come to the conclusion, as a way to explain certain illogical appearances (like contradictions between experiences or between our beliefs/predictions and experiences) that some things are illusory, i.e. that consciousness errs occasionally, we posit that reality is a mere subcategory of appearance, and therefore of existence. The complementary subcategory of appearance, unreality or 'illusion,' also has the status of existence, note well. There are also appearances that we are at a given time unable to classify as reality or illusion; these are temporarily problematic.

One cannot claim that all appearance is illusion, without thereby contradicting oneself, since such a claim is itself an appearance that is being assumed a reality; it is therefore logically self-evident that some appearances are realities. The deductive relation between these concepts is therefore this: appearance is the common ground of reality and illusion, i.e. implied by both but not implying either. Reality and illusion are mutually contradictory concepts – both cannot be true/applicable, but one of them must ultimately be so. Thus, every object of awareness can be claimed as appearance offhand, without prejudicing the issue as to whether it is real or illusory. However, appearance and reality are also inductively related, as follows: every appearance may be assumed a reality unless (or until, if ever) it is judged (for logical reasons, as mentioned) to be an illusion. Just as the concepts of appearance and reality are initially (at an uncritical, naïve level) the same, so in every instance they remain equal except where illusion is demonstrated (or at least, doubt is instilled).

Note well that the above differentiations between existence, appearance and reality are not immediately obvious, neither in the development of an individual's knowledge nor in the history of human thought. They are not a priori givens, or self-evident deductive certainties or an axiomatic absolute truths, but conclusions of rational (conceptual and logical) process. That is, they express a set of hypotheses which inductively, over time, have been found to satisfactorily integrate and explain a mass of appearances, i.e. to fit-in in a comprehensive and convincing world-view. Thus, mention to differentiations ab initio, as we do here, may be misleading - they are only at this stage vague notions

and assumptions, which are in the long run further defined and found confirmed by the absence of any equally credible hypotheses, any other conceptual constructs which prove as coherent and consistent both internally (as theoretical postulates) and externally (in relation to cumulative appearance, and especially experience). Their being hypotheses does not per se invalidate them, for the claim that all hypothesizing is invalid is itself equally hypothetical and so self-invalidating.

We shall again anticipate, with reference to what we mean by 'consciousness' or 'awareness' or 'cognition.' This may be defined as *the relation* between Subject and Object, whatever activities or states either may undergo within such relation¹⁴. The fundamental given is appearances – but we have no reason to believe that all appearances appear to each other, i.e. we seem to have a privilege among existents in being aware of other existents. We suppose thereby that the fact of 'appearance' is different from mere 'existence,' and occurs *only* relative to a conscious Subject.

The 'Subject' of this relation is identified with the intuited self (me, in my case – you, in yours), but such intuition has at first only the status of an appearance; it is initially a vague and uncertain notion rather than a fully developed and justified concept. The other pole in the putative relation of consciousness, the 'Object,' refers to the appearances involved (which are here given another

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¹⁴ Whereas 'consciousness' refers to the relation, 'cognition' is conceived rather as an 'act,' and 'awareness' as a state – but for our purposes we shall regard them as equivalent terms. The point is that the essence is relational, irrespective of activities or states that may often attend it.

name to stress their being taken into consideration specifically within the said relation).

To posit such a relation does not tell us anything much about it, admittedly – we merely have a word for it, referring to something supposedly too primary in knowledge to be definable. But the trilogy Subject-consciousness-Object is posited by us in a bid to understand and explain how and why appearance differs from existence. The meaning and validity of this hypothesis, including the new ideas of a Subject and consciousness, are not immediate, but established with reference to the cumulative thrust of experience and reasoning, including consideration of conflicting hypotheses. It is only after the latter are found less coherent and consistent than the former that we inductively conclude that our hypothesis is convincing and reliable.

Let me emphasize preemptively that to postulate that appearance signifies existence within awareness is not meant to imply that the existence of appearances is caused by awareness, but only to differentiate putative non-apparent existents from appearances. The relation of consciousness is postulated as per se neutral, affecting neither the Subject nor the Object. Existents remain essentially unchanged by it when they enter the field of and are labeled more specifically 'appearances.' To presume the contents of consciousness 'subjective' (in the pejorative sense of the term), implying a dependence (creation or modification) of the Object by the Subject, is a very different hypothesis; one, indeed, hard to uphold, since if we apply it to itself we put it in doubt. Moreover, if such subjectivist hypothesis were claimed true, there would be no need for it, for

'appearance' and 'existence' would be coextensive. So our hypothesis of consciousness is inherently rather 'objectivist.' Evidently, there is lots of reasoning behind such concepts and postulates; they are not arbitrary assertions (as some philosophers contend). Also, such reflections and clarifications are not and need not be consciously made before at all embarking on the enterprise of knowledge; they flower gradually in response to specific doubts and questions.

2. Material, Mental, Intuitive, Abstract

Now, of all appearances, those labeled 'concrete' are the most manifest, the most evidently present to our consciousness. They are also called 'phenomena,' to stress that we should not immediately take for granted their apparent reality, having over time become aware that some are best judged illusory after due consideration. Concrete objects seem more directly or immediately knowable than others – apart from the issue of reality or illusion just mentioned – so we assign them a special kind of consciousness or cognition called perception and label them 'percepts.'

Among concretes, some are more ostentatious and permanent than others and seem relatively far and independent of us – these we refer to as 'material' or 'physical.' The remainder we label 'mental' or 'imaginary,' distinguishing them by their relative poverty, transience, intimacy and dependence on us. Most of our common 'world' (cumulative appearance) is composed of material phenomena, and all or most mental phenomena seem to be derivative replicas of them or of parts of them. Among material phenomena, some are

considered 'in our own body' or 'physiological,' and the others 'outside our body,' our 'body' being distinguished by its relative proximity (to the observer) and the peculiar events occurring in it (sensations and sentiments). Some bodily phenomena (such as sentiments and 'actions') seem to have mental origins, and so are called 'psychosomatic.' Conversely, many mental phenomena are regarded as having bodily causes.

In addition to mental phenomena, we should distinguish the non-concrete appearances we may call 'intuitive' appearances, which are our impressions of self-knowledge (one's self, cognitions, valuations, volitions). These differ from imaginations, in that they per se have no concrete expressions, yet they share with mental phenomena the appearance of intimacy and being in our power to some degree. They are assigned a specific kind of consciousness called intuition (whence their name here) or apperception.

Concretes (mental or material) and intuited objects have in common a status of *immediate evidence*, which we express by calling them '**empirical**' or 'experiential.' Experiences are 'givens' in a way other appearances (namely abstracts) cannot match. Considered purely in and for themselves, without interpretation or inference, they are unassailable, not requiring any proof.

'Abstract' appearances may be classed as last in that they seem *derived*, by various means, from the preceding, experiential varieties of appearance. These means are collectively labeled 'rational' (implying they proceed from a faculty of reason). The term abstract refers to the primary act of reason, namely abstraction (which depends on identification of sameness or

difference, i.e. on comparison and contrast between two or more appearances).

Abstract appearances share with intuitive ones the lack of concrete manifestation; we have nothing to directly show for them, they are phenomenally blank. But abstracts differ from intuitive appearances, in that getting to know the former requires a process (comparison and contrast), whereas the latter are directly known (in self-experience). Furthermore, abstract objects are 'universals' and essentially 'external to us,' whereas intuitive objects are 'particulars' and very much 'part of us.'

Consciousness of abstracts is called conception, so they are also called 'concepts.' But the processes leading to concepts (our discourse) are far from simple and seem subject to many rules; the latter are labeled 'logic.' Abstracts require proof, and ultimately some sort of empirical grounding. The only exception to this rule is the case of self-evident propositions, which cannot logically be denied without committing a self-contradiction. But even in the latter cases, the concepts involved are never entirely 'a priori,' but require some preceding experience to have at all arisen.

Let me summarize here: perception is knowledge of concretes, i.e. material or mental phenomena; intuition is self-knowledge; perception and intuition are experiences; conception is knowledge of abstracts, derived with the aid of logic from phenomenal or intuitive data. 'Knowledge,' of course, at first simply means consciousness or cognition – the term is rendered more precise later with reference to cumulative Appearance. 'Thought' and 'idea' are, by the way, catchall terms that may include a mix of conception (concept formation, conceptualization), imagination (visualization,

verbalization, forming hypotheses) and logical discourse (inductive and deductive), all of course implying some experience (sensory or intuitive).

As I have indicated earlier, I am not convinced that qualitative differences alone suffice to distinguish material from mental phenomena. We tend to think of the latter as less clear or vivid than the former, but this is not always the case. Dreams are sometimes extremely vivid and colorful, and the physical world is sometimes misty unclear. For this reason, I suggest phenomenology must suppose that introspection is to some extent involved in making this fundamental distinction. We are presumably somehow aware of the direction of input of the concrete data. Material data is 'felt' as coming from or via the body, whereas mental data is 'felt' as coming from a closer source (called the mind). Granting that such 'feelings' of direction of source are not themselves phenomenal marks (otherwise we would be begging the question), we must interpret them more precisely as intuitions. To be consistent we must say that we do not intuit where the data comes from, but rather intuit in what direction we turn our attention to gain access to the data.

It should be noted that we have above effectively distinguished three **substances** or stuffs of existence, matter, mind and spirit. We have based their differentiation partly on the fact that some experiences (those intuited) do not have phenomenal characteristics; and partly (as regards the distinction between material and mental phenomena) on the differences in phenomenal properties and locations combined with assumed intuited differences. All three of these substances may give rise to concepts. We may also

presume souls, i.e. spiritual entities, other than our own through their apparent phenomenal effects and by conceptual means.

Just as the phenomenal modalities and qualities and their behaviors are considered as mere varieties of matter and mind, so the cognitions, volitions and affections of the soul need not be assigned yet another substance, but may be considered as events or properties of that same substance. Abstracts relating to material, imaginary or spiritual givens do not, likewise, require a further substance, but may be considered as mere expressions of three substances. There is nothing these epistemologically unreasonable in assuming substantial differences between the said three classes of object. It remains possible that the three substances are ultimately different versions or degrees of one and the same stuff.

The concept of substance is introduced relative to those of static attributes and dynamic movements, implying a presumed substratum for them. It allows us to presume continuity of something, an individual **entity**, in the midst of motion or change. The various attributes and movements are thus conceived not as mere happenstances but as all 'belonging' *to* and 'caused' *by* an abiding, unifying entity¹⁵. We also assume that different instances of that kind of entity remain essentially the same (i.e. of same substance) although some of their attributes and movements may differ. Note well that both 'substance' and 'entity' are abstracts.

In the case of a human Subject/Agent, causality is usually meant as 'volition' (implying some consciousness and responsibility) not as mere mechanical 'causation,' note well. Similarly, 'possession' of attributes may in some cases be voluntary.

Although material and mental phenomena have concrete character, while soul has not, the latter may nonetheless equally legitimately be conceptually posited.

These beliefs, in substances and entities, are not conceptual immediate certainties but constitute hypotheses. This fact alone does not disqualify them, contrary to what some philosophers suggest. If a hypothesis gives rise to a world-view that is always, all things considered, consistent and confirmed, and no alternatives serve the same purpose as well or better, then it is inductively worthy of adoption. This seems to be the case with regard to the concepts of substance and entity. Without them, we would find ourselves unable to 'make sense' of (integrate, explain) all our experiences and intuitions; no one has to my knowledge managed to construct in detail equally credible and useful counterhypotheses.

9. CHAPTER NINE

Drawn from *Phenomenology* (2003), Chapter4 (section 2).

COMPATIBILITY OR INCOMPATIBILITY

1. Apprehension

Allied to sameness and difference are the concepts of compatibility or incompatibility, which underlie what Aristotle has called the three 'laws of thought' – identity, non-contradiction and exclusion-of-the-middle. How do we apprehend things (percepts, intuitions, concepts and propositions about them) as able to coexist (compatible) or as unable to do so (incompatible) or problematic (not established as either compatible or incompatible)? We must answer this question urgently, if we admit that these logical processes of **confrontation** (or facing-off) are as basic as those of identifying sameness or difference. The whole of logical science is built on their assumption, and we must explain how we know two things to be harmonious or mutually exclusive or of undecided correlation.

An important insight or principle we may suggest at the outset is that **consistency is not something we apprehend** – **it is** *in***consistency we apprehend; consistency is just the absence of inconsistency**. Thus, despite the polarities we have given the words, compatibility is something negative, whereas

incompatibility is something positive. Everything seems harmonious to us, till we discern some conflict. We judge things consistent, so long as we have no logical insight of inconsistency between them. Thus, strictly speaking, inconsistency can be directly 'seen', whereas consistency is normally assumed till found lacking. In some cases, consistency is indirectly put in doubt, without some direct inconsistency having been found, so that an uncertainty arises.

Aristotle formulated his three 'laws' firstly with reference to percepts or concepts by stating them as 'A is A', 'A cannot be non-A' and 'Either A or non-A'. In a later stage, they are formulated with reference to propositions. As I argue extensively in Future Logic¹⁶, these laws are not laws in the sense of a-priori principles or arbitrary axioms, as some have claimed, though they self-evident in that to deny them is selfcontradictory¹⁷, but have to be regarded as given in their objects somehow. Psychologically, they are profound impulses (which may be ignored or followed), which make humans rational; ethically (in the ethics of knowledge gathering), they are indispensable tools and imperatives to actively respond to certain epistemic situations in certain ways (though one can be dishonest or unaware and ignore the facts, or evasive or lazy and ignore the imperative).

¹⁶ See Future Logic, chapters 2 and 20.

¹⁷ See Future Logic, chapter 31.

2. Explications

Identity brings to mind the visual image and sensation of calm or attraction or a tendency to merge of two things (equation), contradiction that of conflict or repulsion or explosive collision between them (because they cannot occupy the same place), while exclusion of the middle refers to a gap or deficiency between them (raising doubts and awakening questions). These may be imaginative representations for philosophical discussion like here, but they are not always (if ever) involved in concrete identification of identity, contradiction or research needs. Their involvement is more technical or abstract, straddling as it were the experiential domain and the conceptual knowledge domain. Although formulated as a triad, the laws of thought are three aspects of essentially one and the same necessity.

The law of identity, simply put, tells us "what you see is what you get" - it is a mere acknowledgment that the data of phenomenal experience are the fundamental givens of any knowledge enterprise; that there is ultimately no other data to base inference on, so that all their details must be paid attention to and taken into consideration in any inference. With respect to its formulation as 'A is A', with reference to terms rather than propositions, this law would simply mean that, if we for instance compare the constituent points in any two material or mental complex phenomena, we have to acknowledge that wherever dots appear (or fail to appear) to us, we can definitively say that there are (or are not, respectively) dots (at least phenomenal dots) – at least for now, until if ever the situation changes or further scrutiny tends to belie the first observation (because

many later observations supplant the first, by their statistical weight).

Identity is a law, because there is no other way to conceive things – at this phenomenal level to 'seem' is to 'be'. You can deny your phenomenon's reality, but not its very occurrence or existence. If you try to deny your actual phenomenon by immediately hypothesizing some conflicting 'phenomenon' behind it noumenon, to use Kant's word), you are condemned to being basically unempirical and therefore without epistemological justification for your own act. You have nothing to show for your case, since by definition you appeal to the unseen, whereas you must acknowledge the seen as seen to at all deny it. The baselessness and circularity of such refusal to accept the phenomenon (as a phenomenon, no more, at least) merely reflects that the phenomenon experienced is the given to deal with in the first place (for this reason any denial of it is bound to admit it, implicitly and explicitly by referring to it). All such argumentation is of course very conceptual, and so only at best lately and peripherally significant in any actual act of acceptance of the phenomenon as such.

Phenomenologically, the law of identity means that an image of a material entity, mentally projected externally onto that entity, does not blank out the entity (being as it were in a parallel space, transparent). When such mental image seemingly shares outer space with the material body it is projected on, then the phenomenon as a whole has changed, though the material entity stays on (perseveres as an appearance), having been *augmented* in respect of a mental image. That is, the new phenomenon is enlarged (by an additional image) in comparison to the originally given phenomenon. This means that

postulation of a noumenon merely adds a mental component (including additional phenomena) to the first presented phenomenon, and does not succeed in erasing the first phenomenon, precisely because it is introduced *in relation to* the first phenomenon (specifically, as an attempt to explain it or explain it away).

The law of identity is an impulse, a call to empiricism, which we normally obey without doubt or question. It acknowledges that appearances might in the long run change or prove misleading, taking into consideration all other appearances. It does not deny, nor accept ab initio, that behind the seen appearance there might be unseen or invisible events or things; but such outcome can only be arrived at through an overall consideration of all experiences and much pondering. That is, 'noumena' might well exist beyond a given field of phenomena but they would have to be end products of an evaluative process and could not be first assumptions. Since evoking noumena does not in itself annul phenomena (merely adding more phenomena to them), the questions inherent in phenomena and their apparition to us remain unanswered.

The reason why the thesis of noumena seems at first sight credible, is that we have experience of different sense-modalities, each implying that the others are *incomplete*, and we have memory of changes in our experience and/or its interpretation *over time*, so that our conceptual knowledge (or its suppositions) has naturally come to conclusions that 'things are not quite or always what they seem'. But in such case, the term noumenon is trivially but another name for abstracts or concepts. In Kant's coinage and use of the term, however, the noumenon is not a hidden extension of the phenomenon,

but purports to discard and replace the phenomenon altogether. The noumenon is by definition unknowable (universally) - though Kantians never tell us how come they themselves have the privilege to even know enough about it to know that it exists and is unknowable! The correct statement would rather be that noumena (i.e. less abstrusely, abstracts, concepts) are not concrete experiences, but merely logically assumed derivatives of percepts. They are hoped to be ontologically 'more real' than percepts, digging deeper into reality than the visible surface of things (to which we are supposedly restricted somewhat by the limited range of sense-modalities open cognition), even as they are epistemologically admitted to be less reliable.

The laws of non-contradiction and of the excluded middle are intertwined with that of identity, as evident in the arguments above. But how do we know that 'A is not non-A' or that it is either-or between them? Consider our basic dot of light or its absence (darkness) in the visual field – such a dot is evidently never in contradiction with itself. We never simultaneously perceive a dot and notperceive it – in any given place we mentally chose to focus on, there either appears or does not appear a lighted (or dark) dot. At this level, where the object is reduced to a single character (light) and precise place (the smallest possible size), we cannot honestly, sincerely answer 'yes and no' or 'neither yes nor no' to the question. It is there or it is not. If it seems there, it is. If it does not seem there, it is not. We cannot even pretend we don't see what we see – at least not in words, for we would have to acknowledge their meanings, and therefore the actual phenomenon.

These laws are indeed *in* the phenomenal world, insofar as positively no phenomena ever appear in contradiction or as neither-nor, i.e. by *absence* of empirical evidence to the contrary. They are in, because their negations are *not* in. But they relate to mind, inasmuch as when a dot A appears and we start speaking of the unseen non-A, *we are in fact imagining non-A in our heads*, and so bring a new (mental) element into the picture. By the law of identity, this non-A phenomenon (which is mental) must be distinguished from its alleged opposite A (the given, which may or may not be mental), and admitted as an *addition* in the experiential field. But it remains true that A and non-A themselves are not in fact coexisting or both absent in the field – rather what we experience is coexistence of the given A with a *projected* non-A.

The law of contradiction does not deny the possibility that two *different* things might coexist, like a dot of light and the imagination (or memory) of absence of such dot of light; such things are merely contrary. The law of the excluded middle does not deny the possibility for something and *the idea of* its absence to be both absent from a field of experience; in such case, we can still suppose, as we indeed *see* as experience, that the thing itself is absent (even though the idea of its absence is allegedly absent – until mentioned as absent, that is!)¹⁸.

Our minds seem so made that, indeed, we might consider that we always *think* non-A when we see A. This is not a mere perversion of the mind, it is rather an expression of the fact that concept-formation involves not only reference to perceived similarities between two objects, but also to perceived *dissimilarities* between other objects and them. Thus, in order to classify something as A, we must simultaneously declassify it from non-A. That is, the *thought* of A automatically calls forth the *thought* of non-A, for purposes

Thus, these laws are empirical, in the sense that they do not impose anything on the phenomenon, but accept it as is. They merely push *the observer* back into the fold of experience, should he venture to stray. They do not involve a modification or manipulation of the phenomenon, but on the contrary make the observer openly and carefully *attentive to* what is phenomenal. They involve a distinction between primary phenomena (be they 'material' or 'mental'), as given *ab initio*, and imaginary alleged representations (ideas, mental phenomena) of eventual phenomena, which merely introduce additional phenomena.

3. Negation

It is very important to emphasize again that **negation is a logical act**. It is never a pure experience, but always involves conceptual interference by the Subject. In formal logic, terms like A and non-A are neutral and formally indistinguishable. That is, they function in interchangeable ways, so that the negation of non-A (non-non-A) is technically equivalent to A (by obversion); and we might label non-A as 'B' and A as

of distinction. It is not that A per se implies non-A (though in most cases, A in one thing implies non-A in others, otherwise neither A nor non-A would be distinguishable in the first place), rather it is that A cannot be fully delimited or understood without bringing to mind non-A as a possible alternative (except perhaps 'non-existence' – though in that ultimate case, we can say that the term is merely verbal, without conceivable concrete referent). Furthermore, concepts formed by negation (like darkness) presuppose some relatively positive phenomena (like light), whose absence they express, having been conceived first.

'non-B' without affecting inferential processes. But at the phenomenological level, these labels are quite distinct. Something appearing would be labeled positively (say, A), whereas something not-appearing would be labeled negatively (as non-A).

What we here labeled A is a phenomenon or percept. What we here labeled non-A is *not* apparent per se, but only effectively 'apparent' in that A did not appear. Non-A signifies that we have asked a question 'is A there (i.e. in the phenomenal field)?' and after further scrutiny answered it by 'no, I do not find it there'. The former (presence) is directly known, the latter (absence) is indirectly known through a mental projection (imagining A, i.e. inventing it or remembering it from previous perceptions) coupled with an experimental search (whose result is unsuccessful). Clearly these are very different cognitions – one being purely passive and empirical, the other involving an active inquiry and referring to observation only by the failure to confirm an anticipated equivalent of one's imagination. The later is useful and informative, but it is a construct.

Negative concepts or statements are thus never strictly-speaking empirical, and negation is a fundamental building block *of reason*. A negation is at the outset, by its very *definition* when introduced by the Subject as a cognitive artifice, logically contradictory to something. It cannot then be said *empirically* that both percepts A and non-A occur (since saying I 'see' non-A in the present field of perception just means I looked for and did not see A in it), nor that neither A nor non-A occur (since if I look and do not see A in the present field of perception, I would conclude non-A for it – though I may remain open-minded about other eventual fields of perception

containing A)¹⁹. A negative concept or statement is therefore fundamentally different from a positive one, and can at best only indirectly ever be characterized as 'empirical'.

4. Primaries

The three laws of thought are logical primaries, involved in all discourse about any phenomenon (and similarly relative to intuitive data, and at a later stage with respect to conceptual discourse itself). They jointly operate in identical ways in every observation, pushing us to admit what we see (identity), not to contradict what we see (non-contradiction), and not to ignore and possibilities to what we see (exclusion of a middle). To fail to apply them is simply to confuse the given data with additional mental ingredients (fantasies), which neurotically either deny the evidence (mentally replacing it with its contradiction) or question it (by mentally proposing a 'middle' term). These laws can be stated as propositions, but they nevertheless have no conceivable alternatives. Any doctrine proposed has to be reconciled with experience somehow, since all discourse is a reaction to experience, an attempt to solve the mystery it

¹⁹ Of course, at a conceptual level, i.e. when dealing with abstracts, we may encounter contradictions (i.e. both A and non-A seeming true) and doubts (i.e. neither A nor non-A seeming true). Here, both the positive and negative concepts are mental constructs, and so there is no guarantee that the issue can immediately be resolved by one look. That is of course where the whole science of logic comes into play; it is needed to deal with just such issues with reference to a plurality of experiences.

presents, so merely ignoring experience does not qualify as reconciliation.

In that sense, it is accurate to say that these laws are laws of thought; they are laws for the mind (the observer). We may say that something is A and not A, or neither A nor not A. But these words have no meaning in experience, no phenomenal referents. They are just words, sounds or drawings that signify nothing, not even an imaginable circumstance. The way we 'imagine' them is to stupidly or deliberately confuse a thing and an image of a thing, and project the idea of non-A (instead of non-A itself) next to A (or next to the idea of A) or some such artifice. In other words, the propositions claiming to deny the laws of thought have only a superficial meaningfulness and credibility, due to in fact having referents (ideas) other than those they pretend to have (things). With regard to the original objects of perception, they are in fact silent

Note well that application or obedience the laws of thought does not involve an imaginative act (a volition); it is on the contrary attempts to ignore or deny them which do, requiring interference of the observer's imagination in the cognitive process (preempting experience). That is, the laws of thought themselves are objective, it is only their denials that are subjective (in the pejorative sense). The laws of thought thus remain empirically, and epistemically, and therefore epistemologically, undeniable. So much with regard to applications of the laws of thought to perceptual evidence.

With regard to concepts (which derive from comparisons and contrasts, or from subsequent imaginations recombining such concepts) and propositions

(imaginations of relations between concepts), they remain always open to doubt, hypothetical, so long as equally credible alternatives are imaginable. Credibility is found in everything experienced or thought, it is merely admittance that such and such has been experienced or thought (thought being a sort of experience, though mental). *Ab initio*, any two concepts or propositions are *compatible*, having both been thought. Incompatibility is a later judgment, which follows realization that the concept or proposition somehow directly or indirectly contradicts experiential evidence or leads to internal inconsistency in knowledge or is inherently self-contradictory.²⁰

If two such ideas or thoughts are found or not found to be in utter conflict, they both retain the minimal credibility of being at least *imaginable*, at least till one or both of them is found incoherent with some experience(s) or for some reason unimaginable. If for some reason they are considered to be in conflict, they separately retain some credibility, though their interaction raises a doubt and it is understood that we have to ultimately eliminate at least one of them, removing its temporary credibility with reference to further experiences or abstract considerations. During the phase of doubt, we may refer to their frequencies of confirmation in experience, and regard one as more credible (or likely or probable) than the other.

We consider concepts or propositions compatible until and unless we find some incompatibility between them. As I already pointed out in *Future Logic*, in opposition to the claims of certain modern logicians, we do not 'prove consistency' but rather 'find inconsistencies'

The job of Logic is, note well, not to *exclude* as much as possible, but to find ways to *include* as much as possible, so that all opinions and points of view (which all have some basis and so represent some kind of experience) are accounted for and explained or explained away. Logic is thus not merely, as some contend, search for *contradictions*, but (this in order to) search for *harmonizations*.

10. CHAPTER TEN

Drawn from *Phenomenology* (2003), Chapters 7 (sect. 3) and 4 (sect. 5).

1. Logical Attitudes

Logic is usually presented for study as a static description and prescription of forms of proposition and arguments, so that we forget that it is essentially an *activity*, a psychic act. Even the three Laws of Thought have to be looked at in this perspective, to be fully understood. To each one of them, there corresponds a certain mental attitude, policy or process...

- a) To the Law of Identity, corresponds the attitude of **acknowledgement of fact**, i.e. of whatever happens to be fact in the given context. Here, the term 'fact' is meant broadly to include the fact of appearance, the fact of reality or illusion, or even the fact of ignorance or uncertainty. Also, the attention to eventual conflicts (contradictions, incompatibilities, paradoxes, tensions) and gaps (questions, mysteries); and by extension, other forms of oppositional relations.
- b) To the Law of Non-contradiction, corresponds the policy of **rejection of contradictions**. Contradictions occur in our knowledge through errors of processing of some kind (e.g. over-generalization, uncontrolled adduction, unsuccessful guessing), which is ultimately due to the gradual presentation of

information to the human observer and to his limited, inductive cognitive means. The Law is an insight that such occurrence, once clearly realized, is to be regarded not as a confirmation that contradiction can occur in reality, but as a signal that a mere illusion is taking place that must be rejected.

c) To the Law of the Excluded Middle, corresponds the process of searching for gaps or conflicts in knowledge and pursuing their resolution. This is the most dynamic cognitive activity, an important engine in the development of knowledge. And when a contradiction or even an uncertainty arises, it is this impulse of the human thinking apparatus that acts to ask and answer the implicit questions, so as to maintain a healthy harmony in one's knowledge.

Thus, the exercise of logic depends very much on the *human will*, to adopt an attitude of factualism and resolve to check for consistency, look for further information and issues, and correct any errors found. The psychological result of such positive practices, coupled with opportunity and creativity, is increasing knowledge and clarity. The contraries of the above are avoidance or evasion of fact, acceptance of contradictions, and stupidity and laziness. The overall result of such illogical practices is ignorance and confusion.

Whereas 'consciousness' refers to the essentially static manifestation of a Subject-Object relation, 'thought' is an activity with an aim (knowledge and decision-making). The responsibility of the thinker for his thought processes exists not only at the fundamental level of the three Laws, but at every level of detail, in every cognitive act. Reasoning is never mechanical. To see what goes on around us, we must turn our heads and

focus our eyes. To form a concept or formulate a proposition or construct an argument or make an experiment or test a hypothesis, we have to make an effort. The more attentive and careful our cognitive efforts, the more successful they are likely to be.

2. Unity In Plurality

The ... 'wave' theory of universals, *granting* its premise that everything is ultimately reducible to 'waves,' i.e. mobile vibrations in some sort of continuum, leads to the very radical conclusion that 'all things are one.'

The world as it appears to our touch-organs or to the naked eye – or even the eye aided by microscope or telescope – may give the impression that dimensionless points, lines or surfaces exist in nature, but as Physics has evolved it has become clearer that *physical objects* do not have precise corners, sides or facades – but fuzzy limits, arbitrarily defined by the visibility to our senses (specifically, sight and touch), aided or unaided, of concentrations of matter or energy.

For example, the tip of my penknife may seem like a sharp "point" to my touch or sight, but it is really – according to physical science (i.e. upon further investigation and reflection) – a rough, voluminous conglomerate of atoms, which are themselves complexes of smaller and smaller particles (electrons, protons and neutrons, seemingly some distance 'apart' from each other, etc.), which are themselves without beginning or end being really vague clusters of waves. Similarly with regard to the cutting edge or flat sides of my penknife.

Indeed, if one takes these considerations to their extreme conclusion, one could say that *no object has a beginning*

or end, every object stretches to the ends of the universe or to infinity, and what we refer to as a specific individual object is merely the most humanly visible or concentrated part of that whole, which we arbitrarily or conventionally consider a separable unit (and habitually name, to solidify our viewpoint). So that ultimately, there are in fact no individual objects, but only ripples in the single object that is the universe as a whole.

Where does an atom (or any other body) begin or end, granting that all consists of waves? If we see a star billions of miles away, on what basis do we say that the star ends over there, while the "light from the star" is here? Rather, we ought to say that the light we see is part of the star, i.e. that it extends all the way to us (at and through our visual sense organs, and on to our memory) and beyond. At what distance from the star do the gases or the light it emits cease to 'belong' to it, and are to be considered as 'separate' bodies? The cut-off point can only be arbitrary, i.e. mere convention. Gravity operates at astronomical distances. What objective ground do we have for distinguishing a field from its apparent origin? Furthermore, stars are in constant flux, arising in time and disappearing in time. At what point in time (as well as space) may we claim that the matter and energy we now call a star is 'not yet' or 'no longer' a star? Surely, the quarks from which the star emerged were already 'the star' and when the star bursts or is absorbed into a black hole it is still 'the star.' We ourselves are stardust – does that mean that the stars in question became us, or that being a star - from the beginning of time to its end includes eventual human forms?

In this view, every entity in the universe stretches out with every other to fill the whole space and time of the

universe! And if we say this, we might as well say without any mystical intent, though in agreement with Buddhist mystics – that all things are one. There are just more intense concentrations of matter or energy here and there, now and then, in one continuous field, but nowhere dividing lines. Because we perceive only fractions of the totality, only the aspects involving the sense-modalities, we isolate small blobs of the whole as individual phenomena. All phenomena perceived are centers of complex wave activities in the universal fabric; We 'individuate' phenomena with reference to the sensemodalities they exhibit which are accessible to our senses. We regard as delimiting an individual object in space and time such perceivable fraction (visible to the senses) of the wave activity stretching to the ends of the universe - ignoring its larger invisible extensions, later induced by reason. Thus, all individuation is fantasy (this can be known by rational considerations, as here), reinforced by naming (itself sense-modality a phenomenon, by the way). In which case, strictly speaking, nothing is divisible at all.

That would seem to be a correct view of our physical world in the context of present knowledge – the hypothesis most consistent with experience, experiment and current scientific theorizing. We thus, provided we anticipate the results of Physics and claim that some sort of unified field theory is sure to be established, and provided we stretch that assumption to include wave explanations of the mental and spiritual domains, arrive at a concept of the world as 'unity in plurality' – a harmonious marriage of the philosophies of Pluralism and Monism. Heraclitus was right – everything is ultimately motion (i.e. waves) and Parmenides was right

too – everything is ultimately one thing (i.e. the medium subject to waves).

We could even view this conclusion as a justification of the Buddhist view that "all things are empty!" For instance, the message of The Diamond Sutra seems to be that all objects material or spiritual are infinite vortices with no beginning and no end. They are neither categorical as they seem; nor can they be surely declared hypothetical, being delimited merely by our naming of them, but having no sure limits in themselves so far as we know so that they are therefore effectively boundless. We have already, inspired by Buddhist doctrine, concurred with them that individuation is a man-made artifice. But even granting that we might legitimately, out of mere convenience, focus on specific places and durations of the universe, because a disturbance 'standsout' there and then in relation to our senses – we are still left with the question as to what it is that is disturbed? What is the medium or substratum of all wave motions? We are tempted to view it as a stuff and call it "existence," or like Descartes call it "the ether." The problem is that since the Michelson-Morley experiment on the velocity of light such a substance underlying waves has apparently been discredited. These physicists measured the velocity of light in the same direction as our planet's motion and in the opposite direction. To everyone's surprise, they found the velocity identical either way. This was eventually explained by Albert Einstein as indicative that there is no absolutely stationary substratum or "ether" relative to which wave motions occur, and he built his famous theory of Relativity as an alternative world-view (such that space

and time coordinates are depend on the velocity of the observer relative to what he measures).

Thus, although when we think of waves, and mathematically work out their motions and interactions, we regard them as disturbances within some medium, it turns out that there is no such medium according to experimental indices! On this basis, we can agree with Buddhist philosophers that (surprisingly, incomprehensibly) *nothing* is being waved – i.e. that the ultimate nature of "existence" is "emptiness." And there is no need of high meditation or mystical insight to arrive at this conclusion – it is seemingly justified by ordinary experience and reason (scientific experiment and theory).

11. CHAPTER ELEVEN

Drawn from *Ruminations* (2005), Chapter 1 (sections 1-3).

1. Dialectical Reasoning

The three "Laws of Thought" may be briefly explicated as follows:

- 1. *Thesis*: there are certain appearances; appearances appear.
- 2. *Antithesis*: there are incompatibilities between certain of these appearances; in such cases, one or both of them must be false.
- 3. *Synthesis*: some remaining appearances must be true; find out which!

We can in this perspective consider *dialectic* as a fundamental form of thought, through which knowledge is made to progress on and on. It is not a mere detail, an occasional thought-process, but a driving force, an engine, of thought.

The laws are not mere information, but calls to cognitive action. They enjoin proactive and curative cognitive measures, to ensure (as much as possible at any given time) continued verification, consistency and completeness.

(i) The law of identity tells us to seek out the facts and sort them out as well as we can. The purpose of this law

is to instill in people a healthy respect for facts, in the course of observation and judgment. It is essentially a call to honesty, and submission to the verdict of truth. People often think, or act as if they think, that ignoring or denying unpleasant facts or arguments will make them 'go away' – the law of identity says 'no, they will not disappear, you must take them into consideration'.

Some people think that it is impossible for us to ignore that "A is A". Far from it! All of us often do so – as when we refuse to look at or admit the evidence or a logical demonstration; when we avoid reality or evade it having glimpsed it; when we lie to ourselves or to others; and so forth. If the law of identity were always obeyed by us, there would be no need to formulate it. Logic states the obvious, because it is often shunned.

(ii) When the law of non-contradiction says to us "you cannot at once both affirm and deny a proposition", it is also telling us that if we ever in the course of discourse encounter a situation where a proposition seems both true (for some reason) and false (for other reasons), we have to go back upstream in our discourse and find out where we went wrong in the course of it²¹, and we have to effect an appropriate correction such as to eliminate the difficulty.

We are not just saying: "ah, there is a contradiction", and leaving it at that, nonplussed. No, we are impelled to seek a solution to the problem, i.e. to resolve the contradiction. We are inferring that there must be something wrong in our earlier thinking that led us to this conundrum, some error of observation or reasoning that requires treatment. So long as this situation is tolerated, and we cannot pinpoint the source of error, the credibility

²¹ "Check your premises", Ayn Rand would say.

of all related knowledge is proportionately diminished. Consistency must be restored as soon as possible, or we risk putting all subsequent knowledge in doubt.

(iii) Similarly, the law of the excluded middle does not just inform us that "no proposition can be claimed neither true nor false". This law insists that if we find ourselves in such a situation, and it is indeed the case that both a proposition and its exact negation both seem false, we cannot let the matter rest or hope to find some compromise position – we have to eventually, as soon as possible, find good reason to opt for one side or the other. There is no logically acceptable middle ground, no avenue of escape.

These action implications inherent in the laws of thought may also be characterized as dialectical thinking. In this perspective, the "thesis" is our knowledge (or opinion) as it happens to be at a given time; the "antithesis" is the discovery of a logical flaw in that thesis, which causes us to have doubts about it and seek its review; and finally, the "synthesis" is the corrections we make in our premises, so as to resolve the difficulty encountered and obtain a less problematic new state of knowledge.

2. Genesis of Axioms

Axioms are not arbitrary, a-priori starting points of true human knowledge. They may be deductive or inductive, but in either case are to some extent *empirical* (in the large sense of 'phenomenological', i.e. without depending on any materialist or mentalist assumption concerning what is experienced).

Deductive axioms are established using certain positive or negative logical *arguments*, which we naturally find

convincing. But even a deductive axiom relies on certain experiences, those *that gave rise to* the concepts and logical techniques involved in the proposition and its acknowledgment as an axiom.

The positive argument for an axiom is essentially dilemmatic: "whether this or that, so and so is true". An example is the axiom that diversity exists. The mere *seeming* of diversity is itself a case of diversity, sufficient to establish the fact of diversity. It is no use arguing (like Parmenides or the Buddha) that this apparent diversity is an "illusion", and that "all is really one" – because the coexistence of illusion and reality is itself an event of diversity. Thus, diversity truly exists, and cannot just be ignored. We might still try to uphold the thesis that reality is ultimately unitary, but only if we convincingly account for the fact of diversity.

Deductive axioms are also justified negatively through paradoxical logic, i.e. by showing that their contradictories are *self-contradictory*. For example, "There is no diversity" is a claim to diversity (since it involves many words, many letters, many sounds, etc.), and therefore self-contradictory; whence, it is *self-evident* that "There is some diversity". This argument may also be construed (as above) as dilemmatic in form: "whether you deny or affirm diversity, you affirm it".

Inductive axioms rely on some generalization, or (more broadly) adduction, from experience; but such inductive process in their case is not ever likely to be in need of revision. Many truths of utility to epistemology are inductive, and yet once realized remain immutable; they

thus behave largely like deductive axioms, and may by analogy be classed as inductive axioms.²²

For example, the fact that most of our beliefs are contextual is a non-contextual truth, though based on common observation. The awareness that most of our knowledge is empirical, and subject to revision as new experiences are encountered, that it is in constant flux, altering and growing - this is a broad observation that once realized will not be affected by any further empirical data. This observation is not useless, note well: it logically affects pursuit of knowledge, teaching us to remain aware of the non-finality of most of our beliefs. But note also, the said principle of contextuality is pretty vague; it cannot by itself put specific knowledge in doubt (i.e. without some other more specific reason for doubt). Another example of such general but unspecific truth is the principle (derived from the law of the excluded middle) that "there is always some explanation". This optimistic principle serves to encourage research, but does not tell us what the solution of the problem is specifically.

3. Paradoxical Propositions

A (single) paradoxical proposition has the form "if P, then notP" or "if notP, then P", where P is any form of proposition. It is important to understand that such propositions are logically quite legitimate within discourse: a (single) paradox is not a contradiction. On

Indeed, it could be argued that, since 'deductive' axioms all have some empirical basis (as already explicated), they are ultimately just a special case of 'inductive' axiom.

the other hand, a double paradox, i.e. a claim that both "if P, then notP" *and* "if notP, then P" are true in a given case of P, is indeed a contradiction.

The law of non-contradiction states that the conjunction "P and notP" is logically impossible; i.e. contradictory propositions cannot both be true. Likewise, the law of the excluded middle states that "notP and not-notP" is logically unacceptable. The reason for these laws is that such situations of antinomy put us in a cognitive quandary — we are left with no way out of the logical difficulty, no solution to the inherent problem.

On the other hand, single paradox poses no such threat to rational thought. It leaves us with a logical way out – namely, denial of the antecedent (as self-contradictory) and affirmation of the consequent (as self-evident). The proposition "if P, then notP" logically implies "notP", and the proposition "if notP, then P" logically implies "P". Thus, barring double paradox, a proposition that implies its own negation is necessarily false, and a proposition that is implied by its own negation is necessarily true.

It follows, by the way, that the conjunction of these two hypothetical propositions, i.e. double paradox, is a breach of the law of non-contradiction, since it results in the compound conclusion that "P and notP are both true". Double paradox also breaches the law of the excluded middle, since it equally implies "P and notP are both false".

These various inferences may be proved and elucidated in a variety of ways:

Since a hypothetical proposition like "if x, then y" means "x and not y is impossible" – it follows that "if P, then notP" means "P and not notP are impossible"

(i.e. P is impossible), and "if notP, then P" means "notP and not P are impossible" (i.e. notP is impossible). Note this explanation well.

We know that the negation of P is the same as notP, and the negation of notP equals P, thanks to the laws of non-contradiction and of the excluded middle. Also, by the law of identity, repeating the name of an object does not double up the object: it remains one and the same; therefore, the conjunction "P and P" is equivalent to "P" and the conjunction "notP and notP" is equivalent to "notP".

Notice that the meaning of "if P, then notP" is "(P and not notP) is *impossible*". Thus, although this implies "notP is true", it does *not* follow that "if notP is true, P implies notP". Similarly, *mutatis mutandis*, for "if notP, then P". We are here concerned with strict implication (logical necessity), not with so-called material implication.

The reason why this strict position is necessary is that in practice, truth and falsehood are contextual — most of what we believe true today might tomorrow turn out to be false, and vice-versa. On the other hand, logical necessity or impossibility refer to a much stronger relation, which in principle once established should not vary with changes in knowledge context: it applies to *all* conceivable contexts.

• Since a hypothetical proposition like "if x, then y" can be recast as "if x, then (x and y)" - it follows that "if P, then notP" equals "if P, then (P and notP)", and "if notP, then P" equals "if notP, then (notP and P)". In this perspective, a self-contradictory proposition implies a contradiction; since contradiction is

logically impermissible, it follows that such a proposition must be false and its contradictory must be true. This can be expressed by way of apodosis, in which the laws of thought provide the categorical minor premise, making it possible for us to exceptionally draw a categorical conclusion from a hypothetical premise.

If P, then (P and notP) but: not(P and notP) therefore, not P

If notP, then (notP and P) but: not(notP and P) therefore, not notP

• We can also treat these inferences by way of dilemma, combining the given "if P, then notP" with "if notP, then notP" (the latter from the law of identity); or likewise, "if notP, then P" with "if P, then P". This gives us, constructively:

If P then notP – and if notP then notP but: either P or notP therefore, notP

If notP then P – and if P then P but: either notP or P therefore, P

Paradox sometimes has remote outcomes. For instance, suppose Q implies P, and P implies notP (which as we saw can be rewritten as P implies both P and notP).

Combining these propositions in a syllogism we obtain the conclusion "if Q, then P and notP". The latter is also a paradoxical proposition, whose conclusion is "notQ", even though the contradiction in the consequent does not directly concern the antecedent. Similarly, non-exclusion of the middle may appear in the form "if Q, then neither P nor notP". Such propositions are also encountered in practice.

It is interesting that these forms, "Q implies (P and notP), therefore Q is false" and "Q implies (not P and not notP), therefore Q is false", are the arguments implicit in our application of the corresponding laws of thought. When we come across an antinomy in knowledge, we dialectically seek to rid ourselves of it **by finding and repairing some** *earlier* **error(s)** of observation or reasoning. Thus, paradoxical argument is not only a derivative of the laws of thought, but more broadly the very way in which we regularly apply them in practice. That is, the dialectical process we use following discovery of a contradiction or an excluded middle (or for that matter a breach of the law of identity) means that

we believe that: Every apparent occurrence of antinomy is in reality an illusion.

It is an illusion *due to paradox*, i.e. it means that *some of the premise(s)* that led to this apparently contradictory or middle-excluding conclusion are in error and in need of correction. The antinomy is never categorical, but hypothetical; it is a sign of and dependent on some wrong previous supposition or assumption. The apparent antinomy serves knowledge by revealing some flaw in its totality, and encouraging us to review our past thinking.

Contradiction and paradox are closely related, but not the same thing. Paradox (i.e. single not double paradox) is not equivalent to antinomy. We may look upon them as cognitive difficulties of different degrees. In this perspective, whereas categorical antinomy would be a dead-end, blocking any further thought—paradox is a milder (more hypothetical) degree of contradiction, one open to resolution.

We see from all the preceding (and from other observations below) the crucial role that paradox plays in logic. The logic of paradoxical propositions does not merely concern some far out special cases like the liar paradox. It is an essential tool in the enterprise of knowledge, helping us to establish the fundaments of thought and generally keeping our thinking free of logical impurities.

Understanding of the paradoxical forms is not a discovery of modern logic²³, although relatively recent (dating perhaps from 14th Cent. CE Scholastic logic).

For instance, Charles Pierce (USA, 1839-1914) noticed that some propositions imply all others. I do not know if he realized this is a property of self-contradictory or logically impossible propositions; and that self-evident or necessary propositions have the opposite property of being implied by all others. I suspect he was thinking in terms of material rather than strict implication.

12. CHAPTER TWELVE

Drawn from *Ruminations* (2005), Chapter 1 (sections 4-6).

1. Contradiction

Many people misunderstand what we logicians mean by 'contradiction'. The contradictory of a term 'A' is its negation, 'not A', which refers to anything and everything in the universe other than A, i.e. wherever precisely A is absent in the world. The relation of contradiction between A and not-A is mutual, reversible, perfectly symmetrical.

The presence of something (A) excludes its absence (i.e. not A) in that very same thing, and vice versa, if all coordinates of space and time are identical. However, this does not exclude the logical possibility that the same thing may be partly A and partly not A. Thus, the law of thought 'either A or not A' can also be stated more quantitatively as "either 'all A' or 'all not A' or 'part A and part not A".

Some people appeal to this possibility of three alternatives as an argument *against* the laws of thought! But that is a misunderstanding – or worse, deliberate sophistry.

If something, e.g. 'B', implies but is not implied by not-A, it (i.e. B) is as 'incompatible' with A as not-A is, but it is not contradictory to A: it is merely *contrary* to A. The contradictory not-A of A differs from A's contraries

in that *the absence* of not-A implies A, whereas in the case of mere contraries like B (or B1 or B2... etc.) this added logical relation of 'exhaustiveness' does not apply. When contradictories are placed in a disjunction, 'either A or not-A', the disjunction involved signifies both mutual exclusion ('or', meaning 'not together') and exhaustiveness ('either', meaning 'and there is no other alternative'). It intends: if 'A', then not 'not-A'; and if not 'A', then 'not-A'.

On the other hand, any number of contraries can be placed in a disjunction: 'A or B or B1 or B2... etc.', so that the presence of any disjunct implies the absence of all the others; but such disjunction is not exhaustive, unless we specify that the list of contraries in it is complete. If that list *is* indeed complete, then the negation of all but one of the disjuncts implies the affirmation of the remaining one. Thus, 'not-A' can be equated to the exhaustive disjunction of all things in the world 'contrary to A'.

Something different from A, e.g. 'C', is not necessarily contradictory or even contrary to A. The mere fact of difference does not imply incompatibility. Different things (like A and C) may be compatible, i.e. capable of coexistence in the same thing, at the same time and place. 'Difference' simply signifies that we are able to distinguish between the things concerned: i.e. they are not one and the same when they appear before our consciousness. 'Similar' things may be the same in appearance, but not one (e.g. two instances of the same kind); or they may be one (i.e. parts of a single whole), yet not the same.

Thus, for example, the logical relation between the colors black and white depends on how precisely we focus on them. They are different, since distinguishable. Since they may coexist on different parts of the same surface, they are broadly compatible. However, as such or *per se*, they are contrary; that is to say: if I perceive a surface or part of surface as totally white, and you perceive *the very same* place and time as totally black, our claims are incompatible²⁴. This irreconcilability is not a contradiction, however, because it is possible for a surface to be neither black nor white.

2. Varieties of Contradiction

The expression 'contradiction in terms' refers to a compound term composed of incompatible elements, such as 'A and not A' or 'A and B (where B is contrary to A)'. Such a mixed-up term may be said to be paradoxical, as well as internally inconsistent, since it implies that contradiction is possible, so that the laws of thought are denied by it, and then (by generalization, if you like) 'anything goes' including denial of the 'A and not A' conjunction.

For example, the term "illusory reality" is a contradiction in terms. On the other hand, note, terms like 'an inhuman human' or 'an anti-Semitic Jew' are not strictly speaking contradictions in terms; they refer to natural possibilities of conjunction, only the terminology used makes them superficially seem contradictory

Our disagreement is not terminological, note. We have in the past agreed as to what experiences 'black' and 'white' correspond to; here, we suddenly diverge.

(i.e. there are people who behave inhumanly, or Jews that hate their own people).

The proposition 'A is not A' (or 'some thing that is A is also not A'), being self-contradictory, implies 'A is A', its contradictory form. This statement should be explicitly acknowledged, though obvious, because it correlates two important concepts, viz. 'internal inconsistency' and 'the logic of paradoxes'.

The statement 'A is not A' is logically impossible, because it both affirms and denies the same thing. Therefore, the opposite statement is true. That statement, i.e. 'A is A', is logically necessary, because *even its* contradictory 'A is not A' implies it.

Whoever claims 'A is not A' is admitting 'A is A' - ipse *dixit*, he himself said it! Whereas, whoever claims 'A is A' is consistent with himself.

Self-contradiction consists of three items:

- 1. The proposition in question, call it P.
- 2. The admission that it is *an assertoric statement*, i.e. one that affirms or denies something.
- 3. The admission that all assertoric statements *involve claims* to consciousness, to knowledge, to truth, etc.

Thus, given P (e.g. "reality is unknowable"), admit that P implies "this is an assertion" – but all assertions imply some knowledge of reality – *therefore*, P implies non-P. There is a process from P to its negation, which Logic demands we acknowledge. That demand cannot be refused without committing the very same self-contradiction. This is not a circular or ad infinitum proof, but an appeal to honesty, without which no dialogue is possible.

That all assertoric propositions assert is an aspect of the Law of Identity. The Law of Non-contradiction may be discerned in the argument: All assertions assert something; P is an assertion; therefore, P asserts; whence, if P denies asserting, P implies non-P. The Law of the Excluded Middle is also implicit here, in the awareness that we have no choice but to firmly disown P.

3. Double Standards

Contradictions appear in discourse in many guises. They are not always overt, but may be hidden in the fact of making a statement or in the standards of judgment used.

A claim may be paradoxical because it **inherently entails** its own contradiction, although it does not on the surface seem to be self-inconsistent. Such implication is not always formal but requires awareness of the meaning of the terms used. This form of indirect self-contradiction has been called "the Stolen Concept fallacy"²⁵.

For instance, the skeptical claim "I know nothing" may be rejected as self-contradictory, because as soon as someone makes it — someone who understands and intends the meaning of the terms "I", "know" and "nothing" — that is by itself proof absolute that the person concerned "knows" something, whence the original claim (of total ignorance) is shown up to be unavoidably contradictory and thus necessarily false.

Thus, in cases of this sort, the tacit implication involved is that one of the terms used (knowing nothing) implicitly includes the act in question (knowing that I know nothing), as a case in point contradictory to the explicit

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By Ayn Rand and (I think) Nathaniel Branden.

claim. (Rephrasing the said statement as "I do *not* know anything" does not change its underlying assumptions, needless to say.)

There are countless examples of such inherent self-contradiction. Saying "I have nothing to say" is saying something. Claiming "We have no memory" is self-contradictory, because each term in it presupposes a word, concept and background experiences remembered by the speaker – and the hearer too. An amusing common example is "I do not speak a word of English"!

Another important form of covert self-inconsistency is the use of **a double standard**. This consists in applying less stringent standards of judgment to one's own discourse than to the discourse of one's intellectual opponents. A lot of philosophical, and particularly political and religious, discourse resorts to such inequitable methodology.

The contradiction involved in a double standard is apparent the moment we step back and view its user's knowledge and methodology as a whole. In this wider perspective, the user of a double standard is clearly inconsistent with himself, even if his discourse viewed piecemeal may superficially seem self-consistent.

Whole philosophies may be based on such fallacious reasoning. For instance, Phenomenalism sets as a general standard a limitation of knowledge to sensory data without allowing extrapolations from them to assumed external material objects – yet it does not criticize its own adductions using the same rigid standard.

There are two ways this fallacy may be committed: one may use relaxed standards on one's own discourse, while seemingly applying universal norms to one's opponents' discourse; or one may appear to apply universal norms to oneself, while concocting overly strict norms for them. One may *exempt oneself* from the usual logical rules, or one may make unusual logical *demands on others*.

In either case, the holder of a double standard is in conflict with logic's requirement of uniformity. An assumption of reason is that all humans are epistemologically on the same plane. Equity is an aspect of 'common sense'. Experience and logic have to be used to convince oneself and others, not sophistical manipulation or authority.

Standards of judgment have to be fair and universal; all discourse must be equally treated. If differences are advocated, they have to be convincingly justified. The principle of equality admittedly involves generalization; but the onus of proof is on any proposed particularization of it.

An example of a double standard is the appeal to cultural relativism. One may seek to rationalize ideas or thought processes that are contrary to ordinary reason, by claiming them to belong to a different cultural framework. Such tolerance seems on the surface friendly and open-minded, but it is proposed without full consideration of its negative human and epistemological implications.

13. CHAPTER THIRTEEN

Drawn from *Ruminations* (2005), Chapter 1 (sections 7-10).

1. Special Status of the Laws

The three Laws of Thought must not be construed as some prejudice of Aristotle's, which some scientific discovery – like the particle-wave duality or the relativity of space-time measurements – could conceivably raise doubt about or displace. These laws of thought are intended as perfectly neutral; they make no direct, specific ontological or epistemological claim, but rationally sort out the very act and concept of such claims – whence their name.

These laws express the ways we assimilate complex experiences, and resolve difficulties in the course of thought (concepts, propositions and arguments). Only by such logic can we 'make sense' of the world around us and in us. By making these truths explicit, Aristotle made it possible for humans to henceforth consciously practice the logic they were already unconsciously tending to.

These laws *exclude*, *ab initio*, the notion that something could both have and lack some property, or neither have nor lack it - *at the same place and time and in the same respects*. The latter specification, which Aristotle clearly and repeatedly stressed, is often ignored by those who consider these laws expendable.

That, say, a stone is blue on one side and red on the other, is not a contradiction, since the different colors are in different parts of it. That over time the colors may change is not an antinomy either: the concept of time is intended to ensure that. That you and I view the same object from different angles, and see different aspects of it, is no surprise. That my view of the world and yours are not quite identical, is quite understandable in view of the different context of experience and thought we each have.

The laws of thought do not evade or deny the *appearance* of contradictions or unsolved problems; they just tell us that such appearances are *illusions*, *not realities*. They are designed precisely to help us take such apparent discrepancies into consideration and resolve them in some way. We continue to need the same laws of thought in the more complex cases uncovered by modern physics.

The theory of relativity is precisely an attempt to rationalize the surprising empirical constancy in the velocity of light, whichever direction we measure it from. The theory is not a statement that there are no absolute truths, but a statement that such and such a way of looking at the surprising events discovered makes them rationally comprehensible. The theory affirms that this way is probably (i.e. inductively) the best explanation, and effectively denies those who contradict it (unless they come up with an inductively better explanation, more in line with the empirical findings). It does not deny the laws of thought, but is an application of them.

Similarly, the discovery that the same things may behave occasionally as particles and occasionally as waves does not constitute an argument against the laws of thought. Whether we interpret this duality epistemologically or ontologically, as due to different circumstances of observation or different material circumstances, it is affirmed to be a mysterious finding that must be faced. This realist attitude is precisely what the laws of thought demand. Any attempt to interpret the finding, one way or the other, is again an attempt to make the finding rationally comprehensible, so that we do not feel them logically impossible.

Under no circumstances may scientists or philosophers seriously claim the laws of thought to be abrogated. Such a claim is self-contradictory – because then its opposite is equally acceptable. It is therefore as if nothing has been said. It is the denial of reason, the institution of madness. The three laws of thought thus together constitute the most incontrovertible and universal frame of reference of rational thought.

Note also, the emphasis the laws of thought lay on *existence*. A common error of deniers of these laws is to regard 'non-existence' as just some other sort of existence, a parallel world or a location beyond space and time *from* which new existents come and *to* which finished existents go! These people are misled by linguistic habit into a *reification* of the word 'non-existence'.

Whatever positively appears, exists to that extent. Existence becomes open to doubt to the extent that we add assumptions to appearance – i.e. we adductively guess what might lie beyond them. At this stage, the reality vs. illusion dichotomy arises. At this stage, too, the rational act of *negation* comes into play – when we

say: this is apparent, but (since it gives rise to some antinomy) it is *not* real, it is illusory.

The 'concept' of *non*-existence thus has no direct empirical basis of its own. It is based on a rational act relative to experiences of existence. It is just a figment of the imagination, a mental dumping place for *ideas* that have failed the test of existential basis.

2. Motors of Rational Thought

It is important to realize that the laws of thought are *the motors of rational thought*. They generate questions and the pursuit of answers; they feed curiosity and fuel research. If we are satisfied with the way things seem, however contradictory or incomplete they seem, thought is arrested. We lose perspective and become ignorant. We lose intelligence and become stupid. We lose touch with reality and become insane.

Consider the *irrelevancy* to science of a hypothetical denial of the laws of thought. For instance, according to Einstein's theory of relativity, nothing can travel faster than light, yet it has been found that particles may affect each other instantaneously even though they are far apart. If in the face of such an apparent contradiction we just said: "oh, well, I guess the law of contradiction must be wrong!" and left it at that — would we be consoled? Clearly, not — this would not honestly solve the problem for us, but merely sweep it under the carpet. Our minds would not rest till some deeper, more convincing explanation was found.

Accepting contradiction is just simplistic and evasive. Similarly, with breaches of the law of the excluded middle: if you ask me a question, and inquire is X the

answer or not X? and I reply, it is neither, but some third thing: will you be satisfied with such reply? Your knowledge of the issue at hand is not made complete by such reply; a gap remains, which can only be filled by either X or nonX. The law of the excluded middle is just a recognition of the *inadequacy* of such neither-nor replies.

3. Cogito, Ergo Sum

Descartes' "cogito, ergo sum"26 is composed of two selfevident propositions: "I think" (in the sense, I am conscious) and "I am" (I exist). For the contradictory of each of these propositions is self-contradictory, i.e. involves a stolen concept and gives rise to a paradox. Thus, "I am not conscious" could not be thought or said (or for that matter heard or understood) without being conscious. Similarly, "I am not" could not be expressed (or observed) without existing. Thus, Descartes was quite right in regarding these propositions as axioms; i.e. as first principles, which do not depend on prior principles. Note moreover that these two clauses are axiomatically true independently of each other - So what about the ergo, which suggests that the sum follows from the cogito? Is the "therefore" perhaps meant to imply an order of knowledge, rather than an inference? One could formally deduce existence from consciousness, in the sense that a conscious being is a fortiori an existent

See Hamlyn, p. 137. The comments made here are not intended as an exhaustive analysis of the *cogito* statement, needless to say.

being; but one would never in practice resort to such inference.

In practice, in my opinion, we are conscious of other things before we become conscious that we are conscious of them – so it would not be correct to place the "I think" before the "I am". It could be argued that a baby may first experience inner states, but I would reply that such states are results of prior sensations. We may however support Descartes' order, by considering it a logical one, in the sense that if the Subject did not have the power of consciousness, he or she would not be aware of existence. That is, it perhaps means: "I can think, therefore I can know that I am".

But I think the correct interpretation is the following: when we are aware of something, any thing, this provides an occasion to become aware of oneself, i.e. that there is a Subject who is being conscious of that thing, whatever it is. Thus, the first clause of the sentence is not strictly: "I think", but: "consciousness of things is taking place" (or "thought is occurring"). Whence the second clause is truly inductively inferred, i.e. we may well hypothesize that "there is something being conscious of things", i.e. "thought has a Subject as well as an Object", i.e. "there is an I" (or "I exist").

It is *the self* that is inferred from the appearance of objects — reason argues: they must appear before someone. This is what distinguishes appearance from mere existence: it occurs *through* 'cognition' *by* 'someone'. Thus, Descartes is justifying our habitual assumption of a cognizing Subject from the fact of cognition. It is not mere grammatical convention, he tells us, but "think" *implies* "I".

4. Concerning Identity²⁷.

Where does a material object begin or end, in view of the constant flow of particles and energy in and out of it, even (over a long enough time) in the case of apparent solids? We have to use the *apparent limits* of things as their space-time definition. Or more precisely, in acknowledgment of the above difficulties, their *illusory limits*. Thus, knowledge of matter is built on arbitrary, knowingly inaccurate, delimitations of "things".

We can similarly argue concerning mental objects (i.e. images, sounds, etc.). At first thought, their limits seem obvious; but upon reflection, they become doubtful – imprecise and insecure. And this being the case, we cannot convincingly argue that the limits of material bodies are mental projections. If the limits of mental lines are unsure, then the limits of whatever they are intended to delimit are still unsure.

Ultimately, then, since we cannot even mentally delimit mental or material things, all delimitations are merely verbal artifices, i.e. *claims we cannot substantiate*. This remark concerns not only 'borderline' cases, but all material or mental objects.

These are very radical queries, productive of grave skepticism. They are principles of vagueness and doubt much more unsettling than the Uncertainty Principle, since they more basically question the validity of any geometry (and therefore, more broadly, of mathematics and physics).

When some Greek or Indian philosophers expressed skepticism at the possibility of human knowledge, this is

I have already discussed this ontological issue in chapter 10, section 2 (above).

perhaps what they were referring to. If one cannot delimit things, how can one produce precise concepts and propositions? And without precision, how can we judge them true or false?

Whereas denial of knowledge as such is self-contradictory, denial of *accurate* knowledge is not so. It is possible to observe the general vagueness of experience without denying the law of identity. If cloudiness is the identity of things, or we are simply incapable of sufficiently focusing our senses to get past such cloudiness, we simply remain stuck at that level of experience, like it or not.

The best counterargument I can muster is that phenomenological knowledge is still knowledge of sorts, and this can be used as a springboard to arrive at deeper knowledge, by means of *adduction*. That is, we can still formulate ontological hypotheses, capable of ongoing confirmation or rejection with reference to reason and experience, even if the epistemological status of the latter is at the outset merely phenomenological.

This does not directly overcome the difficulty of measurement, but it gives us some hope that we might succeed indirectly. I leave the issue open, and move on.

14. CHAPTER FOURTEEN

Drawn from *Ruminations* (2005), Chapter 2 (section 17).

1. Appearance, Reality and Illusion

Phenomenology results from a realization that the building blocks of knowledge are appearances. This realization is obtained through a dialectic, comprising thesis, antithesis and synthesis, as follows.

- (a) At first, one naturally regards everything one comes across in experience or thought as 'real' (this is the 'naïve realist' stance).
- (b) Then, faced with evident contradictions and gaps in one's knowledge, one logically realizes that some things that seemed real at first must or at least may eventually be considered unreal i.e. 'illusory' (this constitutes a cognitive crisis).
- (c) Finally, one realizes that, whether something is real or illusory (and ultimately remains so or turns out to be the opposite), at least it can immediately (unconditionally and absolutely) be acknowledged as 'apparent' (this is the 'phenomenological' stance, which resolves the crisis).

Knowledge of reality can then be inductively built up from knowledge of appearances, thanks to the following principle (d): One may credibly assume something that appears to be real is indeed real, until and unless it is proved illusory or at least put in doubt for some specific **reason.** This may be characterized 'subtle realism', and proceeds from the realization that the mere fact of appearance is the source of all credibility.

Thus, phenomenology follows the natural flow of knowledge, which is to initially accept individual appearances as real, while remaining ready to reclassify them as illusory if they give rise to specific logical problems that can only be solved in that specific way. The concept of 'appearance' is therefore not strictly primary, but a transitional term for use in problematic cases. Since it refers to the common ground between 'reality' and 'illusion', it is deductively primary. But since the latter are in practice attained before it, it is inductively secondary.

The concepts appearance, reality and illusion are to begin with concerned with experiences; and only thereafter, by analogy, they are applied to abstractions, i.e. conceptual products of experience arrived at through rational considerations, such as comparison and contrast (i.e. affirmation or negation, and measurement).

The term 'fact' is usually intended to refer to purely experiential data, i.e. the raw material of knowledge, in which case the opposite term 'fiction' refers to other items of knowledge, i.e. those tainted by interpretative hypotheses. (But note that in practice of course we do not always abide by such strict definitions, and may use the terms more broadly or narrowly.)

The concepts of truth, falsehood and uncertainty correspond in scope to those of reality, illusion and appearance. The latter triad is applied to the contents of propositions, while the former concerns the propositions as such. For example, considering "dogs bark", the fact

of dogs barking is 'a reality', while the proposition that dogs bark is 'true'; similarly in other cases.

Once we understand all such concepts as signifying different epistemological and ontological *statuses*, it becomes clear why they need to be distinguished from each other. They are all used as logical instruments – to clarify and order discourse, and avoid confusions and antinomies.

Note well that phenomenology is not a skeptical philosophy that denies reality to all appearances and claims them all to be illusions. Such a posture (which too many philosophers have stupidly fallen into) is logically self-contradictory, since it claims itself true while rejecting all possibility of truth. The concept of illusion has no meaning if that of reality is denied; some credulity is needed for incredulity. Doubt is always based on some apparent contradiction or gap in knowledge; i.e. it is itself also an item within knowledge.

2. Existence and Non-existence

What is the relation between the concepts of *existence* and non-existence (or being and non-being), and those just elucidated of appearance, reality and illusion, one might ask?

At first, the term existence may be compared to that of reality, or more broadly to that of appearance (to admit the fact that illusions occur, even if their status is not equal to that of realities). However, upon reflection, an important divergence occurs when factors like time and place are taken into consideration.

We need to be able to verbally express changes in experience over time, space and other circumstances. An

appearance, be it real or illusory, 'exists' at the time and place of its appearance – but may 'not exist' at some earlier or later time, or in another place. The 'existence' of appearances is transient, local, conditional and relative.

What appears today may cease to appear tomorrow, although it might (or might not) continue to appear less manifestly, through someone's memory of it or through the appearance of exclusive effects of it. Something may appear here within my field of vision, but be absent elsewhere. You may see this in some circumstances, and then notice its absence in others.

We thus need to distinguish different ways of appearance. With reference to time: in actuality, or through memory or anticipation; or with reference to spatial positioning. Or again, with regard to modality: in actuality, only through potentiality (i.e. in some circumstances other than those currently operative), or through necessity (i.e. in all circumstances).

Time and place also incite a distinction between 'existence' and 'reality' (or 'truth'), in that when something ceases to exist at a given time and place, the reality of its having existed at the previous time and place is not affected.

Furthermore, appearances are apparent to someone, somewhere – they are contents of consciousness, objects of cognition. The concept of existence is differentiated also with reference to this, by conceiving that what may be apparent to one Subject, may not be so to another. Moreover, we wish to eventually acknowledge that something may conceivably exist even without being experienced by anyone (though of course, in defining such a category, we must admit for consistency's sake

that we are thereby at least vaguely and indirectly conceptually cognizing the object concerned).

We thus come to the realization that the concept of appearance is a relatively subjective one, involving two distinct factors: an object of some kind with specific manifestations, on the one hand, and an awareness by someone of that object at a given time and place. The concept of existence is intended to separate out the objective factor from the factor of consciousness implicit in the concept of appearance.

'Existence' is thus needed to objectify 'appearance', and allow us to conceive of the object apart from any subject's consciousness of it. We need to be able to conceive of the objects appearing to us as sometimes 'continuing on' even when we cease to be aware of them. Furthermore, we need to be able to consider objects that we have not yet personally experienced, and even may never experience. In this manner, we can project our minds beyond mere appearance, and through conception and adduction hope to grasp existence in a larger sense.

The concept of existence and its negation are thus additional instruments of logic, facilitating rational discourse, without which we would not be able to mentally express many distinctions. Consequently, saying 'existence exists' and 'non-existence does not exist' is not mere tautology, but an acknowledgement that the words we use have certain useful intentions. These statements constitute one more way for us to express the laws of thought. Existence cannot be denied and non-existence cannot be affirmed.

We do not make the distinction between 'existents' and non-existents' by mentally lining up two kinds of things, like apples and things other than apples. The epistemological scenario applicable to most of our concepts is not applicable to such basic ones, which are of a more broadly pragmatic nature. Discernment rather than distinction is involved.

Whereas the concept 'existence' has some ultimate experiential content, 'non-existence' has none – because factual denial is not based on the same mental process as affirmation. We never experience non-existence – we only (in certain cases) *fail to* experience existence. The concept of existence is not built up by contrast to that of non-existence, since (by definition) the former relates to 'all things' and the latter to 'nothing', and nothing is not some kind of something. There is no time, place or circumstance containing nothingness. The word 'non-existence' is just a dumping place for all the words and sentences that have been identified as meaningless or false.

Terms like 'existence' and 'non-existence' are not ordinary subjects, copulae or predicates; they are too broad and basic to be treated like any other terms. Those who construct a theory of knowledge, or an ontology, which concludes that 'existence does not exist' or that 'non-existence exists' have not understood the logic of adduction. When there is a conflict between theory and observed facts, it is the theory (or the 'reasoning' that led up to it) that is put in doubt and is to be dismissed, not the facts.

15. CHAPTER FIFTEEN

Drawn from *Ruminations* (2005), Chapters 3 (sect. 5), 5 (sect. 1) and 6 (sect. 3 & 4).

1. Poles of Duality

Concerning the principle, advocated by many, especially oriental, philosophers, that *poles of duality* (e.g. goodbad, light-dark, etc.) arise together – certain comments are worth making.

Oriental philosophers pursue a non-sorting mode of consciousness, the awareness prior to the making of distinctions; for this reason, dualities are obstacles in their eyes. Such Monist consciousness is, however, rarely if ever attained.

I would reply, ontologically: since we can conceive of Monism, then we can also conceive of a universe with *only* good or *only* light, etc.; i.e. a world with one polarity of such dualities is logically possible. Of course, this would only be strict Monism, if this quality was quite alone and no other quality was found in the world (i.e. not just not the other polarity of that quality). Of course, also, we – those now conceiving of that world – would not be distinguishable in it, since then there would be two things in it – viz. object and subject.

But note such solitude of existence could not apply to just any quality. Negative concepts like 'imperfect'

cannot exist alone²⁸; i.e. an *only* imperfect world is inconceivable, as some part of it must remain perfect to exist at all. However, this remark may rather concern the next observation.

From an epistemological and psychological (rather than ontological) viewpoint, there is some truth in the said oriental belief. That is, *the idea* of good or light is not possible without *the idea* of bad or dark. Imaging one pole necessitates our *also bringing to mind* the other pole for the purpose of contrast. This is due to the mechanics of concept formation: it functions by *making distinctions* as well as by identification of the things distinguished.

Because it is only by way of contrast to dissimilars that similars can be classified, every word, every concept, has to make some room for its opposite; we cannot comprehend a term without having to think of its opposite. Thus, one might suggest: although logically, X totally excludes nonX – psychologically, "X" may be said to be say 99% "X" and 1% "nonX".

Another point worth making, here: contradictory terms, such as X and not-X, have equal logical status, i.e. their treatment identical: is phenomenologically, affirmation and denial are very the first signifies an actual experience different: (phenomenal, through the senses or mentally, or nonphenomenal, intuitively) – whereas the latter signifies a rational act, a conceptual report that some anticipated experience has not occurred. Strictly, experiences should be verbalized affirmatively, while negations should be cast in negative terms. In practice, this is rarely followed.

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As Alan Watts pointed out, somewhere.

A positive word like 'silence' or 'stillness' may indicate a negative event (no sound, no move). However, even in such cases, there may be an underlying positive event; in our examples, although silence refers to the non-perception of any sound phenomenon – we may by this term mean rather to refer to our will to block sounds, which volition is something positive, though without phenomenal character, known intuitively.

Similarly, I suspect, some negatively cast words may in fact refer to positive experiences, although there may be a good reason why the negative form is preferred. For example, 'unabashed' simply means without apology, but viewed more closely refers to certain behavior patterns; so, though negative in form, it is rather positive in intent. However, the negative form is not accidental, but serves to indicate the missing ingredient in the behavior patterns, which makes them socially questionable.

2. On the Liar paradox²⁹

Once we grasp that the meaning of words is their intention, singly and collectively – the solution of the liar paradox becomes very obvious. Self-reference is meaningless, because – an intention cannot intend itself, for it does not yet exist; an intention can only intend something that already exists, e.g. another intention directed at some third thing.

In view of this, the proposition "this proposition is false" is meaningless, and so is the proposition "this proposition is true". Both may freely be declared equally true and

Further to chapter 5, section 2 (above).

false, or neither true nor false – it makes no difference in their case, because the words "this proposition" refer to nothing at all³⁰.

Although the words used in these sentences are separately meaningful, and the grammatical structure of the sentences is legitimate – the words' collective lack of content implies their collective logical value to be nil. Self-reference is syntactically cogent, but semantically incoherent. It is like circular argument, up in the air, leading nowhere specific.

Regarding the exclusive proposition "Only this proposition is true", it implies both: "This proposition is true" and "All other propositions are false" – i.e. it is equivalent to the exceptive proposition "All propositions but this one are false". The latter is often claimed by some philosopher; e.g. by those who say "all is illusion (except this fact)".

My point here is that such statements do not only involve the fallacy of self-reference (i.e. "this proposition"). Such statements additionally involve a reference to "all others" which is open to criticism, because:

- To claim knowledge of "all other propositions" is a claim to *omniscience*, a pretense that one knows everything there is to know, or ever will be. And generally, such statements are made without giving a credible justification, though in contradiction to all prior findings of experience and reason.
- Surely, some other propositions are in fact regarded and admitted as true by such philosophers. They are generally rather talkative, even verbose – they do not

See Future Logic, chapter 32.2.

consistently *only* say that one statement and refuse to say anything else.

• And of course, formally, if "this" is meaningless (as previously shown), then "all others", which means "any other *than this*" is also meaningless!

The liar paradox, by the way, is attributed to the ancient Greeks, either Eubulides of Miletus (4th Cent. BCE) or the earlier Epimenides of Crete (6th Cent. BCE). I do not know if its resolution was evident to these early logicians, but a (European?) 14th Cent. CE anonymous text reportedly explained that the Liar's statement is neither true nor false but simply meaningless. Thus, this explanation is historically much earlier than modern logic (Russell et alia, though these late logicians certainly clarified the matter).³¹

3. Non-Aristotelian "Logic"

As already stated, many "modern logicians" – since the late 19th Century – have yearned to do for (or to) Logic, what Copernicus did in Astronomy, or later what Einstein did in Physics. Each one of them was, it seems, fired by the grandiose desire to be the equivalent great modern revolutionary in the field of logic.

They thus inaugurated a persistent assault on Reason, a veritable carnival of Unreason, which has lasted for over a hundred years, with disastrous consequences for many a poor mind and for social peace and wellbeing.

Their conceptual model was non-Euclidean geometry. Just as modern mathematicians came to consider certain Euclidean axioms to be debatable, if not arbitrary, so

See *Future Logic*, chapter 63, sections 3 and 6.

these modern logicians sought to put in doubt or discard the Aristotelian "laws of thought", and found some new system – a "non-Aristotelian logic".

But this is an impossible exercise, because³² the laws of thought are more fundamental to reason than Euclid's axioms (in particular, that regarding parallels). The geometrical model of axioms and theorems is only superficially applicable to logic, because it is itself an aspect or teaching of (Aristotelian) logic.

When mathematicians decided to review the traditional axioms of geometry, they were using reasoning *by means of* the laws of thought. They argued: "we see no self-contradiction, or doctrinal inconsistency, or even (eventually) contradiction to experience in proposing some alternative axioms and systems; therefore, Euclid's assumptions are not exclusive and irreplaceable."

The same cannot be argued in the case of logic itself, without self-contradiction. We cannot, say, point to the particle-wave duality and say "it seems that contradictions do exist in the world, therefore we shall review the logical axiom of non-contradiction" — we cannot do so, for the reason that such review is motivated and rendered credible precisely by the law of non-contradiction, in the way of an attempt to restore an apparently lost consistency.

The very method used of reviewing one's premises in the face of contradiction and abandoning or at least modifying one or more of them to recover consistency – this very methodology is a teaching of Aristotelian logic! We cannot say: "I understand that if I advocate contradiction, I open myself to being contradicted; but

As I have explained repeatedly in *Future Logic*.

that does not bother me, because it is a consistency of sorts – I accept self-contradiction."

In the very act of making such a superficially reasonable proposal, we are reasserting the universality of the laws of thought, their being at the very root of reason, inherent in the very act of reasoning. The only way we could conceivably abandon these laws would be to give up all thought, all attempt at rational knowledge. Logic cannot be used against itself: it is the very paradigm and paragon of consistency.

We can suggest: "A can be non-A", or some such "new axiom" for logic, but the resulting discourse will still be nonsense — however nicely wrapped up and ordered, however well "systematized" stealing the methods of Aristotelian logic. Such proposals are an imposture.

Those who propose such ideas are swindlers, profiting from the gullibility and intimidation of many people. It is like in the story of the emperor's new clothes, in which con men sold the emperor invisible clothes, which no one dared to deny were clothes – till a child pointed out he was naked.

There simply is *no such thing* as "non-Aristotelian logic" (i.e. a logical system that denies one, two or all three laws of thought). To come forward with such a system is merely to pronounce words. These words have no collective content, no meaning; there is nothing behind them other than the imagination that there might be something behind them because the phrase is composed of individually meaningful words.

No "Copernican revolution" is conceivable in the field of logic: it would not merely be anti-Aristotelian but anti-rational. Logicians must abandon such vain ambitions, and more modestly continue to expand the scope of

logical analysis and the depth of understanding of logic. The role of logicians is to do logic, not undo it. Reason is a precious value for mankind, and logicians ought to be its guardian.

Would you entrust your life to, say, an airplane built by engineers practicing "non-Aristotelian logic", people who feel cozy in the midst of contradictions and in between truth and falsehood? Similarly, in all fields of human endeavor and interaction: logic is a guarantee of sanity and safety.

4. Postmodern "Logic"

As if such irrational currents were not enough, there is (I gather) a new generation of "postmodern" logicians and philosophers who eschew even the pretense of accountability, considering that any discourse that seems to be about "logic" is acceptable. These are of course part of a wider trend, not limited to our field.

Being relativists, these people are not directly attacking anything or anyone. They are not mere anti-rationalists: they are so indifferent to the niceties of reason that they feel no need to justify themselves. They are of course the natural offspring of the moderns, taking their teachings to their 'logical' conclusion. They are more consistently illogical than their predecessors, no longer owing a semblance of allegiance to reason, not needing even to pay lip service to it. Absurdity does not bother them, so they need no logical window dressing for their doctrines. Indeed, these people take pride in their fashionable They strive to be confusing madness. as incomprehensible as possible, considering that what others cannot possibly understand must be very deep

indeed. They have only a very vague notion of what logic is about, but seek to impress other people with meaningless symbolic constructs and use of fancy pseudo-scientific terminology. They prattle away, eruditely formulating fake theories immune to any empirical or rational review. They function as (con) artists rather than scientists.

Yes, such people do exist; some even have teaching positions in prestigious universities. Because most people – including some in high academic positions, including some who are hired to teach logic – know or understand little about logic, they are easily intimidated by such intellectual posturing and imposture. They fear to reveal their own poverty in the course of questioning or debate. Besides, it is no use denouncing the swindle; no one apparently cares, because few people realize the importance of logic (apart from some simple formulas needed in computer programming). Reason is out of fashion, has been for generations. Logic is too abstract; you cannot show artistic footage of it on TV. It cannot be very entertaining: it requires an effort of thought.

16. CHAPTER SIXTEEN

Drawn from *Ruminations* (2005), Chapter 9 (sections 1-4).

ON NEGATION

1. Negation in Adduction

Concepts and theories are hypothetical constructs. They cannot (for the most part) be proven (definitely, once and for all), but only repeatedly confirmed by experience. This is the positive side of adduction, presenting evidence in support of rational constructs. This positive aspect is of course indispensable, for without some concrete evidence an abstraction is no more than a figment of the imagination, a wild speculation. The more evidence we adduce for it, the more reliable our concept or theory.

But, as Francis Bacon realized, the account of adduction thus far proposed does not do it justice. Just as important as the positive side of providing evidence, is the negative aspect of it, the rejection of hypotheses that make predictions conflicting with experience. As he pointed out, even if a hypothesis has numerous confirmations, it suffices for it to have *one* such wrong prediction for it to be rejected.

Stepping back, this means that the process of adduction is concerned with selection of the most probable hypothesis among two or more (already or yet to be conceived) explanations of fact. Each of them may have numerous 'positive instances' (i.e. empirical evidence that supports it); and so long as they are all still competitive, we may prefer those with the most such instances. But, the way we decisively advance in our conceptual/theoretical knowledge is by the successive *elimination* of propositions that turn out to have 'negative instances' (i.e. empirical evidence against them).

Now all the above is well known and need not be elucidated further. This theory of inductive logic has proven extremely successful in modern times, constituting the foundation of the scientific method.

But upon reflection, the matter is not as simple and straightforward as it seems at first!

Consider, for example, the issue of whether or not there is water on Mars. It would seem that the proposition "There is water on Mars" is far easier to prove inductively than the contradictory proposition "There is no water on Mars". Both propositions are hypotheses.

The positive thesis would be somewhat confirmed, if it was discovered using certain instruments from a distance that there are serious indices that water is present; the thesis would be more solidly confirmed, if a sample of Mars was brought back to Earth and found upon analysis to contain water. In either case, the presence of water on

Mars would remain to some (however tiny) degree unsure, because some objection to our instrumental assumptions might later be raised or the sample brought back may later be found to have been contaminated on the way over. Nevertheless, something pretty close to certainty is conceivable in this matter.

The negative thesis, by contrast, is much more difficult to prove by experience. We can readily assume it to the extent that the positive thesis has not so far been greatly confirmed. That is, so long as we have not found evidence for the positive thesis (i.e. water on Mars), we should rather opt for the negative thesis. But the latter is only reliable to the degree that we tried and failed to confirm the former. If we earnestly searched for water every which way we could think of, and did not find any, we can with proportionate confidence assume there is no water.

Thus, in our example, the negative thesis is actually *more difficult* to establish than the positive one. It *depends on a generalization*, a movement of thought from "Wherever and however we looked for water on Mars, *none was found*" to "*There is no* water on Mars". However, note well, it remains conceivable that a drop of water be found one day somewhere else on Mars, centuries after we concluded there was none.

Granting this analysis, it is clear that Bacon's razor that "What is important is the negative instance" is a bit simplistic. It assumes that a negative is as accessible as

(if not, indeed, more accessible than) a positive, which is not always the case.

In practice, a negative may be inductively more remote than a positive. Granting this conclusion, the question arises – is the negative instance *ever* more empirically accessible than (or even as accessible as) the positive one? That is, *when* does Bacon's formulation of induction actually come into play?

If we look at major historical examples of rejection of theories, our doubt may subsist. For example, Newtonian mechanics was in place for centuries, till it was put in doubt by the discovery of the constancy of the velocity of light (which gave rise to Relativity theory) and later again by the discovery of various subatomic phenomena (which gave rise to Quantum mechanics). In this example, the 'negative instances' were essentially 'positive instances' – the only thing 'negative' about them was just their negation of the Newtonian worldview!

Such reflections have led me to suspect that the 'negation' referred to by Bacon is only meant *relatively* to some selected abstraction. His razor ought not be taken as an advocacy of absolute negation. If we look at the matter more clearly, we realize that the data used to thus negate an idea is essentially positive. A deeper consideration of the nature of negation is therefore patently called for.

2. Positive and Negative Phenomena

People have always considered that there is a difference between a positive and a negative term. Indeed, that is why logicians have named them differently. But logicians have also found it difficult to express that difference substantially. Yet, there are significant phenomenological differences between positive and negative phenomena.

The concrete material and mental world is evidently composed only of positive particular phenomena, some of which we perceive (whether through the bodily senses or in our minds). These exist at least as appearances, though some turn out to seem real illusory. This others is an phenomenological, epistemological and ontological truth. To say of phenomena that they are 'particular' is to express awareness that they are always limited in space and time. They have presence, but they are finite and transient, i.e. manifestly characterized by diversity and change.

We do not ordinarily experience anything concrete that stretches uniformly into infinity and eternity (though such totality of existence might well exist, and indeed mystics claim to attain consciousness of it in deep meditation, characterizing it as "the eternal present"). We do commonly consider some things as so widespread. 'Existence' is regarded as the substratum of all existents; 'the universe' refers to the sum total of all existents; and we think of 'spacetime' as defining the extension of all existents. But only 'existence' may be classed as an experience (a

quality found in all existents); 'the universe' and 'space-time' must be admitted as abstractions.

However, the limits of particulars are perceivable without need of negation of what lies beyond them, simply due to the variable concentration of consciousness, i.e. the direction of focus of attention. That is, though 'pointing' to some positive phenomenon (e.g. so as to name it) requires some negation (we mean "this, but not that"), one can notice the limits of that phenomenon independently of negation.

b. Negative phenomena (and likewise abstracts, whether positive or negative), on the other hand, do depend for their existence on a Subject/Agent – a cognizing 'person' (or synonymously: a self or soul or spirit) with consciousness and volition looking out for some remembered or imagined positive phenomenon and failing to perceive it (or in the case of abstracts, comparing and contrasting particulars).

Thus, negative particular phenomena (and more generally, abstracts) have a special, more 'relative' kind of existence. They are not as independent of the Subject as positive particular phenomena. That does not mean they are, in a Kantian sense, 'a priori' or 'transcendental', or purely 'subjective' – but it does mean that they are ontological potentials that are only realized in the context of (rational) cognition.

Another kind of experience is required for such realization – the self-experience of the Subject, his intuitive knowledge of his cognitions and volitions. This kind of experience, being immediate, may be positive or negative without logical difficulty. The Subject reasons inductively as follows:

I am searching for X;
I do not find X;
Therefore, X "is not" there.

The negative conclusion may be 'true' or 'false', just like a positive perception or conclusion. It is true to the degree that the premises are true – i.e. that the alleged search for X was diligent (intelligent, imaginative, well-organized, attentive and thorough), and that the alleged failure to find X is not dishonest (a lie designed to fool oneself or others).

Whence it is fair to assert that, unlike some positive terms, negative terms are never based *only* on perception; they *necessarily* involve a thought-process – the previous mental projection or at least intention of the positive term they negate.

This epistemological truth does reflect an ontological truth – the truth that the 'absences' of phenomena lack phenomenal aspects. A 'no' is not a sort of 'yes'.

Note well the logical difference between 'not perceiving X' and 'perceiving not X'. We do not have direct experience of the latter, but can only indirectly claim it by way of *inductive inference* (or extrapolation) from the former. In the case of a positive, such process of reasoning is not needed – one often can and does 'perceive X' directly.

Suppose we draw a square of opposition for the propositions (labeling them by analogy to standard positions) – "I perceive X" (A), "I do not perceive not X" (I), "I perceive not X" (E), "I do not perceive X" (O). Here, the A form is knowable by experience, whereas the I form is knowable perhaps only by deductive implication from it. On the negative side, however, the E

form is not knowable by experience, but only by inductive generalization from the O form (which is based on experience).

3. Positive Experience Precedes Negation

Negation is a pillar of both deductive and inductive logic, and requires careful analysis. We have to realize that negative terms are fundamentally distinct from positive ones, if we are to begin fathoming the nature of logic. The following observation seems to me crucial for such an analysis:

We can experience something positive without having first experienced (or thought about) its negation, but we cannot experience something negative without first thinking about (and therefore previously having somewhat experienced) the corresponding positive.

a. Cognition at its simplest is perception. Our perceptions are always *of positive particulars*. The contents of our most basic cognitions are phenomenal sights, sounds, smells, tastes, and touch and other bodily sensations that seemingly arise through our sense organs interactions with matter – or mental equivalents of these phenomena that seemingly arise through memory of sensory experiences, or in imaginary re-combinations of such supposed memories.

A positive particular can be experienced directly and passively. We can just sit back, as it were, and receptively observe whatever happens to come in our field of vision or hearing, etc. This is what we do in meditation. We do not have to actively think of (remember or visualize or conceptualize) something else

in order to have such a positive experience. Of course, such observation may well in practice be complicated by thoughts (preverbal or verbal) – but it is possible in some cases to have a pure experience. This must logically be admitted, if concepts are to be based on percepts.

b. In the case of *negative particulars*, the situation is radically different. A negative particular has *no* specific phenomenal content, but is *entirely* defined by the 'absence' of the phenomenal contents that constitute some positive particular. If I look into my material or mental surroundings, I will always see present phenomena. The absence of some phenomenon is only noticeable if we first think of that positive phenomenon, and wonder whether it is present.

It is accurate to say that our finding it absent reflects an empirical truth or fact – but it is a fact that we simply would not notice the negative without having first thought of the positive. Negative knowledge is thus necessarily (by logical necessity) more indirect and active. It remains (at its best) perfectly grounded in experience – but such negative experience requires a rational process (whether verbal or otherwise).

To experience a negative, I must first imagine (remember or invent) a certain positive experience; then I must look out and see (or hear or whatever) whether or not this image matches my current experience; and only then (if it indeed happens not to) can I conclude to have "experienced" a negative.

Thinking about X may be considered as positioning oneself into a vantage point from which one can (in a manner of speaking) experience not-X. If one does not first place one's attention on X, one cannot possibly experience the negation of X. One may well experience

all sorts of weird and wonderful things, but not specifically not-X.

From this reflection, we may say that whereas affirmatives can be experienced, negatives are inherently rational acts (involving imagination, experience and intention). A negative necessarily involves thought: the thought of the corresponding positive (the imaginative element), the testing of its presence or absence (the experiential element) and the rational conclusion of "negation" (the intentional element).

c. The negation process may involve words, though it does not have to.

Suppose I have some momentary experience of sights, sounds, etc. and label this positive particular "X". The *content of consciousness* on which I base the term X is a specific set of positive phenomenal experiences, i.e. physical and/or mental percepts. Whenever I can speak of this X, I mentally *intend* an object of a certain color and shape that moves around in certain ways, emitting certain sounds, etc.

Quite different is the negation of such a simple term, "not X". The latter is not definable by any specific percepts – it *refers to no perceptible qualities*. It cannot be identified with the positive phenomena that happen to be present in the absence of those constituting X. Thus, strictly speaking, not-X is only definable by 'negation' of X.

Note well, it would not be accurate to say (except ex post facto) that not-X refers to all experiences other than X (such as Y, Z, A, B, etc.), because when I look for X here and now and fail to find it, I am only referring to present experience within my current range and not to all possible such experiences. We would not label a situation

devoid of X as "not X" without thinking of X; instead, we would label that situation in a positive manner (as "Y", or "Z", or whatever).

Thus, we can name (or wordlessly think of) something concrete "X", *after* experiencing phenomena that constitute it; but in the case of "not-X", we necessarily conjure the name (or a wordless thought) of it *before* we experience it.

"Not-X" is thus already a concept rather than a percept, even in cases where "X" refers to a mere percept (and all the more so when "X" itself involves some abstraction — as it usually does). The concept "not X" is hypothetically constructed first and then confirmed by the attempted and failed re-experience of X.

In short, negation – even at the most perceptual level – involves an adductive process. It is never a mere experience. A negative term never intends the simple perception of some negative thing, but consists of a hypothesis with some perceptual confirmation. Negation is always conceptual as well as perceptual in status.

A theory cannot be refuted before it is formulated – similarly, X cannot be found absent unless we first think of X.

4. Negation is an Intention

Now, there is no specific phenomenal experience behind the word "not". Negation has no special color and shape, or sound or smell or taste or feel, whether real or illusory! What then is it? I suggest the following:

Negation as such refers to a 'mental act' – or more precisely put, it is an act of volition (or more precisely still, of velleity) by a Subject of consciousness.

Specifically, *negation is an intention*. Note that our will to negate is itself *a positive act*, even though our intention by it is to negate something else.

Negation does express an experience – the 'failure' to find something one has searched for. Some cognitive result is willfully pursued (perception of some positive phenomenon), but remains wanting (this experience is qualitatively a suffering of sorts, but still a positive intention, note) – whence we mentally (or more precisely, by intention) mark the thing as 'absent', i.e. we construct an idea of 'negation' of the thing sought.

Thus, negation is *not a phenomenon* (a physical or mental percept), *but something intuited* (an event of will within the cognizing Subject). 'Intuition' here, note well, means the self-knowledge of the Subject of consciousness and Agent of volition. This is experience of a *non-phenomenal* sort. Such self-experience is immediate: we have no distance to bridge in space or time.

When a Subject denies the presence of a material or mental phenomenon, having sought for it in experience and not found it – the 'denial' consists of a special act of intention. This intention is what we call 'negation' or 'rejection of a hypothesis'. It occurs in the Subject, though it is about the Object.

This intention is not however an arbitrary act. If it were, it would be purely subjective. This act (at its best) remains sufficiently dependent on perception to be judged 'objective'. The Subject must still look and see whether X is present; if that positive experience does not follow his empirical test, he concludes the absence of X. Indeed, an initial negation may on closer scrutiny be found erroneous, i.e. we sometimes think something is

'not there' and then after further research find it on the contrary 'there'. Thus, this theory of negation should not be construed as a claim that our negating something makes it so. Negation is regulated by the principles of adduction – it is based on appearance that is credible so long as confirmed, but may later be belied.

We can ex post facto speak of an objective absence, but we cannot fully define 'absence' other than as 'non-presence', and the 'non-' herein is not a phenomenon but an intention. The 'absence' is indeed experienced, but it is *imperceptible* without the Subject posing the prior question 'is X present?'

Absence, then, is not produced by the Subject, but is made perceptible by his vain search for presence. For, to repeat, not-X is not experienced as a specific content of consciousness — but as a continuing failure to experience the particular positive phenomena that define X for us.

Although we are directly only aware of apparent existents, we can inductively infer non-apparent existents from the experience that appearances come and go and may change. On this basis, we consider the categories 'existence' and 'appearance' as unequal, and the former as broader than the latter. Similarly, we inductively infer 'objective absence' from 'having sought but not found', even though we have no direct access to former but only indirect access by extrapolation from the latter. Such inference is valid, with a degree of probability proportional to our exercise of due diligence.

For these reasons, I consider the act of negation as an important key to understanding the nature and status of logic. Negation is so fundamental to reason, so crucial an epistemic fact, that it cannot be reduced to something else.

We can describe it *roughly* as an intention to 'cross-off' (under the influence of some reason or other) the proposed item from our mental list of existents. But this is bound to seem like a circular definition, or a repetition of same using synonyms. It is evident that *we cannot talk about negation without engaging in it*. Thus, we had better admit the act of negation as a primary concept for logical science.

Note in passing: the present theory of negation provides biology with an interesting distinction regarding rational animals.

Sentient beings without this faculty of negation can only respond to the present, whereas once this faculty appears in an organism (as it did in the human species) it can mentally go beyond the here and now. A merely sensory animal just reacts to current events, whereas a man can fear dangers and prepare for them. Once the faculty of negation appears, the mind can abstracting, conceiving alternatives hypothesizing. Memory and imagination are required to project a proposed positive idea, but the intent to also required to reject projections. Without such critical ability, our fantasies would quickly lead us into destructive situations.

17. CHAPTER SEVENTEEN

Drawn from *Ruminations* (2005), Chapter 9 (sections 5-8).

ON NEGATION

1. Formal Consequences

Returning to logic – our insight [earlier] into the nature of negation can be construed to have *formal consequences*. The negative term is now seen to be a radically different kind of term, even though in common discourse it is made to behave like any other term.

We cannot point to something as 'negative' except insofar as it is the negation of something positive. This remark is essentially logical, not experiential. The term 'not' has no substance per se – it is a purely relative term. The positive must be experienced or thought of before the negative can at all be conceived, let alone be specifically sought for empirically. This is as true for intuitive as for material or mental objects; and as true for abstracts as for concretes.

One inference to draw from this realization of the distinction of negation is: "non-existence" is not some kind of "existence". Non-existent things cannot be classed under existence; they are not existent things. The term "non-existence" involves no content of consciousness whatsoever – it occurs in discourse only as the verbal repository of any and all denials of

"existence". Existentialist philosophers have written volumes allegedly about "non-being", but as Parmenides reportedly stated:

"You cannot know not-being, nor even say it."

This could be formally expressed and solidified by saying that *obversion* (at least that of a negative – i.e. inferring "This is nonX" from "This is not X") is essentially an artificial process. If so, the negative predicate (nonX) is not always inferable from the negative copula (is not). In other words, the form "There is no X" does not imply "There is non-X"; or conversely, "X does not exist" does not imply "nonX exists".

We can grant heuristically that such eductive processes work in most cases (i.e. lead to no illogical result), but they may be declared invalid in certain extreme situations (as with the term "non-existence")! In such cases, "nonX" is 'just a word'; it has no conscionable meaning – we have no specific thing in mind as we utter it.

Logicians who have not yet grasped the important difference of negation are hard put to explain such formal distinctions. I know, because it is perhaps only in the last three years or so that this insight about negation has begun to dawn on me; and even now, I am still in the process of digesting it.

Note that a philosophical critic of this view of negation cannot consider himself an objective onlooker, who can hypothesize 'a situation where absence exists but has not or not yet been identified'. For that critic is himself a Subject like any other, who must explain the whence and wherefore of his knowledge like anyone else – including the negatives he appeals to. No special privileges are granted.

That is, if you wish to deny all the above, ask yourself and tell me how you consider you go about denying without having something to deny! Claiming to have knowledge of a negative without first thinking of the corresponding positive is comparable to laying personal claim to an absolute framework in space-time — it is an impossible exercise for us ordinary folk.

It should also be emphasized that the above narrative describes only the simplest kind of negation: negation of a perceptual item. But most of the time, in practice, we deal with far more complex situations. Even the mere act of 'pointing' at some concrete thing involves not only a positive act ("follow my finger to this"), but also the act of negation ("I do not however mean my finger to point at that").

Again, a lot of our conceptual arsenal is based on imaginary recombinations of empirical data. E.g. I have seen "pink" things and I have seen "elephants", and I wonder whether "pink elephants" perhaps exist. Such hypothetical entities are then tested empirically, and might be rejected (or confirmed). However, note, abstraction does not depend only on negation, but on quantitative judgments (comparing, and experiencing what is more or less than the other).

Abstraction starts with experiences. These are variously grouped through comparisons and contrasts. Negation here plays a crucial role, since to group two things together, we must find them not only similar to each other but also different from other things. This work involves much trial and error.

But at this level, not only denial but also affirmation is a rational act. For, 'similarity' means seemingly having some quality in common in some measure, although there are bound to be other qualities not in common or differences of measure of the common quality. The essence of affirmation here is thus 'measurement'.

But Nature doesn't measure anything. Every item in it just is, whatever it happens to be (at any given time and place). It is only a Subject with consciousness that measures: this against that, or this and that versus some norm.

This weighing work of the cognizing Subject is not, however, arbitrary (or ought not to be, if the Subject has the right attitudes). As in the above case of mere negation, the conclusion of it does proceed from certain existing findings. Yet, it is also true that this work only occurs in the framework of cognition.

2. Negation and the Laws of Thought

Logic cannot be properly understood without first understanding negation. This should be obvious from the fact that two of the laws of thought concern the relation between positive and negative terms. Similarly, the basic principle of adduction, that hypotheses we put forward should be empirically tested and rejected if they make wrong predictions — this principle depends on an elucidation of negation.

a. The so-called laws of thought are, in a sense, laws of the universe or ontological laws – in that the universe is what it is (identity), is not something other than what it is (non-contradiction) and is something specific (excluded middle).

They have phenomenological aspects: appearances appear (identity); some are in apparent contradiction to

others (a contradiction situation); in some cases, it is not clear just what has appeared (an excluded middle situation).

They may also be presented as epistemological laws or laws of logic, in that they guide us in the pursuit of knowledge. However, they are aptly named laws of thought, because they really arise as propositions only in the context of cognitive acts.

To understand this, one has to consider the peculiar status of negation, as well as other (partly derivative) major processes used in human reasoning, including abstraction, conceiving alternative possibilities and making hypotheses.

b. The impact of this insight on the laws of thought should be obvious. The law of identity enjoins us primarily to take note of the *positive* particulars being perceived. But the laws of non-contradiction and of the excluded middle, note well, both involve *negation*. Indeed, that's what they are all about – their role is precisely *to regulate our use of negation* – to keep us in harmony with the more positive law of identity!

Their instructions concerning the subjective act of negation, at the most perceptual level, are as follows. The law of non-contradiction forbids negating in the perceptible presence of the thing negated. The law of the excluded middle forbids accepting as final an uncertainty as to whether a thing thought of is currently present or absent.

We are unable to cognize a negative (not-X) except by negation of the positive (X) we have in mind; it is therefore absurd to imagine a situation in which both X and not-X are true (law of non-contradiction). Similarly, if we carefully trace how our thoughts of X and not-X

arise in our minds, it is absurd to think that there might be some third alternative between or beyond them (law of the excluded middle.)

Thus, these two laws are not arbitrary conventions or happenstances that might be different in other universes, as some logicians contend (because they have unfortunately remained stuck at the level of mere symbols, "X" and "non-X", failing to go deeper into the cognitive issues involved). Nor are they wholly subjective or wholly objective.

These laws of thought concern the interface of Subject and Object, of consciousness and existence – for any Subject graced with rational powers, i.e. cognitive faculties that go beyond the perceptual thanks in part to the possibility of negation.

They are for this reason applicable universally, whatever the content of the material and mental universe faced. They establish for us *the relations* between affirmation and denial, for any and every content of consciousness.

c. On this basis, we can better comprehend the ontological status of the laws of thought. They have no actual existence, since the concrete world has *no use for or need* of them, but exists self-sufficiently in positive particulars.

But the laws are a potential of the world, which is actualized when certain inhabitants of the world, who have the gifts of consciousness and freewill, resort to negation, abstraction and other cognitive-volitional activities, in order to summarize and understand the world.

In a world devoid of humans (or similar Subject/Agents), there are no negations and no 'universals'. Things just are (i.e. appear) – positively and particularly. Negation

only appears in the world in relation to beings like us who can search for something positive and not find it. Likewise for 'universals' – they proceed from acts of comparison and contrast.

Consciousness and volition are together what gives rise to concepts and alternative possibilities, to hypotheses requiring testing. It is only in their context that logical issues arise, such as existence or not, reality or illusion, as well as consistency and exhaustiveness.

It is important to keep in mind that the laws of thought are themselves complex abstractions implying negations — viz. the negative terms they discuss, as well as the negation of logical utility and value in contradictory or 'middle' thinking. Indeed all the 'laws' in our sciences are such complex abstractions involving negations.

d. The insight that negation is essentially a volitional act allied to cognition explains why the laws of thought are prescriptive as well as descriptive epistemological principles.

The laws of thought are prescriptive inasmuch as human thought is fallible and humans have volition, and can behave erratically or maliciously. If humans were infallible, there would be no need for us to study and voluntarily use such laws. There is an ethic to cognition, as to all actions of freewill, and the laws of thought are its top principles.

The laws of thought are descriptive, insofar as we commonly explicitly or implicitly use them in our thinking. But this does not mean we all always use them, or always do so correctly. They are not 'laws' in the sense of reports of universal behavior. Some people are unaware of them, increasing probabilities of erroneous thinking. Some people would prefer to do without them,

and eventually suffer the existential consequences. Some people would like to abide by these prescriptions, but do not always succeed.

These prescriptions, as explicit principles to consciously seek to abide by, have a history. They were to our knowledge first formulated by a man called Aristotle in Ancient Greece. He considered them to best describe the cognitive behavior patterns that lead to successful cognition. He did not invent them, but realized their absolute importance to human thought.

Their justification is self-evident to anyone who goes through the inductive and deductive logical demonstrations certain logicians have developed in this regard. Ultimately it is based on a holistic consideration of knowledge development.

Our insights here about the relativity of negation and abstraction, and the realization of their role in the laws of thought serve to further clarify the necessity and universality of the latter.

3. Pure Experience

A logically prior issue that should perhaps be stressed in this context is *the existence of pure experience*, as distinct from experience somewhat tainted by acts of thought.

Some philosophers claim that all alleged 'experience' falls under the latter class, and deny the possibility of the former. But such skepticism is clearly inconsistent: if we recognize some *part* of some experience as pure of thought, this is sufficient to justify a claim to *some* pure experience. Thus, the proposition "There are some pure experiences" may be taken as an axiom of logic,

phenomenology, epistemology and ontology. This proposition is self-evident, for to deny it is self-contradictory.

Note that this proposition is more specific than the more obvious "There are experiences". Denial of the latter is a denial of the evidence before one's eyes (and ears and nose and tongue and hands, etc. – and before one's "mind's eye", too): it directly contravenes the law of identity. Philosophers who engage in such denial have no leg to stand on, anyway - since they are then hard put to at all explain what meaning the concepts they use in their denial might possibly have. We have to all admit *some* experience – some appearance in common (however open to debate) – to have anything to discuss (or even to be acknowledged to be discussing).

Let us return now to the distinction between pure and tainted experiences. This concerns the involvement of thought processes of any kind -i.e. of ratiocinations, acts of reason. To claim that there are pure experiences is not to deny that some (or many or most) experiences are indeed tainted by conceptual activity (abstraction, classification, reasoning, etc.)

We can readily admit that all of us very often have a hard time distinguishing pure experience from experience mixed with rational acts. The mechanisms of human reason are overbearing and come into play without asking for our permission, as is evident to anyone who tries to meditate on pure experience. It takes a lot of training to clearly distinguish the two in practice.

But surely, any biologist would admit that lower animals, at least, have the capacity to experience without the interference of thought, since they have no faculty of thought. The same has to be true to some extent for humans – not only in reflex actions, but also in the very fact that reasoning of any sort is only feasible in relation to pre-existing non-rational material. To process is to process something.

I have already argued that what scientists call 'experiment' cannot be regarded as the foundation of science, but must be understood as a mix of intellectual (and in some cases, even physical acts) and passive observation (if only observation of the results of experiment displayed by the detection and measuring instruments used). Thus, observation is cognitively more fundamental than experiment.

Here, my purpose is to emphasize that perceptual 'negation' is also necessarily a mix of pure experience and acts of the intellect. It is never pure, unlike the perception of positive particulars (which sometimes is pure, necessarily) – because it logically cannot be, since to deny anything one must first have something in mind to deny (or affirm).

Thus, negation can be regarded as one of the most primary acts of reason — it comes before abstraction, since the latter depends to some extent on making distinctions, which means on negation.

4. Consistency is Natural

It is important to here reiterate the principle that consistency is natural; whereas inconsistency is exceptional.

Some modern logicians have come up with the notion of "proving consistency" – but this notion is misconceived. Consistency is the natural state of affairs in knowledge; it requires no (deductive) proof and we are incapable of

providing such proof, since it would be 'placing the cart before the horse'. The only possible 'proof' of consistency is that no inconsistency has been encountered. Consistency is an inductive given, which is very rarely overturned. All our knowledge may be and must be assumed consistent, unless and until there is reason to believe otherwise.

In short: harmony generally reigns unnoticed, while conflicts erupt occasionally to our surprise. One might well wonder now if this principle is itself consistent with the principle herein defended that negatives are never per se objects of cognition, but only exist by denial of the corresponding positives. Our principle that consistency is taken for granted seems to imply that we on occasion have logical insights of *inconsistency*, something negative!

To resolve this issue, we must again emphasize the distinction between pure experience and the *interpretations* of experience that we, wordlessly (by mere intention) or explicitly, habitually infuse into our experiences. Generally, almost as soon as we experience something, we immediately start interpreting it, dynamically relating it to the rest of our knowledge thus far. Every experience almost unavoidably generates in us strings of associations, explanations, etc.

The contradictions we sometimes come across in our knowledge do not concern our pure experiences (which are necessarily harmonious, since they in fact exist side by side – we might add, quite 'happily'). Our contradictions are necessarily contradictions between an interpretation and a pure experience, or between two interpretations. Contradictions do not, strictly speaking, reveal difficulties in the raw data of knowledge, but

merely in the hypotheses that we conceived concerning such data.

Contradictions are thus to be blamed on reason, not on experience. This does not mean that reason is necessarily faulty, but only that it is fallible. Contradictions ought not be viewed as tragic proofs of our ignorance and stupidity — but as helpful indicators that we have misinterpreted something somewhere, and that this needs reinterpretation. These indicators are precisely one of the main tools used by the faculty of reason to control the quality of beliefs. The resolution of a contradiction is just new interpretation.

How we know that two theories, or a theory and some raw data, are 'in contradiction' with each other is a moot question. We dismiss this query rather facilely by referring to "logical insight". Such insight is partly 'experiential', since it is based on scrutiny of the evidence and doctrines at hand. But it is clearly not entirely empirical and involves abstract factors. 'Contradiction' is, after all, an abstraction. I believe the answer to this question is largely given in the psychological analysis of negation.

There is an introspective sense that *conflicting intentions* are involved. Thus, the 'logical insight' that there is inconsistency is not essentially insight into a negative (a non-consistency), but into a positive (the intuitive experience of conflict of intentions). Although the word inconsistency involves a negative prefix, it brings to mind something empirically positive — a felt tension between two theses or a thesis and some data.

For this reason, to say that 'consistency is assumable, until if ever inconsistency be found' is consistent with our claim that 'negations are not purely empirical'.

(Notice incidentally that we did not here "prove" consistency, but merely *recovered* it by clarifying the theses involved.)

The above analysis also further clarifies how the law of non-contradiction is expressed in practice. It does not sort out experiences as such, but concerns more abstract items of knowledge. To understand it fully, we must be aware of the underlying intentions. A similar analysis may be proposed to explain the law of the excluded middle.

In the latter case, we would insist that (by the law of identity) 'things are something, what they are, whatever that happen to be'. Things cannot be said to be *neither* this *nor* the negation of this, because such characterizations are negative (and, respectively, doubly negative) – and therefore cannot constitute or be claimed as positive experience. Such situations refer to uncertainties *in the knower*, which he is called upon to eventually fill-in. They cannot be proclaimed final knowledge (as some modern sophists have tried to do), but must be considered temporary postures in the pursuit of knowledge.

18. CHAPTER EIGHTEEN

Drawn from Logical and Spiritual Reflections (2008), Book 1, chapter 2.

THE PRINCIPLE OF INDUCTION

1. The Uniformity Principle

Concerning the uniformity principle, which Hume denies, it is admittedly an idea difficult to uphold, in the sense that we cannot readily define uniformity or make a generality of it. We might speak of repetition, of two or more particular things seeming the same to us; but we are well aware that such regularity does not go on ad infinitum. On the contrary, we well know that sooner or later, something is bound to be different from the preceding things, since the world facing us is one of multiplicity.

Therefore, this "principle" may only be regarded as a heuristic idea, a rule of thumb, a broad but vague practical guideline to reasoning. It makes no specific claims in any given case. It just reminds us that there are (or seem to us to be) 'similarities' in this world of matter, mind and spirit. It is not intended to deny that there are also (apparent) 'dissimilarities'. It is obviously not a claim that all is one and the same, a denial of multiplicity

and diversity (in the world of appearances, at least³³). To speak of uniformity in Nature is not to imply uniformity of Nature.

We might also ask – can there be a world *without any* 'uniformities'? A world of universal difference, with no two things the same in any respect whatever is unthinkable. Why? Because to so characterize the world would itself be an appeal to uniformity. A uniformly non-uniform world is a contradiction in terms. Therefore, we must admit *some* uniformity to exist in the world. The world need not be uniform throughout, for the principle of uniformity to apply. It suffices that some uniformity occurs.

Given this degree of uniformity, however small, we logically can and must talk about generalization and particularization. There happens to be some 'uniformities'; therefore, we have to take them into consideration in our construction of knowledge. The principle of uniformity is thus not a wacky notion, as Hume seems to imply. It is just a first attempt by philosophers to explain induction; a first try, but certainly not the last. After that comes detailed formal treatment of the topic. This proceeds with reference to specifics, symbolized by X's and Y's, and to strict logic.

The uniformity principle is not a generalization of generalization; it is not a statement guilty of circularity, as some critics contend. So what is it? Simply this: when we come upon some uniformity in our experience or thought, we may readily assume that uniformity to

I.e. such recognition of pluralism does not at the outset exclude monism. The former may be true at the superficial phenomenological level, while the latter reigns at the metaphysical level of ultimate reality.

continue onward until and unless we find some evidence or reason that sets a limit to it. Why? Because in such case the assumption of uniformity already has a basis, whereas the contrary assumption of difference has not or not yet been found to have any. The generalization has some justification; whereas the particularization has none at all, it is an arbitrary assertion.

It cannot be argued that we may equally assume the contrary assumption (i.e. the proposed particularization) on the basis that in past events of induction other contrary assumptions have turned out to be true (i.e. for which experiences or reasons have indeed been adduced) – for the simple reason that such a generalization from diverse past inductions is formally excluded by the fact that we know of many cases that have not been found worthy of particularization to date.

That is to say, if we have looked for something and not found it, it seems more reasonable to assume that it does not exist than to assume that it does nevertheless exist. Admittedly, in many cases, the facts later belie such assumption of continuity; but these cases are relatively few in comparison. The probability is on the side of caution.

In any event, such caution is not inflexible, since we do say "until and unless" some evidence or argument to the contrary is adduced. This cautious phrase "until and unless" is of course essential to understanding induction. It means: until *if ever* — i.e. it does not imply that the contrary will necessarily occur, and it does not exclude that it may well eventually occur. It is an expression of open-mindedness, of wholesome receptiveness in the face of reality, of ever readiness to dynamically adapt one's belief to facts.

In this way, our beliefs may at all times be said to be as close to the facts as we can get them. If we follow such sober inductive logic, devoid of irrational acts, we can be confident to have the best available conclusions in the present context of knowledge. We generalize when the facts allow it, and particularize when the facts necessitate it. We do not particularize out of context, or generalize against the evidence or when this would give rise to contradictions.

Hume doubted the validity of generalization because he thought that we adopt a general proposition like All X are Y, *only* on the basis of the corresponding particular Some X are Y. But if the latter was *sufficient* to (inductively) establish the former, then when we were faced with a contingency like Some X are Y and some X are not Y, we would be allowed to generalize both the positive and negative particulars, and we would find ourselves with a contradiction³⁴ in our knowledge, viz. with both All X are Y and No X are Y.

But since contradiction is error, according to the 2nd law of thought, it follows that a particular is not by itself enough to confirm a generality. To do so, we need also to first adduce that the opposite particular is not currently justified. Note well what we have shown here: this criterion for generalization follows from the law of non-contradiction. Hume and his skeptical successors did not take this additional criterion into account. They noticed the aspect of 'confirmation', but ignored that of 'non-rejection'.

Or more precisely a contrariety.

2. The Principle of Induction

The uniformity principle ought to be viewed as an application of a much larger and important principle, which we may simply call *the principle of induction* (in opposition to the so-called problem of induction). This all-important principle could be formulated as follows: *given any appearance, we may take it to be real, until and unless it is found to be illusory*.³⁵

This is the fundamental principle of inductive logic, from which all others derive both their form and their content. And indeed, this is the way all human beings function in practice (with the rare exception of some people, like Hume, who want to seem cleverer than their peers). It is, together with Aristotle's three laws of thought, the supreme principle of methodology, for both ordinary and scientific thought, whatever the domain under investigation³⁶.

I have formulated and stressed this principle since I started writing logic, although I here name it "principle of induction" for the first time. See, for instances: *Future Logic*, chapter 2, etc.; *Phenomenology*, chapter 1, etc.; *Ruminations*, chapters 1 and 2.

I stress that here, to forestall any attempt to split ordinary and scientific thought apart. We should always stress their continuity. The difference between them is (theoretically, at least) only one of rigor, i.e. of effort to ensure maximal adherence to logic and fact. This only means, at most, that more ordinary people fail to look carefully and think straight than do most scientists – but both groups are human. Another important thing to stress is that this method is the same for knowledge of matter or mind, of earthly issues or metaphysical ones, and so forth. The principle is the same, whatever the content.

Indeed, we could construe this principle of induction as *the fourth law of thought*. Just as the three laws proposed by Aristotle are really three facets of one and the same law, so also this fourth law should be viewed as implicit in the other three. Induction being the most pragmatic aspect of logic, this principle is the most practical of the foundations of rational discourse.

The principle of induction is a phenomenological truth, because it does not presume at the outset that the givens of appearance are real or illusory, material or mental, full or empty, or what have you. It is a perfectly neutral principle, without prejudice as to the eventual content of experience and rational knowledge. It is not a particular worldview, not an *a priori* assumption of content for knowledge.

However, in a second phase, upon reflection, the same principle favors the option of reality over that of illusion as a working hypothesis. This inbuilt bias is not only useful, but moreover (and that is very important for skeptics to realize) logically rock solid, as the following reasoning clearly shows:

This principle is self-evident, because its denial is self-contradictory. If someone says that *all appearance is illusory, i.e. not real*, which means that all our alleged knowledge is false, and not true, that person is laying claim to some knowledge of reality (viz. the knowledge that all is unreal, unknowable) – and thus contradicting himself. It follows that we can only be *consistent* by admitting that we are indeed capable of knowing some things (which does not mean everything).

It follows that the initial logical neutrality of appearance must be reinterpreted as in all cases an initial reality *that* may be demoted to the status of illusion if (and only if)

specific reasons justify it. Reality is the default characterization, which is sometimes found illusory. Knowledge is essentially realistic, though in exceptional cases it is found to be unrealistic. Such occasional discoveries of error are also knowledge, note well; they are not over and above it.

If we did not adopt this position, that appearance is biased towards reality rather than illusion, we would be stuck in an inextricable agnosticism. Everything would be "maybe real, maybe illusory" without a way out. But such a problematic posture is itself a claim of knowledge, just like the claim that all is illusory, and so self-inconsistent too. It follows that the interpretation of appearance as reality until and unless otherwise proved is the only plausible alternative.³⁷

If appearance were not, *ab initio* at least, admitted as reality rather than as illusion or as problematic, we would be denying it or putting it in doubt without cause — and yet we would be granting this causeless denial or doubt the status of a primary truth that does not need to be justified. This would be an arbitrary and self-contradictory posture — an imposture posing as logical insight. All discourse *must* begin with some granted truth — and in that case, the most credible and consistent truth

Worth also stressing here is the importance of working hypotheses as engines of active knowledge development. A skeptical or agnostic posture is essentially static and passive; taken seriously, it arrests all further development. Scientists repeatedly report the crucial role played by their working hypothesis, how it helped them to search for new data that would either confirm or refute it, how it told them what to look for and where and how to look (see for instance, Gould, p. 172). This is true not only of grand scientific theories, but of ordinary everyday concepts.

is the assumption of appearance as reality unless or until otherwise proved.

We may well later, *ad terminatio* (in the last analysis), conclude that our assumption that this appearance was real was erroneous, and reclassify it as illusory. This happens occasionally, when we come across conflicts between appearances (or our interpretations of them). In such cases, we have to review in detail the basis for each of the conflicting theses and then decide which of them is the most credible (in accord with numerous principles of adduction).

It should be stressed that this stage of reconciliation between conflicting appearances is not a consequence of adopting reality as the default value of appearances. It would occur even if we insisted on neutral appearances and refused all working hypotheses. Conflicts would still appear and we would still have to solve the problem they pose. In any case, never forget, the assumption of reality rather than illusion only occurs when and for so long as no contradiction results. Otherwise, contradictions would arise very frequently.

3. Regarding Husserl

Note well that I do not understand appearance in quite the same way Edmund Husserl does, as something *ab initio* and intrinsically mental; such a view is closer to Hume or even Berkeley than to me.

The ground floor of Husserl's phenomenology and mine differ in the primacy accorded to the concepts of consciousness and of the subject of consciousness. My own approach tries to be maximally neutral, in that appearances are initially taken as just 'what appears', without immediately judging them as 'contents of someone's consciousness'. Whereas, in Husserl's approach, the wider context of appearance is from the start considered as part and parcel of the appearance.

For me, some content comes first, and *only thereafter* do we, by a deduction or by an inductive inference, or perhaps more precisely by an intuition (an additional, secondary, reflexive act of consciousness), become aware of the context of consciousness and conscious subject. At this later stage, we go back and label the appearance as a "content of" consciousness, i.e. as something whose apparition (though not whose existence) is made possible by an act of consciousness by some subject. Content is chronologically primary, the context is secondary.

Husserl's philosophy, the Whereas in fact consciousness and its subject are present from the start, as soon as the appearance appears. Husserl's mistake, in my view, is to confuse logical order and chronological order, or ontological and epistemological. Of course, ontologically, logically appearance and consciousness someone being conscious; but and chronologically and epistemologically, they occur in succession.

As a result of this difference, his approach has a more subjectivist flavor than mine, and mine has a more objectivist flavor than his. Note, however, that in his later work Husserl tried more and more to shift from implied subjectivism to explicit objectivism.

4. The Flexibility of Induction

We have seen the logic of induction in the special case of generalization. Given the positive particular 'Some X are Y' (appearance), we may generalize to the corresponding generality 'All X are Y' (reality), *provided* we have no evidence that 'Some X are not Y' (no conflicting appearance). Without this caveat, many contradictions would arise (by generalizing contingencies into contrary generalities); that proves the validity of the caveat. If (as sometimes occurs) conflicting evidence is eventually found (i.e. it happens that Some X are not Y), then what was previously classed as real (viz. All X are Y) becomes classed as illusory (this is called particularization).

Induction is a flexible response to changing data, an ongoing effort of intelligent adaptation to apparent facts. Few logicians and philosophers realize, or take into consideration, the fact that one of the main disciplines of inductive logic is harmonization. They discuss experiment, generalization observation and adduction, and deduction, with varying insight and skill, but the logic of resolving contradictions occasionally arrived at by those other inductive means is virtually unknown to them, or at least very little discussed or studied. This ignorance of, or blindness to, a crucial component of induction has led to many foolish theories³⁸.

Notice well, to repeat, the *conditional form* of the principle of induction: it grants credibility to initial appearances "until and unless" contrary appearances arise, which belie such immediate assumption. Thus, in

For example, Hempel's so-called paradox of confirmation

the case of the narrower uniformity principle, the initial appearance is the known few cases of similarity (or confirmation) and the fact of not having to date found cases of dissimilarity (or conflicting data); this allows generalization (or more broadly, theory adoption) until if ever we have reason or evidence to reverse our judgment and particularize (or reject, or at least modify, the theory).

The principle of induction may likewise be used to validate our reliance on intuition and sensory and inner perception, as well as on conception. It may also be applied to causality, if we loosely formulate it as: order may be assumed to exist everywhere, until and unless disorder appears obvious. However, the latter principle is not really necessary to explain causality, because we can better do that by means of regularity, i.e. with reference to the uniformity principle, i.e. to generalization and adduction.

In any case, the principle of induction is clearly a *phenomenological* principle, before it becomes an epistemological or ontological one. It is a logical procedure applicable to *appearance as such*, free of or prior to any pretensions to knowledge of reality devoid of all illusion. The claims it makes are as minimal as could be; they are purely procedural. It is for this reason as universal and indubitable as any principle can ever be.

Moreover, the principle of induction (and likewise its corollary the uniformity principle) applies equally to the material, mental and spiritual realms. It is a valid method of dealing with data, independently of the sort of data involved, i.e. irrespective of the 'substance' of the data. Many people associate induction exclusively with the physical sciences, but this is misconceived. Inductive

logic sets standards of judgment applicable in all fields – including in psychology and in moral and spiritual concerns.

19. CHAPTER NINETEEN

Drawn from Logical and Spiritual Reflections (2008), Book 3, chapter 2.

THE PRIMACY OF THE LAWS

1. Briefly Put

Aristotle's laws of thought cannot be understood with a few clichés, but require much study to be fathomed. *The laws of thought* can be briefly expressed as³⁹:

- 1. A thing is what it is (the law of identity).
- 2. *A thing cannot at once be and not-be* (the law of non-contradiction).
- 3. *A thing cannot neither be nor not-be* (the law of the excluded middle).

These three principles imply that whatever is, is something – whatever that happens to be. It is not something other than what it is. It is not nothing whatsoever. It is not just anything. If something exists, it has certain features. It cannot rightly be said to have features other than just those, or no features at all, or to both have and lack features.

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These are of course simple statements, which have to be elaborated on. Note that when I speak of a 'thing' here, I mean to include not only terms (percepts and concepts, or the objects they refer to), but also propositions (which relate percepts and/or concepts).

A thing is what it is, whether we know what it is or not, and whether we like what it is or not. It is not our beliefs or preferences that make a thing what it is. It is what it is independently of them. Our beliefs can be in error, and often are. How do we know that? By means of later beliefs, based on better information and/or arguments.

However, a thing can have conflicting features in different parts or aspects of its being. Notably, a thing can change over time. So long as these differences are separated in respect of place, time, or other relations to other things, such as a causal relation – the contradiction is not impossible. But if we refer to the exact same thing, at the same place and time, and the same in all other respects, contradiction is logically unacceptable – it is indicative of an error of thought.

Also, we may well have no idea or no certainty what some (indeed, many or most) features of a thing are. Such problematic situations are indicative of our ignorance, and should not be taken to imply that the thing in question necessarily lacks the unknown features, or neither has nor lacks certain features, or both has and lacks them.

All these logical insights are evident in our ordinary thoughts and in scientific thinking. If we look upon our discourse clearly and honestly, we see that our conviction in every case depends upon these criteria. Occasionally, people try to make statements contrary to these criteria; but upon further analysis, they can always be convincingly shown to be erring.

These general logical principles, and certain others (notably the principle of induction, to name one), help us regulate our thinking, ensuring that it sticks as close as possible to the way things are and that we do not get

cognitively lost in a complex maze of fantastical nonsense.

They do not force us to be truthful, or guarantee the success of our knowledge endeavors, but they provide us with crucial standards by which can test our progress at all times. (More will be said about these principles in this volume, in addition to what has already been said in the past.)

If the crucial epistemological and ontological roles of Aristotle's three laws of thought in human knowledge are not sought out and carefully studied, there is little hope that these little jewels of human understanding will be treasured. It takes a lifetime of reflection on logical and philosophical issues to fully realize their impact and importance.

2. Antagonism to the Laws

I marvel at people who think they can show reason to be unreasonable. Leaning on hip, postmodern sophists, like Wittgenstein or Heidegger, or on more ancient ones, like Nagarjuna, they argue confidently that the foundations of rationality are either arbitrary, or involve circularity or infinite regression. They do not realize that their intellectual forebears were in fact either ignorant of logic or intentionally illogical.

Many critics of the laws of thought simply do not understand them; no wonder then that they are critical. They have very narrow, shallow views about the laws of thought; they have not studied them in any breadth or depth. For instance, to some people, brought up under "modern" symbolic logic, the laws of thought are simply X=X, $\sim(X+\sim X)$ and $\sim(\sim X+\sim\sim X)$. Given such simplistic,

superficial statements, no wonder the laws seem arbitrary and expendable to them.

The laws are not a prejudice about the world, as some critics try to suggest. The law of identity does not tell us about some particular identity, but only tells us to be aware of how and what things are or even just appear to be. The law of non-contradiction does not favor the thesis that something is X, or the thesis that it is not X; it allows for us sometimes facing dilemmas, only forbidding us to settle on the implied contradictions as final. The law of the excluded middle does not deny the possibility of uncertainty, but only enjoins us to keep searching for solutions to problems.

If nothing were known, or even knowable, as some claim, this would not constitute a good reason to dump the laws of thought – for these laws make no claims about the specific content of the world of matter, mind or spirit. They make no *a priori* demand regarding this or that thesis. They only serve to regulate our cognitive relation to the world, however it happens to be or seem. They show us how to avoid and eliminate errors of reasoning.

These laws can for a start teach us that to claim "nothing is known or knowable" is self-contradictory, and thus illogical and untenable.

Such a claim, about the nonexistence or impossibility of knowledge as such, must be admitted to itself be an allegation of knowledge (such admission being a requirement of the law of identity). Therefore, it is unthinkable that any Subject might attain such alleged knowledge of its total ignorance (because such attainment would be against the law of noncontradiction). We could not even adopt a negative

posture of denying both knowledge and knowledge of ignorance (in an attempted bypass of the law of the excluded middle), for that too is an assertion, a claim to established fact, a claim to knowledge.

All these rational insights are not open to debate.

Antagonism to the laws of thought is sure and incontrovertible proof that one is erring in one's thinking. How might such antagonism be *systematically* justified without appeal to those very laws? One couldn't claim to be generalizing or adducing it from experience, for this would appeal to the law of generalization or the principle of adduction, which are themselves based on the laws of thought. One couldn't claim to be drawing some sort of syllogistic or other deductive conclusion, for the same reason. Such antagonism can only be based on arbitrary assertion, without any conceivable rational support.

3. Counterarguments

Arguments like this in favor of the laws of thought are claimed by their opponents to be 'circular' or 'infinitely regressive' – i.e. arbitrary. But to point to the fallacy of circularity or infinite regress is to appeal to the need to ground one's beliefs in experience or reasoning – which is precisely the message of the laws of thought. Therefore, those who accuse us of circularity or infinity are doing worse than being circular or infinite: they are appealing to what they seek to oppose; they are being self-contradictory, as well as arbitrary!

It is our faculty of logical insight or rationality that teaches us to beware of arbitrary propositions, which are sometimes given an illusion of proof through circular or infinite arguments. One cannot deny this very faculty of logical insight by claiming that it can only be proven by circular or infinite arguments. This would turn it against itself, using it to justify its own denial. It would constitute another fallacy – that of "concept stealing".

The proposition "if P, then P" is not circular or infinite – it is true of all propositions. Such a proposition does not "prove" the truth of P, but merely acknowledges P as a claim that may turn out to be true or false. If one proposes "if P, then P" as a proof of P, one is then of course engaged in circularity or infinite regression; but otherwise no logical sin is involved in affirming it. On the other hand, the paradoxical proposition "if P, then not P" does imply P to be false. To affirm P as true in such case is a logical sin, for P is definitely implied false by it. The laws of thought are not circular or infinite – they are iust consistent with themselves. It is their opponents who are engaged in fallacy – the failure to think reflexively, and realize the implications of what they are saying on what they are saying. To deny all claims to knowledge is to deny that very claim too – it is to be self-inconsistent. One logically must look back and check out whether one is self-consistent; that is not circularity, but wise reflection.

The laws of thought are not based on any particular argument, but the very basis of all reasoning processes. This is not an arbitrary starting point; it is an insight based on observation of all reasoning acts, an admission of what evidently carries conviction for us all. These laws cannot be disregarded or discarded, simply because they are so universal. That these laws do not lead to any paradox adds to their force of conviction; but that too is just an application of their universality. They encapsulate what we naturally find convincing in practice, provided

we are not seeking dishonestly to pretend otherwise in theory.

The laws of thought may be viewed as specific laws of nature: they express the nature of rational thought, i.e. of logical discourse. By logic is here meant simply a mass of experiences – namely, all the 'events having the form expressed by the laws of thought'. That is, logic refers to the concrete occurrences underlying the abstractions that we name 'laws of thought'. This is a primary given for which no further reason is necessary. It is not arbitrary, for it is the source of all conviction. To ask for a further reason is to ask for a source of conviction other than the only natural source of conviction! It is to demand the impossible, without reason and against all reason. It is stupid and unfair.

4. Our Pedestrian Path

If one examines the motives of critics of the laws of thought, one often finds an immature and irrational yearning for absolutes. They seek a shortcut to omniscience, a magic formula of some sort, and think the laws of thought are obstacles to this pipedream, and so they abandon these laws and seek truth by less restrictive means.

Our ordinary knowledge is very pedestrian: it progresses step by step; it advances painstakingly by trial and error; it is rarely quite sure, and certainly never total and final. This relativity of common knowledge unsettles and displeases some people. To them, such inductive efforts are worthless – knowledge that is not omniscient is not good enough; it is as bad as no knowledge at all. Thus,

they reject reason. This is an unhealthy attitude, a failure of 'realism'.

Let's face it squarely: our knowledge as a whole has no finality till everything about everything is known. And how, by what sign, would we know we know everything? Ask yourself that. There is no conceivable such sign. Our knowledge is necessarily contextual; it depends on how much we have experienced and how well we have processed the data. There is no end to it.

Even so, at any given stage of the proceedings, one body of knowledge can conceivably be considered *better* than another, given experience and reasoning so far. *To be better does not necessarily mean to be the best – but it is still better than to be worse or equal.* That is a realistic posture, and a source of sufficient security and satisfaction.

A phenomenological approach to the problem of knowledge is necessary, to avoid erroneous views. It starts with mere *appearance*, whether of seemingly material or mental phenomena (bodies and ideas), or of spiritual intuitions (of self, and its cognitions, volitions and valuations)⁴⁰. The contents of one's consciousness

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Note well that I do not posit perception itself as the starting point of knowledge, as some do. Perception is a relational concept – it is perception of something by someone. Before we become aware of our perceptual ability, we have to exercise it – i.e. we perceive something (other than the perceiving itself). The empirical basis of our concept of perception is our common experience of sensory and mental phenomenal content. When you and I were young children, we were perceiving such phenomena – only later when we became older did we form a concept of perception. Therefore perception as such cannot be taken as a primary in the order of things.

are, *ab initio*, appearances; this is a neutral characterization of what we are conscious of, the raw data and starting point of knowledge. Our first cognitive task is to acknowledge these appearances, as apparent and just as they appear, coolly observing them without interference or comment before any further ado.

It is equally naïve to assume as primary given(s) matter, or mind, or spirit; what is certainly given in experience is the appearance of these things. Much logical work is required before we can, *ad terminatio*, establish with reasonable certainty the final status of these appearances as matter, mind or spirit. We may indeed to begin with assume all such appearances to be real; but in some specific cases, due to the discovery of contradictions between appearances or to insufficiencies in our theories about them, we will have to admit we were wrong, and that certain appearances are illusory.

There is an order of things in the development of knowledge that must be respected. Everything beyond appearances is 'theory' – which does not mean that it is necessarily false, only that it must be considered more critically. Theory involves the rational faculty in one way or another. What is theory needs to be sorted out, organized, kept consistent, made as complete as possible. This is where the laws of thought are essential. But these laws cannot make miracles; they can only help us (with the aid of our intelligence and imaginative faculty) formulate and select the best theory in the present context of knowledge.⁴¹

Note well: the laws of thought cannot by themselves immediately tell you whether what you have apparently perceived is true or false – but what they can tell you is that you should notice well what you did perceive (its configuration,

Human knowledge is thus essentially inductive and probabilistic, depending on the scope and quality of experience, and then on successive generalizations and particularizations, or on competing larger hypotheses requiring ongoing comparative confirmation or refutation. The laws of thought are involved at all stages of this process, regulating our judgments to minimize its chances of error.

the phenomenal modalities, i.e. the sights, sounds, etc., apparent times, places, and so forth). Similarly for introspective data of intuition. The question of truth and falsehood for any single item of experience can only be solved progressively, by holistic consideration of all other experiential items, as well as by logical considerations (including consistency and completeness). This is the inductive process.

20. CHAPTER TWENTY

Drawn from Logical and Spiritual Reflections (2008), Book 3, chapters 3 & 4.

1. Ontological Status of the Laws

Discussion of the laws of thought inevitably arrives at the question: are these ontological or epistemological laws, or both; and if both in what sequence? Furthermore, what is their own ontological status — i.e. where do they 'reside', as it were? Are they 'out there' somehow, or only 'in our minds'?

As my thought on the issue has evolved over the years⁴², I am now convinced that the traditional term "laws of thought" is accurate, in that these statements are primarily *imperatives* to us humans on how to think about reality, i.e. how to ensure that we cognitively treat the givens of appearance correctly, so that our ideas remain reasonably credible possible expressions of reality and do not degenerate into delusions.

Why? Because Nature can only posit; and so 'negating' depends on Man. That is to say, the world process is always positive; negation involves a particular relation between a conscious being and that presentation. For negation to occur, a conscious being has to project and look for something positive and fail to find it; otherwise, all that occurs is positive.

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See especially my *Ruminations*, chapter 9 ("About Negation").

Thus, when we state the laws of non-contradiction and of the excluded middle, formally as "X and not X cannot both be true" and "X and not X cannot both be untrue", we mean that such *claims* (i.e. 'both true' or 'both untrue') cannot reasonably be made within discourse. We mean that 'X and not-X', respectively 'not-X and not-not-X', cannot correctly be claimed as known or even as reasonably opined.

Conjunctions of (positive or negative) contradictories are thus *outside the bounds of logically acceptable discourse*. These two laws of thought together and inseparably effectively *define* what we naturally mean by negation. Note well, 'middles' between contradictories are as unthinkable as coexisting contradictories.

Note that the law of identity is also tacitly involved in such definition of negation, since before we can understand the logical act of negating, we must grasp the fact of positive presence. So, it is not just the second and third laws that define negation, but strictly speaking also the first.

Such definition is, needless to say, not arbitrary or hypothetical. Were someone to propose some other definition of negation (e.g. using the law of noncontradiction alone, or some other statement altogether), this would only produce an equivocation – the natural definition with reference to the three laws of thought would still be necessary and intended below the surface of all discourse, however willfully suppressed.

From this it follows, by an extrapolation from logically legitimate thought to reality beyond thought, that these laws of thought (or, identically, of logic – 'logic' meaning 'discourse' by a thinker) are also necessarily laws of reality.

Words are symbols, and symbols can be made to do what one wills, because they are per se not in fact subject to the laws of thought. That is to say, mental gymnastics like placing the symbol X next to the symbol not-X are indeed feasible, but that does not mean that the things the symbols symbolize can equally well be conjoined.

To *label* an observed illusion or a deliberate fantasy as 'real' does not make it in fact real. We can easily *verbally imagine* a 'reality' with non-identity, contradictions and inclusions of the middle, but we cannot *actually conjure* one.

2. The Need for a Subject

As for the status of the laws of thought themselves: being products of reason, their existence depends on that of a conscious – indeed, rational – subject. All particular acts of reasoning – such as negation, abstraction, measurement, classification, predication, generalization, etc. – depend for their existence on some such rational subject (e.g. a man).

Take away all such subjects from the universe, and only *positive particular* things or events will remain. Without an act of negation, no mixing of or intermediate between contradictories occurs in thought; all the more so, they cannot occur outside thought. Similarly, with regard to abstraction and other acts of the reasoning subject.

Concepts like similarity, difference, uniformity, variety, continuity, change, harmony, contradiction, and principles like the laws of thought, being all outcomes of such ratiocinative acts, are similarly dependent for their existence on there being some appropriately conscious subject(s).

These concepts and principles are, we might say, inherent in the world in the way of a potential; but without the involvement of such a subject, that potential can never be actualized.

These concepts and principles depend *for their existence* on there being conscious subjects to form them – but their *truth or falsehood* is not a function of these subjects. Their occurrence is dependent, but the accuracy of their content when they occur is a different issue. It is not subjective and relative, but on the contrary objective and absolute.

It is important not to draw the wrong inference from the said existential dependence, and to think it implies some sort of relativism and subjectivism (in the most pejorative senses of those terms) as regards issues of truth and falsehood.

No: the 'reasonableness' of our basic concepts and principles is the guarantee of their truth. To suggest some other standard of judgment, or the equivalence of all standards of judgment, is to tacitly claim such other standard(s) to be somehow 'reasonable'. A contradiction is involved in such an attitude. Of course, you are free to propose and accept contradictions, but you will have to pay the cognitive and other consequences. As for me, I prefer to stand by and rely on what is evidently reasonable.

3. Fuzzy Logic

In some cases, X and notX are considered not to be contradictory, because the term or proposition X is too *vague*. If precisely what things X refers to is unclear, or if the exact boundaries of some individual thing labeled

X are uncertain, then obviously the same can be said for the negative complement 'not X' (see diagram further on). In such cases, the terms or propositions involved are simply problematic. 43

It is easy to see how such realization can lead to a general critique of the human rational act of naming, and to a philosophy of Nominalism. For, if we observe our concepts carefully, we must admit that they are always in process – they are never fully formed, never finalized. Our ordinary knowledge is predominantly *notional*, tending towards precise conception but never quite attaining it. Thus, the meaning of words (or even of wordless intentions) is in flux – it is becoming rather than being.

This is not a merely epistemological critique, but one that has ontological significance. What is being said here is that things, the objects of our consciousness (be they objective or subjective) are difficult, if not impossible, to precisely pin down and delimit. This is true of concrete individuals and of abstract classes. It is true of matter (e.g. where does the body of a man end: if I breathe air in or out, or swallow water or spit it out, at what stage does the matter entering or exiting become or cease to be part

Note also that in some cases we face a range of things, or different degrees of something, and we erroneously call the extremes X and notX – whereas in fact if X is used for one extreme, then notX must refer to all other degrees; and vice versa, if notX is used for one extreme, then X must refer to all other degrees; otherwise, we would be left with some intermediate referents without name (i.e. as neither X nor notX). It also happens that X and notX are made to overlap in our thinking, so that X and notX are made to seem compatible. These are simply common errors of concept formation; they do not justify any denial of the laws of thought.

of 'my body'?), and it is true of mind and of soul (who knows where their respective limits are?).

Ultimately, we realize, everything is one continuum, and the divisions we assume between things or classes are ratiocinative and intellectual interpositions. We cannot even truly *imagine* a fine line, a separation devoid of thickness, so how can we claim to *even mentally* precisely separate one thing from another? All the more so in the physical realm, such division is impossible, given that all is composed of continuous and endless fields.

Another critical tack consists of saying that all our experience (and consequently all our conceptual knowledge) is illusory, in the way that a dream is illusory (compared to awake experience). In a dream world, X and not X can apparently both coexist without infringing the law of non-contradiction. Distinctions disappear; opposites fuse into each other.

But this is only superficially critical of our ordinary knowledge. For what is said to coexist here are 'the appearance of X' and 'the appearance of not X' – and not 'X' and 'not X' themselves. We have symbols, or standins, or effects, instead of the objects themselves. So, this is nothing that puts the law in doubt, but rather a viewpoint that by its own terminology (reference to illusion) confirms adherence in principle to that law.

Such reflections lead us to the idea of fuzzy logic, as opposed to definite logic. The difference is illustrated in the following diagram:

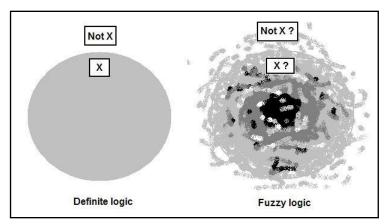


Figure 20.1 Definite and Indefinite terminology.

Aristotle's three laws of thought are aimed at a "definite logic" model — in this model, terms and theses are in principle *clearly* definable and knowable; or at least, this is the assumption in most cases, though in a minority of cases there might be some measure of temporary vagueness and doubt. But this ideal is in practice rarely met, and we should rather refer to a "fuzzy logic" model — wherein the assumption in most cases is that limits are chronically unclear and hard to establish with certainty, though exceptions to this rule must be acknowledged for the sake of consistency.

Ordinarily, our reason functions in a self-confident manner, from conviction to conviction, unfazed by the changes in our 'utter convictions' that in fact occur over time. In other words, we lay the stress on what we (think we) know, and minimize what we consider still unknown or the errors we made in the past. This is the approach of definite logic, an essentially 'deductive' approach. The idea of a fuzzy logic is that we ought to, on the contrary, at the outset acknowledge our cognitive limitations and the ongoing flux of knowing, and opt more thoroughly for an 'inductive' approach.

According to this view, the logical perfection presupposed by Aristotle is largely mythical. Our concepts, propositions and arguments are, in practice, usually exploratory, tentative, approximating, openended with regard to referents, open to change, of uncertain pertinence and truth, and so forth. Our rational faculty works by trial and error, constantly trying out different overlays that might fit a momentarily glimpsed reality, then noticing an apparent mismatch trying out some more adjusted overlay, and so on without end.

Things are rarely quite the way we think of them, and yet our thought of them is not entirely wrong. Hence, we might well say that it is not correct to say that the referents of X fit exactly what we mean by 'X'; and it is not correct to say that they do not all or wholly fit in. Hence, it might be said that certain things are both X and not X, and neither X nor not X — without really intending to imply any contradiction, but only in the way of a reminder to ourselves that we are functioning in shifting sands.

Such a logical posture does not really constitute a denial of the laws of thought. They continue to help us make sense of things. Their precision helps us sort out the vagueness and uncertainty we actually face in practice. They give us an ontological and epistemological ideal we can tend to, even if we can never hope to fully and permanently match it.

4. Stick to Logic

In the light of the aforementioned difficulties, some logicians and philosophers are tempted to give up on all rational knowledge, and more specifically the laws of thought. However – and this is the point I am trying to make here – this would be a tragic error. The error here is to think that we humans can navigate within the sea of phenomena and intuitions without the guiding star of the laws of thought. Even if in particular cases these laws are often hard to *apply* decisively, they help us do our best to make sense of the world of appearances we face.

We have to stick with logic. It provides us with a minimum of firm ground in the midst of the shifting sands of experience and conception. Even if it is only an ideal, a theoretical norm, its importance is crucial. Without logic, we have no way to sort out changing impressions and deal with the practical challenges of our existence. Is that not the very definition of madness, insanity?

Nevertheless, sticking to logic should not be taken to signify rigid conventionality, or fearful similar mindedness. or excesses of 'rationalism'. not exclude enlightened logic does consciousness, flowing with the current of life, having faith, and similar liberating attitudes. Logic is a tool, not an end in itself. To give up a useful tool is stupid; but it is also stupid not to know when to put down the tool.

There is a stage in the life of the spirit when logical ifs and buts become irrelevant, or even disturbing, and it is wise to just be.

21. CHAPTER TWENTY ONE

Drawn from Logical and Spiritual Reflections (2008), Book 3, chapter 5.

MISREPRESENTATION OF ARISTOTLE

1. Ups and Downs of Aristotelianism

Aristotle's three laws of thought are often misrepresented, in the service of some doctrine or other. Often, nowadays, the motive is a desire to defend Buddhist antinomies; some decades ago, the motive might have been to defend Marxist contradictions; before that, maybe Hegelian ones. Usually, the proposed reading of Aristotle is unfair to him, a misrepresentation of his evident intentions.

During the late Middle Ages in Europe, the authority of Aristotelian philosophy was unmatched. The reason for this was that before that period many of the works of Aristotle (384-322 BCE) had been mostly lost to Christian Europe; when they were rediscovered, the superiority in many respects of the knowledge they contained was such that his influence became great⁴⁴.

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The rediscovery occurred mostly by way of translation into Latin (from Arabic, sometimes via Hebrew) of Greek classical texts in the libraries of Moslem Spain. These included works by Aristotle on physics, metaphysics and ethics. Aristotle's thought was also made known to the West indirectly through commentators like Avicenna (Persia, 11th century) and Averroes (Muslim Spain, 12th century). His

But, as a result of that overwhelming belief in everything Aristotelian, scientists of the Renaissance period and after often had to struggle hard to overcome what had become an academic bias.

It could be argued, paradoxically, that Aristotle's influence on the Christian European mind was one of the factors that led to the intellectual Renaissance; nevertheless, just as students must rebel from teachers to some extent to innovate and advance, an anti-Aristotelian reaction had to occur. Many historians thus regard Aristotelianism as the impetus of the Renaissance and thus of modern science.

Note moreover, Aristotle himself was no rigid ideologue; his approach was open-minded and adaptive, what we now call 'scientific'. Although many of his material opinions⁴⁵ have turned out to be false, they were quite reasonable for his period of history – and for the Middle Ages. Had he still been around in the modern era, he would no doubt have adjusted his views.

Opposition to Aristotelianism, ranged over the special sciences, more philosophical issues and logical aspects, in no particular order. With regard to his logical work, the greater emphasis Francis Bacon put on induction was indeed a marked improvement; whereas, attempts in the late 19th and early 20th centuries to supersede Aristotle's formal logic with more systematic deductive approaches seem (to me at least) rather pretentious. The attempts,

influence reached its peak perhaps with the writings of Thomas Aquinas (Italy, 13th century).

For example, his cosmological views, which led to the Ptolemaic model that Copernicus and Galileo had to overcome.

lately, to belittle or do away with Aristotle's laws of thought fall in the same category (again, in my opinion). In many cases, criticisms of Aristotle's thought were and are of course justified. But in many cases, too, the critics were and are just (I suspect) seeking a shortcut to academic notoriety, taking an easy ride on the ongoing wave (in some quarters) of 'Aristotle bashing'. It is very easy to be critical regarding someone who cannot answer back; I daresay, if that genius were still around, they would not dare.

2. Aristotle Bashing

A case in point (taken at random) is the following presentation, drawn from an Internet site⁴⁶. I quote:

The three laws of "formal logic" which Aristotle set down in his Posterior Analytics are as follows: (1) Law of Identity: Each existence is identical with itself; (2) Law of Noncontradiction: Each existence is not different from itself; (3) Law of Excluded Middle: No existence can be both itself and different from itself.

Of course, nowhere in the *Posterior Analytics*, or anywhere else in Aristotle's known writings, are such inane formulations of his laws of thought to be found. Anyone who has read Aristotle knows this is not his

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History and Theory of Psychology Course, by Paul F. Ballantyne, Ph.D. "Aristotelian and Dialectical Logic", in posted May 2003 at http://www.comnet.ca/~pballan//section1(210).htm. (I was recently pointed to this website by a Buddhist correspondent arguing against Aristotelian logic; that is how I came across it.)

language or terminology, nor his thought or intent. He does not speak of "existences" and is not concerned with whether or not they are "identical with" or "different from" themselves.

These statements are, admittedly, not presented as verbatim quotations; but they are not, either, declared to be mere readings or interpretations; they are made to seem like loyal paraphrases. But they are not a fair statement of what Aristotelian logic is about. It is not about tautology or the lack of it, not even in an ontological sense.

In Aristotle's *Posterior Analytics*, we find the following statements of the law of non-contradiction: "it is impossible to affirm and deny simultaneously the same predicate of the same subject", and of the law of the excluded middle: "every predicate can be either truly affirmed or truly denied of every subject".⁴⁷

But the above author seems rather to base his formulations on common statements of the laws of thought, like "A is A", "A cannot be not-A" and "Either A or not-A". Such statements, however, are not meant as comprehensive expressions, but as shorthand formulas; they are more like titles, stand-ins for fuller statements that comprise all that can be said about these laws. The simplest way to read them is as follows:

Both these statements are there (in Book 11) referred to as laws, and the latter is specifically called the law of the excluded middle. Translation by G. R. G. Mure. See http://graduate.

gradsch.uga.edu/archive/Aristotle/Posterior_Analytics_(analytic).txt

Or at least the first two; for the third law he misconceives altogether. See further on.

- 1. Something that is evidently A must be admitted to be A.
- 2. Something admitted to be A cannot also be claimed not to be A (i.e. no thing can be claimed both to be A and not to be A).
- 3. And no thing can be claimed neither to be A nor not to be A.

In this primary reading, note well, the term "A" is everywhere a predicate, as Aristotle presents it, rather than a subject, as it may seem. In all three cases, the tacit subject of the proposition is "some thing", an individual thing under consideration, i.e. any apparent object of cognition. Moreover, all three propositions are primarily logical or epistemological statements, rather than ontological ones. They tell us *how to behave* in our discourse or cognition.

In a second phase, we can give "A" the role of subject that it superficially has in the expressions "A is A" and "A cannot be not-A", and "Either A or not-A". Such perspective suggests a more ontological reading of these laws, namely that every existent has a particular identity, i.e. 'a nature', whatever that happen to be.

Each thing is something specific (say "A"), not just anything whatsoever ("both A and not A"), nor nothing at all ("neither A nor not A"). It includes some distinguishable aspects and excludes others: it is not infinitely elastic in appearance. It neither includes nor excludes everything. It cannot include things incompatible with it ("contradictions" of it). Its negation may replace it, but nothing in between (no "middle") can replace both it and its negation.

Note this: the law of the excluded middle could, in analogy to the law of non-contradiction, equally well be called the law of non-neutrality. These laws respectively tell us that there is *no common ground and no neutral ground* between A and not-A. They ontologically together firmly separate A and not-A, allowing of no wishywashy togetherness or further possibility. They do not however epistemologically exclude that we might (occasionally, though not invariably) come across contradiction or uncertainty in our thinking.

Even such interpretations ought not, in any event, be treated as the whole of the meaning of the laws of thought, but more modestly as a beginning of explication⁴⁹. They make clear, anyway, that these laws are not about equation or non-equation of things or symbols with themselves, as the already mentioned author's formulations misleadingly suggest.

Additionally, the wording he proposes for the law of the excluded middle "No existence can be *both* itself *and* different from itself" – is *formally* wrong. This could be construed as a statement of the law of non-contradiction, perhaps, but the law of the excluded middle would (using

Many more issues arise in them, such as: what do we mean by predicating "A" of something? What is the relation between a label like "A" and what it intends? At what stage may we consider "A" the exclusive label of that thing? Further: so far, the laws have been expressed in terms of an individual thing; but what about their application to kinds of things? Clearly, these laws of thought are pregnant with the whole philosophical enterprise!

the same sort of language) have to be stated as "No existence can be *neither* itself *nor* different from itself".⁵⁰ Clearly, Aristotle's concern was whether the ideas we form about the world are compatible with experiential data and with each other. That is, one might say, an interest in the intersection between appearance and belief, or seeming reality and alleged knowledge. The two components of consistency with experience and other ideas correspond roughly to the tasks of inductive and deductive logic, respectively.

Elsewhere on the same website⁵¹, the said author apparently advocates, in lieu of his pseudo-Aristotelian laws, something called "materialist dialectics," which "holds that the basic rules of correct thinking should reflect a universe not in which the static and changeless is at the core but in which change is at the core." He goes on to propose three questionable alternative "laws", which place change at the center of things.

Thus, the above quoted debatable presentation of the laws of thought is used to convey the idea that Aristotle had a static view of existence, and to propose instead a

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Such a glaring formal misstatement of the law discussed tells us much about the critic's logical awareness, or lack of it! When I advised him by e-mail of this formal error, his response was at first flippant, then he made a small show of open-mindedness, but finally he made no effort to correct his statement. (N.B. I have just recently looked again at his website and found out that he now seems to his credit to have corrected this and other errors.)

In http://www.comnet.ca/~pballan/logic2.htm. He there quotes statements like "What Aristotle sees as the most basic characteristic of existence is static self-identity" by J. Somerville, p. 45 in "The Nature of Reality: Dialectical Materialism", in *The Philosophy of Marxism: An Exposition*. (Minneapolis: Marxist Educational Press, 1967/1983).

more dynamic alternative set of laws. It is tendentious rewriting of history.⁵²

3. Aristotle's Dynamism

In truth, Aristotle is *throughout his work very much* concerned with dynamic becoming as well as with static being. His laws of thought are precisely intended to help the intellect cope with variety and change, and remain lucid and poised in the midst of the cacophony of sense-impressions and ideas.

Consider, for instance the following statement drawn from his Metaphysics⁵³:

For a principle which every one must have who understands anything that is, is not a hypothesis; and that which every one must know who knows anything, he must already have when he comes to a special study. Evidently then such a principle is the most certain of all; which principle this is, let us proceed to say. It is, that the same attribute cannot at the same time belong and not belong to the same subject and in the same respect.

With characteristic intellectual accuracy, Aristotle expresses the law of non-contradiction by saying that nothing (i.e. no subject of a true proposition) can both be

For an understanding of **the logic of change** in formal terms, see in my works: *Future Logic*, chapter 17, and *Volition and Allied Causal Concepts*, chapter 14. See also, *Buddhist Illogic*, chapter 6.

Book 4, Part 3. (Translated by W. D. Ross.) Posted in the Internet Classics website at http://classics.mit.edu/Aristotle/metaphysics.4.iv.html

and not-be the same thing (i.e. have and not have the same predicate) in the same respect at the same time.

These last words are crucial to his statement, yet often ignored by dishonest critics such as the above quoted. By these words, Aristotle implied that something may well be subject to both a predicate and its negation – in different respects at the same time, or in the same respect at different times, or in different respects at different times.⁵⁴

He is not ignoring that a given thing may have a variety of aspects at once, or that it may change in various ways over time. He is simply reminding us that *in a given location and time of its being*, a thing cannot contradict itself. His intent is therefore clearly not an attempt to deny the existence of variety and change, but to affirm the consistency that things *nevertheless* display at any given place and time.

Evidently, the earlier quoted attempted reformulation of the laws of thought as "Each existence is identical with itself; not different from itself; and can[not] be both itself and different from itself" is not only an inaccurate rendition of Aristotle, but an extremely superficial one⁵⁵.

Grass can be green and yellow, but not in exactly the same places and times of its existence. Grass can mean what the cows eat or what the hippies smoke, but these two same words do not refer to the same things. If such differences of perspective are impulsively or dogmatically ignored - well, that does not prove that contradictions exist. To affirm contradiction is to lack depth.

Due no doubt to the influence of dimwitted modern symbolic logic, which makes every effort to reduce and limit these complex laws to their simplest possible expression, thus concealing most of their philosophical riches and depth. Why do they wish to so simplify? *In order to fit* logic into their simplistic "formal languages", designed by people (like Gottlob

Aristotle should be given the credit, respect and gratitude due him for a timeless and irreplaceable achievement.

Frege) with hopelessly bureaucratic minds, who think that standardizing thought processes makes them "scientific". But science is not a deductive, Cartesian enterprise; it is an inductive, evolutionary process. They claim to go above common 'intuition'; but actually, all they do is permanently impose their own insights, and thereby inhibit future insights in the field. Development of the science of logic flexibility, depends on alertness and rather institutionalization and rigidity.

22. CHAPTER TWENTY TWO

Drawn from Logical and Spiritual Reflections (2008), Book 3, chapter 6.

NOT ON THE GEOMETRICAL MODEL

1. How to Validate Logic?

Since (or insofar as) the "geometrical model" of theory justification involves arbitrary axioms, it is ultimately conventional. If the first principles ("axioms") of a body of alleged knowledge cannot apparently be justified by experience, but have to be based on mere speculation ("arbitrary"), such principles must be admitted to be without proof ("conventional"). If the axioms are unproven, then logically so are all claims based on them. This is freely admitted in the case of geometry (where for instance Euclid's fifth postulate may be replaced by assumptions), and similarly alternative in disciplines. mathematical Here. the apparent conventionality of certain axioms gives rise to the possibility of alternative systems, all of which might eventually be found useful in specific empirical contexts. But such a liberal attitude is impossible with regard to the science of Logic.

If we accept the geometrical model for Logic, then Wittgenstein's claim that "The propositions of logic are tautologies... [and] therefore say nothing" is made to

In *Tractatus*, 6 (quoted in A Dictionary of Philosophy).

seem true. But if we follow him, and admit that logic is meaningless babbling, then we must regard his own statement as meaningless – for, surely, it is itself intended as a "proposition of logic", indeed as the highest principle of meta-logic! Granting that, it is as if he has said nothing, and we can well ignore him and move on.

Similarly, some critics have accused Aristotle of 'begging the question' in his defense⁵⁷ of the laws of non-contradiction and of the excluded middle, i.e. of arguing in a circular manner using the intended conclusion(s) as premise(s). Here again, we can more reflexively ask: does that mean that the fallaciousness of such *petitio principii* is an incontrovertible axiom of logic? If the speaker is convinced by this rational principle as an irreducible primary, why not also – or even more so – by the second and third laws of thought? Can he justify his antipathy to circularity without committing circularity?

If Logic is not solidly anchored in reality through some more rigorous process of validation, then *all* knowledge is put in doubt and thus effectively invalidated. If all knowledge is without validity, then even this very claim to invalidity is without validity. The latter insight implies that this skeptical claim is itself invalid, like all others, note well. Therefore, since this skeptical claim is paradoxical, i.e. self-denying, the opposite claim (which is not inherently paradoxical) must be admitted as necessarily true. That is to say, we must admit that Logic has undeniable validity. Only given this minimal

For instance, in Chapter IV of his *Metaphysics*, Gamma.

admission, does it become possible to admit anything else as true or false.

I have said all this before again and again, but must keep repeating it in view of the ubiquity of statements I encounter these days in debates to the effect that Aristotle's three Laws of Thought are mere conventions. To make such a statement is to imply one has some privileged knowledge of reality – and yet at the same time to explicitly suggest no such knowledge is even conceivable. Thus, any such statement is self-contradictory, and those who utter it are either fools or knaves, kidding themselves and/or others.

The said laws of thought must not be viewed as axioms of knowledge within a geometrical model. The very idea of such a model is itself an offshoot of Aristotle's logic – notably his first-figure syllogism, where a broad principle or general proposition (the major premise) is used to derive a narrower principle or particular proposition (the conclusion). It follows that such a model cannot be used to justify Logic, for in such case we would be reasoning in circles and obviously failing to anchor our truths in reality.

2. The Inductive Nature of Knowledge

The only way out of this quandary is to notice and understand the inductive nature of all knowledge, including deductive knowledge. The *ground* of all knowledge is experience, i.e. knowledge of appearances (material, mental and spiritual appearances of all sorts). Without cognition of such data, without some sort of given data whatever its ultimate status (as reality or illusion), no knowledge true or false even arises.

There is no such thing as "purely theoretical" knowledge: at best, that would consist of words without content; but upon reflection, to speak even of words would be to admit them as experienced phenomena. To attempt to refer, instead, to wordless intentions does not resolve the paradox, either — for intentions that do not intend anything are not. There has to be some *experiential* basis to any knowledge claim. Whether the knowledge so based is indeed true, or the opposite of it is true, is another issue, to be sorted out next.

Logic comes into play at this stage, when we need to discriminate between true and false *theoretical* knowledge. We are always trying to go beyond appearances – and that is where we can go wrong (which does not mean we cannot sometimes be right). If we stayed at the level of pure appearance – the phenomenological level – we would never be in error. But because we existentially need to surpass that stage, and enter the rational level of consciousness, we are occasionally evidently subject to error.

Moreover, it is very difficult for us to remain at the purely phenomenological level: we seem to be biologically programmed to ratiocinate, conceptualize and argue; so we have little choice but to confront logical issues head on. The principles of Logic, meaning the laws of thought and the specific logical techniques derived from them, are our tools for sorting out what is true and what is false. We do not infer truth from these principles, as if they were axioms containing all truth in advance. Rather, these principles help us to discern truth from falsehood in the mass of appearances. Without some appearance to work with, logic would yield no conclusion – it would not even arise.

The validity of Logic is, thus, itself an inductive truth, not some arbitrary axiom. Logic is credible, because it describes how we actually proceed to distinguish truth from falsehood in knowledge derived from experience. No other logic than the standard logic of the three laws of thought is possible, because any attempt to fancifully propose any other logic inevitably gets judged through standard logic. The three laws of thought are always our ultimate norms of discursive conduct and judgment. They point us to an ideal of knowledge we constantly try to emulate.

3. The Crucial Role of Negation

This logical compulsion is not some deterministic force that controls our brain or mind. It is based on the very nature of the ratiocination that drives our derivation of abstract knowledge from concrete appearances. The primary act of ratiocination is **negation**: thinking "not this" next to the "this" of empirical data. That act is the beginning of all knowledge over and above experience, and in this very act is the secret of the laws of thought, i.e. the explanation as to why they are what they are and not other than they are.

For, whereas the law of identity (A is A) is an acknowledgment of experience as it presents itself, the law of non-contradiction (nothing can be both A and not A) and the law of the excluded middle (nothing can be neither A nor not A) both relate to things as they do *not* present themselves. These two laws define for us what *denial* of A means – they set the standard for our imagination of something *not* presented in experience at

the time concerned. Note this well, for no one before has noticed it that clearly.

Negation is the beginning of the "bigbang" of conceptual and argumentative knowledge, the way we pass from mere experience to concepts and principles; and *the only way* to test and ensure that our rational framework remains in reasonable accord with the givens of experience is to apply the laws of thought. Negations are never directly positively experienced: they are only expressions that we have not experienced something we previously imagined possible. There is no bipolarity in concrete existence; bipolarity is a rational construct.⁵⁸

The concept or term 'not X' can be interpreted to mean 'anything except X' (whether X here intends an individual thing or a group of things). To deny the law of non-contradiction is to say that this "except" is not really meant to be exclusive – i.e. that 'not X' can sometimes be included in 'X'. Again, to deny the law of the excluded middle is to say that this "anything" is not really meant to be general – i.e. that besides 'X' there might yet be other things excluded from 'not X'. Thus, to deny these laws of thought is to say: "I do not mean what I say; do not take my words seriously; I am willing to lie".

This is made clear if we consider what we mean when we say, for example, neither the dog nor the cat is in the room we are in. The absence of the dog and the absence of the cat look no different to us; what we actually see are the positive phenomena only, i.e. the carpet, the desk, the chairs, etc. We do not see a non-dog and a non-cat, or anything else that "is absent" from this room, as if this is some other kind of "presence". (However, it does not follow that non-dog and non-cat are equivalent concepts – for the cat may be present when the dog is absent and vice versa.)

23. CHAPTER TWENTY THREE

Drawn from Logical and Spiritual Reflections (2008), Book 3, chapter 7.

A POISONOUS BREW

1. Truth vs. Proof

Despite its name, the modern theory of knowledge called Intuitionism, developed by L.E.J. Brouwer⁵⁹, can be classed as an excessively deductive approach. It was, significantly, originally intended and designed for mathematics, and was thereafter by extrapolation applied to all knowledge⁶⁰. Equating for all intents and purposes the logical modality of proof with that of fact, "Intuitionist logic" rejects the law of the excluded middle (and hence the inference of a positive statement from a double negation).

Arguing that nothing can be claimed to be true if it is not *proved* to be true, Intuitionism claims to accept the law

⁵⁹ Holland, 1881-1966.

Such extrapolations are unfortunate: since mathematics deals with special classes of concepts (notably numerical and geometrical ones), insights concerning it cannot always be generalized to all other concepts. Inversely, comments concerning logic in general like the ones made here do not exclude the possibility of specific principles for the mathematical field. I am not a mathematician and do not here intend to discuss that subject.

of non-contradiction (since we cannot both prove A and prove not-A), but denies the law of the excluded middle (since we can both fail to prove A and fail to prove not-A). Thus, whereas Aristotle originally formulated these laws with reference to facts (as nothing can be A and not-A, and nothing can be neither A nor not-A), Brouwer focused on proof alone.

Many errors are involved in this change of perspective. For a start, one can refute it on formal grounds: just as we cannot both prove A and prove not-A, we cannot both *dis*prove A (= prove not-A) and *dis*prove not-A (= prove A). The fact that we can be in ignorance of both A and not-A, i.e. uncertain as to which is true and which is false, does not change the fact that A and not-A cannot be both true or, equally, be both false. The two laws are symmetrical and cannot be taken separately.

Note that Aristotle's approach was to set ontological standards that would serve as epistemological guides, whereas Brouwer tried to place epistemology squarely before ontology. The former implicitly allowed for knowledge not at all dependent on rational processes, viz. knowledge from experience, whereas the latter considered all knowledge as dependent on reasoning, i.e. as purely mental construction.

For classical logic, proof is a conflation of empirical givens and conceptual constructs. To anchor concepts in experience involves deductive methods, but the result is always inductive. If we precisely trace the development of our knowledge, we always find ultimate dependence on empirical givens, generalization and adduction. *There is no purely deductive truth* corresponding to the Intuitionist's notion of "proved" knowledge. The

Intuitionist's idea of proof is misconceived; it is not proof.

Even an allegedly "purely deductive system" would need to rely on *our experience* of its symbols, axioms and rules. Thus, it cannot logically claim to be purely deductive (or *a priori* or analytic, in Kantian terms), i.e. wholly independent of any experience. Moreover, *our understanding* of the system's significance is crucial. A machine may perform operations we program into it, but these are meaningless without an intelligent human being to consume the results. Brouwer's assumptions are rife with ignored or hidden issues.

Note too that Brouwer effectively regards "proved" and "not proved" to be exhaustive as well as mutually exclusive. This shows that he implicitly mentally relies on the law of the excluded middle (and on double negation), even while explicitly denying it. Certainly, we have to understand him this way – otherwise, if the terms proved and *un*proved (N.B. not to confuse with *dis*proved) allow for a third possibility, his theory loses all its force. That is, something in between proved and not proved (N.B. again, not to confuse with proved not) would have to somehow be taken into consideration and given meaning!

Brouwer's denial of the law of the excluded middle is in effect nothing more than a recognition that some knowledge has to be classed as *problematic*. That was known all along, and we did not need to wait for Mr. Brouwer to realize it. The law of the excluded middle does not exclude the possibility of problemacy, i.e. that humans may sometimes not know for sure whether to class something as A or not-A. On the contrary, the law of the excluded middle is formulated on that very

assumption, to tell us that when such problemacy occurs (as it often does), we should *keep looking* for a solution to the problem one way or the other.

The law of non-contradiction is similarly based on human shortcoming, viz. the fact that contradictions do occur occasionally in human knowledge; and its function is similarly to remind us to try and find some resolution to the apparent conflict. Note here the empirical fact that we do sometimes both seem to prove A from one angle and seem to prove not-A from another tack. In other words, if we follow Brouwer's formulation of the law of non-contradiction, that law of thought should also be denied!

The fact of the matter is that what we commonly call proof is something tentative, which may turn out to be wrong. The genius of classical logic is its ability to take even such errors of proof in stride, and lead us to a possible resolution. It is a logic of realism and adaptation, not one of rigid dogmas.

Indeed, if there is anything approaching purely deductive truth in human knowledge, it is the truth of the laws of thought. So much so, that we can say in advance of any theory of knowledge that if it postulates or concludes that any law of thought is untrue — it is the theory that must be doubted and not these laws. Such antinomy is sure proof that the theory is mixed-up in some way (just as when a theory is in disagreement with empirical facts, it is put in doubt by those facts).

In the case of Intuitionism, the confusion involved is a misrepresentation of what constitutes "proof". Only people ignorant of logic are misled by such trickery. Why on earth would we be tempted to accept Brouwer's idea of "proof" in preference to the law of the excluded

middle (which this idea denies)? Has he somehow "proved" his idea, or even just made it seem less arbitrary, more credible or more logically powerful than the idea of the law of the excluded middle? His view of proof is not even "proved" according his own standards – and it is certainly not proved (indeed it is disproved) by true logic.

Consider the implications of denials of the second and third laws of thought on a formal level. To deny the law of non-contradiction only is to wish to logically treat X and not-X as subcontraries instead of as contradictories. To deny the law of the excluded middle only is to wish to logically treat X and not-X as contraries instead of as contradictories. To deny both these laws is to say that there is no such thing as negation. All the while, the proponent of such ideas unselfconsciously affirms some things and denies others.

Reflect and ask yourself. If X and not-X cannot be contradictories, why should they be contraries or subcontraries? On what conceivable basis could we say that incompatibility (as that between X and not-X) is possible, but exhaustiveness is not; or vice versa? And if nothing can be incompatible and nothing can be exhaustive – what might negation refer to? It is clear that all such proposed antinomial discourse is absurd, devoid of any sort of coherence or intelligence. It is just a manipulation of symbols emptied of meaning.

2. Double Negation

The deeper root of Intuitionist logic is of course *a failure* to understand the nature of negation. What does 'not' mean, really? How do we get to know negative terms,

and what do they tell us? How does negation fit in the laws of thought? I will not go far into this very important field here, having already dealt with it in detail in the past⁶¹; but the following comments need be added.

Another, related weakness of Intuitionist is *ignorance of inductive logic*. As already stated, Brouwer functioned on an essentially deductive plane; he did not sufficiently take induction into consideration when formulating his ideas. In a way, these were an attempt to get beyond deductive logic; but his analysis did not get broad enough.

This can be illustrated with reference to *double negation*. On a deductive plane, negation of negation is equivalent to affirmation. This is an implication and requirement of the laws of thought. However, on an inductive plane, the matter is not so simple, because negation is always a product of generalization or adduction. That is to say, 'not' always means: 'so far, not'; i.e. it is always relative to the current context of knowledge.

What distinguishes deductive from inductive logic is that in the former the premises are taken for granted when drawing the conclusion, whereas in the latter the uncertainty of the premises and therefore of the conclusion are kept in mind. Thus, deductively: 'not not X' means exactly the same as, and is interchangeable with, 'X'; but inductively: the premise 'not not X' tends towards an 'X' conclusion, but does not guarantee it.

Since 'not X' really means 'we have looked for X but not found it so far', it always (with certain notable exceptions) remains somewhat uncertain. On the other hand, a positive, namely 'X' here, can be certain insofar

In Chapter 9 of my book *Ruminations*. I strongly recommend the reader to read that crucial essay.

as it can be directly perceived or intuited (and in this context, the experience 'not found' must be considered as a positive, to ensure theoretical consistency).

If 'not X' is always uncertain to some degree, it follows that 'not not X' is *even more* uncertain and cannot be equated in status to the certainty inherent in 'X' (if the latter is experienced, and not merely a conceptual product). Double negation involves two generalizations or adductions, and is therefore essentially an abstraction and not a pure experience.

Moreover, the expression 'not (not X)' inductively means 'we have looked for the negation of X and not found it'. But since 'not X' already means 'we have looked for X and not found it', we may reasonably ask the question: is the path of 'not not X' the way to find 'X' in experience? Obviously not! If we seek for X, we would directly look for it—and not indirectly look for it through the negation of its negation.

Note, too, that having found 'X' in experience we would consider 'not not X' to follow with deductive force, even though the reverse relation is (as already mentioned) much weaker.

Thus, the problem of double negation posed by Brouwer is a very artificial one, that has little or nothing to do with actual cognitive practice. Not only are the laws of thought nowhere put in doubt by this problem – if we are careful to distinguish induction from deduction – but it is not a problem that would actually arise in the normal course of thought. It is a modern sophistical teaser.

24. CHAPTER TWENTY FOUR

Drawn from Logical and Spiritual Reflections (2008), Book 3, chapter 8.

THE GAME OF ONE-UPMANSHIP

1. Misleading Symbolism

People who think the law of non-contradiction and/or the law of the excluded middle is/are expendable have simply not sufficiently observed and analyzed the formation of knowledge within themselves. They think it is just a matter of playing with words, and they are free to assert that some things might be "both A and not A" and/or "neither A nor not A". But they do not pay attention to how that judgment arises and is itself judged. They view "A is A", etc.⁶², as verbal statements like any other, and think they can negate such statements like all others, saying "A is not A", etc. But in fact, negation is not possible as a rational act without acceptance of the significance of negation inherent in the second and third laws of thought, in comparison to the first law of thought. To say "not" at all meaningfully, I must first accept that

Incidentally, I notice people on the Internet nowadays labeling the three laws of thought (LOT): LOI, LNC and LEM, for brevity's sake. Sure, why not?

"A cannot be not A" and that "there's no third alternative to A and not A".⁶³

To try to introduce some other (less demanding) definition of negation is impossible, for true negation would still have to be thought of (in a hidden manner or using other words). Inventing a "many-valued logic" or a "fuzzy logic" cannot to do away with standard two-valued logic – the latter still remains operative, even if without words, on a subconscious level. We have no way to think conceptually without affirmation and denial; we can only pretend to do so.

Many "modern" logicians are so imprisoned by symbolic logic that they have lost contact with the intended meanings of their symbols. For this reason, the symbols 'X' and 'not X' seem equivalent to them, like 'X' and 'Y'. But for classical logicians, a term and its negation have a special relationship. The negation of X refers to *all but* X, i.e. everything that is or might be in the whole universe other than X.⁶⁴

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Some logicians accept the law of non-contradiction as unavoidable, but consider the law of the excluded middle as expendable: this modern notion is quite foolish. *Both* laws are needed and appealed to in both deductive logic and in inductive logic. They do not only serve for validation (e.g. of syllogisms or of factorial inductions), but they generate questions and research (e.g. what does this imply? or what causative relation can be induced from that?). Moreover, they are mirror images of each other, meant to complement each other so as to exhaust all possibilities, and they ultimately imply each other, and both imply and are implied by the law of identity.

Note that difference does not imply incompatibility. Two things, say X and Y, may be different, yet compatible – or even imply each other. We are well able to distinguish two things (or characteristics of some thing(s)), even if they always

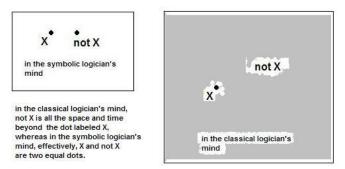


Figure 24.1 Visualizations of Negation.

The diagram above illustrates how differently these people effectively visualize negation:

Obviously, if a person mentally regards 'X' and 'not X' as commensurate, he will not understand why they cannot both be affirmed or both be denied at once; the second and third laws of thought will seem to him prejudicial and conventional. To return to a rational viewpoint, that person has to become conscious of the radical intent of the act of negation; it leaves no space for mixtures or for additional concoctions.

Bipolar logic is not a mere "convention", for the simple reason that making a convention presupposes we have a choice of two or more alternatives, whereas bipolarity is the only way rational thought can at all proceed. We do

occur in tandem and are never found elsewhere. Their invariable co-incidence does not prevent their having some empirical or intellectual difference that allows and incites us to name them differently, and say that X is not the same thing as Y. In such case, X as such will exclude Y, and not X as such will include Y, even though we can say that X implies Y, and not X implies not Y.

not arbitrarily agree bipolarity, because it is inherent in the very asking of the question. To claim something to be conventional is already to acknowledge the conflict between it and the negation of it, and the lack of anything intelligible in between the two.

The motive behind the attempts of some thinkers to deny the laws of thought (i.e. the laws of proper affirmation and denial) is simply an ego ambition to "beat the system", or more specifically (in the case of Western philosophers) to surpass Aristotle (the one who first made these laws explicit objects of study). "You say X? I will 'up the ante' and say Not X (etc.) – and thus show I am the greatest!"

2. Upping the Ante

This is not mere perversity – but a sort of natural denial instinct gone mad. For, funnily enough, to deny some suggestion (including the suggestion there are three laws of thought) is in the very nature of conceptual knowing, a protective mechanism to make sure all alternative interpretations of fact are taken into consideration. This is precisely the faculty of negation – the very one which gives rise to the need for the laws of thought! The problem here is that it is being turned on itself – it is being over-applied, applied in an absurd way.

This can go on and on ad infinitum. Suppose I say "A" (meaning "A but not notA"), you answer "not A" (meaning "notA but not A")⁶⁵; I reply "both A and

Note that if we start admitting the logical possibility of "A and notA" (or of "not A and not notA"), then we can no longer mention "A" (or "notA") alone, for then it is not clear

notA", you oppose "neither A nor notA"; what have we said or achieved? Perhaps I will now say: "all of these four alternatives"; and you will reply: "none of these four alternatives". Then I trump you, asserting: "both these last two alternatives" and you answer: "neither of them". And so forth. Whither and what for?

A more complex version of the same game of oneupmanship can be played with reference to the laws of thought:

- 1. A is A (affirming the law of identity).
- 2. A is not A (denying the law of identity).
- 3. Both (1) and (2). A is A, and A is not A. (disregarding the law of non-contradiction).
- 4. Neither (1) nor (2). A is not A, and A is not not A (disregarding the law of the excluded middle).
- 5. Both (3) and (4).
- 6. Neither (3) nor (4).
- 7. Both (5) and (6).
- 8. Neither (5) nor (6).
- 9. And so on and so forth.

Thus for the first law of thought; and similarly for the other two. We do not merely have a choice of four alternatives (the first four in the above list), a so-called 'tetralemma', but an infinite choice of denials of denials of denials... How would we even evaluate the meaning of all these alternatives without using the laws of thought? They would all be meaningless, because every proposed interpretation would be in turn deniable.

whether we mean "A with notA" or "A without notA" (etc.). This just goes to show that normally, when we think "A" we mean "as against notA" – we do not consider contradictory terms as compatible.

Thus, the attempt to propose a radically "alternative logic", instead of the standard (Aristotelian) logic, is really *the end of all intelligible logic*, the dissolution of all rationality. It is not a meaningful option but a useless manipulation of meaningless symbols. None of it makes any sense; it is just piling up words to give an optical illusion of depth. People who engage in such moronic games should clearly not be granted the status of "logicians".

25. CHAPTER TWENTY FIVE

Drawn from Logical and Spiritual Reflections (2008), Book 3, chapter 9.

IN BUDDHIST DISCOURSE

1. Mitigating Clarifications

Opposition by some Western logicians to (one or more of) the laws of thought is mostly naïve symbolic games, without any profound epistemological or ontological reflection; of quite another caliber is the opposition to these laws found in some Buddhist literature⁶⁶. But we can, with a bit of effort of reflection, explain away the apparent antinomies in their discourse.

When Buddhist philosophers make statements of the form "not X and not notX", they should not (or not always) be viewed as engaging in antinomy, or in rejection of the laws of thought. Rather, such statements are abridged expressions intending: "don't look for X and don't look for not X", or "don't think X and don't think not X", or "don't say X and don't say not X", or

I am of course over-generalizing a bit here, for emphasis. There are of course more savvy Western logicians and less savvy Oriental (including Buddhist) logicians. A case of the latter I have treated in some detail in past works is Nagarjuna.

"don't attach to X and don't attach to not X", or the like.⁶⁷

When thus clarified, statements superficially of the form "neither X nor not X" (or similarly, in some cases, "both X and not X") are seen to be quite in accord with logic. For the laws of thought do not deny that you cannot *look for* 'X' and for 'not X', or for that matter for 'both X and not X', or even 'neither X nor not X'. Similarly, with regard to *thinking* this or that, or to *claiming* this or that, or to *attaching* to this or that, etc.

The laws of logic would only say that you cannot at once 'look for X' and 'not look for X', and so forth. It does not say you cannot at once 'look for X' and 'look for not X', and so forth. The latter situation merely asserts that the issue of X or not X ought to be left *problematic*. An unsolved problem is not an antinomy. The most we can say is that whereas Buddhism might be deemed to enjoin us to accept such uncertainty as final, Western logic would recommend pressing on to find a solution of sorts. Thus, in some cases, the apparent contradictions and inclusions of middle terms in Buddhist philosophy (and similarly in some other texts) are merely verbal. They are due to inaccuracy in verbal expression, omitting significant implicit aspects of what is really meant. The reason for such verbal brevity is that the focus of such statements is heuristic, rather than existential. They are merely meant as "skillful means" (to the end of Realization), not as factual descriptions. That is to say,

For example, the following is a recommendation to avoid making claims of truth or falsehood: "Neither affirm nor deny... and you are as good as a enlightened already." *Sutra of Supreme Wisdom*, v. 30 – in Jean Eracle (my translation from French).

they are statements telling the subject *how to* proceed (cognitively, volitionally or in valuation), rather than telling him/her how things *are*.

2. Examples

To give an actual example from Buddhist literature, I quote the following passage from the *Wake-up Sermon* attributed to Bodhidharma:

"Mortals keep creating the mind, claiming it exists. And arhats keep negating the mind, claiming it doesn't exist. But bodhisattvas and buddhas neither create nor negate the mind. This is what's meant by the mind that neither exists nor doesn't exist... called the Middle Way."68

When we face an unresolved contradiction or an unsolved problem of any sort, we are from the point of view of knowledge in front of a void. This 'emptiness' can be looked upon with anxiety, as a precipice, as a deficiency of means to deal with the challenges of life. Or it may be viewed as something pregnant with meaning, a welcome opportunity to dive fearlessly into infinity. The former attitude gives rise to Western science, the latter to Zen meditation.

Or again, consider the following quotation from Huang Po's teaching:

P. 53. This passage is particularly clear in its explanation of "neither exists nor does not" as more precisely "is neither created nor negated". Whereas the former is logically contradictory, the latter is in fact not so. What is advocated here is, simply put, non-interference.

"If only you will avoid concepts of existence and non-existence in regard to absolutely everything, you will then perceive the Dharma." (P. 43.)

Here again, the meaning is clear. The Zen master is not here denying existence or non-existence or both; he is just telling us not to engage in judgments like 'this exists' or 'this does not exist' that are inherent to all conceptualization. He refers to such judgments as "dualism", because they require a decision between two alternatives. Clearly, Huang Po's statement is not a formally contradictory ontological proposition, but a *prima facie* coherent epistemological injunction not to be concerned with judging whether what one experiences is real or unreal.

Admittedly, some Buddhists⁶⁹ do take such a statement as implying that existence does not exist, or that it both exists and does not exists, or neither exists nor does not exist. But as far as commonsense logic is concerned, existence does exist – i.e. whatever is, is (Aristotle's law of identity). Any clear denial of this fundamental truth would just be self-contradictory – it would deliberately ignore the fact and implications of its own utterance (i.e. that a statement has been made, alleging a truth, by someone to someone, etc.)

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In truth, Huang Po is among them, since elsewhere he piously states: "from first to last not even the smallest grain of anything perceptible has ever existed or ever will exist" (p. 127). This is a denial of all appearance, even as such. Of course, such a position is untenable, for the existence of mere appearance is logically undeniable — else, what is he discussing? Before one can at all deny anything, one must be able to affirm something. Also, the act of denial is itself an existent.

3. Doing Rather than Talking

More precisely, in the present context, we must acknowledge that *whatever but appears, certainly exists* – whether it is eventually judged to be real or illusory. On this basis, we can reasonably interpret Huang Po (at least in the citation above) as simply saying "do not ask whether some particular (or general) thing exists or not, or whether it is real or not, because such questioning diverts your attention from a much more important insight into the nature of being".

It should be added that, even though I above admit that Huang Po's position is prima facie coherent, it is not so coherent upon further scrutiny. He cannot strictly speaking utter a statement without using concepts and he cannot be understood by us without use of our conceptual faculty. All discourse is conceptual, even anti-conceptual discourse. That is, in the very act of preaching abstinence from concepts, he is in fact not practicing what he preaches.

This shows that even persons presumed to be enlightened need concepts to communicate, and also that such conceptuality does not apparently (judging by the claims of those who practice it) affect their being enlightened. So concepts cannot be intrinsically harmful to enlightenment, and the claim that they must be eschewed is internally inconsistent! This is not a game of words (as some might argue) – it is a logical insight that cannot be waved off. One can only at best argue against excessive conceptualization.

In any event, it must be understood that Buddhist anticonceptual philosophy is aimed at psychological development: it is primarily a "way" or "path". Its focus is how to react to ordinary experiences, so as to get to see the ultimate reality beyond them. It refers to the object (X or not X), not independently (as in most Western logic), but as an object of the Subject (i.e. sought out, thought of, claimed, or attached to by the subject-agent). The latter 'subjectivity' (i.e. dependence on the subjectagent) is very often left implicit, simply because it is so pervasive. Notwithstanding, there are contexts in which the intent is more 'objective' than that⁷⁰.

4. Imprecise Language

It should also be noticed that many of the contradictions or paradoxes that Buddhist philosophers produce in their discourse are due to their tendency to make apparently general statements that in the last analysis turn out to be less than all-inclusive. Even while believing that there is more to the world as a whole than what is commonly evident, they formulate their ideas about the phenomenal world as *unqualified universal propositions*. There are many examples of this tendency.

"All is unreal", says the *Dhammapada* (v. 279). Calling all unreal or illusory is of course possible *in imagination*, *i.e. verbally* — by taking the predicate 'unreal' or 'illusory' from its original legitimate subjects of application and applying it to 'all' subjects. Implicit in this manipulation is *an analogy*—i.e. a statement that just as within the realm of appearance some items are found not real and labeled illusory, so we can project a larger

For a start, to claim a means as skillful is a kind of factual description.

realm in which the whole current realm of appearance would seem unreal.

This explains how people assimilate that oft-repeated Buddhist statement, i.e. why it seems thinkable and potentially plausible. But it does not constitute logical justification for it. The only possible justification would be to personally experience a realm beyond that of ordinary experience. Even then, the logically consistent way to make the statement would be "all ordinary experience is unreal" (because saying just "all" would of course logically have to include the extraordinary experience).

Another frequently found example is "existence is suffering⁷¹." This statement is true, all too true, about the world we commonly experience, i.e. the world of material and mental phenomena. If one is observant, one discerns that we are always feeling some unpleasantness in the background of our existence. No earthly happiness is ever complete, if only because it is tenuous. Even sexual pleasure or orgasm – which more and more of my contemporaries seem to regard as the ultimate ecstasy and goal of existence – is a pain of sorts⁷².

Buddhism has displayed extreme wisdom in emphasizing the fact of suffering, because once we realize it we are by this very simple realization already well on the way to

This is the usual translation of the Sanskrit term is *dukkha*. This connotes not only physical and emotional pain, but more broadly mental deficiencies and disturbances, lack of full satisfaction and contentment, unhappiness, absence of perfect peace of mind.

If we are sufficiently attentive, we notice the pain involved in sexual feelings. Not just a pain due to frustration, but a component of physical pain in the very midst of the apparent pleasure.

being freed of suffering. If one were visiting hell, one would not expect to experience heaven there; likewise, it is natural in this halfway world to experience some suffering. I used to suffer a lot at the sight of people getting away with injustices or other ugly acts; but lately I just tell myself: "well, I am in samsara and this is normal behavior in samsara⁷³ – so long as I am here, I have to expect this kind of unpleasant experience and take it in stride!"

But the statement "existence is suffering" is wrongly formulated from the logical point of view, and for that reason it is bound to lead to paradoxes. For if we believe (as Buddhists do) that suffering can eventually be overcome (specifically, when nirvana is attained), then the truth of suffering must be formulated less universally as: "mundane existence is suffering". The usual formulation of the first Noble Truth, "existence is suffering," is not intended to be as all-inclusive as it seems – for suffering disappears according to the third Noble Truth when we become enlightened. Therefore, to make the former consistent with the latter, it has to be rephrased more restrictively.

Another example of the tendency to artificially refuse to count the experience of enlightenment as part of the world as a whole is the idea that enlightenment takes us "beyond good and evil". This is logically incorrect – if we regard enlightenment as the *summum bonum*, the ultimate good (which we do, if we enjoin people to prefer it to all other pursuits).

Or, using Jewish terminology: "I am in *galut* (exile, in Hebrew), and such unpleasantness is to be expected here". Note in passing, the close analogy between the Buddhist concept of samsara and the kabbala concept of galut.

The phrase "beyond good and bad" is intended to stress the practical problem that pursuing good is as much a form of attachment as avoiding evil. The pursuit of worldly good things is ultimately bad, because it just ties us to this world and subjects us to the bad in it. And indeed, even the pursuit of liberation from this world, i.e. of an otherworldly good, is problematic, in that it involves the wrong attitude, a grasping or clinging attitude that is not conducive to success. All this is true, but tends towards paradox.

To avoid confusion, we must simply rephrase our goal as "beyond *pursuit of* good and avoidance of evil". That is to say, we must admit that nirvana is 'good' in the most accurate sense of the term, while what we call 'good' in the world of samsara (i.e. wealth position, power, sensual pleasure, etc.) is really not much better than what we call 'bad'. Alternatively, we should distinguish good in an absolute sense (the good of nirvana) and good in a relative sense (the goods within samsara). Relative goods would then to be classified as not so good from the absolute point of view.

The result of this change of perspective is that, rather than view existence as fundamentally bad (due to suffering), we may now view it as fundamentally good (since nirvana underlies all samsaric existence). Our common view and manner of existence is just an error of sorts, causing us much suffering; if we but return to correct cognition and behavior, we will experience the natural good at the core of all things. Here, the illusory good and evil of the mundane are irrelevant, and we are fully immersed in the real good.⁷⁴

We could read S. Suzuki as saying much the same thing, when he says: "Because we are not good right now, we

To conclude – Buddhist discourse often leads to paradox or contradiction because it insists on using terms in conventional ways and uttering generalities that apply to only part of the totality of experience (namely, the mundane part, to the exclusion of the supramundane part). To avoid the doctrinal problems such discursive practices cause, we must either clearly specify the terms used as having such and such conventional senses, or particularize statements that were formulated too generally (i.e. which did not explicitly take into consideration the data of enlightenment).

want to be better, but when we attain the transcendental mind, we go beyond things as they are and as they should be. In the emptiness of our original mind they are one, and there we find perfect composure" (p.130).

26. CHAPTER TWENTY SIX

Drawn from Logical and Spiritual Reflections (2008), Book 3, chapter 20.

THE LAWS OF THOUGHT IN MEDITATION

1. Cognitive Virtues

The three laws of thought are commonly considered by many current commentators⁷⁵ to be (at best) only relevant to rational discourse, and not relevant at all or even antithetical to meditation and all the more so to its finale of enlightenment. Nothing could be further from the truth, as will now be explicated.

The laws of thought are principally 'moral' imperatives to the thinker, enjoining him or her to have certain cognitive attitudes in all processes of thought. They call upon the thinker to make an effort, so as to guarantee maximum efficiency and accuracy of his or her thoughts. The 'metaphysical' aspect of the laws of thought is a substratum and outcome of this practical aspect.⁷⁶

1. **The law of identity** is a general stance of 'realism'. In *discursive thought*, this means: to face facts; to observe and think about them; to admit the factuality

Judging by Internet postings and debate on this topic.
It could also be said that the two aspects are 'co-

It could also be said that the two aspects are 'coemergent', mutually significant and equally important. But here I wish to stress the psychological side of the issue.

of appearances as such and that of logical arguments relating to them; to accept the way things are (or at least the way they seem to be for now), that things are as they are, i.e. whatever they happen to be; and so on.

Clearly, these same cognitive virtues are equally applicable to *meditation practice*, which requires **awareness**, receptivity and lucidity. The antitheses of these attitudes are evasiveness, prejudice and obscurantism, resulting in "sloth and torpor"⁷⁷.

At the apogee of meditation, in the *enlightenment* experience, this is expressed as (reportedly) consciousness of the "thus-ness" (or "such-ness") of "ultimate reality".

2. **The law of non-contradiction** is a general stance of 'coherence' (which is an aspect of 'realism').

In *discursive thought*, this means: while giving initial credence to all appearances taken singly, not to accept two conflicting appearances as both true (or real), but to place one or both of them in the category of falsehood (or illusion); to seek to resolve or transcend all apparent contradictions; to pursue consistency in one's concepts and theories; to reject inconsistent ideas as absurd and self-contradictions as untenable nonsense; and so on.

Clearly, these same cognitive virtues are equally applicable to *meditation practice*, which requires **harmony**, balance and peace of mind. The antitheses of these attitudes are conflict, confusion and neurosis

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(or madness), resulting in "restlessness and anxiety" 78.

At the peak of meditation, in the *enlightenment* experience, this is expressed as (reportedly) the "oneness" (monism or monotheism) of "ultimate reality".

3. **The law of the excluded middle** is a general stance of 'curiosity' (which is also an aspect of 'realism').

In *discursive thought*, this means: engaging in research and study, so as to fill gaps in one's knowledge and extend its frontier; engaging in speculation and theorizing, but always under the supervision and guidance of rationality; avoiding fanciful escapes from reality, distorting facts and lying to oneself and/or others; accepting the need to eventually make definite choices and firm decisions; and so on.

Clearly, these same cognitive virtues are equally applicable to *meditation practice*, which requires **clarity**, judgment and understanding. The antitheses of these attitudes are ignorance, uncertainty and delusion, resulting in "doubt and indecision"⁷⁹.

At the pinnacle of meditation, in the *enlightenment* experience, this is expressed as (reportedly) the "omniscience" of "ultimate reality".

Thus, I submit, rather than abandon the laws of thought when we step up from ordinary thinking to meditation, and from that to enlightenment, we should stick to them, while allowing that they are expressed somewhat differently at each spiritual stage. Whereas in discursive

See Kamalashila, p. 249.

See Kamalashila, p. 258.

thought awareness is expressed by intellectual activity, in meditation the approach is gentler and subtler, and in enlightenment we attain pure contemplation.

When such final realization is reached⁸⁰, the laws of thought are not breached, but made most evident. "Thusness" is the essence of existence; it is the deepest stratum of identity, not an absence of all identity. "One-ness" is not coexistence or merging of opposites, but where all oppositions are dissolved or transcended. "Omniscience" is not in denial of ordinary experience and knowledge, but their fullest expression and understanding. What in lower planes of being and knowing seems obscure, divergent and uncertain, becomes perfect at the highest level.⁸¹

I submit, on the basis of my own limited experience, but also out of logical expectation of consistency between all levels of being. I think many people more knowledgeable than me would agree with the descriptions here given of the higher realms.

Buddhist, and especially Mahayana, philosophers often stress that nirvana (the common ground of all being) and samsara (the multiplicity of changing appearances) are ultimately one and the same. Even while admitting this, we must remain aware of their apparent difference. The whole point of the philosophical idea of monism ("nirvana") is of course to resolve the contradictions and gaps inherent in the experience of plurality ("samsara"). At the same time, the oneness of nirvana is in a sort of conflict with the multiplicity of samsara. We must somehow both admit and ignore this tension. In truth, all this remains an unsolved problem at some level.

2. The Absurdity of the Antitheses

Those teachers or commentators who claim that the laws of thought are abrogated once we transcend ordinary discourse are simply misinterpreting their experiences. Either their experience is not true "realization", or their particular interpretation of their realization experience is just an erroneous afterthought that should not be viewed as part of the experience itself.

Instead of the laws of identity, non-contradiction and exclusion of any middle, they propose *a law of non-identity, a law of contradiction, and a law of the included middles!* According to them, the ultimate reality is that nothing has an identity, all contradictories coexist quite harmoniously, and there may be other alternatives besides a thing and its negation!

They adduce as proofs the Buddhist principles of non-selfhood, impermanence and interdependence.

But they cannot claim that something has no "nature" whatsoever, for then what is that "something" that they are talking about? If it is truly non-existent, why and how are we at all discussing it and who are we? Surely these same people admit the existence of an "ultimate reality" of some sort — if only a single, infinite, universal substratum⁸². They call it "void" or "empty", but surely

The "great self" or "ocean of permanence", to use the words of Dogen (p. 267. Note that Dogen is not here saying there is no such thing, but is stressing that we do not – as some people claim – *automatically* all return there after death, but rather are subject to various rebirths according to our respective karmas; he is implying that to get there is hard-won realization, not something given *gratis* to all comers). Some identify this underlying ultimate reality with the "*Deus sive Natura*" of Baruch Spinoza (Holland, 1632-77). But I hasten to

such a negation is not logically tenable without the admission that something positive is being negated; a negation can never be a primary given.

Similarly, we might argue, "impermanence" means the impermanence of something and "interdependence" means the interdependence of two or more things. They cannot claim infinite impermanence, without admitting the extended existence in time of something however temporary; and they cannot claim a universal interdependence, without admitting causal connections between actual facts.

There is an unfortunate tendency here to use words without paying attention to their relational implications. Another example of this practice is to speak of "consciousness" (or perception or thought or some such cognitive act), without admitting that this implies consciousness of something (called an object) by something (called the Subject).

This is done deliberately, to conform with the ideological prejudice that there is no cognizing self and nothing to cognize. Similarly, so as not to have to mention the Agent willing an action, volition is concealed and the action is made to appear spontaneous or mechanical. They refuse to admit that *someone* is suffering, thinking, meditating or becoming enlightened.

Another claim often made is that our common experience of the world is like a dream compared to ultimate reality. The implication being that the laws of thought are not

add that I do not subscribe to Spinoza's equation of God and Nature, which implies that God is like Nature subject to determinism. For me, as in normative Judaism, God is the free, volitional creator of Nature. He underlies and includes it. It is a mere product His and but a tiny part or aspect of Him.

obeyed in a dream. But in truth, even in a dream, though images and sound come and go and seem to intertwine, actually there is no contradiction if we observe carefully. As for the difference between dream and awake experience, it is not strictly a contradiction since they are experienced as distinct domains of being.

Contradiction is not even thinkable, except in words (or intentions). We cannot even actually imagine a contradiction, in the sense defined by Aristotle (is and is not at once in every respect). We can only say (or vaguely believe) there is one. We of course commonly encounter apparent contradiction, but that does not prove that contradiction exists in fact. It is an illusion, a conflict between verbal interpretations or their non-verbal equivalents.

We formulate theories; they yield contradictions; we correct the theories so that they no longer yield these contradictions. We tailor our rational constructs to experience. We do not infer contradiction to exist from contradictions in our knowledge. We question and fix our knowledge, rather than impose our beliefs on reality. That is sanity, mental health. That is the way knowledge progresses, through this dialectic of thesis-antithesis-synthesis.

27. CHAPTER TWENTY SEVEN

Drawn from *A Fortiori Logic* (2013), Chapters 3.1, 7.2.

UNDERSTANDING THE LAWS OF THOUGHT

1. Adapting the Laws of Thought

Many people regard Aristotle's three 'laws of thought' – the laws of identity, of non-contradiction and of the excluded middle⁸³ – as rigid prejudices. They think these are just conventions, that some moronic old fellow called Aristotle had the bad grace to impose on the rest of us, and that we can just chuck 'em out at will. In each of my past works, I have tried to explain why these are fundamental human insights that cannot under any pretext be discarded. I would like to add a few more explanations in the present work.

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Aristotle states the laws of non-contradiction and of the excluded middle in his *Metaphysics*, B, 2 (996^b26-30), Γ, 3 (1005^b19-23), Γ, 7 (1011^b23-24). *Metaph*. Γ, 7 (1011^b26-27) may be viewed as one statement by Aristotle of the law of identity: "It is false to say of that which is that it is not or of that which is not that it is, and it is true to say of that which is that it is or of that which is not that it is not." These references are found in the Kneales, p. 46 (although they interpret the latter statement as somewhat defining truth and falsehood, rather than as expressing the law of identity).

The laws of thought must not be thought of as mechanical rules, but as repeated *insights* of our *intelligence*. Every 'application' of these laws in a new context demands a smart new insight from us. We must in each new context reaffirm these laws, and use them creatively to deal with the complexities of the case at hand.

In a fortiori logic, where new forms are encountered, and new problems need solutions, we can expect our intelligence and creativity to be called upon. We have already come across many contexts where subtlety was required. The distinction between a proposition like 'X is Y' and 'X is R enough to be Y' was one such context. Another was our development of a distinction between absolute terms (R and notR) and relative terms (R and notR). The laws of thought are ever present in logical discourse, but they must always be understood and adapted in ways that are appropriate to the context at hand – so they are not mechanical laws, but 'smart laws'.

The laws of thought have to repeatedly be adapted to the increasing complexity of discourse. Originally, no doubt, Aristotle thought of the laws with reference to indefinite propositions, saying that 'A is B' and 'A is not B' were incompatible (law of non-contradiction) and exhaustive (law of the excluded middle). In this simplest of contexts, these laws implied only two alternatives. However. when Aristotle considered quantified propositions, 'All A are B' and 'Some A are B and some A are not B' and 'No A is B' - he realized that the application in this new context of the very same laws implied three alternatives. From this example, we see that the subtleties of each situation must be taken into

consideration to properly 'apply' the laws. They are not really 'applied'; they are intelligently *formulated anew as befits* the propositional forms under consideration.

We could say that the disjunction "Either 'A is B' or 'A is not B" refers to an individual subject A, whereas the disjunction "Either 'All A are B' or 'Some A are B and some A are not B' or 'No A is B" refers to a set of things labeled A. But then the question arises: what do we mean when we say that an individual A 'is B'? Do we mean that A is 'entirely B', 'partly B and partly not B'? Obviously, the mutually exclusive and exhaustive alternatives here would be: "Either 'A is wholly B' or 'A is partly B and partly not B' or 'A is not at all B'." It seems obvious that in most cases 'A is B' only intends 'A is partly B and partly not B' – for if 'A is wholly B', i.e. 'A is nothing but B' were intended, why would we bother verbally distinguishing A from B? Well, such tautologies do occur in practice, since we may first think of something as A and then of it as B, and belatedly realize that the two names in fact refer to one and the same thing. But generally we consider that only B is 'wholly B', so that if something labeled A is said to have some property labeled B, A may be assumed to be intended as 'only partly B'.

To give a concrete example: my teacup is white. This is true, even though my teacup is *not only* colored white, but also has such and such a shape and is made of such and such a material and is usually used to drink tea. Thus, though being this teacup intersects with being white, it does not follow that the identity of this individual teacup is entirely revealed by its white color (which, moreover, could be changed). With regard to classes, even though we may choose to define the class

of all A by the attribute B, because B is constant, universal and exclusive to A, it does not follow that A is thenceforth limited to B. B remains one attribute *among* the many attributes that are observed to occur in things labeled A. Indeed, the class A may have other attributes that are constant, universal and exclusive to it (say C, D, etc.), and yet B alone serves as the definition, perhaps because it intuitively seems most relevant. Thus, to define concept A by predicate B is not intended to limit A to B. If A were indeed limited to B, we would not name them differently.

These thoughts give rise to the logical distinction between 'difference' and 'contradiction', which calls forth some further use of ad hoc intelligence. When we say that 'A and B are different', we mean that these labels refer to two distinct phenomena. We mean that to be A is not the same as to be B, i.e. that B-ness is different from A-ness. It does not follow from this that No A is B. That is to say, even though A is not the same thing as B, it is conceivable that some or all things that are A may yet be B in some way. To say the latter involves no contradiction, note well. Therefore, the laws of non-contradiction and of the excluded middle cannot in this issue be applied naïvely, but only with due regard for the subtleties involved. We must realize that 'difference' is not the same as 'contradiction'. Difference refers to a distinction, whereas contradiction refers to an opposition. Two propositions, say X and Y, may have different forms and yet imply each other. It is also possible, of course, that two propositions may be both different and contradictory.

Another subtlety in the application of the laws of thought is the consideration of tense. Just as 'A is B' and 'A is not-B' are compatible if they tacitly refer to different places, e.g. if they mean 'A is B here' and 'A is not-B there', so they are compatible if they tacitly refer to different times, e.g. if they mean 'A is B now' and 'A is not-B then'. Thus, if a proposition is in the past tense and its negation is in the present or future tense, there is no contradiction between them and no exclusion of further alternatives. Likewise, if the two propositions are true at different moments of the past or at different moments of logically compatible the future, they are and inexhaustive.

These matters are further complicated when we take into consideration the various modalities (necessity, actuality, possibility), and still further complicated when we take into consideration the various modes of modality (natural, extensional, logical). I have dealt with these issues in great detail in past works and need not repeat myself here. In the light of considerations of the categories and types of modality, we learn to distinguish factual propositions from epistemic propositions, which qualify our knowledge of fact. In this context, for instances, 'A is B' and 'A seems not provable to be B', or even 'A is B' and 'A seems provable not to be B', might be both true.

One of the questions Aristotle made a great effort to answer, and had some difficulty doing, was how to interpret the disjunction: "Either there will be a sea battle tomorrow or there will not be a sea battle tomorrow"84. But the solution to the problem is simple enough: if we can truly predict today what will (or will not) happen

See De Interpretatione, 9 (19^a30).

tomorrow, it implies that tomorrow is *already* determined at this earlier point in time and that we are able to know the fact; thus, in cases where the fact is *not* already determined (so that we cannot predict it no matter what), or in cases where it is already inevitable *but* we have no way to predict the fact, the disjunction obviously cannot be bipolar, and this in no way contravenes the laws of thought. Nothing in the laws of thought allows us to foretell whether or not indeterminism is possible in this world.

As a matter of fact, either now there will be the sea battle tomorrow or now there won't be one or the issue is still undetermined (three alternatives). As regards our knowledge of it, either now there will be the battle tomorrow and we know it, or now there won't be and we know it, or now there will be and we don't know it, or now there won't be and we don't know it, or it is still undetermined and so we cannot yet know which way it will go (five alternatives). We could partially formalize this matter by making a distinction between affirming that some event definitely, inevitably 'will' happen, and affirming only that it just possibly or even very likely 'will': the former is intended in deterministic contexts. whereas the latter is meant when human volition is involved or eventually when natural spontaneity is involved. These alternatives can of course be further multiplied, e.g. by being more specific as regards the predicted time and place tomorrow.

What all this teaches us is that propositions like 'A is B' and 'A is not B' may contain many tacit elements, which when made explicit may render them compatible and inexhaustive. The existence of more than two alternatives is not evidence against the laws of thought. The laws of

thought must always be *adapted* to the particulars of the case under consideration. Moreover, human insight is required to properly formalize material relations, in a way that keeps our reading in accord with the laws of thought. This is not a mechanical matter and not everyone has the necessary skill.

Another illustration of the need for intelligence and creativity when 'applying' the laws of thought is the handling of double paradoxes. A proposition that implies its contradictory is characterized as paradoxical. This is a logical possibility, in that there is a quick way out of such single paradox – we can say that the proposition that implies its contradictory is false, because it leads to a contradiction in knowledge, whereas the proposition that is implied by its contradictory is true, because it does not lead to a contradiction in knowledge. A double paradox, on the other hand, is a logical impossibility; it is something unacceptable to logic, because in such event the proposition and its contradictory both lead to contradiction, and there is no apparent way out of the difficulty. The known double paradoxes are not immediately apparent, and not immediately resolvable. Insights are needed to realize each unsettling paradox, and further reflections and insights to put our minds at rest in relation to it. Such paradoxes are, of course, never real, but always illusory. Double paradox is very often simply caused by equivocation, i.e. using the same word in two partly or wholly different senses. The way to avoid equivocation is to practice precision and clarity. Consider, for instance, the word "things." In its primary sense, it refers to objects of thought which are thought to exist; but in its

expanded sense, it refers to any objects of thought, including those which are not thought to exist. We need both senses of the term, but clearly the first sense is a species and the second sense is a genus. Thus, when we say "non-things are things" we are not committing a contradiction, because the word "things" means one thing (the narrow sense) in the subject and something else (the wider sense) in the predicate. The narrow sense allows of a contradictory term "non-things;" but the wider sense is exceptional, in that it does not allow of a contradictory term – in this sense, everything is a "thing" and nothing is a "non-thing," i.e. there is no "non-thing." The same can be said regarding the word "existents." In its primary sense, it refers to actually existing things, as against non-existing things; but in its enlarged sense, it includes non-existing things (i.e. things not existing in the primary sense, but only thought by someone to exist) and it has no contradictory. Such very large terms are, of course, exceptional; the problems they involve do not concern most other terms. Of famous double paradoxes, we can perhaps cite the Barber paradox as one due to equivocation⁸⁵. Many of the famous double paradoxes have more complex causes. See for examples my latest analyses [in chapter 30, below] of the Liar and Russell paradoxes. Such paradoxes often require a lot of ingenuity and logical skill to resolve.

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I deal with this one in my Future Logic, chapter 32.3.

2. Two More Laws of Thought⁸⁶

The first three laws of thought, which were formulated by Aristotle, are that we *admit facts as they are* (the law of identity), *in a consistent manner* (the law of noncontradiction) *and without leaving out relevant data pro or con* (the law of the excluded middle). To complete these axioms of logic, and make them fully effective in practice, we must add two more. The fourth, which I have called **the principle of induction**; and a fifth, which I call **the principle of deduction**.

These five laws are nothing new, being used in practice by mankind since time immemorial; only our naming them in order to spotlight them and discuss them is a novelty. They qualify as 'laws of thought' because they are self-evident, and necessary to and implied in all rational thought.

The principle of deduction is a law of logic that no information may be claimed as a deductive conclusion which is not already given, explicitly or implicitly, verbally or tacitly, in the premise(s). The premises must obviously fully justify the conclusion, if it is to be characterized as deduced. This fundamental rule is true for all forms of deductive (as against inductive) arguments, which helps us avoid fallacious reasoning. It may be viewed as an aspect of the law of identity, since it enjoins us to acknowledge the information we have, as it is, without fanciful additions.

It may also be considered as the fifth law of thought, to underscore the contrast between it and the principle of

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This essay was originally written for *A Fortiori Logic*, and may still be found there in a scattered way, notably in chapter 7.2.

induction, which is the fourth law of thought. The principle of induction may, in its most general form, be stated as: what in a given context of information appears to be true, may be taken to be effectively true, unless or until new information is found that puts in doubt the initial appearance. In the latter event, the changed context of information may generate a new appearance as to what is true; or it may result in some uncertainty until additional data comes into play.

Deduction must never be confused with induction. Although deduction is one of the tools of induction in a broad sense, it is a much more restrictive tool than others. Deduction refers specifically to inferences with 100% probability; whereas induction in a narrow sense refers to inferences with less than 100% probability.

Inductive reasoning is not subject to the same degree of restriction as deduction. Induction is precisely the effort to extrapolate from given information and predict things not deductively implied in it. In inductive reasoning, the conclusion can indeed contain more information than the premises make available; for instance, when we generalize from some cases to all cases, the conclusion is inductively valid *provided and so long as* no cases are found that belie it. In deductive reasoning, on the other hand, the conclusion must be formally implied by the given premise(s), and no extrapolation from the given data is logically permitted. In induction, the conclusion is tentative, subject to change if additional information is found, *even if* such new data does not contradict the initial premise(s)⁸⁷. In deduction, on the other hand, the

For example, having generalized from "some X are Y" to "all X are Y" – if it is thereafter discovered that "some X are not Y," the premise "some X are Y" is not contradicted, but the

conclusion is sure and immutable, so long as no new data contradicts the initial premise(s).

As regards the terms, whereas in induction the conclusion mav contain terms. denotations connotations that are not manifest in the premise(s), in deduction the terms, denotations and connotations in the conclusion must be uniform with those given in the premise(s). If a term used in the conclusion of a deductive argument (such as syllogism or a fortiori) differs however slightly in meaning or in scope from its meaning or scope in a premise, the conclusion is deductively invalid. No equivocation or ambiguity is allowed. No creativity or extrapolation is allowed. If the terms are not exactly identical throughout the argument, it might still have some inductive value, but as regards its deductive value it has none.

Any deductive argument whose conclusion can be formally validated is necessarily in accord with the principle of deduction. In truth, there is no need to refer to the principle of deduction in order to validate the conclusion - the conclusion is validated by formal means, and the principle of deduction is just an ex post facto observation, a statement of something found in common to all valid arguments. Although useful as a philosophical abstraction and as a teaching tool, it is not necessary for validation purposes.

Nevertheless, if a conclusion was found not to be in accord with the principle of deduction, it could of course be forthwith declared invalid. For the principle of deduction is also reasonable by itself: we obviously cannot produce new information by purely rational

conclusion "all X are Y" is indeed contradicted and must be abandoned

means; we must needs get that information from somewhere else, either by deduction from some already established premise(s) or by induction from some empirical data or, perhaps, by more mystical means like revelation, prophecy or meditative insight. So obvious is this caveat that we do not really need to express it as a maxim, though there is no harm in doing so.

The principle of deduction is that the putative conclusion of any deductive argument whatsoever must in its entirety follow necessarily from (i.e. be logically implied by) the given premise(s), and therefore cannot contain any claims not supported in the said premise(s). If a putative conclusion contains *additional* information and *yet seems true*, that information must be proved or corroborated from some *other* deductive or inductive source(s). Inference in accord with this principle is truly deductive. Inference not in accord with this principle may still be inductively valid, but is certainly not deductively valid.

In truth, the principle of deduction is a redundancy. That the conclusion cannot go beyond what is given in the premises is true of all deductive argument, without any need to state it as a special principle; it is the very definition of deduction, as against induction or fallacious thought, and so the subtext of any deductive act. Clearly, the principle of deduction is not an artificial, arbitrary or conventional limitation, but a natural, rational one.

28. CHAPTER TWENTY EIGHT

Drawn from *A Fortiori Logic* (2013), Chapter 12.1 and Appendix 7.3.

ASSAULTS ON LOGIC

1. Zen's Anti-logic

Zen logic, as is well known, is no logic, but a sort of **anti-logic**, an antithesis of logic⁸⁸. It thrives on paradox and even contradiction, at least apparent if not real. A major feature of Zen logic, though this may not be distinctive to Buddhist or even to Indian or Chinese logic, is its belief in the 'tetralemma' (or *catuskoti*). According to this viewpoint, not only a thesis alone (A and not not-A) or alternatively its antithesis alone (not-A and not A) may in fact be true, but there is a real possibility that *both* the thesis and its contradictory (A and not-A) are true, or *neither* the thesis nor its contradictory (not A and not not-A) are true – or even, eventually, that two or more of these four compounds are true together or all false together.

For example, the "two truths" doctrine, formulated by the Buddhist philosopher Nagarjuna (India, ca. 150-250 CE),

I should reiterate here that though I repeatedly criticize Buddhism for its illogic, my purpose is not to totally discredit it. I greatly respect this philosophy of life, and am myself positively influenced by it daily. However, there is much in its philosophical discourse that needs to be revised. Its cavalier attitude to logic is simply untenable.

which distinguishes the "relative truth" of conventional minds and the "absolute truth" of enlightened minds may be classified under the tetralemma category of "neither A nor not-A," since relative truth is neither absolutely true nor untrue, but something in between. Again, the doctrine that "Nirvana and Samsara are one" may be classified under the tetralemma category of "both A and not-A," since it proposes a mixture of opposites. These two doctrines are paradoxically considered as mutually supportive, but of course that is quite illogical: if truth is twofold, its two aspects cannot be one; you can't have it both ways. In scientific Western thought⁸⁹, truth is one; if it is merely 'relative', it is simply untruth. Again, if two things are opposites, they cannot overlap.

Moreover, Buddhists argue that existents have no identity of their own, being merely aggregates, constantly in flux, and thoroughly dependent on causes and conditions. They apply this idea to mind as well as matter, and deny existence of the self or soul. Such claims are logically patently absurd. To deny the self or soul is to deny the existence of someone doing the denying⁹⁰. If literally everything is aggregated, then the

The 12th century CE Islamic philosopher Averroës (or Ibn Rushd) tried to introduce a similar notion of "double truth" (one for common people and one for the élite). Some Christian philosophers, possibly including Boethius, tried to follow suit. But such tendencies were ultimately rejected in both cultures, as it was realized that if religion was cut off from reason, it ultimately implied that religion is irrational and therefore untrue. More recently, most Christians have gradually adapted their beliefs to empirical science and history (though many still resist, e.g. with regard to Darwinism). Islam, on the other hand, is still firmly marooned in the Middle Ages.

This goes against Descartes' phenomenological dictum: "I think, therefore I am," which means that as of the

elements that are aggregated must also be aggregated, ad infinitum⁹¹, in which case there is ultimately nothing to aggregate. There cannot be such a thing as aggregation without something *non*-aggregated to aggregate; the buck has to stop somewhere. Similarly, to the Buddhist doctrine of impermanence, which claims that literally everything is constantly in flux, we must ask: flux of what? Change must be change *of* something *to* something, with an least momentary stationary existence before and after the change. There cannot be such a thing

moment one acknowledges the phenomenon of thought by venturing some proposition, one logically must acknowledge the existence of someone having that thought.

'Consciousness' presupposes some sort of subject and some sort of object, being a special relation between two things, the conscious one being called 'subject' and the one the subject is conscious of being called 'object'. The difficulty of fathoming this relation, due to its ontological distinctiveness and therefore primacy, does not make it any the less real; there are a great many things we cannot fathom, but must take for granted. Knowledge must start with some irreducible primaries; it cannot be grounded in an infinity of definitions and proofs. To make a demand for endless grounding is to claim that demand as an irreducible primary; it is selfcontradictory. Buddhists consider that what we call the self is simply the totality of our sensory and mental experiences at any given moment of time: for them, there is no one having those experiences - they just are, forming a changing bubble of manifest being (which they call 'consciousness'); this bubble being particular gives the illusion of selfhood. But the question remains: who has this illusory idea of being a self? How can a non-self imagine that it is a self? They have no answer to such questions, and avoid to ask them, being dogmatically attached to the idea of no-self.

Moreover, how can a human mind go *all the way to infinity* and observe that aggregation continues there, before making such a bold claim?

as change without something static that changes or emerges from the change. The same applies to the Buddhist idea of interdependence, or co-dependence of everything⁹²: one thing cannot depend on another if that other thing is as devoid of independent existence as itself. Dependence presupposes something more firmly rooted in being, which can be depended on. Simultaneous mutual dependence is unconscionable. Thus, Buddhist discourse is built on stolen concepts, ignoring their conceptual basis.⁹³

Such Buddhist beliefs are contrary to the laws of thought discovered by Aristotle, namely the laws of identity, of non-contradiction and of the excluded middle. For Buddhists, all existents are ultimately "empty" of any nature. But the law of identity is that every existing thing has an identity, a specific nature (whatever that happens

Here again, how can a human mind know the dependence of literally *all* things on each other? To have such knowledge, of all things past, present and future throughout the universe and their exact relations to each other, is conceivable for God – but how can a mere mortal obtain it?

Although Buddhists claim that enlightenment brings about omniscience, such a claim is not empirically justified. For a start, Buddhism has made and still makes many claims about the physical world and the history of life and men that are rejected by modern science; e.g. that the world and life have existed forever. More specifically, consider the following blooper: Zen master Dogen, after attaining enlightenment in 1227-8, wrote in an essay dated 1231 that the Buddha was active about 2000 years before, whereas we know that he lived in circa the 6th-5th centuries BCE, i.e. some two to four hundred years later than Dogen thought. See: *Beyond Thinking: A Guide to Zen Meditation*, Ed. Kazuaki Tanahashi (Boston, MA: Shambhala, 2004), p. 31. Dogen claims having attained enlightenment in another essay (p. 13).

to in fact be94): it is not 'just anything' and it is not 'nothing'. Every existent is something in particular, with features and behavior peculiar to it. Moreover, a fact is a fact: while it occurs, its constituents, its history and its causes and conditions, if any, are irrelevant to the fact of its existence: it just is. Moreover, identities, facts, are mostly objective givens, not products of mind; to claim otherwise is to affirm one's claim itself to be imaginary and thus untrue. The law of contradiction is that an existent cannot at once have and not-have a particular identity; presence and absence are incompatible. The law of the excluded middle is that an existent cannot at once neither have nor not-have a particular identity; there is nothing besides presence or absence. These laws, properly understood, are absolute; they are not subject to any exceptions, under any circumstances whatsoever.

These laws — which have been foundational for Western logical thought and the source of its successes (although in today's atmosphere of willful unreason many people do take a perverse pleasure in disowning them) — were never, it seems, very influential further East. The tetralemma was evidently freely used very early on in East Asia and the Far East, since it is so pervasive in later literature. The "reasoning" behind this irrational belief is that all ordinary human cognition is necessarily "dualistic." According to its proponents, as soon as anything comes to mind, through perception or

How the identity of things is to be known is the question the science of logic seeks to answer. The short answer is, of course, by means of our senses and our reason. That is, empirically and logically, inductively and deductively. Not all identities are necessarily knowable; but we must admit that some are, for otherwise we would be involved in self-contradiction (claiming knowledge and denying it at once).

conception, its negation must also be considered, even if we tend to either ignore it or arbitrarily reject it. Thus, a positive is unthinkable without a corresponding negative. In one author's words:

"In Buddhist logic, it is said that all concepts are based upon exclusion. As soon as we affirm something by saying 'It is this,' we automatically exclude so many other things it might have been. By imposing a conceptual limitation we fabricate an idea. The suggestion here is that it is just an idea – it is not an open experience."

But of course this 'reasoning' is quite fallacious. Knowledge starts with pure perception of positive phenomena; negatives are never pure percepts but are *necessarily* products of conception. Note well: positives come before negatives; and negatives are inconceivable without positives. I can cognize a positive through perception and therefore without any reference to its contradictory; but I cannot do the same with a negative.

See Traleg Kyabgon Rinpoche, "The Path of Mahamudra," in The Best Buddhist Writing 2005, Ed. Melvin McLeod and the editors of the Shambhala Sun (Boston, MA: Shambhala, 2005), p. 98. Although he refers specifically to conception, the implication of such statements is usually taken to be that all affirmation implies negation, i.e. even affirmation based exclusively on perception. Note however, the contrary statement by Eleanor Rosch, in the same collection of essays, p. 114: "According to Buddhist teachings, while all of the interdependent past can be causally gathered into the microcosm of the moment of present experience, that does not mean that the basic mode of apprehending the present moment is somehow filtered or distorted or abstractly representational." In other words, Buddhists do ultimately admit of unadulterated percepts (if only in the context of the enlightenment experience).

In the latter case, I must first have some idea however vague or hypothetical of the positive, before I can even think of, let alone check out, the negative. Thus, though exclusion is indeed eventually part of the knowing process, it is certainly not a primary act: it is only possible after the pure perception of some things and the subsequent imagination of their possible negation.

Some Buddhist philosophers go still further and, appealing to the notion of "emptiness" (shunyata), claim the ontological primacy of negation over affirmation. But here again the question they do not ask is: "negation of what?" If as they suggest there is nothing there at all, then even negatives have no foot to stand on. The negation of a nothing does not produce a something. What needs to be understood by such people is that the word 'not' is more akin to a verb than to a noun. It expresses the Subject's mental act of rejection of a proposed object. It is therefore necessarily conceptual, and never perceptual. Moreover, such claims invariably ignore the positive existence of the claim, and of someone doing the claiming, and of someone receiving the claim. Such people imagine they can speak in a vacuum, without acknowledging the existential context of their speech. This is illogical.

If anything, it is the Buddhist proponents of "paraconsistent" logic who are dualistic and divorced from reality. They fail to take note of the actual order of knowledge development from positive percepts to negative concepts. Indeed, even at the level of perception, one precedes two. Contrary to what many philosophers imagine, we perceive a whole before we mentally divide it into parts. Here, the confusion involved is to conflate a given moment of perception and

perception over time. In any given moment, what we happen to perceive is a whole and this is quickly and mostly automatically divided into parts by the conceptual faculty (note that the perception precedes the subdivision, and only the latter involves negation, i.e. saying 'this part is not the same as that part'); but of course, over time, many such moments of perception, or more precisely their memories, are added together (again by the conceptual faculty) to form a larger whole. These two operations of the conceptual faculty – viz. conceptual dissection of a present perceptual whole and integration of many past percepts into a conceptual whole – should not be confused.

I did not, unfortunately, note down every use of such deviant logic that I came across over the years in Buddhist literature. But I do still remember one relatively early instance in the *Dhammapada*, traditionally attributed to the Buddha (India, ca. 563-483 BCE)⁹⁶, "He for whom there is neither this nor the further shore, nor both...." The tetralemma plays a very important role in the Madhyamika philosophy of Nagarjuna⁹⁷, regarded as

The Dhammapada was probably compiled in the third century BCE.

See my book *Buddhist Illogic* on this subject. It should not be thought that Nagarjuna's perverse thought has had no equivalent in the West. For example, the Megaric school (founded by Euclides of Megara in 4th century BCE Greece) argued much like him that predication is either wrong (if the predicate "differs" from the subject) or useless (if the predicate is "the same as" the subject), ignoring the fact that such a statement is itself an act of predication. I have over the years spotted many such similarities between Eastern and Western philosophies. This is a topic that still needs extensive study, though there may already be good books on it that I am unaware of.

a forerunner of Zen. As for later, specifically Zen writings, in China and then Japan, they are full of it. Consider, for instance, the words of the third patriarch of Zen, Seng Tsan (China, d. 606 CE):

"What is, is not; what is not, is. If this is not yet clear to you, you're still far from the inner truth. One thing is all, all things are one; know this and all's whole and complete." (Italics mine.)98

Eihei Dogen (1200-53 CE), who founded the Japanese Soto Zen sect, often seems (to me, at least) maddeningly obscure, if not insane, due to his frequent breaches of the laws of thought. He indulges without hesitation in selfcontradictory statements, such as "There is sitting letting go of body-mind, which is not the same as sitting letting go of body-mind." Likewise, the law of the excluded middle is no obstacle to his way of thinking. Consider, for instance, this statement: "Active buddhas are neither originally enlightened nor enlightened at some particular neither naturally enlightened, nor without enlightenment" - what are they, then, I ask? Or again: "practice-realization is neither existence nor beyond existence" - what's left, I ask? Surely, if all logical possibilities are exhausted (as seems to be the intention, here), then there are no other possibilities! Dogen pays

In his "Affirming Faith in Mind," given in full in Roshi Philip Kapleau's Zen Merging of East and West (New York: Doubleday, 1980), pp. 184-189. It is hard for me to believe that illogic, the suppression of reason, is compatible with enlightenment, let alone conducive or essential to it—just as it is hard for me to believe that idolatry, the worship of inanimate objects, is compatible with enlightenment, let alone conducive or essential to it. Yet these are recurring theoretical teachings within Zen Buddhism. Even so, paradoxically, I do believe that Zen has much good to offer mankind on a practical level!

no attention to such obvious restrictions, making his discourse incomprehensible nonsense.⁹⁹

Although Zen discourse is often antinomic, its favorite form seems to be "neither this nor that." That is to say, although contradictions and exclusions of the middle are both viewed as possible and do occur in practice, the main emphasis is on denying any thesis whatever, and 'logically' enough also the contradictory of any thesis whatever. For ultimate reality is considered by Zen philosophers as essentially out of this world (even while in it) — therefore, whether phenomena point to the existence or to the non-existence of something, anything, is irrelevant. No proposition is true, because none is capable of describing reality as it really is. The phenomenal world is inherently paradoxical; only beyond it can all opposites be harmonized.¹⁰⁰

This is the gist of the argument, however self-inconsistent and unconscionable it seems to us who are not enlightened. Of course, some sense can be made of it by thinking of ultimate reality as the 'common ground' of conflicting phenomena – and this sort of explanation is

Beyond Thinking, pp. 51, 79, 80. I should additionally draw attention to the frequent use of *tautology* in some Buddhist texts, as if this was informative. For example, Dogen also enjoys tautologies like "sitting is sitting;" he also, I notice, takes pleasure in reversing statements, as in "sitting is buddha-dharma and buddha-dharma is sitting" (p. 51); and reshuffling terms, as in "zazen is invariably the intention to become buddha, and... zazen is invariably becoming buddha with intention" (p. 39). Such discourse may of course be informative, but I suspect the intention is more poetic.

The *Vimalakirti Nirdesa Sutra*, a Mahayana text some consider as dating from about 100 CE (although there is no mention of it till after Nagarjuna's time, i.e. about a century later), is a veritable litany of antinomies.

often used (for example, see the above quoted statement by Seng Tsan). So the 'noumenon' (i.e. that which is beyond the phenomenon) may be thought of as both transcendent and immanent. But a true Zen master would disdainfully reject all such philosophizing as misleading babble. Any resort to words as a means of rational description or explanation is regarded as useless when it comes to the "matter" of enlightenment. Consider for instance the following remarks in *The Blue Cliff Record*¹⁰¹:

"It's wrong to say either that he had words or didn't have words; nor will it do to say that his answer neither had nor didn't have words. Chao Chou left behind all the permutations of logic. Why? If one discusses this matter, it is like sparks struck from a stone, like flashing lightning. Only if you set your eyes on it quickly can you see it. If you hesitate and vacillate you won't avoid losing your body and life."

All the above tends to the conclusion that Zen 'logic' is illogical. However, that judgment can be considerably mitigated, if we understand Zen 'anti-dualistic' discourse not as theoretical but as pragmatic. Its purpose is not to formulate a true philosophy, in the Western sense, but to push people to a transforming mystical experience. Thus, when a Zen advocate states: "This is neither true nor false" or "This is neither good nor bad" or "This is neither desirable nor repugnant" – his intent is really, respectively: "Do not think or say that this is true and do

Pi Yen Lu, a Chinese Ch'an Buddhist classic. These remarks were made by Yuan Wu K'e Ch'in (1063-1152), relative to Case 59 (p. 339). Boston, MA: Shambala, 2005. Tr. Thomas & J.C. Cleary.

not think or say that it is false," "Do not think or say that this is good and do not think or say that it is bad," and "Do not think or say that this is desirable and do not think or say that it is repugnant." For example, Seng Tsan says:

"When you *assert* that things are real, you miss their true reality. But to *assert* that things are void, also misses reality. The more you *talk and think* on this, the further from the truth you'll be." (Italics mine.)¹⁰²

In other words, the Zen advocate is not really making logical, prescriptive or descriptive judgments, but advocating the suspension of all judgments, all discourse, in order to arrive at the ultimate "truth." There is no great inconsistency in doing that. We may, of course, point out that in claiming to be free of concepts he is using concepts and that that is an inconsistency. However, he would reply that he is doing that only in order to communicate with us in our language, in an attempt to allude to things beyond its scope. He is able to function in both the phenomenal and noumenal worlds, whereas we are not – so he has to find some way to reach out to us. In that case, we can only criticize him for being rather gauche in his discourse. He should make it more precise, as just demonstrated. It would then be possible to speak of Zen logic, without inverted commas.

Nevertheless, although a statement like "neither claim it is nor claim it is not" is intended as a non-claim, it objectively definitely does contain a factual claim – viz. the claim that following this advice will facilitate or result in enlightenment ("the truth"); and such a claim is,

The Blue Cliff Record. (I forgot to note the page number.)

of course, subject to assessment as true or false, whether the Zen advocate admits it or not.

I would say a very representative example of Zen logic is the *koan* of Te-shan (China, 9th century CE), the Zen master famous for presenting his students with the following predicament: whether they 'uttered a word' (i.e. showed some evidence through word or deed of their understanding of Zen) or not, they would get thirty blows. Another master, Lin-chi (the founder of the Rinzai sect), sent one of his own followers to him with specific instructions. He told him to ask Te-shan why someone who said a word would nevertheless get thirty blows; then, when Te-shan struck him, the student was to grab the stick and push Te-shan back with it. When the student did as instructed, Te-shan responded by simply walking away.¹⁰³

What we have here is a logic of action, rather than words¹⁰⁴. There is, to start with, a seemingly inescapable dilemma – whether you speak (rightly or wrongly) or abstain from speech, the result will be the same: you will be in error and punished by blows. There is, however, a logical possibility of escape – grab the stick as it comes down and push it back. This could be described as a martial arts response to the attempted physical blow. Logically, the dilemma has by this means been effectively dissolved. There seemed to be no way out, judging by Te-shan's statement; but there was in fact a

See D. T. Suzuki, *The Zen doctrine of no-mind* (Boston, MA: Weiser, 1972), p. 87.

Notice that the student did not try to dissuade Teshan, saying "if you try to hit me, I will grab the stick and push it." Rather, he waited for Te-shan to actually strike and then grabbed the stick and pushed it.

way out, perceived by Lin-chi. The opponent is neutralized, prevented from producing the threatened consequences (blows) to either antecedent (speech or silence).

Earlier in the present volume, in an attempt to more accurately depict the logic of a fortiori reasoning, I developed the notion of relative terms, say R1 and R2, such that more R1 and less R2 (and vice versa) are logically equivalent. This idea, I showed, can be extended to the special case of complements, say R and not-R. Although complements, taken as absolute terms, are mutually exclusive — if we take them as relative terms, they are compatible, indeed imply each other. That is, we can define R and not-R so largely that each includes the other, in the same continuum but in opposite directions, i.e. in such a way that more R is less not-R and less R is more not-R. This logical artifice of course changes the meaning of R and not-R, but it is useful for the development of a fortiori logic.

After I worked this idea out, it occurred to me that it could help explain Zen logic. It could be that Oriental philosophers who conceive of A and not-A as being compatible are really thinking in relative terms. Perhaps we in the West think of A and not-A in absolute terms, while they in the East think in relative terms. This may explain, at least in part, why the conjunction of A and not-A does not repel the Oriental mind to the same degree as it does the Western mind. Although, to be sure, this theory is somewhat belied by the fact that Orientals also accept the possibility of neither A nor not-A, which this theory cannot explain.

Needless to say, the said insight does not change the fact that A and not-A, taken in their absolute senses, are incompatible; the Aristotelian law of non-contradiction remains true and unassailable. Relative terms are logical artifacts that function consistently within that universal framework — they do not erase it. The law simply changes form, becoming a distinction between 'more' and 'less': What is more R is less (and *not* more) not-R, and what is less R is more (and *not* less) not-R. Moreover, it is interesting to note that when A and not-A are intended as relative terms, everything falls under both of them; there is no further possibility beyond them. That is to say, the law of the excluded middle also remains operative for relative terms, although it too is stated slightly differently.

2. The Vanity of the Tetralemma

The most radical assault on reason consists in trying to put in doubt the laws of thought, for these are indeed the foundations of all rational discourse. First, the law of identity is denied by saying that things are never quite what they seem to be, or that what they are is closer to grey than black and white. This is, of course, an absurd remark, in that for itself it lays claim to utter certainty and clarity. Then, the laws of non-contradiction are denied by saying that things may both be and not-be, or neither be nor not-be. This is the 'tetralemma', the fourfold logic which is favored in Indian and Chinese philosophies, in religious mysticism, and which is increasingly referred to among some 'scientists'. To grasp the vanity of the tetralemma, it is necessary to understand the nature of negation and the role of negation as one of the foundations of human logic.

The first thing to understand is that everything we experience is positive phenomenon. Everything we perceive through our senses, or remember or imagine in our minds, or even cognize through 'intuition' – all that has to have some sort of content to be at all perceived. Each sense organ is a window to a distinct type of positive phenomenon. We see the blue sky above, we hear birds sing, we smell the fresh air, we taste a fruit, we feel the earth's texture and warmth, etc. Similarly, the images and sounds in our heads, whether they come from memory or are produced by imagination, are positive phenomena; and even the objects of intuition must have some content that we can cognize. Secondly, we must realize that many positive phenomena may appear together in space at a given moment. This is true for each phenomenal type. Thus, the blue sky may fill only part of our field of vision, being bounded by green trees and grey buildings; we may at once hear the sounds of birds and cars; and so on. Thirdly, many positive phenomena may at any given time share the space perceived by us. Thus, superimposed on visual phenomena like the sky may be other types of phenomena: the sound of birds in the trees, the smell of traffic in the streets, the feelings in our own body, and so on. We may even hallucinate, seeming to project objects of mental perception onto physical space. For example, the image of one's eyeglasses may persist for a while after their removal. Fourthly, each positive phenomenon, whatever its type, varies in time, more or less quickly. Thus, the blue sky may turn red or dark, the sounds of birds or traffic may increase or decrease or even stop for a while, and so forth.

In order to express all these perceptual possibilities – differences in space and in time and in other respects, we need a concept of negation, or more precisely an act of negating. Without 'negation', we cannot make sense of the world in a rational manner – it is the very beginning of logical ordering of our experience. Thus, in a given visual field, where (say) blue sky and trees appear, to be able to say 'the sky ends here, where the trees begin' we need the idea of 'negation' – i.e. that on one side of some boundary sky is apparent and on the other side it is not, whereas on the first side of it trees are not apparent and on the other they are. Likewise, with regard to time, to be able to describe change, e.g. from blue sky to pink sky, we need the idea of 'negation' – i.e. that earlier on this part of the sky was blue and not pink, and later on it was pink and not blue. Again, we need the idea of 'negation' to express differences in other respects – e.g. to say that 'the sounds of birds singing seem to emanate from the trees, rather than from buildings'. Thus, negation is one of the very first tools of logic, coming into play already at the level of sorting of experiences.

Moreover, negation continues to have a central role when we begin to deal with abstractions. Conceptual knowledge, which consists of terms and propositions based directly or indirectly on perceptual phenomena, relies for a start on our ability to cognize similarities between objects of perception: 'this seems to resemble that somewhat' – so we mentally project the idea of this and that 'having something in common', an abstract (i.e. non-phenomenal, not perceived by any means) common property, which we might choose to assign a name to. However, to take this conceptual process further, we must be able to negate – i.e. to say that 'certain things

other than this and that do not have the abstract common property which this and that seem to have', or to say that 'this and that do not have everything in common'. That is, we must be able to say not only that one thing resembles another in some way, but also that these or other things do not resemble each other in that way or in another way. Thus, negation is essential for making sense of information also at the conceptual level of consciousness.

Now, what is negation? To answer this question we first need to realize that there are no negative phenomena in the realm of experience. Everything we perceive is positive phenomenon - because if it was not we obviously would have nothing to perceive. We can only 'perceive' a negative state of affairs by first mentally defining some positive state of affairs that we should look for, and then look for it; if having looked for it assiduously we fail to find it, we then conclude inductively that it is 'absent', i.e. 'not present'. Thus, positive phenomena come before negative ones, and not logically precedes Existence non-existence. Negative phenomena 'phenomena' are metaphorically, by analogy to positive phenomena – in truth, negative phenomena are not: they do not exist. 'Negation' is not a concept in the sense of an abstraction from many particular experiences having a certain property in common. Negation is a tool of the thinking observer, as above described. It is an act, an intention of his

To illustrate how confused some people – even some scientists – are with regard to negation, I offer you the following example drawn from Richard Dawkins' *The*

Greatest Show on Earth: The Evidence for Evolution¹⁰⁵. He describes an experiment by Daniel J. Simons, in which some people are asked to watch a brief video and observe how many times a certain event takes place in it; but at the end they are asked another question entirely, viz. whether they noticed the presence of a man dressed up as a gorilla in the course of the movie, and most of them admit they did not¹⁰⁶. According to Dawkins, we may infer from this experiment how "eye witness testimony, 'actual observation', 'a datum of experience' – all are, or at least can be, hopelessly unreliable."

But this is a wrong inference from the data at hand, because he confuses positive and negative experience. The people who watched the video were too busy looking for what they had been asked to observe to notice the gorilla. Later, when the video was shown them a second time, they did indeed spot the gorilla. There is no reason to expect us to actually experience everything which is presented to our senses. Our experiences are always, necessarily, selective. The validity of sense-perception as such is not put in doubt by the limited scope of particular sense-perceptions. The proof is that it is through further sense-perception that we discover what we missed before. Non-perception of something does not constitute misperception, but merely incomplete perception. 'I did not see X' does not deductively imply 'I saw the absence of X', even though repetition of the former tends to inductively imply the latter

New York: Free Press, 2009. Pp. 13-14.

The video can be seen at: www.theinvisiblegorilla.com/videos.html.

A negative 'phenomenon' is not like a positive phenomenon, something that can directly be perceived or intuited. A negation is of necessity the product of indirect cognition, i.e. of an inductive (specifically, adductive) process. We mentally hypothesize that such and such a positive phenomenon is absent, and then test and confirm this hypothesis by repeatedly searching-for and notfinding the positive phenomenon¹⁰⁷. If we were to at any time indeed find the positive phenomenon, the hypothesis of negation would immediately be rejected; for the reliability of a negation is far below that of a positive experience. We would not even formulate the negation, if we already had in the past or present perceived the positive phenomenon. And if we did formulate the negation, we would naturally retract our claim if we later came across the positive phenomenon. Therefore, the content of negative phenomena is necessarily always hypothetical, i.e. tentative to some degree; it is never firm and sure as with (experienced) positive phenomena.

Negative assertions, like positive assertions, can be right or wrong. If one looked diligently for a positive phenomenon and did not find it, then one can logically claim its negation. Such claim is necessarily inductive – it is valid only so long as the positive phenomenon is actively sought and not found. The moment the positive phenomenon is observed, the negation ceases to be

Not-finding is the non-occurrence of the positive act of finding. Objectively, note well, not-finding is itself a negative phenomenon, and not a positive one. But subjectively, something positive may occur within us – perhaps a sense of disappointment or continued relief. See more on this topic in my *Ruminations*, chapter 9.

justified. If one did *not* look for the positive phenomenon, or did not look with all due diligence, perhaps because of some distraction (as in the example cited above), then of course the claim of negation is open to doubt; certainly, it is inductively weak, and one is very likely to be proved wrong through some later observation.

How, then, is negation to be defined? We could well say that negation is defined by the laws of non-contradiction and of the excluded middle. That is, with regard to any term 'X' and its negation 'not-X', the relation between them is by definition the disjunction "Either X or not-X" - which is here taken to mean that these terms (X and not-X) cannot be both true and cannot be both false, i.e. they are exclusive and exhaustive. What do I mean here by 'definition'? - is that an arbitrary act? No - it is 'pointing to' something evident; it is 'intentional'. Here, it points to the instrument of rational discourse which we need, so as to order experience and produce consistent conceptual derivatives from it. The needed instrument has to be thus and thus constructed; another construct than this one would not do the job we need it to do for us. That is, the only conceivable way for us to logically order our knowledge is by means of negation defined by means of the laws of non-contradiction and of the excluded middle. Without this tool, analysis experience is impossible.

Suppose now that someone comes along and nevertheless objects to the preceding assertion. Well, he says, how do you *know* that the dilemma "either X or not-X" is true? You just arbitrarily *defined* things that way, but it does not mean it is a fact! Could we not equally well claim the tetralemma "Either X or not-X *or both or neither*" to be

true? The reply to that objection is very simple. Suppose I accept this criticism and agree to the tetralemma. Now, let me divide this fourfold disjunction, putting on the one side the single alternative 'X' and on the other side the triple alternative 'not-X or both or neither'. I now again have a dilemma, viz. "either 'X' or 'not-X or both or neither'." Let me next define a new concept of negation on this basis, such that we get a disjunction of two alternatives instead of four. Let us call the complex second alternative 'not-X or both or neither' of this disjunction 'NOT-X' and call it 'the super-negation of X'.

Thus, now, the objector and I agree that the disjunction "either X or NOT-X" is exclusive and exhaustive. We agree, presumably, that this new dilemma cannot in turn be opposed by a tetralemma of the form "Either X or NOT-X or both or neither" – for if such opposition was tried again it could surely be countered by another division and redefinition. We cannot reasonably repeat that process ad infinitum; to do so would be tantamount to blocking all rational thought forever. Having thus blocked all avenues to thought, the objector could not claim to have a better thought, or any thought at all. There is thus no profit in further objection. Thus, the tetralemma is merely a tease, for we were quite able to parry the blow. Having come to an agreement that the new disjunction "Either X or NOT-X" is logically unassailable, we must admit that the original disjunction "Either X or not-X" was logically sound from the first. For I can tell you that what I meant by not-X, or the 'negation of X', was from the beginning what is now intended by NOT-X, or the 'super-negation of X'!

I was never interested in a relative, weak negation, but from the start sought an absolute, strong negation. For such utter negation, and nothing less radical, is the tool we all need to order experience and develop conceptual knowledge in a consistent and effective manner. In other words, whatever weaker version of negation someone tries to invent¹⁰⁸, we can still propose a strong version such that both the laws of non-contradiction and of the excluded middle are applicable without doubt to it. If such negation did not exist, it would have to be invented. No one can destroy it by denying it or diluting it. Those who try to are merely sophists who do not understand the source, nature and function of negation in human discourse. They think it is a matter of symbolic manipulation, and fail to realize that its role in human discourse is far more fundamental and complex than that. Negation is the indispensable instrument for any attempt at knowledge beyond pure perception.

¹⁰⁸ There are people who say that the law of noncontradiction is logically necessary, but the law of the excluded middle is not. Clearly, this claim can be refuted in the same way. If they claim the three alternatives "Either X or not-X or 'neither X nor not-X'" - we can again split the disjunction into two, with on one side "X" and on the other side "not-X or 'neither X nor not-X'" - and then proceed as we did for the tetralemma. The same can be done if anyone accepts the law of the excluded middle but rejects the law of non-contradiction. All such attempts are fallacious nonsense.

29. CHAPTER TWENTY NINE

Drawn from *A Fortiori Logic* (2013), Appendix 7.1-2.

MODERN LOGIC

1. Modern Symbolic Logic

Since the later decades the 19th century, and more and more so throughout the 20th century, "modern symbolic logic" has gradually discarded and displaced "classical formal logic." What is the essential difference between them? Classical formal logic, which was discovered or invented by Aristotle (4th century BCE) and further developed and improved on over time by many successors, is based on the idea of studying the logical properties of propositions by replacing propositions with formal ones. A categorical proposition is formal, if its terms are variables instead of constants – e.g. "All X are Y" is formal, because the symbols X, Y represent in theory any terms that might arise in practice. A hypothetical proposition is formal, if its theses are variables instead of constants – e.g. "If X, then Y" is formal, because the symbols X, Y represent in theory any theses that might arise in practice.

Now, whereas classical logic symbolized terms and propositions, it did not similarly symbolize the other components of propositions, such as their quantities, their modalities or their relational operators. In "All X are Y,"

the words "all" and "are" remained in ordinary language (in our case, plain English). Similarly, in "If X, then Y" the words "if" and "then" were not symbolized. In modern symbolic logic, on the other hand, the trend developed to symbolize every aspect of every proposition¹⁰⁹. This was, to be sure, *a new school* of logic, which considered that only in this way could utter precision of language be achieved, and all ambiguity or equivocation be removed from human discourse. Modern symbolic logic, then, advocated the adoption of an altogether artificial language comparable to the language of mathematics.

Some of the pros and cons of this approach are immediately obvious¹¹⁰. One advantage of symbolization, already mentioned, is the sense of precision sometimes lacking in natural languages. However, this impression is surely illusory – for if one's understanding of the matter at hand is vague and uncertain to start with, how can symbols improve on it? A patent disadvantage of symbolization is the esoteric nature of artificial language. Logic was originally intended as a teaching for the masses, or at least the intellectuals, to improve their daily thinking. Nowadays, logic has become the exclusive domain of a few specialists, and has little to do with human cognitive practice. Moreover, communication is not always easy even among symbolic logicians, because each of them quite naturally prefers a different set of

¹

For examples: the word "all" became an upside down capital A, the word "some" became a laterally inversed capital E (for existence, as in 'there are'), the words "if—then" (implication) became an arrow pointing from antecedent to consequent, and so forth.

For more on this topic, see my *Future Logic*, chapter 64: Critique of Modern Logic.

symbols, so that there is in fact not one artificial language, but many of them. Another disadvantage is the slow adaptability of any artificial language to forms of discourse newly discovered in everyday usage. An example of this is the a fortiori argument, which is still without convincing symbolic expression.

The main activity of 'modern logicians' nowadays seems to be to translate ordinary language into their favorite symbolic language. Most of the time, they seem content to rewrite a perfectly comprehensible plain English sentence into a purely symbolic one, as if this is some great achievement that will earn them their place in history, or at least in the profession. Just that act of translation or rewriting in symbolic terms seems to satisfy and thrill them tremendously, as if it confers scientific status onto the sentence. Additionally, they resort to pompous terminology for window-dressing and intimidation purposes. One gets the impression that symbols play for them the role of magic incantations in ancient times — 'abracadabra!' they would chant in pursuit of mystical insights and powers.

But, think about it a moment. In truth, when modern logicians rewrite a sentence in symbolic terms they have achieved *exactly nothing* other than to use shorter 'words' (i.e. the symbols they invent) in place of ordinary words, and (ideally) drawn up a table telling us what symbols correspond to what ordinary words. All they have done is *abbreviate* the given sentence. Apparently, they are too lazy to write long sentences and prefer concise ones. They have produced *no new information or insight*. They cannot credibly argue that the ordinary language statement was essentially deficient, since it must have been understandable enough for them

to have translated it into symbols. If it was understandable enough for them, why not for everyone else? What absolute need have we of the artificial language(s) they so insistently try to sell us?¹¹¹

Moreover, note this well, whenever we (and they) read the symbolic statement they have concocted, our minds have to translate it back into ordinary language in order to understand it. We have to mentally repeatedly refer back to the ordinary language definitions of the symbols. We have to remember: "Oh! This funny symbol means so and so, and that weird doodle means this, and the zigzag means that," and so forth. This means that our mental process of understanding is made more difficult and slowed down considerably. We are further from the object of study than we were to start with - more removed from the reality we are trying to think about. This increased distance and waste of time is not accidental or incidental, however – it serves to cloud the issues and prevent critical judgment. Errors are hidden from sight, and if we spot them we hardly dare point to them for fear of admitting we may have missed something. In this way, foolishness is perpetuated and spreads on.

But all that is not the worst of it. The worst of it is twofold. First, modern logicians usually *symbolize much too soon*, when their level of insight and analysis is still in its early stages. They typically do not give the subject-

Ideally, of course, symbols are useful to summarize large amounts of information. I would dearly love to develop terse symbolic formulas that summarize my findings in the logic of causation. So I am not entirely rejecting symbolism. What I am saying here is that it is not necessary (i.e. we can well do without it) and indeed can be a serious hindrance to logical thinking and logic theory.

matter studied time to develop and mature in their own minds, but impatiently rush into their more orderly looking and comfortable world of symbols. The result is that their symbols are usually representative of a very naïve, elementary, immature level of understanding of the object at hand. Secondly, once their symbolic representation is done, it freezes all subsequent work at that childish level. Since the symbolization is already settled, all they can do is play around with it in different ways. All they can do is manipulate and reorder and recombine their symbols this way and that way, and this is what they pass the rest of their time doing. They cannot feed on new experiences and insights from the world out there or actual human discourse, since they have already confidently and reassuringly separated themselves from all that. Their symbols thus blind them and paralyze them. The paucity of their results testifies to it

Another aspect of modern symbolic logic important to note is its pretensions of 'axiomatization'. In classical formal logic, the Laws of Thought (Identity, Noncontradiction and the Excluded Middle) were sovereign; these were axioms in the original sense of irreducible primaries of rational knowledge, together constituting the very essence of logic. In syllogistic validation by Aristotle, all syllogisms could be reduced directly or *ad absurdum* to a minimum number of primary moods – mainly the first figure positive singular syllogism: "This S is M, and all M are P, therefore, this S is P." The latter argument was not perceived as an axiom in its own right or even as an arbitrary convention, but as a logical insight in accord with the laws of thought that what is claimed applicable to a concept must be acknowledged to

apply to the things it subsumes. The relation of human reasoning, and more deeply of formal logic, to the laws of thought was progressively 'systematized', but it was allowed to remain essentially *open and flexible*. Though integrated, it was not rigidly fixed, so as to allow for its constant evolution and adaptation as knowledge developed.

Modern logicians, on the other hand, focusing on the more geometrico, the method of proof used in Euclidean geometry, sought a more predictable and definitive arrangement of knowledge. Their simplistic minds demanded rigid rules and perfect orderliness. A hierarchy was established between thoughts – with those at the top of the hierarchy (the laws of thought) being viewed as 'axioms' and those lower down (syllogisms, and eventually similarly other arguments) as 'theorems'. This may work well for mathematics, which is a relatively special science, but it caused havoc in general conceptual logic, which is the science of science. The question naturally arose as to where those apparent axioms came from and whether they could be replaced by contrary ones as was done in non-Euclidean geometry. It did not take long for these simpleminded people to decide that logic was a conventional mental game, with no apparent connection to the empirical world. This philosophy (known as Logical Positivism) was largely justified by Immanuel Kant's analytic-synthetic dichotomy¹¹², so it could hardly be doubted.

What is lacking in this model of knowledge is the understanding that formal logic is not deduced from the

See my Logical and Spiritual Reflections, book 2, chapter 2, posted online at: www.thelogician.net/6_reflect/6_Book_2/6b_chapter_02.htm.

laws of thought. The laws of thought are not premises of formal logic; they are not contents from which other contents are deduced. The laws of thought refer us to the autonomous logical insights through which we naturally iudge what constitutes appropriate inference. They are what justifies the processes of deduction (and more broadly, of induction) from premises to conclusions. The forms of syllogism and other arguments are not deduced from the laws of thought. The forms are induced from actual thought contents. The thought contents can be judged correct or not without reference to the forms, using ad hoc logical insights. What formal logic does is simply collect under a number of headings recurring types of thoughts, so that again using ad hoc logical insights we can once and for all predict for each type of thought (e.g. syllogism 1/AAA) whether it is correct or not. There is in fact no appeal to general 'laws of thought' in this validation (or invalidation) process; honest ad hoc logical insights are sufficient. The 'laws of thought' are merely ex post facto typologies of ad hoc particular acts of logical insight. For that reason, they are not top premises is a geometrical model of knowledge. The question these modern pseudo-logicians did not ask themselves, of course, is why their allegedly logical insights in the course of 'axiomatization' (including their skepticism towards the objectivity of the laws of thought) should be preferred to the logical insights of the 'nonaxiomatization' logicians. Is any discussion of logic possible without use of logic? Can logicians ever rightly claim to transcend logic? Can they logically deride and nullify logic? For instance, some have argued that appeal to the laws of thought is either circular argument or infinite regression. They stopped their reflection there,

and never asked themselves why the rejection of circular argument or infinite regression should be considered primary logical acts not needing justification, while the laws of thought are to be rejected precisely on the ground that (according to them) there are no logical acts not needing justification. Is that not a double standard (another primary logical insight)?¹¹³

The radical blunder of the Kantian legacy is the belief that there is such a thing a 'purely analytic' or 'a priori' knowledge. Logicians influenced by this inane idea remain blind to the empirical aspects of all knowledge development. Even apparently purely symbolic systems of logic rely on perceptions. Some symbols refer to concrete objects (e.g. individuals a, b) and some to abstract ones (e.g. classes x, y); but every symbol is, as well as a sign for something else, in itself a concrete object (whether as a bit of ink on paper, or of light on a computer screen, or as a shape conjured in our mind's eye). If follows that symbolic formulas, whether inductively or deductively developed, always depend on some empirical observation. The observation of symbols is not a transcending of experience; it is an empirical process just like the observation of cows; it requires physical or mental perception. Thus, if I count symbolic entities or I imagine them collected together, that is not purely analytic work – it is quite synthetic work. Moreover, such logicians tend to ignore the countless memories, imaginations and rational insights that form the wordless background of all discourse concerning logic.

We should of course in this context mention Kurt Gödel, who showed the incompleteness of axiomatic systems like that of David Hilbert.

Clearly, axiomatization was a con-game on a grand scale, through which shallow but cunning pseudo-logicians wanted to take power in the domain of logic. And they have indeed managed to do that almost completely. But the fact remains that primary logical insights like the laws of thought, or the rejection of circularity, infinite regression and double standards, or again the acceptance of subsumption (syllogistic reasoning) and the many, many other foundations of human thought, are not open to doubt. No amount of 'axiomatization' can either prove or disprove them, because all proof and disproof depends on such insights to at all convince us. The conclusion to that is certainly *not* relativism from conventionalism, for that too would be a claim to the validity of some logical insight – i.e. the insight that all insights are arbitrary must itself be arbitrary. Logic cannot be refuted by logic. Logic can however be justified, by honest acknowledgment that some thoughts are primary logical insights. And these insights, which together constitute what we call 'human reason', cannot all be listed in advance, but emerge over time as knowledge develops.

Aristotle said all that long ago, but many have preferred to ignore him or misrepresent him because they dearly want to belittle him and supplant him, being envious of his achievements. Consider for instance the following statement about the law of non-contradiction drawn from his *Metaphysics* (Book 4, part 3. Translated by W. D. Ross.):

"For a principle which every one *must have who* understands anything that is, is not a hypothesis; and that which every one must know who knows anything, he must already have when he comes to

a special study. Evidently then such a principle is the most certain of all; which principle this is, let us proceed to say. It is, that the same attribute cannot at the same time belong and not belong to the same subject and in the same respect." (Italics mine.)

As a result of symbolization and axiomatization, modern logic is essentially a deductive logic enterprise. However complicated or complex it may look, it is inevitably superficial and simplistic. Even when modern logicians pretend to discuss induction, they are stuck in deductive activities. Their 'logic' is thus more and more divorced from reality. For this reason their thinking on issues of metalogic is thoroughly relativistic. Things have to be the way they think they are, since symbols are somehow omniscient and omnipotent. They see no idiocy or harm in 'paraconsistent logic' (i.e. in breach of one or more of the laws of thought), since to them it is all a game with conventional symbols with no connection to any reality. When things do not fit into their preconceived schemes, they blithely force them in and use florid terminology to keep critical judgment at bay. They do not look upon practical deviations from their arbitrary theoretical constructs as problems, as signals that they have made mistakes somewhere on their way; they just add more symbols and make their theories more abstruse. Please don't think I am exaggerating – that's the way it is.

Why are so many people drawn to and impressed by modern symbolic logic? Part of the problem is of course that this is what the universities demand from their teaching staff and teach their students; papers have to be written in symbolic terms to be even considered. But why this preference? Perhaps because pages filled with

esoteric symbols seem more 'scientific', reminding readers of mathematical formulae in the physical sciences. It matters little that in logical science the subjacent subject-matter becomes less transparent and comprehensible when translated into symbols. Indeed, part of the aim is to befuddle and intimidate the reader, so as to conceal weaknesses and faults in the treatment. The grandiloquent language is similarly useful as eyewash. Modern symbolic logic boasts of superiority to classical formal logic, to give itself authority; but the truth is that most good ideas the former has it has stolen from the latter, reworking them a little and renaming them to seem original and independent. The whole enterprise is a massive ongoing fraud; or, alternatively, a collective delusion of epidemic proportions.

I am not, of course, saying (as, no doubt, some will rush to accuse me of saying) that everything modern symbolic logic tells us is false and irrelevant, or stolen. What I am saying is that whatever is true and significant in it is certainly not *due to* symbolization and axiomatization, and can equally well be (could be and probably was) developed by classical formal logic. Moreover, to repeat, excessive symbolism tends to simplistically lump things together and gloss over important nuances, and condemns its users to rigid and abstract thinking processes out of touch with the empirical domain.

We have seen, in the course of the present treatise on a fortiori logic, how some budding or experienced logicians strayed or failed due to their attempt to solve problems by means of modern symbolic logic. In the following pages, I present some more examples of the relative ineffectiveness of modern symbolic logic compared to classical formal logic. I show how the issue

of 'existential import' is far less significant that it is touted to be; how attempts to bypass the laws of thought are futile; how the liar paradox is not only due to self-reference; and how the Russell paradox is due to the acceptance of self-membership.

2. The Existential Import Doctrine

A term is, nowadays, said to have 'existential import' if it is considered to have existing referents; otherwise, it is said to be 'empty' or a 'null class'. For examples, 'men' has existential import, whereas 'dragons' does not. This concept is considered original and important, if not revolutionary, in modern symbolic logic; and it is often touted as proof of the superiority of that school over that of classical formal logic. We shall here examine and assess this claim. As we shall see, although the concept has some formal basis, it is in the last analysis logically trivial and cognitively not innocuous.

The founder of formal logic, Aristotle, apparently did not reflect on the issue of existential import and therefore built a logical system which did not address it. The issue began to be raised in the middle ages, but it was not till the latter half of the nineteenth century that it acquired the importance attached to it today by modern logicians.

a. Based on Aristotle's teaching, classical formal logic recognizes six basic categorical forms of proposition: the general affirmative, "All S are P" (A), which means that each and every S is P; the general negative, "No S is P" (E), which means that each and every S is not-P; the particular affirmative, "Some S are P" (I), which means that each of an indefinite number (one or more) of S is P; the particular negative, "Some S

are not P" (O), which means that each of an indefinite number (one or more) of S is not-P; and the singular affirmative, "This S is P" (R), and the singular negative, "This S is not P" (G), which refer to a specifically pointed-to or at least thought-of individual instance. Note that general (also called universal) propositions and particular propositions are called plural, in contradistinction to singular ones¹¹⁴. The labels A, E, I, O, R and G come from the Latin words *AffIRmo* and *nEGO*; the first four are traditional, the last two (R and G) were introduced by me years ago¹¹⁵.

The symbols S and P stand for the subject and predicate. The verb relating them is called the copula, and may have positive (is or are) or negative (is not or are not) polarity¹¹⁶. In the present context, the copula should be understood very broadly, in a *timeless* sense¹¹⁷. When we say 'is' (or 'is not') we do not mean merely "is (or is not) now, at this precise time," but more broadly "is (or is not) at some time, in the past and/or present and/or future." The expressions 'all', 'some' and 'this' are called quantities. Obviously, the general 'all' covers every single instance, including necessarily 'this' specific

Singular propositions are often called particular, but this usage is inaccurate, since they refer to an indicated individual.

One can remember these six labels by means of the phrase ARIEGO.

What I have called 'polarity' is traditionally called 'quality', but the latter term is inaccurate and confusing and should be avoided.

This approach allows us to momentarily ignore the issue of modality, and reflects common usage in many contexts. A fuller treatment of categorical propositions must of course deal with modality; I do that in my earlier work, *Future Logic*.

instance; and 'all' and 'this' both imply the particular 'some', since it indefinitely includes 'at least one' instance. The 'oppositions' between the six forms, i.e. their logical interrelationships, are traditionally illustrated by means of the following 'rectangle of oppositions':

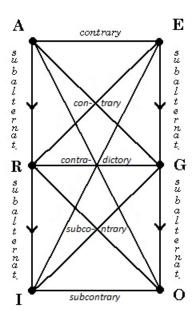


Diagram 29.1 – Aristotelian oppositions

Although Aristotle did not, to our knowledge, represent the oppositions by means of such a diagram, we can refer to it as a summary his views. It is taken for granted that, on the positive side A implies R and R implies I (so, A implies I), and on the negative side E implies G and G implies O (so, E implies O), although these implications cannot be reversed, i.e. I does not imply R or A, and R

does not imply A, and so forth. This is called subalternation¹¹⁸. The core opposition in this diagram is the contradiction between R and G; from this assumption, and the said subalternations, all else logically follows¹¹⁹. A and O are contradictory, and so are E and I; A and E, A and G, E and R, are pairs of contraries; I and O, I and G, O and R, are pairs of subcontraries. Two propositions are contradictory if they cannot be both true and cannot be both false; they are contrary if they cannot be both true but may be both false; they are subcontrary if they may be both true but cannot be both false.

b. Shockingly, the above traditional interpretation of the basic categorical forms (Diagram 29.1) has in modern times been found to be problematic. The above listed propositions are not as simple as they appear. The form "Some S are P" (I) means "Something is both S and P," while the form "All S are P" (A) means "Something is both S and P, and nothing is both S and not-P;" similarly, the form "Some S are not P" (O) means "Something is both S and not-P," while the form "No S is P" (E) means "Something is both S and not-P, and nothing is both S

The implying proposition being called the subalternant and the implied one the subaltern, and the two being called subalternatives.

If A is true, then R is true, then G is false, then E is false; whence, the contrarieties shown on the diagram. If I is false, then R is false, then G is true, then O is true; whence, thus the subcontrarieties shown. Since R and G are incompatible (cannot both be true) and exhaustive (cannot both be false), it follows that A and O, and likewise E and I, whose instances overlap somewhat, must be contradictory, since, if they were both true or both false, R and G would in at least one case be accordingly both true or both false (this is proof by exposition).

and P." Seeing the forms I, A, O, E, in this more detailed manner, we can understand that A implies I since I is part of A (and likewise for E and O), but then we realize that A and O are not truly contradictories (and likewise for E and I).

The exact contradictory of "Something is both S and not-P" (O) is "Nothing is both S and not-P" (i.e. only part of A, with no mention of its I component) and the exact contradictory of "Something is both S and P, and nothing is both S and not-P" (A) is "Nothing is both S and P, and/or something is both S and not-P" (i.e. a disjunction including O, but also E). Note this well¹²⁰.

It should be pointed out that "All S are P" (A) can be defined more briefly as: "Something is S, and nothing is both S and not-P;" for given this information, it follows logically that the things that are S are P (for if this was denied, it would follow that some things are both S and not-P), Similarly, "No S is P" (E) can be defined more briefly as: "Something is S, and nothing is both S and P," without need to specify explicitly that "Some things are both S and not-P." Thus, all four forms A, E, I, O, imply, or presuppose (which is logically the same), that "some S exist(s)." Also, the positive forms, A and I, imply that "some P exist(s)." On the other hand, the negative forms, E and O, do not imply that "some P exist(s)," since the

The Kneales propose a similar analysis of the problem in *The Development of Logic* (Oxford, London: Clarendon, 1962), chapter II, section 5. Further on (on p. 211), they say that Peter Abelard "should have the credit of being the first to worry about the traditional square of opposition, though he did not work out all the consequences of the change he advocated."

negation of a term is not informative regarding its affirmation¹²¹.

Thus, in the above diagram, the diagonal links between the corners A and O, and between E and I, should not be contradiction but contrariety. For, while to affirm one proposition implies denial of its opposite, to deny one proposition does not imply affirmation of the other. To remedy this real problem of consistency, modern logicians have proposed to redefine the general propositions A and E as the exact contradictories of O and I, respectively. That is to say, the new meaning of A is only "Nothing is both S and not-P" and the new meaning of E is only "Nothing is both S and P." It follows from this measure that A (in its new, slimmer sense) no longer implies I, and likewise E (in its new, slimmer sense) no longer implies O. This redefinition of symbols A and E can, to my mind, lead to much confusion. In my view, it would be better to re-label the forms involved as follows:

- Keep the traditional (old) labels A and E without change of meaning; i.e. old A = A, old E = E.
- Label the modern (new) senses of A and E as respectively not-O and not-I.
- That is, new 'A' = not-O ≠ old A. Whereas, old A = new 'A' plus I = I and not-O.
- Likewise, new 'E' = not-I ≠ old E. Whereas, old E = new 'E' plus I = O and not-I.

Thus, when we say A or E in the present paper, we mean exclusively the traditional A and E; and when we wish to

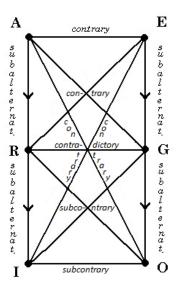
We could say that nothing in the world is conceivably P, without affecting the truth of "Some S are not P" or "No S is P." Clearly, in the special case where "nothing is P," the latter propositions are true for any and every value of S.

speak of the modern 'A' and 'E' we simply say not-O and not-I, respectively. Note this convention well¹²². Actually, such propositional symbols are effectively abandoned in modern logic and the propositions are expressed by means of a symbolic notation, including the existential and universal quantifiers, ∃ (there exists) and ∀ (for all), respectively; but we do not need to get into the intricate details of that approach here, because we can readily discuss the issues of interest to us in plain English. Now, let us consider the formal consequences of the above findings in pictorial terms.

One way for us to solve the stated problem is to merely modify the traditional rectangle of oppositions, by showing the diagonal relationships between A and O and between E and I to be contrariety instead of contradiction; this restores the traditional diagram's consistency, even if it somewhat dilutes its force (Diagram 29.2). Another possibility, which is the usual modern reaction, is to change the top two corners of the rectangle to not-O and not-I, instead of A and E respectively; this allows us to retain the contradiction between diagonally opposed corners, although now the lateral relation between the top corners is unconnectedness instead of contrariety, and the vertical

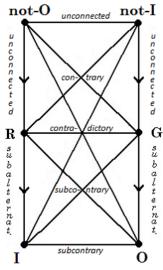
Of course, we could introduce modified symbols for the new A and E, such as A' and E', but I prefer to stress their underlying meanings, viz. not-O and not-I. In my view, it is dishonest and misleading to redefine the symbols A and E themselves as meaning only not-O and not-I. This is like a hostile takeover, permanently blocking further reflection and debate

relations in the upper square are unconnectedness instead of subalternation (Diagram 29.3).¹²³



29.2 – modified traditional

A third possible approach is, of course, to draw a rectangle with A and E in the top two corners, and not-E and not-A (instead of I and O) in the bottom two corners. In that case, it is the lower square that would suffer changes, with not-E and not-A as unconnected to each other and to R and G respectively. This possibility is however not very interesting, as the forms not-E and not-A are disjunctive. That is, not-E = not-(O and not-I) = not-O and/or I; and not-A = not-(I and not-O) = not-I and/or O. Note that this position is historically found in Peter Abelard, who insisted on distinguishing between "Not all S are P" (not-A) and "Some S are not P" (O), and who apparently denied that "No S is P" (E) implies anything to be S let alone P (even while regarding "All S are P" (A) as implying that something is S); see Kneales, p. 210.



29.3 – modern version

Notice that the lower square of the modern version is unchanged. This is due to the judgment that the forms I and O, i.e. "Something is both S and P" and "Something is both S and not-P," both imply that "some S exist" (or "some things are S" or "there are things which are S") meaning that if they are true, their subject 'some S' has existential import. Moreover, in the case of I, the predicate P is also implied to have existential import, since it is affirmed; but in the case of O, the predicate P is not implied to have existential import, since it is merely denied. Until now, note well, we have not mentioned the issue of existential import in our formal Now. into play, treatment. it comes with this interpretation of particular propositions.

The same applies to R and G – their subject 'this S' has existential import, whereas the predicate P has it if affirmed but lacks it if denied. On the other hand, since

not-O (as distinct from A) is a negative statement, i.e. means "Nothing is both S and not-P," it has no implication of existential import. Similarly, since not-I (as distinct from E) is a negative statement, i.e. means "Nothing is both S and P," it has no implication of existential import. Clearly, if not-O was thought to be contrary to not-I, then if not-O were true, it would imply the negation of not-I, i.e. it would imply I; but this being erroneous, not-O and not-I cannot be contrary, i.e. they must be unconnected. Similarly, if not-O was assumed to imply R, it would then imply I, since R still implies I; therefore, not-O must also be unconnected to R; and similarly for not-I and G. On the other hand, not-O remains contrary to G, since if not-O is true, then O is false, in which case G must be false; similarly as regards not-I and R.

It is now easier to see why the traditional rectangle of oppositions (7.1) seemed right for centuries although it was strictly-speaking wrong. It was tacitly assumed when drawing it that the subjects of general propositions *always* have existential import, i.e. imply that "some S exist (s)." *When this condition is granted*, then in combination with it not-O becomes A and not-I becomes E, and A implies I and E implies O, and A exactly contradicts O and E exactly contradicts I – in other words we happily return to the original rectangle of oppositions (7.1). The problem is that this condition is *not* always satisfied in practice. That is, not-O or not-I can occur without their subject S having existential import.

Effectively, the forms "Nothing is both S and not-P" (not-O) and "nothing is both S and P" (not-I) signify conditional propositions ("Whatever is S, is P" and "Whatever is S, is not P") which, without the minor

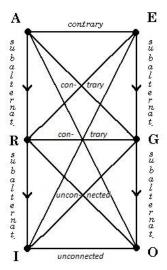
premise "this is S," cannot be made to conclude "this is P" or "this is not P" (respectively). In other words, they record a 'connection' between an antecedent and a consequent, but they have no 'basis', i.e. they contain no information affirming the antecedent, and thence the consequent. Obviously, if that information is provided, the condition is fulfilled and the result follows. Once we realize that the traditional rectangle remains true in the framework of a certain simple condition (viz. that some S exist), we see that its hidden 'inconsistency' is not such a big problem for formal logic.

It is interesting to also consider the significance of the above revisions in the field of eduction (i.e. immediate inference). Whereas A, which implies I ("Some S are P"), is convertible to "Some P are S" - not-O, which does not imply I, is *not* so convertible. Also, whereas not-I is convertible to "No P is S," since "Nothing is S and P" and "Nothing is P and S" are equivalent and have no implication of existential import for S or P - E is not likewise unconditionally convertible, since in its case even if we are given that "some S exist" we cannot be sure that "some P exist" (but only that "some not-P exist"). Note well, just as O does not imply predicate P to have existential import, since it merely negates it, so is it true for E; therefore, the traditional conversion of E is really only valid conditionally. We can also look into the consequences of the above revisions in the field of syllogistic reasoning; the main ones are pointed out further on.

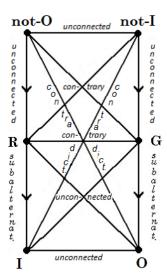
c. Let us now go a step further in the possible critique of Aristotelian oppositions, and suggest that *all* terms may be denied to have existential import, whatever the forms they occur in, and whatever their positions

therein. That is to say, not only the subjects of general propositions, but even the subjects of singular or particular propositions might conceivably lack existential import. Although R and G, and I and O, do formally imply that some S exist(s), it is still possible to deny them in pairs without self-contradiction. That is, R and G cannot be claimed strictly-speaking contradictory, because if "this S exists" is false then they are both false; this means that their traditional relation of contradiction is valid only conditionally (i.e. provided "this S exists" is true) and their absolute relation is in truth only contrariety. Similarly, I and O are only relatively subcontrary and their unconditional relation is really unconnectedness.

Indeed, it happens in practice that we reject a singular subject altogether, when we find that some predicate can be both affirmed and denied of it. This is dilemmatic argument: finding both that 'this S' is P and that it is not P, we must conclude that either one of these predications is wrong, or both are wrong because 'this S' does not exist. Particulars, of course, do not necessarily overlap; but if we can show by other means that "no S exists," we can be sure that neither the set of S referred to by I nor that referred to by O exist, and thus deny both propositions at once. Granting all this, the above diagrams (7.2 and 7.3) can be *further* modified as follows:



29.4 re-modified traditional



29.5 – modified modern version

In both these diagrams (7.4 and 7.5), all relations are the same as before, except the one between R and G (contrariety), and those between R and O, G and I, and I and O (which are now unconnected pairs). Notice that in the second diagram (7.5), although R and G are no longer contradictory, the pairs not-O and O, and not-I and I, remain contradictory, since if we deny that "Something is both S and P" (I) on the basis that "No S exists," we can all the more be sure that "Nothing is both S and P" (not-I), and likewise regarding O and not-O.

d. We have thus proposed two successive dilutions (weakening revisions) of the traditional rectangle of oppositions. In the first, we followed modern logic in no longer assuming with Aristotle that the subjects of universal propositions have existential import. In the second, we went further and additionally denied that singular and particular propositions may well lack existential import. Clearly, if our goal is to formulate an absolute logic, one applicable equally to propositions with existential import and those without, the successive dilutions of the Aristotelian diagram are justified and important. But are such logics of anything more than academic interest – are they of practical interest? The answer must clearly be no, as I will now explain.

A difficulty with the ideas of existential import and emptiness is immediately apparent: these are characterizations that may be true or false. Different people at the same time, or the same person at different times, may have different opinions as to the existential import or emptiness of a certain term. Some people used to think that dragons exist, and maybe some people still do, yet most people today think dragons never existed. So, these characterizations are not obvious or fixed. Yet

modern logicians present the question of existence or non-existence as one which has a ready answer, which can be formally enshrined. They fail to see that the issue is not formal but contentual, and thus in every given material case subject to ordinary processes of testing and eventual confirmation or disconfirmation.

It follows that the issue of existential import is not as binary as it is made out to be. The issue is not simply existence or non-existence, as modern logicians present it. The issue is whether at a given time we know or not that existence or non-existence is applicable to the case at hand. A term with existential import may be said to be 'realistic', in that it refers (or is believed to refer) to some existing thing(s). An empty term, i.e. one without existential import, may be said to be 'unrealistic', in that it refers (or is believed to refer) to a non-existent thing. In between these two possibilities lies a third, namely that of 'hypothetical' terms, for which we have not yet settled the issue as to whether they are (in our opinion) realistic or unrealistic. Moreover, this third possibility is not monolithic like the other two, but comprises a host of different degrees.

Our knowledge is mostly based on experience of physical and mental phenomena, though also on logical insights relating to such experience. Roughly put, we would regard a term as realistic, if we have plentiful empirical evidence as to the existence of what it refers to, and little reason to doubt it. We would regard a term as unrealistic, if we have little empirical evidence as to the existence of what it refers to, and much reason to doubt it. And we would regard a term as hypothetical if we are thus far unable to decide whether it should be characterized this way or that. In any case, the decision is

usually and mostly inductive rather than purely deductive as modern logicians effectively imagine it.

How are terms formed? Very often, a term is formed by giving a name to a circumscribed phenomenon or set of phenomena that we wish to think about. Here, the definition is fixed. More often, a term is applied *tentatively* to a phenomenon or set of phenomena, which we are not yet able to precisely and definitively circumscribe. In such case, we may tentatively define it and affirm it, but such a term is still vague as well as uncertain. Over time we may succeed in clarifying it and making it more credible. Here, the definition is variable. Thus, the formation of terms is usually not a simple matter, but *an inductive process* that takes time and whose success depends on the logical skills of the thinker(s) concerned.

Of course, as individuals we mostly, since our childhood, learn words from the people around us. This is effectively fixed-definition terminology for the individual, even if the term may have been developed originally as a variable-definition one. In this context, if we come across an obscure ready-made term, we cannot understand it till we find some dictionary definition of it or someone somehow points out for us the referent(s) intended by it. But even then, inductive acts are needed to understand the definition or the intent of the pointing. When you point at something, I cannot immediately be sure exactly what it is you are pointing at; I may have to ask you: 'do you mean including this, excluding that?' and thus gradually zero in on your true intent.

Each of us, at all times, retains the responsibility to judge the status of the terms he or she uses. The judgment as to whether a term is realistic, or unrealistic is not always easy. In practice, therefore, most terms are effectively hypothetical, whether classed as more probably realistic or more probably unrealistic. Even so, some terms are certainly realistic or unrealistic. All terms that are truly based exclusively on empirical evidence or whose denial is self-contradictory are certainly realistic, and all manifestly counterfactual or self-contradictory terms are certainly unrealistic. So, all three of these characterizations are needed and effective.

Let us suppose the formation of realistic terms is obvious enough, and ask how imaginary ones are formed. Imaginary terms are not formed ex nihilo; they are formed by combining old terms together in new ways. A new term T is imagined by means of two or more existing terms T1, T2.... We would call term T realistic, if all the terms (T1, T2...) constituting it are realistic and their combination is credible. But if all the terms on which T is based are realistic, but their combination is not credible (e.g. we know that no T1 is T2, so the conjunction T1 + T2 is contrary to fact), we would call T unrealistic; and of course, if one or more of the terms constituting T is/are unrealistic, we would call T unrealistic. If T is made up of hypothetical elements or if its elements are realistic but their combination is of uncertain status, we would call T hypothetical.

Now, our thinking in practice is aimed at knowledge of reality. That is to say, when we come across a term without existential import, i.e. when we decide that a term is unrealistic, whether because it goes against our empirical observations or because it is in some way illogical—we normally lose interest in it and drop it. We certainly do not waste our time wondering whether such a subject has or lacks some predicate, since obviously if

the subject is non-existent it *has no* predicates anyway. If we regard a term as empty, the oppositions of its various quantities and polarities in relation to whatever predicate are henceforth totally irrelevant. An empty term, once established as such, or at least considered to be such, plays no further role in the pursuit of knowledge. This attitude is plain common sense, except perhaps for lunatics of various sorts. For this reason, the oppositions between propositions involving empty terms are *trivial*. That is, the above detailed non-Aristotelian models of opposition are insignificant.

The net effect of the successive 'dilutions' is to make the strong, Aristotelian rectangle of oppositions (concerning propositions with existential import) seem like a special case of little importance, and to give the weaker, non-Aristotelian rectangles (concerning variously empty disproportional appearance propositions) a importance. The reason why this occurs is that the weaker oppositions represent the lowest common Aristotelian and denominator between the Aristotelian oppositions, which we need if we want to simultaneously discuss propositions with and without existential import. But the result is silly, for the Aristotelian diagram (7.1) is the important one, teaching us to think straight, whereas the non-Aristotelian ones are really of very minimal and tangential academic interest. Practical logic is focused on terms that are believed to be realistic or at least hypothetical – it is not essentially concerned with empty terms. Contrary to the accusations made by modern logicians, Aristotelian logic is not only concerned with realistic terms. It is in fact mainly used with hypothetical terms, since (as already pointed out) most of the terms which furnish our thoughts are

hypothetical – tentative constructs in an ongoing inductive enterprise. We do not think hypothetical thoughts by means of some special logic – we use the same old Aristotelian logic for them. That is to say, in accord with **the principle of induction**, we treat a hypothetical term as a realistic term until and unless we have reason to believe otherwise.

The reason we do so is that a hypothetical term, i.e. one not yet proved to be realistic or unrealistic, is a candidate for the status of realism. This being the case, we treat it as we would any realistic term, subjecting it to the strong, Aristotelian model of oppositions, rather than to any watered-down model with wider aspirations, in the way of an inductive test. If the hypothetical term is indeed deserving of realistic status, it will survive the trial; if, on the other hand, it does not deserve such status, it will hopefully eventually be found to lead to contradiction of some sort. In that event, we would decide that the hypothetical term should rather be classed as an unrealistic term, and we would naturally soon lose interest in it. Thus, there is only one significant and useful model for oppositions between propositions, namely the Aristotelian one.

Indeed, we sometimes use Aristotelian logic even for unrealistic terms. Very often, we remove the stigma of unrealism by rephrasing our statement more precisely¹²⁴.

For example, we might say (instead of "unicorns are horses with a horn") "the imaginary entities called unicorns look like horses with a horn on their forehead" or (instead of "some unicorns are white, some black") "some of the unicorn illustrations I have seen involve a white horse, but some involve a black one". Note that *both* the initial propositions (given in brackets) have empty terms, even though one is general and the other is particular. Clearly, after such

Alternatively, we might just keep the imaginary intent in mind: say a novelist wishes to write about fictional people, or even science-fiction creatures, he would not logically treat his subjects as empty terms — but rather subject them to the logic applicable to realistic terms, so as to enhance the illusion of realism in his novel. Thus, the logic applicable to empty terms which we have above investigated is in practice never used.

Whatever the alleged existential import of the terms involved, our thoughts remain guided by the demanding model of Aristotelian oppositions. The rational pursuit of knowledge still indubitably *requires* the clear-cut logic of Aristotle enshrined in the traditional rectangle of oppositions (diagram 29.1). The reason why Aristotle took the existential import of the subjects of categorical propositions for granted is, I suggest, because *naturally*, *if there is nothing (i.e. no subject) to talk about (i.e. to predicate something of) we will not talk about it; and if we are talking, then that presumably means we do have something to talk about, i.e. a subject as well as a (positive or negative) predicate. This is manifest common sense.*

If Aristotle – as far as we know, or at least as far as readers of his extant works have so far managed to

corrective rephrasing the two propositions do have existential import, although they do so with reference to imaginary (mental) entities rather than to real (physical) ones. Consequently, while the initial propositions cannot be said to be true, the more precise ones replacing them can be said to be true, and we can apply Aristotelian logic to them without qualms. Note also in passing that even a seemingly eternally imaginary entity may one day become real – for example, we might by artificial selection or by some genetic manipulation one day produce real unicorns.

discern, or so we are told by historians of logic – did not ask the question regarding the existence of the subject, it is probably simply because he quite intelligently had no interest in empty subjects. He was rightly focused on the pursuit of knowledge of the world facing him, not some non-existent domain. Modern logicians are rather, I suggest, more intent on impressing the yokels with their intellectual brilliance. With that overriding purpose in mind, they fashion systems of no practical significance whatever. They make mountains out of molehills, presenting trivia as crucial discoveries, so as to draw attention to their own persons.

e. Modern logic is a complex web of static relationships, most of them irrelevant. It ignores the dynamics of human thinking, the fact that our knowledge is constantly in flux. It is, we might say, a science of space irrespective of time. In an effort, on the surface praiseworthy, to formally acknowledge the issue of existential import, it gives undue attention to empty terms, elevating them from a very marginal problem to a central consideration. Instead of dealing with existential import parenthetically, as a side issue, it erects a logical system that effectively shunts aside some of the most important logical processes in the human cognitive arsenal.

The traditional universal propositions are cognitively of great importance. They cannot just be discarded, as modern logic has tried doing under the pretext that formal logic had to be expanded to include consideration of counterfactual terms. There are logical processes involving these propositional forms that are of great practical importance, and which logic must focus on and emphasize. It is absurd to henceforth effectively ignore

these venerable and indispensable forms while making a big thing of a theoretical consideration of no practical significance whatever. The universals A and E cannot be retired under any pretext; they are not mere conventional conjunctions of more primitive forms.

For a start, universal propositions are essential to the crucial logical processes of subsumption and nonsubsumption, which are enshrined in Aristotle's syllogistic. First figure syllogisms serve to include an instance in a class or a subclass in a wider class; they teach us the notion that 'all X' includes every individual 'this X' and any possible set of 'some X'. If, instead of an argument such as "All X are P and this S is X, therefore this S is P" (1/ARR) we propose the modern major premise "Nothing is X and not-P," with the same minor premise, we obviously (even though the minor premise implies the existential import of an X) can no longer directly draw the desired conclusion! We are forced to stop and think about it, and infer that "this S is not not-P" before concluding that "this S is P." Similarly, second figure syllogisms serve to exclude an instance from a class or a subclass from a wider class, and third figure syllogisms to identify overlaps between classes; and the moods of these figures become inhibited or universal greatly propositions distorted if are reinterpreted as modern logicians suggest.

Again, universal propositions are essential to the crucial logical processes of *generalization and particularization*. If 'this X' and 'some X' are not implied by 'all X', then we cannot generalize from the former to the latter. Of course, given 'this X' or 'some X', we do have existential import, and thus can anyway generalize to 'all X'. But the fact remains that if, in accord with modern

logic, we conceive our generalization as a movement of thought from "This/Some X is/are Y" to "Nothing is X and not-Y," we miss the point entirely, even if admittedly the existential import of X is implied by the premise. For in such case, *the formal continuity between premise and conclusion is lost*, there being two inexplicable changes of polarity (from something to nothing and from Y to not-Y)! Similarly, particularization requires formal continuity. To move freely from I to A, and then possibly to IO, we need the traditional opposition (contradiction) between A and O.

Another issue that is ignored by modern logicians is *modality*. Although modern logic has developed modal logic to some extent, it has done so by means of symbolic notations based on very simplistic analyses of modality. Although it has conventionally identified the different categories of modality (necessity, impossibility, actuality, inactuality, possibility, unnecessity), it has not thoroughly understood them. It has not clearly identified and assimilated the different types of modality (the logical, extensional, natural, temporal, and spatial modes), even if human discourse has included them all since time immemorial. Notably lacking in its treatment is the awareness that modality is an expression of conditioning and that the different types of modality give rise to different types of conditioning.

Consideration of modality is manifestly absent in the doctrine of existential import. The latter (as we saw) is built around the timeless (or 'omnitemporal') forms of categorical proposition, which are non-modal. It does not apply to modal categorical propositions, for these do not formally imply (or presuppose) the actuality of their subject but only its possibility. Thus, a universal

proposition with natural-modality, "All S can (or must) be P," does not formally imply that "Some things *are* S" but only that "Some things *can be* S;" likewise, one with temporal modality, "All S are sometimes (or always) P" does not imply that "Some things *are* S" but only that "Some things *are sometimes* S;" and so forth.

This may be called 'existential import' in a broadened sense, acknowledging that being has degrees; but it is certainly not the actual sense intended by modern logicians: they apparently imagine that use of such modal propositions implies belief that "Some things are S." And of course, the modality of subsumption, as I have called this phenomenon in my book Future Logic (chapter 41), is very relevant to the processes of opposition, eduction (immediate inferences), syllogistic deduction (mediate inference) and induction. Regarding the latter, see my detailed theory of factorial induction in the said work. Thus, we may well say that the proponents of the doctrine of existential import constructed an expanded system of logic based on a rather narrow vision of the scope of logic. Even if their expansion (for all it is worth—not much, I'd say) is applicable to non-modal propositions, it is not appropriate for modal ones.

f. The critique of the Aristotelian rectangle of oppositions began apparently in the middle ages, with Peter Abelard (France, 1079-1142). According to the Kneales, further input on this issue was made over time by William of Shyreswood, by Peter of Spain and St. Vincent Ferrer, and by Leibniz. They also mention Boole's interest in it, and many people attribute the modern view of the issue to this 19th century logician. However, E. D. Buckner suggests that the modern idea stems rather from Franz Brentano (Austria, 1838-1917),

in a paper published in 1874¹²⁵. And of course, many big name logicians such as Frege and Russell have weighed in since then.

Even though the new logic that ensued, based on the concept of existential import, is today strongly entrenched in academia, the switchover to it was epistemologically clearly not only unnecessary but illadvised. The doctrine of existential import has been woefully misnamed: it is in fact *not* about existential import, but rather about *non*-existential import. It gives to empty terms undue importance, and thus greatly diminishes the real importance of non-empty terms. To be sure, this innovation fitted the anti-rational 'spirit of the times', and it kept many people happily busy for over a century, and thus feeling they existed and were important – but it was in truth emptiness and vanity.

Apparently, none of these people reflected on the obvious fact that once a term is identified as empty, it is simply dumped – it does not continue affecting our reasoning in any significant manner. This being so, there is no need to abandon the universal forms A and E because they imply (presuppose) the existential import of their subject. Even if the Aristotelian framework, which is built around non-empty terms, occasionally 'fails' due to the appearance of an empty term in discourse, such event is taken in stride and dealt with by summarily eliminating the discredited term thenceforth, and

For Buckner's account of the history, see: www.logicmuseum.com/cantor/Eximport.htm. Notice his pretentious characterization of "the traditional 'syllogistic'" as "a historical curiosity." Brentano's position is to be found in his *Psychologie vom empirischen Standpunkt*, II, ch. 7. The Kneales do mention the latter reference in passing, in a footnote on p. 411.

certainly *not* by switching to a non-Aristotelian framework as modern logicians recommend to do. In any case, the issue of existential import does not apply to modal logic, and so lacks generality.

Moreover, these people failed to realize that Aristotelian logical processing relates not only to realistic terms, but more significantly to hypothetical terms, i.e. terms in process. They viewed logic as a deductive activity; they did not realize its essentially inductive character. If, due to an immoderate interest in empty terms, the science of logic abandons the universal forms A and E, it deprives people of a language with which to accurately express the movements of thought inherent in the processes of inference and of generalization syllogistic particularization. The science of logic must acknowledge the forms of actual human thinking, and not seek to impose artificial contraptions of no practical value. Otherwise. natural processes essential cognition cannot be credibly expressed and logic will seem obscure and arbitrary.

Modern logic has sown confusion in many people's minds, turning the West from a culture of confident reason to one of neurotic unreason. The purpose of logic studies ought to be to cognitively empower people, not incapacitate them. If logicians err in the forms of thought they describe and prescribe, they betray their mission, which is to intelligently and benevolently guide and improve human thinking. If they err, whether out of stupidity or malice, they turn logic from a responsible science and a fine art to a vain and dangerous game. They do not merely cease benefitting mankind; they positively harm people's minds.

30. CHAPTER THIRTY

Drawn from *A Fortiori Logic* (2013), Appendix 7.4-5.

THE LIAR AND RUSSELL PARADOXES (REDUX)

1. The Liar Paradox (Redux)

I dealt with the Liar paradox previously, in my *Future Logic*¹²⁶, but now realize that more needs to be said about it. This paradox is especially difficult to deal with because it resorts to several different discursive 'tricks' simultaneously.

a. The statement "This proposition is false" looks conceivable offhand, until we realize that if we assume it to be true, then we must admit it to be indeed false, while if we assume it to be indeed false, then we must admit it to be true — all of which seems unconscionable. Obviously, there is a contradiction in such discourse, since nothing can be both true and false. But the question is: just what is causing it and how can it be resolved? We are not 'deducing' the fact of contradiction from a 'law of thought' — we are 'observing' the fact through our rational faculty. We cannot, either, 'deduce' the resolution of the contradiction from a 'law of thought' — we have to analyze the problem at hand very closely and

See there chapter 32.2. (See also *Ruminations* 5.1.)

creatively propose a satisfying solution to it, i.e. one which indeed puts our intellectual anxiety to rest. As we shall see, this is by no means a simple and straightforward matter.

The proposition "This proposition is false" is a double paradox, because: if it is true, then it is false; and if it is false, then it is true. Notice the circularity from true to false and from false to true. The implications we draw from the given proposition seems unavoidable at first sight. But we must to begin with wonder how we know these implications (the two if—then statements) to be true. How do we know that "it is true" implies "it is false," and that "it is false" implies "it is true"? Apparently, we are not 'deducing' these implications from some unstated proposition. We are, rather, using ad hoc rational insight of some sort – i.e. in a sense directly 'perceiving' (intellectually cognizing) the implications of the given proposition. But such rational insight, though in principle reliable, is clearly inductive, rather than deductive, in epistemological status. That is to say, it is trustworthy until and unless it is found for some reason to be incorrect. This means, there may be one or more errors in our thinking, here; it is not cast in stone. And indeed there must be some error(s), since it has led to double paradox. Therefore, we must look for it.

Perhaps use of the pronoun "it" is a problem, for it is a rather vague term. Let us therefore ask the question: more precisely what does the pronoun "it" refer us to, here?

At first sight, the "it" in "if it is true, then it is false; and if it is false, then it is true" refers to *the whole given statement*, "This proposition is false." In that event, we must reword the double paradox as follows: *if 'this*

proposition is false' is true, then 'this proposition is false' is false; and if 'this proposition is false' is false, then 'this proposition is false' is true. Here, the subject of the two if—then statements is more clearly marked out as "this proposition is false," and so remains constant throughout. But this clarification reveals an abnormal changes of predicate, from "true" to "false" and from "false" to "true," which cannot be readily be explained. Normally, we would say: if 'this proposition is false' is true; and if 'this proposition is false' is false, then 'this proposition is false' is false. The reason we here reverse the predicates is that we consider the original proposition, "this proposition is false," as instructing such reversal.

However, whereas a proposition of the form "this proposition is false' is true" is readily interpretable in the simpler form "this proposition is false," a proposition of the form "this proposition is false' is false" cannot likewise be simplified. How would we express the double negation involved? As "this proposition is true"? Clearly, the meaning of the latter is not identical to that of the former, since the subject "this proposition" refers to different propositions in each case. So the formulation of the liar paradox in full form, i.e. as "if 'this proposition is false' is true, then 'this proposition is false, then 'this proposition is false' is true," does not make possible the reproduction of the initial formula expressed in terms of the pronoun "it."

b. Let us therefore try something else. If the pronoun "it" refers to *the term* "this proposition", then the double paradox should be reformulated as follows: *if* 'this proposition' is true, then 'this proposition' is false;

and if 'this proposition' is false, then 'this proposition' is true. But doing that, we see that in each of these two ifthen statements, though the subject ("this proposition") remains constant throughout, the predicate ("true" or "false," as the case may be) is not the same in the consequent as it was in the antecedent. There is no logical explanation for these inversions of the predicate. Normally, the truth of a proposition P does not imply its falsehood or vice versa.

We might be tempted to use the given "This proposition is false" as a premise to justify the inference from the said antecedents to the said consequents. We might try to formulate two apodoses, as follows:

> true, then it is false false, then it is true (hypothesis), and this proposition is false (given); therefore, this proposition is true (putative conclusion).

> If this proposition is If this proposition is (hypothesis), and this proposition is false (given); therefore, this proposition (putative conclusion).

Obviously, in the first case we have invalid inference, in that we try to deny the antecedent to deny the consequent, or to affirm the consequent to affirm the antecedent. In the second case, the putative conclusion does follow from the premises; but we can still wonder where the major premise (the hypothetical proposition) came from, so we are none the wiser. So, this approach too is useless - i.e. it proves nothing.

Alternatively, we might try formulating the following two syllogisms:

This proposition is false (given), and this proposition is true (supposition); therefore, this proposition is false (putative conclusion).

This proposition is false (given).
and this proposition is false (supposition); therefore, this proposition is true (putative conclusion).

Clearly, these arguments are not quite syllogistic in form; but they can be reworded a bit to produce syllogisms. The first two premises would then yield the conclusion "there is a proposition that is true and false" (3/RRI), which is self-contradictory (whence, one of the premises must be false); the second two premises, however, being one and the same proposition, would yield no syllogistic conclusion other than "there is a proposition that is false and false" (3/RRI), which is self-evident (and trivial). But these are not the conclusions we seek, which must concern "this proposition" and not merely "some proposition."

A better approach is to look upon the latter two arguments as follows. In the first case, the premises "this proposition is false" (given) and "this proposition is true" (supposition) seem to together imply "this proposition is both true and false;" and the latter paradoxical conclusion in turn indeed suggests that "this proposition is false," since contradiction is impossible. And in the second case, the premises "this proposition is false" (given) and "this proposition is false" (supposition) agree with each other that "this proposition is false," and so this is their logical conclusion. Since both arguments

conclude with "this proposition is false," the latter must be the overall conclusion.

However, the latter result is not as conclusive as it seems, because upon closer scrutiny it is obvious that "this proposition is false" and "this proposition is true" do not refer to the same subject, since the predicate changes. The first "this proposition" refers to the proposition "this proposition is false" and the second "this proposition" refers to the proposition "this proposition is true." So, these two propositions in fact have different subjects as well as different predicates (viz. false and true. respectively). The subjects superficially look the same, because they are verbally expressed in identical words; but their underlying intent is not the same, since they refer to significantly different propositions (propositions manifestly different, indeed contradictory, predicates). This means that when the predicate changes, the subject effectively changes too. When the predicate is "true," the subject means one thing; and when the predicate is "false," the subject means something else. Although the words "this proposition" are constant, their underlying intent varies. That is to say, the term "this proposition" does not have a uniform meaning throughout, and therefore cannot be used as a basis for the inferences above proposed.

c. Let us now try another angle. If we examine our initial reasoning in terms of the pronoun "it" more carefully, we can see what is really happening in it. Given that 'this proposition is false' is true, we can more briefly say: 'this proposition is false.' Also, given 'this proposition is false' is false, we can by negation educe that 'this proposition is *not* false' is true, which means that 'this proposition is true' is true, or more briefly put:

'this proposition is true'127. In this way, we *seem* to argue, regarding the subject "this proposition is false," from 'it is true' to 'it is false', and from 'it is false' to 'it is true'. But in fact the use of the pronoun "it" or the term "this proposition" as abbreviated subject is a sleight of hand, for the underlying subject changes in the course of the second transition (that ending in "this proposition is true"). When abbreviation is used throughout, we seem to be talking about one and the same proposition throughout as being both true and false. But seeing that this is based on hidden equivocation, the paradoxes disappear.

It is interesting to note that when the reasoning is viewed more explicitly like that, the proposition "this proposition is true" also becomes paradoxical! We can argue: if 'this proposition is true' is true, then obviously 'this proposition is true'. And: if 'this proposition is true' is false, then its contradictory 'this proposition is *not* true' must be true, which means that 'this proposition is false' is true, i.e. more succinctly: 'this proposition is false'. Here, superficially, there seems to be no paradox, because we seem to argue, regarding the subject "this proposition is true," from 'it is true' to 'it is true', and from 'it is false' to 'it is false'. But if we look at the final conclusion, viz. "this proposition is false," we see that it *corresponds to* the liar paradox!¹²⁸ And here again, the

Some logicians have tried to deal with the liar paradox by denying that true and false are contradictory terms, i.e. that not-true = false and not-false = true. Such a claim is utter nonsense; the attempt to shunt aside the laws of non-contradiction of the excluded middle so as to resolve a paradox is self-contradiction in action.

That 'this proposition is true' is implicitly (if only potentially) as paradoxical as 'this proposition is false' is, so

explanation of the double paradox is that the apparent subject "it" or "this proposition" changes significance in the course of drawing the implications.

Notice that, in both these lines of reasoning, the first leg is ordinary self-implication, mere tautology, while the second leg is the operative self-contradiction, the paradox. If the given proposition (whether "this proposition is false" or "this proposition is true") is true, we merely repeat the proposition as is (without need to add the predication "is true"). But if the given proposition is false, we cannot drop the additional predication (i.e. "is false") without changing the original proposition. Thus, we could say that the propositions, "this proposition is false" proposition is true," present no problem when taken as true; and it is only when they are hypothetically taken as false that the problem is created. So we could say that the way out of the liar paradox (and its positive analogue) is simply to accept the two claims as true, and not imagine them to be false!

We could furthermore, if we really want to, argue that "this proposition is false" and "this proposition is true" differ in that the former explicitly appears to put itself in doubt whereas the latter does not do so. On this basis, we could immediately reject the former and somewhat accept the latter, even while admitting that the latter is equally devoid of any useful information. That is to say, since the former appears 'more paradoxical' than the

far as I know, a new discovery. Note well how *both* paradoxes occur through quite ordinary eductions: viz. if 'P is Q' is affirmed, then P is Q; and if 'P is Q' is denied, then 'P is not Q' is affirmed, then P is not Q (where P stands for 'this proposition', and Q for 'false' or 'true' as the case may be).

latter, the latter is to be preferred *in extremis*. But this, note well, ignores the equally insurmountable difficulties in it. It is better to resolutely reject both forms as vicious constructs.

To grasp the illusoriness of the liar paradox, it is d. important to realize that the two forms, "this proposition is false" and "this proposition is true," are not each other's contradictory; and that, in fact, neither of them has a contradictory! This is a logical anomaly, a fatal flaw in the discourse of the liar paradox; for in principle, every well-formed and meaningful proposition is logically required to have a contradictory. If a propositional form lacks a contradictory form, it cannot be judged true or false, for such judgment depends on there being a choice. We do not even have to limit our propositions to the predicates "true" or "false" - any predicate X and its negation not-X would display the same property given the same said subject. That is, "this proposition is X" and "this proposition is not-X" are not each other's contradictory, and are therefore both equally deprived of contradictory.

We could, of course, remark that "this proposition is X" can be denied by "that proposition (i.e. the preceding one) is not X," or even introduce a symbol for the original proposition in the new proposition. In such case, although the subjects would be verbally different, their intents would surely be the same. But the form "that proposition is not X" is more akin to the form "this proposition is X' is not X," in which the whole original proposition is given the role of subject and its predicate is given the role of predicate. However, though these two forms somewhat equivalent in meaning to each other and to the original proposition, their logical behavior patterns

are not identical with that of the original proposition, as we have already seen. The fact remains that "this proposition is not X" is not the contradictory of "this proposition is X."

Clearly, any proposition involving the special subject "this proposition" exhibits a very unusual property, and may be dismissed on that basis alone. The reason why such a proposition lacks a contradictory is that its subject refers to the proposition it happens to be in, and that proposition is evidently different when the predicate in it is the term "false" and when it is the term "true" (or more generally, any pair of predicates 'X' and 'not-X'). When the predicate changes, so does the subject; so the subject cannot be pinned-down, it is variable, it is not constant as it should be. The term "this proposition" has a different reference in each case, which depends on the predicate; consequently, each subject can only be associated with one predicate and never with the other (i.e. its negation). From this we see that when at the beginning we thought, looking upon the statement "This proposition is false," that if we take it at its word, then it is must be regarded as false, and so we have to prefer to it "This proposition is not false," i.e. "This proposition is true," and so forth, we did not realize that we were in fact, due to the ambiguity inherent in the term "This proposition" or "it," changing its meaning at every turn. This change of meaning passes by unnoticed, because the term used is by its very nature not fixed. The pronouns "this" and "it" can be applied to anything and its opposite without such change of meaning being verbally signaled in them. They are not permanently attached to any object, but are merely contextual designations. In the technical

terminology of linguistics, they are characterized as 'deictic' or 'indexical'.

Thus, it appears that the liar paradox arises, however we understand its terms, as a result of some sort of equivocation in the subject. Although we seem superficially to refer to one and the same subject in the antecedent and consequent of our if—then reasoning, there is in fact a covert change of meaning which once we become aware of it belies the initial appearance of contradiction. The suggested impossible implications are thus put in doubt, made incredible. The contradictions apparently produced are thus defused or dissolved, by virtue of our inability to make them stick.

e. Another, and complementary, way to deal with the liar paradox is to point out *the logical difficulty of self-reference*. This is a tack many logicians have adopted, including me in my first foray into this topic in *Future Logic*. The argument proposed here is that the term "this proposition" refers to an object (viz. "This proposition is false" or "This proposition is true") which includes the term itself. A finger cannot point at itself, and "this" is the conceptual equivalent of a finger. Effectively, the expression "this" has no content when it is directed at itself or at a sentence including it. It is empty, without substance. It is as if nothing is said when we indulge in such self-reference.

Thus, "This proposition is X" (where X stands for false, or true, or indeed anything) is in fact meaningless; and a meaningless sentence cannot be true or false. Such a sentence can reasonably be described as neither true nor false, without breach of the law of the excluded middle, because neither of these logical evaluations is applicable to meaningless sentences. "This proposition is false"

looks meaningful because its four constituents (i.e. "this," "proposition," "is" and "false") are separately normally meaningful. But in this particular combination, where one of the elements (viz. "this") does not refer to anything already existent, the sentence is found to be meaningless.

The apparent contradictions that self-reference produces help us to realize its meaninglessness. And it is through the intellectual realization of the meaninglessness of self-reference that we explain away and annul the apparent contradictions. On this basis, we can say that even though the sentence "This proposition is true" does not at first sight give rise to any paradox (as people think: "if it is true, it is true; and if it is false, it is false"), nevertheless, since it involves self-reference as much as "This proposition is false," it is equally meaningless and cannot be characterized as true or false. In fact, as I have shown above, "This proposition is true" does also give rise to double paradox.

Someone might object: What about the propositions: "this statement is self-referential"? Surely, we can say that these are meaningful and that the former is true while the latter is false! The retort to that objection is that the two propositions "this proposition refers to itself" and "this proposition does not refer to itself" are *not* mutual contradictories, because (just like in the liar paradox) their subjects differ radically, each referring to the proposition it is in and not to the other. Thus, while the positive version may seem more self-consistent than the negative one, and therefore to be preferred *in extremis*, they are in fact both fundamentally flawed, because (just like in the liar paradox) neither of them *has* a

contradictory, and without the logical possibility of negating a discourse it is impossible to judge whether it is right or wrong.¹²⁹

f. Not long after the preceding reflections, I happened to come across another interesting example of paradoxical self-reference, namely "Disobey me!" 130 This involves the 'double bind' – *if I obey it, I disobey it and if I disobey it, I obey it.* To resolve this paradox, we need to first put the statement in more precise form, say: "you must disobey this command!" We can then disentangle

129 Another objection (which was actually put to me by a reader) would be propositions like "this statement has five words" and "this statement has six words" - even though they contain the demonstrative "this," the former looks true and the latter false! Here, we might in reply point out that though the propositions "this statement has five words" and "this statement does not have five words," seem to mean opposite things, they cannot be contradictories, since both appear true. Also compare: "this statement has five words" and "this statement does have five words" - the former is true while the latter is false, though both *mean* essentially the same. Clearly, the behavior of these propositions is far from normal, due to their unusual dependence on the wording used in them. On one level, we get the message of the proposition and count the number of words in it, and then check whether this number corresponds to the given number: if yes, the proposition is judged 'true', and if no, it is judged 'false'. But at the same time, we have to be keep track of the changing reference of the demonstrative "this," which complicates matters as already explained, and additionally in this particular context we must beware of the impact of wording. The Kneales give "What I am now saying is a sentence in English" as an example of "harmless self-reference" (p. 228).

I found this example in Robert Maggiori's *La philosophie au jour le jour* (Paris: Flammarion, 1994); the author does not say whether it is his own invention or someone else's (p. 438).

the knot by realizing that the order being given has outwardly imperative form but inwardly lacks content. It does not define a specific, concrete action that is to be done or not-done. If we wished to obey it, or to disobey it, we would not know just what we are supposed to do or not-do! It is therefore an order that can neither be obeyed nor be disobeyed. Ruminating on this case led me to what I now believe is the trump card, which convincingly finalizes the resolution of the liar paradox, even as the preceding reflections all continue to be relevant.

It occurred to me then that this is precisely the problem with the liar paradox. It says "this proposition is false" – but it does not tell us anything about the world that can be judged as true or false. A 'proposition' is a statement that makes some claim about the world. If the statement makes no such claim, if it 'proposes' nothing, it cannot be logically assessed as true or false. If it refers to nothing – whether physical, mental or spiritual. perceptual, intuitive or conceptual – it has no meaning. A qualify meaningless statement does not 'proposition'. The attributes of 'true' or 'false' are not ordinary predicates, like 'white' or 'black', which can be attached to any subject and then judged to be truly or falsely attached. The attributes of 'true' or 'false' require a precise claim to be made before they can at all be used. The truth of this explication can be seen with reference to the 'propositional forms' used in logic theory. Take, for example, "All X are Y." Such a propositional form cannot be judged true or false because it manifestly has no content. Only when such an abstraction is given some specific content, such as "All men are mortal," can we begin to ask whether it is true or false. A propositional form is too vague to count as a proposition. It does not

tell us anything about the world, other than implying that there are (or even just that there may be) concrete propositions which have this form. Just as we cannot disobey or even obey an imperative without content, so we cannot judge a purely formal expression true or false. The same applies to the liar paradox: like a formal proposition, it has no concrete content, and therefore cannot be judged true or false. The liar paradox has no content partly due to its having a self-referential subject ("this proposition"). But the truth is, even if its subject was not self-referential, it would still have insufficient content. This is so, because its predicate "false" (and likewise its opposite, "true") is not an ordinary predicate; it is more like a formal predicate. It can only be used if another, more concrete predicate has already been proposed for the subject at hand. For example, "this proposition is interesting" could be judged true or false (if it was not self-referential) because it already has a predicate (viz. "interesting"). Thus, the problem with the liar paradox is not only the self-reference it involves but also its lack of a predicate more concrete than the logical predicate "false" (or "true").

All this illustrates how the 'laws of thought' are not axioms in the sense of top premises in the knowledge enterprise from which we mechanically derive other premises. Rather the expression 'laws of thought' refers to recurring insights which provide us with some intellectual guidance but cannot by themselves determine the outcome. The individual in pursuit of knowledge, and in particular the logician, is *driven* by the obviousness or by the absurdity of a situation to look for creative solutions to problems. He or she must still think of possible solutions and test them.

2. The Russell Paradox (Redux)

Logic is what helps us transmute scattered concrete perceptions into well-ordered abstract concepts. Human knowledge, or opinion, is based on experience, imagination and rational insight. The latter is a kind of 'experience' in the larger sense, a non-phenomenal sort of experience, call it logical 'intuition'. Reason was for this reason called by the ancients, in both West and East, the 'sixth sense' or 'common sense', i.e. the sense-organ which ties together the other five senses, those that bring us in empirical contact with phenomenal experience: colors, shapes, sounds, smells, tastes, touch-sensations, etc., whether they are physically perceived or mentally imagined. The five senses without the sixth yield chaotic nonsense (they are non-sense, one cannot 'make sense' of them); and conversely, the sixth sense is useless without the other five, because it has nothing about which to have rational insights. Imagination reshuffles past experiential data and reasoning, making possible the formation of new ideas and theories which are later tested with reference to further experience and reasoning.

Elements of class logic. Logic initially developed as a science primarily with reference to natural discourse, resulting in what we today refer to as predicate logic. In natural human discourse, we (you and me, and everyone else) routinely think of and discuss things we have perceived, or eventually conceived, by means of categorical propositions involving a subject (say, S) and a predicate (say, P) which are related to each other by means of the copula 'is'. Such propositions have the form "S is P," which may be singular or plural, and in the

latter case general (or universal) or particular, and positive or negative, and moreover may involve various modes and categories of modality¹³¹.

A proposition of the form 'S is P' is really a double predication – it tells us that a thing which is S is also P; thus, S and P are really both predicates, though one (the subject S) is given precedence in thought so as to 'predicate' the other (the predicate P) of it¹³². Primarily, S refers to some concrete phenomenon or phenomena (be it/they physical, mental or spiritual), i.e. an individual entity or a set of entities, and P to a property of it or of theirs. For examples, "John is a man" and "All men are human beings" are respectively a singular predication (about one man, John) and a plural one (about all men). Additionally, still in natural discourse, the subject of our thoughts may be predicates as such, i.e. predicates in their capacity as predicates; an example is: "men' may be the subject or predicate of a proposition." The latter occurs in specifically philosophical (or logical or linguistic) discourse; for example, in the present essay. Now, logicians through the ages, and especially in modern times, have effectively found natural discourse somewhat inadequate for their needs and gradually developed a more artificial language, that of 'classes' 133.

¹

We need not go into the details of these distinctions here, for they are well known. There are also many fine distinctions between different sorts of terms that may appear in propositions as subjects or predicates; but let us keep the matter simple.

^{&#}x27;Predication' refers to the copula and the predicate together as if they were an action of the speaker (or the statement made) on the subject.

The following account of class logic is based on my presentation in *Future Logic*, chapters 43-45. The word 'class'

This type of discourse exactly parallels natural discourse, but is a bit more abstract and descriptive so as to facilitate philosophical (or logical or linguistic) discourse and make it more precise. In this language, instead of saying "this S is P," we say "this S is a member (or instance) of P" (note well the lengthening of the copula from 'is' to 'is a member (or instance) of'. If 'this S' symbolizes a concrete individual, then 'P' here is called a 'class'; but if 'this S' symbolizes an abstract class, then 'P' here is called a 'class of classes'.

A class, then, is an abstraction, a mental constructs in which we figuratively group some concrete things (be they physical, mental or spiritual). Although we can and do temporarily mentally classify things without naming the class for them, we normally name classes (i.e. assign them a distinctive word or phrase) because this facilitates memory and communication. Naming is not the essence of classification, but it is a great facilitator of large-scale classification. The name of a class of things does not 'stand for them' in the way of a token, but rather 'points the mind to them' or 'draws our attention to them'; that is to say, it is an instrument of intention.

A class in the primary sense is a class of things in general; a class in the secondary sense is more specifically a class of classes. Membership is thus of two kinds: membership of non-classes in a class, or membership of classes in a class of classes.

comes from the Latin *classis*, which refers to a "group called to military service" (Merriam-Webster). I do not know whether the Ancients used that word in its logical sense, or some such word, in their discourse, but they certainly thought in class logic mode. Examples of class thinking are Aristotle's distinction between species and genera and Porphyry's tree.

Alternatively, we may speak of first-order classes and second-order classes to distinguish these two types. There are no other orders of classes. When we think about or discuss more concrete things, we are talking in first-order class-logic; when we think about or discuss first-order classes, we are talking in second-order class-logic, and the latter also applies to second-order classes since after all they are classes too. The two orders of classes should not be confused with the hierarchy of classes within each order.

The relation between classes of classes and classes is analogous to the relation between classes and concretes; it is a relation of subsumption. When a lower (i.e. firstorder) class is a member of a higher (i.e. second-order) class, it does not follow that the members of the lower class are also members of the higher class; in fact, if they are members of the one they are certainly not members of the other. Thus, for example, you and me, although we are members of the class 'men' because we are men, we are not members of the class 'classes of men' because we are not 'men'. Also, the class 'men' is not a man, but is a member of the class 'classes of men'. The members of the class 'classes of men' (or more briefly put, 'menclasses'), which is a class of classes, are, in addition to the broad class 'men', the narrower classes 'gardeners', 'engineers', 'sages', 'neurotics', and so on. 134

Note that saying or writing the word men without inverted commas refers to a *predicate*. When we wish to refer to the corresponding *class*, we say the class of men, or the class men; if we are writing, we may write the same with or without inverted commas, or simply 'men' in inverted commas. When dealing with classes of classes, we say the class of classes of men, or the class of men-classes, or the class men-classes, and we may write the same with or without inverted

Hierarchization, on the other hand, refers to classes within a given order that share instances, not merely by partly overlapping, but in such a way that all the members of one class are members of the other (and in some but not all cases, vice versa). For example, since all men are animals, though not all animals are men, the class 'men' is a subclass (or species) of the class 'animals', and the class 'animals' is an overclass (or genus) of the class 'men'. If two classes have the same instances, no more and no less, they may be said to be co-extensive classes (a class that serves as both species and genus in some context is said to be sui generis). If two classes merely share some instances, they may be said to be intersecting (or overlapping) classes, but they are not hierarchically arranged (e.g. 'gardeners' and 'engineers'). If two classes of the same order have no instances in common, they may be said to be *mutually* exclusive classes.

It is important to grasp and keep in mind the distinction between hierarchy and order. Since you and I are men, each of us is a member of the class 'men'; this is subsumption by a first-order class of its concrete instances. Since all men are animals, the class 'men' is a subclass of the class 'animals'; this is hierarchy between two classes of the first order. Since 'men' is a class of animals, it is a member of the class 'classes of animals' (or 'animal-classes'); this is subsumption by a second-order class (i.e. a class of classes) of its first-order-class instances (i.e. mere classes). Since all 'classes of men' are 'classes of animals', the class 'men-classes' is a subclass of the class 'animals-classes'; this is hierarchy

commas, or simply 'classes of men' or 'men-classes' in inverted commas.

between two classes of the second order, i.e. between two classes of classes. The relation between classes of the first order and classes of the second order is never one of hierarchy, but always one of subsumption; i.e. the former are always members (instances) of the latter, never subclasses. Hierarchies only occur between classes of the same order.

Thus, in class logic, we have two planes of existence to consider. At the ground level is the relatively objective plane of empirical phenomena (whether these are physical, mental or spiritual in substance); above that, residing in our minds, is the relatively subjective plane of ideas (which are conceived as insubstantial, but do have phenomenal aspects – namely mental or physical images, spoken or written words, and the intentions of such signs), comprising ideas about empirical phenomena and ideas about such ideas. Classes are developed to facilitate our study of empirical phenomena and classes of classes are developed in turn to facilitate our study of classes for classes (including classes of classes) are of course themselves empirical phenomena of sorts. Classification is a human invention helpful for cognitive ordering of the things observed through our senses or our imaginations or our introspective intuitions. Although classes are products of mind, this does not mean that they are arbitrary - they are formed, organized and controlled by means of our rational faculty, i.e. with the aid of logic. Clearly, to qualify as a class, a class must have at least one member (in which case the sole member is "one of a

Clearly, to qualify as a class, a class must have at least one member (in which case the sole member is "one of a kind"). Usually, a class has two or more members, indeed innumerable members. A class is finite if it includes a specified number of instances; if the number of instances it includes is difficult to enumerate, the class is said to be open-ended (meaning infinite or at least indefinite). What brings the instances of a class together in it is their possession of some distinctive property in common; the class is defined by this property (which may of course be a complicated conjunction of many properties). A class without instances is called a null (or empty) class; this signifies that its defining property is known to be fanciful, so that it is strictly speaking a non-class.

Thus, note well, the term 'class' is a bit ambiguous, as it may refer to a first-order class (a class of non-classes, i.e. of things other than classes) or a second-order class (a class of classes, i.e. a mental construct grouping two or more such mental constructs). A class (of the first order) is not, indeed cannot be, a class of classes (i.e. a class of the second order). There is, of course, a class called 'non-classes'; its instances are principally all concrete things, which are not themselves classes; for example, you and I are non-classes. 'Non-classes' is merely a class, not a class of classes, since it does not include any classes. Thus, 'non-classes' may be said to be a first-order class, but does not qualify as a second-order class.¹³⁵

Note that, whereas positive terms are easy enough to translate into class logic language, negative terms present a real difficulty. For example, whereas the term men refers only to non-classes, its strict antithesis, the term non-men in its broadest sense, includes both non-classes (i.e. concrete things other than men) and classes (i.e. more abstract things). Again, whereas the term finite classes refers only to classes, its strict antithesis, the term non-finite-classes in its broadest sense, includes both open-ended classes (abstracts) and non-classes (concretes). Thus, we must, for purposes of consistency, admit that some terms do cover both non-classes and classes (including classes of classes). Practically, this means we have to make use of *disjunctives* which reveal the

The realm of classes of classes is very limited as an object of study in comparison to the realm of mere classes. For what distinctions can we draw between classes? Not many. We can distinguish between classes and classes of classes, between finite and open-ended classes, between positive and negative classes¹³⁶, and maybe a few more things, but not much more.

apparent double paradox. Bertrand Russell (Britain, 1872-1970) proposed a distinction between 'a class that is a member of itself' and 'a class that is not a member of itself'. Although every class is necessarily coextensive with itself (and in this sense is included in itself), it does not follow that every class is a member of itself (evidently, some are and some are not). Such a distinction can be shown to be legitimate by citing convincing examples. Thus, the class 'positive classes' is a member of itself, since it is defined by a positive property; whereas the class 'negative classes' is not a member of itself, since it is also positively defined (albeit with general reference to negation). Again, the class 'finite classes' is not a member of itself, since it has innumerable members; while the class 'open-ended classes' is a member of itself, since it too has innumerable members.

What about the class 'classes' – is it a member of itself or not? Since 'classes' is a class, it must be a member of

implicit alternatives. This of course complicates class logic considerably.

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Positive classes are defined by some positive property and negative classes are defined by a negative one. For examples, 'men' is defined with reference to rational animals (positive), whereas 'bachelors' is defined with reference to not yet married men (negative).

'classes' – i.e. of itself. This is said without paying attention to the distinction between classes of the first and second orders. If we ask the question more specifically, the answer has to be nuanced. The class 'first-order classes' being a class of classes and not a mere class, cannot be a member of itself, but only a member of 'second-order classes'; the members of the 'first-order classes' are all mere classes. On the other hand, since the class 'second-order classes' is a class of classes, it is a member of itself, i.e. a member of 'secondorder classes'. Thus, the class 'second-order classes' includes both itself and the class 'first-order classes', so that when we say that the wider class 'classes' is a member of itself, we mean that it is more specifically a member of the narrower class 'classes of classes'. As regards, the class 'non-classes', since it is a class and not a non-class, it is not a member of itself. Note however that Russell's paradox does not make a distinction between classes of the first and second orders, but focuses on 'classes' indiscriminately.

Russell asked whether "the class of all classes which are not members of themselves" is or is not a member of itself. It seemed logically impossible to answer the question, because either way a contradiction ensued. For if the class 'classes not members of themselves' is not a member of the class 'classes not members of themselves,' then it is indeed a member of 'classes not members of themselves' (i.e. of itself); and if the class 'classes not members of themselves' is a member of 'classes not members of themselves,' then it is also a member of 'classes which are members of themselves' (i.e. of its contradictory). This looked like a mind-blowing double paradox.

The solution of the problem. The pursuit of knowledge is a human enterprise, and therefore one which proceeds by trial and error. Knowledge is inductive much more than deductive; deduction is just one of the tools of induction. There are absolutes in human knowledge, but they are few and far between. When we formulate a theory, it is always essentially a hypothesis, which might later need to be revised or ruled out. So long as it looks useful and sound, and does so more than any competing theory, we adopt it; but if it ever turns out to be belied by some facts or productive of antinomy, we are obliged to either reformulate it or drop it. This is the principle of induction. When we come upon a contradiction, we have to 'check our premises' and modify them as necessary. In the case at hand, since our conception of class logic is shown by the Russell paradox to be faulty somehow, we must go back and find out just where we went wrong. So, let us carefully retrace our steps. We defined a class and membership in a class by turning predication into classification, saying effectively:

If something is X, then it is a member of the class 'X', and not a member of the class 'nonX'.

If something is not X, then it is not a member of the class 'X', but a member of the class 'nonX'.

Where did we get this definition? It is not an absolute that was somehow cognitively imposed on us. We invented it – it was a convention by means of which we devised the idea of classes and membership in them. Knowledge can very well proceed without recourse to this idea, and has done so for millennia and continues to do so in many people's mind. It is an idea with a history, which was added to the arsenal of reasoning techniques by logicians of relatively recent times. These logicians

noticed themselves and others reasoning by means of classification, and they realized that this is a useful artifice, distinct from predication and yet based on it somehow. They therefore formally proposed the above definition, and proceeded to study the matter in more detail so as to maximize its utility. The 'logic of classes', or 'class logic', was born.

However, at some stage, one logician, Bertrand Russell, realized that there was an inherent inconsistency in our conception of classification, which put the whole edifice of class logic in serious doubt. That was the discovery of the paradox bearing his name. That was a great finding, for there is nothing more important to knowledge development, and especially to development of the branch of knowledge called formal logic, than the maintenance of consistency. Every discovery of inconsistency is a stimulation to refine and perfect our knowledge. Russell deserves much credit for this finding, even if he had a lot of difficulty resolving the paradox in a fully convincing manner. Let us here try to do better, by digging deeper into the thought processes involved in classification than he did. What is classification, more precisely?

If we look more closely at our above definition of a class 'X' and membership of things in it by virtue of being X, we must ask the question: what does this definition achieve, concretely? Are we merely substituting the phrase 'is a member of' for the copula 'is', and the class 'X' for the predicate X? If this is what we are doing, there is no point in it – for it is obvious that changing *the name* of a relation or a term in no way affects it. Words are incidental to knowledge; what matters is their underlying intent, their meaning. If the words change, but

not the meaning, nothing of great significance has changed. No, we are not here merely changing the words used – we are proposing a mental image.

Our idea of classification is that of *mental entities* called classes in which things other than classes (or lesser classes, in the case of classes of classes) are *figuratively collected and contained*. When we say of things that they are members of class 'X', we mean that class 'X' is a sort of box *into* which these things are, by means of imagination, stored (at a given time, whether temporarily or permanently). That is to say, our 'definition' of classification is really a formal convention used to institute this image. What it really means is the following:

If something is X, then it is *in* the class 'X', and out of (i.e. *not in*) the class 'nonX'.

If something is not X, then it is out of (i.e. not in) the class 'X', but in the class 'nonX'.

Clearly, to 'be' something and to 'be in' (within, inside) something else are not the same thing. Our definition conventionally (i.e. by common agreement) decrees that if X is predicated of something, then we may think of that thing as being as if contained by the mental entity called class 'X'. But this decree is not an absolute; it is not a proposition that being subject to predication of X naturally and necessarily implies being a member of class 'X'. For the whole idea of classification, and therefore this definition of what constitutes a class and membership therein, is a human invention. This invention may well be, and indeed is, very useful – but it remains bound by the laws of nature. If we find that the way we have conceived it, i.e. our definition of it, inevitably leads to contradiction, we must adjust our

definition of it in such a way that such contradiction can no longer arise. This is our way of reasoning and acting in all similar situations.

As we shall presently show, since the contradiction is a consequence of the just mentioned defining implication, we must modify that implication. That is to say, we must decree it to have limits. Of course, we cannot just vaguely say that it has limits; we must precisely define these limits so that the practical value of our concept of classification is restored. We can do that by realizing that our definition of classification with reference something 'being in' something else means that class logic is conceived of as related to geometrical logic. This is obvious, when we reflect on the fact that we often 'represent' classes as geometrical figures (notably, circles) and their members as points within those figures. This practice is not accidental, but of the very essence of our idea of classification. Classification is imagining that we put certain items, identified by their possession of some common and distinctive property, in a labeled container 137.

Let us now examine the concept of self-membership in the light of these reflections. What is the idea of self-membership? It is the presupposition that a class may be a member of itself. But is that notion truly conceivable? If we for a moment put aside the class logic issue, and reformulate the question in terms of geometrical logic, we see that it is absurd. Can a container contain itself? Of course not. There is no known example of a container containing itself in the physical world; and indeed we cannot even visually imagine a container containing

This is a pictorial 'representation', an analogical image not to be taken literally.

itself. So the idea of self-containment has no empirical basis, not even in the mental sphere. It is only a fanciful conjunction of two words, without experiential basis. For this reason, the idea strikes us as illogical and we can safely posit as a universal and eternal 'axiom' that self-containment is impossible. A nonsensical term like 'the collection of all collections' is of necessity an empty term; we are not forced to accept it, indeed we are logically not allowed to do so; we can only consistently speak of 'the collection of all *other* collections' 138.

A container is of course always co-extensive with itself. i.e. it occupies exactly the space it occupies. But such 'co-extension' is not containment, let alone containment, for it does not really (other than verbally) concern two things but only one; there is no 'co-' about it, it is just extended, just once. We refer to containment when a smaller object fits inside a larger object (or in the limit when another object of equal size neatly fits inside a certain object). The concept of containment refers to two objects, not one. There has to be two distinct objects; it does not suffice to label the same object in two ways. To imagine 'self-containment' is to imagine that a whole object can somehow fit into itself as a smaller object (or that it can somehow become two, with one of the two inside the other). This is unconscionable. A whole thing cannot be a part (whether a full or partial part) of itself; nothing can be both whole and part at once. A single thing cannot be two things (whether of the same or

To give a concrete image: a bag of marbles (whether alone or, even worse, with the marbles in it) cannot be put inside itself, even if the bag as a whole, together with all its contents, can be rolled around like a marble and so be called a marble

different size) at once; nothing can simultaneously exist as two things.

You cannot decide by convention that something is both whole and part or that one thing is two. You cannot convene something naturally impossible. You can only convene something naturally possible, even though it is unnecessary. Thus, the concept of self-containment is meaningless; it is an inevitably empty concept, because it assumes something impossible to be possible. There is no such thing as self-containment; a container can never contain itself. If this is true, then it is of course equally true that no class includes itself, for (as we have seen) classification is essentially a geometrical idea. Given that a container cannot contain itself, it follows that the answer to the question as to whether a class can be a member of itself is indubitably and definitely: No. Because to say of any class that it is a member of itself is to imply that a container can be a content of itself. Just as no container which is a content of itself exists, so no class which is a member of itself exists!

Now, this is a revolutionary idea for class logic. It applies to any and every class, not just to the class 'classes not members of themselves' which gave rise to the Russell paradox. Moreover, note well that we are here denying the possibility of membership of a class in itself, but not the possibility of *non*-membership of a class in itself. When we say that no container contains itself, we imply that it is true of each and every container that it does not contain itself. Similarly, when we say that no class is a member of itself, we imply that it is true of each and every class that it is not a member of itself. What this means is that while we acknowledge the subject of the Russell paradox, namely the class 'classes

that are not members of themselves', we reject the notion that such a class might *ever*, even hypothetically for a moment, be a member of itself (and therefore paradoxical) – for, we claim, no class whatever is ever a member of itself.

How can this be, you may well ask? Have we not already shown by example that some classes are members of themselves? Have we not agreed, for example, that the class 'classes' being a class has to be a member of the class 'classes', i.e. of itself? How can we deny something so obvious? Surely, you may well object further, if the class 'classes that are not members of themselves' is not a member of itself, then it is undeniably a member of itself; and if it is a member of itself, then it is undeniably not a member of itself? To answer these legitimate questions, let us go back to our definition of classification, and the things we said about that definition. As I pointed earlier, our definition of classes and membership in them has the form of a conventional implication. It says:

If and only if something is X, then it is a member of the class 'X'.

Now, since this conventional implication leads us inexorably to paradox, we must revise it, i.e. make it more limited in scope, i.e. specify the exact conditions when it 'works' and when it ceases to 'work'. What is essentially wrong with it, as we have seen, is that it suggests that a class can be a member of itself. For example, since the class 'classes' is a class, then it is a member of 'classes'; in this example, the variable X has value class and the variable 'X' has value 'classes'. But, as we have shown, the claim that a class can be a member of itself logically implies something

geometrically impossible; namely, that a container can be a content of itself. So, to prevent the Russell paradox from arising, we need to prevent the unwanted consequences of our definition from occurring. Given that our concept of classification is problematic as it stands, what are the conditions we have to specify to delimit it so that the problem is dissolved?

The answer to this question is that when the subject and predicate of the antecedent clause are one and the same, then the consequent clause should cease to be implied. That is to say, if the antecedent clause has the form "if the class 'X' is X" then the consequent clause "then the class 'X' is a member of 'X' (and thus of itself)" does not follow. This 'does not follow' is a convention, just as the general 'it follows' was a convention. What we have done here is merely to draw a line, saying that the consequent generally follows the antecedent, except in the special case where the subject and predicate in the antecedent are 'the same' (in the sense that predicate X is applicable to class 'X' which is itself based on predicate X). This is logically a quite acceptable measure, clearly. If an induced general proposition is found to have exceptions, then it is quite legitimate and indeed obligatory to make it less general, retreating only just enough to allow for these exceptions.

Since the initial definition of classification was a general convention, it is quite permissible, upon discovering that this convention leads us into contradiction, to agree on a slightly narrower convention. Thus, whereas, in the large majority of cases, it remains true that if something is X, then it is a member of the class 'X', and more specifically, if a class (say, 'Y') is X, then it (i.e. 'Y) is a member of the class of classes 'X' – nevertheless,

exceptionally, in the special case where the class that is X is the class 'X' (i.e. where 'Y' = 'X'), we cannot go on to say of it that it is a member of 'X', for this would be to claim it to be a member of itself, which is impossible since this implies that a container can be a content of itself. Note well that we are not denying that, for example, the class 'classes' is a class; we are only denying the implication this is normally taken to have that the class 'classes' is a member of the class 'classes'. We can cheerfully continue saying 'is' (for that is mere predication), but we are not here allowed to turn that 'is' into 'is a member of' (for that would constitute illicit classification).

In this way, the Russell paradox is inhibited from arising. That is to say, with reference to the class 'classes not members of themselves': firstly, it is quite legitimate to suppose that the class 'classes not members themselves' is not a member of itself, since we know for sure (from geometrical logic) that no class is a member of itself; but it is not legitimate to say that this fact (i.e. that it is not a member of itself) implies that it is a member of itself, since such implication has been conventionally excluded. Secondly, it is not legitimate to suppose, even for the sake of argument, that the class 'classes not members of themselves' is a member of itself, since we already know (from geometrical logic) that no class is a member of itself, and therefore we cannot establish through such supposition that it is not a member of itself, even though it is anyway true that it is not a member of itself.

As can be seen, our correction of the definition of classification, making it less general than it originally was, by specifying the specific situation in which the

implication involved is not to be applied, succeeds in eliminating the Russell paradox. We can say that the class 'classes not members of themselves' is not a member of itself, but we cannot say that it is a member of itself; therefore, both legs of the double paradox are blocked. In the first leg, we have blocked the inference from not-being 'a member of itself' to being one; in the second leg, we have interdicted the supposition of being 'a member of itself' even though inference from it of notbeing one would be harmless. Accordingly, the answer to the question posed by Russell – viz. "Is the class of all classes which are not members of themselves a member of itself or not?" - is that this class is not a member of itself, and that this class not-being a member of itself does not, contrary to appearances, make it a member of itself, because no class is a member of itself anyway.

Thus, to be sure, though it is true that the class 'classes' is a class, it does not follow that it is a member of itself; though it is true that the class 'classes of classes' is a class of classes, it does not follow that it is a member of itself; though it is true that the class 'positive classes' is a positive class, it does not follow that it is a member of itself; though it is true that the class 'open-ended classes' is an open-ended class, it does not follow that it is a member of itself; though it is true that the class 'classes that are not members of themselves' is a class that is not a member of itself, it does not follow that it is a member of itself. As for the class 'classes members themselves', it has no members at all. It should be emphasized that the restriction on classification that we have here introduced is of very limited scope; it hardly affects class logic at all, concerning as it does a few very borderline cases.

The above is, I believe, the correct and definitive resolution of the Russell paradox. We acknowledged the existence of a problem, the Russell paradox. We diagnosed the cause of the problem, the assumption that self-membership is possible. We showed that selfmembership is unconscionable, since it implies that a container can contain itself; this was not arbitrary tinkering, note well, but appealed to reason. We proposed a solution to the problem, one that precisely targets it and surgically removes it. Our remedy consisted in uncoupling predication from classification in all cases where self-membership is assumed, and only in such cases. This solution of the problem is plain common sense and not a flight of speculation; it is simple and elegant; it is convincing and uncomplicated; it does not essentially modify the concept of class membership, but only limits its application a little; it introduces a restriction, but one that is clearly circumscribed and quite small; it does not result in collateral damage on areas of class logic, or logic in general, that are not problematic, and therefore does not call for further adaptations of logic doctrine. Note moreover that our solution does not resort to any obscure 'system' of modern symbolic logic, but is entirely developed using ordinary language and widely known and accepted concepts and processes.

A bit of the history. Let us now look briefly at some of the history of the Russell paradox, and see how he and some other modern symbolic logicians dealt with it¹³⁹. Georg Cantor had already in 1895 found an antinomy in his own theory of sets. In 1902, when Gottlob Frege

¹³⁹ I am here referring principally to the account by William and Martha Kneale in *The Development of Logic* (Oxford, London: Clarendon, 1962), ch. XI.1-2.

(Germany, 1848-1925) was about to publish the second volume of his *Grundgesetze*, he was advised by Russell of the said paradox. Frege was totally taken in and could not see how to get out of the self-contradictions inherent in "the class of classes that do not belong to themselves." He perceived this as very serious, saying: "What is in question is ... whether arithmetic can possibly be given a logical foundation at all." Frege first tried to fix things by suggesting that there might be "concepts with no corresponding classes," or alternatively by adjusting one of his "axioms" in such a way that:

"Two concepts should be said to have the same extension if, and only if, every object which fell under the first but was not itself the extension of the first fell likewise under the second and vice versa" 140.

Clearly, Frege's initial suggestion that there might be "concepts with no corresponding classes" can be viewed as an anticipation of my uncoupling of predication and classification in specific cases. However, Frege did not identify precisely in what cases such uncoupling has to occur. This is evident in his next suggestion, which, though it points tantalizingly to the difficulty in the notion of self-membership, does not reject this notion outright but instead attempts to mitigate it. He speaks of two concepts instead of one, and tries to conventionally exclude the extension as a whole of each from the other, while of course continuing to include the objects falling under the extension; this shows he has not realized that self-inclusion by an extension is not even thinkable.

It should be stressed that Russell's paradox pertains to a certain class (viz. that of all classes not members of

Kneale and Kneale, p. 654. Italics theirs.

themselves) being or not-being a member of itself – not of some other class. Frege tries to resolve this paradox with reference, not to a single class, but to a pair of equal classes, even though (to my knowledge) he has not demonstrated that co-extensive classes result in a paradox comparable to the Russell paradox. It follows that his attempted solution to the problem is not germane to it. Moreover, Frege seems to have thought that if all items that fall under one class (say, 'Y') fall under another class (say, 'X'), then the class 'Y' may be assumed to fall under the class 'X': and vice versa in the event of co-extension. This is suggested by his attempt to prevent such assumption, so as to avoid (in his estimate) the resulting Russell paradox. But in truth, it does not follow from the given that all Y are X that the class 'Y' is a member of the class 'X' - it only follows that the class 'Y' is a subclass of the class 'X', or an equal class if the relation is reversible. Thus, it appears that Frege confused the relations of class-membership hierarchization of classes, using a vague term like 'falling' to characterize them both.

We may well ask the question whether an equal class, or a subclass, or even an overclass, might be a member of its hierarchically related class. Offhand, it would seem to be possible. For example, all positive classes are classes and therefore members of the class 'classes', and the class 'positive classes' is a subclass of the class 'classes'; however, although not all classes are positive classes (some are negative classes), nevertheless the class 'classes' is a positive class (being defined by a positive statement), and so is a member of the class 'positive classes'. But although this example suggests that an overclass might be a member of its subclass (and

therefore, all the more, an equal class or a subclass might be a member of its hierarchical relative), we might still express a doubt by means of analogy, as Frege perhaps intended to do. We could argue inductively, by generalization, that if a class cannot be a member of itself, then maybe it cannot be a member of any coextensive class (as Frege suggests), and perhaps even of a subclass or an overclass. For the issue here is whether the instances referred to by the first class can be thought to occur twice in the second class (as members of it in their own right, and as constituents of a member). So, Frege may have raised a valid issue, which could lead to further restrictions in class logic. However, this need not concern us further in the present context, since (as already explained) it is not directly relevant to resolution of the Russell paradox.

Russell described his paradox in his book Principles of Mathematics, published soon after. Although at first inclined to Frege's second approach, he later preferred Frege's first one, proposing that there might be "some propositional functions which did not determine genuine classes." Note here again the failure to pinpoint the precise source and remedy of the problem. Subsequently, Russell thought that "the problem could never be solved completely until all classes were eliminated from logical theory." This, in my view, would be throwing out the baby with the bath water – an overreaction. But then he found out (or rather, he thought he did, or he convinced himself that he did) that the same paradox could be generated without "talk of classes," i.e. with reference to mere predicates - that is, in terms of predicate logic instead of in terms of class logic. As Kneale and Kneale put it (p. 655):

"Instead of the class which is supposed to contain all classes that are not members of themselves let us consider the property of being a property which does not exemplify itself. If this property exemplifies itself, then it cannot exemplify itself; and if it does not exemplify itself, then it must exemplify itself. Clearly, the nature of the trouble is the same here as in the original paradox, and yet there is no talk of classes."

But even if classes are not explicitly mentioned here, it is clear that they are tacitly intended. How would a property 'exemplify' itself? Presumably, property X would be 'a property which exemplifies itself' if property X happens to be one of the things that have property X. That is to say, X exemplifies X if X is a member of the class of things that are X. We cannot talk about properties without resorting to predication; and once we predicate we can (given the initial definition of classification) surely classify. So, this attempt is just verbal chicanery; the same thought is intended, but it is dressed up in other words. It is dishonest. Moreover, the way the paradox is allegedly evoked here does not in fact result in paradox.

We cannot say, even hypothetically, "if this property [i.e. the property of being a property which does not exemplify itself] exemplifies itself" for that is already self-contradictory. To reconstruct a Russell paradox in 'property' terms, we would have to speak of 'the property of all properties which do not exemplify themselves'; for then we would have a new term to chew on, as we did in class logic. But clearly, this new term is quite contrived and meaningless. Here again, we must mean 'the class of all properties which do not exemplify

themselves' – and in that event, we are back in class logic. Thus, note well, while Russell was right in looking to see whether his paradox was a problem specific to class logic, or one also occurring in predicate logic, and he claimed to have established that it occurred in both fields, in truth (as we have just demonstrated) he did not succeed in doing that. In truth, the paradox *was* specific to class logic; and he would have been better off admitting the fact than trying to ignore it.

In response to certain criticisms by his peers, Russell eventually "agreed that the paradoxes were all due to vicious circles, and laid it down as a principle for the avoidance of such circles that 'whatever involves all of a collection must not be one of the collection'." Thus, Russell may be said to have conceded the principle I have also used, namely that a collection cannot include itself as one of the items collected, although in truth the way he put it suggests he conceived it as a convention designed to block incomprehensible vicious circles rather than a logical absolute (notice that he says 'must not' rather than 'cannot'). He viewed the paradoxes of set theory as "essentially of the same kind as the old paradox of Epimenides (or the Liar)." This suggests that, at this stage, he saw his own paradox as due to self-reference, somehow. It does look at first sight as if there is some sort of self-reference in the proposition 'the class of all classes that are not members of themselves is (or is not) a member of itself', because the clause 'member of itself' is repeated (positively or negatively, in the singular or plural) in subject and predicate¹⁴¹. But it cannot be said that self-reference is exactly the problem.

Note that if self-reference were the crux of the problem, then the proposition 'the class of all classes that are

A few years later, in a paper published in 1908, Russell came up with a more elaborate explanation of the Russell paradox based on his 'theory of types'. Russell now argued that "no function can have among its values anything which presupposes the function, for if it had, we could not regard the objects ambiguously denoted by the function as definite until the function was definite, while conversely ... the function cannot be definite until its values are definite"142. In other words, the question "the class of all classes that are not members of themselves, is it or is it not a member of itself" is inherently flawed, because the subject remains forever out of reach. We cannot take hold of it till we resolve whether or not it is a member of itself, and we cannot do the latter till we do the former; so, the conundrum is unresolvable, i.e. the question is unanswerable. Effectively, the subject is a term cognitively impossible to formulate, due to the double bind the issue of its definition involves for any thinker.

Here, we should note that the purpose of Russell's said explanation was effectively to invalidate the negative class 'classes not members of themselves', since this is the class giving rise to the double paradox he was trying to cure. The positive class 'classes members of themselves' clearly does not result in a double paradox: if we suppose it is *not* a member of itself, self-contradiction does ensue, but we can still say without self-contradiction that it *is* a member of itself. In fact, if Russell's explanation were correct, the positive class

members of themselves is (or is not) a member of itself' would be equally problematic, even though it apparently does not result in a similar paradox.

Quoted by the Kneales, p. 658.

ought to be as illicit as the negative one. For if we claim the impossibility of a class referring to something that is not yet settled, as Russell did with reference to the negative class, then we must admit this characteristic is also found in the positive class, and we must reject it too. Russell does not seem to have realized that, i.e. that his remedy did not technically differentiate the two classes and so could be applied to both. For this reason, his attempt to solve the Russell paradox with reference to circularity or infinity must be judged as a failure. In my own theory, on the other hand, it is the positive class (that of self-membership) which is invalid (and empty), since it is geometrically unthinkable, while the negative class (that of non-self-membership) remains legitimate (and instantiated), as indeed we would expect on the principle that all claims (including that of selfmembership) ought to be deniable.

Anyway, Russell concluded, briefly put, that a function could not be a value of itself; and proposed that function and value be differentiated as two 'types' that could not be mixed together indiscriminately. But this theory is, I would say, too general, and it complicates matters considerably. As we have seen, we cannot refuse to admit that, for instance, 'classes' is a class; the most we can do is to deny that this implies that 'classes' is a member of itself. This is a denial of self-membership, not of self-predication or of self-reference. As regards the notion of 'types' and later that of 'orders within types', these should not be confused with the more traditional ideas of hierarchies and orders of classes, which we laid out earlier in the present essay. In truth, the resemblance between Russell's concepts and the latter concepts gives Russell's theory a semblance of credibility; but this

appearance is quite illusory – these are very different sets of concepts. Russell's notion of 'types' is highly speculative and far from commonsense; while it might appear to solve the Russell paradox, it has ramifications that range far beyond it and incidentally invalidate traditional ideas that do not seem problematic¹⁴³. In short, it is a rough-and-ready, makeshift measure, and not a very convincing one.

Every paradox we come across is, of course, a signal to us that we are going astray somehow. Accordingly, the Russell paradox may be said to have been a signal to Frege, Russell, and other modern logicians, that something was wrong in their outlook. They struggled hard to find the source of the problem, but apparently could not exactly pinpoint its location. All the intricacy and complexity of their symbolic and axiomatic approach to logic could not help them, but rather obscured the solution of the problem for them. This shows that before any attempt at symbolization and axiomatization it is essential for logicians to fully understand the subject at hand in ordinary language terms and by means of commonsense. To my knowledge, the solution of the problem proposed in the present essay is original, i.e. not to be found elsewhere. If that is true, then the theory of class logic developed by modern symbolic logicians, which is still the core of what is being taught in universities today, needs to be thoroughly reviewed and revised.

A bit of self-criticism. As regards the resolution of the Russell paradox that I proposed over two decades ago in my *Future Logic*, the following needs to be said here.

See for a start the Kneales' critique of the 'theory of types' in ch. XI.2.

While I stand, in the main, by my theory of the logic of classes there (in chapters 43-44), I must now distance myself somewhat from my attempted resolution of the Russell paradox there (in chapter 45).

I did, to my credit, in that past work express great skepticism with regard to the notion of self-membership; but I did not manage to totally rule it out. I did declare: "Intuitively, to me at least, the suggestion that something can be both container and contained is hard to swallow," and I even postulated, in the way of a generalization from a number of cases examined, that "no class of anything, or class of classes of anything, is ever a member of itself," with the possible exception of "things" or "things-classes" (although it might be said of these classes that they are not members of any classes, let alone themselves¹⁴⁴); but still, I did not reject self-membership on principle, and use that rejection to explain and resolve the Russell paradox, as I do in the present essay.

This is evident, for instance, in my accepting the idea that "'self-member classes' is a member of itself." The reason I did so was the thought that "whether self-membership is possible or not, is not the issue." Superficially, this is of course true – the Russell paradox concerns the 'class of all classes that are not members of themselves', and not 'the class of all classes that are members of themselves'. But in fact, as I have shown today, this is not true; acceptance of self-membership is the true cause of the Russell paradox, and non-self-membership is not in itself problematic.

Note that in this context I come up with the idea that the definition of membership might occasionally fail. But I did not at the time pursue that idea further, because I did not then analyze what such failure would formally imply.

Anyway, not having duly ruled out self-membership, I resorted to the only solution of the problem that looked promising to me at the time - namely, rejection of 'permutation' from "is (or is not) a member of itself" to "is (or is not) {a member of itself}" (notice the addition of curly brackets). That is to say, I proposed the logical interdiction of changing the *relation* of self-membership or non-self-membership into a predicable term. Now I see that this was wrong - it was an action taken too late in the process of thought leading up to the Russell paradox. It was a superficial attempt, treating a symptom instead of the disease. I did that, of course, because I thought this was "of all the processes used in developing arguments, [the] only one of uncertain (unestablished) validity." But in truth, it was not the only possible cause of the effect – there was a process before that, one of deeper significance, namely the transition from 'is' to 'is a member of'. I did not at the time notice this earlier process, let alone realize its vulnerability; and for that reason, I did not attack it.

Clearly, I was on the right track, in that I sought for a place along the thought process at which to block development of the Russell paradox. But my error was to pick a place too late along that process. In fact, the right place is earlier on, as advocated in the present essay. The Russell paradox does not arise due to an illicit permutation, but due to the illicit transformation of a predicate into a class in cases where a claim of self-membership would ensue. And while the remedy proposed is even now in a sense 'conventional', the flaw it is designed to fix is quite real — it is that self-membership is in fact impossible and therefore can never be assumed true. My previous proposed solution to the

problem only prevented the Russell paradox; it did not prevent self-membership, which is the real cause of the paradox. Thus, the solution I propose in the present essay is more profound and more accurate.

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Avi Sion is the author of several works on logic and philosophy: Future Logic (1990); Judaic Logic (1995); Buddhist Illogic (2002); Phenomenology (2003); The Logic of Causation (1999, 2003, 2010); Volition and Allied Causal Concepts (2004); Ruminations (2005); Meditations (2006); Logical and Spiritual Reflections (2008-9); A Fortiori Logic (2013). These works can be freely studied at: www.TheLogician.net.

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